Modeling the Asian Fish Sector: Issues, Framework, and Method

Madan M. Dey
Roehlano M. Briones
Mahfuzzudin Ahmed

The WorldFish Center, Batu Maung, Penang, Malaysia

Abstract. The paper presents a detailed approach to modeling supply and demand of the Asian fish sector. It discusses the salient features of the fish sector in Asian countries that need to be incorporated in a comprehensive model of fish supply and demand, as well as the usefulness of supply and demand modeling for disaggregated impact analysis. Empirical implementation through econometric estimation is outlined, based on the normalized quadratic profit function for supply, multi-stage budgeting and the quadratic LA-AIDS for demand, and modified Armington aggregation for foreign trade. The model structure and equations (in general form) for computing equilibrium and generating market projections on supply, demand, price, and foreign trade are stated and explained.

Key Words: supply, demand, disaggregated impacts, economic model

1. Introduction

Recent decades have witnessed a major transformation in global demand for and supply of fish. Per capita consumption has nearly doubled from 8 kg. per capita consumption in 1950 to almost 16 kg. per capita in 1999. Fish exports from developing countries have already surpassed their traditional exports of crops and meat. These changes have also been observed in Asia, a large contributor to global consumption and production of fish.

One of the most significant developments has been the rapid growth of aquaculture: over the past decade, aquaculture in Asia has posted an annual growth of 12%, with Asia now accounting for 91% of global aquaculture output (FAO, 2002).

Underlying these transformations are structural factors such as technological change and policy reform. Better techniques in fish breeding, aquaculture, and capture fisheries provide new opportunities for expanded output. Trade liberalization has also opened up bigger markets for consumption and production of fish. At the same time, pressures on aquatic resources have raised concerns about the sustainability of production trends; consequently, new institutional and policy regimes have been established to promote resource conservation.

However, little is known about the magnitudes of impact these structural factors have had on prices, production, and earnings in the fish sector. There are of course a number of food sector models have been developed for analyzing agricultural trends; in particular the impact of technology and policy changes on Asian agriculture has been well-studied (Evenson et. al., 1993; Huang and Chen, 1999). Unfortunately, fish is typically absent in such models, despite the importance of fish in the well-being of the poor, providing one billion people daily sustenance and 150 million people employment (Pinstrup-Anderson and Pandya-Lorch 1999). This simply reflects the usual but unwarranted omission of fish and other aquatic products in food security analysis (James 1994; Williams 1996, 1999).

A notable exception is the extended IMPACT model (International Model for Policy Analysis of Agricultural Commodities and Trade) of the International Food Policy Research Institute (IFPRI). The incorporation of fish in this model is the outcome of collaboration between the WorldFish Center, IFPRI, and the Food and Agriculture Organization (Delgado et. al., 2001). It is a valuable initial step for quantitative modeling of fish demand and supply.

Further modeling work is however necessary for two reasons. First, the extended IMPACT model uses synthetic elasticities to characterize supply and demand for fish; instead, these elasticities should be based on production and consumption data. Second, the extended IMPACT adopts very broad commodity groups to model fish; fish however is a highly heterogeneous commodity. Production requirements and consumer preferences vary widely.
across species groups. Moreover, as fish is normally consumed as a whole or in pieces (rather than as fillet), preferences also vary depending on size, color, etc. A simple comparison of prices across countries and species (based on 1995 survey data) is suggestive of this diversity: for example, the price of common carp ranges from as low as USD 0.65/kg. in India to as much as USD 1.07/kg. in China; within China, low value fish (silver carp) is priced at only USD 0.65/kg, but high value fish can fetch up to USD 1.70/kg (Dey et. al., 2002).

A more useful description of fish sector trends clearly requires projections for specific fish types. For equity analysis, it is essential to determine trends for fish types of which the poor are major consumers or producers. Disaggregated analysis would also serve as an informative guide for resource allocation within the fish sector, such as directions for capital investment, development financing, and research priorities.

To address these issues, the WorldFish Center is constructing a multimarket fish sector model under its project on Demand and Supply of Fish in Asia. The model draws its parameters from a large-scale data set on fish production and consumption in nine Asian countries. The model disaggregates fish into its major types, production categories, and market destinations. It shall be capable of generating numerical projections on prices, quantities, and the welfare of fish-related sectors, based on probable and alternative scenarios for demographic, technological, and institutional changes.

This paper is a technical description of the model structure. The rest of the paper is organized as follows: Section 2 presents salient features of supply and demand as well as the basic structure of the model. Section 3 discusses the functional forms and estimation of model parameters. Section 4 presents the model closure and the simulation model for making market projections. Section 5 concludes with some remarks on model implementation.

2. Background and overview of the model

The effect of technology and policy

The aim of the model is to analyze the effect of technology and policy on the fish sector. The basic framework for examining this effect is outlined in Figure 1.

Figure 1. Basic framework for fish sector modeling
The left column of boxes traces the flow of the analytical process, while the right column expands on the contents of the first. The preliminary step is to perform a background analysis for characterizing technologies and policies affecting the fish sector. With this background analysis, shocks that feed into the basic model can be identified. These shocks mainly take the form of indices for technological change and quantifiable policy variables; other shocks that may be important in explaining fish sector trends are also identified. The model for analyzing the impact of these shocks takes the form of a demand and supply system. The system can trace the effect of these shocks on prices and quantities, disaggregated by species group and destination market (i.e. exports and imports); further disaggregation allows analysis of distributional impact by economic class (i.e. consumers, producers, factor suppliers, or income group).

The Demand and Supply project has already undertaken an extensive background analysis of the fish sector. Based on the foregoing framework, model construction proceeds in three steps: The first step involves specifying the basic model structure, down to the level of the functional form of the equations. The second step entails estimation of the functional forms, thus fleshing out a computable form of the model. The third applies this model to make numerical projections up to 2015. The first step is handled by this paper; the remaining steps shall be taken up by subsequent papers of the Demand and Supply project.

Salient features of the fish sector

Salient features of the fish sector should be incorporated in a realistic supply and demand model. The production side covers a diverse set of activities, spanning a variety of species, aquatic zones and production categories. The major zones are inland and marine; further distinctions can be made by species group (i.e. marine-pelagic and marine-demersal). The most important production categories are capture fisheries and aquaculture. Within each category further classification is possible. Aquaculture can be distinguished by aquatic zone (freshwater, brackish water, or marine water), species cultured, and scope of output (i.e. monoculture versus polyculture). Capture fisheries can be distinguished by fishing gear (i.e. small boat, trawlers, etc.).

