Title: TRANSPIRATION COOLING OF A ROTATING DISK

Heat transfer coefficients were experimentally determined for a transpiration-cooled rotating disk. Comparison of the experimental results to a theoretical prediction by Sparrow and Gregg is made.

Air was injected into an air environment in this study. The heat transfer surface used was a four-inch disk constructed of fine mesh, porous stainless steel. Temperature of the rotating surface was measured with an infrared radiometer, accurate to one-half to two degrees Fahrenheit, depending on the temperature. The radiometer was calibrated each day by focusing on a stationary surface of material identical to the rotating disk. Temperature of the stationary calibrator surface was measured with calibrated thermocouples.

The effect of both injection rate and rotational rate was investigated. Nominal injection rates of 0.35, 0.44, 0.73, and 1.04 lb/min ft² and rotational Reynolds numbers of 19,000, 26,000, 39,000 and 51,000 were used. Within the range of variables studied, it was found that transpiration decreased the heat transfer coefficient from a rotating disk by as much as 80% compared to the value expected for
a non-porous disk.

Results of the experiments were correlated within approximately ten per cent by the equation

$$\log_{10} \ Nu = -0.505 \ \frac{\rho_w}{\rho_\infty} H_w - 0.519$$

where Nusselt number is defined as $h(v/\omega)^{1/2}$ and the injection parameter $H_w$ is

$$H_w = \frac{V_{zw}}{(\omega v)^{1/2}}.$$

The theoretical calculations of Sparrow and Gregg inadequately predicted the experimental results because the analysis assumed constant properties. If, however, the injection parameter $H_w$ used by Sparrow and Gregg were modified with a density ratio $\frac{\rho_w}{\rho_\infty}$ as indicated above, then the theoretical prediction and the equation above correlate the data equally well over the range $0.3 < \frac{\rho_w}{\rho_\infty} H_w < 1.3$. 
TRANSPERSION COOLING OF A ROTATING DISK

by

WILLIAM PRIMO COSART

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Approved:

Redacted for privacy

Professor of Chemical Engineering
in charge of major

Redacted for privacy

Professor of Chemical Engineering
in charge of major

Redacted for privacy

Head of Department of Chemical Engineering

Redacted for privacy

Dean of Graduate School

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>INTRODUCTION</td>
</tr>
<tr>
<td>II</td>
<td>THEORY AND PREVIOUS WORK</td>
</tr>
<tr>
<td></td>
<td>Previous Work</td>
</tr>
<tr>
<td></td>
<td>Flow Near A Rotating Disk</td>
</tr>
<tr>
<td></td>
<td>The Effect of Nearby Stationary Surfaces</td>
</tr>
<tr>
<td></td>
<td>Heat and Mass Transfer From a Rotating Disk</td>
</tr>
<tr>
<td></td>
<td>Transpiration Cooling</td>
</tr>
<tr>
<td></td>
<td>Theory</td>
</tr>
<tr>
<td></td>
<td>Equations of Change</td>
</tr>
<tr>
<td></td>
<td>Energy Balance on the Rotating Disk</td>
</tr>
<tr>
<td>III</td>
<td>EXPERIMENTAL EQUIPMENT AND INSTRUMENTATION</td>
</tr>
<tr>
<td></td>
<td>Source of Air</td>
</tr>
<tr>
<td></td>
<td>Rotary Union</td>
</tr>
<tr>
<td></td>
<td>Rotating Pipe</td>
</tr>
<tr>
<td></td>
<td>Heat Transfer Surface</td>
</tr>
<tr>
<td></td>
<td>Guard Heater</td>
</tr>
<tr>
<td></td>
<td>Enclosure</td>
</tr>
<tr>
<td></td>
<td>Temperature Measurement</td>
</tr>
<tr>
<td></td>
<td>Thermocouples</td>
</tr>
<tr>
<td></td>
<td>Infrared Radiometer</td>
</tr>
<tr>
<td>IV</td>
<td>EXPERIMENTAL PROCEDURE</td>
</tr>
<tr>
<td></td>
<td>Calibration</td>
</tr>
<tr>
<td></td>
<td>Heat Transfer Measurement</td>
</tr>
<tr>
<td>V</td>
<td>EXPERIMENTAL RESULTS</td>
</tr>
<tr>
<td></td>
<td>Experimental Data</td>
</tr>
<tr>
<td></td>
<td>Correlation of the Data</td>
</tr>
<tr>
<td></td>
<td>Discussion of Results</td>
</tr>
<tr>
<td>VI</td>
<td>CONCLUSIONS</td>
</tr>
<tr>
<td>VII</td>
<td>SUGGESTIONS FOR FURTHER WORK</td>
</tr>
<tr>
<td></td>
<td>BIBLIOGRAPHY</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>II-1</td>
<td>Rotating Disk Coordinates</td>
</tr>
<tr>
<td>III-1</td>
<td>Schematic Flow Diagram Indicating Variables Measured</td>
</tr>
<tr>
<td>III-2</td>
<td>Details of Rotating Apparatus</td>
</tr>
<tr>
<td>III-3</td>
<td>Typical Calibration Curves For Radiometer</td>
</tr>
<tr>
<td>V-1</td>
<td>Relative Sizes of Porous and Non-Porous Areas</td>
</tr>
<tr>
<td>V-2a</td>
<td>Effect of Reynolds Number and Injection On Heat Transfer</td>
</tr>
<tr>
<td>V-2b</td>
<td>Effect of Reynolds Number and Injection On Heat Transfer</td>
</tr>
<tr>
<td>V-3a</td>
<td>Dependence of Nusselt Number On Reynolds Number</td>
</tr>
<tr>
<td>V-3b</td>
<td>Dependence of Nusselt Number On Reynolds Number</td>
</tr>
<tr>
<td>V-4a</td>
<td>Nusselt Number Dependence On Injection Parameter $H_W$</td>
</tr>
<tr>
<td>V-4b</td>
<td>Nusselt Number Dependence On Injection Parameter $H_W$</td>
</tr>
<tr>
<td>V-4c</td>
<td>Nusselt Number Dependence On Injection Parameter $\frac{\rho_W}{\rho_\infty} H_W$</td>
</tr>
<tr>
<td>V-4d</td>
<td>Nusselt Number Dependence On Injection Parameter $\frac{\rho_W}{\rho_\infty} H_W$</td>
</tr>
<tr>
<td>V-5a</td>
<td>The Ratio $h/h_o$ versus $R_T$</td>
</tr>
<tr>
<td>V-5b</td>
<td>The Ratio $h/h_o$ versus $\frac{\rho_W}{\rho_\infty} R_T$</td>
</tr>
<tr>
<td>Appendix Figure</td>
<td>Description</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>D-1</td>
<td>Extrapolation of Heat Transfer Coefficient To Zero Injection</td>
</tr>
<tr>
<td>F-1</td>
<td>Decay of Pressure In Sealed Piping System</td>
</tr>
<tr>
<td>Appendix</td>
<td>Table</td>
</tr>
<tr>
<td>----------</td>
<td>-------</td>
</tr>
<tr>
<td>A-1</td>
<td></td>
</tr>
<tr>
<td>B-1</td>
<td></td>
</tr>
<tr>
<td>B-2</td>
<td></td>
</tr>
<tr>
<td>C-1</td>
<td></td>
</tr>
<tr>
<td>D-1</td>
<td></td>
</tr>
</tbody>
</table>
I. INTRODUCTION

Transpiration cooling is a means of cooling by injection of a fluid through a porous surface. During the last twenty years, transpiration cooling has been the subject of scores of scientific and engineering studies. High interest has been shown in the topic from both the practical and theoretical standpoint.

Transpiration cooling is of theoretical interest in fluid mechanics because fluid injected into a boundary layer obviously alters the flow conditions near the wall. Since temperature and concentration profiles are dependent on the velocity field, the entire character of the boundary layer is affected. Moreover, the stability of the flow, i.e. its tendency to change from laminar to turbulent flow as well as its tendency to separate from the solid boundary, is also affected.

The practical importance of transpiration cooling is understood by visualizing the following situation: A surface is swept by a very hot fluid, for example exhaust gases exiting through a rocket nozzle. If the fluid is extremely hot, the mechanical strength of the surface will be degraded, perhaps leading to failure. If, however, the surface were porous, so that a small amount of cool gas may be injected through it, then the surface is cooled in two ways: First, the entering gas cools the surface as it passes through the pores. Second, after injection into the main stream, the coolant remains close to the surface, providing
an insulative blanket between the hot stream and the surface. Due to this possibility of protecting materials exposed to very hot environments, interest has been stimulated in studying the reduction of heat transfer rates (and also skin friction) by transpiration in such applications as combustion chamber walls, gas turbine blades and disks, rocket motor nozzles, hypersonic ram-jet intakes, etc.

The particular geometry chosen for the study of transpiration cooling reported herein is the rotating disk. The rotating disk itself has been the subject of many investigations, for practical as well as theoretical reasons. From the practical standpoint, the rotating disk represents a mechanically simple model for many complex items of equipment, such as turbine disks, fan blades, or centrifugal pumps. From the theoretical standpoint, the disk is of interest since the boundary layer induced by its rotation exhibits three dimensional flow which can be treated by the full Navier-Stokes equations (i.e., no terms are left out as in the two-dimensional boundary layer equations).

Because of the extensive previous study of transpiration cooling, and also the considerable interest in the rotating disk, the present research stands at the confluence of two important streams of boundary layer work. It is hoped that some light is shed on both.
II. THEORY AND PREVIOUS WORK

A brief description of the work of others is appropriate in order to establish the position which the present work holds in the continuity of scientific investigation into the subjects of transpiration cooling and boundary layer flow in rotating disk systems. Such a description will also suggest areas which need further investigation, as discussed in the final chapter of the thesis.

