MICROCOMPUTER SIMULATION FOR
SUBSURFACE WATER POTENTIAL ON HILLSLOPE

by

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Microcomputer Simulation for Subsurface Water Potential on Hillslope

ABSTRACT: This research paper focuses upon subsurface flow because of its dominant influence on the other types of hillslope water flow processes. The basic theory of water movement is quickly discussed in order to build the reader's general background knowledge. With this background, the journey starts into the poorly understood realm of subsurface flow mechanisms where some qualitative relationships and hypotheses are discussed.

These concepts are used in an elementary computer simulation model within appropriate soil physics theory. This model can demonstrate the effect of major controlling factors; soil, bedrock and topography; on steady-state water potential in a hillslope segment to the field hydrologist.

INTRODUCTION

The processes by which precipitation moves from a particular point on a hillslope to the stream channel is poorly understood. There is a desire to explain the mechanisms of hillslope response in a scientific sense. For applied scientists and engineers, there is a need for techniques to better predict runoff from hillslopes. The path by which water reaches the base of a hillslope depends upon such controls as climate, geology, topography, soils, vegetation and land use. In various parts of a watershed, different processes may generate water flow at the hillslope base. The relative importance of the various processes may also differ from hillslope to hillslope in a watershed. Researchers have recognized that there are essentially three main processes that feed streams. These are overland flow, subsurface flow (or interflow) and groundwater flow. An understanding into the nature of the subsurface flow regime is important for understanding the runoff generated by any of these three mechanisms.

Computer simulation models have been proposed for better understanding of other watershed hydrology problems. Simulation appears to provide cost effectiveness and high flexibility when compared to field studies. Freeze (1978) explains the five limitations of physically based mathematical models which are the base for computer simulation. The first limitation is presented by listing the assumption used in the theoretical development of the model. For example, the model might assume laminar flow in a non-swelling soil. If the soil swells, the theoretical assumption is causing an error. The second limitation results from model failure to simulate an actual mechanism that occurs on a natural hillslope. With the variability of topography, soils, vegetation and precipitations, generalizations about the exact timing and sequence of events that takes place are difficult to identify or quantify. The third limitation is simply insufficient data which is commonplace in field hydrologic work. This limitation includes insufficient samples for accurate prediction (Baker 1978). The fourth limitation is lack of computer capacity needed for proper model operation. A choice of the microcomputer provides a limit of the storage capacity for model operation. The fifth limitation is discussed by Stephenson and Freeze (1974) as the failure to properly adjust model output to real field data. These limitations affect the effectiveness and the use of a model.

The objective of this computer simulation is to provide information about outflow and pressure potential changes under hillslope segment conditions to field personnel and hydrology students. An emphasis is placed on the transformations to represent a hillslope segment in a compatible form with simulation on a microcomputer. The computer capacity, data base, current knowledge and solution technique in literature provide the restrictions which limit the simulation.

THEORY OF WATER MOVEMENT

A brief review of theoretical considerations will make it possible to understand specific subsurface flow principles from these concepts. Important concepts include energy state, hydraulic gradient, head loss, hydraulic conductivity and types of flow.

The energy state controls the movement of water through the soil material. Water always flows from high to low potential. Total water potential of soil material is defined by the sum of gravity, pressure, osmotic and overburden potentials. Generally, overburden and osmotic potential are small and can be neglected (Hillel, 1971). Saturated soils have a positive pressure potential, the free water surface is considered zero, and unsaturated soils have negative pressure potentials. These water potentials can be expressed as the hydraulic head with units in centimeters of water equivalent. The total hydraulic head equals hydraulic head due to pull of gravity plus hydraulic head due to weight of water if saturated or gas pressure at a point (pressure potential).

The potential gradient that exists in soil material is the driving force that results in movement. The difference between the two unequal potential energies divided by the distance between them expresses this gradient. Hillel (1971) states that the hydraulic gradient is the head loss per unit distance in the direction of flow. This head loss is the energy being lost by frictional resistance. Most soils have considerable resistance.

The ratio of the flux to the hydraulic gradient, with units similar to velocity, defines hydraulic conductivity. It is composed of two parts, intrinsic permeability of the soil and fluidity of the fluid. Permeability is changed by varying porosity of the medium. The fluidity changes with viscosity of water. Hydraulic conductivity is affected by the soil structure, temperature, texture, total porosity, and particularly pore size distribution of a given soil (Hillel, 1971). It varies greatly with pressure potential, but remains relatively constant after saturation. When the soil is at or near saturation, all pores in the soil matrix are contributing to the flow. This "saturated" flow can be several orders of magnitude greater than the unsaturated flow when water is present only in the smaller pores and in water films.

Darcy's law states that the specific discharge is equal to the product of the hydraulic conductivity and the hydraulic gradient. This specific discharge calculation which is incorrectly called flow velocity and is only accurate for laminar flow. Inertial forces during saturated flow are no longer negligible when compared to viscous forces, and turbulent flow results. The Reynolds number is the ratio of inertial to viscous forces, and can be used to determine the onset of tuberlence. In soils, Hillel (1971) states that laminar flow remains only as long as Reynolds number is less than one.

Unsaturated flow also obeys Darcy's Law. Unsaturated flow has a different hydraulic conductivity than saturated flow since the largest pore sizes are not contributing. The hydraulic conductivity varies with the change in the negative pressure head. Horton and Hawkins (1965) state that the percolation of water through the soil is accomplished throughout the flow path by downward displacement of water previously held by the soil at field capacity. In unsaturated flow, water moves in films and menisci formed between soil particles, and not through air filled pores (Hillel, 1971).