Distinctions output by species produced are problematic; in capture fisheries catch is typically multi-species, while aquaculture can adopt polyculture of several fish types. Input use may be difficult to separate and apportion to each species produced. Specification of production response must recognize the pervasiveness of multiple outputs.

After production comes the downstream treatment of fish. Many fish types are simply marketed in fresh form through a network of traders, with the retailer being the point of contact with the consumer. Other fish types are processed prior to retailing. Finally, some fish is exported to foreign markets in either fresh or processed form.

On the consumption side, while consumption preferences are heterogeneous, for each country it is typically possible to identify major fish types. As earlier mentioned, analysis of equity impact requires a distinction between high value and low value fish types. For household demand of fish, most of the consumption occurs at home. For some countries though, fish consumption in restaurants may be very popular. The characteristics of fish demand for consumption away from home is different from demand for consumption at home; for example, consumption at home can select over a wide array of fish in the market, whereas consumption at a restaurant may face a limited the range of choice. These differences should be recognized in modeling household demand. Household demand furthermore should be recognized as one of the sources of demand; industry demand may be important for fish that are processed as fishmeal, for use as feed for livestock and the aquaculture subsector.1

Lastly, fish demand may distinguish in terms of place of origin as well as place of purchase. For the former, demand can be met either by domestic or foreign production, i.e. imports. For the latter, a critical distinction is between urban and rural markets. The distributional system as well as magnitudes of retail prices can vary quite widely between these areas.

Basic model structure

A market may be represented graphically by a network of supply and demand curves (Figure 2), with each curve corresponding to an equation of the market model within each country. The equations are divided into a consumer core, a producer core, and a trade core (Sadoulet and de Janvry, 1995). Suppose for each country there are N fish types indexed by i, defined identically for production and consumption. The supply curve is

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1 In the nine countries studied, demand for fish from the manufacturing sector (e.g. for glue, oil, etc.), are negligible.
divided into a domestic and foreign component; the domestic component is derived from the producer core. Let \( Q_s \) denote domestic supply of fish type \( i \); then

\[
Q_s = Q_s(p, V) \tag{2.1}
\]

Here \( Q_s \) sums all individual producers’ supply functions, which may fall into different categories (i.e. capture fishers and aquaculturists). Individual supply behavior is predicated on profit maximization, which implies that output respond to price changes; hence \( p \) is a vector of market prices \( p_i, i = 1, 2, \ldots, N \). Meanwhile \( V \) is a vector of all exogenous variables that can potentially affect fish supply. For domestic supply, shifts may be induced by changes in technology and policy. Note that the impact of technology may depend on the production category, i.e. the supply shift for a particular innovation may differ between aquaculture and capture fisheries.

Production entails transformation of inputs into outputs. Input use is also decided according to profit maximization, which implies that input and supply decisions respond to both input and output prices. We may therefore trace a demand curve for input \( j \), under the factor market column; for market demand \( X_j \) we may write

\[
X_j = X_j(p, V) \tag{2.2}
\]

Note that factor market supply can be horizontal or upward sloping, hence the dashed line in the graph under the factor market column. If fish production contributes negligibly to total factor demand, then the factor supply curve may well be horizontal. Equation (2.2) incorporates this assumption, as only output prices are endogenous, and factor prices can be moved into the vector of exogenous variables. Supply shifts in the product market also act as demand shifters in the factor market. Meanwhile for primary inputs, shifters are also present for input supply; for example, labor supply can be shifted by population growth, or by the expansion of outside employment opportunities.

Aside from primary inputs, fish production uses intermediate inputs; for aquaculture, one of the intermediate inputs is fish feed. This feed may be partly sourced from low value fish obtained from domestic fisheries. The top graph in the factor market column depicts this input-output relation within the producer core.

Meanwhile, the demand curves correspond to the system of demand equations. The household component of fish demand is estimated from the consumer core. Letting \( Qd \) represent total demand for fish type \( i \), we can write:

\[
Qd = Qd(p, Z) \tag{2.3}
\]

The quantity of fish demand depends on fish prices and a vector of exogenous or shift factors denoted by \( Z \). For households, demand response to fish price is explained by utility maximization. The shift variables meanwhile include aggregate household income, population size, size of the urban sector, and other consumption-related factors.

The producer and consumer cores respectively concern domestic production and consumption of fish. Fish imports and exports are modeled separately in a trade core. Imports are motivated by demand for foreign supply of fish, while exports constitute domestic supply of fish to meet foreign demand. Let \( IM_i \) and \( EX_i \) respectively denote imports and exports of fish type \( i \). Clearly, imports and exports each respond to both domestic and foreign prices. Import and export functions be specified as:

\[
IM = IM(p, Z), \quad EX = EX(p, V) \tag{2.4}
\]

Here \( p \) denotes domestic prices; foreign prices are incorporated in \( V \) and \( Z \), the vectors of exogenous variables. Fixing world prices amounts to imposing the small country assumption, i.e. the domestic economy’s size is negligible compared to world markets. The assumption also allows each country model to be treated separately from those of the other countries.

**Model closure**

Model closure involves bringing together demand and supply to solve for market equilibrium. Within a multimarket setting, the \( N \) markets must reach equilibrium (or “clear”) simultaneously, by a specific
configuration of \( N \) market prices. Graphically, this is the set of prices at which supply and demand curves intersect. A formal statement of the equilibrium conditions is:

\[
Q_s_i(p, V) - EX_i(p, V) = Qd_i(p, Z) - IM_i(p, Z),
\]

\( i = 1, 2, \ldots, n. \)

Given these equilibrium identities, equilibrium values of \( p \), denoted \( p^e \), can be computed. Alternative values of \( V \) and \( Z \) yield different sets of equilibrium values for \( p^e \). Market projections can be generated by evaluating supply, demand, and net import functions at the equilibrium prices. The relationship between equilibrium prices and exogenous factors can be represented by the vector function,

\[
p^e = p^e(V, Z).
\]

By incorporating a technology and policy index into \( V \), one can conduct impact analysis of technical change and policy shifts. If the exogenous variables contain no lagged endogenous values (e.g. prices and quantities in the previous periods), then the model is essentially static. Nevertheless, dynamic projections can be made by denominating the exogenous variables by period, and imposing model closure within each period. That is, comparative statics may be interpreted as a type of dynamics – a common assumption in multimarket and applied general equilibrium analysis.
Figure 2. General Framework for Demand and Supply in the Fish Sector Model
Equation (2.7) is a time-denominated version of (2.6):

\[ p'_t = p'(Z_t, X_t) \]

Projections of exogenous variables will have to be generated from outside the model. Accuracy of projections will then depend on accuracy of these exogenous variable projections, as well as the faithfulness of the model in reflecting the actual structure of the fish sector. The latter will require rigorous extraction of supply and demand behavior from production and consumption data, which is the subject of the next section.

