A contraction of domestic grain supplies, following a period of low prices and acreage retirement programs in the early 1970s, combined with a large increase in export demand due to a world shortage of grain, resulted in a sharp increase in feed grain prices in 1973-74. Grain prices remained elevated through 1976. Feed prices, over this period, severely aggravated the cyclical variation in beef prices and production from late 1973 to the present. The purpose of this study is to examine and quantify the linkage between feed grain prices and long-run production of beef cattle.

A theoretical framework is developed in this study to examine the sequential linkage between feed grain prices, feedlot demand for feeder cattle, feeder cattle prices, and breeding inventories. A number of criteria are established for the design and validation of a quantitative model.
Secondary data, maintained by the USDA, and least squares techniques are used to estimate a reduced form of the theoretical linkage. A quarterly model of the feeder market is estimated to examine the impact of feed grain prices on feeder market prices. An annual model of cow and replacement heifer inventories is estimated to consider the potential long-run impact of feed grain price effects in the feeder market. Several formal hypothesis tests are constructed to evaluate the estimating equations. The estimating equations are placed in a recursive block, and are solved for the historical values of the exogenous parameters. The results of the historical simulation are compared to actual feeder market prices and breeding inventories between 1970 and 1978 to validate the complete model. The model followed feeder market prices through both major price swings and maintained good predictive accuracy through most of the tracking period. The model also followed the growth and liquidation of breeding inventories over the tracking period. However, the model underestimated the magnitude of breeding herd liquidation.

The model is solved for an alternative set of feed costs to estimate the impact of increased feed costs between 1973 and 1976. Cross-price elasticities, calculated from the historical and alternative feed cost simulations, indicated that over 75 percent of the variation in the price of feed is transmitted to the feeder market. Furthermore, the results show that feeder calf prices are more sensitive to feed grain price effects than heavier feeder cattle. This suggests that cow-calf producers may have absorbed the greatest loss in returns, due to increased feed costs. The estimated loss in forage producer returns,
from cattle placed in feedlots in 23 states between 1973 and 1977, exceeded 14 billion dollars, 20 percent of the value of total beef production for this same period. This decline in returns to cow-calf producers resulted in a decline in breeding production. By the beginning of 1979, the loss in breeding capacity, projected by the simulation, reached over 20 percent.

From the results of this study, it may be concluded that relatively elastic feedlot demand for feeder cattle and inelastic forage sector supplies transmits feed grain price effects to forage producers. This fact may hold some short-run benefits for both feedlot producers and consumers during periods when feed grain prices rise. However, the long-run response in breeding production may ultimately result in consumers paying higher prices for less beef, a problem which has surfaced in 1979. The potential long-run impact of changes in feed prices on the beef industry raises some serious problems with respect to agricultural policy designed to support prices in the grain sector. Programs which do not provide control of high prices are a distinct disadvantage to both beef producers and consumers. The costs and benefits of alternative policies is an area which warrants continued research.
An Econometric Model of the Effects of Feed Grain Prices in the Feeder Cattle Market and Breeding Inventories

by

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An Econometric Model of the Effects of Feed Grain
Prices in the Feeder Cattle Market
and Breeding Inventories

I. INTRODUCTION

Feed Grain and Beef Prices

A sequence of historical events over the past decade have initiated a new round of interest and research concerning the effects of feed grain price on U.S. beef production and prices. Beginning in 1971 and 1972, agricultural policies designed to improve feed grain prices through acreage controls were re-established. Production of feed grains decreased in the United States between 1972 and 1973. A world shortage of feed supplies led to a large increase in export demand for U.S. feed grains in 1973 and 1974. Between the 1971 to 1974 crop years, the average annual price of corn received by farmers in the United States increased from $1.08 per bushel to $3.03 per bushel. This era was highlighted, and further aggravated, by the now famous "Russian Grain Deal of 1975."

This tripling of the cost of feed grains, combined with a relatively elastic demand for beef products, resulted in a substantial decline of returns to cattle feeders. A USDA estimate of net returns to the cost of feed and feeder, for a typical cornbelt-cattle feeding operation, declined from $5.29 cwt. in the first half of 1972, to -$3.45 cwt. in the second half of 1974.
This decline in feeding returns resulted in a sharp decline in feeder cattle prices. The average price of feeder cattle, 500-700 lbs. in Kansas City, declined from $53.17 cwt. in 1973 to $33.91 cwt. in 1975.

This decline in feeder cattle prices was felt severely by the cow-calf producers who, in 1975, started to reduce production. The liquidation of breeding stock placed more animals in slaughter which, in turn, further depressed prices. This contraction of the beef breeding herd continued through 1978.

Today, total cattle numbers are down over 21 million head since 1975, and feed grain prices are reduced from their 1974 level. The price of choice slaughter steers, 900-1100 lbs. in Omaha, have climed to $65.42 cwt., in the first quarter of 1979, a sizable increase from the annual average of $39.11 cwt. in 1976. The high price of beef, coupled with the expected cyclical variation in beef prices, has alarmed consumers and some government agencies.

In light of the events of the past decade, a number of questions concerning the effects of feed grain prices on U.S. beef production have re-emerged with new significance. What would be the expected impact of alternative grain policies on the cow-calf producer, feedlot operator, and the long-run supply of cattle? How long does it take for an impact to work its way through the beef system? What is the effect of unstable feed grain prices on beef prices over time? Can a policy be developed to stabilize feed prices to benefit both the grain and cattle producer? These, and similar questions, should be of interest
to persons in, and associated with, the beef industry, policy makers, and ultimately, the consumer.

These issues have not been ignored by previous researchers. An evaluation of prior work can aid in establishing a better perspective of the current felt need for improved understanding of the theoretical relationship between feed grain and the beef industry, and properly specified quantification of these relationships.

**The Problem**

Previous researchers have considered the impact of feed grain prices on beef production and prices. Early supply studies simply specified slaughter cattle supply as a function of live cattle price and feed price. These studies observed zero or negative elasticities of supply with respect to live cattle prices and zero or positive elasticities of supply with respect to feed prices (Reutlinger, 1966). Reutlinger resolved this apparent contradiction to economic theory by estimating a model in which steer, heifer, and cow slaughter were estimated separately. These relationships were re-examined and refined by a number of authors (Hayenga and Hacklander, 1970; Tryfos, 1974; and Nelson and Spreen, 1978).

Langemeier and Thompson (1967) reviewed early demand studies, and estimate a simultaneous equation supply and demand model of the beef industry. In this model, they estimate cross-price elasticities for feed grain and cattle prices.
The models developed prior to 1974 were estimated from a period of relatively stable grain prices. Simple models appeared adequate, and the magnitude of feed grain price effects may have been understated.

More complex models of the livestock industry were developed in more recent years (Rahn, 1973; Freebairn and Rausser, 1975; Man, et al., 1976; and Arzac and Wilkinson, 1979). Individual components of these large models, designed specifically for the beef sectors, remained relatively simple. Freebairn and Rausser correctly specified calf production as a function of feeder calf prices. However, they fail to incorporate feed grain prices in the estimation of feeder cattle prices. Arzac and Wilkinson specify a relationship between feed grain prices and feeder cattle prices. The explanatory power of their model is derived from specifying feeder steer prices as a function of non-fed beef prices. One should expect a high correlation between grass-fed feeder steers weighing 500 to 700 pounds and grass-fed slaughter cattle weighing approximately 700 to 900 pounds. This correlation does not constitute an adequate explanation.

The relationship between feed grain prices and feedlot demand for feeders, feedlot demand and feeder cattle prices, and feeder cattle prices and breeding production provides a potentially strong link between the feed grain market and long-run beef production. Proper specification of this linkage may serve as a powerful tool in beginning to explain the impact of shifts in either supply or demand in the grain market on the beef industry. More specifically, correct specification could assist in assessing the impact of feed grain prices on forage-based producers in the U.S., and the impact of feed grain prices on long-run supplies of beef cattle.
Objectives

The primary objective of this research is to develop and quantify the relationships between feed grain prices, feeder cattle prices, and breeding cattle inventories. A secondary objective of this study is to estimate the impact of changes in feed grain prices on prices of feeder cattle of different weights, and breeding and replacement inventories of beef cows. To adequately meet these objectives, a number of criteria must be met. First, a theoretical framework is needed to identify the sequence of casual relationships which link feed grain prices and beef production. Second, a quantitative model must be designed from the theoretical framework, within the limitations of available secondary data. And, last, the quantitative model requires validation. This may take the form of hypothesis testing, or more subjective analysis, such as historical tracking ability.
II. THEORETICAL FRAMEWORK

Overview

The effects of feed grain prices on prices of feeder cattle and long-run production of beef cattle can be explored within the framework of the relevant grain and livestock markets. Figure 2.1 is a schematic diagram of the relationships which will be examined. Two producing sectors, forage and feedlot, have been identified as primary components of the beef production system. Within the forage sector, two subsectors are specified, the cow-calf or breeding subsector, and the backgrounding or stocker subsector. Three markets have been chosen to reflect the interaction of the production sectors, the slaughter, feeder, and grain markets. Each of these components will be defined in the following sections.

Figure 2.1. Primary Production and Related Markets
The Slaughter Market

The slaughter market is a critical component of the beef marketing system. The slaughter market links live animal production with consumer demand for beef products.

Slaughter market demand for beef cattle is derived from consumer demand. Determinants of consumer demand include income, relative prices of other products, consumer tastes, preferences, and attitudes (Nix, 1978). Demand in the retail markets is for a highly diverse set of products, such as various cuts and grades of meat and beef by-products. Highly differentiated consumer demand is reduced to a limited set of characteristics, such as weight, sex, and grade, defined for the live animal. Buyers of cattle for the meat packing and processing industry transmit this aggregated consumer demand for beef to the supplier of live animals through price offers in the slaughter market. The transmission of demand in the processing sector is a mirror image of the transformation of live animal supplies into the wide variety of final beef products.¹

For this study, a real and effective consumer preference for grain-fed beef is assumed to exist. This assumption is affirmed in sensory evaluations (Wheeling, 1975). This preference is revealed in retail demand and transmitted to the slaughter market through a system of grades. Grain-fed beef tends to grade good or better, grass-fed beef

tending to grade less than good (Ginn, 1977). Consumer preference for grain versus grass-fed beef is effectively differentiated in the slaughter market, resulting in a price premium for grain-fed beef.

Slaughter market supplies of grain finished cattle are marketed from the feedlot sector. Cattle marketed for slaughter, from the forage sector, are the market supplies of grass-fed beef. The total quantity of beef cattle supplied in the slaughter market reflects long-run production of animal numbers. The relative quantities of grain and grass-fed beef supplied reflect:

1. Price differentials for grain versus forage-fed beef.
2. The relative costs of producing grain versus forage-fed beef. A major cost of producing grain-fed beef are feed grains.

**The Feed Grain Market**

Feed grains comprise a large portion of the diet used to grow and finish cattle in feedlot production. Hence, feed grains are defined as a factor of feedlot production. A dependent relationship may exist between feed grain supplies and feedlot production.

Two assumptions are made with respect to the feed grain market. First, short-run feed grain prices are determined independently of feedlot demand for feed grains. Restated, in the short-run, feed grain supplies are perfectly elastic with respect to beef production. Second, feed grain prices are assumed to establish general price levels for all harvested feeds. Feed grain prices are taken to establish a factor price for all feed in the feedlot sector.
The Feedlot Sector

The feedlot sector may be defined as the activities associated with the production of grain-fed slaughter cattle. The feedlot sector purchases feed and animal inputs in the production of slaughter beef (grading good or better). Presently, over 60 percent of domestic beef production consists of grain finished beef (Ginn, 1977). The relatively large contribution of the feedlot sector, in total beef production, may be attributed to the following factors: One, with respect to current levels of beef production, extensive feedlot production may be a part of the least cost alternative (Brokken, 1975); two, large supplies of feed grains available for feedlot production; three, an effective consumer preference for grain-fed beef.

Feedlot sector demand for animal inputs is a major component of the market demand for non-breeding inventories held in the forage sector. Bruce Ginn states:

Once the past ten years, an average of over 55% of feeder cattle supplies on January 1 were placed on feed during the year. As the major source of demand, feedlots represent a primary determinant of feeder cattle prices. (Ginn, 1977, pp. 98-99).

Feedlot sector demand for feeder cattle creates a potential linkage between feed grain prices and forage production. Feedlot sector demand for feeder cattle is, in part, a function of the cost of feed. Feeder cattle supplies are determined, for the most part, by current and past levels of breeding production.
A feedlot sector profit function is a useful tool for examining the origin of feeder cattle demand (Brokken, 1975). Let the following profit function be specified for the feedlot sector:

\[ \pi_h = W_p (P_s - P_p) + W_g (P_s - FC - C_o) \]  \hspace{1cm} \text{Eq. 2.20}

where:

- \( \pi_h \) = profit per finished animal
- \( W_p \) = purchase weight of the feeder animal
- \( P_s \) = price per unit weight of the finished animal
- \( P_p \) = price per unit weight of the feeder animal
- \( W_g \) = total weight gained in the feedlot
- \( FC \) = feed costs per unit of \( W_g \)
- \( C_o \) = costs of all other inputs per unit \( W_g \)

The profit equation may be solved for the purchase price by rearranging Equation 2.10:

\[ P_p = P_s + \frac{W_g (P_s - FC - C_o) - \pi_h}{W_p} \]  \hspace{1cm} \text{Eq. 2.11}

If the price parameters of Equation 2.11 are replaced with expected prices and the profit per head is dropped from this equation, an expected break-even price maybe expressed:

\[ BE = P_s^* + \frac{W_g (P_s^* - FC^* - C_o^*)}{W_p} \]  \hspace{1cm} \text{Eq. 2.12}

where:

- \( BE \) = an expected break-even price, and:
- * denotes expectation
The expected break-even price may be used as a proxy for feedlot sector demand for feeder cattle.

The direction of an effect on the break-even price, due to a change in the expected price of finished cattle or feed costs, is indicated by the following partial derivatives:

$$\frac{\partial BP}{\partial p^*} = 1 + \frac{W}{W_p} > 0 \quad \text{Eq. 2.13}$$

$$\frac{\partial BP}{\partial FC^*} = -\frac{W}{W_p} < 0 \quad \text{Eq. 2.14}$$

From Equation 2.13 it may be seen that an increase in the price of finished cattle, $P^*_s$, will result in an increase in the break-even price. Therefore, a positive relationship is expected between the price of finished cattle and the demand for feeder cattle of a given market weight, $W_p$. An increase in the expected cost of feed, resulting from an increase in feed grain prices, may result in a decrease in the demand for feeder cattle of market weight, $W_p$. However, both the effect of a change in the price of finished cattle (Equation 2.13) and the effect of a change in feed costs (Equation 2.14) on the break-even price, contain the weight terms, $W_g$ and $W_p$. The magnitude of the change in the break-even price is proportional to the ratio of the total weight gained in the feedlot, $W_g$, and the weight of the feeder animal purchased from the forage sector, $W_p$.

These relationships are somewhat oversimplified. For example, feed efficiency is dependent on animal weight and animal age.
Through adjusting the average purchase weight, the feedlot sector may control, in part, the impact of a change in market prices. The feedlot sector is flexible, or elastic, due to the ability to increase or decrease the relative contribution of the sector in producing a given output of finished beef. An increase in the average purchase weight will decrease the ratio of feed grains to forage utilized to produce a given level of grain finished beef because more growth will take place outside the feedlot when the ratio is nearly all forage. A decrease in the average purchase weight will increase the feed grain to forage ratio.

Total weight gain, and therefore finishing weights, are also variable parameters of feedlot production. However, in order to achieve the price premium for grain feeding (a grade of good or better), and to avoid price penalties for excess fat, the total weight gain in the feedlot sector is subject to technical constraints. For a given purchase weight, feedlot gain may be restricted to a range of weights. Within this range, a number of factors may establish feedlot production. For a given purchase weight, let feedlot gain, \( W_g \), be fixed at \( W_g^* \); where \( W_g^* \) is the mean value of \( W_g \) with respect to all other parameters. Then:

\[
W_g^* = f(W_p) \quad \text{such that;} \quad \frac{\partial W_g^*}{\partial W_p} < 0
\]

Equation 2.13 may be rewritten as:

\[
BE = P^* + \frac{f(W_p)(P^* - FC^* - C_g^*)}{W_p} \quad \text{Eq. 2.16}
\]
Since $W_p$ and $f(W_p)$ are inversely related functions, the following conditions may be noted. If the net return to weight gained in the feedlot is positive:

1. The break-even price increases as the purchase weight decreases.
2. The break-even price decreases as the purchase weight increases.

If the net return to the weight placed in the feedlot is negative:

1. The break-even price increases as the purchase weight increases.
2. The break-even price decreases as the purchase weight decreases.

The effects of a change in feed grain prices on the break-even price, with respect to the purchase weight, may be demonstrated:

\[
BE = \frac{f(W_p)(P^* - FC^* - C^*)}{W_p}
\]

Eq. 2.17

Taking the partial of $BE$, with respect to $W_p$, yields:

\[
\frac{\partial BE}{\partial W_p} = \frac{f^2(W_p)(P^* - FC^* - C^*)W_p - f(W_p)(P^* - FC^* - C^*)}{W_p}
\]

Eq. 2.18

The necessary and sufficient conditions needed to determine the sign of the partial are:

given: $f^1(W_p) < 0$ implies;

\[(f^1(W_p)W_p - f(W_p^2)) < 0\]

thus;

\[\frac{\partial BE}{\partial W_p} > 0 \text{ iff } (P^* - FC^* - C^*) < 0\]

and;

\[\frac{\partial BE}{\partial W_p} < 0 \text{ iff } (P^* - FC^* - C^*) > 0\]
Taking the cross partial with respect to feed costs yields:

$$\frac{\partial BE}{\partial W_p \partial FC} = \frac{(-1)(f'(W_p)W_p - f(W_p))}{W_p^2} > 0 \quad \text{Eq. 2.19}$$

From Equation 2.19, it may be implied that the impact of a change in feed costs on feeder cattle demand is inversely related to the market weight of the feeder. If feed costs (feed grain prices) increase, then the negative impact on feeder cattle demand will increase as the market weight of the feeder decreases. If feed grain prices decline, then the positive impact on feeder cattle demand will increase as market weight of the feeder decreases. If feed grain prices decline, then the positive impact on feeder cattle demand will increase as market weights decrease.

This relationship between market weight and feed prices reflects the fact that, as market weights increase, total weight added in the feedlot sector decreases. Therefore, the quantity of feed grains demanded by the feedlot sector may also decline. The impact of a change in feed costs decreases as market weights for feeder cattle increase.

It is evident that feeder cattle demand is comprised of a series of interdependent price and quantity schedules over a range of market weights, where the range of market weights is divided into discrete weight intervals. This system of demand schedules allows the feedlot sector to adjust the average market weight as well as the total quantity of animals demanded.

