

## AN ABSTRACT OF THE THESIS OF

Benjamin T. Pharn for the degree of Master of Science in Nuclear Engineering presented on March 29, 2019.

Title: Incorporating Statistical Uncertainty into Nuclear Material Diversion Pathway Analysis using Probabilistic Risk Assessment

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A Level 1 probabilistic risk assessment (PRA) is applied to a plutonium button-making process, but in regards to the risk of diversion (or theft) of special nuclear material (SNM) rather than the risk of mechanical failure. The main objective of the project was to identify the relative vulnerability of areas in the process, with the motive of improving nuclear safeguards analysis by developing a quantitative measure for safeguards.

The unavailability of data concerning the failure rates of safeguards measures necessitated the use of random sampling through Python. Multiple distributions were considered to investigate the effect of the chosen distribution on the system's vulnerability. The relative distribution of diversion probabilities persists despite the change in the chosen distribution, with one area consistently being the most vulnerable and a different area consistently being the least vulnerable. Ideas for improvements and future work are also discussed.

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Incorporating Statistical Uncertainty into Nuclear Material Diversion Pathway  
Analysis using Probabilistic Risk Assessment

by

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I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

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Benjamin T. Pharn, Author

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# Incorporating Statistical Uncertainty into Nuclear Material Diversion Pathway Analysis using Probabilistic Risk Assessment

## 1 INTRODUCTION

Since its beginning, nuclear technology has been irrevocably linked to weapons of mass destruction and devastation. Any person familiar with the general history of the world recognizes the terms “nuclear” and “atomic” and immediately think about the bombing of Hiroshima and Nagasaki. While those bombings are the first and last times a nuclear device has been used on a country, it is understandable that the global populace is wary of nuclear technology.

One of the biggest fears across the world is that a nuclear weapon will be used again. While the world in general does not see nuclear weapons in a positive light, there are many state and non-state actors (e.g. terrorist organizations) that may seek to acquire and/or use a nuclear weapon. Some of these state actors already possess nuclear weapons and are recognized by the international community as nuclear weapon states. Others do not yet possess nuclear weapons but desire them, either for defense (through deterrence) or for use (against enemies). In order to ensure that these actors do not unlawfully acquire nuclear weapons, it is important that the global nuclear industry does its best to safeguard special nuclear material (SNM), defined as plutonium, the isotope uranium-233, or the isotope uranium-235, that could be used to develop nuclear weapons [4].

According to the International Atomic Energy Agency (IAEA), the objective of nuclear safeguards is “to deter the spread of nuclear weapons by the early detection of the misuse of nuclear material or technology” [1]. Most safeguards efforts are

focused on international safeguards at the state level, but it is also important to examine domestic safeguards and ensure that diversion and misuse does not occur at the facility level.

The purpose of this project is to develop a methodology that can quantitatively assess the effectiveness of domestic safeguards. This would be accomplished by taking a pre-existing quantitative methodology and adapting it for domestic safeguards to produce information about the probability of diversion from that safeguards system.

Four methodologies were examined. The first methodology is the Diversion Path Analysis (DPA). Developed in 1978 by the US Department of Energy (DOE), it was designed to evaluate the risk of diversion of a safeguards system. DPA was not chosen for two reasons: despite its use of mathematical formulas and numerical values, it is strictly a qualitative methodology, and there is a distinct lack of subsequent literature [3].

The second methodology is the Probabilistic Assessment of Safeguards Effectiveness (PASE) technique. Developed in 1991, it was a coordinated effort between the IAEA and the Australian Support Programme to employ probabilistic methods in the design of nuclear safeguards for large reprocessing plants. The PASE technique was not chosen for two reasons: it relies on a set of programs which are not publicly available, and, like DPA, also has a distinct lack of subsequent literature [5].

The third methodology is the Separations and Safeguards Performance Model (SSPM). In 2012, Sandia National Laboratory published a report detailing SSPM, which they developed for analyzing integrated safeguards and security systems. It makes use of Matlab Simulink to track mass flow rates of nuclear material, model

MC&A and physical protection, and simulate diversion by a non-violent insider. This would result in an analysis of the probability of diversion [6]. Comparisons between SSPM and this project's methodology are further discussed in Section 4.2.

The fourth methodology is Probabilistic Risk Assessment (PRA), which is the chosen methodology for this project. There are several reasons for this decision. While the methodology can be applied in non-nuclear fields, PRA was designed with nuclear problems in mind. It is a reliable methodology that has provided important, actionable safety insights and lessons, and the U.S. Nuclear Regulatory Commission (NRC) encourages the use of PRA in all nuclear regulatory matters [7]. This project seeks to take PRA, normally used for nuclear safety analysis, and apply its methodology to nuclear security in order to assess the vulnerabilities of a given process or system to diversion of special nuclear material.

For this project, PRA is performed using the SAPHIRE program and the results are processed in Python [8]. SAPHIRE is recognized in the nuclear industry as a reliable tool for performing PRA. Python is a powerful programming language used by professionals all over the world. These two software tools and their specific functions for this project are briefly described in this chapter.

## **1.1 Literature Review**

This section is intended to provide some background on both nuclear safeguards and PRA and discuss the tools used to apply PRA to nuclear safeguards. A brief history of nuclear safeguards and PRA is given, followed by a discussion of their significance to the project. Finally, the use of SAPHIRE, Python, and DPA in this project is described.

### ***1.1.1 Nuclear Safeguards***

The concept of nuclear safeguards began with Eisenhower’s “Atoms for Peace” speech in 1953, and the subsequent birth of the IAEA in 1957 marks the beginning of global nuclear security. As an organization independent from the United Nations, the IAEA’s mission is two-fold: promote the growth of peaceful uses of nuclear technology and suppress the spread of military uses of nuclear technology [9]. This mission was greatly enhanced by the signing of the Treaty on the Non-Proliferation of Nuclear Weapons (NPT) in 1968. A total of 191 member states of the U.N. joined the NPT, mutually agreeing to do their part to prevent the growth and spread of nuclear weapons.

Although it was never universally accepted (four members of the United Nations have never agreed to the NPT and one member has withdrawn from it), more countries have ratified the NPT than any other non-proliferation agreement [10]. The NPT attempts to further the peaceful uses of nuclear technology while limiting the growth of nuclear arms. Specifically, nuclear weapon states (NWS) will not “transfer to any recipient whatsoever nuclear weapons . . . and not in any way to assist, encourage, or induce any non-nuclear-weapon State to manufacture or otherwise acquire nuclear weapons” [11]. Non-NWS will not “receive the transfer from any transferor whatsoever of nuclear weapons . . . not to seek or receive any assistance in the manufacture of nuclear weapons” [11]. In addition, each state agrees to accept nuclear safeguards in order to prove to the other members of the Treaty that they are properly adhering to their obligations. The IAEA would make individual agreements with each state, dependent on their respective levels of nuclear technology and expertise, to ensure that each state would have fair and thorough safeguards [11].

Since then, the IAEA safeguards system has evolved to become a critical part of global nuclear security. In 2017, 181 states had some level of safeguards agreement with the IAEA. 127 states had both comprehensive safeguards agreements (CSAs) and additional protocols (APs) in place, while 46 states had CSAs but no APs. The 5 NWS had voluntary offer safeguards agreements and APs in place, and the remaining 3 states chose to maintain the old safeguards agreement based on INFCIRC/66/Rev.2, a document that detailed the IAEA's safeguards system in 1968 [12]. A CSA is a formal agreement between a state and the IAEA, where the state agrees to accept IAEA safeguards for all peaceful nuclear activities and the IAEA will verify that nuclear material is not diverted to make nuclear weapons [13, 14]. An AP is a document that complements a CSA by granting additional tools and power to the IAEA and significantly improving its ability to inspect and verify [15]. CSAs and APs serve the purpose of legally granting access to the IAEA to inspect the nuclear facilities, which in turn provide a neutral third party that can confirm that a state is acting in line with the NPT. It should be noted that CSAs are only made by non-NWS, as they are intended for *peaceful* nuclear activities. NWS instead have made voluntary offer safeguards agreements, which applies to facilities the state has voluntarily offered for safeguarding and the IAEA has selected for the application of safeguards. This distinction allows NWS to protect military nuclear secrets while still fulfilling their obligations to the NPT [14].

The IAEA defines safeguards as a set of technical measures that help the IAEA to independently verify a state's legal obligation to the NPT. The implementation of IAEA safeguards follows an annual cycle with four main processes, shown in Figure 1. The first process is the collection and evaluation of safeguards-relevant

information, which the IAEA reviews to evaluate a state’s consistency with its declarations about its nuclear program. The second process is the development of a safeguards approach for the state, including measures to achieve the technical objectives for verification. The third process is the planning and execution stage, where the IAEA develops a plan specifying the safeguards activities for both the state and the IAEA, implements the plan, and reevaluates the plan if adjustments are necessary. The fourth (but not final) process is the analysis stage, where the IAEA makes conclusions about the state’s fulfillment of its legal obligation to the NPT and provides credible assurance that the state is abiding by its obligations [1].

This definition of safeguards covers safeguards measures at the state level, or



Figure 1: Main steps in safeguards implementation [1]

international safeguards. For the U.S. in particular, there is a second definition of safeguards. The NRC defines domestic safeguards as “ensuring that special nuclear material within the United States is not stolen or otherwise diverted from civilian facilities for possible use in clandestine fissile explosives and does not pose

an unreasonable risk owing to radiological sabotage” [16]. The focus of the research described in this thesis falls within this context of “domestic safeguards”.

A system of safeguards can be divided into three basic subsystems: physical protection, material control, and material accountability. Physical protection covers measures such as mechanical or electronic locks and doors. Material control consists of instrumentation, such as seals, cameras, and detectors, to detect and prevent unauthorized movements of nuclear material. Material accountability involves thorough documentation of permitted movements and inventories of nuclear material in order to detect if nuclear material is being diverted. By comparing the inventories with the records, it is possible to detect diversion of nuclear material. If diversion is detected, the records can provide accurate information to aid in its timely location [17].

Nuclear safeguards fill an important role in modern society. The threat of nuclear weapons is not to be taken lightly, and numerous measures have been taken to reduce the possibility of a nuclear detonation. However, the nature of nuclear security implies that these measures are untested and unproven; qualitative only, as we have no publicly known occurrences of attempted theft. We cannot be confident that these safeguards can detect or prevent diversion if we do not have any historical data on their effectiveness. In order to answer that question, we will briefly examine the history of nuclear safety, which ran into the same concerns in its early stages of development.

### ***1.1.2 Difficulties of Early Nuclear Safety Analysis***

The development of PRA had a rocky start. In 1953, General Electric Hanford’s statistics director wrote a memorandum proposing a probabilistic approach

to safety. It described a “chain of events” of small malfunctions and mistakes in a reactor that could lead to an accident occurring. These events could be individually examined and then combined to obtain the probability of that accident. However, GE struggled with their research, and it wasn’t until the late 1960s when the development of this approach made some headway. At this time, the nuclear industry became interested in fault trees, which was a relatively new methodology that had seen use in the aerospace and airline industries. GE changed tracks and started researching how to apply fault tree analysis for nuclear safety [18].

As research of fault trees progressed, problems with their application in the nuclear field were observed. Due to the infancy of the method, the numbers used in fault trees had very large uncertainties. This led many people in the nuclear industry to have doubts about the reliability of the technique [18]. Even the WASH-740 report in 1957, the first report about the risk of a civilian nuclear power plant, had dramatically large ranges for the estimated casualties and financial costs due to uncertainty [19]. This uncertainty came from the lack of reliable or quantitative science, forcing the analysis to be based off expert judgment and opinion. The disappointing results of the WASH-740 report led to a follow-up study that was known as WASH-1400 [18].

WASH-1400, also known as the “Reactor Safety Study” or the “Rasmussen Report”, introduced the methodology that would later develop into PRA. WASH-1400 built on its predecessor’s methodology by incorporating event trees. WASH-740 relied solely on fault trees, which could not sufficiently characterize complex accident sequences. By making use of event trees, WASH-1400 was able to rectify this issue. This enabled WASH-1400 to make sufficiently satisfactory conclusions about the risk that nuclear power plants posed to the public [20]. However, WASH-1400 still

suffered from the same problems of insufficient supporting data and large uncertainties in the probability estimates used, which remained a major criticism [18]. In response to this, the WASH-1400 stated that there is a difference between reliability assessment, which relies on highly accurate data, and risk assessment, which does not. In risk assessment, the objective is not the *absolute* magnitude of risk, but the *relative* magnitude of risk compared to the normal level of risk. The reason it is sufficient to use data with any level of accuracy is because the results must be examined “to see if they are meaningful” [20].

### **1.1.3 Probabilistic Risk Assessment**

PRA is a systematic methodology used to assess the risk of mechanical failure in a given system [2]. Risk is defined by the NRC as the probability of an accident and the consequences of the accident if it occurs [21]. PRA can be simply described as a risk assessment methodology that answers three basic questions:

1. What are the possible steps that can lead to an undesirable outcome?
2. What is the probability or likelihood of an undesirable outcome occurring?
3. If an undesirable outcome occurs, what are the potential consequences?

The PRA method involves creating a model of a mechanical system, asking these three questions, and determining the answers. Once these questions are thoroughly answered, the risk of the system is known and its vulnerabilities are identified [2].

There are several benefits to using PRA for risk assessment. PRA provides a consistent, quantitative measure of the risks in a given system. It considers both mechanical influences and human reliability when assessing risk and explicitly

includes uncertainty when performing calculations. It also presents a measure we can use to compare and rank the absolute or relative importance of system components, allowing the comparison of two or more systems that are significantly different. All of these allow PRA to provide a quantitative way to judge the overall health and safety of an engineered system [2].

PRA has three levels which differ in the scope of the analysis. A Level 1 PRA looks only for the probability or frequency of an accident. A Level 2 PRA considers the immediate short-term effects that happen if an accident occurs. A Level 3 PRA includes an analysis of the long-term consequences that may occur as a result of the effects [22]. For nuclear power plant applications, a Level 1 PRA evaluates the probability of core damage. A Level 2 PRA enhances a Level 1 PRA by considering radioactivity and the operation of the containment system in order to estimate the the amount and type of radioactivity release from containment, i.e. containment failure. A Level 3 PRA takes this information, the material release magnitude, and examines the offsite consequences, e.g. dose to the public, early and cancer fatalities, contamination of the land, etc [22,23]. Since the purpose of this project is to adapt a Level 1 PRA methodology, the remainder of this section will go into more detail about Level 1 PRA but not Level 2 or Level 3 PRA.

A Level 1 PRA is made up of three types of components: initiating events, event trees, and fault trees. An initiating event is an event that triggers a response from the system and has the potential to progress into an accident. There can be multiple initiating events for a single system, and each one needs to be analyzed in order to accurately assess the risk [22].

An event tree shows the various accident sequences that stem from a single initiating event. It also designates the state of the system at the end of each

accident sequence. Each of the system's components is represented on the event tree, and the reliability of each component contributes to the probabilities of the accident sequences. An event tree's overall purpose is to show how an initiating event can progress into an accident [22].

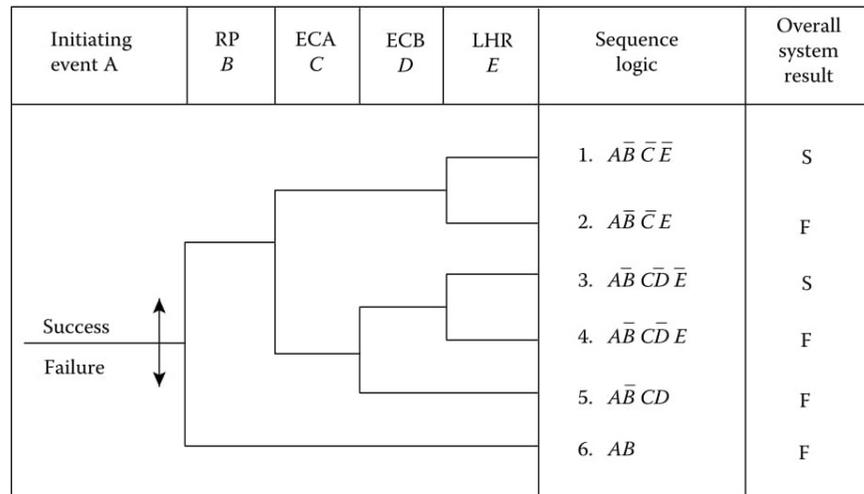


Figure 2: Sample Event Tree [2]

Figure 2 is a sample event tree. The events are as follows:

RP = Operation of the reactor-protection system to shut down the reactor

ECA = Injection of emergency coolant water by pump A

ECB = Injection of emergency coolant water by pump B

LHR = Long-term heat removal

The model begins at the initiating event A and progresses from left to right. At each branch, a decision is made about whether this event occurs, depending on the component's reliability. If the component successfully functions (i.e. the event does not occur), the sequence moves upwards. If the component fails to function (i.e. the event occurs), the sequence moves downwards. This continues until the

sequence reaches the end of the tree.

Each sequence has a logical representation and an overall system result, or endstate. The sequence logic indicates whether or not an event has occurred, and the endstate indicates whether the overall system has succeeded or failed. For example, the probability of sequence 1 is

$$A * \bar{B} * \bar{C} * \bar{E}$$

which can also be stated as the occurrence of event A and the non-occurrence of events B, C, and E. This sequence leads to a successful result, meaning the system has not been compromised.

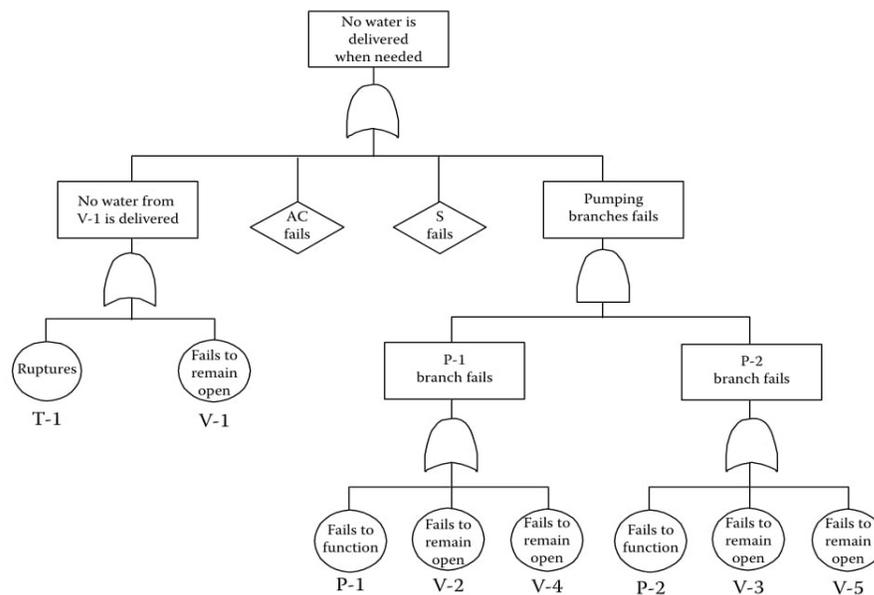


Figure 3: Sample Fault Tree [2]

A fault tree models the functions and response of a operation to calculate the probability of that operation failing through the use of logic gates. Figure 3 is an example of a fault tree. Fault trees can be used qualitatively to show the possible

combinations of component failures that cause the overall operation to fail. They can also be used quantitatively to calculate the probability of the operation failing. The purpose of a fault tree is to determine the overall probability of failure of a system based on the probabilities of failure of its components [22].

Fault trees can incorporate both mechanical and human components. Graphically, they make use of various symbols to indicate the logic at each of its branches. A list of some symbols is shown in Figure 4.

There are three types of symbols: events, gates, and transfers. Events are the component failures that can contribute to the occurrence, or failure, of the top event. The term “top event” is used to differentiate the operation being modeled from “basic events”, which are the various components within the operation. Gates prevent the further development of events unless a condition specific to the gate, its logic, is fulfilled. Transfers indicate that the section of the fault tree is developed elsewhere. This can be used to represent duplicate branches or to indicate a complex branch that benefits from having its own fault tree for evaluation.

A completed fault tree can be analyzed and divided into cut sets. Cut sets are collections of basic events that, when they occur, cause the top event to occur. A minimal cut set is the smallest possible collection of basic events, such that each event is necessary in order for the top event to occur. For example, the minimal cut sets from the earlier example are {T-1}, {V-1}, {P-1, V-3}, and {P-1, V-5} [22].

Cut sets are important for identifying and delineating combinations of basic events that cause failure, which may be more difficult to notice in complex fault trees. This also allows us to recognize common causes, where a single condition or event causes multiple basic events. This knowledge helps us locate vulnerabilities in the system, both quantitatively and qualitatively, as probabilities are

### Primary Event Symbols

	BASIC EVENT—A basic event requiring no further development
	CONDITIONING EVENT—Specific conditions or restrictions that apply to any logic gate (used primary with PRIORITY AND and INHIBIT gate)
	UNDEVELOPED EVENT—An event which is not further developed either because it is of insufficient consequence or because information is unavailable
	EXTERNAL EVENT—An event which is normally expected to occur

### Intermediate Event Symbols

	INTERMEDIATE EVENT—An event that occurs because of one or more antecedent causes acting through logic gates
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### Gate Symbols

	AND—Output occurs if all of the input events occur.
	OR—Output occurs if at least one of the input events occurs
	EXCLUSIVE OR—Output occurs if exactly one of the input events occurs
	PRIORITY AND—Output occurs if all of the input events occur in a specific sequence (the sequence is represented by a CONDITIONING EVENT drawn to the right of the gate)
	INHIBIT—Output occurs if a single event input to produce output only if a CONDITIONING EVENT input is met
	Not—OR—Output occurs if at least one of the input events does not occur
	Not—AND—Output occurs if all of the input events do not occur

### Transfer Symbols

	TRANSFER IN—Indicates that the tree is developed further at the occurrence of the corresponding TRANSFER OUT (e.g., on another page)
	TRANSFER OUT—Indicates that this portion of the tree must be attached at the corresponding TRANSFER IN

Figure 4: List of Fault Tree Symbols [2]

calculated from cut sets and design flaws can be identified when similar cut sets are compared [22].

In PRA, fault trees are used to determine the failure probabilities of events in an event tree. Cut sets can be used in the event tree's sequence logic to determine the exact ways each endstate is formed. For example, if event B has 5 cut sets, event C has 2 cut sets, event D has 3 cut sets, and event E has 6 cut sets, the list of cut sets for sequence 4 would be

$$A * C_1 * E_1$$

$$A * C_2 * E_1$$

$$A * C_1 * E_2$$

$$A * C_2 * E_2$$

$$A * C_1 * E_3$$

$$A * C_2 * E_3$$

and so on, where each variable would be replaced by the respective groups of component failures. Each sequence has a set of cut sets that lead to its endstate, and would be listed in a report for further analysis.

Probabilistic risk assessment is a straightforward methodology that can provide reliable results. While the bulk of the work lies in developing the event and fault trees, depending on their complexity, performing the actual assessment may be very time-consuming and tedious work. The use of computer processing power allows for the tedious work to be done by machines, making the overall process easier and faster.

#### **1.1.4 Introduction to SAPHIRE and Python**

SAPHIRE is a computer program designed to perform PRA. SAPHIRE stands for “Systems Analysis Programs for Hands-on Integrated Reliability Evaluations”. It was designed for the Nuclear Regulatory Commission by the Idaho National Laboratory, who continue to develop the software. SAPHIRE facilitates the work needed to perform a PRA by taking advantage of computer processing power to create graphical event and fault trees and significantly reduces the analysis time [24]. A user can supply the basic event data and build the event trees and fault trees, and then make the computer solve the trees, perform uncertainty analyses, and generate reports for further analysis. While SAPHIRE was designed with nuclear applications in mind, the program is flexible enough to be used to analyze any complex system, facility, or process [8].

SAPHIRE can be used to perform a Level 1 PRA, a Level 2 PRA, or (to a limited extent) a Level 3 PRA. It has numerous features and functions that aid in the development of a PRA, which are listed below [8].

- Graphical fault tree construction
- Graphical event tree construction
- Rule-based fault tree linking
- Fast cut set generation
- Fault tree flag sets
- Failure data
- Uncertainty analysis
- Cut set editor, slice, display, and recovery analysis tools
- Cut set path tracing
- Cut set comparison

- Cut set post-processing rules
- Cut set end state partitioning
- End state analysis
- User-defined model types
- User-defined basic event attributes

One of the key features of SAPHIRE is its ability to generate cut sets and cut set reports. SAPHIRE is able to quickly generate and list cut sets from the model. These lists can be filtered to show certain cut sets that fit a specific criteria, such as those from a particular fault tree, that reach a particular endstate, or have a specified probability or greater. Once the list of cut sets has been finalized, SAPHIRE can publish the list as a Cut Set Report to a variety of file types. This makes it possible to use the Cut Set Report for further analysis [25].

Python is a well-established, high-level programming language [26]. It is fast and powerful, being able to “achieve superior results in significantly shorter timescales” when compared to other modern programming languages such as Java or C [27]. Python is simple, easy to learn, and easy to debug, making it particularly attractive for beginning programmers. Python supports all major operating systems and supports modules and packages, allowing users to develop code for specific uses and easily reuse or share software [26]. In this project, Python is used to handle both the input and output data of SAPHIRE.

### ***1.1.5 Diversion Path Analysis Methodology***

DPA is an evaluation methodology developed by the DOE to rank and classify the operations of a nuclear-related process by the risk of diversion, or theft, that each operation presents. It can “determine the vulnerability of the material

control and material accounting (MC&MA) subsystems to the threat of SNM by a knowledgeable insider” and evaluate the capability of the subsystems to detect the loss of SNM. Using DPA, facility personnel can systematically determine:

- How, from a adversary’s point of view, to covertly acquire SNM and conceal the theft
- How soon, if ever, the theft would be noticed
- What modifications, if any, could eliminate or reduce the severity of the vulnerability [3]

A DPA specifically addresses the diversion of SNM by a person who has access to the process area and/or the material, i.e. an Insider. It is not meant to assess the threat of diversion by an outside agent, the threat of sabotage to the facility, or the threat of dispersal of the SNM. It also does not address the removal of the SNM from the facility site; the fact that SNM can be removed from its authorized location should be enough cause for alarm [3]. In this way, it is similar to a Level 1 PRA in that it is only concerned with the risk of an event occurring rather than a Level 2 or Level 3 PRA, which considers the potential consequences of the event.

A DPA evaluates the MC&MA subsystems of a process but does not address the physical protection subsystem. It presumes that physical protection methods such as locks fail when an Insider employs deceit and/or stealth to divert SNM. It also does not address diversion by upper-level management of the facility, as their access to the MC&MA subsystems may provide them with the ability to completely conceal any diversion and escape detection, thus going unnoticed by a DPA [3].

The DPA methodology consists of five stages, as shown in Figure 5. These stages can be summarized as: learn and gather information about the process, ex-

amine, organize, and classify the process and dividing it into smaller unit processes, assess and analyze the risk of diversion from the unit processes, collecting and sorting the results to determine the findings and recommendations, and documenting everything into a final report [3].

The second and third stages are of particular importance. The “Process Characterization” stage involves the division of the process into smaller unit processes with the purpose of simplifying the following “Analysis” stage. A unit process can be defined as a segment of the overall Process where: the SNM physically or chemically changes; a material flow starts, ends, or merges with another flow; or significant material accounting information is generated. The description of each unit process includes information on the material flows, the information flows, and the personnel responsibilities involved in the unit process. Essentially, each unit process needs to be fully and accurately described so that the analysis can be done properly [3].

The “Analysis of Diversion Paths” stage involves the members of the DPA team mentally stepping into the shoes of the adversary and looking for diversion paths. No assumptions are made about the adversary’s intelligence, motivation, or rationality, only that the adversary believes they can successfully divert SNM. The flow chart used to guide the DPA team is shown as Figure 6. Every component of each unit process is examined in order by the DPA team to discern what, if any, paths an adversary can use to divert SNM [3].

These paths are known as specific diversion paths (SDPs). Once an SDP is identified, the DPA team determines: the first abnormal situation (where the SNM is recognized as missing) guaranteed to occur, the person who will observe that abnormal situation; the maximum detection time for that abnormal situation, any

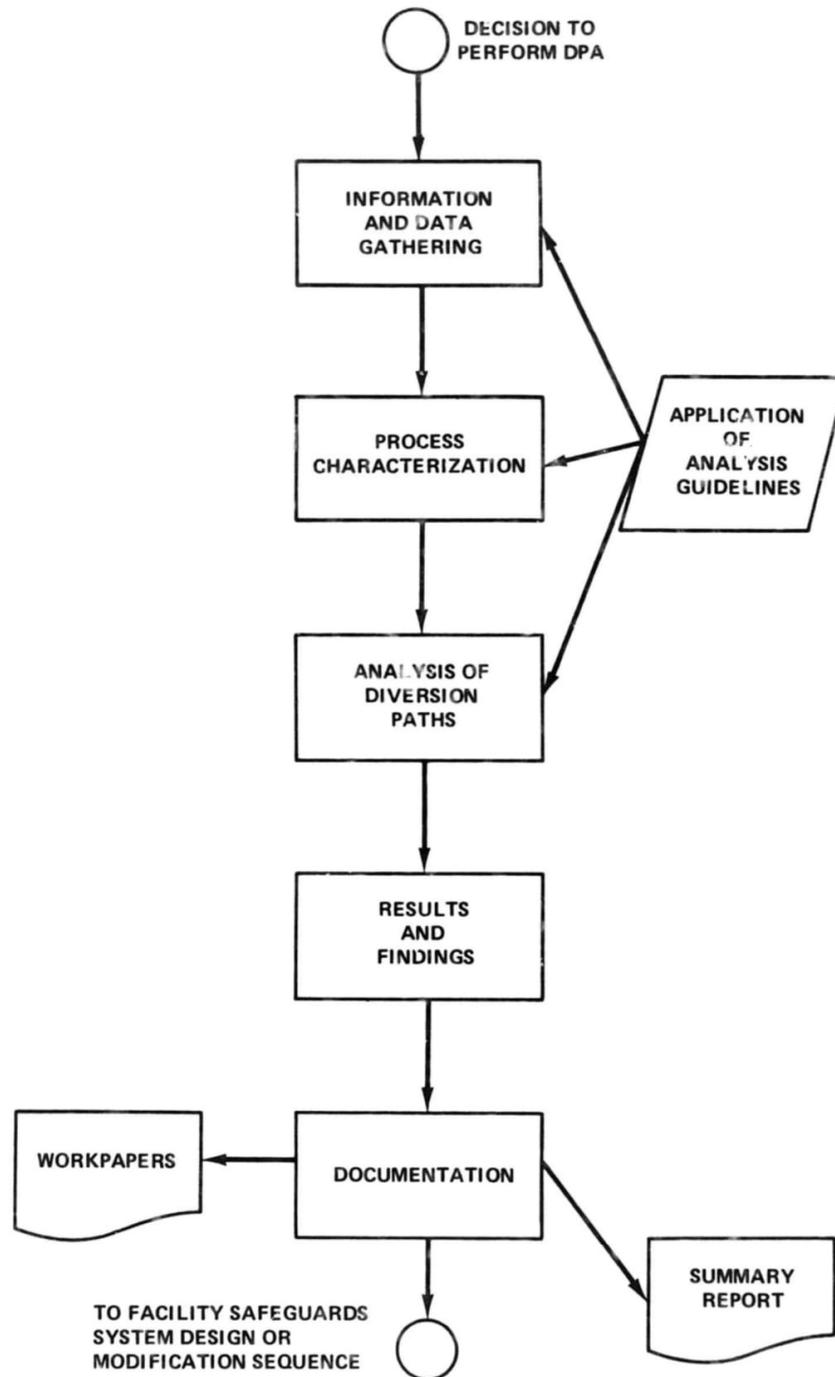


Figure 5: Basic Steps of Diversion Path Analysis [3]

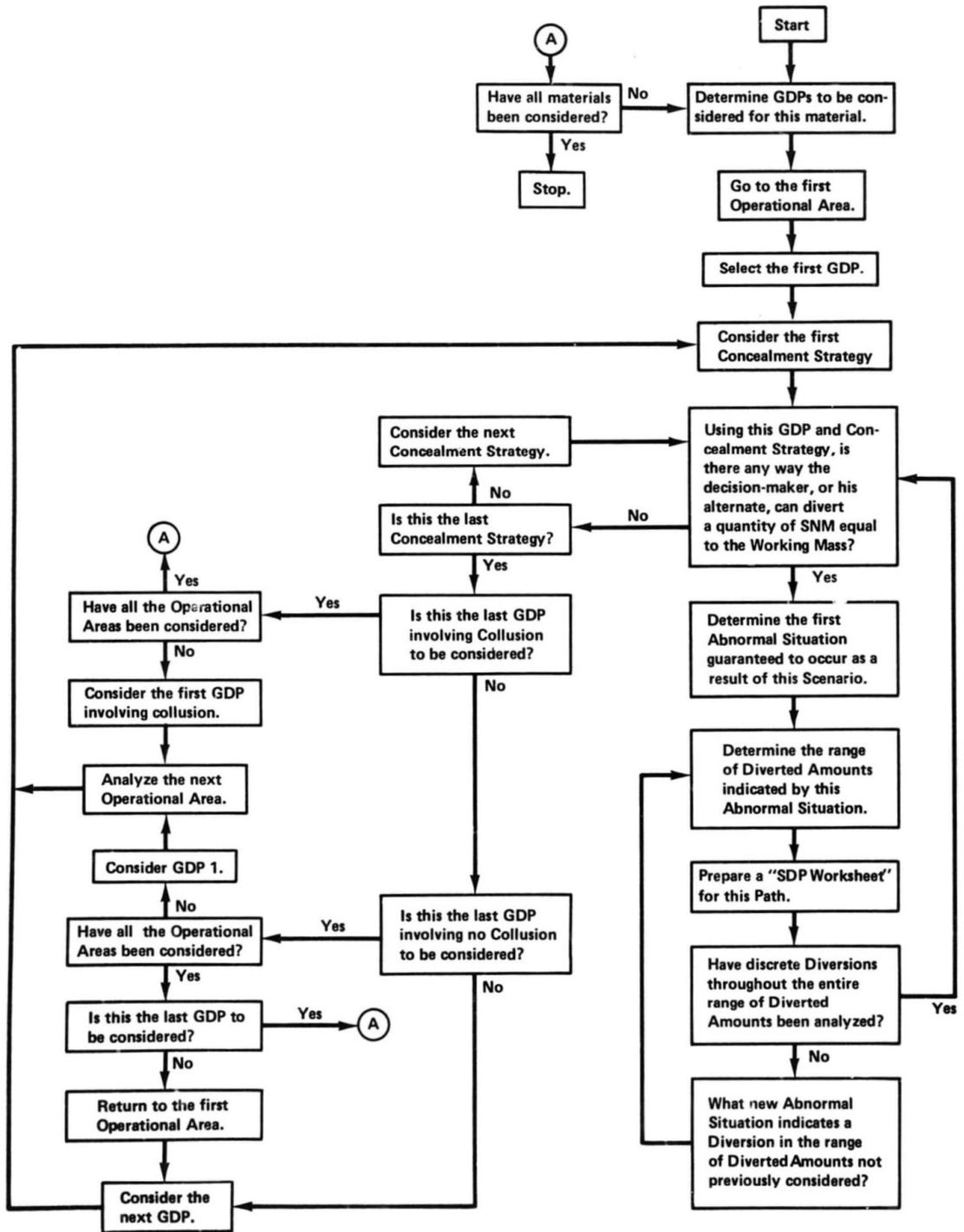


Figure 6: Diversification Path Analysis Flowchart [3]

possible Innocent Cause (where the abnormal situation occurs but not because of diversion) for that abnormal situation, and if possible, a minor modification which would eliminate the SDP or significantly reduce the detection time. This information is recorded on a separate worksheet for each SDP and collected for further analysis [3].

The concept of the SDP is based on the general diversion paths (GDPs), which are used to “assure uniformity among analyses from different facilities and provide all DPA teams with a common basis for performing a thorough analysis” [3]. Both the GDPs and the SDPs are characterized by six diversion path parameters:

- Material attractiveness
- Diverted amounts
- Deceit by records
- Deceit in removal
- Number of insiders, and
- Type of insider

Each parameter has several attributes, each of which have been assigned a relative weight factor. These relative weight factors denote the relative complexity of diversion, with the exception of material attractiveness where the factor denotes the relative attractiveness of the SNM. They can be multiplied together to form the relative path weight, which is used by DPA to rank the SDPs and assess the relative vulnerabilities of the Process [3].

The relative weight factors and the relative path weight should not be viewed as measures of the probability of diversion, only as relative measures of the difficulty

of diversion [3]. This is a critical caveat and has important implications for this project. These implications are discussed in Section 2.3.1.

The final product of a DPA is a report that collects the vulnerabilities of the MC&MA subsystems. It consists of two parts: a workpaper documentation and a summary documentation. The workpaper documentation contains all of the information, results, analysis, and other referenced data that support the conclusions made by the DPA. The summary documentation provides an overview of the security posture of the analyzed process, including any assumptions made, the diversion paths identified, the proposed modifications to the process, and the recommendations made [3].

#### **1.1.6 Research Objectives**

The main goal of this project is to incorporate statistical uncertainty to DPA using PRA. In support of this goal, several research objectives have been defined that direct the project's focus and analysis.

1. Develop a model to quantitatively characterize the probability of diversion from a given system.
2. Identify the areas that are the most vulnerable and the least vulnerable to diversion.
3. Determine the extent of the effect of random sampling on the results through statistical analysis.

The first objective is the central motive of the project. By quantitatively characterizing a system, it will be possible to make meaningful comparisons between two or more unlike systems, such as whether or not one system is objectively

better than another at preventing diversion. The second objective ties into the first, as the intention of this methodology is to identify the vulnerabilities of a system. This would allow the analyst to determine areas that are in need of attention or improvement. The third objective stems from the project's reliance on random sampling, which is discussed in Section 2.3. The use of random sampling means it is necessary to examine its significance on the results. These research objectives help guide the direction of the research and strengthen the significance of the project.

## 2 METHODS

### 2.1 Introduction

The goal of this project is to incorporate statistical uncertainty into diversion pathway analysis using probabilistic risk assessment. When PRA is used to analyze a mechanical system, the system is modelled in a way that the only variables are the failure probabilities of the safety subsystems and components. It should be logical that one can apply this same concept to the material control & accounting systems. The failure of safety components can be replaced by the failure of MC&A measures. A fault tree can be used to determine the overall failure rate of an area's security, and an event tree can determine the frequency that SNM is diverted. In this chapter, we discuss the methods we took to fulfill our objective.

### 2.2 Building a Model

The first step in performing PRA is to create a model of a system. We modelled the process described in volume 2 of the DPA Handbook [28]. We made this decision for two reasons. The first reason is that the process was used in an example showcasing DPA. Using the same process would allow us to make use of the information in the example. The second reason is that we were severely limited in what processes we could model. We did not have access to information about actual processes, nor did we have the knowledge and expertise necessary to fabricate a realistic one. As a result, the process used in the example DPA would be the same process used in this example PRA.

### **2.2.1 Example Process**

The process is a production line where plutonium metal bars are processed into buttons for storage or shipping. This process is a fictitious process, due to classification concerns, and was designed for use in an example for DPA. [28]. The fact that this example does not model an actual process does not detract from the validity of it, however. This topic is further discussed in Section 4.1.1.

Each bar is a single solid piece of plutonium metal, weighing 600 grams and measured at 200 mm by 50 mm by 3 mm. Each bar has a 5-digit serial number hand-scribed into it [28]. These bars are stored in an undefined location outside of the process until they are used.

The process consists of four operations: Receiving, Bar Chopping, Casting, and the Vault. Receiving is where a specified number of bars are transferred into the Receiving Box by the Receiving Box Operator. The bars remain in the Receiving Box until they are needed by the Bar Chopping Operator, who transfers one bar at a time into the Bar Chopping Box and then chops the bar into small pieces. These pieces are then placed in a sealed bag and then into a sealed canister before being transferred to the Casting Box. Once four cans are in the Casting Box, the Casting Operator collects, mixes, and casts the plutonium contents together into a single 2400 gram button. Once the button is complete, a 0.5 gram sample is drilled from the cast, and both the button and the sample are bagged and canned separately before being delivered to their respective locations. The sample is sent to the Lab, while the button is sent to the Vault for storage [28].

The Handbook did not fully describe the measures used to safeguard the Process. There were no descriptions about the material control measures, but the material accounting measures were briefly detailed. Each transfer and process of

the plutonium metal includes a report or a log that each Operator is responsible for filling out accurately. This information could be the serial number of each bar used or the weight of the bar pre- and post-operation. Once these workpapers are filled out, they are collected by the Foreman and reviewed [28].

The analysis done in the DPA Handbook Volume 2 includes the final set of reports of the DPA, as described in section 1.1.5. These reports are the source of the data and information used in this project.

### **2.2.2 Event Tree**

In order to develop the event tree, it is necessary to understand both the process and the material and information flows of the process. One of the easiest ways to do this is to create a flowchart. Since the DPA methodology involves documenting the material and information flows in diagrams, it is a simple step to re-assemble the given information into a block diagram.

There were many pages in the report that indicated the flow of material (SNM) and/or information (MBA). In creating the block diagram, we relied on what we believed was the clearest and most thorough pages of information, which were the “Material, Information, and Activities Worksheet(s)”. There was one of these worksheets for each operational decision maker (the worker in a given Unit Process in charge of handling the MBA documents). Each worksheet listed the incoming and outgoing flows of both the SNM and the MBA documents, including the sources and destinations for each item. Using these worksheets, we were able to make a block diagram with little hassle, shown as Figure 7. The grey boxes are the Unit Processes, and the materials and information are on the left and right halves respectively. The block diagram was compared with the various flow diagrams also

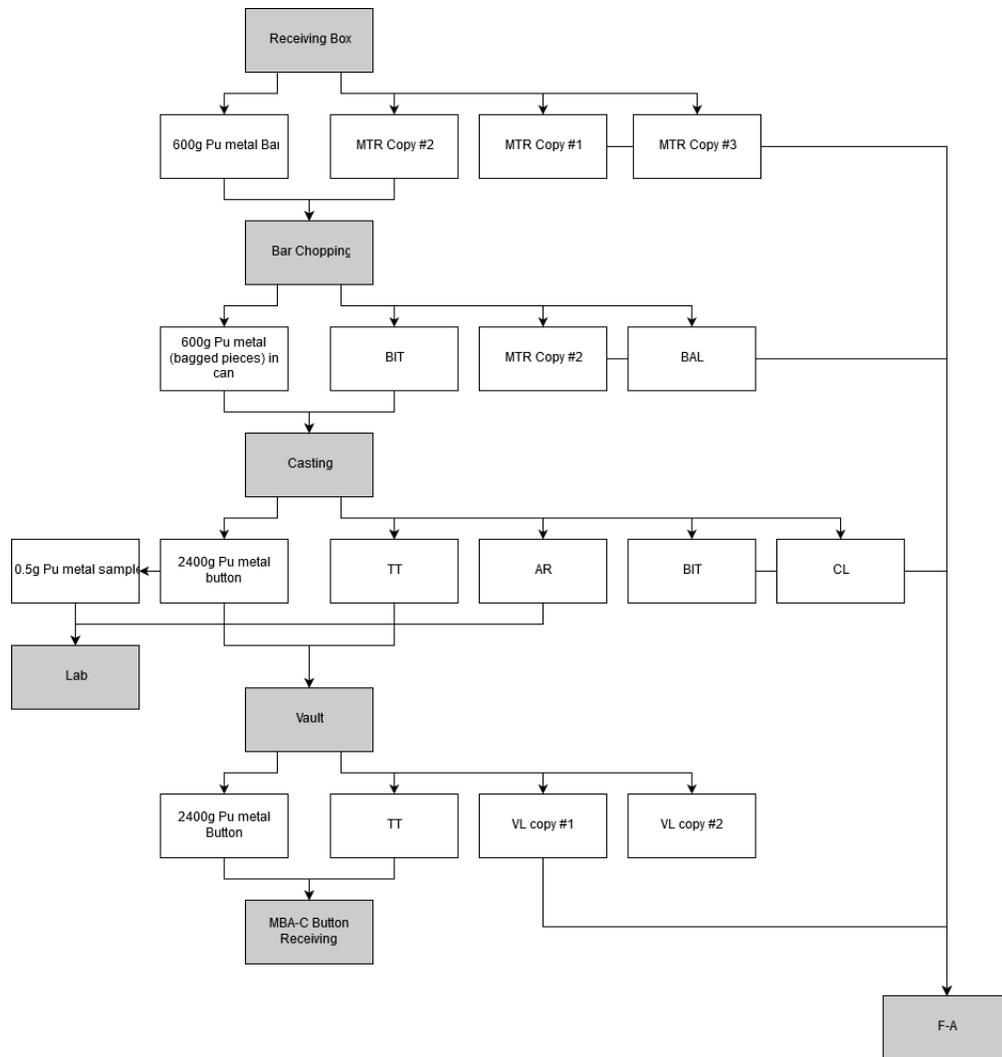


Figure 7: DPA Block Diagram

included in the DPA report to verify its accuracy.

Completing the block diagram made it an easy matter to design the event tree. The resulting event tree is shown as Figure 8. The Initiating Event is the beginning of the process, where the plutonium bar enters the system. Each event is a unit process where SNM has the potential to be diverted. A success means that the plutonium bar is able to traverse through the respective unit process safely, whereas a failure means that some or all of the plutonium was diverted.

In this design, the MC&A measures are placed in the fault trees. Taking this into consideration, each individual area or unit process was split into two top-level events, prefixed by either “MC-” or “MA-”. This allows us to distinguish between the Material Control subsystem and the Material Accounting subsystem. It is important that these two subsystems are distinguished in some manner. Although they are closely linked, they are two distinct and independent systems with unique measures. There could be situations where the Material Control measures are unable to prevent Diversion but the Material Accounting was sufficient to catch it. The reverse is also possible: where the Material Accounting detects a Diversion but none has occurred. These of course are in addition to the situations where both sets of systems either succeed or fail to prevent Diversion from occurring. Since each system can arrive at a result independent of the other, it is necessary that they remain separate from the other.

This model assumes that *any* diversion of SNM is unacceptable. This simplifies the model in two ways. The first is that if any event occurs, it leads directly to a failure endstate; no other calculations need to be made. The second is the implication that an adversary does not attempt to divert SNM from more than a single area. These simplify the model by removing unnecessary branches in the

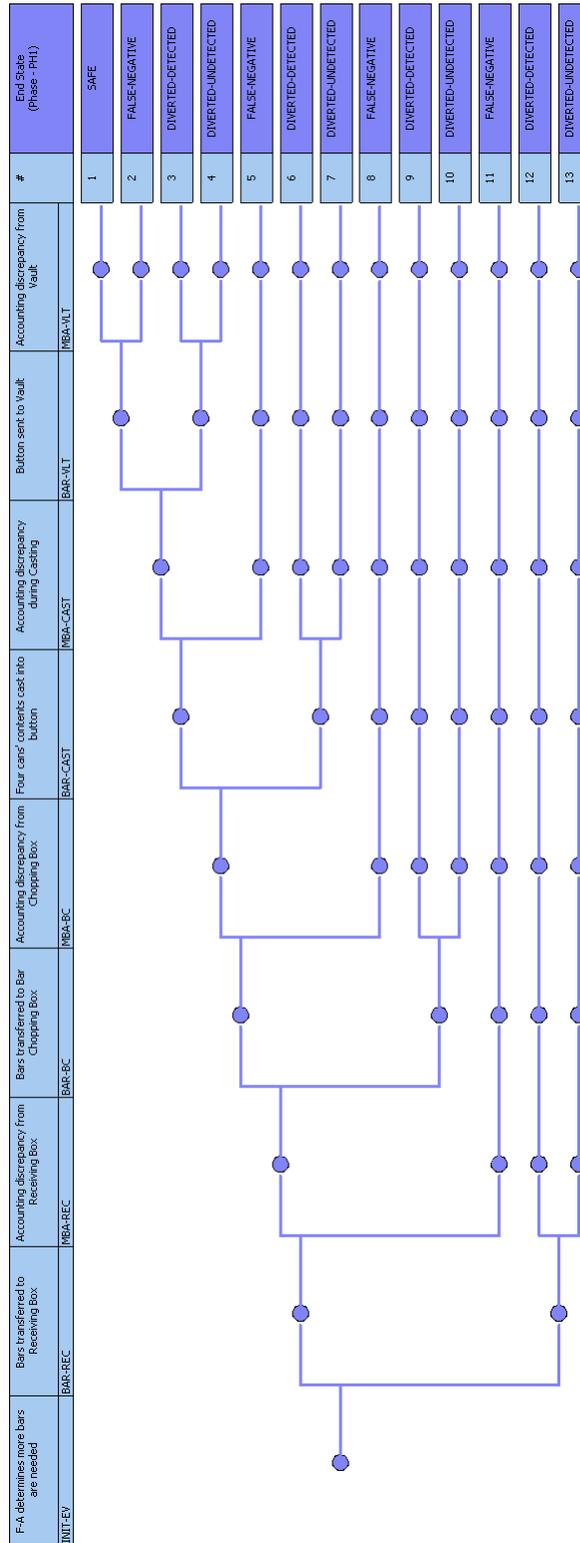


Figure 8: Event Tree

event tree.

### 2.2.3 *Fault Tree*

After the design of the event tree comes the design of the fault trees. The unit processes are the systems that will be analyzed using fault trees: one each for the Material Control subsystem (MC) and the Material Accounting subsystem (MA).

For the MC events, we substituted the use of basic events based on component failures with the use of basic events based on the SDPs from the example DPA. This decision was made due to the lack of information on the material control measures. The MC system for each unit process became a black box that we could not examine or replicate, a necessary task for designing a fault tree. However, by making use of SDPs, we can ignore those details and directly examine the outcomes.

DPA looks at each unit process and identifies every possible scenario where SNM can be diverted. This can be written as

$$\sum_{i=1}^n ARPW_i \tag{1}$$

where  $n$  is the range of all SDPs in a given area.

If we assume that the SDPs represent every possible path of diversion, we can logically say that if any diversion occurs, it must follow one of the SDPs. This means we know every possible cause of failure for each area. Then, if each SDP has a probability of occurrence, it is possible to use the SDPs in a fault tree as basic events. As an example, Figure 9 shows the fault tree for the MC-REC event.

The usage of SDPs will only be applicable to the MC- events, however, as the SDPs represent the physical removal of SNM. A different design is required for the

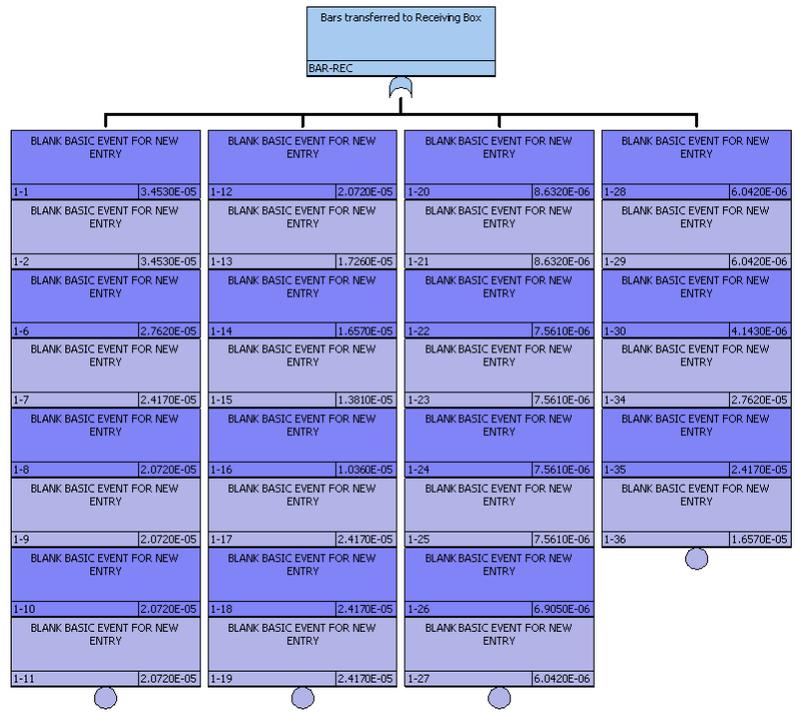


Figure 9: Fault Tree (MC-REC)

MA- events. The MA- events represent the failure of a person in verifying the relevant records. SAPHIRE has several failure models built into the program, one of which is the SPAR-H Human Reliability Analysis failure model. This methodology, and its use, are discussed in 2.3.2.

### 2.3 Probability Data

With the model finalized, the last issue is determining the probabilities of failure. The DPA did not assign the SDPs any probabilities of occurrence, so our solution was to generate our own probabilities.

Table 1: ARPW Values

SDP	ARPW	SDP	ARPW	SDP	ARPW	SDP	ARPW
1-1	1	1-25	0.219	2-14	0.4	3-16	0.5
1-2	1	1-26	0.2	2-15	0.3	3-17	0.4
1-3	0.1	1-27	0.175	2-16	0.3	3-18	0.4
1-6	0.8	1-28	0.175	2-17	0.25	3-19	0.3
1-7	0.6	1-29	0.175	2-18	0.15	3-20	0.3
1-8	0.6	1-30	0.12	2-19	0.6	3-21	0.3
1-9	0.6	1-34	0.8	3-1	1	3-22	0.219
1-10	0.6	1-35	0.7	3-2	0.875	3-23	0.175
1-11	0.6	1-36	0.48	3-3	0.875	3-24	0.15
1-12	0.6	2-1	1	3-4	0.875	3-25	0.6
1-13	0.5	2-2	1	3-5	0.875	3-26	0.7
1-14	0.48	2-3	0.1	3-6	0.875	4-1	1
1-15	0.4	2-4	0.1	3-7	0.263	4-2	1
1-16	0.3	2-5	0.875	3-8	0.7	4-3	0.875
1-17	0.7	2-6	0.875	3-9	0.7	4-4	0.875
1-18	0.7	2-7	0.875	3-10	0.7	4-5	0.875
1-19	0.7	2-8	0.875	3-11	0.6	4-6	0.875
1-20	0.25	2-9	0.8	3-12	0.6	4-7	0.875
1-21	0.25	2-10	0.8	3-13	0.6	4-8	0.875
1-22	0.219	2-11	0.7	3-14	0.6	4-9	0.8
1-23	0.219	2-12	0.6	3-15	0.5	4-10	0.8
1-24	0.219	2-13	0.5	4-11	0.7		

### 2.3.1 Random Sampling

Generating probability data made use of the ARPW values linked to each SDP. As stated in Section 1.1.5, the ARPWs are not intended to represent the likelihood that a given SDP will be used to divert SNM. However, they do represent a relative value of attractiveness to the adversary. Assuming all SDPs are equally feasible, we can use these values as a probability distribution function and randomly sample from it to determine the probability of failure. The reliability of an SDP can be calculated using

$$P_{SDP} = \frac{ARPW_{SDP}}{\sum_{i=1}^n ARPW_i} * P_f \quad (2)$$

where  $P_f$  is a randomly sampled probability of failure. The ARPW values for the SDPs can be found in Table 2.3.1.

For example, the MC-REC event is assigned a probability of 1%. When the MC-REC event occurs, REC<sub>1</sub> has a 6.9056% chance of being the chosen diversion path or cause of failure. Using Eq. 2, we can see that the probability of REC<sub>1</sub> is 0.069056%, and we can assign this probability to the basic event representing REC<sub>1</sub>. This would be repeated for all basic events in each MC event.

The task of random sampling was performed by Python, specifically using the NumPy package. Each sample is one probability that an area will fail to prevent diversion. Since there are 4 areas in the process, there are 4 samples per data set, and 1000 data sets per distribution. Multiple distributions were sampled from, but each sample in a given set was drawn from the same distribution, albeit independently. Further discussion about this can be found in Section 4.2.

We assumed that realistic probabilities of failure would be very close to 0, so we drew samples from an exponential distribution [29]. We sampled from three distributions, varying the parameter  $\beta = \{1, 2, 3\}$ . The exponential probability distribution function used by NumPy is

$$f(x; \frac{1}{\beta}) = \frac{1}{\beta} \exp(-\frac{x}{\beta}) \quad (3)$$

The parameter  $\beta$  is the mean, standard deviation, and the scale parameter of the distribution. As  $\beta$  increases, the shape of the distribution changes, increasing the likelihood of sampling larger values. This is shown in Figure 10.

We also drew samples from a truncated normal distribution in order to draw comparisons between results from an exponential distribution and a truncated normal distribution [30]. We are not able to use a regular normal distribution because it would have values below 0, and negative probabilities are meaningless in the context of this project.

