AN ABSTRACT OF THE THESIS OF

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This study investigated the beliefs and classroom practices of four teachers who had used graphing calculators in their teaching for a minimum of three years prior to the study and at least one prior year in the teaching of second year algebra. The persistence criteria, prior experience utilizing graphing calculators in their teaching, was designed to provide an investigation of established beliefs and practices. A case study approach involving detailed examination of the four teachers was used. The data collected and analyzed included interviews, observational fieldnotes, videotapes of classroom observations, and documents. Upon the completion of data collection detailed descriptions of the beliefs and classroom practices of the individual teachers were created. Additional analysis included exploration of the consistencies and discrepancies within individual teacher's beliefs and practices, exploration of the consistencies among teachers, and comparisons of teachers' professed beliefs and demonstrated practices to the constructivist theory and visions for the use of graphing calculators.

A high degree of consistency was found between the teachers' beliefs and classroom practices, both when graphing calculators were in use and when they were not. Particularly notable were the consistency between the espoused belief in the importance of assisting students in making connections and the observed emphasis on connections between concepts being presented and concepts previously explored. It

was found that teachers' experiences outside of the classroom, especially interaction with other teachers, played a significant role in the process of bringing beliefs and practices into agreement. These experiences served as factors in development of beliefs and practices and as stimulators for reflection, the central element in the process of developing an integrated structure of beliefs and practices.

The use of graphing calculators was found to focus on learning to use the tool to do mathematics and not as a tool to learn mathematics. While the focus on using the graphing calculator as a tool to do mathematics was not consistent with the constructivist approach to teaching and the visions for the use of graphing calculators, it was consistent with the teachers' view of algebra as the foundation for the study of higher mathematics.

Beliefs and Classroom Practices of Teachers Who Persist in the Use of Graphing Calculators in the Teaching of High School Algebra

By

Martha Eleanor VanCleave

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I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

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Martha Eleanor VanCleave

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I owe a debt of gratitude to the four teachers who participated in this study,. Their willingness to give their precious time for interviews, allow me to observe their teaching, and freely share their beliefs made this research possible. These teachers demonstrated dedication to their students and a love for teaching. I have learned much from them.

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DEDICATION

This thesis is dedicated to my loving partner Lou
and my wonderful children Ben and Jenny.

Beliefs and Classroom Practices of Teachers who Persist in the Use of Graphing Calculators in the Teaching of High School Algebra

CHAPTER ONE
THE PROBLEM

Introduction

Spurred by concerns about the state of mathematics education voiced in A Nation at Risk (National Commission on Excellence in Education, 1983) and Educating Americans for the Twenty-First Century (National Science Board Commission on Precollege Education in Mathematics, Science, and Technology, 1983) and reinforced by the National Education Goals adopted in 1990 by the President and the governors of the United States, the current movement to reform mathematics education has taken shape. Teachers are being urged to implement discovery learning, mathematics laboratory activities, individualized instruction and other changes from the traditional lecture discussion approach to teaching mathematics. The National Council of Teachers of Mathematics' (NCTM) Curriculum and Evaluation Standards for School Mathematics (hereafter referred to as Curriculum Standards) (1989) calls for such reform:

> A variety of instructional methods should be used in classrooms in order to cultivate students' abilities to investigate, to make sense of, and to construct meanings from new situations; to make and provide arguments for conjectures; and to use a flexible set of strategies to solve problems from both within and outside mathematics. (p. 125)

Similar recommendations can be found in the National Research Council's (NRC) Everybody Counts : A Report to the Nation on the Future of Mathematics Education (1989) and in Reshaping School Mathematics: A Philosophy and Framework for Curriculum (1990) from the Mathematical Sciences Education Board (MSEB) of the NRC.

The use of calculators and computers in the teaching and learning of mathematics is prominent in discussions on and recommendations for the reform of mathematics education. While mathematicians have been using electronic computation for four decades, the power of technology has been available to most teachers in the classroom for only two decades. The breakthrough for education has come in the last decade as personalized computers and hand-held computers (super calculators) have become widely available (Kaput, 1992). In recognition of the increased availability and versatility of computer technology, the NCTM's Curriculum Standards (1989) has recommended the incorporation of technology into the teaching and learning of mathematics in the schools saying that the standards for grades 9 to 12 are based on the assumption that "scientific calculators with graphing capabilities will be available to all students at all times" (p.124). The NCTM's Professional Standards for Teaching Mathematics (hereafter referred to as Professional Standards) (1991) states that "the teacher of mathematics, in order to enhance discourse, should encourage and accept the use of computers, calculators, and other technology" (p. 52). The NRC (1989) emphasizes the importance of incorporating computer technology into the teaching of mathematics to mirror the real world of mathematical investigation. In Reshaping School Mathematics: A Philosophy and Framework for Curriculum (1990), the MSEB of the NRC presents a set of principles for reform. The second principle states that "calculators and computers should be used throughout the mathematics curriculum" (p. 37).

The vision for utilizing technology in the teaching of mathematics includes changes in classroom practices. The MSEB (1990) states that "the proper use of technology requires new approaches to teaching mathematics in which students will be much more active learners" (p. 39). Pea (1987) suggests that technology can be used as a tool for developing conceptual fluency, for mathematical exploration, for integrating different mathematical representations, for learning how to learn, and for learning problem solving methods. The ease of viewing the graph of an algebraic expression of a function with the graphing calculator, thus connecting the algebraic expression with the graphical representation, has the potential of furnishing concrete links between geometry and algebra (Demana & Waits, 1990). Boyd, Ross, and DeMarios (1993) have suggested that consistent use of computer or calculator demonstrations can educate students to a view of mathematics as a dynamic rather than static field, that the use of technology permits instructors to introduce experimentation into the mathematics classroom, and that technology can be used to implement a discovery approach to learning mathematics. The use of technology in the classroom can enable the instructor to "be a facilitator of a student's thought process, and not simply a source of knowledge" (Lomen, 1993, p. 13).

Graphing technology, computers or super calculators capable of generating graphs of algebraic expressions and performing other numerical and programming tasks, is one form of computer technology gaining popularity in the teaching of high school mathematics. Proponents of graphing technology suggest that its use in the teaching of precalculus mathematics facilitates changes in the ways in which mathematics is taught and learned in classrooms. These visions for the incorporation of graphing technology in classrooms are consistent with the constructivist theory that underlies the recommendations for reform as detailed by the NCTM's Curriculum Standards (1989) and Professional Standards (1991).

Constructivism holds that all knowledge is constructed by the individual. Mathematical knowledge is constructed, at least in part, by reflective abstraction. Reflective abstraction refers to Piaget's concept of the process of interiorizing physical operations on objects. As a set of objects is manipulated, one interiorizes properties of mathematical operations rather than objects, thus acquiring an implicit understanding of mathematical concepts. Constructivism requires the existence of cognitive structures that are activated in the processes of construction. These cognitive structures account for the constructions, that is they explain the result of cognitive activity. Further, these cognitive structures are under continual development. Purposive activity induces transformation of existing structures (Noddings, 1990). Mathematical learning from a constructivist perspective occurs when students construct knowledge from their experiences by adapting their cognitive structures through reflective abstraction.

Constructivism has implications for teaching as well as for learning. When constructivism is applied to the issue of teaching, the assumption that one can simply pass information on to a set of learners and expect that understanding will result must be rejected. Students must learn to construct powerful ideas, ideas that the student believes and that have internal consistency, ideas that are in agreement with experts and can be reflected on and described, ideas that can act as a foundation for the construction of further constructions, guide future actions, and be justified and defended. In order for students to construct powerful ideas, instruction must be inherently interactive (Confrey, 1990). Additionally, constructivism depends on the autonomy of the learner. The learner must be responsible for and have control over his learning.

Moving from the theoretical use of graphing technology and its benefits to the classroom where teachers actually implement the new technology, the incorporation of graphing technology in the classroom must be seen as more than simply adding a new tool to the existing practices of teachers. If the visions for utilizing graphing technology are to be realized, teachers' classroom practices must match the visions of the recommendations for reform. Thus implementing graphing technology in the high school mathematics classroom can be considered as a curricular innovation requiring teachers who use a traditional style of teaching to change, not just add to, their existing practices.

Statement of the Problem

As graphing calculators have been introduced into high school mathematics teaching, a number of studies have been conducted examining the effects of their use on student achievement. Several studies showed that students using graphing technology performed significantly better on measures of procedural knowledge (Ruthven, 1990; Dunham 1993). Some studies found no significant differences between groups on these types of measures (Gesshel-Green, 1986; Rich, 1993). In two studies it was found that students not using graphing technology outperformed students using technology on certain procedural tasks (Giamati, 1991; Rich, 1993). Fewer studies have attempted to measure students' conceptual understanding of precalculus mathematics. These studies either found that students using graphing technology were better able to understand and interpret graphs than students not using graphing technology (Browning, 1991; Taylor, 1991) or that there was no significant difference between the groups' abilities in interpreting graphical information (Ruthven, 1990). While these studies have attempted to address the issue of the value of the change for students, the results have not offered a convincing argument for the incorporation of graphing technology in the teaching of mathematics. In addition to the lack of consistently significant improvement in student performance with the use of graphing technology, other limitations of the studies include small sample sizes, lack of control over the curriculum especially between the experimental and control groups, the variety of different curriculum materials used, and the differences in approaches to testing when graphing technology was used in teaching. Research on curriculum change indicates that research designed to compare student outcomes between different models of a curriculum change can lead to inconsistent or no significant difference due to the phenomenon of mutation, the adaptation of the innovation to each particular individual or site (Berman & McLaughlin, 1978).

Limited research has been conducted in relation to other changes or effects of utilizing graphing technology in the teaching of high school mathematics. In a study conducted to examine teacher and student behaviors in classrooms where graphing technology was being used, Farrell (1990) attempted to establish that there were differences between teachers' behaviors in these classrooms when technology was in use and when graphing technology was not in use. Predetermined categories were used to classify and analyze the teacher behaviors observed during an experimental implementation of the Calculator and Computer Precalculus (C^2PC) materials. Farrell concluded that there were differences between the classroom activities when graphing technology was in use and the activities when it was not in use. Informal observations in the classrooms where technology was being used in studies to measure its effect on student performances suggested some changes in student and teacher behaviors. Gesshel-Green (1986) and Rich (1993) suggested that students using graphing technology engaged in more explorations of mathematical ideas. Boers-VanOosterum (1990) indicated that students using technology were better able to apply their knowledge in new situations. Rich (1993) observed that teachers asked higher order questions when teaching with graphing technology. In a study of the implementation of programmable graphing calculators in the teaching of calculus, Jost (1992) found that the use of these calculators was compatible with interactive or inquiry-oriented methodologies. Further, the programmable calculator was found to be a vehicle for reform in teaching mathematics because it made computations and types of problems accessible to students that previously were not easily incorporated.

While these studies have contributed to knowledge on the effects of using graphing calculators in the teaching of mathematics, the studies have approached the issue of educational change by looking at the results of the change. The implication is that if teachers, contemplating the change, can see the benefits of the change, they will be inclined to adopt the innovation. Unfortunately, these studies do not provide enough evidence to convince skeptics. The potential benefits of using graphing technology as an instructional tool indicated by these studies, including improved problem solving abilities and improved conceptual understanding, are long term for students. A longitudinal study would be required to document these benefits. Additionally, educators now realize that how teachers interpret and implement curricula is influenced significantly by their knowledge and beliefs (Clark & Peterson, 1986; Romberg & Carpenter, 1986). Therefore, attempting to substantiate the effects of implementing graphing calculators is not sufficient.

Carpenter (1988) proposed a model for research and curriculum development that is based on the premise that teaching is a problem solving activity in which classroom instruction is based on teachers' decisions. Teachers' decisions were presumed to be based upon their knowledge and beliefs as well as their assessment of students' knowledge. Teachers' beliefs about mathematics and its teaching played a significant role in shaping the teachers' characteristic patterns of instructional behavior (Thompson, 1992). Teachers' approaches to mathematics teaching depended fundamentally on their systems of beliefs, in particular on their conceptions of the nature and meaning of mathematics, and on their mental models of teaching and learning mathematics (Ernest, 1989). Differences in teachers' classroom behaviors were found to be related to differences in beliefs (Thompson, 1984).

Teachers' conceptions are deeply rooted. Change must therefore be seen as a long-term process resulting from the teacher testing alternatives in the classroom, reflecting on their merits and making connections to one or more alternatives. Thompson (1992) suggested that case studies of teachers who had made desired changes could be used intentionally to prompt other teachers to reflect on their own beliefs and practices related to these changes.

A survey of high school mathematics departments conducted by the author (see Appendix A for the form and a summary of the results of the survey), found schools that introduced graphing calculators in the teaching of high school mathematics tended to continue the practice. The duration of use of graphing calculators among the respondents to the survey ranged from a new practice, the first year of use, to nine years of use with an average of three to four years. Some schools responded that graphing calculators had never been used in the teaching of mathematics. The results of the survey indicated that once a decision was made to incorporate graphing calculators in the teaching of mathematics, the practice was continued. All schools where graphing calculators had been introduced into the teaching of mathematics at some time indicated that graphing calculators were still in use. With evidence that the use of graphing calculators in the teaching of high school mathematics has developed into an established teaching practice, case study research on teachers who have incorporated their use is warranted. An examination of the beliefs and practices of teachers persisting in the utilization of graphing calculators will provide information that can be used to understand the characteristics of classrooms in which the use of graphing calculators is an established practice and in prompting others to incorporate graphing calculators into their teaching.

The NCTM's Curriculum Standards call for the use of graphing calculators at all levels of high school mathematics. Most of the research that has been conducted concerning graphing calculators at the high school level has involved precalculus courses because the initial implementations were at this level. Little is known about the use of graphing calculators in other high school courses. The survey conducted by the author indicated that implementation of graphing calculators in the schools began with precalculus and calculus courses and progressed downward through the curriculum including graphing calculators in the teaching of second year algebra followed by first year algebra. Gradually, graphing calculators are being introduced into the teaching of most high school mathematics courses. Because of the study of complex functions in precalculus, this course was understandably the initial recipient of graphing technology enrichment.

However, a limited number of high school students enroll in precalculus and few high school mathematics teachers teach this course. Algebra II is the course in which the curriculum goes beyond the simple graphing of linear equations and begins to explore more complex graphical representations of functions and systems of equations. The use of graphing calculators in the teaching of Algebra II has the potential of reaching many students at a pivotal point in their study of mathematics.

This study examined the beliefs and practices of teachers who had persisted in the use of graphing calculators in the teaching of Algebra II. These teachers (persistors) were defined to be teachers who were in at least their fourth year of using graphing calculators in their teaching. By examining the beliefs and practices of persistors, the study did not intend to examine the beliefs that led teachers to attempt the implementation of graphing technology, but rather the beliefs and practices of those who had integrated graphing calculators into their teaching.

Teachers' beliefs concerning mathematics and its teaching, the role of graphing calculators in the teaching of Algebra II, appropriate teaching practices in a curriculum utilizing graphing calculators, benefits to students from utilizing graphing calculators, curriculum implications of the use of graphing calculators, and teachers' roles in a graphing calculator enriched classroom were examined. The teachers' beliefs were compared with these teachers' observed classroom practices. Since change is a process, the beliefs and practices of teachers incorporating graphing calculators into their teaching undergo some changes during the implementation process. By examining persistors, it was believed that a degree of stability would exist in their beliefs and practices thus enabling a better examination of the relationships between these beliefs and practices. Comparing persistors' beliefs and practices to the visions and recommendations for the technology revealed the degree to which the visions were being realized and the degree to which teachers using graphing calculators espoused the constructivist theories on which the recommendations were formulated. The examination of teacher's beliefs and practices when utilizing graphing technology in the teaching of second year algebra focused on the following questions:

(1) What are the classroom practices of teachers who have persisted in the use of graphing calculator technology in their teaching?

(2) What are the beliefs of teachers who have persisted in the use of graphing calculator technology in their teaching?

(3) What is the relationship between teachers' professed beliefs and their classroom practices?

(4) Do teachers who persist in the use of graphing calculators do so from a constructivist perspective?

(5) Are the activities found in these classrooms consistent with the goals of the current curriculum reform movement?

Significance of the Study

If teachers do not share in the visions for the use of graphing calculators, or even perceive of them as being minimally influential in affecting learning, it is unlikely they will implement these practices (Brown & Baird, 1993). In order for teachers to choose to teach according to the visions for the use of graphing calculators in high school mathematics, teachers must believe that these visions are valuable. The results of this study contribute to the base knowledge on the beliefs of teachers using graphing calculators. Teachers share the vision only when they are convinced of its efficacy. Since teachers' characteristic patterns of behavior are functions of their views, beliefs, and preferences, any attempt to improve the quality of mathematics education must begin with an understanding of the conceptions held by teachers and how these conceptions are related to their instructional practices. This study provides an understanding of the relationships between the beliefs and practices of teachers persisting in the use of graphing technology. The findings of this study can assist teacher education programs by describing the variety of beliefs that support the use of graphing technology.

The use of graphing calculators has been encouraged from the constructivist perspective. Is the constructivist perspective the only perspective/belief system that supports the use of graphing technology? If so, how does the use of graphing calculators reflect constructivist beliefs? The information this study provides about the consistency between teachers' beliefs and practices and the constructivist perspective can assist in the design of programs that encourage the use of graphing calculators.

Studies suggest that because teachers, especially preservice teachers, do not possess rich constructs about mathematics and its teaching, they may be able to envision only limited curricular objectives or teaching styles. With limited abilities to envision a variety of curricular objectives and teaching styles, these teachers may be handicapped in realizing the visions for curricular innovations (McGalliard 1982), particularly the incorporation of graphing technology. The NCTM's Professional Standards (1991) emphasize that "teachers need a rich, deep knowledge of the variety of ways mathematical concepts and procedures may be modeled" (p. 151). By investigating the use of graphing calculators in the teaching of Algebra II this study explored an environment familiar to a larger number of high school teachers. Although the environment is familiar the practices and beliefs described in this study may be new to many teachers. Therefore, findings of this study can be used to assist teachers in understanding the beliefs and practices of others and in enriching their belief systems.

The examination of instructional practices of persistors in the use of graphing calculators in the teaching of Algebra II makes a valuable contribution. Little research has been done at this level. Most of the research on the use of graphing calculators has been conducted at the high school precalculus level or above. This study explored integration of graphing calculators at a lower level of the mathematics curriculum. Are the practices that have been utilized at the precalculus level found in algebra classes? Additionally, much of the research conducted at the precalculus level has involved curriculum materials developed to utilize graphing technology. Much less attention has been paid to developing curriculum at the high school algebra level. What are teachers' beliefs and practices in this situation? Answers to these questions contribute to further studies on the incorporation of graphing calculators and other innovations in the teaching of high school mathematics.

As graphing technology has been integrated into the teaching of mathematics, questions have been raised concerning the effect of this technology on the curriculum. Of major concern are procedures, often time consuming, that technology can accomplish quickly. Because the technology can quickly perform procedures on which teachers and students have spent much time in the past, should these procedures be left to the technology and no longer taught to students or must students still learn these procedures because they are important in the development of students mathematical abilities and understanding? Understandings of the ways in which graphing calculators are being used as established teaching practices in Algebra II classrooms contributes to the ongoing process of evaluating and revising the high school mathematics curriculum.

The teacher must be the center of any change made in the curriculum, including implementation of any innovation, because the teacher orchestrates the curriculum. Teachers' beliefs about teaching are an important element in the process of changing curriculum and teaching practices. Exploring and documenting the beliefs and practices of persistors in the use of graphing calculators in the teaching of Algebra II assists in developing curriculum, in helping others make changes in their teaching, in preparing prospective teachers, and in understanding relationships between beliefs and practices. The knowledge gained from this study concerning how and why graphing calculators are being used in the teaching of Algebra II contributes to the understanding of the potential of graphing calculators as a tool for realizing the visions of reform in mathematics education.

CHAPTER TWO
REVIEW OF THE LITERATURE

Introduction

This study investigated the beliefs and classroom practices of teachers who had persisted in the use of graphing calculators in the teaching of Algebra II. Descriptions of the beliefs and practices of teachers persisting in the use of graphing calculators were written from interview and observational data. Relationships between teachers' professed beliefs and their practices were explored. The study also examined the consistencies between these teachers' demonstrated beliefs and practices and the theoretical foundations for and desired benefits from the implementation of graphing technology.

Background research and theories in three major areas related to this study are presented in this chapter. The first area reviewed is the existing classroom research on the use of graphing technology in the teaching of high school precalculus mathematics. The studies reviewed in this section serve to provide background concerning the type of research that has been conducted on the use of graphing technology. The second area of research and theory deals with curriculum change. This area includes research and theories on curriculum change and the role of teachers' thinking in the change process. The purpose of this section is to establish the role of teachers' beliefs in the process of change. Research is included on teachers' beliefs about the incorporation of computers in classrooms to explore teachers' thinking and lay a foundation for a direction of research on teachers' thinking about the use of graphing calculators. The third section of this review focuses on teachers' thinking, establishes the role of teachers' beliefs in their thinking and discusses the relationship between teachers' thinking and their classroom practices. The relationship between teachers' beliefs and instructional practices is explored in studies on teachers' beliefs and practices in mathematics.

Graphing Technology in the Teaching of Precalculus Mathematics

The movement to use graphing technology in the classroom has been described in three phases: first a few groups and individuals warn that the old system is not working and make recommendations to use graphing technology, second the "gurus of graphing" emerge and attract others to their ways of using technology by providing workshops, minicourses, inservices and articles, and third the use of graphing technology becomes the established way (Dunham, 1993). During this movement, those who followed the "graphing gurus" have attempted to provide evidence to the masses that graphing technology is the established way by conducting research on the effects of using graphing technology on student performance.

The Effects of Graphing Technology on Student Performance

Gesshel-Green (1987) conducted a study on the effects of using the graphics software PLOT with Algebra II students. The study included one control class and one experimental class, both using the same textbook and completing the same textbook exercises. The experimental class spent seven class sessions in a computer lab using the software to analyze families of equations and solve equations and systems of equations. Analysis of pretest, posttest, and retention test scores for both groups indicated no significant difference in achievement between the groups. Gesshel-Green did observe that many students in the experimental group displayed a higher level of motivation and some students who had difficulty with symbol manipulation methods were successful in the computer lab using the graphics software. The experimental group, with the computer software facilitating the graphing of families of equations, spent more time exploring the relationships evident in the comparison of graphs and engaged in "what if" questions. More interaction among students in the experimental class was observed as compared to a more competitive spirit exhibited in the control class. This study was limited by the small sample size, only three classes all from the same school, were compared and the lack of documentation of observations concerning differences in students' behavior.

Boers-Van Oosterum (1990) investigated the understanding of variables and their uses by students in traditional and computer-intensive algebra courses. Two traditional algebra (TA) classes taught by the same teacher and three computer-intensive algebra classes (CIA) at a different high school taught by three different teachers were involved in the study. The TA classes were taught with a teacher-directed approach in which student participation was limited to note taking, solving problems during seatwork, and answering a few questions from the teacher. The CIA classes demanded that students study concepts in applied problem situations that they explored graphically, with the use of tables of values and successive approximations, or in symbolic form. MuMath (a computer software package that is capable of generating graphs, tables of values, and symbol manipulation) was utilized extensively. Analysis of pretest and posttest results augmented by interview data indicated that CIA students had a richer understanding of the concept of variable and were better able to model problem situations and translate from one representation to another while TA students had more context-bound knowledge. In this study the differences in the curriculum and schools as well as the use of computer technology must be considered as factors affecting student performance. The pre- and posttest utilized in these studies appeared to have only face validity.

In a study on the influence of graphing calculator use on translation from graphic to symbol form, Ruthven (1990) compared the performances of students who had unrestricted use of graphing calculators and students who did not have regular access to graphing calculators. This research took place in England where the students using graphing calculators were involved in the Graphics Calculators in Mathematics project designed to utilize graphing calculators throughout the two-year advanced level (upper secondary) mathematics course. Comparison classes were following the same course of study without access to graphing calculators. At the end of the first year of the course a test covering standard function families and variation of functions was administered to students in both project and comparison classes in four schools. Items were designed to allow students to use their calculators while testing competencies for which there were no automatic graphing calculator procedures; thus students using graphing calculators had no direct advantage. It was decided to allow students to use technology normally available to them in doing mathematics as it was deemed to be of little value to examine students' performances under unduly artificial circumstances. Ruthven found that students with access to graphing calculators performed better on tasks requiring students to supply an algebraic equation for a graph but not on items requiring them to extract information from verbally-contextualized graphs. These results were attributed to the increased use of graphic approaches in solving problems and the development of new concepts possible with the use of graphing calculators. This use of graphic approaches was thought to strengthen both specific and general relationships between graphic and symbolic forms. Additionally, the availability of graphing calculators was thought to improve the quality of information available to students particularly by facilitating the checking of solutions reached using a non-calculator method and to improve the prospects of success by reducing uncertainty and diminishing anxiety on the part of students leading indirectly to improved performance. Ruthven observed that the influence of graphing calculators depended as much on the way in which they were used in the classroom as on simple access to their use. While this study did allow students to utilize the technology they had been accustomed to in their study during the testing, there was no evidence provided for the validity of the tests. The results were also tainted by use of the wrong unit of analysis in analyzing the data from this study.

Giamati (1991) conducted a study to examine the effects of using graphing calculators on high school precalculus students' understanding of functions and their graphs. Four high school precalculus classes, two control and two experimental, studied the same material on transformations of functions utilizing teacher designed worksheets to accompany their text. The experimental classes had graphing calculators to use, in pairs, during classroom discussions and to generate the graphs for worksheets. Graphing calculators were not available for use on homework or tests. Results of the analysis of data from a pretest, two posttests, and the researcher's observations of experimental classes indicated that there was no significant difference in overall performance between the experimental and control groups. Further analysis indicated that the control groups performed better on tasks requiring the sketching of graphs and on concepts related to stretches, shrinks, and translations. Giamati linked these results to the lack of use of tables of values with the experimental group, indicating that tables of values must be explicitly taught, and the difficulties encountered in teaching about stretches, shrinks, and translations with graphing calculators. While care was taken to ensure equivalence of the curriculum materials used by experimental and control groups, the effect of students working in pairs was not considered when analyzing the results of this study. Other factors that could have contributed to the results found but not considered in the study were the differences between teachers and the lack of randomness in the samples. Additionally, the wrong unit of analysis was used and the validity of the tests was not established. Furthermore, not allowing students who had utilized graphing calculators in their study to use them on the test may have confounded the results.

Browning (1990) conducted a study designed to measure a hypothesized increase in understanding of functions and their graphs by high school students participating in the Calculator and Computer Precalculus (C^2PC) Project. The C^2PC project was a modified precalculus curriculum utilizing calculators and computers to increase the access to graphs and emphasize the correspondence between the numerical and algebraic representations of functions with their graphical counterparts. The researcher designed instrument to measure students' levels of understanding of graphs was administered as both a pretest and posttest to five C^2PC classes and to one traditional precalculus class that served as the control group. A random sample of the pretests was used in a cluster analysis to determine the levels of understanding. Four levels of understanding were established that became increasing complex, requiring more knowledge, interpretation, and more complex problem solving strategies. Validity of the level structure was reinforced by a comparison between pretest clusters and posttest results that showed that the level structure was essentially preserved for both C^2PC and control groups. The results of the posttest indicated that the majority of the control group students remained at or below Level 2 while the majority of the C^2PC students reached Level 3 or Level 4. In analyzing the results of the tests, Browning found that the use of graphing technology within the precalculus classroom provided increased student understanding of graphs. Unfortunately, no indication was given that the curricula utilized by the two groups were equivalent. Also, the test may have emphasized concepts developed in the C^2PC classes but not in the traditional class introducing a bias. The differences in size between the experimental and control groups are also an area of concern in evaluating the results of this study.

In order to investigate the relationship between students' understanding of algebraic concepts and their use of computer/calculator graphing utilities, Taylor (1991) conducted a study involving high school precalculus students and the C^2PC materials. The study involved three intact high school classes, one using the C^2PC materials and two using traditional materials. The Graphing Levels Test developed by Browning (1990) with five additional questions related specifically to knowledge of quadratic equations was used to measure students' understanding of graphs and their understanding of quadratics. The test was administered as a posttest; no pretest was administered. Multiple Analysis of Variance on the two groups, control and C^2PC, and levels of understanding indicated a significant difference between groups at Level 3 on the Levels of Understanding test. The difference favored the C^2PC group. The C^2PC group also performed better on the questions designed to measure understanding of quadratics. From these results the researcher concluded that the C^2PC group was functioning at a higher level of graphing understanding and held a better understanding of quadratics. Unfortunately, the C^2PC materials included study of quadratics while the traditional classes did not study quadratics. Further, no attempt was made to establish the validity of the instrument for this study. Because of these weaknesses, it was not possible to conclude from this study that the better performance of the C^2PC group was a result of the use of C^2PC materials or graphing utilities.

Rich (1991) investigated the ways in which the use of a graphing calculator as a teaching tool affected precalculus students' learning of functions and related concepts and teachers' methods and beliefs. Three classes using the C^2PC materials at two schools and three comparison classes at each school were involved in the study. Data were gathered concerning comparison and C^2PC classes at one school through periodic classroom observations, periodic completions of a systematic classroom observation instrument, a conventional algebra posttest, and periodic interviews with students from each class. At the second school the posttest was administered and teachers were interviewed at the completion of the school year. All teachers were asked to complete a questionnaire dealing with their experiences. Analysis of posttest results from the second school only, the one at which there were no observations, showed no significant difference between the C^2PC and comparison groups. An item analysis indicated differences in the responses of the two groups on 15 of the 35 items. The C^2PC group scored better on 10 items, 8 of these items were graphic in nature, dealing with scale and matching graphs with equations. The other two items on which the C^2PC group scored better dealt with function concepts such as domain, range, and intercepts. The five questions on which the comparison group scored better were computational in nature with the exception of one trigonometric simplification problem. The differences found in the item analysis were attributed to differences in teaching approaches. Analysis of the interview data revealed differences between the groups in the ways students approached problems. Students in the C^2PC group knew the basic shapes of many graphs before graphing them and showed a broader understanding of functional reasoning and the relationship between algebraic and geometric representations. From the classroom observations the researcher characterized the comparison classes as following the homework-lecture-homework model with students listening, asking questions, and taking notes. The C^2PC classes included student participation and exploration with students contributing to the discussions by making observations, conjectures, or proposing alternative solutions. In these classes the teacher lectured less and listened more as the year progressed. Teachers' responses to the interviews and questionnaire indicated that the graphing calculators provided an alternative method for problem solving, an environment for exploration, frequent access to graphs, and experience working with equations and graphs. More realistic applications could be explored because "real numbers" could be used. Teachers found that topics of increasing and decreasing functions and local extrema could be taught intuitively and that their approach to testing changed. This study was complicated with different data gathered at different sites making analysis complex. Results of the analysis of test results would be more valuable if test data had been collected from all sites. The differences found were questionable because the incorrect unit of analysis was utilized.

Data from field tests of the C^2PC materials was compiled and analyzed by Dunham (1993). The field testing involved over 2000 students at 86 high schools and 40 colleges. Teachers of the C^2PC classes were volunteers who received training in the use of project materials. The Calculus Readiness Test (CRT) was administered as a pretest and posttest to intact classes, both C^2PC classes and comparison classes. Analysis of the results of the pretest scores showed no significant differences between C^2PC classes and comparison classes. Analysis of the results of the posttest, using the pretest as a cofactor, indicated that the C^2PC classes significantly outperformed the comparison classes. The C^2PC classes were allowed to use graphing utilities on both the pretest and posttest. Results of this study indicated that students using the C^2PC materials and graphing utilities were better prepared for calculus than students in comparison classes. Of the students whose pretest scores indicated that they were not prepared for calculus, almost twice as many students from C^2PC classes (72.8%) as comparison students (40.8%) demonstrated calculus readiness on the posttest. No specific information was given concerning the nature of the comparison classes. Without knowing more about the curriculum and teaching practices in the comparison classes, it is difficult to interpret the findings of this study.

Teacher and Student Behaviors with Graphing Technology

Farrell (1989) conducted a study designed to explore the teaching and learning activities that occur when graphing calculators and computers are integrated into a precalculus curriculum. The classrooms under investigation were involved in a one-year field testing of the C^2PC curriculum. Care was taken to emphasize that the purpose of the study was to describe what was happening in the classrooms and not to make any judgments. In each of the six classrooms involved in the study, six consecutive videotaped class sessions were studied. Each taped class session was coded by two qualified observers using a modified version of the Systematic Classroom Analysis Notation (SCAN) matrix that identified teaching activities, demands placed on students, number of questions by the teacher, number of questions by the students, lesson segments (differentiated by unique goals), technology in use, classroom roles, student behaviors, and pupiling (student activities inferred from observable student behaviors).

In reporting the findings of the study, comparisons were made between the segments (five-minute sections of the viewed videotaped class sessions) of classroom observations when technology was in use (56% of the segments) and the segments when technology was not in use (44% of the segments). Students were found to exhibit a wider variety of roles more frequently, including explainer, fellow investigator, and manager, when technology was in use than when it was not in use. Students' activities also shifted with the use of technology showing less didactic behaviors, although still present, and more symbolizing and problem solving behaviors than were present without the use of technology. Evidence was also found that teachers' roles shifted with the use of technology. During segments including the use of technology, teachers exhibited the role of consultant more often and the roles of task setter and explainer less often than they did during segments that did not include the use of technology. Only a slight shift in teaching activity was observed during the use of technology with more time spent on exercise, consolidation, practice and investigation and less time spent on exposition than when technology was not in use.

The results of this study were confounded by the comparisons made between the use of technology and non-use of technology within the same classrooms. Better comparisons might have been made between technology-enriched classrooms and non-technology classrooms using the same or similar curriculum. Additionally, the use of the SCAN may have limited the richness of the results. Using predetermined categories to describe the observed behaviors in a case study experiment could have prevented the researcher from discovering some of the subtle characteristics of these classrooms.

Conclusions from Studies on the Use of Graphing Technology

Eight studies involving high school precalculus courses attempted to compare the performance of an experimental group (using computer or graphing calculator technology to assist in instruction and problem solving) with a group taught in a traditional manner (using a lecture-based curriculum that stressed memorization of rules and computational skills). Five of the studies compared students' achievement on procedural, computational, or symbol-manipulation measures. The results of these five studies were mixed. Two studies (Gesshel-Green, 1986 and Rich, 1993) found no overall difference in achievement, two found significant differences favoring the experimental group (Ruthven, 1990 and Dunham, 1993), and one found significant differences in some areas that favored the control group (Giamati, 1991). The remaining three studies used instruments designed to probe students' understanding of the concepts taught. Two of these studies explored students' levels of graphing understanding (Browning, 1990 and Taylor, 1991). Both studies found differences favoring the experimental group, however the difference was only significant in one of the two. The third study probing student understanding of concepts found significant differences in students' responses between the traditional and computer enriched-curricula (Boers-Van Oosterum, 1990). Ruthven's (1990) study, that found a significant difference in favor of the experimental group on procedural items, found no difference between experimental and control groups on interpretation items.

Through item analysis and augmentation of the paper and pencil test results with interview data and classroom observations, strengths and weaknesses of the students in technology-enriched classrooms and traditional classrooms were assessed. Students in the experimental sections displayed better abilities to interpret graphs and relate graphs to their functions (Boers-Van Oosterum, 1990; Ruthven, 1990; Browning, 1990; Taylor, 1991; Rich, 1993; Dunham, 1993) except for one study (Giamati, 1991) in which the group not using technology performed better on tasks relating functions and their graphs. Students in experimental sections considered different aspects of graphs and talked about global features of the graphs including domain, increasing and decreasing behavior, and asymptotic and local behaviors (Rich, 1993). Students using technology could better model problem situations (Boers-Van Oosterum, 1990) and learned to use both graphical and algebraic methods to solve problems (Rich, 1993). Richer understandings of the concept of and uses of variables were held by students in the experimental group (Boers-Van Oosterum, 1990). Students using technology displayed better abilities on items that could utilize graphs in their solution (Ruthven, 1990; Dunham, 1993). Use of technology provided students access to a greater variety of approaches for solving and checking their work (Ruthven, 1990). Results indicated that the technology-enriched curriculum better prepared students for calculus (Dunham, 1993). One study (Dunham, 1993) showed students using technology performed better on non-graphing (computational) items, while students using the traditional curriculum performed better on computational items in only one study (Rich, 1993). In all other studies examining computational abilities, no significant differences were found between the groups. Indications that students not using technology held better understanding of specific transformations including shrinks, stretches, and vertical and horizontal translations were found in one study (Giamati, 1991). In this study, the group using technology omitted the use of tables when studying graphs of functions. This omission may have contributed to the differences found, an important result of the study that should be considered when implementing technology in the future.

Observational data indicated that students using the technology engaged in more explorations (Gesshel-Green, 198; Rich, 1993) and were better able to apply their knowledge in new situations (Boers-Van Oosterum, 1990). It was observed that teachers using the technology tended to ask more higher order questions (Rich, 1993). Evidence was found to indicate a shift in the roles of both teachers and students in technology-enriched classrooms (Farrell, 1989). The shift for teachers was seen to move away from the task setter and explainer toward the role of consultant while students were seen to shift toward more use of the roles of explainer, fellow investigator, and manager. Student activity was seen to be less didactic with more emphasis on symbolizing and problem solving behaviors.

When the differences found in these studies were examined several issues caused them to be regarded speculatively. In only one of the studies, Dunham (1993), was the correct unit of analysis used. It is probable that the differences found in the other studies would not have been significant if the correct unit of analysis had been used. Students were allowed to use technology on achievement instruments in only two of the studies (Ruthven, 1990; Dunham, 1993). In all other studies the students were not permitted to use the graphing technology. While not using graphing technology had the merit of making the test the same for all students, it was a questionable practice in light of the treatments that used the technology as a tool for doing mathematics. As Ruthven (1990) argued, not using the graphing technology forced students, accustomed to its use, to do mathematics "under unduly artificial conditions" (p. 438).

The issue of validity of the instrumentation was important when interpreting the results. Most studies did not report the validity or reliability of the instrument or reported only face validity. Even when validity was reported, there was a question that the same instrument could be valid for two different treatments. Borg and Gall (1989) suggested that when different treatments are used content validity should be carefully checked for all treatments. No study indicated an attempt to assess content validity for all treatments.

Perhaps the greatest weakness of most of these studies was that the comparison groups used radically different curricula. While the differences found were attributed to the use of technology in the teaching of precalculus mathematics, the issue of differences in curriculum cannot be discounted. Only Gesshel-Green (1986) and Giamati (1991) made an attempt to insure that the two approaches covered the same content. In these two studies specific content was taught using two different teaching methods, one including technology and one using a traditional lecture-approach. In both of these studies little or no differences were found between the two groups. The differences found were attributed to the lack of use of tables with the group using technology. The remaining five studies actually compared experimental, technology-enriched curriculum to traditional, lecture based curriculum.

In spite of the weaknesses of these studies, the results indicated a trend towards improvement of students' performance when computer and calculator graphing technology was used in teaching precalculus mathematics. Particularly noteworthy was the improved access to more complicated and involved situations that the technology provided.

Students using the technology were more able to use a variety of approaches in solving problems without losing computational and procedural skills perhaps indicating that the use of graphing technology will improve students' problem solving abilities when used in conjunction with methods that stress the importance of computational and procedural skills.

Some evidence existed to show that the use of graphing technology affected student and teacher behaviors. Students were found to exhibit a wider variety of roles when technology was in use while teachers were found to shift on the continuum away from the lecture approach toward a more discovery-oriented approach to teaching. The limited scope of studies conducted in this area leaves many questions waiting to be explored.

Curriculum Change

The Process of Curriculum Change

The studies examined in the preceding section indicated that the implementation of graphing calculators must be seen as not a mere addition to the existing curriculum, but rather as change in the curriculum. Educational (curriculum) change as described by Fullan (1982) can be seen as a multi-phase process with three broad phases (Figure 1). Phase I, adoption, was the process that lead up to and included the decision to proceed with a change. Phase II, implementation or initial use (usually the first two or three years of use), involved the first attempts and experiences of trying to put an idea into practice. Phase III, continuation, referred to the stage when the innovation became a routine part of the system or disappears. Continuation was an extension of the implementation phase in which the innovation was sustained beyond the first year or two. The non-linearity of the process was indicated by the double arrows. These double arrows indicated that events at one phase could feed back to alter decisions taken at previous stages, that then proceeded to work through the following stages in an interactive way.

Adoption ⟷ Implementation ⟷ Continuation ⟷ Outcome

Figure 1. A simplified overview of the change process.

Teachers in classrooms are the ones who actually implement any curricular change. These teachers "want, need, and benefit from tangible, relevant program materials that have been produced and tested in real classroom situations" (Fullan, p. 60). Since the essence of educational change consisted of learning new ways of thinking, it followed that staff development was one of the most important factors related to change in practice. Implementation was a process of resocialization, the foundation of which was interaction. Effective training approaches combined concrete teacher-specific training activities, ongoing continuous assistance and support during the process of implementation, and regular meetings with peers and others (Fullan, p. 67). The quality of working relationships among teachers was strongly related to implementation. Collegiality, open communication, trust, support and help, interaction and morale were all closely related. The amount of time required to make change could not be overlooked. The Study of Dissemination Efforts Supporting School Improvement (DESSI) (Huberman & Miles, 1984) research showed that time teachers spent on implementing an innovation was strongly related to change in practice resulting from the implementation attempt. Innovation could not be added on but must have been integrated into a regular part of the working schedule of teachers involved. McLaughlin (1989) added that change strategies rooted in the natural networks of teachers, in their professional associations, may have been more effective than strategies from other sources. Reforms or policies that engaged the natural networks of teachers supported change efforts in a more sustained fashion.

Continuation can be thought of as another adoption decision. The quality of the implementation phase directly impacted the continuation phase. Berman and McLaughlin (1978) found that projects that were not implemented effectively were discontinued and only a minority of those implemented were continued beyond the period of federal funding. Additional reasons for lack of continuation included lack of interest, inability to fund a project using district funds, lack of money for staff development and staff support for continuing and new teachers, lack of support at the central district office, and lack of support at the school level by the principal. Implementation strategies that focused on reliance on outside consultants included one-shot, pre-implementation training, pay for training, and formal, summative evaluation. These strategies were found to be ineffective because they failed to provide the on-going and sometimes unpredictable support teachers needed, excluded teachers from project development, and signaled a mechanistic role for teachers (McLaughlin, 1989).

During the implementation process, the innovation was often adapted to each user, site or context. This adaptation resulted in the implementation of many different innovations in the name of one. This phenomenon called mutation (Berman and McLaughlin, 1978) led to inconsistent or no significant difference results in research designs comparing student outcomes between different models or innovations implemented by different teachers at different sites.

Teachers and Curriculum Change

Roberts (1984) described the "theory-practice" interface as the point of convergence between the developer's world with intentions for hypothetical students and the teacher's world of specific teaching designed for known, real, but unique students. The teacher "sees" the curriculum developer's world through his or her own perspective, so that the developer's viewpoint about aims, the nature of learning, and knowledge may not be shared by the teacher, and are thus read differently, or may not even be seen in the curriculum materials. Russell (1984) argued that the scientific paradigm for studying curriculum change implied that logic alone could influence teachers to make changes. However, he added that "personal convictions about the value... may well be the strongest elements in decisions to teach in that fashion" (p.118).

Olson's (1981) study of the ways teachers used the materials of a particular innovation, the English Schools Council Integrated Science Project (SCISP), revealed that teachers encountered dilemmas when the methods of the innovation were at odds with their customary methods of teaching. In the study, teachers were asked to discuss their work with the innovative SCISP curriculum project. Eight teachers participated in a three month study. SCISP was chosen because it asked teachers to take seriously the discussion of values issues in science in the context of integrated subject matter that was thought likely to create dilemmas for teachers. Coping with the demands of the project provided a context in which teachers could talk generally about demands of teaching and specifically about how they resolved dilemmas associated with their use of the project materials. A dilemma arose when doctrinal commitments of SCISP were at odds with those of the teacher. It was in relation to these dilemmas that teachers were able to articulate the meaning they attached to what they did. Kelly's grid technique was used to allow the investigator to confront teachers with a "picture" of their thinking about classroom activity, and particularly about relationships with the students. Each teacher was interviewed for a period of four hours on four occasions, over a period of three months. The last two interviews were devoted to construct elicitation and a probing follow-up interview. The quantitative analysis of the grid data (obtained in the interviews) indicated that an important common and underlying construct in the practical language of teachers was that of classroom influence.

A dilemma that teachers faced with the SCISP materials concerned reduced classroom influence as a result of attempts to conform to the project doctrine. Reduced influence came from the project features such as: free ranging discussion episodes, downplaying in the design the importance of content in science teaching and examination preparation, requiring teachers to instruct outside of their discipline. Teachers were unaccustomed to talking about the effects of teaching in terms of students achieving certain levels of problem solving skill, as in the SCISP materials. Rather teachers were accustomed to measuring student achievement in terms of notebooks accumulated and content learned as measured by examination results. Teachers were aware that how they wanted to proceed was at odds with the project, yet they believed that what they did was more reliable for accomplishing the goals in which they believed. Analysis of the interview data revealed that teachers believed in two forms of high teacher influence, "teacher as prime mover" and "teacher as navigator." Teachers used emphatic, positive language in describing the high teacher influence forms, yet perceived a dilemma in their attachment to high influence teaching. Teachers were unclear when they attempted to describe low influence teaching suggesting that teachers lacked the language to talk about low influence teaching. Since teachers were not facile in the use of language about low influence teaching, it was not surprising that they were tentative when it came to describing what happened to students as a consequence of low influence teaching. The low influence teaching seemed to involve the abdication from teaching as the teachers in this study saw it. As a result of their lack of belief in low influence teaching, teachers translated the materials into high influence teaching to which they were accustomed.

Olson (1981) concluded that in order for the innovation to result in change of practice, there must be a dialogue concerning the innovation between the innovator and the teacher. By trying new ideas and discussing them with the innovators, teachers built the experiential base from which more powerful language was developed. Innovators must understand the dilemmas that teachers faced and why dilemmas were resolved as they were. Teachers needed to understand the potential for new ideas and assess their value. Through a dialectical approach the innovation acted as an heuristic device for probing value systems, instructional arrangements, and classroom practice.

In a study to examine teachers' continued use of a successfully implemented innovative nutrition education curriculum, Lewis (1988) emphasized the importance of the teacher as the implementer. The study utilized a rubric of partnership evaluation. Teachers were an essential ingredient in the implementation from the beginning with support provided by the "partners." All the "partners" (teachers, administrators, school food-service staff, external coordinators, the funding agency, and the evaluation team) were encouraged to participate in decisions about the study. Summer workshops were held each of the three years of the study for all the participants. Teachers were encouraged to fit the activities into the courses they were already teaching. Three major instruments, a teacher questionnaire, a teacher interview schedule, and a teacher rating scale, were used to explore teacher characteristics, teacher perception of internal support, and teacher perception of external support as determinants of continued-use of the innovation. Twenty-one teachers who had been involved in the three year implementation of the innovation were included in the continued use study. These teachers were described as being a representative sample of the population.

The teacher questionnaire revealed that 81 percent of the teachers' continued use of the innovation at the same level as practiced during the three year implementation study. Further the results of the questionnaire and interviews revealed that teachers who continued the use did so because the activities were important and effective instructional materials and fit into the courses they were teaching. Teachers discontinued using the innovation because the activities no longer fit into the courses they were teaching, with students they were teaching, or with the need to find new materials to prevent boredom. Teachers rated internal support from the school and district administration low and indicated that the external support they received helped them to utilize the materials more effectively. Lewis concluded that in the beginning stages of continued use, teachers' perceptions of the importance of the innovation were the determining factor. Teachers' perceptions of importance may be derived from the perceived effectiveness of the innovation and how well it fit into the courses they were teaching. The innovation's fit appeared to be a major reason that teachers believed the innovation was important.

These studies confirmed the importance of the teacher in the implementation and adaptation of curricular innovations. Evidence from these studies suggested that teachers' beliefs about their teaching situation were important to their decisions about how and why they utilized innovations. Further support for the importance of teachers' beliefs in the curriculum change process were found in studies concerning the implementation of computer technology.

Teachers and the Implementation of Technology

Olson and Eaton (1987) conducted a 12-month research product, funded by the Ontario Ministry of Education, to investigate how teachers were using computers in the classroom and why they were using them in these ways. They conducted eight case studies with a variety of different applications including creative writing, graphics, geographical simulations, remediation in elementary math and language arts, and elementary French. These studies revealed two distinct patterns of computer use: teaching computer awareness as a new school subject and using the computer as an instructional tool to teach existing school subjects.

The eight case-study schools included elementary, intermediate, and senior schools, all in the same metropolitan school district. The teachers' experience with computers ranged from absolute novice to night-school instructor at a nearby university, but all had deliberately sought the opportunity to incorporate computer use into their classroom practice. Preliminary interviews with the schools' principals to ascertain the background to computer use in each school were followed by a series of interviews with the teachers concerned. Videotapes of their students using computers, analyses of commonly occurring computer-related situations using Kelly's repertory grid technique in which teachers were asked to categorize the situations and then construe their response, and a Computer Use Journal that each teacher was asked to keep for a one-week period, all provided instances of actual classroom practices that became the basis for further analysis and discussion with the teachers.