For the flow process, Darcy's Law is combined with a continuity principle (Law of Conservation of Matter). The continuity principle in one dimensional case states that the rate of change of specific discharge in a horizontal direction must equal the change in volumetric water content with time. The flow equation is derived by substitution of Darcy's Law in the continuity principle (see Appendix). If a steady state flow exists, the change in volumetric water content with time is zero.

SUBSURFACE FLOW MECHANISMS

For substantial subsurface flow to occur in a hillslope, there are four basic requirements. First, there must be vertical permeability discontinuities in the soil horizons or the underlying weathered bedrock to concentrate flow. Second, there must be a reasonable gradient slope to provide potential energy to the infiltrating water. Third, a pore size distribution with a fairly high hydraulic conductivity must exist somewhere in the soil material. Finally, the interaction between rainfall and infiltration must supply sufficient water to the flow generating zone. If these conditions are not present, water may also move to a stream channel as overland flow or regional groundwater. The best way to examine these points is to review some concepts.

Hewlett and Hibbert (1963) provide a good physical model which was a large concrete box placed on a 40% slope and filled with a sandy loam subsoil. Their model demonstrated regional saturated groundwater aquifers are not the source of stream flow during nonstorm periods on steep, forested slopes. The real source was unsaturated flow which supplied water for two months after the initial input of water.

Regression analysis showed linear trends in logarithm transforms of model data for two separate periods. The first 36 hours after wetting was the time when the large marcopores were emptied, and water collected on the cement base. There was a free water surface. Unsaturated flow was occurring throughout the remainder of the soil. After this period, there was a transition period. By the fifth day all the large pores were drained except for a very small saturated zone near the outlet. A new regression was estimated for the drainage of the micropores. Hewlett and Hibbert concluded the narrow groundwater bodies along the stream channel are not a source, but rather a conduit through which unsaturated flow passes to enter the stream channel.

Relating Hewlett and Hibbert's (1963) model to a real hillslope, one can say that water leaves the soil mantle through a permanently saturated zone at the base of a slope. During a storm period, the saturated zone grows upslope and up the soil profile (Weyman, 1973). Water moves with the gradient into the saturated zone from surrounding unsaturated soil (Dunne 1970). This expanding-contracting saturated wedge appears to be a general occurrence. In the saturated wedge, flow conforms to Darcy's Law. The discharge at the base of the slope is related to the form of the wedge.

Infiltrating water moves vertically downward in the unsaturated state for most cases. The classical view of infiltrating water is a concept where water moves from a saturated zone under a water ponded soil surface through a wetting zone and transmission zone to the wetting front (Hillel, 1971). Whipkey (1967) states this is not true for hillslope soils he studied. The soil profile remains unsaturated until water accumulates at the profile base or restricting horizon where saturated zone is produced. Whipkey and Kirby (1978) theorize that downslope saturated lateral flow then results.

Conceptually, the first stage of drainage is dominated entirely by saturated lateral flow operating in non-capillary pores. After a transition period, unsaturated flow supplies water to small saturated zones near the slope base. The lateral movement operates only in the capillary pores. In the transition period, Weyman (1973) states that the saturation zone is contracting so rapidly that no equilibrium between supply and removal exists.

Soil wetness is invariably higher near the stream channel due to the migration of unsaturated flow downslope. This causes water yields to vary between different positions on a slope. The saturated portion of the hillslope shrinks and expands, depending on the amount of rainfall and the antecedent soil wetness. During a storm, the expansion of saturated zones along lower portions of a hillslope is observed by the growth of small rivulets on hillslopes. This process happens when the subsurface flow from upslope exceeds the capacity of the profile to transmit water.

Flow by displacement is accepted as the mechanism which allows hillslopes to produce large amounts of subsurface relatively quickly in soils (without non-Darcian macropores). This concept of translatory flow is based on Horton and Hawkins (1965) study. The direct flow first seen in a channel was the water that displaced in the hillslope soil profile by infiltrating rain water from the present occurring storm. This rain water ends up temporarily stored. Hewlett and Hibbert (1967) found that translatory flow affects direct throughflow in the lower and midslope areas, while upslope water migrates slowly in "pulses."

Large biological (i.e. earth worm holes) and structural macropores are very important to water movement (Parker and Jenne, 1967). This macropore flow is probably turbulent which means Darcy's Law does not apply. These macropores can cause a rapid response in storm hydrographs from hillslopes. Large amounts of water can move through these openings without appreciably wetting the soil mass (Bouma and Dekker, 1978). Aubertin (1971) found that root channels, formed in place by decomposition, often act as stable conduits for rapid water movement. His study showed that in fine textured soils, old root channels are abundant and last for a long time, when compared with those in coarser textured soils. Earthworms, squirrels, moles, and shrews can produce these large macropores beside plant roots. There is still confusion on how such large amounts of water can be concentrated so quickly. Apparently, rainfall collects in depressions formed by the channel making process and is funneled downward.

There are two parts to the hydraulic conductivity when soils contain these types of macropores; that of soil matrix, and that of the cracks and channels. In fine textured soils, rapid subsurface flow in macropore filled soils is not related to soil texture. The soil is not always completely wetted (Bouma and Dekker, 1978). In contrast, for coarser textured soils, textural macropores allow rainfall to enter and permeate the soil mass as a whole.