PREVIOUS WORK will be described under the headings:

a. Flow Near a Rotating Disk
b. The Effect of Nearby Stationary Surfaces
c. Heat and Mass Transfer From a Rotating Disk
d. Transpiration Cooling

Following this summary of previous work, a more detailed synopsis of one particular reference will be given (Sparrow and Gregg [57]), since itformulates the theoretical foundation for the experimental work carried out in the present investigation. Finally, an energy balance on the rotating disk system will be given, which provides the basis for converting the experimental data obtained to convective heat transfer coefficients. The THEORY portion of the chapter will thus be presented under two headings:

a. Equations of Change
b. Energy Balance on the Rotating Disk
1. **PREVIOUS WORK**

a. **Flow Near a Rotating Disk**

One of the first theoretical investigations into flow induced by a rotating disk was by von Karman [65] in 1921. By assuming axial symmetry and constant fluid properties and that the radial and tangential velocities of the fluid could be normalized into similar profiles by dividing by the tangential velocity of the disk, the Navier-Stokes equations were reduced to a set of ordinary differential equations. The independent variable was a normalized axial distance, radial distance having been eliminated. The similarity assumption has since been verified experimentally.

In 1934, Cochran [11] noted and corrected an error in von Karman's analysis. He then re-solved the set of equations using a different technique. Near the disk, the dimensionless velocities were expanded in a power series in terms of the normalized distance variable. At very large distances from the disk, a decaying exponential series was assumed. These solutions were then matched, choosing the constants in the expansions so that velocities and derivatives coincided at intermediate points.

Improvements on Cochran's solution have been given by
a number of investigators. With the advent of modern
digital computers, Ostrach and Thornton [45] recalculated
the solutions to the momentum equations using finite
difference techniques. Schwiderski and Lugt [52] con-
verted the differential equations into Volterra integral
equations, numerically solving the latter. Benton [3]
solved the time-dependent equations by a power series
expansion in \(\omega t\). His solution was valid only at small \(t\),
however, as pointed out by Homsey and Hudson [28], who
extended the work to approach steady state. They found
a single overshoot in tangential velocity which decayed
after about two revolutions of the disk.

Turbulent flow has also been studied. Assuming
velocities near the wall followed a one-seventh power
distribution, von Karman [65] tried to calculate tur-
bulent velocity profiles. Goldstein [23] recalculated
the profiles using a logarithmic distribution. Sub-
sequently, Dorfman [19] adjusted the parameters in the
logarithmic profile to approach experimental results
more closely. More recent work by Cham and Head [8] and
Cooper [13] utilized Prandtl's mixing length theory, modi-
fied by an exponential damping factor to cause the mixing
length to disappear at the wall. In the outer portion of
the boundary layer, Cham and Head used an "entrainment
factor" (with an experimentally determined parameter) to determine turbulent viscosity, while Cooper used an "intermittency", the numerical value of which was borrowed from two-dimensional boundary layer work. Both investigations permitted reasonably accurate calculations of velocity profiles.

Early experimental work measured moment (i.e. drag) coefficients and velocity profiles. Goldstein [24], Schlicting [51], and Dorfman [19] summarized and compared much of this work. Theodorson and Regier [61] and Gregory and Walker [25] measured laminar velocity profiles. The former group also measured velocities in the turbulent regime, and determined that transition from laminar to turbulent boundary layer occurred at Reynolds numbers \( \frac{\omega R^2}{v} \) from 125,000 to 310,000, depending on the roughness of the disk. Cham and Head [8] reported laminar flow up to \( \text{Re} = 185,000 \), stable vortex formation from 185,000 to 285,000, and fully turbulent flow above 285,000. These results gave quantitative expression to earlier, qualitative observations by Smith [54], who observed output patterns from a hot-wire anemometer. Smith reported three regimes of flow: laminar, oscillatory, and turbulent.

Theoretical investigations into the stability of flow on a rotating disk include those of Stuart [59] and
Rogers and Lance [50], both concluding that suction through a porous surface tends to stabilize the boundary layer. Without suction, Rogers and Lance reported that flow about a disk rotating in an environment which is also rotating can be stable only if both are rotating in the same direction. However, the addition of suction could provide stability even if disk and environment rotated in opposite directions.

The effect of both suction and injection was studied by Sparrow and Gregg [57] in a paper which formulated the theoretical background for the experimental investigation reported in this thesis. Working with the differential equations of momentum and continuity derived by von Karman, plus equations of energy and continuity of species, Sparrow and Gregg presented solutions which included non-zero axial velocity at the disk surface. Assuming constant fluid properties and Pr = Sc = 0.7, velocity, temperature, and concentration profiles as well as Nusselt numbers were given for several values of suction and injection. A more complete description of the equations used and results obtained will be given under THEORY in the latter part of this chapter.

b. The Effects of Nearby Stationary Surfaces

Flow in the vicinity of a rotating disk within an
enclosure has interested many investigators because the system models in an elementary way gas turbine disks and other machinery of practical importance. Within the context of the present investigation, at least a qualitative understanding of the effect of stationary surfaces is important in order to assure that nothing interferes with the normal flow due to a rotating disk in an "infinite environment".

Early work on flow caused by a rotating disk in an enclosure by Schultz-Grunow [52] and Daily and Nece [14], as well as the very recent studies by Tomlan and Hudson [64] and Lehmkuhl and Hudson [35], described the development of three boundary layers: one on the rotating disk, another on the surface opposite the disk, and a third on the surface normal to the disk.

The third boundary layer was found not to interfere with the first two, so long as the diameter of the surrounding enclosure was sufficiently greater than the disk; only ten percent greater diameter was necessary in the liquid system studied by Lehmkuhl and Hudson. Measurements in air by Richardson and Saunders [49] indicated that after 40 per cent of the disk radius outward from the edge, a fully developed radial jet has developed, decaying in a manner such that the product of velocity and radius are constant. Chanaud [9], however, in an
article published in 1971, measured velocity profiles and found similarity after about ten per cent of the disk radius away from the edge. After two to four radii, velocities were very low and no interference with heat transfer measurements would be expected. The box forming the environment for the rotating disk described in the present research is square in cross section, each edge 5.6 times the diameter of the transpired disk, thus eliminating possible interference.

Flow patterns in an enclosure depend significantly on axial clearance between the disk and the stationary surface opposite the disk. When the two surfaces are very close, the two boundary layers merge into couette flow. For greater separation, the flow is visualized as two individual boundary layers separated by a central core of fluid rotating as a body at constant angular velocity. The value of the angular velocity is some fraction of that of the rotating disk, and depends on the clearance. Extrapolation of the data of Daily and Nece [14] indicates decay to zero core velocity at a separation somewhat less than one disk diameter. Calculations by Tomlan and Hudson [63] indicated substantial though rapidly decaying interaction at one diameter. Mass transfer results by Kreith, Taylor, and Chong [34] with a naphthalene disk in air indicated only a ten per cent lowering of Sherwood number with separation of
Measuring the dissolution of cinnamic acid in water, Lehmkuhl and Hudson [35] concluded that the parameter determining interference with mass transfer is not the ratio of clearance to diameter, but rather Reynolds number based on clearance. They found no effect on Sherwood number for \( \frac{\omega H^2}{v} > 50 \). Care must be used in extrapolating this conclusion quantitatively to other systems, for the Schmidt number of the system studied was approximately 600, indicating a mass transfer boundary layer much thinner than the momentum boundary layer. Flow visualization studies by the same authors indicated interaction of hydrodynamic boundary layers below a clearance Reynolds number of 430.

Many other investigators have reported theoretical and experimental results for flow [2, 6, 12, 30, 39, 40], heat transfer [41, 44, 47, 48], and mass transfer [33, 38] in enclosures, but most included also the effect of forced radial inflow or outflow through a central opening in the stator and the periphery. The one article which does not consider such auxiliary flow is by Jawa [30], who used such unusual nomenclature (and who failed to cite several obvious precursors to his work) that the value of his publication is difficult to ascertain.

In conclusion, it can be said that the apparatus
used for the work described in this thesis was designed to avoid interference of the surroundings with the flow and heat transfer from the rotating disk. Clearance between the disk and the surface opposite it was greater than ten diameters and the smallest clearance Reynolds number was greater than seven million. Thus, regardless of the study used to base the conclusion on [14], [34], or [35], it is apparent that no interference is expected.

c. Heat and Mass Transfer From a Rotating Disk

The rotating disk geometry has been the subject of many energy and mass transfer investigations. Because it is a model of the gas turbine disk, heat transfer has been studied extensively, and because it provides a simple and effective tool for measuring reaction rates, diffusivities, and electrochemical transference numbers in liquid systems (see Levich [36] for example), mass transfer has also received much attention. Most of the work discussed below will deal with heat transfer, since the present investigation is limited to heat transfer in air, with a Prandtl number of 0.7. A few mass transfer results will be included for cases with Schmidt number not widely different from unity since, in those cases, the mass transport boundary layer thickness is the same order of magnitude as the momentum transport layer. This
provides situations with behavior reasonably analogous to the present case.

An early heat transfer study was by Wagner [66] in 1948. After correcting von Karman's velocity profiles for laminar flow and assuming constant properties, Wagner presented integral expressions for the temperature distribution. He concluded that Nusselt number, defined as \( \frac{h\delta}{k} \), where \( \delta = \left( \frac{v}{\omega} \right)^{\frac{1}{3}} \), was equal to 0.34 and independent of Reynolds number.

