INVESTIGATION, ANALYSIS AND DESIGN OF AN EXPERIMENT TO TEST PONDING

LOADS ON FLEXIBLE ROOF SYSTEMS

by

Duncan Stark

A PROJECT

Submitted to

Oregon State University

University Honors College

In partial fulfillment of
the requirements for the
Degree of

Honors Baccalaureate of Science in Civil Engineering (Honors Scholar)

Presented June 4, 2008
Commencement June 2008

AN ABSTRACT OF THE THESIS OF

Duncan Stark for the degree of Honors Baccalaureate of Science in Civil Engineering presented on June 4, 2008. Title: Investigation, Analysis and Design of an Experiment to Test Ponding Loads on Flexible Roof Systems.

Abstract Approved: ____

Dr. Christopher Higgins

While damages due to ponding are common and expensive, provisions for design and construction of flat roof systems lack guidance regarding these loads. The goal of this research is to present methods of designing or building flat roofs that are less vulnerable to this condition. An investigation of the literature, building codes and design specifications has been undertaken. It was found that although ponding theory has expanded to cover many areas in the literature, details within design specifications are still lacking. The effect of ponding loads has been studied through numerical analysis. This investigation confirmed the basic published theory, and also presents new results. It was found that for a roof close to the ponding stability limit, slope does not provide large benefits until it is steeper than many commercial roofs. A design of a full-scale, realistic experiment has been completed. This design calls for loading open web steel joists under ponding loads to failure. It is hoped that through this test, the contribution of the ponding effect to the total load can be determined and the results will provide better design and construction methods for this type of roof system.

Key Words: Ponding, Roof, Collapse, Stability, Joist

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APPROVED

Mentor, representing Civil Engineering

Committee Member, representing Civil Engineering

Committee Member, representing Civil Engineering

Head, School of Civil Engineering

Dean, University Honors College

I understand that my project will become part of the permanent collection of Oregon State University, University Honors College. My signature below authorizes release of my project to any reader upon request.

Duncan Stark, Author

ACKNOWLEDGEMENT

I owe thanks to many people for help with this project. First, thanks to Dr. Higgins for his guidance and patience with me on this project. Thanks also to Dr. Higgins, Dr. Miller and Dr. Scott for their invaluable instruction over the last two years in the structural engineering classes in my program.

Thanks to the Honors College for their guidance through my four years at OSU, and for the opportunity to undertake this research. Thanks also to both the Honors College and the College of Engineering for allowing me to participate in the Opportunity Plus program.

Thanks to Margaret Mellinger and the staff at the library who helped me find Academic Search Premier, Compendex, and a variety of information I would have been very frustrated without.

Finally, thanks to the industry professionals who have, and hopefully will continue to help me with this project. Thanks to John Ashbaugh and Steven Schaefer Associates, Richard Davis and FM Global, Tim Holtermann and Canam, and Perry Green and the Steel Joist Institute.

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PREFACE

This is my Honors College Thesis project. Through the Opportunity Plus program at Oregon State, offered by the College of Engineering and the University Honors College, I will be continuing this research and expanding this work into a Master's Thesis. Expected completion of physical tests, data analysis and the thesis is about one year from now.

All units are English. This is for simplicity, convenience, and to prevent error in conversion. Design, experimentation, and results will all be done in English units, so this is appropriate.

INTRODUCTION

The most commonly used loads in structural engineering include wind, rain, snow, earthquake, live and dead loads. Also included in the ASCE Minimum Design Loads publication are loads due to hydrostatic loads, flood loads and earth pressure loads (ASCE and SEI, 2005). Despite consideration of a variety of loads, most building codes and design specifications, provide only minimal guidance to design and construction professionals on the effects of ponding. The lack of dedicated space in code is not reflective of the importance or intricacy of this type of loading. Ponding related roof collapses are common, destructive, and potentially life threatening. They often occur without warning, and can be difficult to predict (Blaauwendraad, 2007). They have occurred on roofs made of a variety of materials, including wood, steel, concrete and aluminum (Haussler, 1962) (Moody and Salama, 1967). Failures due to these loads have occurred across America, in both northern and southern regions, regardless of climate. This type of loading and the continued collapse of engineered roof systems under such conditions demand more research, a better understanding of the phenomena, and more prescriptive design provisions in building codes.

Definition

The ponding condition can be defined simply as progressive deflection and resulting accumulation of load until either stability or collapse is reached. In a typical scenario, a nearly flat roof will collect a certain amount of load in the form of snow or standing water, which will cause deflection. Assuming water is available, it can fill this deflection to a certain height (to at least the height of the supports), and the deflection will create a still larger volume for the water to fill. As more water flows in, the deflection increases, and the water level continues to rise. This process can continue to one of three end cases. First, the roof system could reach stability, in which case excess water will flow out over the edges, leaving collected water to eventually drain or dry out later. Second, the roof system could approach stability, but reach an overload condition before stability, and fail because the loads are too large. Third, in the most dangerous case, the roof deflections could become large and unbounded rapidly so that the roof system will never reach stability. In this case, the roof will fail eventually due to overload.

Ponding Stability

There are two phenomena that lead to failure under ponding loads: overload due to load amplification, and instability. While the overload condition will be tested, as it is more common, stability is also investigated. Ensuring stability of a roof system is not a simple matter, as the literature demonstrates. Many factors play a role, including the effects of two way systems, support conditions, sloped roofs, camber, and the general geometry of the system. The work done in the area has shown that ponding stability or instability can be determined, and there are various methods of doing so. The most simple and widely cited ponding stability criterion was initially published by Robert Haussler in 1962, for a flat, simply supported beam. This generally represents the worst case, and a safe way to ensure stability. It is reproduced in modified form here:

$$\pi^4 EI > \gamma BL^4 \tag{1.1}$$

Where E is the modulus of elasticity, I is the moment of inertia, L is the length, B is the spacing between beams and γ is the density of the fluid ponding on the roof. It is worth noting that the ponding problem is purely geometric. In general, the stability of a system will depend on the properties of the members and their layout in the system. The properties that determine stability are internal to the system and do not include external factors, such as the initial load.

If a system is stable, then as the load and deflections increase, they will approach a limit. As pointed out earlier, this limit may be above or below the critical load that will fail the structure, but if an infinitely strong, yet flexible system is assumed, then a stable system will come to equilibrium and not fail. If a system is unstable, the load and deflection will increase unboundedly, until failure is reached. In this case, if an infinitely strong, yet flexible system is assumed, then it will simply deflect infinitely far. This means that for an unstable system with water available, any initial imperfection or deflection that allows water to begin to collect will be catastrophic.

Causes of Ponding

Ponding loads can be caused by either rain or snow loads. It is common for snow on a roof to melt as heat passes through the building membrane, which can lead to the ponding effect. Additionally, snow on a roof often acts as a sponge, absorbing rainfall, and increasing the loads on a roof. Rain after a snowstorm may produce some of the heaviest loads a roof will experience, and can lead to ponding.

Several things must be present in a roofing system for it to be susceptible to ponding loads. First, it must be a relatively flexible roof. Without this quality, the roof will not deflect enough to collect additional water to create a ponding situation. Also, a roof must either be relatively flat, or sloped with some form of a parapet that allows collection of runoff water. Each of these properties will allow water to pond, and initiate deflections that may continue to failure. Other issues that can exacerbate the problem include blocked, misplaced, or missing drains or scuppers, and initial sag due to mechanical units or other unexpected dead loads. One problem to be aware of is that often, drains are placed near columns (Kaminetzky, 1991). This can be a problem because as the roof deflects under load, the points at columns will be the high points, and there is little sense in providing a drain at a high point.

Over the last century, there has been a trend in construction towards stronger materials. By using high strength materials such as steel, more efficient, long span roofs made of smaller, shallower and more slender building elements have been possible (Bohannan and Kuenzi, 1964). This trend is epitomized in the efficiency provided by open web steel joists: very slender elements made of strong but ductile materials can lead to very efficient but very flexible structural units. All of these properties serve to make it increasingly flexible, which can increase the chances of ponding loads. While they allow for more efficient designs, high strength materials and flexible roofs require careful attention to detail to prevent ponding.

Prevention

It seems it would be a simple matter to ensure that a roof was stable and strong enough to withstand these loads, yet buildings continue to collapse under ponding loads. The problem in practice is that systems that are stable under the criteria provided in the literature and in the design specifications still experience a degree of the ponding effect. A beam that is close to the critical ratio will be subjected to an amplification of the loads it experiences. A beam that is stable and strong enough to hold loads will still deflect, allowing larger loads to collect on the system. As will be seen subsequently, this amplification factor is not accounted for in roof systems that provide a slight pitch. The two simplest ways to avoid ponding are to either increase the pitch of the entire roof, or to provide more drainage in better locations (midspan), and conduct regular maintenance and inspection of the drainage systems. Both of these options will help to limit the water that collects on the roof and can help to prevent ponding loads. While not a cure for the problem, providing additional camber to steel joists or to the roofing system will help to reduce the effects of ponding loads. A cambered roof will collect water first at the edges, instead of at midspan, which produces much smaller bending moments and stresses in the system. This can easily be the difference between a failed and a safe roof.

Data Collection

The first thing any researcher will find regarding structural failures is that it is incredibly difficult to get data. It is hard to find any relevant, important, accurate data at all, let alone a comprehensive collection of information on the subject. It seems as though failures do not like exposure. In an article published in June 1981, a forward looking author wrote about the lack of available information on structural failures (ENR, 1981):

> "Large-scale structural failure is a nightmare that haunts the construction industry. The financial devastation, the demolished reputations and the loss of life that could result from collapse have troubled the sleep of probably every architect, engineer, contractor or owner at some time.
> This frightening quality of failures almost guarantees that they will continue to happen. Fear, embarrassment and the gag of interminable lawsuits have kept information on failures from traveling quickly enough, what little of it gets into general circulation at all.
> The way to dispel a nightmare is to attack it with hard fact, with eyes open wide and the mind alert…
> …A more promising development is the Engineering Performance Information Center. Its developers hope eventually to set up repositories for information on all types of failures, in a standardized format that would permit the comparisons necessary to develop an understanding of how failures can be prevented. This availability of complete and accurate information could be the first step towards shaking the dread of collapse."

The result of this work was the Architecture and Engineering Performance Information Center, established at the University of Maryland in 1982. The center no longer exists in this form, and could not be found elsewhere. It is likely that the "Fear, embarrassment and gag of interminable lawsuits" kept support from reaching the volume required to make it useful. A data center as described here would be incredibly valuable, and could lead to fewer structural failures in the future. It remains to be seen, however, if such an institution will succeed.

The best information obtained regarding failures, roof collapse and ponding loads came from the Factory Mutual Insurance Company (FM Global). They provide public data sheets on their webpage regarding the safety of a variety of commercial buildings and equipment. FM Global Public Data Sheet 1-54 provides information relevant to structural roof collapses, and some important statistics. An employee was also contacted for more specific information.

According to FM Global statistics, more than 1700 roof failures occurred over the twenty years from 1977 to 1996 (FM Global, 2006). FM Global states that the primary cause of overloading that leads to these failures is ponding of water in roof depressions. Their statistics show that the majority of these failures occur on flat roofs, and that blocked or inadequate drainage systems are a large contributor to the ponding problem. In a phone conversation with an employee at FM Global, it was noted that roof collapse is a serious problem, and that roof failures are typically very expensive, but that the number of deaths is relatively very small. It was also pointed out that the majority of roof collapses are due to snow and rain, and that collapse is a much larger problem in the southern states, as rainfall intensity is higher, and resistance to loads is often lower, due to lower design snow loads.

FM Global also provided financial data on the costs involved in roof collapse. The data provided is representative of all roof collapses that were insured by FM Global, and provides the data both by number of failures and by the costs of those failures. This data shows that the average cost of a roof failure is around $770,000, which illustrates how costly these failures are. In the first set of data, the failures are divided by the type of load; in the second set, they are divided by type of construction. From the first set, it can be seen that the two most damaging loads, by expense, are snow and rain (both contributors to the ponding effect). From the second set of data, it can be seen that the two most damaged roofing systems, by cost, are metal buildings and steel decking on a steel frame, indicating that flexible materials more often lead to failures. Together, this data indicates that the ponding effect is a very strong contributor to roof collapse.

Roof Collapse Overload Losses By Type of Load 1986-2005

Probable Overload Cause	No. of Losses	Indexed Gross 2007$
SNOW, ICE, HAIL	730	$588,739,011
RAIN, ETC	255	$219,910,829
FIXED EQUIPMENT LOAD	16	$40,473,920
MISCELLANEOUS OVERLOAD	90	$33,761,599
CEMENT, SAND	15	$13,511,047
SNOW, ICE EQUIPMENT OVERLOAD	21	$12,988,698
STORAGE	55	$10,788,188
MISCELLANEOUS MATERIAL	4	$4,526,282
SAWDUST, CHIPS	9	$4,434,785
TEMPORARY EQUIPMENT LOAD	16	$4,006,022
Grand Total	1,211	$933,140,380

Table 1: Roof Collapse Data by Load

Roof Collapse Overload Losses By Type of Construction 1986-2005

Type of Construction	No. of Losses	Indexed Gross 2007$
All Metal Buildings	116	$369,027,896
Steel Deck on Steel	141	$207,629,811
Not Classified by Construction Type	779	$181,675,505
Concrete on Steel (Exposed)	22	$49,147,899
Boards on Joists	81	$42,292,682
Plank on Timber or Steel	24	$30,942,251
Plywood on Laminated Beam	23	$25,914,381
All Concrete (No exposed steel)	11	$14,050,841
Plank on Laminated Timber	7	$8,990,095
Miscellaneous	7	$3,469,019
Grand Total	1,211	$933,140,380

Table 2: Roof Collapse Data by Construction

To put this data in perspective, it is important to note that it only represents losses from the companies FM Global ensures, which include about one third of S&P 1000 companies. FM global does not track deaths in their statistics, as they are only a facilities insurance agent. According to other estimates, however, roof collapses cause about 20 deaths yearly (Senteck, 2008). They also lead to huge financial costs and delays to companies, which could force some smaller companies to close.

Case Study: New OSU Energy Center

The best way to get a good practical understanding of how these types of roofs (steel deck, steel joist) are put together is to look at an actual example. There just happens to be a great example of this type of construction on the OSU campus now. The new Energy Center, which is a replacement and upgrade to the old facility, will have about 23 thousand square feet of building space and produce enough energy to power about half of the campus. This facility provides an interesting example of steel joist roof design. A typical roof will be pitched to the edge so that rain runs off into gutters. This roof, however, is pitched in both directions, so the rain from either side collects in the middle of the building.

The structure will use a steel deck roof supported on steel joists. The membrane roofing system will consist of a 2 ply SBS modified bitumen roofing system on ½" Georgia Pacific DensDeck insulation. Based on the design drawings, the joists of the highest roof are 30' 16K9 joists spaced at 7.5', and are welded to their supports. These joists are shorter than typical, but their strength is representative of roof loads in the area. Included here are photos of the facility during construction.

November 15, 2007
The joists have been installed, and some decking has begun to go up:

November 30, 2007
All of the steel decking has been installed:

February 25, 2008
Insulation and roofing are being installed:

BACKGROUND

Literature Review

Ponding has become a more important design consideration recently as a result of increased strength of materials available for construction, which leads to more flexible roofing systems. Although roof collapses have been a major concern for quite some time, collapse due to this specific load scenario was not studied until the 1960s. In the following, technical literature will be reviewed and summarized. Note that the material comes from a variety of sources, so variables are defined differently in different places. For this reason, all variables will be defined with the equations containing them.

The first paper written on the topic was published in 1962 by Robert W. Haussler (Haussler, 1962). In this paper, the author begins by assuming that the roof structure is a simply supported beam, and that deflections can be approximated by a half sine wave. Many authors use this approximation, as it makes the mathematics much simpler, and is only slightly conservative. He also assumes that the ponding fluid is not held by any wall, but only rises to the level of the supports. Using this as a starting point, he finds that for a stable system under water loads:

$$\frac{EI\pi^4}{L^4} > \gamma \qquad (2.1)$$

Where E is the modulus of elasticity, I is the moment of inertia per inch of width, L is the length and γ is the density of the fluid. If a roof is flat, provided with adequate drainage, and meets this stability requirement, then it will be safe from ponding loads. He also states that any roof built on an adequate slope will not experience ponding loads, as water will simply run off.

Haussler provides a very simplified method for calculating the required slope for a safe roof. He suggests the designer choose an initial slope, then use local rainfall data to estimate a depth at the low end. Assuming this depth is constant across the roof (a very conservative and simple assumption), an end rotation can be calculated. This rotation can then be used as a conservative value for the safe pitch of the roof.

Finally, Haussler notes that the analysis of complex roof structures (those with primary and secondary members) could be handled by using the sum of individual deflections. A designer could apply a 5 psf load (approximately one inch of water), then sum the resulting deflection of each system. If this deflection is greater than an inch, then ponding will probably be a problem. He also considers long span systems, and concludes that the common code live load limit of a fraction of the length (live load deflection limited to L/360) is meaningless with respect to ensuring ponding stability. The equation Haussler arrived at, shown above, is not dependant on the live load at all. A better limit to ensure ponding stability would be a ratio of deflection to load (1/2 inch per 5 psf).

Two years later, analysis of ponding loads superimposed on existing load cases was done (Bohannan and Kuenzi, 1964). The authors began by assuming linear elastic behavior and a sinusoidal deflected shape. Using energy methods, the authors determine that the work done by the load will be less than the energy in the beam if:

$$\frac{EI\pi^4}{a^4} > k \tag{2.2}$$

Where E is the modulus of elasticity, I is the moment of inertia, a is the length and k is the unit weight of the fluid times the beam spacing. They conclude that if the inequality is not satisfied, then the work done by the load will be greater than the bending energy, and the beam is unstable. This is essentially a confirmation of the work of Haussler. The authors continue, however, to expand the work to the case of an original distributed load in addition to the ponding load due to the deflection. The midspan deflection resulting from both loads can be calculated as:

$$\Delta = \frac{5w_0 * a^4}{384EI\left(1 - \frac{ka^4}{EI\pi^4}\right)} \tag{2.3}$$

Where w_0 is the initial uniform distributed load and all other variables are as defined above. Note that this equation is simply a combination of the critical ponding criteria and the deflection due to a uniform distributed load. It is also good to notice that as a system approaches the limits for stability as defined in equations 2.1 and 2.2, this expression goes to infinity, and that the ponding effect amplifies the deflection due to initial loads by the factor:

$$\frac{1}{1 - \frac{ka^4}{EI\pi^4}} \tag{2.4}$$

As a result, the stresses in the materials are also increased by the same factor. The authors also go on to solve the problem for the case of a point load with additional ponding effects, and they repeat the analysis for both loading cases under fixed end conditions instead of the simply supported case. The theory was then tested with small aluminum beams. The experiment was set up with three cases. In the first, the total deflection should have been twice that under dead load alone, in the second, four times, and in the third case, the beam was designed to be unstable and deflect to infinity or failure. The experiment verified the theory. The largest difference between the theory and the results of the experiment was the discrepancy between the theoretical and actual deflections under uniform loading, signaling that the greatest uncertainty is not in the ponding theory.

Less than a year later, a paper regarding the failure due to overload of these simply supported flat roofs under ponding loads was published (Chinn, 1965). The author expands on the problem of overload of stable roofs. For a first step, Chinn determines that the final deflection of a beam under ponding loads is:

$$D = \frac{d}{1 - \frac{\gamma L^4}{\pi^4 EI}} \tag{2.5}$$

Where d is the initial deflection, γ the fluid density times the beam spacing, L the length, E the modulus of elasticity of the material and I the moment of inertia. As in the Kuenzi and Bohannan paper, it is clear that as the system approaches the limits of the requirements for a stable system as outlined in equations 2.1 and 2.2, the final deflection will go to infinity. Chinn then solves for the maximum stress in a beam under ponding loads:

$$F = \frac{M_0 c}{I} + \frac{\gamma L^2 \pi^2 Ecd}{\pi^4 EI - \gamma L^4} \tag{2.6}$$

Where M_0 is the moment due to the initial loads, c is the distance from the neutral axis to the extreme fiber, and all other variables are as defined above. The maximum stress can be calculated by this equation and compared to the yield stress of the material to check for possible failure. This assumes that a beam will fail when it becomes inelastic. This equation allows an engineer to calculate when a stable system will fail due to overload, and could be used by a designer to choose an appropriate value for the moment of inertia of a member to prevent this failure mode.

The theory was then expanded to consider the effects of a two way system (Marino, 1966). Until now, all equations only considered a one way system bending independently of the supports. This paper treated the system as one with secondary members holding the load and supported by primary members that collect the load and transfer it to columns.