For four of the eight case-study teachers "doing computers" became a new, unofficial school subject. Built into the ways these teachers described this new subject were notions about its purpose, its scope and sequence, ways in which the subject might be learned and/or taught. Unlike other school subjects, "doing computers" was one with which the teachers had no professional training, regardless of how much they knew about or could use computers themselves. The teachers generally had little experience and few resources upon which to draw. Learning about the computer required students to share access to a costly and scarce tool, one that might, for many reasons, refuse to work. These "newness" aspects of the innovation challenged existing classroom practices and the established relationship between teacher and students, and blurred traditional classroom roles. It was these aspects of computer use, not the technological aspects, that concerned teachers the most. These four case-studies revealed teachers attempting to cope with a modern, unfamiliar technology using familiar, well-tried routines and responses. But the unanticipated elements of the innovation, such as imprecise negative feedback and unpredictable student responses to the material, meant that the well-tried methods did not always work.

The teachers in this study were not required to "implement" any specific program. They decided what to do and what resources they needed. The teachers all "volunteered" to become involved in what became a time consuming and frustrating process. These teachers explored images of how the computer might function in their classroom. All expressed satisfaction at what they had accomplished, but also spoke about not being able to carry out some of their plans. These teachers discovered that there were some aspects of what they had wanted to do that could not be done. By making these discoveries, they were able to adjust their activities to what they could manage and test their ideas within a realistic framework. Through the interview process, it became clear that the teachers had used their experiences of innovative activity to begin to reflect critically on their practice - to ask questions both about what they normally do and what they were trying to do that was new.

These reflexive experiences with innovation provided information about how teachers coped with innovation. Conducting classroom activities in a new way was an extremely complex process. Common responses of teachers to this complex process provided information about the routine and novel elements of an innovation. Incorporating elements of the innovation within the familiar activities of well-established routines was an important issue. Teachers could not be expected to suddenly abandon their practices in favor of teaching activities quite remote from that to which they were accustomed. The process was not one of substituting one practice for another, but of subjecting existing practice to a challenge posed by another well-conceived practice. The effect of the challenge was to provide reasons to modify the existing practice through a process of critical comparison.

This study supported what Olson found in an earlier study (Olson, 1981) concerning teacher influence. In the case of computers as a subject, teachers isolated the computer activities allowing them to proceed without affecting the ongoing "official" work of the class. The teacher maintained a "modern" posture by incorporating computers into the classroom while maintaining high influence teaching in the "official" work of the class thus actually increasing influence over the class. In the case of the computer as a teaching tool, the teachers integrated the computer into familiar teaching routines, not risking dramatic changes in teaching styles that might undermine their ability to cover curriculum effectively. These teachers did not risk their influence so their influence over the core of their work remained secure. The power of the computer as a teaching tool did, however, disturb teachers concerned about their own role and influence in the classroom. These teachers asked if their classroom routines were such that the potential for the computer might be realized. When teachers asked this question, they were calling into question the basis of those routines, their very essence as teachers, and their capacity to represent themselves to their students (Olson, 1988). In this reflexive conception of change, teachers had a key role to play because it was they who were required to find a way of making new ideas work: it was through teachers taking new ideas seriously that innovators could assess what new ideas meant in practice. Talking to teachers about these new ideas helped understanding of what the rational basis for practice was and how new ideas fit into the overall framework of teacher intention.

In a study based on in-depth interviews with 76 teachers from 10 sites around the country Wiske et. al. (1988) examined the effects on teachers of the challenges and opportunities provided by computer technology. The study explored how and why teachers used computers, what training and support had been available to teachers, and what influence teachers had and might have on technology and on how it was used. The most frequently mentioned barrier to using educational technology was a lack of access to appropriate preparation and support. Teachers indicated that the training did not prepare them to integrate the computer into their teaching, that it did not include enough time for them to become comfortable with the software, that it did not include follow-up support to help them "troubleshoot" during the early implementation stages and that the training experience was not tailored to the teachers' needs.

The teachers surveyed felt that the use of computers had enabled them to present ideas in new ways, to include new topics and to teach traditional topics more thoroughly. Teachers who thought that using computers had shifted their teaching approach most often mentioned that computers had helped them change from the traditional lecture approach to serving as a coach or facilitator of student learning. These teachers who said that their teaching approach changed also observed their students move from memorization of facts and algorithms to active inquiry with more open-ended problems. Teachers who reported little effect on their teaching tended to have already been committed to "discovery learning" approaches and found computers a natural extension of their store of teaching tools.

Teachers who seemed most satisfied with their uses of educational technology were often the beneficiaries of several layers of support. These layers of support included on-site aides to assist with logistics, district-wide support staff, colleagues with whom to exchange strategies and build an atmosphere that supported collegiality and experimentation, building principal support, and district level support for developing clear priorities. The majority of teachers needed considerable assistance and encouragement to learn how to incorporate technology into their classrooms.

The findings of this study were based on in-depth interviews, many conducted by telephone. No apparent attempt was made to substantiate responses with other data sources. While these issues should be considered, the results of this study served to further substantiate the importance of teachers' decision making in the process of implementation of technology.

Kerr (1991) conducted a pair of studies on the implementation of computer technology in three metropolitan school districts. The studies described the place educational technology had (or perhaps more accurately was coming to have) in the thought and practice of working classroom teachers. The studies characterized teachers' thinking about technology within the framework of concerns about teaching, their intents for classroom practice, and their actual work with technology as it became available. The first study was a set of interviews and observations over a one-year period in the three school districts. The second was a formal evaluation study of a specific technology-based program that began at about the same time as the first set of interviews.

The interviews focused on teaching practice and the place of technology in that practice. Some of the teachers interviewed were also observed in their classrooms. With the intention of discerning the place technology had in the thinking of teachers, one of the interview questions asked teachers to "identify five milestones that marked changes in how you thought about teaching." Technology specifically figured in only four of the 20 responses to this question. In no case was it the first item mentioned and in no case was much loading attached to the mention. When asked what place technology had in their thoughts about teaching over the years, some teachers used the word "tool" to describe their images, while others talked about the "need to do things better" or the possibility of capitalizing on the novelty of computers to "liven it up in the classroom." Most of the teachers seemed to agree that technology did not have a profound effect, all they needed were chalk, a board, and students. Further probing on how teachers envisioned classroom activity and the place there for technology revealed that teachers saw variety and the potential of opening up specific new teaching approaches, but continued to emphasize that technology generally played only a minor role in their thinking about what happened in their classrooms. The theme of cautious adoption was stressed by several teachers with comments like, "it needs to make things easier, I don't want it if it interferes with learning or creates a hurdle" and "I'm not a pioneer, as I become comfortable, I incorporate it."

In contrast to their statements about the role of technology in their thinking about teaching and vision for the classroom, most of the teachers in the interview study indicated that the presence of technology had affected the way in which their classrooms were organized and in their roles in the classroom. These teachers commented that there was more activity, especially in small groups, and that there was a move toward the teacher as a facilitator or helper with less "front-of-class" teaching. There was a sense that using technology resulted in a fundamental redistribution of power and authority in the classroom. Teachers were able to restructure their role in ways that led to more flexibility, the opportunity to do more things and different things in the classroom. Only a few teachers indicated that technology had not changed their classrooms.

The teachers in the evaluation study exhibited somewhat more developed visions of technology's place in the classroom, understandable based on the experiences of these teachers in an environment both rich in hardware and supportive in terms of the help and advice made available by the district. Several of these teachers tended to have a vision of technology as a tool to use in pursuit of their own goal of promoting individual learning by students in the classroom. Others, however, saw the shift to a technologically enriched classroom and the possibilities it provided for dealing with students individually as a wrenching experience. These teachers saw that technology allowed for a classroom environment that was not their customary teacher-centered approach. The need for change was apparent, although not necessarily comfortable for them.

The teachers in the evaluation study reported that the impact of technology in their classrooms was consistent with their visions. All teachers in this study were in agreement that the use of computers had significantly altered their ways of organizing and handling classes. The teachers found that technology had enabled them to give students choices, have students work in groups, and required that not all students be doing the same thing at the same time. These changes did not come easily to all. When asked to reflect on what the incorporation of technology into the curriculum required of teachers, the need to go slowly was stressed. The focus was more on changes in teaching style and approach than on specific training on either hardware of software use. Teachers emphasized the importance of trying to keep one's image of teaching open and flexible while constructing new ways of thinking about classroom reality with technology.

Kerr concluded with two visions of the place of technology in teaching. All teachers saw themselves as teachers first and users of technology second. Most of the teachers in the interview study described technology as a "tool"; the vision was technology as a lever, a way of increasing efficiency. The second vision, present predominantly among those in the evaluation study, was that technology might become a fulcrum for broader educational change, a point on which teaching practice could consciously shift in new directions. The difference in vision between the two studies may be explained by the amount of time and support the teachers in the evaluation study had with the use of technology. The more experience teachers had, the more they began to believe that technology provided a fulcrum for change in teaching practice.

In Jost's (1992) study of the implementation of a calculus curriculum using programmable graphing calculators, five pilot sites were studied in a comparative case study approach focusing on in-depth interviews with the teachers at the sites as the main data source supplemented with analysis of documents (lesson plans, teacher journals, or other teacher-generated documents), classroom observations, and field notes from training workshops. Teachers' use of the technology could be expressed on a continuum. At one end of the continuum was the limited use made by the traditional lecturer who viewed the graphing calculator as a computational tool and was more concerned with making certain that the prescribed content was covered by the course. The other end of the continuum was the teacher who employed an interactive, inquiry-oriented style of presenting new information in which students were encouraged to question and actively participate in instruction. In this setting the graphing calculator became a natural part of the classroom activity for both the teacher and students. Students learned to use the graphing calculator as a learning tool.

A number of changes were noted in the curriculum as a result of implementing the use of technology in the classroom. Information about the use of the calculator, estimation techniques needed for determining if an answer was reasonable and for selecting ranges for graphing, and the inclusion of more realistic problems and examples were all benefits of the use of technology. The graphing calculator also opened up the possibility of teaching more in depth. The use of the graphing calculator necessitated changes in the types of problems used on tests and a reevaluation of objectives for students. Instructors were able to show more examples and students were able to solve more problems, more realistic problems, and problems that could be solved using alternate, graphical methods. Complicated functions whose graphs were tedious could be studied in more detail with the use of the technology. Students whose teachers used an inquiry, discovery, or interactive approach seemed to acquire a more intuitive understanding of calculus. Teachers found that the use of the calculator generated more group interactions. The use of the graphing calculator had a greater impact on how the curriculum was taught than on what was taught.

The findings of this study indicated that teachers did not make dramatic changes in their teaching styles. Teachers who had an interactive or inquiry style used the calculator more. Most of the teachers involved did not change their beliefs on teaching or learning as a result of implementing the graphing technology. Two teachers indicated that the use of the graphing technology raised questions concerning the introduction of new topics and the understanding and communication of certain topics. These teachers tended to be reflective professionals. The teachers involved in this implementation study all reported their school administration as being supportive. Most reported that there were considerable communications and positive interaction with other teachers at their schools. They also indicated that the training workshops and interaction with other teachers involved in the implementation were positive experiences.

Summary of Curriculum Change Literature

Curriculum change is a process in which the teacher plays a crucial role. Specific innovations can be designed for teachers, however the final implementation will be adapted by the teacher to fit into the teacher's world. In designing innovations, developers ought to proceed in dialogue with teachers, who will adapt any innovation.

In making curricular changes, the dilemmas faced by teachers need to be understood, while teachers need to understand the potential for new ideas and assess their value. A dialogue between developer and teacher can facilitate such understandings. Teachers as implementers tend to fit an innovation into their existing classroom structures and continue use of an innovation if they believe in its value and effectiveness.

The degree to which teachers reflected on their practices and the possibilities for an innovation are related to the success of implementation. It is in this reflexive conception of change (Olson & Eaton, 1987) that the key role of the teacher in the change process is illustrated. Teachers must find a ways of making new ideas work. Teachers find support important in the process (Lewis, 1988; Wiske et al, 1988).

Teachers' beliefs about an innovation impact the ways in which they make changes in their practices. While change in practice takes time, teachers who are reflective find more questions arising concerning their teaching practices (Jost, 1992). Beliefs about the role of the implementation of technology, as an example of innovation, may change with experience (Kerr, 1991).

Teachers' Thinking

The literature reviewed in the preceding section indicated that teachers' thinking was a key component in the curriculum change process. The following literature serves to explore teachers' thinking. Special emphasis is placed on the role of teachers' beliefs in their thinking. Research on teachers' beliefs and their classroom practices in mathematics illustrates the use of the theories about teachers' thinking and beliefs in investigating classroom practices.

Research on teacher's thinking was launched by Jackson's (1968) book Life in Classrooms reporting the results of his attempt to describe and understand the mental constructs and processes that underlie teacher behavior. In 1974 the National Conference on Studies in Teaching was convened to create an agenda for the future research on teaching. Panel Six of this conference produced a report (National Institute of Education, 1975) that developed a rationale for a proposed program of research on teachers' thought processes. In this report the panelists argued for the necessity of research on teachers' thinking in order to understand the process of teaching:

> It is obvious that what teachers do is directed in no small measure by what they think. Moreover, it will be necessary for any innovations in the context, practices, and technology of teaching to be mediated through the minds and motives of teachers. To the extent that observed or intended teacher behavior is "thoughtless," it makes no use of the human teacher's most unique attributes. In so doing, it becomes mechanical and might well be done by a machine. If, however, teaching is done and, in all likelihood, will continue to be done by human teachers, the question of the relationships between thought and action becomes crucial. (p. 1)

In the time period since the Panel Six report was written, research on teachers' thought processes has grown into a respected field of research on teaching.

Clark and Peterson (1986) divided teachers' thought processes into three main categories: (a) teacher planning (preactive and postactive thoughts), (b) teacher's interactive thoughts and decisions, and (c) teacher's theories and beliefs. Of particular interest to this study is the relationship between teachers' theories and beliefs and their instructional practices.

Teachers' Beliefs and Instructional Practices

Using the Repertory Grid Technique, Munby (1984) explored the beliefs and principles of one science teacher. He argued that in order to understand how a teacher might deal with an innovation, one must first understand the teacher's beliefs and principles. The focus of the qualitative method used in this study was on providing an individual teacher with opportunities to talk about fundamental beliefs and principles and on certifying the integrity of what emerged.

The teacher involved in this study, a female, had taught life science, general science, health, and physical education in grades six through eight during the preceding six years. During the year of the study she was teaching grade seven life science and grade eight earth science. She held a B.S. degree in biology and physical education and was certified to teach earth and life science. The teacher volunteered to participate in the study requiring two interviews. During the first interview, details of the teacher's professional background and experience were established. The remainder of the interview was used to elicit the teacher's beliefs about teaching. Statements that described generally what sorts of activities characterized her teaching, in her perspective, were recorded on cards by the researcher. The teacher then read over the recorded statements to ensure that her descriptions were preserved. These statements were labeled "elements." Next, the teacher was asked to group the elements in any way she wished. The actual way in which the cards were grouped was not important, rather the assumption was that the ways in which she characterized the cards within each group and distinguished one group from another substantially represented how she thought about her teaching. As the teacher discussed each group, the terms and phrases used to distinguish and characterize the groups were noted and she was asked to explain any that were unclear. These terms and phrases were labeled "constructs." A grid was then constructed, listing the elements on one axis and the constructs on the other. The teacher was asked to rate the association each element statement had for her with each construct phrase. A three-point scale, "3" definitely associated, "2" neutral, and "1" definitely not associated, was used for the ratings.

A factor analysis was used to analyze the construct-element grid in which the constructs were treated as variables with the elements being treated as subjects. The assumption was that variables (construct phrases) could be factored with expectation that the variables that exhibited some commonality would be grouped together. The factor analysis resulted in six groups of construct statements.

During the second interview, the teacher was asked to discuss why the statements in each of the six groups were grouped together. She was also asked to comment on where the central idea of each group might have originated. From the transcripts of this interview, the data collected through the previous interview, and the factor analysis, an effort was made to characterize the more significant beliefs of the teacher that drove her professional practice. The discussion during the second interview constituted in part the context for the labels of the groups and triangulation for the principles expressed. From the interviews and analyses, the teacher was characterized as being dominantly concerned for the students' confidence and increasing ability to handle information independently. She appeared to strive toward meeting these concerns by invoking instructional principles that arose from her own experiences, not from formal coursework. The origin of her principles appeared to be pragmatic not theoretical.

Munby concluded that the usefulness of this information was specific. The information was used to explain why this teacher used curriculum materials as she did, why she chose to adopt or reject certain instructional approaches. The knowledge gained about this teacher helps in the understanding of the particularities of unique professional practice.

Individual profiles of the conceptualizations of geometrical knowledge communicated through instruction, aims in teaching geometry, and evaluative assessments of students for four high school geometry teachers were constructed by McGalliard (1983). Extensive observations, interviewing, and teachers' written responses were used to collect data from which the profiles were formulated. A high degree of consistency between the conceptions of geometry and their instructional practices while teaching geometry was found. Based on their dualistic conceptions of mathematics, the teachers acted in "authoritative" ways regarding the content of their lessons, adopted a "right versus wrong" stance, and emphasized the use of rules without explanations or justifications.

The teachers emphasized the importance of memorizing answers and taking notes in class, thus promoting a belief in external authority as the source of mathematical justification. However, the teachers professed the belief that mathematics, especially geometry, helped promote students' logical thought processes. This discrepancy between instructional practices and professed beliefs may be explained by their statements concerning the necessity to complete the syllabus, urgency about preparing students for the next math course, and a need to cover the subject matter in consultation with other teachers. Apparently, the teachers allowed their desires to insure a smooth running course to overshadow their belief that geometry helps promote students' logical thought processes. By examining both the teachers' professed beliefs and their instructional practices, McGalliard found that there is a relationship between beliefs and instructional practices. Inconsistencies between the teachers' beliefs and practices appeared to be related to the constraints of their teaching situation.

Thompson's (1984) study investigated the conceptions of mathematics and mathematics teaching held by three junior high school teachers and examined the relationship between their conceptions and practices. Each teacher in the study was observed daily teaching a mathematics class over a four week period. During the first two weeks, the researcher conducted observations only. The initial two weeks of observations were designed to acquaint the researcher with the social context of the teacher and to allow the researcher to generate conjectures about what the teacher's conceptions might be, providing a sense of direction for future probing. From the initial two weeks of observations the researcher made inferences that led to a tentative characterization of the teacher's conceptions based only on instructional practices. This approach was designed to avoid the potential influence that the teacher's professed beliefs might have on the investigator's sensitivity to the different events observed. During the second two weeks, each observed lesson was followed by an interview. The interviews provided the opportunity to test the accuracy of the inferences made by eliciting relevant information. The inferred and the professed conceptions were then examined for consistency. In addition to the observations and interviews, six written tasks were administered during the study. These six tasks were designed to elicit information about teachers' beliefs about mathematics teaching and conceptions of mathematics. All new data obtained were examined in light of data previously collected. Tentative hypotheses and inferences were made from the collected data providing new foci for subsequent observations and interviews.

These case studies revealed differences among the teachers in specific beliefs, views, and preferences regarding mathematics and its teaching. In general the differences in the teachers' instructional practices could be related to differences in their prevailing views of mathematics. The teachers held differing views about the nature of mathematics ranging from a rather static body of knowledge consisting of logically interrelated topics to a more dynamic view of mathematics whose essential processes were discovery and verification. Variations existed within these views that contributed to differences in instructional practices. Teachers' views about the locus of control in the teaching process varied from a belief that students learned best by doing and reasoning about mathematics, to a view that the teacher's role was to demonstrate the procedures that the students were to use in performing the tasks in the daily assignments on which they worked independently, to a view that it was the teacher's responsibility to direct and control all classroom activities.

The integratedness of a teacher's beliefs and views was identified as the extent to which the beliefs interrelated and interacted to modify each other. Of the three teachers in this study, one teacher did not have an integrated conceptual system. Her view of mathematics as "cut and dried" was not consistent with her references to mathematical activities that called into play creativity and inventiveness. Apparently, these contradictory beliefs were held in isolation allowing her to mold her instructional practices in a manner that reflected her "cut and dried" image of mathematics. The inconsistencies lay in the relationship between her expressed views about mathematics teaching (using creative activities) and her instructional practice that was primarily a lecture approach.

The teacher who demonstrated the most integrated system of beliefs about mathematics and mathematics teaching often qualified her beliefs in light of her teaching experience and other views she had expressed. These references to her experiences and other beliefs were an indication of the reflectiveness of this teacher. The reflectiveness of this teacher and the integratedness of her beliefs indicated a relationship between the reflectiveness of the teacher and the integratedness of that teacher's belief system. This integratedness of conceptions seemed to contribute to consistency between professed views and instructional practices.

The relationship between teachers' conceptions of mathematics and mathematics teaching is a complex one. The findings of this study indicated that teachers' beliefs, views, and preferences about mathematics and its teachings, whether they are consciously or unconsciously held, played a significant role in shaping the teachers' characteristic patterns of instructional behavior. The teachers in this study also demonstrated that they held conceptions about teaching that were general and not specific to the teaching of mathematics. In some cases, these conceptions about teaching in general took precedence over other views and beliefs specific to the teaching of mathematics.

Cases studies of three high school teachers were conducted by Grant (1984) to investigate their beliefs about the purpose of mathematics teaching, the conditions of mathematics learning, and the nature of mathematics. The study also investigated the extent to which these beliefs were reflected in the teachers' practices. Data were gathered over a six-week period through classroom observations and conversations with the teachers. Grant found, in general, that the teachers' beliefs were congruent with their teaching practices. In one case, deviations from stated beliefs occurred when the teacher focused on time constraints in respect to course coverage. In another case incongruity between stated beliefs and beliefs in practice occurred through gaps the teacher found between his expectations and the actual results of his teaching behavior. All three of the teachers involved indicated that the study had a positive effect on their tendency to reflect on their teaching. The teachers indicated that they had not previously reflected on their teaching with any degree of seriousness.

Cooney (1985) and Brown's (1986) study of a beginning teacher's view of problem solving revealed conflicts between the teacher's idealism and the reality of classroom practice. Interviews were conducted with this teacher seven times during the winter and spring of his preservice training. The position was taken that preservice teachers were not likely to have well-articulated theories about teaching, but they may have implicit theories that could be revealed given appropriate stimuli. Episodes varying in open-endedness, voice of the expected response (e.g., sometimes the teacher assumed he was responding as himself, sometimes as someone else), and in realism were used to elicit information about the teacher's implicit beliefs about teaching mathematics. The first two interviews dealt entirely with preselected episodes such as: describe a particular anecdote during your student teaching, or if you could be another person when teaching, whom would you pick, why? The third and fourth interviews focused on elaboration of discussions from the first two interviews and on episodes suggested by those discussions. During the fifth interview, after reviewing transcripts of the first four interviews, the teacher was asked to identify his statements found in the transcripts that captured what he felt were important aspects of his beliefs about mathematics and its teaching.

During the sixth interview, a clustering technique was used to structure the teacher's beliefs. The statements identified in the fifth interview were written on cards by the investigators. The teacher was asked to group the cards into categories of his own choosing. The criteria used to group the cards was entirely the teacher's own. Once the cards were grouped, the teacher was asked to create a title or heading for each group and a brief sentence to capture the essence of what the individual cards seemed to be expressing. The clustering was done to help identify statements and subsequently beliefs that the teacher thought were important. The statements made by the teacher combined with the titles and descriptors played a major role in the analysis of what he believed about mathematics and its teaching. The final interview focused on the origins of the teacher's ideas and beliefs. Although the intent of the study was not to focus on the teacher's beliefs about problem solving, his repeated references to problem solving indicated it was central to his view of mathematics and its teaching. Problem solving was, therefore, made the primary focus of subsequent analyses.

During the following summer a report was written, based on the seven interviews, that attempted to capture the essence of the teacher's beliefs. In the fall the teacher was given a copy of the report and asked for his reaction to it. The purpose of this inquiry was to help validate the investigators' impressions of the teacher's beliefs. The teacher indicated that the report captured what he was all about, but he was more interested in using the report as a basis for talking about his first weeks of teaching. The teacher had been teaching for several weeks in a small high school at which he was the only mathematics teacher. Beginning a week after the inquiry interview, the investigators observed the teacher's classes on nine consecutive mornings. Stimulated recall interviews were conducted based on the preceding classroom observations. A more general interview was conducted at the conclusion of the observation period. Several students from the teacher's classes were also interviewed.

This beginning teacher professed beliefs that the principal activity of mathematics was problem solving and that a central point of teaching problem solving was teaching heuristics, yet his instructional practices did not always reflect these beliefs. He expressed frustration over the extensive time demands of a problem-solving orientation and confessed that "it is much easier to teach by the book, so to speak, and leave heuristics out completely" (Cooney, p. 330). He did attempt to actualize his beliefs in problem solving by beginning some lessons with interesting problems he created from his experiences. In spite of his introductory problem solving approach, he reverted to instructing his students to follow fixed procedures. The treatment of the solution method was anything but problematic. The reactions of the students were discouraging to the teacher. When the teacher attempted to introduce problems that he viewed as "interest creators," the students indicated that their time was being wasted. On another occasion the students were unable to make the connections between some experiments with dice and the object of the lesson. The teacher's view of problem solving as recreational or extracurricular created difficulties for the integration of problem solving into his teaching. This teacher seemed to have a notion that problem solving was a layer of mathematics that could be separated from the content. Further, he saw that teaching creatively, using problem solving, was hard and that it was easier to fall back onto teaching by the book. The implication was that the content of the book was nonproblematic and that to teach creatively required the creation of a new curriculum.

The beginning teacher in this study was faced with the dilemma of balancing authoritarianism and problem solving. This dilemma revealed the tensions that existed between the teacher's idealism and the reality of classroom life. The ways in which the realities of the classroom affected the teacher were not clear. The outcome of the struggle between ideals and realities influenced, even determined whether professional objectives were realized.

An investigation of the relationship between teachers' conceptions of mathematics and teaching and their level of dogmatism was conducted by Kesler (1985). Four high school algebra teachers were each studied for five weeks. Data collection was based on participant observation, audiotaped records of the teaching sessions, fieldnotes, nonstandard interviews with the teachers, and two written instruments. Kesler found that teachers' conceptions of teaching and mathematics were related to their instructional behavior. The teachers' conceptions of mathematics differed, ranging from a dualistic conception to a multiplistic/relativistic conception. Similarly, the teachers' instructional practices differed, ranging from strict authoritarian to an inquiry mode of presentation. The two teachers who held dualistic conceptions of mathematics demonstrated instructional practices that were consistent with their beliefs while the instructional practices of the two teachers who held multiplistic/relativistic conceptions of mathematics were not consistent with their beliefs. The findings of this study indicated that in spite of differences in beliefs and similar differences in instructional practices between teachers, consistency can exist between beliefs and practices.

Carpenter (1988) presented a general model for research and curriculum development (Figure 2) that served as the framework for a program of research conducted by Carpenter, Fennema, and Peterson based on the premise that teaching is problem solving.

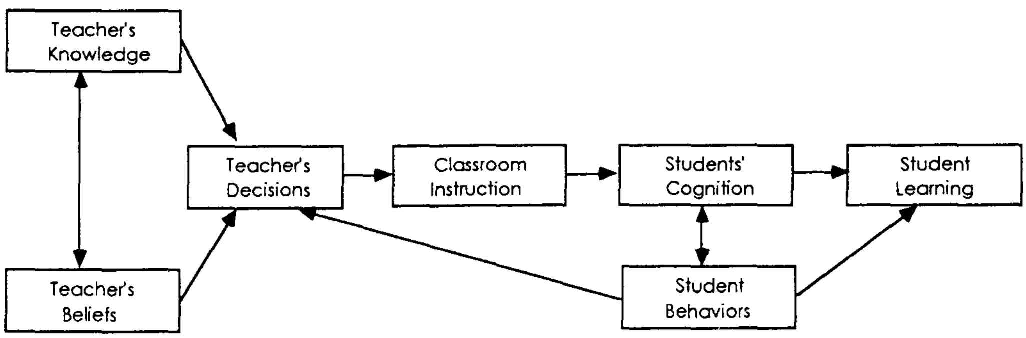


Figure 2. Model for research and curriculum development.

This model assigned a central role to teachers' and students' thinking. In this model classroom instruction was based upon teachers' decisions. As indicated in the model, teachers' decisions are presumed to be based upon their knowledge and beliefs as well as their assessment of students' knowledge through their observation of students' behavior.

The researchers applied this model to the study of instruction in addition and subtraction in first grade. All aspects of the model were analyzed. Of particular interest to the current study were the methods employed for the study of teachers' beliefs and classroom instruction. Teachers' beliefs were assessed in terms of some of the fundamental assumptions underlying the constructivist perspective and the researchers' analysis of how it should be applied to instruction. The researchers constructed four belief scales: (1) from the belief that children construct knowledge to the belief that children receive knowledge, (2) from the belief that instruction should facilitate children's construction of knowledge to the belief that the teacher should present knowledge, (3) from the belief that skills should be related to understanding and problem solving to the belief that skills should be taught in isolation, and (4) from the belief that the natural development of children's mathematical ideas should provide the basis for sequencing instruction to the belief that the sequence of instruction should be based on the formal structure of mathematics. Teachers' beliefs were evaluated using these four scales through observations and structured interviews.

Classroom instruction was studied using separate coding systems for teachers' actions and students' behaviors. The coding systems included categories for mathematics content and the strategies used to solve problems that were derived from previous analysis of children's solutions to addition and subtraction problems. The coding was able to pick up the relative emphasis on word problems and distinguish between four distinct categories of word problems. The coding system for teachers' behaviors was also designed to identify teachers' attempts to diagnose their students' understandings. The coding system was able to distinguish between teachers' actions that focused on answers to problems and teachers' actions that focused on the processes that students used to get answers.

The primary thesis of the model of research and curriculum development used by Carpenter, Fennema and Peterson was that teaching was problem solving. Rather than attempting to derive prescriptions for teaching, the focus was on teachers' knowledge and beliefs and how teachers solved problems of instruction.

Three experienced middle school teachers participated in a study conducted by Shaw (1989) to examine the relationship between teachers' ideal and actual beliefs about understanding. The teachers were selected from an inservice mathematics education course that emphasized teaching for understanding. Data were collected through daily observations, daily interviews, and three questionnaires from which the teachers' beliefs were inferred. During the three week observational period the teachers were given frequent opportunities to respond to the researcher's analysis of their beliefs. Convictions of how the teacher would like to teach for understanding and ideally would like students to learn constituted ideal beliefs. Actual beliefs consisted of convictions of how the teacher actually needed to teach for understanding and how students actually needed to learn. Shaw found that teachers held ideal clusters of beliefs about understanding that were different from their actual clusters of beliefs. He identified several contextual factors that kept the teachers from incorporating their ideal beliefs in the classroom: how the teachers learned mathematics, how they had been teaching mathematics, their students' backgrounds and goals for learning mathematics, standardized tests, administrative demands, textbooks, and time.

Ernest (1989) made a distinction between the teacher's thought processes such as planning, interactive decision making, and reflection, and the thought structures of the teacher including the knowledge, beliefs, and attitudes stored in the mind of the teacher. He presented a model of the permanent but ever-changing and growing body of knowledge, beliefs, and attitudes of the mathematics teacher as the sources of the constructs, relations, procedures, and strategies through which the teacher's thought processes operated. Because the focus of the current study was on teachers' beliefs and their relationship to instructional practice, the portion of Ernest's model dealing with beliefs is discussed. Beliefs, in this model, included conceptions of the nature of mathematics, models of teaching and learning mathematics, and principles of education.

Teachers' conceptions of the nature of mathematics were their belief systems concerning the nature of mathematics as a whole. While such belief systems formed the basis for a philosophy of mathematics, some views held by teachers might not have been developed into articulated philosophies. Teachers' conceptions of the nature of mathematics did not have to be consciously held views, they might have been implicit philosophies. Three major philosophies of mathematics had been observed in the teaching of mathematics. First was the view that mathematics is a dynamic, problem-driven continually expanding field of human inquiry. This view, referred to as the problem-solving view, held that mathematics is not a finished product, its results remain open to revision. The second prevalent view was of mathematics as a static but unified body of knowledge, consisting of interconnecting structures and truths. Mathematics is static and is discovered but not created. This view was the Platonist view. The third view, the instrumentalist view, was that mathematics is a useful but unrelated collection of facts, rules, and skills.

Teachers' beliefs about the nature of the teaching and learning of mathematics constituted their models of teaching and learning mathematics. These models had a powerful impact on the way in which mathematics was taught in the classroom. Ernest presented six simplified models of mathematics teaching based on the types and ranges of teaching actions and classroom activities found in prototypical mathematics classrooms. These six models are: (1) the pure investigational, problem posing, and problem solving model, (2) the conceptual understanding enriched with problem solving model, (3) the conceptual understanding model, (4) the mastery of skills and facts with conceptual understanding model, (5) the mastery of skills and facts model, and (6) the day to day survival model. Given the contextual constraints that must be accommodated within any school situation, the teacher's mental model of mathematics teaching was the key determinant of how mathematics was taught.

Teachers' mental models of the learning of mathematics were closely associated with their models of the teaching of mathematics. Teachers' mental models of the learning of mathematics consisted of their views of the process of learning mathematics, what behaviors and mental activities were involved on the part of the learner, and what constituted appropriate and prototypical learning activities. Two key constructs, on which the range of models of learning mathematics were based, were: a view of learning as the active construction of knowledge as a meaningful connected whole versus a view of learning mathematics as a passive reception of knowledge; and the development of autonomy and the student's own interests in mathematics versus a view of the learner as submissive and compliant. Using these two constructs, six simplified models of learning mathematics were described: (1) student's exploration and autonomous pursuit of own interests model, (2) student's constructed understanding and interest driven model, (3) student's constructed understanding driven model, (4) student's mastery of skills model, (5) student's linear progress through curricular scheme model, and (6) student's compliant behavior model. The teacher's model of learning mathematics, as it was realized in the classroom, was an important factor in a student's experience of learning mathematics.

Teachers' principles of education were the general values, beliefs, and principles underpinning their views of the aims and purposes and nature of education. Teachers possessed specific principles concerned with the teaching of mathematics such as a commitment to give every student the experience of success and confidence in mathematics. The effect a teacher's principles exerted on teaching depended very much on the extent to which the teacher's beliefs and actions formed an integrated whole. For principles to be effective they must have been linked with the teacher's models of teaching and learning as well as with their actual practices of teaching. Principles, beliefs, and actions were linked through planning and reflection.

Teachers' views of mathematics provided a basis for their mental models of the teaching and learning of mathematics. Views of the nature of mathematics were likely to correspond to views of its teaching and learning. For example, the instrumentalist view of mathematics was likely to be associated with the transmission model of teaching and the students' compliant behavior and mastery of skills. Other such associations were conjectured. Teachers' mental or espoused models of teaching and learning mathematics, subject to the school context, were transformed into classroom practice (enacted model). The espoused and enacted models of teaching and learning held by a teacher could differ. There were three possible causes for these differences. First was the depth of the espoused beliefs. If espoused beliefs were not richly connected to other beliefs and knowledge, only a limited basis for their enactment existed. The second possible cause for differences between espoused and enacted models of teaching was teachers' levels of consciousness of their own beliefs and the extent to which teachers' reflected on their practice of teaching. The third possible cause for these differences was the social context including the expectations of others, especially teachers and administrators.

The beliefs and behaviors of four third and fourth grade teachers were investigated by Carter (1992). She found that the teachers had four fundamental common beliefs about how children learn mathematics: (1) children learn mathematical concepts by manipulating or visualizing concrete materials, (2) children learn arithmetic through specific sequenced steps, (3) children learn mathematics through practice and repetition, and (4) children learn mathematics best when they feel good about themselves and experience success in mathematics. The teachers had one or more factors associated with each belief that they considered when planning mathematics lessons. They demonstrated a variety of classroom behaviors that were concomitant although not always congruent with each belief. Discrepancies between teachers' classroom behaviors and their beliefs were most commonly because of the pressures of time and curricular expectations. Teachers tended to rely on the textbook rather than build upon the strength of their own convictions and beliefs about how children learn.

Change in Teachers' Beliefs and Instructional Practices

Thompson's (1988) study was designed to document changes in the conceptions of mathematical problem solving of 16 elementary school teachers over a three-week summer course on problem solving and a year of teaching problem solving in their classrooms. The summer course focused on principles of heuristic teaching in mathematics. The main purpose of the course was to enhance teachers' confidence and competence in solving problems, in the use of heuristics, and in the use of pedagogical techniques for enhancing students' problem-solving performance and mathematical thinking skills. During the class sessions, time was devoted to posing and solving problems. Initially the focus was on modeling the use of heuristics in solving nonroutine problems. Eventually, the teachers led class discussions on the problems that had been posed. Additionally, time was spent dealing with pedagogical methods and issues related to problem solving. Issues dealt with in these discussions included the role of the teacher in problem solving, planning, using instructional resources, evaluating students' problem-solving performance, and using assessment methods and instruments that support a problem-solving teaching approach. Often the problems posed and solved entered into the discussions on pedagogical issues. Teachers were given readings throughout the course.

Data were gathered through three questionnaires that were administered at the beginning and end of the summer course and at the end of the school year following the course, teachers' daily journal entries, informal interviews, classroom observations, and four follow-up sessions held throughout the school year. Teachers indicated that the readings and discussions provided them with terminology that enabled them to make distinctions among categories of problems. They indicated that this terminology was useful, especially in planning for the inclusion of different types of problems in their teaching. Teachers indicated an emerging notion of problem solving as a general process for generating mathematical knowledge. Teachers reported feeling more confident to teach problem solving and more knowledgeable of ways to help students.

From the questionnaire administered at the beginning of the summer course, data were obtained on the teachers' problem solving teaching. Of the 16 teachers, four reported teaching problem solving approximately once a week, two utilizing supplementary materials. Four teachers indicated having taught a separate unit on problem solving. The remaining eight teachers indicated that they did not teach problem solving per se, but occasionally assigned word problems from the textbook. Observations were arranged so that each teacher was observed teaching a nonroutine problem, a word problem, and a class in which students were engaged in some independent or small-group problem-solving activity. Results of observations of 14 of the teachers were available, two teachers were transferred midyear. The data from the observations and teachers journals showed that six teachers taught problem solving in a systematic way, allowing for some type of daily activity in problem solving. Three of the remaining eight teachers taught problem solving two or three times a week. The remaining five teachers taught problem solving on an irregular basis. While there were some changes in the problem-solving teaching practiced by these teachers, some teachers remained rigid in their approaches to teaching problem solving.

In discussing the results of this study Thompson pointed out that one feature of the course that seemed essential for broadening the teachers' conceptions of the nature of problem solving was their active involvement in solving a wide variety of problems and reflecting on their attempts to solve them. The modeling of teaching techniques followed by discussion of the rationale for their selection and use, as well as the readings provided for the teachers served to provide opportunities for the teachers' involvement and reflection. This study found that it was possible for teachers to make changes in their instructional practices. A connection was found between these changes in instructional practices, changes in teachers' conceptions, and reflection by teachers on their conceptions and instructional practices.

Through research attempting to coordinate a constructivist view of learning mathematics with the practice of teaching with the purpose of analyzing children's mathematical learning, Cobb, Wood, and Yackel (1990) engaged in an examination of the changes in beliefs of a classroom teacher. The study was designed to analyze young children's mathematical learning in a classroom where instruction was broadly compatible with constructivism. The research took place in second grade classrooms.

The first year of the study involved a single classroom. The classroom teacher was a teacher/researcher. In the spring prior to the study, the researchers met with the teacher weekly to orient her to the aims of the research. Through these weekly meetings the researchers discovered that the teacher held beliefs about her teaching that were in conflict with the design of the study. The researchers engaged the teacher in dialogue concerning her teaching practices and encouraged her to conduct interviews with her current students that would reveal the conflicts. Through these experiences the teacher began to realize that her established teaching style might be problematic. As the study progressed, the teacher worked to resolve the conflicts she found between her established practices and classroom norms she came to believe were desirable. The attempts made by the teacher to resolve these conflicts provided learning experiences for her. Further learning opportunities arose as she encountered unanticipated problems and made observations that were surprising to her. In order to make sense of what she saw in the classroom, the teacher had to reorganize her beliefs about the teaching and learning of mathematics. Through experience and reflection this teacher made changes in her beliefs and instructional practices.

The next phase of the research involved inducting additional teachers into the program. Based on their experiences with the initial teacher, the researchers planned opportunities for the new teachers to discover conflicts between their established classroom practices and the project. The researchers provided a summer institute during which the teachers had opportunities to begin to question their current practices and thus had a reason to consider an alternative approach. Additionally, examples of the teaching of the initial project teacher, via videotape and demonstration, were provided. Opportunities were also provided for the new teachers to experiment with these techniques. During the school year, the researchers provided support to the new teachers through classroom visits and a series of working sessions focused on specific teachers' concerns. The provision of these opportunities for the newly inducted teachers to interact with the project was consistent with the researchers' belief that attempts to influence teachers' knowledge and beliefs would not be at their most effective unless they drew upon teachers' first-hand experiences.

Cobb, Wood, and Yackel found that it was important to help teachers develop personal, experientially-based reasons and motivations for reorganizing their classroom practices. They found that beliefs and practice were dialectically related. Beliefs were expressed in practice. Problems or surprises that were encountered in practice gave rise to opportunities to reorganize beliefs.

In a study of the beliefs and instructional practices of college instructors during the initial implementation of graphing calculators into the teaching of first term calculus, Barton (1995) found that beliefs concerning the utilization of graphing calculators and the teaching of calculus could change. The study involved observations and interviews with five college instructors beginning the implementation process. These instructors were observed approximately weekly during the first term of graphing calculator utilization.

They were also interviewed formally before and after the term and briefly through informal conversations taking place throughout the term. Through these interviews and conversations the instructors' beliefs were ascertained both before and after their initial experiences with graphing calculators. The beliefs held by the instructors before they utilized the graphing calculators in their teaching were compared with their beliefs at the end of the term.

Barton found some change in the beliefs of the instructors, especially among those who had been most skeptical about the value of the graphing calculators. Instructors who had been skeptical about the use of the graphing calculator before utilization found their use to be beneficial and worthwhile. Differences in the use of the graphing calculator were also found with different teaching approaches. Theoretical and procedural approaches did not incorporate the calculator in the lesson as much as investigatory or conceptual-oriented approaches.

Barton found that extensive training in operating the calculator and incorporating the technology tool when teaching was important for the teachers. She concluded that sharing of teaching experiences as well providing further training were essential to the successful implementation of graphing calculators in the teaching of college calculus.

Summary of Research on Teachers' Beliefs and Instructional Practices

Studies on teachers' beliefs about mathematics teaching included investigation of teachers' views about the nature of mathematics, the teaching of mathematics, and the learning of mathematics. These studies showed that teachers held a variety of different beliefs (Thompson, 1984; Kesler, 1985). These beliefs were held at conscious or unconscious levels.

When teachers' beliefs and instructional practices were examined together, there was often consistency between beliefs and practices (McGalliard, 1983; Thompson, 1984; Grant, 1984; Carter, 1992). The differences in the beliefs of teachers were related to differences in their instructional practices (Thompson, 1984). Conflicts and discrepancies did occur between the professed or ideal beliefs of teachers and their instructional practices. There were a number of factors that contributed to the discrepancies between beliefs and practices. Curricular constraints including pressure to cover a certain course content (Grant, 1984; Shaw, 1989; Carter, 1992), time pressures (Cooney, 1985; Brown, 1986; Carter, 1992), and students' backgrounds and expectations (Cooney, 1985; Brown, 1986; Shaw, 1989) were contextual factors that contributed to these discrepancies. These discrepancies were sometimes reflected in a dependency on the textbook rather than reliance on convictions or beliefs (Cooney, 1985; Brown, 1986; Carter, 1992). Conflicting clusters of beliefs could be held in isolation (Thompson, 1984) making it possible for a teacher's instructional practices to be consistent with some beliefs and inconsistent with others.

Reflectiveness was shown to be related to the degree of integratedness of a teacher's beliefs (Thompson, 1984). Reflectiveness could be facilitated in teachers (Grant, 1984; Thompson, 1988; Cobb, Wood, & Yackel, 1990). By increasing the reflectiveness of teachers it was possible to improve the level of consistency between beliefs and instructional practices.

The relationship between teachers' beliefs and instructional practices was not linear. There appeared to be a dialectical relationship between teachers' beliefs and instructional practices (Figure 3). Teachers' beliefs appeared to act as filters through which teachers

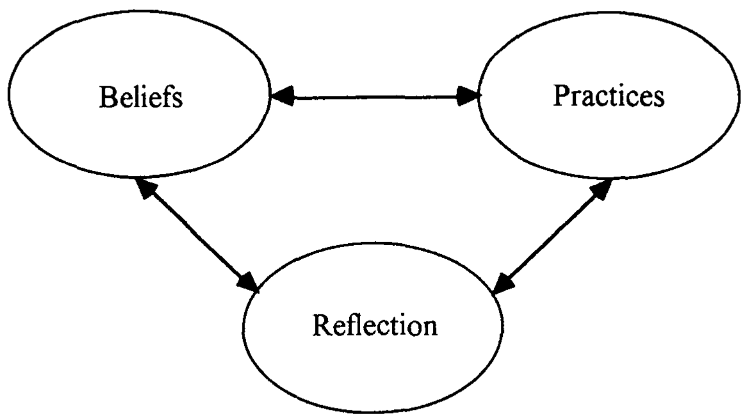


Figure 3. The dialectical relationship between beliefs, practices, and reflection

interpreted and ascribed meaning to their experiences. At the same time, a teacher's beliefs and views seemed to originate in and be shaped by experiences in the classroom including their practices (Cobb, Wood, & Yackel, 1990). Teachers appeared to evaluate and reorganize their beliefs through reflective acts (Thompson, 1984), some more so than others. Thus, teachers' beliefs, instructional practices, and reflection on beliefs and practices interacted shaping one another.

Research on the effects of graphing calculators on students' achievement in precalculus mathematics did not provide enough evidence of improvement in students' performance for teachers to change their established teaching practices. The research did indicate that there were some benefits to incorporating graphing calculators into the teaching of mathematics, both for students and teachers. Changes in teachers' classroom practices were indicated in some of the studies on the incorporation of graphing technology. When utilizing new technology required teachers to make changes in their established practices, the beliefs of the teacher played an important role in determining the extent to which the teacher would make any changes. Teachers' instructional practices were related to their beliefs about mathematics, its teaching, and learning. By reflecting on these beliefs and experiencing unexpected situations, teachers could reorganize their beliefs. Reorganization of their beliefs could lead teachers to make changes in their classroom practices.

CHAPTER THREE
DESIGN AND METHOD

Introduction

This study explored the beliefs and classroom practices of high school Algebra II teachers who have persisted in the use of graphing technology, incorporating its use into their teaching. The beliefs of these teachers concerning the role of graphing calculators in the teaching of high school algebra, appropriate teaching practices in a curriculum utilizing graphing calculators, benefits to students from utilizing graphing calculators, curriculum implications of the use of graphing calculators, and teachers' roles in a graphing calculator enriched classroom were examined. Teachers' beliefs were compared with their observed classroom practices in order to explore the relationships between teachers' professed beliefs and practices in a graphing calculator enriched classroom. In addition, a rich description of the classroom activities and teacher-student interactions found in these settings was developed. The examination of teacher's beliefs and practices when utilizing graphing technology in the teaching of second year algebra focused on the following questions:

(1) What are the classroom practices of teachers who have persisted in the use of graphing calculator technology in their teaching?

(2) What are the beliefs of teachers who have persisted in the use of graphing calculator technology in their teaching?

(3) What is the relationship between teachers' professed beliefs and their classroom practices?

(4) Do teachers who persist in the use of graphing calculators do so from a constructivist perspective?

(5) Are the activities found in these classrooms consistent with the goals of the current curriculum reform movement?

The investigation of these questions called for a qualitative research design enabling the researcher to explore the setting in detail and look for features that might be overlooked in a study designed to examine certain specific predetermined qualities, characteristics, or activities.

The Subjects

In order to examine the relationships between teachers' beliefs and instructional practices and develop a rich description of the instructional practices of teachers who had incorporated graphing calculators into their teaching, the criteria of having used graphing calculators in teaching a high school mathematics course for a minimum of three years previous to the 1995-96 academic year and in Algebra II for at least one prior year was employed. Changes can and do occur in teachers' beliefs and practices during the implementation of a new innovation (Kerr, 1991; Jost, 1992; Barton, 1995). Thus, the criteria of persistence in the use of graphing calculators was designed to allow examination of established teachers' beliefs and instructional practices rather than those during an implementation period which might be in a state of transition.

In order to compare and contrast the beliefs and practices of teachers dealing with the same general curricular expectations, all teachers involved in the study were teaching the same course, second year algebra. In the fall of 1994 the researcher conducted a survey of graphing technology use at high schools within the researcher's area. According to the data from the survey (see Appendix A for the survey and a summary of the results), graphing calculators were being used in the teaching of mathematics courses ranging from Pre-Algebra to Calculus. Graphing calculators were first introduced in Precalculus and Calculus courses with use spreading downward through the curriculum with common use in Algebra II and less common use in Algebra I. Most of the existing studies on graphing calculator use were conducted in Precalculus and Calculus classes, however the enrollment in these classes is much smaller than in lower level courses and fewer teachers teach these higher level courses. In order to study high school mathematics teachers who were more representative of high school mathematics teachers in regard to their teaching assignments, this study focused on teachers utilizing graphing calculators in the teaching of Algebra II or equivalent year-long courses.

High school mathematics teachers were eligible to participate in the study based upon meeting the persistence criterion in the use of graphing calculators in their teaching, having experience in teaching Algebra II with graphing calculators, their willingness to participate in the study, and the willingness of their school district to allow their participation. Additionally, the study required that graphing calculators be available to all students in Algebra II classes at all times. This requirement was designed to assure that there would be a degree of consistency in the ways in which graphing calculator were incorporated in the teaching of Algebra II.