CONTROLLING FACTORS

Subsurface flow can be highly variable on a hillslope. It can occur in a range of conditions from unsaturated flow to saturated flow in completely saturated soil profile. There are four major factors controlling subsurface flow: topography, bedrock, soil, and precipitation patterns.

Topography is a simple starting point. If a slope is very steep, convex slopes, it can be considered as one that possibly generates substantial subsurface stormflow. Conversely, concave slope with marshy riparian zones would undoubtedly have variable saturated zone that produces overland flow near the stream channel. Steep slopes tend to have soils that are permeable and shallow. With large hydraulic gradients, high subsurface specific discharge can result. This is not usually the case for gentle slopes, since they have deep soils, lower gradients, and less permeable soils (Whipkey and Kirby, 1978).

A more difficult parameter to understand is that of bedrock. The permeability is often difficult to evaluate since the extent of point fracturing, faults, and bedrock micro relief are not visible and vary from spot to spot. One can determine the hydraulic conductivity in the laboratory, but in the field it can be very different (Megahan, 1973). The effects of bedrock permeability can have on subsurface are varied depending on the site conditions.

The soil itself is a crucial factor in affecting the timing of subsurface flow. If there are large channels in soil profile, the hydraulic conductivity of the soil matrix is of secondary concern. These pores do away with translatory flow since gravity pulls the water through them very quickly. Without these large, non-Darcian macropores, the matrix conductivity is of the utmost importance to subsurface flow. Shallow profiles are better suited to saturated lateral transmission of water. Whipkey (1967) showed how differences in horizon permeabilities and antecedent moisture levels in the profile greatly affect subsurface flow.

The precipitation pattern for a given hillslope can also determine the extent of saturated flow and the timing of subsurface flow. For example, high intensity storms on low permeable soils lead to overland flow since water does not infiltrate fast enough.

SIMULATION MODEL

The model is a theoretical approach to subsurface water movement in an unsaturated-saturated hillslope segment. The segment size and shape is fixed by the data input of internal condition matrix. The use of a finite difference method causes the true shape of slope to be approximated in a stair step pattern. The first computational step involves the input of problem data, boundary conditions, and use of flow equation for the calculation of pressure potentials within the defined region. The second step is the input of new boundary conditions which simulates a time step. This causes the first step to repeat. The final step is a net outflow estimation.

Flow Process

The steady state water flow assumption is made on the basis that the calculation is made during a very short period of time in comparison to the main time frame. The other assumptions are heterogenous, anisotropic soil with a predictive pattern within the hillslope. The equation becomes:

$$\frac{\partial}{\partial X}\left(K(P)_X\left(\frac{\partial P}{\partial X}\right)\right)+\frac{\partial}{\partial Z}\left(K(P)_Z\left(\frac{\partial P}{\partial Z}+1\right)\right)=0 \qquad 1$$

where

P is pressure potential

X is horizontal distance

Z is vertical distance

K(P) is hydraulic conductivity as a function of pressure potential

The boundary conditions are:

1) No vertical inflow
2) No vertical outflow
3) Side conditions are defined by data inputs

Finite difference method is used to solve this equation for the following reasons: 1) the method is fundamentally very simple, 2) the geometry of the slope is an appropriated way maintained in the solution, 3) the solution is obtainable on microcomputer, and 4) a number of techniques are available for solving this equation type. A relaxation technique is used in this model. This technique allows a factor between one and two to be placed in an important calculation. This shortens the time to arrive at a solution.

The hydraulic conductivity can be in the unsaturated state. Brooks and Corey (1966) describe the empirical method which uses saturated hydraulic conductivity and the shape of the moisture release curve of undisturbed soil samples. Their equation estimates unsaturated conductivity as a function of pressure potential in the following manner.

$$K(P) = KS\left(\frac{PE}{P}\right)^{n} \qquad 2$$

K(P) = unsaturated conductivity

KS = saturated conductivity

PE = air entry potential

P = pressure potential of the soil

$n = 2 + 3/B$

Both PE and B are found by plotting moisture release data on a log-log scale and fitting a straight line to the data. Examples are presented in Table 1.

Water Balance

The analysis of vertical and horizontal moisture movement is based on the use water balance equation as the initial side boundary conditions change. The general equation is:

$$E = R - D + S \qquad 3$$

E = evaporatransportation

R = precipitation

Table 1. Soil Data for Hillslope Simulation

Depth (cm)	B	N	PE (cm)	Porosity %	Saturated Hydraulic Conductivity (cm/hr) Vertical	Horizontal
15	3.0	3.000	1.0	60.0	139.6	122.5
30	3.1	2.968	1.2	58.0	138.1	124.0
75	5.5	2.526	8.6	48.9	111.25	88.80
115	7.6	2.395	15.2	43.4	33.01	28.41

D = net drainage

S = change in soil moisture storage

These variables are more complex. For example, S expresses the change in total water content of all soil material in the profile to a given depth between two times (t_1, t_2).

If E = 0 and P = 0, the equation becomes after rearrangement of terms:

$$D = S \qquad 4$$

Since boundary conditions are input as pressure potential, Brooks and Corey (1966) provide the methodology to convert this data to water content as follows:

$$\frac{W}{WS} = \left(\frac{P}{PE}\right)^{-1/B} \qquad 5$$

where

W = water content at pressure P

WS = water content at air entry (assumed total saturation)

B & PE are the same is in equation 2

The water balance equation is used to estimate the net water movement from the sample profiles. The total water moving through the sample profile is estimated by use of field data and an empirical technique used by Hewlett and Hibbert (1963). The model does not estimate total water movement through a given profile because of this need for site specific data.