In a two way system, the primary elements hold up the secondary elements. The secondary elements are more closely spaced, and have less strength than the primary members. The critical secondary member is the one at the center of the span of the primary member because it will be at the lowest elevation, thus incurring the greatest load. The author assumed that all of the primary members will deflect together so that a single bay can be analyzed as a unit, and that all deflections are sine waves. He also assumed that a theoretically stable system will not fail in overload conditions. From his analysis, Marino concludes that:

$$\Delta_w = \frac{\alpha_p\left(\Delta_0 + \frac{\pi}{4}\delta_0\right) + \frac{\pi}{4}\alpha_p\alpha_s\left(\delta_0 + \Delta_0\right)}{1 - \frac{\pi}{4}\alpha_p\alpha_s} \tag{2.7}$$

And:

$$\delta_w = \alpha_s\left(\delta_0 + \frac{\pi^2}{8}\Delta_0\right) + \frac{\pi^2}{8}\alpha_p\alpha_s\frac{\Delta_0 + \frac{\pi}{4}\delta_0 + \frac{\pi}{4}\alpha_s\delta_0 - \frac{2}{\pi}\alpha_s\delta_0}{1 - \frac{\pi}{4}\alpha_p\alpha_s} \tag{2.8}$$

Where Δ is the midspan deflection of a primary member, and δ is the midspan deflection of the critical secondary member. Subscript w indicates after the fluid load, subscript 0 indicates before the fluid load. The parameters α are defined in terms of flexibility constants:

$$\alpha_s = \frac{C_s}{1 - C_s} \tag{2.9}$$

$$\alpha_p = \frac{C_p}{1 - C_p} \tag{2.10}$$

And these flexibility constants are defined in terms of the properties of the system, reflecting the critical ponding criteria already outlined in previous literature:

$$C_s = \frac{\gamma S L_s^{\,4}}{\pi^4 E I_s} \tag{2.11}$$

$$C_p = \frac{\gamma L_s^{\,4} L_p}{\pi^4 E I_p} \tag{2.12}$$

Where s indicates secondary and p primary, S indicates the spacing between secondary elements, L the length of the members, E the modulus of elasticity, I the moment of inertia, and γ the density of the fluid.

Marino went on to make simplifying assumptions that make these equations easier to work with, and, using a factor of safety of 1.25, creates design aides based on the important properties of these systems. This analysis is now the basis of the AISC steel manual check for ponding. Marino's design aides are included in the AISC code appendix 2: design for ponding. Marino concludes by stating that the easiest method of preventing this type of collapse is to provide sufficient slope to adequate drainage. He claims that 1/8 inch per foot should be sufficient, but notes that roof drainage can be complex and should be analyzed in more detail for roofs of this pitch.

Soon thereafter, the theory was expanded to cover several variations on the ponding problem (Moody and Salama, 1967). The authors expand the theory to include beams with different support conditions, ponding loads on plates, and they are the first to draw a connection between the ponding problem and steady state vibrations.

They begin by restating Haussler's inequality for a simply supported beam, rearranged to identify the critical stiffness. The authors go on to calculate the critical stiffness for beams and plates with varied supports. Throughout, the authors use superposition, a set of differential equations and impose the appropriate boundary conditions. They solve the problem of the critical stiffness under ponding loads for a beam that is simple-fixed, fixed-fixed, continuous over three supports with fixed ends, and continuous over any number of supports with simple supports. They also solve the problem for plates simply supported on all edges, simply supported on two edges and fixed on the others, fixed on all edges, and continuous over several simple supports. The results are summarized in this table:

TABLE 1.—SUMMARY OF THE MINIMUM CRITICAL STIFFNESSES FOR BEAMS AND PLATES

Boundary conditions	Minimum critical stiffness
(a) Beams	
Simple-Simple	$(E\,I)_{cr} = \frac{\bar{\gamma} L^4}{\pi^4}$
Simple-Fixed	$(E\,I)_{cr} = \frac{\bar{\gamma} L^4}{(3.927)^4}$
Fixed-Fixed	$(E\,I)_{cr} = \frac{\bar{\gamma} L^4}{(4.73)^4}$
Continuous over 3-supports (outside ends are fixed)	$(E\,I)_{cr} = \frac{\bar{\gamma} L^4}{(3.927)^4}$
Continuous over any number of simple supports	$(E\,I)_{cr} = \frac{\bar{\gamma} L^4}{\pi^4}$
(b) Plates	
Simple, four sides	$D_{cr} = \frac{\gamma}{\pi^4\left(\frac{1}{a^2} + \frac{1}{b^2}\right)^2}$
Simple along two opposite sides of length b, and fixed along two sides of length a	$D_{cr} = \frac{\gamma}{\left[\left(\frac{5.789}{b}\right)^2 - \left(\frac{\pi}{a}\right)^2\right]^2}$
Four sides fixed each of length a	$D_{cr} = \frac{\gamma_a{}^4}{1.303.76}$
Continuous over any number of spans. All edges are simply supported	$D_{cr} = \frac{\gamma}{\pi^4\left(\frac{1}{a^2} + \frac{1}{b^2}\right)^2}$

Figure 1: Critical stiffnesses by boundary conditions (Moody and Salama, 1967)

Where $\bar{\gamma}$ is the unit weight times the beam spacing, L is the length, and a and b are the edge dimensions of a plate. In their work, the authors also note that the ponding problem is analogous to steady state forced vibrations. They relate the idea of the critical stiffness to the natural frequency of the member. This is useful, they assert, because there has been much more work done on the problem of steady state vibrations than ponding, so to relate the two would open up additional approaches for study of the ponding problem. In this analogy, the critical stiffness is analogous to harmonic vibration: as the period of a forcing function approaches the natural frequency, deflection becomes unbounded, just as deflection becomes unbounded when the stiffness of a beam or plate equals the critical stiffness. From this analogy, it is concluded that the critical stiffness can be calculated if the natural frequency of the beam or plate is known:

For Plates:

$$\left(\frac{\gamma}{D}\right)_{cr} = \frac{m\omega^2}{D} \tag{2.13}$$

For Beams:

$$\left(\frac{\gamma}{EI}\right)_{cr} = \frac{m\omega^2}{EI} \tag{2.14}$$

Where γ is the density of the liquid, D and EI represent flexural rigidity, m is the mass per unit length or unit surface area, and ω is the natural frequency. The authors conclude by comparing the critical stiffness value for beams to the Euler buckling load for columns, and suggest that it should be used similarly as a critical design value.

More authors began attempting to create simple aides for designing for ponding (Sawyer, 1967). Donald Sawyer starts by re-deriving Haussler's original inequality. Sawyer sets the ponding critical stiffness criteria equal to a new value he terms the Criterion Ratio:

$$R = \frac{\gamma B L^4}{\pi^4 EI} \tag{2.15}$$

If the Criterion Ratio (R) is greater than unity, then Sawyer calls the beam supercritical. If the criterion ratio is equal to one, the beam is critical, and if it is less than that, it is subcritical.

It is understood that supercritical beams will fail with sustained rain or snow that allows the roof to continually deflect and collect load. This analysis of supercritical beams is only applicable for a set amount of water, that is, if conditions are such that water is not continually entering the system. Based on the criterion ratio and the design plots provided, a designer should be able to calculate maximum moments, maximum deflection, and maximum weight. The plots are shown here:

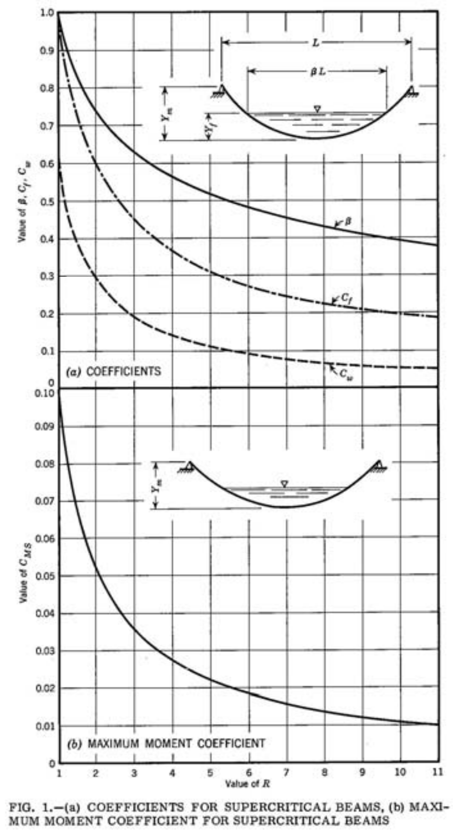

Figure 2: Design guide (Sawyer, 1967)

After pulling these values from the plots, the important properties of the supercritical beam can be calculated:

$$W_f = C_{ws}\gamma BY_s L \tag{2.16}$$

$$Y_m = \frac{W_f}{C_{ws}\gamma BL} \tag{2.17}$$

$$Y_f = C_f Y_m \tag{2.18}$$

$$M_m = C_{ms}\gamma BL^2 Y_m \tag{2.19}$$

Where W_f is the total weight of the load, Y_m is the maximum deflection, Y_f is the midspan liquid depth, M_m is the maximum moment. Y_s is the end depth of the liquid, γ is the density of the liquid, B is the spacing, L is the length of the beams and the parameters C_{ws}, C_f and C_{ms} are from the charts. This analysis is somewhat limited in the fact that it only applies to the situation where a set amount of liquid sits on the structure. For this specific case, this method makes the calculations simpler from a design standpoint.

It is more interesting, however, to study subcritical beams to determine when they will or will not fail, especially because most practical beams are subcritical. A general solution should allow for any depth, initial camber or sag, pitch, and include the effects of a two way system. The author constructs some curves that help identify parameters regarding subcritical beams. The use of this chart requires the designer to calculate both the Criterion Ratio, as well as a parameter, α, as defined individually in each plot, based on the degree of camber of the beam. From this chart, a designer can find C_y for cambered or non-cambered beams, and C_w and C_m for cambered beams:

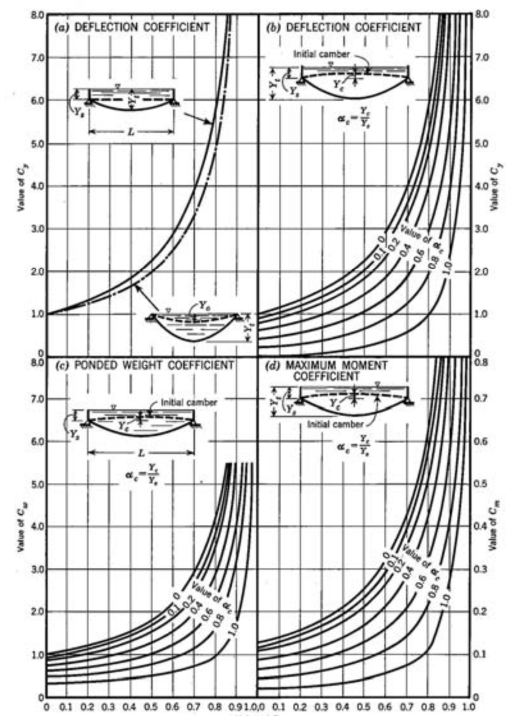

FIG. 2.—(a) DEFLECTION COEFFICIENT FOR INITIALLY FLAT BEAMS, (b) DEFLECTION COEFFICIENTS FOR CAMBERED BEAMS, (c) PONDED WEIGHT COEFFICIENTS FOR CAMBERED BEAMS, (d) MAXIMUM MOMENT COEFFICIENTS FOR CAMBERED BEAMS, (e) TOTAL REACTION COEFFICIENTS FOR SLOPED BEAMS, (f) MAXIMUM MOMENT COEFFICIENTS FOR SLOPED BEAMS

Figure 3: Design guide (Sawyer, 1967)

From these parameters, important properties of the subcritical beam can be calculated:

$$W_f = C_w \gamma B Y_s L \tag{2.20}$$

$$Y_t = C_y Y_s \tag{2.21}$$

$$M_m = C_m \gamma B Y_s L^2 \tag{2.22}$$

Where W_f is the total weight of the load, Y_t is the midspan deflection, M_m is the maximum moment, Y_s is the height of the liquid above the supports, and C_y, C_w and C_m are values pulled from the plots. The values are important because they will let a designer determine whether a beam will fail under ponding loads, even if it is part of a subcritical system.

It is clear that initial upward camber is beneficial to preventing ponding from occurring, but the author notes that caution should be used. Camber should not be used to replace the additional benefits provided by increasing the beam stiffness. This is because as the depth of water approaches the height difference provided by the initial camber, the rate of deflection increases rapidly. For this reason, some roofs could perform well in some events, but fail completely in only slightly different circumstances, depending on how close to this tipping point the system gets.

Sawyer also provides charts that allow a designer to calculate the maximum shear and moments in a beam on a slope, which is useful, as many sloped roofs should also be checked for ponding problems. He notes that in the current AISC specifications, (1963 Ed.) the check for ponding stability was disregarded for anything but a completely flat roof. He points out that some sloped roofs, if the slope is shallow enough, will still experience the ponding effect, and it is unacceptable to ignore this loading because a nominal pitch is specified. Sawyer argues that if it is reasonable to expect the water level to rise above the high end of the roof by at least one half of the depth at the low end, then the roof should be treated as flat. The charts he provides again require the user to calculate the Criterion Ratio and a parameter α based on the initial camber. The charts are shown here:

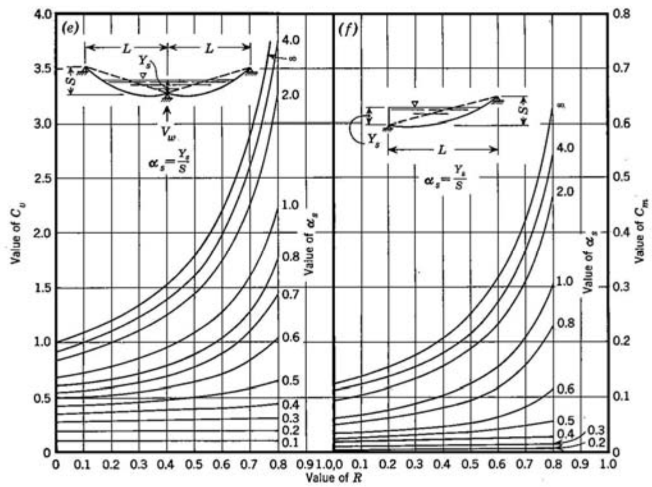

FIG. 2.—CONTINUED

Figure 4: Design guide (Sawyer, 1967)

Based on the values for the coefficients C_v and C_m from these plots, the maximum shear and moments can be calculated as follows:

$$M_m = C_m \gamma B Y_s L^2 \tag{2.23}$$

$$V_f = C_v \gamma B Y_s L \tag{2.24}$$

Sawyer goes on to discuss roof systems under ponding loads. Roof systems are more complicated than the simple one member case for several reasons. The variables are essentially compounded and interact in various ways. Sawyer treats the system in pairs of framing members. In each pair, he assigns a host (supporting members) and a parasite (supported members), and uses the properties of the parasite to modify those of the host. His procedure calls for the modification of the host Criterion Ratio by a factor of the parasitic member's C_w. First, R values are calculated. Next, starting at the top of the system, a C_w value is found for the parasite, and multiplied by the host's R to find the host's effective R value. This new R is then used in the next iteration when the host is treated as the parasite. In this way, the modifications of the Criterion Ratio compound, and a system that looks sound at first glance may by further analysis not be adequate. This method is more involved than the one presented by Marino for two way systems.

Later, Salama and Moody expanded their study of beams and plates (Moody and Salama, 1967) to those with a nonlinear response (Moody and Salama, 1969). Following a complex analysis, they outlined an iterative procedure for calculating the response of these elements. They conclude that for these nonlinear-elastic members, the initial load is an important factor on the final response, which is in contrast to what other authors have shown for linear-elastic beams and plates. It is doubtful that much of this work would be useful in a design situation, as materials are generally taken as linear elastic. The authors outline a complex iterative analysis technique, but provide no simple method of analysis.

That same year, an article was published that investigates subcritical beams with various loading conditions and the effect of initial imperfections on the ponding factor (Adams, Chinn and Mansouri, 1969). The authors begin with the usual assumptions, and analyze a simply supported beam with a fluid filling the depression formed in the middle of the span. They solve the governing fourth order linear non-homogeneous differential equation, and arrive at the same equation Haussler published years earlier. The authors provide equations for the maximum deflection, maximum moment, and beam end rotations for beams with ponded water superimposed with a point load, a distributed load, applied end moments, and nothing. The equations published are long and numerous; they will not be reprinted here.

The authors go on to investigate the effects of initial sag and crookedness on ponding loads. They express the deflection in a Fourier sine series, which shows that the critical ponding factor is not dependant on the type of loading. They point out that a numerical solution would require truncating the series to the dominant term to get an approximate value of the internal forces. A more accurate method would be to treat the loads from the liquid in the depression separately from everything else. It has been shown that deflection is linear with initial imperfections and loads, so this analysis would work by superposition of all sources of deflection (Moody and Salama, 1967).

Again, engineers began trying to make the analysis simpler and more suitable for design use, this time for two way systems (Burgett, 1973). The author simplified the existing plots and equations, which were based on the work of Marino, and produced just two simple equations. Roof framing systems were identified as stable if:

$$C_p + 0.9C_s \leq 0.25 \tag{2.25}$$

$$I_d \geq 25S^4 10^{-6} \tag{2.26}$$

Where I_d is the moment of inertia of the deck, S is the spacing, and C_p and C_s are defined:

$$C_p = \frac{32L_s L_p{}^4}{10^7 I_p} \tag{2.27}$$

$$C_s = \frac{32SL_s{}^4}{10^7 I_s} \tag{2.28}$$

Where L is span length, I moment of inertia, and the subscripts p and s represent the primary and secondary systems, respectively. Burgett also included graphical representations of these expressions for both deck and framing checks. This approach has now become part of the AISC code, in appendix 2, design for ponding, and is called the simplified design for ponding.

The same year, a paper was published that focused specifically on truss behavior under the loads (Chao, 1973). The author studies a specific type: warren, pin connected, simply supported trusses. Using a set of differential equations, Chao solves for the joint displacements in the x and y directions for every node of the truss. The solution for the nodal displacements lists the displacements as functions of several variables: several parameters, a, defined below, the number of panels in the truss, n, and an arbitrary constant C.

$$a_0 = \frac{\gamma s d^2}{2 A_w E_w} \sec(\Theta) \csc^2(\Theta) \tag{2.29}$$

$$a_1 = \cot(\Theta) \tag{2.30}$$

$$a_2 = 2 \frac{A_w E_w}{A_t E_t} \cos^3(\Theta) \tag{2.31}$$

$$a_3 = 2 \frac{A_w E_w}{A_b E_b} \cos^3(\Theta) \tag{2.32}$$

$$a_4 = a_2 \tan(\Theta) \tag{2.33}$$

$$a_5 = a_3 \tan(\Theta) \tag{2.34}$$

Where γ is the fluid density, s is the spacing of the trusses in the one way roof, d is the width of a panel, A is the cross sectional area, E is Young's Modulus, and Θ is the angle between chord and web members. The subscript w is for web, t for top chord, and b for bottom chord. The truss geometry and parameters are illustrated in figure five:

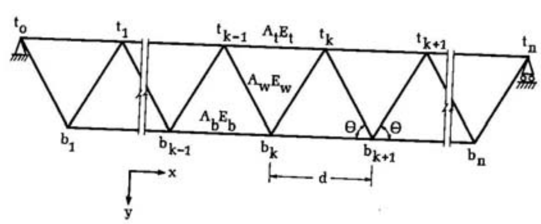

Figure 5: Truss diagram (Chao, 1973)

With these parameters established, the solution for the nodal displacements is:

$$u_t(k) = C a_0 a_4 \cot\left(\frac{\pi}{2n}\right)\left(\cos\left(\frac{k\pi}{n}\right) - 1\right) \tag{2.35}$$

$$u_b(k) = -C a_0 a_4 \cot\left(\frac{\pi}{2n}\right) - C a_0 a_5 \csc\left(\frac{\pi}{2n}\right)\cos\frac{(k-0.5)\pi}{n} \tag{2.36}$$

$$v_b(k) = C\cos\left(\frac{\pi}{2n}\right)\left(4\sin^2\left(\frac{\pi}{2n}\right) - a_0 a_2\right)\sin\left(\frac{(k-0.5)\pi}{n}\right) \tag{2.37}$$

$$v_t\left(k\right)=4C\sin^2\left(\frac{\pi}{2n}\right)\sin\left(\frac{k\pi}{n}\right) \tag{2.38}$$

Where u and v are the displacements in the x and y directions, respectively. The parameter k is an index that represents the number of the panel point node. The subscripts t and b indicate either the top or bottom chord. Chao goes on to determine a stability condition requirement for trusses under ponding loads. He defines this condition in terms of the parameter β:

$$\beta=1-\frac{1}{4}\left(a_0\left(2-a_2\right)+\left(a_0^{\,2}\left(2-a_2\right)^2+16a_0\left(a_2+a_3\right)\right)^{\frac{1}{2}}\right) \tag{2.39}$$

Based on this value of β, stability is mathematically assured if:

$$\beta<\cos\left(\frac{\pi}{n}\right) \tag{2.40}$$

By making some simplifying assumptions, this equation can be shown to be equivalent to the stability equation other authors have found (equations 2.1, 2.2, 2.15 etc.) These simplifying approximations are shown to be reasonable for large values of both n and the ratio A_wE_w/A_tE_t. Chao was the first to note that the typical 15% reduction the critical ponding load for joists, which is reflected in the AISC code as a 15% reduction in the moment of inertia. Chao asserted that this may not be an appropriate reduction, and that the effect of n and A_wE_w would be better criteria for adjusting the critical load.