From the responses to the graphing technology use survey (see Appendix A), 14 schools were identified that met the persistent graphing calculator use in the teaching of Algebra II criterion. Correspondence with the mathematics departments of these schools was conducted during August and September 1995 soliciting potential participants for the study. Additional data about the curricular materials used, scheduling of classes, and general data about the potential participants (gender, experience teaching high school mathematics, age) were solicited to be considered in the selection process. Additional referrals from teachers in these schools expanded the pool of available candidates for the study.

In order to develop a diverse sample and a manageable study size, four teachers were selected to participate from the pool of eligible teachers. Two teachers were selected from the same small, suburban school district in a large metropolitan area. Choosing two teachers from a school district provided comparisons of teachers utilizing identical curricular materials and teaching under the same school district and community expectations. Conducting the study with teachers in three areas with different demographic characteristics allowed for comparisons of beliefs and practices in settings that utilized different curricula and operated within differing community value systems. Data collected concerning each teacher, while unique, contributed to form a set of findings that can be applied in a variety of situations. The observations and analysis of one teacher's classroom practices and beliefs confirmed by observations and analysis of a second, third, or fourth teacher where it is suspected that the same results should occur, will support the application of the results for a much larger number of similar situations (Bogden & Bicklen, 1992; Yin, 1989).

Selection of schools at which the study was conducted took place in conjunction with the selection of the specific teachers who were the subjects of the study. All teachers involved in the study met the persistence criteria, were teaching Algebra II or an equivalent course utilizing graphing calculators, and had previous teaching experience with graphing calculators in the Algebra II course. A diverse sample of teachers were selected by choosing teachers and schools that provided variety in the following: (1) curricular materials used (one school used Algebra 2 with Trigonometry, published by Prentice Hall, the other schools used Advanced Algebra, published by Scott Foresman (hereafter referred to as the Chicago materials)); (2) location (one school was from a mid-sized city, two from a suburban school district, and one from a private school drawing from a large urban area); (3) daily class schedules (one school followed a traditional 50 minute per class per day schedule, two followed a 90 minute per class session every other day, and the fourth followed a combined 50 minute three days a week, 90 minute one day a week schedule); (4) gender of teachers (two females and two males); (5) teaching experience of teachers (teaching experience ranged from 17 to 34 years of experience); (6) type of school (three public schools, one private). The sample included teachers at schools located within a convenient distance from the researcher to allow research at more than one location during the same time period.

Permission to conduct the study at each school was obtained from the appropriate administrators in each school district. Approval of the building principal and district administration to pursue the study were obtained. In one of the schools the teacher involved made all the necessary contacts with the administrators. In the second school, a meeting was held with the teacher, the headmaster, and the researcher at which the purpose of the study was discussed and permission was granted. In the third setting, both schools from the same district, a letter explaining the purpose of the study was sent to a district administrator (see Appendix B). Permission was granted for the study with the understanding that parental permission for classroom videotaping would be secured. Parental permission slips were distributed to and collected from students in each of these classes (see Appendix C). Each teacher involved signed an informed consent (see Appendix D) before data collection began.

Method

The review of research examining the effects of using graphing technology on student performance has shown that research on student outcomes cannot control or account for the multitude of variables such as differences in teaching style and choice of curriculum materials involved in the implementation of graphing calculators. The focus of this research instead was to ask "why" and "how" questions about the implementation of graphing calculators; thus, a case study approach was best suited for the research. Through in-depth interviews and classroom observations, the beliefs and classroom practices of teachers persisting in the use of graphing calculators were examined in detail. The case study approach has the advantage of allowing the discovery of events or processes that might be missed with more superficial methods such as standardized techniques for surveying classroom interactions (Biddle & Anderson, 1986; Yin, 1989).

In addition to utilization of three districts and four teachers for this research, multiple sources of data collection were employed to provide evidence that the conclusions drawn were not subject to the biases of the researcher and to assure the accuracy of the findings. Several forms of data collection (in-depth interviews, observations, and collection of documents) were utilized. The data gathered by one method was used to check the accuracy of data gathered in another way. Interviews, the primary source of information about teachers' beliefs, were audiotaped or videotaped depending on the nature of the interview. All interviews were open-ended in nature seeking to elicit both facts and opinions from the participants. Direct observation by the researcher was used as a primary data source for information about teachers' practices. These observations, approximately half of which were videotaped, served to provide detailed information about the activities in the classrooms under investigation. Documents used in the classrooms were collected. Documents in this study were used to corroborate and augment data from other sources. The use of these multiple sources of evidence provided a means of exploring convergence between evidence from each source. After data collection began, information from one data source was used to suggest new questions for study which were investigated using the other data sources.

For this study, each teacher was considered as a single case. The data collection and analysis for each teacher was conducted separately. Data collection and analysis for more than one teacher was conducted simultaneously, however the data for each teacher was analyzed separately. Once all data for all teachers was analyzed separately, similarities among and differences between the teachers were analyzed.

Data Sources

Data collection from three sources, interviews, teachers' documents, and direct observation, took place in three phases. The initial phase was a pre-observational interview during which data were collected concerning the background of the teacher and the characteristics of the school. The observational phase was the second phase of data collection. During the observational phase, data were collected primarily through observations by the researcher of the teacher in the classroom. During this phase, informal interviews were also conducted with the teacher and documents were collected. The final phase involved a pair of interviews designed to assess the explicit and implicit beliefs of the teacher and to allow the teacher the opportunity to comment on and clarify the results of the initial analysis of data. All interviews and observations were taped and the tapes were transcribed for analysis. Fieldnotes were taken at each interview and observation and were transcribed as soon after taken as possible to enable the researcher to interpret and augment any comments made in the notes.

Background Interview

A focused interview took place with each participating teacher before the period of classroom observations. This interview served to introduce the researcher and the purpose of the study to the participant, to obtain background information on the participant, and to acquaint the researcher with the participant and the participant's teaching situation. This interview was audiotaped and transcribed to assure that all information was captured accurately.

The background interview focused on establishing a working relationship between the researcher and the participant. Specific interview questions (see Appendix E) were designed to obtain background information about the teacher including preservice training, prior experiences in teaching, current teaching situation, inservice training, professional involvement outside the classroom, the teacher's training experiences related specifically to the implementation of graphing calculators in teaching. Additional questions were designed to acquaint the researcher with the teacher's classroom practices. Several questions explored the importance the teacher placed on the use of technology in teaching. Each interview varied depending on the responses given to specific questions and the direction the researcher pursued based upon specific responses. While the teacher's beliefs were not the focus of this interview, any comments the teacher made about classroom practices were recorded. When reviewing transcripts of this interview, specific statements about mathematics, teaching, and using graphing calculators made by the teacher were recorded by the researcher on 3" x 5" cards to be used for clustering of beliefs.

Observational Phase

Teachers' classroom practices were the focus of data collection during the observational phase. Data were collected through observations in the classrooms, informal interviews, and documents. During the observational phase, all class sessions of at least one class taught by each teacher were observed with documents utilized for that class collected and catalogued. Additional classes taught by each teacher were observed as possible in the researcher's schedule. Fieldnotes were taken by the researcher throughout the observation period.

Observations. Each teacher's classroom practices were observed for approximately a four-week period including one complete unit. For teachers who were teaching more than one section of Algebra II, one section was chosen as the primary focus of the research. Each section was observed in order to broaden the observation data base. In order to capture classroom practices that utilized the graphing calculator, specific units of study that lent themselves to the use of graphing calculators were observed. Three of the teachers were observed teaching units on systems of equations. The fourth teacher was observed teaching a unit on functions.

Observations began as soon as possible after the completion of background interviews. One entire unit of study or a minimum of two weeks of class sessions were videotaped including introductory and assessment activities. Fieldnotes were taken during each session. Observations were scheduled so that the researcher observed several class sessions before the beginning of the unit that was videotaped. The videotapes and fieldnotes were transcribed. The focus of the videotape was the teacher in order to make a record of the teacher's naturally occurring classroom practices. For each class videotaped, the taping began before the class began and stopped after the class was dismissed. This procedure allowed the capture of not just the planned classroom activities, but also the incidental interaction between teacher and students. The videotape was used for analysis of the events which emphasizes nonverbal as well as verbal behavior. As Erickson and Wilson (1982) suggested, because peoples' understandings of the purpose of the study and comfort with the use of equipment ease the nervousness they might experience with the process, care needed to be taken before and during the initial observation to explain the purpose of the study to the students in the class and the camera was positioned so that it was as unobtrusive as possible. Each day's class sessions were taped on a separate tape and labeled with the teacher's name, the class(es) taped, and the date. The transcript made of each tape indicated both the audio content and the nonverbal activity observed. Fieldnotes and observer's comments were prepared from the observations. The process of gathering information and drawing conclusions about the activities in the classrooms under study was enhanced by the ability to view and review the videotapes.

Informal observation interviews. During the observation period, the researcher conducted informal interviews with the teacher whenever a need arose to check the researcher's understanding of the teacher's practices. During these interviews the following type of questions were used:

1) As I observed your classroom I noticed.... Do you consider this a regular part of your teaching?

2) If yes, describe why you use this type of activity. If no, can you explain why this activity occurred and why it is not a regular part of your teaching?

The purpose of these interviews was to check the developing description of the instructional practices of a persistent user of graphing calculators. Statements made by the teacher during these informal interviews were included in the statements used for investigation of the teacher's beliefs in the belief clustering interview.

Documents. Documents reflecting the teacher's classroom practices were collected from each teacher during the period of observations. The documents included handouts, quizzes, and tests. These documents were collected for the entire period during which observations were made. Documents were marked with the teacher's name, date, and class in which they were used so that they could be cross-referenced to the data obtained from the videotapes and fieldnotes. Evidence of the teacher's classroom practices obtained from these documents were used to triangulate information from other sources. These documents were not used as primary evidence to support the presence of any classroom activity. Documents were not assumed to contain a completely accurate representation of classroom activity and teachers' beliefs. Each document was evaluated to determine the purpose for which it was written and to be critically interpreted in light of data from other sources. The use of documents was verified through references made to them in the videotaped sessions and the observations of the researcher. These documents also served to support observational data about assessment techniques.

Belief Interviews

A series of formal interviews was conducted with the teacher beginning at least two weeks after the completion of the observations. The purpose of the first of these interviews, the belief clustering interview, was to elicit the teacher's beliefs about mathematics, teaching, and using graphing calculators. The next interview was used to clarify and refine the researcher's profile of the teacher's professed beliefs. The belief clustering was videotaped to assure that all non-verbal as well as verbal information was captured. The verification interview was audiotaped. All interviews were transcribed.

Belief clustering interview. The belief clustering interview, conducted approximately two weeks after the completion of the classroom observations, focused on exploring the teacher's explicit and implicit beliefs and conceptions about the nature of mathematics, teaching mathematics, and using graphing calculators in teaching. What a teacher considered to be desirable goals of the mathematics programs, his or her role in teaching, the students' role, appropriate classroom activities, desirable instructional approaches and emphases, legitimate mathematical procedures, and acceptable outcomes of instruction all contributed to a teacher's conception of mathematics teaching (Thompson, 1992). In order to assure that the profile of the teacher's beliefs described by the researcher reflected the beliefs that were paramount to the teacher and not what the teacher said based on some predetermined set of possible beliefs, an open-ended interview was conducted

(Munby, 1984). A clustering technique based on techniques used by Cooney (1985), Brown (1986) and Munby (1984) was utilized to elicit the teacher's beliefs. Teachers were asked to talk about their teaching practices. Based on the data collected from the background interview, the informal interviews, and the classroom observations, the researcher prepared a set of 3" x 5" cards on which statements made by the teacher concerning teaching practices and observed practices had been recorded. The statements were descriptive statements about the teacher's classroom practices. The 3" x 5" cards were given to the teacher, who read the cards and was given the opportunity to include additional statements or alter any statement present if there were features of the teacher's classroom activities the teacher felt the researcher had not recorded accurately. A record was made of any changes made to the collection with a reason given for the change. The teacher was given the opportunity to remove any cards from the collection, the reason for removing any card was noted by the researcher. Once the teacher agreed that the collection of cards were an accurate representation of the teacher's classroom practices, the cards were used to provide a vehicle for the teacher to talk about beliefs.

Utilizing a method of clustering adapted from Cooney (1985) and Brown's (1986) studies of a teacher's beliefs about teaching problem solving, the teacher was instructed to group the cards any way desired, thus the criteria used for the grouping was entirely of the teacher's own making. The teacher was then asked to create a title or heading for each group of cards and a brief sentence to capture the essence of what the individual cards seemed to be expressing. The teacher then discussed the heading and the cards grouped under that heading. This discussion allowed the teacher to express beliefs about mathematics and teaching mathematics that could not be inferred solely from observations in the classroom.

The purpose of this clustering was to help identify statements, and subsequent beliefs, that the teacher thought were important. These statements and headings played an important part in the analysis of what the teacher believed about mathematics and the teaching of mathematics. By examining the statements on the cards that were grouped together and the statements made about the groups of cards, underlying beliefs of the teacher emerged. The assumption was that the teacher would group the statements in a way that represented something substantial about the teacher's beliefs about teaching (Munby, 1984).

Belief verification interview. The next interview was used to refine and validate the profile of the teacher's beliefs that the researcher developed from information obtained in the prior interviews. This interview took place during the summer, allowing the researcher time for an initial analysis of the data obtained in the previous interviews and observations. An outline of the teacher's beliefs was prepared by the researcher prior to this interview. During the validation interview the researcher probed for details about the teacher's beliefs, seeking to clarify any inconsistencies or uncertainties found in the initial analysis. Since beliefs tend to be held in clusters isolated from one another, inconsistencies appeared between and among teachers espoused beliefs. Because of the isolation of clusters of beliefs from one another, the inconsistencies within the belief system may not create a problem for the teacher (Thompson, 1992), however the researcher probed for details about what may have influenced the teacher in the development of such beliefs.

In order to facilitate discussion, the researcher took the statements used in the belief clustering interview and grouped them in ways that reflected the teacher's beliefs concerning mathematics, the teaching and learning of mathematics, teaching, and the use of graphing calculators as determined by the initial analysis of prior interviews and observations. The teacher was then asked to discuss how these statements reflected personal beliefs in each area. The same outline was followed for each interview (see Appendix F), but the statements included in each area were unique for each teacher.

Data Analysis

Data analysis proceeded in three stages with different foci. The foci of the stages of analysis were: (1) description of beliefs and classroom practices of persistent users of graphing calculators, (2) relationships between the beliefs and practices of teachers using graphing calculators, and (3) comparisons of theoretical foundations for the use of graphing calculators with actual beliefs and practices. While the techniques utilized for the different stages of analysis varied and the questions being explored in each differed, the analyses were not independent of one another. Analysis done in each stage was utilized in the other stages to refine and expand the findings.

Description of Beliefs and Classroom Practices

Initial data analysis centered around the following questions:

(1) What are the classroom practices of teachers who have persisted in the use of graphing calculator technology in their teaching?

(2) What are the beliefs of teachers who have persisted in the use of graphing calculator technology in their teaching?

As answers to these questions emerged from the analyses of interviews and observational data, descriptions for each participant were formulated. Data from successive observations and interviews were used to enhance, expand, and verify the descriptions being developed.

Each transcribed interview tape was combined with fieldnotes and the observer's comments to create a record of the data for each interview. Data from the first interview was used to create a background profile of the teacher which included information concerning the teacher's educational background, teaching experiences, and professional activity. Information concerning the teacher's classroom practices obtained from this initial interview was recorded and used in eliciting the teacher's beliefs about mathematics, teaching, and the use of graphing calculators.

During the observation phase of data collection a record of each observation was created by combining the transcript of the videotape, researcher's fieldnotes, and the documents collected. The researcher also kept a notebook for each teacher in which comments and impressions were recorded. The process of inductive analysis described by

Marshall and Rossman (1989), where categories emerged from the data, was used to develop coding categories for classroom activities as the records of observations were analyzed. The process of developing coding categories required reading and rereading the transcripts of observations, fieldnotes, and researcher's comments as well as viewing and reviewing videotapes. During this process, salient, grounded categories of activities demonstrated by the participating teachers were identified. In order to test the validity of the categories of teacher activities, the researcher checked findings with the teacher through the informal observation interviews. Information from these informal interviews also contributed to the formulation of coding categories.

The data for each teacher were analyzed separately but coding categories were developed on an ongoing basis. The coding categories developed for the first teacher analyzed were used in subsequent coding and analysis. As the analysis continued additional categories emerged. Care was taken to reflect on and review the analysis done on previous teachers and sessions and incorporate newly emerging categories in the coding of such data. As the data were being coded, the process of developing descriptions of the teaching practices of each began. Details of the teachers' instructional practices and specific ways in which the graphing calculator was incorporated into the teaching of Algebra II were the focus of the descriptions.

From the interview using the clustering technique (Cooney, 1985; Brown, 1986) to explore the beliefs of the teacher, titles and sentences provided by the teacher were analyzed and categorized to create an initial profile capturing the essence of the teacher's beliefs. As categories emerged from the analysis of one teacher's beliefs, the same category names were used in the analysis of other teachers in order to facilitate comparison in a later phase of the analysis. Data obtained from the verification interview were analyzed, additional categories were created as necessary and the initial profile of teacher's beliefs was modified to reflect this analysis.

Relationships between Teachers' Beliefs and Classroom Practices

The second area of data analysis focused on the question: What is the relationship between teachers' professed beliefs and their classroom practices? A constant comparative method of data analysis was utilized to answer this question. The steps in the constant comparative method, as given by Bogden and Bicklen (1992) were utilized: begin collecting the data; formulate initial categories of focus based on key issues, recurrent events, or activities in the data; collect additional data that provide many incidents of the categories while searching for diversity under the categories; write about the categories being explored, attempting to describe and account for all the incidents in the data while continually searching for new incidents; work with the data and emerging model to discover basic relationships between beliefs and practices; and continue sampling, coding and writing as the analysis focuses on the core categories.

Ongoing analysis of the data was essential to the qualitative research method. In developing theory about the relationships between teachers' instructional practices and their professed beliefs, the researcher was constantly searching for consistencies and discrepancies between the description of the teacher's instructional practices and the emerging profile of the teacher's beliefs. The coding categories developed to describe a teacher's instructional practices were compared with the coding categories emerging for the profile of teacher's beliefs. As questions arose about the relationships between the two sets of descriptions, the researcher made comments in the notebook being kept on the teacher. The researcher attempted to clarify these issues by reviewing the data collected. With the availability of videotaped classroom observations and belief interviews, the researcher also reviewed previously observed sessions in order to seek confirmation of newly emerging theories in previously analyzed sessions.

The analysis of data in the area of the relationship between teachers' beliefs and classroom practices extended beyond the search for consistencies and discrepancies within the data for individual teachers to an exploration of the consistencies among teachers. In reflecting on the data about an individual teacher, new material was used to broaden the theory and was integrated into the developing theory. This process of reflection and integration necessitated a continual review of the findings related to each teacher.

Comparisons of Theoretical Foundations with Actual Beliefs and Practices

Finally, the analysis compared teachers' professed beliefs and demonstrated practices with the constructivist theory and theoretical benefits of utilizing graphing technology in high school algebra. Questions that were explored in this comparison included: (1) Do teachers who persist in the use of graphing calculators do so from a constructivist perspective and (2) are the activities found in these classrooms consistent with the goals of the current curriculum reform movement? The completed descriptions of the teachers' beliefs and practices were compared to the constructivist theory and goals of the current curriculum reform movement. Consistencies and discrepancies were discussed in relationship to the developing theory concerning teachers' beliefs and practices.

Triangulation of Data

The external validity, that is the transferability or generalizability of finding to other populations or settings, is often seen as a weakness of qualitative research. This study's generalizability was enhanced by the triangulation of multiple sources of data. According to Marshal and Rossman (1989) "Triangulation is the act of bringing more than one source of data to bear on a single point" (p. 146). Additionally, as Yin (1989) states,

> The most important advantage of using multiple sources of evidence is the development of converging lines of inquiry, a process of triangulation... Thus, any finding or conclusion in a case study is likely to be much more convincing and accurate if it is based on several different sources of information following a corroboratory mode. (p. 97)

In this study collecting numerous forms of data, transcripts of interviews and observations, fieldnotes from interviews and observations, documents from teachers, and researcher's comments, made it possible to check and recheck developing theory. The availability of videotapes for review of the activities in classrooms, in addition to the transcripts of these videotapes, served as an additional means of checking and rechecking. In addition to the multiple forms of data, the inclusion of multiple teachers, each studied separately, provided for another means of checking, rechecking and expanding the developing theory.

The data gathered from interviews, especially from the belief clustering interview that utilized the clustering technique, was the most heavily weighted in formulating the descriptions of teachers' beliefs. Data from observations, especially the transcripts of the videotapes, were utilized to check and augment the descriptions of teachers' beliefs. Descriptions of classroom practices were formulated based most heavily on the data from observations of classroom activities but were augmented and checked through the informal observation interviews. Documents collected during the observation period were used to corroborate data obtained through the primary data sources.

As the data for this study were collected, a file and a notebook for each teacher was maintained. The file contained all transcripts of audiotapes, videotapes and fieldnotes, and researcher's comments on interviews and observations. The notebook contained researcher's comments about emerging theories, discrepancies and consistencies between classroom practices and stated beliefs, and areas for additional inquiry. All audiotapes, videotapes, and original fieldnotes were archived and available for future reference. The availability of these files and notebooks for the analysis of data assisted the researcher in correcting for possible biases and strengthened the overall validity and reliability of the study.

Description of the Researcher

All collected data was filtered through the researcher. It was therefore important to deal with the researcher's own biases. Thompson (1992) emphasized this saying, "It is important that researchers make explicit to themselves as well as others, the theory or theories of teaching and learning and conceptualizations of the nature of mathematics with which they are approaching the study of mathematics teachers' beliefs" (p. 130). One method used by the researcher was to keep a daily journal where personal assumptions, experiences, and reflections were recorded. Extra precautions were taken by the researcher when analyzing data to acknowledge personal perceptions and experiences with respect to the research and to seek conflicting evidence and alternate hypotheses to assist in transcending potential biases. The numerous types of data collected and variety of teachers being studied also helped the researcher to confront and limit personal assumptions and bias. A brief description of the researcher is provided to assist the reader in assessing the perspective from which the data were collected and analyzed.

The researcher has been teaching mathematics at a small liberal arts college in the Northwest for 16 years. Prior to that the researcher spent six years teaching high school mathematics (as a substitute teacher and in short-term teaching positions). The researcher received a Bachelor of Arts degree majoring in mathematics with preparation for teaching certification in advanced mathematics from the liberal arts college at which she now teaches. The researcher obtained a Master of Science in Education with core work in mathematics at a state college in the same state.

The researcher's teaching involved lower division mathematics courses including algebra, trigonometry, finite mathematics with introductory calculus, statistics, and mathematics content courses for elementary teachers. Occasionally, the researcher has taught an upper-division special topics course in operations research. Additionally, the researcher has advised students preparing to teach mathematics in secondary schools and has taught graduate courses for educators on the use of computers in education and the use of graphing technology in teaching high school mathematics.

The researcher began using graphing calculators in her teaching of statistics in the fall of 1993 and has continued in this practice utilizing first the TI-81, then the TI-82, and now the TI-83. The use of graphing calculators was incorporated into her teaching of finite math with calculus in the spring of 1995 and has continued. Now, the researcher utilizes the graphing calculator in the teaching of courses beginning with college algebra and trigonometry and extending throughout the curriculum. The researcher has encouraged colleagues at the college where she teaches to incorporate the use of graphing calculators and other technology into their teaching. Her colleagues have utilized graphing calculators in the teaching of statistics, college algebra, trigonometry, and finite math with calculus. *Mathematica* is also utilized by the researcher and her colleagues. The researcher utilizes *Mathematica* primarily for the preparation of teaching materials such as overheads of three-dimensional graphs. Other members of the mathematics department incorporate the use of *Mathematica* in the teaching of calculus, linear algebra, and numerical methods.

In order to assess her beliefs concerning the teaching of mathematics, the researcher was interviewed by a graduate student using the belief clustering interview protocol developed for this study. The researcher prepared a set of 20 cards with statements describing practices she utilized in the teaching of finite mathematics, a course in which she used the graphing calculator. In sorting the cards, the researcher created five groups (Figure 4). The order in which she discussed the groups of cards revealed her beliefs about the roles of the instructor and the students in the learning process, the nature of mathematics, and the use of graphing calculators in teaching mathematics.

Students work in groups	Concept development	Student involvement	Graphing calculators	The old stand-by

Group 1 Students work in groups
All group members agree on correct answers for group problems.
Students work in small groups.
I am available to groups by try not to give answers.
Students assist one another in analysis of errors.
Students discuss solutions of homework with classmates.

Group 2 Concept development
I summarize key ideas.
I put a list of key ideas and vocabulary on the board.
I present solutions to unanswered group questions.
I develop new concepts by showing examples.
I use graphs in the development of concepts.

Group 3 Student involvement
Students make suggestions and provide input to chalkboard solutions
Students ask questions.
I direct student explorations.
Students use a guided discovery activity.

Group 4 Graphing Calculators
Students use graphing calculators to demonstrate solutions of problems.
I provide symbolic rationale for conclusions drawn [from graphing calculator explorations].
Graphing calculators are used to produce graphs quickly.

Group 5 The old stand-by
I just show students "the way to do the problem."
I rush explanations because of time constraints.
I do the problems on the board.
Students depend on me to provide the magic.

Figure 4. The researcher's card sorting.

The first group of cards she discussed were titled "students work in groups" and reflected her belief in the importance of providing students the opportunity to "direct their own work with guidance from the instructor." The importance of the instructor presenting content was captured in the second group of cards which were described as "concept development." The researcher indicated her belief that the instructor was responsible for providing structure and content saying, "I develop concepts by letting students know what the important ideas are and showing examples to illustrate those ideas." Emphasis on the important ideas being presented illustrated the researcher's belief in the structure of mathematics and the role of the instructor in conveying that structure to the students. The importance of interaction between instructor and students was captured in the third group of cards titled "student involvement." The inclusion of the statements, "Students make suggestions and provide input to chalkboard solutions," and "Students ask questions," in this group indicated the importance the researcher placed on creating an interactive learning environment in which both instructor and students participated. In discussing the group of cards titled "graphing calculators," the researcher indicated her belief in the value of the graphing calculator for learning mathematics. Her statement, "Graphing calculators provide another avenue to explore math," indicated a belief in exploration as a vehicle for learning mathematics. The final group of cards was titled, "the old stand-by" and was described as "represent[ing] the way mathematics has been taught in the past and we fall back on when we are out of time." This statement reflected a desire to change her teaching practices from the traditional lecture-based presentation while acknowledging that the process of changing had not been completed. Reflecting on the collection of cards, the groups into which they had been sorted, and the order in which she discussed them, the researcher described a continuum in her teaching practices and beliefs about teaching from "the importance of students being actively involved" to "I depend on the old tried and true."

CHAPTER FOUR
ANALYSIS OF DATA

Introduction

The purpose of this study was to examine the beliefs and classroom practices of teachers who had persisted in the use graphing calculators in the teaching of high school Algebra II. The study further explored the relationships between these beliefs and practices and their relationships to the recommendations for the use of technology in the teaching of high school mathematics. Four high school mathematics teachers participated in this study by completing a background interview, a belief clustering interview, and a belief verification interview. These teacher were also observed extensively by the researcher during the teaching of a unit suitable for use of the graphing calculator.

All four of the teachers completed the background interview at the commencement of their participation in the study. Classroom observations of all teachers took place during the same fall and winter. Three of the teachers were observed teaching a unit on systems of equations. The fourth teacher was observed teaching a unit on functions. The periods of observation for the four teachers overlapped but no more than two teachers were being observed at any time. The belief clustering interview was conducted with each teacher within six weeks of the conclusion of the classroom observations. The belief verification interviews were conducted with all teachers during the summer at a location away from the schools where they taught.

The four teachers (two female and two male) involved in this study were teaching full-time in mathematics departments of their schools. In addition to their Algebra II classes, they were teaching one or more other mathematics classes including at least one of the following: Algebra I, Precalculus, and Calculus. The teachers were at different schools, three public and one private. Two of the public schools were in the same school district. The three public schools had enrollments between 1000 and 1500. The private school, a K-12 school, had between 250 and 300 students in the high school. Two of the teachers were teaching from the same textbook: *Advanced Algebra,* published by Scott, Foresman (Senk, Thompson, & Viktora, 1990), part of the University of Chicago School Mathematics Project (Chicago series). A third teacher was teaching from the field trial version of the second edition of the same text. The fourth teacher was teaching from the textbook: *Algebra II with Trigonometry*, published by Prentice Hall (Hall & Fabricant, 1993). The three teachers who were teaching the same unit, systems of equations, were teaching from different textbooks. All four teachers used TI graphing calculators in their teaching. Two used the TI-85 and two used the TI-82. Except for one teacher whose students were required to use the TI-82, the teachers dealt with a variety of different calculators in their classrooms including TI-81, TI-82, TI-85, and HP-48G.

Individual Profiles

In order to answer the questions guiding this study it was necessary to describe classroom practices of each teacher and each teacher's beliefs concerning mathematics, the teaching of mathematics, and how students learn mathematics. Individual profiles developed for each teacher used as sources the background interview, classroom observations, belief clustering interview, and belief verification interview. Each profile begins with a description of the teacher including details about the teacher's school, academic background, teaching experience, and use of graphing calculators. The classroom practices of each teacher are then analyzed including specific details on the use of graphing calculators. The profile continues with the teacher's beliefs concerning mathematics, the teaching of mathematics, classroom structure, and how students learn mathematics based on the belief interviews. Pseudonyms are used for the teachers and schools to assure the anonymity of the teachers. Summaries of the profiles of classroom practices and beliefs concerning mathematics, its teaching, and student learning that emerged are described. Triangulation of all four teachers' data supported the descriptions of the summary of beliefs and practices.

Mr. Lorenz

Initial contact with Mr. Lorenz occurred in a telephone call during which he indicated he was teaching Algebra II and was willing to discuss the study. A meeting took place a few days later during which Mr. Lorenz agreed to participate in the study.

The daily schedule at Mr. Lorenz's school, Central High, was a standard 54-minute per period model with six class periods per day. Third period was an extended period to allow time for school wide announcements and the viewing of Channel One. Fourth period allowed two lunch options with class time occurring either before or after lunch. An early bird period before first period and late bird period after sixth period were scheduled every day for optional activities.

During the years that Mr. Lorenz had taught at the school, he had seen a change in the students at the school from "a fairly high motivated, high socio-economic group" to a "more mixed group" reflecting the more racially and socially diverse population of the area. Mr. Lorenz's classes were all honors or higher level, so he characterized his students as "kind of where the school used to be."

A total of 15 teachers taught mathematics classes that fall, several of whom also taught in other departments. In addition to Mr. Lorenz, two teachers were teaching a total of three sections of Algebra II. Mr. Lorenz was the only Algebra II teacher making extensive use of the graphing calculator in this course. The other two teachers used the graphing calculators only occasionally and were not as experienced in its use as Mr. Lorenz. At Central, graphing calculators were available to all teachers, but there were not enough to use in every class every day, so some priorities were set. It was the expectation that from Algebra II on, graphing calculators would be used in all classes. Classroom sets of TI-85, TI-81, and HP-48G calculators were available for use. Mr. Lorenz used the TI-85 in his teaching. Other Algebra II teachers used the TI-81. HP-48G calculators were used in Calculus. Students were not required to use a specific type of calculator nor were they required to purchase a graphing calculator. The students who owned their graphing calculators had a variety including TI-81, TI-82, TI-85, and HP-48G. The school only needed to furnish calculators for a third of the students. These students had calculators available to them during every class and could check one out for overnight use from the mathematics office at the end of a school day. When the current textbook for Algebra II , *Algebra 2 With Trigonometry* (Hall & Fabricant, 1993), was adopted the department considered only texts that made use of the graphing calculator.

Mr. Lorenz characterized the school administration as "fairly progressive, quite involved in education reform." At Central High there was a great deal of site-based management, according to Mr. Lorenz, "in that all the major decisions that are made that effect the whole school are made by the staff within administrative parameters, of course." He emphasized that, "the staff is very involved in the direction of the school." The decision making in the school was affected by the political environment as well. Mr. Lorenz noted that, "we have some very active interest groups that tend to be our checks and balances."

<u>Background</u>. Mr. Lorenz had been teaching mathematics for 23 years. He held an interdisciplinary Master's degree in mathematics, computer science, and education. Mr. Lorenz began college with the intention of becoming an engineer. At the end of his freshman year he transferred. "I guess I was kind of re-focused because my intent since I was in middle school was to be a teacher. I have come from a family of teachers. That was the thing I always wanted to do." His undergraduate education experience had a profound impact on his teaching. "A college professor that I had ... just kind of created the image of who I wanted to be and so I ended up patterning an awful lot of what I did after what I saw him do." He characterized his formal preparation in mathematics as emphasizing the "system and the art and how it all fit together."

Mr. Lorenz completed his undergraduate program with an internship at Central High teaching chemistry and career explorations. He then taught mathematics for seven years at a high school in a nearby, smaller, more rural city. At that school Mr. Lorenz gained experience teaching a variety of mathematics classes. "I was hired as their advanced math teacher. The first year teaching I taught the top classes.... But, then I taught the whole spectrum by the time I left seven years later." While he was teaching in this first position, he completed his Master's degree at a state university.

Mr. Lorenz then spent three years working in construction and other ventures. Regarding this experience, he noted, "the three years off, really immersed me in another area and gave me the opportunity to look at what I was teaching and the way it was being used." He decided to return to teaching when there was an opportunity to return to Central High.

Along with teaching mathematics courses, Mr. Lorenz had periodically taught computer courses. At the time of the study, he was department coordinator for the mathematics department, teaching two sections of Honors Algebra II and one section of Pre-Calculus. Mr. Lorenz was serving as chair of the Twenty-First Century Committee in the school.

In addition to the formal coursework required for his degrees, Mr. Lorenz had participated in two terms of cooperative education training which "made a lot of difference." He participated in an Applied Math Training Program in Waco, Texas, "where everything was applied," after which he became a trainer in applied mathematics for the school district. Additionally, he stated that, "the number of workshops and stuff would be almost impossible to list," including workshops and training at the district and state level in mathematics and teaching philosophies. During the period of observation, Mr. Lorenz attended a district inservice called Maximize Students' Performance to Full Potential. Regarding his professional activity he commented, "I'm not much of a reader, so I do more listening than reading. While I get some professional journals, they aren't a big part of what I do. I think probably it's just connection with people and peers." Mr. Lorenz felt the current reform movement had certainly affected his teaching.

> We aren't alone anymore. What used to be pretty much you figured out what you wanted to do, you went in your room and closed the door, and you taught, did what you wanted to do. And now I think that it's a whole lot more open environment than it's ever been before. And I think that's helped a lot of us to see things broader.

Mr. Lorenz shared that the school district in which Central High was located had an active department coordinator group with mathematics department coordinators from other schools. This group had recently organized a day long mathematics inservice with a speaker followed by small group sessions. An active coordinator group in the school worked "together a lot, resolving classroom problems."

Introduction to and thinking about graphing calculators. When discussing the use of graphing calculators, Mr. Lorenz indicated he had always used calculators in his teaching since "I got the first one on a grant for $60, ... a four function calculator." Using graphing calculators just seemed like the natural thing to do. "The tool became available, it made easier some of the things we'd been doing, it made it easier for kids to see some of the things we were doing. It just seemed like the right thing." He had been introduced to graphing calculators when he attended a workshop put on by the district mathematics department coordinators' group. This workshop occurred soon after the introduction of graphing calculators into the teaching of mathematics.

> It was before school started. I don't remember how many years ago, quite a few years ago. But the day before the teachers reported to school, we had one of the teachers from [another school in the district] who was very interested in it [the use of graphing calculators.] She put together a workshop to train teachers on how to use it.

At that point, Mr. Lorenz realized that he was ready to make a change. He decided that the graphing calculator was "what I'm going to use in the classroom."

While Mr. Lorenz, as department coordinator, was influential in the decision to utilize graphing calculators at Central in the teaching of courses from the Algebra II level on up through the curriculum, it was a staff (mathematics department) decision. The department made decisions as a group, including the decision to direct the use funds toward the use of graphing calculators. The funds available to the department included money raised through fund-raisers.

Mr. Lorenz's ongoing use of the graphing calculators was facilitated by the economic level of the area where the school was located that enabled many parents to provide graphing calculators for their children. "If we were in a different environment where we had to furnish a calculator for every kid in the classroom, I'm sure we wouldn't have moved as fast as we have." The use of the graphing calculator had an impact on Mr. Lorenz's teaching. "When it came into being I think it forced change in the way we taught things because it changed what was important. It certainly changed the amount of time you spent doing some of the tasks that you spent most of your time doing before." He had found that keeping up with the students and their discoveries on and about the graphing calculators was a challenge. The development of new models of calculators, including the introduction of the HP-48G in the teaching of calculus at Central, challenged Mr. Lorenz to keep up with the demands of his students. But, there were rewards as well, like being able to do computations and display graphs that were previously so difficult. Understanding the graphs of polynomial functions was one area where students could see why and what the graph did by using the graphing calculator.

> All those theorems that you used to build to try to narrow it down, and now you can narrow it down on the calculator in no time. The theorems are still important, but now that the kids can see, like the intermediate value theorem, why that, what that does. Because they can pick a number here, and pick a number here and see they have to have a zero in between.

Professed beliefs about mathematics and the teaching of mathematics. Mr. Lorenz described mathematics as "a language and a tool that we use to understand the world around us." For him, algebra could be defined as "**the** language of higher mathematics." In discussing these concepts with students he found that for students who were not planning to continue in their study of mathematics it was important to make connections to their world. "The kid probably asks... so, why do I have to learn this stuff if I'm not going to [study] higher mathematics... I think it's a way, I try to relay it as a way that they can describe common every day things that happen." He indicated that real world examples like understanding variables in accounting or material in their science classes were areas where algebra was useful. Students learn, Mr. Lorenz believed, "by doing. They have to do it."

Classroom practices. Mr. Lorenz's classes, both morning and afternoon sections of Honors Algebra II, were observed for a period of four weeks including an introductory period of three days prior to the introduction of the new unit of study, solving systems of equations, and continuing through the presentation and assessment for the unit. While both classes were observed, the observations of the morning class served as the primary focus of this portion of the study with information from observations of the afternoon class providing additional data.

Mr. Lorenz had an established structure for the class which was followed on most days. He arrived a few minutes before class began, making calculators available for students to borrow and stationing himself outside the classroom door to greet students as they entered. He often interacted with students as he began taking roll, talking with them about recent activities. He cultivated a friendly atmosphere in his classroom. After the bell rang, Mr. Lorenz provided the students with a starter activity of some sort with which they would be engaged while he took roll. These starter activities included reviewing tests or quizzes he returned, discussing the current assignment with classmates, working on a problem displayed on the overhead projector, and preparing for group presentations. Correction and discussion of the assignment covering the previous day's lesson followed the starter activity. Generally, Mr. Lorenz read the answers to the assigned problems from the teacher's edition of the textbook, a group of answers were read and then questions were asked on those problems. Throughout the discussion of the assignment, Mr. Lorenz depended on questions from students to direct the explanation and review. After reviewing the assignment, he moved on to the new material for the day.

In the presentation of the new material, connections were made to either the previous day's assignment or the starter problem students had been given at the beginning of class. As Mr. Lorenz led the students through the presentation of the new material, he often suggested an approach and asked students what the result would be, thus involving the class even though they had not previously been exposed to the particular techniques being presented. When a problem was completed, Mr. Lorenz reviewed the process explaining the reason and procedure for each step. Throughout, students asked questions for clarification and posed optional strategies. Mr. Lorenz responded to the students' questions and incorporated their insights into his explanations. After the presentation and explanation of the new material, students spent the remainder of the class working either on a specific problem which was then discussed or on the assignment for the section covered. An assignment sheet was distributed at the beginning of the unit listing assignments for each section covered. Unless a change was announced by Mr. Lorenz, students completed the assigned problems. While students worked, Mr. Lorenz circulated throughout the room, interacting with individual students and occasionally directing a comment to the entire class. As class concluded, Mr. Lorenz made certain that borrowed calculators were returned and reminded students they could check out calculators for overnight use.

Student involvement was essential to Mr. Lorenz in his teaching. Whether discussing assignments or presenting new material, he consistently asked questions and waited for student responses. For example, when introducing the concept of a solution to a system of linear equations, he began class by providing a discovery activity in which students followed a step-by-step procedure (displayed from an overhead transparency) for finding the coordinates of the point at which the graphs of two linear equations intersected. The instructions explained how to use the graphing calculator and TRACE to find the coordinates of the point. After the students had completed the activity, Mr. Lorenz discussed the concepts involved. He defined the solution to a system of linear equations as "a pair of x, y's that I can plug into both of those equations that makes them true." He then asked where that point had been found in the activity they had completed and waited for a response. When a student responded "intersection of the lines" he repeated the student's response and went on to elaborate and explain why that was the correct answer.

When responding to a student's question concerning a problem in an assignment, Mr. Lorenz involved that student in the process. In the following example Mr. Lorenz used the student's explanation of the method he had used to solve the problem to guide the discussion of the correct solution to the problem.

Student: Could I get 34?

Mr. Lorenz: (while writing the original problem on the overhead) How did you get started?

Student: I took the top one and I cross multiplied.

Mr. Lorenz: Okay, so the new equation would be $15y - 5 = 6x + 16$.

Student: Then I took the bottom one and multiplied by two.

Mr. Lorenz: So the new equation is $x + y = 6 + x - y$.

Student: And then I got them into standard form, moved the 6x over [on the top equation.]

Mr. Lorenz: Okay, you went to standard form, so you got $-6x + 15y = 21$.

Student: Then I did the same thing on the bottom.

Mr. Lorenz: So, you had $0x + 2y = 6$.

Student: Yeah and then I thought it was no solution because there wasn't any x.

Mr. Lorenz: Let's ask ourselves a question. When is this true? [indicating the equation $0x + 2y = 6$]

[waits]

Student: $y = 3$

Mr. Lorenz: Yeah, this is true when $y = 3$. Is it always true? No, because if I put $y = 8$ in there it's not going to be true. It has to always be true to be infinite solutions. Okay, so this one fell out before you had to do addition. That's going to happen sometimes. When it does, take advantage, use that, and plug it back in saving a step.

In this discussion, Mr. Lorenz utilized the method begun by the student to solve the problem. The student explained what he had done and what he had been thinking when he answered the problem incorrectly. Mr. Lorenz was then able to build on the student's explanation and resolve the problem.

Building connections between new material and prior learning was a facet of Mr. Lorenz's teaching. In this unit, several methods were developed for solving a system of equations. As each new method was presented, a problem was used which had already been solved using a known method. This way Mr. Lorenz built a connection between what had been taught and the new material. He also thought it was important for students to understand the value of having multiple ways of approaching a problem. When he asked the students why they would ever want to learn another way to solve a problem, a student responded, "Because it is easier." Mr. Lorenz agreed and added, "I used to be in construction and I never went to the job with one hammer. The more tools you have the more choice - choose the one that works the best for your situation." When reviewing the solution of a linear programming problem, Mr. Lorenz sketched the graph of the feasible region then discussed finding the coordinates of the vertices with the class. The coordinates of the two vertices which fell on the axes were easily found, but finding the other vertices required more work and provided an opportunity to emphasize the variety of methods available to the students.

> How do I locate this one? It looks like (1, 2), but my graph is not very accurate. I want to make sure that I get this point located right. What could I do to locate it? [repeating student response.] I could put both lines on the calculator and use trace. What other tools do I have? [pause] Doesn't that point have to satisfy these two equations? [repeating student response] I could use simultaneous equations on my calculator. Go back and put these two in simultaneous, that will tell me this point. How else?... Remember you have all those tools to use: addition method, substitution method, graphing, simultaneous, the TRACE button. Any of them will help you with this problem.

Mr. Lorenz encouraged students to learn and use a variety of approaches to solving problems. He also emphasized the value of understanding the process used to solve a specific problem so that the process could be adapted to other situations and problems.

When applying the four methods for solving systems of linear equations to the solution of applied problems, Mr. Lorenz discussed examples as patterns for solving similar problems. "I want to look at the examples your book gives for application problems. I'm going to talk you through some of these because they are patterns you're going to need to do today's assignment." As he continued to discuss these problems, he connected the examples to students' experiences outside the classroom. When solving a river current problem he asked if students could recall their experiences. "Have any of you ever rowed in a river or a stream, or canoed? You feel that [the effect of the current] real quickly. Or even swimming against the current, the same idea...." Then he continued with his discussion of the solution to the problem by recalling previously solved problems.

> These are going to be DIRT problems, distance equals rate times time. [The] same kind of problem as we were doing before, but now you've got two equations instead of one, distance equals rate times time twice. What are we looking for to get an equation out of DIRT problems? What are the key words in DIRT problems that tell us what the equation might be? Find it in this one.

Consistent with his belief in the importance of student involvement, he waited more than eight seconds for a student response. He then built upon that student response to continue the discussion of the solution to the problem.

Part of understanding the process of solving problems for Mr. Lorenz was finding and correcting errors. As he taught students to solve systems of equations, he emphasized the importance of checking the correctness of their work.

> You've done a lot of algebra here and your answer may not work in one or the other of the equations because of some error you've made in that algebra.... Question, what do you do if it doesn't check?... Check for sign errors. Then go back and check your work. Go through your work, check for sign errors, addition errors, multiplication errors, those kinds of things. If it still doesn't work, the third step I would suggest is to put that to one side and start all over. If you've gone through your work once or twice and can't find a mistake, you're going to go through again and overlook the same mistake again and again. So after you've given it a good effort, take your scratch paper and rework that problem from scratch to see if maybe you've overlooked the same thing time and time again.

He also emphasized the importance of recognizing when something was wrong in the solution process. Mr. Lorenz recognized that something was wrong when he was graphing the feasible region for a linear programming problem. He had written one of the constraints incorrectly so when he sketched the graph, there was no feasible region.

> Hmm, something isn't working right because I have an inconsistent system.[looks back at the problem] Okay we forgot one thing.... How did I know that I made a mistake? All of a sudden I went to graph that [the original constraint] and just stopped.... The one that I had originally was an impossible situation so I could see I had made an error some place.

By recognizing that he had made a mistake and discussing it with the class, Mr. Lorenz modeled good problem solving practices for his students.

Mr. Lorenz's assessment techniques displayed the importance he placed on student responsibility for their learning, group work, and individual accountability. At the culmination of each unit, students were expected to have a notebook containing all assignments for the unit as well as corrections for the test on the previous unit, handouts, and other activities. In discussing the daily assignments kept in the notebooks, Mr. Lorenz regularly reminded the students to go back over the assignment especially if they had not completed it correctly. He had several reasons for having students review their work.

> It gives them an opportunity to get the full points. But, also, if they didn't complete that assignment there's probably some problem they're having there. Maybe the second time through they'll find it. Maybe the second time through that they haven't asked, they'll ask [about what they are not able to do].

Students were given a scoring rubric that was used to grade the notebooks. For some units, Mr. Lorenz evaluated the notebooks. Other times, students evaluated their own notebooks. Mr. Lorenz indicated that having students evaluate their own notebooks made them responsible for the process.

Group work was an important piece of the classroom for Mr. Lorenz. During the unit observed, students worked in groups on two assignments and a quiz. The two assignments covered application problems. The first one required a group presentation of the solution to a selected problem. The second group assignment included a formal write-up of the problem including a clear explanation of the solution. After the students had worked in their groups on these two assignments, giving them a chance to develop a working relationship, a group quiz was given. The design of this quiz demonstrated Mr. Lorenz's notion of interdependence and individual responsibility. Each group was required to complete all five problems on the quiz and each student was required to have at least two completed problems on his paper. There was not enough time allowed for an individual student to complete all five problems. In this way, students were required to solve problems individually and depend on their group for the solutions to other problems.

Giving students an opportunity to ask questions and get additional assistance was important to Mr. Lorenz. Before the unit test he arranged a time, outside of class, for students to meet with him for review. Students completed an end of unit test designed to assess their grasp of the concepts and their ability to accurately solve problems. In discussing the results of the test, Mr. Lorenz emphasized the importance of being able to assess for oneself the accuracy of the answers. As before, Mr. Lorenz encouraged students to take individual responsibility for their learning and assessment of what they had learned.

Mr. Lorenz prepared for class sessions by selecting problems for starter activities, often utilizing prepared overhead transparencies produced by the textbook publishers. When a problem arose in class, he spent time preparing a solution to the problem that he displayed on a hand-made overhead. This type of preparation enabled him to efficiently address questions that had arisen. His plans for each class session were not elaborate, consisting primarily of a notation of the section to be covered and the assignment to be made. He often depended on questions from students to lead into the discussion of the next topic. "Isn't it great when students supply the lead into the next lesson. So often it works out so well." His years of experience provided him with the background to be able to present the material without detailed planning. Occasionally, however, class did not go as anticipated and there was not sufficient time to complete all that he had planned necessitating a change of plans.

> We were going to go into our formal write-ups today, but we've got some answers we need to get first. So, what we're going to do is go to the review assignment. It's the one listed. Do the review assignment. Tomorrow will be a review day and we'll put together these constraints [referring to the problem left unsolved] and you'll still do formal write-ups. It's going to postpone them a day. The test won't be until Thursday.

Mr. Lorenz displayed enough flexibility in his planning that he was able to change the schedule based on the needs of the class and the circumstances that arose.

Use of graphing calculators in teaching. Mr. Lorenz utilized graphing calculators in a variety of ways in his teaching. In introducing the unit on solving systems of equations, a discovery activity was used which presented the students with a visual representation of the solution to a system of two linear equations. Students were familiar with the use of graphing calculators for graphing linear equations, but they had no experience with solving systems of equations. The activity served two purposes: it introduced students to the concept of the solution of a system of linear equations and it expanded the students' familiarity with the graphing calculator and ways it could be used in the solution of problems.