We sampled from six distributions, varying the mean  $\mu = \{0, 1\}$  and the stan-

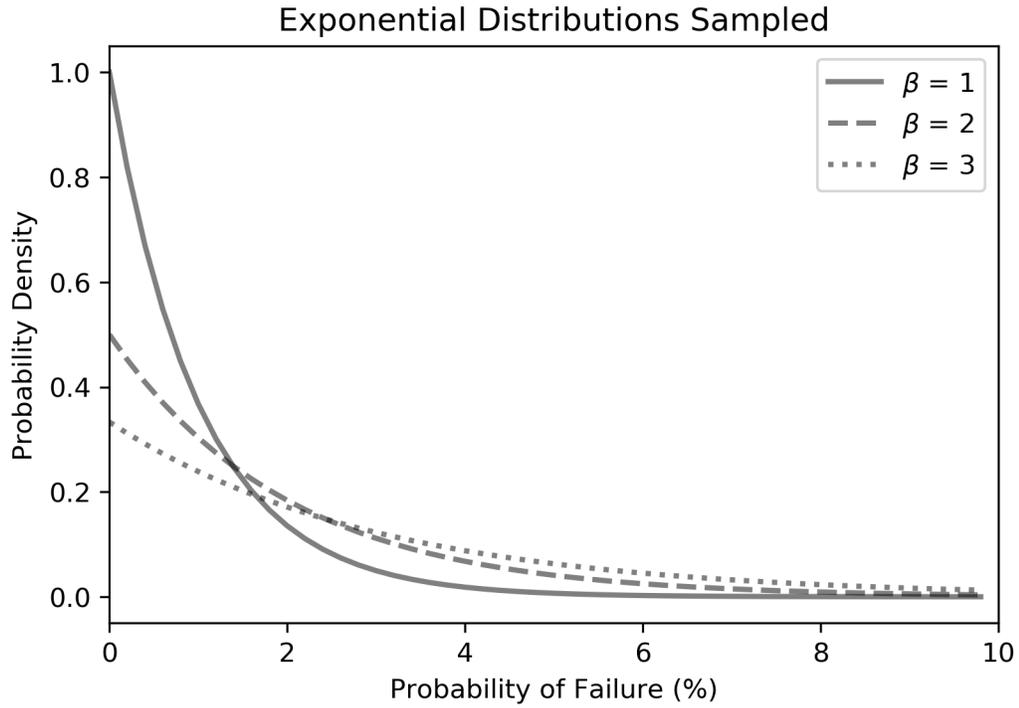


Figure 10: Exponential Distribution Sampled

dard deviation  $\sigma = \{1, 2, 3\}$ . The normal probability distribution function used by NumPy is

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right) \quad (4)$$

Much like the exponential distribution, increasing either the mean or the standard deviation also increased the likelihood of sampling larger values. This is shown in Figure 11.

Truncating the normal distribution changes the probability distribution function, however, since samples are not being drawn/accepted from part of the distribution. In other words, the regular normal distribution is in the range of  $(-\infty, \infty)$ , but the truncated normal distribution is in the range  $f(x) \in (0, \infty)$ . In order to

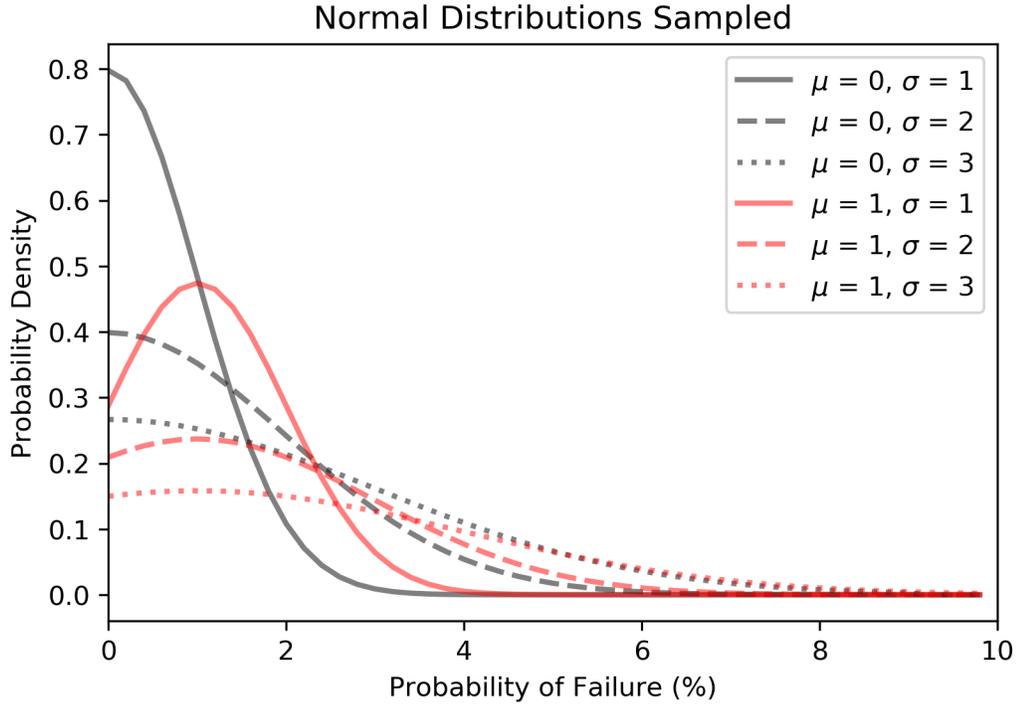


Figure 11: Normal Distributions Sampled

determine the new distribution, we can use

$$g(x) = \frac{f(x)}{F(b) - F(a)} \quad (5)$$

where  $f(x)$  is Equation 4, the range  $(a, b]$  is  $(0, \infty]$ , and  $F(x)$  is the cumulative density function, or

$$F(x) = \frac{1}{2} \left[ 1 + \operatorname{erf} \left( \frac{x - \mu}{\sigma\sqrt{2}} \right) \right] \quad (6)$$

If we rewrite Equation 5 with Equations 4 and 6 and values  $a = 0, b = \infty$ , we get

$$g(x) = \frac{\frac{1}{\sigma\sqrt{2\pi}} \exp \left( -\frac{(x-\mu)^2}{2\sigma^2} \right)}{1 - \frac{1}{2} \left[ 1 + \operatorname{erf} \left( \frac{-\mu}{\sigma\sqrt{2}} \right) \right]} \quad (7)$$

or

$$g(x) = \frac{2}{\sigma\sqrt{2\pi}} \frac{\exp \left( -\frac{(x-\mu)^2}{2\sigma^2} \right)}{1 + \operatorname{erf} \left( \frac{\mu}{\sigma\sqrt{2}} \right)} \quad (8)$$

Equation 8 with the parameters  $\mu = 0, \sigma = 1$  is plotted in Figure 12.

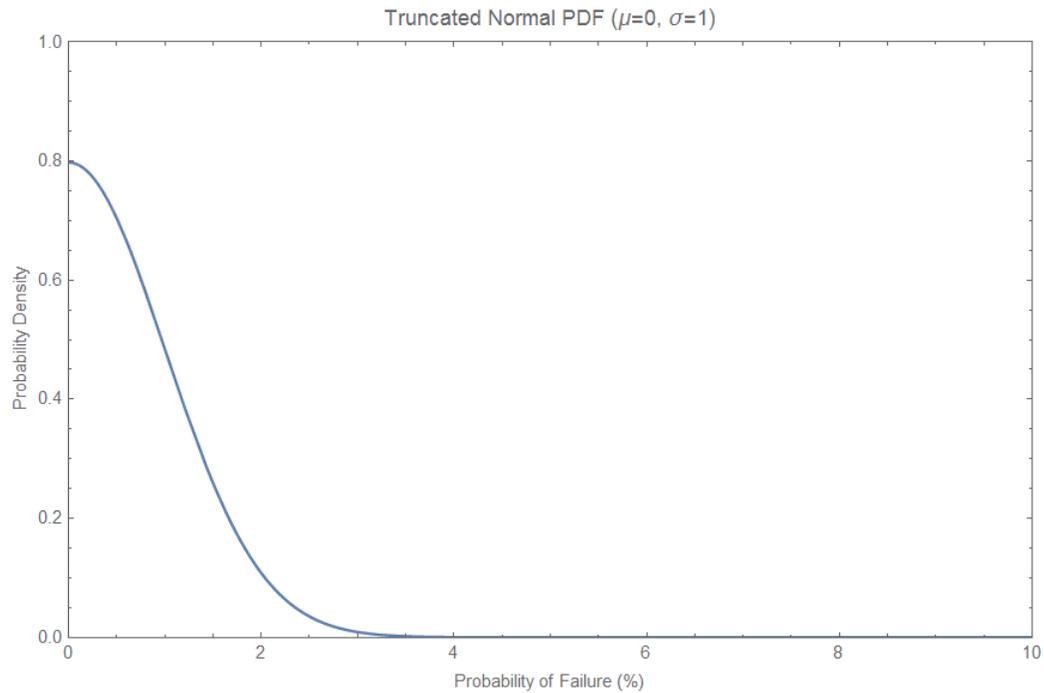


Figure 12: Truncated Normal PDF

In the context of Python, the method used to truncate the distribution was simply to check each value as it was sampled. If the value was zero or negative, the sample was discarded and resampled. The probability distribution function being sampled was not altered in any way.

### **2.3.2 SPAR-H Human Reliability Analysis**

The SPAR-H HRA model is an easy-to-use human reliability analysis method used by the NRC to identify human error probabilities. It examines several factors that influence a person's ability to perform, shown in Figure 13. Analysts complete a relatively straightforward worksheet, which can be found in Appendix A, and use it to calculate the probability of human error [31].

Since the Material Accounting subsystem consisted solely of paper documen-

PSF	Diagnosis Levels	Action Levels	Range of Influence
Available time	5	5	.01 to failure
Stress/Stressors	3	3	1 to 5
Complexity	3(4)*	3	0.1 to 5
Experience/Training	3	3	0.5 to 3
Procedures	4	4	0.5 to 50
Ergonomics/HMI	4	4	0.5 to 50
Fitness for Duty	3	3	1 to failure
Work Processes	3	3	0.8 to 5

\*The number in parentheses = the number of levels associated with LP/SD

Figure 13: SPAR-H Performance Shaping Factors

tation and accounting, we examined the accounting documentation handled in each area and applied engineering judgment to assign appropriate diagnoses. Since SPAR-H relies on examining real human beings, most diagnoses were left as “Nominal”; the only performance shaping factor changed was the “Complexity” factor. In addition, the complexity of the documentation does not vary between iterations, so the probabilities calculated remain constant over each iteration.

## 2.4 Python Processing

In addition to random sampling, Python had several other uses. We used Python to create the probability distribution functions, i.e. assign each SDP its proper probability, and then write files that could be imported into SAPHIRE. We also plotted the random samples from each distribution alongside their respective distributions in order to verify that the sampling was in line with the actual distributions. These plots can be found in Appendix A.

Once the data is imported and SAPHIRE has completed the analysis, the cut set reports are generated. These reports are fed back into Python, where the results are organized and plotted. The Python scripts can be found in Appendix A.

## 2.5 Summary

This chapter described the steps taken in incorporating statistical uncertainty in diversion pathway analysis through probabilistic risk assessment. The process used in the DPA Handbook was modelled in SAPHIRE, and both the process itself and the SAPHIRE model were briefly described. The use of Python to generate probability data was also explained.

## 3 RESULTS

### 3.1 Introduction

While the goal of this project is to incorporate statistical uncertainty into nuclear material diversion pathway analysis, the goal of the analysis itself is to identify which areas in a system are vulnerable to diversion. While the significance of this project does not lie in the actual results of the analysis, the results are necessary when considering if further development of this concept methodology has merit. In this analysis, a total of nine distributions were randomly sampled from using Python. Each distribution had 1000 sets of random samples, which were fed into SAPHIRE. The resulting cut set reports were tabulated, analyzed, and plotted. The results are discussed below.

### 3.2 Exponential Distribution Sampling

Three exponential distributions were considered, where the scale parameter  $\beta$  was chosen to be  $= \{1, 2, 3\}$  in Equation 3. 1000 data sets were taken from each distribution. Four types of bar plots were made to highlight different areas of interest. The y-axis is the list of 166 cut sets (excluding the “Safe” cut set), and the x-axis is the average probability of occurrence for the cut sets. (0,0) is located at the bottom left corner. The black bars on each rectangle indicate a confidence interval of 95% [32].

The results from the exponential distributions are presented graphically below as well as in Appendix A. The values have also been tabulated and included in Appendix A. All plots use a log scale, and each type of plot shares the same

x-range to better compare and contrast the results.

All of the plots show that the results have a consistent shape. Figs. 14-16 shows that changing the scale parameter  $\beta$  affects the absolute probabilities of occurrence but does not change the overall shape of the results. This is expected, as the basic event probabilities are constant fractions within an area. The random sampling only changes the overall probability of an area, and does not influence the distribution of basic events.

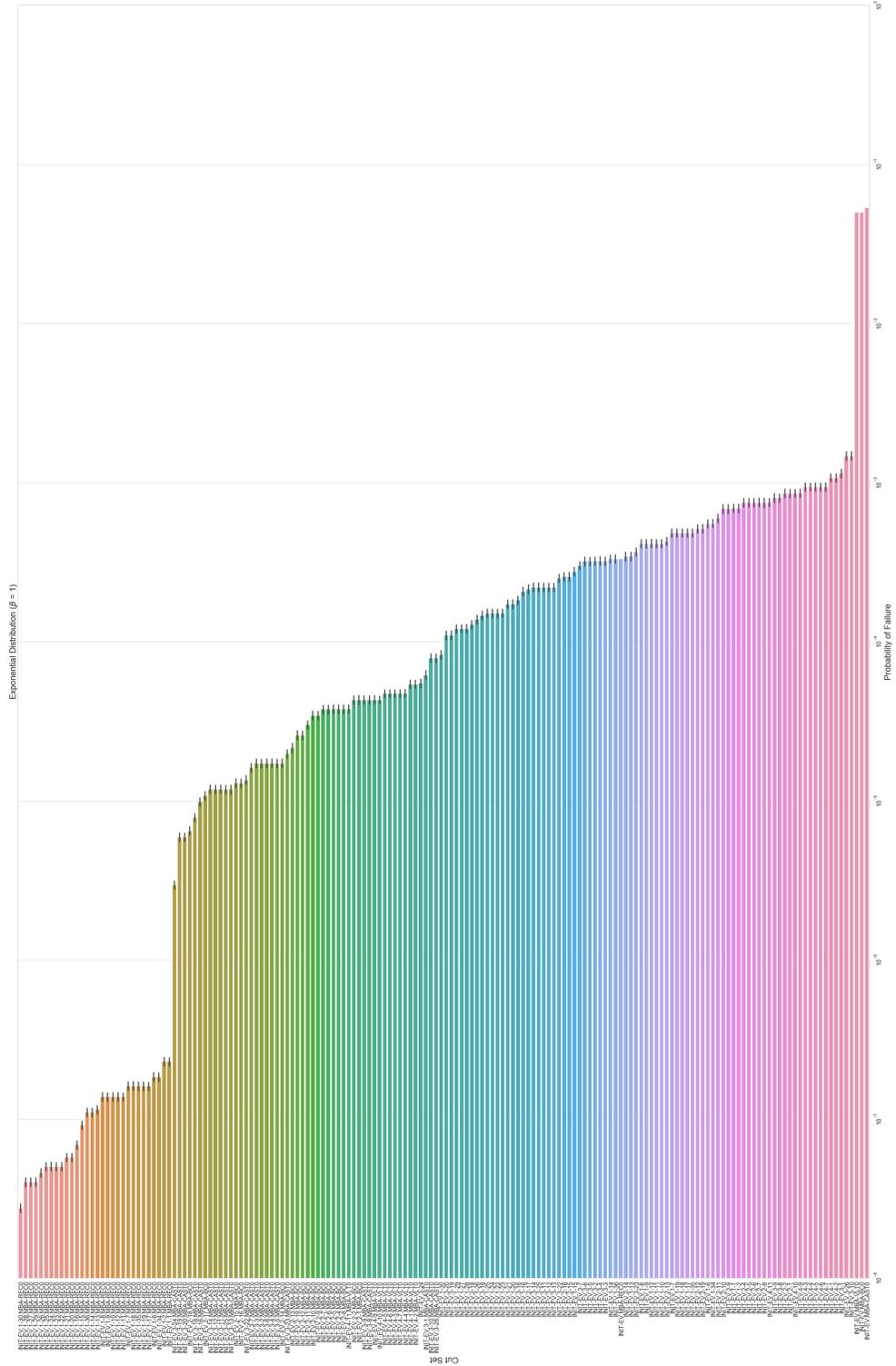


Figure 14: Exponential Distribution ( $\beta = 1$ ): Overall

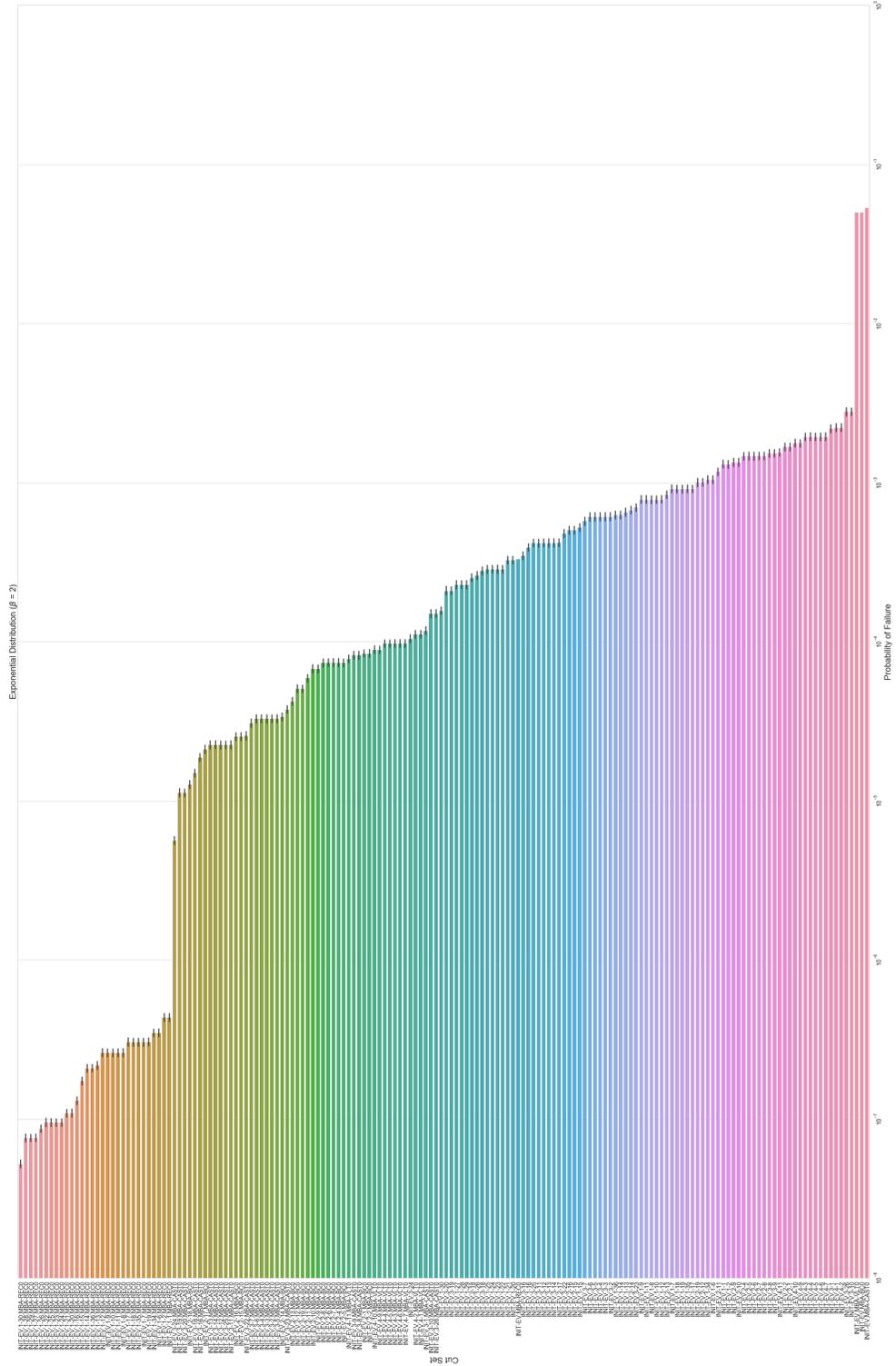


Figure 15: Exponential Distribution ( $\beta = 2$ ): Overall

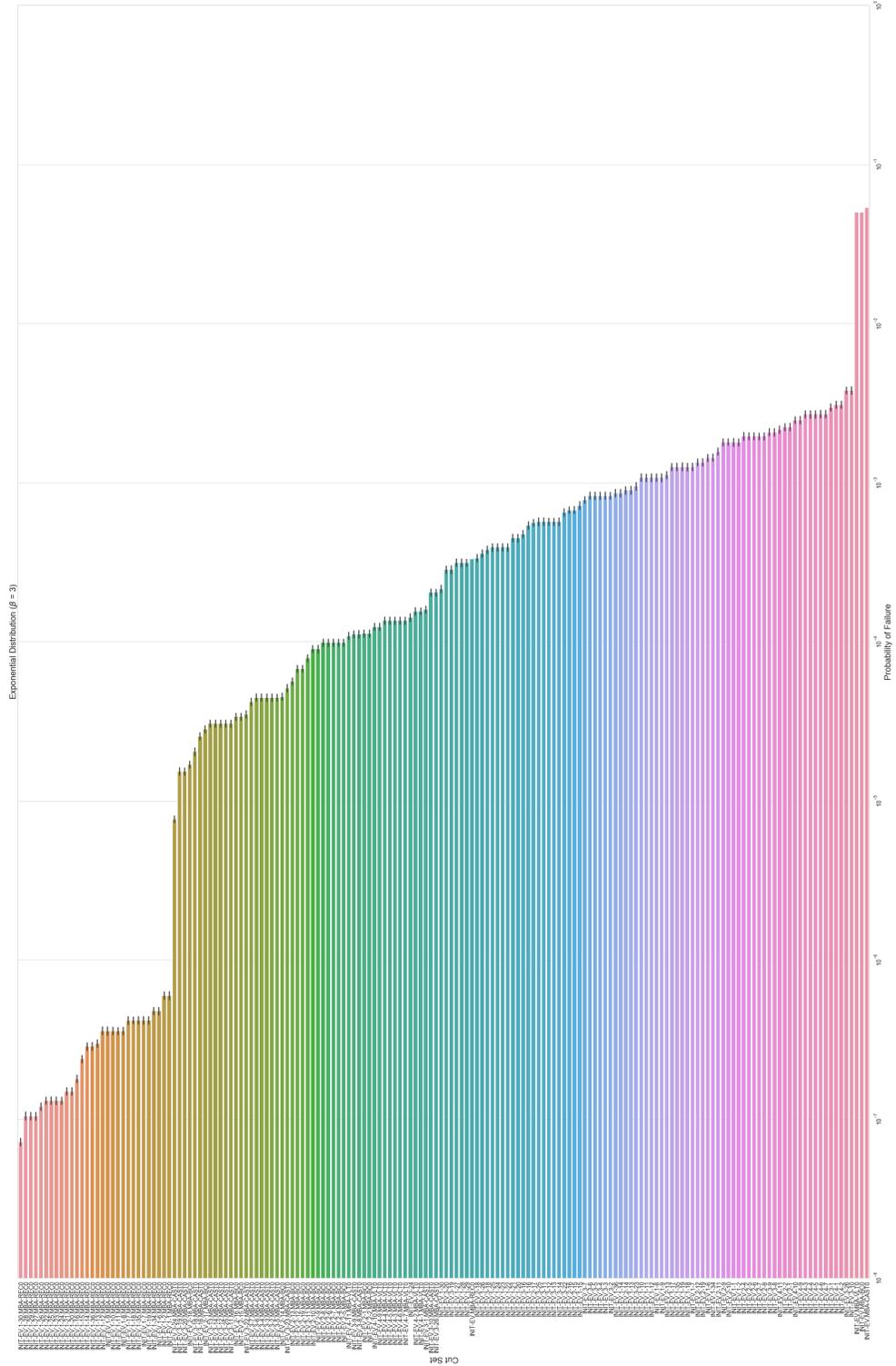


Figure 16: Exponential Distribution ( $\beta = 3$ ): Overall

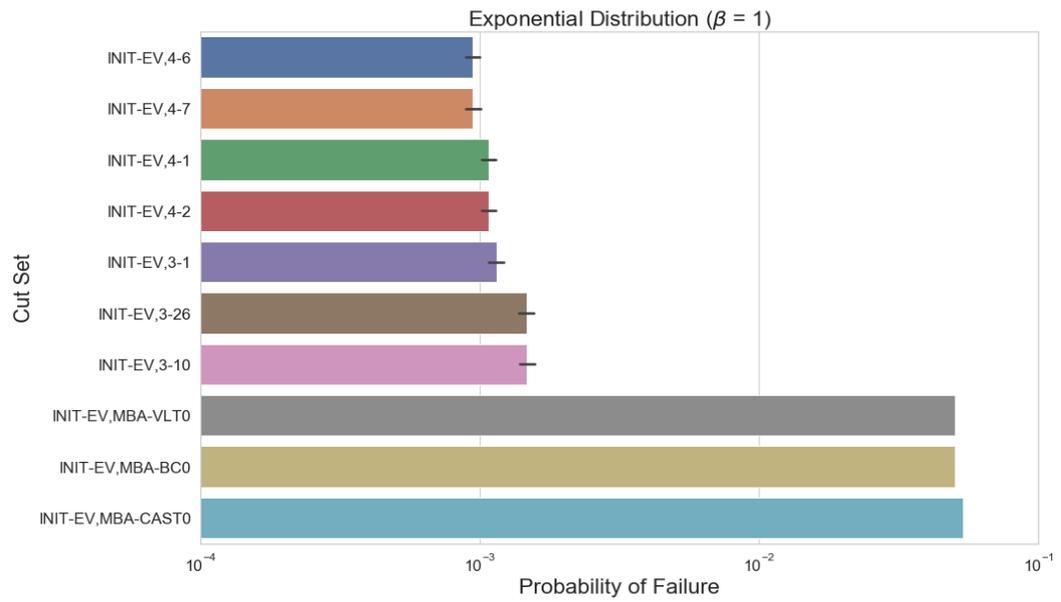


Figure 17: Exponential Distribution ( $\beta = 1$ ): Largest

Figure 17 shows the 10 cut sets with the highest probabilities of occurrence, while Figure 18 shows the 10 cut sets with the smallest probabilities of occurrence.

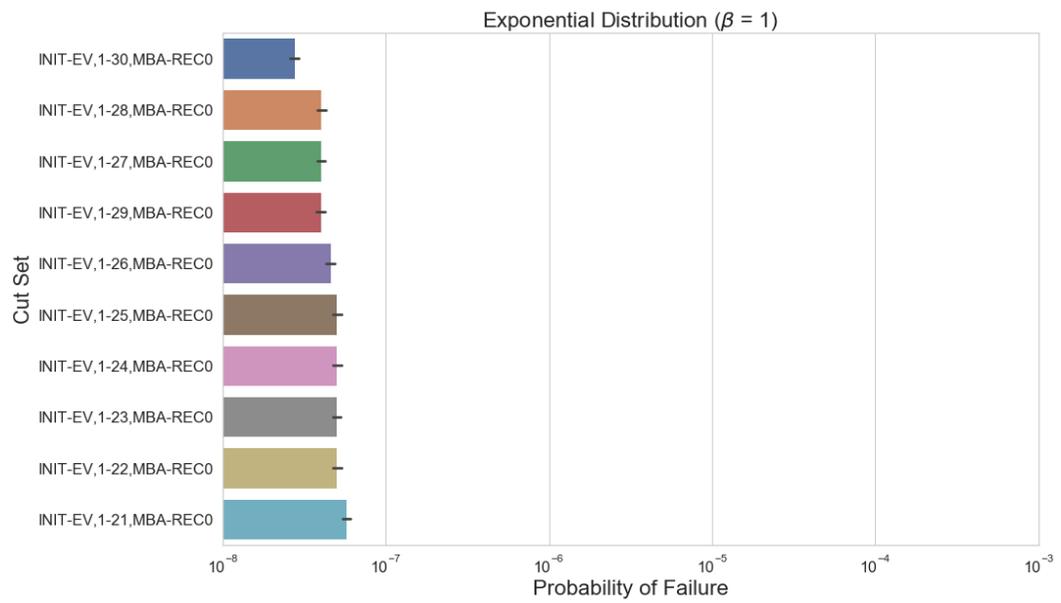


Figure 18: Exponential Distribution ( $\beta = 1$ ): Smallest

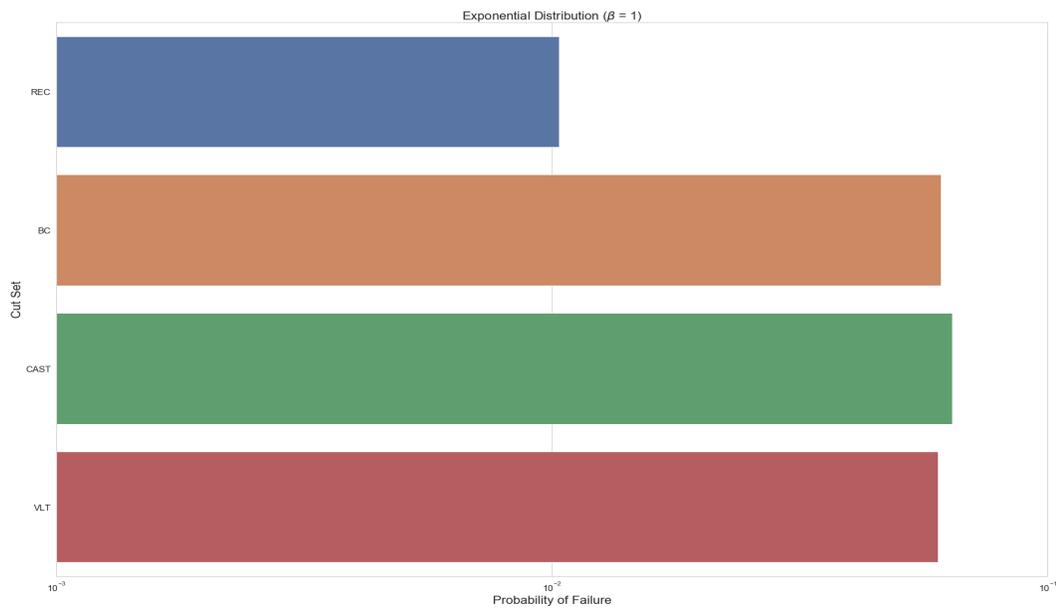


Figure 19: Exponential Distribution ( $\beta = 1$ ): By-Area

For Figure 19, the cut sets were summed to show the full probability that an area will have diversion.

Among the three distributions, there are four cut sets that remain static. These four cut sets include only the MBA events, which have static probabilities of occurrence. This is expected, as the probabilities for these four basic events were determined through SPAR-H and are not randomly sampled in any way.

Using SPAR-H to calculate the human error probability has a significant impact on the final results, since cut sets are the product of the probabilities of basic events. For example, the average probability of the “MBA-REC0” basic event is about two orders of magnitude smaller than the probabilities of the other MBA events. This means all cut sets that include “MBA-REC0” will invariably have lower probabilities of occurrence than other cut sets. Figs. 20-37 were made by omitting the cut sets that include an MBA event; they are otherwise identical to the original.

The most apparent difference between Figure 14 and Figure 20 is the overall shape of the plot. The “No MBA” plots make it apparent that the range of average probabilities is relatively small, which was difficult to see in the original plots. The issue of having identical probabilities for multiple groups of basic events is also made more apparent in these plots.

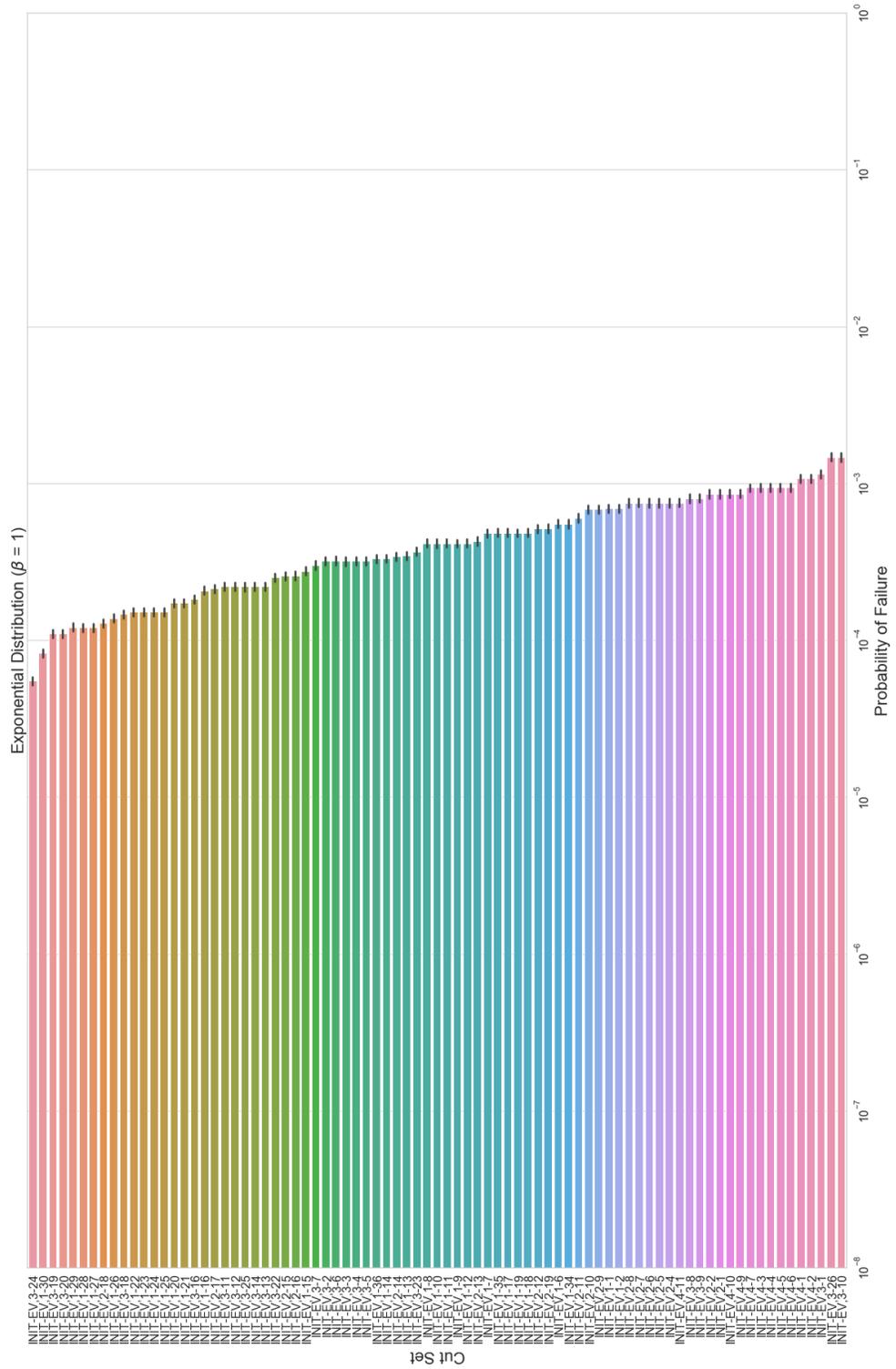


Figure 20: Exponential Distribution ( $\beta = 1$ ): Overall, No MBA

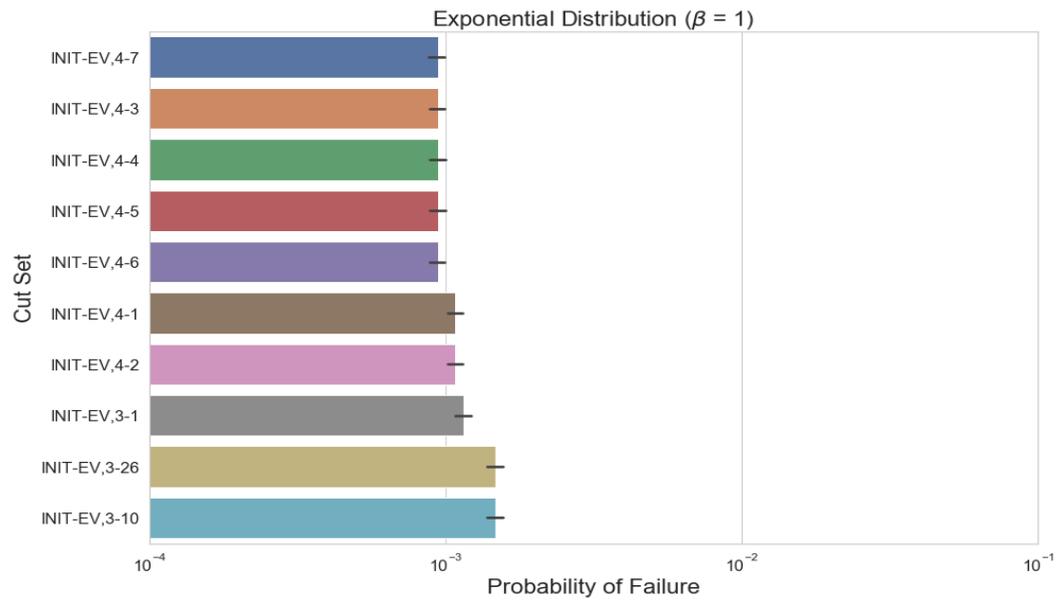


Figure 21: Exponential Distribution ( $\beta = 1$ ): Largest, No MBA

Figure 21 shows the 10 cut sets with the highest probabilities of occurrence, while Figure 22 shows the 10 cut sets with the smallest probabilities of occurrence.

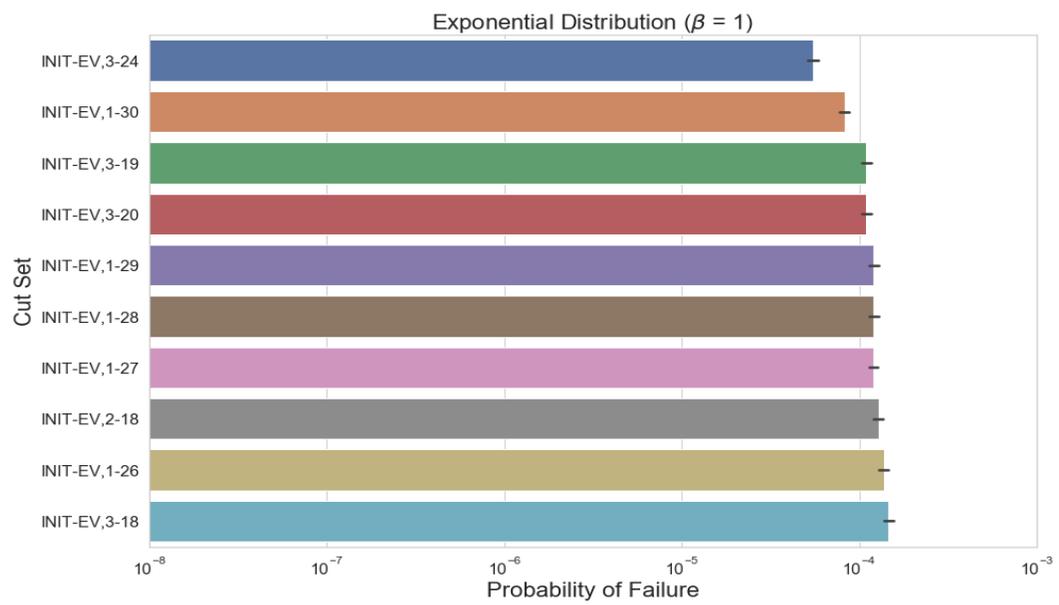


Figure 22: Exponential Distribution ( $\beta = 1$ ): Smallest, No MBA

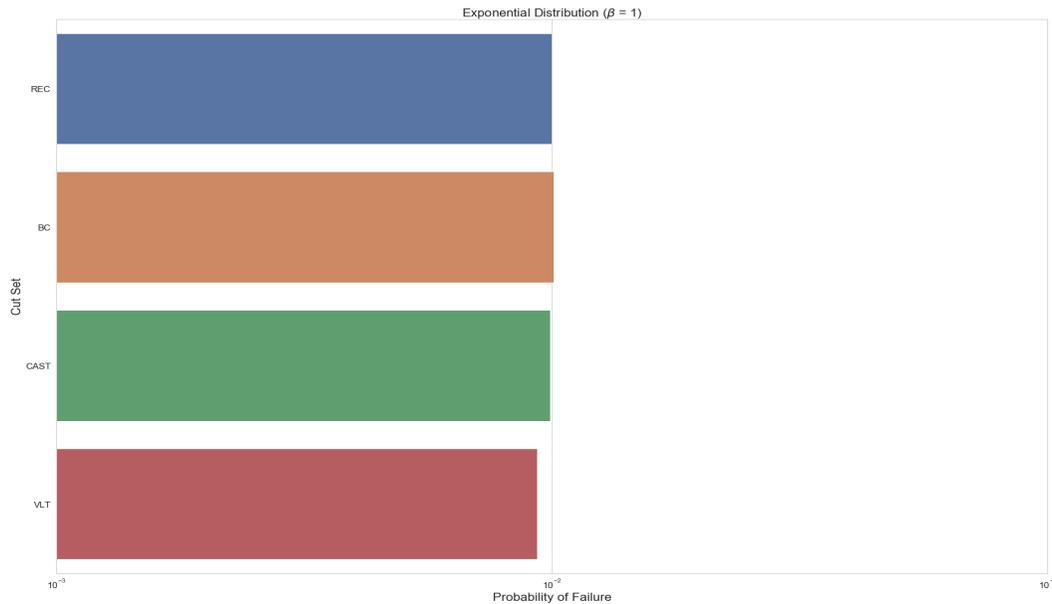


Figure 23: Exponential Distribution ( $\beta = 1$ ): By-Area, No MBA

Figure 23 reveals that the omission of MBA events drastically changes the shape of the plot. In Figure 19, the average probability for the REC area is about one order of magnitude smaller than the CAST, BC, and VLT areas.

However, Figure 23 shows a much more balanced “staircase” shape. In addition, the average probability of the REC area has increased, while the average probabilities of the CAST, BC, and VLT areas have decreased. This is attributed to the influence of the MBA events. The “MBA-REC0” event weighed down the average probability of the REC area, while the “MBA-CAST0”, “MBA-BC0”, and “MBA-VLT0” events pulled up the average probabilities of their respective areas.

By comparing Figure 19 and 23 or the values in Table A, we see that removing the MBA events reduced the difference between the REC area and the other three areas from a factor of 10 to a factor of 3 at most. This analysis is significant because it reveals how much the MBA events contribute to the probabilities of diversion.

### 3.3 Normal Distribution

Six normal distributions were considered, with specified means  $\mu = \{0, 1\}$  and standard deviations  $\sigma = \{1, 2, 3\}$  in Equation 4. 1000 data sets were taken from each distribution. Four types of bar plots were made to highlight different areas of interest. The y-axis is the list of 166 cut sets (excluding the “Safe” cut set), and the x-axis is the average probability of occurrence for the cut sets. (0,0) is located at the bottom left corner. The black bars on each rectangle indicate a confidence interval of 95%.

The plots of the results from the normal distributions are presented in Appendix A. The plots have been omitted from this chapter for brevity, as they are nearly identical to the earlier plots and do not reveal any additional insights. The only notable difference is that the normal distribution plots for  $\mu = 0$  are consistently smaller than the exponential distribution plots. Tabulated results can be found in Appendix A.

### 3.4 Analysis of Results

The analysis shows that the VLT area is the most vulnerable, since it has the highest average probability of diversion. However, the most likely paths of diversion belong to the CAST area, as SDP 3-10 and 3-26 consistently have the greatest individual probabilities of diversion. We also see that the REC area is the least vulnerable area in general, as well as the specific cut sets with the least likely paths of diversion.

The omission of MA events shows us that material accounting has a significant impact on the ability to detect diversion of material. The paths that include MA

events make up both the highest and the lowest probabilities of diversion.

Finally, we recognize that despite random sampling and varied distributions, the shape and distribution of the results remain consistent. This is due in part to the fact that the ARPW values are fixed and discrete, which affects the distribution of the cut sets within a particular area.

## 4 CONCLUSIONS

### 4.1 Introduction

The goal of this project was to incorporate statistical uncertainty into diversion pathway analysis by using probabilistic risk assessment. This goal was achieved, although there were some considerations that needed to be made in regards to this project. In this chapter, those considerations are described and proposals for future work in this methodology are discussed.

#### 4.1.1 *Project Considerations*

A significant portion of the data used in this project came from the work done by the authors of the DPA Handbook [3,28]. Many of the assumptions and concerns related to their work carry over to this project.

One significant concern is that ARPWs were not intended to be used as probabilities. Their purpose is to show the relative attractiveness of the SNM being diverted and the relative difficulty in doing so. This is not necessarily the same as the likelihood that a certain SDP will be used. For example, two of the parameters used in the calculation of the ARPWs are the material attractiveness (which changes depending on the isotope, the physical description, and the dose rate) and the amount that can be diverted. This means that SNM that is completely undesirable can have a similar ARPW as SNM that is very valuable, as long as the other parameters make up for the difference.

Despite this, the ARPWs are still the most accurate set of data available regarding the probability of failure and diversion from a system. Much like the way early

nuclear safety analyses relied on experts to make up for the lack of reliable data, the work done in this project is relying on the data generated through DPA [18]. The lack of empirical data certainly does mean the conclusions are correct, but that does not diminish the feasibility of this application.

Another significant concern is the generation of probability data. The usage of random sampling implies that the actual values of the results have no meaning, since they were essentially fabricated. However, the purpose of the analysis is not to find the probabilities of diversion, but to identify which areas in the system are vulnerable and require attention. In addition, it has been shown that as long as the data is relatively similar, the differences in the results are negligible. Hauptmanns examined the impact of differences in reliability data, and found that “the identification of design weaknesses and of components with large contributions to the expected frequency of the undesired event is not hampered by differences in reliability data, although the absolute values naturally differ” [33].

For this project, we made the logical assumption that any realistic values for failure probabilities of material control measures would be very low. This would provide reasonably accurate values that would be further enhanced through statistical analysis of large data sets. As a result, this analysis has proven successful in identifying areas of vulnerability, even though the actual values are not exact.

The use of SPAR-H in determining the failure probabilities for the MA events may also have affected the accuracy of the results. The calculation of human error probability by SPAR-H is formulaic, and we have removed any uncertainty by not varying the factors.

DPA specifically addresses diversion by an insider, not a thief. This means that using DPA as a base for our project limits us to people within the system

itself. While most facilities *should* be secure enough that an outsider cannot enter facility grounds, this is still something that must be considered when performing this analysis.

DPA does not address the physical protection subsystem. It presumes that an insider has the ability to bypass any physical protection measure. This affects how a system can be modelled for PRA. As mentioned in Section 2.2.3, the lack of information of the material control measures forced the design of the fault trees to differ from conventional PRAs.

## 4.2 Future Work

All of the concerns in Section 4.1.1 can be addressed by not using DPA in the analysis. The main reason for using DPA is the lack of usable information for this project, summarizable as two concerns: not having a real system to analyze and not having data on the failures of probability for material control measures. Future developments following this work should address one or both of these concerns before progressing. Incorporating these would significantly strengthen the validity of this type of analysis.

We could also use multiple distributions in the same data set. One design choice made was to limit the choices of distributions in a given data set to a single distribution, e.g. a data set sampled from the same normal distribution 4 times. If we instead allowed a data set to sample from any of the distributions available, we could see how the results would be impacted.

One important aspect of safeguards, both domestic and international, that was not incorporated into the model is time. In both types of safeguards, the time from diversion to detection is a key detail that must be known for a complete un-

derstanding of the risk of diversion. For example, a diversion path that is detected within minutes or hours is less dangerous than a diversion path that remains undetected for days, weeks, or months. This is critically important for international safeguards, and would be necessary if this methodology were to be adapted for international safeguards rather than domestic safeguards.

A direction for future work could be to perform a level 2 or level 3 PRA and expand the scope of the analysis. Different types of SNM can have different consequences if diverted, so adapting and performing a level 2 or level 3 PRA would improve the secondary goal of identifying areas that are vulnerable to diversion. A level 2 PRA could incorporate the physical protection measures, and a level 3 PRA could incorporate details such as the time to detection and the quantity and type of material diverted.

There are several differences between SSPM and the usage of DPA/PRA, and the SSPM methodology has some features that could be incorporated into future developments of this project's methodology. The calculation of probability of failure for the material control measures make use of a Page's Test, which is a statistical test using a number of variables used in this case to simulate material loss detection. The use of the Page's Test, other similar tests, could replace the use of DPA to improve upon the results in this thesis.

SSPM also properly models all three safeguards and security subsystems: physical protection, material control, and material accounting. The electrochemical reprocessing plant modelled is significantly more in-depth than the process from the DPA Handbook. In addition, the steps the insider takes to divert material are detailed and cover the full path from diversion to escape.

One major strength of this project's methodology compared to SSPM is the use

of PRA. The results from SSPM only show the outcomes of simulated diversion attempts, e.g. 30 attempts resulting in 27 failures and 3 successes [6]. The results from using PRA would not only show the frequency of successful diversion but identify the paths that are the most likely (and the least likely) to cause diversion. While PRA may have some difficulty replicating some of the features of the Matlab Simulink model, this one detail is a significant strength that distinguishes this project's methodology from SSPM.

### 4.3 Overall Conclusions

Although there are some imperfections in this project, we feel that this project was successful in proving that incorporating statistical uncertainty in diversion pathway analysis through probabilistic risk assessment is feasible. While we weren't able to make an ideal model, we were still able to show correlations between the failure probabilities of components in a subsystem and the overall vulnerability of that subsystem.

A majority of the errors in this project's methodology can be attributed to the raw data used for the analysis, rather than the adaptation of the analysis. While this does not guarantee this project's methodology can work, it does strengthen confidence in further research.