3. The empirical model

The empirical content of the model is fleshed out by specifying the functional form of supply and demand equations, as well as estimating the numerical values of the parameters stated within each form. This discussion focuses on the functional specification as well as the estimation procedure for the producer, consumer, and trade cores. Notation for the following designates coefficients generically by constants \( a, b, c \); in different equations they correspond to different numerical parameters.

The producer core

Estimation of the supply curve begins with data from the individual production unit. The technology of production may be described by a set of feasible “netputs”, i.e. a vector of quantities of net outputs (positive netput) and net inputs (negative netput). The netput convention is particularly useful for describing multi-output, joint-input production, i.e. aquaculture, inland capture, and marine capture may designate different production categories characterized by their own unique netputs.

Let \( M \) denote the number of elements of a generic netput vector \( [q, -x] \), where \( q \) is a \( 1 \times N \) vector of positive net outputs while \( x \) is the \( 1 \times (M - N) \) vector of net inputs. In the short run, the \( x \) vector would contain only variable inputs. Corresponding to the outputs and inputs are their respective producer prices, which we may collect into a price vector \( [p, w] \). Then profit is simply the sum of vector products \( s_p q - w x \). Technology, parametric prices, and conditioning variables (incorporating fixed factors, production techniques, biophysical characteristics, etc.) together determine maximum profit, and therefore output supply and input demand.

To estimate output supply and input demand, one approach would be to estimate the production function directly, and then apply profit-maximization to compute the relevant functions. Supply and factor demand would then depend on prices and \( v \), a vector of conditioning variables. This however runs into complications when production uses joint inputs, as in the case of fish production. One may instead sidestep output-input estimation entirely and proceed with the profit function, which relates maximum profit directly to the parametric prices and conditioning variables.

The profit function approach to multi-output analysis of fisheries has been taken by Squires (1987), Kirkley and Strand (1988), and others. Note that the profit function approach must be applied to a product category which includes outputs that are jointly produced. Due to differences in technology and output composition, estimation should proceed separately for marine-capture, inland-capture, and aquaculture. This implies that most fish types would fall under one production category. Marine-capture can be further disaggregated depending on data availability (i.e. marine-pelagic, marine-demersal, etc.) Meanwhile for aquaculture, one of the input demands to be estimated should be demand for fish feed.

It is helpful to first normalize prices by calculating a vector \( P \), defined as:

\[ P = (1/p_m)^* [p, w]. \]

Here \( p_m \), the price of the \( m \)th netput, is arbitrarily chosen as the numeraire. The first part of \( P \) consists of \( N \) elements corresponding to normalized output prices, while the second consists \( M - N - 1 \) elements corresponding to the non-numeraire factor prices. Normalized profit \( \pi \) which is nominal profit divided by \( p_m \), may be written as a profit function \( \pi = \pi(P, v) \) given maximizing behavior.

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Denote elements of vectors \( P \) by \( p_i \). The profit function approach assumes that \( \pi \) is differentiable, then applies Hotelling’s Lemma to derive the output supply and input demand as follows:

\[
- \frac{\partial \pi}{\partial P_i} = q_{s_i}(P, v), \quad i = 1, 2, \ldots, N \\
\frac{\partial \pi}{\partial P_i} = -x_{i}(P, v), \quad i = N + 1, N + 2, \ldots, M - 1.
\]

The foregoing implies estimation of output and input quantities against left hand side variables consisting of prices and conditioning variables.

A specific form for fitting fish production data is the *normalized quadratic profit function*; this form has frequently been applied to the joint agricultural production (Ball, et. al. 1997; Shumway et. al. 1987). To state it, let there \( L \) be the number of variables determining \( \pi \), i.e. there are \( L - (N - 1) \) conditioning variables, each denoted by \( v_i \). Then the normalized quadratic form is:

\[
\pi = a_0 + \sum_{i=1}^{L} a_i P_i + \sum_{i=M+1}^{L} a_i v_i + \frac{1}{2} \left( \sum_{i=M+1}^{L} \sum_{j=M+1}^{N} b_{ij} P_i P_j + \sum_{i=M+1}^{L} \sum_{j=M+1}^{N} c_{ij} v_i v_j \right)
\]

(3.1)

(3.2.a) \quad q_{s_i}^h = a_i + \sum_{j=M+1}^{L} b_{ij}^h P_j + \sum_{j=M+1}^{L} c_{ij} v_j + e_i^h; \quad i = 1, 2, \ldots, N.

(3.2.b) \quad x_i^h = -\left( a_i + \sum_{j=M+1}^{L} b_{ij}^h P_j + \sum_{j=M+1}^{L} c_{ij} v_j + e_i^h \right); \quad i = N + 1, N + 2, \ldots, M - 1.

The supply and input demand functions are linear in prices. The elasticities are easily computed from the parameters of the foregoing equations. The numeraire demand is:

\[
q_{e_i}^h = \left[ a_i + \sum_{j=M+1}^{L} b_{ij}^h P_j + \frac{1}{2} \left( \sum_{i=M+1}^{L} \sum_{j=M+1}^{N} b_{ij} v_i v_j \right) \right] + \sum_{j=M+1}^{L} c_{ij} v_j
\]

(3.2.c) \quad x_{e_i}^h = \left[ a_i + \sum_{j=M+1}^{L} b_{ij}^h P_j + \frac{1}{2} \left( \sum_{i=M+1}^{L} \sum_{j=M+1}^{N} b_{ij} v_i v_j \right) \right]

Once (3.2.a) and (3.2.b) have been estimated, the parameters can then be applied to (3.2.c); the intercept term may be obtained by calibration.

The consumer core

Household demand is premised on utility maximization. The following is a method of practical estimation patterned after Dey (2000). Specification of household demand amounts to estimating, at the level of the household, the following system of equations:

\[
q_{d_i} = f(p, y, z); \quad i = 1, 2, \ldots, N
\]
Here $qd_i$ denotes per capita quantity demanded for commodity $i$. $f$ is generic function notation, $p$ is a vector of consumer prices, $y$ is per capita household income, and $z$ is a vector of household characteristics. Fish consumption occurs within a context of choice over a wide array of consumer goods. These other goods must somehow be incorporated in the estimation, without losing focus on fish demand. One approach would be to classify consumer goods into commodity aggregates. Theoretically, this requires the underlying utility function to be separable according to these aggregates. Optimization can then proceed according to a multi-stage budgeting framework. An example would be a three-stage framework, as diagrammed in the following utility tree (Figure 3).