Another way Mr. Lorenz used the graphing calculator was as a tool to check one's work. After demonstrating the graphical solution of a linear inequality using paper and pencil, Mr. Lorenz discussed the assignment with students by using the overhead graphing calculator display unit to show the correct graphs. Mr. Lorenz began the demonstration by simply producing the correct graphs to show students what the answers were to the assignment. Because it was not possible to distinguish, on the graphing calculator, between a strict inequality and an inequality that included equality, Mr. Lorenz discussed whether each graph should have a dashed line, for strict equality, or a solid line in the cases where equality was included. As Mr. Lorenz produced the graphs, students began to ask questions concerning the use of the graphing calculator. Students were attempting to produce the graphs he was displaying on their calculators. At this point, Mr. Lorenz responded that students should just check their answers.

After showing the solutions to several problems and answering questions concerning the solutions, Mr. Lorenz talked the students through the process of finding the solution to the next problem with the graphing calculator. As he pushed buttons on the calculator, he told the students which buttons he was pushing. Graphing the solution to an inequality required the use of the DRAW menu and the SHADE function which were unfamiliar to the students. Mr. Lorenz explained how to maneuver through the DRAW menu and what to enter into the SHADE function. He talked the students through the complete process required to produce the graph of the solution for an inequality, giving them hints about how to figure out what to do as he proceeded.

Mr. Lorenz's expertise was primarily with the TI-85. He was usually able to assist students with other calculators. When the students with HP-48 calculators asked about using their calculators for solving inequalities he responded that he had run out of time the night before, but that he brought the manual with him and he would work with them on it. When students asked about using the TI-81 and 82 he directed them verbally, explaining the differences between the use of their calculators and the TI-85. Throughout the discussion, Mr. Lorenz asked questions, waiting for student responses before continuing, and troubleshooting for students who were not producing the desired results on their calculators. He was able to make suggestions and offer advice for most students' difficulties. After working through several problems with the students, discussing what the desired outcome should be and then what to do to produce that result, Mr. Lorenz had thoroughly explained the syntax of the SHADE function. In this way, Mr. Lorenz had accomplished two goals: he had shown the students how to use the graphing calculator to verify the accuracy of the work they did by hand and he had increased their expertise in the use of the graphing calculator.

Emphasis on the limitations of the graphing calculator in the solution of problems was a part of Mr. Lorenz's teaching. When teaching about the solution to a system of inequalities, he explained how to use the SHADE function to solve a system of inequalities in which one inequality contained y greater than an expression and the other inequality contained y less than some other expression. This type of system of inequalities lent itself to the use of the SHADE function because one of the inequalities would form the lower boundary of the shaded region and the other would form the upper boundary. By proceeding to a system in which both inequalities were of the form y less than an expression, Mr. Lorenz was able to show the limitation of this method for finding the solution for a system of inequalities. Even though it would be difficult to use the graphing calculator to find the full solution to the problem, Mr. Lorenz pointed out that the graphing calculator could still be used to produce graphs of the equations that corresponded to the two inequalities. In this way, the graphing calculator could be used as a tool to help in finding the solution even though it was not possible to find the entire solution with the graphing calculator. Extending the use of the graphing calculator to the solution of linear programming problems was done only briefly with one example for which Mr. Lorenz graphed the equations that corresponded to the constraints. He found the use of the graphing calculator cumbersome and did not encourage its use for the solution of linear programming problems.

> My suggestion is that you graph this, probably on paper is your best bet, because you're going to have difficulty graphing this [on the graphing calculator]. Those [graphing equations and using SHADE on the graphing calculator] are tools you can use, but I think you're best served with this particular problem on paper.

In addition to utilizing its graphing capabilities for solving systems of equations, Mr. Lorenz also utilized the graphing calculator's matrix capabilities. The TI-85 calculator had a built in program SIMULT for finding the solution to a system of linear equations. This program utilized matrices to solve the system, but the matrix operations were not apparent to the user. Students had not previously studied matrices, so Mr. Lorenz prepared an overhead transparency that gave a brief description of matrix notation. Having given a brief explanation of matrices and the matrix equation which lead to a solution for a system of linear equations, Mr. Lorenz demonstrated, using the graphing calculator overhead display unit, the use of SIMULT on the TI-85. He then explained how to input the required data into the TI-85 and instructed students to "play with it." Before the demonstration on the TI-85, Mr. Lorenz distributed handouts to students using the HP-48 giving step-by-step instructions for a similar method for solving a linear system on their calculators. Students using TI's other than 85's needed to enter the entire system into their calculators as matrices. Mr. Lorenz guided these students through the steps to find the solution, telling them which keys to use and where they were located. After explaining to all students how to utilize their calculator's matrix capabilities to solve systems of linear equations, Mr. Lorenz circulated through the room, assisting students with their calculators as they worked on the day's assignment.

Throughout the demonstration and discussion of utilizing the graphing calculators' matrix capabilities to solve systems of equations, no attempt was made to explain how the calculator was using the matrices to find the solution. Rather, students were expected to accept the method as an alternate for solving the system and trust the calculator. When a student came to him with a calculator displaying an error message he told the student that the calculator was giving him a good error message, not one that meant the student had done something incorrectly. Mr. Lorenz reminded the student that not all systems had solutions. He told the student that the calculator display of SINGULAR MATRIX was trying to tell him something. He suggested that the student take a problem from another assignment in order to figure out what the error message meant. Mr. Lorenz was demonstrating that the graphing calculator had capabilities the students could utilize to solve problems without understanding the methods being employed by the calculator. In this way, the graphing calculator was extending the students' abilities.

Mr. Lorenz encouraged students to use their graphing calculators in the solution of problems both on daily assignments and tests. When students used the graphing calculator to complete a problem, Mr. Lorenz was not satisfied with just an answer. "If you get a problem that you can use your calculator, use it. Write a line or two to describe what the graph looks like." He wanted students to be able to explain what they were doing even when they were using the graphing calculator as a tool.

Belief clustering interview. Mr. Lorenz was presented with 37 cards containing statements based on comments he had made in previous conversations and on observations of his classroom practices. The cards had been shuffled so that the statements were in no particular order. After indicating that he was comfortable that these cards accurately described his teaching he sorted them into four groups arranged in a two by two array (Figure 5). The group numbers were added by the researcher for clarity following the order in which Mr. Lorenz discussed them. When asked if any cards needed to be added or deleted his first response was, "this stack is too big," indicating Group 1. After further review of the statements, Mr. Lorenz asked to have a card added, which would be "more implied than specified as often as it should be," saying "the kids are expected to read the text." This statement was written on a card which he added to Group 2.

Group 1	Group 2
Group 4	Group 3

Group 1 Things I do

#I learn as much as I can about the graphing calculator and take it to the kids.
#I chat with students about their activities before and after class.
#You see me running out of time.
I show students how to solve the problems
I read some the students' reflections in class.
I am available to help students while they are working in class.
I demonstrate that there may be more than one way to solve a problem.
I answer students questions about a quiz.
I hold help sessions for students before tests.
I demonstrate how to use the graphing calculator.
I read the answers to the homework problems.

# denotes subgroup "for myself" added by Mr. Lorenz

Group 2 Individual Expectations for Kids

Students keep a notebook for each chapter.
Students tell the teacher how they solved a problem.
We have individual focus where you're just working all by yourself.
Students are encouraged to go back and correct their assignments after they've been discussed in class.
Notebooks are required to be organized and easy to use.
+Students are allowed a make-up or retest.
+Students take quizzes over the major points.
+*Students are expected to read the text.
+Assignments are occasionally picked up.
+Students take a chapter test at the end of each chapter.
+Students write reflections about what they have studied.
+Students use a scoring rubric to score their own notebooks.

*denotes statement added by Mr. Lorenz
+denotes subgroup " for assessment" added by Mr. Lorenz

Group 4 Group Structure

Students take a group quiz.
We do a couple of performance tasks.
Students are intentionally crowded for time on the group quiz.
Students talk to each other about the solutions to problems on a quiz that has been returned.
We use the small group setting with groups of 3 or 4 students.
Students prepare formal write-ups of problems they have solved in their groups.
Students present the solutions to problems they have solved in their groups.

Group 3 Class Atmosphere

Students are asked what tool they have for solving a specific problem.
Students point out mistakes I have made in solving a problem.
Students supply the lead in to the next lesson through their questions.
We use the large group almost like a lecture setting.
There is an interest and questioning atmosphere.
Students are required to take the ATPAC [Atlantic-Pacific High School Mathematics League exam].
Kids come into class with questions about the topic they've been working on.

Figure 5. Mr. Lorenz's card sorting.

Mr. Lorenz sorted the cards according to the structure of the statements, limiting his sort to broad categories. The categories he chose were Teacher Action, Student Action, Classroom Activity, and Group Structure. His sort did not initially consider why certain actions occurred, however he did eventually separate the cards in two of the groups into subgroups that dealt more with the purpose of the action rather that the action itself. Even when discussing the cards and the groups, he did not include underlying motivation for the actions.

Mr. Lorenz titled Group 1 "Things I Do," saying, "I'm a little surprised that the stack is this large. It's kind of eye-opening sometimes to, when you think you're teaching in a method where the kids do everything, to find out how many things you're really doing." His surprise at the size of this group of cards caused him to reflect on the large number of teacher actions in what he had considered a student centered classroom. After reducing the size of this group of cards by removing three that he said were, "for myself, to get ready for the kids, how I use time; where these [the remainder] are more procedural kinds of things." He continued to describe his conception of his teaching, "My own picture of the class is that I'm kind of guiding them through and they're doing everything. With the pile so large it made me stop and think, maybe I'm doing more of the chalk-talk type things and less of the coaching activity than I realize."

The second group he titled, "Individual Expectations for the Kids." He indicated these statements "refer to things they [students] have to do themselves in order to be successful in the class." The card about the students being expected to read the textbook was added to this group. All the cards in this group referred specifically to student actions. As he continued to discuss these cards, he separated them into two smaller groups. The statements marked with + in Figure 5, Mr. Lorenz called "assessment activities," which are "done for the benefit of assigning a grade." The other group of student action cards were, "directed more towards the understanding."

> For example the notebook. This one is scoring the notebook. This one says they are required to keep it [the notebook] in an organized manner. I'd separate them out as this [keeping the notebook in an organized manner] is useful to them [for understanding] and this [scoring the notebook] is for putting a score on for assessment purposes.

He did not feel that the subtitles learning and assessment were totally appropriate for these cards because "there's learning in assessment, too, but probably not the main focus."

Mr. Lorenz regarded the remaining two groups of cards, those at the bottom of the array, as dealing with the atmosphere and structure of the class. Group three he titled, "Class Atmosphere," indicating they "set the tone for everything." The final group of cards were titled, "Group Structure," which Mr. Lorenz considered as "either implied or directed group structure." All of the cards in this pile referred to students working in groups, either what they were doing or how the groups were organized. Mr. Lorenz did not make a distinction between the actual group activities involving students and the mechanics of organizing the groups when he sorted these cards.

Belief verification interview. Mr. Lorenz's belief interview took place at his home during the summer after initial analysis of classroom observations and beliefs had been completed. For this interview, the statements used in the belief clustering interview were separated by the researcher into statements that reflected Mr. Lorenz's beliefs about mathematics, the teaching of mathematics, the structure of the classroom, and how students learn mathematics.

The first set of statements, reflecting beliefs about mathematics were:

Students are asked what tools they have for solving a specific problem.

The teacher demonstrates more than one way to solve a problem.

Students write reflections about what they have studied.

The teacher reads some of the student's reflections to the class.

Mr. Lorenz responded to these statements about his teaching and their relationship to his beliefs about mathematics by indicating that mathematics was "a tool that you use to solve real world problems." While it is important to teach the "beauty of mathematics," he indicated that it was a lot more important to teach "how I can use this and how I can use these tools when I get out of here." Mr. Lorenz attributed the emphasis on mathematics as a tool in his teaching in part to his experience with computer programming and participation in the Applied Math training program.

The next set of statements Mr. Lorenz discussed were related to his beliefs about teaching mathematics:

We use the large group almost like a lecture setting.

The teacher reads the answers to the homework problems.

The teacher shows students how to solve the problems.

Students supply the lead in to the next lesson through their questions.

The teacher learns as much as the teacher can about the graphing calculator and takes it to the kids.

The teacher demonstrates how to use the graphing calculator.

The teacher answers students questions about a quiz.

The teacher is available to help students while they are working in class.

The teacher holds help sessions for students before tests.

We use the small group setting with groups of 3 or 4 students.

Students take a group quiz.

Mr. Lorenz' first response was that he did a lot of different things. In reflecting on these statements he shared an experience he had had in an attempt to utilized discovery learning in a precalculus class. He was utilizing an approach in which he would introduce a topic just a little bit and then have the students do some problems. He would then follow up the next day by working the problems in great detail. He indicated that one student felt he was not teaching her anything, but he believed that he was accomplishing two things. "I wanted them to discover the system of mathematics. And I wanted them to learn how to dig things out on their own and use me as a resource instead of a lecturer." Utilizing the small group structure for Mr. Lorenz became important because it gave students an opportunity to "teach each other, almost better [than he could] because sometimes we don't talk their language."

Mr. Lorenz's use of reading homework assignment answers in class, when the teacher was focused on having the students discover the system of mathematics and explore concepts on their own, became more than just reading the answers to the practice the students did the previous night. If the assignment was just practice, then reading the answers was reinforcement and a few scattered questions needed to be answered. But if students were exploring mathematics on their own, questions would arise so that "reading the answers usually involved working about half of the problems, or having the students work half of the problems. I don't always do them myself." Mr. Lorenz felt that when questions arose it indicated a level of commitment by the students and an ownership of the mathematics.

The statements related to Mr. Lorenz' beliefs concerning the structure of the classroom were:

The teacher chats with students about their activities before and after class.

Students take a chapter test at the end of each chapter.

Students are allowed a make-up or retest.

Assignments are occasionally collected.

Students use a scoring rubric to score their own notebooks.

Students are required to participate in the Atlantic Pacific (ATPAC) competition.

Mr. Lorenz began expanding on these statements by emphasizing his concern for making connections between mathematics and the real world. When the teacher is able to relate to the student's world, it opens the door. "Students learn better from someone they can relate to a little bit." Furthermore, he emphasized that what was really important was not when a student was able to do the mathematics, but that a student was able to do it. Mr. Lorenz felt that it was not important to be able to pass a test, but to be able to demonstrate that you had the knowledge required to complete the task. Being able to figure out how to use the skills (that they practiced in assignments) together to solve a problem outweighed being able to complete assignment set after assignment set correctly. Having students at advanced levels (Algebra II and above) maintain and score their own notebooks reflected this philosophy because the responsibility was on the students.

The final set of statements in this interview were related to beliefs concerning how students learn mathematics.

Students are expected to read the text.

There is an interest and questioning atmosphere (in the classroom).

Kids come into class with questions about the topic they've been working on.

Students talk to each other about the solutions to problems on a quiz that has been returned.

Students tell the teacher how they solved a problem.

Students point out mistakes the teacher has made in solving a problem (on the board or overhead).

Students prepare formal write-ups of problems they have solved in their groups.

Students present the solutions to problems they have solved in their groups.

Students keep a notebook for each chapter.

Notebooks are required to be organized and easy to use.

Students are encouraged to go back and correct their assignments after they've been discussed in class.

We have individual focus where you're just working all by yourself.

Students take quizzes over the major points.

Students are intentionally crowded for time on the group quiz.

We do a couple of performance tasks.

Reflecting on these statements, Mr. Lorenz noted the inclusion of drill and practice, trial and error, and learning from your mistakes, all of which he considered to be part of his understanding of how students learn. Overall, he felt that often it is not clear how students learn, "we're almost just shotgunning it, trying a little bit of everything, and hope that this fits for this kid and this fits for this kid over here, to make those connections." It was clear that not knowing how to make the connections for all students was frustrating for Mr. Lorenz. But, the connections were sometimes made. The connection might be with some concept or technique completed previously in the class he was teaching or in another class, or "with something that you or I can't see in any way. It's just the 'ah, hah,' that happens." The utilization of groups, according to Mr. Lorenz, could allow some students to make connections by seeing how another student understands rather than how the teacher sees it. Furthermore, allowing a student to talk to another student "reinforces ideas and [they] build on one another."

Another thread that Mr. Lorenz saw in these statements was the importance of organization, "just to be able to organize your thoughts. When kids don't understand a topic, you look at the way they're organizing what they're doing. You think, well, if I could just get them to change, and order things, they could see their own connections." As he reflected on the need for students to learn organization as a part of learning mathematics, he mentioned the state open-ended assessment test as an "extrema of organization where you're actually graded on your organization as much as on whether you solve the problem." These comments led Mr. Lorenz to reflect back on his thinking about the teaching of mathematics and the effect of external standards and testing on his teaching.

> We'd always been teaching to what we think the colleges want.... [Now,] we're almost teaching to the test [the state assessment].... As the state brings all these things in line, my fear is that we're going to teach only to the test. I'm going to have to make some choices somewhere along the line. There's this really neat thing that I do, that students enjoy and I think there's some real value in, but I have three more content standards that I have to cover this year. And so, I'm afraid that some of that good stuff we do, that may not match 100% with some of the standards that we have, is going to be thrown out.

The tension between teaching the system and art of mathematics that he valued and the need to meet the externally imposed standards was real.

Finally, Mr. Lorenz added the importance of attendance to the composite of how students learn. "You have to be there to learn." For him, attendance was part of the larger notion of the student's responsibility for learning.

> We can talk all we want about teaching students, but if the students aren't willing to learn and aren't receptive and aren't willing to accept some of the basic responsibilities such as getting to class on time, and bringing a few essential tools, and shutting up and listening for a little while, it doesn't make a whole lot of difference what we do.

Concerning the use of graphing calculators, Mr. Lorenz reiterated his view of the graphing calculator as a tool to use when you need it. The way in which the use of the graphing calculator was taught had changed since its introduction. "I am finding less and less need to teach the basics [of how the graphing calculator works] because they are coming in with them." Furthermore, he saw that teaching the use of the graphing calculator was easier because the students "have absolutely no fear of them." There was a new complication though. "They're three screens past where they need to be and you have to bring them back to the screen you're working on in order to solve the problem." Another graphing calculator teaching issue for Mr. Lorenz was related to the multitude of models now available. His response to students with a variety of different graphing calculators was to become proficient with one or two models and encourage students with other models to share their information and learn to utilize the graphing calculator manual.

> I'm teaching primarily to two calculators. I can usually help them with just about anything that comes up. If they have one of the others, we're either going to have to go to this student who is good at it, we're going to have to go back and start digging it out of the manual, or we're going to have to say, "Okay, come in, bring your calculator, bring your manual, and we'll sit down and work on it and figure it out together."

Beyond the way in which his teaching the use of the graphing calculator had changed, Mr. Lorenz's teaching of the course content had changed with the inclusion of the graphing calculator. The complexity of the problems had changed.

> Not too long ago you had to make sure the problem worked out nicely. Kids weren't going to have near enough time to solve the problem if it came out with fractions, weird decimals, and all that. You don't have to do that so much anymore, so you can work more real situation problems than you could before.

The use of the graphing calculator facilitated the teaching of more mathematics "because you have the power to be able to do things so much faster." He particularly referred to the ability of the graphing calculator to produce tables of values for functions which then allowed time to explore "extensions and applications and ways that they're going to use that." The use of the graphing calculator put a lot heavier demand on test writing as well. "If you are going to allow kids to use the graphing calculator, then you better know what you are testing." Mr. Lorenz emphasized the importance of realizing when students could just push buttons on their graphing calculators to solve a problem and when they would have to demonstrate understanding and mastery of the mechanics. Sometimes, he said, "you can just take one number out and put in a letter and make them write it down." But, it was essential to know before the test was given, what was being tested and how students could utilize their graphing calculators to complete the tasks. Fairness, making sure students with different models of graphing calculators were able to complete the test in comparable manners, contributed to the demand on Mr. Lorenz in the writing of tests.

Summary of beliefs. For Mr. Lorenz, there were two important parts to be incorporated into his definition of mathematics. Mr. Lorenz believed that the use of mathematics as a tool to solve problems was the most important facet for students. He emphasized the variety of tools mathematics provided for solving problems and made connections between the tools of mathematics and the problems of the real world. He also believed that the beauty of mathematics should be taught. Part of this beauty was the system of mathematics that he wanted students to discover on their own.

In order to teach mathematics, Mr. Lorenz believed it was necessary to use a variety of approaches. He believed in a student-centered approach to teaching in which he was guiding students and serving as a resource for them rather than supplying all the information. He employed a group structure to facilitate student-to-student teaching because he believed that there were times when students could teach each other. At other times, Mr. Lorenz recognized that students needed expert information and so he presented material to them. Even when presenting material, he believed in actively involving students. To facilitate this student involvement he posed questions and waited for responses. Another technique Mr. Lorenz utilized to promote student involvement in the development of new material was to present the material in response to students' questions. His belief in the system of mathematics and its use as a tool to solve real world problems was reflected in his belief that teaching mathematics required making connections to what students had learned previously and to real world problems.

Mr. Lorenz believed that students learned best when the teacher could relate to the student's world which he did by showing an interest in their activities outside of class. While he felt that it was unclear how students learned, making connections was an important piece of the puzzle. Furthermore, students needed to take individual responsibility for their own learning. This individual responsibility included attending class, being prepared, and putting forth individual effort. Making expectations clear to students was part of teaching for Mr. Lorenz. He also believed that organization was helpful to student learning. Being able to discuss with other students was beneficial, too. He emphasized the importance of students discovering mathematics for themselves. Most importantly, he believed that students had to do mathematics in order to learn it.

Consistency between practices and beliefs. Overall, Mr. Lorenz' practices and beliefs were consistent. His flexibility in adapting his plans to student questions and problems that arose showed his overall concern for the students and their needs. This concern for the students was consistent with his belief in a student-centered classroom. In reflecting on his classroom practices he expressed some concern over the level of observed teacher activity in what he considered a student-centered class. Teacher activity does not necessarily indicate that the class is not student-centered. The analysis of the data on Mr. Lorenz' classroom practices clearly demonstrated that while the direction of the class and the material covered were teacher-directed, the activities in the classroom were tailored to the needs of the students and the teacher was responsive to their input. Mr. Lorenz' concern regarding the level of teacher-activity may have reflected a belief that discovery and student-centered activities do not require active teacher involvement.

Ms. Shade

Ms. Shade was contacted after a referral from the department chair at her school. During an initial meeting with her in the mathematics office, she indicated a willingness to participate in the study. Before formal participation began, a meeting with the Headmaster of the school took place during which he agreed to allow Ms. Shade to participate in the study.

Church School, where Ms. Shade taught, was a private kindergarten through twelfth grade, church related-school. The school was divided into a lower school, kindergarten through fifth grade; middle school, sixth through eighth grades, and upper school (ninth through twelfth grades). The schedule for the upper school was a blend of 50-minute class sessions and 60 - 65 minute class sessions. On Monday, Wednesday, and Friday all seven classes met for 50 minutes each with a gathering time after the first class period (approximately 15 minutes) on these mornings. On Tuesday and Thursday class sessions were 60 - 65 minutes. Only first, third, fifth, and seventh periods met on Tuesday. On Thursday, second, fourth, and sixth periods met. On both Tuesday and Thursday there was an activity period at the end of the day. On Tuesday morning there were Advisory and Chapel time periods and on Thursday morning there were Advisory, Meeting Time and an X period designated for enrichment activities. The entire upper school ate lunch together daily in the dining hall. All of the mathematics teachers at the school had desks in one office. There was space in the office for students to study, consult with teachers, and use the computer.

Ms. Shade described the students at Church School as "B or better students. A couple of years ago, three-fourths of that graduating class had a B average or better." The students were expected to enter at or above grade level. In their freshman class one-fifth to one-fourth would be in Algebra I, about the same portion in Advanced Algebra, and the remainder of the class in Geometry. Ms. Shade described Church School as having "an ethos that it's okay to do well, that you're not embarrassed if you do well." The teachers at Church School, according to Ms. Shade, were "highly qualified individuals." She considered individual attention for students from teachers to be a major feature of the environment at the school. "Kids who start off weak often times will grow because of the individual attention teachers can give them. Having classes of 15 means that I can know the kids within a month, and know who they are, what they need."

Several other teachers at Church School were also teaching Advanced Algebra during the term that the study took place. All sections were taught with the use of the graphing calculator, but Ms. Shade had the most experience with its use. Because of the nature of the school, it was possible to require all students to purchase the same graphing calculator. All students were using the TI-82 in Advanced Algebra. The textbook used was the Chicago series *Advanced Algebra* published by Scott Foresman (Senk, Thompson & Viktora, 1990).

Ms. Shade characterized the administration as supportive of the staff. The Headmaster "has totally supported the faculty and staff development" with time and money. "They want us to develop as people and as teachers. I don't think you can ask for much more support."

Background. Ms. Shade was in her seventeenth year of teaching, having taught at Church School for the past 10 years. She had taught mathematics exclusively with all but one course at the high school level. In discussing her educational background, Ms. Shade began by saying that she never wanted to be a teacher. "I just thought a teacher was a woman's job and I wanted to be different. So, I got my Bachelor's in math, I got the teacher's certificate on the side because I always thought it was something I should do."

After completing her BS in Mathematics at a private university, Ms. Shade went to graduate school. Graduate school in mathematics was a possibility, but instead she chose a program in student personnel work at a public university where she completed a Master's degree. Ms. Shade then took a job in admissions at a university on the East coast, but she decided the job was not for her. "I didn't like the job at all. I knew after a day I didn't like it, I knew after two weeks I didn't like it. Finally, after six months I'd had it." Then she found a long-term substitute position in a public school. In her words, "from the first day in class, from that very first day of subbing in someone else's class, I loved teaching."

Ms. Shade taught for several years in that school district before moving to the Northwest. After moving, Ms. Shade taught at an urban public high school for five years. While there she taught the full range of high school mathematics courses from General Math through Calculus. She had been at Church School for the last 10 years. When reflecting on the differences between teaching in a private school and a public school Ms. Shade commented that she "missed helping the students who really need the help" at the public school. She found that at Church School, "we help B students very well.... They're the ones who probably learn the most from us, because in the public schools they're the ones who tend to get lost. They don't ask the questions, they just sit there and do their work. And here, they'll move." She had also found that the pressure and expectations from parents was more intense at the private school than they had been at the public school. She felt that she couldn't "just teach without worrying about mommy or daddy sometimes." When she began teaching at Church School she felt she was "going to have to live up to expectations" which she did not think "affected the way I was teaching. But it affected the way I was thinking about things from day to day."

While at Church School, Ms. Shade spent several years serving as Dean of Students and Student Activities, teaching only one or two mathematics classes. She agreed to serve as Dean because "I felt I would be helping to fulfill a need of the school." But, after three years as the Dean, she knew she wanted to go back to teaching full-time.

> I knew I was giving up one major thing which was the freedom to be my own boss. When I'm in my classroom, I'm doing my own thing. When I was Dean, I was reporting to the Head and reporting to the Headmaster.... I always thought I was doing a disservice to my teaching that whole time. I never thought I was quite as on.... And then I went back full-time to teaching. I'm just much happier.

She had continued teaching Calculus while serving as Dean. Ms. Shade was teaching two sections each of Calculus and Advanced Algebra and one section of Algebra the fall of the study. She was also coaching basketball and serving as reader for the Advanced Placement (AP) Calculus exams.

During her time at Church School, Ms. Shade had been able to attend and make presentations at numerous national mathematics meetings, attend the Critical Thinking Skills Seminar, and twice participate in the Exeter Conference on Secondary Mathematics and Technology. She had also completed a number of graduate and continuing education courses from a nearby state university. When discussing her preparation for teaching and inservice experiences, Ms. Shade indicated that the education course work she had taken had been of little value; rather she had learned the most from more experienced teachers with whom she taught.

> For example, back in Algebra I, the first time I taught that, I went to the teacher who'd taught it for years and said, "you know factoring has always been easy for me, I can just see it. And this other way of doing all the different pairs...is too lengthy." And she showed me the neatest algorithm that I still show the kids.

Introduction to and thinking about graphing calculators. The introduction of graphing calculators into the teaching of calculus had been an opportunity for Ms. Shade to approach her teaching of calculus in a new way. "I was ready to change classes. I had started saying that maybe someone else wants to teach Calculus. It was getting too routine for me. And the calculators just changed that whole thing. And so for the last five, six years, it's [teaching calculus] new for me." Ms. Shade was first introduced to graphing calculators when several students brought in Casio graphing calculators. She "thought they were neat" so she convinced the school to buy her a TI-81. Gradually, as she learned to use the graphing calculator, she introduced it to her calculus classes. During this introductory phase Ms. Shade "would go to the sessions [at mathematics teachers' meetings], that would be the biggest place where [she] learned things. It would mainly be the conferences, fighting to get into the calculator sessions. And there weren't many of those either." During the first few years, she was the only one at Church School using the calculators, but gradually Ms. Shade had encouraged others to incorporate graphing calculators into their teaching.

Ms. Shade found that with the graphing calculators, students could master concepts and not be restricted to learning techniques.

> Like this matrix stuff we did today. It would have taken us two weeks to master that stuff. Instead we can master the concept. I started seeing right away [when using graphing calculators], we weren't going to have to deal with technique, or little algebra mistakes, we could deal with understanding the theory behind it. I think that's the biggest reason I've stayed with it.

Using graphing calculators had moved her teaching to a different level of thinking. Now, students could spend less time mastering techniques and more time solving problems like systems of equations. "I see people learning on a whole different level.... We learned a little about matrices, now let's see what we can do with them.... Part of that is what is most important, what you can do with it, not always how you can do it."

Figuring out what needs to stay in the curriculum, what needs to be emphasized with the use of the graphing calculator, was what Ms. Shade saw as the next big challenge. In her opinion, it should be the teachers, not the textbook publishers, who made the decisions about calculators and curriculum. "I don't want the publishers to sit down; I don't want a few writers to sit down; I want the teachers to sit down and say, 'Where do we need to go? What are we going to take out of the curriculum now that we have a calculator? And what are we going to really emphasize on the calculator'." Through her experiences as a grader for AP Calculus exams, she had been able to spend time with other calculus teachers discussing the curriculum and what kinds of questions should be on the AP exams. She felt the same type of discussions needed to take place among teachers of advanced algebra and trigonometry.

Additionally, Ms. Shade found much to learn about using graphing calculators. The manuals for the graphing calculators were often not helpful. But, as she had found with her teaching in general, the best way to learn about graphing calculators, Ms. Shade indicated, was to exchange ideas and learn from others. "I'd like to go into a room with about 15 people where we all share what we've got [what we do with graphing calculators]." Learning from students who often had the time to experiment with their graphing calculators and were excited to share what they had found was another way Ms. Shade had been able to expand her graphing calculator skills and knowledge.

Professed beliefs about mathematics and teaching mathematics. Ms. Shade defined mathematics as "a study of numbers and number concepts that allow us to do other things in the world." The study of mathematics also contributed to the development of thinking skills. To Ms. Shade, "the biggest thing that the study of mathematics offers people is the development of thinking skills. It's not the facts and data, but the development of a thinking pattern that's going to help them [students] for life."

Ms. Shade looked at algebra in two ways depending on the goals of the students. For students who continued to study more mathematics, algebra provided a foundation. "For a student that wants to go on in math or engineering, these [algebraic] are the skills that you need for what you're going to do, the pure skills that you need, in addition to the thinking skills that we're developing." In contrast, students for whom advanced algebra was a terminal course were able to benefit from the discipline and thinking skills developed.

> For a weaker student, I'm hoping that part of the discipline of having to learn these things, part of the process of having to pull it all together is what's important to them. They may never remember [the details] but if they can hold it together for a chapter and see a picture, then I've helped them think through a project for life.

While Ms. Shade believed that students learned in varied ways, she felt that doing was essential to learning.

> There are kids who can sit there and conceptualize and say, "Yes I understand." I guess a common thread is you have to do to learn. The brightest kids who just sit there and conceptualize [at the Advanced Algebra level], when they get to Calculus can still conceptualize, but they can't do a thing. They don't have the algebra skills. So if you're really going to learn mathematics, I think you have to do it.

For students who did not understand, especially at the basic levels of General Math and Algebra I, hands on material was beneficial. "I think hands on is important because they've got to; again it's that doing." Even at the Calculus level, labs were "a way to bring in some hands on approaches." Doing the mathematics included "trying to do a project," and "applying it to real-life situations."

Classroom practices. The first of Ms. Shade's two Advanced Algebra classes was observed for one entire unit plus the period after the unit was completed which was review for the semester final exam. Several days during the teaching of the next unit were also observed. The second of Ms. Shade's classes was observed for a portion of the observation period. Data from the second class was used to enhance the description of the classroom practices observed.

Ms. Shade's class routine began with students correcting their assignments from overhead transparencies of the answers that had been copied from the teacher's edition of the text. While she believed correcting assignments was important and she utilized the overhead transparencies in an attempt to move quickly through the process, she was frustrated with the results. As students corrected their work, the answers they copied from the overhead were troublesome for Ms. Shade.

> It's supposed to be a time saver. They write so much down [from the overhead display]. First of all, it's not a problem I cared about and they've wasted all this time writing it down. Secondly, if they write it all down and I do care about it, then they've written more, so that I don't know what they didn't know.

After students compared their work to the answers displayed, Ms. Shade responded to their remaining questions on the assigned work. She then proceeded into a discussion of new material. In the presentation of new material Ms. Shade used a variety of techniques including demonstration lecture, interactive dialogue with students, and discovery problems. As time allowed, class concluded with students working either individually or in pairs. Ms. Shade varied her approach to her involvement with students during times that they were working on problems. Generally, if students were working individually she remained at her desk and had students come to her for assistance. If students were working in pairs, she tended to circulate, checking their progress. As students were dismissed, Ms. Shade reminded them of the assignment and expectations for the next class.

Ms. Shade utilized an interactive dialogue with students when presenting new material that built upon their prior knowledge while expanding it to include new concepts and techniques. When presenting the concept of the solution to a system of equations, she began by having students recall revenue and cost problems they had solved previously.

> Ms. Shade: Who remembers what we did with revenue and cost functions? What did we do with revenue and cost functions?
>
> Student: We found where they intersected.
>
> Ms. Shade: We found where they intersected. And what was the reason for finding where they intersected?
>
> Another student: It's the break-even point.
>
> Ms. Shade: It's the break-even point. Okay, revenue and cost have a break-even point and it was their point of intersection. Okay so if I were to solve for where two places are equal, what do I look for when I graph that?
>
> Student: [Where they] intersect.

Beginning from students' recollection of the solution to a revenue-cost problem as the point of intersection of the two functions, Ms. Shade built to the general concept of the solution to a system of linear equations. In this development, she directed the students as they worked specific problems in their notes as examples of the method she was presenting. Because students were knowledgeable about graphing, Ms. Shade was able to lead them through the method without doing an example prior to their attempts. She did review the correct solution, but only after students had completed the problem for themselves. In this way Ms. Shade created a situation where students extended their own mathematical understanding.

Another technique Ms. Shade utilized in presenting new material was to demonstrate with an example, allowing students to ask questions and eliciting insights from them as she proceeded. During this process she emphasized definitions and her expectations of the students.

> Ms. Shade: We're going to learn three methods, two new methods to solve systems of equations. What are systems of equations? When I say that, what does it mean? What does that mean when I say systems of equations? [pause]
>
> Student [called by name]: You want to find where they intersect.
>
> Ms. Shade: We want to find where they intersect. To solve a system of equations we want to find where they intersect. We, basically, have three methods. We actually have four methods. I'm going to explain one more to you. [listing them on the overhead] We have one, graphing; two..., substitution...; three is what we call the linear combination method; ... and the fourth will be matrix methods which we will actually learn tomorrow.

After reviewing the four methods that students had learned for graphing linear equations, three by hand and the fourth with the graphing calculator she explained that for the test she expected them to be able to find the solution to a system of equations by graphing without the graphing calculator.

> A lot of what we are going to do is going to be on the calculator, but one problem [on the test] will be where I see you graphing by hand and you explaining what you are doing step by step. One of those methods, and I'm not going to care which one you choose, but you need to have a working knowledge of one of those methods.

Ms. Shade's expectations for her students included not only being able to complete problems without the use of the graphing calculator, but also being able to explain what they were doing. She taught the two methods for solving linear equations, substitution and linear combination, during this class session by first outlining the steps needed to complete each method then presenting examples to the class. As Ms. Shade demonstrated these two methods using examples, she involved the students by asking questions that relied on recall of previously learned methods and checking for understanding of what she was doing.

> I'm going to solve this one for y, and I get this [writing on overhead.] What I want to do now is use substitution. Here's a y down here. I'm going to substitute what y is in the first equation into the y of the second equation. Does that make sense? [Brief pause, followed by completion of the process on the overhead.] Everybody see what I did? [Continues to work the problem through to the solution.] Am I going too fast or are you still with me?

After completing the problem Ms. Shade asked a student to explain what she had done, making certain that there was understanding.

Allowing students to discover, for themselves, a method for solving a problem was a feature of Ms. Shade's teaching. When introducing linear programming, she changed entirely the normal routine by announcing that students should "get out your calculators and pencils" at the beginning of class time. Students chose their own partners for the activity and Ms. Shade introduced the problem by having a student read it aloud. After defining the concept of a feasibility region she discussed with the students where they might find the maximum or minimum value of the profit.

> Ms. Shade: If I look at the feasibility region, where do you think my maximum or minimum might occur if I'm trying to look at profit? Where do you think my maximum or minimum might occur? Anywhere in that region or at specific points in that region?
>
> Student: On the lines.
>
> Ms. Shade: On the lines, okay. And more specifically, what parts of the lines?
>
> Student: Intersection.
>
> Ms. Shade: At the intersections of the lines. So remember you learned how to use INTERSECT on you calculator. You're going to play with that after you graph all of these.

After introducing the problem and discussing briefly the theory involved, Ms. Shade outlined a process for students to follow. "So, your first goal is to write all the equations.

Your second goal is to graph all the equations. The third goal is to find where all the intersections are, and those are your possible max/min points. And your fourth thing will be to determine your max/min." Students knew how to accomplish each individual goal, but had never put them all together in one problem. Knowing this, Ms. Shade encouraged them as they began, "This should not necessarily be easy for you to do. It's a thinking problem today. Okay, go to it."

The remainder of the class, Ms. Shade circulated through the room spending time with individual pairs of students, observing their work and answering their questions. Occasionally, she addressed a comment to the entire class. "Everybody listening? Once you do your graph, I would take the back of your paper and figure out what you're going to shade. You get a bigger picture if you draw it on the back. Take some time and think about what you want to shade." As she inspected their work, she offered suggestions that led them to correct errors they had made. Her suggestions included comments like "your window is probably not set up right," "It's the second one that's way off," and "Change your scale," giving students information while still allowing them to solve their own dilemmas. When she saw that most students had reached the stage of finding the points of intersection of the boundaries of the feasible region she made a sketch on the board and reminded them how to complete the problem.

> So when you're finding points of intersection, one here, one here, one here, one here, one here; looks like we have five points of intersection, I want all of those written out. Then your last question is to test which one of those five gives us our maximum profit. Most of you did an excellent job getting this far.

Throughout the process, Ms. Shade encouraged and advised the students. After class she commented, "I was surprised at how well they did. I like doing it before I teach it. Last year they really panicked." For Ms. Shade, encouraging the students to complete the problem without having first demonstrated the entire process was valuable for building confidence and learning techniques.

Providing students with time to work in class was important to Ms. Shade. This work time allowed her to observe the students at work and to provide additional assistance as required. Occasionally, Ms. Shade had the students work problems at the board, either in pairs or individually. After several days had been spent working on linear programming problems, Ms. Shade assigned pairs, distributed copies of a problem, and sent the students to the board to complete the problem. She instructed the students to work together. "You're working on it with your partner. You both need to keep track of the complete answer. So, one of you might be writing on the board and one of you might be writing on here [the handout sheet] so you can share your results." As students worked Ms. Shade circulated around the room, observed each pair's work, and offered assistance tailored to the needs of each pair. The assistance she offered included asking students directed questions such as,

> Are you writing a new variable?
>
> How does that relate to what you already know?
>
> You need an equation, what does that equal?
>
> You guys want to split up that last one so you can graph them independently, can you graph those two independently of each other?
>
> If you have x plus y equals anything, is that going to be a vertical or horizontal line? ... It's going to be what kind of line?

These questions were designed to move the students either from a difficulty they were having or to the correction of an error they had made. For other students who needed more assistance, Ms. Shade offered specific instructions. When a student was having difficulty getting started she assisted him by pointing out the part of the problem he had skipped. Later Ms. Shade returned and provided step-by-step instruction like, "Set m equal to ... yes. What I'd do is.... How are you doing? Is that this one here? What you did is.... What you should have done ...." With students working at the board, Ms. Shade was able to observe exactly what each student or pair was doing in the process of solving a problem and intervene as necessary to direct them to a correct solution. Knowing how individual students approached problems and what they did or did not understand was important to Ms. Shade.

In addition to the informal assessment of students' work that took place as students worked in class, Ms. Shade assessed students work through daily assignments, quizzes and chapter tests. Daily assignments were collected and evaluated by Ms. Shade. She primarily looked for completion of the work and areas where students needed assistance. Quizzes and tests were graded for accuracy and completeness. Before each quiz or test, Ms. Shade reviewed the material over which they would be tested. "Tomorrow you've got a 35 point quiz on 5.1-5.6, everything we've reviewed." As they left class, she reminded them again what would be covered. "I'm assuming you're leaving here comfortable with linear combination, substitution, graphing, and matrices. You're excused. Be ready for the quiz tomorrow." When returning the quiz, Ms. Shade spent time making sure students understood the expectations and criteria used to evaluate their work.

> Anytime you do a graph we need to make sure I can totally tell your picture. Some of you lost points because I was having to guess. You weren't really clear... so, I'm trying to guess, did they mean where it overlapped? Be really clear when you do graphs like this [for systems of inequalities] that I know where your answer is.

She also expressed her disappointment in their performance in some areas. "I have to tell you that I was really disappointed with the class's work on [number] 14 because 14 is pretty much identical to the problem we spent at least 10 minutes with up on the board." Because students had performed poorly on this problem, she discussed the errors they had made and advised students who had missed this and other problems that they should rework them. Another concern Ms. Shade expressed to her students was the lack of attention to checking their answers. "A lot of you forgot to do the check. But, a lot of you did the check, and you had the wrong answers and you checked it correctly. [Your check showed that your answer was correct.] What happened?" Further, she pointed out that if they discovered an error when they checked, they could just explain what they found and would not be penalized for the wrong answer.

> Someone got the wrong answer completely, but they got full credit for the problem. Here's why, they'd done the whole problem by hand and gotten the wrong determinant..... The rest of the problem was correct AND she came up and told me what her mistake was and I said don't redo the problem as long as everything else you carried out was correct. She recognized she had made a mistake. I didn't have her redo the whole problem because she knew what to do.

For Ms. Shade the process students used to solve problems, recognizing their errors, and knowing how to correct their work were as important as finding the correct answers. Quizzes and tests also served as opportunities for students to learn according to Ms. Shade. Having students review and correct their work was one way she emphasized the importance of using quizzes as a means for learning.

Use of graphing calculator in teaching. Ms. Shade made extensive use of the graphing calculator in teaching. Every day of the unit observed, the graphing calculator was used or its use was discussed by Ms. Shade or the students. She encouraged students to learn how to use the calculator in as many ways as possible. During the observation period, the graphing, table, computation, matrix, and evaluation features of the calculator were utilized. Ms. Shade emphasized the usefulness of the graphing calculator while maintaining the importance of understanding the processes and concepts involved. Students were expected to know when and how to utilize their graphing calculators.

Ms. Shade: When you put it in your calculator, what's x and what's y?

Student: I thought you said we had to draw it.

Ms. Shade: I did, but I didn't say you couldn't use another tool. I didn't say you couldn't plug it into you TI - graph link and give me a picture.

When assisting the students in extending their knowledge of the graphing capabilities of the calculator for finding the solution of a system of equations, Ms. Shade utilized the interactive dialogue technique. She asked the students key questions that needed to be answered in order to utilize the graphing calculator.

How am I going to enter those into my calculator?

What do you have to think about before you push GRAPH?

We sort of played with this by hand, let's go to window and try this together.

What kind of window do we want?

> [After displaying a graph], What do you see there?... Where does the action occur in this picture?

When students began commenting on the results they were seeing, Ms. Shade continued asking questions, based on their input. She asked students why certain results had occurred.

> Ms. Shade: Someone's got this nice little graph here, okay, and then she's got these nice little squares here.
>
> Student: I think she's got STATPLOT
>
> Ms. Shade: What do you have to do with STATPLOT to get those off there?
>
> Student: Turn the plots off.

After taking care of similar problems which had occurred, Ms. Shade continued with the problem by discussing the use of the arrow keys to move the cursor near the point of intersection, the use of ZOOM to get even closer and finally the use of TRACE to find coordinates which actually satisfied at least one of the equations. Ms. Shade demonstrated a comprehensive knowledge of the features of the TI-82 calculator. She did not have to be concerned with other calculators because all students were required to use the TI-82.

Ms. Shade encouraged students to explore the graphing calculator on their own time by providing extra credit for students who found new ways to use their calculators without her instruction. "One piece of extra credit is to find out something else on this calculator that will find you the point of intersection." The next day a student had found the INTERSECT feature and was able to demonstrate it to the class.

Ms. Shade introduced students to new ways of utilizing their graphing calculators by walking them through the process, step-by-step. As she did a new type of problem, she dictated the steps and waited for students to complete each step before continuing. For example, after explaining how to convert a system of three equations in three unknowns to a matrix equation and then how to manipulate the matrix equation to isolate the solution, she provided step-by-step instructions on the use of the graphing calculator.

> Find MATRIX [pause]; go to EDIT [pause]; change your thing to 3 by 3 [pause]; next to MATH [pause]; go to EDIT push ENTER [pause]. Enter these numbers [indicating matrix written on board] this one, the given matrix. Then go 2nd QUIT [pause]; MATRIX [pause]; NAME [pause]. Did you all put it in A? [Pause] Press ENTER. You should have A with brackets. Which key do I use to get the inverse? [Pause for response.] Push x to the -1 key, press ENTER. Did you get this answer?

In this way, Ms. Shade led the students through the steps necessary to find the inverse of a three by three matrix using the graphing calculator. She continued to complete the process of evaluating the matrix equation for a system of equations using the graphing calculator in the same step-by-step manner, waiting for students to complete each step and answering their questions as she proceeded.

The graphing calculator provided a means for creating teaching materials for Ms. Shade to use in explaining the solutions to linear programming problems. With the use of the TI-graph link, she prepared overhead transparencies that showed both the equations used and the resulting graph of the feasible region. She was able to label both the vertices and the lines so that she could explain to the students how the results displayed represented the problem being solved. Ms. Shade's use of the graphing calculator in this way was more than teaching the students how to use the graphing calculator to solve a problem, rather she was using the graphing calculator to enhance her presentation of the material.

Students were required to solve problems on tests and quizzes both with and without the use of the graphing calculator. Problems for which the graphing calculator could not be used included those for which there was no way to utilize the graphing calculator and those for which specific instructions were given requiring methods other than the use of the graphing calculator. Problems for which the graphing calculator was required included problems that students did not have adequate skills to complete without its use. Additionally, there were problems for which using the graphing calculator was one of several choices. In these cases, students were expected to explain how they had utilized the graphing calculator.

Ms. Shade used the graphing calculator in every facet of her teaching. She used it to demonstrate the solution of problems and for preparing teaching materials. The graphing calculator was a tool which students were encouraged to use both in their daily work and on tests to complete and check their work.

Belief clustering interview. Ms. Shade was presented with 23 cards containing statements that were based on statements she had made in previous conversations and on observations of her classroom practices. The cards had been shuffled so that the statements were in no particular order.

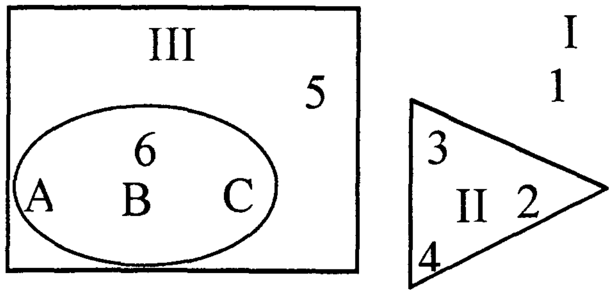
As she read through the cards, Ms. Shade questioned the intent of the card saying, "I depend on the kids [to figure out how to make something work on the TI.]" She did not agree that she depended on the students, but that she did sometimes say, "Why don't you guys figure this out, or have you thought about..." Actually, this statement was based on her statement in an earlier interview concerning the way in which she had learned to use the graphing calculator and the importance she placed upon input from students in the process. Ms. Shade indicated the importance she placed on assessing students' conceptual understanding when she expressed surprise at the presence of the statement referring to students going to the board and explaining problems to the class. She indicated that having students work at the board was important and she felt that she did not utilize this strategy often enough. In her response to the statement indicating that the final exam contained multiple-choice questions, Ms. Shade expressed further concern for evaluating students' conceptual understanding, "I would have liked observations on the test. Do I ask enough conceptual questions? Do I miss the boat on those kind of things?" As she sorted the cards, Ms. Shade arranged them in nine groups. When she had placed all the card in these groups, Ms. Shade merged two groups and rearranged the positions of two other groups. She later grouped three of the groups under a single title. The resulting sort included six groups, numbered one through six, with three subgroups in group six (Figure 6).