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- egrated Reliability Evaluations (SAPHIRE) Version 8*, vol. 3, Idaho National Laboratory, Idaho (2011).
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# Appendices

## A Tabulated Results



2-4	$7.50 \cdot 10^{-4}$	$1.47 \cdot 10^{-3}$	$1.96 \cdot 10^{-3}$	$5.80 \cdot 10^{-4}$	$1.22 \cdot 10^{-3}$	$1.77 \cdot 10^{-3}$	$9.61 \cdot 10^{-4}$	$1.54 \cdot 10^{-3}$	$2.08 \cdot 10^{-3}$
2-5	$7.50 \cdot 10^{-4}$	$1.47 \cdot 10^{-3}$	$1.96 \cdot 10^{-3}$	$5.80 \cdot 10^{-4}$	$1.22 \cdot 10^{-3}$	$1.77 \cdot 10^{-3}$	$9.61 \cdot 10^{-4}$	$1.54 \cdot 10^{-3}$	$2.08 \cdot 10^{-3}$
2-6	$7.50 \cdot 10^{-4}$	$1.47 \cdot 10^{-3}$	$1.96 \cdot 10^{-3}$	$5.80 \cdot 10^{-4}$	$1.22 \cdot 10^{-3}$	$1.77 \cdot 10^{-3}$	$9.61 \cdot 10^{-4}$	$1.54 \cdot 10^{-3}$	$2.08 \cdot 10^{-3}$
2-7	$7.50 \cdot 10^{-4}$	$1.47 \cdot 10^{-3}$	$1.96 \cdot 10^{-3}$	$5.80 \cdot 10^{-4}$	$1.22 \cdot 10^{-3}$	$1.77 \cdot 10^{-3}$	$9.61 \cdot 10^{-4}$	$1.54 \cdot 10^{-3}$	$2.08 \cdot 10^{-3}$
2-8	$7.50 \cdot 10^{-4}$	$1.47 \cdot 10^{-3}$	$1.96 \cdot 10^{-3}$	$5.80 \cdot 10^{-4}$	$1.22 \cdot 10^{-3}$	$1.77 \cdot 10^{-3}$	$9.61 \cdot 10^{-4}$	$1.54 \cdot 10^{-3}$	$2.08 \cdot 10^{-3}$
2-9	$6.85 \cdot 10^{-4}$	$1.35 \cdot 10^{-3}$	$1.79 \cdot 10^{-3}$	$5.30 \cdot 10^{-4}$	$1.11 \cdot 10^{-3}$	$1.62 \cdot 10^{-3}$	$8.79 \cdot 10^{-4}$	$1.41 \cdot 10^{-3}$	$1.90 \cdot 10^{-3}$
2-10	$6.85 \cdot 10^{-4}$	$1.35 \cdot 10^{-3}$	$1.79 \cdot 10^{-3}$	$5.30 \cdot 10^{-4}$	$1.11 \cdot 10^{-3}$	$1.62 \cdot 10^{-3}$	$8.79 \cdot 10^{-4}$	$1.41 \cdot 10^{-3}$	$1.90 \cdot 10^{-3}$
2-11	$6.00 \cdot 10^{-4}$	$1.18 \cdot 10^{-3}$	$1.57 \cdot 10^{-3}$	$4.64 \cdot 10^{-4}$	$9.75 \cdot 10^{-4}$	$1.41 \cdot 10^{-3}$	$7.69 \cdot 10^{-4}$	$1.23 \cdot 10^{-3}$	$1.66 \cdot 10^{-3}$
2-12	$5.14 \cdot 10^{-4}$	$1.01 \cdot 10^{-3}$	$1.35 \cdot 10^{-3}$	$3.98 \cdot 10^{-4}$	$8.36 \cdot 10^{-4}$	$1.21 \cdot 10^{-3}$	$6.59 \cdot 10^{-4}$	$1.06 \cdot 10^{-3}$	$1.43 \cdot 10^{-3}$
2-13	$4.28 \cdot 10^{-4}$	$8.41 \cdot 10^{-4}$	$1.12 \cdot 10^{-3}$	$3.32 \cdot 10^{-4}$	$6.97 \cdot 10^{-4}$	$1.01 \cdot 10^{-3}$	$5.49 \cdot 10^{-4}$	$8.82 \cdot 10^{-4}$	$1.19 \cdot 10^{-3}$
2-14	$3.43 \cdot 10^{-4}$	$6.73 \cdot 10^{-4}$	$8.97 \cdot 10^{-4}$	$2.65 \cdot 10^{-4}$	$5.57 \cdot 10^{-4}$	$8.08 \cdot 10^{-4}$	$4.40 \cdot 10^{-4}$	$7.06 \cdot 10^{-4}$	$9.51 \cdot 10^{-4}$
2-15	$2.57 \cdot 10^{-4}$	$5.05 \cdot 10^{-4}$	$6.73 \cdot 10^{-4}$	$1.99 \cdot 10^{-4}$	$4.18 \cdot 10^{-4}$	$6.06 \cdot 10^{-4}$	$3.30 \cdot 10^{-4}$	$5.29 \cdot 10^{-4}$	$7.13 \cdot 10^{-4}$
2-16	$2.57 \cdot 10^{-4}$	$5.05 \cdot 10^{-4}$	$6.73 \cdot 10^{-4}$	$1.99 \cdot 10^{-4}$	$4.18 \cdot 10^{-4}$	$6.06 \cdot 10^{-4}$	$3.30 \cdot 10^{-4}$	$5.29 \cdot 10^{-4}$	$7.13 \cdot 10^{-4}$
2-17	$2.14 \cdot 10^{-4}$	$4.20 \cdot 10^{-4}$	$5.61 \cdot 10^{-4}$	$1.66 \cdot 10^{-4}$	$3.48 \cdot 10^{-4}$	$5.05 \cdot 10^{-4}$	$2.75 \cdot 10^{-4}$	$4.41 \cdot 10^{-4}$	$5.94 \cdot 10^{-4}$
2-18	$1.28 \cdot 10^{-4}$	$2.52 \cdot 10^{-4}$	$3.37 \cdot 10^{-4}$	$9.95 \cdot 10^{-5}$	$2.09 \cdot 10^{-4}$	$3.03 \cdot 10^{-4}$	$1.65 \cdot 10^{-4}$	$2.65 \cdot 10^{-4}$	$3.56 \cdot 10^{-4}$
2-19	$5.14 \cdot 10^{-4}$	$1.01 \cdot 10^{-3}$	$1.35 \cdot 10^{-3}$	$3.98 \cdot 10^{-4}$	$8.36 \cdot 10^{-4}$	$1.21 \cdot 10^{-3}$	$6.59 \cdot 10^{-4}$	$1.06 \cdot 10^{-3}$	$1.43 \cdot 10^{-3}$
3-1	$1.15 \cdot 10^{-3}$	$2.19 \cdot 10^{-3}$	$2.97 \cdot 10^{-3}$	$9.45 \cdot 10^{-4}$	$1.81 \cdot 10^{-3}$	$2.78 \cdot 10^{-3}$	$1.51 \cdot 10^{-3}$	$2.37 \cdot 10^{-3}$	$3.13 \cdot 10^{-3}$
3-2	$3.21 \cdot 10^{-4}$	$6.12 \cdot 10^{-4}$	$8.30 \cdot 10^{-4}$	$2.64 \cdot 10^{-4}$	$5.06 \cdot 10^{-4}$	$7.75 \cdot 10^{-4}$	$4.22 \cdot 10^{-4}$	$6.62 \cdot 10^{-4}$	$8.75 \cdot 10^{-4}$
3-3	$3.21 \cdot 10^{-4}$	$6.12 \cdot 10^{-4}$	$8.30 \cdot 10^{-4}$	$2.64 \cdot 10^{-4}$	$5.06 \cdot 10^{-4}$	$7.75 \cdot 10^{-4}$	$4.22 \cdot 10^{-4}$	$6.62 \cdot 10^{-4}$	$8.75 \cdot 10^{-4}$
3-4	$3.21 \cdot 10^{-4}$	$6.12 \cdot 10^{-4}$	$8.30 \cdot 10^{-4}$	$2.64 \cdot 10^{-4}$	$5.06 \cdot 10^{-4}$	$7.75 \cdot 10^{-4}$	$4.22 \cdot 10^{-4}$	$6.62 \cdot 10^{-4}$	$8.75 \cdot 10^{-4}$
3-5	$3.21 \cdot 10^{-4}$	$6.12 \cdot 10^{-4}$	$8.30 \cdot 10^{-4}$	$2.64 \cdot 10^{-4}$	$5.06 \cdot 10^{-4}$	$7.75 \cdot 10^{-4}$	$4.22 \cdot 10^{-4}$	$6.62 \cdot 10^{-4}$	$8.75 \cdot 10^{-4}$
3-6	$3.21 \cdot 10^{-4}$	$6.12 \cdot 10^{-4}$	$8.30 \cdot 10^{-4}$	$2.64 \cdot 10^{-4}$	$5.06 \cdot 10^{-4}$	$7.75 \cdot 10^{-4}$	$4.22 \cdot 10^{-4}$	$6.62 \cdot 10^{-4}$	$8.75 \cdot 10^{-4}$
3-7	$3.01 \cdot 10^{-4}$	$5.75 \cdot 10^{-4}$	$7.80 \cdot 10^{-4}$	$2.48 \cdot 10^{-4}$	$4.75 \cdot 10^{-4}$	$7.29 \cdot 10^{-4}$	$3.97 \cdot 10^{-4}$	$6.22 \cdot 10^{-4}$	$8.22 \cdot 10^{-4}$
3-8	$8.03 \cdot 10^{-4}$	$1.53 \cdot 10^{-3}$	$2.08 \cdot 10^{-3}$	$6.61 \cdot 10^{-4}$	$1.27 \cdot 10^{-3}$	$1.94 \cdot 10^{-3}$	$1.06 \cdot 10^{-3}$	$1.66 \cdot 10^{-3}$	$2.19 \cdot 10^{-3}$
3-9	$8.03 \cdot 10^{-4}$	$1.53 \cdot 10^{-3}$	$2.08 \cdot 10^{-3}$	$6.61 \cdot 10^{-4}$	$1.27 \cdot 10^{-3}$	$1.94 \cdot 10^{-3}$	$1.06 \cdot 10^{-3}$	$1.66 \cdot 10^{-3}$	$2.19 \cdot 10^{-3}$
3-10	$1.47 \cdot 10^{-3}$	$2.80 \cdot 10^{-3}$	$3.81 \cdot 10^{-3}$	$1.21 \cdot 10^{-3}$	$2.32 \cdot 10^{-3}$	$3.55 \cdot 10^{-3}$	$1.94 \cdot 10^{-3}$	$3.03 \cdot 10^{-3}$	$4.01 \cdot 10^{-3}$
3-11	$2.20 \cdot 10^{-4}$	$4.19 \cdot 10^{-4}$	$5.69 \cdot 10^{-4}$	$1.81 \cdot 10^{-4}$	$3.47 \cdot 10^{-4}$	$5.32 \cdot 10^{-4}$	$2.90 \cdot 10^{-4}$	$4.54 \cdot 10^{-4}$	$6.00 \cdot 10^{-4}$
3-12	$2.20 \cdot 10^{-4}$	$4.19 \cdot 10^{-4}$	$5.69 \cdot 10^{-4}$	$1.81 \cdot 10^{-4}$	$3.47 \cdot 10^{-4}$	$5.32 \cdot 10^{-4}$	$2.90 \cdot 10^{-4}$	$4.54 \cdot 10^{-4}$	$6.00 \cdot 10^{-4}$
3-13	$2.20 \cdot 10^{-4}$	$4.19 \cdot 10^{-4}$	$5.69 \cdot 10^{-4}$	$1.81 \cdot 10^{-4}$	$3.47 \cdot 10^{-4}$	$5.32 \cdot 10^{-4}$	$2.90 \cdot 10^{-4}$	$4.54 \cdot 10^{-4}$	$6.00 \cdot 10^{-4}$
3-14	$2.20 \cdot 10^{-4}$	$4.19 \cdot 10^{-4}$	$5.69 \cdot 10^{-4}$	$1.81 \cdot 10^{-4}$	$3.47 \cdot 10^{-4}$	$5.32 \cdot 10^{-4}$	$2.90 \cdot 10^{-4}$	$4.54 \cdot 10^{-4}$	$6.00 \cdot 10^{-4}$
3-16	$1.83 \cdot 10^{-4}$	$3.49 \cdot 10^{-4}$	$4.74 \cdot 10^{-4}$	$1.51 \cdot 10^{-4}$	$2.89 \cdot 10^{-4}$	$4.43 \cdot 10^{-4}$	$2.41 \cdot 10^{-4}$	$3.78 \cdot 10^{-4}$	$5.00 \cdot 10^{-4}$
3-18	$1.47 \cdot 10^{-4}$	$2.80 \cdot 10^{-4}$	$3.80 \cdot 10^{-4}$	$1.21 \cdot 10^{-4}$	$2.31 \cdot 10^{-4}$	$3.54 \cdot 10^{-4}$	$1.93 \cdot 10^{-4}$	$3.02 \cdot 10^{-4}$	$4.00 \cdot 10^{-4}$
3-19	$1.10 \cdot 10^{-4}$	$2.10 \cdot 10^{-4}$	$2.85 \cdot 10^{-4}$	$9.05 \cdot 10^{-5}$	$1.73 \cdot 10^{-4}$	$2.66 \cdot 10^{-4}$	$1.45 \cdot 10^{-4}$	$2.27 \cdot 10^{-4}$	$3.00 \cdot 10^{-4}$
3-20	$1.10 \cdot 10^{-4}$	$2.10 \cdot 10^{-4}$	$2.85 \cdot 10^{-4}$	$9.05 \cdot 10^{-5}$	$1.73 \cdot 10^{-4}$	$2.66 \cdot 10^{-4}$	$1.45 \cdot 10^{-4}$	$2.27 \cdot 10^{-4}$	$3.00 \cdot 10^{-4}$
3-22	$2.51 \cdot 10^{-4}$	$4.79 \cdot 10^{-4}$	$6.51 \cdot 10^{-4}$	$2.07 \cdot 10^{-4}$	$3.97 \cdot 10^{-4}$	$6.08 \cdot 10^{-4}$	$3.31 \cdot 10^{-4}$	$5.19 \cdot 10^{-4}$	$6.86 \cdot 10^{-4}$



1-25,MBA-REC	5.03 · 10 <sup>-8</sup>	9.53 · 10 <sup>-8</sup>	1.31 · 10 <sup>-7</sup>	4.10 · 10 <sup>-8</sup>	7.84 · 10 <sup>-8</sup>	1.15 · 10 <sup>-7</sup>	6.39 · 10 <sup>-8</sup>	1.07 · 10 <sup>-7</sup>	1.36 · 10 <sup>-7</sup>
1-26,MBA-REC	4.60 · 10 <sup>-8</sup>	8.71 · 10 <sup>-8</sup>	1.19 · 10 <sup>-7</sup>	3.74 · 10 <sup>-8</sup>	7.16 · 10 <sup>-8</sup>	1.05 · 10 <sup>-7</sup>	5.84 · 10 <sup>-8</sup>	9.74 · 10 <sup>-8</sup>	1.24 · 10 <sup>-7</sup>
1-27,MBA-REC	4.02 · 10 <sup>-8</sup>	7.62 · 10 <sup>-8</sup>	1.05 · 10 <sup>-7</sup>	3.27 · 10 <sup>-8</sup>	6.26 · 10 <sup>-8</sup>	9.20 · 10 <sup>-8</sup>	5.11 · 10 <sup>-8</sup>	8.52 · 10 <sup>-8</sup>	1.08 · 10 <sup>-7</sup>
1-28,MBA-REC	4.02 · 10 <sup>-8</sup>	7.62 · 10 <sup>-8</sup>	1.05 · 10 <sup>-7</sup>	3.27 · 10 <sup>-8</sup>	6.26 · 10 <sup>-8</sup>	9.20 · 10 <sup>-8</sup>	5.11 · 10 <sup>-8</sup>	8.52 · 10 <sup>-8</sup>	1.08 · 10 <sup>-7</sup>
1-29,MBA-REC	4.02 · 10 <sup>-8</sup>	7.62 · 10 <sup>-8</sup>	1.05 · 10 <sup>-7</sup>	3.27 · 10 <sup>-8</sup>	6.26 · 10 <sup>-8</sup>	9.20 · 10 <sup>-8</sup>	5.11 · 10 <sup>-8</sup>	8.52 · 10 <sup>-8</sup>	1.08 · 10 <sup>-7</sup>
1-30,MBA-REC	2.76 · 10 <sup>-8</sup>	5.22 · 10 <sup>-8</sup>	7.17 · 10 <sup>-8</sup>	2.25 · 10 <sup>-8</sup>	4.29 · 10 <sup>-8</sup>	6.31 · 10 <sup>-8</sup>	3.50 · 10 <sup>-8</sup>	5.84 · 10 <sup>-8</sup>	7.43 · 10 <sup>-8</sup>
1-34,MBA-REC	1.84 · 10 <sup>-7</sup>	3.48 · 10 <sup>-7</sup>	4.78 · 10 <sup>-7</sup>	1.50 · 10 <sup>-7</sup>	2.86 · 10 <sup>-7</sup>	4.21 · 10 <sup>-7</sup>	2.33 · 10 <sup>-7</sup>	3.90 · 10 <sup>-7</sup>	4.95 · 10 <sup>-7</sup>
1-35,MBA-REC	1.61 · 10 <sup>-7</sup>	3.05 · 10 <sup>-7</sup>	4.18 · 10 <sup>-7</sup>	1.31 · 10 <sup>-7</sup>	2.50 · 10 <sup>-7</sup>	3.68 · 10 <sup>-7</sup>	2.04 · 10 <sup>-7</sup>	3.41 · 10 <sup>-7</sup>	4.33 · 10 <sup>-7</sup>
1-36,MBA-REC	1.10 · 10 <sup>-7</sup>	2.09 · 10 <sup>-7</sup>	2.87 · 10 <sup>-7</sup>	8.98 · 10 <sup>-8</sup>	1.72 · 10 <sup>-7</sup>	2.52 · 10 <sup>-7</sup>	1.40 · 10 <sup>-7</sup>	2.34 · 10 <sup>-7</sup>	2.97 · 10 <sup>-7</sup>
2-1,MBA-BC	4.31 · 10 <sup>-5</sup>	8.46 · 10 <sup>-5</sup>	1.13 · 10 <sup>-4</sup>	3.34 · 10 <sup>-5</sup>	7.01 · 10 <sup>-5</sup>	1.02 · 10 <sup>-4</sup>	5.53 · 10 <sup>-5</sup>	8.87 · 10 <sup>-5</sup>	1.20 · 10 <sup>-4</sup>
2-2,MBA-BC	4.31 · 10 <sup>-5</sup>	8.46 · 10 <sup>-5</sup>	1.13 · 10 <sup>-4</sup>	3.34 · 10 <sup>-5</sup>	7.01 · 10 <sup>-5</sup>	1.02 · 10 <sup>-4</sup>	5.53 · 10 <sup>-5</sup>	8.87 · 10 <sup>-5</sup>	1.20 · 10 <sup>-4</sup>
2-4,MBA-BC	3.77 · 10 <sup>-5</sup>	7.40 · 10 <sup>-5</sup>	9.87 · 10 <sup>-5</sup>	2.92 · 10 <sup>-5</sup>	6.13 · 10 <sup>-5</sup>	8.89 · 10 <sup>-5</sup>	4.84 · 10 <sup>-5</sup>	7.76 · 10 <sup>-5</sup>	1.05 · 10 <sup>-4</sup>
2-5,MBA-BC	3.77 · 10 <sup>-5</sup>	7.40 · 10 <sup>-5</sup>	9.87 · 10 <sup>-5</sup>	2.92 · 10 <sup>-5</sup>	6.13 · 10 <sup>-5</sup>	8.89 · 10 <sup>-5</sup>	4.84 · 10 <sup>-5</sup>	7.76 · 10 <sup>-5</sup>	1.05 · 10 <sup>-4</sup>
2-6,MBA-BC	3.77 · 10 <sup>-5</sup>	7.40 · 10 <sup>-5</sup>	9.87 · 10 <sup>-5</sup>	2.92 · 10 <sup>-5</sup>	6.13 · 10 <sup>-5</sup>	8.89 · 10 <sup>-5</sup>	4.84 · 10 <sup>-5</sup>	7.76 · 10 <sup>-5</sup>	1.05 · 10 <sup>-4</sup>
2-7,MBA-BC	3.77 · 10 <sup>-5</sup>	7.40 · 10 <sup>-5</sup>	9.87 · 10 <sup>-5</sup>	2.92 · 10 <sup>-5</sup>	6.13 · 10 <sup>-5</sup>	8.89 · 10 <sup>-5</sup>	4.84 · 10 <sup>-5</sup>	7.76 · 10 <sup>-5</sup>	1.05 · 10 <sup>-4</sup>
2-8,MBA-BC	3.77 · 10 <sup>-5</sup>	7.40 · 10 <sup>-5</sup>	9.87 · 10 <sup>-5</sup>	2.92 · 10 <sup>-5</sup>	6.13 · 10 <sup>-5</sup>	8.89 · 10 <sup>-5</sup>	4.84 · 10 <sup>-5</sup>	7.76 · 10 <sup>-5</sup>	1.05 · 10 <sup>-4</sup>
2-9,MBA-BC	3.45 · 10 <sup>-5</sup>	6.77 · 10 <sup>-5</sup>	9.03 · 10 <sup>-5</sup>	2.67 · 10 <sup>-5</sup>	5.61 · 10 <sup>-5</sup>	8.13 · 10 <sup>-5</sup>	4.42 · 10 <sup>-5</sup>	7.10 · 10 <sup>-5</sup>	9.56 · 10 <sup>-5</sup>
2-10,MBA-BC	3.45 · 10 <sup>-5</sup>	6.77 · 10 <sup>-5</sup>	9.03 · 10 <sup>-5</sup>	2.67 · 10 <sup>-5</sup>	5.61 · 10 <sup>-5</sup>	8.13 · 10 <sup>-5</sup>	4.42 · 10 <sup>-5</sup>	7.10 · 10 <sup>-5</sup>	9.56 · 10 <sup>-5</sup>
2-11,MBA-BC	3.02 · 10 <sup>-5</sup>	5.92 · 10 <sup>-5</sup>	7.90 · 10 <sup>-5</sup>	2.33 · 10 <sup>-5</sup>	4.90 · 10 <sup>-5</sup>	7.11 · 10 <sup>-5</sup>	3.87 · 10 <sup>-5</sup>	6.21 · 10 <sup>-5</sup>	8.37 · 10 <sup>-5</sup>
2-12,MBA-BC	2.59 · 10 <sup>-5</sup>	5.08 · 10 <sup>-5</sup>	6.77 · 10 <sup>-5</sup>	2.00 · 10 <sup>-5</sup>	4.20 · 10 <sup>-5</sup>	6.09 · 10 <sup>-5</sup>	3.32 · 10 <sup>-5</sup>	5.32 · 10 <sup>-5</sup>	7.17 · 10 <sup>-5</sup>
2-13,MBA-BC	2.15 · 10 <sup>-5</sup>	4.23 · 10 <sup>-5</sup>	5.64 · 10 <sup>-5</sup>	1.67 · 10 <sup>-5</sup>	3.50 · 10 <sup>-5</sup>	5.08 · 10 <sup>-5</sup>	2.76 · 10 <sup>-5</sup>	4.44 · 10 <sup>-5</sup>	5.98 · 10 <sup>-5</sup>
2-14,MBA-BC	1.72 · 10 <sup>-5</sup>	3.38 · 10 <sup>-5</sup>	4.51 · 10 <sup>-5</sup>	1.33 · 10 <sup>-5</sup>	2.80 · 10 <sup>-5</sup>	4.06 · 10 <sup>-5</sup>	2.21 · 10 <sup>-5</sup>	3.55 · 10 <sup>-5</sup>	4.78 · 10 <sup>-5</sup>
2-15,MBA-BC	1.29 · 10 <sup>-5</sup>	2.54 · 10 <sup>-5</sup>	3.39 · 10 <sup>-5</sup>	1.00 · 10 <sup>-5</sup>	2.10 · 10 <sup>-5</sup>	3.05 · 10 <sup>-5</sup>	1.66 · 10 <sup>-5</sup>	2.66 · 10 <sup>-5</sup>	3.59 · 10 <sup>-5</sup>
2-16,MBA-BC	1.29 · 10 <sup>-5</sup>	2.54 · 10 <sup>-5</sup>	3.39 · 10 <sup>-5</sup>	1.00 · 10 <sup>-5</sup>	2.10 · 10 <sup>-5</sup>	3.05 · 10 <sup>-5</sup>	1.66 · 10 <sup>-5</sup>	2.66 · 10 <sup>-5</sup>	3.59 · 10 <sup>-5</sup>
2-17,MBA-BC	1.08 · 10 <sup>-5</sup>	2.11 · 10 <sup>-5</sup>	2.82 · 10 <sup>-5</sup>	8.34 · 10 <sup>-6</sup>	1.75 · 10 <sup>-5</sup>	2.54 · 10 <sup>-5</sup>	1.38 · 10 <sup>-5</sup>	2.22 · 10 <sup>-5</sup>	2.99 · 10 <sup>-5</sup>
2-18,MBA-BC	6.46 · 10 <sup>-6</sup>	1.27 · 10 <sup>-5</sup>	1.69 · 10 <sup>-5</sup>	5.00 · 10 <sup>-6</sup>	1.05 · 10 <sup>-5</sup>	1.52 · 10 <sup>-5</sup>	8.29 · 10 <sup>-6</sup>	1.33 · 10 <sup>-5</sup>	1.79 · 10 <sup>-5</sup>
2-19,MBA-BC	2.59 · 10 <sup>-5</sup>	5.08 · 10 <sup>-5</sup>	6.77 · 10 <sup>-5</sup>	2.00 · 10 <sup>-5</sup>	4.20 · 10 <sup>-5</sup>	6.09 · 10 <sup>-5</sup>	3.32 · 10 <sup>-5</sup>	5.32 · 10 <sup>-5</sup>	7.17 · 10 <sup>-5</sup>
3-1,MBA-CAST	6.17 · 10 <sup>-5</sup>	1.18 · 10 <sup>-4</sup>	1.60 · 10 <sup>-4</sup>	5.08 · 10 <sup>-5</sup>	9.73 · 10 <sup>-5</sup>	1.49 · 10 <sup>-4</sup>	8.12 · 10 <sup>-5</sup>	1.27 · 10 <sup>-4</sup>	1.68 · 10 <sup>-4</sup>
3-2,MBA-CAST	1.72 · 10 <sup>-5</sup>	3.29 · 10 <sup>-5</sup>	4.46 · 10 <sup>-5</sup>	1.42 · 10 <sup>-5</sup>	2.72 · 10 <sup>-5</sup>	4.17 · 10 <sup>-5</sup>	2.27 · 10 <sup>-5</sup>	3.56 · 10 <sup>-5</sup>	4.70 · 10 <sup>-5</sup>
3-3,MBA-CAST	1.72 · 10 <sup>-5</sup>	3.29 · 10 <sup>-5</sup>	4.46 · 10 <sup>-5</sup>	1.42 · 10 <sup>-5</sup>	2.72 · 10 <sup>-5</sup>	4.17 · 10 <sup>-5</sup>	2.27 · 10 <sup>-5</sup>	3.56 · 10 <sup>-5</sup>	4.70 · 10 <sup>-5</sup>
3-4,MBA-CAST	1.72 · 10 <sup>-5</sup>	3.29 · 10 <sup>-5</sup>	4.46 · 10 <sup>-5</sup>	1.42 · 10 <sup>-5</sup>	2.72 · 10 <sup>-5</sup>	4.17 · 10 <sup>-5</sup>	2.27 · 10 <sup>-5</sup>	3.56 · 10 <sup>-5</sup>	4.70 · 10 <sup>-5</sup>
3-5,MBA-CAST	1.72 · 10 <sup>-5</sup>	3.29 · 10 <sup>-5</sup>	4.46 · 10 <sup>-5</sup>	1.42 · 10 <sup>-5</sup>	2.72 · 10 <sup>-5</sup>	4.17 · 10 <sup>-5</sup>	2.27 · 10 <sup>-5</sup>	3.56 · 10 <sup>-5</sup>	4.70 · 10 <sup>-5</sup>
3-6,MBA-CAST	1.72 · 10 <sup>-5</sup>	3.29 · 10 <sup>-5</sup>	4.46 · 10 <sup>-5</sup>	1.42 · 10 <sup>-5</sup>	2.72 · 10 <sup>-5</sup>	4.17 · 10 <sup>-5</sup>	2.27 · 10 <sup>-5</sup>	3.56 · 10 <sup>-5</sup>	4.70 · 10 <sup>-5</sup>
3-7,MBA-CAST	1.62 · 10 <sup>-5</sup>	3.09 · 10 <sup>-5</sup>	4.19 · 10 <sup>-5</sup>	1.33 · 10 <sup>-5</sup>	2.55 · 10 <sup>-5</sup>	3.92 · 10 <sup>-5</sup>	2.13 · 10 <sup>-5</sup>	3.34 · 10 <sup>-5</sup>	4.42 · 10 <sup>-5</sup>
3-8,MBA-CAST	4.32 · 10 <sup>-5</sup>	8.23 · 10 <sup>-5</sup>	1.12 · 10 <sup>-4</sup>	3.55 · 10 <sup>-5</sup>	6.81 · 10 <sup>-5</sup>	1.04 · 10 <sup>-4</sup>	5.69 · 10 <sup>-5</sup>	8.91 · 10 <sup>-5</sup>	1.18 · 10 <sup>-4</sup>

3-9,MBA-CAST	$4.32 \cdot 10^{-5}$	$8.23 \cdot 10^{-5}$	$1.12 \cdot 10^{-4}$	$3.55 \cdot 10^{-5}$	$6.81 \cdot 10^{-5}$	$1.04 \cdot 10^{-4}$	$5.69 \cdot 10^{-5}$	$8.91 \cdot 10^{-5}$	$1.18 \cdot 10^{-4}$
3-10,MBA-CAST	$7.90 \cdot 10^{-5}$	$1.51 \cdot 10^{-4}$	$2.05 \cdot 10^{-4}$	$6.50 \cdot 10^{-5}$	$1.25 \cdot 10^{-4}$	$1.91 \cdot 10^{-4}$	$1.04 \cdot 10^{-4}$	$1.63 \cdot 10^{-4}$	$2.16 \cdot 10^{-4}$
3-11,MBA-CAST	$1.18 \cdot 10^{-5}$	$2.25 \cdot 10^{-5}$	$3.06 \cdot 10^{-5}$	$9.73 \cdot 10^{-6}$	$1.86 \cdot 10^{-5}$	$2.86 \cdot 10^{-5}$	$1.56 \cdot 10^{-5}$	$2.44 \cdot 10^{-5}$	$3.22 \cdot 10^{-5}$
3-12,MBA-CAST	$1.18 \cdot 10^{-5}$	$2.25 \cdot 10^{-5}$	$3.06 \cdot 10^{-5}$	$9.73 \cdot 10^{-6}$	$1.86 \cdot 10^{-5}$	$2.86 \cdot 10^{-5}$	$1.56 \cdot 10^{-5}$	$2.44 \cdot 10^{-5}$	$3.22 \cdot 10^{-5}$
3-13,MBA-CAST	$1.18 \cdot 10^{-5}$	$2.25 \cdot 10^{-5}$	$3.06 \cdot 10^{-5}$	$9.73 \cdot 10^{-6}$	$1.86 \cdot 10^{-5}$	$2.86 \cdot 10^{-5}$	$1.56 \cdot 10^{-5}$	$2.44 \cdot 10^{-5}$	$3.22 \cdot 10^{-5}$
3-14,MBA-CAST	$1.18 \cdot 10^{-5}$	$2.25 \cdot 10^{-5}$	$3.06 \cdot 10^{-5}$	$9.73 \cdot 10^{-6}$	$1.86 \cdot 10^{-5}$	$2.86 \cdot 10^{-5}$	$1.56 \cdot 10^{-5}$	$2.44 \cdot 10^{-5}$	$3.22 \cdot 10^{-5}$
3-16,MBA-CAST	$9.85 \cdot 10^{-6}$	$1.88 \cdot 10^{-5}$	$2.55 \cdot 10^{-5}$	$8.10 \cdot 10^{-6}$	$1.55 \cdot 10^{-5}$	$2.38 \cdot 10^{-5}$	$1.30 \cdot 10^{-5}$	$2.03 \cdot 10^{-5}$	$2.69 \cdot 10^{-5}$
3-18,MBA-CAST	$7.88 \cdot 10^{-6}$	$1.50 \cdot 10^{-5}$	$2.04 \cdot 10^{-5}$	$6.48 \cdot 10^{-6}$	$1.24 \cdot 10^{-5}$	$1.90 \cdot 10^{-5}$	$1.04 \cdot 10^{-5}$	$1.63 \cdot 10^{-5}$	$2.15 \cdot 10^{-5}$
3-19,MBA-CAST	$5.91 \cdot 10^{-6}$	$1.13 \cdot 10^{-5}$	$1.53 \cdot 10^{-5}$	$4.86 \cdot 10^{-6}$	$9.32 \cdot 10^{-6}$	$1.43 \cdot 10^{-5}$	$7.78 \cdot 10^{-6}$	$1.22 \cdot 10^{-5}$	$1.61 \cdot 10^{-5}$
3-20,MBA-CAST	$5.91 \cdot 10^{-6}$	$1.13 \cdot 10^{-5}$	$1.53 \cdot 10^{-5}$	$4.86 \cdot 10^{-6}$	$9.32 \cdot 10^{-6}$	$1.43 \cdot 10^{-5}$	$7.78 \cdot 10^{-6}$	$1.22 \cdot 10^{-5}$	$1.61 \cdot 10^{-5}$
3-22,MBA-CAST	$1.35 \cdot 10^{-5}$	$2.58 \cdot 10^{-5}$	$3.50 \cdot 10^{-5}$	$1.11 \cdot 10^{-5}$	$2.13 \cdot 10^{-5}$	$3.27 \cdot 10^{-5}$	$1.78 \cdot 10^{-5}$	$2.79 \cdot 10^{-5}$	$3.69 \cdot 10^{-5}$
3-23,MBA-CAST	$1.97 \cdot 10^{-5}$	$3.77 \cdot 10^{-5}$	$5.11 \cdot 10^{-5}$	$1.63 \cdot 10^{-5}$	$3.12 \cdot 10^{-5}$	$4.78 \cdot 10^{-5}$	$2.60 \cdot 10^{-5}$	$4.08 \cdot 10^{-5}$	$5.39 \cdot 10^{-5}$
3-24,MBA-CAST	$2.95 \cdot 10^{-6}$	$5.63 \cdot 10^{-6}$	$7.65 \cdot 10^{-6}$	$2.43 \cdot 10^{-6}$	$4.66 \cdot 10^{-6}$	$7.14 \cdot 10^{-6}$	$3.89 \cdot 10^{-6}$	$6.10 \cdot 10^{-6}$	$8.06 \cdot 10^{-6}$
3-25,MBA-CAST	$1.18 \cdot 10^{-5}$	$2.25 \cdot 10^{-5}$	$3.06 \cdot 10^{-5}$	$9.73 \cdot 10^{-6}$	$1.86 \cdot 10^{-5}$	$2.86 \cdot 10^{-5}$	$1.56 \cdot 10^{-5}$	$2.44 \cdot 10^{-5}$	$3.22 \cdot 10^{-5}$
3-26,MBA-CAST	$7.90 \cdot 10^{-5}$	$1.51 \cdot 10^{-4}$	$2.05 \cdot 10^{-4}$	$6.50 \cdot 10^{-5}$	$1.25 \cdot 10^{-4}$	$1.91 \cdot 10^{-4}$	$1.04 \cdot 10^{-4}$	$1.63 \cdot 10^{-4}$	$2.16 \cdot 10^{-4}$
4-1,MBA-VLT	$5.41 \cdot 10^{-5}$	$1.12 \cdot 10^{-4}$	$1.55 \cdot 10^{-4}$	$4.57 \cdot 10^{-5}$	$9.42 \cdot 10^{-5}$	$1.38 \cdot 10^{-4}$	$7.20 \cdot 10^{-5}$	$1.13 \cdot 10^{-4}$	$1.61 \cdot 10^{-4}$
4-2,MBA-VLT	$5.41 \cdot 10^{-5}$	$1.12 \cdot 10^{-4}$	$1.55 \cdot 10^{-4}$	$4.57 \cdot 10^{-5}$	$9.42 \cdot 10^{-5}$	$1.38 \cdot 10^{-4}$	$7.20 \cdot 10^{-5}$	$1.13 \cdot 10^{-4}$	$1.61 \cdot 10^{-4}$
4-3,MBA-VLT	$4.73 \cdot 10^{-5}$	$9.77 \cdot 10^{-5}$	$1.36 \cdot 10^{-4}$	$4.00 \cdot 10^{-5}$	$8.24 \cdot 10^{-5}$	$1.21 \cdot 10^{-4}$	$6.30 \cdot 10^{-5}$	$9.86 \cdot 10^{-5}$	$1.41 \cdot 10^{-4}$
4-4,MBA-VLT	$4.73 \cdot 10^{-5}$	$9.77 \cdot 10^{-5}$	$1.36 \cdot 10^{-4}$	$4.00 \cdot 10^{-5}$	$8.24 \cdot 10^{-5}$	$1.21 \cdot 10^{-4}$	$6.30 \cdot 10^{-5}$	$9.86 \cdot 10^{-5}$	$1.41 \cdot 10^{-4}$
4-5,MBA-VLT	$4.73 \cdot 10^{-5}$	$9.77 \cdot 10^{-5}$	$1.36 \cdot 10^{-4}$	$4.00 \cdot 10^{-5}$	$8.24 \cdot 10^{-5}$	$1.21 \cdot 10^{-4}$	$6.30 \cdot 10^{-5}$	$9.86 \cdot 10^{-5}$	$1.41 \cdot 10^{-4}$
4-6,MBA-VLT	$4.73 \cdot 10^{-5}$	$9.77 \cdot 10^{-5}$	$1.36 \cdot 10^{-4}$	$4.00 \cdot 10^{-5}$	$8.24 \cdot 10^{-5}$	$1.21 \cdot 10^{-4}$	$6.30 \cdot 10^{-5}$	$9.86 \cdot 10^{-5}$	$1.41 \cdot 10^{-4}$
4-7,MBA-VLT	$4.73 \cdot 10^{-5}$	$9.77 \cdot 10^{-5}$	$1.36 \cdot 10^{-4}$	$4.00 \cdot 10^{-5}$	$8.24 \cdot 10^{-5}$	$1.21 \cdot 10^{-4}$	$6.30 \cdot 10^{-5}$	$9.86 \cdot 10^{-5}$	$1.41 \cdot 10^{-4}$
4-9,MBA-VLT	$4.33 \cdot 10^{-5}$	$8.93 \cdot 10^{-5}$	$1.24 \cdot 10^{-4}$	$3.66 \cdot 10^{-5}$	$7.54 \cdot 10^{-5}$	$1.11 \cdot 10^{-4}$	$5.76 \cdot 10^{-5}$	$9.01 \cdot 10^{-5}$	$1.29 \cdot 10^{-4}$
4-10,MBA-VLT	$4.33 \cdot 10^{-5}$	$8.93 \cdot 10^{-5}$	$1.24 \cdot 10^{-4}$	$3.66 \cdot 10^{-5}$	$7.54 \cdot 10^{-5}$	$1.11 \cdot 10^{-4}$	$5.76 \cdot 10^{-5}$	$9.01 \cdot 10^{-5}$	$1.29 \cdot 10^{-4}$
4-11,MBA-VLT	$3.78 \cdot 10^{-5}$	$7.82 \cdot 10^{-5}$	$1.09 \cdot 10^{-4}$	$3.20 \cdot 10^{-5}$	$6.60 \cdot 10^{-5}$	$9.68 \cdot 10^{-5}$	$5.04 \cdot 10^{-5}$	$7.89 \cdot 10^{-5}$	$1.13 \cdot 10^{-4}$
MBA-REC	$3.33 \cdot 10^{-4}$								
MBA-BC	$5.03 \cdot 10^{-2}$								
MBA-CAST	$5.37 \cdot 10^{-2}$								
MBA-VLT	$5.03 \cdot 10^{-2}$								
REC	$1.69 \cdot 10^{-4}$	$3.16 \cdot 10^{-4}$	$4.32 \cdot 10^{-4}$	$1.39 \cdot 10^{-4}$	$2.61 \cdot 10^{-4}$	$3.81 \cdot 10^{-4}$	$2.14 \cdot 10^{-4}$	$3.53 \cdot 10^{-4}$	$4.47 \cdot 10^{-4}$
BC	$1.65 \cdot 10^{-3}$	$1.92 \cdot 10^{-3}$	$2.11 \cdot 10^{-3}$	$1.58 \cdot 10^{-3}$	$1.83 \cdot 10^{-3}$	$2.03 \cdot 10^{-3}$	$1.73 \cdot 10^{-3}$	$1.95 \cdot 10^{-3}$	$2.15 \cdot 10^{-3}$
CAST	$1.37 \cdot 10^{-3}$	$1.57 \cdot 10^{-3}$	$1.72 \cdot 10^{-3}$	$1.33 \cdot 10^{-3}$	$1.49 \cdot 10^{-3}$	$1.68 \cdot 10^{-3}$	$1.44 \cdot 10^{-3}$	$1.60 \cdot 10^{-3}$	$1.75 \cdot 10^{-3}$
VLT	$4.97 \cdot 10^{-3}$	$5.80 \cdot 10^{-3}$	$6.43 \cdot 10^{-3}$	$4.85 \cdot 10^{-3}$	$5.55 \cdot 10^{-3}$	$6.19 \cdot 10^{-3}$	$5.23 \cdot 10^{-3}$	$5.82 \cdot 10^{-3}$	$6.52 \cdot 10^{-3}$
REC-NOMBA	$3.33 \cdot 10^{-4}$	$6.32 \cdot 10^{-4}$	$8.67 \cdot 10^{-4}$	$2.71 \cdot 10^{-4}$	$5.19 \cdot 10^{-4}$	$7.63 \cdot 10^{-4}$	$4.23 \cdot 10^{-4}$	$7.07 \cdot 10^{-4}$	$8.98 \cdot 10^{-4}$
BC-NOMBA	$5.60 \cdot 10^{-4}$	$1.10 \cdot 10^{-3}$	$1.47 \cdot 10^{-3}$	$4.34 \cdot 10^{-4}$	$9.11 \cdot 10^{-4}$	$1.32 \cdot 10^{-3}$	$7.19 \cdot 10^{-4}$	$1.15 \cdot 10^{-3}$	$1.55 \cdot 10^{-3}$

CAST-NOMBA	$4.31 \cdot 10^{-4}$	$8.23 \cdot 10^{-4}$	$1.12 \cdot 10^{-3}$	$3.55 \cdot 10^{-4}$	$6.81 \cdot 10^{-4}$	$1.04 \cdot 10^{-3}$	$5.68 \cdot 10^{-4}$	$8.90 \cdot 10^{-4}$	$1.18 \cdot 10^{-3}$
VLT-NOMBA	$9.33 \cdot 10^{-4}$	$1.93 \cdot 10^{-3}$	$2.68 \cdot 10^{-3}$	$7.88 \cdot 10^{-4}$	$1.63 \cdot 10^{-3}$	$2.39 \cdot 10^{-3}$	$1.24 \cdot 10^{-3}$	$1.94 \cdot 10^{-3}$	$2.78 \cdot 10^{-3}$