DISCUSSION

This model of hillslope water movement is based on a rather arbitrary and hypothetical selection of soil and topographic conditions.

The side boundary conditions are input before each calculation as a desired pattern of soil profile pressure potentials. From the input information, a pressure potential distribution is calculated for the hillslope segment. By inputting new boundary conditions, a second pressure potential distribution is generated. Then two outputs are compared and interpretted. The quality of information discovered is dependent on the data interpreter's ability. Hence a specific example of output is not discussed within the body of this communication.

The problem of accounting for natural variability of necessary input data is common to all simulations. The arbitrary selection of hypothetical data allows the advantage of systematic assignment of data where field data could be incomplete and highly variable. Problems arise in the selection of hydraulic conductivity, air entry potentials and soil "B" values to meet desired soil profile conditions. Air entry potentials and soil "B" values are estimated from moisture retention data in the literature. Hydraulic conductivity patterns are also found in the literature or soil resource reports. The boundary conditions which implicitly reflect infiltrating rainfall or water moving laterally in the soil are patterned after tensiometer and piezometer data gathered in a field study. An operational error occurs when the saturated horizontal and vertical hydraulic conductivities are not placed in their respective matrix in a pattern which approximates the desired slope.

The arrangement and organization of the model is established for the purpose of modelling an "as is" field problem. For this purpose, the saturated hydraulic conductivity matrices and other soil conditions remain constant throughout simulations on one hillslope. This stops field personnel from systematically varying one parameter while holding the others constant. This use of the scientific method which is a good study procedure is misleading to field personnel who do not encounter textbook examples in their work.

The main data output is the pressure potential distributions of the hillslope segment. The position of saturated flow is located by the positive values in this output. A comparison of two consecutive pressure distributions is used to stimulate a one time step. This allows the estimation of net water outflow from each sample profile which is printed out after the second pressure distribution.

This modelling effort is limited by the lack of a mathematical hillslope discharge theory. Scientists are unable to generalize the exact timing and sequence of physical events creating subsurface flow in a hillslope into a mathematical equation. Hewlett and Hibbert (1963) provide a state of science study which uses a statistical method for determining hillslope outflow in time. This type of relationship is only valid under the conditions at which the data was gathered. The use of this type of relationship limits the conditions under which the simulation model can be used. This is not a desired quality of a physical based mathematical model.

Even in its elementary form, the model explains significant hydrological implications such as the effect of spatial variable hydraulic conductivity on the water movement in a hillslope segment. In one simulation the effect of impervious bedrock or a restricting zone is demonstrated by changing the boundry conditions and observing the patterns in the calculated pressure potentials. The controlling factors of soil depth and topography are easily evaluated for their effect on a hillslope segment when changed in a series of simulations.

This model can aid field personnel and hydrology students in understanding the interacting functional relationships governing water movement in a hillslope during varying circumstances encountered in the field. The model relates the possible formulation of hydrologically important factors such as hydraulic conductivity and pressure potential to real physical mechanisms in a mathematical way. This provides a better understanding of the phenomenon than shown with empirical estimation technique.

An introduction of real microtopographical and soil physical processes is a challenging learning experience for students in hillslope hydrology. In fact, the need to introduce soil physical processes and effects of microtopography is one of the most challenging tasks in hillslope hydrology today.

REFERENCES

Aubertin, G. M. 1971. Nature and extent of macropores in forest soils and their influence on subsurface water movement. USDA Forest Service Report Paper NE-192. 33 pp.

Baker, F. G. 1978. Variability of hydraulic conductivity within and between nine Wisconsin soil series. Water Res. Res. 14: 103-108.

Bouma, J. and L. W. Dekker. 1978. A case study on infiltration into dry clay soil. I. Morphological observation. Geoderma 20: 27-40.

Brooks, R. H. and A. T. Corey. 1966. Properties of porous media affecting fluid flow. J. Irrig. Drain., Proc. ASCE 92(IR2): 61-88.

Dunne, T. 1970. Runoff production in a humid area. USDA ARS-41-160 pp. 1-108.

Freeze, R. A. 1978. Mathematical Models in Hillslope Hydrology. In M. J. Kirby (ed) Hillslope Hydrology John Wiley and Sons, New York.

Hewlett, J. D. and A. R. Hibbert. 1963. Moisture and energy conditions within a sloping soil mass during drainage. J. Geophys. Res. 68: 1081-1087.

Hewlett, J. D. and A. R. Hibbert. 1967. Factors affecting the response of small watershed to precipitation in humid areas, In W. E. Sooper and H. W. Hall (eds) Proceedings of the Intermational Symposium on Forest Hydrology Pergamon, New York p. 275-290.

Hillel, D. 1971. Soil and Water - Physical Principles and Process, Academic Press, New York.

Horton, J. H. and R. H. Hawkins. 1965. Flow path of rain from the soil surface to the watertable. Soil Sci. 100: 377-383.

Megahan, W. F. 1973. Role of bedrock in watershed management, ASCE, Proc. Irrig. and Drain. Div. Specialty Conference at Fort Collins, Colo, April 22-24, p. 449-470.

Parker, G. G. and E. A. Jenne. 1967. Structural failure of western highways caused by piping. In Symposium on Subsurface Drainage, Nat. Pres. Council Nat. Acad. Sci./Nat. Acad. Engr. Publication #1534.

Weyman, D. R. 1973. Measurements of the downslope flow of water in a soil. J. Hydrol. 20: 267-288.