When the net returns to cattle feeding are positive, feedlot sector demand for feeder cattle is stronger for weaned calves and light
feeders, relative to heavier animals. The technical restriction of a continuous product flow may result in a continued demand for heavy feeder cattle. However, when returns to grain feeding are positive, returns may be maximized at the lowest possible market weights of feeder cattle (the greatest possible total weight gained in the feedlot sector). The feedlot sector may increase the average finishing weight to increase the weight gained in the feedlot sector. However, given the growth characteristics of cattle, feed efficiency declines as the finishing weight increases. Price penalties for excessive fat may also restrict the upward flexibility of the finish weight.

A number of characteristics of feedlot sector demand may be implied from Equations 2.12 through 2.19. These characteristics define the theoretical model of feedlot sector demand for feeder cattle. In summary:

(1) The prices of feed grain are inversely related to the demand for feeder cattle of a given market weight (Equation 2.14).

(2) As feed grain prices increase, the demand for heavy feeder cattle may increase relative to the demand for light feeder cattle (Equation 2.19).

These characteristics of feedlot sector demand for feeder cattle may be given another interpretation. First, as feed costs increase, the feedlot sector may reduce the production of fed beef through reduced total demand for feeder cattle. Second, in response to increased feed costs, the feedlot sector may reduce the relative contribution of the feedlot sector in producing a given level of fed beef through increased demand for heavy feeder cattle, relative to light feeder cattle.
Feedlot sector demand for feeder cattle comprises one part of the feeder market. The determination of prices and quantities in the feeder market requires a consideration of feeder cattle supplies. Potential supplies of feeder cattle are held in the forage sector.

The Forage Sector

The forage sector may be defined as the joint activities of the cow-calf and backgrounding subsectors. The activities of the cow-calf subsector are associated with breeding production. The backgrounding subsector may be defined as the production of animal weight on non-breeding inventories of forage cattle. The sector definition of forage production is based on the interaction between the cow-calf and backgrounding subsectors. If the cow-calf subsector is defined as the production of weaned calves through the maintenance of breeding inventories, then weaned calves are marketed outside the forage sector or transferred to the backgrounding subsector. Since the subsector activities are not mutually exclusive, the transference of animal inventories within the forage sector may not require an open market exchange. This internal transfer of calves, not intended for breeding, between subsectors may be reflected in cattle markets. However, the definition of cow-calf versus backgrounding inventories of calves is not perfectly clear. Hence, sector definitions of cow-calf and backgrounding production would be weak. A second form of interaction between cow-calf and backgrounding production may also contribute to a combined sector definition. Cow-calf and backgrounding production share a common set of resources, the forage base.
The forage base is a land intense, composite input of residual feeds comprised of crop aftermath, range pasture, and other land not demanded for more intense production (Jacobs, 1977). The aggregate forage base is a highly heterogeneous factor of production. Consideration of the capacity, cost structure, and the effects of externalities, such as weather, on the forage base is severely limited.

The cost structure of the forage base is highly variable among regions. Midwestern production is typified by many small herds held as a supplementary enterprise with respect to crop production. The allocation of land, labor, and capital to forage production, and the costs associated with this allocation, are not independent of primary Midwest farm enterprises. In contrast, western rangeland production is typified by a fewer number of large herds where cattle production is often a sole enterprise. A single factor, such as precipitation, may have large effects on western production. Overwintering costs, which may constitute a critical cost, especially to western producers, may not be a consideration of producers in the south.

It may not be possible to develop an aggregate model of the forage base, which takes into account the supply and demand characteristics of the forage base. However, for a given capacity and cost structure of the forage base, it may be possible to examine the basic relationships between:

1. The cow-calf and backgrounding subsectors.

---

3 This problem is addressed by Vic Jacobs in a series of publications: Farm Management Newsletter: Beef Forage Series, 29, 30, and 31, Cooperative Extension Service, University of Missouri, Lincoln University (1977).
(2) The forage sector and the other components of the primary production system.

The Cow-Calf Subsector (Long-run Production)

The cow-calf subsector may be defined as the production of animal numbers through the maintenance of a breeding population. Management of the aggregate breeding population may be considered as the long-run production decision. To construct a framework for the long-run production decision, the physical activities of the subsector must be examined. The production activities of the cow-calf subsector may include:

(1) Maintenance of an active breeding population.
(2) Maintenance of a breeding replacement population.

Marketing activities of the cow-calf subsector include:

(1) Marketing of animals culled from the breeding population.
(2) Marketing of calves not withheld in the replacement population.

The relationship between production and marketing activities, in the cow-calf subsector, give rise to the basic structure of long-run production. A simple set of equations may be used to demonstrate the following points. First, the long-run production decision may be reduced to two decision parameters. One, the cull rate, and two, the rate at which calves are withheld for breeding, the withholding rate. Second, given existing subsector inventories, the long-run production

\[ \text{Equation} \]

4 Under the assumption that male and female breeding stock are held in constant proportions.
decision determines market supplies of cow-calf products and future production levels. Third, current levels of breeding production are a function of a series of past production decisions.

Market supplies of cows may be expressed as:

\[ S_{\text{cow}} = CR \cdot I_B \]  
Eq. 2.20

where:

- \( S_{\text{cow}} \) = quantity of cows supplied for slaughter, and;
- \( CR \) = the cull rate
- \( I_B \) = inventories of active breeding cows.

The market supply of calves may take the form of:

\[ S_C = (1 - WR)(I_C) \]  
Eq. 2.21

where:

- \( S_C \) = quantity of calves supplied, and;
- \( WR \) = the rate at which calves are withheld for breeding, and;
- \( I_C \) = inventory of weaned calves.

Market supplies of cows and calves are determined by existing inventory levels, the cull rate, and the withholding rate.

Production of calves may be specified as:

\[ I_{B_{t+1}} = C \cdot (I_B_t) \]  
Eq. 2.22

where:

- \( C \) = calving rate, assumed to be a constant.

Inventory of active breeding cows may be expressed as:
\[ I_B = (I_{B_{t-1}})(1 - CR_t) + (I_{r_t}) \]  \hspace{1cm} \text{Eq. 2.23}

where:

\[ I_r = \text{inventory of mature replacement heifers.} \]

In a similar manner, mature replacement stock may be specified as:

\[ I_r = WR(I_{c_{t-2}}) \]  \hspace{1cm} \text{Eq. 2.24}

Breeding production, as measured by cow-calf subsector inventories, is determined by past inventories and the two decision parameters, the cull and withholding rates. The interdependence of past and present long-run decisions on future production may be illustrated by expanding the inventory equations for the active cow herd:

\[ I_{B_{t-1}} = (I_{B_{t-2}})(1 - CR_t) + (I_{c_{t-2}})(WR_{t-2}) \]  \hspace{1cm} \text{Eq. 2.25}

\[ = (I_{B_{t-2}})(1 - CR_{t-1}) + (I_{c_{t-3}})(WR_{t-3}) \]

\[ + (1 - CR_t)(I_{B_{t-3}})(c)(WR_{t-2}) \]

\[ = \ldots \]

It may be noted that the current decision is constrained by existing inventory levels and the effect of the current decision will be carried over successive production periods through inventories of cows and replacement heifer inventories.

The effect of the current decision on future production is a form of inertia, which greatly reduces the flexibility of the cow-calf sector to shift production in response to short-run changes in market prices. This inertia may restrict production response in the cow-calf subsector
to perceived changes in long-run market prices. Expected long-run prices may tend to weigh current price changes with long-term price trends. Hence, an extended set of market information (market prices) may enter into the long-run production decision.

Market prices which affect the determination of the cull and replacement rate may be classified into three groups. One, calf prices, as product prices, which determine the present value of calf inventories and the expected capital value of calves withheld for breeding; two, cow prices, which may reflect the current versus expected salvage price for capital inventories of cows; three, other non-fed cattle prices which determine opportunity costs for alternative forms of production on the forage base. The major alternative to breeding production is backgrounding, since forage resources may be converted, in part, from breeding production to backgrounding. Therefore, expected prices for backgrounding subsector products, such as heavy feeder cattle, may be relevant to the long-run production decision in establishing the opportunity costs of breeding production.

The Backgrounding Subsector

Backgrounding is the production of forage-fed beef, marketed as feeder or slaughter cattle. In comparison with the cow-calf subsector, the backgrounding subsector utilizes the forage base to produce weight as opposed to animal numbers (Jacobs, 1977). However, cow-calf and backgrounding activities may not be mutually exclusive with respect to individual operators. The marketing activities between the forage subsectors may not be restricted to open market exchange. However, for
a given production capacity of the forage base, a tradeoff exists between cow-calf and backgrounding production. An increase in the relative utilization of forage in cow-calf production will yield additional animal numbers. However, the average market weight of forage-fed cattle must decline. An increase in the relative usage of the forage base in backgrounding will yield greater market weights for forage-fed cattle. In this case, however, the total number of calves produced must decline. This tradeoff between breeding and backgrounding production exists for each individual producer and is independent of the cost structure or the capacity of the forage base. Vic Jacobs (1977) defines this tradeoff as a forage subsector production mix, where the production mix is a specific combination of breeding and backgrounding production. For a fixed capacity of forage resources, the forage subsector production mix may be shifted to produce:

1. greater animal numbers at lower market weights,
2. fewer animal numbers at greater market weights.

The forage subsector production mix reflects the relative demand for animal inventories as capital goods in the cow-calf and backgrounding subsectors. The reservation demand of the cow-calf subsector for breeding stock may determine the production of animal numbers and create an effective demand for the forage base. Inventories of potential feeder cattle, which are demanded by the backgrounding subsector for the continued production of animal weight, also creates a demand for the forage base.

Returns to the forage sector and the relative efficiency of forage production, as a component of the beef industry, depend, in part, on
the forage subsector production mix. The problem of optimizing forage sector production is extremely complex. However, a simple relationship may be assumed to hold true. One, as the price differential between calves and heavier feeder calves increases, returns to background production will decline. The cost of the animal input, weaned calves, is increasing, and the net returns to the weight produced in backgrounding is declining. Hence, as the price differential between feeder calves and heavy feeder cattle increases, the forage subsector production mix may shift toward cow-calf production, yielding greater animal numbers and lower market weights. On the other hand, as the price spread decreases, the forage subsector production mix may shift toward backgrounding, yielding fewer animal numbers at greater market weights.

Differential prices for calves and heavy feeder cattle may be partially explained by a second production mix, the forage-grain production mix. Let the inventories of weaned calves, not withheld for breeding, be marketed to the feedlot sector or transferred to the backgrounding subsector. Further, let the backgrounding subsector market grass-fed beef to the feedlot sector and the slaughter market. These relationships are shown in Figure 2.2.

The market weight of animals marketed from the forage sector may be taken as a function of the time inventories held on the forage base. The average market weight of cattle marketed from the backgrounding subsector is a measure of the relative contribution of the backgrounding subsector in the production of a given quantity of slaughter cattle.
Figure 2.2. A model of primary production.

Consider the animals marketed from the forage sector to the feedlot sector. The average market weight of feeder cattle is a measure of the relative utilization of the forage base in the production of grain finished cattle. This may be contrasted with the portion of total slaughter weight added in the feedlot sector. The forage-grain production mix may be defined as the proportion of slaughter weight added in the forage versus feedlot sectors. As the average market weight for feeder cattle decreases, the weight added in the feedlot sector must increase, and the forage-grain production mix shifts toward grain-based production. An example may illustrate the relationships under examination.
Consider the change in feedlot sector demand for feeder cattle in response to increasing feed costs. One, the total quantity of feeder cattle demanded may decline. Two, the average market weight of feeder cattle demanded may increase. Both these factors will decrease the total weight added in the feedlot and increase the relative contribution of the forage sector in producing slaughter cattle. Hence, the forage-grain production mix may shift toward forage-based production in response to increased feed costs.

The shifts in feedlot sector demand for feeder cattle, cited in the example above, may also affect feeder cattle prices. First, the average price of feeder cattle may decline. Second, the price spread between calves and heavy feeder cattle may decrease. This decrease in the price differential between calves and heavy feeders may signal the forage sector to shift the forage-subsector production mix toward backgrounding. The production of animal numbers may decline and average market weights may increase.

From this example, it may be seen that feeder market prices are important market signals. Absolute and relative price levels in the feeder market may aid in maintaining a balance between backgrounding, breeding, and feedlot production.

The Feeder Market

Market prices and quantities of feeder cattle are determined by interaction of supply and demand in the feeder market. The exchange functions of the feeder market equate forage sector supplies and feedlot sector demand for feeder cattle. Quantities demanded and prices
determined in the feeder market facilitate the determination of both the forage-grain and the forage subsector production mixes of the primary production system.

Pricing in the feeder market is, in part, accomplished through partitioning the feeder market into component markets. Supply and demand relationships, although highly interdependent, are differentiated on the basis of sex, grade, and weight. Partitioning of the feeder market into discrete weight classes allows the exchange function of the feeder market to:

(1) Determine separate prices for calves and heavier feeder cattle.

(2) Adjust the average market weight of feeder cattle.

The determination of feeder calf and heavier feeder cattle prices may allow for the differentiation of cow-calf versus forage sector product prices. The adjustment of average market weight may allow for physical shifts in forage-grain production mix.

If breeding production and backgrounding production are in a competitive relationship with respect to the forage base, then the spread between feeder calves and heavy feeders may constitute a market evaluation of the forage subsector production mix. The price differential may reflect the relative value of forage utilized for breeding versus backgrounding production. Hence, the expected long-run price spread may be a viable parameter of the long-run production decision.

*Feed Grain Price Effects in the Feeder Market*

Market prices for feeder cattle are determined, in part, by the feedlot sector demand for feeder cattle. Therefore, the characteristics
of feedlot demand, with respect to changes in market prices for feed grains, may be transmitted to the market prices for feeder cattle. A model of the feeder market may illustrate these feed grain price effects in the feeder market.

Let the feeder market be partitioned into two mutually exclusive weight classes. In addition, let each weight class have fixed average market weights, \( W_1 \) and \( W_2 \) such that \( W_1 \) is less than \( W_2 \). The interaction of supply and demand within the feeder market will then yield an ordered price-quantity pair for each weight class market equilibrium. The quantity of feeder cattle demanded for a given class may be expressed in the form:

\[
Q_{f_i} = f(P_{f_i}, D)
\]

Eq. 2.30

where:

\( Q_{f_i} = \) the quantity demanded of feeder cattle of average market weight \( W_i \), and;

\( P_{f_i} = \) the price of feeder cattle of average market weight \( W_i \), and;

\( D = \) a set of exogenous factors which affect demand.

The exogenous parameters of demand, \( D \), may be replaced with the break-even price defined in Equation 2.12.

Equation 2.12 may be rewritten in the following form:

\[
BE_i = P_s + \frac{WG_i(P_s - FC_i - CO_i)}{WP_i}
\]

Eq. 2.31

where:
\( \text{BE}_i \) = the break-even price for feeder cattle of weight \( W_i \), as a parameter of demand,

\( P_s \) = the price of finished cattle,

\( W_{g_i} \) = the weight placed in the feedlot sector on feeder cattle of market weight \( W_p \),

\( F_{C_i} \) = the cost per unit \( W_g \) for animals of market weight \( W_i \),

\( C_{o_i} \) = the non-feed costs per unit \( W_g \) for animals of market weight \( W_i \).

The quantity of feeder cattle demanded at average market weight \( W_i \), may be expressed in the form:

\[
Q_{f_i} = f(P_f, \text{BE}_i) \quad \text{Eq. 2.32}
\]

A market model may be specified with the addition of the quantity supplied:

\[
Q_{f_i} = f(P_f, \text{BE}_i) \quad \text{(demand)} \quad \text{Eq. 2.33}
\]

\[
Q_{f_i} = g(P_f, S_i) \quad \text{(supply)}
\]

where:

\( S_i \) = a set of exogenous parameters of supply.

The reduced form of the market model may be written as:

\[
P_f = f_i(\text{BE}_i, S_i) \quad \text{Eq. 2.34}
\]

\[
Q_{f_i} = g_i(\text{BE}_i, S_i) \quad \text{Eq. 2.34}
\]

If the relationship between the break-even price as a proxy for demand and the market price is specified:

\[
\frac{\partial P_f}{\partial \text{BE}_i} > 0 \quad \text{Eq. 2.35}
\]
then, from Equation 2.14, it may be noted:

\[
\frac{\partial P_f}{\partial FC_i} = \frac{\partial P_f}{\partial BE_i} \frac{\partial BE_i}{\partial FC_i} < 0 \quad \text{Eq. 2.36}
\]

Equation 2.36 implies that feed costs (feed grain prices) are inversely related to the market price of a given class of feeder cattle. The market model may be extended to examine the impact of feed grain prices on feeder market prices, relative to market weights.

Consider the price quantity relationships between the two average market weights \(W_1\) and \(W_2\). The cross partial derivative of the break-even price, with respect to feed costs and market weight (Equation 2.19), may be written:

\[
\frac{\partial BE_i}{\partial FC_i} \frac{\partial BE_i}{\partial W_i} = \frac{-f'(W_i)W_i - f(W_i)}{W_i^2} > 0 \quad \text{Eq. 2.37}
\]

Given:

\[
W_1 > W_2 \quad \text{Eq. 2.38}
\]

This implies:

\[
\left(\frac{\partial BE_i}{\partial FC_i}\right)_{W_1} < \left(\frac{\partial BE_i}{\partial FC_i}\right)_{W_2} < 0 \quad \text{Eq. 2.39}
\]

This implies:

\[
\frac{\partial P_f}{\partial FC_i} \cdot \frac{\partial P_f}{\partial BE_i} \cdot \frac{\partial BE_i}{\partial FC_i} \cdot \frac{\partial BE_i}{\partial W_i} < \frac{\partial P_f}{\partial BE_i} \cdot \frac{\partial P_f}{\partial FC_i} \cdot \frac{\partial BE_i}{\partial FC_i} \cdot \frac{\partial BE_i}{\partial W_i} \quad \text{Eq. 2.40}
\]

\[
= \left(\frac{\partial P_f}{\partial FC_i}\right)_{W_2}
\]
An interpretation of the relationship in Equation 2.40 may yield a more powerful relationship with respect to feed grain prices and market prices for feeder cattle. As feed grain prices increase, the negative impact on feeder market prices increase as market weights decrease. As feed grain prices decrease, the positive impact on feeder market prices decrease as market weights increase. Therefore, an increase in feed costs may decrease the absolute price level of a given class of feeder cattle. However, heavy feeder prices may increase relative to feeder calf prices. This may reflect a desired shift in the forage-grain production mix toward forage production of finished cattle.

The change in relative prices, which may signal for a shift in the forage-grain production mix, may also affect long-run production. If an increase in feed costs induces a greater decrease in feeder calf prices, relative to heavy feeder prices, then the price spread between feeder calves and heavy feeder cattle will also decline. This shift in relative prices will increase the opportunity cost of breeding versus backgrounging production.