## B Plots of Cut Sets

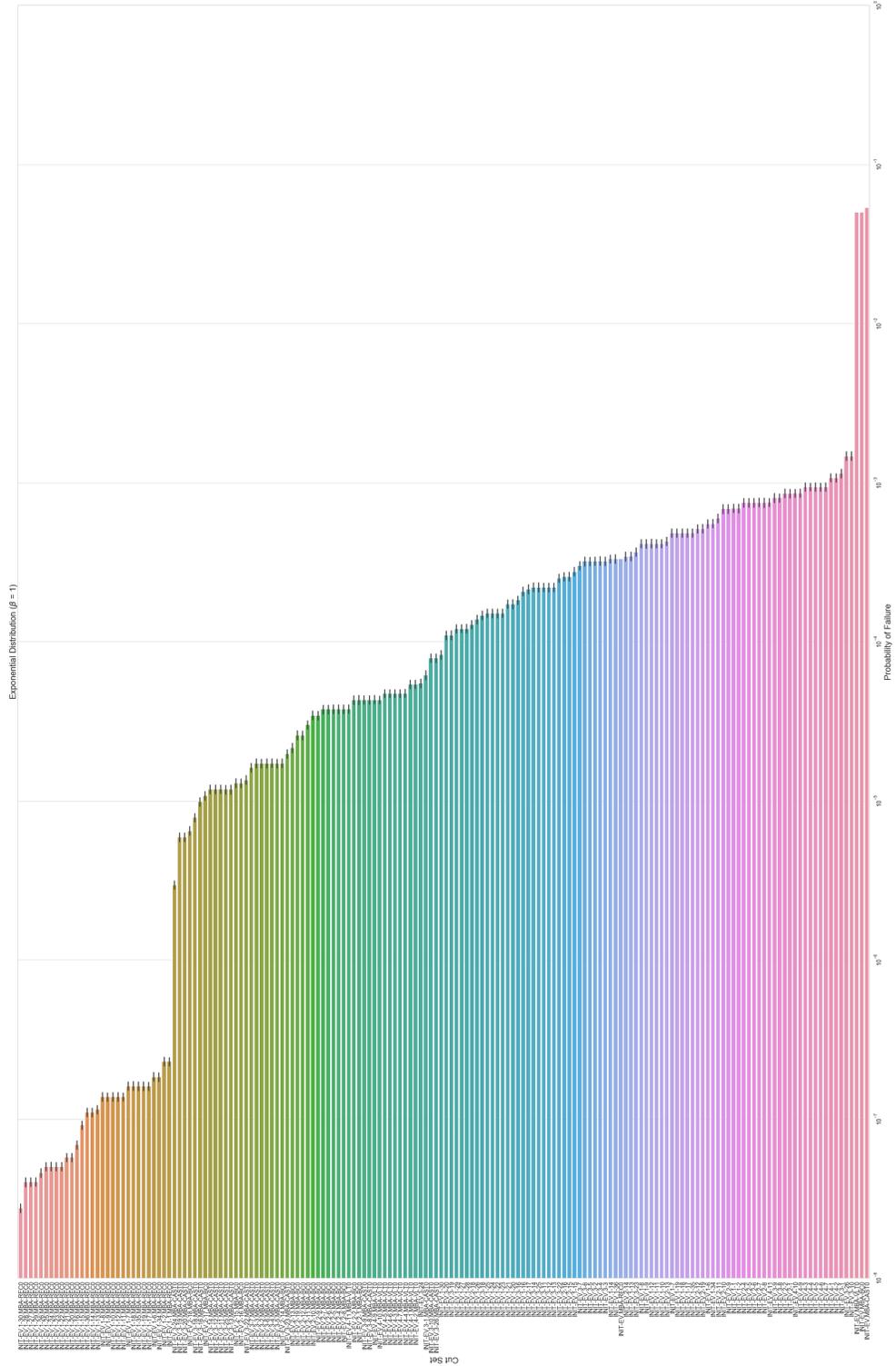


Figure 14: Exponential Distribution ( $\beta = 1$ ): Overall

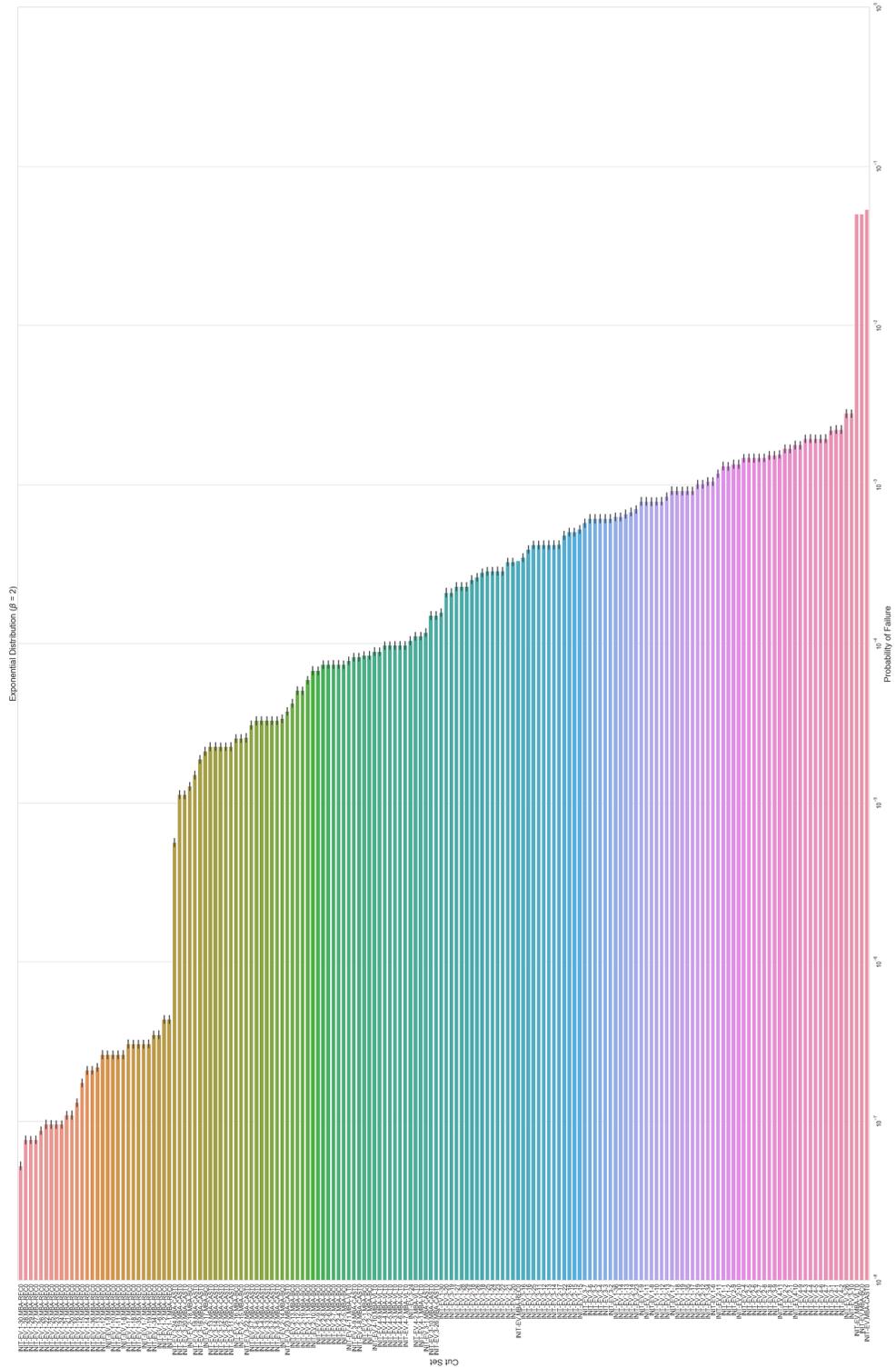


Figure 15: Exponential Distribution ( $\beta = 2$ ): Overall

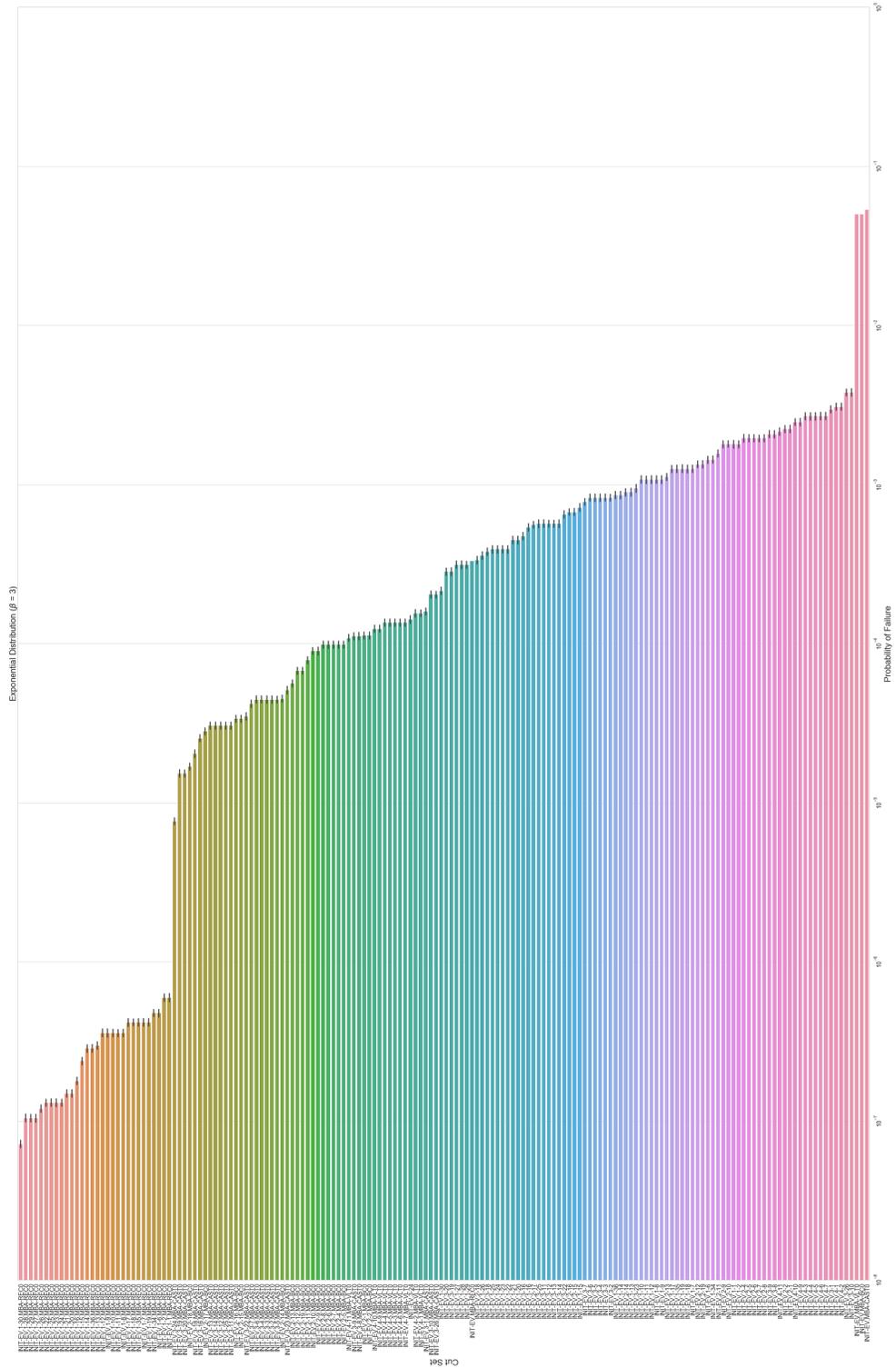


Figure 16: Exponential Distribution ( $\beta = 3$ ): Overall

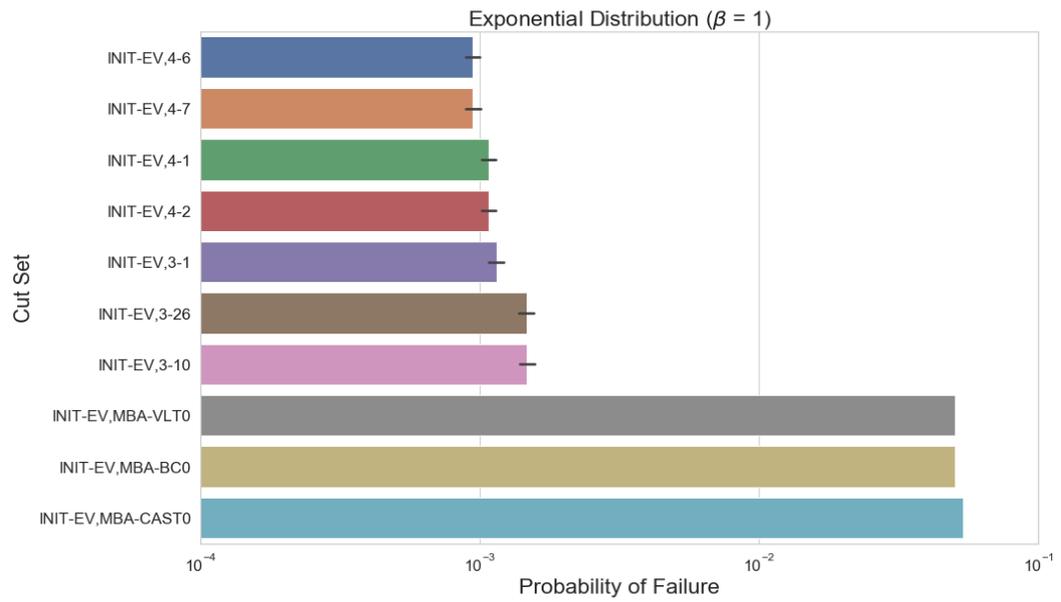


Figure 17: Exponential Distribution ( $\beta = 1$ ): Largest

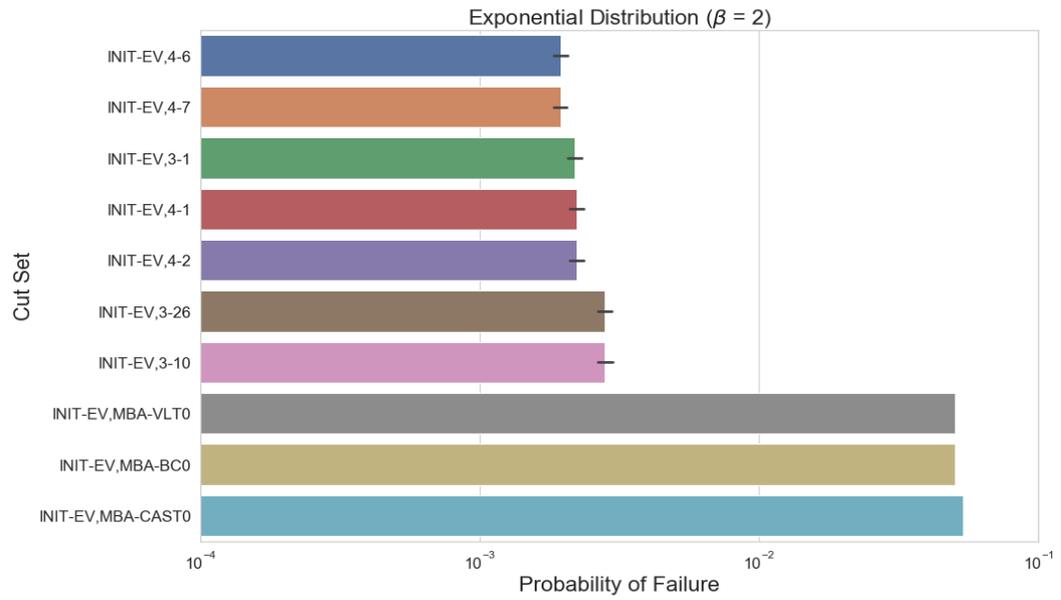


Figure 24: Exponential Distribution ( $\beta = 2$ ): Largest

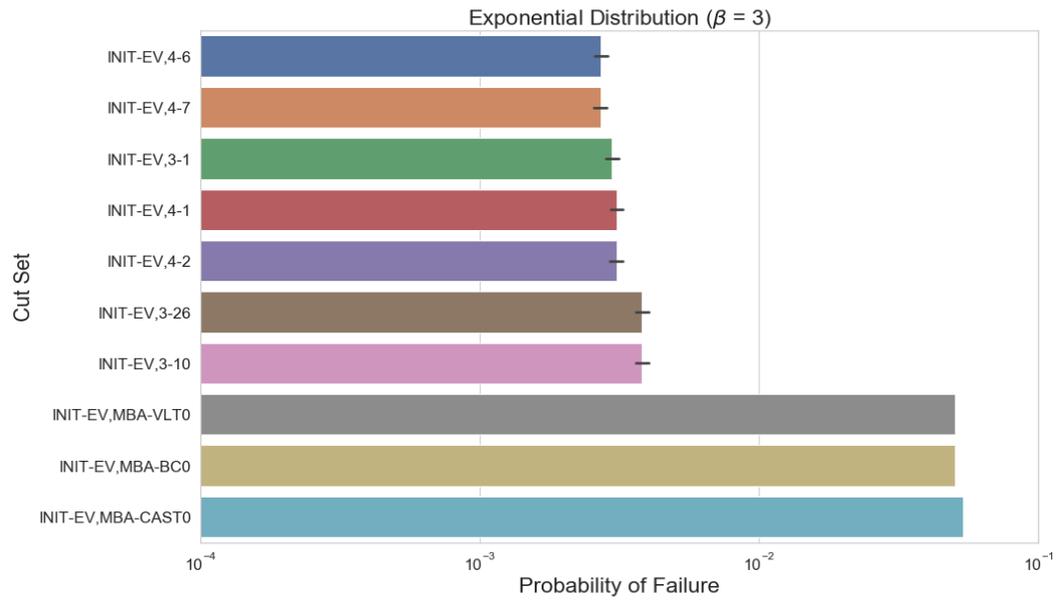


Figure 25: Exponential Distribution ( $\beta = 3$ ): Largest

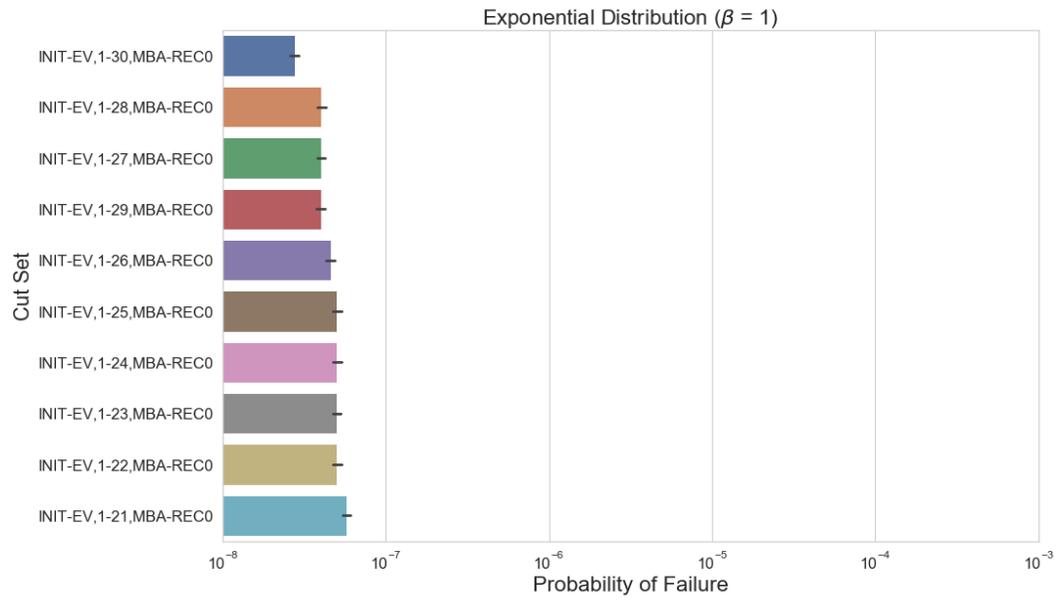


Figure 18: Exponential Distribution ( $\beta = 1$ ): Smallest

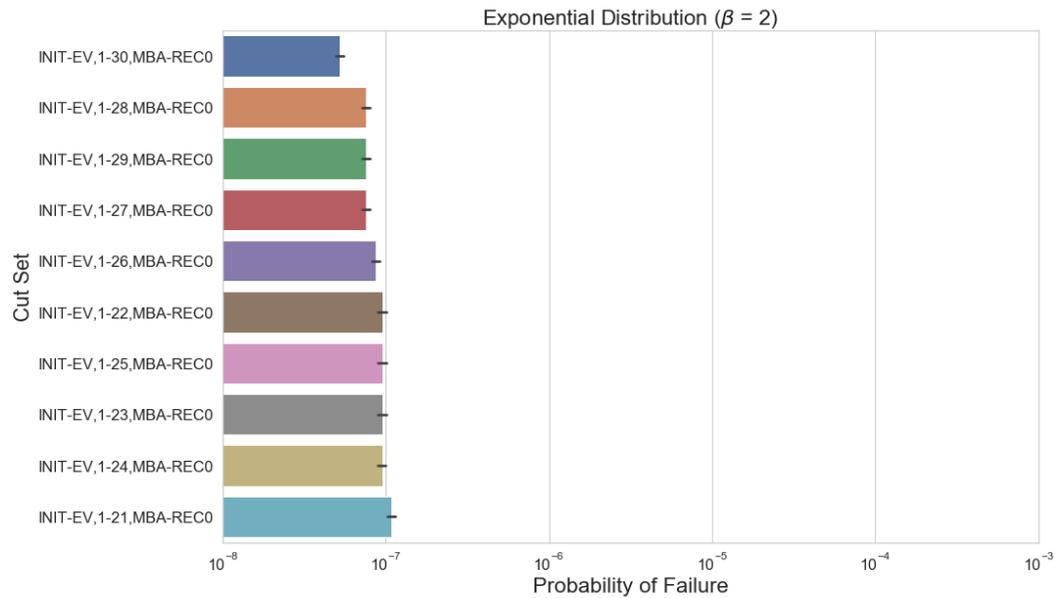


Figure 26: Exponential Distribution ( $\beta = 2$ ): Smallest

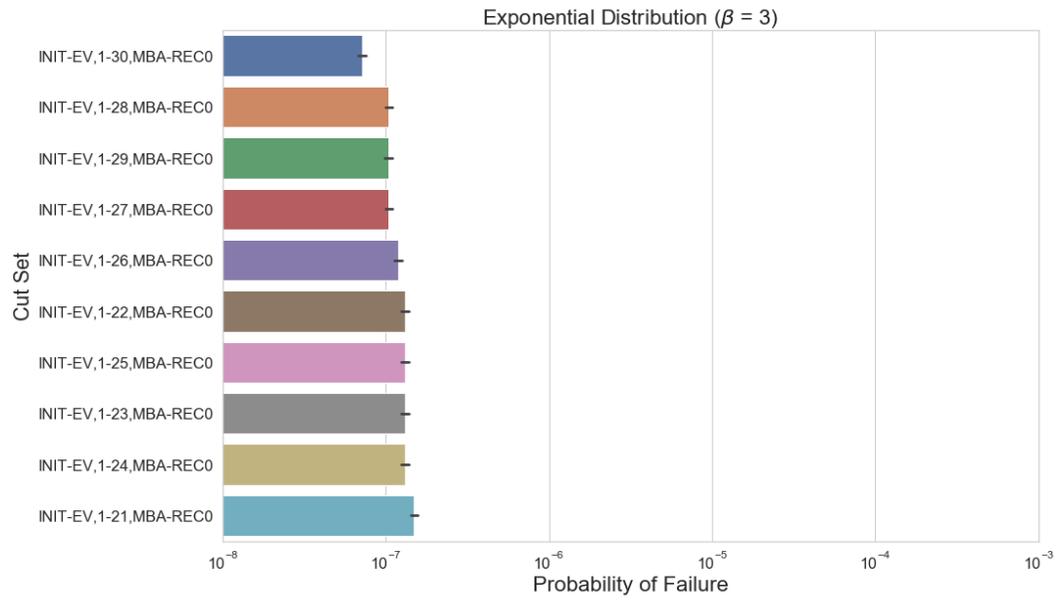


Figure 27: Exponential Distribution ( $\beta = 3$ ): Smallest

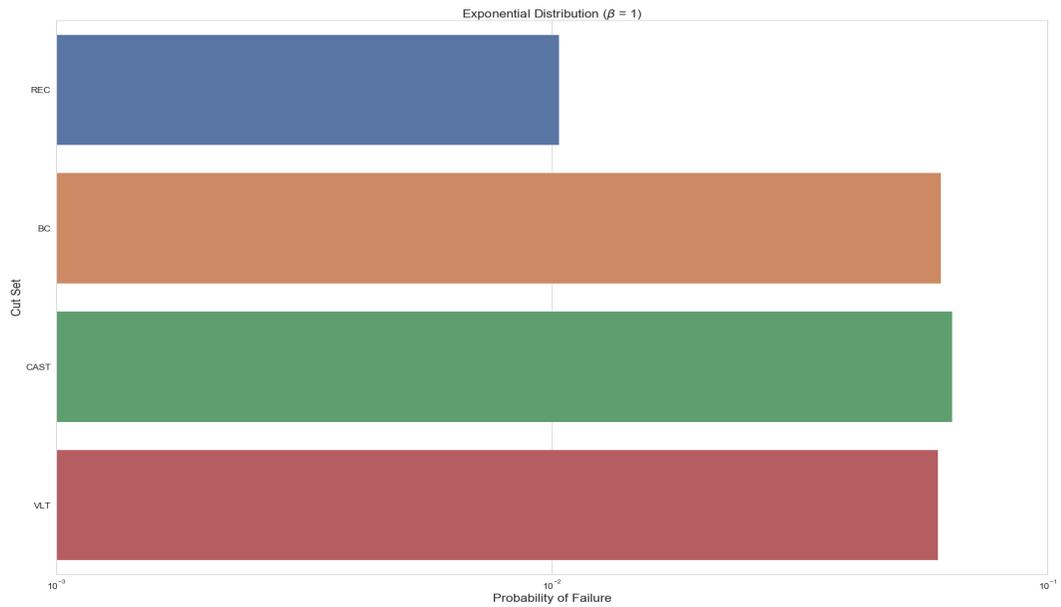


Figure 19: Exponential Distribution ( $\beta = 1$ ): By-Area

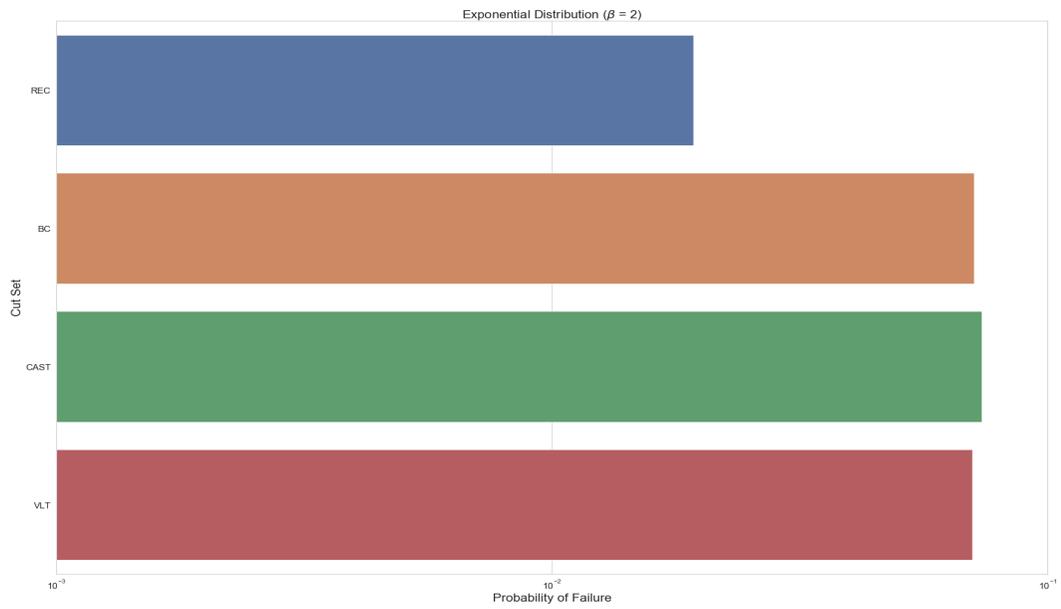


Figure 28: Exponential Distribution ( $\beta = 2$ ): By-Area

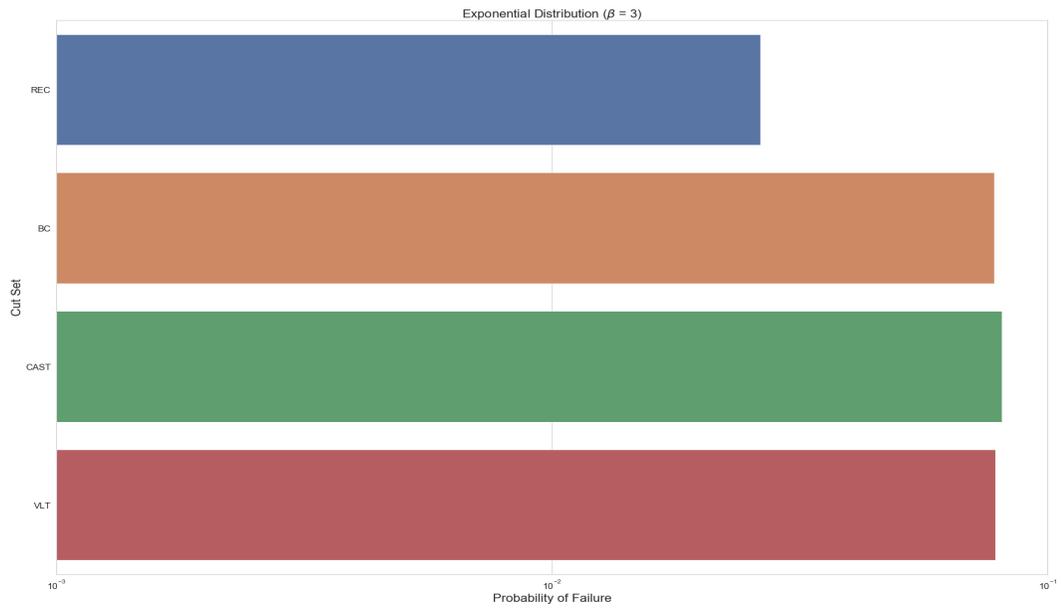


Figure 29: Exponential Distribution ( $\beta = 3$ ): By-Area

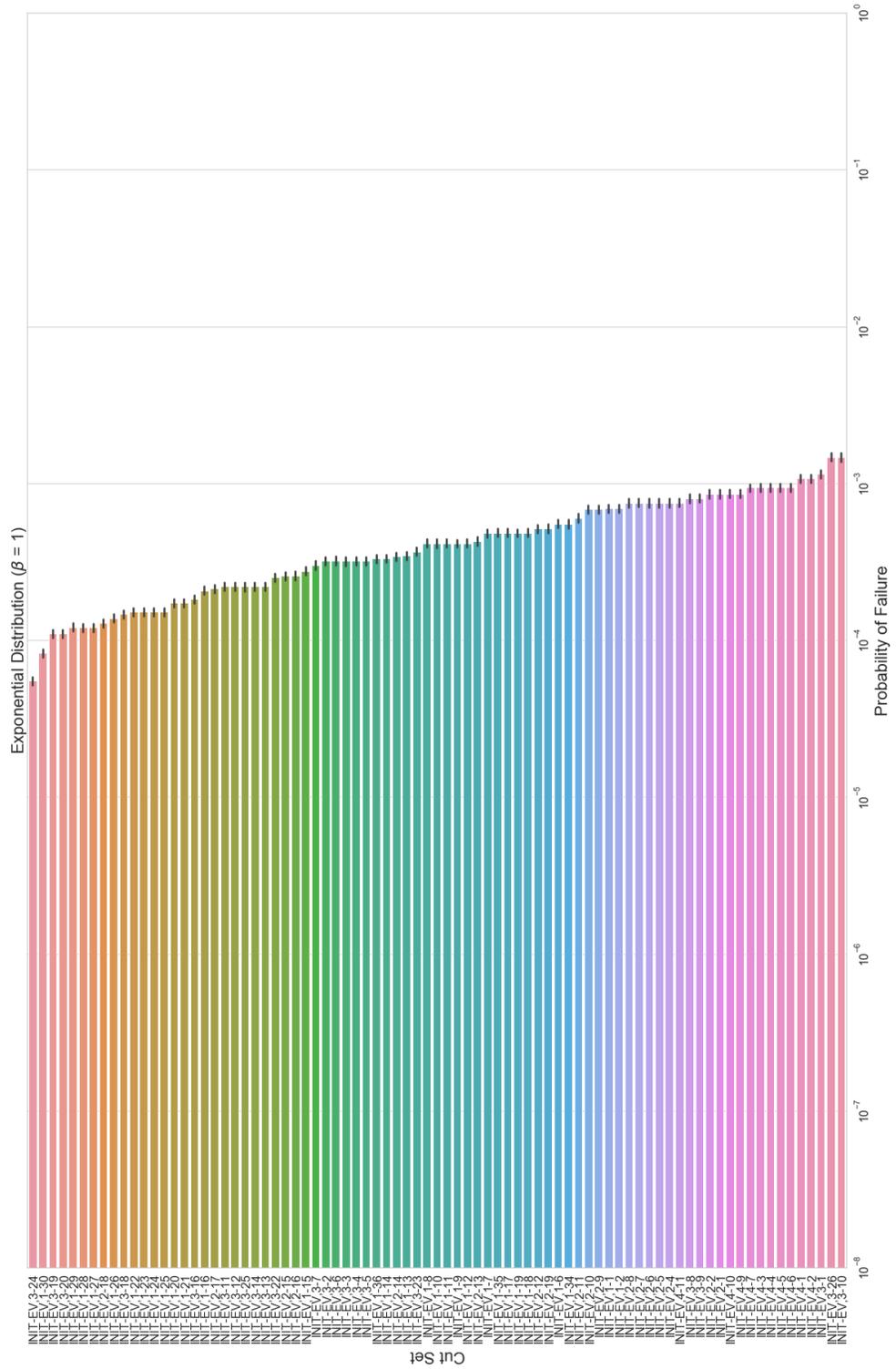


Figure 20: Exponential Distribution ( $\beta = 1$ ): Overall, No MA



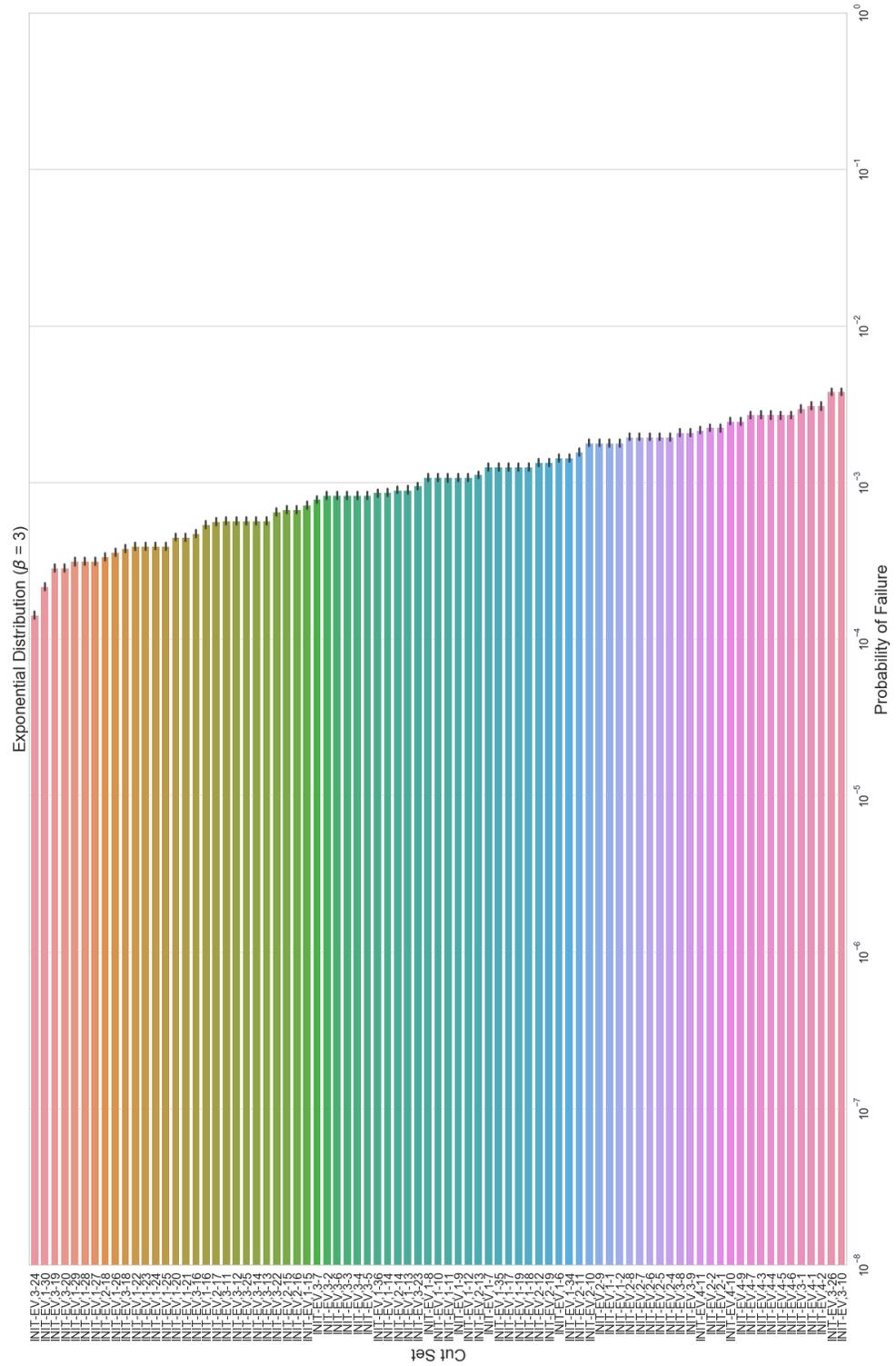


Figure 31: Exponential Distribution ( $\beta = 3$ ): Overall, No MA

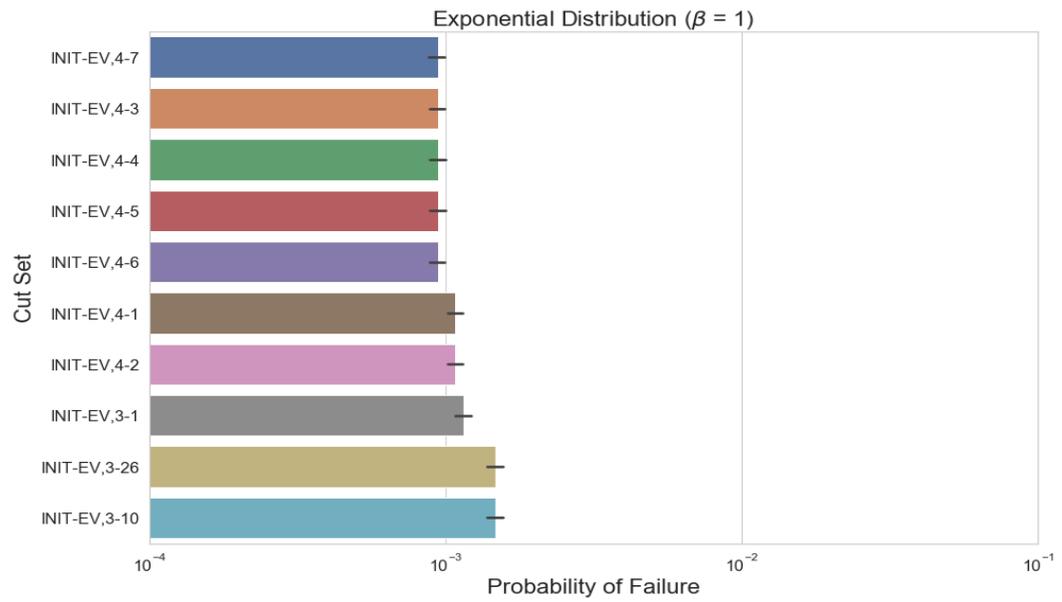


Figure 21: Exponential Distribution ( $\beta = 1$ ): Largest, No MA

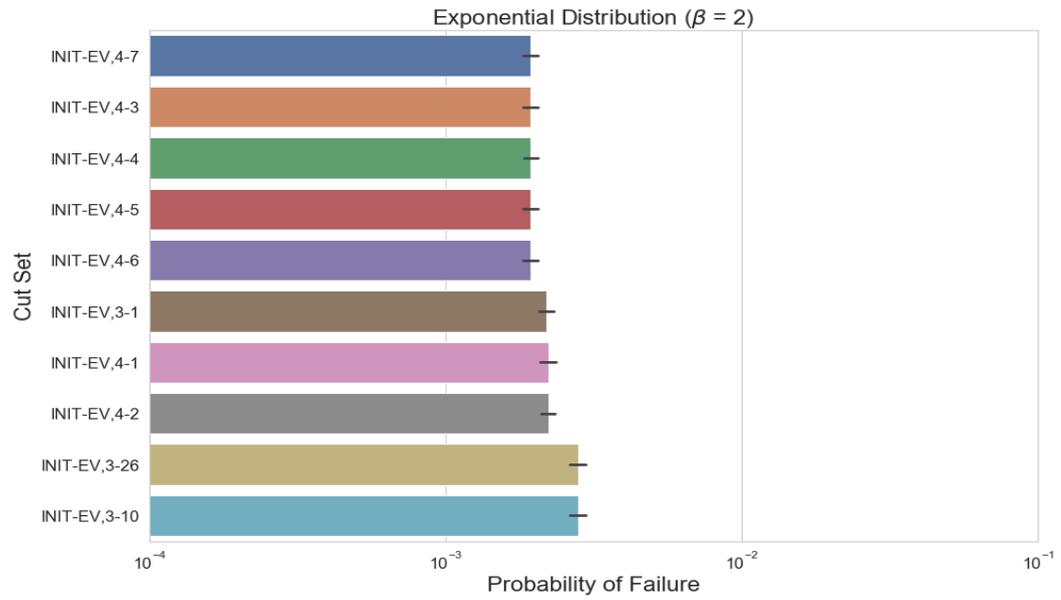


Figure 32: Exponential Distribution ( $\beta = 2$ ): Largest, No MA

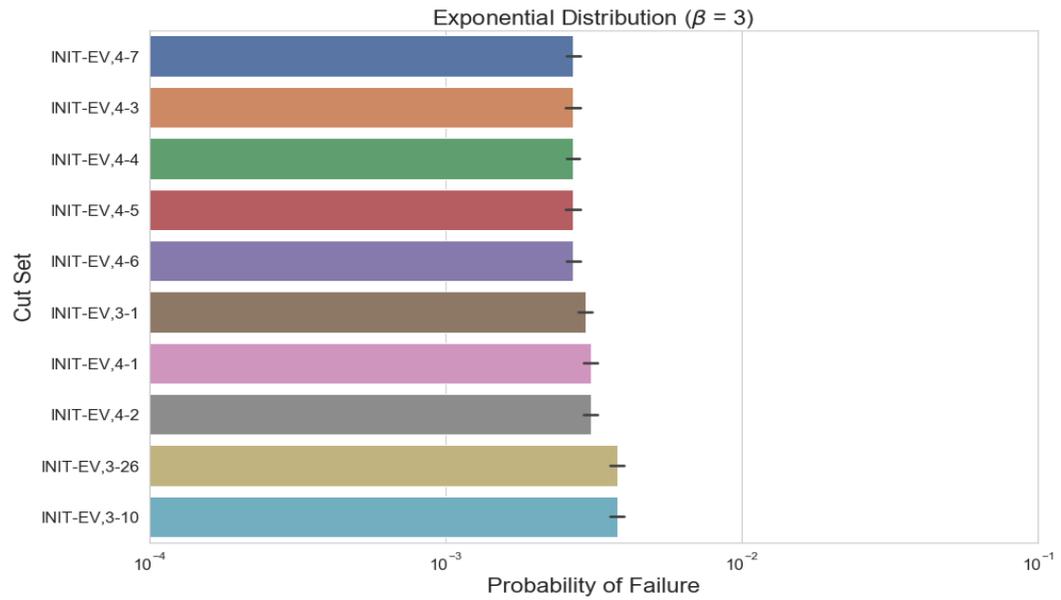


Figure 33: Exponential Distribution ( $\beta = 3$ ): Largest, No MA

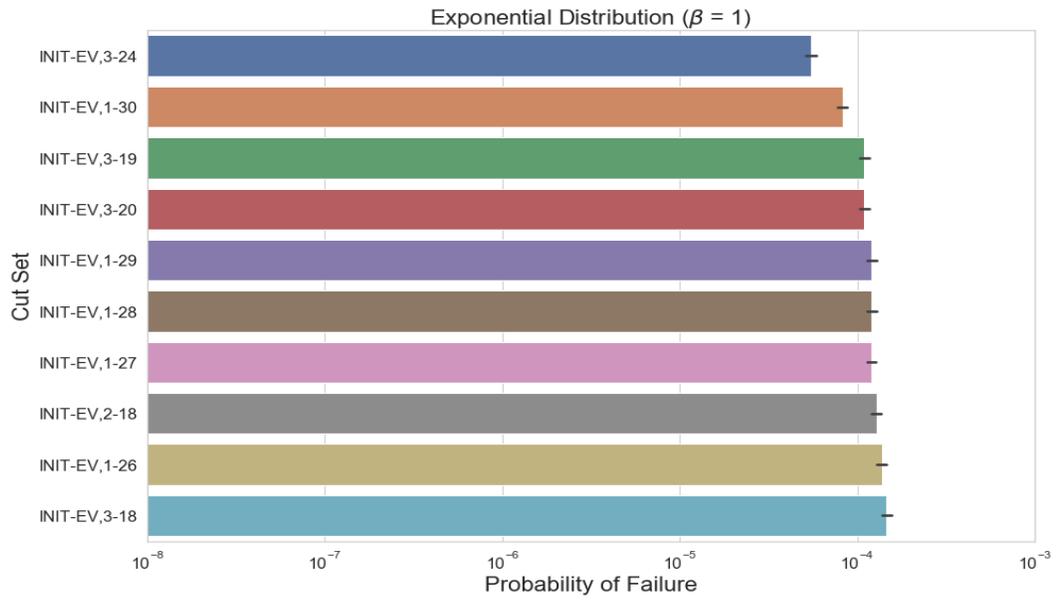


Figure 22: Exponential Distribution ( $\beta = 1$ ): Smallest, No MA

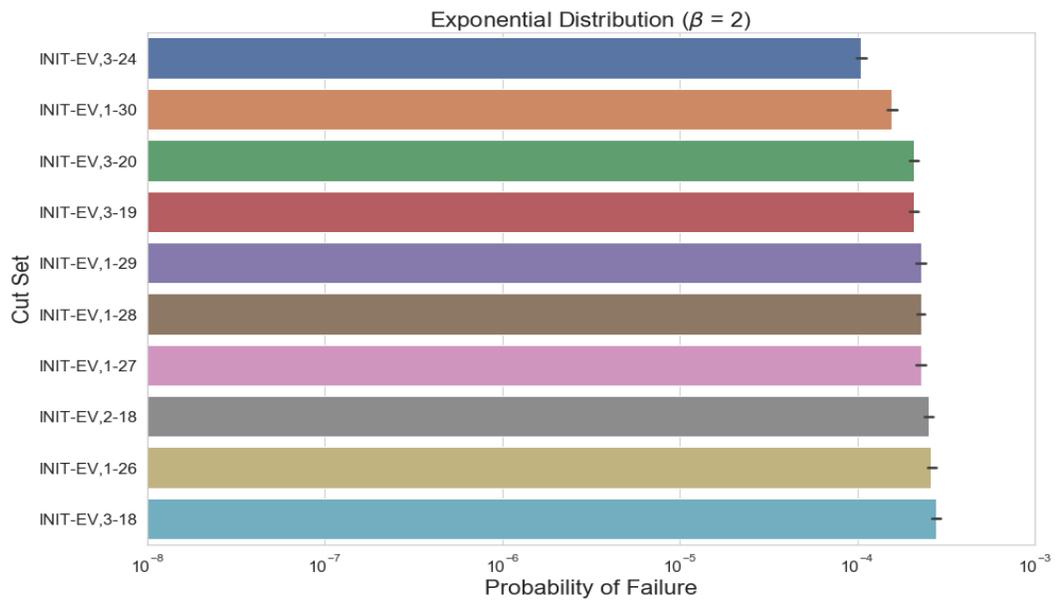


Figure 34: Exponential Distribution ( $\beta = 2$ ): Smallest, No MA

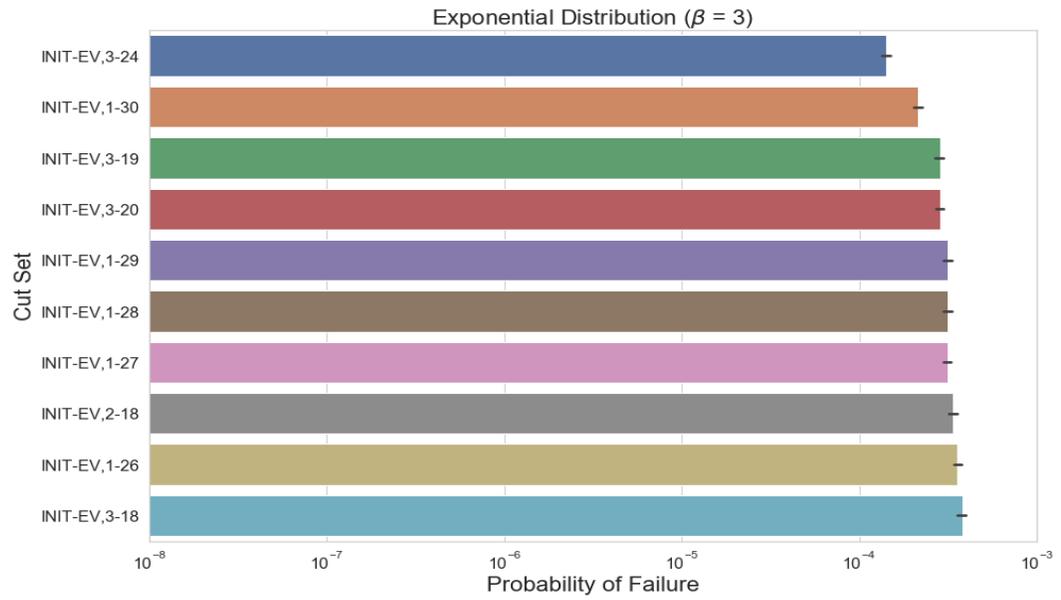


Figure 35: Exponential Distribution ( $\beta = 3$ ): Smallest, No MA

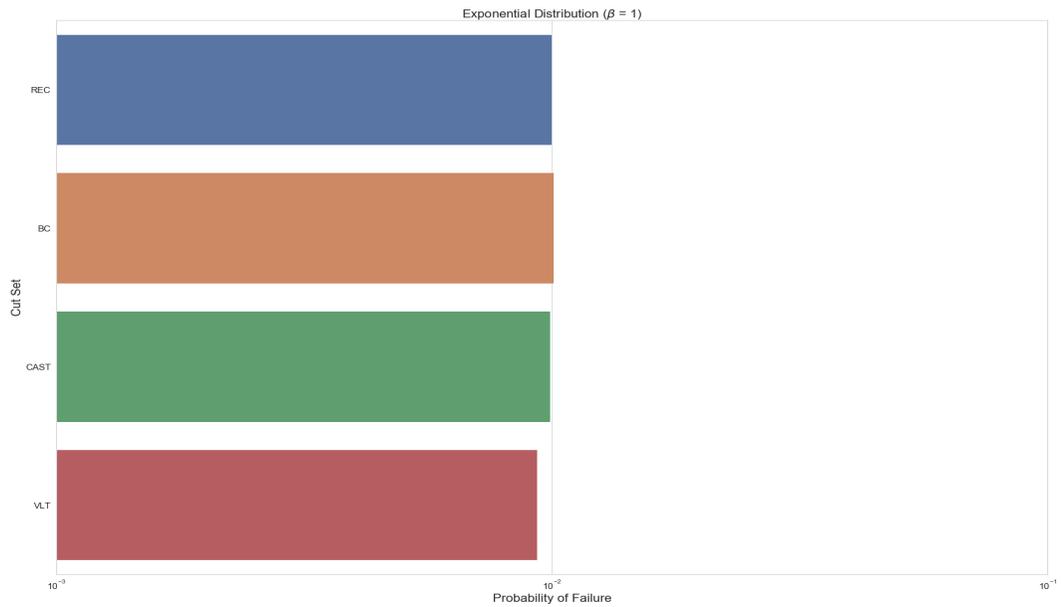


Figure 23: Exponential Distribution ( $\beta = 1$ ): By-Area, No MA

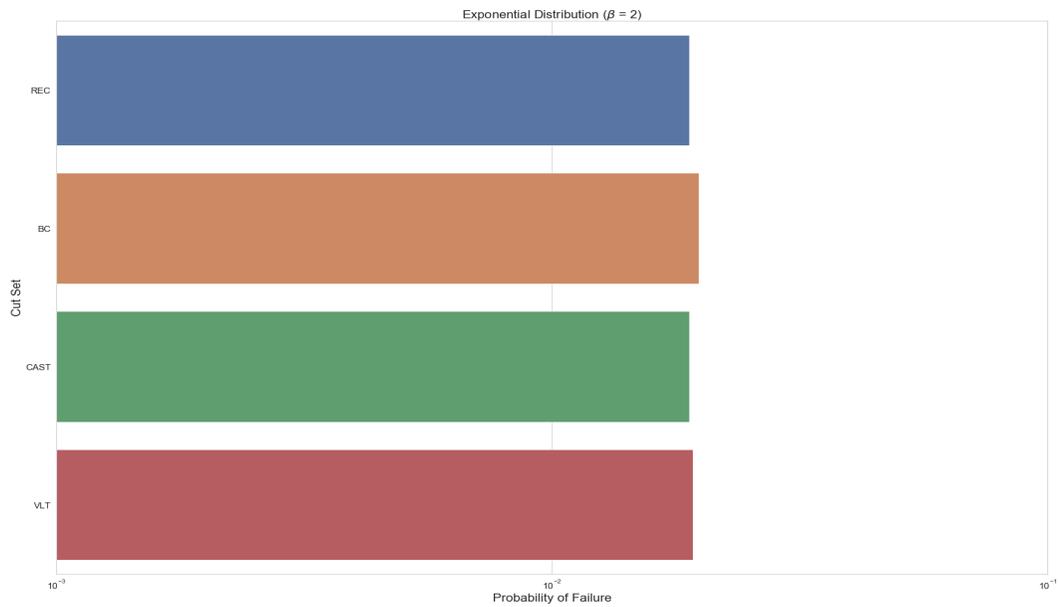


Figure 36: Exponential Distribution ( $\beta = 2$ ): By-Area, No MA

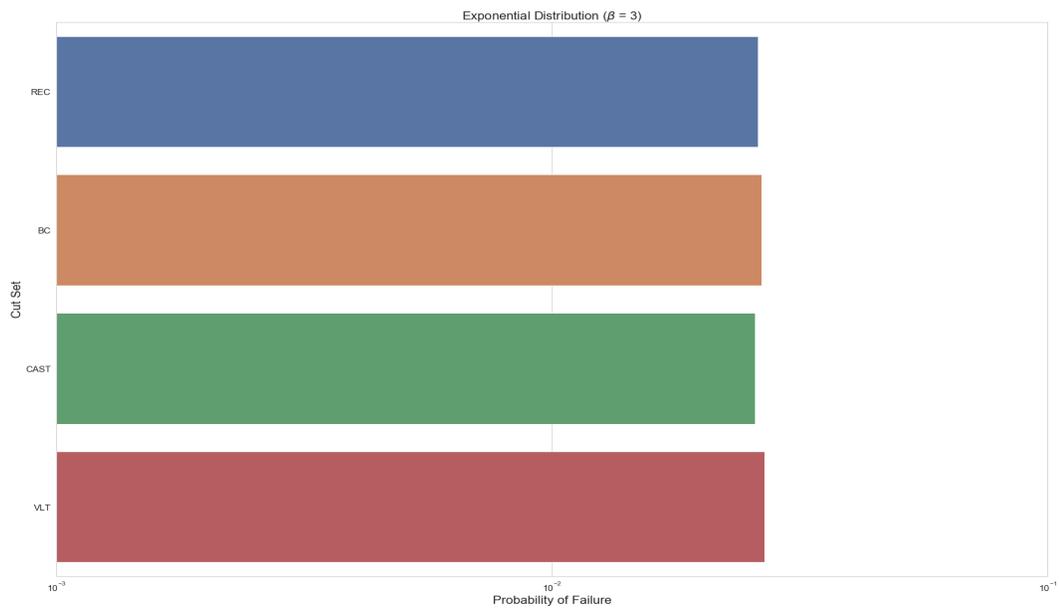


Figure 37: Exponential Distribution ( $\beta = 3$ ): By-Area, No MA

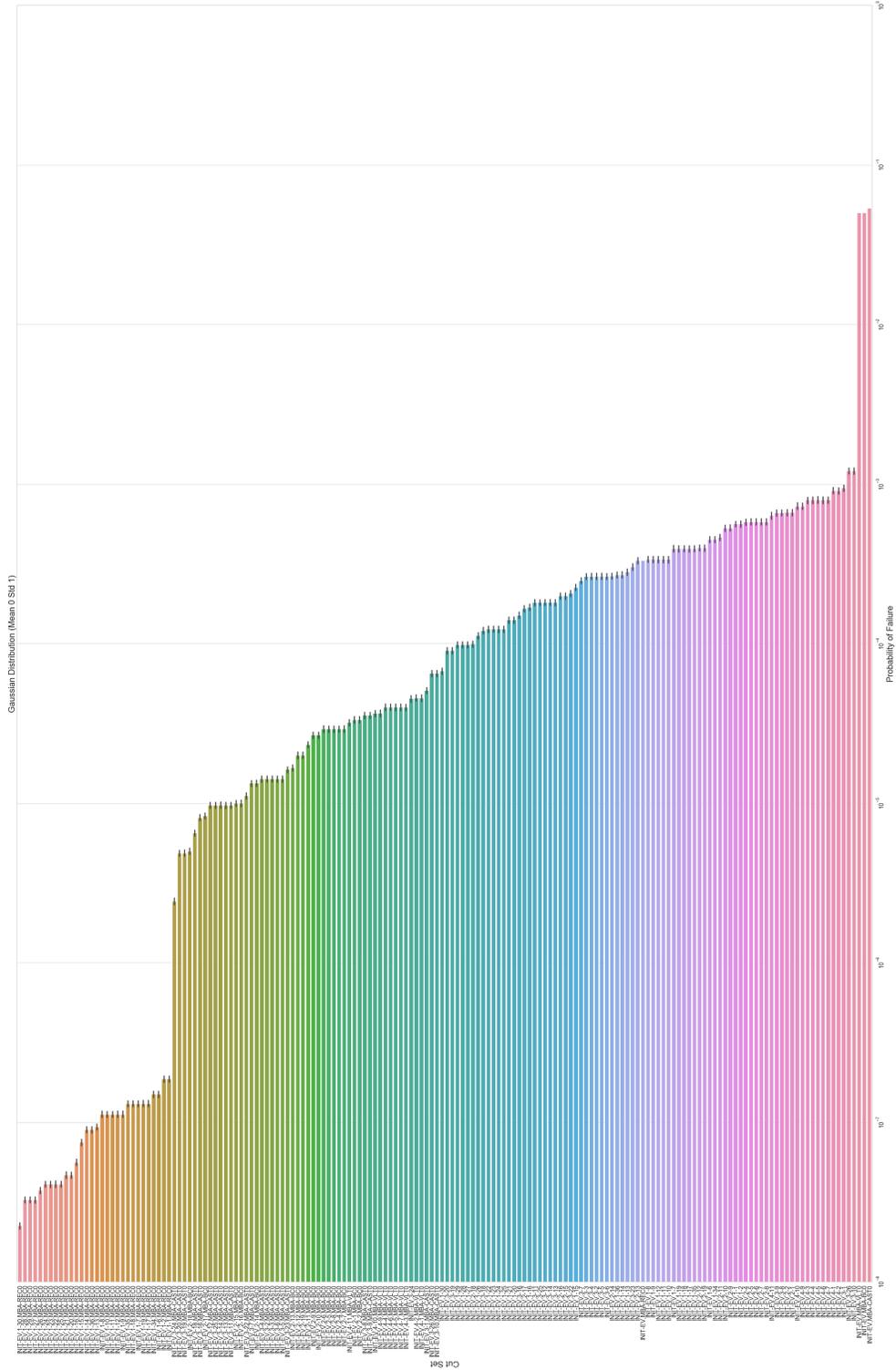


Figure 38: Gaussian Distribution ( $\mu = 0, \sigma = 1$ ): Overall

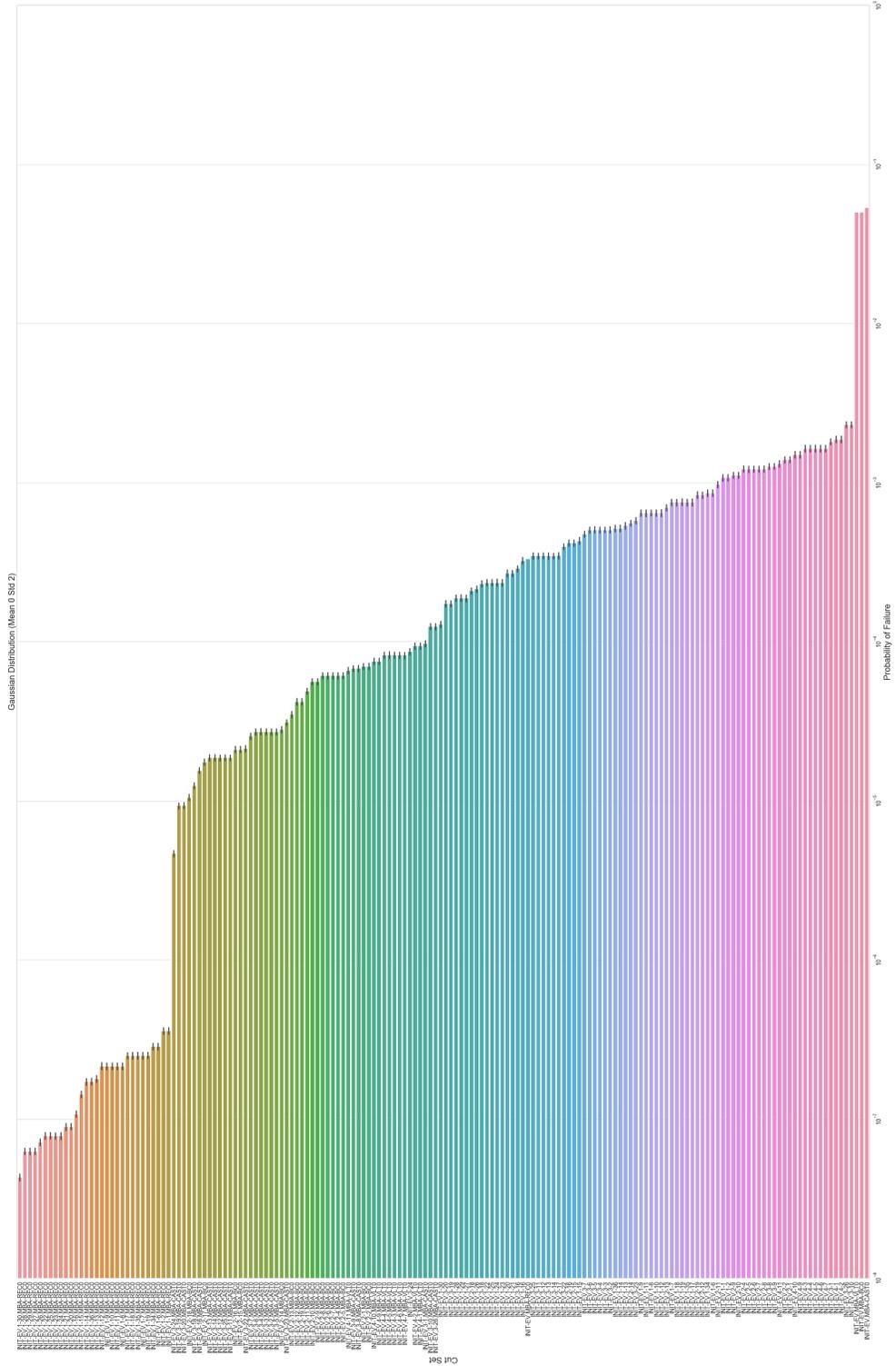


Figure 39: Gaussian Distribution ( $\mu = 0, \sigma = 2$ ): Overall

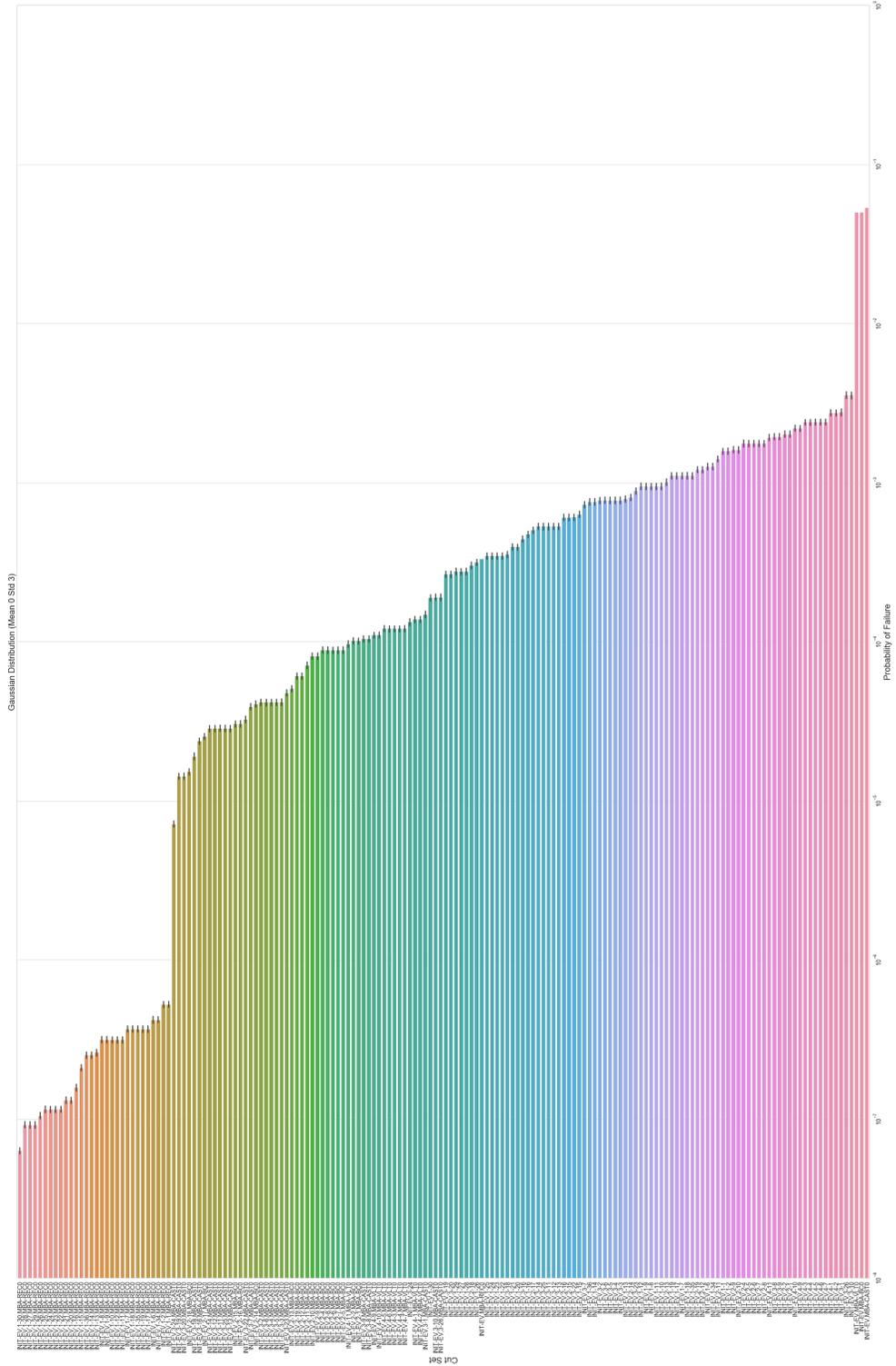