Whipkey, R. Z. 1967. Theory and mechanics of subsurface stormflow, In W. E. Sooper and H. W. Lull (eds) Proceedings of the International Symposium on Forest Hydrology Pergamon New York p. 255-259.

Whipkey, R. Z. and M. J. Kirby. 1978. Flow within the soil. In M. J. Kirby (ed) Hillslope Hydrology John Wiley and Sons, New York p. 121-145.

Appendix A

Continuity principle for two dimension problems can be stated as

$$\frac{\partial V}{\partial X}x + \frac{\partial V}{\partial Z}z = \frac{\partial \theta}{\partial t} \qquad 1$$

Darcy's law for each dimension

$$V_x = \frac{Q}{A}x = \left(K_x\right)\frac{\partial h}{\partial X}x \qquad 2a$$

$$V_z = \frac{Q}{A}z = \left(K_z\right)\frac{\partial h}{\partial Z}z \qquad 2b$$

where $h_x = P$

$h_z = P + G$

Combining the above equation

$$\frac{\partial}{\partial X}\left(K_x \frac{\partial P}{\partial X}\right) + \frac{\partial}{\partial X}\left(K_z \frac{\partial (P+Z)}{\partial Z}\right) = \frac{\partial \theta}{\partial t} \qquad 3$$

For steady state $\frac{\partial \theta}{\partial t} = 0$

The equation is simplified

$$\frac{\partial}{\partial X}\left(K_x \frac{\partial P}{\partial X}\right) + \frac{\partial}{\partial Z}\left(K_z\left(\frac{\partial P}{\partial Z} + 1\right)\right) = 0 \qquad 4$$

Using standard finite difference techniques, we can write

$$\frac{\partial}{\partial X}\left(K_x \frac{\partial P}{\partial X}\right) \approx \frac{1}{\Delta X}\left(\left(K_x \frac{\partial P}{\partial X}\right)_{i+\frac{1}{2},j} - \left(K_x \frac{\partial P}{\partial X}\right)_{i-\frac{1}{2},j}\right) \qquad 5$$

We can further evaluate the bracketed quantity

$$\left(K_x \frac{\partial P}{\partial X}\right)_{i+\frac{1}{2},j} \approx \frac{K_x}{\Delta X}\, i+\tfrac{1}{2},j \left(P_{i+1,j} - P_{i,j}\right) \qquad 6a$$

$$\left(K_x \frac{\partial P}{\partial X}\right)_{i-\frac{1}{2}j} \approx \frac{K_{x\; i-\frac{1}{2},j}}{\Delta X}\left(P_{i,j} - P_{i-1,j}\right) \qquad 6b$$

We can evaluate $(K_x)_{i+\frac{1}{2},j}$ by geometric mean of the form

$$(K_x)_{i+\frac{1}{2},j} \approx SQR\left((K_x)_{i+1,j} * (K_x)_{i,j}\right)$$

If these expressions are substituted in Eq 5, we find the following

$$\frac{\partial}{\partial X}\left(K_X \quad \frac{\partial P}{\partial X}\right) \approx \frac{1}{\Delta X^2}\left[K1(P_{i+1,j}) - K1(P_{i,j}) - K2(P_{i,j}) + K2(P_{i-1,j})\right]$$

where

$$K1 = (K_X)_{i+\frac{1}{2},j}$$

$$K2 = (K_X)_{i-\frac{1}{2},j}$$

Similarly, for the second part of Eq 4

$$\frac{\partial}{\partial Z}\left(K_Z \quad \frac{\partial P}{\partial Z}\right) \approx \frac{1}{\Delta Z^2}\left[K3(P_{i,j+1}) - K3(P_{i,j}) - K4(P_{i,j}) + K4(P_{i,j-1})\right]$$

$$+ \frac{K3-K4}{\Delta Z}$$

where

$$K3 = (K_Z)_{i,j+\frac{1}{2}}$$

$$K4 = (K_Z)_{i,j-\frac{1}{2}}$$

If these are combined and rearranged, we have the following

$$\left(\frac{K2+K1}{\Delta X^2} + \frac{K3+K4}{\Delta Z^2}\right)\left(P_{i,j}\right) = \frac{K1(P_{i+1,j}) + K2(P_{i-1,j})}{\Delta X^2} +$$

$$\frac{K3(P_{i,j+1}) + K4(P_{i,j-1})}{\Delta Z^2} + \frac{K3-K4}{\Delta Z}$$

We can solve for $P_{i,j}$

$$P_{i,j} = \frac{\dfrac{K1(P_{i+1,j}) + K2(P_{i-1,j})}{\Delta X^2} + \dfrac{K3(P_{i,j+1}) + K4(P_{i,j-1})}{\Delta Z^2} + \dfrac{K3-K4}{\Delta Z}}{\dfrac{K1+K2}{\Delta X^2} + \dfrac{K3+K4}{\Delta Z^2}}$$