The impact of an increase in feed grain prices on breeding production may be twofold. First, a decline in the cow-calf sector product price, the price of feeder calves. Second, an increase in the opportunity costs of breeding versus backgrounding production. A decrease in the prices of feed grains may have the inverse effects on long-run production.
Transmission of Feed Grain Price Effects to Long-Run Production

If feed grain prices affect both the general and relative price levels in the feeder market, then the transmission of feed grain price effects to breeding production is dependent on the degree to which feeder market prices are incorporated into the long-run production decision.

The long-run production decision was considered in the cow-calf subsector. Cow-calf subsector inventories of active breeding cows and replacement heifers may provide a basis for both quantitative and theoretical examination of long-run production.

Cow-calf inventories were specific in Equations 2.24 and 2.25:

\[ I_{B_t} = (I_{B_{t-1}})(1 - CR) + I_{r_t} \]
\[ I_{r_t} = (I_{c_{t-2}})(WR) \]

If feeder market prices strongly influence long-run production, then the long-run decision parameters, cull rate, and the withholding rate may be specified as a function of expected feeder prices. Let the cull and withholding rate be expressed as:

\[ CR = f (P_{f*}, W_{1}, (P_{f*} - P_{f*} \text{ W}_{1} - W_{2}) \text{ Eq. 2.50} \]
\[ WR = g (P_{f*}, W_{1}, (P_{f*} - P_{f*}) \text{ Eq. 2.51} \]

On the basis of economic production theory, a positive relationship between production levels and the product prices may imply:
(1) A positive relationship between the price of feeder calves and the withholding rate.

(2) An inverse relationship between the price of feeder calves and the cull rate.

An inverse relationship between production levels and the opportunity costs of alternative forms of production may imply:

(1) A positive relationship between the price spread (between feeder calves and heavy feeder cattle), and the withholding rate.

(2) An inverse relationship between the price spread and the cull rate.

If cow-calf subsector inventories are specified in the following form:

\[ I_{R_t} = h_2(I_{c_{t-2}}, (PC^*_t), (PS^*_t)) \]

\[ I_{B_t} = h_1(I_{B_{t-1}}, I_{r_{t}}, (PC^*_t), (PS^*_t)) \]

where:

\( (PC^*_t) = \) the expected light feeder cattle price

\( (PS^*_t) = \) the expected price spread, \( Pf_{W_1} - Pf_{W_2} \)

then the expected relationship between feeder market prices and long-run production may be expressed as:

\[ \frac{\partial I_B}{\partial (PC^*_t)} > 0 \]  \hspace{1cm} Eq. 2.54

\[ \frac{\partial I_B}{\partial (PS^*_t)} > 0 \]  \hspace{1cm} Eq. 2.55

\[ \frac{\partial I_r}{\partial (PC^*_t)} > 0 \]  \hspace{1cm} Eq. 2.56
Equations 2.54 and 2.56 imply a positive relationship between breeding inventories and the expected price of feeder calves. Equations 2.55 and 2.57 imply a positive relationship between the expected price spread (the price of feeder calves less the price of heavy feeder cattle), and cow-calf subsector inventories.

The inventory model, defined in Equations 2.52 and 2.53, provides a basis to observe the impact of feeder market prices on cow-calf subsector inventories. If the inventory model is combined with the feeder market model, then it may be possible to observe the transmission of feed grain price effects to the cow-calf subsector through the feeder market.

A Combined Model

A set of sequential relationships between feed grain prices and feeder cattle demand, feedlot demand and feeder cattle prices, feeder prices, and breeding inventories has been discussed in this chapter. In the following chapter, a reduced form of this combined model will be quantified.
III. QUANTITATIVE ANALYSIS

Overview

Quantitative analysis in economics is often given to the term, "econometrics". The role of econometrics is stated by Johnston as:

The role of econometrics is the estimation and testing of economic models. The first step in the process is the specification of the model in mathematical form. (Johnston, 1972, p. 5)

Specification of the mathematical form of the model is a transition between the theoretical model and quantitative analysis. The relationships established in the theoretical model may be viewed as a set of prior restrictions on the mathematical format of the model relating feed grain prices to feeder market prices, and feeder market prices to long-run production. For example, an inverse relationship between feed grain prices and feeder market prices is specified in the theoretical model. With respect to the quantitative model, this may be considered as a restriction on the sign of the coefficient, relating feed grain and feeder market prices. The expected sign of the coefficient is less than zero. If the estimated coefficients, relating feed grain and feeder market prices, is consistent with the prior restriction on the sign of the coefficient, then the theoretical and quantitative models are, in part, consistent.

Prior restrictions established in the theoretical model are not sufficient to specify an exact mathematical form of the relationships under consideration. An exact mathematical model is required for
estimation. An assumption must be made concerning the mathematical specification of the relationships.

If a linear relationship is taken to exist between dependent variables, such as feeder cattle prices, and independent variables, such as feed grain prices, then the mathematical form of a given model may be expressed:

\[ y = X\beta + u \]  
Eq. 3.11

where:

- \( y \) = a vector of observations on the dependent variables,
- \( X \) = a matrix of observations on the independent variables,
- \( \beta \) = a vector of true coefficients, and
- \( u \) = a vector of random disturbance terms.

In this form, the prior restrictions on the model may be postulated as a set of hypotheses concerning the sign and relative magnitude of the true coefficients, denoted by the vector \( \beta \). For example, let:

\( \beta_i \) = the true coefficient relating feed grain prices to the average price of all feeder cattle.

The inverse relationship between feed grain prices and feeder cattle prices may take the form of a null hypothesis. Common statistical procedure may be used: the premise to be rejected is formulated as the null hypothesis:

\[ H_0: \beta_i \geq 0 \]

The premise to be accepted may take the form of the alternative hypothesis:

\[ H_A: \beta_i < 0 \]
The linear model may be estimated in the form:

\[ y = X\hat{\beta} + \epsilon \]  

where:

\( \hat{\beta} \) = a vector of estimated coefficients, and;

\( \epsilon \) = a vector of error terms

If the estimated value of the coefficient, \( \hat{\beta}_1 \), leads to rejecting the null hypothesis, then acceptance of the alternative hypothesis implies an inverse relationship between observed feed grain and observed feeder market prices. If the null hypothesis is accepted, then the quantified form of the model must be rejected.

Two additional criteria may be used to evaluate the model. First, the estimation techniques should minimize violations of the underlying assumptions of econometric analysis. Second, the model, as a system of equations, may be used to track a historical period. A subjective evaluation of actual and predicted values may provide insight into the model's ability to describe the relationships being estimated. Subjective evaluation is an important aspect of validating an econometric model. The specification of the linear model and the reliability of the data base must be evaluated without the tools of statistical inference.

The Data Base

This study utilizes secondary data sources provided by the U.S. Department of Agriculture, Economics, Statistics, and Cooperatives Service (ESCS), and Agricultural Marketing Service (AMS). The data
Data series may be classified on a number of criterion. Some series, such as beef cow inventories, are reported as an aggregate figure for U.S. beef production. Other data series are reported for specific markets or for limited sample groups, such as market prices at Omaha. Data available to quantify aggregate market and inventory models may be of two types. First, are data series representing the U.S. in aggregate. Second, are point source data series which may be taken as a proxy for U.S. aggregate data, such as a specific market price taken as an aggregate market price.

Data may also be classified in a second format. First, simple data series used directly as reported, such as specified market prices. Second, composite data series, two or more simple data series combined into a single series, such as aggregating feed grain prices into a feed cost series. Third, synthetic data series, data series which are transformed on the basis of additional information or by assumption. An example of a synthetic series may be a series reflecting feedlot sector profit levels, given an assumed set of sector production constraints. Data series, used to estimate the models under consideration, will be of all three of the above groups.

Data Series for the Feeder Market and Feed Costs

To quantify the relationship between feed grain prices and feeder market prices, it is necessary to construct a model of the feeder market. Four data groups must be quantified:
(1) The exogenous or predetermined parameters of demand.

(2) The exogenous parameters of supply.

(3) The endogenous feeder market prices.

(4) The endogenous quantity demanded in the feeder market.

Two major predetermined parameters of demand are the prices of finished steers and the prices of feed grains. Market prices for cattle, finished steers, and feeder cattle are reported for steers and heifers of various grades. Prices for choice steers, 900 to 1100 pounds, at Omaha, may serve as a proxy for finished cattle prices from 1963 to the second quarter of 1978. Feed grain prices may also be approximated with point source data series. Feed grain prices may be aggregated into a composite series, the cost of feed, as a predetermined parameter of demand. The relative composition of feed grains in the concentrated diets used in the feedlot sector varies geographically. Feedlots are concentrated in regions where feed crops are grown. For example, corn may be the principle grain used in midwestern feedlots, and grain sorghum may be the principle feed grain in the southwest.

A diet may be constructed for the aggregate feedlot sector with four components, feed corn, grain sorghum, alfalfa, and soybean meal.

5 USDA, ESCS. Livestock and Meat Statistics, Statistical Bulletins Nos. 333 (1963); 522 (1973); and Annual Supplements.

5 USDA, ESCS. Livestock and Meat Situation, USDA, ESCS, Quarterly issues, 1978.
Total consumption of these feeds in the feedlot sector are reported annually over the interval 1960 to 1977. Consumption of each feed may be averaged over the interval. The averages may be combined and a relative percentage of each feed in the aggregate diet may be calculated. The results are presented in Table 3.11.

Feed costs of the aggregate diet may be expressed:

\[
FC = (0.397)(PCORN) + (0.202)(PGS) + (0.362)(PAFH) + (0.039)(PSBM)
\]

where:

- \( FC \) = feed costs per ton, and;
- \( PCORN \) = price of corn/ton, Chicago #2 yellow dent, and;
- \( PGS \) = price of grain sorghym/ton, Milo, and;
- \( PAFH \) = price of alfalfa - hay/ton, and;
- \( PSBM \) = price of soybean meal/ton, 44% protein, Decatur.

Average feed costs per hundredweight gain may also be calculated, given the following assumptions. Let the average beginning weight of animals placed in the feedlot sector be 600 pounds. Let the average finishing weight be 1050 pounds, with an average daily rate of gain being 2.4 pounds. Given the net energy for gain and the percentage of dry matter in the aggregate diet (Table 3.11), the total feed requirement in tons of feed per hundredweight gain may be calculated. Utilizing an adaptation of the "California Net Energy System", developed by Ray Brokken,

\[\text{Feed Situation, (1960-77).} \]

USDA, ESCS. Statistical Bulletin #489, (July 1972); Table 489.

The source for these feeds was: USDA, ESCS. "Feed Situation," (1960-78).
<table>
<thead>
<tr>
<th>Feed</th>
<th>Average of grain fed to cattle annually, in 1,000 tons (ATF)</th>
<th>Percentage of feed in aggregate diet (% DIET)</th>
<th>Net energy for gain, Mcal/lb. dry feed (NEG) a/</th>
<th>Net energy for gain, meal/lb. dry feed in diet (DIET NEG)</th>
<th>Percentage dry matter in feed (% DM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn, #2 yellow dent</td>
<td>16,493 b/</td>
<td>.397</td>
<td>.67</td>
<td>.27</td>
<td>.88</td>
</tr>
<tr>
<td>Grain sorghum, MILO</td>
<td>8,407 b/</td>
<td>.202</td>
<td>.56</td>
<td>.11</td>
<td>.89</td>
</tr>
<tr>
<td>Alfalfa hay (mid-bloom)</td>
<td>15,049 b/</td>
<td>.362</td>
<td>.22</td>
<td>.08</td>
<td>.89</td>
</tr>
<tr>
<td>Soybean meal (44% protein)</td>
<td>1,642 c/</td>
<td>.039</td>
<td>.59</td>
<td>.02</td>
<td>.89</td>
</tr>
<tr>
<td>SUM</td>
<td>4,592</td>
<td>1.0</td>
<td>-</td>
<td>.48</td>
<td>-</td>
</tr>
<tr>
<td>AVERAGE</td>
<td></td>
<td>1.0</td>
<td>.51</td>
<td>.48</td>
<td>.89</td>
</tr>
</tbody>
</table>


c/ 14-year average. Source: ESCS, USDA, Statistical Bulletin #489, July 1972; Table 207.
the feed requirement per hundredweight gain, under the above conditions, is .46 tons of the aggregate diet. The average feed cost per hundredweight gain may be expressed:

\[ FC^* = (.46)(FC) \]

Eq. 3.14

where:

\[ FC^* = \text{average feed costs per hundredweight gain.} \]

In either form, feed costs may be considered as an exogenous parameter of demand.

The major predetermined parameters of supply are the inventories of potential feeder cattle held in the forage sector. However, data series are not available for these parameters.

A synthetic series may be created to approximate growth trends in forage sector inventories. The series may take the form of a lagged distribution of the cow herd. The cow herd is reported annually as cows on farms as of January 1, over the interval 1960 to 1964, and cows that have calved, and replacement heifers on farms as of January 1, over the interval 1965 to 1977. The cow herd inventory may be distributed evenly over a quarterly frequency. A lagged moving average may be used to distribute the breeding inventory over the period corresponding to the time when a given generation of cow-calf subsector production reaches average market weights for feeder cattle. A four-quarter moving average of the distributed breeding herd lagged six quarters will correspond to feeder cattle inventories between one and two years old. This moving average

---

8 Ray Brokken. USDA, ESCS, Oregon State University, Corvallis, Oregon. Two internal papers, n.p., n.d.

9 USDA, ESCS. Meat and Livestock Statistics.
may serve as a proxy to the general growth in forage sector inventories of potential feeder cattle, a synthetic aggregate series taken to be an exogenous parameter of supply.

Two endogenous parameters of the feeder market model may be considered. First, the aggregate prices of feeder cattle. Point source data series may be used to quantify three prices. **One:** the average feeder price level as the price of choice feeder steers 500 to 700 pounds at Kansas City; the price of light feeder cattle, or the price of calves as choice feeder steers 400 to 500 pounds at Kansas City; the price of heavy feeder cattle as the price of choice feeder steers 700 to 800 pounds at Kansas City. The interval of the prices is 1963 to the second quarter of 1978.

A corresponding set of series for the quantities of feeder cattle demanded are not available. As a proxy for the quantity of feeder cattle demanded at all weights, feedlot placements in 23 states may be used. Cattle placed on feed, as the endogenous quantity demanded, delimit the minimum frequency quarterly, and the maximum interval from 1964 to the second quarter of 1978 of the feeder market model.

**Quantification of the Long-Run Production Model**

Inventories of beef cows that have calved and replacement heifers 500 pounds and over are reported on an annual basis, over the interval

---


11 USDA, ESCS. *Cattle on Feed,* (1964 to the second quarter of 1978).
1965 to 1977. These inventories are the dependent variables of the long-run production model.

The price variables under consideration are the long-run expected price for calves and expected price spread between feeder calves and heavy feeder cattle. Actual market prices may be converted by simple averages to an annual frequency. However, the mathematical specification of price expectation is unknown (Elam, 1975). A two-year moving average of the price of calves, and the price spread between calves and heavy feeder cattle, may be taken as a simple model of long-run price expectations for calves and the price spread.

With available data base, it may now be possible to specify quantitative models of the feeder market and long-run production.

**Estimation of the General Market Model**

Consider a simple model of the aggregate feeder market:

\[ Q_F = f_1(P_F, D) \] demand, and;

\[ Q_F = f_2(P_F, S) \] supply

where:

\[ Q_F = \text{market quantity of all feeder cattle, and;} \]

\[ Q_F = \text{average market price of all feeder cattle, and;} \]

\[ D = \text{a set of predetermined parameters of demand, and;} \]

\[ S = \text{a set of predetermined factors affecting supply.} \]

The reduced form of the market equations may be written:

\[ P_F = g_1(D, S) \] and;

\[ P_F = g_2(D, S) \]
The reduced form equations identify the market price and quantity as a function of two sets of predetermined parameters of supply and demand. To estimate the general market model, these parameters must be defined.

A common parameter to both supply and demand may be the market price in the previous period. The past market price may be taken as a first approximation of the current equilibrium price.

Two exogenous variables which may affect supply may be defined. First are the inventories of potential feeder cattle held in the forage sector. These variables may be approximated with a lagged distribution of the breeding herd. This variable may reflect the general growth pattern of the forage sector. As a parameter of supply, the lagged breeding herd may be positively related to the quantity supplied. The expected relationship between feeder inventories and the market quantity is positive. With respect to market prices, an inverse relationship is expected. However, feeder market prices have been subject to a strong growth trend, due to inflation which may create a strong positive correlation between feeder market prices and the growth in forage sector inventories of feeder cattle. Therefore, the lagged distribution of the breeding herd may be restricted to the quantity component of the model.

A second factor affecting supplies may be seasonal adjustments. Overwintering costs may be a major cost to many forage producers. In the quarters preceding the winter season, the forage sector supply schedules may shift out, increasing the quantity supplied at each price. This effect may be approximated by third and fourth quarter seasonal dummy variables. The expected relationship between these
variables and the market price is negative; a positive relationship is anticipated with respect to quantity. A second quarter seasonal dummy variable may be negatively related to quantity due to the anticipated increase in capacity of the forage base of the spring and summer months.

Two primary determinants of feedlot sector demand for feeder cattle are the relative profitability of cattle feeding, and the technical considerations in the flow of production. Continuous production over time requires a balance between the purchase of animal inputs and the marketing of finished cattle. The number of cattle marketed in a given period may affect, directly and positively, the number of feeder cattle demanded.