Figure 40: Gaussian Distribution ( $\mu = 0, \sigma = 3$ ): Overall

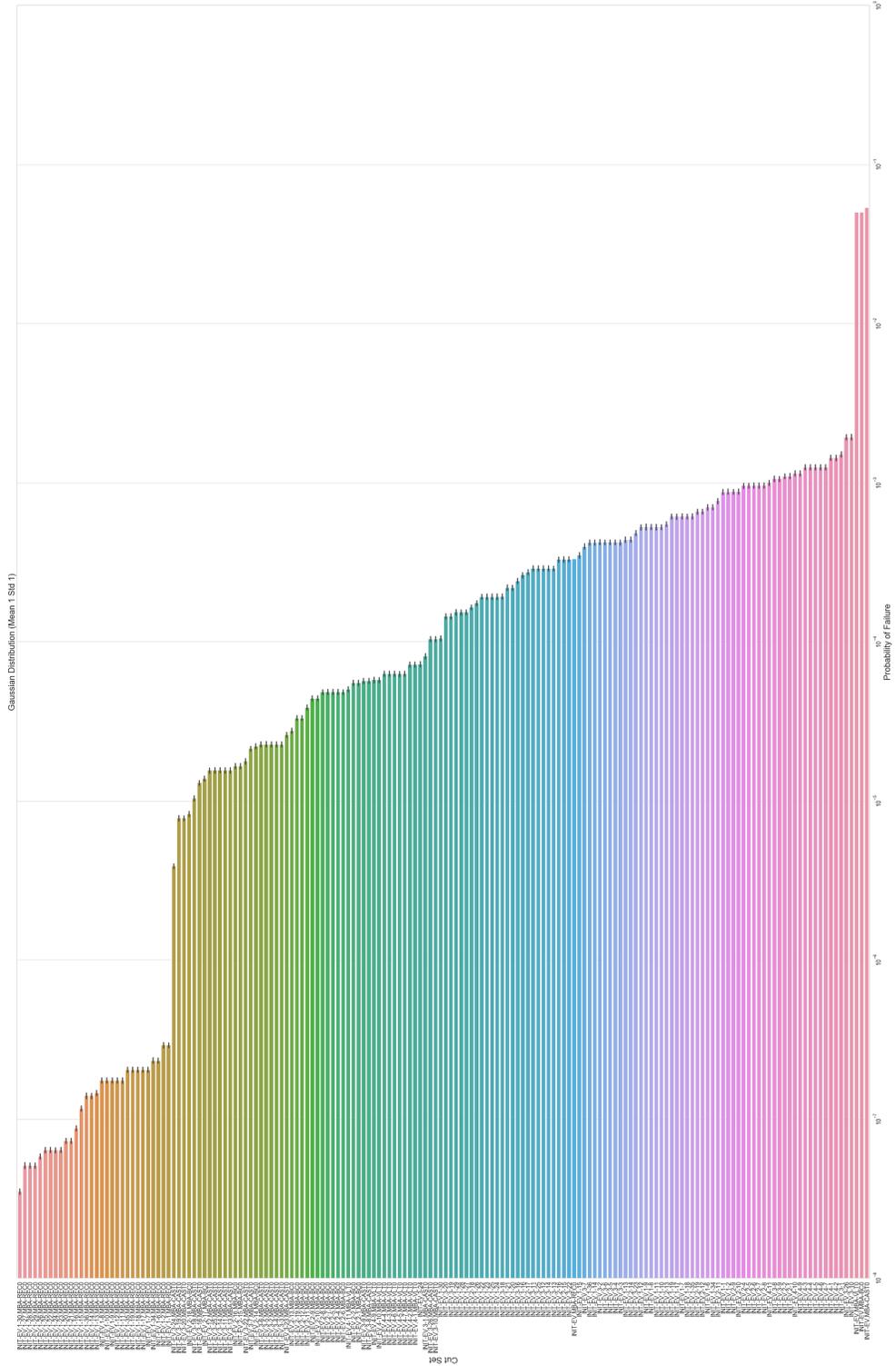


Figure 41: Gaussian Distribution ( $\mu = 1, \sigma = 1$ ): Overall

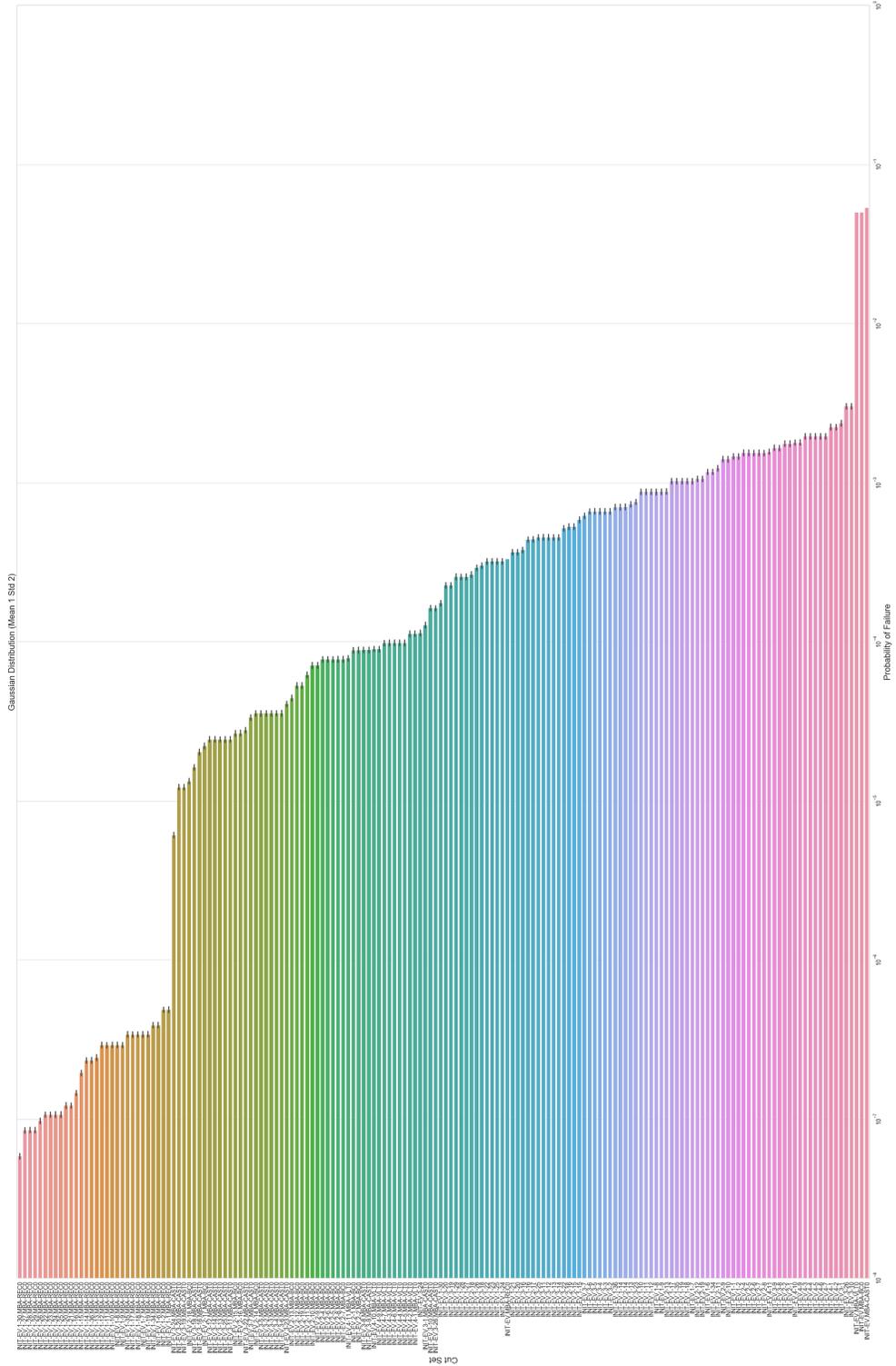


Figure 42: Gaussian Distribution ( $\mu = 1, \sigma = 2$ ): Overall

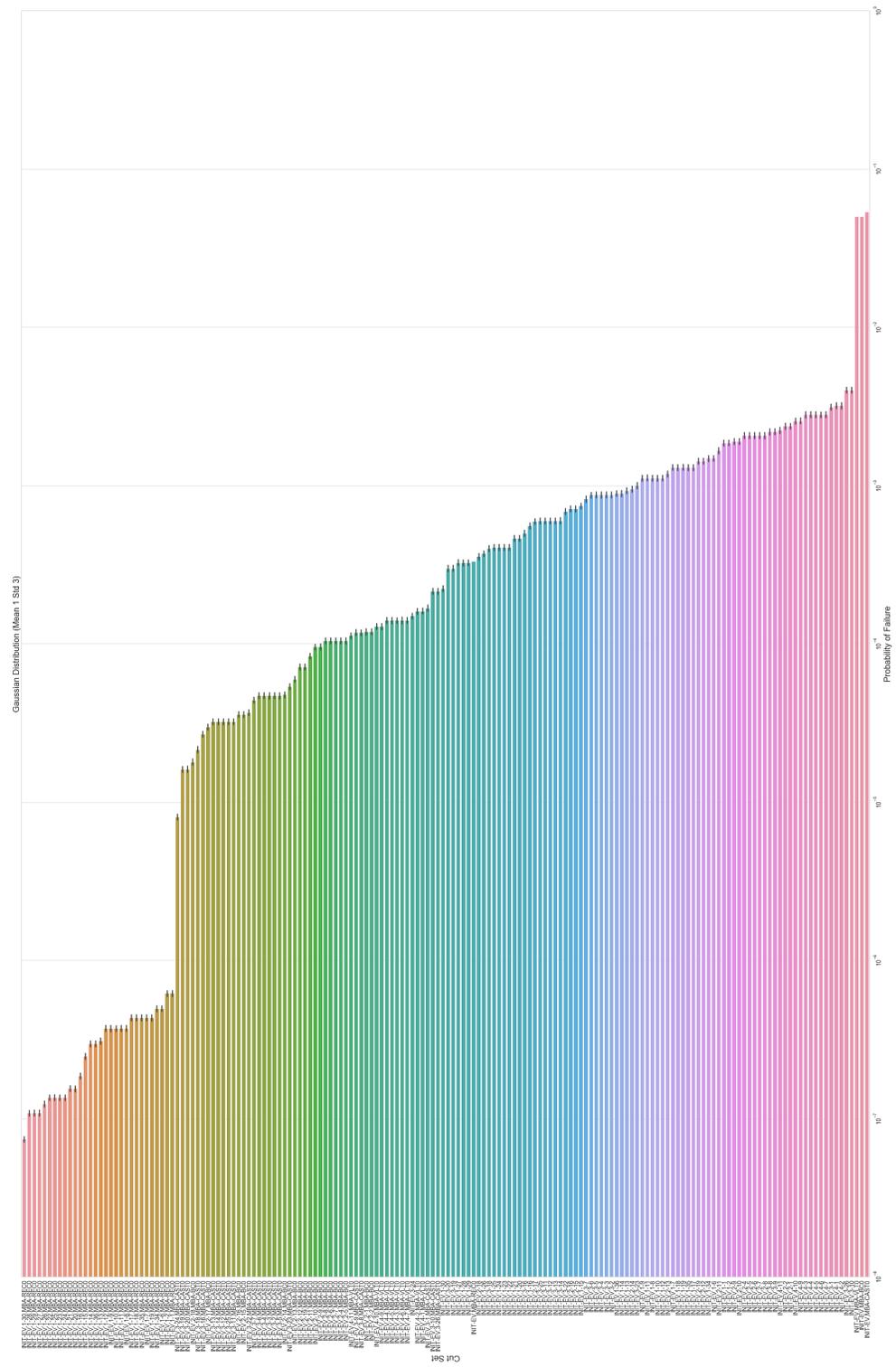


Figure 43: Gaussian Distribution ( $\mu = 1, \sigma = 3$ ): Overall

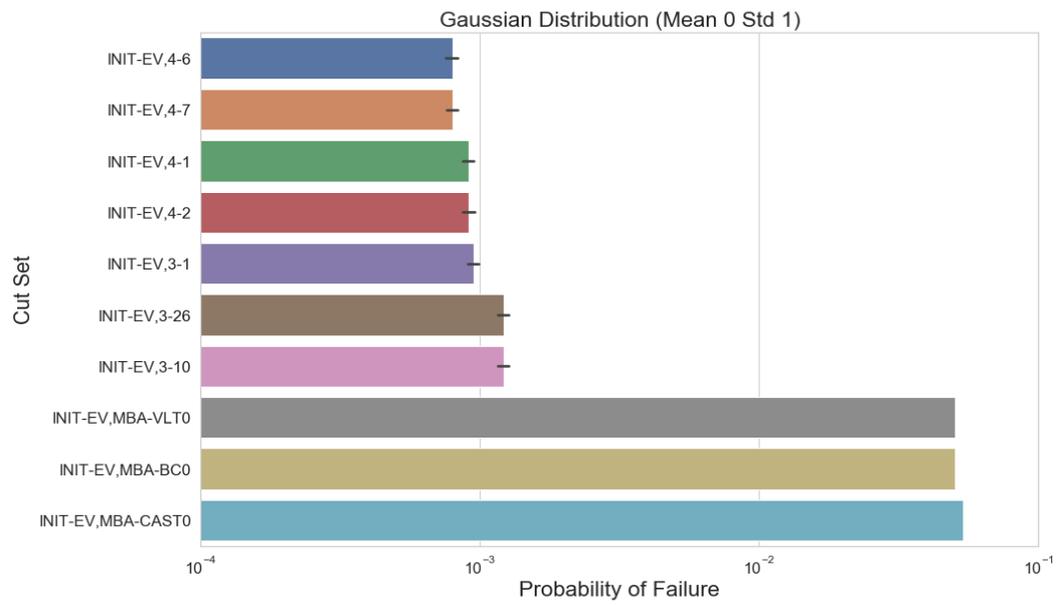


Figure 44: Gaussian Distribution ( $\mu = 0, \sigma = 1$ ): Largest

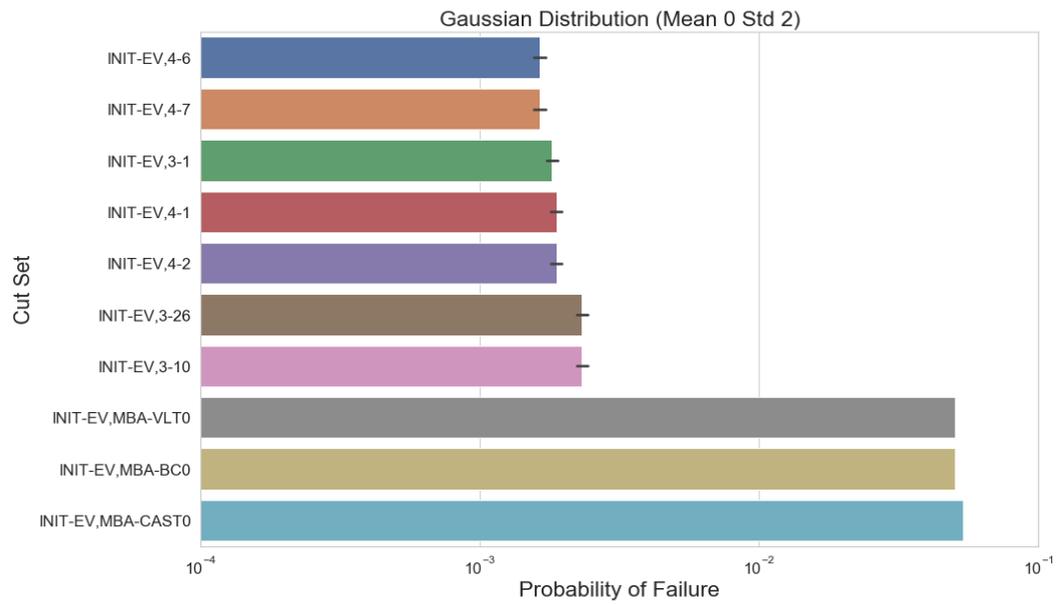


Figure 45: Gaussian Distribution ( $\mu = 0, \sigma = 2$ ): Largest

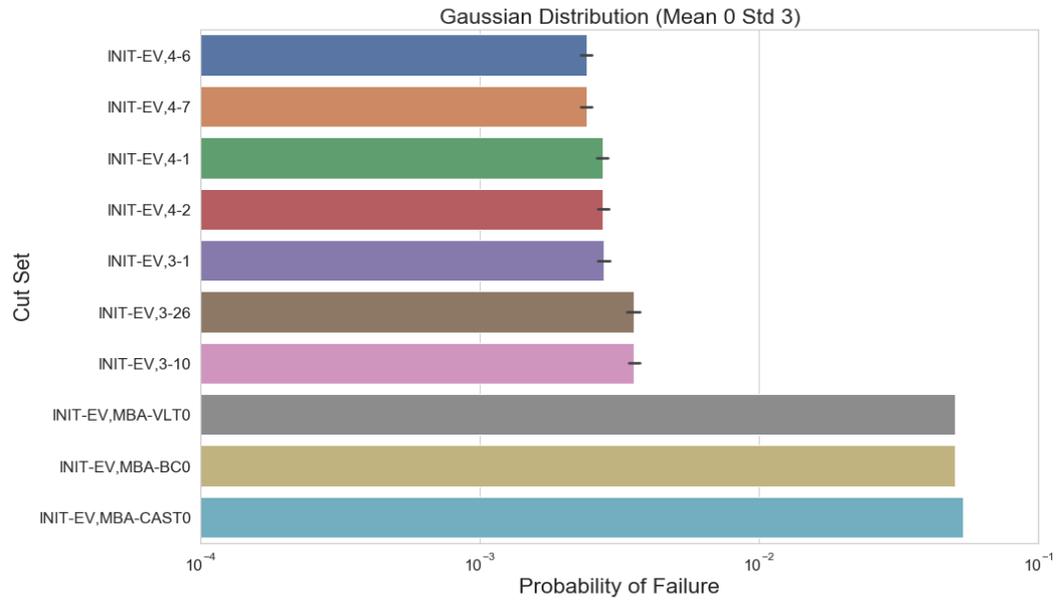


Figure 46: Gaussian Distribution ( $\mu = 0, \sigma = 3$ ): Largest

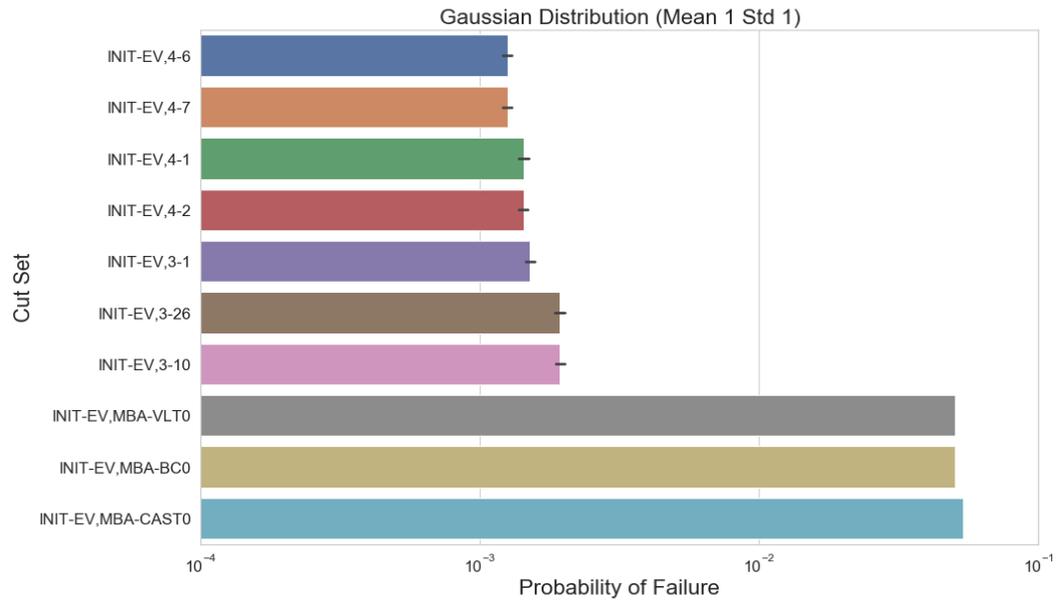


Figure 47: Gaussian Distribution ( $\mu = 1, \sigma = 1$ ): Largest

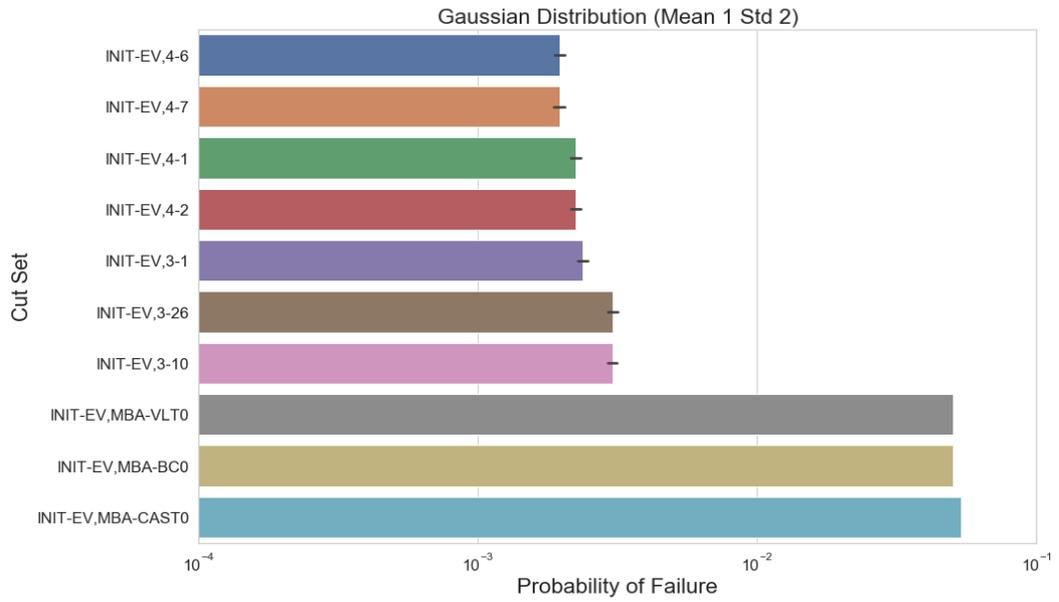


Figure 48: Gaussian Distribution ( $\mu = 1, \sigma = 2$ ): Largest

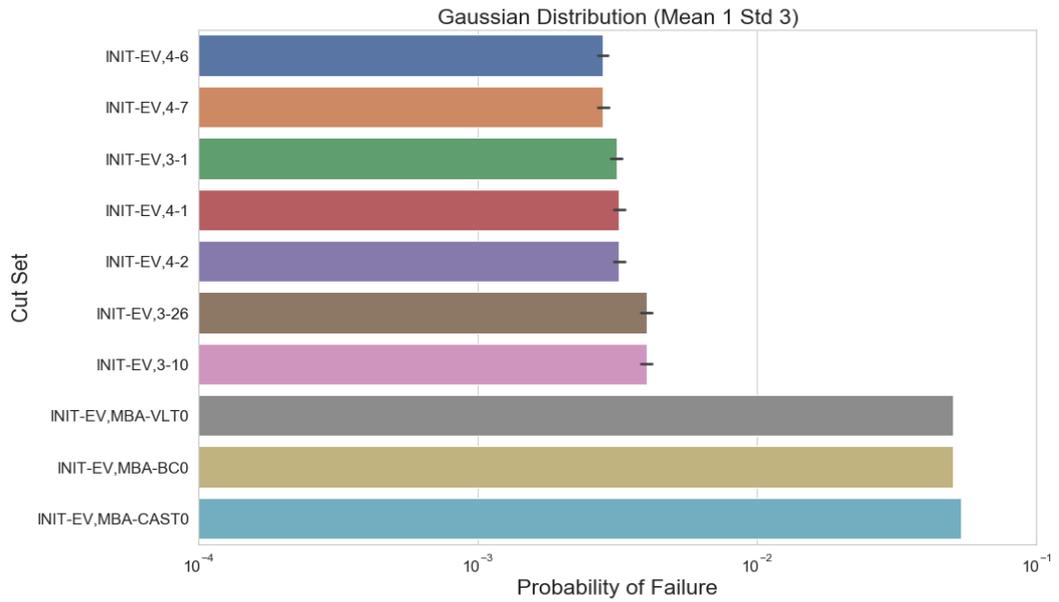


Figure 49: Gaussian Distribution ( $\mu = 1, \sigma = 3$ ): Largest

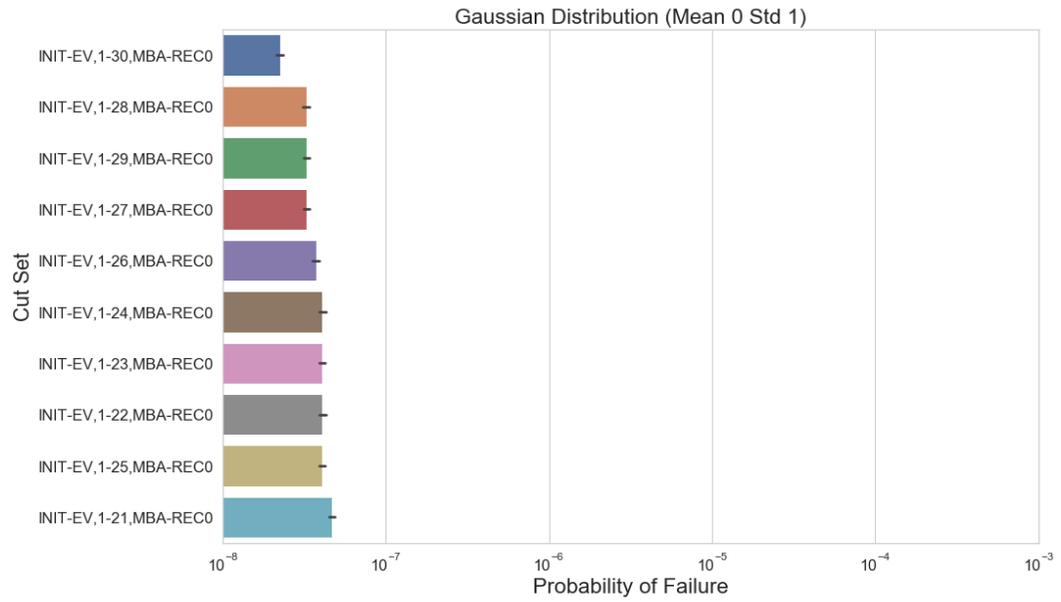


Figure 50: Gaussian Distribution ( $\mu = 0$ ,  $\sigma = 1$ ): Smallest

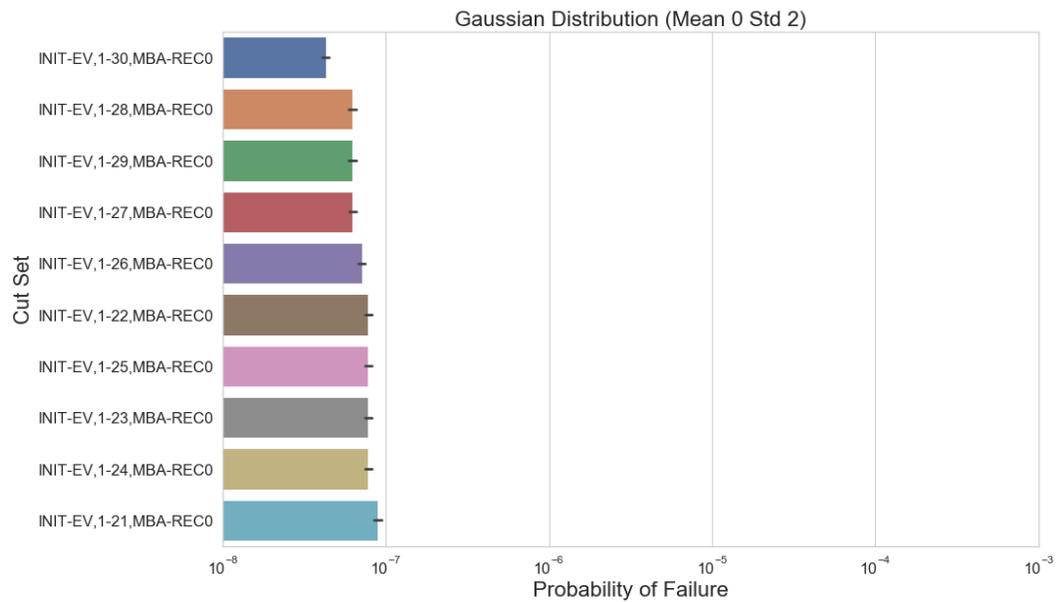


Figure 51: Gaussian Distribution ( $\mu = 0$ ,  $\sigma = 2$ ): Smallest

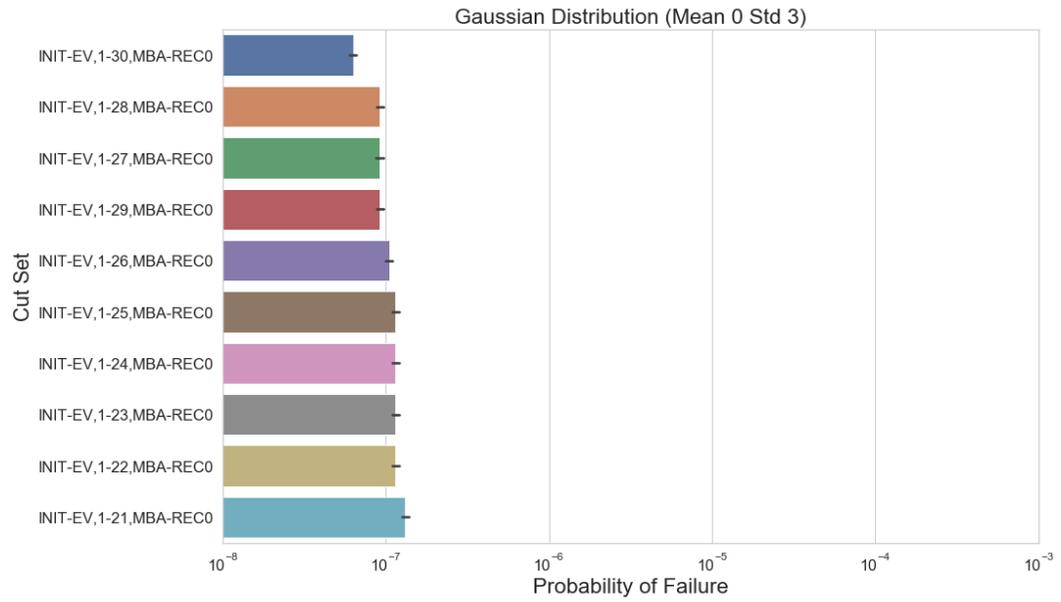


Figure 52: Gaussian Distribution ( $\mu = 0$ ,  $\sigma = 3$ ): Smallest

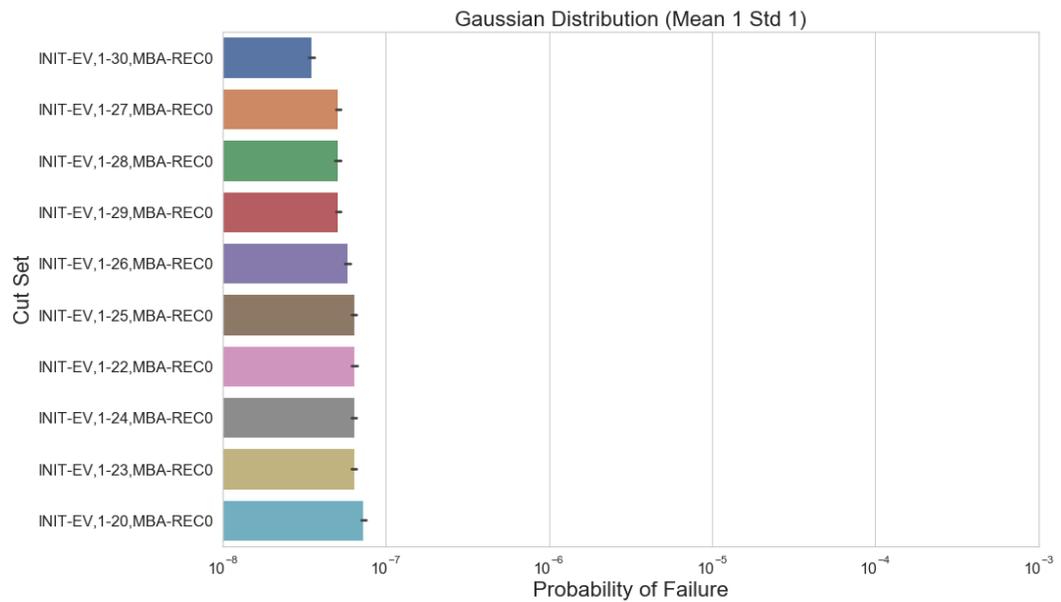


Figure 53: Gaussian Distribution ( $\mu = 1$ ,  $\sigma = 1$ ): Smallest

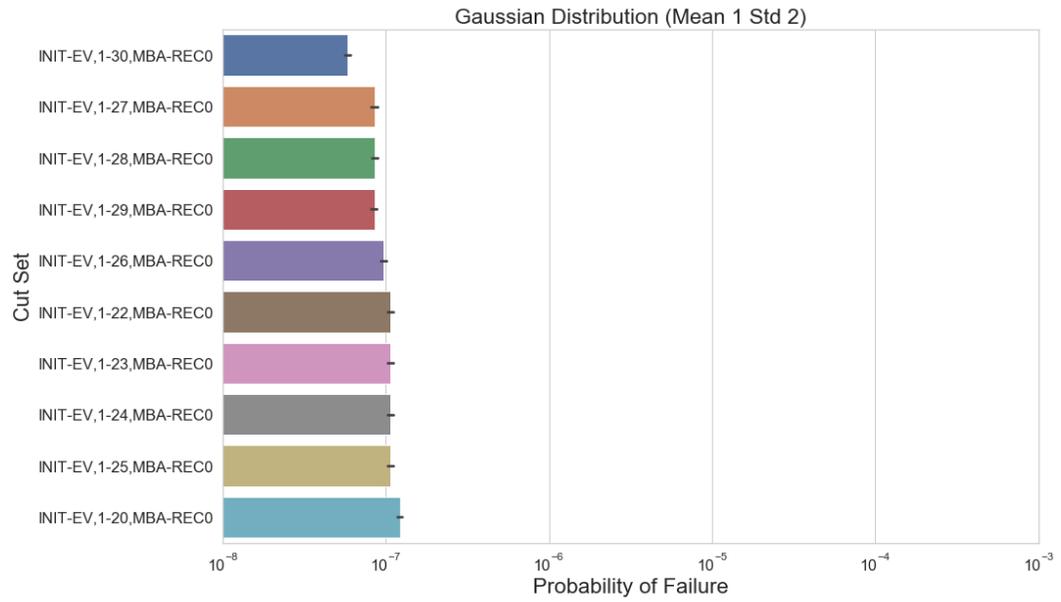


Figure 54: Gaussian Distribution ( $\mu = 1, \sigma = 2$ ): Smallest

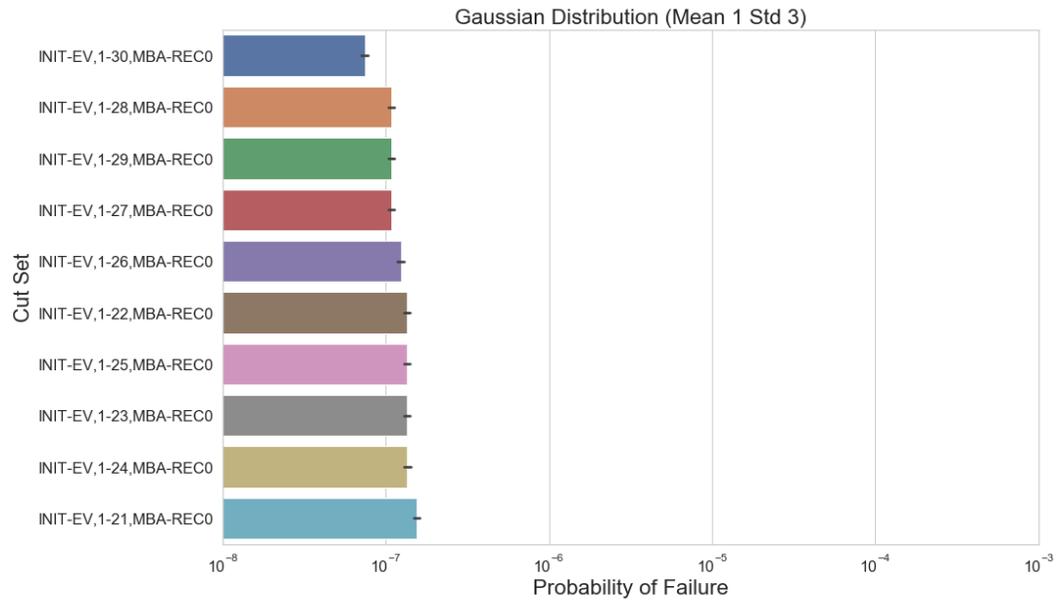


Figure 55: Gaussian Distribution ( $\mu = 1, \sigma = 3$ ): Smallest

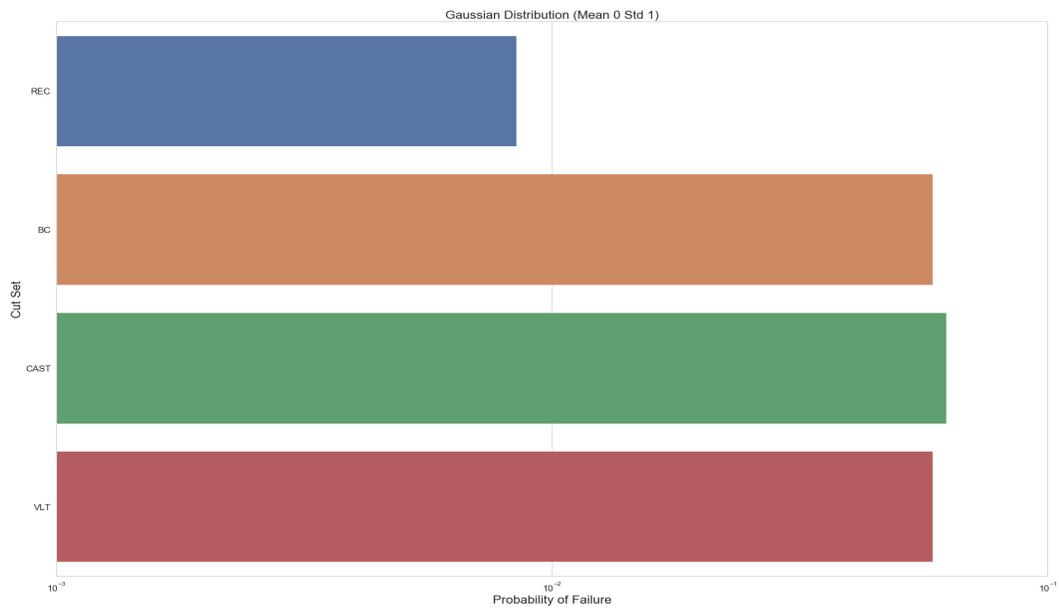


Figure 56: Gaussian Distribution ( $\mu = 0$ ,  $\sigma = 1$ ): By-Area

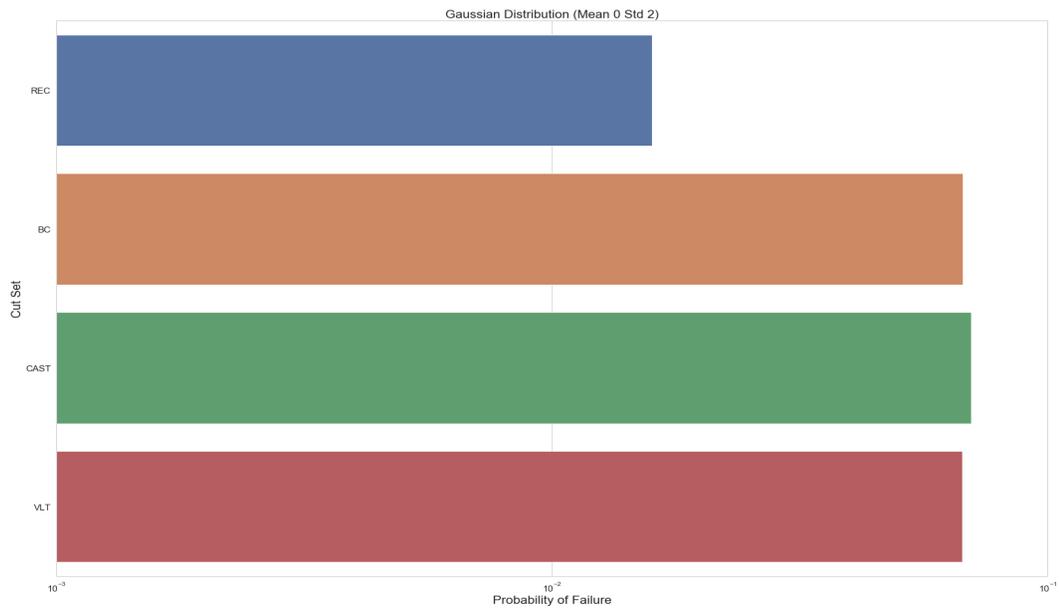


Figure 57: Gaussian Distribution ( $\mu = 0$ ,  $\sigma = 2$ ): By-Area

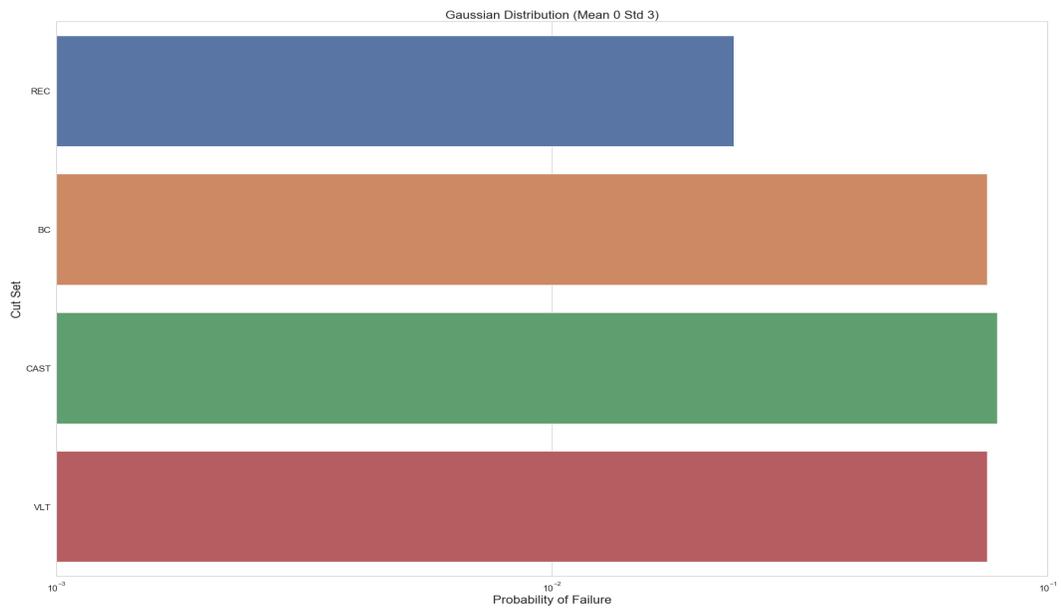


Figure 58: Gaussian Distribution ( $\mu = 0$ ,  $\sigma = 3$ ): By-Area

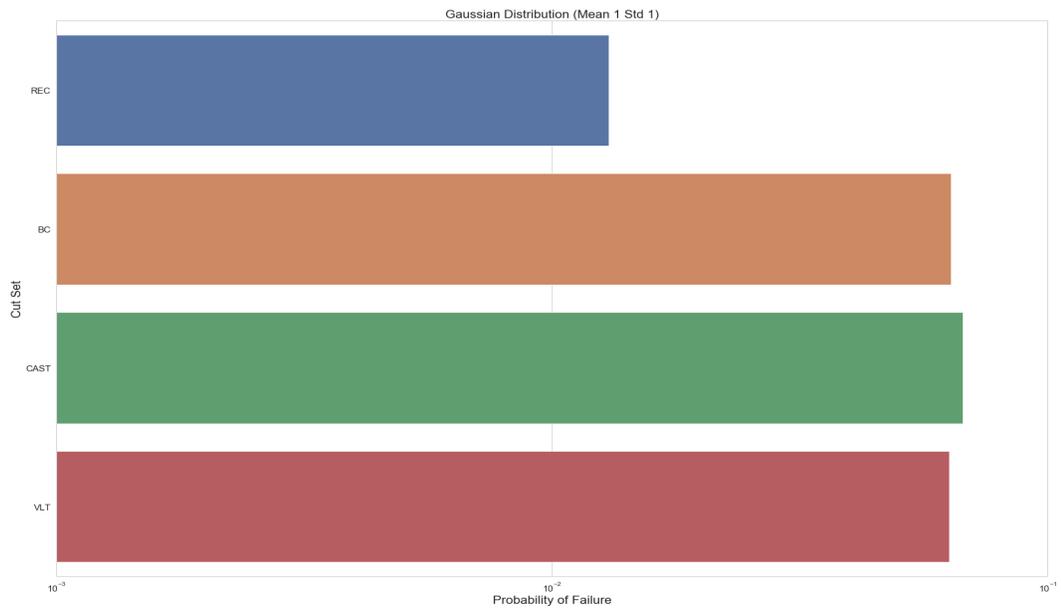


Figure 59: Gaussian Distribution ( $\mu = 1$ ,  $\sigma = 1$ ): By-Area

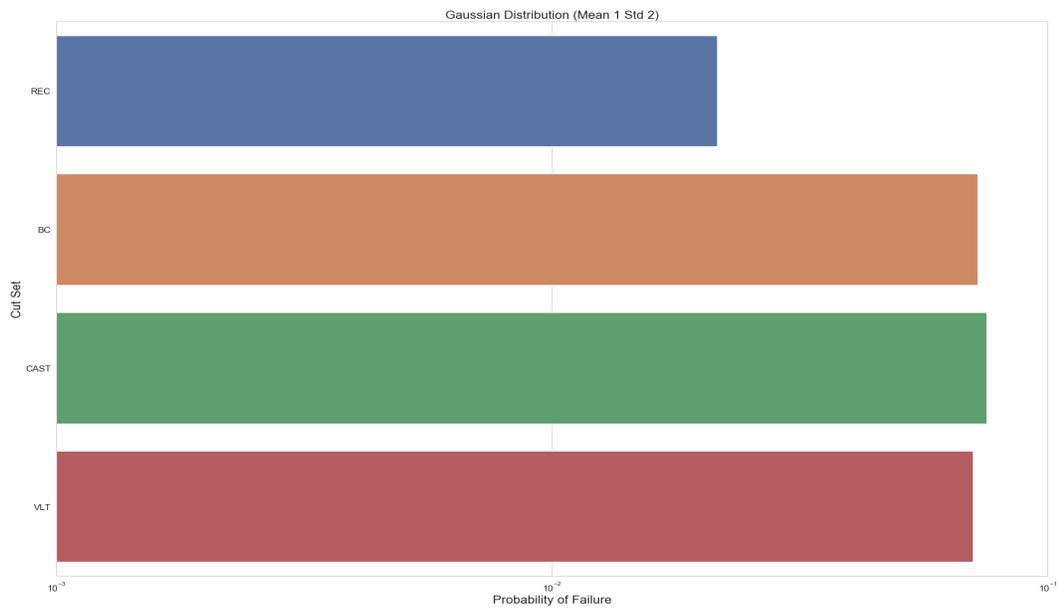


Figure 60: Gaussian Distribution ( $\mu = 1, \sigma = 2$ ): By-Area

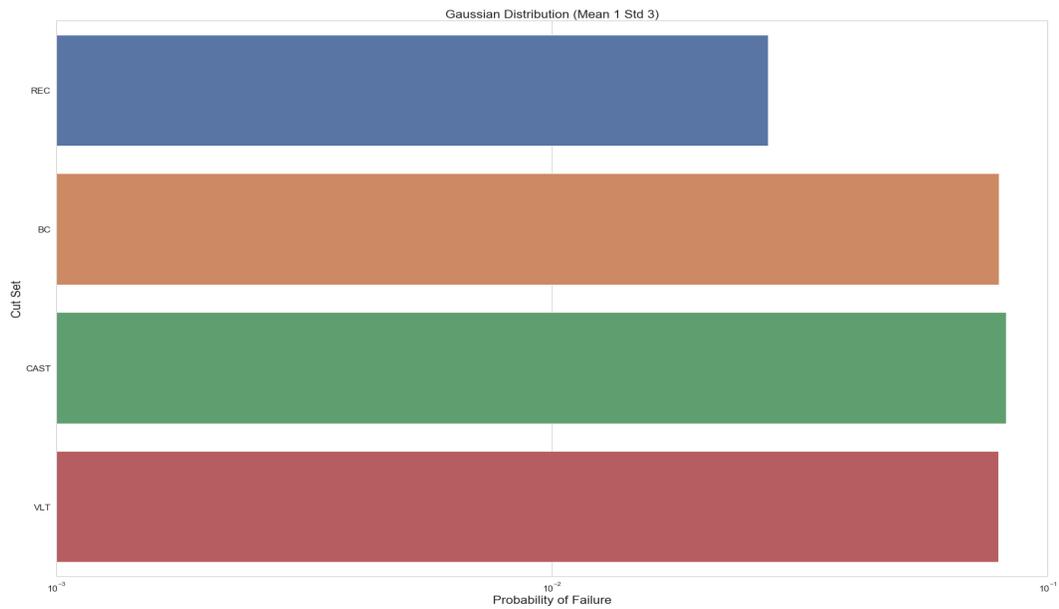


Figure 61: Gaussian Distribution ( $\mu = 1, \sigma = 3$ ): By-Area

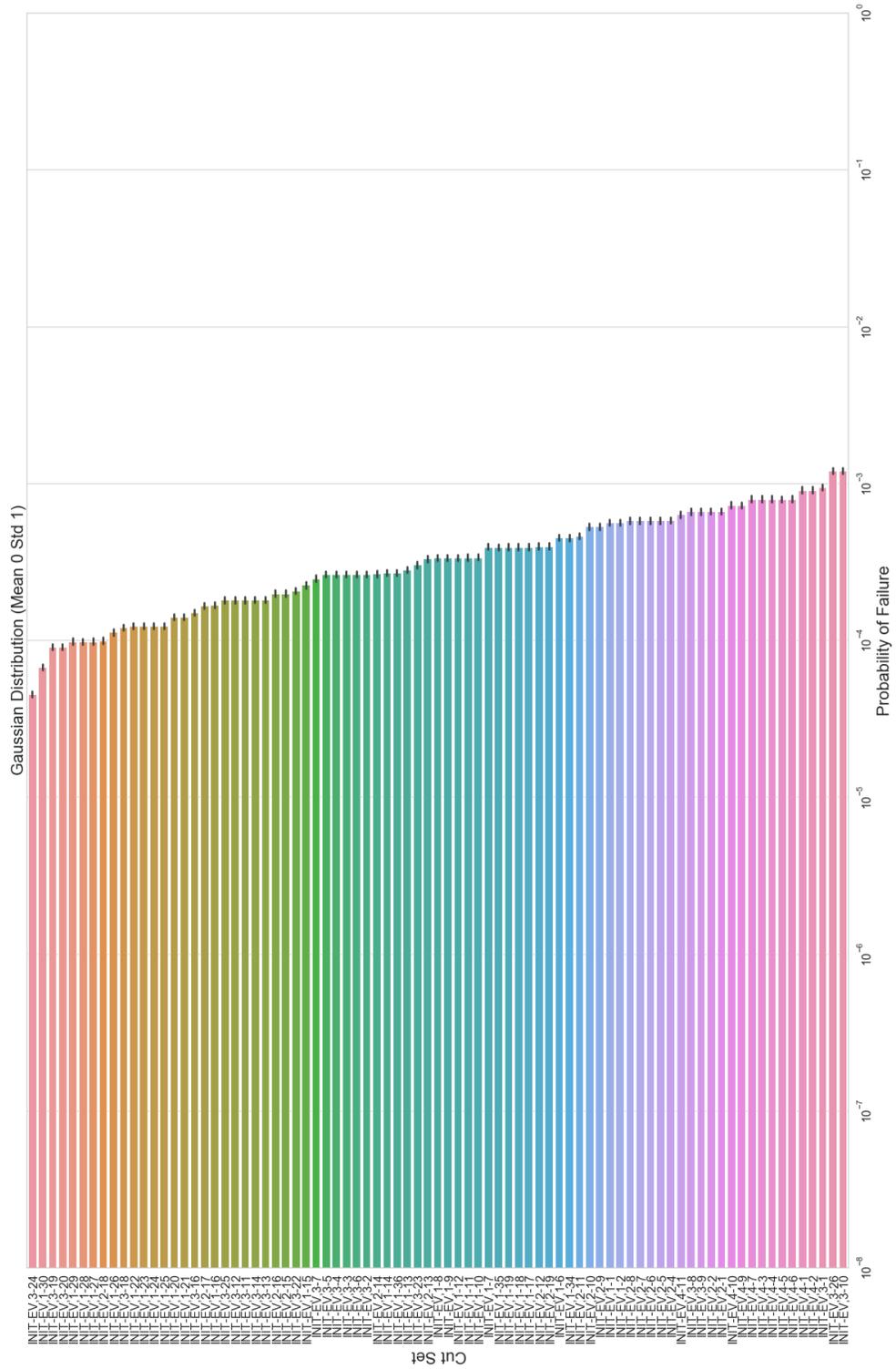


Figure 62: Gaussian Distribution ( $\mu = 0, \sigma = 1$ ): Overall, No MA

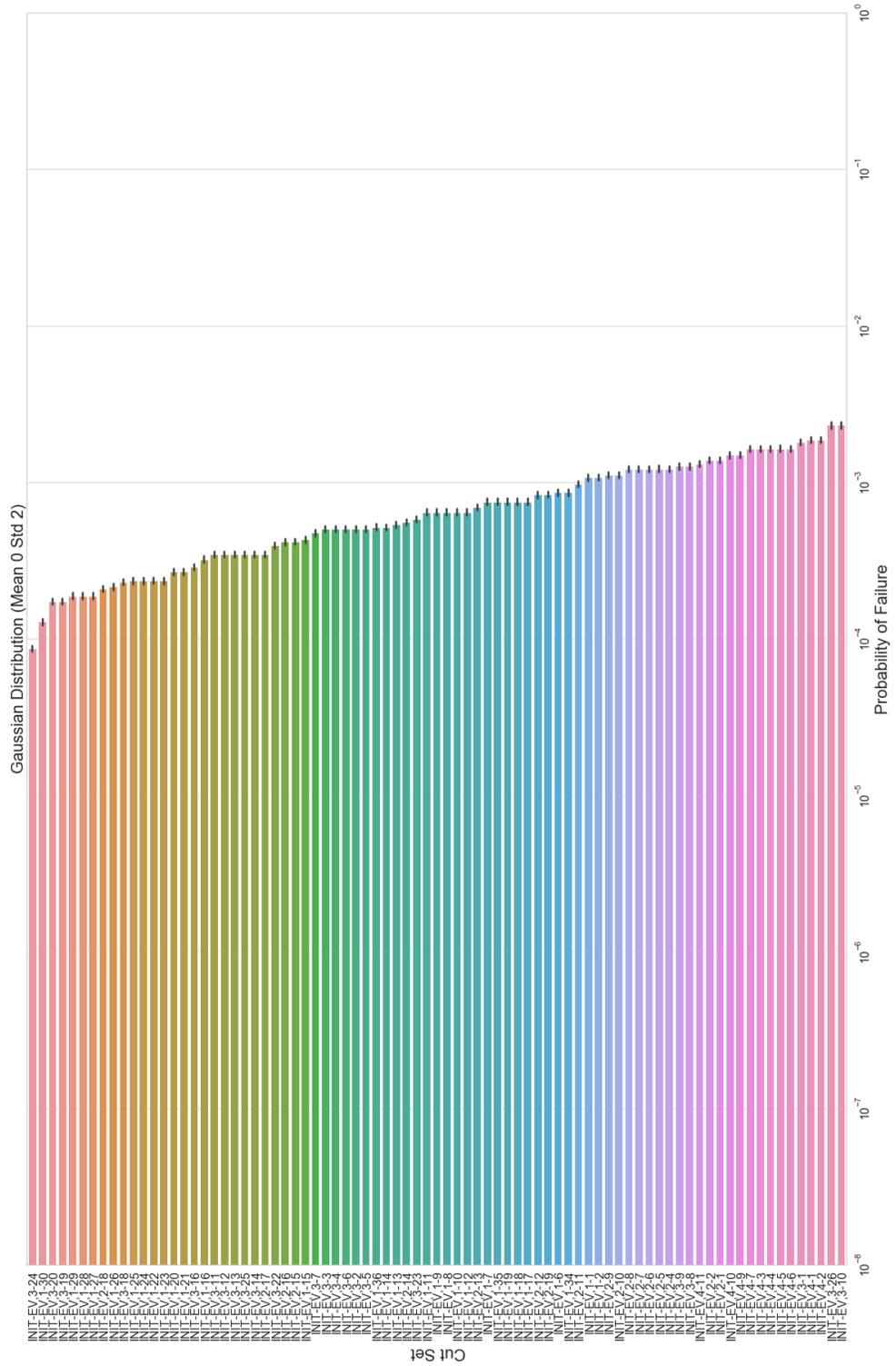


Figure 63: Gaussian Distribution ( $\mu = 0, \sigma = 2$ ): Overall, No MA

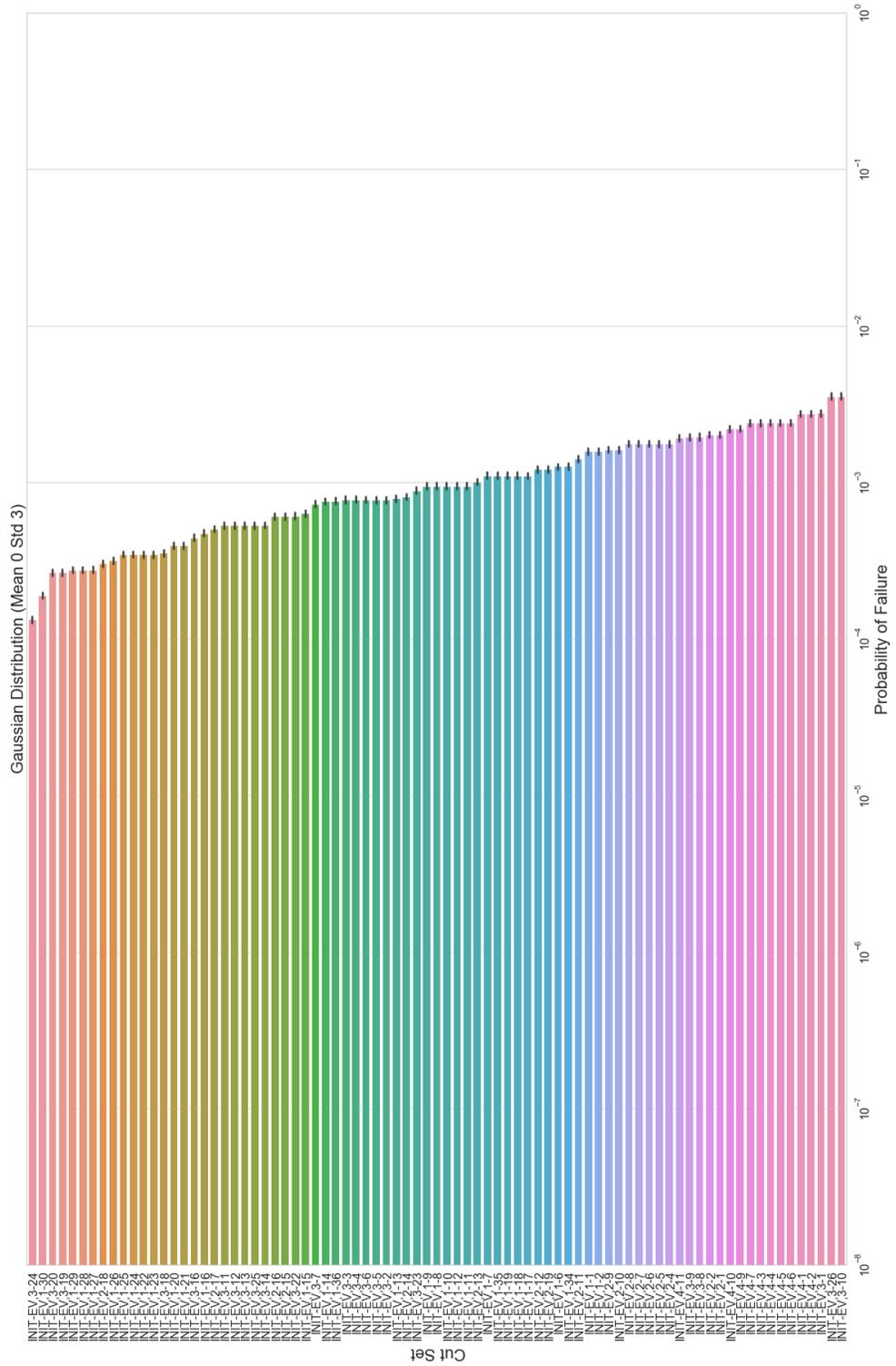


Figure 64: Gaussian Distribution ( $\mu = 0, \sigma = 3$ ): Overall, No MA