I Doesn't fit anywhere else

1. " It doesn't fit any other category so it's there. It's true."
The final exam includes multiple choice questions.

II It's what the student does

2. "This has to do with notetaking."
Students take notes in class
Teacher checks students' study notes for the final exam.



3. "This is all related to homework or a quiz."
The teacher answers students' questions about a quiz, going over the complete solution to each problem.
Students work on their homework.
Students correct their homework from an overhead display of the correct answers.
The teacher works out homework problems for students using the overhead projector or white board.

4. "It's more about the students. So, it's giving them a chance to learn in different ways. One by having to take some risks, one by doing the teaching, one by having to explore the TI. So to me, students working in groups on an activity, those are all exploratory type things."
Students go to the board (individually) and explain how to do problems.
I depend on the kids (to figure out how to make something work on the TI).
Students work in groups on an activity.
Students are offered extra credit if they find out how to find the inverse of a 3x3 matrix.

III What the teacher does

5. "Teacher expectations of myself and of my students."
The teacher shows a student that a different approach to a problem gives a solution that matches the solution found by the teacher.
The teacher talks with students about what they thought about a problem or project.
Students are expected to show their work on homework and exams.
Students do analysis of where they are and pick their best work for their portfolios.
The teacher explains how credit was awarded for partial solutions on test questions.
The teacher calls on students by name.

6 Teacher style

A. "Practical things as I see them"
The teacher prepares overhead transparencies using output from the graphing calculator.
The teacher circulates among the pairs (or individuals) working on an activity.
The teacher writes definitions on the board or overhead.

B. "Choices of methods that I use"
*I try to question the kids in a way that they have to derive an answer.
I try to talk more broadly than the book and expand on examples.
The teacher introduces new material by working problems.
I have students experiment, like with the graphs of consistent, inconsistent, and dependent systems.
The teacher demonstrates how to use the graphing calculator in a new way.
The teacher takes students to the math lab to show them the solution of a problem which is written on the board.
The teacher takes students to the math lab to demonstrate a problem on the computer.
The teacher uses the graphing calculator, reviewing techniques, to answer students' questions.
Activities include applied math problems.
Students draw graphs by hand.

C. " My teaching style allows these, my choices in these happening"
Students do an activity on the topic before the concept has been formally introduced.
Students work in pairs at the board.
The teacher demonstrates the use of the graphing calculator to graph linear programming problems.
A student explains how to find the intersection of two lines using the graphing calculator in a new way.

* denotes added by Ms. Shade to capture the desire to use questioning techniques effectively

Figure 6. Ms. Shade's card sorting

She began discussing the sorted cards, with the card about the final exam, which she had singled out because "it doesn't really fit in any other category. It's there, it's true." The remainder of the cards were in two major clusters related to "what the student does" and "what the teacher does." Ms. Shade divided the cards related to "what the student does" into three areas; notetaking, homework and quizzes, and exploratory activities.

"One has me checking their notes, I put it together with this, the students taking notes in class because I wanted to see what they did in their notetaking. So to me that's important." Ms. Shade emphasized both the importance of students being involved in class through notetaking and her responsibility to check on individual student's study notes. By grouping the cards mentioning the teacher answering questions with the cards concerning students working on and correcting their homework, Ms. Shade connected the students' responsibility to do the work with the teacher acting as a guide through problems with which they were having difficulties. The third area of "what the student does" was what Ms. Shade called exploratory.

> It's giving them a chance to learn in different ways: one by having to take some risks, one by doing the teaching, one by having to explore the TI. Those are all exploratory type things. Even if a kid has to go to the board and explain it, that's an exploratory for them because they haven't really had the opportunity to do it. So, I see that as exploratory.

Through the way in which she sorted and discussed the cards, Ms. Shade emphasized three areas of student activity; notetaking for studying and developing conceptual understanding, homework and quizzes for learning and demonstrating skills, and exploratory activities.

Ms. Shade described the remaining cards as "what the teacher does." She divided these cards into several groups. One group which she described as "teacher expectations of myself and of my students," dealt with communications between the teacher and students. In this group were statements dealing with the teacher knowing students by name, students showing their work, students analyzing and picking out work for their portfolios, the teacher talking to students about their thinking about a project and getting their feedback, and the teacher explaining how credit was awarded in the grading of a test. By grouping these statements together Ms. Shade placed value on communications between students and teacher as part of the learning environment. She felt it was important for students to understand her expectations of them and for them to communicate their experiences to her.

Within the cluster of "what the teacher does," Ms. Shade collected statements under the broad title "teacher style" and then subdivided these into three areas: "practical things...," "choices of methods...," and "my teaching style...." The cards contained some statements about students doing things such as going to the board and explaining, working in pairs, and doing an activity. While she could have put these cards in with "what the student does," she indicated that they represented choices she made in her teaching. For Ms. Shade the variety of student activities that took place in the classroom was a feature of her teaching style. Missing from these cards was a statement related to questioning techniques. "I'm trying to throw more concept questions at them ... am I getting them to understand the concept better?" To include this concern, a card was added that said, "I try to question the kids in a way that they have to derive and answer." This card was added to teacher style group under "choices of methods." As she reflected on the variety included in these groups she commented, "I feel like I'm employing a lot of methods, more than I think I would have thought about." For Ms. Shade, the variety of teaching methods she employed was an important characteristic of her teaching.

Belief verification interview. The belief verification interview was conducted on a summer morning at a picnic table in Ms. Shade's yard. For this interview, the researcher had grouped the statements used in the belief clustering interview according to a preliminary analysis of how they described Ms. Shade's beliefs concerning mathematics, the teaching of mathematics, the structure of the classroom, and how students learn mathematics. Ms. Shade was asked to discuss how each group of statements reflected her beliefs in each area.

The first set of statements, reflecting beliefs about mathematics, were:

Activities include applied math problems.

Students do an activity on the topic before the concept has been formally introduced.

> The teacher questions students in a way that they have to derive an answer.
>
> Students must graph systems of equations by hand, but are encouraged to check their answers with the graphing calculator.
>
> The teacher explains how credit was awarded for partial solutions on test [and quiz] problems.
>
> The teacher talks with students about what they thought about a problem or project.

In responding to these statements Ms. Shade emphasized that mathematics was more than just concepts. "You need to be able to see where it [a mathematical concept] works, how it works in the real world." She used applied mathematics problems and projects to help students make these connections. Ms. Shade also emphasized the importance of the process involved in solving a problem.

> The answer is not the essential part. It's the work they show. And they need to understand what kind of work I am looking for. It's not always what I'm looking for, it's what's appropriate. I could be looking for one thing, but if they can explain where theirs came from, I will give them credit as long as it's logical.

In addition to understanding that mathematics is more than a concept and that solving a problem is more than finding an answer, Ms. Shade emphasized the importance of making connections and thinking about mathematics. She explained a final assignment she gave the students:

> I gave them a page of topics [and said] tell me the differences and similarities between all these type of equations, a quadratic, a cubic, and a linear, and an absolute value. And write about it and give me examples.

For her, this assignment was important because it gave students an opportunity to think about what they had been studying and make connections between the topics. When talking about a school district's decision to utilize standardized tests, Ms. Shade said,

> I would have serious reservations about going back and giving a standardized test. I think I've gone so far beyond that in terms of wanting to have kids be able to explain and write.

A final important characteristic of mathematics for Ms. Shade was "math is fun!"

The following statements were used to stimulate Ms. Shade's discussion of her beliefs about the teaching of mathematics:

The teacher writes definitions on the board.

The teacher introduces new material by working problems.

The teacher prepares overhead transparencies using the output from the graphing calculator.

The teacher demonstrates how to use the graphing calculator in a new way.

The teacher takes the students to the math lab to demonstrate a problem on the computer.

The teacher answers students' questions about a quiz, going over a complete solution to each problem.

The teacher works out homework problems for students using the overhead projector or whiteboard.

The teacher uses the graphing calculator, reviewing techniques, to answer students' questions.

The teachers talks more broadly than the text and expands on examples.

Students experiment, like with the graphs of consistent, inconsistent, and dependent systems.

The teacher circulates among the students as they work, individually or in pairs.

The teacher encourages the students to figure out how to use the graphing calculator in new ways.

Ms. Shade indicated that it was very important to show students how mathematics works. "It doesn't do a lot of good to just say this is something without doing a problem to show them how it works." It's not just how to work the problem, but the thought process behind the procedure that must be demonstrated. "If I go over the complete thought process in my head, then they will enlarge their thought processes. They will see a bigger picture.... I seldom try to go just with an answer. I want them to see, here's all the thinking that I went through on this problem." Ms. Shade was not certain that it was possible to teach students how to think, "but if they will be willing to take those first steps, they will learn them, because the next time they'll be able to take the next steps plus one more." Thus, she believed that learning mathematics, like mathematics itself, was a process of putting together a series of steps, making connections to what the student had previously learned.

Using different perspectives also added to the learning process. Using the graphing calculator when reviewing, Ms. Shade indicated was, "trying to put it all together ... look at it graphically, look at it analytically, look at it process-wise. That's to tie in all the different ways to look at the same problem." Having students work in pairs at the board was an effective way for Ms. Shade to teach because it was "hands on," allowed her to "see everybody so easily," and "the kids' response is so positive." Working in pairs allowed students to have someone to talk to and even though they knew Ms. Shade was watching them, they were not threatened. They knew they were not being tested, but were learning. Although Ms. Shade often circulated through the room when students were working, she thought it was also important to stay at her desk and have students make the effort to ask her a question. "If they come up you know that they really struggled with the problem, probably. If I'm walking around, it's too easy for them to ask a question. So, I tend not to circulate as much for help reasons as to encourage and just see what they're up to."

Regarding how students learn, the following statements were presented to Ms. Shade to stimulate her comments:

Students take notes in class.

The teacher checks students' study notes for the final exam.

Students do an analysis of where they are and pick their best work for their portfolio.

Students draw graphs by hand.

Students work in groups on an activity.

Students work in pairs at the board.

> Students work on their homework.
>
> Students are expected to show their work on homework and exams.
>
> A student explains how to find the intersection of two lines using the graphing calculator in a new way.
>
> Students go to the board individually and explain how to do problems.
>
> Students correct their homework from an overhead display of the correct answers.
>
> The teacher shows a student that the approach he used to solving a problem, although different than the one used by the teacher, gives a solution that is equivalent to the one found by the teacher.
>
> Students are offered extra credit for finding out how to do a type of problem not included in their text [find the inverse of a 3x3 matrix].

For Ms. Shade the psychological impact of mathematics on kids was important. "There are so many times that I do so much work on building self-esteem." As she discussed the statements related to how students learn mathematics, Ms. Shade repeatedly emphasized the importance of building students' confidence and reducing stress in the learning environment. Through analyzing their work and making selections for their portfolios, students are "realizing they have learned, they have grown, and they develop confidence." Working in groups on an activity "takes some stress off the students." Playing music during tests "took some stress off the test taking." Working on their homework, "gives them [students] the individual confidence and skill building so that they aren't depending on someone else." Having a student explain to the class something discovered using the graphing calculator or go to the board and explain how to do a problem was particularly beneficial. "The kid's building confidence because they're up speaking in front of a group. They're using the graphing calculator so we're having success on them using something.... You're saying, 'This is good.' So, I feel I'm encouraging them to keep trying to do things."

While recognizing the importance of building students' confidence in their abilities to learn mathematics, Ms. Shade also recognized that students learn in a variety of different ways. "What does it say? If you do, you learn this much; if you write, you learn this much; and if you listen, you learn this much. And we know that listening is way down on there." She encouraged notetaking while she talked to "add a level to their learning." She taught using a variety of colors on both the overhead and white board and encouraged students to use different colors in their notes or "go back and highlight what they think are the important parts." She also worked with them to organize their notes because "it's a way for them to pull all these individual pieces together."

Ms. Shade provided students opportunities to work in groups on activities or in pairs at the board during class which "moves them ... and movement is important." Through working together, students "learn that they can talk about math. They get to hear other people's ideas which broadens their thinking." These learning opportunities for students also provided opportunities for students to receive feedback. "I can give them immediate feedback, which I think is pretty important, or they can get immediate feedback from looking at somebody else's work - see if they're doing it accurately." In discussing how students learn mathematics, Ms. Shade continued to emphasize the importance of the process. "Showing their work is essential to me in that, that's what's important is the process that they are learning, not the answer." In conclusion, Ms. Shade acknowledged that learning mathematics was a process that took practice.

> You really can't learn it [mathematics] by just doing it on the weekends. It's having to do it constantly. I really believe that you have to listen to the teacher. You have to try the homework. Then you have to listen to the teacher again, go over the homework. And then, when you try the homework again it will make more sense. I don't think you can listen to the teacher, go home and do the homework perfectly.

Ms. Shade emphasized the importance of the graphing calculators in her teaching for "giving the kids a visual representation." For example, in Calculus "I can teach a calculus concept and there's a volume program that will show the rotation on the rectangles. The kids can see what's happening." She had, however, realized that there were some drawbacks to utilizing graphing calculators.

> There's still a fine line with the calculator in terms of overuse.... They didn't want to learn the concept, they just wanted to figure out [the answer]. They would grunge through it on their calculators, somehow, and end up with the

> answer. But they didn't really know what they did, they were just doing keys and they made sense and all of a sudden the answer came up. But they probably couldn't reproduce it.

In spite of the possibility of overuse of the graphing calculator, Ms. Shade continued to stress its value. When teaching about shifting curves, "you ask them what it is going to look like, then you experiment." The opportunities to use such questioning strategies outweighed the possible overuse. She had also found that it was important to stress mechanics, such as working with fractions, and incorporate skill building exercises into her teaching. In conclusion, Ms. Shade found, "for me the graphing calculator also does what they say which is be able to look at things numerically, graphically, and analytically. And I think that's a good way to be able to look at math from all those directions."

<u>Summary of beliefs</u>. The idea of mathematics as a process was central to Ms. Shade beliefs about mathematics and teaching. Her beliefs about mathematics included the importance of concepts but emphasized that the structure connecting the concepts was equally important. Knowing where and how a concept fit into the structure was essential to the understanding of mathematics. Learning mathematics, therefore, required development of conceptual understanding as well as reflection on the structure of mathematics and the connections between concepts.

Ms. Shade's beliefs about teacher and student roles in the classroom were consistent with her beliefs about the nature of mathematics and the learning of mathematics. The teacher's role in the learning process was to serve as a guide by communicating expectations, showing how mathematics works, demonstrating the thought processes required to apply mathematical concepts, making the connections between concepts in order to pull the concepts together into the structure of mathematics, providing multiple perspectives to assist students in making connections, and having fun. Ms. Shade believed in the value of knowing the students and understanding the variety of learning styles they employed in order to effectively guide the learning process. Attention to building student self-esteem and confidence were also important to her description of the teacher's role in the learning process.

Students were also responsible for participating in the learning process. Ms. Shade believed that effective learning required students to communicate their mathematical understanding through asking questions, explaining, working in groups, and exploring new concepts. Learning mathematics required students to make connections between their prior knowledge and new concepts as well as among concepts. Reflection, practice, and exploration were, in Ms. Shade's view, all means for making these connections.

In Ms. Shade's view, the graphing calculator was valuable for learning mathematics. She emphasized the usefulness of the graphing calculator as a tool for students to see mathematical concepts, explore ideas, and answer questions. She valued the graphing calculator for the ability it provided to explore mathematical concepts numerically, graphically, and analytically. Additionally, she recognized the importance of emphasizing understanding and mastery of the mechanics [algebraic and arithmetic skills] required to solve problems mathematically. Utilization of the graphing calculator was not, in her view, to be substituted for mastery of algebraic and arithmetic skills.

<u>Consistency between beliefs and practices</u>. Ms. Shade's beliefs and practices showed a high degree of consistency. Her belief in the importance of understanding the structure of mathematics and the connections between mathematical concepts was demonstrated throughout her teaching as she made connections between new material and prior knowledge. The value she placed on understanding the needs of individual students was clear in the relationships observed in the classroom. Asking directed questions of individual students displayed both her knowledge of the students and her ability to frame questions that provided guidance. There was, for Ms. Shade, a conflict between her belief in the role of the teacher as a guide and her practice of leading the class. As she acknowledged in conversation, she was accustomed to being in control in the classroom. The role of "guide on the side" requires the teacher to allow students more freedom to explore, ask questions, and find their own answers. Ms. Shade attempted to allow

students this freedom, however, she was not always successful in her attempts. The value she placed on communication and reflection in learning mathematics were exemplified in her teaching and assessment practices where she required students to provide explanations and utilize their notes taken during class in preparation for tests and quizzes.

Ms. Dancer

Ms. Dancer was recommended for the study by a teacher from another high school who was not using graphing calculators in the teaching of advanced algebra. When contacted by phone, Ms. Dancer indicated her willingness to discuss participation in the study. The school district in which she was teaching required formal approval from the district administration and parent permission for classroom video taping. Permission was secured in response to a letter (see Appendix B) to a district administrator. Parental permission slips (see Appendix C) were distributed to students and returned prior to the beginning of observations.

In the school district, Lakeshore, where Ms. Dancer taught there were two high schools, Lake at which she taught and Shoreview. There were also two middle schools in the district, one feeding each high school. The mathematics curriculum was district-wide with the expectation that all schools followed the same curriculum. Both high schools used a block schedule. The schedule consisted of eight class periods meeting for 90 minute sessions. On "A" days, periods one through four met; on "B" days, periods five through eight met. Each morning there was a 20 minute teacher/student contact time. "A" and "B" days alternated throughout the school year and the mathematics classes lasted the full year. Ms. Dancer's Advanced Algebra class met eighth period, the afternoon class on "B" days. At Lake High there was a mathematics office where all mathematics teachers had their desks and files. The teachers shared the classrooms, so most teachers did there preparation work and met with students in the mathematics office. The shared workspace fostered dialogue among the teachers and encouraged shared planning for common courses.

Ms. Dancer described the students at Lake High as relatively homogeneous, middle to upper class, highly motivated with highly educated parents. She indicated that the students had high expectations with 85 percent of the students continuing on to college. She described the community as "a fairly small town, everybody knows everybody." Ms. Dancer felt that the fact that she lived in the community was important. "I live in the community.... I know a lot of the parents; it's hard to get out of the grocery store without at least one conversation about kids. But I like that." She described the faculty as "strong, well educated, generally older." She felt that the staff was stable because most of them had been at the school longer than she had experienced at other schools, but "maybe that could be a negative because we don't have as much youth as we should maybe have." She felt that the administration was fairly supportive but there was pressure from the state legislature. "You sort of feel more of an outside threat than an inside threat here."

Several other teachers were teaching Advanced Algebra at Lake High when the study took place, but no other teacher was available to participate in the study. All sections used the graphing calculator. The TI-82 was used for demonstrations but students were permitted to use different graphing calculators. Students in Ms. Dancer's class used TI and HP calculators. The textbook being used by Ms. Dancer was *Advanced Algebra* published by The University of Chicago (Senk et al, 1993). She was pilot testing the new edition of the text while other teachers in the district were continuing with the previous edition.

Background. Ms. Dancer was in her twenty-fourth year of teaching, having taught in the northeast, midwest, southeast, and northwest. She began teaching in a small, private secondary school (seventh through twelfth grades) in the northeast where she taught "everything from seventh grade English to twelfth grade physical science, a lab course." She had completed her Bachelor's degree in history with a "heavy minor" in mathematics including computer programming, but had not had any teacher training. After this initial teaching experience Ms. Dancer completed a Master of Arts in Teaching (MAT) in history. She described the MAT program, one of only two in the country at that time, as "more application and out there doing it and applying the academic learning you got in your subject core area, less philosophy of education." In this program, she spent one year taking courses and working in the lab school followed by a summer of classes. In the second year, she taught three-fifths time in a Chicago inner-city public school with seminars at the university to "pull together the experience, understand it better, and deal with it better." After completing her MAT, she spent another year teaching in an inner-city school, a private school with mostly non-English speaking students. Ms. Dancer then spent four years teaching in a prep school in the southeast during which time she made the decision "that I didn't want to teach history anymore, but that I wanted to go straight math. I found that kids liked math better at the high school level because they saw a future, something they needed out of it." When she moved to the northwest, Ms. Dancer was required to complete additional coursework in order to qualify for a teaching license. She taught for six years in a junior high school in a neighboring suburban school district before joining the faculty at Lake High. Ms. Dancer was in her tenth year at the school where she had taught computer programming as well as mathematics courses. Computer programming was no longer being taught, so Ms. Dancer was now teaching mathematics exclusively. At Lake School she had taught most of the Chicago series courses including Transition Math, Algebra I (both first and second editions), Geometry, Advanced Algebra, and Analysis (the Functions, Statistics and Trigonometry text). She had been on panels for the University of Chicago as an "experienced teacher" sharing with new teachers on the use of the textbooks. At the time of the study she was teaching an Intermediate course for students who "struggled in algebra and need more" between Algebra I and Geometry. This course did not utilize a Chicago text. She was also teaching the Analysis course.

In addition to the formal coursework she had completed, Ms. Dancer often attended local and regional mathematics conferences and occasionally a national conference at which she participated in workshops. She had participated in training sessions at the district level conducted by outside experts, calculator workshops, and training on the HP 38G and HP 48. Reading journals was also a part of Ms. Dancer's ongoing teacher development. She felt that the "unique setting in the mathematics office" where "we do a lot of discussion" contributed to her development. One of the teachers from Lake High was on the national board of directors for the National Council of Teachers of Mathematics (NCTM) and brought back information about trends in teaching. In the mathematics office the teachers were able to discuss these ideas, plan together, and try to apply the new ideas in their teaching. Ms. Dancer was also part of a group from the district who wrote and received funding for a grant to investigate National Science Foundation (NSF) curriculum projects. The goals of the grant were to find out what was available and to explore alternatives to the district's Algebra I and junior high curriculum with which there was dissatisfaction. "We're trying to see what's available; and get a feel for the direction things are moving and how we should move with that; and to see if there are some materials we want to adopt out there." Ms. Dancer had also been involved in a pilot project using computer assisted instruction (CAI) in Algebra I. She was one of two teachers selected to teach using the donated computer system connected to an outside server. "From my perspective it was a disaster. And it was really tough to be part of something you know was not working and you had to stick it out for nine months. It brought out the worst in students. We had parent complaints - justified. But we had a commitment to see it through for the year."

Introduction to and thinking about graphing calculators. The use of graphing calculators in her teaching "made sense" to Ms. Dancer. She had been teaching computer programming and utilizing computers in the classroom, graphing calculators seemed like the next step. Now, she "couldn't imagine at this point teaching it [mathematics] otherwise." While she had attended a number of workshops conducted by other teachers and the Math Learning Center, she indicated that "a lot of the graphing calculator has been self-taught." Within the department there were individuals with expertise using the TI-85 and the HP. Ms. Dancer focused primarily on the use of the TI-81 and 82. All of the teachers shared information and techniques with each other.

Ms. Dancer emphasized the visual quality of the calculator as well as the different learning modes and ease of manipulation possible with the calculator as reasons for pursuing its use in her teaching.

> I think it opens up so many more areas of math for kids. The visual quality...that we tried to reproduce on the board [couldn't] match having them key something in and see it visually. I think the learning is so much greater with the calculator because it pulls in so many different learning modes. And it allows us to spend less time on the manipulation of numbers and go more to the outcomes so they can see where they're headed.

The array of graphing calculators available and in use by students in her classes could hinder the process. "Our purpose is not so much in teaching how to use the machine but in using the machine to do the math. And we spend a lot of frustration in how to use it when it's a different machine, different tool." In spite of the frustrations, she felt that there were more "ah-hah's" for students utilizing graphing calculators. She commented that in the unit she had just completed on matrices, the calculator enabled students to complete the manipulations more easily "so once they got the answer, then they could interpret the answer. And that's where we want them to be." Ms. Dancer saw that the use of graphing calculators enabled her students to go beyond finding the answers to problems to the next level of interpreting the answer found.

Professed beliefs about mathematics and teaching mathematics. Mathematics was "an ordering of our work, a way to explain the processes, a way to measure and order the processes that go on around us," according to Ms. Dancer. She saw mathematics tied to sciences and humanities, not as a subject that stood alone. For her mathematics was "a tool to understand everything else." Her beliefs about mathematics included an emphasis on the importance of mathematics and the need to "understand the whole picture." She wanted students to be able to see that whole picture. Ms. Dancer felt that algebra was crucial to mathematics because "we need to explain in equation form the relationship[s]." Algebra provides the tool to make equations that then explain the relationships in the world.

Ms. Dancer was concerned that for many students, learning mathematics meant they had to "memorize processes." She believed that they had to attach what they were learning to what they already knew. "We have to build certain key concepts into the curriculum as things to attach to." Because students have a variety of different ways of attaching, she saw it as her job to help them make the connections and fill in the gaps.

> They have a whole range of ways of attaching. You can tell from their questions ... who's thinking along what lines. So, they have to keep building layers. And I see my job partially as helping them make the connections. Something they've already learned and understand to something new and finding the gaps that need to be filled.

Classroom practices. Ms. Dancer's Advanced Algebra class was observed over an eight week period, interrupted by winter storms and the winter vacation period. Initial classroom observations took place at the conclusion of the unit on matrices. The following unit, solving systems of equations, was observed in its entirety and served as the basis for this description of Ms. Dancer's classroom practices. Because of the storms and vacation period, the four weeks of class sessions devoted to the unit on systems of equations were separated by a nearly three week period during which there were no classes. Four class sessions took place before the break and six class sessions occurred after the break. Additionally, two of the class sessions were shortened to less than half their normal 90 minutes because of adverse weather conditions and early dismissal of students.

Ms. Dancer's teaching was characterized by intentional planning of class sessions including the exact material to be presented and the activities to be completed by students. As she presented new material, Ms. Dancer referred to notes containing problems that had been specifically selected to connect new material to previously discussed material, develop the ideas being presented, and explore applications of the material. Class time was organized to divide each 90 minute period into shorter segments designed to keep students actively involved in the learning process. While there was not a precise routine in Ms. Dancer's classroom, she structured classes to include a warm-up activity, an overview of the day during which she communicated her expectations, time for review of the material presented during the previous class, presentation of new material, and time for students to work individually or in small groups.

The warm-up activities Ms. Dancer used were often a means for reviewing the material presented during the previous class or covered in the homework assignment. After students had read the section in their text on inequalities in one variable and worked the exercises as a homework assignment, the warm-up activity was a worksheet requiring students to solve and graph single-variable inequality problems. As Ms. Dancer discussed the worksheet, she indicated the intentionality of her planning when, after discussing the graph of the solution to the problem $x > 2$ and $x < 5$, she commented about the problem $x > 2$ or $x < 5$.

> Okay, now, how about the next one where it says or? Isn't that the same thing? I purposely made it sort of follow. Here's one [indicating the graph of $x > 2$], here's two [indicating the graph of $x < 5$.] When I lay one on top of the other, do I care about the overlap? No, I want to take it all. I think of or, this is my own little way of thinking..., as a big basket and we take all the answers from both parts and put them into the final answer.

Her discussion of the problem clearly demonstrated the difference between the "and" problem and the "or" problem. Later in the same discussion she again indicated her intentionality when she commented, "I tried to put in every possibility." Ms. Dancer had created the warm-up activity to serve as a comprehensive review of the material the students had studied as they prepared for class. She had carefully selected the problems on the worksheet so the concepts were developed from one problem to the next. At the conclusion of the discussion she summarized her intentions for the activity when she said,

> That's pretty much 5.1 [the section covered]. Are there any problems that we didn't cover that were on your homework that you think are still important, that you still don't understand? I think we hit most of the questions [pause during which no questions are asked.] Then, in the interest of time, let's move on, and we'll come back next class and I'll ask again if there's anything on 5.1 you just don't get.

Ms. Dancer felt that it was important for students to know what to expect both in terms of what they would be required to do and what they would be required to understand. She communicated these expectations to her students in a variety of ways. At the beginning of each unit she distributed a schedule for the unit that included the dates numbered day 0, day 1, day 2, and so on; anticipated class activities including homework review, sections over which the teacher would lead discussion, and assignments to be completed during class; quizzes and tests; and homework assignments. During each class session, either at the beginning of class time or after the warm-up activity had been completed, Ms. Dancer gave an overview of the day including the concepts that would be covered, the activities that would take place, and her specific expectations of the students. On one occasion she commented, "I like this stuff, [systems of equations] and I think you will, too, if you can focus on it for a bit here." In this statement she expressed both her feeling about the topic and her expectations of the students. Because of the adverse weather conditions and changing school schedule, Ms. Dancer's plan for the unit had to be revised several times. Although she indicated some frustration about constraints these revisions placed on her delivery of the material, she encouraged her students by saying, "We're trying to accomplish a lot today, more than usual and it's going to require a real commitment and intention on your part to stay focused so that we can do all this." Later, in the same class when she felt it was necessary to shift the focus from review and explanation of the material that had been covered during the previous class to the new material, she indicated her concern that students were not comfortable enough with the first material to move on. "Well, we have a dilemma here and our dilemma was caused by the storm. Looking through the next four days; 5.9 and 5.10, the new material, can't wait. We'll leave Thursday to pick up pieces. Our goal is to finish the chapter today. You can do it, I have confidence in you guys." By clearly indicating expectations and confidence in their abilities, Ms. Dancer encouraged her students to strive to meet the expectations.

In her teaching, Ms. Dancer emphasized both the ability to carry out a variety of techniques in solving problems and comprehension of the concepts underlying the techniques. When presenting the matrix method for solving a system of equations, Ms. Dancer began with a discussion about determinants:

> Ms. Dancer: Last night when you were doing you assignment you read about something called determinants. Does anybody remember what the determinant was?
>
> Student: It's like the denominator.
>
> Ms. Dancer: It's the denominator? Where?

Student: Inside the matrix.

Ms. Dancer: Okay, so if we start off with a, b, c, d as our matrix, what would the determinant for that matrix be?

Student: a times d minus c times b

Ms. Dancer: 'ad - cb' Should we put bc here? Okay, that's the determinant. Now, determine, I've heard that word. What is determined by the determinant? Unless somebody just chose this word.

Student: You take d, -c, -b, a and the determinant to get the inverse.

Ms. Dancer: Determines the inverse. Okay, so we're going to use this. You said this guy [the inverse matrix] is going to be what? Shall we write it out? What is it going to be?

Student: d, -c, -b, a

Ms. Dancer: d, -c, -b, a Okay, is that it for the inverse?

Students: No, you need the denominator.

Ms. Dancer: Now we need the determinant under here for all of them, don't we? ad - bc [writes 'ad-bc' under all four entries in the matrix]. What's getting determined about this? Back to that question. Anybody want to just give it a try? Belinda?

Belinda: Well, it tells whether, um, there's an inverse or not because if it's zero there's no inverse.

Ms. Dancer: If this is zero, and I'm going to use D, capital D, for the determinant. Or, I'll use Det = 0, then what do we know?

Belinda: Then there's no inverse.

Ms. Dancer: Okay, if it's not zero, what do we know?

Belinda: There is one.

This discussion demonstrated the importance Ms. Dancer placed on understanding the underlying concepts. She was not satisfied for the students to simply know the formula for finding the inverse of a matrix. Rather, she required that they understand the role the determinant played in finding the inverse. Ms. Dancer's emphasis on the vocabulary used in mathematics and its connection to common English usage was also apparent in this discussion when she made the connection between the definition of determine and the role of the determinant in finding the inverse. The emphasis on making connections between the new material being explored, in this case the concept of a determinant and its connection to the inverse, and students' prior knowledge was a consistent focus of Ms. Dancer's teaching practices.

Ms. Dancer's development of the matrix method for solving a system of linear equations continued with a discussion of how to write the system in matrix form. Student involvement was a key element of this discussion. After writing two matrices on the board, Ms. Dancer checked her students' recollection of multiplying matrices by asking a series of questions.

> Ms. Dancer: First of all, can we multiply them? They look a little different. Judy, what's the dimension of our first matrix?
>
> Judy: Two by two.
>
> Ms. Dancer: Dimension of our second one, Annie?
>
> Annie: Two by one.
>
> Ms. Dancer: Are these compatible, could we multiply them?
>
> Students: Yep.
>
> Ms. Dancer: What's the dimension of the answer matrix?
>
> Students: Two by one.
>
> Ms. Dancer: So, two means two rows or two columns?
>
> Students: Two rows.
>
> Ms. Dancer: Two rows and one column. Okay, anybody got my first element up here on the top? Sandy?
>
> Sandy: I don't know if this is right, but 3x - 1y.

> Ms. Dancer: Well, we could put 3x - 1y, I'll stick the 1 in because you said 1. And the next one?
>
> Sandy: 5x + 2y.
>
> Ms. Dancer: Anybody disagree? Did you get that just by multiplying?

Ms. Dancer's careful planning of her development of the concept was clear in this discussion as she combined the introduction of a new technique with the verification of students' understanding of a previously mastered skill. The brief review of matrix multiplication also served to involve students directly in the development of the new technique and to make a concrete connection with material they had studied previously. Additionally, the trust between the students and Ms. Dancer was apparent when the student, Sandy, was willing to volunteer an answer even though she did not know for certain that it was correct.

Having worked with the students to demonstrate that the coefficient-variable side of a system of equations could be written in matrix form, Ms. Dancer continued the discussion by wondering if the entire system could be written in matrix form.

> That's interesting, that looks like part of a system of equations to me. Here's the first part of my first equation. Here's the first part of my second equation. Right? So, if all of this multiplied together gives me what Sandy just said, "3x - 1y and 5x + 2y." This product gives me this. Then, could I create a problem, full problem, like we've been looking at, systems of equations, by putting the right half of my equation over here? [writing the constant column matrix -6, -10 to the right of the matrices that have been discussed with an = connecting them.] ... Haven't I written the same thing here, in matrix form? This is matrix form of a system and this is just the standard form of a system. Aren't these equivalent to the same thing?

Ms. Dancer had developed the relationship between the matrix form of the system and the standard form with a clear, step by step, demonstration that included student participation. Now, she was ready to lead the students through a demonstration of how to solve the matrix system.

> Ms. Dancer: Now watch what I'm going to do. I'm just going to work with matrices because I know my calculator can do a lot with matrices and this may be a quick way to solve this system. I'm going to come back up here to my original problem, my matrix form. This is the matrix form and I want to get x

and y alone because that's what we do for solving a system of equations. We get x = and y =. So, if I can just come up with a matrix that I can multiply that would undo this matrix or give me the identity matrix here, then I would be left with just x and y alone on the left side. Do you know any matrix that would undo this matrix and get me back to the identity matrix?

Student: Use the thing with the denominator.

Ms. Dancer: The denominator, you mean the determinant, what do you mean by denominator?

Student: I was thinking of using that thing with ad - bc in the denominators.

Ms. Dancer: You mean the determinant. So, you're going to find the inverse of this. Now, from last class, the inverse times the matrix will equal what?

Student: 1, 0, 0, 1.

Ms. Dancer: Which is the equivalent of one in matrices, right? So, if we could find the inverse of this guy and multiply the left side, what should we be left with?

Student: x and y.

Ms. Dancer: x and y, just what we want...Now, I multiplied the left-hand side of the matrix form of this system by this inverse, is that going to change my problem at all, multiplying the left side?

Student [not heard by Ms. Dancer]: You have to multiply both sides.

Ms. Dancer: Is it legal to just move in and multiply by something? I can take a balanced equation like a seesaw, equal things on both sides, and I can multiply one side by something and they will still be equal?

Students: You have to multiply both sides.

Ms. Dancer: Okay, I need to multiply this side as well as that side. I can't just multiply one side and expect to have it balance. So, this inverse needs to multiply over here, too.

Throughout this lesson, Ms. Dancer involved students by asking them questions they could answer from their previous mathematical experience. The connections she made to previously studied material were clear. Even though students had never encountered a matrix equation, Ms. Dancer led them through the solution process by demonstrating its similarity to the process of solving other equations. Her reference to the image of an equation as a balanced seesaw demonstrated Ms. Dancer's ability to utilize a variety of approaches to mathematical understanding including a visual model.

The variety of approaches to the teaching and learning of mathematics found in Ms. Dancer's teaching, in addition to the strategies already discussed, included cooperative work situations and episodes during which students were encouraged to hypothesize about mathematics. Students worked cooperatively on two occasions, in groups on an exploration activity and in pairs on a portion of the test over the unit. The exploration activity required the student groups to work both individually and cooperatively. After the concept of a matrix inverse had been introduced through a teacher led demonstration, students were assigned to groups of four and given a worksheet on matrices and reflections. In the groups each student was required to explore a different reflection by writing out the matrix for the reflection, finding the inverse of the matrix, multiplying the two matrices together, finding the image of a set of points under the reflection matrix, and finding the image under the inverse of the reflection matrix. After each student carried out these tasks, students discussed their individual problems as a group, summarized the results, and applied what they found to another problem. Throughout the time period allowed for working in groups, students interacted frequently with the members of their group and occasionally with members of other groups. Ms. Dancer also circulated throughout the room, answering questions and assisting both individuals and groups. It appeared that the students had previous experience working in groups and were able to manage their time and stay on task. Because students were required to apply the concept illustrated in the individual explorations to an additional problem, both the individual and cooperative aspects of group work were incorporated in this activity.

Ms. Dancer allowed students to choose their own partners for the completion of the linear programming portion of the unit test. This portion of the unit test was scheduled for and given on a day when the weather caused an early closure of the school. The class session was shortened and there were several distractions including an announcement concerning the early closure and adverse weather conditions. In spite of the circumstances, students worked throughout the period on the linear programming problem, interacting with their partners and working toward a solution. As students worked on their tests, Ms. Dancer made several comments to the observer. "I like pair testing," she said, "they really focus. Even the social ones settle down." Unfortunately, by the end of the period, most students had not finished the problem. Ms. Dancer recognized that students needed more time, collected what they had completed, and allowed them to complete the problem during the next class session. Extending the time allowed for completion of the problem into the next class session demonstrated Ms. Dancer's flexibility and understanding of students.

In addition to the hypothesizing and drawing of conclusions students were required to do in the cooperative, exploration activity on matrices and reflections, students were encouraged to hypothesize and use their experience at other times. When introducing the idea of the solution to a system of equations, Ms. Dancer followed the discussion of graphing the solutions to single-variable inequalities with a problem involving two linear equations. By writing the two equations vertically and enclosing them with a brace on the left-hand side, she defined the problem as an "and" problem. "So," she said, "I am looking for the ordered pair here, remember overlap for and, so, the same ordered pair for both." After brainstorming with the class about methods for finding the ordered pair that would work for both equations, Ms. Dancer decided to graph the two equations. Looking at the completed graphs of the two linear equations, Ms. Dancer asked, "Does anybody have a feeling for what value of x and what value of y is the common solution?" When a student replied, "Where they cross," Ms. Dancer used the student's insight to proceed to find the solution to the system. This exchange demonstrated Ms. Dancer belief that students needed to be involved in theorizing about mathematics and exploring the validity of their theories.

When first discussing the idea of the inverse of a matrix, Ms. Dancer again encouraged students to hypothesize, saying, "Now, without looking in your book, without thinking about it, without any previous understanding of this, do you have kind of a feel for what the inverse matrix would be? The one that I could multiply this by and end up with 1 or 1,0,0,1? Do you have any idea?" The students were confronted with a matrix for a stretch and asked to make a guess, based on their "feel." The level of trust Ms. Dancer had developed and the degree of confidence students had in their mathematical abilities was clear when a student offered a guess. The student's guess was affirmed by Ms. Dancer response, "This is her guess, and this is good." When the first guess did not work, another student asked, "Can I guess?" Ms. Dancer accepted the second student's guess, which turned out to be the correct response. Through this episode, the importance Ms. Dancer placed on involving students in thinking about mathematics and encouraging their progress were shown by her approval of student "guesses."

The importance Ms. Dancer placed on student understanding was demonstrated throughout her teaching by her practice of asking students to raise their hands in response to her inquiries about their level of comprehension. After working through several solutions of systems using the substitution method, Ms. Dancer asked, "How many of you feel pretty comfortable about substitution and you could do it now?" After checking for raised hands, she continued,. "Okay, let's draw a line on your papers and go to [the] linear combination [method] now." Later, after working through several examples with the linear combination method, she asked, "Anybody got a problem with how to do those?" When there was no indication of a problem, she proceeded, "Okay, let's try some tricky problems, see if I can trick you." Each time Ms. Dancer made a transition in the class, she first checked to be certain that there was understanding. The one exception to her practice of checking for understanding before making a transition came on a day when she asked, "How many are understanding?" When only students sitting in the front of the room raised their hands she commented, "Maybe the front row is good. If you sit up here you understand." On this occasion, however, she moved ahead with new material saying, "Well, this is a teacher's dilemma. I want you to understand, but we're going to have to move on right now. This new stuff I think you'll understand." In this situation, the need to complete the unit of study in a timely fashion and work around the schedule changes caused by adverse weather conditions forced Ms. Dancer to proceed with new material before she was certain that students had reached a level of comfort with the previous material.

In addition to the informal assessment of students' understanding found throughout Ms. Dancer's teaching, formal assessment included quizzes and tests. During the unit on systems of equations there was one quiz and a two-part unit test. Both the quiz and part one of the test were designed to assess students' understanding of vocabulary and concepts as well as ability to solve systems of equations using the variety of techniques studied in the unit. The use of graphing calculators was allowed on all assessments. As previously discussed, part two of the test was completed by pairs of students and consisted of finding the solution to a linear programming problem.

Use of graphing calculators in teaching. Ms. Dancer's use of graphing calculators in her teaching was a natural extension of her use of a variety of methods and tools. She incorporated the use of graphing calculators through worksheets, demonstrations, and student investigation. In each case, the graphing calculator was used as a tool to do mathematics. While how to use the calculator was taught, it was what the graphing calculator enabled the student to do that was emphasized.

In the unit on the solution of systems of equations, the graphing calculator was first utilized when the graphing method of solution was being explored. Even though Ms. Dancer had very carefully graphed a system of equations on the board, it was still difficult to determine the point of intersection of the two lines.

Ms. Dancer: So, my system here of drawing the graphs isn't too effective.

Student: Use our calculators.

Ms. Dancer: We have to use our calculators. What will the calculator do for us?

Student: Draw.

Ms. Dancer: Will it? Want to try it? Who needs a graphing calculator?

As she set up the graphing calculator display, Ms. Dancer questioned the students about their previous experience with graphing on their calculators. Having determined that most students had experience with graphing linear equations she proceeded with her demonstration.

Ms. Dancer: Okay, how do I graph 2 equations? If I want both y = 5x and y = 3x +1, what do I do?

Student: Go up to the "y=."

Ms. Dancer: Okay.

Student: And then you enter.

Ms. Dancer continued to elicit student input as she produced a graph of the system of equations on the overhead display. The demonstration included a review of the use of the ZOOM menu and a discussion of the WINDOW dimensions. Having graphed the system, she proceeded to demonstrate the use of BOX in the ZOOM menu followed by TRACE to approximate the point of intersection of the two equations. Ms. Dancer included a discussion of pixel size and the limitations of the graphing calculator's display that resulted in the need to approximate the solution to the system when using the ZOOM-TRACE method. Throughout the demonstration she responded to student inquiries, retraced her steps when the result was not exactly what she desired, and presented the use of the graphing calculator as a natural extension of the paper and pencil methods used previously.

The graphing calculator also enabled students to experiment and explore. After Ms. Dancer had demonstrated the use of ZOOM and TRACE to find the point of intersection a student asked if CALCULATE could be used to find the exact point of intersection. Ms. Dancer's response was very candid, "I admit, I'm not an expert on CALCULATE." In spite of her admitted lack of experience, Ms. Dancer demonstrated the use of CALCULATE. As she proceeded, she included the student who had initiated the discussion.

Ms. Dancer: Now, from here what should we do?

Janice: Put the tracer on the first line.

Ms. Dancer: How do you know?

Janice: Up there [indicating the top of the display window]

Ms. Dancer: '1st CURVE,' is that a question? Well, let's just experiment. That's the nice thing, we can try, and if it doesn't work, we can try again. So, you're saying hit ENTER

Janice: Yeah

Ms. Dancer: Okay. Did anything happen? It went to '2nd CURVE,' it says my position down here. Now what?

Janice: ENTER

Ms. Dancer: Hit ENTER again. What does the 'guess' mean?

Janice: You hit ENTER again and it gives you the answer.

Ms. Dancer: Hit it once more. So, we aren't sure what this means. But, we'll go [and try it out]. Oh! Is that our intersection? Does it work in both equations? We can always check our answer, check it in both equations and see if it works. Is this better than drawing by hand?

Through her demonstration, Ms. Dancer not only taught the students how to use the graphing calculator to find the point of intersection of the two equations, she also discussed the limitations of the graphing calculator to accurately display a graph and modeled an experimental approach when investigating the use of CALCULATE. Her willingness to acknowledge her lack of experience with a specific function of the calculator and articulate the process of experimentation showed the students a useful method of approaching a new learning situation. Students were then given a calculator worksheet containing problems to be solved using the graphing calculator.

During the next class session, in a review of the use the graphing calculator to find the solution to system of equations, questions arose about the use of CALCULATE and INTERSECT to which Ms. Dancer was unsure of the answer. She turned the situation into a learning experience by offering the class a challenge.

You know the intersection button is something I haven't spent a lot of time with. I know it exists. I know that you could use it by pushing ENTER a bunch of times. But, here's an extra credit opportunity [for] the first person who's willing to go to the manual, read about INTERSECT. And, come and make an oral report so that everyone can learn about what it does mean to

> push [the entries for] 1st CURVE, 2nd CURVE, GUESS. There's probably some nice power in there we could use if we knew what it did.

In addition to providing a student an opportunity for extra credit, this episode demonstrated that Ms. Dancer was not the keeper of all information to be dispensed to the students. She was allowing a student to provide information to the class, a further example of her desire to move students from the traditional passive recipient role to an active participant role. A student did investigate the use of the INTERSECT feature and explain the procedure during the next class session.

The power of the graphing calculator as a tool for doing mathematical manipulations was explored by Ms. Dancer in the sections on matrix inverses and the use of matrices to solve systems of equations. After a detailed discussion of the algebraic method for finding the inverse of a matrix and an introduction to the matrix form of a system of equations Ms. Dancer wondered about the use of the graphing calculator.

> Ms. Dancer: I'm using matrices here to speed up my operation. So, I wonder if my calculator will find the inverse for me? Hmm, has anybody figured that out?
>
> Student: Mine does.
>
> Ms. Dancer: Yours does, okay. Well, she's got an HP. I wonder if our 81 and 82 will find an inverse? First, we will have to tell our calculator what our original matrix is. So, on your calculators right now, will you enter this 2 by 2 matrix.... We could do it by hand if asked to do that on the test, but now we're going to see if the calculator can do it and save us a bunch of time.

With verbal instructions, symbols written on the board, and the location of calculator keys pointed out on the chart of the TI-82 hanging on the bulletin board; Ms. Dancer led the students through the process of finding the inverse of a matrix using the graphing calculator. When the resulting inverse matrix turned out to include long, repeating decimal values, she was not concerned, reminding the students that finding the inverse was not the final goal. Returning to the problem from which the necessity to find the inverse arose, she continued with the solution of the matrix form of the system of equations. Having found the solution to the system of equations she summarized the process saying,

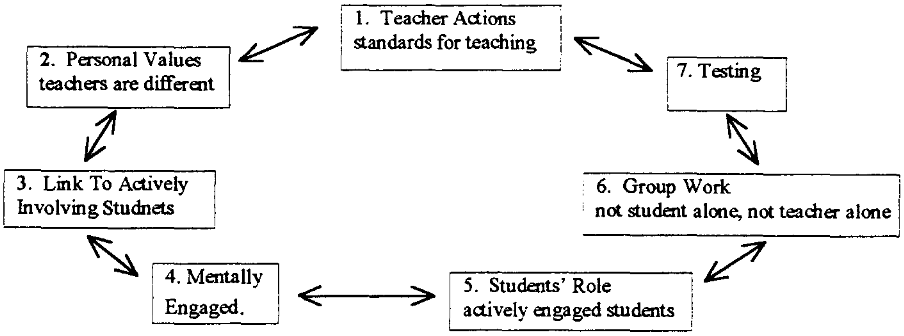
> I really don't care [that this was a strange inverse.] The calculator will have the inverse. All I need to do to find the solution for x and y is set up a matrix with coefficients of my original x and y for my equations. [That is] my first matrix. Set up a matrix of my constants for the second [matrix.] Multiply the inverse of the first [matrix] times the second [matrix.] A inverse times B on my calculator will give me the value of x and y.

Ms. Dancer could simply have taken out the overhead graphing calculator display unit and demonstrated the use of matrices for solving a system of linear equations. That was not her approach. Instead, as discussed earlier, she carefully explained the algebraic process and then led the students through the use of their graphing calculators to find the solution. The power of the graphing calculator to carry out the calculations was apparent when the inverse needed to find the solution was "strange." With the use of the graphing calculator, a problem which required some tedious arithmetic was reduced to a series of matrix manipulations. There was no secret or "magic box" approach; all the algebra was carefully explored and then the power of the calculator was utilized.

Belief clustering interview. The belief clustering interview was conducted with Ms. Dancer in her classroom at the end of a school day approximately 10 days after the conclusion of the period of observations. Ms. Dancer was given 42 cards containing statements based on comments she had made in previous conversations and on observations of her classroom practices. The cards had been shuffled so that they were in no particular order. As she read through the cards, Ms. Dancer asked that several statements be changed slightly so that they more accurately reflected what she perceived to have occurred in the classroom. Specifically, she asked that the word "nurture" be changed to "encourage" making the statement read "I'll encourage discussion with my candy." On another card the words "from homework" were added so that the card read "Questions from homework are sorted out and answered." When reflecting on the content of the cards as a group, Ms. Dancer was concerned that the importance of testing at the end of a unit might not be emphasized enough. A card was added that read "Tests are reviewed after they've been graded," to capture her belief that tests should be a vehicle for learning, "even if it's just through the going over afterwards."