Profit levels in the feedlot sector are taken to be a second major determinant of feedlot sector demand for feeder cattle. Two major exogenous factors affecting returns to the feedlot sector are the prices of finished cattle and the cost of feed. An approximation of expected feedlot sector profit levels may be the current returns on animals marketed from the feedlot sector. Feed costs and the purchase of animal inputs comprise about 87 percent of total feedlot expenses.\(^{12}\)

For the general level of returns to the feedlot sector, a profit model may be specified:

\[
AP = \text{\(AW_p(PS-AP_F) + AW_g(PS-FC^*)\)}
\]

Eq. 3.15

\(^{12}\)Computed from "Corn Belt Cattle Feeding." Livestock and Meat Situation, (Selected issues, 1972-1978).
where:

\[ AP = \text{average profit for all feeder cattle}, \]
\[ AW_p = \text{average purchase weight}, \]
\[ PS = \text{price of finished cattle in the current period}, \]
\[ AP_F = \text{average purchase price for all feeder cattle over the past production periods}, \]
\[ AW_g = \text{average weight added in the feedlot sector}, \]
\[ FC^* = \text{average cost of feed per hundredweight gain over the last production period}. \]

Letting the average market weight of feeder cattle equal 600 pounds, and the average weight added in the feedlot equal 450 pounds, average profit may be expressed as:

\[ AP = 6.0(PS-AP_F) + 4.5(.46)(FC^*) \]

Eq. 3.16

Given the prior restrictions concerning the effect of feed grain prices:

\[ \frac{\partial P}{\partial FC} < 0 \]

Eq. 3.17

If the market price is specified as a function of average profit, then:

\[ \frac{\partial P}{\partial AFC} = \frac{\partial P}{\partial AP} \frac{\partial AP}{\partial AFC} \frac{\partial P}{\partial AFC} + \frac{\partial P}{\partial AFC} \cdot -W_g < 0 \]

Eq. 3.18

implies that:

\[ \frac{\partial P}{\partial AFC} > 0 \]

Eq. 3.19

Hence, if the true linear reduced form of the price model is expressed as:

\[ P_F = B_{10} + B_{11}AP + \ldots + u \]

Eq. 3.20
then the expected value of \( B_{10} \) is less than zero. This may be stated as a formal hypothesis:

\[
H_0: B_{12} \geq 0 \\
H_A: B_{12} < 0
\]

Consistent estimates of the price model may be obtained with two-stage least squares.\(^{13}\) The structural form of the estimated price model may be given by:

\[
P_F = \hat{B}_{10} + \hat{B}_{11}A + \hat{B}_{12}Q_F + \hat{B}_{13}Q_3 + \hat{B}_{14}D4 + \epsilon_i
\]

Eq. 3.21

where:

\( \hat{Q}_F \) = the estimated quantity demanded,

\( Q_3 \) = third quarter seasonal dummy,

\( Q_4 \) = fourth quarter seasonal dummy.

The quantity demanded of the structural equation is replaced in the second stage estimating the equation by the estimated values of cattle placed on feed (as proxy to the market quantity). Cattle placed on feed is estimated as a function of the exogenous variables of the first stage. The estimating equations for the first stage are presented in Table 3.20.

The ordinary least squares (OLS) estimate of the first stage market quantity, as a function of the predetermined variables, yields coefficients of the appropriate signs. At the 99 percent confidence level,

a critical "t" value of 2.02 indicates all the coefficients are different from zero. An adjusted $R^2$ value of .942 indicates a good fit between the model and the sample data.

A Durbin-Watson statistic (Durbin, 1951), of 1.99 shows no evidence of serial correlation at the 99 percent confidence level. Multicollinearity may enter into the OLS model estimates. Variance inflation factors for the forage sector inventory variable and the fed cattle marketed variable are relatively high. A ridge regression for $k$, equal to the Lawless-Wang estimate of .0063, is presented in Table 3.21. Very little difference exists between the OLS and ridge estimates, possibly due to the high signal to notice ratio indicated by the $R^2$ value (Hoerl and Kennaryd, 1970; Brown and Beattie, 1975).

The small changes in the regression coefficients and the relatively low variance inflation factor for the variable of interest, 3.11 for average profit, may not warrant the use of the ridge model for the first stage regression.

The estimated values of the market quantities may be calculated from the OLS estimate and applied to the second stage estimate of the average market price. The second stage estimating equations are presented in Table 3.21.

The two-stage least squares estimate of the price model yields coefficients of the appropriate sign. The use of the calculated "t" statistic, as a test of significance, or for hypothesis testing in two-stage least squares estimates, requires the assumption that the calculated "t" is distributed as a "t" statistic, which is not valid.
Table 3.20 Feedlot Placement Estimating Equations (Quarterly, 1964 to Second Quarter 1978, First Stage Equations for the Quantity Demanded)

<table>
<thead>
<tr>
<th>Value</th>
<th>Constant</th>
<th>AP</th>
<th>FS</th>
<th>MKT</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Adjusted R²</th>
<th>Durbin-Watson Statistic</th>
<th>Lawless-Wang k value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ordinary Least Squares</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.942</td>
<td>1.99</td>
<td>.0063</td>
</tr>
<tr>
<td>Coefficient</td>
<td>-1121.2</td>
<td>7.65</td>
<td>.1010</td>
<td>.715</td>
<td>-322.4</td>
<td>309.4</td>
<td>3021.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard error</td>
<td>412.5</td>
<td>1.11</td>
<td>.020</td>
<td>.079</td>
<td>133.1</td>
<td>135.0</td>
<td>135.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;t&quot; statistic, (T)</td>
<td>-2.71</td>
<td>6.86</td>
<td>5.05</td>
<td>9.08</td>
<td>-2.42</td>
<td>2.29</td>
<td>22.27</td>
<td></td>
<td>Standard error of the regression = 361.2</td>
<td></td>
</tr>
<tr>
<td>Variance inflation factor (VIF)</td>
<td>-3.11</td>
<td>38.67</td>
<td>97.77</td>
<td>2.01</td>
<td>1.92</td>
<td>1.95</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Ridge Regression k = 0.0063</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.942</td>
<td>1.99</td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>-1084.1</td>
<td>7.60</td>
<td>.1010</td>
<td>.710</td>
<td>-330.9</td>
<td>298.0</td>
<td>2998.1</td>
<td></td>
<td>Standard error of the regression = 361.3</td>
<td></td>
</tr>
<tr>
<td>Standard error</td>
<td>411.8</td>
<td>1.11</td>
<td>.020</td>
<td>.078</td>
<td>132.4</td>
<td>134.3</td>
<td>135.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;t&quot; statistic, (T)</td>
<td>-2.68</td>
<td>6.83</td>
<td>5.05</td>
<td>9.05</td>
<td>-2.50</td>
<td>2.22</td>
<td>22.21</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ k = \frac{\hat{\sigma}_u^2}{R^2 \Sigma_1^2} \]

where: \( k \) = Lawless-Wang estimate of the ridge \( k \) value, and;
\( P \) = the number of explanatory variables, and;
\( \hat{\sigma}_u^2 \) = the estimated variance of the regression, and;
\( R^2 \) = unadjusted \( R^2 \), and;
\( \Sigma_1^2 \) = sum of squares of the dependent variable, mean corrected.


\[ VIF = \frac{V(\hat{\theta}_i) \Sigma_1^2}{\hat{\sigma}_u^2} \]

where: \( V(\hat{\theta}_i) \) = sample variance of the regression coefficient, and;
\( \Sigma_1^2 \) = sum of squares of the \( i \)th variable, and;
\( \hat{\sigma}_u^2 \) = estimated variance of the regression.

Monte Carlo studies suggest that the distortion is usually, but not always, small (Kmenta, 1971, p. 584). If the calculated "t" is taken to be distributed as a t distribution, then a critical "t" of 2.03 indicates all the coefficients are significantly different from zero at the 99 percent confidence level.

The inclusion of a lagged endogenous variable requires the use of a Durbin "h" statistic for testing for serial correlation. For large samples, the "h" statistic is approximately distributed as z, the standard normal deviate (Durbin, 1970). At the 99 percent confidence level, the critical z value is approximately 1.95. The "h" statistic value of 1.75 is insufficient to reject the hypothesis of zero autocorrelation at the 99 percent confidence interval. However, an autoregressive correction of the two-stage model does not yield consistent estimates. Therefore, the autoregressive model, presented in Table 3.21, may not be acceptable (Kmenta, 1971, p. 589).

Multicollinearity may reduce the precision of the two-stage least square estimates. High variance inflation factors are estimated for the endogenous variable (Q_F, 158.9), and the estimated lagged endogenous variable, P_F, is moderately high, 41.19. However, the calculated Lawless-Wang value for k is low, .0025. The second stage ridge regression estimate for k, equal to .0025, is almost identical to the second stage OLS estimates. The high R^2 value may reduce the corrective value of the ridge procedure (Brown, 1977).

The second stage OLS model may be the preferred model. A relatively high adjusted R^2 indicates a good fit between the model and the sample. Since the model is recursive with one current endogenous
### Table 3.21 Average Feeder Price Estimating Equations (Quarterly 1964 to Second Quarter 1978, Second Stage Equations)

<table>
<thead>
<tr>
<th>Value</th>
<th>Constant</th>
<th>First stage ordinary least square estimates of cattle placed on feed as quantity demanded (Qf)</th>
<th>Average profit per head (AP)</th>
<th>Average feeder-steer price in the last quarter (Pf-t-1)</th>
<th>3rd Qtr. seasonal dummy variable (Q3)</th>
<th>4th Qtr. seasonal dummy variable (Q4)</th>
<th>Cochrane-Orcutt estimate of the autoregressive parameter (Rho)</th>
<th>Adjusted R² (R²)</th>
<th>Durbin-Watson statistic (D.W.)</th>
<th>Lawless-Wang estimate of the ridge k value (L.W.)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Two-Stage Least Squares (Second Stage)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.972</td>
<td>1.56</td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>-3.27</td>
<td>0.00096</td>
<td>0.052</td>
<td>.908</td>
<td>-2.95</td>
<td>-4.37</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard error</td>
<td>1.38</td>
<td>0.0004</td>
<td>0.006</td>
<td>0.036</td>
<td>0.512</td>
<td>1.40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;t&quot; statistic (T)</td>
<td>-2.37</td>
<td>2.22</td>
<td>8.78</td>
<td>25.42</td>
<td>-5.81</td>
<td>-3.35</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance inflation factor (VIF)</td>
<td>-</td>
<td>158.9</td>
<td>5.18</td>
<td>41.19</td>
<td>1.58</td>
<td>11.80</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Two-Stage Least Squares with Autocorrelation Correction</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.973</td>
<td>1.93</td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>-3.29</td>
<td>0.0010</td>
<td>0.053</td>
<td>.896</td>
<td>-2.99</td>
<td>-4.81</td>
<td></td>
<td>.229</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard error</td>
<td>1.67</td>
<td>0.0005</td>
<td>0.006</td>
<td>0.044</td>
<td>0.496</td>
<td>1.58</td>
<td></td>
<td>.145</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;t&quot; statistic (T)</td>
<td>-1.97</td>
<td>2.00</td>
<td>8.61</td>
<td>20.33</td>
<td>-6.03</td>
<td>-3.04</td>
<td></td>
<td>1.58</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Two-Stage L.S. With Ridge Regression k = .0025</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.972</td>
<td>1.56</td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>-3.15</td>
<td>0.00096</td>
<td>0.050</td>
<td>.905</td>
<td>-2.95</td>
<td>-4.49</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard error</td>
<td>1.36</td>
<td>0.0004</td>
<td>0.006</td>
<td>0.035</td>
<td>0.512</td>
<td>1.38</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;t&quot; statistic (T)</td>
<td>-2.32</td>
<td>2.28</td>
<td>8.72</td>
<td>25.86</td>
<td>-5.81</td>
<td>-3.31</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---


variable, the two-stage least squares estimates are consistent and asymptotically efficient. The inclusion of a lagged endogenous variable may lead to biased estimates of the coefficient (Kmenta, 1971, p. 586). If the calculated t is assumed to follow a t distribution, the hypothesis concerning the sign of the average profit coefficient may be tested. The critical t for 53 degrees of freedom at the 99.95 percent confidence level is approximately 2.67. The calculated t for average profit is approximately 8.78. Therefore, the hypothesis

\[ H_0: B_{12} \leq 0 \]

\[ H_A: B_{12} > 0 \]

may be rejected at the 99.95 percent confidence level, and the alternative hypothesis accepted.

The theoretical model and quantitative models, relating the effect of feed grain prices on the general level of prices in the feeder market, are consistent. The effect of feed grain prices on feeder market prices may be subjected to further examination.

**Estimation of Relative Price Levels in the Feeder Market**

A second hypothesis was developed from the theoretical model which considered the impact of feed grain prices on relative feeder market prices with respect to market weight. Let the feeder market be partitioned into three mutually exclusive weight classes with a market price and quantity determined for each class:
Class I (Animals < 500 lbs.)
Class II (Animals 500 to 699 lbs.)
Class III (Animals > 699 lbs.)

With respect to relative changes in feeder market prices, the outside classes, I and III, may tend to show opposing movements in market prices. Relative price movements may tend to pivot about the center price, Class II.

Changes in the Class II price may tend to reflect changes in the general price level. Let the center price be taken as equivalent to the general price level, \( P_F \), defined in the previous section. Consider a market model:

\[
q_i = f_i(P_i, P_F, RD) \quad \text{demand} \quad \text{Eq. 3.22}
\]

\[
q_1 = g_1(P_1, P_F) \quad \text{supply} \quad \text{Eq. 3.23}
\]

\[
q_3 = f_3(P_3, P_F, RD) \quad \text{demand} \quad \text{Eq. 3.24}
\]

\[
q_3 = q_3(P_3, P_F) \quad \text{supply} \quad \text{Eq. 3.25}
\]

where:

\( q_i = \) quantity demanded in Class I,
\( P_i = \) price of feeder cattle in Class I,
\( RD = \) set of factors affecting relative demand,
\( q_3 = \) quantity demanded in Class III,
\( P_3 = \) price of feeder cattle in Class III.

The reduced form equations yield a price model:

\[
P_1 = f_1^*(P_F, RD) \quad \text{Eq. 3.26}
\]

\[
P_3 = f_3^*(P_F, RD) \quad \text{Eq. 3.27}
\]
The prices of light and heavy feeder cattle are specified as a function of the general price level and a set of parameters affecting the relative demand for feeder cattle. However, the general price level is an endogenous variable. Hence, the true reduced form price equations may be written:

\[ P_1 = f_1^{**}(D, S, RD) \]  
Eq. 3.28

\[ P_2 = f_2^{**}(D, S, RD) \]  
Eq. 3.29

where:

- **D** = the set of predetermined variables which affect the general level of demand,
- **S** = the set of predetermined variables which affect the general level of supply.

The set of predetermined parameters which affect relative demand, and the set of exogenous parameters which affect the general level of demand, are not mutually exclusive. Two predetermined variables which affect relative demand are, again, the price of finished cattle and the price of feed, or feed grain prices. The relationship between feed costs and finished cattle prices, as a parameter of relative demand, may be expressed as a ratio:

\[ \text{RFS} = \frac{FC}{PS} \]  
Eq. 3.30

where:

- **RFS** = the feed-steer price ratio,
- **FC** = feed costs per ton,
- **PS** = price of finished steers.

The prior restriction on the relative effect of feed grain prices may be stated:
If the relative prices, \( P_1 \) and \( P_3 \) are specified as a function of the feedsteer price ratio, then:

\[
\frac{\partial P_1}{\partial RFS} \cdot \frac{\partial RFS}{\partial FC^*} < \frac{\partial P_3}{\partial RFS} \cdot \frac{\partial RFS}{\partial FC^*}
\]

Eq. 3.32

This implies:

\[
\frac{\partial P_1}{\partial RFS} \cdot \frac{1}{PS} < \frac{\partial P_3}{\partial RFS} \cdot \frac{1}{PS}
\]

Eq. 3.33

This further implies:

\[
\frac{\partial P_1}{\partial RFS} < \frac{\partial P_3}{\partial RFS}
\]

Eq. 3.34

If the true linear model of the structural price model is given by:

\[
P_1 = B_{11}P_F + B_{12}RFS
\]

Eq. 3.35

\[
P_3 = B_{31}P_F + B_{32}RFS
\]

then, the relative magnitudes of the true coefficients, relating the feed-steer price ratio to the prices \( P_1 \) and \( P_2 \), may take the form of the following hypothesis:

\[
H_0: \hat{B}_{12} \geq \hat{B}_{32}
\]

\[
H_A: \hat{B}_{12} < \hat{B}_{32}
\]

Consistent estimates of the true coefficients may be obtained with two-stage least squares. The structural model may be expressed as:

\[
P_1 = B_{11}\hat{P}_F + B_{12}RFS
\]

Eq. 3.37

\[
P_3 = B_{31}\hat{P}_F + B_{32}RFS
\]

Eq. 3.38
where:

\[ \hat{P}_F = \text{the estimated values of the general price level, } P_F, \]

form the first stage.

The first stage estimates of \( \hat{P}_F \) may be obtained from the second stage estimating equation for \( P_F \), developed in the previous section, Table 3.21.

The second stage model estimations for light and heavy feeder cattle are presented in Tables 3.30 and 3.31, respectively.

The two-stage least squares estimate of the light feeder price model yields coefficients of the expected sign. If the calculated "t" is assumed to have a t distribution, then at the 99 percent confidence level, a critical t of 2.02 for a one tailed test, indicates all the variables are significantly different from zero.

Multicollinearity may not be an estimation problem. Variance inflation factors for the endogenous variable \( P_F \) (9.06), and for the feed-steer price ratio (9.15) are low. The Lawless-Wang estimate of the ridge k value is small, (.0025).

Serial correlation is indicated by the Durbin-Watson statistic value, (.83). At the 99 percent confidence level, the test statistic value is well below the critical value of 1.32. A two-stage least square model, with autoregressive correction, is printed in Table 3.34. Estimates of the coefficients are consistent. At the 99 percent confidence interval, all the coefficients are of the appropriate sign and significantly different from zero. The relatively large value of the coefficient for the autoregressive parameter Rho (.0590), and a respective "t" value of 5.16 confirm the serial correlation problem in the uncorrected model. The corrected value of the Durbin-Watson
Table 3.30 Feeder Calf Price Estimating Equations (Quarterly, 1964 to Second Quarter of 1978)

<table>
<thead>
<tr>
<th>Value</th>
<th>Average price of feeder cattle endogenous variable ($P_F$)</th>
<th>Feeder-steer price ratio ($RFS$)</th>
<th>Cochrane-Orcutt autoregressive parameter ($\rho$)</th>
<th>Adjusted $R^2$ ($R^2$)</th>
<th>Durbin-Watson statistic (D.W.)</th>
<th>Lawless-Wang estimate of ridge k value (L.W.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two-Stage Least Squares$^a/$</td>
<td>1.15</td>
<td>-2.61</td>
<td>-</td>
<td>.933</td>
<td>.83</td>
<td>.0025</td>
</tr>
<tr>
<td>Coefficient</td>
<td>.030</td>
<td>.698</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard error</td>
<td>37.56</td>
<td>-3.75</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;t&quot; statistic (T)</td>
<td>9.06</td>
<td>9.15</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance Inflation factor (VIF)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two-Stage Least Squares with Autoregressive Correction$^a/$</td>
<td>1.13</td>
<td>-2.14</td>
<td>.590</td>
<td>.954</td>
<td>1.61</td>
<td>-</td>
</tr>
<tr>
<td>Coefficient</td>
<td>.043</td>
<td>.981</td>
<td>.114</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard error</td>
<td>26.57</td>
<td>-2.18</td>
<td>5.16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;t&quot; statistic (T)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a/$First stage is the second stage estimate of the average feeder price, $P_F$. 