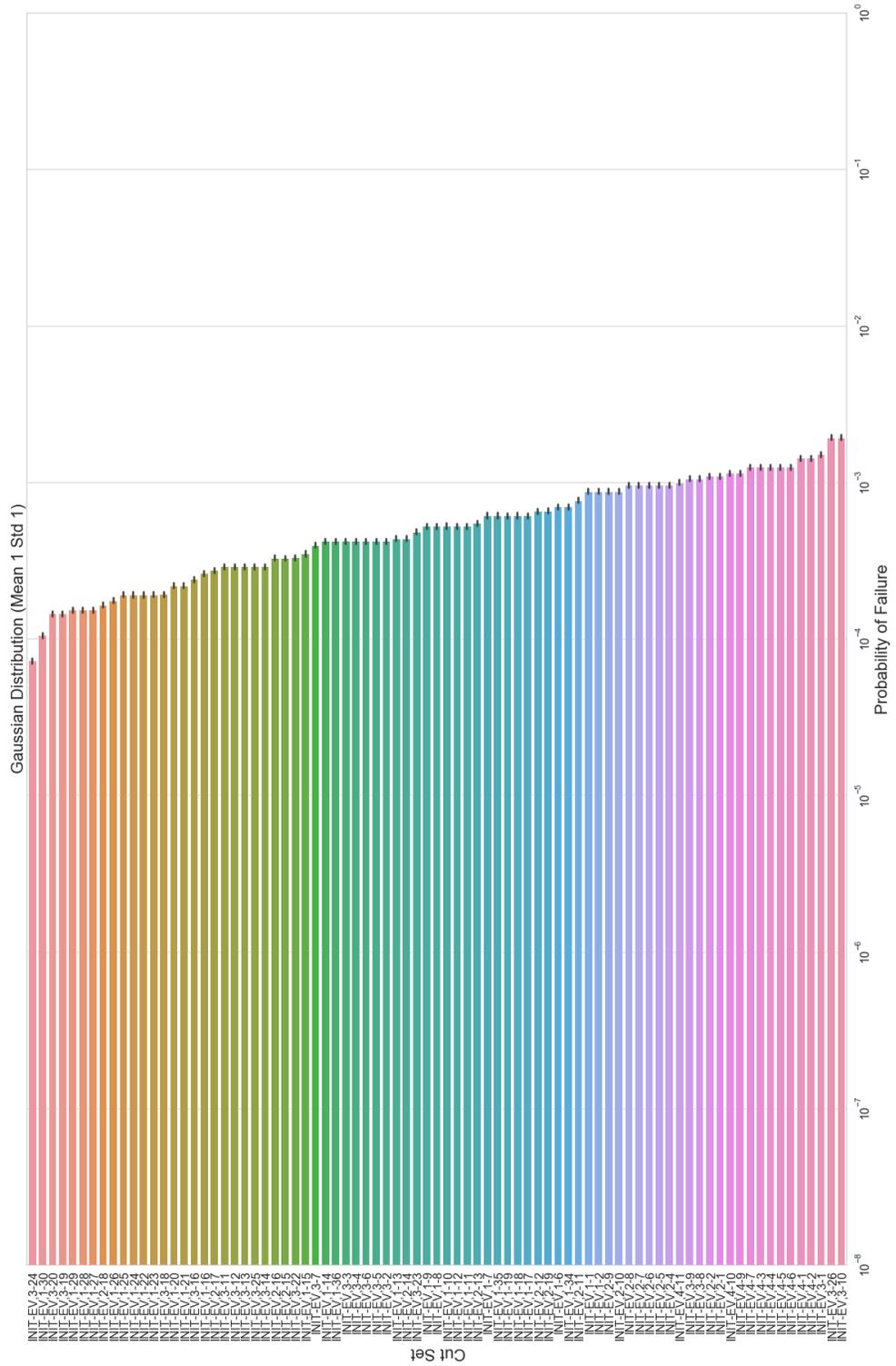


Figure 65: Gaussian Distribution ( $\mu = 1, \sigma = 1$ ): Overall, No MA

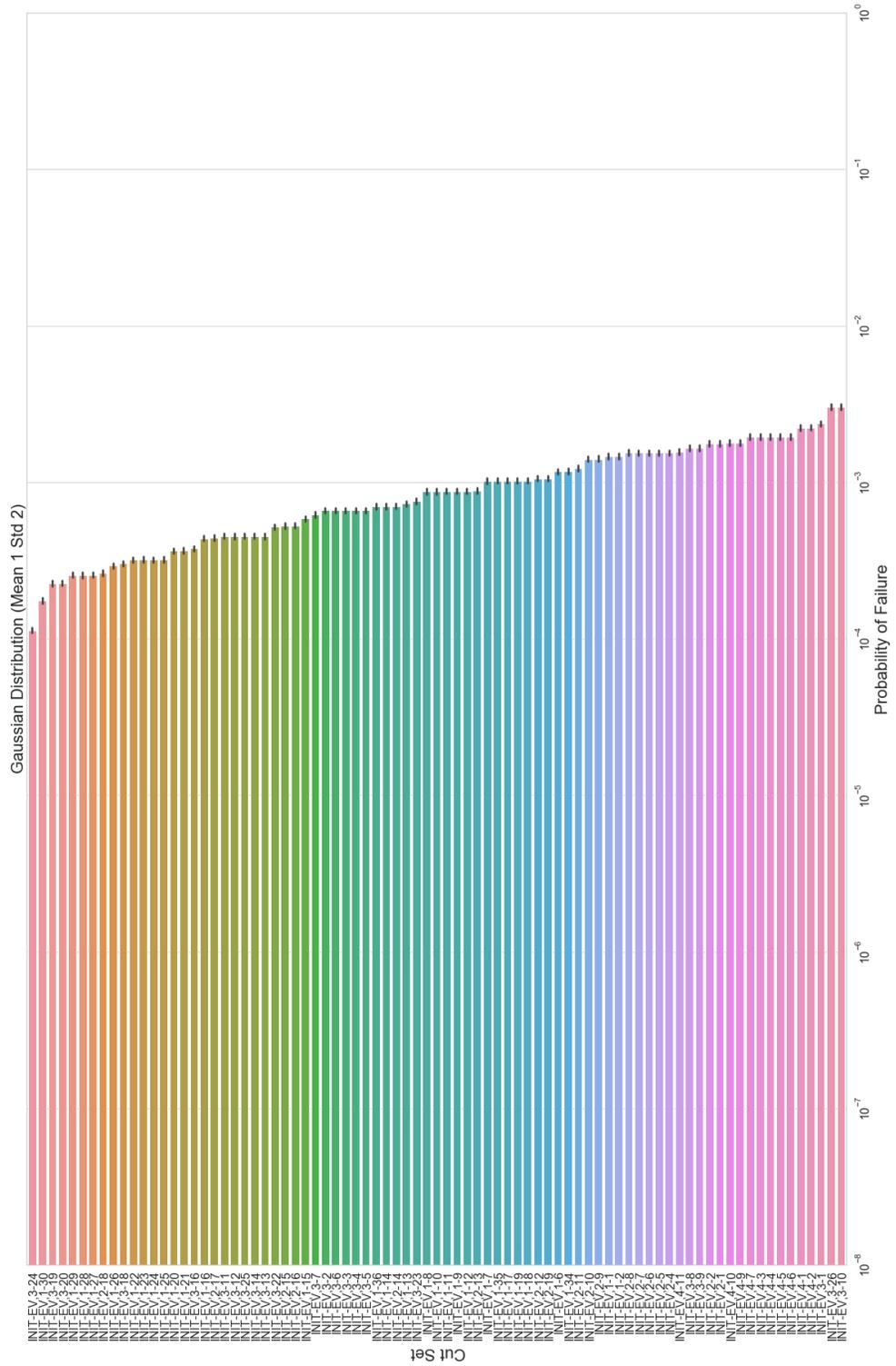


Figure 66: Gaussian Distribution ( $\mu = 1, \sigma = 2$ ): Overall, No MA

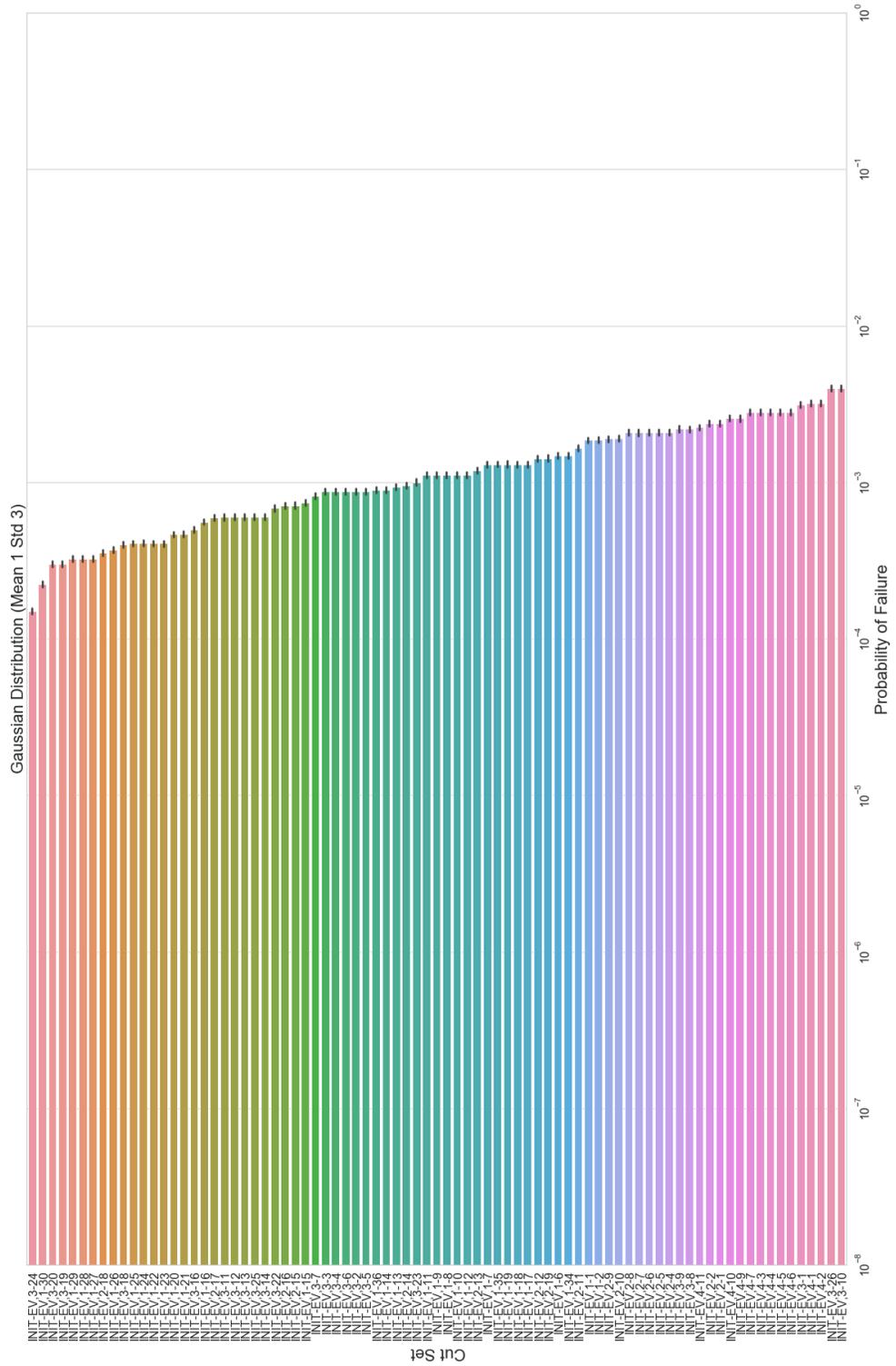


Figure 67: Gaussian Distribution ( $\mu = 1, \sigma = 3$ ): Overall, No MA

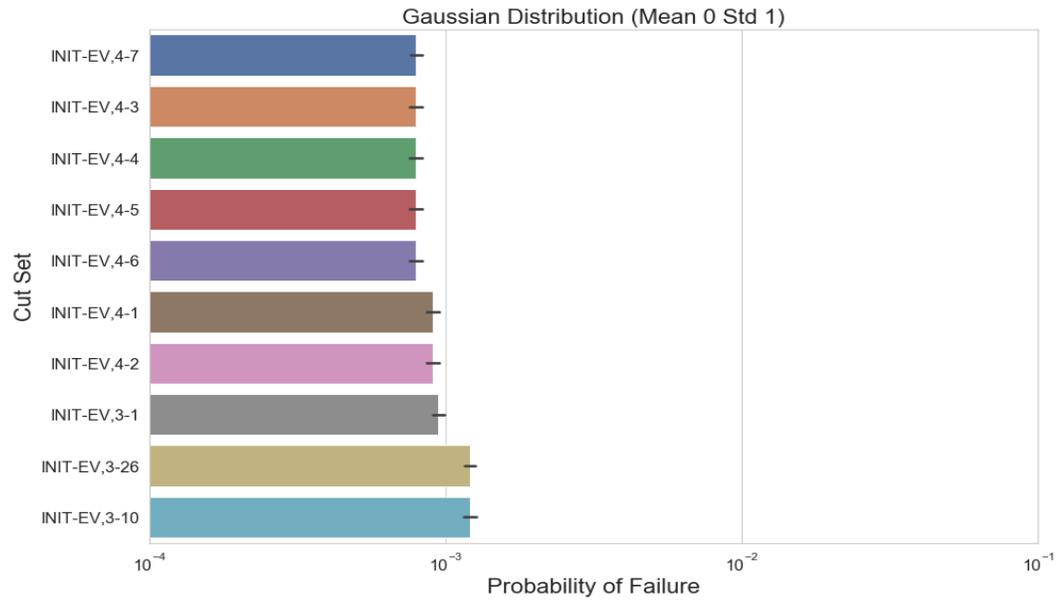


Figure 68: Gaussian Distribution ( $\mu = 0, \sigma = 1$ ): Largest, No MA

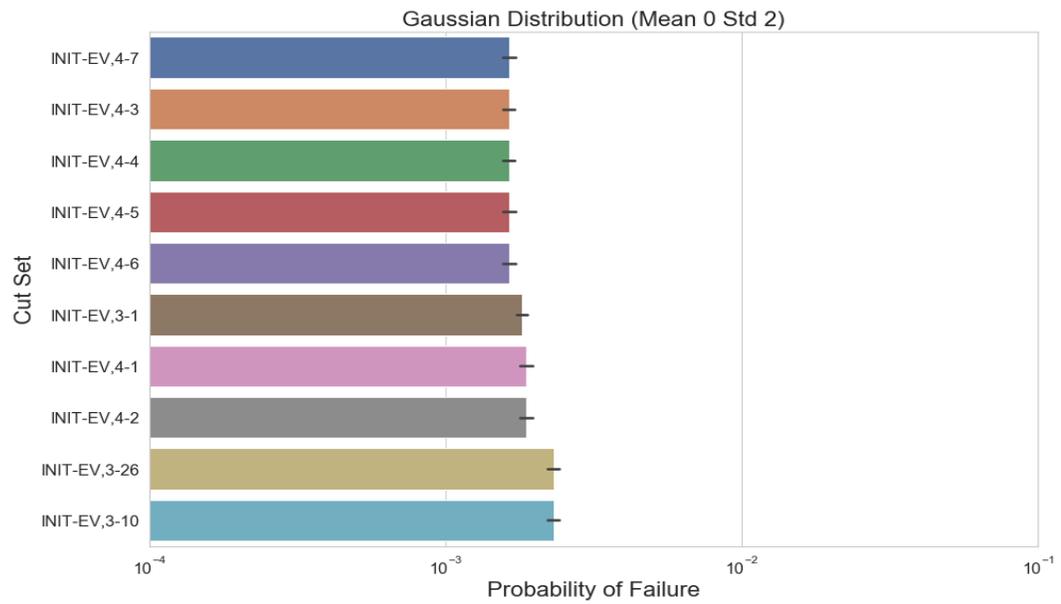


Figure 69: Gaussian Distribution ( $\mu = 0, \sigma = 2$ ): Largest, No MA

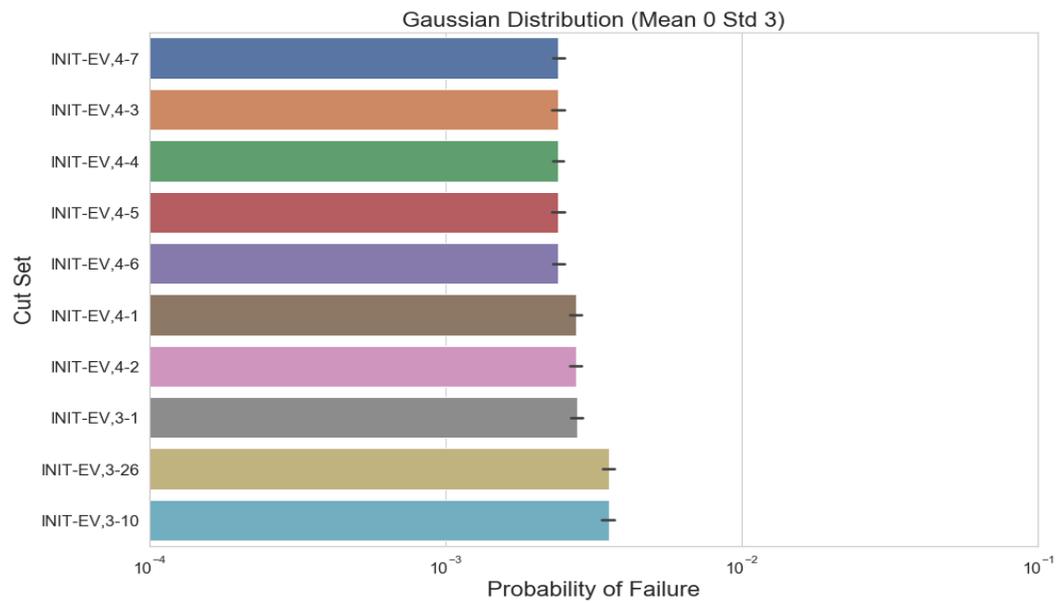


Figure 70: Gaussian Distribution ( $\mu = 0, \sigma = 3$ ): Largest, No MA

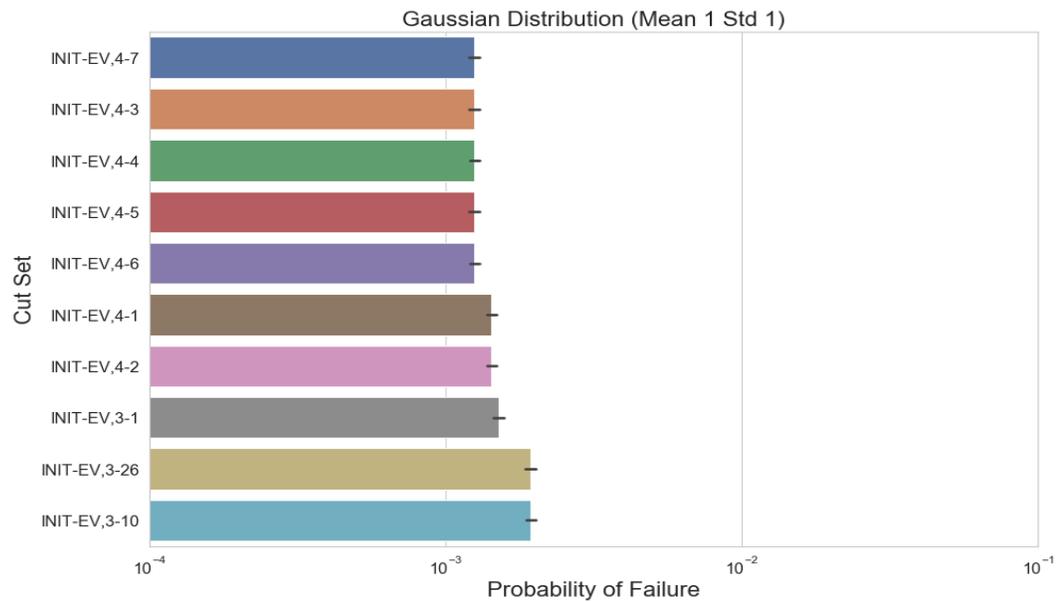


Figure 71: Gaussian Distribution ( $\mu = 1, \sigma = 1$ ): Largest, No MA

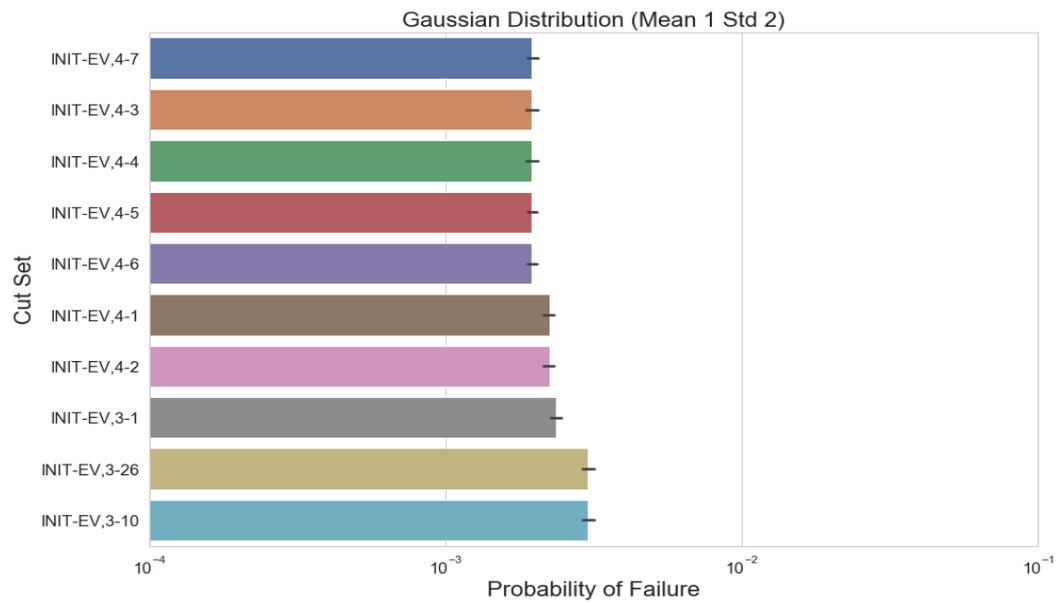


Figure 72: Gaussian Distribution ( $\mu = 1$ ,  $\sigma = 2$ ): Largest, No MA

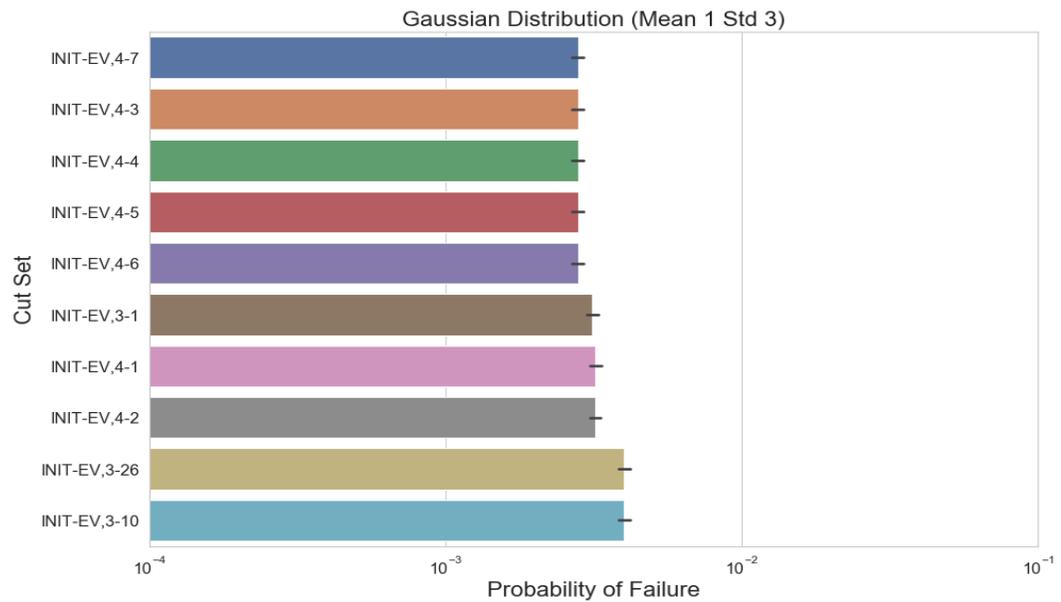


Figure 73: Gaussian Distribution ( $\mu = 1$ ,  $\sigma = 3$ ): Largest, No MA

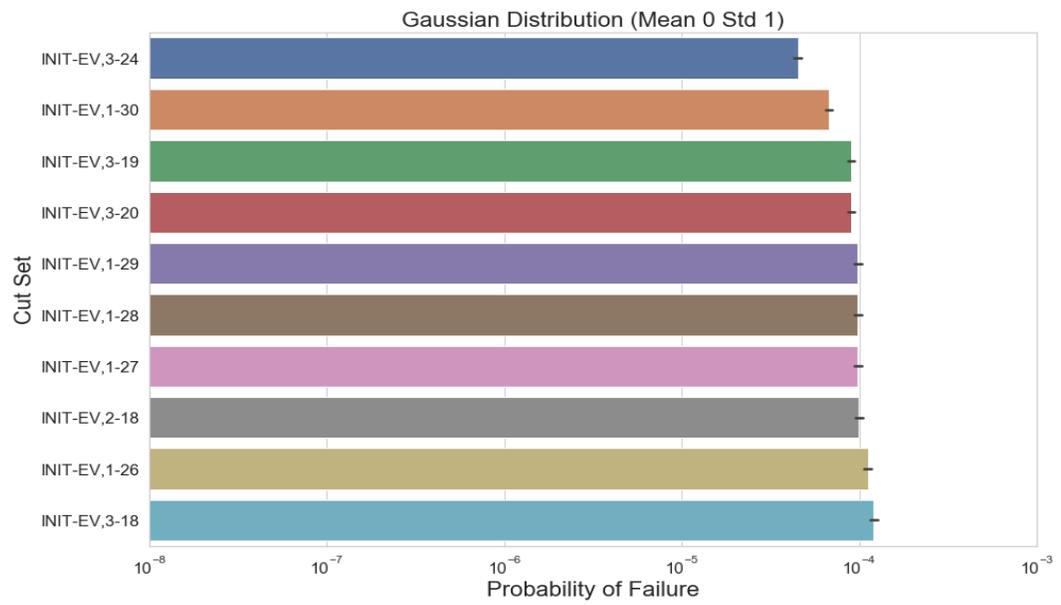


Figure 74: Gaussian Distribution ( $\mu = 0, \sigma = 1$ ): Smallest, No MA

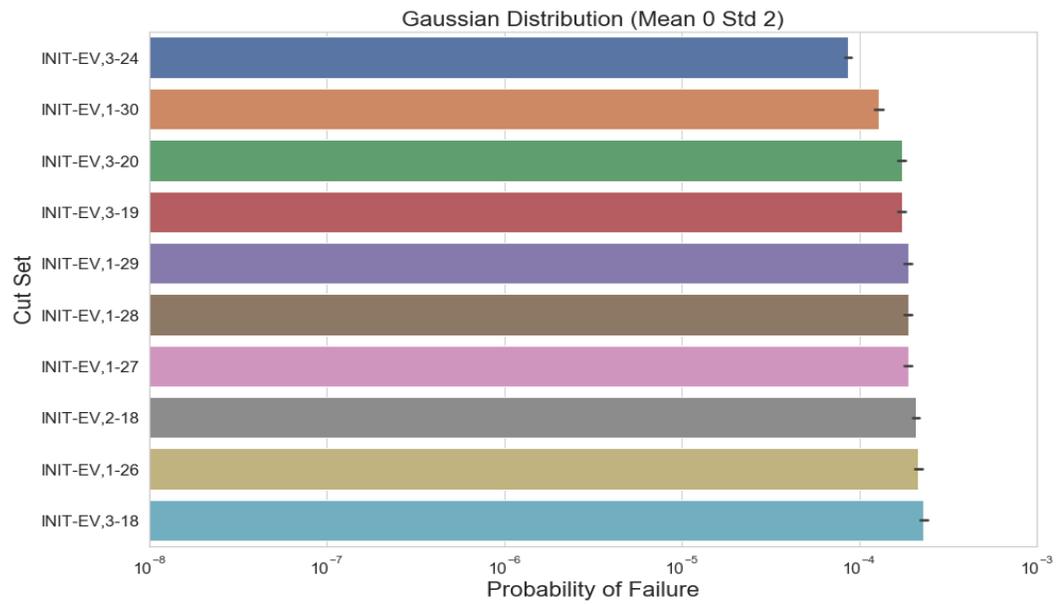


Figure 75: Gaussian Distribution ( $\mu = 0, \sigma = 2$ ): Smallest, No MA

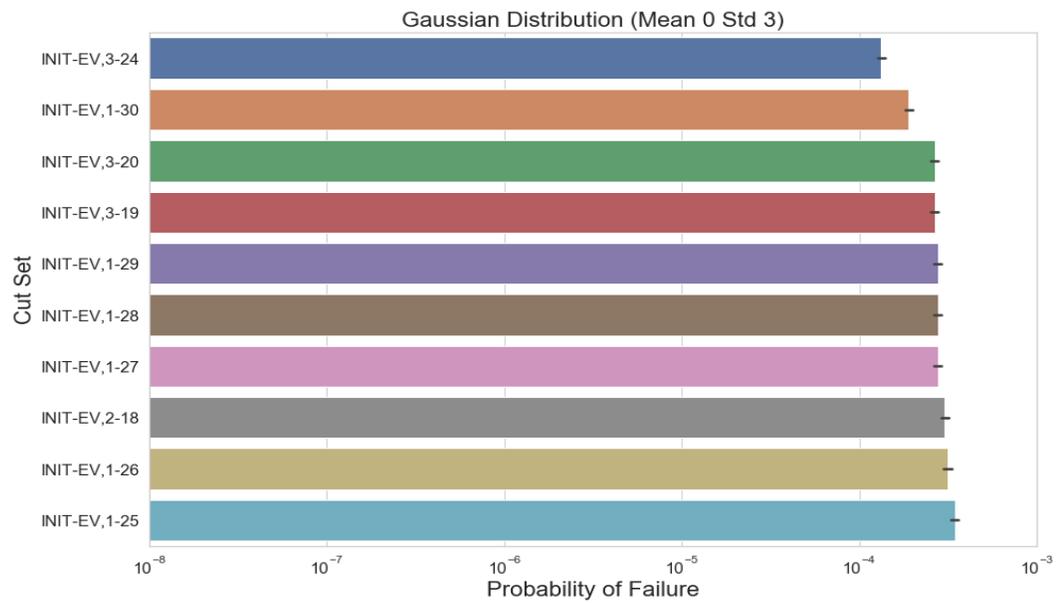


Figure 76: Gaussian Distribution ( $\mu = 0, \sigma = 3$ ): Smallest, No MA

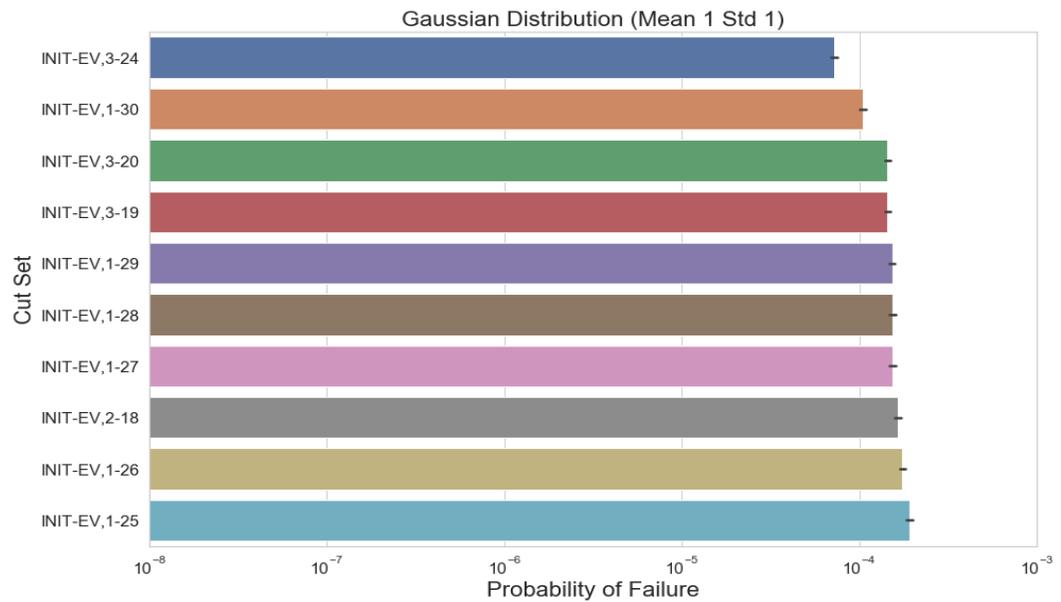


Figure 77: Gaussian Distribution ( $\mu = 1, \sigma = 1$ ): Smallest, No MA

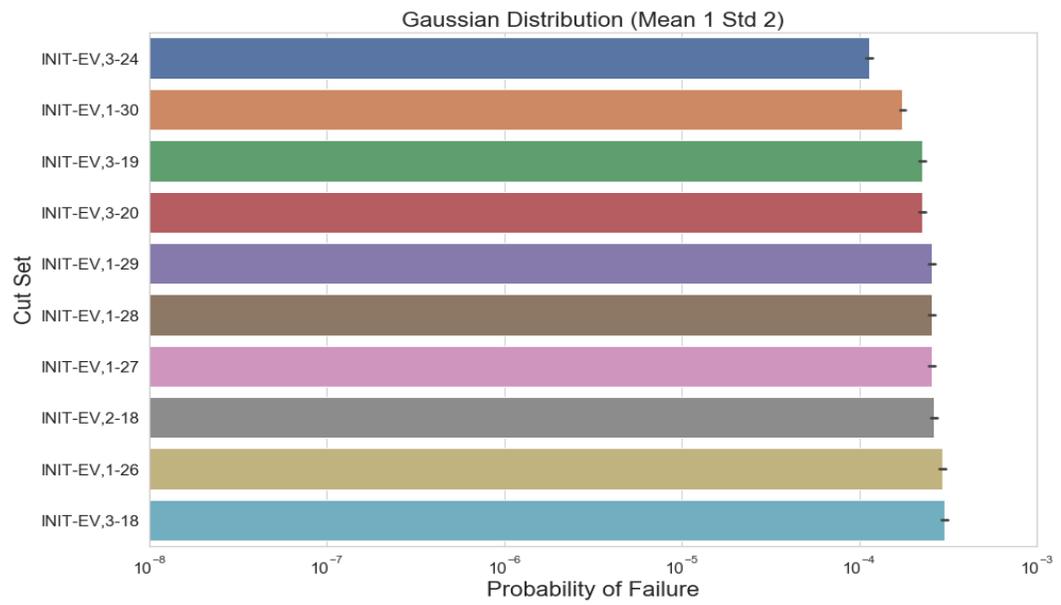


Figure 78: Gaussian Distribution ( $\mu = 1$ ,  $\sigma = 2$ ): Smallest, No MA

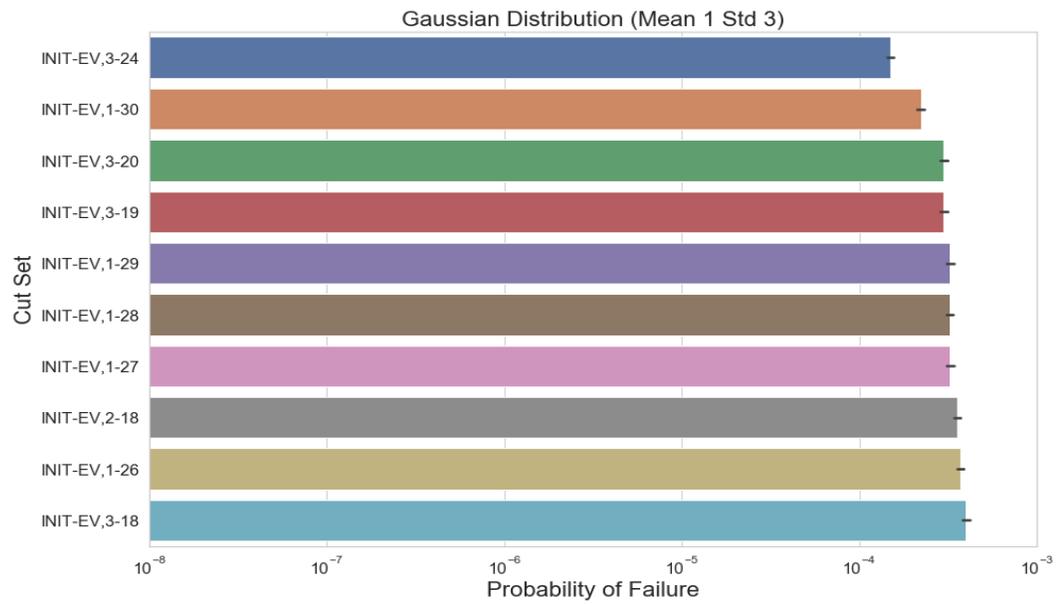


Figure 79: Gaussian Distribution ( $\mu = 1$ ,  $\sigma = 3$ ): Smallest, No MA

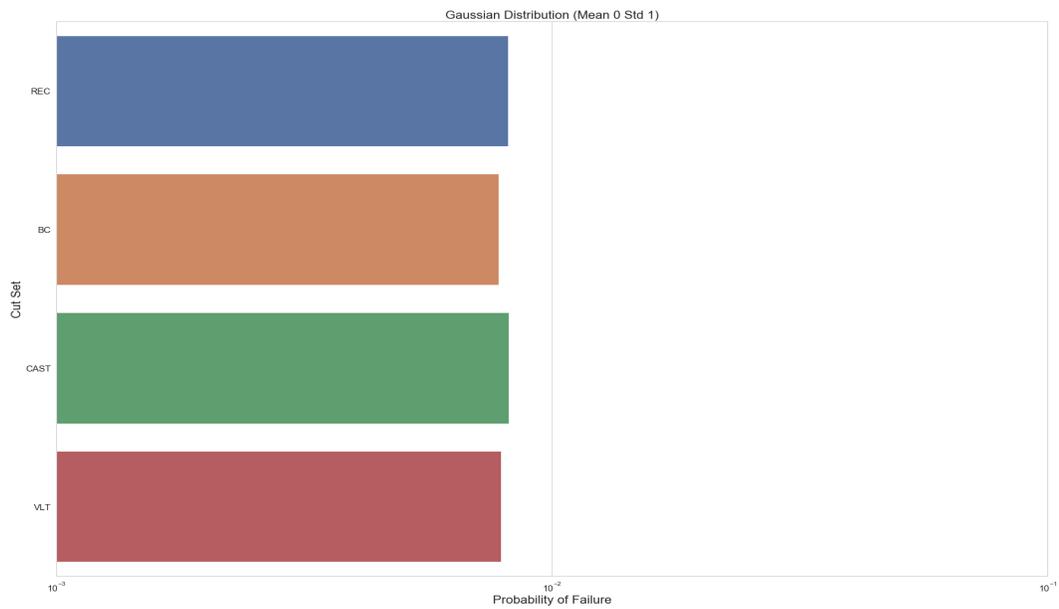


Figure 80: Gaussian Distribution ( $\mu = 0, \sigma = 1$ ): By-Area, No MA

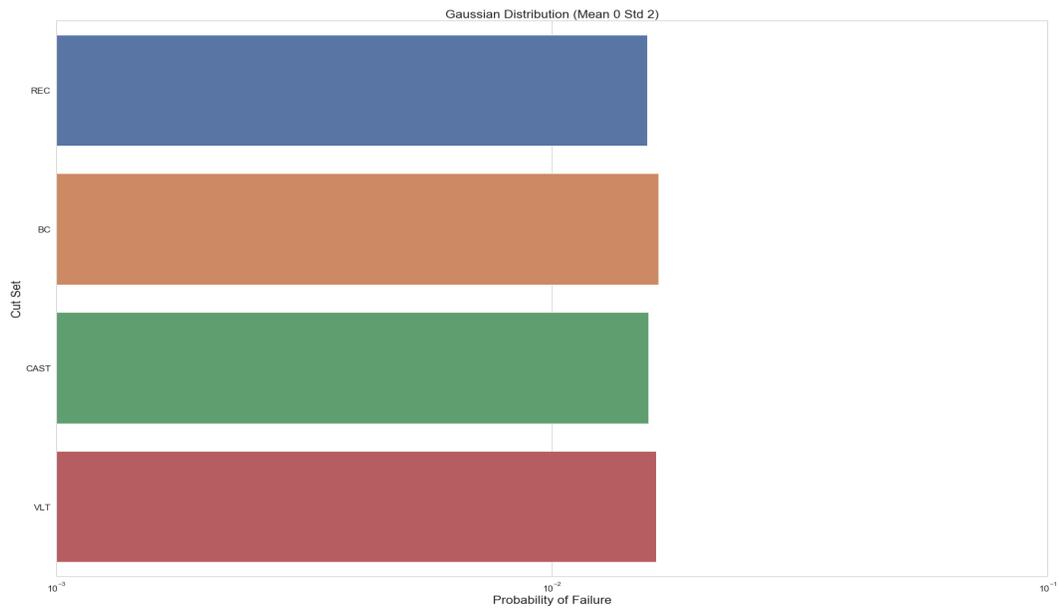


Figure 81: Gaussian Distribution ( $\mu = 0, \sigma = 2$ ): By-Area, No MA

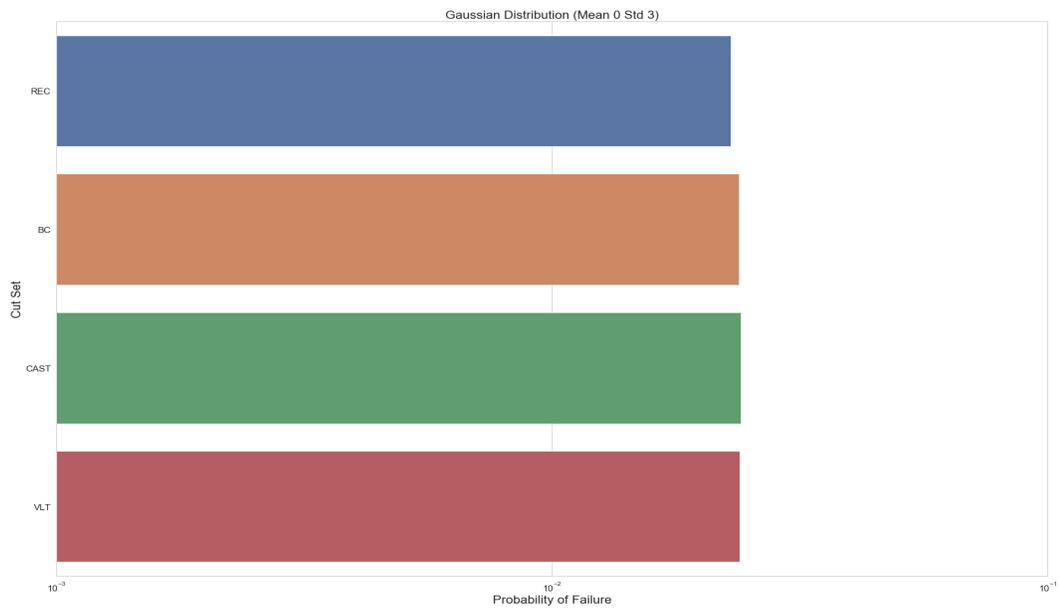


Figure 82: Gaussian Distribution ( $\mu = 0, \sigma = 3$ ): By-Area, No MA

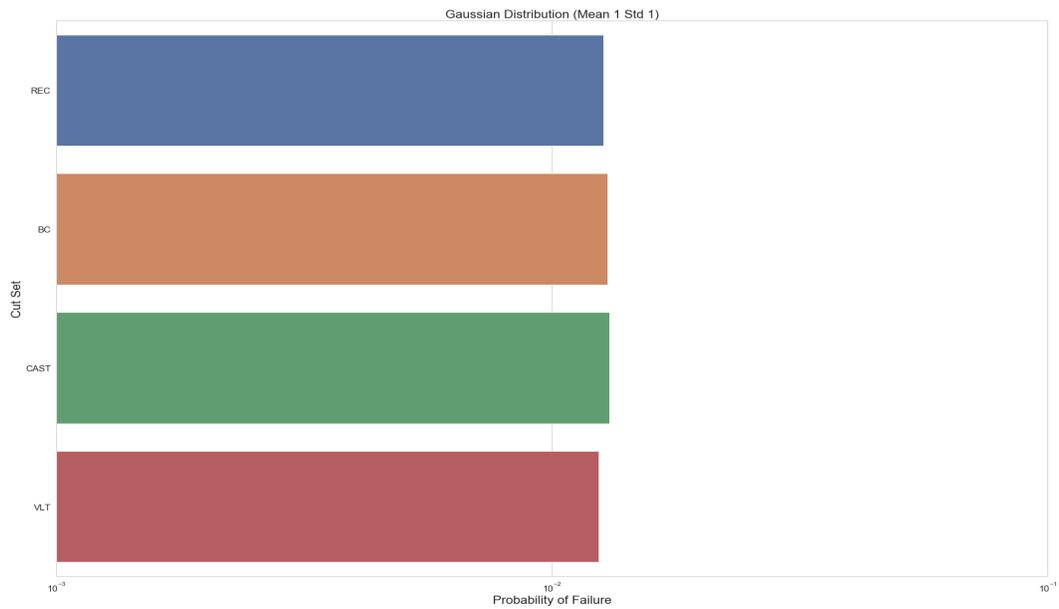


Figure 83: Gaussian Distribution ( $\mu = 1, \sigma = 1$ ): By-Area, No MA

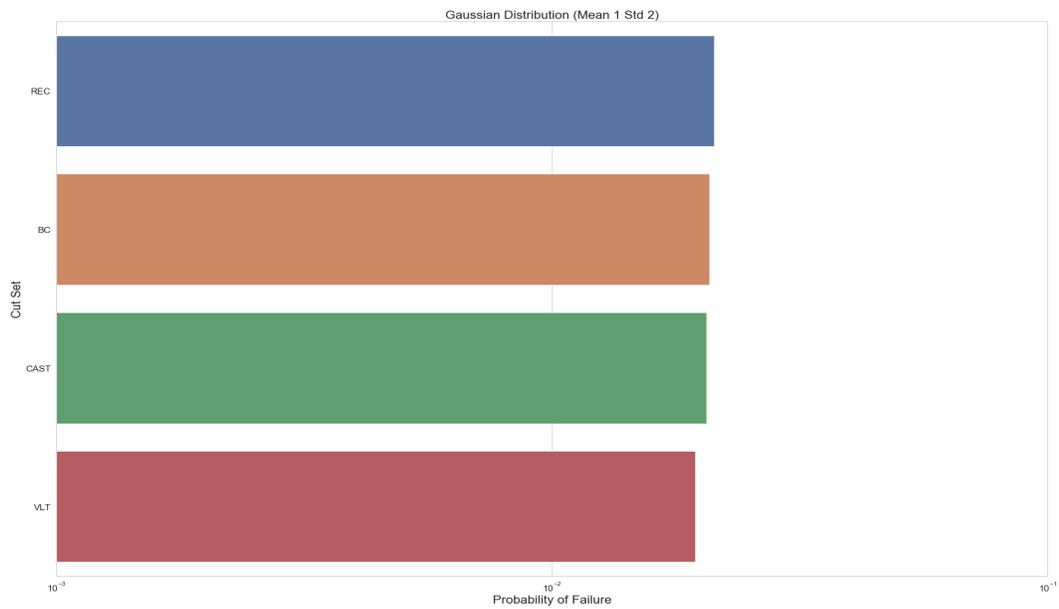


Figure 84: Gaussian Distribution ( $\mu = 1$ ,  $\sigma = 2$ ): By-Area, No MA

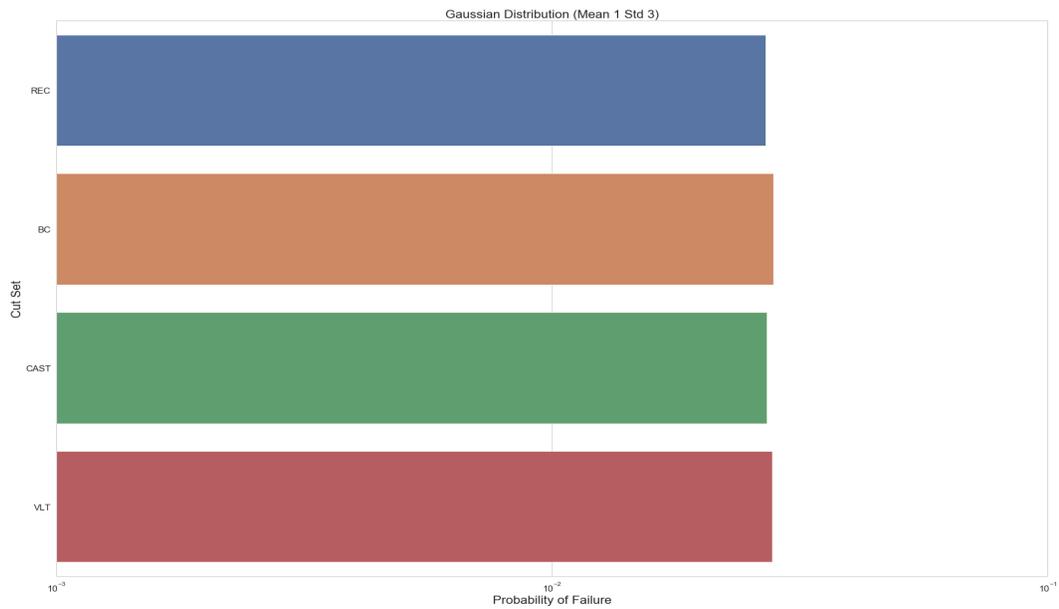


Figure 85: Gaussian Distribution ( $\mu = 1$ ,  $\sigma = 3$ ): By-Area, No MA

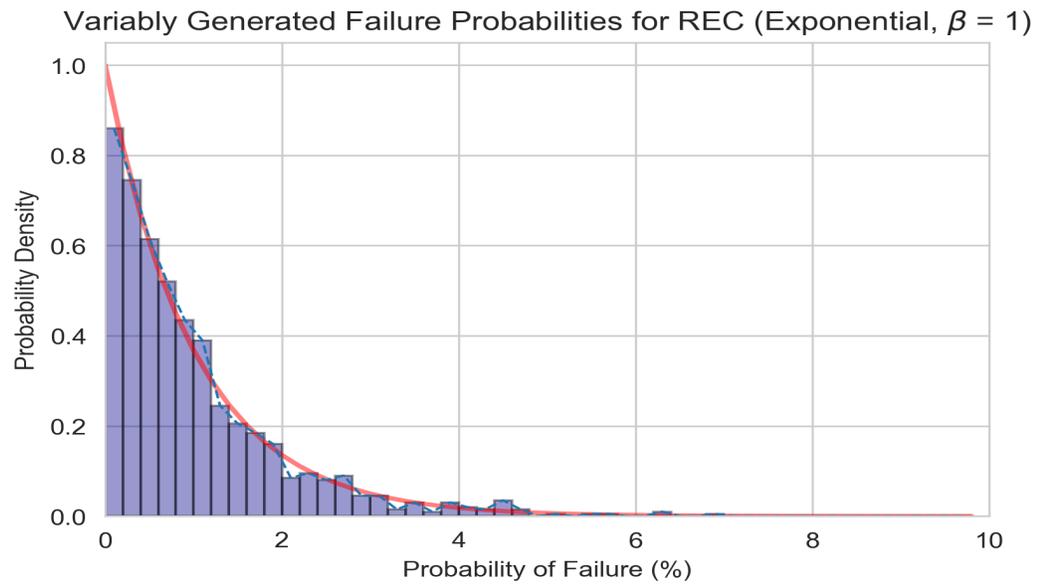
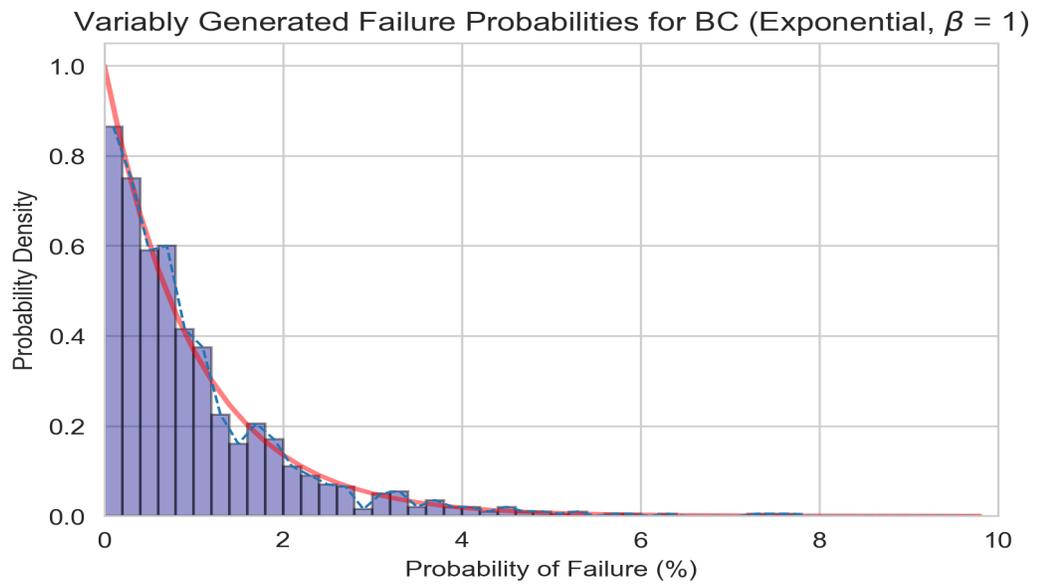
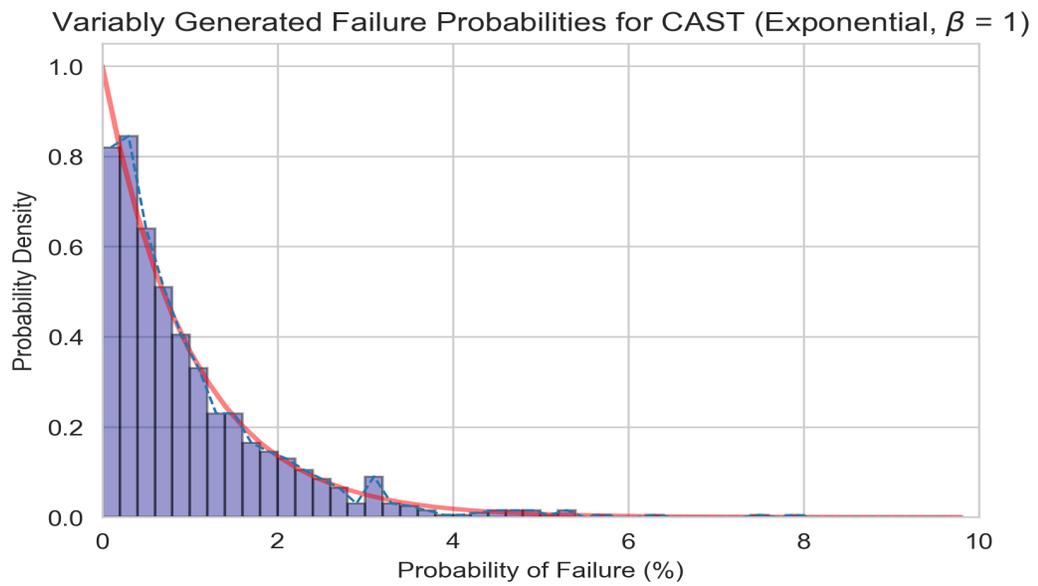
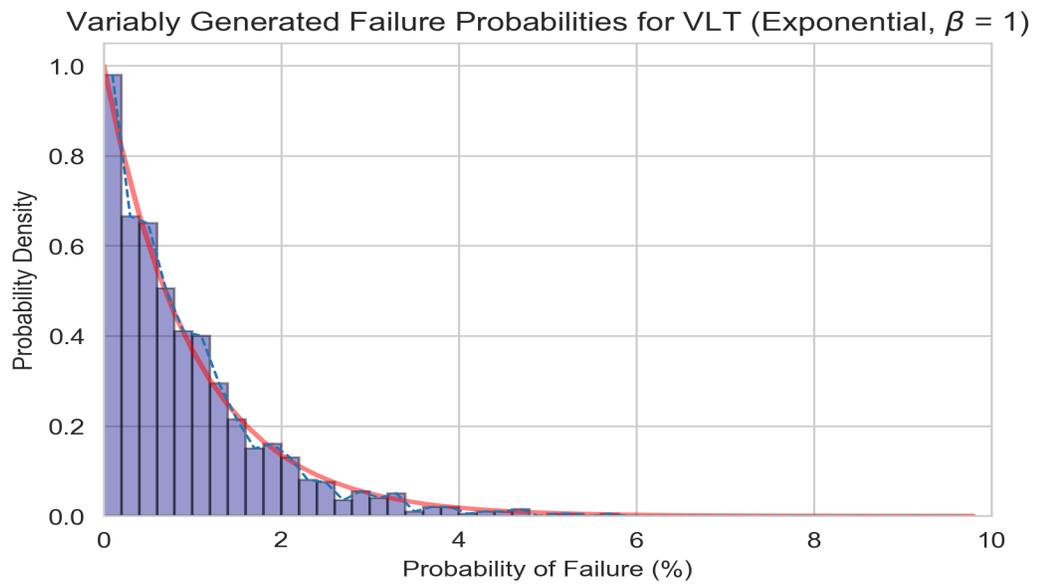
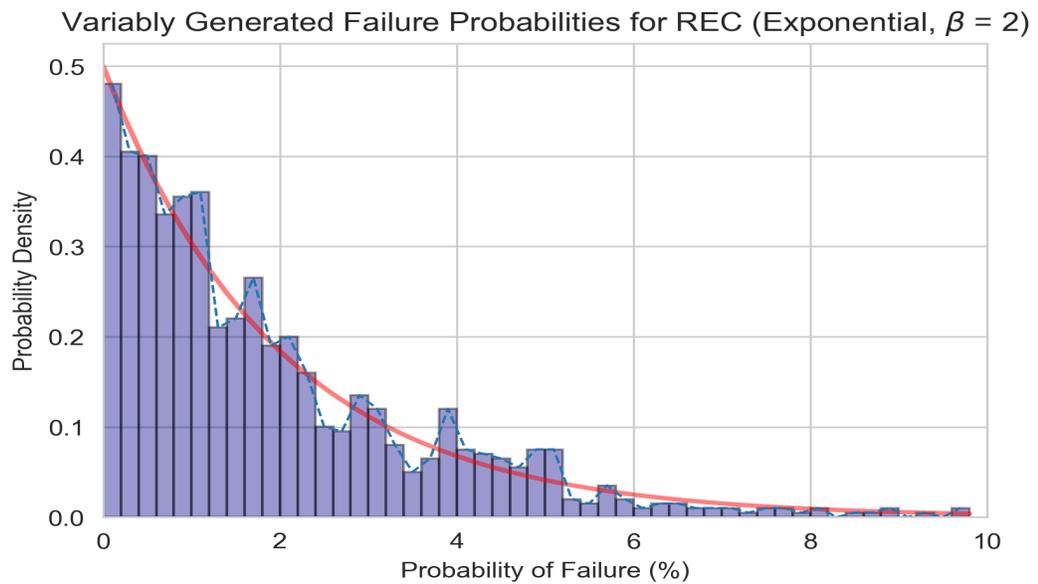
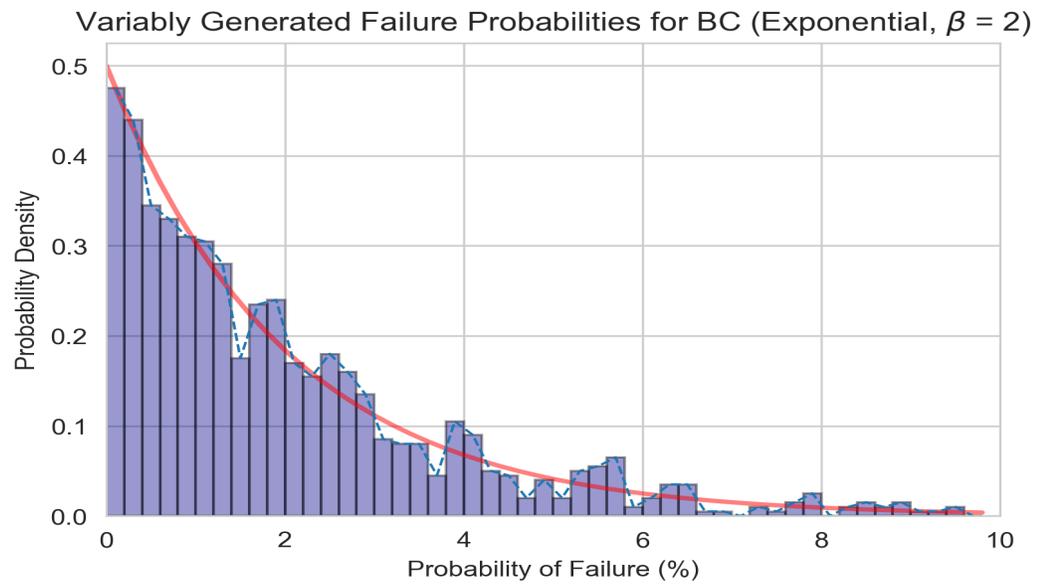
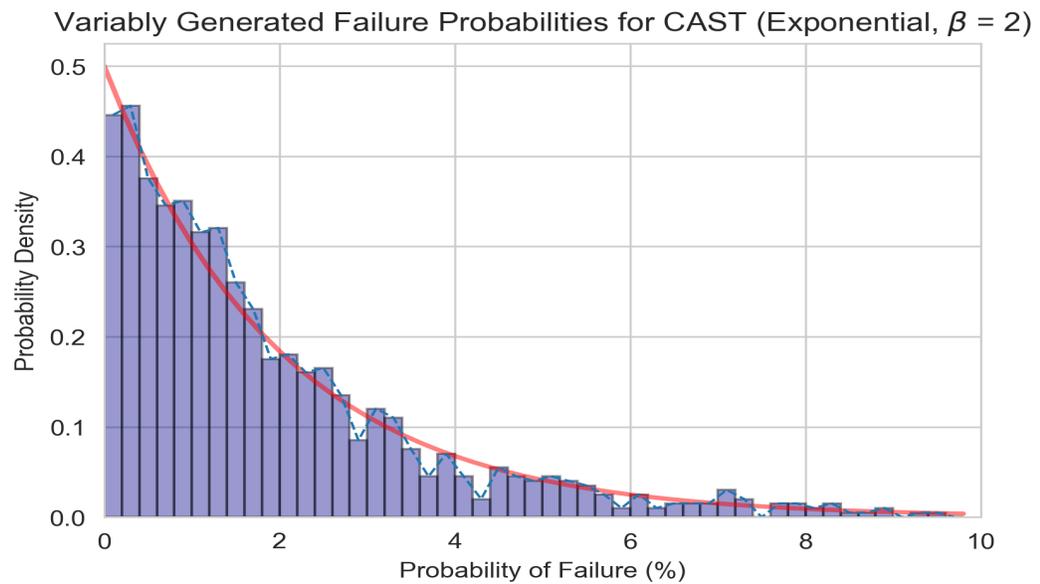