Where

V=specific discharge

θ=volumetric water content

t=time

Q=volume of water

A=cross sectional area

h=hydraulic head

P=pressure potential

G=gravity potential

K=hydraulic conductivity

X=horizonal direction

Z=vertical direction

i=horizontal node location in the finite difference map

j=vertical node location in the finite difference map

APPENDIX B

RADIO SHACK TRS-80 BASIC PROGRAM

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10 REM  HILLSLOPE WATER MOVEMENT- STAGE 1 PROGRAM
20 CLS:DEFINT L,E,I,J,M,N:INPUT"NAME OF THIS PROBLEM";A$:C$="+####"
30 LPRINT A$:MN=0:RELAX=1.0:RR=RELAX:IX=15:IZ=15:T=1
40 INPUT"WHAT ERROR LIMIT DO YOU WANT(IN CM)?";ER
45 INPUT"VERTICAL SOIL DEPTH IS";D
50 D=D/IZ:PRINT"IX=15,IZ=15 FOR AN ELEMENT IN EACH MATRIX (CHANGE AT LINE 20 AND 40)"
60 INPUT" HILLSLOPE SEGMENT TOTAL SIZE (IN CM) IN X (FIRST) AND Z DIRECTIONS IS";S,R
70 M=S/IX:N=R/IZ:DIM F(M+1,N+1),KH(M+1,N+1),KV(M+1,N+1),H(M+1,N+1),B(D+1),A(D+2),U(5,
10),KA(D+1),KC(D+1)
80 DIM HZ(D+1),WS(D+1),W(D+1),PE(D+2):FORJ=1TON:FORI=1TOM:KV(I,J)=0:KH(I,J)=0:NEXT I,
J
85 INPUT"TYPE 0 FOR SPATIAL VARIABLY IN HYDRAULIC CONDUCTIVITY";G:IF G=0 GOTO100
90 FORI=1TOM:FORJ=1TON:PRINT"IS HYDRAULIC CONDUCTIVITY ZERO (K=0) AT";I,J
92 INPUT"TYPE 0 FOR YES, 3 FOR NO";G:IF G=0 GOTO98
96 INPUT"VERTICAL HYDRAULIC CONDUCTIVITY (CM/HR) IS";KV(I,J)
97 INPUT"HORIZONTAL HYDRAULIC CONDUCTIVITY (CM/HR) IS";KH(I.J)
98 NEXTJ,I:GOTO135
100 FORE=1TOD:PRINT"HORIZONTAL HYDRAULIC CONDUCTIVITY (CM/HR) AT";15*E:INPUTKC(E)
102 INPUT"VERTICAL HYDRAULIC CONDUCTIVITY (CM/HR)";KA(E)
110 E=0:FOR I=1TOM:FORJ=1TON:PRINT"IS HYDRAULIC CONDUCTIVITY ZERO (K=0) AT";I,J:INPUT
"TYPE 0 FOR YES,3 FOR NO";G:IF G=0 GOTO130
120 E=E+1:KH(I,J)=KC(E):KV(I,J)=KA(E)
130 NEXT J:E=0:NEXTI
135 FORI=1TOM:FORJ=1TON:H(I,J)=0:NEXTJ,I
140 FOR E=1TOD:PRINT"AT ";15*E;"CM AIR ENTRY POTENTIAL -CM"; :INPUT PE
160 G=1:PRINT"AT DEPTH ";E*15;"CM, SOIL B VALUE IS(ENTER AS 1/B)"; :INPUT B
180 PRINT"AT";15*E;"CM POROSITY IS"; :INPUT P:G=1
200 WS(E)=P:PE=-ABS(PE):PE(E)=PE:B(E)=B:A(E)=2+3*B(E):NEXT E
210 PE(0)=PE(1):A(0)=A(1):PE(D+1)=PE(D):A(D+1)=A(D)
240 FORL=1TOM:FORE=1TOD:F(L,E)=0:NEXTE,L
250 E=0:Y=Y+1:PRINT"ENTER CONDITIONS AT SAMPLING PROFILE ";Y;"(WHAT I POSITION?)"; :I
NPUT L
260 FORE=1TOD:PRINT"AT ";15*E;"CM HEAD VALUE IS"; :INPUT F(L,E):NEXTE
270 U=0:E=0:FORE=1TOD
280 P=F(L,E):IF P<PE(E) THEN W(E)=WS(E)*(P/PE(E))^(-B(E)) ELSE W(E)=WS(E)
290 U=U+W(E)*IZ:NEXTE
300 U(Y,T)=U:IF Y=2 GOTO320
310 INPUT"DO YOU WANT TO INPUT ANOTHER SAMPLE SITE? (TYPE 0 FOR YES)";G:IF G=0 GOTO25
0
320 FORE=1TOD:HZ(E)=(F(1,E)-F(M,E))/(M-1):NEXT E
330 INPUT"J POSITION FOR SURFACE IN 1 COLUMN";J:L=1:I=1:FORE=1TOD:H(I,J)=F(L,E):J=J+1
:NEXT E
340 FORI=2TOM-1:PRINT"J POSITION FOR SURFACE BOUNDRY IN";I;" COLUMN"; :INPUT J:L=I
350 FORE=1TOD:F(L,E)=F(L-1,E)-HZ(E):H(I,J)=F(L,E):J=J+1:NEXTE
360 J=0:NEXT I
370 INPUT"J POSITION FOR SURFACE IN LAST COLUMN";J:L=M:I=M:FORE=1TOD:H(I,J)=F(L,E):J=
```