More analysis was published on the topic of two way systems (Avent and Stewart 1975). The stated goal of the paper was to come up with an analysis method that was more accurate than the work of Marino, but more efficient for design use by the typical engineer. The general approach was the formulation of a set of differential equations solved by Fourier series analysis. The result of this analysis is an inequality that provided a check for the stability of the primary members. As the authors point out, the stability of the secondary members should still be checked by the same criteria that other authors have published. By these calculations, the primary members of a two way system are stable if the following holds true:

$$J < \frac{\pi^3\left(108\sigma_k^{\,2} - \left(3-\sigma_k\right)^3 H\right)}{36\left(2-\sigma_j\right)\left(3-\sigma_k\right)^2 H} \tag{2.41}$$

Where H is the Criterion Ratio as defined by Sawyer, and σ values are defined as:

$$\sigma_j = 1 - \cos\left(\frac{\pi}{n}\right) \tag{2.42}$$

$$\sigma_k = 1 - \cos\left(\frac{\pi}{m}\right) \tag{2.43}$$

Where n is the number of bays parallel to the secondary members and m is the number of bays parallel to the primary numbers. This solution provides a simple check for ponding stability in the primary members and is, according to the authors, more accurate than any other previous approach. The authors also go on, using the same method, to find the maximum moment in a primary member. The equation they developed was a double summation, and would take time to use as an office tool. When used, however, it would help a designer determine whether a member will fail from typical load combined with ponding, even if it meets the stability criteria.

Richard Avent published another article, this time on his own, the following year. He analyzes the deflection of steel joists under loads, including ponding loads (Avent, 1976). He notes that the deflection of these structural units is often important, and that not much work has been done on the subject. He analyzes the idealized warren truss, which is the configuration used in most joists today. The configuration as illustrated by the author is:

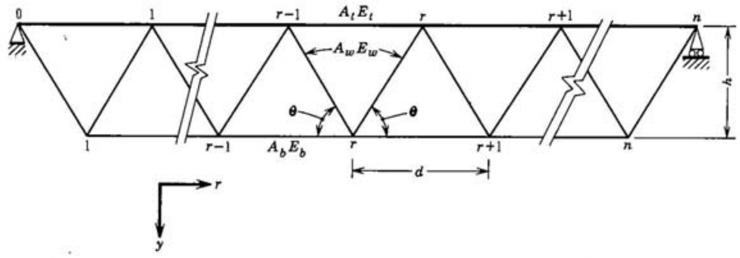

Figure 6: Joist diagram (Avent, 1976)

The author began by improving what had been the equation for calculating the effective moment of inertia in a way he claimed was much more accurate than previous methods. He used this and calculated equations that govern the motion of each of the nodes in the truss with increasing load. The resulting equations resemble those published by Chao in 1973. There are equations for the displacement of a node on each axis, for top and bottom nodes. The results produced the same stability criteria for joists as found by Chao. The author noted that stability can be determined, but that designers should calculate deflection and stresses to ensure that a stable system does not fail. To increase the ease of these calculations, the author determined simple equations to be used in design that very closely approximate the maximum chord and web member forces. The maximum top or bottom chord force is:

$$F_{\max} = \frac{M_s}{h(1-G)} \tag{2.44}$$

And the maximum web bar force is:

$$S_{\max} = \frac{V_s \sin\theta}{1-G} \tag{2.45}$$

Where M_s is the maximum moment due to non-ponding loads, V_s is maximum reaction due to non-ponding loads, h and Θ are as defined in figure six, and G is the Criterion Ratio as defined by Sawyer. These equations provide simple estimates for the forces experienced by the members in a warren truss, and should be useful to anyone steel joists designers.

Thus far, treatment of ponding loads on sloped roofs has been minimal. Bin Chang and Ken Chong presented a paper on this topic to the World Congress on Space Enclosures in 1976 which resulted in an almost identical paper published the next year in the Forest Products Journal (Chang and Chong, 1976) (Chang and Chong, 1977). In the paper, the authors assume that the height of ponded water at the low end of the sloped roof is zero, allowing for water to collect only in the deflected shape below the low support point. This is limiting in that the analysis only allows for this single load case. The results of this analysis show that the deflection due to the ponding effect is dependent on the initial loads and deflection. However, no results are given as to how the stability of such a system changes from that of the flat case.

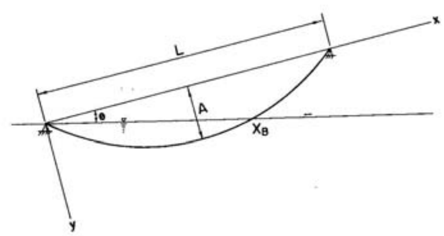

Figure 7: Sloped beam ponding setup (Chang and Chong, 1977)

Based on this geometry, the authors determine that the deflection due to ponding loads only, y_p, can be expressed as a function of the total deflection, A, the angle theta, the length, L, the stiffness, EI, and the density times the spacing, λ:

$$EIy_p = \frac{\lambda L^4}{1440}\left(14\left(A\right) - 9L\Theta\right) \tag{2.46}$$

It can be shown that when the angle is zero, this expression reduces to that found by Chinn (equation 2.5). It should also be noted that by increasing the angle, the deflection due to ponding is decreased. In fact, if the angle is increased to 14A/9L then there will be no deflection due to ponding. Because A is typically very small compared to L, the angle required to eliminate ponding deflection effects is typically very small. In general, a slight pitch should be sufficient to avoid these loads. This equation allows some insight to the ponding problem on sloped roofs, but is limited, as it does not provide an explicit equation for the stability criteria of a sloped roof. More work could be done in this area.

When a new set of stability equations designed for office use were formulated and became candidates for inclusion in the specifications, and some engineers spent some time evaluating them (Carter and Zuo, 1999). The source of the new equations is cited as a letter from author K. P. Milbradt to an AISC representative in February 1995. The equations proposed by Milbradt are candidates for replacement of the analysis based on the work of Marino in the AISC code (Marino, 1966). It is suggested that these equations may provide greater ease and accuracy, as they are calculation based, as opposed to Marino's graphically based solution. The proposed equations for checking the primary and secondary systems (respectively) are as follows:

$$C_p \leq 1.04 - 0.97C_s - 1.27\frac{f_0}{F_y} \tag{2.47}$$

$$C_s \leq 1 - 1.07C_p - 1.25\frac{f_0}{F_y} \tag{2.48}$$

Where C_s and C_p are as defined by Burgett in equations 2.27 and 2.28, F_y is the yield stress, and F_o the maximum extreme fiber stress in the member due to all loads except ponding (Burgett, 1973). The authors' conclusion regarding the comparison of Milbradt's equations with those of Marino is that they are close but different. No conclusions about relative accuracy were drawn.

In his discussion of the article, Milbradt argues that his equations should replace both the ponding analysis based on the work of Marino and the simplified method based on the work of Burgett (Milbradt, 2000). The argument is that his equations are more accurate than the simplified ones, and because his method is calculation based, it is easier and better than Marino's method. Milbradt also discusses the effect of f_0, residual stresses, and the trouble with calculating effective moments of inertia for joists. He argues that the equation provided in the Steel Joist Institute

Design Manual 3 only represents an average approximation and in some cases is unconservative (SJI, 1971). Milbradt suggests that all of this should be included in the commentary of the AISC code. These equations have yet to show up in the AISC specifications.

A paper that presented and discussed ponding loads and a numerical model was presented to the second European conference on steel structures, in Prague (Colombi and Urbano, 1999). The authors present no new results here, but the paper leads to a published article the following year that presents a new, interesting method of analyzing ponding loads (Urbano, 2000). In this paper, the author treats a beam under ponding loads as two equal length beams connected by a spring at midspan:

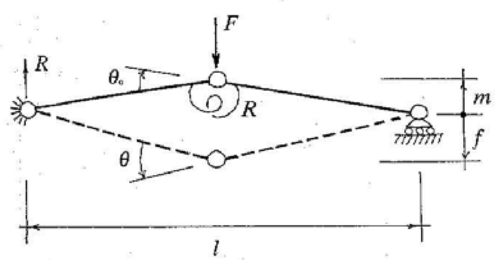

Fig. 2. Static model with one degree of freedom

Figure 8: 1 DOF bar-spring model (Urbano, 2000)

The author defines a factor he terms the influence coefficient, α, which is a property of the system and defined as the ratio of the displacement f due to a corresponding applied force F. Based on this value and some simple algebra and geometry, Urbano derives equations for the displacement of the system, as well as the moment carried in the spring:

$$f_0 = \alpha F_0 \tag{2.49}$$

$$f = f_0 \left(\frac{1 - \frac{m}{f_0}}{1 - Y\alpha} \right) \tag{2.50}$$

$$M_0 = \frac{Yh_0 l}{4} \tag{2.51}$$

$$M = M_0 \frac{1 - mY\alpha / f_0}{1 - Y\alpha} \tag{2.52}$$

Where f, F, m and l are as defined in the diagram. The naught subscript indicates a value that is due to loads before ponding effects occur. Alpha is as defined above, Y is the unit weight of the fluid times the spacing of the beams, and h is the height of the water on the system. For this system, Urbano determines that the critical value for the ponding effect occurs when $\alpha Y = 1$. For a system to be stable, it should be ensured that this value is less than one by whatever factor of safety may be appropriate.

Urbano goes on to incorporate the typical code serviceability requirements of restricting deflection to some fraction of the length into his equations. This is interesting, but adds little to his ideas. He also adds the effect of shear on the deformation by repeating the analysis with three springs:

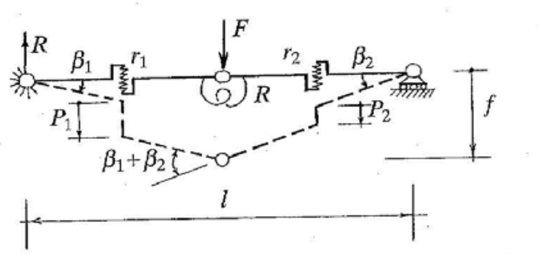

Fig. 6: Static model with three degrees of freedom

Figure 9: 3 DOF bar-spring model (Urbano, 2000)

Based on this analysis, he finds that the deflection can be calculated by the equation:

$$f = \frac{Yh_0\left(\frac{1}{2r} + \frac{l^2}{16k}\right)}{1 - Y\left(\frac{1}{2r} + \frac{l^2}{16k}\right)} \tag{2.53}$$

Where r is the spring constant for the springs in shear, and k is the spring constant for the spring in flexure. He also continues to expand these ideas to a two way roofing system:

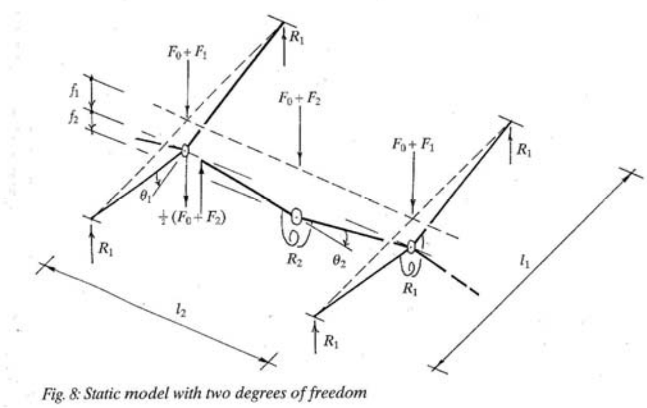

Figure 10: 2-way bar-spring model (Urbano, 2000)

Based on this analysis, Urbano calculates the ratios of moments due to additional ponding load to moment due to initial load for both framing systems as functions of the influence coefficients and Y. This is equivalent to the amplification factors discussed previously (Bohannan and Kuenzi, 1964). The factor amplifies both the displacements and the moments equally. The factors are solved for explicitly, and plots are provided for increased visual comprehension and simplicity:

$$\frac{M_1}{M_{01}} = \frac{f_1}{f_{01}} = \frac{1-\alpha_{22}Y/2}{1-2\alpha_{11}Y-\alpha_{22}Y+\alpha_{11}\alpha_{22}Y^2} \tag{2.54}$$

$$\frac{M_2}{M_{02}} = \frac{f_2}{f_{02}} = \frac{1}{1-2\alpha_{11}Y-\alpha_{22}Y+\alpha_{11}\alpha_{22}Y^2} \tag{2.55}$$

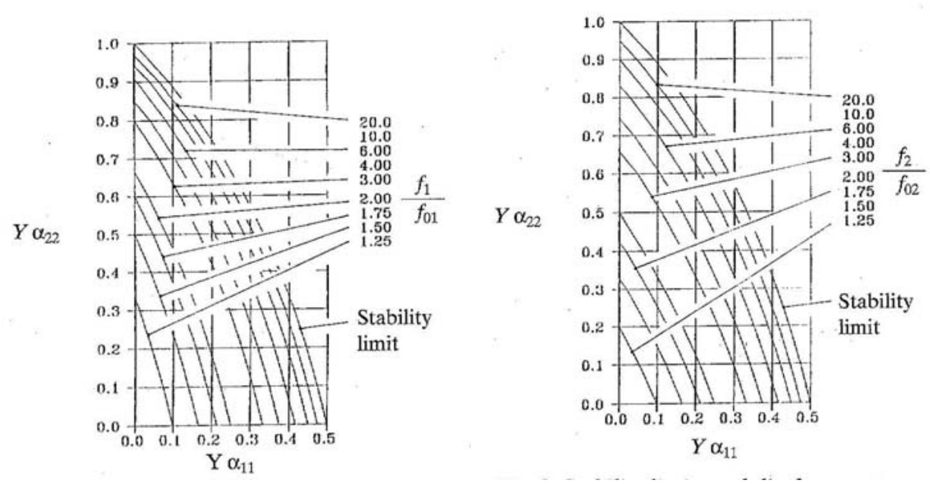

Figure 11: Design guides (Urbano, 2000)

This analysis is good because it is the most comprehensive analysis provided in a single source. The results are equations and graphs that are simple and easy to understand and use. The only drawback is that the author provides no indication of how accurate his initial assumption of a spring connecting two beams is. The equations are simple enough for design use, but need to be evaluated for accuracy. In practice, constants would need to be derived for the spring coefficients and the influence coefficient, so more work is required before this approach can be useful.

Work has been done on members with different end conditions, but it took quite a while before the ideas were expanded to cantilevered members. This is eventually done so that designers can take advantage of the benefits of a cantilevered system derived from moments balancing each other better and smaller overall deflections (Bergeron, Green and Sputo, 2004). The authors begin by defining a variable n as the ratio of the deflection of a simply supported system to the deflection of another system (cantilevered in most of this paper) under the same loading conditions. They define the parameter C_p, as used in previous literature (Burgett, 1973), as:

$$C_p = \frac{32 L_s {L_p}^4}{10^7 n I_p} \tag{2.56}$$

The authors then proceed to outline a method for determining n. They begin by showing that the midspan deflection of a cantilever is approximately equal to the maximum, and use this as a simplifying assumption. Based on the following diagram, the maximum deflection will be at midspan, but will be less than for the simply supported case due to the negative moments caused by the point load on the cantilevered end.

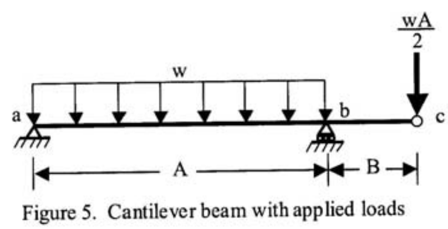

Figure 12: Cantilevered end (Bergeron, Green and Sputo, 2004)

Based on this methodology, it is then shown that the value of n can be calculated by the equation that follows:

$$n = \frac{1}{\left(1 - \frac{2.463BA}{C^2}\right)} \tag{2.57}$$

This is the appropriate value of n for this condition only. The authors go on to calculate the value of n for a beam with both ends cantilevered with point loads:

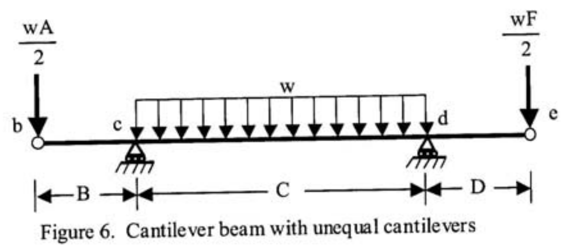

Figure 13: Two cantilevered ends (Bergeron, Green and Sputo, 2004)

$$n = \frac{1}{\left(1 - \frac{2.463(BA + DF)}{C^2}\right)} \tag{2.58}$$

The authors have provided equations for appropriate stiffness factors for two common cantilever setups. This allows designers to take advantage of the additional capacity of the cantilever system, and eliminates some of the unnecessary conservatism in the code on this topic.

In another paper, the concept of partial ponding (ponding due to a given amount of water) was expanded (Colombi, 2005). Instead of water accumulating while a roof deflects, the water simply moves as the load changes and the deflected shape is adjusted. The author begins with an analysis of the traditional ponding problem, and based on the simply supported beam with residual camber, as shown, he produces an equation for the deflected shape of the beam under water loads:

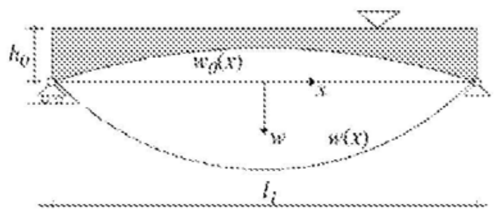

Fig. 3. Simple supported beam under full ponding effect.

Figure 14: Full ponding (Colombi, 2005)

$$w(x)=\frac{h_0}{2}\left(\frac{\cosh(\omega x)}{\cosh\left(\frac{\omega l_1}{2}\right)}+\frac{\cos(\omega x)}{\cos\left(\frac{\omega l_1}{2}\right)}-2\right)-\frac{m}{1-\left(\frac{\omega l_1}{\pi}\right)^4}\cos\left(\frac{\pi x}{l_1}\right) \tag{2.59}$$

Where all variables are as defined in the diagram and m is the residual precambering parameter, the height of the beam at midspan over the straight line. The solution of the partial ponding problem is also found:

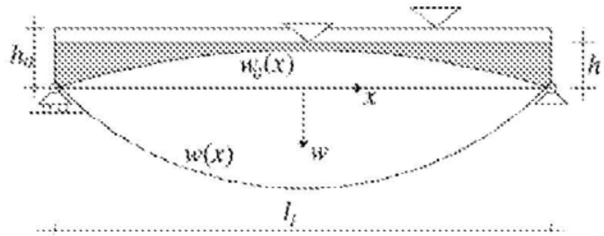

Fig. 6. Simple supported beam under partial ponding effect.

Figure 15: Partial ponding (Colombi, 2005)

$$w(x) = \frac{h_0}{2} g \omega l_1 \left(\frac{\cosh(\omega x)}{\cosh\left(\frac{\omega l_1}{2}\right)} + \frac{\cos(\omega x)}{\cos\left(\frac{\omega l_1}{2}\right)} - 2 \right) - \frac{m}{1 - \left(\frac{\omega l_1}{\pi}\right)^4} \cos\left(\frac{\pi x}{l_1}\right) \qquad (2.60)$$

The author then goes on to outline the numerical approach he will use to solve some of the problems in the rest of the paper. He uses an iterative solution technique that evaluates the initial deflection due to the initial load, and then calculates the subsequent deflection due to the additional ponding load. He divides the surface into a grid to facilitate this analysis, and calculates the deflection of every section of the grid to determine an overall deflected shape. After outlining the procedure used to set up the numerical analyses, Colombi runs through three examples of how the analysis works in practice.

The partial ponding condition is important, as it represents a large portion of what happens in the field. Often, a set amount of water will collect on a roof during a rainstorm, and will remain for some time afterward. It is concluded that the partial ponding condition cannot lead to ponding instability, however, as only a set amount of water is allowed to collect. The deflected shape equations produced and the numerical analysis procedure described are the most useful results of this analysis.