Millsaps and Pohlhausen [43] included viscous dissipation in their study and assumed a parabolic temperature distribution on the disk. Unfortunately, they used an incorrect form of the energy equation, negating much of the value of their work. Oudart [46] attempted separation of variables for the energy equation, assuming \( T(r,z) = T_1(z) + T_2(r) \). His results for \( T_2 = 0 \) gave \( \text{Nu} = 0.27 \), very close to Millsaps and Pohlhausen's value of 0.28.

Early theoretical work dealing with heat transfer in turbulent flow over a disk was by Davies [15]. He assumed a one-seventh power velocity distribution and calculated heat transfer coefficients using the Reynolds analogy. His results agreed fairly well with the measurements of Cobb and Saunders [10], described below. Davies and Baxter [16] discussed the case with a power law radial
distribution of disk temperature.

Experimental investigations were carried out by Young [68] and Cobb and Saunders [10]. Both studies indicated that natural convection was important below Re = 40,000 for their disks. Quantitative conclusions from Young's work are not warranted, however, due to several sources of heat loss which were not adequately accounted for. The results of Cobb and Saunders for laminar flow could be correlated by Nu = 0.36, within ten per cent. At Re = 20,000, natural convection appeared to increase energy loss by about ten per cent. In the turbulent regime, the data tended asymptotically toward Nu = 0.015 Re$^{0.3}$.

Richardson and Saunders [49] indicated that Cobb and Saunders results should be corrected upwards about ten per cent due to an error in ambient temperature measurement. An additional correction was made for natural convection, since in some experiments wall-to-ambient temperature differences reached 140°F. The laminar flow data were correlated by the equation

$$\frac{\text{Nu}}{\text{Re}} = 0.40 \left(1 + \frac{\text{Gr}^\frac{1}{2}}{\text{Re}} \right)$$

with Nu defined as before, and Grashof number defined with disk radius as a typical length. As pointed out by Richardson and Saunders, the ratio Gr/Re$^2$ arises when the
equation of motion is made non-dimensional.

Other work dealing with natural convection used the ratio $Gr/Re^2$ to correct for buoyancy effects. For example, Fox [21] calculated a correction to the work of Hering and Grosh [27], a study of heat transfer from a rotating cone. With a cone half-angle of $\pi/2$, this reduces to the rotating disk case. Under these circumstances, $Nu$ as defined before had a negligible correction for $Gr/Re^2 = 0.01$ and about a ten per cent increase for $Gr/Re^2 = 0.1$. For $Gr/Re^2 = 0.5$, the correction was nearly 70 per cent. These corrections are nearly four times those proposed by Richardson and Saunders, the latter basing their information on experiment rather than computer calculation. In either case, the results indicate that the present research is unaffected by natural convection, since in no case did $Gr/Re^2$ exceed 0.0012.

Further work concerning the rotating cone geometry includes that of Tien [62] and Fukusako et al. [22]. Tien considered the case of non-uniform surface temperature, including power-law distribution and a step-change in surface temperature. The latter case was also studied by an analogous mass transfer experiment, using a cone partially covered with naphthalene. Fukusako et al. considered only uniform surface temperatures, but included
the possibility of blowing and suction as well. Calculations were carried out using power series expansions for velocity profiles. The accuracy of the work is dependent upon errors involved in truncation of the series, but this topic was not discussed in the paper.

Other investigators used naphthalene mass transfer experiments for rotating disk studies. Kreith, Taylor, and Chong [34] studied both laminar and turbulent flow with an eight inch disk. Results corroborated calculations using the technique of Wagner [66], altering the numerical values to account for a Schmidt number of 2.4 rather than a Prandtl number of 0.7. The effect of placing a stationary plane parallel to the disk was also studied (as described previously in this review).

Recalculation of the heat transfer case using modern computers has been carried out by Ostrach and Thornton [45], who included the possibility of variable physical properties (but reported results only for the case of \( \rho u_0 = \text{constant} \)), Lugt and Schwiderski [37], who converted the differential equations to integral equations before calculating temperature profiles and Nusselt numbers, Tien and Tsuji [63], who considered a disk rotating in an otherwise uniform flow field directed normally to the disk, and Andrews and Riley [1], who studied unsteady state heat transfer.
The work of Sparrow and Gregg [57] has been mentioned previously, and will be discussed in more detail in a later section.

d. Transpiration Cooling

A thorough review of the hundreds of individual contributions to the widely studied field of transpiration cooling is beyond the scope of this paper. Fortunately, there exist already several excellent review papers, for example Gross et al. [26] in 1961, Dewey and Gross [18] 1967, and three very broad papers in 1972 by Mills and Wortman [42], Wortman, Mills, and Hoo [67], and Kays [31].

Gross [26] summarized the early work in two-dimensional boundary layers with transpiration cooling. Results are given for constant property analyses, giving velocity profiles, \( \frac{1}{2}C_fRe^\frac{1}{2} \), \( Nu_xRe_x^{-\frac{1}{2}} \), and recovery factor as a function of a blowing parameter \( \frac{\rho_w V_w}{\rho_e U_e} Re_x^{\frac{1}{2}} \). For variable properties, ignoring thermal diffusion and Soret effects, previous results of many investigators were brought together and correlated with a simplified method of accounting for foreign gas injection. The recommendation was to correct friction coefficient and Stanton number results for air injection with the molecular weight ratio \( (M_{air} / M_{gas})^{1/3} \).
Subsequent experiments showed this to be an over-simplification (see Sparrow et al. [58], for instance), but the paper served the excellent purpose of collecting the existing information and causing additional thought and experimentation.

The lengthy paper in 1967 by Dewey and Gross [18] tabulated numerical results for a very wide range of calculations for two-dimensional and axi-symmetric flow. Included were the effect of viscous dissipation, injection, acceleration, angle of attack, and physical property variation.

Kays [31] summarized the work of over a decade in the Heat Transfer Laboratory at Stanford. The effect of injection and suction of air into (or out of) the turbulent boundary layer over a flat plate was studied extensively. Results were correlated using the Prandtl mixing length theory, modified with a simple damping function

\[ \& = 0.4 \mathrm{yD} \]

\[ D = \frac{y^+}{B^+}, \quad y^+ \leq B^+ \]

\[ D = 1.0, \quad y^+ > B^+ \]

where \( B^+ \) depends on both the pressure gradient and the injection rate. \( B^+ \) was chosen to force the calculated velocity profiles to match experimental ones at \( y^+ = 80 \).

This technique was capable of matching friction coefficient
and Stanton number results even for such difficult experimental situations as step functions in injection, step functions in pressure gradient, and combinations of those conditions.

Mills and Wortman [42] reported the results of calculations for binary boundary layers at a two dimensional stagnation point, including thermal diffusion effects. They defined heat transfer coefficient $g_h$ as the energy transferred, divided by an enthalpy driving force ($H_e - h_{es}$), where $H_e$ is the total enthalpy of the external flow, and $h_{es}$ is the enthalpy of a gas of external flow composition, but at surface temperature. Defining $B_h = \dot{m}/g_h^*$, where $\dot{m}$ is total mass flux at the surface and $g_h^*$ is $g_h$ for zero injection, the results were correlated by

$$g_h = \frac{a_{hi} B_h}{g_h^* \exp(a_{hi} B_h) - 1}$$

The factor $a_{hi}$ is a species correction factor and is given by

$$a_{hi} = 1.3 \left( \frac{M_{air}}{N_i} \right)^{1/3} \left( \frac{\gamma}{C_{Pi}} \right)^{1/2} \left( \frac{2.5 R}{T} \right)$$

2. THEORY

a. Equations of Change

The article by Sparrow and Gregg [57] forms the theoretical basis for the experiments conducted by this author and
reported herein. It should be emphasized that this section, which develops and transforms the equations of change, is not original with this author, but rather is an exposition of the work of Sparrow and Gregg.

The mathematical expressions which state the laws of conservation of momentum, energy, and mass follow. As written here, the equations are valid for an incompressible, Newtonian fluid, the properties of which are independent of temperature and composition. The fluid is a two-component mixture. Viscous dissipation is assumed negligible. In cylindrical coordinates, as depicted in Figure II-1, one obtains:

Conservation of Momentum:

\[
\begin{align*}
\frac{dv_r}{dt} - \frac{v_0^2}{r} &= -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left( \nabla^2 v_r - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} - \frac{v_r}{r} \right) \\
\frac{dv_\theta}{dt} + \frac{v_r v_\theta}{r} &= -\frac{1}{\rho r} \frac{\partial p}{\partial \theta} + \nu \left( \nabla^2 v_\theta + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{v_\theta}{r^2} \right) \\
\frac{dv_z}{dt} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \nabla^2 v_z
\end{align*}
\]

[II-1] [II-2] [II-3]

Conservation of Energy:

\[
\frac{dT}{dt} = \alpha \nabla^2 T
\]

[II-4]
Figure II-1. Rotating Disk Coordinates
Conservation of Mass:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r v_r \right) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0 \]  \hspace{1cm} \text{[II-5]}

Conservation of Component 1:

\[ \frac{dW_1}{dt} = D \nabla^2 W_1 \]  \hspace{1cm} \text{[II-6]}

In the above equations, the following operators were used:

\[ \frac{d}{dt} = v_r \frac{\partial}{\partial r} + \frac{v_\theta}{r} \frac{\partial}{\partial \theta} + v_z \frac{\partial}{\partial z} \]

\[ v^2 = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \]

By following the lead of von Karman [65], the following assumptions are made:

1. Angular symmetry \( \frac{\partial}{\partial \theta} = 0 \)

2. The velocity, temperature, and concentration profiles are similar, i.e., by suitable transformation, all \( v_r \) profiles, for example, can be collapsed onto a single curve.