The total budget of the consumer is first divided into food and nonfood expenditure. With fish and animal protein products separable, food expenditure is then subdivided into various categories of food, including the aggregate category of fish. Finally, fish expenditure is then allocated by the consumer into the various fish types. A similar three-stage framework can be identified for the case illustrated in Figure 4, where fish and other animal protein products are not separable.

The foregoing framework can be stated as follows: for a household, let $fdex$ and $fshex$ respectively denote per capita food and fish expenditure. Let $pfsh$ denote an aggregate price index for fish (computed using $p$), $pofd$ a vector of nonfish food prices, $pfd$ an aggregate index of food prices (computed using $p$ and $pofd$), and $pnfd$ an aggregate price index for nonfood consumer good. Finally, let $z_1$, $z_2$, $z_3$ be distinct vectors of household characteristics.

![Figure 3. Utility tree, fish and animal protein products separable](image3.png)

![Figure 4. Utility tree, fish and other animal protein products not separable](image4.png)
The mathematical expression of the foregoing multistage framework is:

Stage 1: \( f_{dex} = f(pfd, pnfd, y, z) \).

Stage 2: \( f_{shex} = f(pfsh, pofd, f_{dex}, z2) \)

Stage 3: \( q_d = f(p, f_{shex}, z3); i = 1, 2, ..., N. \)

Each stage incorporates a vector of relevant household characteristics. Stage 1 determines food expenditure, based on prices indices for food and nonfood goods; Stage 2 determines fish expenditure, based on food prices and food expenditure from Stage 1; and Stage 3 determines per capita quantity demanded, based on consumer prices, as well as fish expenditure from Stage 2.

Empirical implementation requires specification of functional forms as well as the estimation method for each of the stages. Let the \( h \) superscript denote a household observation. For stages 1 and 2 we posit the following stochastic equations:

\[
(3.3) \quad \ln f_{dex}^h = a_0 + a_1 \ln Pfd^h + a_2 \ln Pnfd^h + b_1 \ln y^h + b_2 (\ln y^h)^2 + \sum c_i z_i^h + e^h
\]

\[
(3.4) \quad \ln f_{shex}^h = a_0 + a_1 \ln Pfsh^h + \sum_{i=2} \ln pfd^h + b_1 \ln f_{dex}^h + b_2 (\ln f_{dex}^h)^2 + \sum c_i z_i^h + e^h.
\]

These are logarithmic specifications with quadratic terms. At each stage, the household characteristics vector contains an urban dummy, to distinguish urban from rural demand by way of an intercept shift.

Least squares regression may be used to estimate (3.3). However, application of this method to (3.4) is problematic, as some households are likely to report zero fish consumption. Such censoring probably results from measurement error: data from household surveys is usually generated by recall, hence even when fish is an important part of the diet (as is the case in Asia), zero observations can arise due to infrequent purchasing of fish. Tobit regression is an appropriate way of dealing with this measurement problem.

Upon fitting (3.3), the estimate of \( f_{dex}^h \) replaces actual data on \( f_{dex}^h \) for fitting (3.4). This instrumental variable technique deals with the endogeneity associated with the choice of expenditure at each level of the utility tree. It also addresses the censoring problem of for some observations reporting zero fish expenditure. The practice has been followed by Pashardes (1993), Balisacan (1994), and others.

For the third stage, we impose the Almost Ideal Demand System or AIDS, proposed by Deaton and Muellbauer (1980). The advantage of AIDS is that it permits exact aggregation of individual consumer demands into market demand, while retaining flexibility of functional form. The AIDS specification is:

\[
(3.5) \quad s_i^h = a_i + \sum_{j=1}^N a_{ij} \ln p_i^h + b_i (\ln f_{shex}^h / P^h) + c_i^h,
\]

Here \( s_i^h \) is the expenditure share of fish type \( i \) in total fish expenditure. Following the instrumental variable technique, \( f_{shex}^h \) is obtained from Stage 2. \( P^h \) is an index of fish prices, for which Deaton and Muellbauer suggest a Stone approximation:

\[
\ln P^h = \sum_{i=1}^N s_i^h \ln p_i^h.
\]

The use of the Stone proxy defines the linear approximate form, or LA-AIDS. A further extension of the AIDS involves insertion of a quadratic expenditure term, as suggested by Blundell et. al. (1993). This yields the quadratic LA-AIDS:
Again, least squares regression is an ill-advised technique for estimation, for two reasons. First, as (3.6) is a system of equations for each household, it is probable that the error terms for each system contain a household-specific component. Hence, the error term is correlated across observations. This can be dealt with by seemingly unrelated regression (SUR), estimation of which relies on maximum likelihood techniques.

Second, with detailed disaggregation, one will probably encounter numerous observations of zero purchases of certain fish types. For example, in the Philippines, low value fish (e.g. anchovies) may be absent from the consumption bundle of a wealthy household. Unlike the case of zero purchases of fish, zero purchases of individual fish items may simply result from household choice. The usual Tobit approach is however unavailable due to the SUR maximum likelihood estimation.

Instead, we follow Heien and Wessels (1990), who apply the modified Heckman procedure. In this approach, censoring is deemed analogous to sample selection as in the standard Heckman. A prior probit estimation (scoring positive fish consumption as 1, and zero otherwise) generates the instrumental variable, which embodies the selection mechanism. This instrumental variable is simply the inverse Mill’s ratio \( imrat^h_i \), computed as follows:

\[
imrat^h_i = \frac{\Phi(prob^h_i)}{1 - \Phi(prob^h_j)} \quad \text{for household } h \text{ consuming item } i;
\]

\[
imrat^h_i = \frac{\Phi(prob^h_i)}{\Phi(prob^h_j)} \quad \text{for household } h \text{ not consuming item } i.
\]

Here \( prob^h_i \) is the probability of zero consumption of fish type \( i \) by household \( j \), estimated by probit regression using the right hand side of (3.8). For simplicity we assume that the scalar \( U^h \), the urban dummy, captures all relevant household characteristics. The final form of the estimating equation for Stage 3 is therefore:

\[
s_i^h = \alpha + \sum_{j=1}^{N} a_{ij} \ln p_j^h + b_{ih} \ln(fshe^h / P^h) + b_{ij} \left( \ln(fshe^h / P^h) \right)^2 + c_{ij}^h imrat^h_i + c_{ij}^h U + \epsilon_i^h i = 1, 2, \ldots, N
\]

Utility maximization imposes a set of restrictions on the parameters of (3.7), namely homogeneity of degree zero in prices and income, symmetry of the Slutsky matrix, and the adding up restriction (budget shares sum to 1). For \( i, j = 1, 2, \ldots, N \) the following are imposed at the estimation stage:

Homogeneity: \( \sum_{j=1}^{N} a_{ij} = 0 \)

Symmetry: \( a_{ij} = a_{ji} \); \( b_{ij} = b_{ji} \)

Adding up: \( \sum_{i=1}^{N} a_i = 1, \sum_{i=1}^{N} b_{ih} = \sum_{i=1}^{N} c_{ih} = \sum_{i=1}^{N} c_{ij} = 0 \)

The ratios in the symmetry restriction hold owing to the quadratic form of (3.7).