Most recently, the methods for approaching a ponding analysis were again expanded. By approaching the problem from a new angle, many problems become simplified (Blaauwendraad, 2007). The author notes that "…true insight appears to be missing on the very nature of the ponding phenomenon." In his paper, he outlines two new ways of approaching the problem of ponding analysis: the piston spring model and the bar spring model for stiff and flexible roof systems, respectively. The difference in the two models is that with stiff roofs, deflections will be small and the roof will likely be completely covered, whereas with a more flexible system, deflections will be larger and water may not completely submerge the roof. These models consider the effects of pitch, camber, slope, and various end conditions on the full ponding problem.

The analysis begins by assuming a sinusoidal deflected shape and accumulated water load. It is then shown that the average accumulated water load is eighty percent of the maximum, and this simplification is used throughout. The author then outlines his piston-spring model for stiff systems:

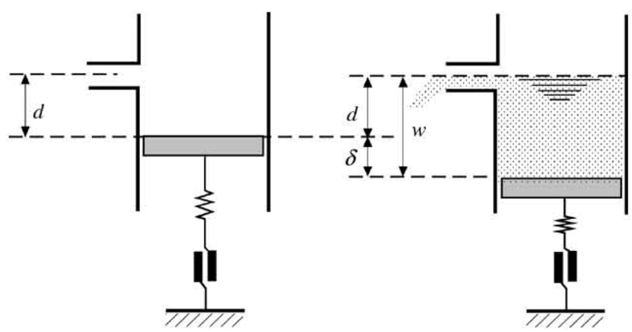

Fig. 3. Piston–spring model of a roof member. Left without, right with water.

Figure 16: Piston-spring model (Blaauwendraad, 2007)

Where d is the original depth of water on the roof, and δ is the average additional load, eighty percent of the maximum in the deflected shape of the roof. He then describes three relevant variables: W, the weight of a meter of water on the roof, D, the spring stiffness, and F_p, the overall total strength of the support structure:

$$W = \gamma a l \tag{2.61}$$

$$D = 96EI / l^3 \tag{2.62}$$

If the support structure remains linear elastic, and F_p is not reached, then the deflection δ can be calculated:

$$\delta = \frac{1}{n-1} d \tag{2.63}$$

Where n is the ratio of D to W. Based on this result, it can be seen that for a very stiff roof, D >> W, the additional deflection and load, δ, will be small. When the ratio approaches unity, however, the additional deflection gets extremely large. This is essentially the same as the original stability inequality published by Haussler (Haussler, 1962). This ratio, n, determines whether a system will be strength dominated or stability dominated. If n is greater than one (D greater than W), then the system will be strength dominated. This is because successive deflections will be smaller, and the system will eventually fail due to simple overload. If n is less than one, then the system is stability dominated and will fail under pure ponding loading conditions. By analyzing the piston-spring system under a force equal to the maximum strength of the system, the author determines that an ultimate value of W can be calculated:

$$\frac{1}{W_u} = \frac{1}{D} + \frac{d}{F_p} \tag{2.64}$$

As W is a function of the fluid density, spacing and length, and because fluid density and length are typically known, this equation essentially limits the spacing of the beams in the system. It is shown that this method can easily include the effects of initial deflection or camber. This is done by using, as before, an average load or loss of load due to these effects of eighty percent of the maximum under the deflected shape. The deflection parameter, d, is modified by eighty percent of the midspan height change due to camber or initial deflection. The solution is also expanded to include the effects of a two or three way roofing system. To do this, the approach is identical, but the formulation of D and W change:

$$\frac{1}{D} = \frac{1}{D_p} + \frac{1}{D_s} + \frac{1}{D_{sh}} \tag{2.65}$$

$$W = \gamma l_p l_s \tag{2.66}$$

Where the subscripts p, s and s_h stand for primary, secondary and metal sheet systems, respectively.

The author also outlines a simple method for performing this analysis on systems with end conditions that are not simply supported. The only change required is that the effective stiffness will be modified by a factor. The factors for several common support conditions are shown:

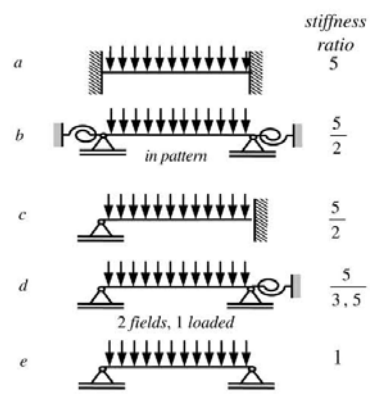

Figure 17: Stiffness ratios (Blaauwendraad, 2007)

The author also outlines a simple method for taking slope into account. If the depth at the low end of the member is d, as before, then the effective depth over the sloped member is:

$$d = d_w - c\alpha l \tag{2.67}$$

Where α is the angle from horizontal and c is a parameter based on how deep the ponded water is. For water that completely submerges the beam, c is one half. For water that does not completely submerge the beam, c is defined by the plot:

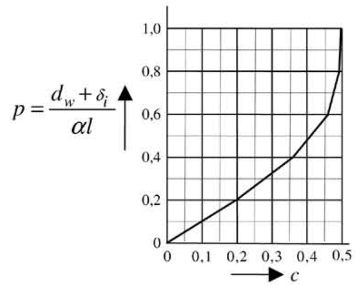

Figure 18: Design guide (Blaauwendraad, 2007)

Blaauwendraad continues by turning to more flexible systems and the bar-spring system as outlined by Urbano (Urbano, 2000), but expands the previous work to treat sloped roof systems. He starts off by defining the location of the spring as at the midspan of the horizontal projection of the submerged portion of the beam, as shown:

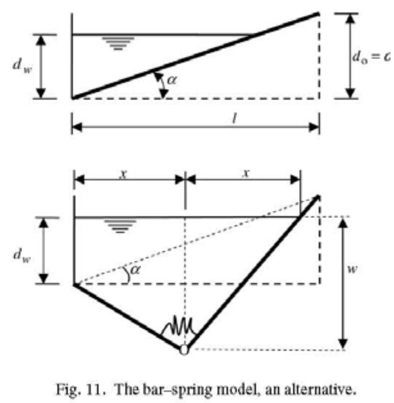

Figure 19: Bar-spring model (Blaauwendraad, 2007)

To complete this analysis, the author begins by finding the rotational stiffness of the spring in terms of E, I, and l, and by treating the entire load due to water as an equivalent point load on the system at the spring. He goes on to check the results of the piston-spring model and the bar-spring model each at the point where water rises to the high end of the member, and finds that they give the same result. Based on the model as it is set up, and a series of algebraic and geometric derivations, the author outlines the results for determining a stable system. He summarizes his findings in the following plots:

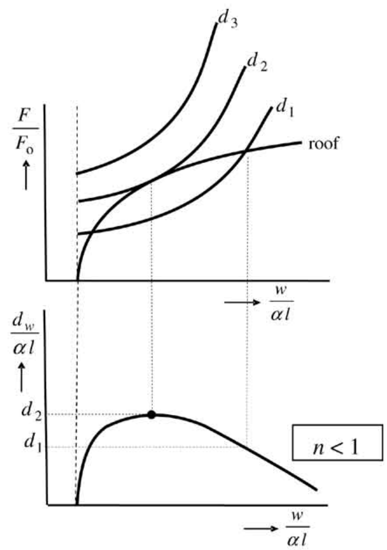

Figure 20: Design guide (Blaauwendraad, 2007)

These plots show that for given values of roof strength or d_w, as the ratio w/αl (equivalent to the ratio of the height of the water at the shallow end to the height of the high end) increases, so does the ratio of final load after ponding to initial load. The curve labeled roof in the diagram indicates the ultimate load allowable on a roofing system in terms of F/F_0, while the curves labeled d indicate the maximum induced load on a roofing system in terms of F/F_0 for a given set of geometric parameters. It can be seen that for low values of d_w such as d_1, stable systems exist, and there are two critical points where systems transition from stable to unstable. For high values of d_w such as d_3, the system will always be unstable. There is a value, labeled here as d_2 that is the critical value for d_w, the highest value it can be while not eliminating the possibility of a stable system. The second plot shows a summary of the curves in the first. Any system that fits under the curve shown is stable, while any system fitting above is unstable.

While these curves are instructive and conceptual, the author provides no simple way of mathematically determining where a system fits on these plots. As with the Chang paper discussed above (Chang and Chong, 1977), work could still be done in the area of providing a simple, accurate, equation based method of determining stability of sloped roof systems. The author finishes by expanding the method for sloped members to include initial camber or sag, then works through some examples. The author has since realized that his method, as outlined in this paper, needs modification. Specifically, his approach to relatively stiff systems (the piston-spring method) is too conservative for some systems. He has a second paper in draft form that would correct this by modification of the formulation of loads and moments in such systems (Blaauwendraad, unpublished).

The theory of ponding loads has covered a lot of ground. It started as a paper that outlined the stability criteria for a flat, simply supported beam under water, and has been expanded to cover two way systems, various support conditions, amplification, slope, initial imperfections, camber, partial ponding, nonlinear response, and uses varied approaches and simplifications. The concepts have been applied to topics as wide ranging as creep, concrete, and trusses. Much knowledge has been published on the topic, yet much still remains to be found. Some of the areas lacking have been identified above, and there are others.

This theory has been slowly produced over more than the last 40 years. It is important, as roof systems do often fail under ponding loads. Typically, heavy rainfall or large storms can create large loads, and the ponding effect will make it much worse. Without designing for ponding, roofs can collapse. It has been pointed out (Chinn, 1980) that the amplification factor, as discussed by Kuenzi and Bohannan (equation 2.4), is the important part of the ponding theory, and the important use of the Criterion Ratio. Most roof systems will be stable by the theory, but all roof designs should include secondary loading due to ponding in the determination of roof loads. All roofs must be designed with this important loading in mind, whether the end result is additional drains or a steeper, stiffer or stronger roof.

While this represents a summary of the theory developed for ponding loads thus far, the ideas have been expanded into some other fields. Probabilistic studies have been done on the reliability of wood members subject to ponding loads and creep (Folz and Foschi, 1990) (Fridley and Rosowsky, 1993). Studies have also been done on ponding effects on floating membranes (Katsikadelis and Nerantzaki, 2003), more details on the effects of initial imperfections on roof behavior (Ahmadi and Glockner, 1984), and mapping flat roofs that may be prone to ponding (Avrahami, Doytsher, Raizman and Yerach, 2007). Other research has been done in the area of hydrology to determine how ponding is affected by specific rain storms (Sawyer, 1968). The work of Marino has been expanded to determine the excess concrete required when pouring on a flexible flooring system due to the ponding effect (Ruddy, 2005). This work is not directly related to the science of ponding loads, but is good background information. Despite this apparent interest in the nature of these loads, full-scale test results have never been reported. This is a motivation for the research at hand. It is hoped that a theory can be confirmed and design simplifications justified so that designers can have useful and practical tools for design of roof systems to resist ponding.

Building Code Review

While there is an abundance of information available on the subject of ponding, most structural engineers do not know this background and the evolution of the field. A small fraction of the information available is published in building codes and design specifications. The following is a summary of what a designer who has done no independent research but uses the codes and specifications will know of ponding loads.

International Building Code (IBC) 2006

The model building code that is used throughout most of the United States has relevant provisions for rain and ponding loads. For roof drainage, the IBC requires that both a primary and a secondary drainage system be provided. For rain loads, the code requires a designer to assume there is standing water at the depth it would reach if the primary drainage system fails. To ensure ponding stability, the code requires that a designer provide adequate slope (at least 1/4 on 12) or else verify adequate stiffness to prevent ponding. For guidance on these calculations, the IBC refers designers to section 8.4 of ASCE 7.

American Society of Civil Engineers (ASCE) 7-05

This guide provides information collected by experts in the field of structural engineering, and provides guidelines for structural designers. These guidelines require that two independent drainage systems be provided, each with the same capacity. It also requires that design of a roof system provide adequate strength to hold standing water to the height it would reach if the primary system failed. For stability against ponding, section 8.4 requires either a sufficient slope (at least 1/4 on 12), or investigation to ensure adequate stiffness against progressive deflection. It is suggested that the larger of the snow and the rain load be used, and that the primary drainage system should be assumed to be blocked for this investigation. The commentary for this section suggests that the guidelines in the AISC specifications for steel construction be used to perform this investigation.

American Institute of Steel Construction (AISC) Steel Construction Manual, 13th Ed.

Section B3.8 of the AISC specification requires that the ponding problem be considered. It requires that a designer do one of three things to ensure ponding stability. Either adequate slope should be provided (at least ¼ on 12), adequate drainage be provided, or the ponding investigation be performed as outlined in appendix 2. This is more lenient than the requirements in the IBC and the ASCE 7, so those documents will typically control, and providing adequate drainage alone, as allowed by the AISC specification, will not be sufficient to satisfy ponding requirements.

Appendix 2 in the AISC specification is the only place where a general and useful method of investigating ponding is presented in code or specifications. Two independent methods are presented: a simplified, conservative check, and a more in depth, accurate method. The simplified method is taken from Burgett, and allows for a factor of safety of four against instability (Burgett, 1973). When using this method for trusses and joists, it is required that the moment of inertia be reduced by fifteen percent to find the effective moment of inertia. This modification accounts for the part of deflections due to shear deflection, as opposed to that due to bending moment alone. Also, within this method, steel decking should be considered a secondary member when it is supported directly by the primary members alone. The in depth analysis method is taken from Marino (Marino, 1966). These methods are discussed in the literature review and the discussion will not be repeated here.

AISC Design Guide 3, Serviceability Design Considerations for Steel Buildings

AISC publishes design guides in addition to the Steel Construction Manual. Design Guide 3, which contains information relevant to ponding loads, is now in its second edition. It provides a good summary of what is contained in the building codes and the AISC appendix 2, but does not, however expand on any of the ideas or add much to help a designer do a ponding check.

Steel Joist Institute (SJI) Standard Specifications, 42nd Ed.

This document provides a list of standardized steel joists and should be used by anyone specifying a joist, and any company producing standard steel joists. It also provides some requirements on design, fabrication, and erection of steel joists. In section 5.10 of these specifications, the SJI requires that a ponding investigation be performed by the specifying professional, but provides for no method of performing such an investigation.

SJI Technical Digest 3, Structural Design of Steel Joist Roofs to Resist Ponding Loads

This document provides more details on how to perform the investigation required by section 5.10 of the SJI standard specifications. It contains a summary of code related to ponding, and notes that it lacks in some areas, especially for atypical roofing systems. The digest suggests ways of expanding the AISC analysis to fit additional systems. It suggests that a good general procedure for a ponding analysis is that outlined in the AISC specifications, but that more detailed methods are available. This digest presents methods for doing a ponding analysis for members with both flexible and rigid supports.

For roof systems with flexible supports, it is suggested that the AISC method be used, but a special equation is provided for the calculation of F_0, the initial extreme fiber stress. For systems with rigid supports, the digest recommends two checks, one for the capacity of the joist, and one for the capacity of the support, as the bearing seats of the joists are also limited in their capacity. The method starts by calculating the three following values:

$$C_s = \frac{32L^4}{10^7 I_E} \tag{3.1}$$

$$\Delta_C = 0.00042L^2 + 0.0625L \tag{3.2}$$

$$w = w_D + \left(w_R \ or \ w_S\right) \tag{3.3}$$

Where L is the length, S the spacing, I_e the effective moment of inertia, w_d the dead load, w_r the impounded water load, and w_s the snow load. By using these values and estimating a height of water, h, above the supports, the centerline deflection can be calculated:

$$\Delta = \frac{C_S}{1-C_S}\left(0.244w + 1.27h - \Delta_C\right) \tag{3.4}$$

Using this value, both the end reaction and the final maximum load can be calculated:

$$R_1 = SL\left[0.375w + 1.95h + 1.24\left(\Delta - \Delta_C\right)\right] \tag{3.5}$$

$$w_1 = S\left[0.75w + 3.9h + 3.16\left(\Delta - \Delta_c\right)\right] \tag{3.6}$$

These values must then be checked to ensure safety. The distributed load must be less than the capacity of the joist, while the reaction must be less than one half the product of the distributed load and the length. If both of these requirements are met, then the joist is stable and strong enough to support the loads, including the ponding effect.

Other Sources

This seems to be a complete description of what building codes and design specifications require as far as ponding loads are considered. Other design specifications exist, but they do not have any requirements for ponding. The National Design Specification for Wood Construction, the Building Code Requirements and the Specifications for Masonry Structures, and the Building Code Requirements for Structural Concrete require no ponding analysis or investigation. It should make sense that the only really useful general ponding investigation procedures appear in the Steel Construction Manual. Steel allows for long spans and slender elements, leading to flexible roof systems. For this reason, steel construction may be more vulnerable to ponding loads.

Conclusion

In general, design codes require that adequate slope and adequate drainage be provided. The IBC and ASCE 7-05 do not provide a method for investigation of ponding stability; this is published in the AISC Steel Construction Manual instead. An additional method is presented by the SJI in a technical digest, but does not replace or add to the method presented in AISC. For a structural engineer interested in ponding loads, the single section of code that must be known is appendix 2 of the AISC specifications. Both the simplified method and the improved method are good ways to ensure stability, and are taken straight from the literature.

The methods provided in the design specifications are somewhat limited. They work only for flat roofing systems with structural members of the same length, strength and stiffness, with identical adjacent framing plans and simply supported members. The design methodology provided by the AISC specifications could use expansion to make the method more universally applicable to a wider variety of roofs. The biggest problem is that the specifications treat roofs as either flat, or pitched, and assume that if a roof is pitched, then it is safe. Often, pitched roofs can still suffer from ponding loads, and should be investigated correspondingly.

It has also been suggested in the literature that serviceability limit requirements for the deflection to span ratio of roof and floor systems are not as helpful as they could be. It has been suggested that a good replacement to these requirements for roofs where ponding is an issue would be a limit on deflection per unit load (Haussler, 1962). This would be a simple, effective method to eliminate unstable systems from designs. The method would consist of a designer analyzing his roofing system with a live load of five pounds per square foot (approximately the weight of one inch of standing water. Then, if the resulting deflection is greater than an inch, it is clear that the system is dangerous and probably unstable.

NUMERICAL ANALYSIS

The ponding effect is a simple idea that can become complex rather quickly. There are several variables involved, and there are numerous variations on the problem. Because there are a variety of factors influencing this phenomenon, a solution for the deflected shape of a member under ponding loads cannot be easily and accurately found. A closed form solution to the problem would be long, tedious, and difficult, due to varied system properties, loads, and the iterative nature of the problem. Since it would be useful, however, to have a tool that could calculate the deflections under these loads, a computer program has been written to do just that. For the complete code, see the appendices.

Approach

In the simplest case, a prismatic beam, with walls built at the end will accumulate load as water collects first behind the walls, then into the deflected area. This case has been analyzed using a numerical model in MATLAB. For simplicity, linear elastic behavior is assumed, and the program is set up only to analyze beams that are flat, or slope up to the right. The program is set up only to analyze beams that are simply supported and bending only due to bending moments induced by the water loads. Additional assumptions are that the ponding fluid is the only load, the beam is initially perfectly straight, and that the water will always rise to the specified height. Shear deformation contributions to the deflection are ignored.