3. Temperature, concentration, and \( v_z \) are functions only of \( z \), as are \( \frac{v_r}{r} \) and \( \frac{v_\theta}{r} \).

Note: Measured velocity profiles have proven the similarity assumptions for velocity, at least, to be correct. See [8], for example.
von Karman suggested the following change of variables, amplified by Sparrow and Gregg to include temperature and concentration:

Independent Variable:

\[ \eta = z \left( \frac{\omega}{\nu} \right)^{\frac{1}{2}} \]

Dependent Variables:

\[ F(\eta) = \frac{v_r}{r \omega} \quad P(\eta) = \frac{p}{\mu \omega} \]
\[ G(\eta) = \frac{v_0}{r \omega} \quad \theta(\eta) = \frac{T - T_c}{T_w - T_c} \]
\[ H(\eta) = \frac{v_z}{(\omega \nu)^{\frac{1}{2}}} \quad \phi(\eta) = \frac{W_1 - W_{1c}}{W_1 - W_{1c}} \]

After the introduction of these new variables into the general equations of change and the assumption of angular symmetry, the following set of ordinary differential equations is obtained:

Continuity: \[ H' + 2F = 0 \quad \text{[II-7]} \]
Momentum: \[ F'' = HF' + F^2 - C^2 \quad \text{[II-8]} \]
\[ G'' = HG' + 2FG \quad \text{[II-9]} \]
\[ H'' = HH' + P' \quad \text{[II-10]} \]
Continuity of Species: \[ \frac{1}{Sc} \phi'' = H \phi' \quad \text{[II-11]} \]
Energy: \[ \frac{d}{d \eta} \theta'' = H' \theta' \] [II-12]

The primes denote differentiation with respect to \( \eta \).

Sparrow and Gregg eliminated \( F \) from Equations [II-8] and [II-9] by substituting [II-7].

\[ H'''' = HH'' - \frac{1}{2} H'^2 + 2G^2 \] [II-13]

\[ G'' = HG' - H'G \] [II-14]

They subsequently ignored Equation [II-10] since they were not interested in calculating pressure distributions. Thus Equations [II-11, 12, 13 and 14] were solved numerically, subject to the boundary conditions.

At \( \eta = 0 \)

\[ H = H_w \]
\[ H' = 0 \]
\[ G = 1 \]
\[ \theta = 1 \]
\[ \phi = 1 \]

As \( \eta \to \infty \)

\[ H' \to 0 \]
\[ G \to 0 \]
\[ \theta \to 0 \]
\[ \phi \to 0 \]

The results of the calculations were presented as \( F, G, H, \theta, \) and \( \phi \) profiles, each for five values of the
blowing parameter $H_w$. Also tabulated were values of $H''(0)$, $G'(0)$, $\vartheta'(0)$, $\Phi'(0)$, and displacement, momentum, and thermal boundary layer thicknesses for a large number of values of $H_w$.

The significance of the values of the derivatives at the wall was twofold. Numerically, the problem was solved by estimating the initial conditions until the boundary conditions far from the disk were satisfied.

Secondly, the values of $\vartheta'(0)$, for instance, were also $-\text{Nu}$. This result can be seen by the following steps:

$$q = -k \left. \frac{\partial T}{\partial z} \right|_{z = 0}$$  \hspace{1cm} [II-15]

Introducing the definitions of $\eta$ and $\Theta$ into [II-15] results in

$$q = -k \left( T_w - T_\infty \right) \left( \frac{\omega^{\frac{1}{2}}}{\nu} \right) \vartheta'(0)$$  \hspace{1cm} [II-16]

Defining the heat transfer coefficient in the customary way

$$h = \frac{q}{T_w - T_\infty}$$  \hspace{1cm} [II-17]

and using $\delta = (\nu/\omega)^{\frac{1}{2}}$ as a typical length in the Nusselt number,
\[ \text{Nu} = \frac{h \delta}{k} \]

\[ \text{Nu} = \frac{q}{T_W - T_\infty} \frac{(\nu/\omega)^{1/2}}{k} \]  

\[ = -\Phi'(0) \]

A similar derivation for \( -\Phi'(0) \) shows that it is the Sherwood number, or mass transfer Nusselt number.

It should be noted that the purpose of the research reported in this thesis has been to experimentally determine heat transfer coefficients, and compare the experimental results with those predicted by Sparrow and Gregg.

b. \textbf{Energy Balance on the Rotating Disk}

In order to relate the variables measured experimentally (temperatures, flow rates, etc.) to the variable of engineering significance (heat transfer coefficient), an element of the porous surface of the disk is considered. Negligible energy loss radially along the surface of the disk is assumed since guard heaters preclude radial temperature gradients. At steady state, the energy balance for injection of air is as follows:
\[ q_{\text{in}} - q_{\text{out}} = \text{Accumulation} \]
\[ = 0 \text{ at steady state} \]

Thus
\[ q_{\text{in}} = q_{\text{out}} \quad \text{[II-19]} \]

Now
\[ q_{\text{in}} = q_{\text{injection}} = \rho_c \dot{H}_c v_c = n_c \dot{H}_c \quad \text{[II-20]} \]

where \( \dot{H}_c \) represents the enthalpy of the incoming air.

Next
\[ q_{\text{out}} = q_{\text{bulk motion}} + q_{\text{molecular}} + q_{\text{radiation}} \]

or
\[ q_{\text{out}} = q_B + q_M + q_R \quad \text{[II-21]} \]

Consider Equation [II-21] term by term. First
\[ q_B = \rho_w u_w v_w = n_w^\prime \hat{H}_w \]

Next

\[ q_M = -k \left. \frac{dT}{dz} \right|_{z=0} \]

But, since the temperature gradient is not usually determined experimentally, \( q_M \) is rewritten using the standard engineering simplification

\[ q_M = h \left( T_w - T_\infty \right) \]

Finally, assuming gray-body radiation into a complete enclosure,

\[ q_R = \sigma \varepsilon \left( T_w^4 - T_\infty^4 \right) \]

Collecting the terms in Equation [II-21] and equating their sum to \( q_{in} \), one obtains

\[ n_C \hat{H}_C = n_w^\prime \hat{H}_w + h(T_w - T_\infty) + \sigma \varepsilon (T_w^4 - T_\infty^4) \] [II-22]

Recognizing that \( n_C = n_w \), and setting \( \hat{H}_C = \hat{H}_w = C_p CW \left( T_C - T_w \right) \), one may rearrange Equation [II-22], and solve for the heat transfer coefficient:

\[ h = \frac{n_w^\prime C_p CW \left( T_C - T_w \right) - \sigma \varepsilon (T_w^4 - T_\infty^4)}{T_w - T_\infty} \] [II-23]

Equation [II-23] illustrates how \( h \) can be determined from the experimental data. It forms the basis for the data reduction program in Appendix E.
III. EXPERIMENTAL EQUIPMENT AND INSTRUMENTATION

The equipment used to measure heat transfer coefficients for a transpired rotating disk is shown schematically in Figure III-1. It includes:

1. A regulated, measured source of clean, dry, heated air;
2. a leak-proof means of transferring the air to a rotating pipe;
3. a means of rotating the pipe;
4. the heat transfer surface;
5. a guard heater to prevent radial heat loss;
6. a large enclosure; and
7. instrumentation for temperature measurement.

Each of these items will be discussed in detail below.

1. SOURCE OF AIR

The source of air for injection was a 100 psi compressor. The compressed air was dried with CaCl₂ (Drierite packed in two-inch diameter, two-foot long pipe) and passed through an oil trap and two, single stage regulators, set at 25 and 10 psi, respectively.

A needle valve allowed the flow rate to be varied. Actual mass flow rate was measured with a square-edged orifice, calibrated by a previous investigator [20] to ± 0.5%.

The air was heated with two 396-watt, 1/2" x 8' heating tapes.
Figure III-1. Schematic Flow Diagram Indicating Variables Measured
One was wrapped around a 1/2" tube and was regulated by a variable transformer. The other was wrapped around a one-inch stainless steel tube. Temperature control was effected by a Model 50 Proportional Controller manufactured by Radio Frequency Labs, Inc., Boonton, New Jersey, which regulated the power delivered to the second heating tape. A third heating tape was sometimes used as a preheater when higher surface temperatures were required.

From the heater, the air flowed to the rotary union through a flexible teflon hose. A flexible coupling was required to avoid non-axial loads, which would cause wear and leakage in the union.

Firebrick were stacked around the heater farthest downstream and fiberglass insulation was wrapped around the flexible hose. Without this insulation, the temperature of the disk was not sufficiently high to be measured accurately.

2. **ROTARY UNION**

The rotary union provided a seal between the stationary air line and the rotating shaft on which the disk was mounted. It was a Model 1105-29 union, manufactured by Deublin Company, Chicago.

When operating at high temperatures and high speed, the rotary union gradually lost the oil which cooled and lubricated the sealing faces. This problem was overcome by attaching an eye dropper via a flexible plastic tube to the oiler fitting on the union. The eye dropper was then filled with spindle oil and
supported several inches above the union. This afforded a steady gravity feed, replacing the oil as necessary.

A fitting was inserted into the stationary part of the union to allow a thermowell to be placed directly in line with the axis of the rotating pipe leading to the disk. This permitted measurement of the injectant temperature by a stationary thermocouple, thus eliminating the need for slip rings with attendant electrical noise and thermal gradients.

A recurring experimental difficulty was a fatigue failure in the 1/8" OD x 1/16" ID thermowell after one to three days of operation. A 3/16" OD thermowell had no greater success. No further provision was made to prevent the failure. The thermowell was merely replaced when it failed.