As Ms. Dancer sorted the cards it was clear that she was doing more than dividing the cards into groups. After she had arranged all the cards, she reviewed them and placed the cards in a distinct order (Figure 7). "Okay, I feel good about this. This is a circle. So, there's more than just piles, there's a continuum, in my mind that is." Ms. Dancer indicated that there were basics that all teachers needed to do in order to teach a lesson. These basics included teacher actions such as writing instructions, using the chalkboard, and calling on students who their raised hands. She commented, "That, to me, seems like a structure for a new teacher." Beyond what Ms. Dancer felt a teacher must do in the classroom, were teacher actions which depended on personality and relationship to the students. For her, the relationship with her students was important. She used compliments and encouragement to establish a relationship with her students. It was also important, as part of her individual style, to discover the level of understanding among the students by asking questions and encouraging student responses.

Ms. Dancer indicated that the teacher alone could not create a learning environment. It took the whole continuum, including the role of the students, for learning to occur. She felt that there was a link between the teacher's role and the students' role in the classroom. This link was to get the students mentally engaged. Ms. Dancer saw "calling on students who are not actively engaged" and "calling on students without waiting for hands to be raised" as ways to make certain students would be mentally engaged. While getting students mentally engaged was the link, mentally engaged students were a part of the continuum that created a learning environment. Ms. Dancer measured students' mental engagement through their explanations, clear statements of where they were headed, and their ability to interpret an answer. Mental engagement would not sustain the learning environment, students needed to be actively engaged in the process, "doing it all the time." Lively discussion, students understanding the whole picture, and using the graphing calculator to pull it all together were ways in which Ms. Dancer saw students involved in the process.



1. Teacher Action which is barebones minimum that a teacher should do in a mathematics class.
   - Teacher writes instructions for students on board.
   - Teacher uses the chalkboard graph to demonstrate.
   - The new topic is introduced.
   - Teacher demonstrates the use of the graphing calculator using the overhead display.
   - The teacher circulates through the room as students work on a warm-up problem.
   - Students do Masters.
   - I want a change of pace frequently.
   - Students are given the opportunity to earn extra credit.
   - Teacher explains criteria for grading assignments.
   - Students correct their assignment from the overhead display of the correct answers. Teacher calls on students who have raised hands.
   - I try to make sure everybody's feeling comfortable with the previous day's homework.
   - Teacher introduces new material by doing examples on the board.
   - Students do a warm-up problem.
   - Questions from homework are sorted out and answered.
   - *To be an effective teacher takes lots of hard work.
2. I would hold these as personal values. Teachers are different from each other, but still teacher centered. Depends on teacher personality and their relationship with the kids. It takes time before you see that these things are in the long run important.
   - Teacher compliments students on their performance on quiz.
   - Teacher encourages student to "Do your best" on homework.
   - Teacher asks students to hypothesize about a method for completing a new, unknown, type of problem.
   - The teacher asks students to respond (by a show of hands) indicating
     -they understand
     -they kind of follow but have some questions
     -they are lost
   - The teacher asks "how did you get this?"
3. The link is to get everybody engaged, not just physically but mentally..
   - Teacher calls on student who in not actively engaged.
   - Teacher calls on students by name without waiting for hands to be raised.
4. Mentally engaged
   - Once students get the answer (with the graphing calculator) then they could interpret it.
   - I want clear statements (from students) of where they're headed and where they've been.
   - I want teaching from them.
   - Student explains how to use the graphing calculator to find the intersection of two curves by a method other than trace - extra credit awarded.
   - Students write their solutions to homework problems (Masters) on overhead and explain what they did.
5. Student's role - These are actively engaged here.
   - I don't want sleepers.
   - It's real important to me to have the kids involved.
   - I'd rather have lively discussion.
   - I want ah-hah's.
   - I want responses from them.
   - I want them (the kids) to be doing it all the time.
   - I'll encourage discussion with my candy.
   - The graphing calculator helps students pull it together better.
   - It is exciting to understand the whole picture.
6. Group Work - Not the student alone, not teacher alone
   - Each group must discuss and reach conclusions concerning the collection of individual problems completed by group members.
   - Each student in the group has a unique individual problem to complete and share with the group.
7. Testing It is not intended to be at the end, because testing is a learning experience, too. By the students using teacher created exams.
   - Students work in pairs on a linear programming test problem.
   - A quiz, during which students are given time to tap any resource in the room, and then complete the quiz, becomes a learning experience.
   - Tests contain non-calculator and calculator questions.
   - Tests should be a vehicle for learning.
   - *Tests are reviewed after they've been graded.
   - Assessment includes writing about math.

*denotes statement added by Ms. Dancer

Figure 7 . Ms. Dancer's card sorting.

The continuum moved on to group work and testing. Ms. Dancer saw group work as an important part of learning in which neither the student alone nor the teacher alone was responsible. Testing was not intended to be at the end, "because testing is a learning experience, too." In testing, students were using teacher-created exams so it was both the student and the teacher who contributed to the learning.

Ms. Dancer's organization of the statements and description of the schema utilized revealed her well-developed conceptions of teaching mathematics and students' learning. When describing her organization she spoke with confidence and certainty. Her ideas were well-articulated. As she reflected on the completed task of organizing the statements, she indicated that there needed to be a statement added to the group describing the teacher actions which indicated the amount of work required of teachers. The statement, "To be an effective teacher you need to put in a lot of time outside the classroom," was added to this group. This statement was a reflection of Ms. Dancer's dedication to teaching.

Belief verification interview. The belief verification interview was conducted in Ms. Dancer's home on a summer morning. The statements she had organized to describe her teaching in the previous interview had been grouped in a different way by the researcher in an attempt to capture the essence of Ms. Dancer's beliefs concerning mathematics, the teaching of mathematics, the structure of the classroom, and how students learn mathematics. As the interview began, Ms. Dancer commented, "I hope I can remember something about teaching now. I'm in my summer mode." This comment reflects the level of dedication and energy she put into her teaching and her need to renew herself during the summer months.

Ms. Dancer was first asked to respond to how the following statements reflected her beliefs about mathematics:

The teacher wants students to experience ah-hah's.

The graphing calculator helps students pull it all together.

Understanding the whole picture is a major goal.

Assessment includes writing about math.

The teacher explains the criteria used for grading assignments.

In responding to these statements, she emphasized the importance of the whole picture. "Mathematics is more than individual skills, the true understanding comes when all of these puzzle pieces are fit together." She explained her grading criteria to students so that they would understand that "mathematics is knowing how to approach a problem as well as knowing there's an answer." In grading individual work, she evaluated each problem carefully to determine whether a student was able to put all the pieces together to successfully solve a problem. She wanted to determine if "they make one small error which led to the inaccurate answer or did they make multiple errors?" It was always important to show all the work. Ms. Dancer indicated that in this way she could determine if students had developed the reasoning that was a part of mathematics. A part of mathematics was "having enough mathematical experience under your belt that you have a feeling for how to approach a problem." Thus, for Ms. Dancer mathematics was the big picture, made up of lots of little pieces including specific skills which with experience could be put together to solve problem.

To stimulate discussion of her beliefs about the teaching of mathematics, the following statements were used:

The teacher changes the pace of classroom activities frequently.

The teacher strives to have students actively involved.

The teachers sorts out and answers questions from the homework.

The teacher makes sure all students are feeling comfortable with the previous day's homework.

The teacher asks students to respond indicating: they understand, they kind of follow but have some questions, they are lost.

The teacher writes instructions for students on the board.

The teacher asks a student, "How did you get this?"

The teachers encourages lively discussion.

The teacher asks students to hypothesize about a method for completing a new, unknown type of problem.

The teacher calls on students who have their hands raised.

The teacher introduces new material by doing examples on the board.

The teacher uses the chalkboard graph to demonstrate.

The teacher demonstrates the use of the graphing calculator with the overhead display.

The teacher circulates through the room as students work on a warm-up problem.

Tests include both calculator and non-calculator questions.

The teacher reviews a test after it has been graded.

Ms. Dancer summarized her approach to teaching mathematics as, "comfort in the classroom, varied pace, conceptual development, and time to practice." In describing these qualities of teaching mathematics she explained that students "need to feel no anxiety in [the classroom]. They need to feel like they're welcomed and they can be themselves." Within this atmosphere, Ms. Dancer stressed the importance of active involvement where students participated in discussions and were willing to let the teacher know the degree to which they understood the concepts. This feedback enabled her to know "what to do next." When describing the pace of the class, Ms. Dancer explained that "multiple activities [were] real crucial with the 90 minute [class sessions]." In conjunction with varying the pace of the class by incorporating a variety of activities was the emphasis on concept development, both the prior concepts and the new concepts needed to be fully discussed in each class.

> I'll start with examples frequently or with a hypothetical problem. And then we'll walk through that. And then, depending on the difficulty we'll walk through one or two or three more. Maybe hitting it from a variety of angles, with the graphing calculator and the chalkboard graphs. But, they need some modeling. Hopefully it's not always me displaying step one, step two, step three. And sometimes them coming up with - this is one way to approach it.

Ms. Dancer concluded with the importance of practice in learning mathematics. "One thing that's important to me is time in the classroom to practice. I try to give them at least 20 minutes with me wandering around."

The next set of statements Ms. Dancer was asked to discussed were related to her beliefs about the environment in the classroom.

The teacher does lots of work outside of the classroom.

The teacher calls on students by name without waiting for hands to be raised.

The teacher encourages students to "do you best" on homework.

The teacher uses candy to encourage discussion when the group is lethargic.

The teacher calls on a student who is not actively engaged in the lesson.

Students are given the opportunity to earn extra credit.

The teacher compliments students on their performance on a quiz.

The most important part of creating a learning environment for Ms. Dancer was comfort. "I just don't think that kids will even try to learn unless they're comfortable." Comfort was important so that students would do their best. One of the facets of creating a comfortable environment for students in which they would strive to do their best for Ms. Dancer was mutual respect.

> I try to be very positive, have an environment where I think of them as not just students but as individuals with a life beyond the classroom - with humanity. And it's real important to me to treat these kids with respect. I say the first day, "I'm going to treat you with respect for the entire year and I expect you to treat me with respect."

Recognizing that mathematics is not "everybody's thing," she would also try to keep students constantly involved. Calling on students who did not seem to be involved, using candy to encourage discussion, and providing extra credit were ways she tried to "bring them back." When providing opportunities for extra credit it was important to Ms. Dancer that the work be "meaningful math, not just doing another set of problems." She saw extra credit as a way to bring in "that quiet kid who's not participating otherwise" or "reward the kids who want to do more." For her, the way she structured her classroom was a natural outgrowth of her attitude toward students. "I think number one is you like kids; you like to be around them. You respect their ideas, their being."

The last set of statements Ms. Dancer discussed were intended to reflect her views about how students learn mathematics.

> Students are required to make clear statements of where they're headed and where they've been.
>
> A student explains how to use the graphing calculator to find the intersection of two curves by a method other than trace (extra credit is awarded).
>
> Once students get the answer from their calculators, they interpret the answer.
>
> The teacher wants responses from the students.
>
> The teacher does not want "sleepers."
>
> The students are doing all the time.
>
> Tests are designed to be a vehicle for learning.
>
> Students do warm-up problems.
>
> Students are given time during a quiz to tap any resource in the room, then complete the quiz.
>
> Each student in a group is given a unique problem to complete, then shares the problem with the group.
>
> Each group discusses and reaches conclusions concerning the collection of individual problems shared by group members.
>
> Students work in pairs on a problem which is a portion of a test.
>
> Students correct their assignment from an overhead display of the correct answers.
>
> Students write their solutions to homework problems on overheads and explain what they did to the class.
>
> Students do "Masters."

For her the key thing in how students learn was "students are doing all the time." Making sure that students were always "doing" was reflected in the way she structured her classroom. Further, she felt that there were some important aspects to the learning process.

> Learning comes from seeing some models. It comes from putting the pieces together for themselves, which is different from seeing the model. They learn by practice. And the practice has to include some validation. They need to know if their answers are correct.

Practice was an important part of the learning process for Ms. Dancer. "Students can't just listen and try in class. They need to try it on their own without support. That cements the information. The classroom just enables [the learning]." She felt that immediate feedback was essential for the students. Because of the importance of immediate feedback "a high, high priority is tests come back the next class day and homework gets dealt with the next day." She stressed that feedback was important so that students "feel confident that they know what they are doing." While Ms. Dancer utilized groups in her classes, she did so cautiously because she felt that some students tried to hide behind their group. But working in pairs, especially on tests, was valuable. In pairs "they are sharing. They are seeing alternative ways to approach the problem."

Reiterating her belief that "math isn't just getting an answer - that the process is important," Ms. Dancer summarized her beliefs about learning mathematics. "Learning is understanding the process." She felt that immediate feedback, students sharing their approaches to solving problems, and class discussion all provided students with opportunities to hear a variety of approaches to solving a problem and "promote[d] multiple methods of solution."

<u>Summary of beliefs</u>. Mathematics, for Ms. Dancer, was more than a set of individual skills. She believed that understanding of mathematics only occurred when students were able to see how all the individual pieces fit together. Ms. Dancer emphasized that reasoning was a part of mathematics. Individual skills and concepts connected with reasoning and applied to solve problems constituted mathematics in Ms. Dancer's view.

Ms. Dancer's teaching of mathematics was based on the belief that students needed to be active participants in the learning process. She characterized the learning process as a continuum which included teacher and learner as active participants. The teacher's role required hard work and carefully planning in order to engage students and enable them to make the necessary connections between their existing knowledge and new content. Engaging students in the learning process, for Ms. Dancer, included both mental engagement and active engagement. This active engagement required that students think about the mathematics being explored and participate in the development of concepts through exploration, discussion, and practice.

The classroom atmosphere was important in Ms. Dancer's view. She worked to create an environment of mutual respect in which students felt comfortable. Students would strive to do their best only if they felt comfortable, according to Ms. Dancer. Involving all students including the quiet ones, the easily distracted ones, and the ambitious ones. Ms. Dancer found that using a variety of activities and approaches was useful, different approaches appealed to different students. By utilizing variety, she felt she was able to provide each student a point of access to the class.

Students could only learn mathematics, according to Ms. Dancer, when they were able to put the pieces together for themselves. In order to see how the pieces fit together they needed to see good models, but seeing the models was not sufficient. Only by actually doing the work could students learn mathematics. Further, Ms. Dancer believed that students needed validation of their work, immediate feedback, in order to cement or correct their learning.

Consistency between beliefs and practices. Ms. Dancer's beliefs were well developed and clearly articulated. It was clear that her planning and teaching was based on her belief system. Each lesson was carefully planned, developing the concepts from a starting point of prior knowledge, building connections to new concepts, and checking for understanding throughout the development. A variety of activities and approaches were used to engage students at different levels and with different styles. Group work, both in exploration and in testing, was utilized to enable students to share insights and work cooperatively. Students were engaged in hypothesizing and presenting material to the class. The teacher was not the exclusive guardian of information.

The atmosphere of mutual respect and trust Ms. Dancer valued in the classroom was reflected in students' willingness to share hypotheses and speculations. Ms. Dancer's belief in shared responsibility for the teaching-learning process could be seen through her articulation of expectations to her students. In spite of her belief in shared responsibility, she recognized her position of authority when it was required. Ms. Dancer's position of authority was demonstrated when she made the decision to continue with the development of new concepts when students did not indicate full understanding of the concepts that had been presented. This situation created anxiety for Ms. Dancer. She recognized a conflict between her belief in the need for understanding and the need to cover the required curriculum. In order to assuage her anxiety, she encouraged her students, telling them she believed in their ability to understand the material.

Mr. Carpenter

Mr. Carpenter taught in the same district as Ms. Dancer but at the other high school. He was contacted because the district had a common curriculum and it was decided that a comparison between two teachers in that district would provide valuable data to the study. He agreed to participate in the study during an initial discussion in his classroom. Parental permission slips were distributed to the students in the class which would be videotaped. When some students did not return the permission slips, their parents were contacted directly by the researcher before the video taping began to ensure that permission had been obtained from all parents.

At Shoreview High, where Mr. Carpenter taught, there was an office and shared workspace for the mathematics department, but the teachers spent most of their time in individual classrooms where they taught. The schedule for Shoreview was the same as for Lake High with the exception of the days being called "White" and "Blue" instead of "A" and "B." One of Mr. Carpenter's Advanced Algebra classes met on "White" days, during first period, the other two met on "Blue" days during fifth and sixth periods which were the first two class sessions of the day separated by the teacher/student contact time. The first period class was chosen as the primary focus for observation because of the scheduling. Observations of the fifth and sixth period classes were used to augment the descriptions of Mr. Carpenter's classroom practices.

Mr. Carpenter characterized the students at Shoreview as hard working students who wanted to do well. "The big thing about the kids here is they really want to do well. They will work as hard as they possibly can. You can make an assignment and they'll come back, 90% of the class will have done it." While the students as a group were hard working and capable, he had seen a change in the students in his classes in recent years. "Fifteen years ago we probably had 85% of all the kids in the school in mathematics. Now we have over 95%. So, that makes a difference." He had noticed that more students were continuing with mathematics even though they "don't really have the ability." Overall, he indicated that there were some really talented students, but "most of them [were] in the high average." Because these students were willing to work hard "that makes for really good scores on the national tests and stuff." Additionally, there were "a lot of wealthy kids" in the school.

Mr. Carpenter felt that Shoreview "had a really good math staff." There had been some recent changes with several teachers leaving and new, younger teachers joining the faculty. He indicated that the teachers worked well together. One area in which the good working relationship among the teachers was important had to do with classroom structure. Mr. Carpenter characterized himself as "a little bit looser than some of the others." Because the teachers who taught the younger students were "really strict about turning in homework exactly on time and what form it's in" students were well prepared when they reached his classes. He appreciated the way in which the style of these teachers "makes a nice compliment to the way I do things." Additionally, Mr. Carpenter described the working relationship among the mathematics teachers as "very, very good because we prepare tests for each other, worksheets for each other. We talk about how we're going to grade... and all that type of things."

<u>Background</u>. Mr. Carpenter began teaching in a "real small school" coaching three sports, teaching mostly mathematics and one science course. After spending one year in college in his midwestern homestate he transferred to a college in the Pacific Northwest from which he graduated with a degree in PE and a minor in mathematics. He had always played sports in high school and had "planned on getting a double major in mathematics and PE" but had just quit taking mathematics courses. Throughout the early years of his teaching career, Mr. Carpenter participated in numerous summer institutes including

Project Idaho, designed to introduce the "new mathematics" to teachers, and an National Science Foundation summer institute. He also enrolled in summer and evening courses at nearby campuses which lead him to the completion of a Master's of Natural Science degree. During this time he taught a total of nine years in several different schools in the state. At this point in his career, Mr. Carpenter spent a full year in an institute at a state university where he took graduate level mathematics courses with a heavy emphasis on statistics. He did not receive a degree as a result of this program, but equated it with the coursework for a Master's Degree in Mathematics. After this year he moved to another part of the state where he spent five years teaching before moving to Shoreview where he had taught for the past 20 years.

Until recently he had coached in addition to teaching mathematics. Throughout his career the teams he coached had experienced a high degree of success including winning several state wrestling championships. In addition to having taught the spectrum of high school mathematics classes and an occasional physics class, Mr. Carpenter taught at a nearby community college. At the community college he taught two courses each term, primarily statistics, but also calculus and algebra courses.

Throughout his career, Mr. Carpenter indicated that the institutes and advanced courses had an impact on the way he taught. Additionally, he subscribed to and read several magazines and generally attended regional meetings and technology workshops. He indicated that the regional meetings and technology workshops had "been one of the biggest helps in keeping up." Working with colleagues was another way Mr. Carpenter found to stay current in his area. Since the beginning of his career the people with whom he worked had impacted his teaching..

> When I first started to teach, the superintendent there said something that I have always tried to follow. He said, "Now, I expect that any kid that stays, any kid in your class is going to pass." I thought about that. If you're going to pass everybody you got to figure out some way to do it so it's not a gift. And so, for years, I made up a test that everybody in the class was going to get at least 50% on.

Originally, he would prepare tests on which there were enough "Mickey Mouse" problems that any student could get at least 50 percent on the test. More recently, the "prepared materials and things you have now that you can use" had changed the way in which he structured tests. Because of the amount of time it once took to prepare a test, "you'd give a chapter test, that's all you did." Now, he could give "like three worksheets during the chapter, two quizzes, and the chapter test." Now with the availability of preprinted tests Mr. Carpenter had changed his grading in order to maintain his philosophy of making sure that every student could get at least 50 percent on a test. "I give partial credit. If you write out anything at all you get one point. So, I make sure, unless you don't write anything down at all, you get 50."

Perhaps the most significant part of teaching to Mr. Carpenter was the relationships he established with students. It was important to him that students enjoy being in his class. "I really enjoy it when kids [see me in] other places and say hi.... They just remember it [was] a good place to be.... To me that's the number one thing about the way I try to run the class."

Introduction to and thinking about graphing calculators. Mr. Carpenter began talking about the use of the graphing calculator in his classes by saying that it "has made an awful lot of differences because of the way you do things, things you can start dropping." He had begun utilizing them in teaching the Analysis class with a preliminary version (unbound) of Demana and Waits' Precalculus Mathematics A Graphing Approach which was written to utilize the graphing calculators. The graphing calculators in use at that time were CASIO's. "I just followed along. And I liked the way it [the graphing calculator] did it because I've always put stuff on the board. We do it this way, and this way, and how many ways can we think of to do it?" The next year the first paper bound edition of the text was available and he continued to use the CASIO's.

Before the introduction of graphing calculators, Mr. Carpenter had been teaching computer programming. He had written course materials for two semesters of programming classes including programming in both BASIC and Pascal. When the graphing calculators became available, he and his colleagues began teaching programming with them as well. "We had programmed the CASIO, too. But it didn't program in so simple a way [as the TI's.] We did a lot with it, but the TI came out so much easier to use and stuff. And then of course when the 85 came out, that made it for sure the thing to go with it." Because of the ease of programming the TI graphing calculators and the additional features available on the TI-85, it became the standard calculator in use at the school.

As additional materials became available, "we kind of followed along that progression." At about the same time, Mr. Carpenter indicated that the decision was made to utilize the Chicago materials "which stressed using either the computer or the [graphing] calculator." He felt that the graphing calculator was better than the computer because "the kid has the calculator here and at home, wherever he goes." He also realized that "if we'd been in a different school district where the kids didn't have the money and stuff, that might be different."

Several factors contributed to Mr. Carpenter's persistent use of the graphing calculators in his teaching. To begin with "we bought the books that they went with" so the graphing calculators fit into the curriculum. Additionally, "we just enjoyed playing with them." Mr. Carpenter and another teacher from the school went to conferences and workshops and spent the evenings "sit[ting] in our hotel and play[ing] with the calculator." There was also pressure from the outside to use the graphing calculator. "People kept saying we should. There's one thing about schools, schools are made for administrators first, teachers second, and students third."

The variety of models of graphing calculators available had been a source of frustration for Mr. Carpenter. "Now that we're out of the CASIO's and into the TI's it really doesn't make that much difference whether they have a TI-81 or and 85, but it still does a little." More significant to his teaching was the availability of memory on the graphing calculator. "You can write anything in there you want. It's just like having notes. I think that's a frustrating point, because so many of the kids are looking for a crutch instead of what it will actually help them do, a quicker way so you don't have to work so hard." In contrast, Mr. Carpenter saw that the graphing calculator made a positive a contribution to students' level of understanding. "I think it has helped a lot of kids that usually would just go through the manipulation stuff. They understand a lot more of how it works. They can look at it in different ways."

With the use of the graphing calculator he found that it was possible to graph a few simple equations of a certain type by hand, "and just stick the hard ones in the calculator and you know everything." Mr. Carpenter also found that the graphing calculator provided a way of "being able to do it [a problem] in two of three ways."

Professed beliefs about mathematics and teaching mathematics. Mr. Carpenter explained mathematics to his students as "a way to explain the physical universe." He included the idea that mathematics was created by people. "It's just things that people have thought up that allow them to make predictions that turn out to be right." The emphasis on the development of mathematics by individuals was also a part of his description of algebra. "You may never use algebra again, but you know [what] the people who made this up and the people who do use it had to go through." Additionally, he defined algebra as a tool to solve problems and the foundation of higher mathematics. "The problems that people are working on today are a lot more complicated. They take a lot more mathematics to understand, but you've got to start someplace."

In order to learn mathematics, Mr. Carpenter felt that students needed to have good explanations and to practice. "Most kids learn it by practice and repeating and having somebody explain to them that there's a reason for it." Students could benefit from reading the text and thinking about mathematics. "The good kids think about it other times than when they're working on the problems. The text really adds to that, it make them think a little more because it talks about whys."

Classroom practices. Mr. Carpenter's Advanced Algebra classes were observed over a five week period that included the final examination period for the first semester. All three classes were observed for two weeks, the class that was the main focus of this description was observed over the entire period except for the final examination day.

Initial classroom observations took place at the conclusion of the unit on parabolas and quadratic equations and during the review for the semester final examination. The first unit of the second semester, functions, was observed in its entirety and served as the basis for this description of Mr. Carpenter's classroom practices. Seven class sessions, spread over three weeks were devoted to the unit on functions. There was one day of school lost during this time because of icy conditions. An additional day was added to the time allocated to the unit to make up for the missed day.

The atmosphere in Mr. Carpenter's classroom was very relaxed, approaching disorganized. Students spent much of the class time working individually or in small groups on the assignments from the textbook, worksheets provided by Mr. Carpenter, or reviewing returned quizzes or tests. Students were inclined to spend time talking about subjects other than mathematics. Mr. Carpenter encouraged students to stay on task, often by teasing them about what he overheard, making comments such as, "How many are working on math? How many are worried about what you're going to wear to the dance Saturday night? If I were you I'd wear the pink dress. It's so much nicer with your eyes." This teasing created a friendly atmosphere but did little to direct students back to working on their mathematics. There was a constant tension between allowing students the freedom to work on their own and the need to direct them to stay on task and complete the assigned work in order to master the material.

At the beginning of each unit Mr. Carpenter distributed an assignment sheet listing the days for the unit along with the lesson to be read and the problems on which all steps were to be shown. Students were expected to complete all problems from each section. The dates were not included on the assignment sheet, but were posted on the bulletin board. Students were expected to check the posted schedule and keep up with their work. The assignment sheet instructed students that they were to complete the assigned work prior to class. Mr. Carpenter reiterated this need to keep up with their work saying, "Remember, class is not to do homework. Class is to correct homework and learn new things. You should all be caught up when you come to class. If you don't get caught up because you don't get to do it in class, that's too bad."

There was no routine in Mr. Carpenter's classroom. During the time students spent working individually, Mr. Carpenter circulated through the room, responding to students questions and observing the work they were doing. Often he took an individual student or pair of students to the board and explained a problem on which they had questions. Other students observed the work being done at the board. In this way, Mr. Carpenter was able to work with an individual student and provide guidance to other students at the same time. As he circulated, Mr. Carpenter observed how far students had progressed in their work and when he felt there were students who were ready for the next topic or when a student asked a question about the new material, he called the entire class's attention to the board where he explained the new material.

Mr. Carpenter's presentation of mathematical concepts was not formal, but it was precise. He had a level of comfort and experience with teaching the concepts covered in the course that enabled him to present the material in a relaxed manner while stressing the key concepts and common pitfalls encountered by students. When exploring the concept of domain and range, Mr. Carpenter introduced the idea of a function as a machine. This machine representation of a function was not presented in the text. He drew a sketch of a machine with a hopper on the top and a spout on the bottom and said, "A function is like a machine. Something goes in and something comes out." Mr. Carpenter developed the concepts of domain and range of a function by utilizing the machine and eliciting student responses.

Mr. Carpenter: The things that go in, what are they called?

Student: Input.

Mr. Carpenter: This is the input here [indicating the spout on the top of the machine], what goes in. And the things that go in are the domain, members of the domain. And usually we call them x's. What comes out, that's the output. And it is?

Student: y's.

Mr. Carpenter: Y's, members of the range. Now, this machine works under a rule. In this case the rule is $2x + 1$. Okay? And so we can look at what happens. If we put in a three out comes a ?

Student: Seven.

Mr. Carpenter: Seven, and so on. See? What happens if I put in a t? What comes out ?
Student: 2t + 1.

Mr. Carpenter: 2t + 1. What this thing says is, anything that goes in is multiplied by two and then has a one added to it.

In this interchange Mr. Carpenter established, with participation from students, that a function could be represented as a machine with a rule that needed input called the domain and created output called the range. He then connected the idea of a function machine to a real world problem by describing a machine that could make toys from pieces of plastic when a button was pushed. This concrete example was fun for students to think about and gave them a concrete example to which they could connect the abstract idea of a function. Mr. Carpenter continued with his toy making machine analogy, discussing the difference between a relationship and a function.

> Let's say that we have a machine that always makes the same kind of car if I put in the same piece of plastic. Now, red, what? [I get a] red car. Now, green, [I get a] green car. This is a function because if I put something in I know what's going to come out.... Suppose I put in a piece of plastic, turn on [the machine] out comes a little car; another [piece of plastic] out comes a little toy soldier. [I put in] green [out comes a] green soldier.... Not a function, I put in one thing, green plastic. What happens? I get one of two different things. I don't get the same thing every time. That's like the idea of a function, the opposite of it, not a function.

This concrete image Mr. Carpenter created for the concept of a function was an example of his use of a variety of representations in his teaching. In addition to the algebraic representation of a function that he incorporated into this model by introducing a rule for the function machine and the numerical representation of a function that he discussed by showing what happened when specific values were fed into the machine, he added a real world physical representation to which students could make connections. Mr. Carpenter's function machine example was precise, when the machine only produced one type of toy it was a function, when it produced two toys it was not a function. At the same time, the example was light-hearted and fun.

Central to Mr. Carpenter's teaching was his concern for his students and his desire to illicit the best performance possible from each of them. The unit under observation coincided with the beginning of the second semester, giving Mr. Carpenter the opportunity to challenge his students to recommit themselves to achieving their goals in the class.

> Now, you look at your grades, and you have to make up your mind. As the year goes on there's going to be some real hard chapters, like last chapter was. And there's going to some easy chapters like two chapters ago, in the matrices. But, the big deal is, you've got to stick with each one of them and go through. And on the real easy ones, you've got to kill them. On the really hard ones, you've got to stick in there as good as you can and get the best grade you can on it. And it will all work out in the end, if you keep going. But you are the one that has to keep it up.... So, right now, you're starting all over again on your grades, to try to stay up with it.... Now, wherever you were on the [grade] chart, if you didn't get an A, try and see if you can keep pushing yourself and come out five percentage points higher.... So, see if you can push up by doing the homework and get up there.

This little pep talk reminded students of the importance of taking responsibility for their own learning as well as indicating Mr. Carpenter's understanding of the difficulty of the material for some students and the need to set realistic goals.

While Mr. Carpenter acknowledged the variety of students and abilities in the class, he also emphasized and required students to meet mathematically rigorous standards. Understanding and proper usage of mathematical notation and vocabulary were included among the standards he required of his students. In order to communicate these standards, Mr. Carpenter emphasized them in his discussion of problems. In the unit on functions, the notation for a function, finding the domain and range of a function, and the notation for quadrants of the Cartesian plane were all discussed. The following examples illustrate the importance Mr. Carpenter placed on understanding and proper use of mathematical notation and vocabulary.

When first discussing the concept of a function, Mr. Carpenter took time in the presentation to emphasize the meaning of the notation f(x).

> Everybody can say it [f of x], but they don't understand how to use it a lot of times.... F(x), this is just a number. It's a name for a number, just like y is. And that's one of the things we want to catch on to right at the first. If we can catch on to those two things, pairs of numbers [that] are related in some way, if there's some way to pair them together, they are at least a relationship. If

> there is one y for each x, one second number for each first number then it's a function. Otherwise it just stays a relation.

This discussion illustrates the connection Mr. Carpenter made between understanding the notation that represented a function and the definition or concept of a function. Later, in the same presentation, this additional interchange took place concerning the f(x) notation.

> Mr. Carpenter: Look at that statement right there [S(55) = 206.25]... What's nicer [about it] than saying [writes y = 206.25 on the board]?
>
> Student: You know what number you started with.
>
> Mr. Carpenter: That's the nice thing about this notation, S(x). It tells us what number the independent variable was, as well as the dependent. This one [S(55) = 206.25], you don't have to look back to see where you started.

The additional discussion of the f(x) notation in this dialogue further illustrates the importance Mr. Carpenter placed on understanding and proper usage of the notation.

The importance of understanding concepts and making connections was also illustrated in a discussion of domain and range.

> Everybody got down the domain and range words? If I write down x's and y's alphabetically, which comes first? x come first, still? Ok, write down domain and range alphabetically. Which comes first? [domain] So, that's the way they match up, the x's are domain, the y's are range. See that? And so, it helps to keep them straight because they alphabetically match up. And sometimes on a test, you get a little bit stressed and can't remember which is which. And so, the domain is always the x's and the range is always the y's and they're alphabetical like that.

Pointing out something easy for students to remember, the alphabetical order of commonly used variables x and y corresponding to the alphabetical order of domain and range, illustrates the connections Mr. Carpenter aided his students in making between their existing knowledge base and the new concepts being explored.

During the review for the unit exam, a question was discussed which concerned the quadrants of the Cartesian plane. In discussing the correct answer, Mr. Carpenter emphasized the importance he placed on using correct mathematical notation. "[If you write] this [1, 2, 3, 4, you] are wrong because... it might sound the same, first, second, third, fourth, but it's not the same. Just like spelling somebody's name different. It has to be the Roman numerals... to name the quadrants." Although using the numerals 1, 2, 3, 4 for the quadrants would identify them correctly, according to Mr. Carpenter's standards the answer would not be correct because the proper notation had not been used.

Mr. Carpenter was not rigid in his insistence on correct mathematics notation and form. When discussing the rationalization of expressions, the following interchange took place.

> Student: If you just leave it like that [with a square root in the denominator of the expression] on a test, will it be right?
>
> Mr. Carpenter: Oh, I don't know. If somebody changes it [to an answer without a square root in the denominator], then I'll have to take one point off everybody else's. So if somebody wants to get everybody else, just change it.... If you want to be loved as the number one nerd in the class, make sure you rationalize things.

This example illustrates a subjectivity Mr. Carpenter applied in his assessment of student work. While he emphasized the importance of correct notation, Mr. Carpenter also displayed an understanding of his students and their abilities. His concern for their ability to understand and use the correct notation as well as fully understanding the concepts being explored extended beyond the scope of this algebra course. When discussing the interpretation of a graph displayed on a quiz, Mr. Carpenter emphasized the importance of being prepared for the Scholastic Aptitude Test (SAT). "A lot of people asked me [how to interpret this graph]. And, I said, 'You decide and we'll grade it the way you decide.' The big problem is on tests like the SAT, and that kind of thing. You're going to have to make a decision, but they've made a decision." In this example Mr. Carpenter was explaining to students that it was important for them to understand the standards by which others would interpret mathematical notation and symbols so that they could perform as well as possible on exams like the SAT.

In his teaching, Mr. Carpenter focused on the material and concepts being developed for this course, but he also made connections to future classes. A problem in the review asked students to explain what the absolute value of the difference of two numbers represented. After discussing the absolute value of two numbers as a representation of the distance between the numbers on a number line, Mr. Carpenter focused on the importance of the concept. "Now, everybody, mark that in their brain. Then when you get into a calculus class and you see this, it is a real important part in a calculus class. When you get there, say, 'I started thinking about this calculus class clear back in algebra class when I did this little problem'."

Mr. Carpenter considered mathematics as being superior to other subjects and shared this view with his student. Although not intended to be taken seriously by his students, the following comparison did display a bias on his part.

> This is not like English class. In English class all you have to know is why she's wearing that scarlet letter. And then next week you can forget that completely. Yeah, forget that completely and find out why Huck and who is it in on that island. How come they're there? Does anybody remember? [pause] You mean you can't remember! See, that's the difference between English and math. If you forget in math, you can't do the next problem. If you forget in English, you just read the next book and forget it in a week. English is read and forget, read and forget, remember for a week and forget. Math is remember forever or not get the answer. Yeah, it makes math a lot harder.

In addition to displaying a bias against English and toward mathematics, Mr. Carpenter provided his students with a philosophy of mathematics. He described the importance of building mathematics upon the foundation of prior knowledge. This emphasis on the importance of remembering what one had learned before in order to complete the next problem served to help students realize that they needed to do more than do the problems, they needed to master the material because they would need to utilize what they were learning today in the mathematics they studied in the future.

Use of graphing calculators in teaching. Mr. Carpenter's classroom was equipped with an overhead projector and overhead display unit for the graphing calculator that were situated so that the graphing calculator display could be used at any time but did not interfere with the use of the chalkboard when the graphing calculator was not being used. Mr. Carpenter made extensive use of the graphing calculator in his teaching, integrating its use into every class session. The variety of ways the graphing calculator was utilized included displaying graphs, developing concepts, doing computations, using student-written programs, interpreting results, and exploring the features of the calculator. Mr. Carpenter continually encouraged students to utilize their graphing calculators to display graphs and do computations while emphasizing the importance of understanding and accurately interpreting the results obtained from the calculator.

Mr. Carpenter encouraged students to utilize their graphing calculators to display graphs when they were required to produce the graph of a function. When discussing a worksheet designed to examine the graphs of the functions, he asked if students had used their graphing calculators to produce the required graphs. When no students responded affirmatively, he indicated the value of using the graphing calculator to display a graph saying, "On the semester test there was a whole bunch of people that missed a couple of questions and they didn't graph them to take a look at it." The implication of this statement was that if students had utilized their graphing calculators to graph the problems, they would have discovered the errors they made.

Mr. Carpenter continued the discussion of the worksheet, asking what the graph of $g(x) = x^2 - 4x - 2$ would look like. When a student responded that it would be a parabola, Mr. Carpenter commented, "That's really important, to know what it's going to look like before you start." Knowing the general appearance of the graph of a function before using the graphing calculator to display the graph for further analysis was of major importance to Mr. Carpenter. By knowing the general appearance of the graph before using the graphing calculator to examine specific details of the graph or check an answer found algebraically, students would be able to detect errors they made entering the function into the calculator.

Mr. Carpenter did not only remind students of the importance of knowing the general appearance of a graph before using the calculator, he demonstrated the value of this knowledge in his teaching. After a lengthy exploration, including a chalkboard sketch, of the graphs of $g(x) = \sqrt{4x-4}$ and the composite function $f \circ g(x) = \frac{\sqrt{4x-4}+6}{2}$, during which the shapes of the graphs and their approximate locations in the coordinate plane were examined, Mr. Carpenter instructed the class to graph the functions on their graphing calculators while he did the same using the overhead display. "Okay, do it on your calculators. If I were you, I'd put this one [$g(x)=\sqrt{4x-4}$] and this new one [$f\circ g(x)$] on my calculator and see what difference it makes. See if it fits what we thought would happen. We should always think about it first." Having modeled the process of thinking about the graphs before using the calculator to examine them, Mr. Carpenter proceeded to enter the functions into the calculator, emphasizing the details that could be problematic. "4x - 4 inside a [pair of] parentheses [because it is all] inside the square root. $\sqrt{\ }(4x-4)+6$ all inside a [pair of] parentheses because of the bar [in the rational expression $\frac{\sqrt{4x-4}+6}{2}$], divided by 2." Mr. Carpenter emphasized the details described in this example because he knew that students might forget the grouping symbols required to properly enter the functions into the calculator. In this teaching episode, Mr. Carpenter demonstrated two emphases consistently included in his use of the graphing calculator, emphasis on the importance of understanding the underlying algebraic principles including the shapes of fundamental functions in order to be certain that the results obtained from the graphing calculator were accurate and emphasis on the skills required to utilize the graphing calculator to obtain accurate results. During another class session, he summed up the importance of these two facets of utilizing the calculator. "So, be careful. You need to know what they [the graphs] are going to look like so you can tell if the calculator did it right, because the calculator does what you tell it. And, if you tell it wrong, you're in trouble."

Two examples illustrate Mr. Carpenter use of the graphing calculator to develop and reinforce concepts. After the initial discussion of the concept of a function which included the rule representation, he played a game with the class.

> What if I told you... the ordered pairs, could you guess what the rule is? Let's see, let's see if we can. [Talking as he makes selections on the calculator] PROGRAM, NAMES, FUNKY, huh? FUNKY for function. The rule, I'm going to pick the first one I put in my calculator. It says when x = 2, f(x) is a 4. Think. Can you think what the rule might be? Don't say. Can you think? Got an idea? When x is a 5, f(x) is a 25. Who knows? [counts the number of hands that are raised] One more, check your [rule] out. I put in an 8... I get out a 64. What's the rule?

Mr. Carpenter could have played the same game with the class without using the graphing calculator, but he was showing the students that using the calculator could be fun. Having fun with the class was important, but at the same time he was reinforcing the concept of a rule for a function and connection between the function's rule and it's set of ordered pairs.

After using a composition of function machines to develop the concept of the inverse of a function using $f(x) = x^2$ and emphasizing on the interchanging of x's and y's that occurred between a function and its inverse, the concept of the inverse of a function was further explored utilizing the graphing calculator.

Mr. Carpenter: Let's have everyone looking up on the overhead at those three equations [$y1 = x^2$, $y2 = \sqrt{x}$, $y3 = -\sqrt{x}$, $y4 = x$]and tell me what's going to happen?

Student: You're going to get a parabola. And then you're going to get a parabola on its side.

Mr. Carpenter: The first one is the parabola, the parent parabola like we talked about. The second one is the top half of the parabola lying on its side. And the third one is the bottom half [of the parabola lying on its side]. To be exact, this should graph the same thing [as we graphed on the board earlier]. What was the deal with the parabola and this [other] thing" What were they?

Student: Inverses.

Mr. Carpenter: They were inverses of each other. [Turning back to the overhead display.] What's this equation [$y4 = x$] going to do?

Student: Diagonal line.

Mr. Carpenter: Diagonal line, where at? It's going to be this line. It's going to cut right down through and divide this [the angle formed by the x and y-axes] into two equal angles. The line $y = x$, over 1, up 1, over 2, up 2 etceteras.... [Displaying the graphs after setting the WINDOW dimensions to make the graph the same scale (square) along the x and y-axes] Is it doing what we thought it was going to do?

Student: Yes.

Mr. Carpenter: Now, what just happened is very, very important. Can you see what just happened?... What was it?

Student: It's a reflection line.

Mr. Carpenter: Did you hear that? Several people said it's a reflection line.

Using the graphing calculator to display the graphs and student input, Mr. Carpenter demonstrated the relationship between the graph of a function and its inverse, showing that they were reflections across the line y = x. This visual display, produced by the graphing calculator, was an effective tool for demonstrating the reflective property of the line y = x between a function and its inverse.

The exploration of the graphs of a function and its inverse also provided Mr. Carpenter an opportunity to investigate several features of the graphing calculator.

> Now, a fun thing. This has nothing to do with anything but fun. [Talking through a sequence of calculator choices as they are displayed] MORE, FORMAT. One, two, three, look at that fourth thing. It says SEQUENTIAL. What does that mean? In sequence, one after the other. Look at the next one. What does it say? SIMULTANEOUS What does that mean? All at the same time. Watch this.... It's showing all three [the function, the inverse, and the reflection line] at the same time. It's doing the y's for one x all at the same time. So, when it got here, [indicating a point on the x-axis] it did all three of these pixels [indicating the points on the three graphs on the imaginary line x = the value on the x-axis] at the same time. When it moved over here [to a new x-value] it did all three of these pixels at the same time.

This demonstration of the use of SIMULTANEOUS to display the graphs of the function, its inverse, and the reflection line provided the students an additional tool for exploring mathematical concepts via the graphing calculator. Another feature introduced in the same demonstration was DRAWINVERSE. With assistance from a student who had already found the feature. Mr. Carpenter explained how to use the feature.

> Go to the Home Screen. DRAW, MORE, MORE, DRAWINV... 2nd Alpha, Y1. Let's see if that works.... There it comes. Wow! Okay, if we can follow what Peter's instructions were... we can draw the inverse of anything that we can write in there [y1] to start with. It will even draw them [the inverses] when they [the inverses] are not a function. There, see that, all in one step. It draws the inverse even if it isn't a function.

While neither the SIMULTANEOUS nor the DRAWINVERSE features of the calculator were essential to the development of the concepts in the course, both features provided additional tools for students to explore the concept of a function and its inverse and added to their general knowledge about the graphing calculator.

In addition to utilizing the built-in features of the calculator, students were encouraged to add their own programs. The usefulness of having a program for finding the key points of the graph of a parabola arose out of the discussion about finding the range of a quadratic function.

> Mr. Carpenter: The range isn't going to be everything in a parabola. The range is only the y's that are used. And where do they start?
>
> Student: Vertex.
>
> Mr. Carpenter: At the vertex and work up. So, to be able to know what the range is, you have to know what the vertex is. How do you find the vertex?
>
> Student: Put it in vertex form.
>
> Mr. Carpenter: Vertex form [writes $y - k = a(x - h)^2$].

After working through the problem of finding the vertex of the parabola of interest, Mr. Carpenter continued,

> You see, there's a lot of things to remember and when there's a lot of things to remember, we run into mistakes.... Everybody's got that down [how to find the vertex of a parabola], but it takes time and it's easy to forget little parts. Although, one wants to look at it every once in awhile. Let's look at something here [turning on the graphing calculator display.] PROGRAM, NAMES, MORE, MORE.... What do you see that fits into what Don was just talking about?... QUAD Press that button, it says 'Quadratic'....

He demonstrated the quadratic program he had stored in his calculator then asked if any students had their own programs. After assisting the students who volunteered to demonstrate their programs, he encouraged all students to input some type of a quadratic program into their calculators. "I would suggest that you have one of these [quadratic programs] in your calculator.... Now, it doesn't do any good to have the program if you don't know what the pieces mean when you get them." Mr. Carpenter was encouraging his students to maximize their use of the graphing calculator as a tool to make solving problems easier while maintaining the emphasis on the importance of understanding the mathematics underlying the use of the calculator.

Graphing calculator usage was completely integrated into Mr. Carpenter teaching. It was his expectation that all students had their calculators available and ready to use at all times. When working problems at the board that resulted in calculations that could not easily be done mentally he asked for the answer, expecting a student to quickly do the necessary computations on the graphing calculator. With the use of the graphing calculator new questions arose that had to be addressed. On the mid-unit quiz, provided by the textbook publisher, there was a question requiring students to state the domain and range for a relation depicted by a graph. The graph shown was a piece of the $y = \cos x$ extending from $-\pi$ to π. The problem was that the graph stopped where it intersected the x-axis at $-\pi$ and π without having dots or arrows at the ends of the curve, while the axes extended further and had arrows at the ends making it difficult to decide whether the domain of the function was all real numbers or $-\pi < x < \pi$. Mr. Carpenter explained that this uncertainty was directly related to the use of the graphing calculator to display the graphs of functions. " [The graph on the quiz] is like your calculator. When it [the graph on your calculator] runs to the edge of the window, it doesn't say, a big note, I'm going to keep going. And, it doesn't put a big arrow on the end. It just stops. But, we all know it goes on. But, this [not knowing whether the graph stops or keeps going] is a real problem. And the calculator has caused this problem." For Mr. Carpenter, it was important that students be able to transfer what they saw when they used the graphing calculator to what was printed in a text or on a test. He emphasized the importance of understanding both the limitations and the usefulness of the graphing calculator.

Belief clustering interview. Mr. Carpenter was presented with 51 cards that contained statements based on information from previous interviews and classroom observations. The cards had been shuffled so they were in no particular order. As he read and sorted the cards, Mr. Carpenter indicated that they were pretty comprehensive and that "I'd have to think for a while before I could think of anything" to add. After completing his sorting, Mr. Carpenter looked back through each group of cards and arranged them neatly. The cards were arranged in six groups (Figure 8).

As he began discussing his sorting of the cards, the first groups Mr. Carpenter talked about were those he considered to be "obvious." One of these groups included items concerning tests and quizzes which he said were "the ones where they [students] are checked for what they know." To Mr. Carpenter, it was important to assess students' knowledge of the material being taught. The other group of statements he thought were obvious were those that "have to do with something that makes it a little nicer to be there, a little easier to do." By grouping these items together, Mr. Carpenter continued to emphasize the importance he placed on the atmosphere of the classroom and his style of interaction with the students. He said he thought "these all fit together because these are part of me and them, all having a little fun in the deal."

The next group of cards Mr. Carpenter described as "where students help themselves and use other things to help themselves with." In discussing the statements on these cards, he emphasized the importance of students examining their own work, discussing their solutions with others, finding their own errors, demonstrating their abilities to solve problems, utilizing the graphing calculator in the solution of problems, and preparing notes to use on an exam as ways in which students can help themselves learn. He felt it was important for students to recognize their own role in the learning process. He indicated the importance of having students "keep track of where they are and keep going."

While Mr. Carpenter felt it was important for students to realize and be responsible for their role in the learning process, he acknowledged that the largest group of statements were related to what he as the teacher did in the classroom. When discussing the statements he had grouped as "the ones where I thought I did something," he divided them into two subgroups. One of these groups emphasized his role in helping "them learn either the new material or the old, to get it down." The other group contained items that he felt directed the process of student learning. In grouping them this way, Mr. Carpenter indicated the importance of his role in delivering the content of the course as well as guiding the learning process. He felt that he needed to "lead it [the class] sometimes and make them do it just the way I want it done, or at least push them in some sort of

2. Having Fun
It's nice to be be there

4. The Teacher
I did something

5. Student
Programs

6. Calculator
Use

3. Students Help Themselves
calculator use,...

1. Check For What They
Know

1. They are checked for what they know
   Students take a test
   Students take a quiz.

2. Part of me and them having a little fun in the deal.
   Students are encouraged to participate in county and national competitive exams.
   Some students talk about things other than their mathematics.
   The teacher has fun in class.
   Students are assignmed new seats after each exam.
   The teacher plays music in the background as students begin their work.
   The teacher prepares an assignment sheet for each unit.