Even simplifying the problem to a simply supported single linear elastic prismatic beam can get complicated. Identifying the appropriate design approach to this problem took some careful consideration. Problems arise with a simply supported beam for several reasons. First, if a simply supported, sloped beam is loaded with water, then the question arises: are the walls connected to the beam, or are the walls independent of the supports? Each setup presents difficulties and complications. In the case of independent supports:

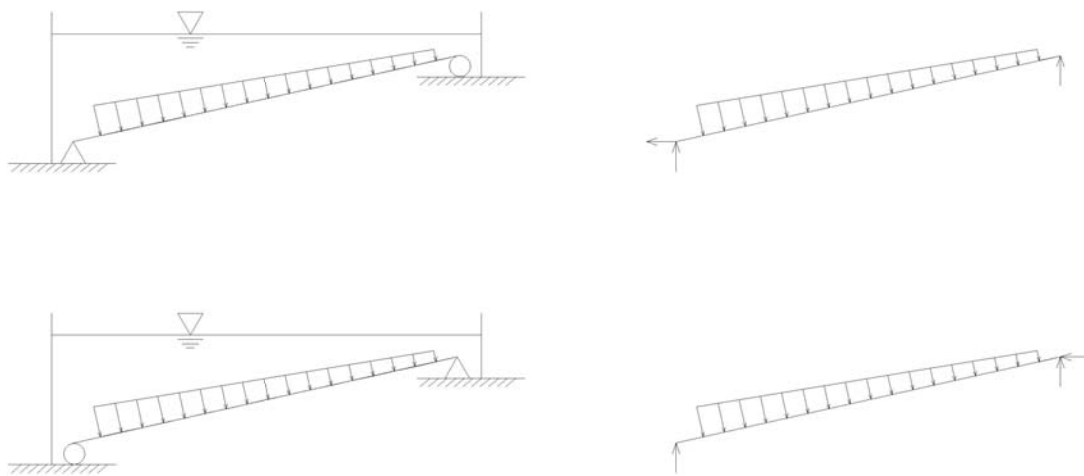

Figure 21: Supports independent of walls

Here it can be seen that if the pinned connection is made at the low end, then the beam will experience tension, and the roller support will be pushed outward. If the pinned connection is made at the top, however, the opposite occurs: the beam experiences compression and the roller support will tend to move inward. These forces will induce second order effects and induce additional bending moments in the beam. These may be small, and would be neglected, but should be noted. In the case of walls attached to the beam:

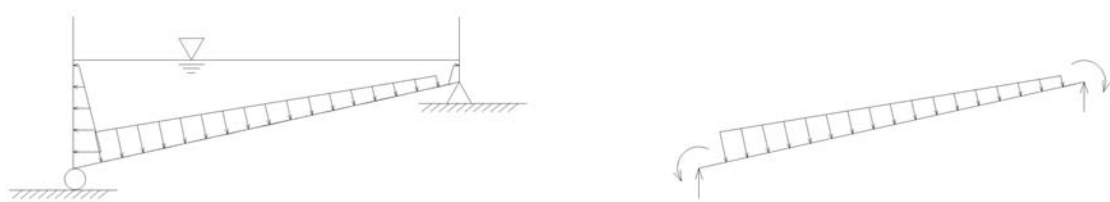

Figure 22: Walls attached to beam

Here it can be seen that the problem of tension/compression in the beam has been eliminated, as the setup has changed, and the horizontal reactions are eliminated as they balance within the tank itself. It is now essentially a solid tank with simple supports. A complication arises here, as the walls will experience pressure themselves. If the beam is isolated, then at the supports, where the walls meet the beam, bending moments will be induced from the walls. As before, this effect is small, and would be neglected in an analysis, but should be noted.

Thus far, all of the designs have appropriately noted that the water pressure acts perpendicular to the surface it rests on, and increases linearly as a function of depth. This is correct, but creates serious complications as the beam deflects. After the first iteration of an analysis, the beam will be deflected:

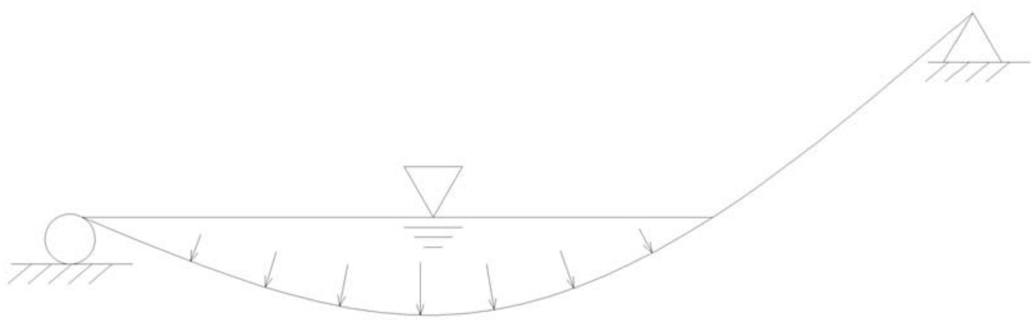

Figure 23: Water pressure on deflected beam

The problem here is that the orientation of the pressure, acting perpendicular to the surface at all points, is difficult to determine. The direction of the resulting forces on the beam will be hard to find, and will change with position along the beam and with each iteration. This effect may be small, depending on the total deflection of the beam, but should be noted.

There are different setups for this design, and there are complications to the analysis. Due to these issues, a simplified case will be chosen for the analysis:

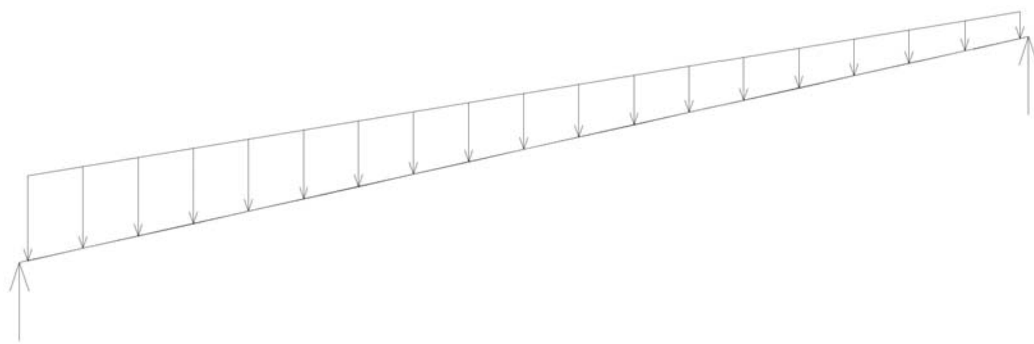

Figure 24: Assumed conditions for numerical analysis

This setup ignores the effects of induced tension/compression on the bending moments in the beam, and ignores the possible end moments from the walls. This design also neglects to consider the water pressure as perpendicular to the surface, instead taking it as a vertical load at all times. This is justifiable by a small angle approximation, as the roofs to be analyzed here are generally on a very shallow pitch. The vertical component of the water pressure should be ρghcos(Θ), but as theta gets small, the vertical component is the same as the perpendicular component: ρgh. This is further illustrated here:

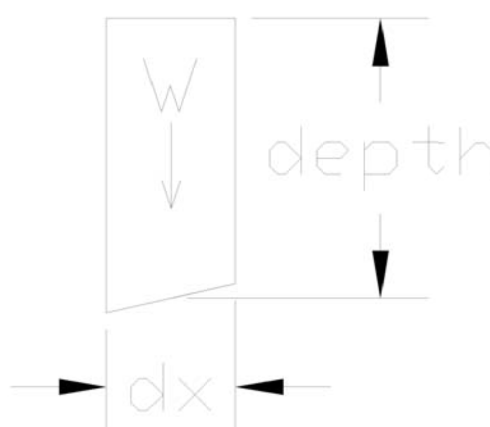

Figure 25: Numerical analysis

The beam will be divided up into very small slices, and each will be loaded with a vertical weight equal to the product of ρgh, the division width, dx, and the beam spacing. Based on this setup, all important internal loads, deflections, and everything else can be calculated.

Method

This program calculates bending moments, rotations, and deflections iteratively using a double numerical integration. The core of the problem is fairly simple. After the input has been collected, the program first finds some of the basic, important values, including the necessary geometric properties and variables, end depths, and more. The program then divides up the beam into very small pieces and calculates the load on each. Based on the locations of these slices and the loads on them, the program can calculate the moment at each slice. Once the moments are known, the program numerically integrates these, then determines and subtracts out the constant of integration. This determines the curvature at each point. The program then integrates again to calculate the deflection at each point. The deflected shape is now known, and the load for the next iteration can be calculated based on this. The program simply repeats this process for the specified number of iterations, and the whole process is repeated for each beam being analyzed.

Variables

There are seven variables that go into the ponding analysis. Most of these are geometric and define the layout of the setup. These include the length, angle, spacing, modulus of elasticity, and moment of inertia. The final two have to do with the load: the density of the ponding fluid and the initial height. The initial height represents the height of water that sits on the beam, and determines the initial load. It is assumed that the water level will remain constant throughout deflection at this height.

Variations

There are four analysis options built in to the program. The first is the basic ponding analysis. The computer takes input regarding the system properties and loads for any number of beams, does the ponding calculations, and outputs the results (numerically) and the deflected shapes (graphically) for each iteration. This analysis will usually make it clear whether the system is stable or not, but will not indicate whether the system will fail under the given loads. The second option in the program checks the strength. By asking the user for a value for the strength of the beam, and comparing this to the moments induced by the loads, the program will determine whether the beam will fail or not.

A third and very useful option has also been built in. It allows the user to input all but one of the variables for a ponding setup, and then determines the value of the last variable that will put the system exactly at the point of instability. This option has been expanded into a fourth, in which the program will repeat the derivation of the critical value as for as many (closely related) setups as desired. This saves a lot of time, and was used to find the output that led to the conclusions of this section. In these analyses, it is important to note that an initial guess at the correct value for the variable is required. Based on the method of analysis, a final result that is more than twice this value will never be found. Results must be checked to ensure that the results are reasonable and that they are less than twice the initial guess.

Excel

This program was written in its first form in an Excel spreadsheet. It is simpler to do the work required to get the program to work in Excel first for a couple reasons. First, Excel provides a simpler, more familiar interface to work in. There was no learning curve as far as syntax. Graphical outputs are built in and simpler than those in MATLAB. Also, it was easier to see the values of the variables in the Excel cells as the work was being done. This allowed debugging, fixes and updates to be much simpler and easier to do.

This Excel spreadsheet does what the core of the MATLAB program does on a basic level, but is very limited in its application. It is limited to ten beams, ten iterations and one hundred divisions of the beams. It is also limited in that it does not do a strength check or find points of stability. All of the variables are input on the first sheet, and each of the next ten sheets is the analysis of one of the beams. The maximum deflections for each iteration and beam are returned to the first page, where they are plotted in a graph. Exploring this spreadsheet provides a good way to understand how the MATLAB program works without having to understand MATLAB code. To see this spreadsheet, see the appendices.

Errors and Accuracy

It should be noted that as the number of iterations increases in a stable system, the successive moments, curvatures, and deflections go to zero. If these values get too small, the computer may round them to zero, leading to division by zero errors. In the MATLAB program, this has been accounted for and should not cause errors, but anyone using the Excel file should be aware of this.

The results from the MATLAB program have been checked against results from the Excel spreadsheet, and hand calculations. The results of the MATLAB program and Excel are identical when MATLAB is using 10 iterations and 100 divisions as Excel does.

Hand calculations can only be done for the first iteration, as successive iterations get complicated for analysis by hand. Also, for the pitched case, the only setup analyzed by hand for checking accuracy was the first iteration of the case where water fills exactly to the high support, as this provides a simple load pattern. For the first iteration of the flat case, when the programs are compared to the hand calculations, the results are very close. It turns out that the error is independent of all but one of the variables (the pitch is the only one that matters here) of the system, and mostly depends on the number of divisions used in the analysis. The results of the analysis of the accuracy provided these results:

Divisions:	100	99	75	50	35	25	20	10	2
% Difference:	0.016	0.00408	0.00711	0.064	0.033	0.064	0.4	1.6	40

Table 3: Errors in Numerical Analysis

Where the divisions are all fairly small numbers, and the percent difference represents the percentage difference between the hand calculated values for maximum displacement from equations as provided in the AISC steel manual, and the value for the maximum displacements from the MATLAB and Excel program.

Based on these numbers, it can be seen that errors are very small for a surprisingly small number of divisions. Also, it can be noted that the error gets smaller at a rate proportional to the square of the rate of the increase in divisions. This can be shown by comparing the errors at 100 and 10 divisions. Another important thing to notice is that an analysis with an odd number of divisions is much more accurate than an analysis with an even number. In fact, if an odd number is used, only half as many divisions are needed. This can be seen by comparing the accuracies of the analyses with 50 and 25 divisions.

Finally, by checking analyses of sloped beams, it was found that this program is slightly less, but still very accurate for sloped beams. Analyses of sloped beams show that the errors seem to be about 3.5 times larger than the corresponding flat case, regardless of slope. Because the MATLAB program allows large numbers of divisions with little problem, this error is insignificant. In some cases, analyses were run with more than 10,000 divisions, which would put the worst case error at less than 1 part in ten million.

Results

The program outputs results both in numerical and graphical data. For the results discussed here, the numerical results are the more useful, but the graphical data often gives a better understanding of the behavior. Shown here are three plots from MATLAB.

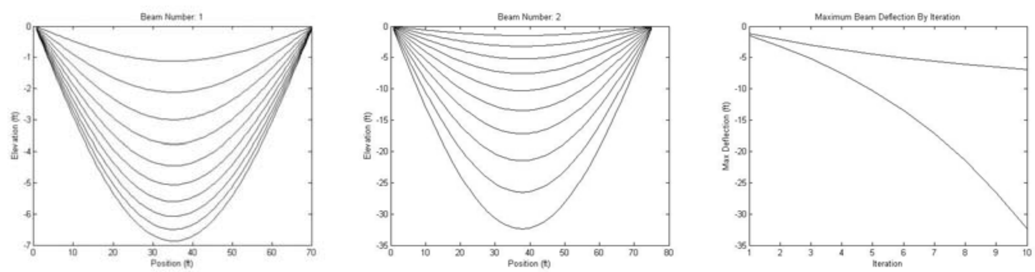

Figure 26: Beams analyzed by numerical analysis

The first plot shown here represents a stable system. The successive deflections get bigger, but the rate of deflection increase decreases. This beam is approaching stability and will not deflect indefinitely. The second plot represents an unstable system. Successive deflections get increasingly larger, and will continue to infinity. The third image represents a simplified version of both the other plots. The top curve represents the stable system, as the total deflection approaches a fixed value. The bottom curve represents the unstable system, as the deflection becomes unbounded.

The results of the program, aside from it being a useful tool itself, are the results of the determination of the critical values of variables that put the system exactly at stability. Based on the results, it has been shown that this numerical analysis and the program created check with the stability criteria in the literature, first presented by Haussler in 1962. The full results are presented in the appendices, but a discussion is presented here.

Using the program option to find a series of critical points, sets of ten such points were found. One variable, the density, was isolated and varied between 100 and 10 pounds per cubic foot. Based on the critical values, results for the critical values of each other variable were found in turn. Based on these, it was confirmed that density is proportional to the moment of inertia and modulus of elasticity, inversely proportional to the spacing and inversely proportional to the fourth power of the length. These proportionalities all reflected the equation published by Haussler (Haussler, 1962).

Both flat and pitched roofs were checked for the effect of the initial depth of water on the ponding stability and it was found that the critical point was not dependant on the initial load at all in either case. For the sloped case, it was found that the relationships between the variables remain unchanged, with the exception of length. The density is still proportional to the modulus of elasticity and the moment of inertia, and inversely proportional to the spacing. The relationship between these variables, the length and the pitch angle is not known quite as well for a non-flat beam. The two variables in question, length and angle, were checked against each other to determine a relationship at critical value for stability. The results look like this:

For 110 pcf density, 6’ spacing, 2’ depth, 500 in^4 moment of inertia, and 29000 ksi modulus of elasticity:

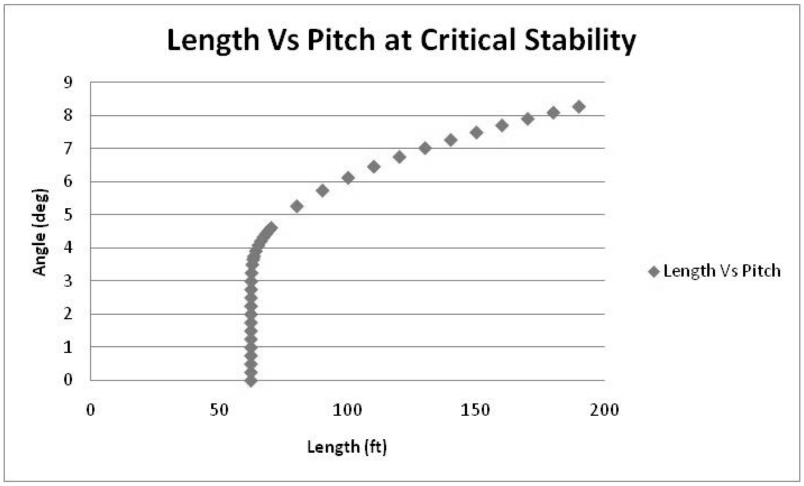

Figure 27: MATLAB output

This typifies the plots of several different sets of the variables. They all have a vertical asymptote that crosses the length axis at the critical length for a flat beam with the other variables constant. It curves up quickly at first, but the growth of the angle with length slows down. This plot does not hit an upper bound, but simply continues up increasingly slowly as the length increases.

This plot illustrates an important fact of the ponding problem. Putting a roof on a pitch will always make it more stable, but it is interesting to note that, in this case, a pitch up to four degrees really doesn't make the allowable length much longer. The real benefits come as the pitch rises above five or six degrees. Building codes and design specifications only require ponding analysis for roofs that have a pitch below ¼ on 12 or about 1.2 degrees. This requirement is based on ideology completely separate from this analysis. The basis of that angle is that for a pitched roof, the water can simply run off. The analysis presented here, however, assumes it cannot. It is more conservative to assume that the water will be blocked at the low end of a roof, and use the analysis presented here.

Conclusions

Based on this analysis, there are seven variables that determine the deflection of a beam under ponding loads, and six of these determine the stability criteria. The result presented by Haussler is verified for the flat case. The sloped case, however, is more complicated. The relationship of the variables length and angle to the other variables is unknown. In two different papers, the authors approach the problem of the sloped roof under ponding loads (Chang and Chong, 1977) (Blaauwendraad, 2007). While they both show that ultimate deflection depends on initial load, neither determines whether stability depends on initial load or not. Also, neither paper presents a simple method for calculating a stability factor. This analysis has shown that the initial load does not affect the stability, and has shown some insight into how the angle affects stability.

It would be good for building codes and design specifications to include the pitch of a roof in the criteria beyond simply to provide a method of drainage. It would be safer to assume that the water draining will be blocked at least to the height of the secondary drainage system, and that this could initiate ponding. Requiring a ponding analysis for sloped roofs to a higher pitch would do a much better job of ensuring safe roof systems.

Future Work

This program works for the conditions, assumptions and design setup used. This does not mean, however, that there are not other things that this program could be expanded to do. A short list of possibly useful additions is provided here. An updated program could:

- Analyze deflections of a joist instead of a beam
- Check serviceability requirements (L/240, etc)
- Allow for variations in loads (Point loads, two sets of loads, uplift etc.)
- Allow for initially cambered beams
- Check either the second order effect or end moments as described above

- Allow for variable elasticity in the beam, as the beam will go inelastic as it approaches failure
- Account for two way systems
- Allow varied support conditions
- Account for shear deformation contributions to deflection
- Be compiled and distributed for use by anyone interested
- Calculate a rate of convergence for stable systems to determine a safety factor

Some of these would be much more useful than others, but all of them are feasible. The one other item that needs more work in the future is the determination of a general equation for the stability condition in the pitched case. Results from the program have been presented for this, but no equation has been found. This would be useful to designers and possibly design code.

EXPERIMENTAL DESIGN

Goals of the Research

While there is a lot of knowledge in the literature regarding ponding loads, not much of it makes its way into the design code. The ponding checks given in the code are brief when compared to the numbers of observed failures and the costs over the past twenty years. Hopefully, through increased research and a better understanding of this loading, roofs will be made safer.

There are several goals for this research. First, it is hoped that it will help create a better understanding of the ponding phenomenon, including how, and in what shape a roof deflects and how the loads are carried. Hopefully, some results will help determine how the roof structure fails, and what type of failure causes the final collapse. Finally, it is hoped that better design and construction methods may be presented for consideration. A better understanding of ponding can reduce the number of failures, reduce costs, and prevent deaths by improving roof designs.

General Design

Materials

The first question when considering the design of this test is: what materials will be used in the roofing system? It seems as though the best option of all roofing systems available is the steel decking on open web steel joist system. There are several reasons for choosing this as the structural system. First, the test is of flexible roof systems. The most flexible systems are made of these materials. Steel itself is very ductile compared to other structural materials. Open web joists made of steel are themselves designed to be long span, slender, efficient elements, which makes them relatively flexible.

Open web steel joists are a great choice of material because of their inherent flexibility, but also because of their popularity. This type of roof is widely used throughout the country. Any large warehouse, office building, or commercial building has a good chance of being made with steel joists and steel decking, especially if the roofs on these structures are relatively flat. Together, these two factors make this the best type of roof to test. It is flexible, and commonly used in practice.

Available Facilities

The long wave channel at the O.H. Hinsdale Wave Research Laboratory at Oregon State University provides an effective setting to test a section of a roof structure under ponding loads. The channel is 15 ft deep, 12 ft wide, and effectively infinitely long. It provides 1 in. holes on grids spaced 12 in. tall and 8 in. wide to bolt supports into the walls at locations spaced at 12 ft along the wall. These will enable support for the test roof system.

These facilities require a few things of the experimental design. First, the length of the span must be a multiple of 12 ft. Second, it must rest on supports that can be bolted into the walls at appropriate locations. Also, the design must be exactly twelve feet wide, as the walls will provide the barrier to hold the water on the roof.