Leakage in the union was tested by removing the disk, replacing the thermocouple tube with a shorter tube, and inserting a rubber stopper in the outlet of the rotating shaft. Total leakage flow through all fittings in the air line after the metering section was too low to be measured directly (<10^-3 lb air/min). An indirect method of estimating leakage (see Appendix F) indicated the loss was less than one-half per cent of the injection flow; therefore, the seal was considered satisfactory.

3. ROTATING PIPE

The heat transfer surface was mounted on a pipe 1-1/8" OD x 1/2" ID x 8 1/2" long. The pipe was mounted in two Fafnir Type
LCJ ball bearings attached to 1/2" thick steel plates. It was driven through a pulley and fan belt by a 1/2 horsepower DC motor. Constant speed was assured by a General Electric Statotrol speed controller. Rotational rate was measured with a Strobotac (General Radio), calibrated before each measurement. Accuracy of the Strobotac is reportedly 1\%.

4. **HEAT TRANSFER SURFACE**

The porous surface consisted of thin, fine-mesh stainless steel wire cloth. The material was 0.005 inch thick Poromesh (Bendix Corporation) 200 x 1400 mesh. It was cut to a diameter somewhat larger than the heat transfer area, then crimped by forcing it into a mold exactly the same dimensions as the aluminum cup over which it was placed. The wire cloth was then fastened to the cup with epoxy adhesive around the outside edge. Within the cup was placed an insulating thickness of transite. A detailed view of the entire rotating apparatus is given in Figure III-2.

In addition to providing support of the porous surface, the cup acted as a plenum chamber of constant pressure beneath the surface. Twelve thicknesses of Whatman Fiberglass filter paper, Grade G/K, provided sufficient pressure drop to ensure a uniform velocity distribution across the disk. Measurements with a velocimeter indicated a variation of \(\pm 3 \, 1/2\%\) from the average at an injection velocity of 1.4 ft/sec on a previous transpired disk of
1. Porous Surface  
2. Inlet Air Thermocouple (Stationary)  
3. Filter Paper  
4. Transite Insulation  
5. Guard Heater (Aluminum)  
6. Heating Tape  
7. Rotary Union  
8. Teflon Flex Hose  
9. Thermowell  
10. Thermocouple Leads

Figure III-2. Details of Rotating Apparatus
essentially the same design as the new disk. Since 0.3 ft/sec was the highest injection velocity used in the heat transfer experiments, actual velocity variation across the disk was no greater than 3 1/2%.

Further indication of both uniformity with respect to radial distance and symmetry, with respect to angular displacement, were several measurements made with the infrared radiometer. With heated air passing through the stationary disk, temperature was measured at the center and at a fixed radius (near the edge of the disk) at four angular positions, displaced π/2.

All readings were within 0.7°F, or approximately within the meter's inherent accuracy at that particular temperature. Actual readings were (°F):

center (before) 180.0
edge 180.4, 179.8, 180.0, 180.0, 179.9 (repeat of first point)
center (after) 179.7

Similar measurements made at other times gave comparable results.

An attempt was made to measure the emissivity of the porous surface, using the method described in references [20] and [60]. However, the measurement was not successful because the accuracy of the calculations depend upon the accuracy of the temperature difference \((T_c - T_w)\). For all runs tried with a stationary disk, the largest value of \((T_c - T_w)\) was 10°F (for one run) and the other values were 4-8°F. Since the accuracy of the instrument was ascertained to be \(\pm 0.5\) to \(\pm 2°F\) (see Appendix C, SOURCES OF ERROR),
the reason for the failure of this technique for measuring emissivity is apparent.

Because this measurement was not successful, a value had to be assumed for emissivity, with a suitable allowance made for the uncertainty this caused in subsequent calculations. (See Appendix C.) The value assumed was 0.3, a value measured by Elzy [20] for the same type of material made by the same manufacturer.

5. GUARD HEATER

To minimize heat losses from the surfaces of the cup other than the porous disk, a guard-heater was placed around it. The guard heater consisted of 3/8-inch of aluminum with a heating tape and insulation around the outside diameter. The inside diameter of the vertical face of the heater was 1/4-inch greater than that of the cup, allowing 1/8-inch clearance. Thermocouples were used to monitor the temperature of the inside surface of the heater. Power to the heating tapes was supplied by a variable transformer allowing the guard heater temperature to be adjusted so that the disk surface showed no radial variation in temperature.

Axial heat losses from the edge of the porous surface were minimized by a piece of transite attached to the top of the guard heater. The transite was carefully placed approximately 1/32-inch below the disk in order for minimal interference with the boundary layer leaving the rotating surface. The inside diameter of the transite was 4.125 inches, compared to 3.90 inches for the porous
disk. This left a thin ring at the outside diameter of the cup exposed directly to the surroundings and not insulated by the transite. This thin, nonporous ring was thus 10.6% of the total surface exposed and required a correction to be applied to the heat transfer data. (See EXPERIMENTAL RESULTS.)

6. **ENCLOSURE**

In order to protect the boundary layer induced by the rotation of the disk from external convection currents, an enclosure 22" x 22" x 40" was placed around the apparatus.

The interior sides (except for the lower two inches) and bottom of the enclosure were solid, thus preventing air currents or light from entering. These surfaces were painted flat black.

While the lower two inches of the sides and the top of the enclosure were permeable to air, they still shielded the interior of the enclosure from convection currents in the room. The lower two inches of the sides were covered with fiberglass, about one-half inch thick. The fiberglass was below the line-of-sight of the heat transfer surface. Over the top of the enclosure were placed three thicknesses of black cloth. This covering and the fiberglass prevented all visible radiation from entering the box (important for the infrared temperature detector, as discussed below), while still permitting the flow induced by the rotating disk (including air injected through the porous surface) to enter and leave the immediate environment.
7. TEMPERATURE MEASUREMENT

a. Thermocouples

All temperatures measured, except that of the rotating disk surface, were determined by calibrated iron-constantan thermocouples, using a Doric integrating digital voltmeter. The voltmeter was checked to 1 μV each day against a specially aged and stable zener diode, which was an integral component within the instrument. The voltmeter was considered accurate to at least 3 μV (0.1°F).

An ice bath, using tap water ice and deionized liquid water and constructed in accordance with Aerospace Recommended Practice 691 [55], was used as a reference temperature for the thermocouple measurements.

Switching between thermocouples was accomplished by a Leeds and Northrup Type 31, 16-position, low noise selector switch. Low thermal solder was used to connect wires to the switch. Also, it was placed in an insulated container to prevent convection currents in the room from causing thermal gradients in the switch.

The iron-constantan thermocouple wire had been previously calibrated in an oil bath against a platinum resistance thermometer. For the calibration, a Leeds and Northrup K-3 Potentiometer was used to measure the emf of the thermocouple, with an ice bath for the reference junction. The potentiometer reading was repeatable within 1 μV (1/30°F). Samples from the beginning and end of a 500 ft. reel of 40 gauge duplex wire (teflon-insulated in a fiber-
glass sheath) were found to give calibration curves identical within 1/30°F. Since all thermocouples used were from the same reel, it was assumed they were of uniform composition and had identical calibrations.

b. Infrared Radiometer

The temperature of the porous surface was measured with a Spartan infrared radiometer manufactured by Irtronics, Inc., Stamford, Connecticut. The instrument was focused visually on the surface where the temperature was to be measured. At 40 inches, its field of view was a circle 0.75 inch in diameter. The lead sulfide infrared detector was sensitive to wavelengths from 1.9 to 2.9 microns. The instrument was connected to a constant voltage transformer to avoid direct connection to line voltage and to stabilize its power source.

To obtain the sensitivity and range required, it was necessary to modify the bias on the lead sulfide detector from the value set by the manufacturer. The emissivity correction was set at "zero" (lowest possible emissivity), and the bias was set at 75 volts. This permitted measurement of temperatures in the range 125°F to 200°F. As illustrated by typical calibration curves given in Figure III-3, sensitivity varied from about 0.1 mv/°F at 130°F to about 0.6 mv/°F at 180°F. The instrumentation output, 0-50 mv, was transmitted by shielded cable to a second Doric integrating digital voltmeter.
Figure III-3. Typical Calibration Curves for Radiometer
Calibration of the radiometer was effected by aiming it at a two-inch diameter stationary disk made of porous material from the same lot as the rotating disk. This disk was mounted in a corner of the enclosure during calibration only, then removed before a rotational run was started.

The stationary calibration disk was the same distance from the radiometer as the rotating disk (40 inches). It, too, had a guard heater wrapped with a heating tape regulated by a variable transformer. Spot welded to the underside of the disk were three thermocouples, one at the center, the other two approximately 3/8" and 1/2" from the center in opposite directions. These thermocouples were between the porous stainless steel cloth and 12 thicknesses of filter paper used to make the flow uniform. The flexible teflon hose normally connected to the rotary union was connected to the calibrator for a calibration run. The power to the guard heater was adjusted to make the disk isothermal, as indicated by the three thermocouples.

Temperature was measured by the radiometer by comparing the infrared radiation emanating from the target to that coming from the interior of the instrument. Therefore, it was important to maintain a constant internal temperature, especially for targets near room temperature. Two steps were taken to maintain the interior at a constant temperature. First, cooling water was circulated through the jacket of the instrument at the rate of approximately ten gallons per hour. The source of the water was a
constant temperature bath (Forma Scientific Corporation), regulated at 70 ± 0.1°F. Second, the instrument was allowed to warm up at least 24 hours before data were taken. During the major period of data acquisition, the instrument was never turned off.

The infrared radiometer was equipped with a telescopic sight to allow visual aiming through the same lens which received the infrared signal. By aiming at a 1/16"-diameter source of light and comparing visual signal with electronic output, it was discovered that the visual aiming point was displaced approximately 1/4" (at 40 inches from the lens) from the center of the infrared aiming point. Therefore, all visual sighting had to compensate for this displacement.