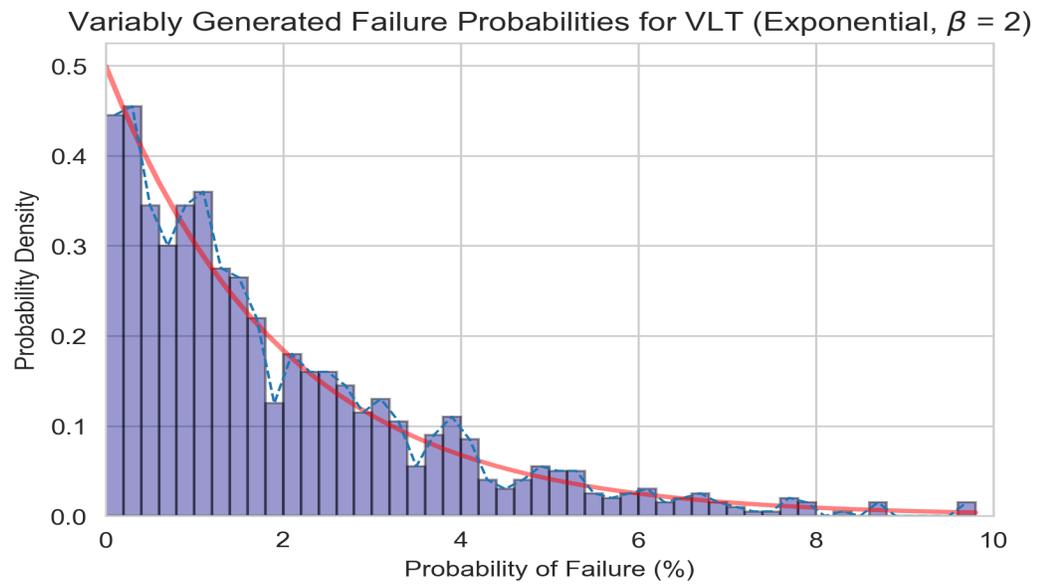
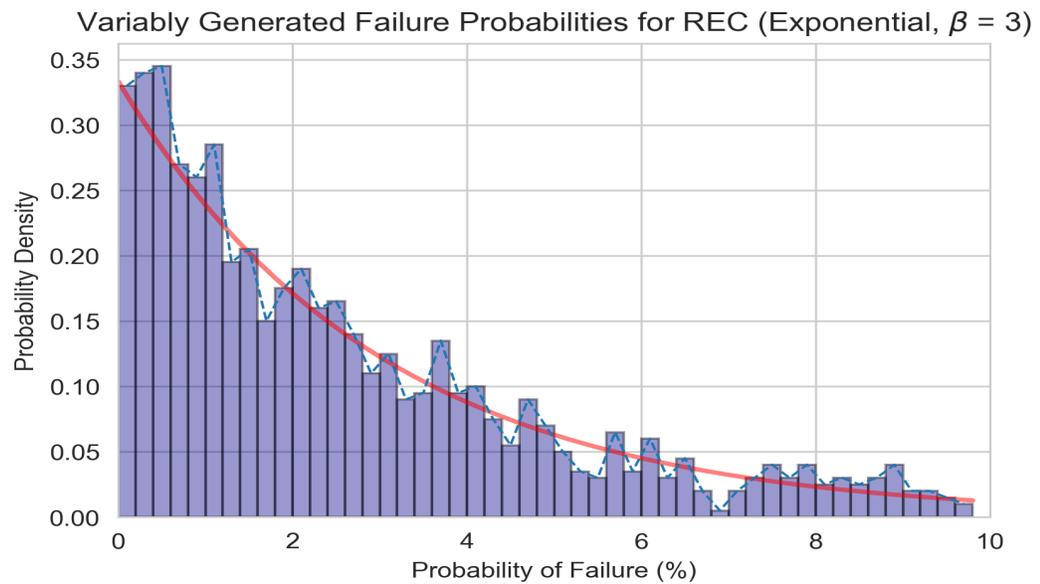
Figure 86: Exponential Distribution ( $\beta = 1$ ): REC

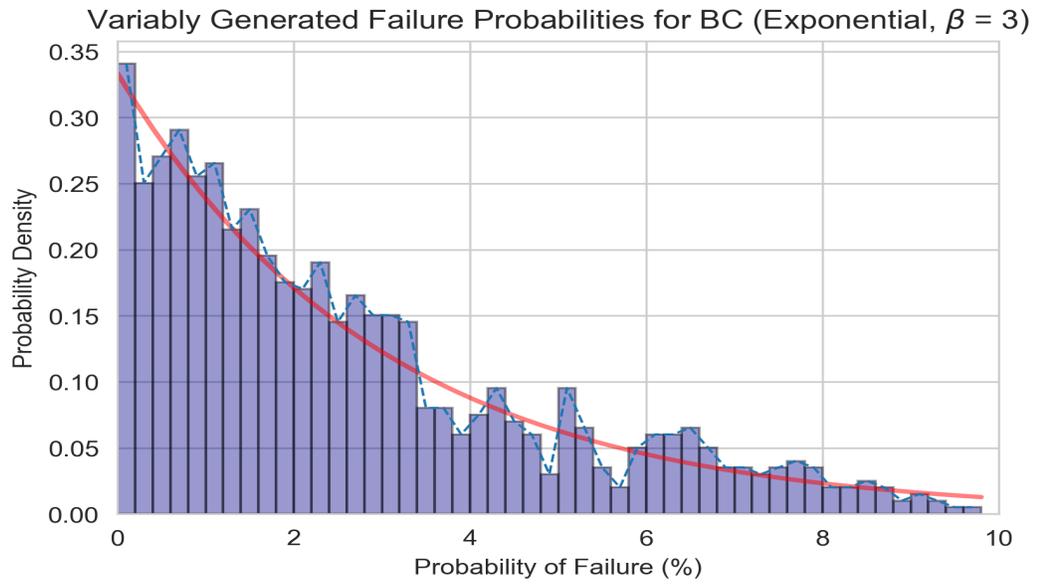
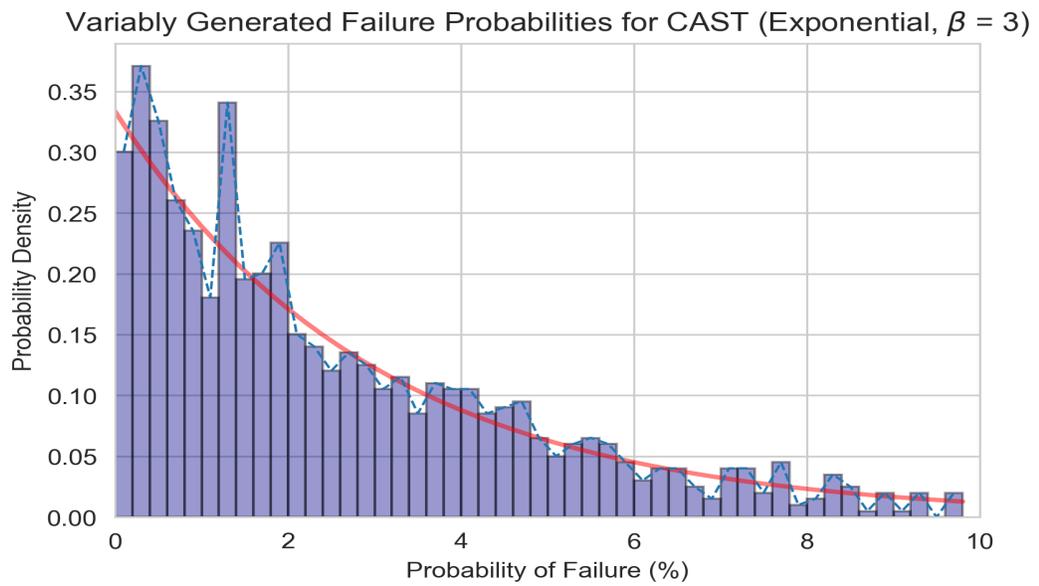
## C Sampled Data Distributions

Figure 87: Exponential Distribution ( $\beta = 1$ ): BCFigure 88: Exponential Distribution ( $\beta = 1$ ): CAST

Figure 89: Exponential Distribution ( $\beta = 1$ ): VLTFigure 90: Exponential Distribution ( $\beta = 2$ ): REC

Figure 91: Exponential Distribution ( $\beta = 2$ ): BCFigure 92: Exponential Distribution ( $\beta = 2$ ): CAST

Figure 93: Exponential Distribution ( $\beta = 2$ ): VLTFigure 94: Exponential Distribution ( $\beta = 3$ ): REC

Figure 95: Exponential Distribution ( $\beta = 3$ ): BCFigure 96: Exponential Distribution ( $\beta = 3$ ): CAST

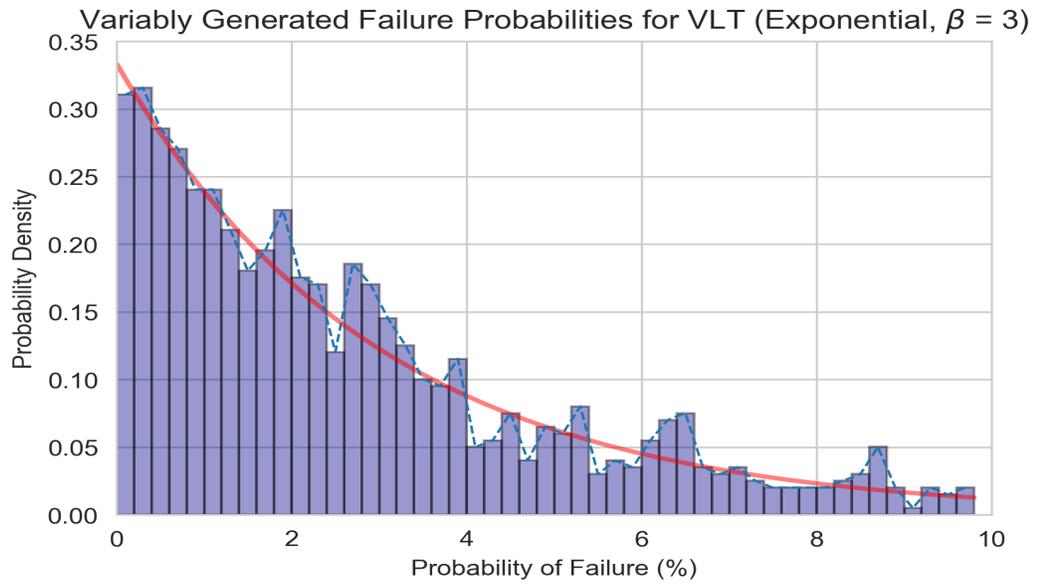


Figure 97: Exponential Distribution ( $\beta = 3$ ): VLT

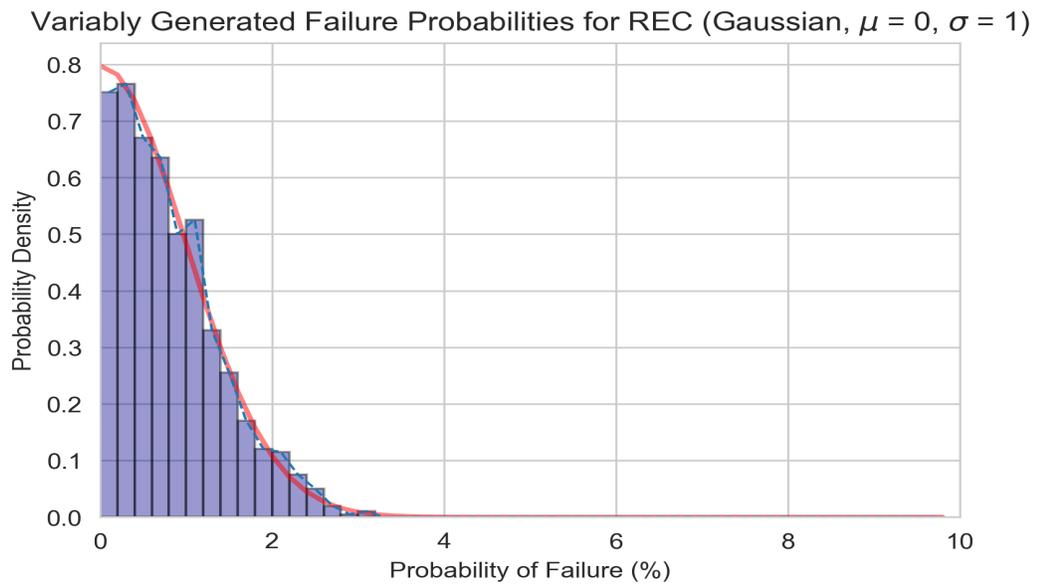
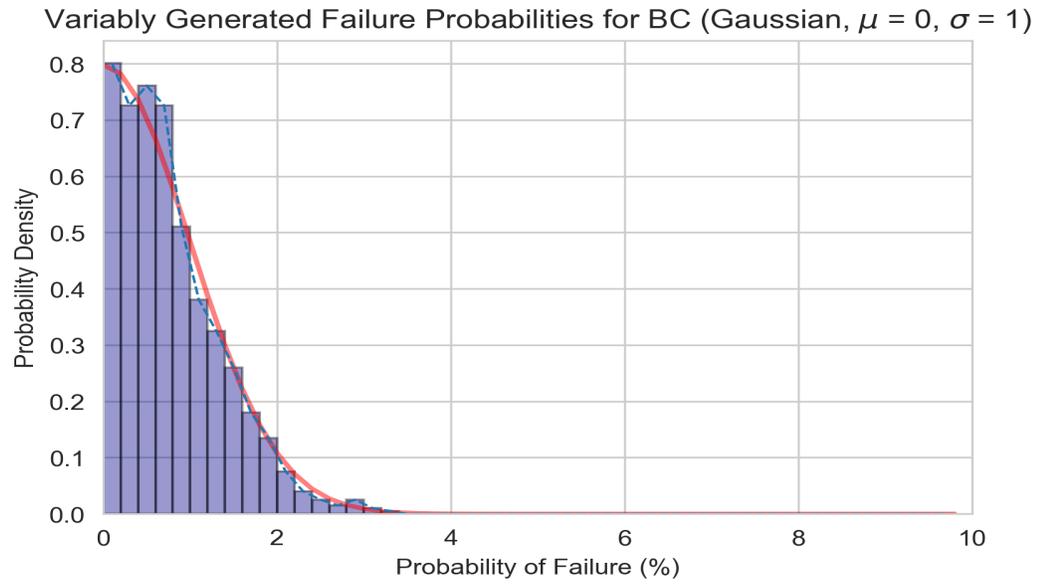
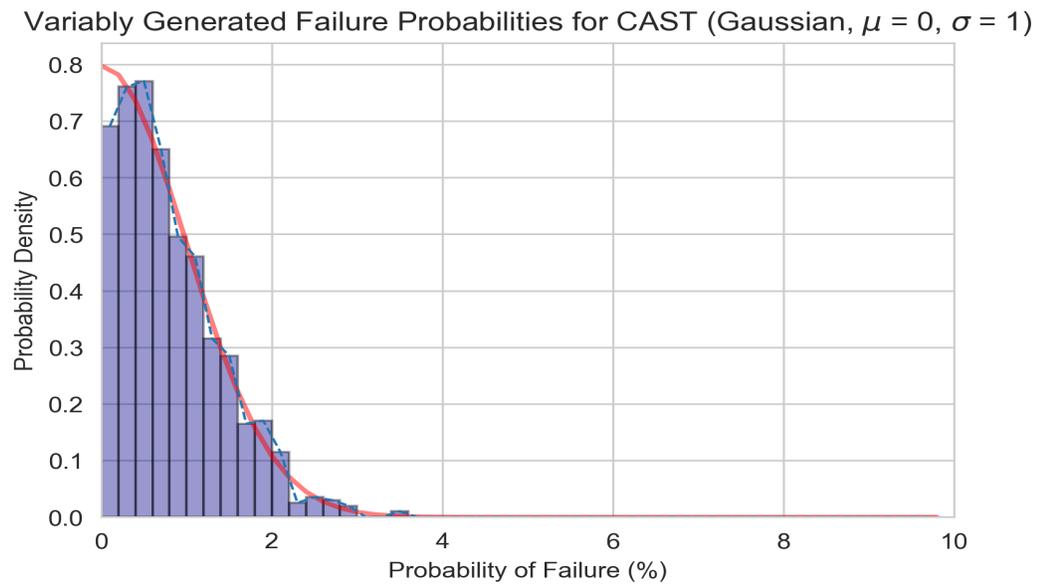
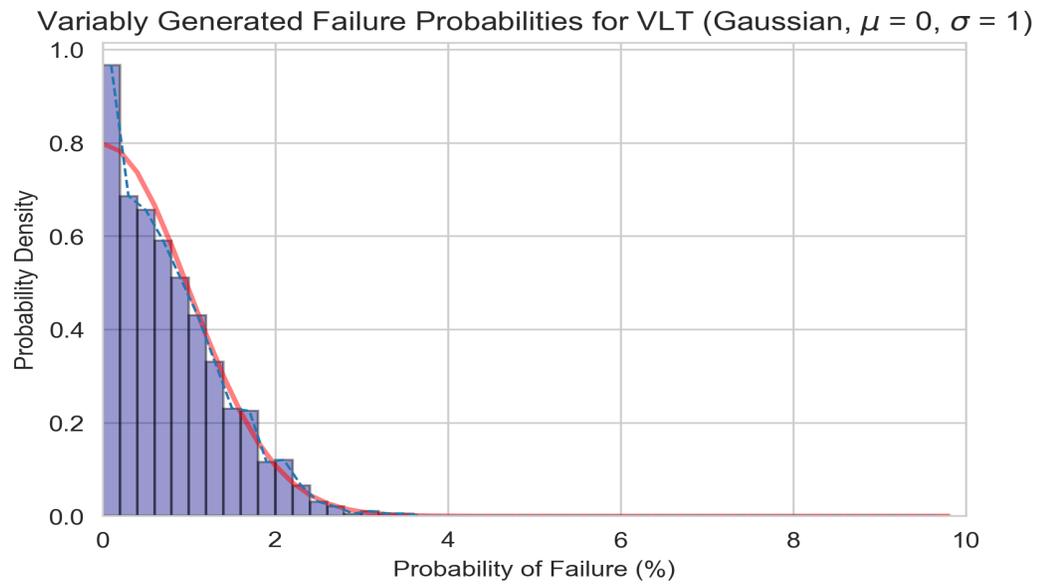
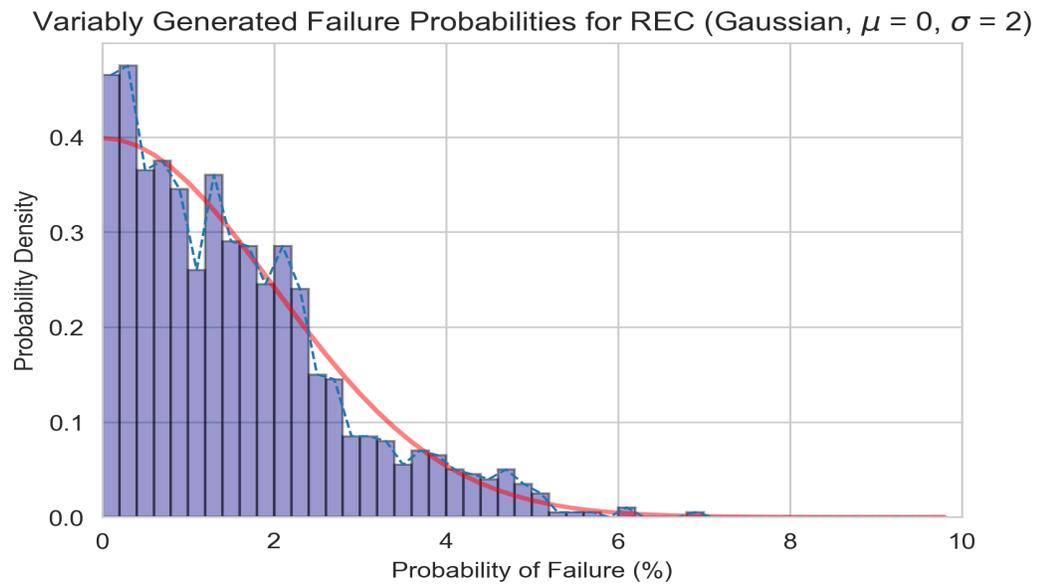


Figure 98: Gaussian Distribution ( $\mu = 0, \sigma = 1$ ): REC

Figure 99: Gaussian Distribution ( $\mu = 0, \sigma = 1$ ): BCFigure 100: Gaussian Distribution ( $\mu = 0, \sigma = 1$ ): CAST

Figure 101: Gaussian Distribution ( $\mu = 0, \sigma = 1$ ): VLTFigure 102: Gaussian Distribution ( $\mu = 0, \sigma = 2$ ): REC

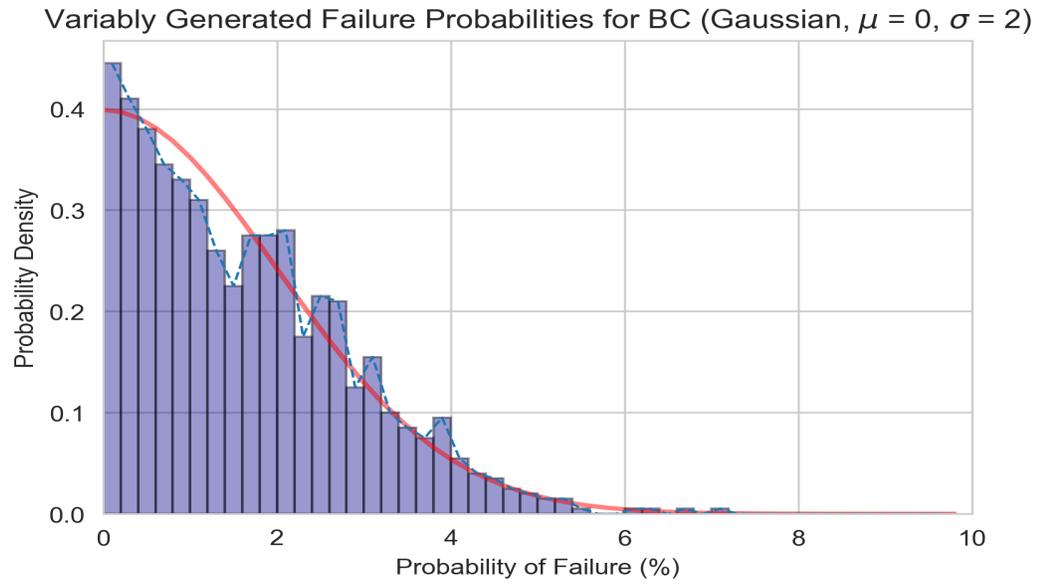


Figure 103: Gaussian Distribution ( $\mu = 0, \sigma = 2$ ): BC

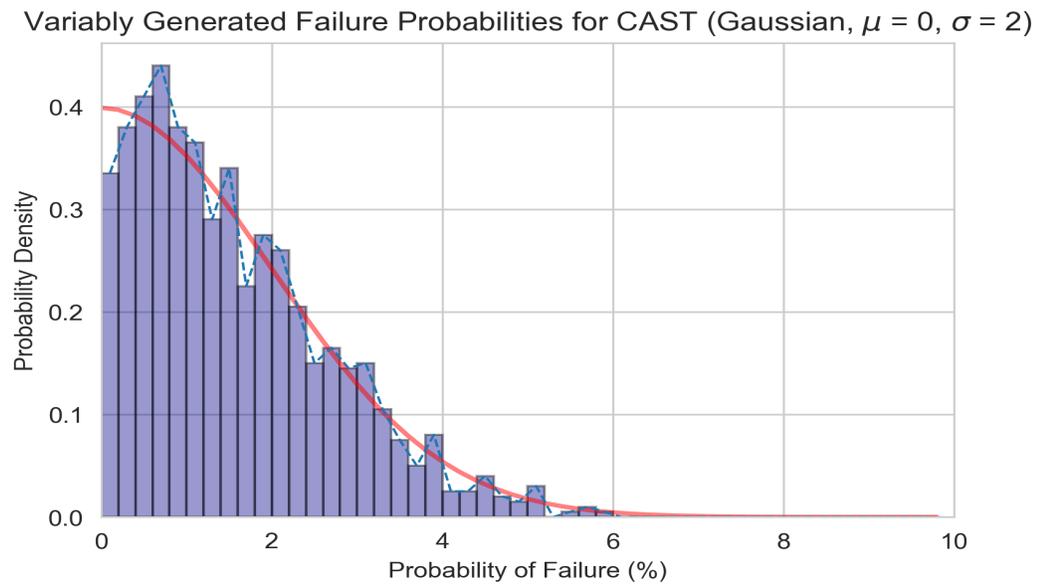


Figure 104: Gaussian Distribution ( $\mu = 0, \sigma = 2$ ): CAST

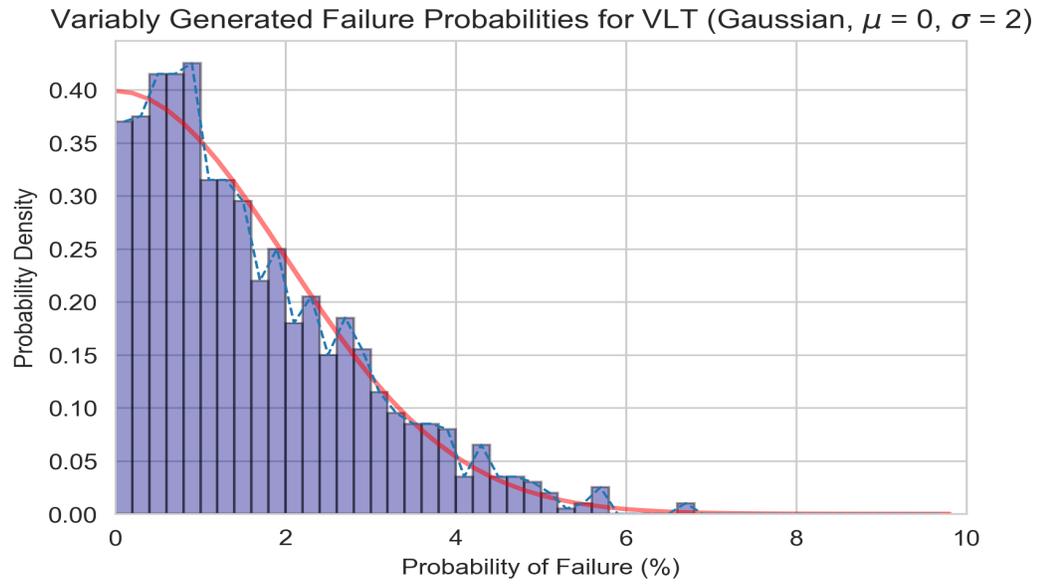


Figure 105: Gaussian Distribution ( $\mu = 0$ ,  $\sigma = 2$ ): VLT

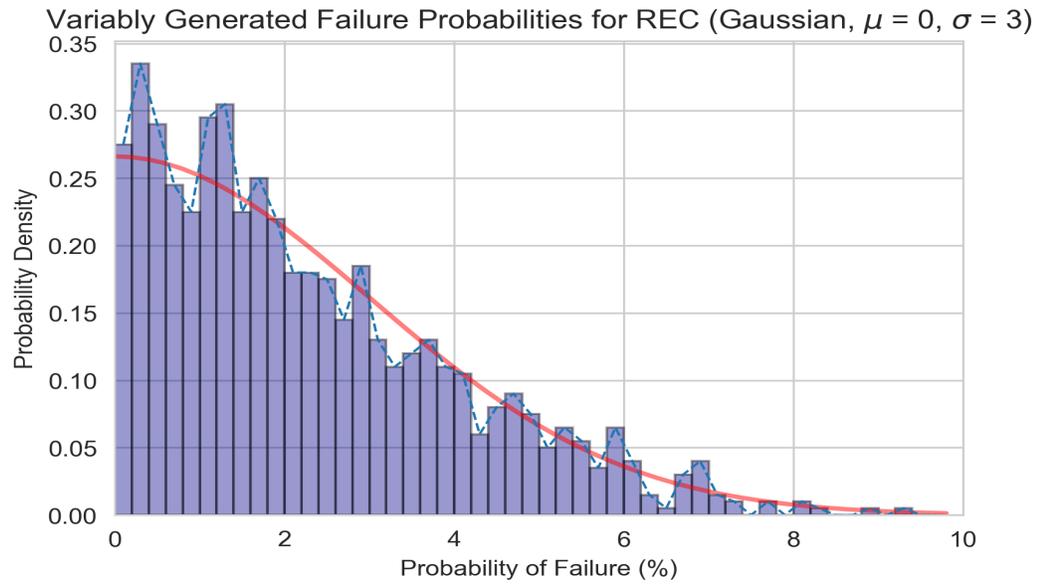


Figure 106: Gaussian Distribution ( $\mu = 0$ ,  $\sigma = 3$ ): REC

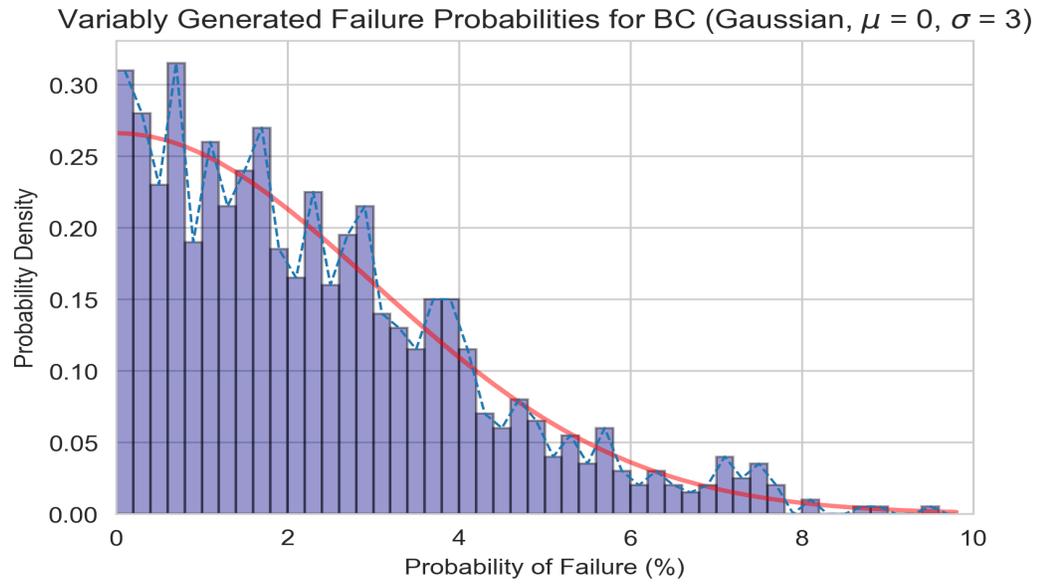


Figure 107: Gaussian Distribution ( $\mu = 0$ ,  $\sigma = 3$ ): BC

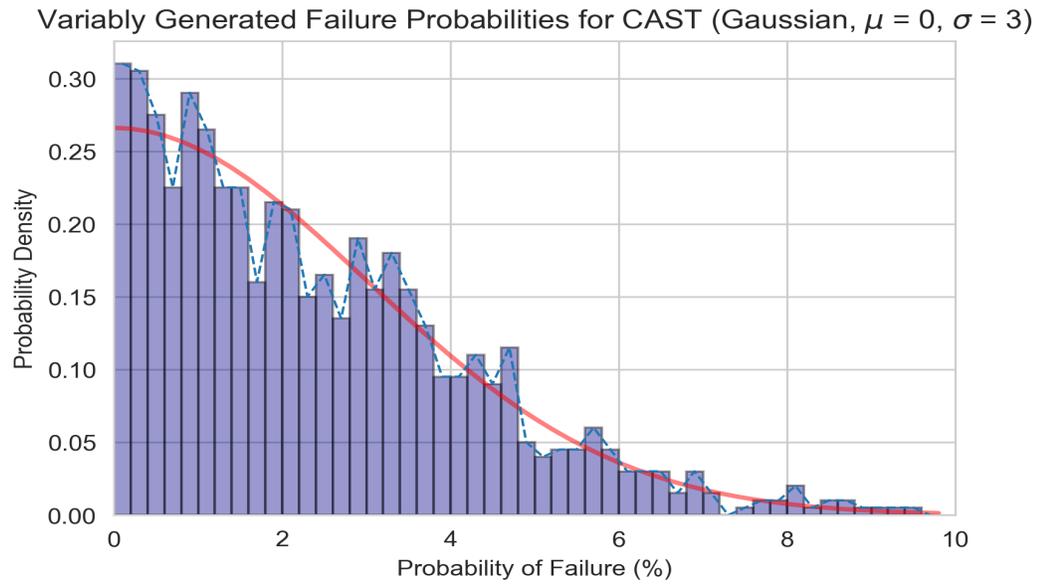


Figure 108: Gaussian Distribution ( $\mu = 0$ ,  $\sigma = 3$ ): CAST

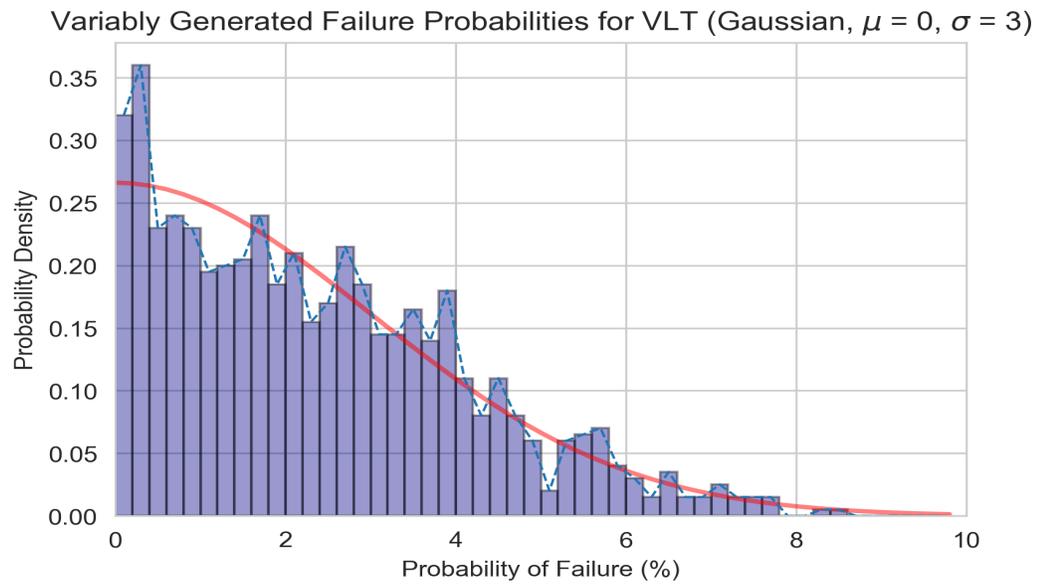


Figure 109: Gaussian Distribution ( $\mu = 0, \sigma = 3$ ): VLT

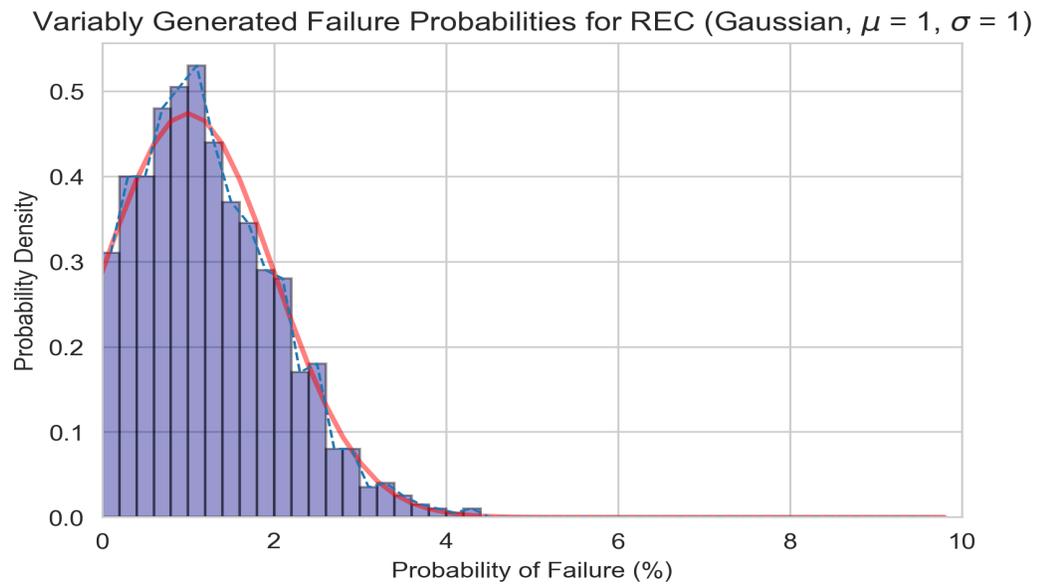


Figure 110: Gaussian Distribution ( $\mu = 1, \sigma = 1$ ): REC

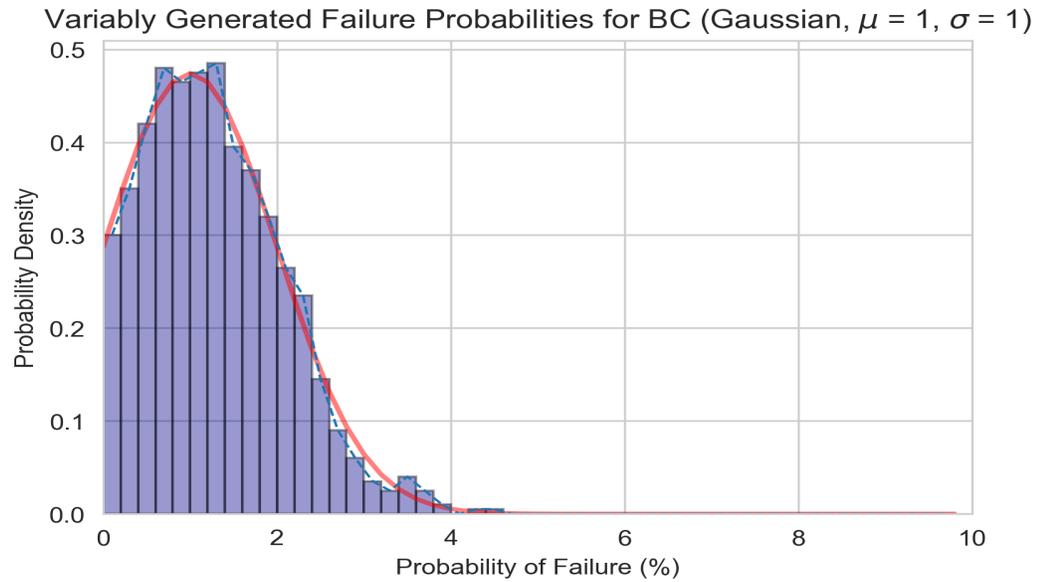


Figure 111: Gaussian Distribution ( $\mu = 1, \sigma = 1$ ): BC

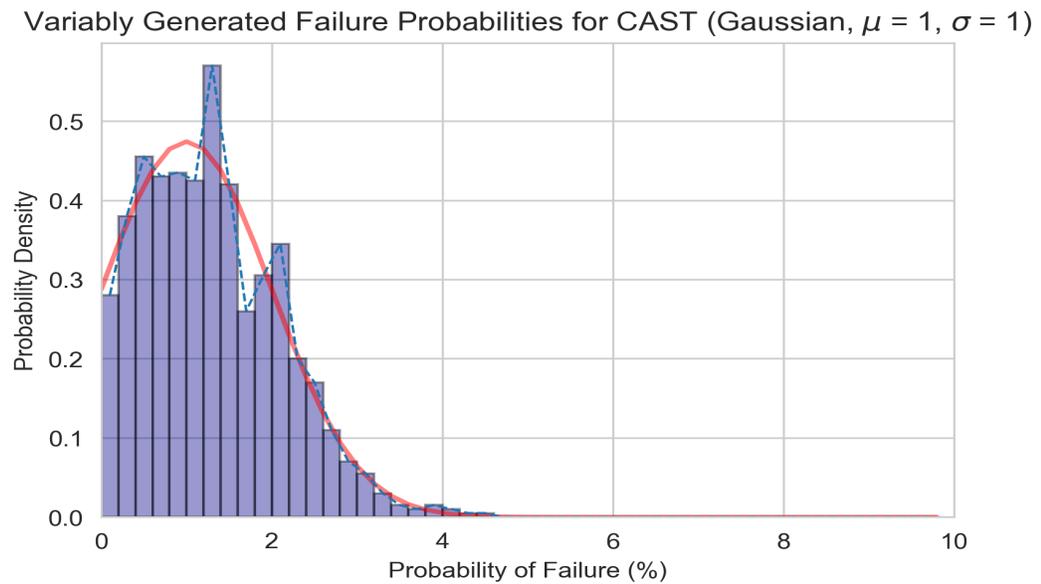


Figure 112: Gaussian Distribution ( $\mu = 1, \sigma = 1$ ): CAST

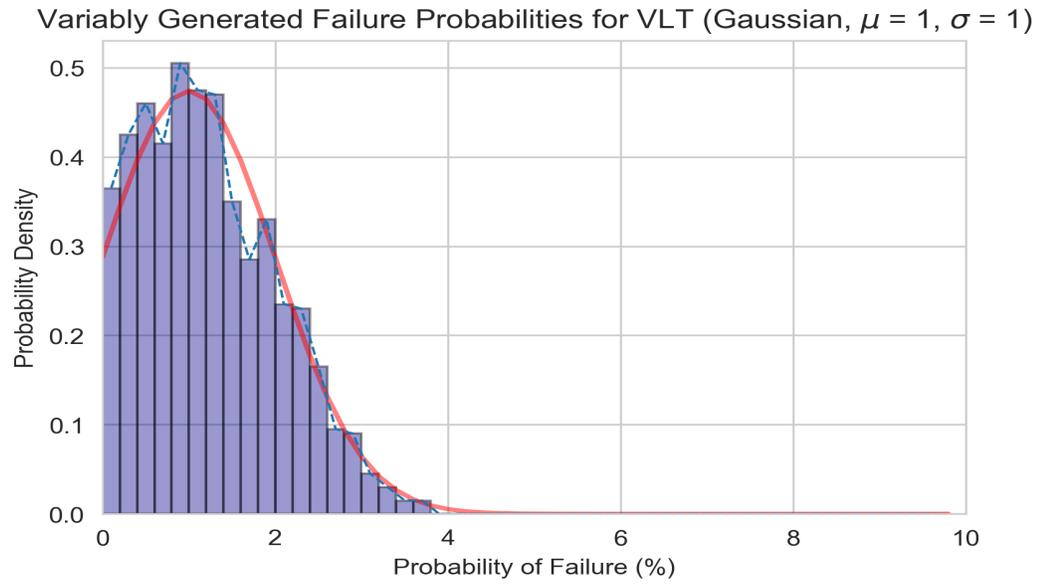


Figure 113: Gaussian Distribution ( $\mu = 1, \sigma = 1$ ): VLT

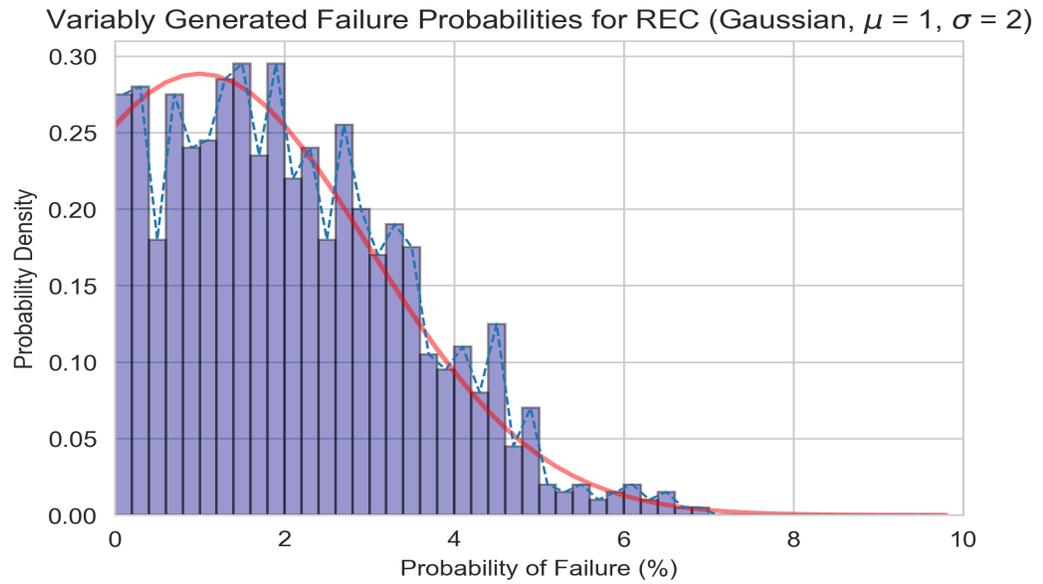


Figure 114: Gaussian Distribution ( $\mu = 1, \sigma = 2$ ): REC

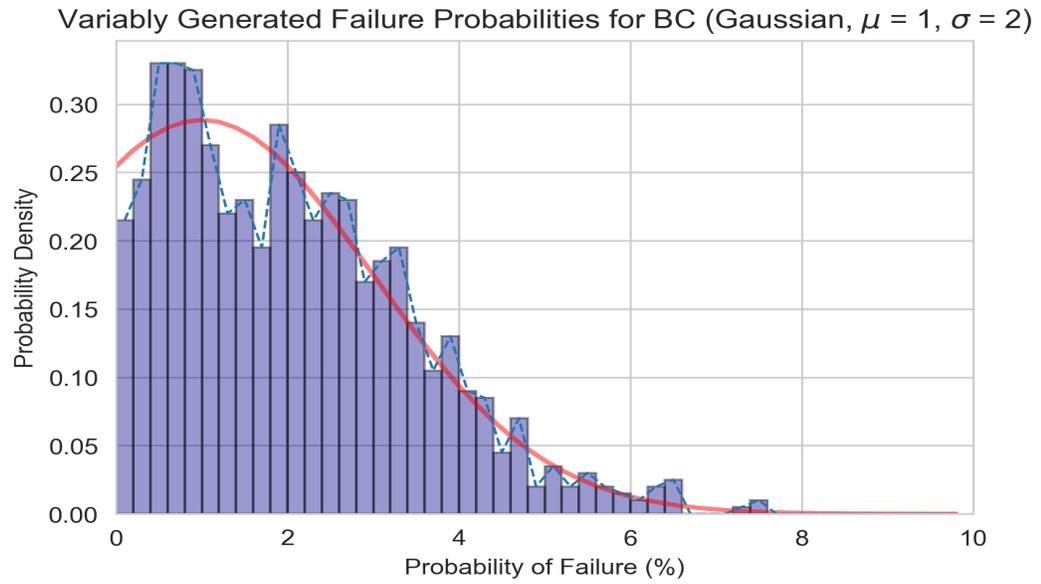


Figure 115: Gaussian Distribution ( $\mu = 1, \sigma = 2$ ): BC

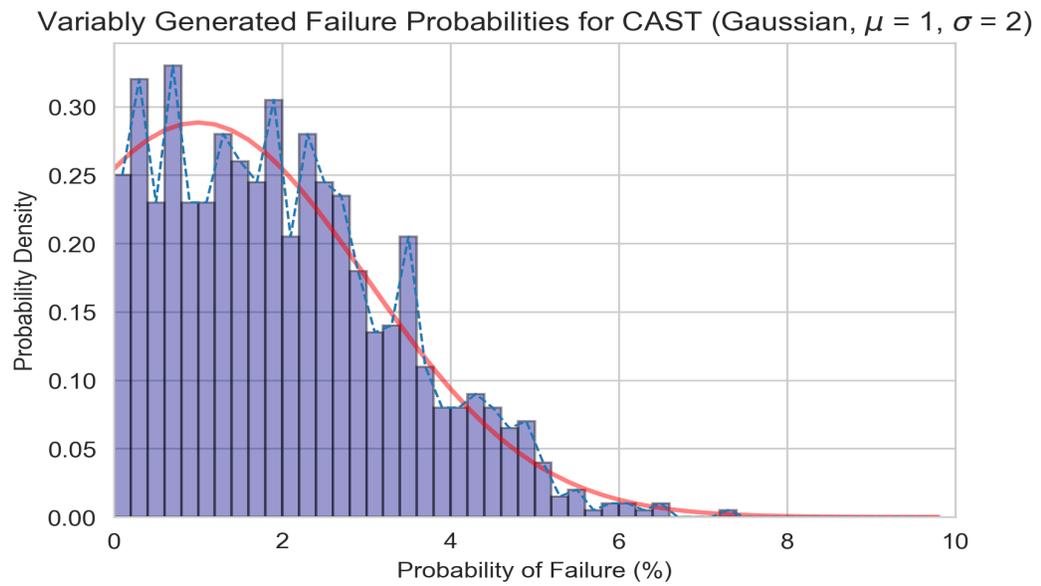


Figure 116: Gaussian Distribution ( $\mu = 1, \sigma = 2$ ): CAST

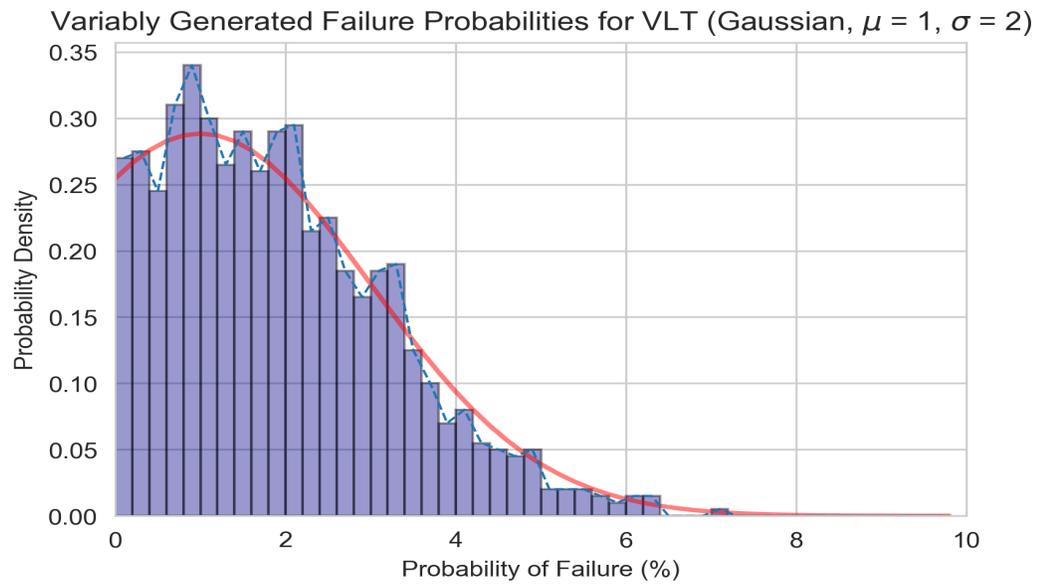


Figure 117: Gaussian Distribution ( $\mu = 1$ ,  $\sigma = 2$ ): VLT

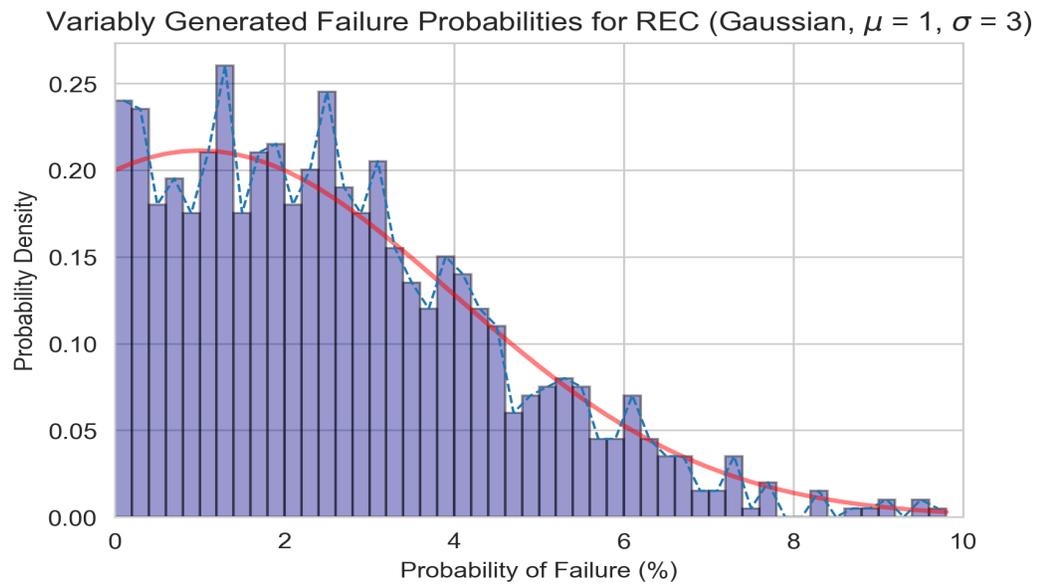


Figure 118: Gaussian Distribution ( $\mu = 1$ ,  $\sigma = 3$ ): REC

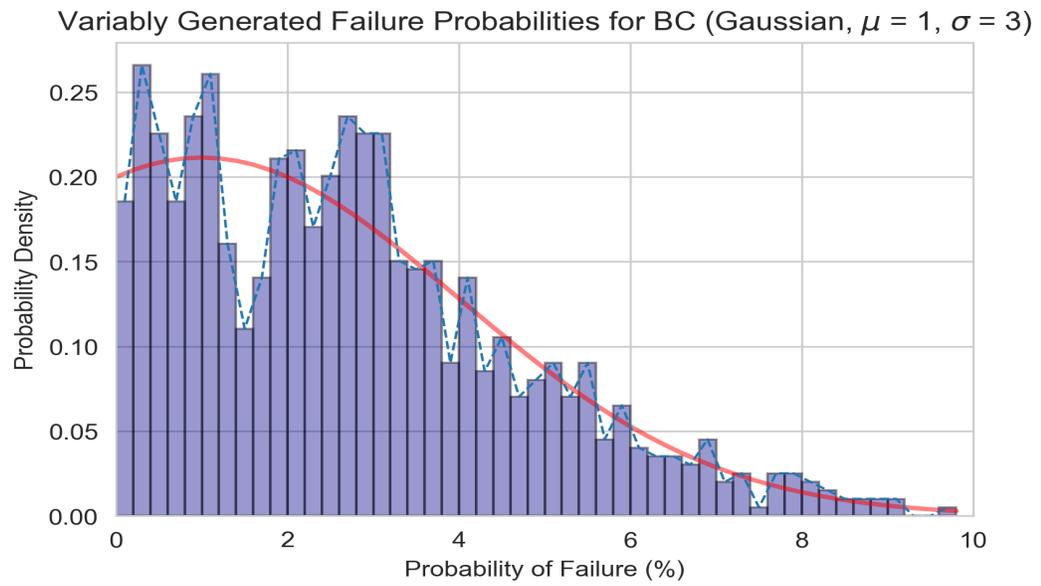


Figure 119: Gaussian Distribution ( $\mu = 1, \sigma = 3$ ): BC

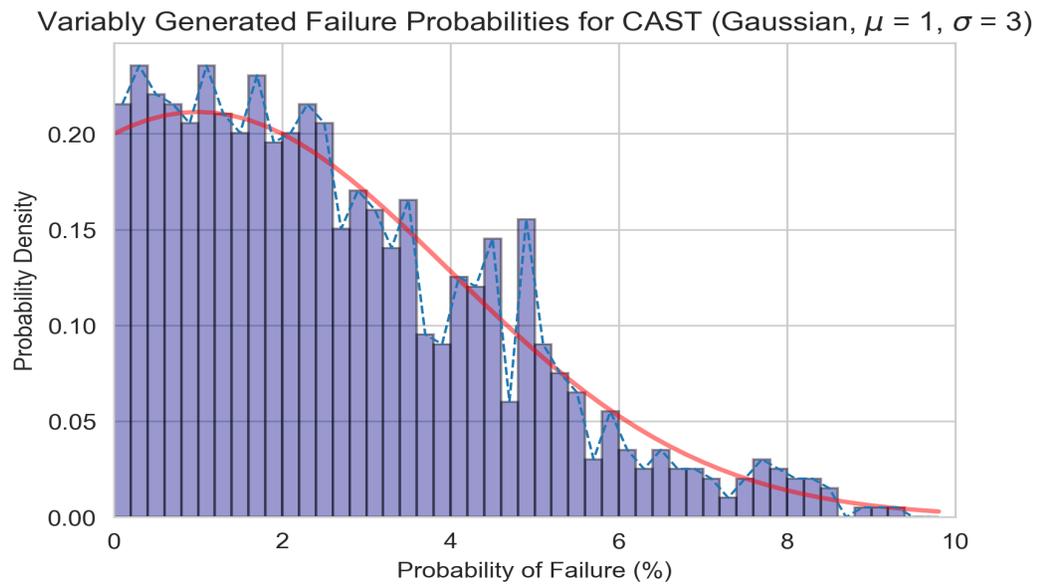


Figure 120: Gaussian Distribution ( $\mu = 1, \sigma = 3$ ): CAST

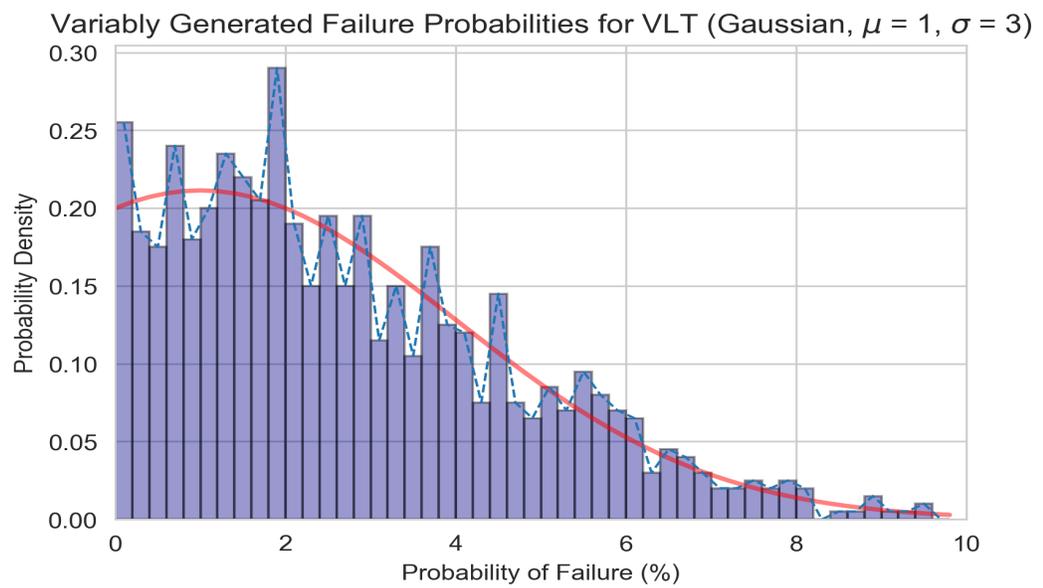


Figure 121: Gaussian Distribution ( $\mu = 1, \sigma = 3$ ): VLT

## D Python Scripts

```

import numpy
import csv
from avariables import *

class MyNumber:
#code to pad 'E+000' to three digits in output text file
#https://stackoverflow.com/questions/39184719/exponent-digits-in-
  ↪ scientific-notation-in-python
    def __init__(self, val):
        self.val = val
    def __format__(self, format_spec):
        ss = ('{0:}' + format_spec + '}').format(self.val)
        if ( 'e' in ss):
            mantissa, exp = ss.split('e')
            return mantissa + 'E' + exp[0] + '0' + exp[1:]
        return ss

def errormessage():
    print("Invalid input, try again.")

def randominput(rand):
    global ftot
    #sample uniform distribution
    #random number between 0-1 (or 0-100%)
    ftot[rand] = numpy.random.rand()
    #divided by 10 to become 0-10%
    ftot[rand] = ftot[rand]*0.1
    #round to 3 decimal places, like 0.01537 (1.537%)
    ftot[rand] = round(ftot[rand], 5)

def gaussianinput(rand, mu, sigma):
    global ftot
    #sample gaussian distribution
    #must be between 0-10
    while ftot[rand] <= 0 or ftot[rand] > 10:
        ftot[rand] = numpy.random.normal(loc=mu, scale=sigma)
    #divided by 10 to become 0-10%
    ftot[rand] = ftot[rand]*0.01
    #round to 3 decimal places, like 0.01537 (1.537%)
    ftot[rand] = round(ftot[rand], 5)