Supports

Open web steel joists are typically modeled as simply supported, but with the addition of a bottom flange extension, they can easily be constructed with fixed ends that resist moments, and this is done in practice. For this experiment, the supports will be pin and roller connections with no moment resistance. This is how joists are typically designed and used, and it simplifies the setup. All of the ponding literature assumes simply supported members except those specifically investigating alternate support conditions. This is also how the supports were modeled in the MATLAB program. These supports will make the structure determinate, which further simplifies the analysis.

Design Background

Open Web Steel Joist Design

Open Web Steel Joists (OWSJ) are proprietary products that are designed and manufactured according to industry standards established by the Steel Joist Institute (SJI). The Institute provides load tables that specify designations and strengths to a variety of joists of a variety of sizes. Any designer can chose steel joists as framing elements, and by finding the required strength, choose a joist from the SJI Standard Specifications. Joists come in several forms. The K-Series joists are the typical framing members that support steel decking. A more specific category of K-Series joist is the KCS joist, which is designed for a wider variety of load cases. Longer spanned, stronger joists are available in the LH and DLH categories. Joist Girders can also be selected, which are designed to be the framing elements that support the joists. These girders are designed to carry several point loads along their length. The K-Series joists are the most typical framing members, and come in designations such as 30K7, indicating a joist that is 30 in. deep and stronger than a 30K6 but weaker than a 30K8. The second number is called the section number, but indicates nothing other than relative size within a family of joist depths.

When these joists are purchased by the contractor for construction, the manufacturer can build the joist in a variety of ways, provided it meets the required strength and a few other requirements from the specifications. In general, joists are fabricated from angle sections and solid round bars. The flanges are generally paired angles, while the web is made from the solid round bars. At the ends, the angle brackets are generally replaced by channel sections to provide a strong bearing seat to resist the vertical end reaction. Joists are typically given a camber during fabrication based on an arc radius of about 3600 feet. This allows the joists to deflect under dead weights and still be flat (SJI and SDI, 2008).

Designing with joists is fairly straightforward. A roof design load is calculated as usual, and a framing plan is drawn. Based on the tributary width each joist will support, the load on each joist can be found in pounds per linear foot. Using the Standard Specifications, joists can be chosen. If a desired strength is not available, or if a joist will be supporting an unusual load pattern (not a uniform load), then custom joists can be ordered. This is done often in practice (SJI and SDI, 2008). There are other considerations to take into account. When a pitched roof is required, it is more economical to pitch the joists themselves rather than the chords. In this case, the span is taken as the diagonal length. Joists are generally not designed for uplift. If this is required, then special joists may have to be ordered. Finally, it is generally more economical for joists to span the longer dimension of a bay, while the joist girders span the short dimension.

Joists are slender and require significant lateral bracing in the form of bridging. Generally, joists are connected to each of the adjacent joists to ensure stability. Bridging requirements are outlined in the Standard Specifications. Based on the length and the section number, the number of rows of bridging can be determined. Bridging is typically horizontal, but based on location of the selected joist in the load table, it may be required to make one set of bridging diagonal. The design of these members is outlined in the Standard Specifications.

By looking at existing structures, it is easy to get an idea of what typical joist framing looks like. Generally, there is decking, supported by joists, supported by joist girders, supported by columns.

Warehouses in the Portland area were investigated to see what range of values is typical. All numbers here are approximate, as exact measurements could not be taken.

- The Fry's Electronics store in Wilsonville has 60 ft joists spaced at 5½ ft.
- The IKEA store in Portland has a complex system of joists and joist girders. There are at least two sets of joists, one being 60 ft joists spaced at about 8 ft, the second being 36 ft joists spaced at 7½ ft. These are all supported on joist girders about 110 ft long, spaced at 77 ft. These are supported on steel columns.
- The Best Buy store in Tualatin has joists that span 50 ft and are spaced at 7 ft.
- The Costco location in Tigard has joists that span 38 ft and are spaced at 4.5'

From this, it is clear that all structures are different, even when made from the same framing system.

Steel Decking

Designing with steel decking is straight forward. If the roof pressure and span conditions are known, then steel decking can be designed. The Steel Deck Institute (SDI) publishes a Design Manual that includes load tables based on span, support conditions and type of decking. Generally, span conditions are decking continuous over three supports (joists), but the design manual allows for one, two or three supports per section of decking. Based on the number of supports and the space between them, four types of decking can be chosen: narrow rib, intermediate rib, wide rib and deep rib. By far the most commonly used is the wide rib decking, but the Design Manual provides load tables for each. These load tables are all unfactored strength, so based on the design methodology, they must be given a factor of safety.

Design Methodology

Ideally, a testing setup would be constructed that would perfectly model a real, practical roofing system. The main idea is to setup a single joist that could be considered typical, instrument it and test it. Based on the results, deflection characteristics, load paths, and the joist behavior could be isolated and analyzed. To set up a single typical joist, the setup must be symmetrical about the joist, to avoid seeing different effects based on uneven distribution of loads. This means that there must be an odd number of joists in the setup. In the available space of twelve feet, either three or five joists must be installed as part of the roofing system. Any more than this is unreasonable because the spacing becomes too small, and one joist is impractical for obvious reasons. With three joists, the spacing will be close to six feet, while with five, the spacing will be closer to three. Because six feet is more typical, and because testing a setup with three joists is less expensive, it seems the best option is to use three joists spaced at almost six feet.

A problem that arises with this or any similar setup is that each joist will see different loads, as the edge joists have only half the tributary area to support. This means that the center, typical joist will deflect more than the others, and that this will induce stresses in the decking and an imperfect deflected shape at each joist based on the differential deflections. In order to fix this and make the roof deflect as a unit, the only option would be to decrease the moment of inertia of the edge joists by one half, to compensate for them seeing only one half of the load. Based on the K-Series load tables in the SJI Standard Specifications, there are several pairs of joists that are the same length, where one has exactly half the moment of inertia the other does.

None of these pairs works for this design, however, because for each, the edge joist with half the moment of inertia also has less than half the strength as the center joist. This means that though the roof section would deflect as a unit, the typical joist (the one the test would be focusing on) would not be the one to fail. The only way to get around this problem would be to order custom joists, which simply may not be possible, as they are being donated. Because of this, the design will call for three identical joists to be used in the setup. There will be interference to the deflections due to the forces from the decking, but this should be workable. This is just one small source of error that may have to be quantified based on the data from the experiment and will be reported in the results.

Another important aspect of the design of the experiment is the support system. The joists should be attached to fairly rigid supports. This also helps the roof deflect as a unit. If the supports deflect independently, then the experiment has become a test of a three way system, which is much more complicated. Ideally, there would be almost no deflection of the supports. Also, the structural supporting system must be much stronger than the roof system itself to preclude failure of the support system from the ponding loads. To ensure there is a significant overstrength of the supports, an additional factor of safety of 2.0 was used for design.

Because supports for systems like these are typically simple supports, and because it simplifies the experiment, the end reactions will make this setup simply supported. This means that one end of the joist will be pinned, and the other will be a roller. As the joist deflects, the ends of the joists will be moving relative to one another. As the top flange is in compression, and because the horizontal length of the joist gets shorter as it deflects, the end of the joist on the roller will move in. Space should be allowed for this support to move, so that neither tension nor compression is induced in the joist.

An ideal ponding test would be to select and test joists that are just at the Criterion Ratio, to determine how well the theory predicts ponding stability in real systems. Unfortunately, the AISC specifications provide a factor of safety against this condition of four, which is relatively large. In order to test this condition outright, very flimsy joists would be required. Again, custom order joists would be required, and they would be purposely, unrealistically, understrength and flexible. This would defeat the purpose of attempting to test a typical roof system. The fact that none of the joists would be unstable against ponding loads in the facilities available demonstrates that joist designs are typically safe for ponding stability. This does not mean, however, that no steel joists will be unstable for ponding loads. Simply by increasing the spacing between joists too much, any joist could become unstable.

This also does not consider the fact that in most roof failures where ponding is a contributing factor, the ponding effect is added to an existing set of loads, and the roof fails due to overload, rather than lack of stability. A more appropriate test may be to examine this additional contribution and failure due to lack of strength. The test would use a typical joist, and load it to failure. As the theory demonstrates, joists in a ponding situation will accumulate additional load due to deflections and water accumulation. The total load is the sum of the uniform load and the contribution from the ponding effect. Instead of testing the stability criteria, the amplification of existing loads (water weight) by ponding will be examined. The deflected shape, load path, and behavior of the joist will also be analyzed. This experiment will investigate the ponding theory and the code provisions, and employs standard joists.

The proposed test will consist of several steps. First, a roofing system must be built that is realistic and possible to test in the facilities available. This system must be supported in the long wave flume at the Wave Research Laboratory, and a pool must be built on top of it to hold the ponding liquid (water). The system will be instrumented, and then loaded with water until it fails. The deflected shape and depth of water will be compared to those expected under normal distributed loads, and the ponding contribution to deflection and failure will be determined. This will allow a discussion of how important the ponding contribution is to the general problem of roof failure, and recommendations can be provided.

Design Calculations

Several aspects of this experiment must be ensured to be safe before the experiment will work. The roof structure must act like a roof, and as a unit. The supports must be strong enough to hold the setup. Calculations have been done to ensure the system will perform as anticipated. The explanation of these calculations follows. For details on the calculations and methodology, see the appendices for the actual hand calculations.

The facilities available limit the joist length to 12 ft increments. Both 48 ft and 60 ft joists are common in practice, and both have advantages for this experiment. In the case of the 48 ft joists, if a sloped test was desired (it may be, but for now, the design is done for the flat case. The design of a sloped experiment would be very similar, and the flat case design would require only minor changes to the supports), then for a ¼ on 12 slope, a 48 ft span would mean the joist would rise exactly one foot. This keeps the supports lined up with the holes in the existing concrete wall, which is convenient. The 60 ft span, however, allows the collection of more data and larger loads, which could be useful when analyzing the data. The 60 ft length was selected as the preferred alternative, and the design will proceed using this value.

Based on the strength of the roof of the new OSU energy center, roofs built in this area should be able to withstand about 46 psf. The experimental design will attempt to achieve this load level. Based on the Standard Specifications, the 60 ft joist that gets closest to this value at slightly less than 6 ft spacing is the strongest available, the 30K12. Based on this joist, all other structural elements in the setup can be sized. The steel deck that most closely matches this load is the SDI IR20 decking.

Based on the loads these joists will be transferring to supporting steel beams, moments can be calculated and the beam sized. It turns out that a very small beam would be satisfactory, even with a significant factor of safety. The design will require, however, that connections between elements be bolted. Based on this, it would be sensible to choose a beam with the largest practical flange. The lightest section with a 10 in. flange was chosen: W10x49. Because it is known that 50 ksi steel will be purchased, the moment capacity tables in the AISC specifications can be used to determine the strength of this section to check. Based on a conservative unbraced length of 12 ft, the strength is 238 kip-ft, which provides a factor of safety of 12.5 against bending.

Next, column sections are required to support this W10 section and connect it to the concrete walls of the long wave flume. Four 21 in. long W12x72 steel column sections are already available at the wave lab, and are practical for the application. To check this column for strength, the unbraced length was taken conservatively as two feet, with a k value of 1.0. Based on the column strength tables in AISC, this steel section provides a compression strength of 953k at 6 ft, which is as short as the tables go. This provides a significant overdesign for the column section.

Additionally, steel braces will be required for the bridging between joists, and to support the walls at the ends of the roof that will contain the water. These walls will be made out of double layered plywood, and will be supported against overturning by small steel angle sections. The loads were calculated for the end supports and found to be negligible for any steel section. The load required to be carried by the bridging elements is similarly small. Based on this, a small angle section (L2x2x¼) was chosen for these elements. An angle section was not chosen because it is particularly strong, but because it will be easier to work with in the field, when building the structure. It is important to note that bridging between joists is required in both directions, which will be difficult for the edge joists, as there are no adjacent joists to attach the bridging to. In these cases, a roller bearing will be bolted to the joists that will enable thrust against the concrete wall, preventing side sway in the whole structure. It is also important to note that based on the chosen joist, four sets of bracing are required for stability of the system, and based on the joist's location in the load tables, the set closest to the center will be cross bracing. To ensure the design meets the specified bracing requirements, a center set of bracing will be added.

Finally, four additional items were checked. The bolts that will support the column sections to the walls were checked for shear capacity. They were found to provide sufficient strength. Also, an estimate of the horizontal deflection of the end of the joist at the roller support was made. This is important to provide adequate space to accommodate the motion of the two points with respect to each other. Based on conservative estimates made by determining the axial force carried in the flange, this deflection should be a little more than one half inch, which can be accommodated. Also, the deflection of the steel beam supporting the joists was calculated, and found to be less than one twentieth of an inch, which is sufficiently small to ignore its influence on the ponding effects. Lastly, the strength of the beam was calculated manually to double check the values from the tables, and was found to be adequate.

Experimental Design

The general testing methodology has been laid out, and the structural elements of the test have been designed. All that remains is to determine how the experiment will be conducted. The application of the load and the instrumentation are both important considerations.

The load will be applied by adding water on top of the roof, which will be made waterproof. On top of the steel deck roofing, insulation such as Georgia Pacific DensDeck will be used, as in typical construction. Over this, a waterproofing membrane will be applied. This membrane will simply rest on the insulation, and wrap up around both the concrete and plywood walls on all four edges. This membrane should not be pulled tight over the roof, as deflections are expected, and slack will be needed where the roof moves relative to the concrete tank walls. This should provide a waterproof barrier throughout the experiment.

The experiment will require instruments to monitor the loads and the response of the specimen. First, a flow meter will be attached to the source that is providing the water to the roofing structure. This will probably provide the most accurate measure of the total volume of water on the system. Second, hooks will be screwed into the roofing at every panel point, so that deflection gages can be attached to the structure. They will be located at panel points to measure the deflection profile along the span. These screws will be sealed using a silicone caulking to ensure the roofing membrane remains watertight. The water height will also be measured.

There will also be load cells under each of the joist end support points. Each joist will be instrumented at both ends, and this data should allow the distribution of the loads to be determined. It is clear that the center joist will carry the largest load, but to what degree is unknown, and these load cells will identify the distribution. The last piece of the instrumentation is a set of strain gages. These will be applied to a variety of members in the steel joist of interest, and possibly to the bridging. These will provide the best method for determining how the loads are transferred through the joist, and where failures are likely to occur. These should be applied last, after all fabrication and especially welds, to prevent residual stress effects. As a flange member may be bending in two directions and carry an axial force, three strain gages will be required to measure all internal loads. Finally, cameras will be set up to videotape the entire test, and to take still images as needed. This may help in determining a mode of failure, and will help maintain a visual record of the event.

It is hoped that through the tests and the failures, all of the instrumentation will be reusable. It is possible that during failure, some of the load cells, for example, may be damaged. If this is the case, then the design may have to be modified for the next test. All together, five things will be measured in this setup. The height of water, deflections of the roof, the total load, the load at each support point, and the strains in many of the joist sections will all be known at the end of the test. With this data, joist behavior will be characterized, including the joist deflection, the strength of the joist, and load distribution among the members. It should also be possible to determine the mode of failure and the contribution of the ponding effect. Based on this data, conclusions and recommendations will be made.

Design Summary

There are three, 60 ft 30K12 joists spaced at 5.6 ft. These are supported on 12 ft W10x49 sections at each end, with web stiffeners under loading points. These W sections rest on ¾ in. plates that are supported on W12x87 sections which are bolted into the existing concrete wall with ¼ in. washer plates. There are 5 sets of cross bracing spaced approximately every 10 ft along the joists; the center is diagonally braced. There are plywood walls at both ends, which are supported by 2x2x1/4 steel L sections. On top of the joists is SDI IR20 steel decking, covered in ½ in. Georgia Pacific DensDeck, covered with a watertight liner.

Technical Drawings of the final experimental design have been completed in AutoCAD. Five of these drawings are included in the appendices. Included are a plan view, elevation view, and an end view of the whole design setup. Also included is a detail of the joist bracing, and a detail of the support structure that shows the column section bolted into the wall, the W section that spans across the tank, the joist end, the wall that is built up to hold the water in, and all the other details of the connections.

Conclusions

This test represents the first of its kind. This will be the first full-scale test of a realistic roof system under ponding loads. It will take advantage of the facilities available at the O.H. Hinsdale Wave Research Lab at Oregon State University as the location of the test. A roofing system made of steel decking supported on steel joists will be tested to failure, in order to determine the contribution of the ponding effect to the failure. The setup will be supported on a relatively strong and stiff steel frame, and will represent a real roof as closely as possible. It is hoped that through this test, more will be understood about the behavior of flexible roof systems and how they behave under ponding loads. Results may enable better design and construction methods to prevent such failures in the future.

Areas for Further Research

While this research is important and stands independently, there are several other related areas that could use further investigation. First, the design outlined here is for the flat case. The sloped case is also important, and research could be expanded into this area. Second, roof ponds are now being induced purposely by design. This can be seen in the trend towards green roofs that have flower beds and soils for plants. When rain falls on a roof like this, the soil will absorb the water and create much larger loads on the roof than in the past. This must be accounted for in design, and it would be interesting to see what magnitude loads result from a typical green roof. Roofs that support water have also been designed because the water acts as an insulator. It will cool the building on hot days and keep it warm through cold nights. It could be interesting to research such roofs and determine average water loads and their effect on the structures. Finally, it would be interesting to study the dynamic response of ponded roofs. How would a roof that has water ponded from a rainstorm or a green roof act in an earthquake? What would be the vertical and horizontal accelerations and how much damage would the sloshing effects cause? There are areas to expand the research of roof ponds.

CONCLUSIONS

Most building codes and design specifications provide minimal guidance for design and construction professionals on the effects of ponding loads. This does not reflect the fact that ponding related roof collapses are common, destructive, and dangerous, and that they have occurred on roofs made of all types of materials. They have occurred throughout the country and improvements are needed to minimize this failure condition.

The theory behind ponding loads, first established by Haussler in 1962, has been expanded to cover a variety of topics. The building codes incorporate simplified versions of ponding analysis for design use and require only that two independent sets of drains and a slope greater than ¼ on 12 be provided to preclude. Beyond this, they refer designers to the ponding equations provided in the AISC specifications. The methods presented there are simple and based on the established theory, but only for flat roofs. More research on ponding loads on nearly flat roofs is needed.

A numerical analysis has been done to investigate the effect of ponding loads on flexible roof systems. This analysis confirmed the published theory by Haussler. It was found, however, that the slope of a roof has an interesting effect. It was shown that real benefits to ponding stability from increasing the pitch of a roof do not come until the roof gets steeper than currently recognized by the code, for the common case where a parapet wall is used. More investigation is needed on this topic.

Despite the work that has been done, full-scale, realistic tests of roof systems have not been reported in the literature. A design for such a test was completed as part of this work. The design calls for loading open web steel joists under ponding loads to failure. It is hoped that through this test, the contribution of the ponding effect to the total load can be determined. The deflected shape, failure mode and load paths in the joists will be identified. This will enhance the understanding of the behavior of flexible roof systems under ponding loads and enable development of better design and construction methods to prevent ponding related collapses in the future.

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APPENDICES

Appendix A: Computer Code

Parts of this report rely on computers to do numerical analyses of beams under ponding loads. The full text of these programs would be included here in the appendices, but it would obviously be inefficient for anyone interested to retype these documents. For any reader interested, the programs (written for use in MATLAB) are available online at: http://oregonstate.edu/~starkdu/

To run this code, open MATLAB, and call the function “Main.” This function will prompt the user to do the rest. As a user, inputting data through the MATLAB interface may get tedious, so an option has been built in to use stock inputs. For quicker, easier and more flexible inputting, change variables in the “InputStock” or “InputStock2,” then tell the program not to use user input.

Appendix B: Results of Numerical Analysis

The following are the results of the MATLAB analysis presented in spreadsheets. Each set of data represents a group of beams at the critical ponding point. In each, one variable, highlighted darker is varied as the independent variable, and another, highlighted lighter, is the resulting dependant variable. Each set is labeled at the far left with two numbers. The first represents which group the results belong to. The first group of analyses is for a flat beam, and show that the relationships found by Haussler are true. The second group of analyses is for a beam at a pitch of five degrees. These analyses show that for a sloped beam, the relationships found by Haussler still apply, with the exception of the variable length. The third and final set of data shows the relationship between a varied angle and the critical length. This data is the basis for the results discussed in the numerical analysis section, and the plot shown there.