Since the detector was sensitive to all radiation it received of appropriate wavelength, it was considered necessary to avoid any direct or reflected light in the room. For this reason, the enclosure was painted flat black inside, all cracks were sealed, and the top was covered with three thicknesses of black cloth to prevent light from entering.

It was found that the instrument was sensitive to mechanical vibration. Hence, it was necessary to mount the radiometer separately from the enclosure, since the enclosure was vibrated slightly by the motor, belt, pulley, etc. The instrument was bolted to a 1/4" steel plate which in turn was mounted on two parallel rails.
The rails were supported at one end by a wall bracket, at the other by a three-inch diameter stainless steel pipe running from floor to ceiling (with cellulose sponges at top and bottom to dampen vibrations). The rails permitted the instrument to slide from the center of the box, over the rotating disk, to one corner, over the stationary disk used for calibration.
IV. EXPERIMENTAL PROCEDURE

The experimental procedure used to acquire data is described below in two parts: the procedure used to calibrate the infrared instrument, and the procedure used to determine heat transfer from the transpired rotating disk.

1. CALIBRATION

Since earlier experimentation with the infrared radiometer had shown its calibration curve of millivolts output versus temperature to vary somewhat from day to day, it was considered necessary to obtain a new calibration on each day of operation. Variation from one day to the next was lessened by leaving the power to the instrument on. (Figure III-3 illustrates the shift encountered from one day to the next.) The instrument was always warmed up at least 24 hours before taking any data. Variation of the calibration on a given day was found to be negligible.

Before each day's operation, the air compressor surge tank was drained of any water or oil accumulated from previous days. After putting the calibrator in place in one corner of the enclosure, the flexible teflon hose leading from the air heater was attached, the air heater and guard heater were turned on at a level appropriate for the lowest temperature required for calibration, and the air valve opened to allow approximately 0.1 lb/min to flow through the two-inch calibrator disk. During warm-up, the ice
bath was filled with cracked ice and distilled water, the radiometer was placed directly above the calibrator, and the enclosure was carefully checked for light leaks. After about 1 1/2 hours, the calibrator was usually isothermal and steady.

Before a reading was taken for the calibration curve, the guard heater was adjusted so that the range of the three thermocouples attached to the underside of the 0.005-inch thick porous stainless steel surface was less than five microvolts. Usually the variation was less than four microvolts, i.e., the average reading was ± 2 μv. This range corresponds to the average temperature ± 0.07°F.

When the disk was determined to be isothermal, thermocouple readings were taken. Radiometer readings were taken over a one to two minute period, then the thermocouple readings were checked. The average temperature never changed significantly (more than two or three microvolts).

The radiometer output was led to a Doric integrating digital voltmeter via a shielded cable. The signal was not absolutely steady, but had considerable electronic noise which was not filterable with a one or ten microfarad capacitor across the terminals of the voltmeter. Readings were recorded as the maximum and minimum digital display observed (repetition rate about one per second) over a period of one to two minutes read to the nearest 0.01 millivolt. The average reading was then taken as the mean of the extremes.

While a better averaging method might have been used, it was
felt that any gain in accuracy would be insignificant compared to the inherent accuracy of the instrument. To illustrate: at the lowest temperature used, the maximum and minimum were typically 1 1/2 to 2 1/2°F above and below the mean. At higher temperatures, the interval was typically ±1/2 to 1°F. It is the author's personal judgement that the true mean must surely lie within one-half of the extreme range. In other words, at the lowest temperatures, the true mean was within 3/4 to 1 1/4°F of the value reported, while at higher temperatures it was within 1/4 to 1/2°F. Further discussion of the accuracy of the radiometer is found in Appendix C.

As mentioned in the description of the infrared instrument under EXPERIMENTAL EQUIPMENT AND INSTRUMENTATION, the radiometric readings were affected by vibration. While the special mounting described earlier dampened this effect materially, some residual effect remained. In order to assure that the calibration was made under the same conditions as the subsequent heat transfer measurements, the rotating disk was revolved at about 2000 RPM to provide vibrations typical of operating conditions. Previous measurements had shown negligible difference from 2000 to 6000 RPM, but usually 2-5% difference in millivolt readings between stationary and 2000 RPM. (Figure III-3 illustrates the effect of vibration on the calibration curve.) Thus calibration readings were made with the rotating disk moving while the radiometer was aimed at the stationary calibrator disk.

Calibration points were taken every 6-8°F from 125-140°F and
every 10-12°F from 140-200°F. It usually required about 30 minutes for new air heater and guard heater settings to bring about steady, isothermal readings on the calibrator disk. Readings were taken at lowest temperatures first, then at successively higher temperatures. Five to six hours were required to complete a calibration.

2. **HEAT TRANSFER MEASUREMENT**

After completing a calibration curve, the flexible hose was immediately connected to the rotary union and air flow and heat were adjusted to the values needed for the first rotational run. During the warm-up period of three and one-half to six hours, other changes in experimental set-up were made. For example, the calibrator was removed and replaced with a plywood dummy to keep light and air currents out. Four thermocouples (those measuring $T_c$, $T_\infty$, and two guard heater temperatures) were attached to a 16-point recorder (altered to scan the four points repeatedly). The recorder was not used for any final measurement, but was very helpful in observing the approach to steady state. Other preparations during warm-up included moving the radiometer to its position over the rotating disk, checking for light leaks, etc. Rotation was started one to two hours after air flow.

As the recorded temperatures approached steady state, infrared readings were also recorded (by hand) and trends observed. Each infrared reading consisted of a maximum and a minimum, as discussed under **CALIBRATION**. Readings were taken:
(a) at the center of the disk,

(b) at the edge of the disk (edge of the 3/4" field of view was within about 1/4" of the edge of the porous disk), and

(c) at the center of the disk.

Adjustments were made in the guard heater to cause center-edge-center readings to coincide as nearly as possible. With all temperatures close to 130°F, the indicated variation across the disk was often 0.5°F. All other runs had apparent variations of 0.3°F or less. Since these variations were less than the accuracy claimed for the instrument, the disk was taken to be isothermal at the average temperature observed.

When the disk appeared to be isothermal and temperatures appeared to be steady, the thermocouples were disconnected from the recorder and reconnected to the thermocouple switch. After all thermocouple readings were taken, the infrared readings were taken again to make certain that they had not changed. If all readings remained essentially unchanged 10-15 minutes later, that run was concluded and a new injection rate or a new rotational rate was started, steady state achieved, etc. After the first run on a given day, successive runs required from one and one-half to three hours to come to steady state.
V. EXPERIMENTAL RESULTS

The results of the research carried out are described under the following sections.

1. **Experimental Data**, which describes how the raw data are presented and how heat transfer coefficients were calculated and corrected;

2. **Correlation of the Data**, which describes how previous investigators have correlated both transpiration and non-transpiration heat transfer data and how the present data are put into similar correlations;

3. **Discussion of Results**, which comments on the scatter of the data around the correlations and compares the present data to theoretical predictions and to previous data for other systems.

1. **EXPERIMENTAL DATA**

The data collected are presented in Appendix A. This tabulation represents the data as taken during experimental runs and converted directly to calculated values by previously obtained calibration charts (see **EXPERIMENTAL EQUIPMENT AND INSTRUMENTATION** for a description of the calibration curves used and their accuracy).

As discussed in **THEORY AND PREVIOUS WORK**, temperature differences and flow rates are converted to heat transfer coefficients by the equation
\[ h = \frac{n_e c_p C_w (T_C - T_W) - \sigma e (T_W^4 - T_{\infty}^4)}{T_W - T_{\infty}} \]  

[\text{V-1}]

Values of \( h \) calculated by Equation [V-1] are also presented in Appendix A, listed as \( h_{\text{uncorrected}} \).

Calculation of \( h \) in this way tacitly assumes that all of the disk which loses heat by convection is porous. However, this was not the case. A thin ring around the outer edge of the disk was in "view" of the surrounding environment, but was not permeable (see Figure V-1). Because of this non-porous area (\( A_{\text{non-porous}} = 0.119 A_{\text{porous}} \)), a correction must be applied to the heat transfer coefficient calculated by Equation [V-1].

Two methods of correcting for the impermeable portion of the disk were used:

(a) Since the outer ring is not porous, it could be argued that the heat transfer coefficient for that area is \( h_0 \), i.e., the coefficient obtained for zero injection.

(b) Since the air injected through the porous disk immediately obtains a radial velocity component (boundary layer thickness \( \delta = (\nu/\omega)^{\frac{1}{4}} \) is a few hundredths of an inch), causing the injectant to flow over the non-porous area, it could be argued that the non-porous area is film-cooled; furthermore that \( h_{\text{film cooled}} = h_{\text{transpiration}} \), and thus the heat transfer coefficient cooled
Figure V-1. Relative Sizes of Porous and Non-Porous Areas
is essentially the same over both the porous and the non-porous areas of the disk.

The two assumptions described above ((a) \( h_{\text{non-porous}} = h_o \) and (b) \( h_{\text{non-porous}} = h_{\text{porous}} \)) represent two extremes. The author feels that assumption (b) represents the physical situation across the top of the disk, but that edge effects (such as those described by Richardson and Saunders [49]) probably induce a heat transfer coefficient higher than one would expect for pure transpiration cooling. Thus it is felt that assumption (a) probably gives a more accurate correction than assumption (b).

Values of heat transfer coefficient corrected in both ways described above are tabulated in Appendix B. All of the correlations which follow are also presented twice, using both sets of values of corrected \( h \). It was found that (See DISCUSSION OF RESULTS below) values for (a) ranged from 6-31% lower than for (b).