Standard error of regression = 2.71

Standard error of regression = 2.25
### Table 3.31 Uncorrected and Autoregressive Model Comparison: Estimates For Feeder Calf Prices

<table>
<thead>
<tr>
<th>Value</th>
<th>Average price of feeder cattle, endogenous variables ($P_f$)</th>
<th>Feeder-steer price ratio, exogenous ($RFS$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-autoregressive (uncorrected) two-stage least squares (2 SLS)</td>
<td>Autoregressive (corrected) two-stage least squares (Ar)</td>
</tr>
<tr>
<td>Coefficient</td>
<td>1.15</td>
<td>1.13</td>
</tr>
<tr>
<td>Standard error</td>
<td>.030</td>
<td>.043</td>
</tr>
<tr>
<td>&quot;t&quot; statistic (T)</td>
<td>37.1</td>
<td>26.6</td>
</tr>
<tr>
<td>Standard error of the regression (S.E. Reg.)</td>
<td>2.71</td>
<td>2.25</td>
</tr>
<tr>
<td>Adjusted $R^2$ ($R^2$)</td>
<td>.933</td>
<td>.954</td>
</tr>
</tbody>
</table>
statistic is 1.61. At the 99 percent confidence level, the hypothesis of zero autocorrelation is accepted \( (d_v = 1.28, d_u = 1.51) \). At the 95 percent confidence level, the test is indeterminent, \( (d_v = 1.45, d_u = 1.68) \). The two models may be compared in Table 3.31. Two significant differences exist between the uncorrected and autoregressive models. One: the corrected standard errors of the coefficients are increased in the autoregressive model. Two: the standard error of the regression is reduced 17 percent in the corrected model. The autoregressive model may be preferred to the uncorrected model, due to the problem of serial correlation. The autoregressive model of light feeder cattle price may be compared to the model estimates for heavy feeder cattle presented in Table 3.32.

The two-stage least squares estimate of the heavy feeder cattle price model yields coefficients of the appropriate sign. However, two major problems exist. One: if the calculated t is assumed to have a t distribution, then, a critical t of 1.30 indicates the coefficient for the feed-steer price ratio is not significantly different from zero at the 90 percent confidence level. The significance level, at which the feed-steer price ratio coefficient is significantly greater than zero, is approximately 81 percent. Two: the Durbin-Watson statistic value (1.12), indicates the presence of serial correlation at 95 percent level \( (d_u = 1.49) \) and the 99 percent level \( (d_u = 1.43) \).

The low confidence level for the feed-steer price ratio does not appear to be the result of multicollinearity. The variance inflation factors for the price ratio (9.05), and the endogenous variable \( \hat{P}_F \) (9.23), are fairly low. The Lawless-Wang estimate of the ridge k value, which
Table 3.32  Heavy Feeder Cattle Price Estimating Equations (Quarterly, 1964 to Second Quarter of 1978)

<table>
<thead>
<tr>
<th>Value</th>
<th>Average price of feeder cattle, endogenous variable ($P_F$)</th>
<th>Feeder-Steer price ratio ($RFS$)</th>
<th>Cochrane-Orcutt autoregressive parameter ($\rho$)</th>
<th>Adjusted $R^2$</th>
<th>Durbin-Watson statistic (D-W.)</th>
<th>Lawless-Wang estimate of the ridge k value (L.W.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.963</td>
<td>1.12</td>
<td>.0013</td>
</tr>
<tr>
<td>Two-State least squares&lt;sup&gt;3/&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>.937</td>
<td>-.40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard error</td>
<td>.019</td>
<td>.43</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;t&quot; statistic (T)</td>
<td>48.83</td>
<td>-.91</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance Inflation factor (VIF)</td>
<td>9.23</td>
<td>9.05</td>
<td></td>
<td></td>
<td>9.23</td>
<td>1.12</td>
</tr>
<tr>
<td>Two-stage least squares&lt;sup&gt;3/&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with autoregressive correction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>.940</td>
<td>-.450</td>
<td>.423</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard error</td>
<td>.026</td>
<td>.593</td>
<td>.130</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;t&quot; statistic (T)</td>
<td>36.45</td>
<td>-.76</td>
<td>3.26</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<sup>3/</sup> First stage is the second stage estimate of the average feeder price, $P_F$. 

Standard error of the regression = 1.70

Standard error of the regression = 1.56
minimizes the mean square error of the regression coefficients is small (.0013). The effect of serial correlation in the uncorrected model may again be to underestimate the standard error of price ratio coefficient. This is indicated by the increased standard error for the price ratio (.593) and a reduced t statistic (-.86) in the autoregressive model. The reason for the low significance level for the price ratio is not a statistical problem. The relative impact of feed costs declines as purchase weight increases. As the purchase weight increases, and the weight added in the feedlot declines, the purchase price becomes an increasingly greater portion of total expenses and the relative cost of feed declines. Hence, the coefficient for the price ratio, given the general price level, should approach zero as the purchase weight increases. Therefore, the significant difference between the coefficient and zero should also decline.

The autoregressive correction of the model yields a Durbin-Watson statistic of 1.92, indicating no serial correlation, at the 99 percent confidence level \( (d_u = 1.45, d_v = 1.68) \), and at the 95 percent confidence level \( (d_u = 1.28, d_v = 1.51) \). The coefficients remain with the proper sign. A comparison of the autoregressive versus the uncorrected model is summarized in Table 3.33.

The coefficients are relatively stable in both models, a change of less than one percent for the endogenous variable, \( P_F \), and a change of 12.5 percent for the price ratio. The standard error of the regression decreased for the corrected model approximately eight percent. The significance of the price ratio declined, due to the underestimation of the standard error in the uncorrected model. The price ratio
Table 3.33 Uncorrected and Autoregressive Model Comparison: Estimates for Heavy Feeder Cattle Prices

<table>
<thead>
<tr>
<th></th>
<th>Average price of feeder cattle, endogenous variables ($P_F$)</th>
<th>Feeder-steer price ratio (RFS)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-autoregressive (uncorrected) two-stage least squares (2 SLS)</td>
<td>Autoregressive corrected two-stage least squares (Ar)</td>
</tr>
<tr>
<td>Coefficient</td>
<td>.937</td>
<td>.940</td>
</tr>
<tr>
<td>Standard Error</td>
<td>.019</td>
<td>.026</td>
</tr>
<tr>
<td>&quot;t&quot; statistic (T)</td>
<td>48.83</td>
<td>.3645</td>
</tr>
<tr>
<td>Standard Error of the regression (S.E. Reg.)</td>
<td>1.70</td>
<td>1.56</td>
</tr>
<tr>
<td>Adjusted $R^2$ ($\bar{R}^2$)</td>
<td>.963</td>
<td>.969</td>
</tr>
</tbody>
</table>
The coefficient is significantly different from zero at the 75 percent confidence level. Due to the effects of serial correlation on the uncorrected model, the autoregressive model may be the preferred estimation for heavy feeder cattle prices.

The autoregressive two-stage estimates for light and heavy feeder cattle prices may be the preferred models. However, to test the hypothesis:

\[ H_0: B_{12} \geq B_{32} \]

\[ H_A: B_{12} < B_{32} \]

The models are not applicable.

**Test of the Relative Price Hypothesis**

To test the relative magnitude of two regression coefficients in separate models, a special form of the market model may be used to construct an F test of the hypothesis:  

\[ F^* = \frac{SSE(R) - SSE(F)}{(n-2) - (n-4)} \]

\[ \frac{SSE(R)}{n-4} \]

Eq. 3.60

where:

- \( F = f \) statistic
- \( SSE(R) = \) sum of residual squares for the reduced model

---

\[ \text{SSE(F)} = \text{sum of residual squares for the full model} \]
\[ n = \text{number of observations}. \]

The full model may be expressed:
\[ P_1 = B_{11}P_F + B_{12}\text{RFS} + \varepsilon_1 \quad \text{Eq. 3.61} \]
\[ P_3 = B_{32}P_F + B_{32}\text{RFS} + \varepsilon_3 \quad \text{Eq. 3.62} \]

The sum of residual squares for the OLS estimates of the full model may be written:
\[ \text{SSR(F)} = \text{SSR}_1 + \text{SSR}_2 \quad \text{Eq. 3.63} \]

where:
\[ \text{SSR}_1 = \text{sum of squared residuals } \varepsilon_1 \]
\[ \text{SSR}_2 = \text{sum of squared residuals } \varepsilon_3 \]

The total number of observations and the degrees of freedom for the full model may be written:
\[ n = m_1 + m_2 \quad \text{Eq. 3.64} \]
\[ \text{DF} = n - r \]

where:
\[ n = \text{number of observations in the full model}, \]
\[ m_1 = \text{number of observation in the } P_1 \text{ model}, \]
\[ m_2 = \text{number of observation in the } P_3 \text{ model}, \]
\[ \text{DF} = \text{degrees of freedom}, \]
\[ r = \text{number of explanatory variables in the full model}. \]

The reduced model may be specified to constrain \( \hat{B}_{12} \) to equal \( \hat{B}_{32} \) in the form:
\[
\begin{bmatrix}
P_{11} \\
P_{12} \\
\vdots \\
\vdots \\
\vdots \\
P_{1m}
\end{bmatrix}
= 
\begin{bmatrix}
RFS_1 & P F_1 & 0 \\
RFS_2 & P F_2 & 0 \\
\vdots & \vdots & \vdots \\
RFS_m & P F_m & 0 \\
RFS_1 & 0 & P F_1 \\
RFS_2 & 0 & P F_2 \\
\vdots & \vdots & \vdots \\
RFS_m & 0 & P F_m
\end{bmatrix}
\begin{bmatrix}
\hat{\delta}_{11} \\
\hat{\delta}_{12} \\
\hat{\delta}_{13} \\
\hat{\delta}_{14} \\
\vdots \\
e_1
\end{bmatrix}
+ 
\begin{bmatrix}
\hat{\delta}_{11} \\
\hat{\delta}_{12} \\
\hat{\delta}_{13} \\
\hat{\delta}_{14} \\
\vdots \\
e_1
\end{bmatrix} = \text{Eq. 3.66}
\]

where:

\( \hat{\delta}_{11} \) = a coefficient of the reduced model equating \( \hat{B}_{12} \) and \( \hat{B}_{13} \),
\( \hat{\delta}_{12} \) = a coefficient of the reduced model,
\( \hat{\delta}_{13} \) = a coefficient of the reduced model,
\( e_1 \) = residuals of the estimates.

The reduced form of the model constrains the feed-steer price ratio to a single coefficient, \( \hat{\delta}_{11} \), by combining the observations on the price ratio and the dependent variables into two vectors. Two separate coefficients are estimated for the average feeder price by partitioning the observations on the independent feeder price variable. The coefficient \( \hat{\delta}_{12} \) corresponds to \( \hat{\beta}_{12} \) of the full model. The coefficient
\( \hat{\delta}_{13} \) corresponds to the coefficient \( \hat{\beta}_{32} \) of the full model. The results of the full and reduced are presented in Table 3.40.

The difference between the sum of squared error terms in the full and reduced models may be used to construct an "F" test of the hypothesis:

\[
H_0: \beta_{12} \geq \beta_{13} \\
H_A: \beta_{12} < \beta_{13}
\]

1) \( F^* = \frac{343.03 - 230.84}{(113) - (112)} = \frac{230.84}{(112)} = 54.44 \)  

2) \( F(.999, 1, 112) = 11.5 \)

Since: \( F^* = 54.44 > F(.99, 1, 112) = 11.5 \)

the null hypothesis is rejected and the alternative hypothesis accepted: \( \beta_{12} > \beta_{32} \) at the 99.9 percent confidence level.

The alternative hypothesis implies that the observed relationship between the feed-steer price ratio and market prices for feeder cattle is sensitive to market weight. The impact of an increase in the price ratio is increasingly negative as market weight decreases. Therefore, an inverse relationship is observed between the feed-steer price ratio and the price spread between feeder calves and heavy feeder cattle.
<table>
<thead>
<tr>
<th>Model</th>
<th>Estimation technique (estimated)</th>
<th>Number of observations (n)</th>
<th>Number of explanatory variable (r)</th>
<th>Degrees of freedom (D.F.)</th>
<th>Sum of squared residuals (S.S.R.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full model........ OLS</td>
<td>116</td>
<td>4</td>
<td></td>
<td>112</td>
<td>230.84</td>
</tr>
<tr>
<td>Reduced model..... OLS</td>
<td>116</td>
<td>3</td>
<td></td>
<td>113</td>
<td>340.03</td>
</tr>
</tbody>
</table>
Estimation of the Long-Run Production Model

Long-run production was defined in the section on the cow-calf subsector (Chapter II), as the management of the breeding herd. The breeding herd may be considered with respect to two inventories, active cows and replacement heifers.

The productivity of a given breeding population may reflect the average age and genetic composition of the herd. Management of the cow and replacement inventories may tend to optimize productivity. However, inventory levels are the primary determinant of cow-calf subsector production. A simple inventory model was developed in the previous chapter (Equations 2.52 and 2.53). Active cow inventories in the current period are specified as a function of the active breeding herd in the previous period, the expected price for calves, and the expected price spread between calves and heavy feeder cattle. The current replacement herd is specified as a function of replacement inventories in the previous period, the expected price of calves, and the expected price spread. The carryover of previous inventories may reflect the inelastic component of breeding production. The expected price of calves and the expected price spread are the feeder market prices under consideration.

Previous studies have specified on-farm inventories as a function of a steer-corn ratio (Reutlinger, 1966), average cattle prices, and an index of feed prices (Tryfos, 1974). A significant and positive relationship between active cow inventories and the steer-corn ratio were obtained by Reutlinger. A positive relationship, with respect to
to average cattle prices and an inverse relationship, with respect to
to the feed cost index, were obtained by Tryfos in estimating on-farm
inventories. However, none of these parameters reflect direct compo-
nents of the long-run production decision. The observed correlation
between these parameters and farm inventories may be attributed to
their effect on the forage sector product prices, the price of calves,
and the prices for heavy feeder cattle. Specification of the inventory
model, as a function of expected price of feeder cattle and the expected
price spread, as a product price and an opportunity cost of cow-calf
subsector production, may better conform to the basic concepts of
production theory.

The true linear form of the model may be expressed as:

\[ I_{R} = B_{11}I_{B_{t-1}} + B_{12}E(P_1) + B E(P_1 - P_3) + u_1 \]  
Eq. 3.70

\[ I_{R} = B_{21}I_{B_{t-1}} + B_{22}E(P_1) + E_{23}E(P_1-P_3) + u_2 \]  
Eq. 3.71

where:

\[ E(P_1) = \text{the expected price of calves}, \]

\[ E(P_1-P_3) = \text{the expected price spread}, \]

\[ u = \text{a random distributance term}. \]

The estimated form of the model may be written:

\[ I_{B} = \hat{B}_{11}I_{B_{t-1}} + \hat{B}_{12}E(P_1) + \hat{B}_{13}E(P_1-P_3) + \epsilon_1 \]  
Eq. 3.72

\[ I_{R} = \hat{B}_{21}I_{B_{t-1}} + \hat{B}_{22}E(P_1) + \hat{B}_{23}E(P_1-P_3) + \epsilon_2 \]  
Eq. 3.73

where:

\[ \epsilon = \text{an error term}. \]
A positive relationship is hypothesized to exist between the expected price of calves (light feeder cattle) and cow-calf subsector inventories. Expected calf prices are taken to be the product price for breeding production. An inverse relationship is expected between the opportunity cost of background production and breeding inventories. The price spread is an inverse measure of the opportunity cost of backgrounding versus breeding production. Therefore, a positive relationship is expected between the price spread and cow-calf subsector inventories. This set of prior beliefs may be expressed as a set of restrictions on the values of the true coefficients in the form of a set of null hypotheses for active cow inventories:

\[ H_0: B_{12} < 0 \]
\[ H_A: B_{12} > 0 \]
\[ H_0: B_{13} < 0 \]
\[ H_A: B_{13} > 0 \]

and for replacement inventories:

\[ H_0: B_{22} < 0 \]
\[ H_A: B_{22} > 0 \]
\[ H_0: B_{23} < 0 \]
\[ H_A: B_{23} > 0 \]

Two major problems exist in estimating the inventory model. First, the functional form of price expectation is unknown. The relative inelasticity of the cow-calf subsector may indicate that price expectations are based on long-term trends in market prices. A moving average is a simple model which may approximate this form of expectation. The
interval for the moving average is also subject to speculation. However, the length of the interval should reflect the time requirements of breeding herd adjustment. Approximately two years may elapse before a weaned heifer, withheld for breeding, enters the active breeding herd, (Guenther, 1975). Therefore, a two-year moving average of light feeder prices and the price spread between light and heavy feeder cattle may approximate the expected price of calves and the expected price spread.

A second estimation problem may be anticipated. Both calf prices and the price spread contain a strong growth trend due to inflation over the estimation period. Calf prices and the price spread may be correlated, due to the growth trend induced by inflation. The increased level of collinearity, between calf prices and the price spread over the estimation interval, may increase the adverse effects of multicollinearity in the estimates. These effects may include:

(1) Increased sample variance of the coefficients.
(2) Increased errors in the coefficient estimates.
(3) Increased sensitivity of the models to specific sets of a sample data (Johnston, 1972, p. 159-168).

A partial remedy may be to remove the growth, due to inflation, from one of the variables. The price spread may be deflated to constant 1972 dollars, with the implicit gross national product deflator to non-durable goods.¹⁵

The inventory model is recursive and may be estimated directly with ordinary least squares. The active cow inventory model estimates are presented in Table 3.50. The replacement herd model estimates are presented in Table 3.52.

Ordinary least squares estimates of active cow inventories yield coefficients for the price variables of the appropriate sign. The adjusted $R^2$ value of .963 indicates a reasonable fit of the model to the sample data. However, the very small sample size of 12 observations creates a problem in evaluating the OLS model for serial correlation. The Durbin "h" statistic, as a test for serial correlation with a lagged dependent variable, is a large sample statistic. Small sample properties of the statistic are unknown and the Durbin-Watson statistic is inappropriate for the model (Durbin, 1970). However, if the h statistic is assumed to be the standard deviate, then a critical z value at the 95 percent confidence level (1.65 for a one-tailed test), and an "h" statistic of 1.14 for the model, does not indicate significant auto-correlation at the 95 percent confidence level. This is confirmed by the low "t" value of the autoregressive parameter Rho in the autoregressive estimates of the cow inventory model. The autoregressive model may be rejected.