3. Students help themselves
   Students put solutins to review problems on the board.
   Students discuss assignments among themselves.
   Students are allowed a sheet of notes for the semester final exam.
   Students turn in homework packets on test day.
   Students check the solutions written on the board by other students.

4. I did something

A. Helping them learn either the new material or the old, to get it down.
   Using a calculator program, the teacher has students try to figure out the rule for a function.
   The teacher asks a specific student a question.
   The teacher does examples on the board to develop new material.
   The teacher discusses cheating on homework.
   The teacher asks questions, calling on students by name.
   The teacher calls on a student whose hand is raised.
   The teacher returns tests (and quizzes) by circulating through the room.
   The teacher circulates around the room working with individual students and small groups.

B. Things I do to direct the learning process and things they do to learn on their own.
   Students are expected to read the text
   Students keep track of the daily schedule of assignment due dates.
   Students return tests to the teacher after they have looked over them.
   As students find errors in other students solutions, they take them back to the teacher.
   Students use graphing calculators on tests.
   The teacher encourages students to utilize their graphing calculators.
   The teacher encourages students to maintain good study habits.
   Students return progress reports signed by parents.
   The teacher encourages students to pay attention.
   Students discuss their tests individually with the teacher at his desk.
   Students are expected to complete the worksheet begun in class.
   The teacher goes over the remainder of the worksheet from the previous class.
   The teacher distributes a worksheet and works several of the problems with the students.
   The teacher calls students to his desk and goes over the review problems (which were on the board) where mistakes were made or solutions could have been more efficient.
   The teacher works assignment problems on the board that the students ask about (for the whole class).
   The teacher goes to the board with one student and explains a problem.
   The teacher demonstrates the use of the graphing calculator using the overhead display.
   Students watch a videotape on the quadratic formula.

5. Students do something on their own (doesn't fit anywhere else)
   Students demonstrate quadratic programs they have in their calculators.

6. We used the calculator
   Before beginning the test, the teacher reviews key points including use of graphing calculator.
   The teacher uses the graphing calculator to explore new concepts.
   Students have been given programs for the graphing calculator such as BTTRPTS.

<u>Figure 8</u>. Mr. Carpenter's card sorting.

directions." He also felt that "it's really important that the students learn to rely on more than just me." Because he believed that he needed to lead the class some of the time and the students needed to learn to not only rely on him, he divided the cards between "things that I do to direct them and things they do to learn on their own."

The final items Mr. Carpenter discussed were related directly to the use of the graphing calculators in his teaching. Here again he felt that it was important that students be able to use the calculators on their own. In fact he separated out a statement concerning students writing their own programs for the graphing calculator as a unique activity in which "students do something on their own" which did not fit with any other group. In addition to the importance of having students utilize the graphing calculator on their own, Mr. Carpenter recognized the special role the graphing calculator played in his teaching because of "how the calculator would help us look at ... [and] do different things."

As the interview concluded, Mr. Carpenter mentioned his frustration with the block scheduling used in the district. He found that with 90 minute class sessions "it's hard to keep them [students] going ... even if you change [the pace with] two or three things." He also found that with the block he was able to teach less "as far as material." He did not agree with the idea that covering less material and teaching it better was appropriate. When "your SAT scores are going like this [up], you can't teach it much better. There's no better, it's already good."

Belief verification interview. This belief verification interview was conducted in a conference room at the community college where Mr. Carpenter taught during the summer. The statements he had discussed in the previous interview had been grouped in a different way to reflect his beliefs regarding mathematics, the teaching and learning of mathematics, and the use of graphing calculators. The first group of statements were intended to reflect Mr. Carpenter's beliefs about mathematics.

> Using a calculator program, the teacher has students try to figure out the rule for a function.

The teacher introduces new material by beginning with an example from the previous lesson and adding to it.

Students use graphing calculators on tests.

Students are allowed a sheet of notes for the semester final exam.

To Mr. Carpenter these statements reflected his belief that "mathematics starts out with little basic materials and puts them together to make something more complex." In building from the little parts to the more complex it was important to be "able to link together the geometry type things, the graphing, and the algebra type." In describing its complexities and the connections between its different aspects, Mr. Carpenter emphasized the hierarchical nature of mathematics which requires reapplication of previously acquired skills and concepts in new contexts. He also acknowledged the possibilities for approaching mathematics and particular problems from a variety of perspectives. He felt it was important to allow "students who think differently or learn differently to make other types of connections when they do problems."

Mr. Carpenter was presented with the following statements to encourage articulation of his beliefs regarding the teaching of mathematics:

The teacher has fun in class.

The teacher does examples on the board to develop new material.

The teacher makes connections between the new material being presented and the material previously studied.

The teacher uses the graphing calculator to explore new concepts.

The teacher distributes a worksheet and works several of the problems with the students.

The teacher works assignment problems on the board that students ask about.

The teacher goes over the remainder of a worksheet begun during the previous class session.

Before distributing a test, the teacher reviews key points including the use of the graphing calculator.

> The teacher demonstrates the use of the graphing calculator using the overhead display.
>
> The teacher returns tests or quizzes by circulating through the room.
>
> The teacher calls students to his desk and goes over the review problems [which were written on the board by the students] where mistakes were made or solutions could be done differently.
>
> The teacher goes to the board with one student and explains a problem.

The idea of building on prior knowledge in the teaching of mathematics was important to Mr. Carpenter.

> I think if you can use examples of something you've previously done and add to them...that you know a start of this came from geometry class even though you're in algebra class, or these two things from algebra and geometry , we're putting them together to do something new...I think it's real important to point those things out.

He thought it was important for students to know "that they aren't just wasting the things that they learned before. And that it's real important to remember the things that we're doing today because they're going to show up somewhere else again." While it was important to point out the connections between new concepts and prior knowledge, Mr. Carpenter also recognized the importance of teaching by example. One way he utilized teaching by example was by preparing worksheets that were organized so that "people go through steps and show other people what they're doing." Additionally, he emphasized that "the answer is only a small part of it." He felt that students needed to "communicate with other people and show them how [they] came up with that answer." Mr. Carpenter thought it was important to work a lot of examples of problems for students so that when students ran into difficulty with problems they could "find out how somebody else would get out of that problem or how they wouldn't haven't run into that difficulty." Thus, he recognized the value of modeling problem solving methods for his students. "I don't think there is any doubt that if you can get people into the mode of thinking and reviewing what they are doing, that they're going to be able to carry that on and do more things." In his teaching, Mr. Carpenter felt it was important to include occasions on which he worked one on one with students. He could work with individual students when they were working on daily assignments or reviewing a test or quiz after it had been returned. By allowing students to work together he could then work individually with students. "I think if you can work one on one with someone you're going to come out a whole lot better." In an individual setting he found that it was sometimes possible to move a student who just wanted to know the quickest way to get the answer "to think out the problem instead of just getting the answer." He summarized his teaching by saying

> I think making everybody participate is really important. And trying to single out individuals to see how they're doing it, and making everybody realize that you don't have to do it exactly one way, but you have to be able to do it again and again that same way you did it. I think it's real important that they realize that there's a lot of ways to solve real simple problems, but you have to start narrowing it down whey they get more complicated. [There are] a lot of ways to do things, but certain basic things they just have to know.

Finally, Mr. Carpenter noted that even though he had solved thousands of equations, he found that "it's still kind of fun to see, the answer really did turn out to be the right answer. It's kind of like a game." He tried to instill that enjoyment in his students.

The statements given to stimulate Mr. Carpenter's discussion about teaching in general were:

> The teacher prepares an assignment sheet for each unit.
>
> Students keep track of the daily schedule of assignment due dates.
> Students are encouraged to participate in county and national competitive exams.
>
> The teacher discusses cheating on homework.
>
> Students return progress reports signed by parents.
>
> The teacher plays music in the background as students begin their work.
>
> The teacher encourages students to pay attention.
>
> The teacher calls on a student whose hand is raised.
>
> The teacher asks questions, calling on students by name.

> Some students talk about things other than their mathematics during work time.
>
> Students take tests and quizzes.

As he read through these statements the first thing Mr. Carpenter mentioned was the importance of organization. "I think it's important to organize and to know where you are going and know where you are in that organization." He felt that students needed to know what lay ahead and that it was important for the teacher to "have a long range goal."

Flexibility was important to Mr. Carpenter as he discussed his views on teaching. While he felt that it was important for students to learn to be "responsible for getting things in," it was also important not to "penalize a kid for being a kid." For him, the flexibility to accept the different styles and personalities of his students while encouraging all of them to excel was important. Giving tests and quizzes, he saw, as a way to teach students to be responsible for learning the material. He also valued rewarding students for their effort. He felt that students who did not perform well in class, but worked hard, should feel success.

> You put on there [the test] problems that you have done in class, that they have done in class, but you put real simple ones. He [a student who does not perform well, but works hard] gets a 50 or a 53 [out of 100] or something of the sort. And now he actually thinks that he's doing something. And I think that's really important. And I think in the end he'll learn more, because now he figures he's got a chance and everything. So, he'll work at it and keep trying to stay up there.

Having fun in the classroom was also important to Mr. Carpenter. "I know teachers that aren't [flexible], and maybe that teaches kids in the long run, but it sure does cost a lot of fun." Parental involvement played a significant role for Mr. Carpenter. He regularly required students to return signed progress reports from their parents for two reasons.

> Number one, it gives their parents some insight into what's going one, that the teacher's trying to have the student do better, to keep track of it, to inform the parent, that kind of thing. But it also lets the student know their teacher thinks that the parent is part of the process.

The next group of statements were intended to reflect Mr. Carpenter' beliefs concerning how students learn mathematics.

Students are expected to read the text.

Students are expected to complete a worksheet begun during class.

Students turn in homework packets on test day.

Students watch a videotape on a specific topic (the quadratic formula).

The teacher asks a specific student a question.

Students put solutions to review problems on the board.

Students check the solutions written on the board by other students.

As students find errors in other students' solutions, they take them back to the teacher (at his desk).

The teacher encourages students to utilize their graphing calculators.

Students have been given programs for the graphing calculator such as BTTRPTS (a program which adjusts the graph viewing window).

Students demonstrate quadratic programs they have in their calculators (which were not distributed by the teacher).

Students are assigned new seats after each exam.

Students discuss assignments among themselves.

Students discuss their tests individually with the teacher at his desk.

Students return tests to the teacher after they have gone over them.

The teacher encourages the students to maintain good study habits.

A consistent theme in Mr. Carpenter's discussion of how students learn mathematics was that they needed to think about the mathematics and how to solve the problems in order to learn.

> I try to tell them they should be thinking ahead, trying to answer the next step before I get to it, or before the rest of the class gets to it, not waiting for somebody else and saying, "Yeah that looks like a good idea." One of the

> ways to do that is just for them to realize that you're going to just randomly call out different people.

Reading the text and trying to figure out how to do the work before it was discussed in class was another example of how Mr. Carpenter encouraged students to think about what they were learning. "It's real important that they read the text, think about the problems themselves, and then get direction on how to work them. Instead of [having] somebody show them how to do every little thing." While it was important that students think through the material for themselves, Mr. Carpenter utilized worksheets to be sure that students worked through all the pieces of each idea explored. "They have to do certain things. They can't skip out and leave out different ones. If you do a worksheet, they have to do the things that are on the sheet ... then everybody's got an idea what to do."

Mr. Carpenter emphasized the importance of talking about mathematics in the learning process. Students should work together while the teacher acts as facilitator. "I think you learn so much more by talking about how to do the problems. I think it's real important (that) the students work on problems together and that the teacher goes around and just kind of facilitates, keeps 'em working, asks 'em how they did things, that type of thing."

The teacher's role of facilitator included having students go to the board and work problems, usually when reviewing for a test. Going to the board facilitated student learning in a number of ways. It provided an opportunity for students to see problems worked correctly and for the teacher to give feedback on the methods used. "A lot of times kids won't ask about certain problems even though they can't do it. So, they see it worked out. [Another] thing, you can see if the students are actually working the problems out with steps and stuff. And if they can't do it you can have them get some help." It also provided an opportunity for discussion of student errors. "When I go up and talk about what they could have done differently, I can say you can see how a person could have come up with this. Or, here's the little mistake. [I can] make the person feel that they still did something worthwhile." Seeing problems worked from different perspectives was an essential piece of student learning. Using videotapes "gives the kids a perspective from somebody else besides yourself" while letting "them see that other people do it in the same way." These videotapes could also help with review because "it puts all the things right together."

Making connections and putting things together was another important aspect of how students learn. Asking students questions could facilitate "individual ownership" which was important because "they know that they're going to be responsible [for knowing the material] sooner or later." Working individually with students and discussing errors made on tests and quizzes was important because "in mathematics, things keep coming back." Mr. Carpenter felt that if students missed something on a test there had to be a reason and it was valuable for students to figure out what they had done.

> When students go over tests, they usually go over the tests with each other and try to find out what the things [they missed] are. Then they come back and talk to me about [it.] I kind of encourage arguing for points because if they know they can argue for a point they go to the trouble of figuring out what they did and doing it [correctly.] That's going to make them do more. They'll work it out and think about what they did before. [That's] really worthwhile.

Finally, Mr. Carpenter believed that students needed to maintain good study habits in order to learn. "Study habits are just part of your organization, part of what you are doing. I think it is important that they keep at it."

Even before the advent of the graphing calculator Mr. Carpenter had incorporated graphic and symbolic approaches in his teaching.

> I've always taught drawing the picture, working out the algebra, etceteras. This [the graphing calculator] just makes it so much easier. You can do it right there, and they can see it show up. I think what the calculator has done is absolutely fabulous. Everybody can have their own. You can have it at home and get it done.

The availability of the calculator for everyone was the factor that Mr. Carpenter thought had made the most difference in his teaching. Looking at graphs and doing complex computations were things he was already doing, but with the graphing calculator they could now be done much more quickly and were accessible to all students. "I don't think the calculator has changed so much how you teach as much as making it simpler."

Summary of beliefs. There was a common idea that ran through Mr. Carpenter's beliefs about mathematics and teaching mathematics, the idea of connections. He saw mathematics as a hierarchical structure in which more complex concepts were built upon simpler concepts. The connections between the simpler concepts and the more complex concepts and the connections between different areas of mathematics were essential elements of the structure of mathematics. Mr. Carpenter also found that there were a variety of perspectives from which mathematical concepts could be approached. In teaching mathematics, he found that it was important to help students build on prior knowledge, making connections between new concepts and previously explored concepts. Mr. Carpenter believed that providing students with a variety of perspectives from which to view a concept, including graphical, numerical, and symbolic, as well as a variety of approaches or processes to utilize in the solution of a single problem assisted students in making the necessary connections and learning the concepts.

Mr. Carpenter held definite beliefs about the role of the teacher and the role of the student in the learning process. Accordingly, he envisioned that the teacher's role was to deliver the content and guide the learning process. He believed that it was important for students to know that there were goals, established by the teacher, toward which they were working. Providing an organization for the learning processes communicated the goals and provided a structure in which students could learn. Inclusion of well-developed and clearly explained examples served as a tool in his delivery of the content. But, Mr. Carpenter believed that students could not learn simply by watching the teacher do examples, they also needed to do the mathematics in order to learn it. The students' role included active involvement and taking individual responsibility for learning. Mr. Carpenter felt that students needed to not rely solely on the teacher to provide the learning, rather they needed to learn to utilize the textbook, other students, and their own thought processes. Reflection on mathematical concepts was an important part of the learning process which students needed to practice individually in order to be successful. Communicating mathematical ideas, talking with other students and the teacher about mathematical problems, was another necessary ingredient of the students' role in the learning process. Mr. Carpenter included assessment of student knowledge as a feature of the learning process.

The teacher and students should have fun along the way, according to Mr. Carpenter. It was important to him that students remember his classroom as a place they enjoyed being and where they learned something. Not all students would utilize the mathematics he taught, but he felt that all students were impacted by the relationships he established with them. He felt that his flexibility and understanding of students as teenagers were important factors that contributed to the atmosphere of his classroom.

Mr. Carpenter was an advocate of the use of the graphing calculator to teach mathematics. He saw it as both an aid to learning and a tool for doing mathematics. Additionally, he felt that exploring and understanding the use of the graphing calculator provided an opportunity to develop student responsibility for and ownership of their learning.

Consistency between beliefs and practices. There was a high degree of consistency between Mr. Carpenter's beliefs and practices. His many year of experience teaching mathematics was clear in both his ability to articulate his beliefs about teaching and his skill in presenting mathematical concepts in a clear, informal, and precise manner. He relied on his years of experience in lieu of extensive planning of each class session. While the result of this reliance on experience seemed to be a somewhat disorganized classroom, he maintained an underlying structure consistent with his belief in the teacher's responsibility to provide an organization to guide students in the learning process.

The importance Mr. Carpenter placed on the connections within the structure of mathematics and in the teaching of mathematics was clear in his teaching. He often made direct reference to other mathematical concepts which were related to the concept under exploration as well as to other areas of students' knowledge, such as alphabetical ordering, in explaining and developing mathematical ideas. Mr. Carpenter worked to demonstrate the connections to the students, not relying on their ability to make all the connections for themselves.

While Mr. Carpenter provided specific examples and made direct reference to the connections that could be made between the material being studied and students prior knowledge, he also encourage student responsibility in the learning process. Allowing students time to work in small groups in the classroom was consistent with his belief in the importance of communicating mathematics. He believed these opportunities for students to work together encouraged individual responsibility in the learning process as well.

Throughout his teaching, Mr. Carpenter interjected opportunities to have fun, both for himself and for his students. Playing a calculator game and joking about what students who were not working on mathematics were talking about were consistent with his belief in the importance of having fun in the classroom. What he thought was fun, however, may not have been fun for all students because sometimes his jokes were at the expense of individual students. Consistent with his belief in the importance of his understanding of teenagers in creating a positive classroom environment was the "pep talk" he gave students at the beginning of the semester. This talk demonstrated his concern about their achievement and reaching their goals. It also demonstrated an understanding of the ways in which teenagers get distracted from their schoolwork. His concern for students was further demonstrated by his approach to assessment in which he created a structure that maintained high standards for top grades, while allowing all students to experience some level of success.

Summary of Individual Profiles

Using the constant comparative method and triangulation of data sources, the individual profiles of all four teachers were analyzed with a focus on revealing common characteristics in their backgrounds, beliefs, and classroom practices. Differences were also revealed. Relationships among common beliefs and practices were analyzed. The following summaries serve to discuss the similarities and differences found.

Backgrounds

All four teachers had extensive teaching experience. Their combined experience included teaching in schools distributed throughout the United States, both public and private, large and small. While the majority of their experience was in teaching high school mathematics, one had taught mathematics at the junior high school level, one was also teaching at a local community college, and three of them had taught computer programming in prior years. All had taught a wide range of mathematics courses from General Math through PreCalculus. Only Ms. Dancer had never taught Calculus. Ms. Shade and Mr. Carpenter were teaching Calculus at the time of the study. Unlike the others who had begun their careers teaching mathematics, Ms. Dancer had begun as a history teacher but had found she preferred teaching mathematics because students found it more practical and meaningful. In addition to teaching, three of the teachers had also coached although only one of them was coaching at the time of the study. One of the teachers, Mr. Lorenz, had left education for a time and worked in construction.

Only Ms. Shade did not have a Master's degree in education, although the other three teachers had distinctly different types of degrees. Ms. Dancer's held a MAT in history, Mr. Lorenz held an interdisciplinary degree in mathematics, computer science, and education, and Mr. Carpenter's degree was in natural science. Mr. Carpenter had also completed a year of graduate level mathematics coursework. Ms. Shade held a Master's degree in student personnel work and had completed graduate level coursework in education but did not hold an advanced degree. Before pursuing teaching, she had worked in student personnel at a public university. Mr. Lorenz and Ms. Shade had administrative experience in their careers. Ms. Shade had served as Dean of Students and Student Activities. Mr. Lorenz was department coordinator and chair of the Twenty-First Century committee in his school at the time of the study.

The four teachers cited a variety of activities that provided for their ongoing professional development including attending local, regional, and national mathematics conferences; participating in and presenting at workshops; taking part in training programs offered in their school districts; and reading journals. Most importantly, all four teachers emphasized the role of other teachers in their professional development. Mr. Lorenz referred to a college professor who served as a role model for his teaching and to the interaction among teachers in the more open environment created by mathematics reform efforts. Ms. Shade reflected on what she had gained by asking a more experienced teacher for advice when she was struggling with presenting a concept. She also valued the interaction among teachers who participated in the grading of AP exams and the dialogue which occurred among participants at workshops on graphing calculator use. For Ms. Dancer it was the interaction that took place in the common office among the mathematics teachers at her school that she found beneficial to her growth as a teacher. Of particular influence was the teacher who was in a leadership role in NCTM. The superintendent at the first school at which Mr. Carpenter taught had a pivotal role in the formation of his philosophy of assessment. It was interaction with his colleagues that enlivened Mr. Carpenter's teaching and helped him to incorporated the use of graphing calculators. The role other educators played in the professional development of these teachers, while different for each teacher, was highly valued by all of them.

Classroom Practices

All four teachers had a high level of confidence and comfort in the classroom. No discipline problems were observed in any of the classes although it was not uncommon for teachers to remind students to stay on task rather than converse. The amount of structure in the classrooms varied widely from the carefully planned variety of activities in each of Ms. Dancer's classes to Mr. Carpenter's relatively unstructured classroom in which students spent much of their time working at their desks while he circulated and answered questions. Between the two extremes were Ms. Shade who included carefully planned teacher demonstrations and student activities as well as interactive lecture and dialogue presentation of material and Mr. Lorenz whose classes followed a routine while being flexible and responsive to students' needs and questions.

Each of the teachers effectively incorporated student questions into their teaching. Mr. Lorenz utilized student questions as a springboard from which to develop new concepts. For each lesson, he seemed to have an outline of the material to be covered that he moved into in response to specific student questions. Mr. Carpenter tended to use student questions as a motivation for explaining material with which students were involved. He did not have a written lesson plan, rather he relied on his experience and knowledge of both students and the mathematics being taught and allowed student questions to motivate his exposition. Both Ms. Dancer and Ms. Shade utilized extensive written plans and encouraged student questions as a means of determining students' levels of understanding and ability to communicate the mathematical concepts being explored. Ms. Shade asked students to demonstrate their understanding by responding to specific questions, or by asking questions when they realized they did not understand. Ms. Dancer was more direct in asking about student understanding. She often asked students to indicate, by a show of hands, their level of understanding of a specific concept. Except when constrained by a tight schedule, she responded to any indicated lack of understanding by answering additional student questions or presenting additional examples. Ms. Dancer also asked direct questions requiring students to demonstrate their understanding.

Presentation of new material was done using a variety of methods both by individual teachers and by the teachers as a group. While all four teachers utilized examples in their presentation, seldom did they follow the "work examples, make an assignment, allow students time to work" model of teaching. There was a high level of teacher-student interaction in all four classrooms during the presentation of new material. Ms. Shade often utilized an interactive dialogue approach which built upon students' prior knowledge. She asked questions requiring students to recall concepts which she then expanded upon to develop new concepts. Ms. Shade displayed a tendency to repeat a question several times, sometimes rewording it slightly, rather than wait for a student to respond to the original question. Mr. Lorenz also involved students in the presentation of new material by asking them questions, particularly as he reviewed a process or technique he had demonstrated. He was disciplined in waiting for student responses to his questions.

On one occasion he waited a full eight seconds for a response when he asked how students could recognize that a problem involved the distance equals rate times time relationship. Because Mr. Carpenter's exposition of material generally occurred in response to specific student questions, there were usually some students who were acquainted with the material being discussed. While these students might have unresolved questions concerning the material, they were often able to answer the questions Mr. Carpenter asked during his presentation. Mr. Carpenter's style was relaxed, he tended to have a casual give and take relationship with the students, he did not always call on individual students, rather he allowed them to volunteer responses. In spite of his casual style, his teaching was mathematically precise and well-articulated. Ms. Dancer's presentation was the most deliberate and yet she effectively involved students in the development of new material. While she carefully planned the material to be presented, she involved students in hypothesizing about the next step in a process, brainstorming about methods of solving a problem, and explaining the reasons for each step of a procedure. If she felt students were not involved she would call on them by name to ensure their involvement. Mr. Carpenter also employed this practice of calling on students who were not actively engaged in the class.

Teaching by example extended beyond simply demonstrating the procedure required for solving a specific type of problem. These four teachers modeled important aspects of learning and using mathematics in their teaching. Ms. Dancer often articulated her thinking about a problem as she worked it through. When she was developing the matrix method of solving a system of equation, she verbalized the similarity she saw between the matrix form of the equation and the standard form. Her ability to express her thoughts provided students with models of thinking about mathematics and communicating mathematical ideas. In his teaching, Mr. Lorenz modeled the important aspect of checking the correctness of one's work, finding errors, and making corrections. When he observed that he had made an error in the process of solving an linear programming problem, he involved the students in the process of uncovering the error by asking them how he knew there was an error and then working back through the problem to find and correct it. Mr. Carpenter stressed the common pitfalls students might encounter in solving problems. By inadvertently making a common error in the process of solving a problem on the board he was able to emphasize the importance of checking through one's work and watching out for the common, easily committed errors.

The practice of connecting the material being studied with students' knowledge and experiences was utilized by all four teachers. Connections were made in different ways and to different facets of students' experiences. Mr. Lorenz stressed the connections between material being presented and concepts previously studied or just explored in a starter activity. He also emphasized the importance of understanding a process being used so that it could be connected to or adapted to fit other situations. Focusing on the connections or applications of mathematical concepts to real-life situations, problems, and experiences was also a feature of Mr. Lorenz's teaching. Ms. Shade built her presentations of material on students prior learning, thus illustrating the connections that could be made between the new concepts and prior knowledge. She focused her teaching on extending students mathematical understanding by assisting them in making the connections required between new concepts and prior learning. Making these connections between new concepts and prior learning was an essential element of Ms. Dancer's teaching as well. One of the ways she succeeded in making connections was by emphasizing the similarity between a new process for solving a problem being presented and a process which had been previously mastered. Recalling the image of an equation as a seesaw which needed to be kept in balance by performing the same operation on both sides of the equation, Ms. Dancer worked through finding the solution to a matrix equation following the same procedures that students knew how to employ in solving an algebraic equation in one variable. Another area in which Ms. Dancer made connections was between students' understanding of the English usage of a term and the mathematical meaning of the term. In discussing the concept of a determinant, she referred several times to the root word, determine, and wondered what a determinant might be determining. This comparison between the mathematical concept of a determinant and the English usage emphasized by Ms. Dancer provided students with a connection between the mathematical concept and an already held conceptual understanding. Mr. Carpenter utilized many of the same types of connections employed by the other teachers. When exploring the concepts of domain and range he made a connection from the alphabetical ordering of the terms domain and range to the alphabetical ordering of the most commonly encountered variables, x and y, to which domain and range correspond. Providing students with the alphabetical order paradigm for recalling the correspondence between x and y variables and domain and range provided students a connection between a mathematical concept and their prior knowledge of alphabetical ordering. Mr. Carpenter's use of the function machine and the corresponding toy making machine in presenting the concept of a function served as a connection to a real-world, physical concept which students could visualize and understand. In this way, he made a connection to a physical concept to enhance understanding. The connections Mr. Carpenter made extended both forward and backwards in time. He recalled concepts students had explored in previous classes and connected new ideas to these concepts thus expanding student understanding. He also emphasized the importance of mastering a concept currently being studied because of the importance it would play in future mathematics courses, thus making connections forward in time. All four teachers emphasized the connections they were making by stressing the importance of understanding and building upon the relationships between the material being studied and that which students had already learned or would learn in the future.

Mr. Lorenz and Mr. Carpenter interacted regularly with students during individual work time. Both teachers circulated through the room as students worked on assigned problems. During these work periods, students often interacted with one another and asked questions of their teacher. Mr. Lorenz generally responded to student's questions by moving to the student's desk and talking quietly with the student. As Mr. Carpenter circulated through the room he would often stop at a student's desk and ask questions about what he saw, effectively checking the student's level of understanding. When a student asked a specific question or Mr. Carpenter found that a student was not able to respond to a question he asked, he would go to the board with the student where he would work through the problem, involving the student in the process. Neither Mr. Lorenz nor Mr. Carpenter spent much time sitting at the teacher's desk when students were working individually.

While there was not much individual work time in Ms. Shade's and Ms. Dancer's classrooms, they both incorporated group work into their teaching. Students worked on solving problems in pairs either at the board or at their desks in Ms. Shade's class. During this time, she circulated, checking on the progress being made by each pair, asking for explanations of what they were doing, and responding to their questions. She made a concentrated effort to interact with each group in order to assess all students' work and understanding on an informal basis. Discovery learning, working on types of problems they had not previously encountered, was also implemented by Ms. Shade using partners. Interdependence was a very important feature of the group work that took place in Ms. Dancer's classroom. All the group work activities that she used required students to share what they did with their partner or partners. Ms. Dancer was available to answer questions, but she served as a resource for the groups, requiring the students to explore and solve the problems under investigation. While she did some explaining when it was needed, the members of the groups primarily depended on each other for the answers to their questions. Students also completed a portion of one exam, the solution of a linear programming problem, working with a partner in Ms. Dancer's class. Mr. Lorenz incorporated group work in a greater number of ways than the others. In addition to working on specific problems in small groups, the groups presented their solutions to the problems to the class, prepared a formal write-up of a problem, and completed an entire quiz. For the group presentation and formal write-up of the solution to a problem, the groups were given time during class to work on the problem and expected to complete the work outside of class. Each group was responsible for a single presentation and write-up. Individual members shared the presentation of the problem by dividing the task into distinct pieces including writing the problem on the board, discussing the solution, and answering questions. The design of the quiz required members of the group to work independently on the completion of at least one problem while depending on other members of the group for solutions to the remainder of the problems. By not allowing sufficient time for individual students to complete all the problems, Mr. Lorenz included both individual accountability and interdependence in the use of cooperative groups.

All four teachers shared a deep level of concern for their students both as students and as individuals. Communication of expectations was one way this concern was demonstrated. The use of printed schedules by all four teachers provided an outline of the material to be covered each day. Students were expected to read the indicated sections in their textbook and complete the assigned problems on a daily basis. Only Ms. Shade collected assignments daily. Mr. Lorenz and Mr. Carpenter required students to turn in all their completed work at the time of the unit test. Ms. Dancer collected assignments throughout each unit, but not necessarily every day. In spite of the timing with which they collected student work, all four teachers emphasized the responsibility of each student to complete and understand each assignment. Mr. Lorenz reviewed assignments daily and encouraged students to correct and review their work to promote understanding. In promoting individual responsibility, the teachers acknowledge the tension between allowing students freedom to develop and demonstrate responsibility and providing sufficient structure to guide their learning.

Each of the teachers worked to create a positive environment in the classroom. Mr. Lorenz greeted students at the door and showed interest in their extracurricular activities. Ms. Dancer encouraged students to perform well and expressed confidence in their abilities to accomplish the tasks presented in each class. Mr. Carpenter gave pep talks encouraging students to set and meet goals. He also displayed a playful attitude, teasing about conversations he overheard and encouraging a friendly competition among the students. Ms. Shade made arrangements to work with students outside of class time to assure their success and worked to develop a positive relationship with each student which was possible because of the relatively small sizes of her classes. These four teachers were committed to providing excellence in teaching and a caring environment.

Use of Graphing Calculators in Teaching

The teachers all taught in schools which incorporated the use of graphing calculators throughout the mathematics curriculum, from second year algebra through Calculus. All the textbooks being utilized supported the use of graphing technology although none depended upon its use. Only Ms. Shade had been the sole catalyst for the use of graphing calculators in her school. She had become interested in their use when students began bringing graphing calculators to school. Ms. Shade had persuaded the school administration to allow her to incorporate graphing calculator into her teaching, first in Calculus and then throughout the curriculum. The use of graphing calculators had been effective in revitalizing her teaching of calculus. For Mr. Carpenter the use of graphing calculators had been an outgrowth of his involvement with computer programming and had been facilitated by participation in piloting an early version of a graphing calculator oriented Precalculus text. His expertise with the use of graphing calculators was facilitated by involvement with a group of colleagues who enjoyed working together to investigate the incorporation of graphing calculators in their teaching. Mr. Lorenz's introduction to graphing calculators had been through a district inservice conducted by a teacher from another school in the district. He found graphing calculators to be a natural progression in the development of tools for teaching and doing mathematics. Ms. Dancer was part of a mathematics department that adopted the use of the graphing calculators. Her background in teaching computer use had enabled her to easily make the transition to the use of graphing calculators. Ms. Dancer was committed to the use of the Chicago series of textbooks which encouraged the use of graphing calculators and had served on panels sponsored by the publishers to present the textbooks to other teachers. All four teachers were enthusiastic about and committed to the use of graphing calculators in their teaching. They had continued using the graphing calculators in their teaching, finding new ways to utilize them and making them an integral part of their teaching, not an add-on imposed from outside.

While all teachers utilized graphing calculators in their teaching and encouraged their students to use them extensively, there were substantial differences in the ways in which the graphing calculators were incorporated. Graphing calculators were fully integrated into Mr. Carpenter's teaching, including having easy access to the overhead display unit at any time during a class session by simply pulling down the screen, plugging in the calculator, and turning on the overhead. For the other three teachers, utilizing the overhead display unit required taking time, either before or during class, to remove the display unit from its carrying case and situate it on the overhead before it could be utilized. Possibly because of the ease with which it could be used, Mr. Carpenter utilized the overhead display unit more frequently than the other teachers. Further, he utilized the display in a greater variety of ways. While all four teachers used the graphing calculator to display and explore the graphs of functions, particularly for finding the solution to system of equations, Mr. Carpenter also demonstrated the use of programs. He used a program he had written to play a game with the class which reinforced the concept of the rule for a function. On another occasion, he demonstrated programs written by students to analyze quadratic functions. The other teachers did not utilize graphing calculator programs during the systems of equations unit. Ms. Shade did provide students with a program for analyzing quadratic functions when she reached the unit on functions that was observed informally.

In addition to utilizing the graphing calculators for displaying graphs of functions, they were utilized for computations by all teachers. Ms. Shade and Ms. Dancer made extensive use of the matrix features of the graphing calculator when teaching the matrix method for the solution of a system of equations. Both teachers emphasized conceptual understanding of the processes being performed by the calculator when the matrix features were utilized by first demonstrating the method algebraically. They then encouraged students to use the graphing calculator to perform the matrix operations, thus emphasizing the power of the graphing calculator for performing complex computations quickly and accurately. When Mr. Lorenz introduced the use of the graphing calculator to solve systems of equations utilizing matrices, he simply instructed students to follow a series of steps that produced the correct solution. Because the unit on matrices followed the unit on systems of equations in the text, the students in Mr. Lorenz's class did not have the background to understand the process, he did not explain how the calculator was producing the results, only encouraged students to utilize the calculator to find the solutions. Unfortunately, this approach did not provide students with sufficient information to understand the result produced when the system was dependent or inconsistent. In these cases, the graphing calculator produced an error message. Mr.

Lorenz's solution to the dilemma was to encourage students to solve the system without the use of the graphing calculator in order to determine how they should interpret the error message. While Mr. Lorenz's incorporation of the matrix features and SIMULT for the solution of a system of equations did not include a conceptual underpinning, it did demonstrate his use of the graphing calculator as a tool to do mathematics. His encouragement of the students to explore the meaning of the error statement they encountered when attempting to find the solution for a dependent or inconsistent system demonstrated another important characteristic demonstrated by all the teachers, students' ability to explore mathematics using the graphing calculator and learn from their explorations without the teachers' direct instruction.

Encouraging students to explore the power of the graphing calculator to do mathematics and to understand the results it produced was demonstrated by all four teachers. Ms. Shade and Ms. Dancer both offered students extra credit for exploring the use of the CALCULATE feature to find the point of intersection of the graphs of two equations. However, their motivations were different. Ms. Shade had demonstrated the use of ZOOM and TRACE to approximate the coordinates of the point of intersection. She then offered extra credit to students who could find another method for finding the point of intersection from the displayed graph. In this way, she encouraged students to explore on their own and provided an opportunity for students to learn from one another rather than from her. For Ms. Dancer the motivation was different. A student inquired in class about the use of CALCULATE to find the point of intersection that had just been found using TRACE and ZOOM. Ms. Dancer admitted to not being acquainted with the CALCULATE feature and offered extra credit to the first student who came prepared to explain the feature to the class. Through this episode, Ms. Dancer demonstrated that she did not hold all the information, that students were able to learn independently, and that students could teach one another. In addition to encouraging students to share programs they had written or loaded into their calculators without his assistance, Mr. Carpenter accepted input from students when demonstrating a new use for the graphing calculator to the class. He was attempting to demonstrate the DRAW INVERSE feature of the calculator, but was having difficulty finding the correct menu when a student volunteered his expertise. This student had explored the DRAW INVERSE feature independently and knew exactly the correct sequence of keystrokes needed to produce the result for which Mr. Carpenter was searching. Not only did Mr. Carpenter accept the student's direction, he showed appreciation for the student's expertise and willingness to share.

The importance of teaching students to use the features of the graphing calculator was apparent in all four classrooms. Several techniques were employed by the teachers when the focus was on teaching students how to use a feature of the calculator. When teaching students how to find the solution of a system of equations from the graph, Ms. Dancer and Ms. Shade checked students' facility with producing the graphs, then demonstrated, using the overhead display, the use of the ZOOM and TRACE features. Mr. Carpenter also used the demonstration approach when introducing students to the use of the SIMULTANEOUS and SEQUENTIAL modes for display of graphs and the DRAW INVERSE feature. When Mr. Lorenz introduced students to the use of the SHADE feature, it was by displaying the solutions to a homework assignment utilizing SHADE and the overhead display unit. He then led the class through the use of SHADE by dictating the step-by-step procedure he was utilizing on the overhead display. The dictation of step-by-step instructions was utilized without the support of the overhead display unit when Ms. Shade and Ms. Dancer led their classes through the graphing calculator procedure for entering a system of equations in matrix form and finding the solution using matrix operations. Ms. Dancer augmented her instructions by pointing out the location of unfamiliar keys on the poster of the graphing calculator displayed on the bulletin board in her classroom. Mr. Carpenter generally relied on the use of the overhead display for instruction on the use of the graphing calculator. He used verbal instructions occasionally, such as when he emphasized the importance of correct placement of parentheses in the entry of a rational expression into the calculator for evaluation.

The greatest divergence in use of the graphing calculator occurred in the teaching of the solution of linear programming problems. Ms. Shade made extensive use of the graphing calculators for solving these problems. During the initial exploration of linear programming, a discovery type activity, she encouraged students to use their calculators to produce the graph of the feasible region and find the coordinates of the vertices. She reminded students of the power of the graphing calculator to perform these tasks accurately and quickly. In the following class sessions, she utilized overheads of the solutions to linear programming problems that she had produced using the graphing calculator and the TI-graph link. She had graphed and labeled the feasible region using the graphing calculator, then downloaded the graph to the computer via the TI-graph link. She was then able to print the graph and create an overhead for class use. Mr. Lorenz made an attempt to utilize the graphing calculator for solving linear programming problems. He did produce graphs of the feasible region and encourage students to do the same, but stopped short of finding the coordinates of the vertices. Rather, he instructed students to utilize other methods of solution for finding these coordinates. Ms. Dancer made no use of the graphing calculator in her teaching of linear programming.

The importance of understanding the results produced by the graphing calculator was emphasized by all the teachers. Mr. Lorenz encouraged students to utilize their graphing calculators when working on assignments and tests, but required them to provide explanations of their answers. The importance of knowing the basic shape of the graph for a function was emphasized by Mr. Carpenter as he utilized the graphing calculator to display and analyze the specific features of the graph in order to detect possible entry errors. If the graph produced was not what the student expected, they could check their work, if they did not recognize that the graph was incorrect, their error would go undetected. Similarly, Mr. Carpenter emphasized the value of the calculator for checking work that was done with paper and pencil. When discussing the results of the semester exam, he pointed out that several students made errors that they would have detected if they had entered the information into their graphing calculators and checked their work. Ms. Shade also encouraged students to use their graphing calculators to check their work. She noted that if the instructions indicated that a problem should be solved without the use of the graphing calculator, it was acceptable to check the solution obtained by using the graphing calculator to solve the problem. As indicated in their comments about checking work and providing explanations, all teachers allowed the use of graphing calculators on tests and quizzes. There were some problems which required the use of the graphing calculator, some problems which did not allow the use of the graphing calculator, and some problems on which the graphing calculator could be utilized, but would not provide a complete solution.

Emphasis on the power of the graphing calculator to perform complicated computations and produce graphs of functions quickly was accompanied by attention to the limitations of the graphing calculator. When discussing a graph presented on a quiz, Mr. Carpenter reminded the students of the assumptions they made about what happened beyond the boundaries of the window of their graphing calculator. While they understood that the graph did not end at the edge of the window, he emphasized the importance of properly conveying that information when they recorded their findings on paper. Ms. Dancer discussed the limitations of the graphing calculator when it came to finding the point of intersection of two lines using the graph. The concept of pixel size and the inability to find a precise answer, even by repeatedly zooming in, were noted. Further, when utilizing the graphing calculator's matrix features to find the solution to a system of equations, Ms. Dancer emphasized the usefulness of the tool as well as the importance of understanding the process. The graphing calculator could assist the student in finding the numerical answer to a problem. Finding the numerical answer was, however, not sufficient. Ms. Dancer required that students also be able to interpret their answers appropriately. Thus, the graphing calculator was a useful tool, but not sufficient, by itself, for doing mathematics.

One other issue arose in the classrooms related to the use of graphing calculators. Both Mr. Lorenz and Ms. Dancer encountered occasions when questions arose, either about a feature on the graphing calculator or a model of graphing calculator, that they did not have the experience to answer. Both teachers were able to handle these questions appropriately, either by offering to spend additional time, outside of class, to assist the student or by encouraging students to explore and present their findings. None the less, these situations did create anxiety for the teachers. Neither Ms. Shade not Mr. Carpenter experienced these types of situations. All students in Ms. Shade's class were required to use the same type of calculator and she was well experienced in its use. In Mr. Carpenter's class, students utilized a variety of different models, but they were all TI's and he had sufficient experience to be able to respond to their questions without difficulty.

Beliefs About Mathematics and Teaching Mathematics

Among these four teachers there was a shared understanding of the structure of mathematics. Mr. Lorenz referred to the beauty of mathematics evident in its structure as something that students needed to discover for themselves. In Mr. Carpenter's view, mathematics was a hierarchical structure built on connections between simpler and more complex concepts and between different areas of mathematics such as algebra and geometry. As Ms. Shade described it, the structure of mathematics was in the connections between concepts as well as in understanding how and where concepts fit. Ms. Dancer added that reasoning and thinking skills were required to connect the skills and concepts that made up mathematics. In addition to the abstract structure and beauty of mathematics, all four teachers viewed mathematics as a tool for solving problems. People had constructed mathematics as a way to figure things out and explain the physical universe according to Mr. Carpenter. In Ms. Dancer words, mathematics provided "an ordering of our world, a way to explain the processes and order the processes that go on around us." Mr. Lorenz described mathematics as a language and tool to understand the world. For Ms. Shade, mathematics allowed us to do things in the world. The notion of mathematics as a duality of the structure and beauty of interconnected concepts together with its usefulness as a tool to understand and explain the world, although expressed in different ways, was an understanding commonly held by the four teachers.

Algebra fit into the structure of mathematics by providing a foundation of tools and thinking skills. Algebra was described as the foundation for the study of higher mathematics. Mr. Carpenter noted that you had to start someplace. Ms. Shade saw that algebra developed skills you needed, both procedural and thinking skills. Mr. Lorenz cited the importance of algebra as the language of higher mathematics, while Ms. Dancer emphasized the importance of algebra as a tool to explain relationships in symbolic equations. While emphasizing different aspects of the study of algebra, the four teachers' views of the importance of algebra to the structure of mathematics were compatible.

Ms. Dancer described the teaching and learning of mathematics as a continuum with the teacher and learner as active participants. While none of the other teachers were able to express their views as cogently as Ms. Dancer, to a large extent they shared similar views on the interaction required between teacher and students in the process. Each described the teacher's role in the process. Providing well-developed, clearly explained examples was the teacher's responsibility in Mr. Carpenter's description. Including examples of where algebra was useful for solving problems outside of the mathematics classroom was a role of the teacher in Mr. Lorenz's view. Developing key concepts to which students could attach meaning was required according to Ms. Dancer who saw hard work and planning as essential for excellence as a teacher. Ms. Shade saw the teacher's role as that of a guide who communicated expectations, demonstrated how mathematics worked, displayed the thought processes used, made connections between the concepts under investigation, and provided multiple perspectives. Utilizing a variety of perspectives in presenting mathematical concepts including graphical, numerical, and symbolic was important to Mr. Carpenter. For Mr. Lorenz, providing students with a variety of approaches for solving a single problem offered additional opportunities to engage students in the learning process. Knowledge of individual students and their varied learning styles contributed to Ms. Shade's description of a teacher. While Mr. Lorenz felt that there were times when it was important for the teacher to supply expert information, the classroom should be student-centered with the teacher acting as a guide and resource but not the supplier of all information. In Mr. Carpenter's view, the teacher's role in guiding the learning process included establishing goals and providing an organization for the class. Ms. Dancer believed that the teacher needed to provide a variety of activities and approaches in order to engage students in the process and enable them to make connections. Helping students to build connections to their prior knowledge was the essence of the teacher's role for Mr. Carpenter. Providing a learning environment in which students were actively engaged in exploring mathematical concepts with emphasis on utilizing multiple perspectives and activities, demonstrating multiple approaches to solving problems, presenting clear examples, and establishing clear goals and expectations without becoming the sole authority and possessor of all knowledge was the role of the teacher in the learning process according to these teachers.

Students also had a role in the learning process. The most consistent aspect of the students' role in learning was doing. All four teachers included the importance of doing mathematics in order to learn mathematics. Ms. Dancer described students' role in the learning process as one of active engagement, both mentally and through active participation. This active participation included thinking about mathematics and participating in exploration, discussion, and practice. Mr. Carpenter felt that students could not truly learn mathematics by simply watching, rather they had to do the mathematics in order to learn it. In his view, students needed to accept individual responsibility for their learning, not relying solely on the teacher, but utilizing the textbook, other students, and their own thinking about the mathematics they were studying. Mr. Lorenz added that individual responsibility in the learning process included attending class, being prepared, and putting forth individual effort. Student-to-student interaction was valuable to students in the learning process. Ms. Dancer believed that students needed to see good models in order to learn, but they had to put the pieces together for themselves. She felt that students needed to actually do the work and that they needed validation of their work, immediate feedback, in order to cement the learning. In addition to doing mathematics, which she saw as essential, Ms. Shade felt that students needed to communicate their understanding of the concepts being explored which they could do through asking questions, providing explanations, and working in small groups. For these teachers, learning was not memorizing the process to be used to solve each type of problem, rather it involved making the connections between concepts, building layers in their understanding of mathematics, and attaching meaning to the processes they were learning. This required individual responsibility and could be fostered through encouraging students to communicate their understanding, practice what they were learning, and reflect on the concepts.

The environment in the classroom was also important in fostering the learning process. Ms. Dancer believed that mutual respect between teacher and students was an essential ingredient. Mr. Lorenz felt that students learned better when they believed that the teacher could relate to their world and was interested in their activities outside the classroom. Mr. Carpenter strove to create a classroom environment that students would look back on and remember as being a fun place to be and a place in which they learned something. Ms. Shade acknowledged the importance of assisting her students in developing confidence and valued her small class sizes which allowed her to know her students. All four teachers valued students as individual teenagers and recognized the complexity of their lives beyond the classroom.

Summary of Relationships

Consistency Between Teachers' Beliefs and Practices

The most striking consistency between the beliefs and practices of these teachers was the importance they placed on making connections. In their teaching, they verbalized and demonstrated the connections they were making between the concepts under discussion and concepts previously explored. When discussing their beliefs concerning mathematics, its teaching, and student learning, they emphasized the importance of connections, from the nature of mathematics as a structured system of connected concepts, to the importance of teachers providing means for students to connect new concepts to prior learning, to the notion of students learning by making connections for themselves between their prior understanding and new information. Recognizing, exploring, discussing, and developing connections between concepts was basic to the beliefs and practices of all four teachers.

There was a tension between teachers' beliefs in the importance of individual student responsibility for learning and the role of the teacher to guide the learning process. For Mr. Carpenter, the importance of individual responsibility was reflected in the large amount of time he allowed for individual work time in the classroom. In contrast and in opposition to his belief in and practice of allowing students to work individually was his belief in the importance of providing organization for student learning. He tended to rely on the provision of a unit schedule and student questions to guide the learning process.

His high level of mathematical understanding and ability to explain clearly and concisely contributed to his ability to create a positive learning environment.

While Ms. Shade espoused a belief in the importance of reflection on mathematical concepts in the building of the connections essential for understanding and learning, in practice she often did not allow students the opportunity for this reflection. When posing questions she had a tendency to repeat or reword a question, effectively interrupting students thoughts. Perhaps she believed that the importance of engaging students in the process by eliciting responses overrode the necessity of time for reflection. She also struggled with letting go of the learning process and allowing students to assume responsibility for their learning. The activity she provided for students to explore a linear programming problem was an example of her attempt to move from her comfortable role of dispenser of knowledge to more of a guiding role.