Data reduction was carried out by a PDP-9 computer, using FOCAL, a conversational language supplied by Digital Equipment Corporation for the PDP-series computers. A listing of the FOCAL program is given in Appendix E.

2. CORRELATION OF THE DATA

In order to correlate the data obtained in the experiments described herein, the results are presented in a number of different ways:

a. Heat transfer coefficients are often correlated by plotting
Nusselt number as a function of Reynolds number. Using variables relevant to the rotating disk geometry (typical length = R, typical velocity = \( \omega R \)), this might appear as

\[
\frac{hR}{k} \text{ vs } \frac{\omega R^2}{v}
\]  

[\text{V-2}]

Previous investigators of heat and mass transfer from solid rotating disks [see 10, 33, 68] have presented their experimental data in this way, as have others investigating heat transfer theoretically [see 66]. Consequently, the grouping \((hR/k)\) for three different injection rates has been calculated and plotted as a function of \(\text{Re} \left( \frac{\omega R^2}{v} \right)\) in Figures V-2a and b. Tabulated results are found in Appendix B.

b. In laminar boundary layer heat transfer, results are sometimes given as \(\text{NuRe}^{-\frac{1}{2}}\) vs. \(\text{Pr}\) (see Knudsen and Katz [32], p. 482, for example). For the disk,

\[
\text{NuRe}^{-\frac{1}{2}} = \frac{h(v/\omega)^{\frac{1}{2}}}{k}
\]  

\[\text{V-3}\]

Instead of disk radius, Sparrow and Gregg have chosen \((v/\omega)^{\frac{1}{2}}\) as the normalizing length variable, referred to as \(\delta\) here. Thus the right hand side in expression [V-3] may legitimately be labeled a Nusselt number. Sparrow and Gregg's analysis shows this Nusselt number to be independent of Reynolds number, a result also found in laminar flow over a flat plate for \(\text{NuRe}^{-\frac{1}{2}}\). Previously, Wagner [66] reached a similar conclusion and calculated \(\text{Nu} = \frac{h\delta}{k}\) for a solid disk.
Figure V-2a. Effect of Reynolds Number and Injection On Heat Transfer

(h corrected, assuming $h_{\text{non-porous}} = h_0$)
Figure V-2b. Effect of Reynolds Number and Injection On Heat Transfer

(h corrected, assuming $h_{non-porous} = h_{porous}$)
To test the validity of the conclusion that $\text{Nu}$ is independent of $\text{Re}$, the variables are plotted in Figures V-3a and b for three different injection rates. Data for other injection rates are not included in this figure because there were too few points to provide a significant test of the hypothesis.

c. In laminar boundary layer calculations which include the effect of transpiration, results are usually presented by plotting $\text{NuRe}^{-\frac{1}{2}}$ as a function of a group of variables called the blowing parameter

$$\frac{\rho_w V_w}{\rho_\infty U_\infty} \text{Re}^{-\frac{1}{2}}$$

(see Gross et al. [26], for example). In terms of disk variables, if $U_\infty$ is replaced by $\omega R$, this grouping becomes

$$\frac{\rho_w V_w}{\rho_\infty (\omega V)^{\frac{1}{2}}}$$

In Sparrow and Gregg's work, a non-dimensional axial velocity $H$ is introduced ($H = \frac{V}{(\omega V)^{\frac{1}{2}}}$). Since their analysis assumes constant properties, $H_w = \frac{V_w}{(\omega V)^{\frac{1}{2}}}$ effectively becomes a blowing parameter.

In order to compare the experimental results of this work to the theoretical results of Sparrow and Gregg, $\text{Nu} = \frac{h_\delta}{k}$ is presented as a function of $H_w$ and as a function of $\frac{\rho_w}{\rho_\infty} H_w$. These results are given for both sets of corrected heat transfer co-
Figure V-3a. Dependence of Nusselt Number On Reynolds Number

\( h \text{ corrected, assuming } h_{\text{non-porous}} = h_0 \)
Figure V-3b. Dependence of Nusselt Number On Reynolds Number
(h corrected, assuming $h_{\text{non-porous}} = h_{\text{porous}}$)
efficients described earlier. They are found in Figures V-4 a, b, c, d.

d. A measure of effectiveness for transpiration cooling is $h/h_0$, where $h_0$ represents the heat transfer coefficient with no injection. Bird, Stewart, and Lightfoot [4], p. 675, call this ratio $\Theta_T$, and plot $\Theta_T$ as a function of a flux ratio $R_T = \frac{\rho W}{\rho_w R_T}$. This plot is for laminar flow over a flat plate.

To permit a comparison of effectiveness of transpiration cooling for a rotating disk to effectiveness for a flat plate, the curve from Bird et al. is reproduced in Figures V-5a and b and data obtained in this work are given on the same graphs. Figure V-5a uses $R_T$ in the abscissa, while figure V-5b uses $\frac{\rho W}{\rho_w R_T}$. Both figures use $h$ assuming $h_{\text{non-porous}} = h_0$. In addition, the rotating disk results predicted by Sparrow and Gregg are also presented.

In order to determine the ordinate for Figure V-5, a value of $h_0$ must be ascertained for each Reynolds number. Two methods for finding $h_0$ suggest themselves:

(1) $h$ may be plotted as a function of injection rate and the resulting curve extrapolated to zero injection. This method is pursued in Appendix D. It is sufficient in this section to note that the data for any single Reynolds number are too sparse to permit credible extrapolation.
\[ \log_{10} \text{Nu} = -0.431 \times H_w - 0.533 \]

Uncertainty in Nu due to uncertainty in \( T_w \)

Uncertainty in Nu due to uncertainty in \( c \)

Uncertainty in Nu due to all factors considered

Sparrow & Gregg (Predicted)

Figure V-4a. Nusselt Number Dependence On Injection Parameter \( H_w \)

(h corrected, assuming \( h_{\text{non-porous}} = h_0 \))
Uncertainty in $\text{Nu}$ due to uncertainty in $T_w$.

Uncertainty in $\text{Nu}$ due to uncertainty in $\sigma$.

Uncertainty in $\text{Nu}$ due to all factors considered.

Sparrow & Gregg (Predicted)

$\log_{10} \text{Nu} = -0.325 H_w - 0.559$

Figure V-4b. Nusselt Number Dependence On Injection Parameter $H_w$

(h corrected, assuming $h_{\text{non-porous}} = h_{\text{porous}}$)
\[ \log_{10} \text{Nu} = -0.505 \frac{\rho W}{\rho_\infty} H_w - 0.519 \]

Uncertainty in Nu due to uncertainty in Tw

Uncertainty in Nu due to uncertainty in \( \sigma \)

Net uncertainty due to all factors considered

Predicted by Sparrow & Gregg (Except for \( \frac{\rho W}{\rho_\infty} \) factor)

Figure V-4c. Nusselt Number Dependence On Injection Parameter \( \frac{\rho W}{\rho_\infty} H_w \)

(h corrected, assuming \( h_{\text{non-porous}} = h_0 \))
\[ \log_{10} \text{Nu} = -0.330 \frac{\rho_w}{\rho_\infty} H_w - 0.549 \]

Uncertainty in Nu due to uncertainty in \( T_w \)

Uncertainty in Nu due to uncertainty in \( \sigma \)

Net uncertainty due to all factors considered

Predicted by Sparrow & Gregg (Except for \( \frac{\rho_w}{\rho_\infty} \) factor)

Figure V-4d. Nusselt Number Dependence On Injection Parameter \( \frac{\rho_w}{\rho_\infty} H_w \)

(h corrected, assuming \( h_{\text{non-} \text{porous}} = h_{\text{porous}} \))
Figure V-5a. The Ratio $h/h_0$ versus $R_T$

(Assumes $h_{\text{non-porous}} = h_0$)
Figure V-5b. The Ratio $\frac{h}{h_0}$ versus $\frac{\rho_w}{\rho_\infty R_T+1}$

(Assumes $h_{\text{non-porous}} = h_0$)
The data of Figure V-4 can be extrapolated to zero injection, giving a value of \( \text{Nu}_0 \) from which \( h_0 \) can be calculated.

The advantage of the second method is that all of the data for all Reynolds numbers are used on the same curve, lending greater confidence to the extrapolation. The values of \( h_0 \) found in this way were used for \( h/h_0 = \Theta_T \) in Figures V-5a and b. Table D-1 includes values of \( \Theta_T \) and \( R_T \).

3. DISCUSSIONS OF RESULTS

Figures V-2 and V-3 illustrate most dramatically the effect of transpiration upon heat transfer. It is apparent from the data that injection through a porous surface can appreciably diminish the rate of heat transfer to or from that surface.

While the scatter of the data for any one injection rate appears to be great due to the scale of the drawing, every individual point falls within 20% from the lines drawn. These lines and the confidence intervals indicated were determined by a linear regression program available in The University of Arizona CDC Computer Library.

The purpose of Figure V-3 is to test the hypothesis that \( \text{Nu} \), defined as \( \frac{h(\nu/\omega)^{\frac{1}{2}}}{k} \), depends only upon injection rate and is independent of \( \text{Re} \). A "t" test (as described in standard statistics textbooks, see Bowker and Lieberman [5] for example) was used to test the hypothesis that the true slope of a line passing through
the data should be zero. For the three injection rates shown, the results of the test at the 90% confidence level are as follows:

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A correct interpretation of the result is: if one desires a 90% assurance that he will avoid incorrectly rejecting the hypothesis that the slope = 0, then that hypothesis must be accepted for all but the highest injection rate. This result is used in Appendix D in a discussion of the determination of $h_0$.