```
J+1:NEXT E
390 PRINT"IF YOU EDIT THE PROGRAM, THE RELAXATION FACTOR CAN BE CHANGED."
400 PRINT MN,S:E=0:MN=MN+1:S=0:FORI=2TOM-1:FORJ=2TON-1:IF H(I,J)>0 THEN RR=1.0 ELSE R
R=1.0
410 IF KV(I,J)=0 THEN GOTO530
412 E=E+1:IF E=1 GOTO530
413 IF E=D GOTO530
420 P=H(I,J):IF P<PE(E) THEN KA=KH(I,J)*(PE(E)/P)^A(E) ELSE KA=KH(I,J)
430 P=H(I+1,J):IF P<PE(E) THEN KB=KH(I+1,J)*(PE(E)/P)^A(E) ELSE KB=KH(I+1,J)
440 P=H(I-1,J):IF P<PE(E) THEN KC=KH(I-1,J)*(PE(E)/P)^A(E) ELSE KC=KH(I-1,J)
450 P=H(I,J):IFP<PE(E) THEN KF=KV(I,J)*(PE(E)/P)^A(E) ELSE KF=KV(I,J)
460 P=H(I,J+1): IF P<PE(E+1) THEN KD=KV(I,J+1)*(PE(E+1)/P)^A(E+1) ELSEKD=KV(I,J+1)
470 P=H(I,J-1):IF P<PE(E-1) THEN KE=KV(I,J-1)*(PE(E-1)/P)^A(E-1) ELSE KE=KV(I,J-1)
480 T1=SQR(KA*KB)/(IX*IX):T2=SQR(KA*KC)/(IX*IX)
490 T3=SQR(KF*KD)/(IZ*IZ):T4=SQR(KF*KE)/(IZ*IZ):T5=((SQR(KF*KD))-(SQR(KE*KF)))/IZ
500 T6=T1*H(I+1,J)+T2*H(I-1,J)+T3*H(I,J+1)+T4*H(I,J-1):T7=T1+T2+T3+T4:X=T6/T7
510 H=H(I,J)-X:IF ABS(H)>S THEN S=ABS(H)
520 H(I,J)=H(I,J)-(H*RR)
530 NEXTJ:E=0:NEXTI:RR=RELAX:IF S>ER GOTO400
540 LPRINT" "
550 PRINT"HORIZONTAL DISTANCE ACROSS THE PAGE. TIME IS";T:LPRINT" ""
560 FORJ=1TON:FORI=1TOM:LPRINT USING C$;H(I,J); :NEXT I:LPRINT" ":NEXT J
570 LPRINT" ":LPRINT" ":IF T<2 GOTO610
580 FORI=1TOY:INPUT"ENTER TOTAL WATER INFILTRATED OVER TIME STEP(CM)";R
590 Q(I)=R-(U(I,T)-U(I,T-1)):LPRINT"RAINFALL OF";R;"CM FOR TIME STEP AT SAMPLE SITE";
I
600 LPRINT"AT SAMPLE SITE,";I;", NET DRAINAGE WAS ";Q(I);" CM/TIME STEP":LPRINT" ":NE
XT I
610 INPUT"TO INPUT NEW DATA FOR TIME STEP,TYPE 0";G:Y=0:T=T+1:MN=0:IF G=0 GOTO250
620 INPUT"TYPE 3 FOR TIME ZERO";G
630 FORJ=1TOT:FORI=1TOY:U(Y,T)=0:NEXTI,J:Y=O:T=1:IF G=3 GOTO250
640 END
```

APPENDIX C

EXAMPLE SOLUTION

JORY 14%

HORIZONTAL DISTANCE ACROSS THE PAGE. TIME IS 1

-16	-16	-16	-16	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0
-22	-22	-21	-21	-15	-15	-15	-15	-15	+0	+0	+0	+0	+0
-27	-25	-24	-23	-20	-19	-19	-18	-17	-15	-14	-14	-14	-14
-30	-28	-27	-25	-24	-23	-22	-22	-20	-19	-18	-18	-18	-18
-33	-29	-28	-27	-26	-26	-26	-25	-23	-22	-21	-21	-21	-21
-29	-27	-27	-26	-27	-27	-27	-27	-26	-25	-25	-24	-24	-28
-26	-24	-24	-25	-25	-25	-25	-25	-25	-26	-26	-26	-26	-30
-20	-21	-22	-23	-23	-22	-22	-22	-22	-23	-24	-24	-25	-29
+0	+0	+0	+0	-19	-19	-19	-18	-18	-20	-21	-21	-21	-24
+0	+0	+0	+0	+0	+0	+0	+0	+0	-18	-18	-17	-17	-17

HORIZONTAL DISTANCE ACROSS THE PAGE. TIME IS 2

-14	-14	-14	-13	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0
-16	-16	-16	-16	-13	-13	-13	-12	-12	+0	+0	+0	+0	+0
-15	-17	-18	-17	-15	-15	-15	-14	-14	-12	-12	-11	-11	-11
-24	-22	-21	-19	-18	-17	-17	-17	-16	-14	-14	-14	-14	-14
-32	-26	-24	-23	-22	-21	-21	-20	-18	-17	-17	-16	-16	-15
-29	-27	-26	-25	-25	-25	-24	-23	-22	-21	-20	-20	-20	-23
-26	-25	-25	-25	-25	-25	-25	-25	-24	-24	-24	-24	-24	-32
-23	-24	-24	-25	-24	-24	-24	-24	-24	-24	-24	-24	-25	-29
+0	+0	+0	+0	-22	-22	-22	-21	-21	-22	-23	-23	-23	-24
+0	+0	+0	+0	+0	+0	+0	+0	+0	-21	-21	-20	-20	-20

RAINFALL OF .56 CM FOR TIME STEP AT SAMPLE SITE 1
AT SAMPLE SITE, 1 , NET DRAINAGE WAS -.709856 CM/TIME STEP

RAINFALL OF .56 CM FOR TIME STEP AT SAMPLE SITE 2
AT SAMPLE SITE, 2 , NET DRAINAGE WAS -.373151 CM/TIME STEP

NEGATIVE SIGN SHOWS WATER STORAGE INCREASE (NO DRAINAGE)

APPENDIX D

PART I - HORIZONTAL HYDRAULIC CONDUCTIVITY MATRIX