Meanwhile the uncompensated demand elasticities can be computed from equation (3.7). For each household, let \( \epsilon_i^h \) be the own- and cross-price elasticities, \( \eta_i^h \) be the elasticity of fish expenditure to food expenditure, \( \eta_y^h \) the elasticity of food expenditure to income, \( \eta_{iy}^h \) the elasticity of fish type \( i \) to income, and \( \Phi^h \) be the probability

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\(^3\) Censoring may also result from seasonality of fish consumption. In this case insertion of seasonal dummies should be able to control this problem.
that consumption of fish is positive. Finally let \( k_{ij} \) be the Kronecker delta, i.e. \( k_{ij} = 1 \) if \( i = j \), \( k_{ij} = 0 \) otherwise. Then:

\[
\epsilon^h_j = \frac{a_i}{s^h_i} - [b_{ij} + 2b_{ij} \ln \left( \frac{fishex^h}{P^h} \right)] \frac{s^h_j}{s^h_i} - k_{ij}
\]

\[
\eta^h_k = b_{ij} + 2b_{ij} \ln \left( \frac{fishex^h}{P^h} \right)
\]

\[
\eta^h = \eta^h \cdot \eta^h \cdot \eta^h
\]

\[
\eta^h = \frac{\partial F}{\partial M} \cdot \Phi
\]

The trade core

The proposed approach to modeling trade is motivated by the following:

- Imported and domestic versions of the same fish type are often found together in retail outlets; awareness of their origin (whether foreign or domestic) hardly matters for fish consumption. Hence, the consumer core equations do not distinguish whether or not the consumed fish is produced domestically, while producer core equations do not distinguish whether or not the output shall be consumed domestically.

- Nevertheless, marketing and other transaction costs may differ in distributing domestically produced versus imported fish.

This model implements a modified Armington (1969) aggregation to incorporate these considerations. The modified Armington developed here is a parsimonious approach towards modeling the marketing stage, where it is applied at the aggregate level of production and consumption. The discussion first considers the case of imports; other cases follow along similar lines. (The index for fish type is suppressed in the following).

The case of imports. Consider a trader, who is trying to meet demand \( Qd \) with a combination of domestic production and imports, corresponding respectively to quantities \( Qdh \) and \( Qm \). Domestic production and imports may be treated as intermediate "inputs" to trade. At demand equilibrium, these inputs are transformed into outputs as a straightforward sum, \( Qd = Qdh + Qm \). This total is sold at a single retail price \( rp \). Let \( p \) denote the domestic producer price, \( wp \) the foreign price; the net revenue per unit of a domestically produced version is

\[
nrp = rp - p.
\]

Meanwhile the net revenue per unit from the foreign-produced version is

\[
nwp = rp - wp.
\]

Then the net revenue \( NR \) from trading is given by

\[
NR = nrp*Qdh + nwpd*Qm.
\]

Meanwhile, trading requires a marketing effort \( T \). Effort is given by the following function:

\[
T = [\alpha Qdh^\rho + (1 - \alpha) Qm^\rho]^\frac{1}{\rho}
\]

The right hand side is in CES form, with the elasticity of substitution \( \sigma \) given by \( \sigma = 1/(1 - \rho) \), and \( \alpha \) being a share parameter.

Let marketing cost be stated as \( mar*T \), where \( mar \) is a constant; then the trader’s profit is given by \( \pi = NR - mar*T \). Consider the problem

\footnote{This condition precludes the standard Armington approach and accounts for the modification developed here.}
(3.8) \[ \text{Max } NR \]

subject to \( \text{mar}^*T = B \) (a constant).

The first order conditions of (3.8) imply:

\[
(3.9) \quad \frac{Q_m}{Q_dh} = \left( \frac{1 - \alpha_{nwp}^{nrp}}{\alpha_{nwp}} \right)^s.
\]

The logarithm of (3.8) leads to an estimating equation:

\[
(3.10) \quad \ln \left( \frac{Q_dh}{Q_m} \right) = a_0 + a_1 \ln \left( \frac{nrp}{nwp} \right).
\]

Equation (3.10) and its variants (Kapuscinski and Warr, 1996) can be estimated given quantity data on imports and domestic production, as well as data for domestic consumer, domestic producer, and world prices. The fitted value of \( a_1 \) is the estimate of \( \sigma \), while estimate for \( \alpha \) is given by \( a_0 / (a_0 + a_1) \).

Denote the ratio \( \frac{1 - \alpha_{nwp}}{\alpha_{nwp}} \) in the right hand term of (3.11), as \( wd \). Solving for the conditional demand for intermediate inputs gives:

\[
Q_{dh} = \frac{1}{\text{mar}} B^* \left[ \alpha^* (wd^* - 1) + 1 \right]^{\sigma/(1-\sigma)}
\]

\[
Q_m = \frac{1}{\text{mar}} B^* wd^* \left[ \alpha^* (wd^* - 1) + 1 \right]^{\sigma/(1-\sigma)}
\]

A convenient normalization would be to express \( Q_{dh} \) and \( Q_m \) in percentage terms, which based on the functional form are constant with respect to \( B \). Denote the share of domestic production in domestic consumption as \( dpdc \); then the conditional input demands imply:

\[
(3.11) \quad dpdc = \frac{Q_{dh}}{Q_{dh} + Q_m} = \frac{1}{1 + wd^*}.
\]

**The case of exports.** Consider now the case of exports. This time the trader acquires domestic good \( Qs \), to meet either domestic or foreign demand; the quantity corresponding to the latter is \( Qx \), while the quantity corresponding to the former is denoted \( Qsh \). Unlike in the case of imports, the trader must now sell at two different prices, namely at the world price and at the domestic retail price. Purchases of the intermediate input are however made at the same producer price. The net world price is therefore:

\[ nwp = wp - p. \]

The net revenue is given by

\[ NR = nrp^*Qsh + nwp^*Qx \]

The marketing effort is given by

\[ T = \left[ \alpha Qsh^p + (1 - \alpha) Qx^p \right]^\frac{1}{p} \]

Define \( wd \) as in the previous case. Application of the maximization problem (3.8) yields an expression for domestic share identical to (3.11):
Interindustry trade. The case of interindustry trade applies when both imports and exports are positive. This mixed case involves a straightforward application of the previous cases to the same industry, except we must now distinguish net prices on the import side from those on the export side.