Multicollinearity does enter into the OLS estimates of the cow inventory model. The variance inflation factors for the lagged inventory variable (36.57), and the calf price (52.62), are relatively large. Hence, the significance of these variables in the OLS estimation may be understated. A ridge regression for "k", equal to the Lawless-Wang estimate of .003, is presented in Table 3.50.
Table 3.50 Cow Inventory Estimating Equations (Annual 1966-1977)

<table>
<thead>
<tr>
<th>Value</th>
<th>Beef cows that have calved in t-1</th>
<th>Two-year moving average of the deflated price spread ( e(P_{t-1}-P_3) )</th>
<th>Two-year moving average of the feeder calf price ( e(P_1) )</th>
<th>Cochrane-Orcutt autoregressive correction parameter (Rho)</th>
<th>Adjusted ( R^2 )</th>
<th>Durbin-Watson statistic (D.W.)</th>
<th>Lawless-Wang estimate of ridge k value (L.W.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( R^2 )</td>
<td>( E_2 )</td>
<td>( L.W. )</td>
</tr>
<tr>
<td><strong>Ordinary Least Squares</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>.868</td>
<td>427.2</td>
<td>109.3</td>
<td>-</td>
<td>.963</td>
<td>2.65</td>
<td>.0031</td>
</tr>
<tr>
<td>Standard error</td>
<td>.038</td>
<td>126.7</td>
<td>49.2</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;t&quot; statistic (T)</td>
<td>22.89</td>
<td>3.37</td>
<td>2.22</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance inflation factor (VIF)</td>
<td>36.51</td>
<td>6.86</td>
<td>52.62</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>First Order Autoregressive Least Squares</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>.873</td>
<td>419.3</td>
<td>104.6</td>
<td>-.380</td>
<td>.964</td>
<td>1.96</td>
<td>-</td>
</tr>
<tr>
<td>Standard error</td>
<td>.032</td>
<td>96.8</td>
<td>39.4</td>
<td>.351</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;t&quot; statistic (T)</td>
<td>27.48</td>
<td>4.33</td>
<td>2.65</td>
<td>-1.08</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Ridge Regression k = .0031</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>.799</td>
<td>353.4</td>
<td>190.9</td>
<td>-</td>
<td>.949</td>
<td>1.89</td>
<td>-</td>
</tr>
<tr>
<td>Standard error</td>
<td>.039</td>
<td>141.3</td>
<td>51.0</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;t&quot; statistic (T)</td>
<td>20.21</td>
<td>2.50</td>
<td>3.74</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 3.51 Evaluation of the Cow Inventory Hypotheses

<table>
<thead>
<tr>
<th>Model</th>
<th>Coefficient</th>
<th>Sample &quot;t&quot; value from estimates</th>
<th>95% Confidence Level&lt;sup&gt;a&lt;/sup&gt;</th>
<th>95% Confidence Level&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordinary least squares</td>
<td>2-year moving average of calf price, $c(P_1)$</td>
<td>2.22</td>
<td>1.833</td>
<td>Reject</td>
</tr>
<tr>
<td>Ordinary least squares</td>
<td>2-year moving average of deflated price spread $c(P_1 - P_3)$</td>
<td>3.34</td>
<td>1.833</td>
<td>Reject</td>
</tr>
<tr>
<td>Ridge regression</td>
<td>2-year moving average of calf price $c(P_1)$</td>
<td>3.74</td>
<td>1.833</td>
<td>Reject</td>
</tr>
<tr>
<td>Ridge regression</td>
<td>2-year moving average of deflated price spread $c(P_1 - P_3)$</td>
<td>2.50</td>
<td>1.833</td>
<td>Reject</td>
</tr>
</tbody>
</table>

<sup>a</sup> One tailed test with 9 degrees of freedom.
<table>
<thead>
<tr>
<th>Value</th>
<th>Inventory of replacement heifers greater than 500 lbs. in t-1</th>
<th>Two-year moving average of calf prices c(P)</th>
<th>Two-year moving average of the deflated price spread c(R-P)</th>
<th>Cochrane-Orcutt auto-regression parameter (Rho)</th>
<th>Adjusted R²</th>
<th>Durbin-Watson statistic (D.W.)</th>
<th>Lawless-Wang estimate of ridge k value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordinary Least Squares</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.911</td>
<td>2.24</td>
<td>.0238</td>
</tr>
<tr>
<td>Coefficient</td>
<td>.687</td>
<td>42.8</td>
<td>154.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard error</td>
<td>.078</td>
<td>18.0</td>
<td>43.6</td>
<td></td>
<td></td>
<td></td>
<td>Durbin &quot;h&quot; statistic = .43</td>
</tr>
<tr>
<td>&quot;t&quot; statistic (T)</td>
<td>8.81</td>
<td>2.38</td>
<td>3.54</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance Inflation factor (VIF)</td>
<td>43.00</td>
<td>62.06</td>
<td>7.16</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First Order Autoregressive Least Squares</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.909</td>
<td>1.77</td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>.724</td>
<td>34.2</td>
<td>163.8</td>
<td>-.342</td>
<td></td>
<td></td>
<td>Standard error of the regression = .382</td>
</tr>
<tr>
<td>Standard error</td>
<td>.074</td>
<td>16.9</td>
<td>37.1</td>
<td>.366</td>
<td></td>
<td></td>
<td>Durbin &quot;h&quot; statistic = .41</td>
</tr>
<tr>
<td>&quot;t&quot; statistic (T)</td>
<td>9.80</td>
<td>2.02</td>
<td>4.42</td>
<td>-.94</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ridge regression k = .58</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.859</td>
<td>1.11</td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>.537</td>
<td>60.1</td>
<td>150.1</td>
<td></td>
<td></td>
<td></td>
<td>Standard error of the regression = 355.2</td>
</tr>
<tr>
<td>Standard error</td>
<td>.054</td>
<td>12.2</td>
<td>44.3</td>
<td></td>
<td></td>
<td></td>
<td>Durbin &quot;h&quot; statistic = 3.24</td>
</tr>
<tr>
<td>&quot;t&quot; statistic (T)</td>
<td>9.80</td>
<td>5.65</td>
<td>3.57</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

²/Large sample test statistic (n > 30) small sample properties unknown.
Three major differences exist between the OLS and ridge regression estimates. First, the coefficient for the lagged endogenous variable is approximately eight percent smaller in the ridge model. Since the lagged cow inventory variable is taken to reflect the inelastic component of breeding production, the ridge regression estimate shows that cow inventories may be somewhat more elastic than indicated by OLS model. Second, the magnitude and significance of calf price coefficient increased and the magnitude and significance of the price spread coefficient decreased in the ridge versus OLS estimates. Although both remain significant at the 95 percent confidence level, the ridge estimates place greater significance on expected calf prices, which is intuitively desirable. Third, the ridge estimate loses some predictive accuracy, indicated by a lower $R^2$ value and 17 percent increase in the standard error of the regression. However, the ridge estimate removes some serial correlation (OLS $h = -1.14$, ridge $h = 0.26$). Both the OLS and ridge estimations of the active cow inventory model may be applied to the hypothesis of the model. The results of the tests are presented in Table 3.51.

At the 95 percent confidence level, for both the OLS and ridge estimating equations, the null hypotheses are rejected and the alternative hypotheses are accepted:

- $H_A: B_{12} > 0$
- $H_A: B_{13} > 0$

Therefore, the results of the quantitative model and the hypotheses stating that feeder market prices, in the form of calf prices and the price spread, are determinants of active breeding inventory levels,
are consistent. However, further grounds for rejecting the hypothesis may be obtained by examining the replacement inventory estimating equations, Table 3.52.

The OLS estimation of the replacement inventory model yields coefficients of the appropriate sign. The adjusted $R^2$ value indicates a relatively good fit between the model and the sample data. The small sample size, 12 observations, does not allow for an accurate test for serial correlation. However, if the "h" statistic is taken to be a standard normal deviate, then an "h" value of .43 indicates no serial correlation at the 95 percent confidence level ($z=1.65$). This is confirmed by the low "t" value, -.94, of the autoregressive correlation parameter, Rho. Multicollinearity may be a problem in the OLS estimate of the replacement inventory model.

The variance inflation factor for the calf price is relatively large, 62.06. Hence, the significance of the calf price may be understated. A ridge regression for "k", equalling .0238 (the Lawless-Wang estimate of k) for the model, is also presented in Table 3.52.

The difference between the OLS and ridge estimates of the replacement heifer equation are similar to those discussed for the cow inventory model. The coefficient for the lagged endogenous variable is 21 percent smaller in the ridge model. While the significance of both price variables is greater in the ridge model, the ridge model again increases the relative significance of feeder calf prices in the model. The ridge estimate yields a poorer fit with the $R^2$ value, decreasing from .911 for OLS to .859 for the ridge regression. The standard error of
<table>
<thead>
<tr>
<th>Model</th>
<th>Coefficient</th>
<th>Sample &quot;t&quot; value from estimates</th>
<th>Critical &quot;t&quot;</th>
<th>95% Confidence Level</th>
<th>Critical &quot;t&quot;</th>
<th>99% Confidence Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordinary least squares</td>
<td>2-year moving average of calf prices, e(P₁)</td>
<td>2.38</td>
<td>1.833</td>
<td>Reject</td>
<td>2.262</td>
<td>Reject</td>
</tr>
<tr>
<td>Ordinary least squares</td>
<td>2-year moving average of deflated price spread e(P₁-P₃)</td>
<td>7.16</td>
<td>1.833</td>
<td>Reject</td>
<td>2.262</td>
<td>Reject</td>
</tr>
<tr>
<td>Ridge regression</td>
<td>2-year moving average of calf prices, e(P₁)</td>
<td>5.65</td>
<td>1.833</td>
<td>Reject</td>
<td>2.262</td>
<td>Reject</td>
</tr>
<tr>
<td>Ridge regression</td>
<td>2-year moving average of deflated price spread e(P₁-P₃)</td>
<td>3.57</td>
<td>1.833</td>
<td>Reject</td>
<td>2.262</td>
<td>Reject</td>
</tr>
</tbody>
</table>

\(^5/\) One tailed test with 9 degrees of freedom.
the ridge regression is 26 percent higher than for OLS. The ridge estimate also shows much higher serial correlation, indicated by an "h" statistic of 3.24 for ridge, as opposed to 0.43 for OLS. Both the OLS and ridge estimating equations may be applied to the hypotheses of the model. The results of the tests are presented in Table 3.53.

At the 99 percent confidence level, the null hypotheses are rejected for the OLS and ridge estimating equation. The alternative hypotheses are accepted:

\[ H_1: B_{22} > 0 \]
\[ A \]
\[ H_A: B_{23} > 0 \]

A significant and positive relationship exists between feeder calf prices, the price spread, and replacement heifer inventories.

The results of the quantitative analyses of long-run production fail to reject the general hypothesis that feeder market prices are parameters of the long-run production decision.

**Model Evaluation**

**Hypotheses Tests**

In the previous sections, six hypotheses were developed and tested to evaluate the specification of the quantitative models. In each case, a null hypothesis was set up to contradict a theoretical restriction placed upon the quantitative analysis. The null hypothesis was rejected in each test in favor of the alternative hypothesis. None of the formal hypothesis tests provided grounds to reject the specification of the
estimating equations. Of the relationships which were examined, the relative price hypothesis allowed the most critical evaluation of the model. Testing of this hypothesis indicated that the quantitative feeder market model maintained the expected relationship between the impact of feed grain prices and market weights. The remaining tests simply indicated that variables of interest were of the proper sign and significantly greater than zero.

Statistical inference is a limited tool for evaluating econometric models. However, more subjective criterion are available. The individual estimating equations may be linked in a series of recursive equations to define the complete model. The complete model may be solved for a set of historical values of the exogenous parameters in order to track a period of history. Actual and predicted values of endogenous variables may be compared over the tracking period.

Historical Tracking

The period between 1970 and 1978 provides a critical test of the tracking ability of the quantitative model. The feeder cattle market moved through a severe price cycle, with feeder calf prices ranging from a high of over $63/cwt. to a low of less than $24/cwt. Over this same period, feed costs increased from an average of $38.52/ton in 1970, to $80.71/ton in 1976.

To evaluate the tracking ability of the model, the six OLS estimating equations for the feeder market and breeding inventories are placed in a recursive block. Five identity equations are required to:
(1) Define the price spread.

(2) Define the expected price of calves and the expected price spread on an annual basis.

(3) Define the calf crop as a percentage of breeding inventories.

(4) Distribute breeding production on a quarterly basis for the quantity equation.

Figure 3.1 is a diagram of the recursive block of six estimating and five identity equations which define the complete model.

Actual and predicted values for cattle placed on feed are presented in Figure 3.20. The placement component of the model follows both cyclical and seasonal variation in actual feedlot placements. The model does not accurately account for the effects of building of liquidation of breeding inventories. This may explain the overestimation of placements in 1973 (herd building), and the underestimate of placements in 1976-77 (herd liquidation). However, the normalized standard error of the placement tracking is only six percent. The model is in phase with most of the trend variation in feedlot placements over the entire period.

Actual and predicted values for the average feeder steer price are presented in Figure 3.21. The model follows the cyclical trend of average feeder steer prices. Both actual and predicted prices peak in the third quarter of 1973, and reach a minimum in the first quarter of 1973. The model recovers quickly from the first quarter of 1975 to

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16The price spread is defined as the price of calves less the price of heavy feeder cattle. Quarterly prices are converted to the annual two-year average of feeder calf prices and the deflated price spread. Breeding inventories are distributed equally each quarter, and then placed in a lagged moving average for the quantity demanded equation.
Figure 3.1
A recursive block of 6 estimating equations and 5 identity equations of the complete model.
Figure 3.20. Actual and predicted feedlot placements, 23 states, 1,000 head, 1970-1978.
the second quarter of 1976. Actual prices over the period recover more slowly. However, the model follows the impact of increasing feed costs and declining slaughter prices in 1976. Both actual and predicted prices begin a strong recovery in 1978. Given the severe variation in actual prices, the endogenous tracking model performs well. The root mean square error of the tracking period is $3.40. Normalized as a percent of the mean, this variation is only eight percent.

The major discrepancies between actual and predicted prices are corrected when the lagged endogenous feeder price is replaced with the actual lagged feeder price. Correcting the endogenous feeder price yields the fitted values of an average feeder price estimating equation, which is compared to actual prices in Figure 3.22. Examining the actual and fitted values indicates that the uncorrected form of the model accumulates past errors. However, this accumulated error does not prevent the model from tracking both major price swings in the third quarter of 1973 and the first quarter of 1975.

The estimating equations for feeder calf and heavy feeder cattle prices are highly dependent upon the endogenous average feeder steer price. The overall performance of the feeder calf and heavy feeder price components of the model closely approximate the average price component. Actual and predicted calf prices are presented in Figure 3.23. Actual and predicted heavy feeder cattle prices are presented in Figure 3.24.

The net mean square error of the heavy feeder tracking run is $3.17, normalized as a percent of the mean, eight percent. The normalized errors of the average and heavy feeder price models are
Figure 3.21 Actual and predicted average feeder price (500-700 lb. choice feeder steer) Kansas City, 1970 to second quarter of 1978.
Figure 3.22 Actual and fitted average feeder price (500-700 lb. choice feeder steer), Kansas City, 1970 to Second quarter of 1978.
Figure 3.23 Actual and predicted calf prices (400-500 lbs. choice feeder steers) Kansas City, 1970 to second quarter of 1978.
Figure 3.24 Actual and predicted heavy feeder prices (700-800 lb. choice feeder steers), Kansas City, 1970-to second quarter of 1978.
the same. The root mean square error for the feeder calf tracking run is considerably higher, $5.41, normalized 13 percent. This is a reasonable increase, given the expected higher volatility of calf prices.

The performance of the feeder market model, in tracking the period from 1971 to the second quarter of 1978, appears acceptable. First, the model is in phase with actual price quantity movements over the tracking period. Second, the model correctly reacted to both major shifts (third quarter of 1973 and first quarter of 1975) of the actual price cycle. Third, the model remained stable over the entire 34-quarter tracking period. Last, given the lack of general stability in the feeder market over this period, the model maintains a reasonable degree of predictive accuracy.

Actual and predicted values for the cow inventory segment of the model are presented in Figure 3.25. The model accurately tracks the growth in breeding cow inventories between 1971 and 1975. Both the model and the actual cow herd begin to decline by 1976. However, the model shows only a slight decline in cow inventories from 1975 to 1976, and a slight increase in cow numbers is predicted for 1978. Actual cow numbers declined steadily from 1975 to 1978. Solving the model with exogenously corrected values of the lagger average feeder price does not greatly improve the model's tracking ability over this period of breeding liquidation, Figure 3.26. The root mean square error for the tracking run is 1,690 head, normalized as a percent of the mean, four percent.
Figure 3.25 Actual and predicted cow inventories on farms, January 1, 1971-1977 (million head).
Figure 3.26 Actual and predicted cow inventories on farms, January 1, 1971-1977 (million head) with endogenous correction of average feeder prices.
Figure 3.27 Actual and predicted replacement heifers on farms, January 1, 1971-1977 (million head).
Actual and predicted replacement inventories are presented in Figure 3.27. The model follows closely the growth in replacement heifer inventories from 1972 to 1975. The model also follows the sharp decline in inventories from 1975 to 1976. Although the model shows a steady decline in replacement heifer inventories, actual inventories declined at a faster rate. The root mean square error for the replacement model is 612,000 head, normalized, nine percent. The normalized error for the replacement model is five percent higher than the cow inventory model. However, replacement inventories show more relative variation than cow inventories over the entire tracking period.

From 1971 through 1977, both components of the inventory model followed the general direction of growth and decline in breeding inventories. However, the model fails to estimate the magnitude of the liquidation phase, 1975-1978. Two factors may contribute to this problem. First, the lagged endogenous values of the inventory equations are expected to reflect the inelasticity of cow-calf production. Over the estimation period, 1966 to 1977, breeding inventories were increasing ten out of twelve years. Hence, the coefficients for the lagged endogenous variables may be too large, creating an internal tendency toward growth. Second, the overall model does not account for the sense of pessimism in long-run producer expectations over the period. Although the models do not follow the magnitude of the breeding herd liquidation, the models did turn breeding inventories downward, from 1975 to 1977.

The combined model followed general price and inventory movements over an extremely unstable period. Given that the individual estimating
equations are consistent with the restrictions discussed earlier, the model may be a useful tool for examining the effects of feed grain prices on the feeder market and breeding production.

**A Comparative Simulation**

Assuming that the complete model adequately describes the relationships between feed grain prices, feeder market prices, and long-run production, some basic questions may be addressed. What was the magnitude of the effect of the 1973-74 increase in feed costs in the feeder market? What impact did increased feed costs have on forage sector returns? How did this affect breeding inventories and long-run supplies of beef cattle?

Questions, such as those above, do not have definitive answers. A number of assumptions are behind any quantitative estimate of feed grain price effects. However, answers to these questions are needed to improve the level of understanding of beef producers, consumers, and policy makers. Policy makers should be aware of the potential impact of grain policy on the beef industry. Forage producers should be aware of the magnitude of feed grain price effects in the feeder market. Breeding inventories were at an all-time high in 1975, two years after the initial increase in feed grain prices. Liquidation of breeding inventories continued through 1978, two years after feed grain prices began to decline and feeder cattle prices began to recover.