```

```

def expinput(rand):
    global ftot
    #sample exponential distribution
    #must be between 0-10%
    while ftot[rand] == 0 or ftot[rand] > 10:
        ftot[rand] = numpy.random.exponential(scale=beta)
    #divided by 10 to become 0-10%
    ftot[rand] = ftot[rand]*0.01
    #round to 3 decimal places, like 0.01537 (1.537%)
    ftot[rand] = round(ftot[rand], 5)

def choices():
    global choice1
    global choice2
    while True:
        try:
            numrepeat = int(raw_input("How many input files?"))
        except ValueError:
            errormessage()
            continue
        else:
            break
    while True:
        try:
            choice1 = int(raw_input("Uniform probabilities (1) or
                ↪ Variable probabilities (2)?"))
        except ValueError:
            errormessage()
            continue
        if choice1 != 1 and choice1 != 2:
            errormessage()
            continue
        else:
            break
    while True:
        try:
            choice2 = int(raw_input("User input (1), Uniform (2),
                ↪ Gaussian (3), or Exponential (4) distribution?"))
        except ValueError:
            errormessage()
            continue

```

```

    if choice2 != 1 and choice2 != 2 and choice2 != 3 and
        ↪ choice2 != 4:
        errormessage()
        continue
    else:
        break
if choice2 == 3:
    while True:
        try:
            mean = float(raw_input("Mean:␣"))
        except ValueError:
            errormessage()
            continue
        else:
            break
    while True:
        try:
            std = float(raw_input("Standard␣deviation:␣"))
        except ValueError:
            errormessage()
            continue
        else:
            break
if choice2 == 4:
    while True:
        try:
            beta = float(raw_input("Scale:␣"))
        except ValueError:
            errormessage()
            continue
        else:
            break

def userInput(calc, oparea):
    global ftot
    global output2
    #user gets to decide for each ftot
    #user input
    if choice2 == 1:
        for x in range(calc+1):
            while True:
                try:

```

```

        ftot[x] = float(raw_input("What is the failure of
        ↪ {}? (0 < x <= 10): ".format(oparea[x])))
    except ValueError:
        errormessage()
        continue
    #check that value is between (0,10]
    if 0 < ftot[x] <= 10:
        #convert 0-10% into decimal format
        #round to 3 decimal places, like 0.01537 (1.537%)
        ftot[x] = ftot[x]*0.01
        ftot[x] = round(ftot[x], 5)
        break
    else:
        errormessage()
        continue
#random sample
    if choice2 == 2:
        output2 = 'Uniform distribution'
        for x in range(calc+1):
            randominput(x)
    elif choice2 == 3:
        output2 = 'Gaussian distribution, mean {}, std {}'.format(
        ↪ mean, std)
        for x in range(calc+1):
            gaussianinput(x, mean, std)
    elif choice2 == 4:
        output2 = 'Exponential distribution, lambda {}'.format(beta)
        for x in range(calc+1):
            expinput(x)

def outputfile():
    global t
    #arpw sum
    rec = a_1 + a_2 + a_6 + a_7 + a_8 + a_9 + a_10 + a_11 + a_12 +
    ↪ a_13 + a_14 + a_15 + a_16 + a_17 + a_18 + a_19 + a_20 +
    ↪ a_21 + a_22 + a_23 + a_24 + a_25 + a_26 + a_27 + a_28 +
    ↪ a_29 + a_30 + a_34 + a_35 + a_36
    bc = b_1 + b_2 + b_4 + b_5 + b_6 + b_7 + b_8 + b_9 + b_10 + b_11
    ↪ + b_12 + b_13 + b_14 + b_15 + b_16 + b_17 + b_18 + b_19
    cast = c_2 + c_3 + c_4 + c_5 + c_6 + c_11 + c_12 + c_13 + c_14 +
    ↪ c_16 + c_18 + c_19 + c_20 + c_24 + c_25 + c_10 + c_23 +

```

$$\rightarrow c_{26} + c_1 + c_7 + c_8 + c_9 + c_{22}$$

$$vlt = d_1 + d_2 + d_3 + d_4 + d_5 + d_6 + d_7 + d_9 + d_{10} +$$

$$\rightarrow d_{11}$$

$$a1_1 = ftot[0]*a_1/rec$$

$$a1_2 = ftot[0]*a_2/rec$$

$$a1_6 = ftot[0]*a_6/rec$$

$$a1_7 = ftot[0]*a_7/rec$$

$$a1_8 = ftot[0]*a_8/rec$$

$$a1_9 = ftot[0]*a_9/rec$$

$$a1_{10} = ftot[0]*a_{10}/rec$$

$$a1_{11} = ftot[0]*a_{11}/rec$$

$$a1_{12} = ftot[0]*a_{12}/rec$$

$$a1_{13} = ftot[0]*a_{13}/rec$$

$$a1_{14} = ftot[0]*a_{14}/rec$$

$$a1_{15} = ftot[0]*a_{15}/rec$$

$$a1_{16} = ftot[0]*a_{16}/rec$$

$$a1_{17} = ftot[0]*a_{17}/rec$$

$$a1_{18} = ftot[0]*a_{18}/rec$$

$$a1_{19} = ftot[0]*a_{19}/rec$$

$$a1_{20} = ftot[0]*a_{20}/rec$$

$$a1_{21} = ftot[0]*a_{21}/rec$$

$$a1_{22} = ftot[0]*a_{22}/rec$$

$$a1_{23} = ftot[0]*a_{23}/rec$$

$$a1_{24} = ftot[0]*a_{24}/rec$$

$$a1_{25} = ftot[0]*a_{25}/rec$$

$$a1_{26} = ftot[0]*a_{26}/rec$$

$$a1_{27} = ftot[0]*a_{27}/rec$$

$$a1_{28} = ftot[0]*a_{28}/rec$$

$$a1_{29} = ftot[0]*a_{29}/rec$$

$$a1_{30} = ftot[0]*a_{30}/rec$$

$$a1_{34} = ftot[0]*a_{34}/rec$$

$$a1_{35} = ftot[0]*a_{35}/rec$$

$$a1_{36} = ftot[0]*a_{36}/rec$$

$$b2_1 = ftot[1]*b_1/bc$$

$$b2_2 = ftot[1]*b_2/bc$$

$$b2_4 = ftot[1]*b_4/bc$$

$$b2_5 = ftot[1]*b_5/bc$$

$$b2_6 = ftot[1]*b_6/bc$$

$$b2_7 = ftot[1]*b_7/bc$$

$$b2_8 = ftot[1]*b_8/bc$$

$$b2_9 = ftot[1]*b_9/bc$$

$$b2_{10} = ftot[1]*b_{10}/bc$$

```
b2_11 = ftot[1]*b_11/bc
b2_12 = ftot[1]*b_12/bc
b2_13 = ftot[1]*b_13/bc
b2_14 = ftot[1]*b_14/bc
b2_15 = ftot[1]*b_15/bc
b2_16 = ftot[1]*b_16/bc
b2_17 = ftot[1]*b_17/bc
b2_18 = ftot[1]*b_18/bc
b2_19 = ftot[1]*b_19/bc
c3_2 = ftot[2]*c_2/cast
c3_3 = ftot[2]*c_3/cast
c3_4 = ftot[2]*c_4/cast
c3_5 = ftot[2]*c_5/cast
c3_6 = ftot[2]*c_6/cast
c3_11 = ftot[2]*c_11/cast
c3_12 = ftot[2]*c_12/cast
c3_13 = ftot[2]*c_13/cast
c3_14 = ftot[2]*c_14/cast
c3_16 = ftot[2]*c_16/cast
c3_18 = ftot[2]*c_18/cast
c3_19 = ftot[2]*c_19/cast
c3_20 = ftot[2]*c_20/cast
c3_24 = ftot[2]*c_24/cast
c3_25 = ftot[2]*c_25/cast
c3_1 = ftot[2]*c_1/cast
c3_7 = ftot[2]*c_7/cast
c3_8 = ftot[2]*c_8/cast
c3_9 = ftot[2]*c_9/cast
c3_22 = ftot[2]*c_22/cast
c3_10 = ftot[2]*c_10/cast
c3_23 = ftot[2]*c_23/cast
c3_26 = ftot[2]*c_26/cast
d4_1 = ftot[3]*d_1/vlt
d4_2 = ftot[3]*d_2/vlt
d4_3 = ftot[3]*d_3/vlt
d4_4 = ftot[3]*d_4/vlt
d4_5 = ftot[3]*d_5/vlt
d4_6 = ftot[3]*d_6/vlt
d4_7 = ftot[3]*d_7/vlt
d4_9 = ftot[3]*d_9/vlt
d4_10 = ftot[3]*d_10/vlt
d4_11 = ftot[3]*d_11/vlt
```



```

    ↪ +000,uu,uu,0.000E+000,1.000E+000, Y, RANDOM_
    ↪ _____\n".format( MyNumber(a1_14)
    ↪ ))
f.write("1-15_____1,_____
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,uu,uu,0.000E+000,1.000E+000, Y, RANDOM_
    ↪ _____\n".format( MyNumber(a1_15)
    ↪ ))
f.write("1-16_____1,_____
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,uu,uu,0.000E+000,1.000E+000, Y, RANDOM_
    ↪ _____\n".format( MyNumber(a1_16)
    ↪ ))
f.write("1-17_____1,_____
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,uu,uu,0.000E+000,1.000E+000, Y, RANDOM_
    ↪ _____\n".format( MyNumber(a1_17)
    ↪ ))
f.write("1-18_____1,_____
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,uu,uu,0.000E+000,1.000E+000, Y, RANDOM_
    ↪ _____\n".format( MyNumber(a1_18)
    ↪ ))
f.write("1-19_____1,_____
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,uu,uu,0.000E+000,1.000E+000, Y, RANDOM_
    ↪ _____\n".format( MyNumber(a1_19)
    ↪ ))
f.write("1-20_____1,_____
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,uu,uu,0.000E+000,1.000E+000, Y, RANDOM_
    ↪ _____\n".format( MyNumber(a1_2)
    ↪ )
f.write("1-20_____1,_____
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,uu,uu,0.000E+000,1.000E+000, Y, RANDOM_
    ↪ _____\n".format( MyNumber(a1_20)
    ↪ ))
f.write("1-21_____1,_____
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,uu,uu,0.000E+000,1.000E+000, Y, RANDOM_
    ↪ _____\n".format( MyNumber(a1_21)

```

```

    ↪ ))
f.write("1-22" + " " * 100 + "1" + " " * 100 + "
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,00,00,0.000E+000,1.000E+000,Y,RANDOM" + " " * 100 + "\n".format( MyNumber(a1_22)
    ↪ ))
f.write("1-23" + " " * 100 + "1" + " " * 100 + "
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,00,00,0.000E+000,1.000E+000,Y,RANDOM" + " " * 100 + "\n".format( MyNumber(a1_23)
    ↪ ))
f.write("1-24" + " " * 100 + "1" + " " * 100 + "
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,00,00,0.000E+000,1.000E+000,Y,RANDOM" + " " * 100 + "\n".format( MyNumber(a1_24)
    ↪ ))
f.write("1-25" + " " * 100 + "1" + " " * 100 + "
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,00,00,0.000E+000,1.000E+000,Y,RANDOM" + " " * 100 + "\n".format( MyNumber(a1_25)
    ↪ ))
f.write("1-26" + " " * 100 + "1" + " " * 100 + "
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,00,00,0.000E+000,1.000E+000,Y,RANDOM" + " " * 100 + "\n".format( MyNumber(a1_26)
    ↪ ))
f.write("1-27" + " " * 100 + "1" + " " * 100 + "
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,00,00,0.000E+000,1.000E+000,Y,RANDOM" + " " * 100 + "\n".format( MyNumber(a1_27)
    ↪ ))
f.write("1-28" + " " * 100 + "1" + " " * 100 + "
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,00,00,0.000E+000,1.000E+000,Y,RANDOM" + " " * 100 + "\n".format( MyNumber(a1_28)
    ↪ ))
f.write("1-29" + " " * 100 + "1" + " " * 100 + "
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,00,00,0.000E+000,1.000E+000,Y,RANDOM" + " " * 100 + "\n".format( MyNumber(a1_29)
    ↪ ))
f.write("1-30" + " " * 100 + "1" + " " * 100 + "

```

```

    ↪ 0.000E+000, {0:9.3e}, 0.000E+000, 0.000E+000, 0.000E
    ↪ +000, 0.000E+000, 1.000E+000, Y, RANDOM
    ↪ \n".format( MyNumber(a1_30)
    ↪ ))
f.write("1-34
    ↪ 0.000E+000, {0:9.3e}, 0.000E+000, 0.000E+000, 0.000E
    ↪ +000, 0.000E+000, 1.000E+000, Y, RANDOM
    ↪ \n".format( MyNumber(a1_34)
    ↪ ))
f.write("1-35
    ↪ 0.000E+000, {0:9.3e}, 0.000E+000, 0.000E+000, 0.000E
    ↪ +000, 0.000E+000, 1.000E+000, Y, RANDOM
    ↪ \n".format( MyNumber(a1_35)
    ↪ ))
f.write("1-36
    ↪ 0.000E+000, {0:9.3e}, 0.000E+000, 0.000E+000, 0.000E
    ↪ +000, 0.000E+000, 1.000E+000, Y, RANDOM
    ↪ \n".format( MyNumber(a1_36)
    ↪ ))
f.write("1-6
    ↪ 0.000E+000, {0:9.3e}, 0.000E+000, 0.000E+000, 0.000E
    ↪ +000, 0.000E+000, 1.000E+000, Y, RANDOM
    ↪ \n".format( MyNumber(a1_6)
    ↪ ))
f.write("1-7
    ↪ 0.000E+000, {0:9.3e}, 0.000E+000, 0.000E+000, 0.000E
    ↪ +000, 0.000E+000, 1.000E+000, Y, RANDOM
    ↪ \n".format( MyNumber(a1_7)
    ↪ ))
f.write("1-8
    ↪ 0.000E+000, {0:9.3e}, 0.000E+000, 0.000E+000, 0.000E
    ↪ +000, 0.000E+000, 1.000E+000, Y, RANDOM
    ↪ \n".format( MyNumber(a1_8)
    ↪ ))
f.write("1-9
    ↪ 0.000E+000, {0:9.3e}, 0.000E+000, 0.000E+000, 0.000E
    ↪ +000, 0.000E+000, 1.000E+000, Y, RANDOM
    ↪ \n".format( MyNumber(a1_9)
    ↪ ))
f.write("2-1
    ↪ 0.000E+000, {0:9.3e}, 0.000E+000, 0.000E+000, 0.000E
    ↪ +000, 0.000E+000, 1.000E+000, Y, RANDOM

```

```

    ↪      \n".format( MyNumber(b2_1))
    ↪  )
f.write("2-10",1,1,
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,0.000E+000,1.000E+000,Y,RANDOM
    ↪      \n".format( MyNumber(b2_10))
    ↪  ))
f.write("2-11",1,1,
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,0.000E+000,1.000E+000,Y,RANDOM
    ↪      \n".format( MyNumber(b2_11))
    ↪  ))
f.write("2-12",1,1,
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,0.000E+000,1.000E+000,Y,RANDOM
    ↪      \n".format( MyNumber(b2_12))
    ↪  ))
f.write("2-13",1,1,
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,0.000E+000,1.000E+000,Y,RANDOM
    ↪      \n".format( MyNumber(b2_13))
    ↪  ))
f.write("2-14",1,1,
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,0.000E+000,1.000E+000,Y,RANDOM
    ↪      \n".format( MyNumber(b2_14))
    ↪  ))
f.write("2-15",1,1,
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,0.000E+000,1.000E+000,Y,RANDOM
    ↪      \n".format( MyNumber(b2_15))
    ↪  ))
f.write("2-16",1,1,
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,0.000E+000,1.000E+000,Y,RANDOM
    ↪      \n".format( MyNumber(b2_16))
    ↪  ))
f.write("2-17",1,1,
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,0.000E+000,1.000E+000,Y,RANDOM
    ↪      \n".format( MyNumber(b2_17))
    ↪  ))

```





```

    ↪ ))
f.write("3-19" + " " * 100 + "1" + " " * 100 + "
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,00,00,0.000E+000,1.000E+000,Y,RANDOM" + " " * 100 + "\n".format( MyNumber(c3_19)
    ↪ ))
f.write("3-2" + " " * 100 + "1" + " " * 100 + "
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,00,00,0.000E+000,1.000E+000,Y,RANDOM" + " " * 100 + "\n".format( MyNumber(c3_2)
    ↪ ))
f.write("3-20" + " " * 100 + "1" + " " * 100 + "
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,00,00,0.000E+000,1.000E+000,Y,RANDOM" + " " * 100 + "\n".format( MyNumber(c3_20)
    ↪ ))
f.write("3-22" + " " * 100 + "1" + " " * 100 + "
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,00,00,0.000E+000,1.000E+000,Y,RANDOM" + " " * 100 + "\n".format( MyNumber(c3_22)
    ↪ ))
f.write("3-23" + " " * 100 + "1" + " " * 100 + "
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,00,00,0.000E+000,1.000E+000,Y,RANDOM" + " " * 100 + "\n".format( MyNumber(c3_23)
    ↪ ))
f.write("3-24" + " " * 100 + "1" + " " * 100 + "
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,00,00,0.000E+000,1.000E+000,Y,RANDOM" + " " * 100 + "\n".format( MyNumber(c3_24)
    ↪ ))
f.write("3-25" + " " * 100 + "1" + " " * 100 + "
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,00,00,0.000E+000,1.000E+000,Y,RANDOM" + " " * 100 + "\n".format( MyNumber(c3_25)
    ↪ ))
f.write("3-26" + " " * 100 + "1" + " " * 100 + "
    ↪ 0.000E+000,{0:9.3e},0.000E+000,0.000E+000,0.000E
    ↪ +000,00,00,0.000E+000,1.000E+000,Y,RANDOM" + " " * 100 + "\n".format( MyNumber(c3_26)
    ↪ ))
f.write("3-3" + " " * 100 + "1" + " " * 100 + "

```

```

    ↪ 0.000E+000, {0:9.3e}, 0.000E+000, 0.000E+000, 0.000E
    ↪ +000, 0.000E+000, 1.000E+000, Y, RANDOM
    ↪ \n".format( MyNumber(c3_3))
    ↪ )
f.write("3-4
    ↪ 0.000E+000, {0:9.3e}, 0.000E+000, 0.000E+000, 0.000E
    ↪ +000, 0.000E+000, 1.000E+000, Y, RANDOM
    ↪ \n".format( MyNumber(c3_4))
    ↪ )
f.write("3-5
    ↪ 0.000E+000, {0:9.3e}, 0.000E+000, 0.000E+000, 0.000E
    ↪ +000, 0.000E+000, 1.000E+000, Y, RANDOM
    ↪ \n".format( MyNumber(c3_5))
    ↪ )
f.write("3-6
    ↪ 0.000E+000, {0:9.3e}, 0.000E+000, 0.000E+000, 0.000E
    ↪ +000, 0.000E+000, 1.000E+000, Y, RANDOM
    ↪ \n".format( MyNumber(c3_6))
    ↪ )
f.write("3-7
    ↪ 0.000E+000, {0:9.3e}, 0.000E+000, 0.000E+000, 0.000E
    ↪ +000, 0.000E+000, 1.000E+000, Y, RANDOM
    ↪ \n".format( MyNumber(c3_7))
    ↪ )
f.write("3-8
    ↪ 0.000E+000, {0:9.3e}, 0.000E+000, 0.000E+000, 0.000E
    ↪ +000, 0.000E+000, 1.000E+000, Y, RANDOM
    ↪ \n".format( MyNumber(c3_8))
    ↪ )
f.write("3-9
    ↪ 0.000E+000, {0:9.3e}, 0.000E+000, 0.000E+000, 0.000E
    ↪ +000, 0.000E+000, 1.000E+000, Y, RANDOM
    ↪ \n".format( MyNumber(c3_9))
    ↪ )
f.write("4-1
    ↪ 0.000E+000, {0:9.3e}, 0.000E+000, 0.000E+000, 0.000E
    ↪ +000, 0.000E+000, 1.000E+000, Y, RANDOM
    ↪ \n".format( MyNumber(d4_1))
    ↪ )
f.write("4-10
    ↪ 0.000E+000, {0:9.3e}, 0.000E+000, 0.000E+000, 0.000E
    ↪ +000, 0.000E+000, 1.000E+000, Y, RANDOM

```





```

else:
    headercheck_1 = headercheck_0[headercheck_2][0]

if choice2 == 3:
    with open('data.csv', mode='ab') as csv_file:
        fieldnames = ['input_prob', 'distrib', 'mean', 'std', '
            ↪ area']
        writer = csv.DictWriter(csv_file, fieldnames=fieldnames)
        if headercheck_1 != 'input_prob':
            writer.writeheader()
        writer.writerow({'input_prob': ftot[0]*100, 'distrib':
            ↪ unifvar, 'mean': mean, 'std': std, 'area': 'REC'})
        writer.writerow({'input_prob': ftot[1]*100, 'distrib':
            ↪ unifvar, 'mean': mean, 'std': std, 'area': 'BC'})
        writer.writerow({'input_prob': ftot[2]*100, 'distrib':
            ↪ unifvar, 'mean': mean, 'std': std, 'area': 'CAST'
            ↪ })
        writer.writerow({'input_prob': ftot[3]*100, 'distrib':
            ↪ unifvar, 'mean': mean, 'std': std, 'area': 'VLT'})

if choice2 == 4:
    with open('data.csv', mode='ab') as csv_file:
        fieldnames = ['input_prob', 'distrib', 'scale', 'area']
        writer = csv.DictWriter(csv_file, fieldnames=fieldnames)
        if headercheck_1 != 'input_prob':
            writer.writeheader()
        writer.writerow({'input_prob': ftot[0]*100, 'distrib':
            ↪ unifvar, 'scale': beta, 'area': 'REC'})
        writer.writerow({'input_prob': ftot[1]*100, 'distrib':
            ↪ unifvar, 'scale': beta, 'area': 'BC'})
        writer.writerow({'input_prob': ftot[2]*100, 'distrib':
            ↪ unifvar, 'scale': beta, 'area': 'CAST'})
        writer.writerow({'input_prob': ftot[3]*100, 'distrib':
            ↪ unifvar, 'scale': beta, 'area': 'VLT'})

#
↪ #####
↪
#
↪ #####
↪

```

```

#loop for entire code
repeatchoice = 'y'
t = 0
while repeatchoice == 'y':
    #calculation variables
    # choices()
    # number of files
    numrepeat = 1000
    # uniform 1 or variable 2
    choice1 = 2
    # user input 1, uniform 2, gaussian 3, exp 4
    choice2 = 3
    mean = 5
    std = 1
    beta = 5
    #uniform or nonuniform probabilities
    for t in range(numrepeat):
        ftot = numpy.zeros(4)
        area = numpy.array(['REC', 'BC', 'CAST', 'VLT'], dtype='|S20
            ↪ ')
        if choice1 == 1:
            output1 = 'Uniform_probabilities'
            disttype = 'Uniform'
            #change "oparea" name for user input
            area[0] = "the_whole_system"
            userinput(0, area)
            #uniform distribution
            ftot = [ftot[0], ftot[0], ftot[0], ftot[0]]
        elif choice1 == 2:
            output1 = 'Variable_probabilities'
            disttype = 'Variable'
            userinput(3, area)
            csvfile(disttype)
            outputfile()
        t = t+1
    # os.rename("{}_{}.txt".format(output1, output2), "{}_{}_{}_{}
        ↪ inputs.txt".format(output1, output2, t))
        if t == numrepeat:
            t = 0
            repeatchoice = 'n'
    # repeatchoice = raw_input("Repeat? y/n \n")

```

```

import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
import numpy as np
import scipy.stats as ss

def plot_exponential(x_range, mu=0, sigma=1, cdf=False, **kwargs):
    '''
    Plots the exponential distribution function for a given x range
    If mu and sigma are not provided, standard exponential is
    → plotted
    If cdf=True cumulative distribution is plotted
    Passes any keyword arguments to matplotlib plot function
    '''
    x = x_range
    if cdf:
        y = ss.expon.cdf(x, mu, sigma)
    else:
        y = ss.expon.pdf(x, mu, sigma)
    plt.plot(x, y, **kwargs)

def plot_normal(x_range, mu=0, sigma=1, cdf=False, **kwargs):
    '''
    Plots the normal distribution function for a given x range
    If mu and sigma are not provided, standard normal is plotted
    If cdf=True cumulative distribution is plotted
    Passes any keyword arguments to matplotlib plot function
    '''
    x = x_range
    a, b = (0 - mu) / sigma, (10 - mu) / sigma
    if cdf:
        y = ss.norm.cdf(x, mu, sigma)
    else:
        y = ss.truncnorm.pdf(x, a, b, mu, sigma)
    plt.plot(x, y, **kwargs)

#bin range
binwidth = 0.2
nbins = np.arange(0, 10, binwidth)

```

```

binmidpt = np.arange(binwidth/2, 10-binwidth, binwidth)

# read in data
#data = pd.read_csv('data.csv')
#data = sns.

# lists
#distributions = ['Uniform', 'Variable']
#scales = [1, 2, 3, 4, 5]
#means = [0, 5, 10]
#stds = [1, 2, 3]
distributions = ['Variable']
scales = [1, 2, 3]
means = [0, 1]
stds = [1, 2, 3]
areas = ['REC', 'BC', 'CAST', 'VLT']

# check for exp or gauss
#while True:
# try:
# dist_type = int(raw_input("Gaussian (1), or Exponential (2)
#     ↪ distribution? "))
# except ValueError:
# print("Invalid input, try again.")
# continue
# if dist_type != 1 and dist_type != 2:
# print("Invalid input, try again.")
# continue
# else:
# break
dist_type = 1

# seaborn histogram
# gaussian
data = pd.read_csv('gaussdata.csv')
for distrib_type in distributions:
    plot_0 = data[data['distrib'] == distrib_type]
    for mean_type in means:
        plot_1 = plot_0[plot_0['mean'] == mean_type]
        for area_type in areas:
            plot_2 = plot_1[plot_1['area'] == area_type]
            for std_type in stds:

```

```

plot_3 = plot_2[plot_2['std'] == std_type]
n, bins, patches = plt.hist(plot_3['input_prob'],
    → bins=nbins, density=True) ## creates histogram
    → array for the midpoint curve
plt.close() ## deletes the plot so it's not mixed
    → with seaborn
sns.distplot(plot_3['input_prob'], hist=True, kde=
    → False, bins=nbins, color = 'darkblue',
    → norm_hist = True, hist_kws={'edgecolor':'black'
    → })
plot_normal(nbins, mean_type, std_type, color='red',
    → lw=2, ls='--', alpha=0.5, label='pdf')
plt.plot(binmidpt, n, linestyle = '--', linewidth =
    → 1)
if distrib_type == 'Uniform':
    title_1 = 'Uniformly'
elif distrib_type == 'Variable':
    title_1 = 'Variably'
else:
    title_1 = 'ERROR'
plt.title('{ } Generated Failure Probabilities for { }
    → (Gaussian,  $\mu$  = { },  $\sigma$  = { })'.format(
    → title_1, area_type, mean_type, std_type))
plt.xlabel('Probability of Failure (%)')
plt.ylabel('Probability Density')
plt.xlim(0, 10)
#plt.ylim(0, 1)
#plt.show()
plt.savefig('Graphs\\Data\\Gauss_Mean_{ }Std_{ }_{ }'.
    → format(mean_type, std_type, area_type), dpi
    → =300, bbox_inches = 'tight')
plt.close()

```

*# exponential*

```

data = pd.read_csv('expdata.csv')
for distrib_type in distributions:
    plot_0 = data[data['distrib'] == distrib_type]
    for scale_type in scales:
        plot_1 = plot_0[plot_0['scale'] == scale_type]
        for area_type in areas:
            plot_2 = plot_1[plot_1['area'] == area_type]
            n, bins, patches = plt.hist(plot_2['input_prob'], bins=

```

```

    ↪ nbins, density=True) ## creates histogram array
    ↪ for the midpoint curve
plt.close() ## deletes the plot so it's not mixed with
    ↪ seaborn
sns.distplot(plot_2['input_prob'], hist=True, kde=False,
    ↪ bins=nbins, color = 'darkblue', norm_hist = True,
    ↪ hist_kws={'edgecolor':'black'})
plot_exponential(nbins, 0, scale_type, color='red', lw=2,
    ↪ ls='--', alpha=0.5, label='pdf')
plt.plot(binmidpt, n, linestyle = '--', linewidth = 1)

if distrib_type == 'Uniform':
    title_1 = 'Uniformly'
elif distrib_type == 'Variable':
    title_1 = 'Variably'
else:
    title_1 = 'ERROR'
plt.title('{ }Generated Failure Probabilities for { }(
    ↪ Exponential,  $\beta = \{ \}$ )'.format(title_1,
    ↪ area_type, scale_type))
plt.xlabel('Probability of Failure (%)')
plt.ylabel('Probability Density')
plt.xlim(0, 10)
# plt.ylim(0, 1)
    ↪ plt.show()
plt.savefig('Graphs\\Data\\Exp#{ }'.format(scale_type,
    ↪ area_type), dpi=300, bbox_inches = 'tight')
plt.close()

```

```

import pandas as pd
import numpy as np
import os
import seaborn as sns
import matplotlib.pyplot as plt
#from scipy import stats
#import xlrd

def plot_data(inputdata, xlen, ylen, loc, xlim1, xlim2):
    global verifycheck
    f, ax = plt.subplots(figsize = (xlen, ylen))
    ax.set_xscale('log')
    sns.set(style = 'whitegrid')
    sns.barplot(x = 'Prob/Freq', y = 'Cut_Set', data = inputdata,
        ↪ order = result['Cut_Set'])
    plt.title('{}'.format(plttitle[i]))
    plt.xlabel('Probability_of_Occurrence')
    plt.ylabel('Cut_Set')
    plt.xlim(xlim1, xlim2)
    plt.savefig('Graphs\\{}-{}.png'.format(loc, plotsave[i]),
        ↪ bbox_inches = 'tight')
    plt.close()
# verifycheck = inputdata.groupby(['Cut Set'])['Prob/Freq'].
    ↪ aggregate(np.mean).reset_index().sort_values('Cut Set')
# verifycheck.to_csv('Graphs\\{}\\{}.csv'.format(loc, plotsave[i])
    ↪ )

zoom = 1

folders = ['B1', 'B2', 'B3', 'mean_0_std_1', 'mean_0_std_2', 'mean_0
    ↪ _std_3', 'mean_1_std_1', 'mean_1_std_2', 'mean_1_std_3']
plttitle = ['Exponential_Distribution_($\beta$=1)', 'Exponential_
    ↪ Distribution_($\beta$=2)', 'Exponential_Distribution_($\
    ↪ \beta$=3)',

```

```

        'Gaussian_Distribution_(Mean_0_Std_1)', 'Gaussian_
        ↪ Distribution_(Mean_0_Std_2)', 'Gaussian_
        ↪ Distribution_(Mean_0_Std_3)',
        'Gaussian_Distribution_(Mean_1_Std_1)', 'Gaussian_
        ↪ Distribution_(Mean_1_Std_2)', 'Gaussian_
        ↪ Distribution_(Mean_1_Std_3)']
plotsave = ['Exp-B1', 'Exp-B2', 'Exp-B3',
            'Gauss-MOS1', 'Gauss-MOS2', 'Gauss-MOS3',
            'Gauss-M1S1', 'Gauss-M1S2', 'Gauss-M1S3']
i = 0

for foldername in folders:
    src = 'Results\\{}'.format(foldername)
    filelist = os.listdir(src)
    outputdf = pd.DataFrame(columns = ('Prob/Freq', 'Cut_Set'))

    for filename in filelist:
# wb = xlrd.open_workbook('{}\{}'.format(src, filename), logfile=
    ↪ open(os.devnull, 'w'))
# df = pd.read_excel(wb, header = 3)
        df = pd.read_excel('{}\{}'.format(src, filename), header =
        ↪ 3)
        df = df.drop([0, 1], axis = 0)
        df = df.drop(['#', 'Case', 'Total_%'], axis = 1);
        outputdf = df.append(outputdf)

        result = outputdf.groupby(['Cut_Set'])['Prob/Freq'].aggregate(np
        ↪ .mean).reset_index().sort_values('Prob/Freq')

        #overall
# plt.xlim(.00000001, 1)
        plot_data(outputdf, 60, 40, 'Overall', .00000001, 1)
        #largest
        result = outputdf.groupby(['Cut_Set'])['Prob/Freq'].aggregate(np
        ↪ .mean).reset_index().sort_values('Prob/Freq')
        result = result.iloc[156:].reset_index()
        checklist = result['Cut_Set'].tolist()
        outputdf1 = outputdf[outputdf['Cut_Set'].isin(checklist)]
# plt.xlim(.001, .1)
        plot_data(outputdf1, 15, 10, 'Largest', .0001, .1)
        #smallest
        result = outputdf.groupby(['Cut_Set'])['Prob/Freq'].aggregate(np

```

```

    → .mean).reset_index().sort_values('Prob/Freq')
result = result.iloc[:10].reset_index()
checklist = result['Cut_Set'].tolist()
outputdf2 = outputdf[outputdf['Cut_Set'].isin(checklist)]
# plt.xlim(.00000001, .000001)
plot_data(outputdf2, 15, 10, 'Smallest', .00000001, .001)
#no mba
outputdf3 = outputdf[~outputdf['Cut_Set'].str.contains('MBA')]
result = outputdf3.groupby(['Cut_Set'])['Prob/Freq'].aggregate(
    → np.mean).reset_index().sort_values('Prob/Freq')
# plt.xlim(.00000001, 1)
plot_data(outputdf3, 30, 20, 'Overall-No-MBA', .00000001, 1)
#by area
conditions = [(outputdf['Cut_Set'].str.contains('1-')), (
    → outputdf['Cut_Set'].str.contains('REC')),
               (outputdf['Cut_Set'].str.contains('2-')), (outputdf
    → ['Cut_Set'].str.contains('BC')),
               (outputdf['Cut_Set'].str.contains('3-')), (outputdf
    → ['Cut_Set'].str.contains('CAST')),
               (outputdf['Cut_Set'].str.contains('4-')), (outputdf
    → ['Cut_Set'].str.contains('VLT'))]
areas = ['REC', 'REC', 'BC', 'BC', 'CAST', 'CAST', 'VLT', 'VLT']
outputdf4 = outputdf
outputdf4['Cut_Set'] = np.select(conditions, areas, default = '
    → ERROR')
result = outputdf4.groupby(['Cut_Set'])['Prob/Freq'].aggregate(
    → np.mean).reset_index().sort_values('Prob/Freq')
checklist = result['Cut_Set'].tolist()
outputdf4 = outputdf[outputdf['Cut_Set'].isin(checklist)]
# plt.xlim(.001, .01)
plot_data(outputdf4, 30, 20, 'By-Area', .0001, .01)

#largest
result = outputdf3.groupby(['Cut_Set'])['Prob/Freq'].aggregate(
    → np.mean).reset_index().sort_values('Prob/Freq')
result = result.iloc[71:].reset_index()
checklist = result['Cut_Set'].tolist()
outputdf5 = outputdf3[outputdf3['Cut_Set'].isin(checklist)]
# plt.xlim(.0001, .01)
plot_data(outputdf5, 15, 10, 'Largest-No-MBA', .0001, .1)
#smallest

```

```

result = outputdf3.groupby(['Cut_Set'])['Prob/Freq'].aggregate(
    ↪ np.mean).reset_index().sort_values('Prob/Freq')
result = result.iloc[:10].reset_index()
checklist = result['Cut_Set'].tolist()
outputdf6 = outputdf3[outputdf3['Cut_Set'].isin(checklist)]
# plt.xlim(.00001, .001)
plot_data(outputdf6, 15, 10, 'Smallest-No-MBA', .00000001, .001)
#by area
conditions = [(outputdf3['Cut_Set'].str.contains('1-')), (
    ↪ outputdf3['Cut_Set'].str.contains('REC')),
              (outputdf3['Cut_Set'].str.contains('2-')), (
    ↪ outputdf3['Cut_Set'].str.contains('BC')),
              (outputdf3['Cut_Set'].str.contains('3-')), (
    ↪ outputdf3['Cut_Set'].str.contains('CAST')),
              (outputdf3['Cut_Set'].str.contains('4-')), (
    ↪ outputdf3['Cut_Set'].str.contains('VLT'))]
areas = ['REC', 'REC', 'BC', 'BC', 'CAST', 'CAST', 'VLT', 'VLT']
outputdf7 = outputdf3
outputdf7['Cut_Set'] = np.select(conditions, areas, default = '
    ↪ ERROR')
result = outputdf7.groupby(['Cut_Set'])['Prob/Freq'].aggregate(
    ↪ np.mean).reset_index().sort_values('Prob/Freq')
checklist = result['Cut_Set'].tolist()
outputdf7 = outputdf3[outputdf3['Cut_Set'].isin(checklist)]
# plt.xlim(.0001, .01)
plot_data(outputdf7, 30, 20, 'By-Area-No-MBA', .0001, .01)

i = i+1
#df.to_csv('output.csv', )

```

```
##code variables
#number of operational areas from [0,sets]
sets = 3

#REC
a_1 = 0.069056
a_2 = 0.069056
a_6 = 0.055245
a_7 = 0.048339
a_8 = 0.041434
a_9 = 0.041434
a_10 = 0.041434
a_11 = 0.041434
a_12 = 0.041434
a_13 = 0.034528
a_14 = 0.033147
a_15 = 0.027622
a_16 = 0.020717
a_17 = 0.048339
a_18 = 0.048339
a_19 = 0.048339
a_20 = 0.017264
a_21 = 0.017264
a_22 = 0.015123
a_23 = 0.015123
a_24 = 0.015123
a_25 = 0.015123
a_26 = 0.013811
a_27 = 0.012085
a_28 = 0.012085
a_29 = 0.012085
a_30 = 0.008287
a_34 = 0.055245
a_35 = 0.048339
a_36 = 0.033147

#BC
b_1 = 0.084926
b_2 = 0.084926
b_4 = 0.074310
b_5 = 0.074310
```

```
b_6 = 0.074310  
b_7 = 0.074310  
b_8 = 0.074310  
b_9 = 0.067941  
b_10 = 0.067941  
b_11 = 0.059448  
b_12 = 0.050955  
b_13 = 0.042463  
b_14 = 0.033970  
b_15 = 0.025478  
b_16 = 0.025478  
b_17 = 0.021231  
b_18 = 0.012739  
b_19 = 0.050955
```

*#CAST*

```
c_2 = 0.096953  
c_3 = 0.096953  
c_4 = 0.096953  
c_5 = 0.096953  
c_6 = 0.096953  
c_11 = 0.066482  
c_12 = 0.066482  
c_13 = 0.066482  
c_14 = 0.066482  
c_16 = 0.055402  
c_18 = 0.044321  
c_19 = 0.033241  
c_20 = 0.033241  
c_24 = 0.016620  
c_25 = 0.066482  
c_1 = 0.347041  
c_7 = 0.091098  
c_8 = 0.242929  
c_9 = 0.242929  
c_22 = 0.076002  
c_10 = 0.444444  
c_23 = 0.111111  
c_26 = 0.444444
```

*#VLT*

```
d_1 = 0.115274
```

```
d_2 = 0.115274  
d_3 = 0.100865  
d_4 = 0.100865  
d_5 = 0.100865  
d_6 = 0.100865  
d_7 = 0.100865  
d_9 = 0.092219  
d_10 = 0.092219  
d_11 = 0.080692
```

**E SAPHIRE Documents**

# HRA Worksheets for At-Power

## SPAR HUMAN ERROR WORKSHEET

Plant: \_\_\_\_\_ Initiating Event: \_\_\_\_\_ Basic Event : \_\_\_\_\_ Event Coder: \_\_\_\_\_

Basic Event Context: \_\_\_\_\_

Basic Event Description: \_\_\_\_\_

Does this task contain a significant amount of diagnosis activity? YES  (start with Part I–Diagnosis) NO  (skip Part I – Diagnosis; start with Part II – Action) Why? \_\_\_\_\_

### PART I. EVALUATE EACH PSF FOR DIAGNOSIS

#### A. Evaluate PSFs for the Diagnosis Portion of the Task, If Any.

PSFs	PSF Levels	Multiplier for Diagnosis	Please note specific reasons for PSF level selection in this column.
Available Time	Inadequate time	P(failure) = 1.0 <input type="checkbox"/>	
	Barely adequate time ( $\approx 2/3$ x nominal)	10 <input type="checkbox"/>	
	Nominal time	1 <input type="checkbox"/>	
	Extra time (between 1 and 2 x nominal and > than 30 min)	0.1 <input type="checkbox"/>	
	Expansive time (> 2 x nominal and > 30 min)	0.01 <input type="checkbox"/>	
	Insufficient information	1 <input type="checkbox"/>	
Stress/ Stressors	Extreme	5 <input type="checkbox"/>	
	High	2 <input type="checkbox"/>	
	Nominal	1 <input type="checkbox"/>	
	Insufficient Information	1 <input type="checkbox"/>	
Complexity	Highly complex	5 <input type="checkbox"/>	
	Moderately complex	2 <input type="checkbox"/>	
	Nominal	1 <input type="checkbox"/>	
	Obvious diagnosis	0.1 <input type="checkbox"/>	
	Insufficient Information	1 <input type="checkbox"/>	
Experience/ Training	Low	10 <input type="checkbox"/>	
	Nominal	1 <input type="checkbox"/>	
	High	0.5 <input type="checkbox"/>	
	Insufficient Information	1 <input type="checkbox"/>	
Procedures	Not available	50 <input type="checkbox"/>	
	Incomplete	20 <input type="checkbox"/>	
	Available, but poor	5 <input type="checkbox"/>	
	Nominal	1 <input type="checkbox"/>	
	Diagnostic/symptom oriented	0.5 <input type="checkbox"/>	
	Insufficient Information	1 <input type="checkbox"/>	
Ergonomics/ HMI	Missing/Misleading	50 <input type="checkbox"/>	
	Poor	10 <input type="checkbox"/>	
	Nominal	1 <input type="checkbox"/>	
	Good	0.5 <input type="checkbox"/>	
	Insufficient Information	1 <input type="checkbox"/>	
Fitness for Duty	Unfit	P(failure) = 1.0 <input type="checkbox"/>	
	Degraded Fitness	5 <input type="checkbox"/>	
	Nominal	1 <input type="checkbox"/>	
	Insufficient Information	1 <input type="checkbox"/>	
Work Processes	Poor	2 <input type="checkbox"/>	
	Nominal	1 <input type="checkbox"/>	
	Good	0.8 <input type="checkbox"/>	
	Insufficient Information	1 <input type="checkbox"/>	

Rev 1 (1/20/04)

Reviewer: \_\_\_\_\_

Plant: \_\_\_\_\_ Initiating Event: \_\_\_\_\_ Basic Event : \_\_\_\_\_ Event Coder: \_\_\_\_\_

Basic Event Context: \_\_\_\_\_

Basic Event Description: \_\_\_\_\_

**B. Calculate the Diagnosis Failure Probability.**

- (1) If all PSF ratings are nominal, then the Diagnosis Failure Probability = 1.0E-2
- (2) Otherwise, the Diagnosis Failure Probability is: 1.0E-2 x Time x Stress or Stressors x Complexity x Experience or Training x Procedures x Ergonomics or HMI x Fitness for Duty x Processes

Diagnosis: 1.0E-2x \_\_\_\_\_ x \_\_\_\_\_ =

**C. Calculate the Adjustment Factor IF Negative Multiple (≥3) PSFs are Present.**

When 3 or more negative PSF influences are present, in lieu of the equation above, you must compute a composite PSF score used in conjunction with the adjustment factor. Negative PSFs are present anytime a multiplier greater than 1 is selected. The Nominal HEP (NHEP) is 1.0E-2 for Diagnosis. The composite PSF score is computed by multiplying all the assigned PSF values. Then the adjustment factor below is applied to compute the HEP:

$$HEP = \frac{NHEP \cdot PSF_{composite}}{NHEP \cdot (PSF_{composite} - 1) + 1}$$

Diagnosis HEP with Adjustment Factor =

**D. Record Final Diagnosis HEP.**

If no adjustment factor was applied, record the value from Part B as your final diagnosis HEP. If an adjustment factor was applied, record the value from Part C.

Final Diagnosis HEP =

Reviewer: \_\_\_\_\_

Plant: \_\_\_\_\_ Initiating Event: \_\_\_\_\_ Basic Event : \_\_\_\_\_ Event Coder: \_\_\_\_\_

Basic Event Context: \_\_\_\_\_

Basic Event Description: \_\_\_\_\_

**Part II. EVALUATE EACH PSF FOR ACTION**

**A. Evaluate PSFs for the Action Portion of the Task, If Any.**

PSFs	PSF Levels	Multiplier for Action	Please note specific reasons for PSF level selection in this column.
Available Time	Inadequate time	P(failure) = 1.0 <input type="checkbox"/>	
	Time available is ≈ the time required	10 <input type="checkbox"/>	
	Nominal time	1 <input type="checkbox"/>	
	Time available ≥ 5x the time required	0.1 <input type="checkbox"/>	
	Time available is ≥ 50x the time required	0.01 <input type="checkbox"/>	
	Insufficient Information	1 <input type="checkbox"/>	
Stress/ Stressors	Extreme	5 <input type="checkbox"/>	
	High	2 <input type="checkbox"/>	
	Nominal	1 <input type="checkbox"/>	
	Insufficient Information	1 <input type="checkbox"/>	
Complexity	Highly complex	5 <input type="checkbox"/>	
	Moderately complex	2 <input type="checkbox"/>	
	Nominal	1 <input type="checkbox"/>	
	Insufficient Information	1 <input type="checkbox"/>	
Experience/ Training	Low	3 <input type="checkbox"/>	
	Nominal	1 <input type="checkbox"/>	
	High	0.5 <input type="checkbox"/>	
	Insufficient Information	1 <input type="checkbox"/>	
Procedures	Not available	50 <input type="checkbox"/>	
	Incomplete	20 <input type="checkbox"/>	
	Available, but poor	5 <input type="checkbox"/>	
	Nominal	1 <input type="checkbox"/>	
	Insufficient Information	1 <input type="checkbox"/>	
Ergonomics/ HMI	Missing/Misleading	50 <input type="checkbox"/>	
	Poor	10 <input type="checkbox"/>	
	Nominal	1 <input type="checkbox"/>	
	Good	0.5 <input type="checkbox"/>	
	Insufficient Information	1 <input type="checkbox"/>	
Fitness for Duty	Unfit	P(failure) = 1.0 <input type="checkbox"/>	
	Degraded Fitness	5 <input type="checkbox"/>	
	Nominal	1 <input type="checkbox"/>	
	Insufficient Information	1 <input type="checkbox"/>	
Work Processes	Poor	5 <input type="checkbox"/>	
	Nominal	1 <input type="checkbox"/>	
	Good	0.5 <input type="checkbox"/>	
	Insufficient Information	1 <input type="checkbox"/>	

Reviewer: \_\_\_\_\_

Plant: \_\_\_\_\_ Initiating Event: \_\_\_\_\_ Basic Event : \_\_\_\_\_ Event Coder: \_\_\_\_\_

Basic Event Context: \_\_\_\_\_

Basic Event Description: \_\_\_\_\_

**B. Calculate the Action Failure Probability.**

(1) If all PSF ratings are nominal, then the Action Failure Probability = 1.0E-3

(2) Otherwise, the Action Failure Probability is: 1.0E-3 x Time x Stress or Stressors x Complexity x Experience or Training x Procedures x Ergonomics or HMI x Fitness for Duty x Processes

Action: 1.0E-3x \_\_\_\_\_ x \_\_\_\_\_ =

**C. Calculate the Adjustment Factor IF Negative Multiple ( $\geq 3$ ) PSFs are Present.**

When 3 or more negative PSF influences are present, in lieu of the equation above, you must compute a composite PSF score used in conjunction with the adjustment factor. Negative PSFs are present anytime a multiplier greater than 1 is selected. The Nominal HEP (NHEP) is 1.0E-3 for Action. The composite PSF score is computed by multiplying all the assigned PSF values. Then the adjustment factor below is applied to compute the HEP:

$$HEP = \frac{NHEP \cdot PSF_{composite}}{NHEP \cdot (PSF_{composite} - 1) + 1}$$

Action HEP with Adjustment Factor =

**D. Record Final Action HEP.**

If no adjustment factor was applied, record the value from Part B as your final action HEP. If an adjustment factor was applied, record the value from Part C.

Final Action HEP =

Reviewer: \_\_\_\_\_

Plant: \_\_\_\_\_ Initiating Event: \_\_\_\_\_ Basic Event : \_\_\_\_\_ Event Coder: \_\_\_\_\_

Basic Event Context: \_\_\_\_\_

Basic Event Description: \_\_\_\_\_

**PART III. CALCULATE TASK FAILURE PROBABILITY WITHOUT FORMAL DEPENDENCE (P<sub>w/od</sub>)**

Calculate the Task Failure Probability Without Formal Dependence (P<sub>w/od</sub>) by adding the Diagnosis Failure Probability from Part I and the Action Failure Probability from Part II. In instances where an action is required without a diagnosis and there is no dependency, then this step is omitted.

$$P_{w/od} = \text{Diagnosis HEP} \quad \underline{\hspace{2cm}} \quad + \text{Action HEP} \quad \underline{\hspace{2cm}} \quad = \quad \boxed{\hspace{2cm}}$$

**Part IV. DEPENDENCY**

For all tasks, except the first task in the sequence, use the table and formulae below to calculate the Task Failure Probability With Formal Dependence (P<sub>w/d</sub>).

If there is a reason why failure on previous tasks should not be considered, such as it is impossible to take the current action unless the previous action has been properly performed, explain here: \_\_\_\_\_

**Dependency Condition Table**

Condition Number	Crew (same or different)	Time (close in time or not close in time)	Location (same or different)	Cues (additional or no additional)	Dependency	Number of Human Action Failures Rule <input type="checkbox"/> - Not Applicable. Why? _____	
1	s	c	s	na	complete	When considering recovery in a series e.g., 2 <sup>nd</sup> , 3 <sup>rd</sup> , or 4 <sup>th</sup> checker  If this error is the <b>3rd error in the sequence</b> , then the dependency is at least <b>moderate</b> .  If this error is the <b>4th error in the sequence</b> , then the dependency is at least <b>high</b> .	
2				a	complete		
3			d	na	high		
4				a	high		
5	nc	s	na	high			
6			a	moderate			
7			d	na	moderate		
8				a	low		
9				na	moderate		
10			d	c	a		moderate
11					na		moderate
12	a	moderate					
13	nc	s			na		low
14					a		low
15			d	na	low		
16	a	low					
17					zero		

Using P<sub>w/od</sub> = Probability of Task Failure Without Formal Dependence (calculated in Part III):

- For Complete Dependence the probability of failure is 1.
- For High Dependence the probability of failure is (1+ P<sub>w/od</sub>)/2
- For Moderate Dependence the probability of failure is (1+6 x P<sub>w/od</sub>)/7
- For Low Dependence the probability of failure is (1+19 x P<sub>w/od</sub>)/20
- For Zero Dependence the probability of failure is P<sub>w/od</sub>

Calculate P<sub>w/d</sub> using the appropriate values:

$$P_{w/d} = (1 + (\underline{\hspace{1cm}} * \underline{\hspace{1cm}})) / \underline{\hspace{1cm}} = \boxed{\hspace{2cm}}$$

Reviewer: \_\_\_\_\_