Nontraded fish. In case fish imports and exports are zero, then the foregoing approach is still applicable to domestic marketing within the fish sector. Consider a trader catering to domestic demand, and suppose \( \alpha \) in \( T \) is equal to 1 (i.e. domestic production is the only source of intermediate inputs for meeting domestic demand). Then

\[
\text{mar}^*T = \text{mar}^*Qd.
\]

That is, marketing effort is a fixed proportion of output sold.

4. Model closure and market projections

The core equations for production, consumption, and trade may all be estimated on the basis of the foregoing methods and functional specifications. However, to generate projections and conduct impact analysis, the following intermediate steps are still required, namely: matching of supply and demand; incorporation of technology and policy; computation of market equilibrium; and finally, generation of projections over a time horizon.

Matching supply and demand

Ideally, the fish types used in supply and demand commodities should be identical. In practice, during estimation, harmonized classification may not be possible, due to the way data is aggregated in the production and consumption sides. On a global level the matching problems are well discussed in Delgado et. al. (2000); similar difficulties on a national level are expected.

The following Table suggests an approach to this matching problem. The first column lists the fish types that are commonly adopted in household consumption data. The second column lists the common fish categories in fish production data. The lines suggest means of matching the categories. When necessary, i.e. if within commodity prices cannot be forced to the same level, ad hoc assumptions may be necessary for a quantitative match, e.g. assuming fixed quantity or value shares of commodity \( i \) within its commodity group. This is especially true for highly aggregated categories such as “low value fish” or “other fish”.

Incorporating technology and policy

In this model, policy and technology are incorporated in the supply side as quantity shifters. The shift is assumed proportional along the quantity axis. This may be represented as a distinction between “actual output” and “effective output”, e.g. technological change (assumed to be factor neutral) may raise actual output given the same effective output (Dixon et. al., 1982). This in turn distinguishes actual from effective price. Alston, Norton, and Pardey (1995, p. 115) give the following example: a farm manager may measure output in kg. per hectare; technological progress raises yield by 10%, hence the effective price per hectare rises by 10%, even though the actual price per kg. is constant. The nominal effective price may be computed as

\[
EP_i = \lambda_i p_i / \lambda_i p_m
\]

where \( \lambda_i \) is the proportional expansion of output due to technological progress or favorable policy shift. Hence, the output supply and input demand functions are:

\[
q_{s_i}^h = \left( a_i + \sum_{j=1}^{M_i-1} b_{ij}EP_j^h + \sum_{j=M_i+1}^{N} b_{ij}v_i^j \right) \lambda_i, \quad i = 1, 2, \ldots, N.
\]
A similar statement follows for the input demands.

**Computing market equilibrium**

Henceforth we shall be referring closely to the simulation model presented in the Annex. The Annex presents the complete variable definitions in each of the cores, subdivided into endogenous and exogenous variables. Note that some of the notation has been altered to maintain a lower-case convention. (Only the number of categories under a classification are written in upper-case, i.e. \( N, K \), etc.) Price and quantity variables are for the most part set to their average values within their applicable core categories (hence the \( h \) superscript is dropped).

The producer core variables introduce \( K \), which is the number of production categories. A new variable that is introduced is \( prrat_i \), the conversion ratio of fresh to processed fish (\( prrat_i \) is set to unity if type \( i \) is marketed fresh). This implies that downstream processing is modeled as Leontieff technology. To obtain aggregate supply, calibration is applied to derive the population of supply units within each category, denoted here as \( pop_i \), i.e. for \( i \) in category \( k \),

\[
pop_i = \frac{Qs_i}{qs_i}.
\]

As long as the sampling frame is valid, the result would be approximately invariant to the \( i \) chosen to calibrate \( pop_i \). Using \( pop_i \) the aggregate quantity supplied can be computed using (C1).

Meanwhile the demand equations are separately computed for urban and rural household categories. As the simulation model endogenizes the fish type shares, it is important to likewise classify the fish price index as endogenous; this is reflected in (D2), (D3), and (D4). Household characteristics as well as the coefficient of the urban dummy has been integrated into the intercept terms in (D6), (D8), and (D10). Average household demand is calculated using (D11). As in the producer core, urban and rural population sizes are calibrated as:

\[
\frac{Qdu}{qdu} = \frac{pop_{u}}{pop} ; \quad \frac{Qdr}{qdr} = \frac{pop_{r}}{pop}.
\]

The trade core calculates shares of domestic components in production (consumption) for imports (exports). If type \( i \) is nontraded then \( dpdc_i = dcdp_i = 1 \). Closure of the trade core includes calculation of equilibrium retail price, which is stated in (T5), (T6), and (T7) by imposing a zero profit condition on the trader for both urban and rural categories. The values of marketing effort parameters \( maru_i \) and \( marr_i \) may first be calibrated by (T7) for \( i\) nontraded; with some plausible adjustment (based on background analysis), the values may then be imputed for the traded fish types.
Table. Practical approaches to matching demand and supply: An example

Household demand

<table>
<thead>
<tr>
<th>Demand for feed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply of feed</td>
</tr>
</tbody>
</table>

Supply

- Culture-freshwater
  - Tilapia
  - Carp/bar
  - Catfish
  - Assorted small fish

- Capture-inland
  - Carp
  - Snakehead
  - Other small fish

- Culture-marine
  - Shrimp
  - Sea bass
  - Grouper

- Capture-marine
  - Treadfin breams
  - Other low value demersal
  - Anchovies
  - Low value pelagic
  - Indo-Pacific mackarel
  - High value pelagic
  - High value demersal
Market equilibrium, as stated in (2.5) is applied in the identities of (C8). The trade quantities are calculated using the shares obtained from the trade core. Note the insertion of an exogenous discrepancy term; this discrepancy may be attributed to a variety of factors, such as consumption away from home.\(^5\)

A numerical solution can be found by programming the Annex equations in a model-solving software package, such as EVIEWS or GAMS. The procedure entails finding solution values for producer prices in (C8), as well as retail prices in (T5) to (T7). From these calculation of quantity demanded, quantity supplied, exports, and imports is straightforward.