The complete model may be used in a comparative simulation to isolate the effects of feed grain prices within the model. The model
may be solved in a historical simulation equivalent to the tracking run. This model solution, for the set of historical feed prices, may be compared to a model solution for an alternative set of feed prices with the remaining exogenous parameters held at their actual values. The differences between the endogenous values of the historical and alternative feed price simulations may be attributed to the differences between feed prices.

The results of the historical simulation are evaluated with respect to the actual values of the endogenous parameters in the tracking run. However, the results of the alternative feed price simulation cannot be validated with historical tracking. Furthermore, a number of implicit assumptions are made in holding the exogenous parameters (finished cattle prices and fed cattle marketings) at their historical values, and in selecting an alternative set of feed costs. In order to evaluate the results of a comparative simulation, these implicit assumptions must be carefully considered.

To isolate the impact of a change in feed prices, all other exogenous parameters must be held at their historical values. This implies that the exogenous parameters are determined independently of the endogenous parameters of the model. There are only two exogenous parameters other than feed prices. The first is fed cattle marketed, a positive parameter of the quantity of feeder cattle demanded. The second is finished cattle prices, a major parameter of the feeder market model. There are three basic sets of endogenous parameters, feeder prices, feedlot placements, and breeding inventories. Fed cattle
marketings are a function of all three endogenous variable sets. However, the impact of this bias may be relatively small. The direct impact of a change in marketings is in the placement component of the model. The model impact of a 100,000 head increase in placements is only a nine cent change in average feeder cattle prices. Hence, if the differences between actual and simulated placements is relatively small, then the bias in feeder prices will be small.

However, the impact of a bias in slaughter market price may be much larger. Two relationships may create a bias in finished cattle prices, shifts in feedlot or breeding production will shift slaughter market supplies. The impact of shifts in feedlot production may be relatively small in the slaughter market. The major determinant of slaughter market supplies is breeding production. This points to the major problem. Slaughter market prices cannot be assumed to be independent of breeding production. However, due to biological restrictions, a period of nearly two years may elapse before a change in breeding inventories affects long-run supplies in the feeder market. Hence, the simulation period may be restricted to a period extending less than two years past the initial divergence between actual and simulated breeding inventories.

Holding fed cattle marketings and finished cattle prices at their historical values will introduce some bias into the simulation results. The differences between actual and simulated feedlot placements and breeding inventories should be carefully considered in the
evaluation of the simulation results. However, the most critical assumptions may involve the selection of an alternative set of feed prices.

The selection of alternative feed prices will establish the magnitude of the impact within the model. To approximate the impact of increased feed costs in the mid-1970's, the difference between historical and alternative feed prices must be a reasonable measure of "high" feed costs. An average of feed prices, over a specific period, provides a constant alternative feed price. However, inflation rates, over the period of interest, will decrease the relative value of a constant price over time. This problem may be solved by inflating the base price with a price index. In Figure 3.30, actual feed prices, and an alternative set of feed prices, are presented. The alternative set of feed prices are generated by inflating the average price of feed from the fourth quarter of 1969 to the third quarter of 1971 ($39.85/ton) with the wholesale price index, as reported in the Economic Report of the President (1978).

In the 1971-72 crop year, differences between historical and simulated feed costs are relatively small. Actual and simulated feed prices converge in 1977. The simulated difference in feed costs shows a shortrun increase in feed costs over the appropriate period, from the fourth quarter of 1972 to the second quarter of 1977. Over this period, historical feed prices averaged $77.27/ton, and alternative prices averaged $52.07/ton, a difference of $25.21/ton. The maximum difference between feed prices occurs in the fourth quarter of 1974, at $46.53/ton.
Figure 3.30  Historical and simulated feed costs, 1970 to the first quarter of 1978.
Solving the model for the historical and alternative feed costs, from the fourth quarter of 1971 to the first quarter of 1978, yields two sets of endogenous values. The results of the comparative simulation for average feeder prices are presented in a set of three figures.

In Figure 3.31, simulated average feeder prices are presented for the historical and alternative feed cost simulations. In Figure 3.32, the simulated difference between feed costs and the simulated difference between average feeder prices are presented. In Figure 3.33, actual average feeder prices and projected feeder prices (actual prices augmented by the simulated difference in feeder prices), are presented.

The graphic results of the simulation for average feeder cattle give a general indication of the impact of increased feed costs. Furthermore, the parallel variation of projected feeder prices (Figure 3.33) gives an indication of changes in slaughter market prices.

Between the fourth quarter of 1972 and the fourth quarter of 1977, the mean price for average feeders is $43.55/cwt. for the historical simulation, and $58.95/cwt. for the alternative simulation, a difference of $15.40/cwt. An arc cross-price elasticity for feed and average feeder prices may be calculated from the simulation results:

$$\varepsilon_F = \frac{\Delta P_F}{(P^*_F + P^*_F)/2} \frac{(FC^* + FC^*)/2}{\Delta FC}$$

Eq. 3.9

where:

$$\varepsilon_F = \text{arc cross-price elasticity for feed and average feeder prices},$$

$$P_F = \text{average feeder price},$$
Figure 3.31 Average feeder steer prices, historical and alternative feed cost simulations (500-700 lb. choice feeder steers, Kansas City, 1970 to the first quarter of 1978.)
Figure 3.32 Differences between the historical and alternative feed price simulations, average feeder steer and feed prices, 1970 to the first quarter of 1978.
Figure 3.33 Actual and projected average feeder prices for the simulation (500-700 lb. choice feeder steers, Kansas City, 1970 to the first quarter of 1978).
FC = feed costs,
\[ \Delta = \text{the average simulated change in}, \]
\[ * = \text{the historical simulation average}, \]
\[ \dot{=} = \text{the alternative simulation average}. \]

For the simulated average of change in feed and feeder prices between the fourth quarter of 1972 and the fourth quarter of 1977, equation 3.9 may be expressed as:

\[ \varepsilon_F = \frac{15.40}{(43.55 + 58.95)/2} \cdot \frac{(77.27 + 52.07)/2 - 25.21}{2} \]

\[ \varepsilon_F = -0.77 \]

The arc price elasticity indicates that 77 percent of the variation in feed prices is transmitted to the feeder market, relative to the simulation means. Given the limitations of the simulation, this cross-price elasticity may be taken as a general estimate of the strong complementary relationship between feed grain prices and feeder market prices.

The results of the comparative simulation for feeder calf and heavy feeder cattle prices are similar to those for average feeder prices. The differences between simulated feed prices and the differences between simulated calf prices are presented in Figure 3.34. Actual and projected calf prices (actual prices augmented by the simulated differences in prices) are presented in Figure 3.35.

Feeder calf prices show a greater response to the simulated change in feed costs. Feeder calf prices average $44.90/cwt. in the historical simulation, and $64.55/cwt. in the alternative simulation, a difference
of $19.65/cwt. The computed arc cross-price elasticity, from equation 3.9, is:

\[
\epsilon_c = \frac{19.65}{(44.90 + 64.55)/2} \cdot \left(\frac{77.27 + 52.07}{2}\right)\frac{-25.21}{-25.21} = -0.92
\]

The cross-price elasticity indicates that 92 percent of the variation in feed prices is transmitted to feeder calf prices relative to the simulation means. This result is consistent with the theoretical consideration of feedlot demand; the impact of a change in feed prices increases as market weight decreases. The cross-price elasticity for calves, -.92, is more negative than the cross-price elasticity for average feeders, -.77.

The differences between simulated feed costs and simulated heavy feeder prices are presented in Figure 3.36. Actual and projected heavy feeder cattle prices are presented in Figure 3.37. Heavy feeder cattle prices show the least response to the lower feed costs of the alternative simulation. Heavy feeder prices averaged $40.90/cwt. for the historical, and $54.72/cwt. for the alternative simulation, a difference of $13.82/cwt. The calculated arc elasticity for feed and heavy feeder prices is -0.74.

The feeder market simulation indicates that a major proportion of the impact of increasee feed costs in 1973-76 was passed to the forage sector through the feeder market. The cross-price elasticities, calculated from the simulation, further suggest that changes in feed prices may induce significant variation in feeder market prices. The comparative simulation maintains the basic relationship between feedlot demand
Figure 3.34 Differences between the historical and alternative feed price simulations, feeder steer, calf, and feed prices, 1970 to the first quarter of 1978.
Figure 3.35 Actual and projected feeder calf prices for the simulation: (400-500 lb. choice steers, Kansas City, 1970 to the first quarter of 1978).
Figure 3.36 Differences between the historical and alternative feed-price simulations, heavy-feeder steer, and feed prices, 1970 to the first quarter of 1978.
Figure 3.37 Actual and projected heavy feeder cattle prices for the simulation (700-800 lb. choice feeder steers, Kansas City, 1970 to the first quarter of 1978).
and market weight; the impact of the alternative set of feed prices declines as market weight increases. However, the potential bias, introduced by holding the non-feed exogenous parameters constant, should be examined.

The difference between actual and simulated feedlot placements, for the alternative feed cost simulation, may yield some indication of the bias in the simulated estimates. Between the fourth quarter of 1972 and the fourth quarter of 1977, actual feedlot placements averaged 6,345,000 head, simulated placements averaged 6,754,000 head, a difference of 409,000 head, or six percent of actual placements. This reflects a relatively small change in feedlot production. Two factors may explain the relatively stable production levels in the feedlot sector. First, feedlot operators may place heavier animals in the feedlot, reducing the weight added while maintaining output. Second, the feedlot sector is able to pass the impact of increasing feed costs to the forage sector through the feeder market. These conditions should hold in reverse for decreasing feed costs. The relatively small change in feedlot production, as indicated by feedlot placements, may produce only a small bias in the results of the simulation.

The results of the feeder market simulation imply a considerable loss in revenues to the forage sector. The difference in forage sector returns, between the historical and alternative simulation, can be approximated. If the average feeder market weight is assumed to be 600 pounds, then forage sector returns may be calculated as feedlot placements times the average feeder steer prices times the average market weight. Using this formula, the difference in forage sector returns
between the historical and alternative feed cost simulations are calculated and presented in Figure 3.34. Figure 3.34 reflects a conservative estimate of the impact of the 1973-76 feed price levels on forage sector revenues. First, feedlot placements are for the top 23 states. Second, returns on culled cows and non-fed slaughter animals are not considered. However, the average difference between simulated revenues, attributable to an average increase in feed costs of $25.21/cwt., is 743 million dollars. On an annual basis, the projected loss in forage revenues averaged 2.97 billion dollars. The total loss for the five-year period, from the fourth quarter of 1972 to the fourth quarter of 1977, is 14.86 billion dollars. These figures demonstrate the potential magnitude of the actual loss in forage sector returns due to increased feed costs between 1973 and 1976. These figures point to the significance of feed grain prices to forage producers. Given the potential impact on forage sector returns, the impact of the 1973-76 change in feed costs may be carried for a number of years in breeding inventories.

The results of the comparative simulation for the inventory component of the model are presented in the next several figures. In Figure 3.40, January 1 cow inventories for the historical and alternative feed cost simulations are presented. Actual and projected cow inventories are presented in Figure 3.41. The graphic results show that the impact of increased feed costs in 1973-74 does not affect active breeding inventories until 1975. This delay of about two years is evident in both the simulated inventories and the decline in actual cow inventories. The effects of decreased feed costs, in the feeder market, result in a sharp increase in cow numbers for the alternative simulation in 1975-76.
Figure 3.38 The projected difference in forage sector returns for the feeder market simulations, 1970-1977.
Figure 3.40. January 1 cow inventories, historical and alternative feed cost simulations, 1971-1977.
Figure 3.41 January 1. cow inventories, actual and projected, for the comparative simulation, 1971-1977.
The inertia of this increase is carried in breeding inventories through the remainder of the simulation. This form of inertia is also evident in the corresponding decline in actual cow inventories for this period.

January 1 replacement heifer inventories for the simulations are presented in Figure 3.42. Actual and projected replacement inventories are presented in Figure 3.43. The graphic results of the simulation for replacement inventories are similar to those for cow inventories. However, a significant increase in replacement inventories occurs in the alternative simulation in 1974-75, only one year after the initial increase in feed costs (1973). This is an indication that replacement inventories are more flexible. The size of the replacement herd is not as dependent on past inventory levels as is the size of the active breeding herd.

The average difference in breeding inventories, resulting from the simulated change in feed costs, is 4.76 million head for cow inventories, and 1.44 million head for replacement heifer inventories, between 1973 and 1978. However, changes in breeding inventories are cumulative. By 1978, the difference between the historical and alternative simulation inventories is 13.28 million head for cow inventories, and 3.97 million head for replacement heifers, a total change in breeding inventories of 17.25 million head. Assuming a calving rate of 80 percent, the projected loss in calf supplies for 1978-79 is 13.8 million head. Between 1975 and 1978, actual breeding inventories declined 10 million head, an estimated loss of eight million head in 1978-79 calf supplies. The difference between the simulated and actual decline in breeding capacity is due to the projected growth in breeding inventories under lower feed prices.
Figure 3.42 January 1 replacement heifer inventories, historical and alternative feed cost simulations, 1971-1977.
Figure 3.43: January replacement heifer inventories, actual and projected for the comparative simulation, 1971-1977.
The magnitude of the simulated impact of lower feed costs between 1973 and 1976, on breeding inventories, may introduce a general bias in the simulation. In 1975, the projected difference in cow inventories is 2.1 million head. By 1976, the difference is 7.8 million head. By the beginning of 1978, increased calf production of the alternative feed price simulation would begin to reach the slaughter market as fat cattle. Hence, a decrease in finished cattle prices may be anticipated. Slaughter market prices are held at their historical values in both simulations. Restricting the feeder market simulation to the end of 1977 may minimize this bias.

However, terminating the simulation at the end of 1977 presents an additional problem. The simulated impact of a change in feed prices, in the feeder market, is terminated before it works completely free of the market. This may be seen in Figures 3.32 (page 101), 3.34 (page 105) and 3.36 (page 107); while actual and alternative feed costs close in 1977: feeder market prices for the alternative feed cost simulation are higher than the historical simulation prices at the end of the simulation in 1977:4. The reason for this lag in the simulated impact of a change in feed prices is twofold. First, in the average profit variable, (defined on page 45) feed costs are computed for the production period, hence, feed prices are lagged one and two quarters. Second, the endogenous average feeder price is lagged one quarter in the average feeder steer price estimating equation (page 51). The results of the tracking simulation indicate that the lagged impact of changes in feed costs in the model does reflect the actual adjustment of the feeder market in response to changing feed
prices. However, some bias is introduced into the cross-price elasticities calculated from the simulation. A relatively small bias may be anticipated in the absolute value of the cross-price elasticities for feed and feeders. If the remaining quarters, in which the historical and alternative feeder market prices close were included in the computation of the cross price elasticities, the average feed price impacts on feeder cattle prices and hence, the absolute value of cross-price elasticities, would be smaller. However, the average difference between actual and historical feed prices would also be smaller, tending to offset this bias. Furthermore, the average impact on feeder market prices is calculated for 22 quarters. The addition of two to three quarters may produce only a small change in the calculated cross-price elasticities.

Another source of model bias cannot be removed by limiting the simulation period. Slaughter market prices were affected by breeding herd liquidation between 1975 and 1979. Liquidation does not occur in the alternative feed cost simulation. However, actual finished cattle prices declined only 3.7 percent between 1974 and 1977, from an annual average of $41.92/cwt. in 1974 to $40.38/cwt. in 1977.

Due to the effects of herd building and increased lot production in the alternative simulation, slaughter market prices may be too high. While the effect may still be relatively small, the simulated response to lower feed costs may be overstated.

Despite the limitations of the simulation, the models provide some interesting information concerning the effects of feed prices on beef
production. Some conclusions which may be drawn from these results are considered in the following chapter.
IV. SUMMARY AND CONCLUSIONS

Summary

Highly unstable beef prices have brought dissatisfaction to both beef producers and consumers over the past several years. Much of the variation seen in beef prices and production may be attributed to changes in feed grain prices. First, grain-fed beef constitutes over 60 percent of cattle marketed for slaughter. Second, feed costs more than doubled between 1971 and 1974.

Changes in feed grain prices which induce expansion or contraction of breeding inventories generate inertia within the cyclical component of beef industry, cow-calf production. For example, an increase in the rate of breeding production temporarily contracts market supplies of beef cattle. Reduced supplies, due to increased demand for breeding stock, may result in increased prices. Higher prices may, in turn, provide an incentive for further expansion. An increase in feed grain prices which initiate breeding herd liquidation may result in increased supplies, lower prices, and continued liquidation. If feed grain prices affect the desired levels of breeding inventories, then feed grain prices may significantly affect beef prices and supplies over an extended period of time. The purpose of this study is to identify and quantify a linkage between feed grain prices and long-run beef production.
Sector analysis is used to identify the structural components and interrelationships of live animal production. Two sectors are defined. First, the feedlot sector, which is assumed to purchase animal inputs, and feed (feed grains), to produce high quality cattle for slaughter. Second is the forage sector. Two subsectors are defined for the forage sector. One: the cow-calf subsector, which encompasses the activities of breeding production. Cow-calf subsector production is the production of animal numbers on the forage base. Long-run supplies of beef cattle originate in the cow-calf subsector. Two: the backgrounding subsector, defined as the forage based production of animal weight on non-breeding inventories.

Three markets are defined for the primary production (live animal) model. First, the slaughter market, which is the linkage between live animal production and consumer demand. Second, is an internal feeder cattle market, which links feedlot sector demand and forage sector supplies of feeder cattle. Third, is the grain or feed market, assumed to be exogenous with respect to beef production.

The sectors and markets in the primary production model provide a basis to examine a sequential linkage between feed grain prices and long-run production. Feed grains are factors of feedlot production. The relative prices of finished cattle and feed grains are determinants of feedlot sector demand for feeder cattle. The quantity of feeder cattle demanded by the feedlot sector is a major component of the total demand for forage production. And, lastly, long-run price expectations for feeder cattle may be parameters of the long-run production decision.
The long-run supply of calves may be based, in part, on price expectation for feeder calves and other feeder cattle.

The potential effects of feed grain prices on feedlot demand for feeder cattle are examined through a feedlot sector profit function. Two discrete weight classes of feeder cattle are considered: feeder calves and heavy feeder cattle. Feedlot demand for a given class of feeder cattle is assumed to be proportional to expected profit levels, or returns to feeding. Two relationships are expected. First, feed costs are inversely related to feedlot demand for feeder cattle of a given weight class. Second, the magnitude of the inverse relationship between feed costs and feeder cattle demand increase as market weight decreases. Hence, an increase in feed grain prices is expected to decrease the demand for feeder calves relative to heavy feeder cattle.

Given a positive relationship between feedlot sector demand and feeder cattle prices, the above relationships may be extended to the feeder market. First, feed grain prices are inversely related to the price of a given class of feeder cattle. Second, feed grain prices are inversely related to the price differential between feeder calves and heavy feeder cattle.