Mr. Lorenz shared the concern about the role of the teacher as a guide rather than a director of learning. While he espoused a belief in student responsibility and desired a student-centered classroom, he found when reflecting on his teaching practices, that he was spending a good deal of time in the expert role and less time in the role of a guide. While Mr. Lorenz assessment of the amount of teacher activity that took place in his classroom was accurate, the motivation for the activity also needed to be analyzed. A student-centered classroom requires a great deal of teacher activity Much of the teacher activity observed in Mr. Lorenz classroom was in fact in response to students. Mr. Lorenz may have been under the assumption that student-directed learning requires less teacher activity than does teacher-directed learning.

In spite of a few areas of divergence between beliefs and practices, these four, experienced teachers showed a strong degree of consistency between their beliefs and practices. Ms. Dancer whose beliefs were the most succinctly articulated also showed the greatest deal of consistency. Perhaps this consistency existed because she had spent more time than the others reflecting on her practices and beliefs. One indication of the reflection Ms. Dancer had done was the organization she utilized in sorting statements during the belief clustering interview. While the other three teachers sorted the cards into piles and discussed their reasons for placing cards together, Ms. Dancer established a relationships between the groups of cards. Her organization of the cards showed the importance she placed on understanding the relationships between her practices and her beliefs.

Consistency Between Teachers' Practices and Constructivist Approach To Teaching

Constructivist theory holds that all knowledge is constructed by the individual. In order for students to construct knowledge they must develop cognitive structures that can be activated and revised to construct new knowledge. Purposive activity induces transformation of students' existing cognitive structures (Noddings, 1990). The teachers involved in this study engaged in purposive activity designed to actively engage students in the learning process. While they may not have described their teaching as designed to transform students cognitive structure, they all recognized the importance of structure, both in mathematics and in their teaching.

Confrey (1990) further describes the constructivist approach to student learning wherein students must learn to construct powerful ideas in which the student believes, which have internal consistency, are in agreement with experts, can be reflected on and described, act as a foundation for further constructions, and can be justified and defended. Each of the teachers in this study engaged in practices consistent with some or all of these principles. Their utilization of student questions and questioning contributed to the construction of ideas which the student could believe and which had internal consistency. By responding to student questions, the students developing structures could be evaluated, verified, and augmented. Ms. Dancer practice of providing validation, immediate feedback, of student work also contributed to students' abilities to construct reliable structures. Providing examples in their teaching was a practice which enabled students to construct ideas which were in agreement with experts. As Mr. Lorenz noted, students sometimes needed to be provided with expert information.

These teachers' view of algebra as the foundation for the study of higher mathematics was consistent with their practices of making connections between the concepts being explored and, as Mr. Carpenter demonstrated, foreshadowing of concepts to be explored in the future. The connections emphasized by the teachers enabled students to construct ideas which could serve as the foundation for further constructions. By repeatedly making connections to prior learning and among the concepts being learned, these teachers embodied the essential constructivist theory of the construction of knowledge.

Concept development was predominantly led by the teachers in these classrooms. Mr. Carpenter depended on students ability to read the textbook, attempt the assignments, and ask questions to initiate exploration of concepts. Once questions about a concept had arisen, Mr. Carpenter then assumed the role of developing the concept. In spite of this teacher led development process, students were actively involved, primarily through questions and responses but also through brainstorming and hypothesizing in Ms. Dancer's class and in providing explanations in Mr. Lorenz's and Ms. Shade's classes. The use of questions by the teachers encouraged students to reflect on their developing constructs, describe what they were thinking, and defend the their conclusions. All of these practices are consistent with constructivist theory.

While the level of teacher-student interaction was high in these classrooms, there was little opportunity for true investigation and exploration. Ms. Dancer and Ms. Shade did provide an occasional activity which fostered exploration and all four teachers encouraged students to explore the use of the graphing calculator on their own. Apart from these few opportunities for mathematical exploration, it was only the brainstorming and hypothesizing in Ms. Dancer presentations that fostered individual student exploration and investigation.

Individual responsibility for learning was encouraged by these teachers. However, they all exerted a level of control and direction over the process which they believed was essential in the learning process. They recognized that students needed to take responsibility, that they could not be passive learners simply absorbing information from an expert. The teachers, as a group, felt that it was there responsibility to direct the learning process by providing structure and goals for their students. When an external structure is imposed, it can be argued that true autonomy is not possible. The importance these teachers placed on creating a structured environment in which students could learn may have taken away from the development of full autonomy for the students.

Consistency Between Teachers' Use of Graphing Calculators and Goals of Reform

The consistency between the teachers' use of graphing calculators in their teaching and the goals of reform varied as the teachers' goals for the integration of the technology. Mr. Lorenz tended to see the use of graphing calculators in second year algebra as an opportunity to teach students how to use the calculators so that they would be well prepared to use them effectively when the reached higher level courses. His focus on teaching students to use the graphing calculator may be why the use in his classroom was centered around displaying graphs and performing computations. He emphasized the power of the graphing calculator as a tool to do mathematics and spent little time using it to explore mathematics. In contrast, Mr. Carpenter fully integrated the graphing calculator into his teaching using it as both a tool to do mathematics and an vehicle for exploring mathematical concepts. His use of the program for guessing the rule of a function and the exploration of the DRAW INVERSE feature were examples of utilizing the graphing calculator for exploration and concept development.

Boyd, Ross, and DeMarios (1993) indicated that one of the potential benefits of using the graphing calculator in the classroom was to move the teacher to a position of facilitator of learning rather than source of knowledge. In encouraging students to investigate and report on the use of CALCULATE to find the point of intersection of two lines, Ms. Shade and Ms. Dancer embodied the facilitator role. Mr. Lorenz also acted in the role of facilitator when he worked with students who had different models of graphing calculators. He acknowledged that he did not have all the answers but that he would assist students in finding answers to their questions.

The only example of utilizing the graphing calculator for true exploration of mathematical concepts occurred in Ms. Dancer class when students engaged in a cooperative group activity. Ms. Shade encouraged students to utilize their graphing calculators when investigating linear programming problems, but she introduced the ideas they would be exploring and discussed how they could use the graphing calculators to find the required solutions. Thus, she did not allow true exploration.

The connection between graphical and algebraic representations was thoroughly developed utilizing graphing calculators in these classrooms. For the three teachers who were teaching the unit on systems of equations, the connection between the graphical representation of the solution for a system of equation, the point of intersection, and the algebraic representation, the values for the variables which satisfied both equations, was emphasized and explored with the use of the graphing calculator. Mr. Carpenter was teaching a unit on functions in which he made extensive use of the graphing calculator to display the graphical representations of the algebraic rules for the functions. He incorporated symbolic, graphical, and numeric representations throughout his teaching and utilized the graphing calculator for emphasis.

Perhaps it is because of the level of the class, second year algebra, but the degree to which graphing calculators were utilized to explore and develop concepts, although present, was minimal. Even in Mr. Carpenter's classroom where their use was fully integrated, there was little investigation done by students using their graphing calculators.

CHAPTER FIVE
DISCUSSION AND IMPLICATIONS

Introduction

This study investigated the classroom practices and beliefs about mathematics and the teaching and learning of mathematics among high school teachers who have persisted in the use of graphing calculators in the teaching of second year algebra. The relationships between the practices and beliefs of these teachers were also examined, as were the relationships between the practices of these teachers and the constructivist approach to mathematics teaching and the goals for reform in mathematics education, especially as these goals pertain to the use of technology in the classroom. The beliefs and classroom practices of the teachers as well as the relationships under investigation were described in Chapter IV. In this chapter the classroom practices and beliefs of these teachers are discussed briefly in order to establish a framework for the discussion of the relationships.

Previous studies have found inconsistencies between teacher's beliefs and practices. Inconsistencies between beliefs and practices have been theorized to be related to reflection on beliefs and practices by the teacher (Thompson, 1984). Few inconsistencies were found between beliefs and practices of the teachers in this study. Possible explanations for the level of congruence between beliefs and practices are explored in the discussion of the relationships between the beliefs and practices of these teachers who have persisted in the use of graphing technology in the teaching of second year algebra.

Relationships between the use of graphing calculators by the teachers in this study and the visions for reform made possible by the incorporation of this technology are discussed. The degree to which the teachers' practices were consistent with the visions for reform and possible reasons for inconsistencies are examined. The goals of the current reform movement, including the incorporation of graphing calculators, are rooted in the constructivist approach to teaching. Consistencies between the classroom practices of the teachers in this study and the constructivist approach are discussed.

The chapter concludes with discussions of limitations of the study, implications, and recommendations for further research. The ways in which the limitations of the study, especially sample size and selection, affected the findings are discussed. The discussion of implications focuses on the role of teacher-to-teacher interactions and their effects on teachers' beliefs and practices. Finally, recommendations for further research are presented.

Classroom Practices

Ernest (1989) presented six simplified models of mathematics teaching based on the types and ranges of teaching actions and classroom activities found in prototypical mathematics classrooms. These six models were: (1) the pure investigation, problem posing, and problem solving model, (2) the conceptual understanding enriched with problem solving model, (3) the conceptual understanding model, (4) the mastery of skills and facts with a conceptual understanding model, (5) the mastery of skills model, and (6) the day to day survival model. These models spanned a continuum of practices from an approach in which teaching is based simply on following a text or scheme, versus an approach in which the teacher supplements or enriches the textbook with additional problems and activities, versus an approach in which the teacher constructs virtually all of the mathematics curriculum materials. When the practices of the teachers in this study were examined, none of the teachers' practices fit neatly into any one of the models described by Ernest.

Three of Ernest's models included conceptual understanding. The focus of the practices in the classrooms of the four teachers in this study was conceptual understanding. A wide variety of teaching activities were utilized to promote this conceptual understanding. The presentation of examples, while serving to demonstrate skills and techniques, included extensive discussion of the underlying concepts and principles involved. The interactive dialogue utilized while presenting examples, with the teachers asking and responding to student questions, further enhanced the promotion of conceptual understanding.

While all four teachers focused on developing conceptual understanding and fostered interaction between the students and teacher, the structure employed by the four teachers varied widely. The study revealed a continuum of structures for the use of class time. At one end of the continuum was carefully planned and organized use of class time with specific examples and learning activities designed to explore specific concepts. The other end of the continuum was a loosely structured use of class time with prescribed content to be addressed but with examples arising from student questions and planned activities limited to periodic worksheets designed to explore specific concepts. While the textbook served as a guide for content and concepts to be explored and a source for assigned work, the teachers relied on their experience, understanding of the concepts, and their students' level of understanding to direct their teaching activities. In addition to relying heavily on teaching materials provided by the textbook publishers, these teachers also created their own materials. The materials created by the teachers were used to augment the textbook materials, emphasize specific aspects of a topic, and provide students with additional exposure to the concepts.

In addition to the focus on conceptual understanding permeating their classroom practices, these teachers consistently emphasized the connections that could be made between the concepts, techniques, and skills being explored and students' prior knowledge. Connections were made between a new technique being presented and a technique previously mastered, between a concept being applied in a new situation and a situation in which the concept had previously been applied, and between a new concept being presented and concepts that the students understood upon which the new concept was developed. Connections were also made to students' experiences outside the mathematics classroom.

While conceptual understanding was central to the classroom practices of these teachers, their classroom practices also emphasized problem solving and thinking skills and the mastery of the facts and skills of algebra. These teachers tended to act more in the role of instructor than of facilitator, though they all acted as facilitator at some times, especially when students were working individually or in small groups. When acting as an instructor, the teachers did not impart information to passive learners, rather they involved students in the development of the concepts. The teachers, however, directed the process, thus serving as instructor more than facilitator. Communication of mathematical ideas was valued in these classrooms as seen in the promotion of interaction between students, the eliciting of student explanations, the valuing of student questions, and the exposition of mathematical concepts verbally as well as symbolically, graphically, and numerically.

Just as Ernest's six models of mathematics teaching spanned a continuum, the practices of these four teachers spanned a continuum. The classroom practices of these four teachers did not place each teacher at a position on the continuum. Rather, the practices of each teacher spanned the continuum of Ernest's model from the conceptual understanding enriched with problem solving, through the conceptual understanding model, to the mastery of skills and facts with conceptual understanding model. Depending on the concept or technique being discussed, each teacher shifted back and forth through the continuum.

Use of Graphing Calculators in Teaching

Farrell (1989) found a slight shift in activity with more time spent on exercise, consolidation, practice, and investigation and less time spent on exposition when technology was in use than when it was not. This shift in activity, although not measured quantitatively, was not found in this study. The teachers expected students to use their graphing calculators for all facets of the study of second year algebra, including exercise, practice, and investigation, but the majority of in-class time was spent on exposition. Teachers spent time demonstrating how to use the graphing calculator in the solution of specific types of problems, how to use the features of the graphing calculator in new and different ways, and how to use features of the graphing calculator that were new to the students. Only on a few occasions did teachers provide opportunities for students to utilize their graphing calculators in investigation. The limited amount of time spent using graphing calculators for investigation was consistent with the limited amount of exploratory activities found in these classrooms. It did not appear that the use of graphing calculators increased the amount of time spent on investigation. The teachers did encourage students to explore and investigate outside of the classroom by providing opportunities for them to share the findings of their independent investigations with their classmates.

Farrell (1989) also found evidence that teachers roles shifted to the role of consultant rather than task setter and explainer when using graphing calculators in the teaching of precalculus. When teachers in this study utilized the graphing calculators for investigation, their roles shifted to that of consultant and advisor rather than explainer, although they maintained the role of task setter as they provided instructions for the investigations. However, since most of the time spent utilizing the graphing calculator was spent in demonstrating its use, teachers remained in the role of explainer the majority of the time.

Beliefs about Mathematics and the Teaching of Mathematics

Because different beliefs about mathematics may have practical outcomes in terms of teachers' choices for classroom practices (Ernest, 1989), teachers beliefs were explored in this study. Three systems of beliefs about the nature of mathematics observed among teachers of mathematics are described by Ernest as the problem-solving view, the Platonist view, and the instrumentalist view. The problem-solving view is a dynamic, problem driven view of mathematics as a continually expanding field of human inquiry. In this view mathematics is not a finished product and its results remain open to revision. The second view, the Platonist view, holds that mathematics is a static but unified body of knowledge, consisting of interconnecting structures and truths. In this view mathematics can be discovered but not created. Third is the instrumentalist view that mathematics is a useful but unrelated collection of facts, rules, and skills. While the beliefs espoused by the teachers in this study do not fit neatly into any of these three views, they combine features of the views described in Ernest's model.

The structure, beauty, and hierarchical nature of mathematics described by the second year algebra teachers is consistent with the Platonist view of mathematics. These teachers viewed algebra as a very specific part of the structure of mathematics, providing a set of tools for solving problems and a language for higher mathematics. Algebra served as the foundation for higher mathematics. These views of algebra are consistent with the Platonist view of mathematics, but these teachers did not limit their beliefs about mathematics to a description of a static body of knowledge. They saw, too, the problem-driven nature of mathematics as they described mathematics as a tool for solving problems. Mathematics was constructed, by people, as a way to explain, understand, and do things in the world. Thus, these teachers held a dualistic belief about the nature of mathematics combining the Platonistic view of the interconnecting structures and truths of mathematics with the dynamic, problem driven field of human inquiry of the problem-solving view.

This study found a consistency between teachers' beliefs about the nature of mathematics and their beliefs about teaching and learning mathematics. They believed in the teacher's role (teaching) as the director of the learning process, providing structure and goals for the classroom and helping students make connections between their existing conceptual understanding and new concepts. Creating an environment conducive to learning was an important feature of these teachers' beliefs about teaching. Additionally, they held that the teacher was not the center of the teaching-learning process, but rather was an active participant along with the students. The teacher did have responsibility for providing expert information, often in the demonstration of examples including multiple approaches to and representations of the solution of problems, but also in the expression of ideas and the modeling of thought processes. In creating a positive learning environment, these teachers believed in mutual respect between teachers and students. They demonstrated an interest in and concern for their students. For them, teaching mathematics included developing students' confidence and having fun.

Learning mathematics required doing mathematics in the views of these teachers. Consistently, they believed that students must be active participants in the teaching-learning process in order to learn. Passively attempting to absorb mathematical concepts or understanding the concepts without being able to perform the techniques and solve the problems was not adequate for learning in the view of these teachers. Students, in the view of these teachers, needed to take individual responsibility for their own learning but required validation from the teacher to ensure that learning took place.

Together these teachers' views of mathematics, its teaching, and learning form a unified, congruent set of beliefs. There is a dualism of views exhibited between the Platonist view of a unified body of interconnected structures and truths and the problem-solving view of a problem driven, dynamic field of human inquiry. The emphasis on multiple approaches to and multiple representations of a problem is consistent with this dualistic view. The emphasis on active participation of students in the teaching-learning process confirms the dualism as does the teachers' role of director of learning and explainer but not center of the process.

Consistency between Beliefs and Practices

This study found a high degree of consistency between the teachers' beliefs and classroom practices, both when graphing calculators were in use and when they were not. Particularly notable were the consistency between the espoused belief in the importance of assisting students in making connections in teaching and the observed emphasis on the connections between concepts and techniques being presented with concepts and techniques previously explored. Connections made in practice extended beyond the mathematics classroom to include connections to students' experiences in the "real world" and their knowledge of other subjects. Thompson (1984) found that the level of congruence between teachers' beliefs and practices was related to their level of reflectiveness. In this study, one of the teachers clearly articulated her views indicating a high level of reflection on her practices and beliefs. While there was evidence to support the theory relating the level of congruence between beliefs and practices to reflectiveness, this study suggested that other factors affect the congruence.

Cobb, Wood, and Yackel (1990) found a dialectic relationship between teachers' beliefs and practices, beliefs were expressed in practice and new experiences or changes in practices gave rise to changes in belief. While data from this study supported this dialect, it suggested that it was not solely reflection on beliefs and practices that lead to congruence. In this study, teachers valued the interactions they had with other teachers. Through these interactions they gained insights into teaching as they discussed beliefs about how students learn and effective teachers teach. Because of the open environment and the shared experiences of the reform movement, they found that they learned from other teachers, by observing them, working with them, and listening to them. The professional development activities in which they participated were primarily experiential: attending meetings of mathematics organizations and conferences on teaching, and participating in workshops on the use of graphing calculators. Administrators in the schools and districts where these teachers taught were supportive and encouraged participation by teachers in activities outside the classroom and incorporation of new approaches in the classroom. This support and encouragement from their administrations made it possible for these teachers to experiment with new ideas and contributed to their professional development. These teachers' beliefs were affected by their experiences both inside and outside the classroom. Their practices reflected these experiences as their beliefs reflected their practices.

This study suggested a more complex relationship between teachers' beliefs and practices. The process of bringing beliefs and practices into agreement required more than reflection by a teacher. An integral part of the process was bringing experiences outside the classroom into the dialectic relationship with practices and beliefs. A new model placing reflection in the center and including experiences in the relationship represents the process required to bring beliefs and practices into agreement (Figure 9). In this model, reflection is central to the development of an integrated structure of beliefs and practices. Additionally, the model includes experiences in the relationship, both as factors in development of beliefs and practices and as stimulators of reflection.

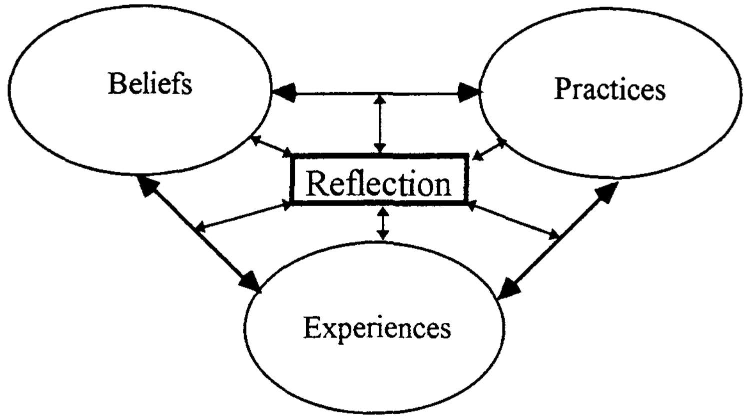


Figure 9. The relationship between beliefs, practices, and experiences.

The experiences in this model include interactions with other educators, a factor found to be significant in shaping the beliefs and classroom practices of this group of second year algebra teachers. The teachers in this study emphasized the role other teachers played in their decision to utilize graphing calculators in their teaching. Once the decision had been made to incorporate graphing calculators, other teachers influenced the development of these teachers classroom practices and beliefs concerning graphing calculators by sharing their experiences, offering support, discussing appropriate uses of graphing calculators, and demonstrating specific techniques. The support they found for the use of graphing calculators, both through experiences with their students and through interactions with other teachers, influenced their beliefs. Convinced that graphing calculators were useful in the teaching of mathematics, these teachers persisted in their use. The role of experiences in the development of teachers' beliefs and practices was not limited to the use of graphing calculators. Teachers cited a number of other experiences involving other teachers and experiences beyond the classroom which had influenced their beliefs and practices.

The role of reflection assumes a central role in the process of bringing beliefs and practices into agreement. The importance of reflection in this process was demonstrated by the teachers in this study. When asked to name five milestones in their teaching careers, all four teachers responded without hesitation. The ease with which they responded indicated a high level of reflection by these teachers on their practices and experiences. They were not asked to discuss the impact of these milestone experiences on their teaching, yet all four teachers indicated the significance of these experiences in molding their teaching practices and beliefs. One teacher referred to time spent working in a profession not related to education as an opportunity to see the relevance of the mathematics taught in the schools. The emphasis on the connection between the mathematics being taught and the "real world" permeated his teaching practices and beliefs. This teacher discussed the impact of his work experience on his teaching practices and beliefs, demonstrating the role reflection played in the process of achieving congruence between beliefs and practices. Examples of the central role of reflection in the process of achieving a high level of congruence between beliefs and practices were found in the descriptions of all the teachers as they discussed the impact their experiences had on shaping their teaching.

The process of incorporating graphing calculators into their teaching illustrated the dialectic model of the relationship between beliefs, practices, experiences, and reflection of these four teachers. As each described their introduction to graphing calculators, they related experiences which led them, through reflection, to make a decision to pursue the use of the graphing calculators in their teaching. These decisions, while different for each teacher, were based on a congruence between the perceived benefits of utilizing the graphing calculator and their existing views on mathematics and the teaching and learning of mathematics. The incorporation of graphing calculators led to changes in the classroom practices of these teachers including increased emphasis on conceptual understanding and incorporation of exploratory activities. In other cases such as the use of graphical, symbolic, and numerical representations of functions; the use of graphing calculators supported teachers' existing practices, providing improved means for students to access the multiple representations. As these teachers incorporated the graphing calculators into their teaching, they sought experiences that would assist them in changing their practices and adjusting their beliefs to accommodate the changes in practices. They attended workshops and conferences where they learned about the use of graphing calculators. They shared their experiences with other teachers and learned from these teachers. Through the process of incorporating graphing calculators into their teaching, these teachers enacted the model of the relationship between beliefs, practices, experiences, and reflection.

Consistency Between Teachers' Use of Graphing Calculators and the Visions for Reform

Pea (1987) suggested that technology could be used as a tool for developing conceptual fluency, for mathematical exploration, for integrating different mathematical representations, for learning how to learn, and for learning problem solving methods. This study found that while the teachers shared these goals for the use of graphing calculators in the teaching of mathematics, the goals were not realized in the teaching of second year algebra. The only use of the graphing calculator in the exploration of a mathematical concept observed was the investigation of the inverse of a function. There was little attention paid to utilizing the graphing calculators for problem solving, in spite of the inclusion of linear programming, a topic for which the use of the graphing calculator is ideal. Only one teacher actively encouraged students to utilize graphing calculators in solving these problems. Lack of expertise appeared to be the major obstacle preventing more widespread use of the technology for problem solving.

One of the theoretical benefits of using the graphing calculator in the classroom was to move the teacher to a position of facilitator of learning rather than source of knowledge (Lomen, 1993). The teachers in this study did not display this shift in roles, although they acted more as explainers than as experts. More attention was paid to teaching students how to utilize the graphing calculator, becoming skilled in the use of a variety of its features, than to actually utilizing the tool to solve problems and explore mathematical concepts. Demonstration of specific features of the calculator and their use in replacing the paper and pencil techniques previously presented was the dominate use of technology. The emphasis was on learning to use the tool, not on exploring mathematical concepts.

Demana and Waits (1990) contended that the ease of viewing the graph of an algebraic expression of a function with the graphing calculator has the potential for furnishing concrete links between geometry and algebra. Utilization of the graphing calculator to make connections between different representations (graphical, numerical, and symbolic) was prevalent in the practices of these teachers. Emphasis on understanding the results produced by the graphing calculator, being able to determine if the graph produced accurately represented the algebraic expression of the function, underscored the importance placed on making connections between graphical and symbolic representations.

Boyd, Ross, and DeMarios (1990) suggested that the use of technology for demonstrations permitted the teacher to introduce experimentation into the mathematics curriculum. This experimentation introduced students to a view of mathematics as a dynamic rather than static field. Such experimentation was not found in this study, either with or without the use of technology. This lack of experimentation was consistent with the teachers' view of mathematics which, while dualistic, tended to place more emphasis on the static structure of mathematics than on the dynamic, problem driven creation of mathematics especially as related to the study of algebra. What experimentation did take place was focused on finding features of the graphing calculator to solve specific types of problems. The emphasis was on using the graphing calculator to replace paper and pencil techniques.

The teachers' use of the graphing calculator was consistent with their view of algebra as a foundation for the study of higher mathematics. As such, their incorporation of graphing calculators into the teaching of second year algebra tended to focus on learning to use the tool to do mathematics rather than on using the tool to learn mathematics. Several teachers indicated that the graphing calculator was utilized for more explorations at higher levels in the curriculum. No observations were made in these classes so use of the graphing calculator for exploration could not be confirmed.

Several additional factors including lack of expertise, variety of models in use, and time constraints, may have contributed to the restricted use of graphing calculators in the teaching of second year algebra. Two of the teachers displayed wider use of the graphing calculators than the other two. Both of these teachers were in situations where students all utilized models of graphing calculators with which the teachers had high levels of expertise. The other two teachers were confronted with students utilizing a variety of models of graphing calculators. These teachers were not experienced in the use of some of the models of graphing calculators that their students were using. Additionally, these teachers did not display as much expertise utilizing the primary model in use. Lack of expertise had several possible causes. One possible cause for lack of expertise was the variety of models of graphing calculators in use. The other possible cause was time constraints. Teachers need time to learn to utilize graphing calculators effectively in their teaching.

In one case, the students did not all own their graphing calculators. Students who did not own graphing calculators were allowed to check them out from the school to use at home, but were required to return them before classes began the following morning. While the students in the other classes were encouraged to explore the calculator outside the classroom, less emphasis was placed on individual exploration in this class. Apparently, an environment in which students have their own graphing calculators, whether they own them or they are provided by the school on a full-time basis, encouraged more exploration by students. Students had the opportunity to explore the graphing calculator on their own initiative and were encouraged to do so by their teachers.

Consistency Between Teachers' Practices and the Constructivist Approach to Teaching

The problem-solving view of mathematics is reflected in the National Council of Teachers Of Mathematics' (NCTM) recommendations for changes in the teaching of mathematics. "Namely, that the processes and strategies of mathematical activity are central, and that the main aim of mathematics teaching is to empower children to become creative and confident problem solvers" (Ernest, 1989, p.21). The teachers in this study exhibited a dualistic view of mathematics, not solely the problem-solving or constructivist view, upon which the recommendations for the current mathematics reform are built. While they shared in some of the beliefs of the problem-solving view, their main aim was not to empower children to become creative and confident problems solvers.

There are, however, features of these teachers' practices that are consistent with the constructivist view. Noddings (1990) contended that constructivism required cognitive structures that were under continual development and could be transformed through purposive activity. While none of the teachers in this study used this kind of language to discuss their beliefs and practices, they all worked to provide students with opportunities and means to make connections between their existing knowledge and the new concepts they were exploring. The teachers' Platonist view of mathematics emphasized its structure and thus enabled them to direct students in constructing their own structures.

Students must learn to construct powerful ideas, ideas that the student believes and that have internal consistency, ideas that are in agreement with experts and can be reflected on and described, ideas that can act as the foundation for the construction of further constructions, guide future actions, and be justified and defended. In order for students to construct powerful ideas, instruction must be inherently interactive (Confrey, 1990). This description of constructivism mirrors much of what occurred in the classrooms of these teachers. With their views on the importance of active participation on the part of learners, they utilized interactive styles of teaching, asking questions, requiring explanations, and responding to students' inquiries. Beyond an interactive style, the teachers encouraged individual responsibility for learning and provided opportunities for students to confirm that their ideas were in agreement with the experts. Further, they encouraged students to communicate mathematical concepts with the teacher and with other students, providing the opportunity to describe, justify, and defend their emerging understanding of mathematical concepts.

Constructivism depends on the autonomy of the learner who must have responsibility for and control over his own learning (Confrey, 1990). While the teachers in this study encouraged individual responsibility for learning, they did not allow students full control of the process. These teachers felt a responsibility to provide a structure and a direction for learning. They controlled the pace and content that was to be learned.

The greatest divergence between the practices of these teachers and constructivism was related to the incorporation of exploratory activities. In an environment designed to allow students the control over their learning and to become creative and confident problem solvers, students must have opportunities to explore and discover mathematical concepts. The teachers in this study did not provide these opportunities. In this way, their practices diverged from the constructivist approach to learning.

All four of these teachers had been teaching for over 15 years. When these teachers were learning mathematics and preparing to become teachers, they were not exposed to the constructivist approach. Their reflections on and descriptions of their teaching experiences and professional development activities indicated a high level of adaptation and change in their practices over the course of their teaching careers. Throughout their careers, these teachers were involved in and sensitive to recommendations for change and reform in the teaching of mathematics. While they demonstrated knowledge of these recommendations and a desire to incorporate changes, consistent with the recommendations, into their teaching, they acknowledged that making such changes was a process. These teachers realized that their teaching practices were changing, but the process of change was not complete. All four teachers expressed desires to make additional changes to their teaching that would incorporate additional discovery activities and encourage student responsibility for learning.

Limitations of the Study

This study was limited by several factors. First, the size and nature of the sample was restricted by the design of the study. Only four teachers were included in the study because of the depth of the exploration of each teacher's beliefs and practices. The criteria of persistent use of the graphing calculator by the teacher and availability of graphing calculators to students at any time limited the population from which the sample was selected. As a result, the teachers in the study all came from schools whose students had relatively high socio-economic standing. Additionally, the geographic diversity was limited because of the selection criteria and the time available to the researcher for travel. Three of the four schools were suburban, in a major metropolitan area. While the fourth school was from the center of a city, it could not be considered an inner-city school because of the size and heterogeneity of the city. Additionally, all four schools were in the same geographic region of a single state. More diversity in socio-economic status of the students and a broader geographic distribution would make the results of the study more representative of the broader population of high schools and high school mathematics teachers. Further, all four teachers had more than 10 years of high school mathematics teaching experience and more than 15 years of teaching experience. All four of the teachers were experienced mathematics teachers with established teaching practices before the introduction of the graphing calculator. The degree to, ease with, and ways in which these teachers had integrated the graphing calculator into their teaching may have been related to their level of experience. The beliefs and practices of the teachers in this study may not be transferable to less experienced teachers. The applicability and value of the findings of this study for less experienced teachers will be discussed in the section of implications later in this chapter.

The descriptions of these teachers' classroom practices and their use of graphing calculators was based on observations of second year algebra classes only. It cannot be assumed that the practices found in other classes taught by these same teachers would be the same as the practices found in their teaching of second year algebra. The consistency of their beliefs and practices indicated a high degree of integration, thus, it is supposed that these teachers practices in other classes would be consistent with their beliefs. The study found that beliefs about algebra were related to practices both when graphing calculators were being used and when they were not. When these teachers utilize graphing calculators in the teaching of other courses, it is likely that their beliefs about the course content would affect their practices in those classes as well. Therefore, conclusions drawn about the ways in which these teachers utilized graphing calculators are only applicable to second year algebra and should not be applied to other courses in the curriculum.

Teachers' beliefs were elicited through several interviews encouraging teachers to talk about their classroom practices. From the statements the teachers made about their practices, the researcher constructed descriptions of their beliefs about mathematics, its teaching, and students' learning. The interviews were designed to provide a minimum of structure so that teachers were free to provide their own organization and structure to their statements. The resulting descriptions of the teachers' beliefs reflected the thinking of the teachers. The primary stimulation for each teacher's discussion of beliefs was a set of statements prepared by the researcher describing activities observed in the teacher's classroom and statements made by the teacher in informal interviews during the observation period. Teachers were encouraged to add statements to the collection before they began discussing them. Even so, it is possible that the teachers held beliefs concerning mathematics, its teaching, and students' learning that did not emerge through the interview process.

The descriptions of teachers' classroom practices were based on observations of a single unit of second year algebra the duration of which ranged from three to four and a half weeks. Teachers' practices could have deviated from their normal practices during the period of the study. The presence of the researcher in the classroom as well as the teachers' perceived goals of the study could have contributed to an altering of their practices. Data collected including unit time lines and worksheets indicated that the teachers conducted the course in the same manner as they had in previous years. Even so, the impact of participation in the study on teachers' practices must be considered when evaluating the results of this study.

Finally, the role of the researcher in the study must be acknowledged. The researcher was the main element in collecting and analyzing data. This study was designed to prevent as many threats to validity as possible, however, the researcher's background, experience, beliefs, and biases still limited the conclusions drawn.

Implications and Recommendations for Future Research

The findings of this study supported existing theories on the implementation of new practices, especially the incorporation of technology, in teaching. McLaughlin(1989) found changes rooted in the natural networks of teachers were more effectively implemented than changes arising from other sources. The teachers in this study incorporated graphing calculators into their teaching with support and encouragement from their colleagues. They attended conferences and meetings of the national, regional, and state mathematics associations where they found information about and training for the use of graphing calculators. A continuation and expansion of this network of support for the use of graphing technology is essential if the recommendation of the National Council of Teachers of Mathematics' (NCTM) Curriculum Standards (1989) that technology be incorporated into the teaching and learning of mathematics in the schools is to be realized.

The teachers in this study persisted in the use of graphing calculators because they were convinced of their value in the teaching of mathematics. Their belief in the value of graphing calculator was based in part on their experiences in teaching (their classroom practices) and on the support they received from their network of teachers and teacher organizations. Additionally, all the teachers in this study noted the role of administrative support for the use of graphing calculators and other innovations in the teaching of mathematics. These teachers were able to be actively involved beyond their classrooms: attending meetings and conferences, participating in workshops, investigating curricular reform, and developing as professionals, because they enjoyed the support of the administrations in their schools. Administrators must realize the importance of, support, and encourage participation in the broader network of mathematics teachers if reform is to take place in their schools.

The teachers in this study were all experienced teachers who possessed rich backgrounds and well developed belief structures. As less experienced teachers consider the incorporation of graphing calculators into their teaching, their beliefs and practices will shape the way in which they make changes in their practices and implement the technology (Thompson, 1992). This study found that teachers' beliefs and practices were influenced by interaction with other educators through dialogue and observation. Because of the importance of others on the development and integration of a teacher's beliefs and practices, this study suggests that mentor relationships be encouraged between new teachers or teachers considering making changes in their practices and experienced teachers or teachers who have already incorporated the desired changes into their practices.

One of the limitations of this study was that observations took place only in the teaching of second year algebra. Expanding the study to a broader range of the curriculum would contribute to the description of practices of teachers who utilize graphing calculators in their teaching. Since teachers can be prompted to make changes in their practices based on reflection on the practices of others (Thompson, 1992), the expansion of the knowledge base would be useful. Studies should be conducted that examine individual teacher's use of graphing calculators in different classes in order to determine if there are differences in use based on course content. Further studies should also be conducted focusing on the use of graphing calculators in the teaching of specific classes. This study indicates that teachers who persist in the use of graphing calculators will continue to expand their use through the curriculum, introducing their use in classes prior to second year algebra. Continuing research needs to be conducted to determine if earlier introduction of the technology changes the way it is utilized in second year algebra and successive courses. Of particular interest is the study of graphing calculator use for mathematical exploration in second year algebra and lower courses. This study found that the focus of the use of graphing calculator in second year algebra was on learning to use the tool rather than on using the tool to learn. As teachers persist in the use of graphing calculators and become more experienced in their use at the second year algebra level and below, will the focus of use shift from learning to use the tool to using the tool to learn?

This study found that teachers using graphing calculators in their teaching of second year algebra emphasized conceptual understanding. Earlier studies indicated that graphing calculators contributed to improved conceptual understanding in the study of precalculus (Browning, 1990; Taylor, 1991; Boers-Van Oosterum, 1990). The development of conceptual understanding is essential to the success of students who continue the study of mathematics beyond algebra. Studies need to be conducted to determine if the goals of improving students' conceptual understanding of mathematics through the use of graphing calculators is being realized. As the use of graphing calculators in the teaching of high school mathematics increases, students' understanding of mathematics may be affected.

Any changes in students' conceptual understanding and view of mathematics will affect the teaching of college mathematics. More needs to be understood about the effects of using graphing calculators on students' conceptual understanding and preparation for college level mathematics courses.

This study did not find that teachers had made changes in the content of second year algebra. But, the issue was mentioned by several of them. One teacher commented that it was important that teachers decided what content should be retained and what should be eliminated from the curriculum. Additional studies need to explore the effect of the use of graphing calculators on the high school mathematics curriculum.

Models of teachers' thinking include content knowledge (Carpenter, 1988). This study did not explore teacher's content knowledge and its relationship to their persistent use of graphing calculators. Studies should be conducted in this area in order to determine if teachers' content knowledge is related to the ways in which they incorporate graphing calculators into their teaching.

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APPENDICES

APPENDIX A
SURVEY OF GRAPHING TECHNOLOGY USE

In September, 1994 a survey was sent to all high school mathematics departments within a convenient geographic distance of the researcher. The purpose of the survey, to collect data about the impact of graphing calculators on the teaching of high school mathematics, was described in a letter accompanying the survey.

Letter Accompanying the Survey

Dear Math Department Chair -

I am collecting data about the impact of graphing calculators and computer graphing technology on the teaching of high school mathematics. Enclosed is a survey which requests information concerning

- the courses in which graphing technology is being used
- the type of graphing technology that is being used
- the amount of time graphing technology has been used in your school

Demographic information about each school is requested on the survey. This information will be used for statistical purposes only.

I will be preparing a summary report of the information obtained through this survey. If you would like a copy of the summary, please mark the box on the survey.

Please complete the enclosed form for your department and send it to:

Martha VanCleave
Math Department
Linfield College
McMinnville, OR 97128-6894

The enclosed envelope is addressed and stamped for your convenience. If you have questions please feel free to call me at: 503-434-2470 or contact me through e-mail at mvcleave@linfield.edu.

Please complete and return the survey regardless of whether you use graphing technology in the teaching of mathematics.

USE OF GRAPHING TECHNOLOGY: CALCULATORS AND COMPUTERS

School ________________________________

School location: __ urban __ suburban __ rural Approximate school enrollment:__________

Type of school: Public: __ 4 year __ 3 year Private: __ church related __ non-church related

Number of full time math teachers: ________ Number of teachers who have one or more math classes but are not full time in math __________

Graphing technology has been used in teaching math classes at this school: ___ over 5 years ____ 3 - 4 yearrs ____ 2 - 3 years ___ 1 - 2 years ___ new this year ___ never
(if never please return survey, make comments on back as desired)

How many of your math teachers have been teachng with graphing technology for: ___ over 5 years ____ 3 - 4 yearrs ____ 2 - 3 years ___ 1 - 2 years ___ new this year

1994-95

COURSES TAUGHT WITH TECHNOLOGY:			USE OF GRAPHING CALCULATORS:					USE OF COMPUTER GRAPHING SOFTWARE:						
Name of Course Textbook Author	Course Content	Number of teachers teaching this course this year	Type(s) of Graphing Calculators	Number of years of use	Demonstration Equipment available	Students purchase	School provides	Type(s) of Computer Software	Demonstration equipment available	Type of lab available	Number of Teachers teaching with graphing technology first year	second year	third year	more than 3 years

Additonal comments on back:

Please send me a copy of the completed summary ☐

Survey Form

Summary of Results of Survey

Surveys were sent to 92 high school mathematics departments. Table 1 contains a summary of the demographic information from the 38 surveys returned.

Table 1

Demographic Responses to Graphing Technology Use Survey

	Number of Schools
Location	
Urban	9
Suburban	16
Rural	13
Enrollment	
Over 1500	6
1000- 1500	9
Under 1000	21
Not reported	2
Type of School	
Public - 4 year	26
Public - 3 year	3
Private - 4 year	9

Table 2 contains a summary of the information about the use of graphing calculators in the 38 schools which returned surveys.

Table 2

Graphing Calculator Use in Schools Responding to Survey

	Number of Schools
Time graphing calculators have been in use	
5 or more years	13
3 to 4 years	13
1 or 2 years	8
Never	4
Courses in which graphing technology is being used	
Calculus	27
PreCalculus	38
Algebra II	23
Geometry	7
Algebra I	10
Pre-Algebra	2
Type of graphing calculators in use	
TI-81/82	29
TI-85	11
Casio	7
HP-28S/48S/48G	10

Among the respondents to the survey, no school indicated that once begun the use of graphing technology had been discontinued. When examining the courses in which schools utilized graphing technology, schools which had utilized graphing calculators for the greatest time tended to have begun the use of graphing calculators in the teaching of Calculus or Precalculus. The initial use of graphing calculutors in Calculus or Precalculus tended to be followed by implementation of graphing calculators in lower level courses including Algebra II, Algebra I, Geometry, and in a few cases Pre-Algebra. Schools which had implemented the use of graphing calculators more recently often introduced their use throughout the mathematics curriuculum.

APPENDIX B
LETTER REQUESTING PERMISSION TO CONDUCT STUDY

District Administrator November 20,
1995
District office address

Ms. Administrator

I have drafted a letter to parents explaining my my proposed research with Ms. Dancer as we discussed on Friday. I have also prepared a permission for videotaping. I have emphasized that students or student work will not be the focus of the research, data collection, or analysis.

Enclosed are two copies of the letter to parents. I have included one copy on my letterhead and one copy on plain paper. If this letter is acceptable to you, it can be duplicated either on my letterhead or on paper of your choosing.

If you need any further information or would like changes made to the letter or permission form I can be reached either at work or at home. I expect to be working at home on Monday and Wednesday of this week and will be in my office on Tuesday.

Office: Math Department
Linfield College
McMinnville, OR 97128
(503)-434-2470

Home: (503)-864-3641

E-mail mvcleave@linfield.edu

I look forward to conducting research with Ms. Dancer at Lake High School.

Martha VanCleave

APPENDIX C
LETTER TO PARENTS AND PERMISSION SLIPS

At the request of the district administrator the following letter was distributed to students in the classes to be videotaped in the Lakeshore district. Students were asked to take the letter to their parents and bring the signed permission slip back to the teacher. The researcher then maintained a file containing the signed permission slips from all students in the classes being videotaped.

Letter to Parents

mm/dd/yyyy

Dear Parents,

I am conducting research on the beliefs and classroom practices of teachers who are utilizing graphing calculators in the teaching of Advanced Algebra. TEACHER has agreed to participate in this study. The study will involve observations of TEACHER'S Advanced Algebra class over a four-week period. I will also conduct several interviews with him. The goal of my research is to explore in depth the ways in which teachers are using graphing calculators and their beliefs about this use.

In order to create a record of the activities that I observe I will be videotaping one complete unit of study during the period of observations. The videotape will focus on the teacher. Class sessions are being videotaped so that I will be able to review the tapes and recall details about the activities which I might not remember without the videotaped record. Students and student work will never be the focus of the observations. Because teachers work with students, students and their activities may occasionally be captured on the tapes. In the analysis of the tapes, it will be the teacher's actions and not the students that will be of concern. Videotapes will be viewed only the researchers involved in this project, Dr. Margaret Niess and myself. All records of the observations and interviews including videotapes will be stored in a locked cabinet in my office. Pseudonyms will be used for schools and teachers participating in this study. No students will be named in the analysis and reporting of the data collected.

This study will contribute to the ongoing effort to improve mathematics education. The findings of this study will be available to the teacher and the school district for use in making decisions about their use of graphing calculators. The goal of the research is to explore how teachers utilize graphing calculators and will not evaluate their use, rather it will describe the variety of uses found in different settings and under different circumstances. The participation of Lake Oswego School District and TEACHER will make a valuable contribution to the understanding of teachers' classroom practices in the teaching of advanced algebra.

Martha VanCleave
Assistant Professor of Mathematics

Permission Slip

I give permission for my son/daughter ________________________________ to be videotaped in the research conducted in the classroom of Ms. Dancer. I understand that the appearance of students in the videotape is incidental and will not be the focus of the taping or analysis of the data collected.

______________________________________Date____________________

signature of parent or guardian

APPENDIX D
INFORMED CONSENT FORM

This study is part of research attempting to document the use of graphing calculators in high school classrooms. This research will provide answers to such questions as: What are the established classroom practices of teachers who have persisted, beyond the experimental stage, in the use of graphing calculators in the teaching of Algebra II (advanced algebra)? Why do teachers choose to utilize graphing calculators in the ways documented? By capturing a complete picture of the practices of teachers who have persisted in the use of graphing calculators in their teaching, this research will provide valuable information needed to support the ongoing effort to incorporate graphing calculators in the teaching of high school mathematics.

Participation will be during the Fall and Winter of the 1995-96 school year. During this time the researcher will conduct several in-depth, open-ended interviews with the teacher and observe Algebra II (advanced algebra) classes taught by the teacher. The observations will take place during every meeting of the class under observation over a four-week period. Approximately two weeks of the classroom observations will be videotaped. The focus of the observations will be the teacher, not the students in the class. One in-depth interview will take place before the period of observation and three in-depth interviews will take place following the period of observation. Informal interviews may take place during the observation period. All interviews will be audio or videotaped. Materials (e.g. handouts, tests, and quizzes) used by the teacher will be collected by the researcher.

This research will provide an in-depth examination of the teacher's classroom practices and the teacher's beliefs about mathematics, its teaching and learning. This in-depth examination may lead the teacher to a new understanding of classroom practices and a better ability to verbalize beliefs. Increased reflection on teaching practices and beliefs is also possible. At times the teacher may experience some difficulty in expressing beliefs, especially if no prior attempt has been made. The increased understanding of and reflection on teaching practices and beliefs may lead some teachers to consider changes in their teaching practices and beliefs. The presence of the researcher as an observor in the classroom may initially create a distrubance which the researcher will work to minimize. Through participation in this research the teacher will be making a contribution to the ongoing process of reform in the teaching of high school mathematics.

Only the researchers will have access to all data collected (interview and observation tapes, observation fieldnotes, and documents). Tapes, fieldnotes, and documents will be stored in a locked cabinet in the office of one of the researchers. Pseudonyms will be used for all teachers and for the schools at which they teach when reporting the results of this research.

Participation is voluntary, refusal to participate will involve no penalty or loss of benefits to which the teacher is otherwise entitled. The teacher may discontinue participation at any time without penalty or loss of benefits to which the teacher is otherwise entitled.

Questions about the research, personal rights, or research-related injuries should be directed to Dr. Margaret L. Niess at 737-1818.

Name ______________________________ Date ______________

APPENDIX E
INTERVIEW QUESTIONS FOR BACKGROUND INTERVIEWS

BACKGROUND INTERVIEW with ______________________________ on __________

How long have you been teaching mathematics?

at this school?

at this level?

Describe this school including:

the students you teach

the teachers you teach with

the administration

any other features you want to mention.

Where did you receive your teacher training?

Describe your training including coursework.

Describe your teaching career including:

the places you have taught

the most rewarding experiences you have had

the biggest disappointments or frustrations you have experienced

Identify five milestones that have marked changes in how you thought about teaching.

Describe training and workshops you have participated in since you began teaching.

Besides the training and workshops mentioned, are there other things you do that relate to your teaching (e.g. attend professional meetings, read journals).

Why did you decide to try using graphing calculators in your teaching?

Describe in more detail training related directly to the use of graphing calculators.

Why did you decide to persist in using graphing calculators in your teaching?

What has been frustrating about using graphing calculators?

What has been rewarding about using graphing calculators?

Describe how you teach mathematics.

On an ideal teaching day, what might I see happening in your classroom?

On a typical teaching day, what might I see happening in your classroom?

On a day you consider unsatisfactory, what might I see happening in your classroom?

Describe the assessment techniques you utilize and explain why you use these techniques.

If a student asked you, what is mathematics, how would you respond?

If a student asked you, what is algebra, how would you respond? (Or how does algebra fit into you description of mathematics?)

How do you think students learn mathematics?

APPENDIX F
OUTLINE FOR BELIEF VERIFICATION INTERVIEW

According to how you have described your beliefs and how you discussed the cards we used in the previous interview session, I would characterize your thinking about mathematics as:

Do you agree with these statements?

Explanation/comments:

What would you say has led you to this way of thinking?

According to how you have described your beliefs and how you discussed the cards we used in the previous interview session, I would characterize your thinking teaching mathematics as

Do you agree with these statements?

Explanation/comments:

What would you say has led you to this way of thinking?

According to how you have described your beliefs and how you discussed the cards we used in the previous interview session, I would characterize your thinking about learning mathematics as

Do you agree with these statements?

Explanation/comments:

What would you say has led you to this way of thinking?

Do you think the use of the graphing calculator has changed the way you teach?
If so, how?

Are there specific teaching techniques you utilize now, with the availability of the graphing calclulator that you did not use befoe you had the graphing calculators?

In what way has the use of the graphing calculator made the greatest impact on your classroom