*Projections over time*

The Annex identifies the variables that are exogenous to the demand-supply system. Some of the important variables to consider are:

**Supply side:**

- Trends in technological progress, or policy shocks (\(\hat{\Lambda}^k\)) Increases in fixed inputs (embedded in \(v_k\))
- Changes in prices of primary and intermediate inputs
- Entry and exit into fish production (affecting \(\text{pop}_k\))

**Demand side**

- Growth of per capita income (affecting \(y_u\) and \(y_r\))
- Inflation rates for non-fish consumer items (affecting \(pfd_i\), \(pnfd_i\))
- Other demographic shifts (embedded in the intercept of the demand equations)
- Population growth in urban and rural areas (affecting \(\text{pop}_u\) and \(\text{pop}_r\))

Given a time horizon (2005 – 2015), projections for each of these exogenous variables will have to be found in order to calculate market prices and quantities for future periods. Plausible scenarios can be constructed with the aid of the background analysis of the fish sector. The impact of structural factors on market trends can be evaluated by specifying alternative plausible time paths for the exogenous variables.

**4. Next steps**

The foregoing has discussed the analytical version of Asian fish sector model, which is the first step of model construction. The second step is to estimate the parameters of the analytical model from fish sector. This step is currently underway for nine Asian countries. The consumer core estimates are nearing completion, while the producer core estimation is on its data processing stage. By mid-year it is expected that estimates of both supply and demand sides of the model would have been finalized. By the end of the year the fish sector model should have been largely completed, up to the stage of initial projections and impact analysis.

**References**


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\(^5\) Ideally consumption away from home should be explicitly modeled. With better information about the magnitudes available data for disaggregation, more detailed modeling of consumption away from home can be undertaken.


Dey, M., F. Paraguas, and M.F. Alam, Cross-country synthesis, Production, Accessibility, Marketing, and Consumption Patterns of Freshwater Aquaculture Products in Asia, 2002.


Moschini, Giancarlo; Moro, Daniele; Green, Richard D, Maintaining and testing separability in demand systems.”American Journal of Agricultural Economics, 76: 61-73, 1994.


Demand and Supply Model

The following pertains to any country. Subscripts and superscripts denote index notation. All market values are in nominal terms. Coefficients in linear form equations are denoted by $a_i$, $b_i$, $c_i$. This notation is shared across equations, but denotes different numerical values, obtained by prior estimation. Fish types are identical for all the cores.

DEFINITIONS

**Producer core**

“Average” denotes mean value over the producer units within a product category. No extra subscript is specified for each of the $K$ production categories.

Endogenous:

$p_i$  
producer price of fish type $i$

$ep_i$  
normalized effective price of $i$th netput entry, entry is a fish product

$q_{si}$  
average quantity supplied, $i$th type, fresh fish

$x_i$  
average input demand, $i$th input type

Exogenous:

$ep_i$  
normalized effective price of $i$th netput entry, entry is a nonfish input

$v_i$  
average of $i$th conditioning variable, producer core regression

$prrat_i$  
conversion ratio for fresh to processed fish ($prrat_i = 1$ for retailed fresh fish)

$\lambda_i$  
technology index, $i$th fish type

$K$  
number of product categories

$N_k$  
number of fish types in $k$th category

$M_k$  
number of netputs in $k$th category

$L_k$  
number of netputs and conditioning variables in $k$th category

**Consumer core**

Households classified into urban and rural categories, identified by the tag “$u$” and “$r$”, respectively. “Average” denotes mean value within urban or rural category.

Endogenous:

$pfshu, pfshr$  
price of aggregate fish commodity

$rpu_i, rpr_i$  
retail price of $i$th fish type

$su_i, sr_i$  
average share of fish of $i$th type in total fish expenditure

$imru_i, imrr_i$  
inverse Mill’s ratio of $i$th type

$sfu, sfr$  
average share of fish in food expenditure

$fshexu, fshexr$  
per capita average fish expenditure

$fdexu, fdexr$  
per capita average food expenditure
pfdu, pfdr
Sfdu, sfdr
stoneu, stoner

price of aggregate food commodity
share of food in average consumption expenditure
per capita average Stone price index for fish

Exogenous:

pfdu, pfdr,
price of ith food item (pfdi = pfsh; pfdi onward are exogenous)

pnfdu, pnfdr
consumer price index of aggregate nonfood commodity

pfothu, pfothr
consumer price index of aggregate food commodity other than fish

pconu, pconr
percentage of sample households actually consuming ith type

y, yr
per capita average household income

N
number of fish types \( N = \sum_{k=1}^{K} N_k \)

Function operator:

Cdf
Cumulative density function operator, normal distribution

Pdf
Probability density function operator, normal distribution

Trade core

An urban-rural distinction is also made in the trade core.

Endogenous:

dpdcu, dpdcr
share of domestic production in domestic consumption, ith fish type

dcdpu, dcdpr
share of domestic consumption in domestic production, ith fish type

Exogenous:

\( \sigma_i \)
elasticity of substitution, foreign and domestic versions of ith fish type

wpu, wr
world market price of ith fish type

\( \alpha_i \)
share of domestic fish expenditure in total fish expenditure

Model closure

Endogenous:

aqsi
market quantity supplied of ith fish type, final output

axi
market input demand, ith input type

pi
market producer price of ith netput entry

aqdu
urban market demand for ith fish type

aqdr
rural market demand for ith fish type

aqdp
industry demand for ith fish type

Exogenous:

popk
number of supply units in kth category (calibrated)

popu, pr
population of urban consumers
population of rural consumers

marui
price margin, retail over producer price, urban market

marr
price margin, retail over producer price, rural market

\( \text{disc}_i \)
statistical discrepancy between demand and supply (as percentage of demand)

EQUATIONS

Producer core
Output supply, fresh fish, type $i$; $k = 1, 2, \ldots, K$ categories

\[(S1) \quad q_s = \left( a + \sum_{j=1}^{M_k-1} b_j e p_j + \sum_{j=M_k+1}^{L_k} b_j v_j \right) * \lambda^i ; \]
\[i = 1, 2, \ldots, N_k;\]

Input demand, non-numeraire; $k = 1, 2, \ldots, K$ categories

\[(S2) \quad x = -\left( a + \sum_{j=1}^{M_k-1} b_j e p_j + \sum_{j=M_k+1}^{L_k} b_j v_j \right) * \lambda^i ; \]
\[i = N_k + 1, N_k + 2, \ldots, M_k - 1\]

Input demand, numeraire; $k = 1, 2, \ldots, K$ categories

\[(S3) \quad x_{\text{mr}} = -\left( a + \sum_{j=1}^{M_k-1} a_{M_k} v_j + \frac{1}{2} \sum_{i=1}^{N_k} \sum_{j=M_k+1}^{L_k} b_j e p_j v_j + \right) \]
\[\left( \frac{1}{2} \sum_{i=1}^{N_k} \sum_{j=M_k+1}^{L_k} b_j v_j v_j \right) * \lambda^i_{\text{mr}}\]

Producer and effective price

\[(S4) \quad e p_i = \frac{p_i \lambda_i}{p_m \lambda_m} ; \quad i = 1, 2, \ldots, N; \quad k \text{ as category containing } i\]

**Consumer core**

These equations (D1) to (D9) are computed separately for urban and rural categories; another set of the following equations should be written with “u” replaced by “r”.