1	1	Density	100	90	80	70	60	50	40	30	20	10
1	1	Spacing	6	6	6	6	6	6	6	6	6	6
1	1	Pitch	0	0	0	0	0	0	0	0	0	0
1	1	Depth	2	2	2	2	2	2	2	2	2	2
1	1	Elasticity	29000	29000	29000	29000	29000	29000	29000	29000	29000	29000
1	1	Inertia	500	500	500	500	500	500	500	500	500	500
1	1	Length	63.5863	65.2834	67.2343	69.5167	72.248	75.6173	79.9555	85.9178	95.0837	113.0742
1	1											
1	1	$\gamma BL^4 / \pi^4 EI$	1.000001	1	1	1.000002	1.000002	1.000002	0.999999	1	1.000001	1.000001
1	1											
1	1	L^4	16347608	18163982	20434478	23353748	27246035	32695250	40868940	54491980	81738054	1.63E+08
1	1	DL^4	1.63E+09	1.63E+09	1.63E+09	1.63E+09	1.63E+09	1.63E+09	1.63E+09	1.63E+09	1.63E+09	1.63E+09
1	1	min	1.63E+09									
1	1	max	1.63E+09									
1	1	% error	0.0003									

1	2	Density	100	90	80	70	60	50	40	30	20	10
1	2	Spacing	6	6	6	6	6	6	6	6	6	6
1	2	Pitch	0	0	0	0	0	0	0	0	0	0
1	2	Depth	2	2	2	2	2	2	2	2	2	2
1	2	Elasticity	42593.06	38333.33	34074.31	29815.28	25555.56	21296.53	17036.81	12777.78	8518.75	4259.028
1	2	Inertia	500	500	500	500	500	500	500	500	500	500
1	2	Length	70	70	70	70	70	70	70	70	70	70
1	2											
1	2	$\gamma BL^4 / \pi^4 EI$	0.999994	1.000005	0.999999	0.99999	1.000005	0.999994	1.000019	1.000005	0.999978	1.00006
1	2											
1	2	D/E	0.002348	0.002348	0.002348	0.002348	0.002348	0.002348	0.002348	0.002348	0.002348	0.002348
1	2	min	0.002348									
1	2	max	0.002348									
1	2	% error	0.008152									

1	3	Density	100	90	80	70	60	50	40	30	20	10
1	3	Spacing	6	6	6	6	6	6	6	6	6	6
1	3	Pitch	0	0	0	0	0	0	0	0	0	0
1	3	Depth	2	2	2	2	2	2	2	2	2	2
1	3	Elasticity	29000	29000	29000	29000	29000	29000	29000	29000	29000	29000
1	3	Inertia	734.37	660.89	587.5	514.03	440.64	367.16	293.77	220.3	146.9	73.43
1	3	Length	70	70	70	70	70	70	70	70	70	70
1	3											
1	3	$\gamma BL^4 / \pi^4 EI$	0.999985	1.00005	0.999978	1.000041	0.999944	1.000053	0.99991	1.000035	0.999808	1.00008
1	3											
1	3	D/I	0.136171	0.13618	0.13617	0.136179	0.136166	0.13618	0.136161	0.136178	0.136147	0.136184
1	3	min	0.136147									
1	3	max	0.136184									
1	3	% error	0.027229									

1	4	Density	100	90	80	70	60	50	40	30	20	10
1	4	Spacing	4.0857	4.5398	5.1062	5.8362	6.8079	8.1702	10.2136	13.6169	20.426	40.8527
1	4	Pitch	0	0	0	0	0	0	0	0	0	0
1	4	Depth	2	2	2	2	2	2	2	2	2	2
1	4	Elasticity	29000	29000	29000	29000	29000	29000	29000	29000	29000	29000
1	4	Inertia	500	500	500	500	500	500	500	500	500	500
1	4	Length	70	70	70	70	70	70	70	70	70	70
1	4											
1	4	$\gamma BL^4 / \pi^4 EI$	1.000124	1.000153	0.999942	1.000035	0.999889	0.999977	1.00006	0.999969	1.000001	1.000018
1	4											
1	4	D*B	408.57	408.582	408.496	408.534	408.474	408.51	408.544	408.507	408.52	408.527
1	4	min	408.474									
1	4	max	408.582									
1	4	% error	0.026433									

2	1	Density	100	90	80	70	60	50	40	30	20	10
2	1	Spacing	6	6	6	6	6	6	6	6	6	6
2	1	Pitch	5	5	5	5	5	5	5	5	5	5
2	1	Depth	2	2	2	2	2	2	2	2	2	2
2	1	Elasticity	29000	29000	29000	29000	29000	29000	29000	29000	29000	29000
2	1	Inertia	500	500	500	500	500	500	500	500	500	500
2	1	Length	79.24	83.88	89.63	96.97	106.65	120.08	140.04	173.01	238.4	433.1
2	1											
2	1	$\gamma BL^4 / \pi^4 EI$	2.411705	2.725351	3.158274	3.786116	4.74833	6.359149	9.410542	16.44191	39.51857	215.2281
2	1											
2	1	L^4	39425560	49503247	64537715	88419811	1.29E+08	2.08E+08	3.85E+08	8.96E+08	3.23E+09	3.52E+10
2	1	DL^4	3.94E+09	4.46E+09	5.16E+09	6.19E+09	7.76E+09	1.04E+10	1.54E+10	2.69E+10	6.46E+10	3.52E+11
2	1	min	3.94E+09									
2	1	max	3.52E+11									
2	1	% error	98.87947									

2	2	Density	100	90	80	70	60	50	40	30	20	10
2	2	Spacing	6	6	6	6	6	6	6	6	6	6
2	2	Pitch	5	5	5	5	5	5	5	5	5	5
2	2	Depth	2	2	2	2	2	2	2	2	2	2
2	2	Elasticity	42593	38334	34074	29815	25556	21296	17037	12778	8519	4259
2	2	Inertia	500	500	500	500	500	500	500	500	500	500
2	2	Length	70	70	70	70	70	70	70	70	70	70
2	2											
2	2	$\gamma BL^4 / \pi^4 EI$	0.999996	0.999988	1.000008	0.999999	0.999988	1.000019	1.000008	0.999988	0.999949	1.000066
2	2											
2	2	D/E	0.002348	0.002348	0.002348	0.002348	0.002348	0.002348	0.002348	0.002348	0.002348	0.002348
2	2	min	0.002348									
2	2	max	0.002348									
2	2	% error	0.011738									

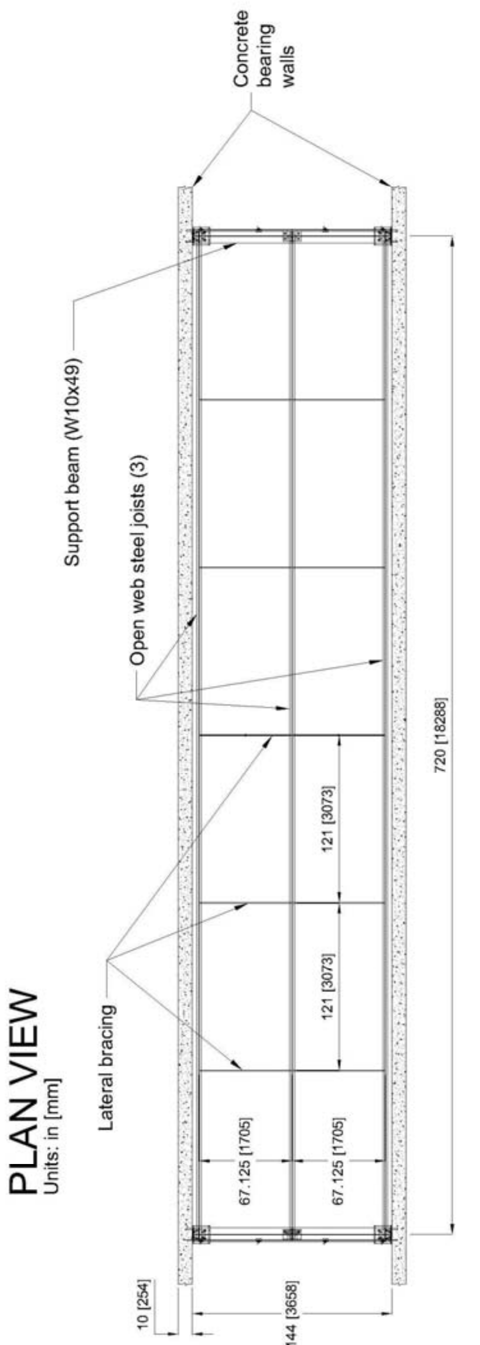
2	3	Density	100	90	80	70	60	50	40	30	20	10
2	3	Spacing	6	6	6	6	6	6	6	6	6	6
2	3	Pitch	5	5	5	5	5	5	5	5	5	5
2	3	Depth	2	2	2	2	2	2	2	2	2	2
2	3	Elasticity	29000	29000	29000	29000	29000	29000	29000	29000	29000	29000
2	3	Inertia	734.37	660.9	587.5	514.03	440.64	367.16	293.77	220.3	146.9	73.43
2	3	Length	70	70	70	70	70	70	70	70	70	70
2	3											
2	3	$\gamma BL^4 / \pi^4 EI$	0.999985	1.000035	0.999978	1.000041	0.999944	1.000053	0.99991	1.000035	0.999808	1.00008
2	3											
2	3	D/I	0.136171	0.136178	0.13617	0.136179	0.136166	0.13618	0.136161	0.136178	0.136147	0.136184
2	3	min	0.136147									
2	3	max	0.136184									
2	3	% error	0.027229									

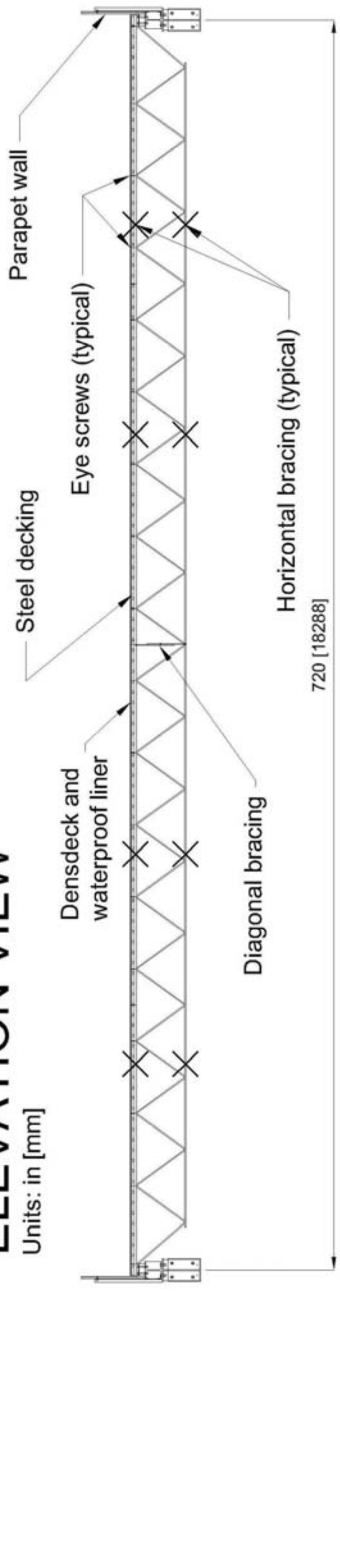
2	4	Density	100	90	80	70	60	50	40	30	20	10
2	4	Spacing	4.0851	4.5392	5.1068	5.8356	6.8097	8.172	10.2118	13.6176	20.4254	40.8527
2	4	Pitch	5	5	5	5	5	5	5	5	5	5
2	4	Depth	2	2	2	2	2	2	2	2	2	2
2	4	Elasticity	29000	29000	29000	29000	29000	29000	29000	29000	29000	29000
2	4	Inertia	500	500	500	500	500	500	500	500	500	500
2	4	Length	70	70	70	70	70	70	70	70	70	70
2	4											
2	4	$\gamma BL^4 / \pi^4 EI$	0.999977	1.000021	1.00006	0.999933	1.000153	1.000197	0.999884	1.000021	0.999972	1.000018
2	4											
2	4	D*B	408.51	408.528	408.544	408.492	408.582	408.6	408.472	408.528	408.508	408.527
2	4	min	408.472									
2	4	max	408.6									
2	4	% error	0.031326									

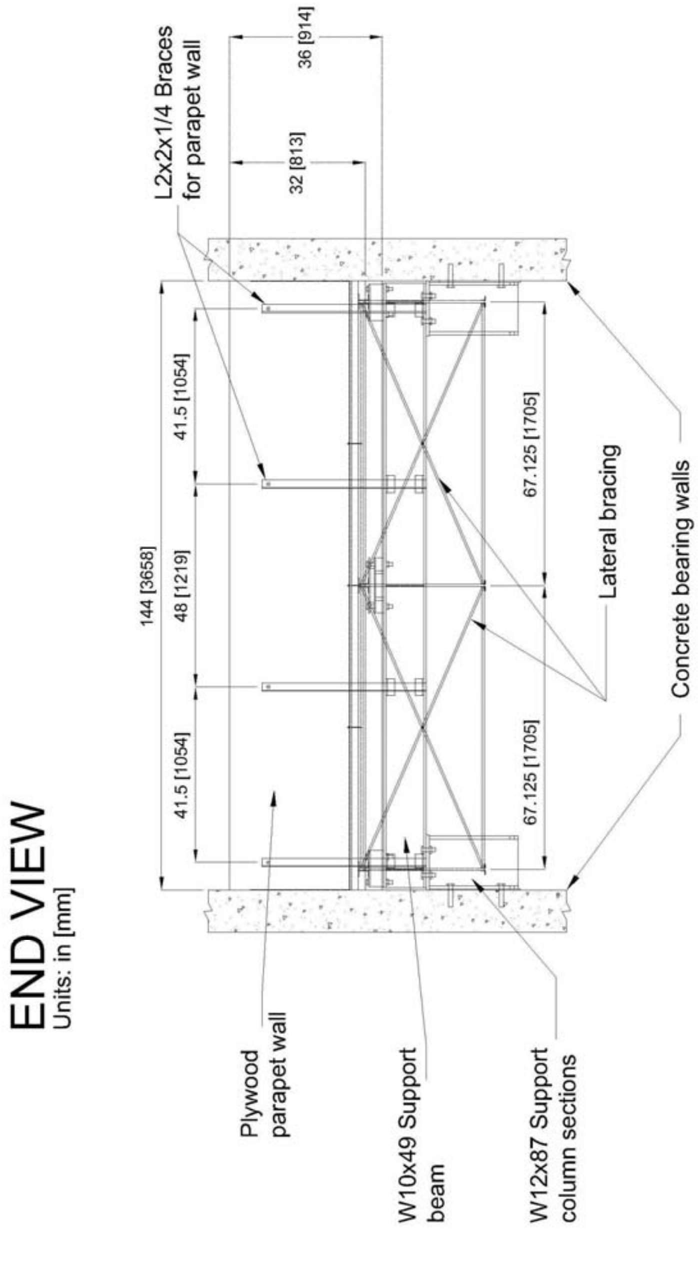
3	1	Density	110	110	110	110	110	110	110	110	110	110	110	110	110	110	110	110	110	110	110	110
3	1	Spacing	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
3	1	Pitch	3.6573	3.91	4.0792	4.2144	4.3302	4.4331	4.5264	4.6127	5.266	5.7392	6.126	6.4588	6.7542	7.0211	7.2659	7.4923	7.7047	7.9037	8.0918	8.2711
3	1	Depth	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	1	Elasticity	29000	29000	29000	29000	29000	29000	29000	29000	29000	29000	29000	29000	29000	29000	29000	29000	29000	29000	29000	29000
3	1	Inertia	500	500	500	500	500	500	500	500	500	500	500	500	500	500	500	500	500	500	500	500
3	1	Length	63	64	65	66	67	68	69	70	80	90	100	110	120	130	140	150	160	170	180	190
3	1																					
3	1	$\gamma BL^4 / \pi^4 EI$	1.059988	1.128909	1.201136	1.276776	1.355933	1.438714	1.52523	1.61559	2.756125	4.414779	6.72882	9.851666	13.95288	19.21818	25.84944	34.06465	44.098	56.19978	70.63646	87.69066

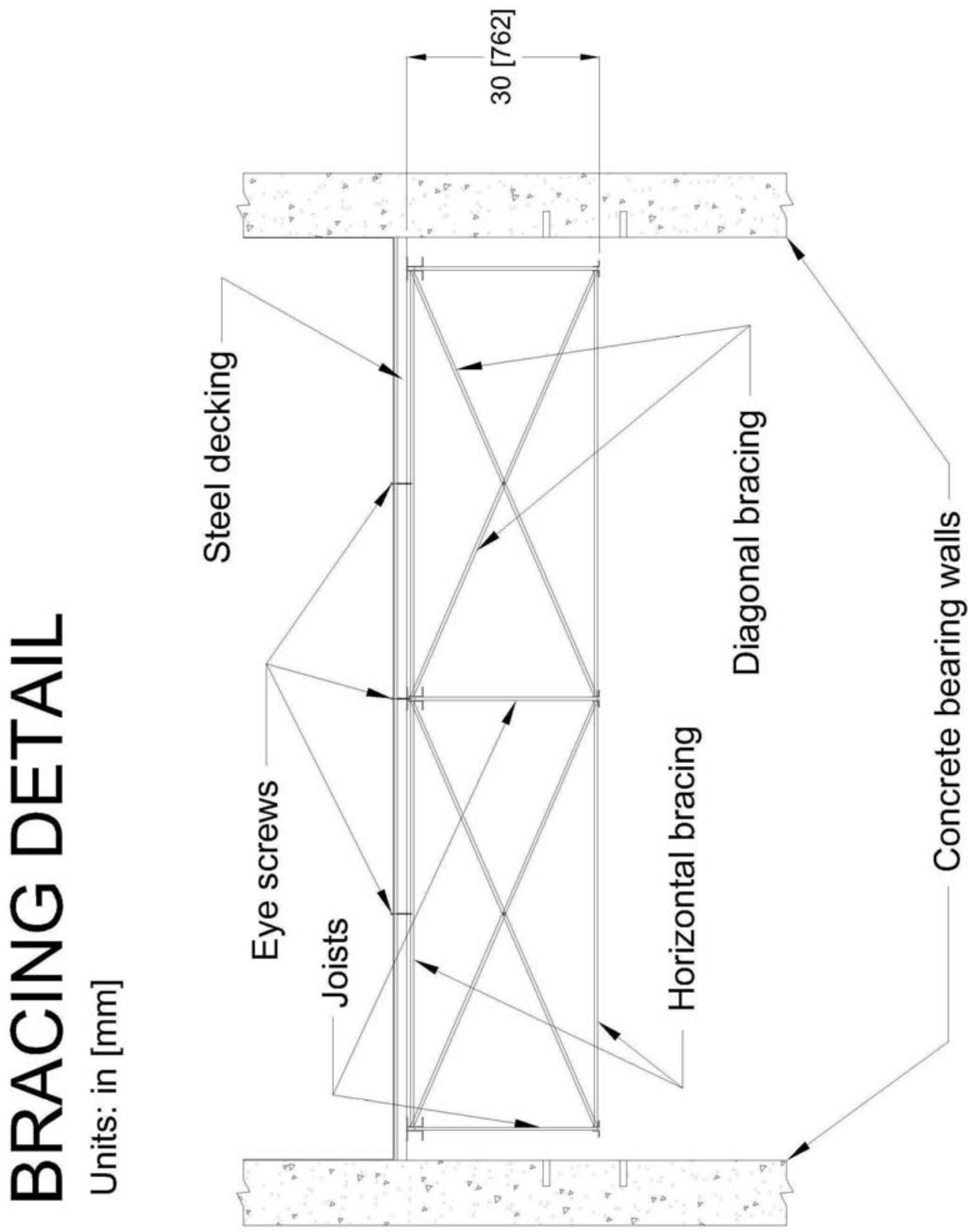
Appendix C: Design Drawings

These drawings are a visual representation of the experimental design. For a better explanation of this, see the experimental design section of the report.