```
+1.220+1.220+1.220+1.220+1.220+1.220+0.000+0.000+0.000+0.000+0.000+0.000+0.000+0.000
+1.330+1.330+1.330+1.330+1.220+1.220+1.220+1.220+1.220+0.000+0.000+0.000+0.000+0.000
+1.440+1.440+1.440+1.440+1.330+1.330+1.330+1.330+1.330+1.220+1.220+1.220+1.220+1.220
+1.600+1.600+1.600+1.600+1.440+1.440+1.440+1.440+1.440+1.330+1.330+1.330+1.330+1.330
+1.850+1.850+1.850+1.850+1.600+1.600+1.600+1.600+1.600+1.440+1.440+1.440+1.440+1.440
+3.850+3.850+3.850+3.850+1.850+1.850+1.850+1.850+1.850+1.600+1.600+1.600+1.600+1.600
+7.650+7.650+7.650+7.650+3.850+3.850+3.850+3.850+3.850+1.850+1.850+1.850+1.850+1.850
+7.300+7.300+7.300+7.300+7.650+7.650+7.650+7.650+7.650+3.850+3.850+3.850+3.850+3.850
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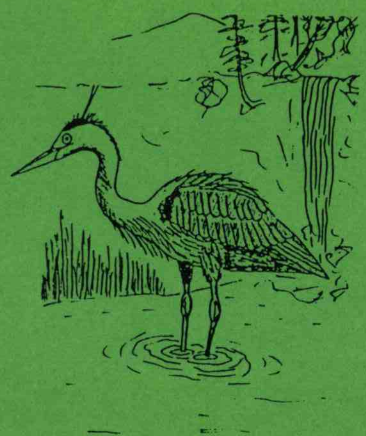
PART II - VERTICAL HYDRAULIC CONDUCTIVITY MATRIX

```
+1.200+1.200+1.200+1.200+1.200+1.200+0.000+0.000+0.000+0.000+0.000+0.000+0.000+0.000
+2.260+2.260+2.260+2.260+1.200+1.200+1.200+1.200+1.200+0.000+0.000+0.000+0.000+0.000
+2.350+2.350+2.350+2.350+2.260+2.260+2.260+2.260+2.260+1.200+1.200+1.200+1.200+1.200
+2.300+2.300+2.300+2.300+2.350+2.350+2.350+2.350+2.350+2.260+2.260+2.260+2.260+2.260
+2.260+2.260+2.260+2.260+2.300+2.300+2.300+2.300+2.300+2.350+2.350+2.350+2.350+2.350
+3.800+3.800+3.800+3.800+2.260+2.260+2.260+2.260+2.260+2.300+2.300+2.300+2.300+2.300
+5.350+5.350+5.350+5.350+3.800+3.800+3.800+3.800+3.800+2.260+2.260+2.260+2.260+2.260
+3.350+3.350+3.350+3.350+5.350+5.350+5.350+5.350+5.350+3.800+3.800+3.800+3.800+3.800
+0.000+0.000+0.000+0.000+3.350+3.350+3.350+3.350+3.350+5.350+5.350+5.350+5.350+5.350
+0.000+0.000+0.000+0.000+0.000+0.000+0.000+0.000+0.000+3.350+3.350+3.350+3.350+3.350
```

T H E   S E C R E T

W I T N E S S   F U N D

The Secret Witness Fund was established in January 1981 by the Audubon Society of Corvallis to try to reduce the intentional harming of protected nongame birds. While the full extent of this problem is not known, it is likely that thousands of protected birds are killed or injured in Oregon each year by people who don't know, or don't care about the consequences.



All birds, including predators such as hawks, owls, herons, and kingfishers are protected by Federal law -- Oregon statutes notwithstanding. The only exceptions are starlings and house (English) sparrows which may be taken at any time. In extreme cases birds may be killed if a depredation permit is first obtained.

Any intentional harm such as shooting, poisoning or trapping is considered a Class A misdemeanor punishable by a fine up to $2500 or one year in prison, or both.

The Secret Witness Fund, consisting entirely of private donations, offers reward money for information leading to conviction of persons responsible for killing, or attempting to kill, nongame birds. Anyone wishing to furnish information may do so anonymously. If a conviction follows, reward money will be paid in the manner suggested by the individual who gave the information. Rewards that

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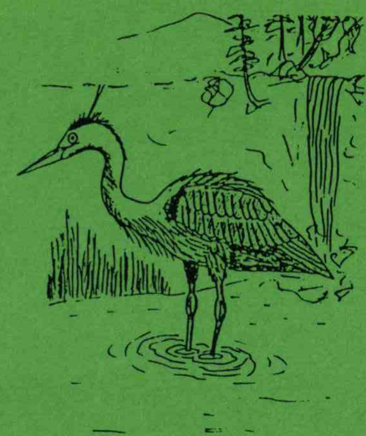

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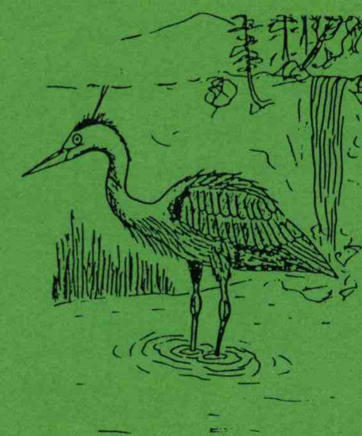

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