Share of $i$th fish type in fish expenditure

\[(D1) \quad s_{ru} = \frac{rpu \ast qdu}{fshe xu}\]

Share of fish in food expenditure

\[(D2) \quad sfu = \frac{fshe xu}{fdexu}\]

Price of aggregate fish

\[(D3) \quad pfshu = \sum_{i=1}^{K} s_{ru} rpu_i\]

Price of aggregate food

\[(D4) \quad pfdu = sfu \ast pfshu + (1 - sfu) \ast pfothu\]

Predicting food expenditure (Stage 1)

\[(D5) \quad \ln fdexu = a_{0u} + a_{1n pfduu} + a_{2n pfshdu} + b_{1} \ln yu + b_{2} (\ln yu)^2\]
Predicting fish expenditure (Stage 2)

(D6) \[ \ln(fshexu) = a_0 + a_1 \ln(pfshu) + \sum_{i=2}^{n} a_i \ln(pfdi) + b_1 \ln(fdexu) + b_2 (\ln(fdexu))^2 \]

Stone price index

(D7) \[ stoneu = \sum_{i=1}^{N} su_i \ln(rpu_i) \]

Probability of positive consumption:

(D8) \[ probu_i = a_{0i} + \sum_{i=2}^{n} a_i \ln(rpu_i) + b_1 (\ln(fshexu - stoneu)) + b_2 (\ln(fshexu - stoneu))^2 \]

Inverse Mill’s ratio:

(D9) \[ \text{imru}_i = \frac{pdf(\text{prob})}{cdf(\text{prob})} \]

LA-QUAIDS share equation (Stage 3)

(D10) \[ su_i = a_{0i} + \sum_{i=2}^{n} a_i \ln(rpu_i) + b_1 (\ln(fshexu - stoneu)) + b_2 (\ln(fshexu - stoneu))^2 + c_{imru}; i = 1, 2, \ldots, N \]

Average household demand

(D11) \[ qdu_i = su_i \times \frac{fshexu}{rpu_i} \]

Trade core

Net prices

(T1) \[ nrpu_i = rpu_i - p_i, \quad nrpr_i = rpr_i - p_i \]

(T2) \[ nwpu_i = rpu_i - wp_i; \quad i \text{ is imported} \]
\[ nwpu_i = wp_i - p_i; \quad i \text{ is exported} \]

Price ratio

(T3) \[ wdu_i = \frac{1 - \alpha_i}{\alpha_i} \times \frac{nrpu_i}{nwpu_i}, \quad wdr_i = \frac{1 - \alpha_i}{\alpha_i} \times \frac{nrpr_i}{nwpr_i} \]

Share of domestically produced fish in total consumption (i is imported)

(T4) \[ dpdcu_i = \frac{1}{1 + wdu_i^{\alpha_i}}, \quad dpdcr_i = \frac{1}{1 + wdr_i^{\alpha_i}} \]

Share of domestically consumed fish in total production (i is exported)

(T5) \[ dcdpu_i = \frac{1}{1 + wdu_i^{\alpha_i}}, \quad dcdpr_i = \frac{1}{1 + wdr_i^{\alpha_i}} \]
Imports, \( i \) is imported

\[
(T6) \quad imu_i = (1 - dcdpu_i) * Qdu_i, \\
imr_i = (1 - dcdpr_i) * qdr_i
\]

Retail price, zero profit, traded fish types, urban

\[
(T5) \quad \left( nrpu_i + nwpu_i * wdu_i^\alpha \right) \left( \alpha \left( wdu_i^{\alpha - 1} - 1 \right) + 1 \right)^{\frac{\alpha}{\alpha - 1}} = maru_i = 0
\]

Retail price, zero profit, traded fish types, rural

\[
(T6) \quad \left( nrpr_i + nwpr_i * wdr_i^\alpha \right) \left( \alpha \left( wdr_i^{\alpha - 1} - 1 \right) + 1 \right)^{\frac{\alpha}{\alpha - 1}} = marr_i = 0
\]

Retail price, zero profit, nontraded fish types:

\[
(T7) \quad nrpu_i - p_i = maru_i, \quad nrpr_i - p_i = marr_i
\]

Model closure

Market supply of fish type \( i \); \( k = 1, 2, \ldots, K \) categories

\[
(C1) \quad aqs_i = prrat_i * qs_i * I_{sk}; \quad i = 1, 2, \ldots, N_k;
\]

Market demand of input type \( i \); \( k = 1, 2, \ldots, K \) categories

\[
(C2) \quad ax_i = x_i * I_{sk}; \quad i = 1, 2, \ldots, N_k;
\]

Urban market demand

\[
(C3) \quad aqdu_i = qd_i * I_{du}; \quad i = 1, 2, \ldots, N
\]

Rural market demand

\[
(C4) \quad aqdr_i = qd_i * I_{dr}; \quad i = 1, 2, \ldots, N
\]

Industry demand

\[
(C5) \quad aqdp_i = ax_j;
\]

\( j \) is a fish type which is an input in fish type \( i \);

\[
aqdp_i = 0 \text{ otherwise}; \quad i = 1, 2, \ldots, N
\]

Total market demand

\[
(C6) \quad aqd_i = aqdu_i + aqdr_i + aqdp_i; \quad i = 1, 2, \ldots, N
\]

Total imports

\[
(C7) \quad imu_i = (1 - dpcdu_i) * aqdu_i; \quad i = 1, 2, \ldots, N
\]

\[
imr_i = (1 - dpcdr_i) * aqdr_i; \quad i = 1, 2, \ldots, N
\]
\[ \text{im}_i = \text{imu}_i + \text{imr}_i; \quad i = 1, 2, \ldots, N \]

Total exports

\[ \text{ex}_u = (1 - \text{dcdpu}_i) \cdot \text{aqdu}_i; \quad i = 1, 2, \ldots, N \]
\[ \text{exr}_i = (1 - \text{dcdpr}_i) \cdot \text{aqdr}_i; \quad i = 1, 2, \ldots, N \]
\[ \text{ex}_i = \text{ex}_u + \text{exr}_i; \quad i = 1, 2, \ldots, N \]

Equilibrium identities:

\[ (1 - \text{disc}_i) (\text{aqd}_i - \text{im}_i) = \text{aq}_i - \text{ex}_i \]