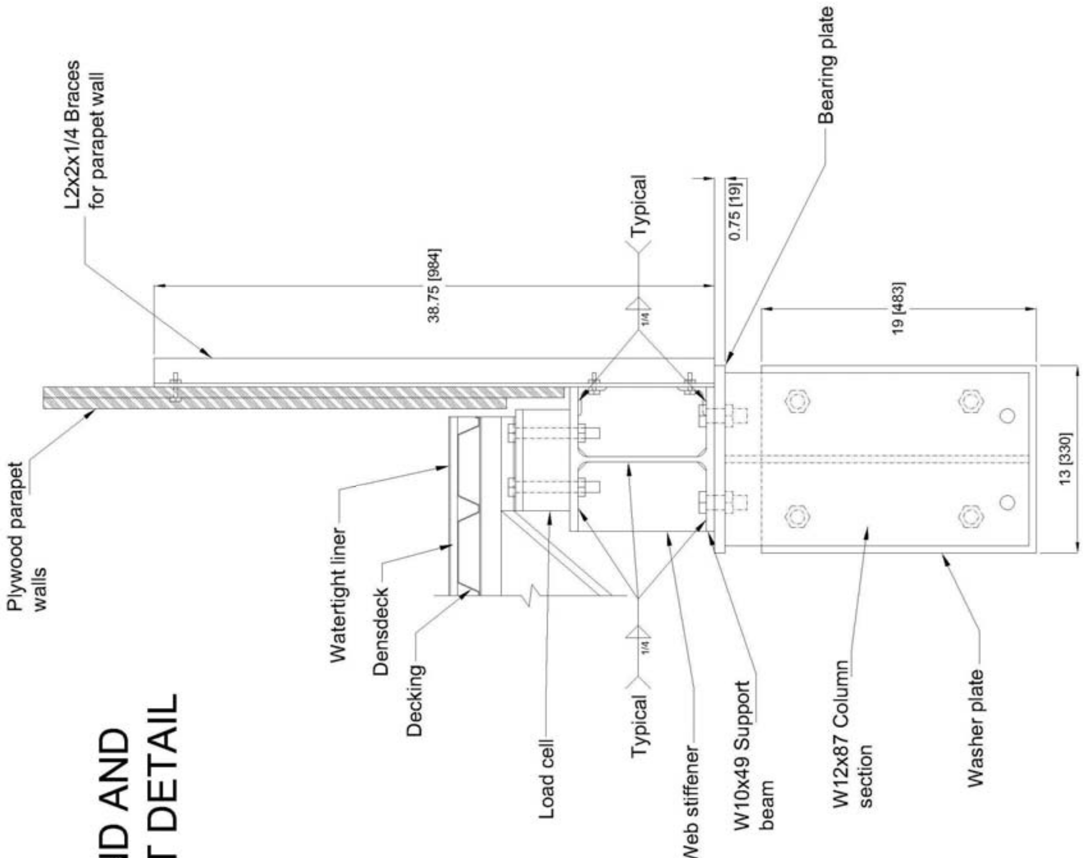

PLAN VIEW
Units: in [mm]
Lateral bracing
Open web steel joists (3)
Support beam (W10x49)
Concrete bearing walls
67.125 [1705]
67.125 [1705]
121 [3073]
121 [3073]
720 [18288]
10 [254]
144 [3658]


ELEVATION VIEW
Units: in [mm]
Parapet wall
Eye screws (typical)
Steel decking
Densdeck and waterproof liner
Horizontal bracing (typical)
Diagonal bracing
720 [18288]


END VIEW
Units: in [mm]
L2x2x1/4 Braces
for parapet wall
32 [813]
36 [914]
144 [3658]
41.5 [1054]
48 [1219]
41.5 [1054]
67.125 [1705]
67.125 [1705]
Lateral bracing
Concrete bearing walls
Plywood
parapet wall
W10x49 Support
beam
W12x87 Support
column sections



BRACING DETAIL
Units: in [mm]
30 [762]
Steel decking
Eye screws
Joists
Diagonal bracing
Horizontal bracing
Concrete bearing walls



JOIST END AND
PARAPET DETAIL
Units: in [mm]

Appendix D: Design Calculations

The calculations contained here are those required to design the setup for the experiment. For a better explanation, see the Design Calculations section of the report.

Choose Joists

- use K-series
- use ASD section of SJI

energy center uses 16K9 at 7.5' c-c for 30' span

we will go 60' span

- max K series span
- fits facility
- allows largest loads (total)

- we need ≅ 6' c-c
  - to fit facility

30' 16K9: 355 plf total strength
10 plf self weight

355 − 10 = 345 plf load allowed

at 7.5' c-c, roof can support $\frac{345 \text{ plf}}{7.5 \text{ ft}} = 46 \text{ psf}$

we will choose something similar: with joists at 5.6' c-c:

$5.6' \cdot 46 \text{ psf} = 258 \text{ plf}$

Choose closest 60' k series joist (also happens to be the strongest): $\boxed{60' \ 30K12 \ \text{at} \ 6' \ c\text{-}c}$

Strength = 262 plf
Self weight = 17.6 plf

allowable load = $(262 - 17.6) \text{ plf} = \boxed{244.4 \text{ plf}}$

this roof will support: $\frac{244.4 \text{ plf}}{5.6'} = \boxed{43.6 \text{ psf}}$

estimate height of water:

From web specs, GP 1/2" DensDeck ≅ 2 psf

water allowed = 41.6 psf

height = $\frac{41.6 \text{ psf}}{62.4 \text{ pcf}} = 2/3' = 8"$ FOS = 2 → 16"

Choose Steel Deck

- use SDI Tables
- conservatively take our C-C distance (Deck span)

  Span = 6'

- Span condition = 2 spans

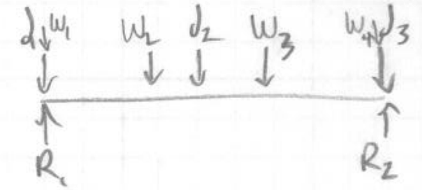
Use Deck Type 1R20

- Intermediate Rib deck, 20 gage steel

49 psf strength

→ Joists should control + break first

Choose Steel I beam Section



J = load from joist

W = load from wood wall and L section (used L2×2×1/4)

other small dead loads are neglected

tributary areas from AutoCAD:

$A_{J1} = A_{J3} = 3.3' \cdot 24' = 79.2\ ft^2$

$A_{J2} = 5.6' \cdot 24' = 134.4\ ft^2$

$J_1 = J_3 = 79.2\ ft^2 \cdot 46.1\ psf = 3652\ lb$

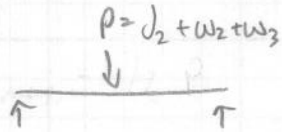
$J_2 = 134.4\ ft^2 \cdot 46.1\ psf = 6196\ lb$

$W_1 = W_4 = 2.3' \cdot 3' \cdot \frac{1.5}{12}' \cdot 35\ lb/ft^3 = 40.0\ lb$
$+ 3.19\ lb/ft \cdot 3ft$

$W_2 = W_3 = 3.8' \cdot 3' \cdot \frac{1.5}{12}' \cdot 35\ pcf + 3.19\ plf \cdot 3ft = 60.0\ lb$

$R_1 = R_2 = \frac{1}{2}(\Sigma J + \Sigma W) = \frac{1}{2}(13700) = 6850\ lb = 6.9K$

assume worst case, J_2, W_2, W_3 all applied at center. Span = 12'



$V_{max} = \frac{1}{2}P = \frac{1}{2}(6316) = 3158\ lb = 3.2K$

$M_{max} = \frac{PL}{4} = \frac{1}{4}(12' \cdot 6316) = 18948\ lb\text{-}ft = 19.0\ k\text{-}ft$

I want a 10" flange for constructability
- try lightest W shape with 10" flange: W10x49

Conservatively, $C_b = 1.0$
$L_b = 12'$

from table 3-10 (AISC) (50ksi Steel)
$M_n = 238$ kft

$FOS = \frac{238}{19} = 12.5$ ✓

from table 3-2 (AISC)
$V_n = 102K$

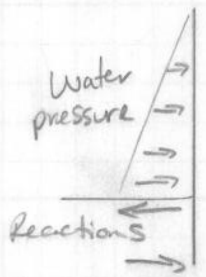
$FOS = \frac{102}{3.2} = 31$ ✓

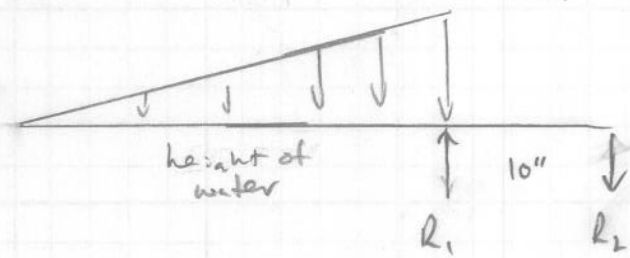
Chose Column Sections.

There are four W12 sections with steel plates welded on top that would probably do well. tough to tell from measurements what section they are, but at least W12x72
They are 21" long.
Conservatively, $L = 2'$
this column is fixed-fixed: Conservatively, however, $k = 1.0$
$KL = 2' \cdot 1.0 = 2'$ conservatively
from table 4-1 (AISC)
at $6' = KL$ $P_n = 953K$
Load = 6.9K + weight from beam = 6.9 + 49 lb/ft · 12ft = 7.5K
$FOS = \frac{953}{7.5} = 127$ ✓

check forces in the "arms" holding up the wooden section:

Worstcase: load all the way to R_1:





estimate depth of water:

Choose roofing:

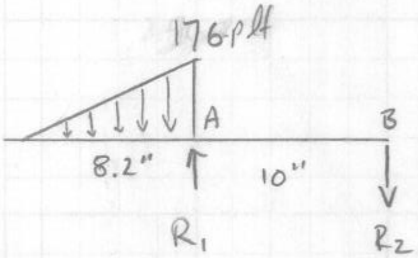
1/2" DensDeck: 2 psf (Georgia Pacific)

2PID-SA1 Modified Bitumen roofing system (John's Manville)

Self adhering 2ply: 1.5 psf

$$46.1\ psf - 3.5\ psf = 42.6\ psf$$

$$\frac{46.2\ psf}{62.4\ pcf} = 0.683\ ft\ water = 8.2''$$



at 8.2" water pressure

$$= \frac{8.2}{12} \cdot 62.4 = 46.2\ psf$$

Worst case: 3.8' tributary area

$$W_{max} = 46.2\ psf \cdot 3.8\ ft = 176\ plf$$

$$\Sigma M_A \rightarrow R_2 = \frac{\frac{1}{2} \cdot 176 \cdot \frac{8.2}{12} \cdot \frac{8.2}{12} \cdot \frac{1}{3}}{\frac{10}{12}} = 16.4\ lb$$

$$\Sigma F_y \rightarrow R_1 = 76.5\ lb$$

M_{max} occurs at A $\rightarrow \frac{10}{12} \cdot 16.4\ lb = 13.7\ ft\ lb$

V_{max} at left of A $\rightarrow (76.5 - 16.4)\ lb = 60.1\ lb$

$\rightarrow$ very small loads ✓

Check all bolts for shear:

① bolts into concrete walls

$load = 7.5K + \text{weight of column section}$

$= 7.5K + 72\ lb/ft \cdot 21''/_{12''/ft} = 7.7K$

– four 1" ϕ bolts

Conservatively, assume all load goes in to one bolt (there are 4)

$R_n = F_n A_b$

F_n from table J3.2

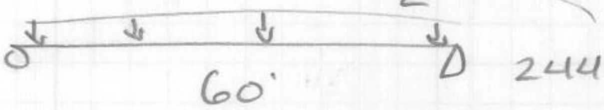
assume A307 bolts : $F_{nv} = 24\ ksi$

$R_n = 24\ ksi \cdot \pi (\cancel{1in})^2 = 75.4K$

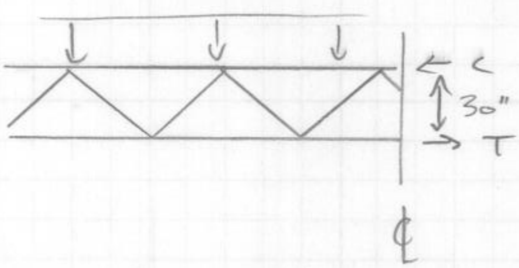
$FOS = \frac{75.4}{7.7} = 9.8$ ✓

② all other bolts hold negligible loads compared to this one

Horizontal defection estimate for top chord at roller



$$M_{midspan} = \frac{WL^2}{8} = \frac{244.4\ plf\ (60ft)^2}{8} = 110\ kft$$



Worst case, top chord carries the midspan compressive force throughout the entire length:

$$C = \frac{M}{z} = \frac{110\ kft}{30'' \cdot \frac{ft}{12''}} = 44K$$

assuming elastic behavior:

$$\sigma = E\varepsilon \qquad \varepsilon = \frac{\Delta L}{L} \qquad \sigma = P/A$$

$$\rightarrow \Delta L = \frac{PL}{AE}$$

assuming A = 4 · smallest angle brackets in AISC

$$A = 4(0.484\ in^2) = 1.936\ in^2$$

$$\Delta L = \frac{44K \cdot 60ft}{29000\ ksi \cdot 1.936 in^2} = 0.56''$$

deflection ok. ✓

check Deflection of W 10×49 section

for worst case (d_2, w_2, w_2 all act at midspan)

$$P = 6316 \text{ lb}$$

$$\Delta_{max} = \frac{PL^3}{48EI} \qquad \text{from AISC, } I_{W10\times49} = 272 \text{ in}^4$$

$$\Delta = \frac{6.316\text{ k}(12 \text{ ft})^3}{48 \cdot 29000 \text{ ksi} \cdot 272 \text{ in}^4} = 2.88\times10^{-5} \ \frac{\text{ft}^3}{\text{in}^2}\cdot\frac{144\text{ in}^2}{\text{ft}^2}$$

$$\Delta = 0.004151' = 0.0498''$$

Very small → O.K. ✓

Calculate Strength Again, for 50ksi W10×49

Yielding

$$M_n = M_p = F_y Z_x$$

$$M_n = (50\ ksi)(60.4\ in^3) = 251 kft \quad (1)$$

LTB:

$$M_n = C_b\left(M_p - (M_p - 0.7 F_y S_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right) \le M_p$$

$C_b = 1$

$M_p = 251\ kft$

$F_y = 50\ ksi$

$S_x = 54.6\ in^3$

$L_b = 12ft = 144 in$

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} = 1.76(2.54in)\sqrt{\frac{29000}{50}} = 107.7in$$

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_x h_o}}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7F_y S_x h_o}{EJc}\right)^2}}$$

$$r_{ts} = \sqrt{\frac{\sqrt{I_y C_w}}{S_x}} = \sqrt{\frac{\sqrt{93.4in^4\ 2070in^6}}{54.6in^3}} = 2.84in$$

$c = 1$

$J = 1.39\ in^4$

$$L_r = 1.95(2.84in)\frac{29000}{(0.7)(50)}\sqrt{\frac{1.39in^4}{54.6in^3 \cdot 9.42in}}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7(50)(54.6)(9.42)}{29000\ \ 1.39}\right)^2}}$$

$$= (4589)\cdot(0.0520)\cdot(1.59)\,in = 379\,in$$

$$M_n = \left(251kft - (251kft - 0.7(50ksi)(54.6in^3))\left(\frac{144in - 107.7in}{379in - 107.7in}\right)\right)$$

$$M_n = 251kft - 91.75kft \cdot 0.1338 = 238kft \quad (2)$$

ELTB:

$$M_n = F_{cr} S_x \leq M_p$$

$$S_x = 54.6 \text{ in}^3$$

$$F_{cr} = \frac{C_b \pi^2 E}{(L_b/r_{ts})^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}}\right)^2}$$

$$F_{cr} = \frac{\pi^2\, 29000\text{ksi}}{\left(\frac{144\text{in}}{2.84\text{in}}\right)^2} \sqrt{1 + 0.078 \frac{1.39\text{in}^4}{54.6\text{in}^3\, 9.42\text{in}} \left(\frac{144\text{in}}{2.84\text{in}}\right)^2}$$

$$F_{cr} = (111.\ \text{ksi})(1.24) = 138 \text{ ksi}$$

$$M_n = F_{cr} S_x = 138\text{ksi} \cdot 54.6 \text{in}^3 = 629 \text{ kft}$$ 3

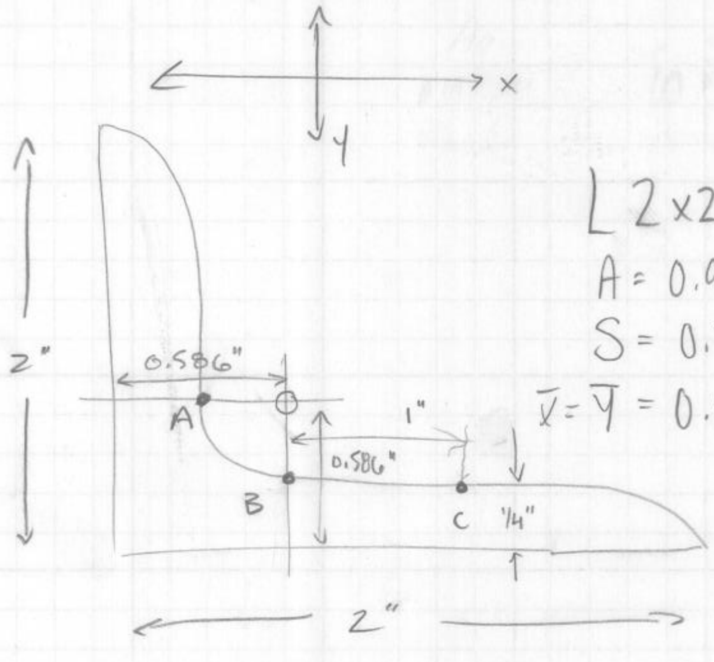
ILTB controls:

$$\phi M_n = 0.9(238\text{kft}) = \boxed{214.2\text{kft}}$$

Shear strength:

$$V_n = \phi V_n = 0.6 A_w F_y = (0.6)(10")(0.34")(50\text{ksi})$$

$$\boxed{V_n = 102 \text{ K}}$$



$L\ 2 \times 2 \times 1/4$

$A = 0.938\ in^2 \quad I = 0.346\ in^4$

$S = 0.244\ in^3$

$\bar{x} = \bar{y} = 0.586$

Strain gages at A, B, C

$\epsilon_A = 0.005$

$\epsilon_B = 0.006$

$\epsilon_C = 0.007$

Find M_x, M_y, P

ϵ_A from M_y, P

ϵ_B from M_x, P

ϵ_C from M_x, M_y, P

$\sigma_A = \epsilon_A E = 145\ KSI$

$\sigma_B = \epsilon_B E = 174\ KSI$

$\sigma_C = \epsilon_C E = 203\ KSI$

$S_{Ax} = \diagup$

$S_{Ay} = \dfrac{I}{(0.586 - 1/4)} = 1.030\ in^3$

$S_{Bx} = 1.030\ in^3$

$S_{By} = \diagup$

$S_{Cx} = \dfrac{I}{(0.586'' - 1/4'')} = 1.030\ in^3$

$S_{Cy} = \dfrac{I}{1''} = 0.346\ in^3$

$$\sigma_C = \frac{P}{A} + \frac{M_x}{S_{Cx}} + \frac{M_y}{S_{Cy}}$$

$$\sigma_B = \frac{P}{A} + \frac{M_x}{S_{Bx}}$$

$$\sigma_A = \frac{P}{A} + \frac{M_y}{S_{By}}$$

$$\sigma_C - \sigma_B = \frac{M_y}{S_{Cy}} = 203\text{ KSI} - 174\text{ KSI}$$

$$203\text{ KSI} - 174\text{ KSI} = \frac{M_y}{0.346\text{ in}^3} \rightarrow M_y = 10.03\text{ Kin} \cdot \frac{\text{ft}}{12\text{ in}}$$

$$M_y = 836\text{ lbft}$$

$$\sigma_C - \sigma_A = \frac{M_x}{S_{Cx}}$$

$$203\text{ KSI} - 145\text{ KSI} = \frac{M_x}{1.030\text{ in}^3} \rightarrow 59.74\text{ Kin} \cdot \frac{\text{ft}}{12\text{ in}} = M_x$$

$$M_x = 4.978\text{ kft}$$

$$\sigma_C = \frac{P}{A} + \frac{M_x}{S_{Cx}} + \frac{M_y}{S_{Cy}} = \frac{P}{0.938\text{ in}^2} + \frac{4.978\text{ kft}}{1.030\text{ in}^3} + \frac{836\text{ lbft}}{0.346\text{ in}^3}$$

$$203\text{ KSI} = \frac{P}{0.938\text{ in}^2} + 58.0\text{ KSI} + 29.0\text{ KSI}$$

$$\rightarrow P = 108.8\text{ K}$$

check:

$$\sigma_A = \frac{P}{A} + \frac{M_y}{S_{Ay}} = \frac{108.8\text{ k}}{0.938\text{ in}^2} + \frac{836\text{ lbft}}{0.346\text{ in}^3} = 145\text{ KSI} \checkmark$$

$$\sigma_B = \frac{P}{A} + \frac{M_x}{S_{Bx}} = \frac{108.8\text{ k}}{0.938\text{ in}^2} + \frac{4.978\text{ kft}}{1.030\text{ in}^2} = 174\text{ KSI} \checkmark$$

→ 3 Strain gages are necessary to measure M_x, M_y, P

(3 independent measurements for 3 unknowns)