Two relationships between feeder market prices and cow-calf inventories are considered. **One:** feeder calf prices, as a product price, are positively related to the capital value of breeding inventories. Expected feeder calf prices are a component of the expected value of withholding inventories for breeding. **Two:** the price spread between feeder calves and heavy feeder cattle is a measure of the relative returns of cow-calf versus backgrounding production to a common
resource, the forage base. The price spread is an inverse measure of the opportunity cost associated with the production of animal numbers versus animal weight.

The relationships developed from the theoretical framework are used to design and evaluate the quantifiable form of the model. In the quantitative analysis, a reduced form of the theoretical linkage, the relationships between feed grain and feeder market prices, feeder market prices and breeding inventories are considered. However, the quantitative model is expected to describe and demonstrate the basic relationships of the theoretical analysis.

Feeder market and cow-calf inventory models are quantified with secondary data, maintained by the USDA. The models are assumed to be linear, and linear estimation techniques are applied. The theoretical relationships are utilized to design null hypotheses for statistical validation of the estimating equations.

The feeder market model consists of two component models estimated quarterly over the interval 1964 to the second quarter of 1978. The first component model is used to examine the general relationship between feed prices, feeder market prices and feedlot placements. Feed grain prices are integrated into an average profit variable. Profit levels are an inverse function of feed costs. Hence, the expected sign of the true coefficient, relating average profit and the average price of feeder cattle, is positive. Two-Stage least squares are applied to estimate the model. At the 99 percent confidence level, the null hypothesis is
rejected, and the alternative hypothesis is accepted: the true coefficient relating average profit and average prices for feeder cattle is positive.

The second component of the feeder market model is used to examine the relationship between feed grain prices and relative prices for feeder calves and heavy feeder cattle. Feed grain prices are integrated into a feed-steer price ratio, a positive function of feed costs. An inverse relationship is expected between the feed-steer price ratio and both feeder calf and heavy feeder cattle prices. However, an inverse relationship between feed costs and the price differential, between feeder calves and heavy feeder cattle, is a more powerful restriction on the model. Hence, the expected value of the true coefficient for the feed-steer price ratio and feeder calf prices is negative and less than the true coefficient relating the feed-steer price ratio to heavy feeder cattle prices. A special form of the price models is estimated to evaluate this hypothesis. An F statistic is constructed, and at the 99 percent confidence level, the null hypothesis is rejected in favor of the alternative hypothesis: the true coefficient of the feed-steer price ratio for feeder calf prices is less than the true feed-steer price ratio coefficient for heavy feeder cattle prices.

The cow-calf subsector inventory model consists of two component equations estimated annually over the interval 1966 to 1978. The first component is the active cow inventory equation. The second component is the replacement heifer inventory equation. Feeder calf prices and the price spread between feeder calves and feeder cattle are integrated into two-year moving averages. The moving average form of feeder calf
prices and the price spread is an approximation of long-run price expectations. Positive relationships are expected between the dependent inventory variables and the independent price variables. At the 95 percent confidence level, the null hypotheses for the cow inventory model are rejected in favor of the alternative hypotheses: the true coefficients for feeder calf prices and the price spread are greater than zero. At the 99 percent level, the null hypotheses for the replacement heifer inventory model are rejected in favor of the alternative hypothesis: the true coefficients for feeder calf prices and the price spread are greater than zero.

All of the estimating equations yielded a good fit with the sample data. Attempts are made to identify and correct violations of the least square assumptions. The best available form of an equation is used to evaluate each hypothesis. However, due to the weakness of hypothesis testing as a tool for model validation, a more subjective evaluation is also made.

The period between 1970 and 1978 provides a severe test of a beef model's tracking ability. A complete feeder market and breeding inventory model is constructed by placing the six estimating equations in a recursive block. Five identity equations are required to transform endogenous variables. Only three exogenous data series are required to solve the tracking model, fed cattle marketings, the price of feed, and the price of slaughter steers. The model is solved for the period 1970 to the second quarter of 1978. Actual and model values of the endogenous parameters are compared.
The model remained stable over the entire tracking period. The feeder market component of the model remains in phase with the cyclical variation in actual feeder placements and prices. The predictive accuracy of the feeder market model is relatively high through most of the historical period. The inventory model followed the general movements in cow and replacement heifer inventories. While the predictive accuracy of the inventory component is relatively high during breeding herd expansion, the model underestimated the extent of breeding herd liquidation. However, given the variation in beef prices and production over this period, it is the author's opinion that the complete model adequately describes the linkage between feed grain prices, feeder market prices, and long-run production. The complete model may provide a tool for examining the impact of recent changes in feed grain prices on the feeder market and long-run production.

The tracking run is a historical simulation of the feeder market and breeding inventories. The historical simulation is compared to an alternative simulation, a model solution for an alternative set of feed prices with fed cattle marketings and slaughter prices held at their historical values.

An alternative set of feed costs is generated by inflating average feed costs between the fourth quarter of 1969 and the third quarter of 1971, with a wholesale price index. Historical feed prices more than doubled between the fourth quarter of 1972 and 1976. The alternative set of feed costs increase at a slower rate with the general economy. The differences between historical and alternative feed costs reflect
a shortrun increase in feed prices between the fourth quarter of 1972 and the fourth quarter of 1977. Over this period, feed prices are an average of $25.21/ton lower in the alternative simulation, 33 percent lower than the simulation period average of actual feed prices, $77.27/ton.

The results of the comparative simulation are presented graphically in chapter three. The feeder market simulation results demonstrate strong complementary linkage between feed grain and feeder market prices. The arc cross-price elasticity (calculated from the fourth quarter of 1972 to the fourth quarter of 1977 means of the simulation parameters) for feed and average feeder prices is -0.77; for feed and feeder calves, -0.92; and, for feed and heavy feeders, -0.74. The cross-price elasticities indicate that a large portion of the impact of a change in feed prices is transferred to the feeder market through the feedlot sector. As a result, the forage sector absorbs much of the impact of increased feed costs in lost revenues.

The difference in forage sector returns between the feeder market simulations is estimated at 2.97 billion dollars per year for a five-year period, a total of 14.86 billion dollars.

The impact of declining returns to cow-calf production is shown in the comparative simulation of breeding inventories. January 1 inventories of mature cows are an average of 2.1 million head larger for the alternative feed cost solution (1973 to 1978). However, the effect of higher feeder market prices in the alternative simulation accumulate in cow inventories. The simulated difference in cow numbers
is 7.8 million head in 1976, 10.8 million head in 1977, and 13.3 million head in 1978. The simulated impact of increased feed costs in 1973-76 reaches a 23 percent loss in breeding capacity by 1978.

Within the limitations imposed upon the simulation results by the assumptions required for the analysis, the comparative simulation gives a fair indication of the potential significance and magnitude of feed grain effects. The results further suggest that increased feed costs, from 1973 to 1976, were a primary force behind the intense cyclical variation in beef prices and production in the last several years. These points are considered in the following section.

Conclusions

This study considers a sequential linkage between feed prices and feedlot demand for feeder cattle, feeder cattle demand and feeder market prices, feeder prices, and breeding inventories. The results of this study indicate that a short-run increase in feed costs can induce and aggregate the cyclical variation in beef prices and production. The significance of this sequential relationship holds a number of implications for beef producers and agricultural policy.

The significance of feed grain price effects in the beef industry owes to the characteristics of supply and demand in the feeder market and the inertia of production response in the cow-calf subsector.

Feedlot sector demand for feeder cattle is relatively elastic. First, feedlots purchase animal inventories, a variable short-term investment of capital, in comparison to forage producers. Second,
feedlot producers may increase or decrease the total weight added in the feedlot by placing lighter or heavier feeders, while maintaining a constant output of grain finished cattle.

In comparison, forage sector supplies of feeder cattle are highly inelastic. First, supplies reflect a long-term capital investment in breeding inventories held on the forage base. Second, the forage base is a land intense factor of production. Feed is a capital factor of feedlot production. Therefore, the capacity of the forage base for holding breeding and non-breeding animal inventories is relatively inflexible.

Highly inelastic supplies and more elastic demand in the feeder market effectively passes most of the impact of a change in feed grain prices to the forage sector through shifts in feedlot demand for feeder cattle. Feed grains are a principal input in the production of slaughter cattle, hence, a change in feed prices may result in a large shift in feedlot demand for feeders. The break-even price for average feeder steers may be used to measure the impact of a change in feed costs on feeder cattle demand. The break-even price may be expressed as:

\[
BE_F = P_S + \frac{Wg(P_S + FC*)}{Wg}
\]

where:

- \(BE_F\) = break-even price for average feeder cattle,
- \(P_S\) = finished cattle price,
- \(Wg\) = weight added in feedlot,
\( Wp = \) feeder purchase weight,
\( FC^* = \) feed costs per weight unit gain.

The change in the break-even price, due to a change in feed costs, may be expressed as:

\[
\Delta \text{BE}_F = \frac{-Wq\Delta FC^*}{Wp} \quad \text{Eq. 4.11}
\]

where:

\( \Delta = \) the change in per hundredweight.

Under the assumption used to calculate average feeder profit levels, the change in the break-even price for average feeders may be expressed as:

\[
\Delta \text{BE}_F = \frac{-5.5 \times 0.48 \Delta FC}{6.0}
\]

where:

\( FC = \) feed costs per ton.

The calculated change in the break-even price, due to a $25/ton increase in feed costs, is $11/cwt. This is a rough measure of how much less feedlot operators would be willing to pay, per hundredweight, for any quantity of 500 to 700-pound feed steers, due to an increase in feed costs. The projected impact of a $25/ton increase in feed costs on average feeder prices may be calculated from the simulation cross-price elasticity for feed and average feeders (-0.77), the average of actual feed costs ($77.21/ton) and actual feeder prices ($43.37/cwt.) for the fourth quarter of 1972 to the fourth quarter of 1977 simulation period. The projected impact of a $25/ton increase in feed costs is a $10.84/cwt. decrease in average feeder prices \((25.00 \div 77.21 \times -0.77 \times 43.37)\).
The actual values of the calculated impacts represent only rough estimates of the impact of a change in feed prices on feedlot demand and prices for feeder cattle. However, these estimates do provide good evidence that the feedlot sector transfers the major impact of a change in feed prices to the forage sector. Furthermore, the response of feedlot sector demand to a change in feed costs and the inelasticity of forage sector supplies of feeder cattle, result in relatively large changes in feeder market prices.

The characteristics of supply and demand in the feeder market force the forage sector to absorb the major impact of a change in feed prices as a windfall gain or loss in revenues. The simulated loss in forage sector returns (on feeder cattle marketed in 23 states, from the fourth quarter of 1972 to the fourth quarter of 1977), attributed to increased feed costs, approaches 15 billion dollars, approximately 20 percent of the total value of beef production, reported by the USDA for this same period.

The cow-calf subsector may have absorbed the greatest loss in revenues due to increased feed prices in the mid-1970's. Calf prices are more responsive to changes in feed prices. The cross-price elasticities for feed and feeders, calculated from the simulation, are: -.92 for calves, -.77 for average feeders, and -.74 for heavy feeders. In response to increased feed costs, cow-calf product prices for weaned calves declined relative to backgrounding subsector product prices. The price spread between feeder calves and heavy feeders average $9.83/cwt. in the alternative feed price simulation, and $4.00 cwt. in the historical simulation, a decline of $5.83/cwt. attributed to higher
feed prices. This result is consistent with two conclusions. First, as feed prices increase, the value of forage utilized to add weight on non-breeding inventories, increases. Second, cow-calf production is less flexible than backgrounding production. Hence, cow-calf subsector supplies are more inelastic in the short-run. However, in the long-run, cow-calf subsector response to the impact of increased feed costs creates the inertia which drives beef prices and production through cyclical patterns of over-contraction and overexpansion. Expansion of breeding production induces further expansion; liquidation of breeding stock induces further contraction. Furthermore, the effect of a change in feed costs, resulting in expansion or contraction of breeding production, is accumulated and carried in breeding inventories. The projected loss in breeding cows, due to increased grain prices from 1973-76, is 2.1 million head in 1975, 7.8 million head in 1976, 10.8 million head in 1977, and 13.3 million head in 1978. This progressive response in breeding inventories to increased feed costs is also seen in the decline in actual inventories between 1975 and 1978. January 1 cow inventories declined 15.2 percent over this period. However, a steady state condition between long-run supply and demand may require continued growth of the breeding herd. The 23 percent decline in simulated inventories may approximate the actual loss in breeding capacity over this period.

The liquidation of breeding inventories, from 1975 through 1978, has resulted in extremely contracted beef supplies in 1979. Current prices reflect this radical shift in market supplies. In the first
quarter of 1979, feeder calf prices exceeded $93/cwt. Between the first quarter of 1975 and the first quarter of 1979, feeder calf prices increased 290 percent, while total production declined approximately 15.2 percent, indicating that cow-calf sector returns are increasing despite less total production. Hence, the impact of increased feed prices in 1973-76 is now being passed upward to the consumer, who is spending more total dollars for less beef. While current conditions in the beef industry are allowing forage producers to recover some of the loss in returns from 1974 to 1977, there is a strong potential for over-expansion.

Current prices reflect the extremely contracted supplies in the beef markets. As breeding producers begin to withhold more animals for future production, market supplies may become more contracted. A period of two to three years may elapse before increased production begins to expand market supplies. Over this entire period, beef prices will remain strong, creating the potential for over-expansion. The effects of increased feed prices in 1973-76 may continue to aggravate the cyclical pattern of beef prices and production for a number of years to come. Breeding producers should exercise caution in expanding production over the next few years. First, as increased production is brought to market, supplies will expand and prices will decline. Second, as prices start to decline, reservation demand in the cow-calf sector may decrease, further expanding market supplies. Third, increased grain prices in 1973-76 induced expanded grain production. Feed prices in 1977 and 1978 reflected increased supplies in the grain
market. However, lower prices over this period may result in declining grain production and increased feed costs in the early 1980's. Breeding producers should be aware of the fact that they are not in a position to capitalize on current prices with increased production.

The potential impact of changes in feed grain prices on live animal beef production give beef producers a vested interest in agricultural policy designed for the grain sector. Consider the effect of a support policy which adds 50 cents to $2/bu. corn. Assuming a proportional, 25 percent, increase in feed prices and using the simulation cross-price elasticity and average price for 500 to 700-pound feeder steers (-.77 and $43.55/cwt. respectively), the projected decline in average feeder prices is $8.38/cwt. (.25 x -.77 x 43.55). Assuming an average market weight of 600 pounds, and using an annual average of feedlot placements between 1972 and 1977 (25.4 million head), the estimated loss in forage sector returns from feedlot placements in 23 states is 1.3 billion dollars per year (8.38 x 6 x 25.4). As a very conservative estimate of the total loss in forage sector returns, this figure is nearly nine percent of the average annual value of total beef production between 1972 and 1977.

The results of this study provide some insight into the effects of alternative agricultural policies on the beef industry. A primary objective of agricultural policy over the past 40 years is to support prices and income in the grain sector. Acreage controls, government-held reserves, and direct payments are three major alternative programs which have been implemented to support the grain sector.
Acreage retirement programs are designed to improve grain prices through contracting supplies in the open market. This is an undesirable policy alternative from the perspective of beef producers, especially those in the forage sector. First, beef producers must absorb significant losses in returns, due to relatively inelastic supplies in beef markets. Second, acreage retirement only provides grain price stability on the low side of the market. Third, contraction of market supplies increases the vulnerability of the grain market to increases in export demand or domestic crop failure. Acreage retirement programs, in the early 1970's, resulted in highly contracted supplies in the 1972-73 crop year. A world shortage of grains sharply increased U.S. export demand over the same period, creating a dramatic increase in feed grain prices. As a result, 1973 legislation shifted the emphasis of agricultural policy from price support to direct payment programs, should the need arise (Brandow, 1976).

Beef producers may benefit from direct payment programs for the grain sector. Direct payments may allow grain producers to maintain production levels during periods of low prices. Direct payments programs may stabilize the grain market at lower price levels by maintaining expanded supplies. Expanded supplies may also tend to dampen the impact of increased export demand or domestic crop failure.

Government reserve programs are another policy alternative designed to affect open market prices of grains. Price support has been the major objective of past reserve programs. Stocks are purchased to contract market supplies. Reserves have been liquidated thorough foreign aid programs and destruction. As in acreage control programs,
this type of reserve policy may substantially reduce returns to live animal beef production. However, government stocks and the maintenance of production levels may reduce the impact of crop failure or increased export demand.

A national reserve program, where stocks are purchased and liquidated on the open market, may be a more viable policy alternative. First, a program of this type may be used to stabilize both high and low feed grain prices, a more equitable solution for beef producers. Second, by buying low and selling high, some of the costs of a reserve program could be deferred. Third, a more stable grain market may also stabilize the feeder market, allowing for more efficient production and less variable returns.

The results of this study provide a good indication of the potential magnitude of the impact of agricultural policy on beef producers. However, to evaluate costs and benefits of alternative policies or different management strategies for national reserve programs, further research is needed. The framework of this analysis needs to be expanded.

Inclusion of an endogenous slaughter market may greatly improve the value of the simulation model as a tool for policy analysis. The bias in a simulation using constant slaughter market prices may be removed. The impact of changes in feedlot or breeding production may be compensated for in the slaughter market. For example, the effects of an increase in feed costs may be followed through a decline in breeding production, a decline in long-run supplies, an increase in slaughter market prices, and, ultimately, a partial recovery of
breeding capacity. The endogenous slaughter market may be differen-
tiated for grain-fed cattle and non-fed cattle (slaughter cows). This
may allow for a more complete evaluation of the impact of feed grain
price changes on feedlot and forage sector returns. The specification
of the breeding inventory equations may be improved. For example cull
prices may be an important and excluded parameter of the long-run
production decision. However the major impediment to modeling forage
based production is a lack of relevant data. There is a need for
disaggregated on farm inventory data, such as data for potential
feeder cattle inventories by weight. More frequent observations are
needed for breeding inventories; for example quarterly observation on
replacement inventories. This information may allow the estimation
of forage sector demand for capital inventories and the impact of a
change in forage sector reservation demand on beef supplies.

Incorporation of a grain market into the model may allow a more
direct approach to evaluating feed grain price effects in the beef
industry. Grain market simulations may be used to generate alternative
feed prices. This may allow for a better evaluation of alternative
grain policies with respect to both beef and grain producers. The
costs and benefits of alternative management strategies of a national
reserve program may be considered.

An underlying goal of this study is to provide a set of sound
structural estimates of a few basic relationships in the beef industry.
The principal relationships, which are considered, are feed prices and
slaughter prices as determinants of feeder market prices and quantities,
and the relationship between feeder market prices and breeding inventories. It is hoped that the results of this study may be integrated with continued research on beef production.
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