In Figures V-4a-d, the data for all injection rates and all rotational speeds are presented together. These figures show less scatter than do the previous ones, demonstrating the advantage of presenting a larger population when attempting to correlate experimental results.

Figures V-4a and b indicate the relationship between heat transfer and injection rate, with rotational rate submerged in both the ordinate \( \frac{h(v/w)^2}{k} \) and the abscissa \( \frac{V_w}{(\omega v)^{1/2}} \). It is apparent that the data can be reasonably well correlated with an equation of the form

\[
\log_{10} Nu = B H_w + C
\]  

[V-4]

where the particular values of the constants depend on the method used to correct for the non-porous surface area. While nearly all the data fall with a ±10% envelope, the envelope does not coincide
with the theoretical prediction by Sparrow and Gregg for a significant portion of the range of injection rate.

The form of the equation chosen [V-4] to correlate the data has no special theoretical basis. However, it is more convenient to plot the data on semi-log coordinates than log-log coordinates, because of the need to extrapolate to zero injection. A logarithmic ordinate was chosen so that equal vertical distances would represent equal per cent error brackets.

It should be recalled that the analysis of Sparrow and Gregg assumed constant fluid properties throughout the environment, regardless of temperature or concentration distributions within the flow. This assumption is obviously invalid for foreign gas injection, where the injectant might have drastically different properties from the ambient fluid. That the assumption of constant properties is inappropriate even for injection of air into air is indicated in Figures V-4c and d.

Figures V-4c and d differ from V-4a and b only in the abscissa. As indicated previously, transpiration heat transfer is commonly correlated with an injection parameter

\[ \frac{\rho_w V_w}{\rho_\infty U_\infty} \text{Re}^\frac{1}{2} \]

which becomes

\[ \frac{\rho_w}{\rho_\infty} H_w \]

when \( \omega R \) replaces \( U_\infty \). Naturally, if one could assume constant properties within the boundary layer, the density ratio would be unity.
Since one cannot assume $\rho_w = \rho_\infty$ even for the moderate temperature excursions encountered in this work, it would seem more appropriate to use $\frac{\rho_w}{\rho_\infty}H_w$ as the injection parameter, rather than simply $H_w$.

As seen in Figures V-4c and d, the data are again adequately correlated by an equation of the form

$$\log_{10} \frac{\rho_w}{\rho_\infty}H_w = B' + C'$$

[\text{V-4}^1]

And again, as in Figures IVa and b, nearly all the points fall within a $\pm 10\%$ envelope. However, in the case of Figure V-4c, a significant portion of the envelope includes the curve predicted by Sparrow and Gregg. Restating the coincidence of the data and Sparrow and Gregg's curve in another way, seventy per cent of the points fall within $12\%$ of the predicted curve.

One further way to illustrate the effect of transpiration on heat transfer is given in Figures V-5a and b. In these figures, $\frac{h}{h_o}$ is presented as a function of a flux ratio $\frac{n_wC_p}{h}$. Figure V-5a shows that heat transfer coefficient can be decreased by as much as $75-80\%$ by injecting air through the surface. It also shows that transpiration affects heat transfer from a rotating disk less than from a flat plate in a uniform stream.

Note that the method of correlating the data illustrated by Figure V-5b indicates a greater displacement from the predicted result than does the correlation of Figure V-4c. This is probably due to the dependence upon $h_o$, an extrapolated value. It should be noted that $h_o$ from an extrapolation of Equation [V-4] is approximately
6% lower than $h_0$ from Sparrow and Gregg. This alone accounts for a portion of the deviation of the experimental from the theoretical result.

A final point of discussion of the data concerns natural convection. For each run, the grouping $\frac{Gr}{Re^2}$ is tabulated in Appendix B, Table B-2. As discussed in THEORY AND PREVIOUS WORK, this grouping is a measure of the interference of natural convection with forced convection heat transfer. According to the experimental results of Richardson and Saunders [10] and the calculations of Fox [21], essentially no effect is to be expected for $\frac{Gr}{Re^2}$ as low as 0.01. Since the highest value for any run in the present experiments was 0.00115, no corrections have been made for natural convection.
VI. CONCLUSIONS

As a direct result of the experimental work described in this paper, the following conclusions can be drawn:

(1) For an injection parameter, the grouping $\frac{\rho_w}{\rho_\infty} H_w$ is more consistent with previous methods of correlating transpiration variables than $H_w$.

(2) The assumption that $h_{\text{non-porous}} = h_0$ provides a correction for the impermeable portion of the disk which brings the data closer to the theoretical prediction than the assumption that $h_{\text{non-porous}} = h_{\text{porous}}$.

(3) Sparrow and Gregg's results need to be tempered with a correction for physical property variation.

(4) The experimental data for transpiration cooling of a rotating disk for injecting air into an air environment may be correlated either with the equation

$$\log_{10} Nu = -0.505 \frac{\rho_w}{\rho_\infty} H_w - 0.519$$

or with Sparrow and Gregg's predicted curve (modified by substituting $\frac{\rho_w}{\rho_\infty} H_w$ for $H_w$) over the range $0.3 \leq \frac{\rho_w}{\rho_\infty} H_w < 1.3$.

(5) Transpiration at a given injection rate affects heat transfer from a rotating disk less than it affects heat transfer from a flat plate.
VII. SUGGESTIONS FOR FURTHER WORK

The next steps for further investigation of transpiration cooling of a rotating disk should be as follows:

1. Possible modification of the apparatus should be investigated to permit a broadening of the range of rotational rate and injection rate.

2. Transpiration cooling of a rotating disk with a foreign gas should be attempted experimentally.

3. A theoretical investigation of transpiration cooling of a rotating disk should be carried out which includes the effect of variable properties and possibly the diffusion-thermo phenomenon (Soret effect). Correlation of results might follow the form suggested by the work of Mills and Wortman [42].

4. The work of Cooper [13] and Cham and Head [8] on the turbulent boundary layer near a rotating disk should be extended to include injection. The work of Kays [31] and Cebeci [7] on the flat plate boundary layer could serve as a model for the work. Both calculational and experimental studies would be appropriate, though experimental difficulties may be insurmountable for the high speeds necessary to attain turbulent flow.
BIBLIOGRAPHY


55. Society of Automotive Engineers. Recommended ice bath for reference junctions. Aerospace Recommended Practice 691, August 1, 1964. 3 unnumb. leaves.


APPENDICES
APPENDIX A

EXPERIMENTAL DATA
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APPENDIX B

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<th>$h_{\text{corr(b)}}$ Btu/hr ft$^2$ °F</th>
<th>$q_{\text{corr(b)}}$ Btu/hr ft$^2$ °F</th>
<th>$q_{\text{corr(b)}}$ Btu/hr ft$^2$ °F</th>
<th>$q_{\text{rad}}$ Btu/hr ft$^2$ °F</th>
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Corr(a) indicates correction for non-porous area determined by $h_{\text{non-porous}} = h_o$.

Corr(b) indicates correction for non-porous area determined by $h_{\text{non-porous}} = h_{\text{porous}}$. 
### Table B-2. Calculated Results.

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<th>( \frac{hR}{k} )</th>
<th>Nu</th>
<th>Nu</th>
<th>( \frac{\rho_w}{\rho_\infty} )</th>
<th>Hw</th>
<th>Gr</th>
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See Table B-1 for a description of corrections applied to the data.
APPENDIX C

SOURCES OF ERROR
Sources of error in the heat transfer coefficients presented in this work can be classified as follows:

1. Experimental error
2. Uncertainty in the value of emissivity
3. Uncertainty in the method of correcting for the impermeable area of the disk

Each of these categories is discussed in turn. A final category, 4. Sum of uncertainties, indicates how the errors discussed in the previous sections are combined statistically to give an estimate of the total uncertainty in $h$.

1. **EXPERIMENTAL ERROR**

Uncertainties in $h$ due to experimental error arise in the measurement of temperatures and in the measurement of flow rate.

a. **Errors in Temperature Measurement**

The effect on calculated values of $h$ due to errors in temperature measurement was determined in the following way: An estimate of the uncertainty of each temperature measurement was made (as described below), resulting in a range of values having a high probability of including the true value. Then the measured value of the temperature, and also the estimated extreme values of the range of the temperature, were used sequentially as inputs for the data reduction program described in Appendix E.
By carrying out this procedure for each experimental run, the sensitivity of $h$ to error in $T_w$, $T_{\infty}$, and $T_{\infty w}$ was determined. Results are presented numerically in Table C-1 and graphically in Figures V-4a, b, c, d by horizontal brackets facing left (see legend).

The values given in Table C-1 are for $\Delta T$ (estimated uncertainty in $T$), $\Delta h$ (resulting from $\Delta T$), and $\frac{\partial h}{\partial T}$ (sensitivity of $h$ to uncertainties in $T$, calculated as the quotient $\Delta h/\Delta T$).

Since errors are combined (see Section 4 below) according to the formula

$$\Delta h_{\text{total}}^2 = \sum_i \Delta h_i^2$$

the values in Table C-1 indicate that the error in $h$ due to uncertainty in $T_{\infty}$ is always less than 12% of that due to certainty in $T_w$.

During the experimental runs, there was uncertainty in the true value of $T_w$ due to uncertainty in the calibration curve and to fluctuations in the millivolt reading of the infrared radiometer. Calibration curve uncertainty was determined by adding the errors of (a) long term drift (estimated as one-half the drift observed over the succeeding 24 hours), and (b) other feasible curves drawn through the same set of calibration points. Error due to electronic noise was assumed to be less than one-half the difference between the highest value observed during any particular measurement and the average value. The estimate of the uncertainty in $T_w$ given in Table C-1 was obtained by combining the above possible errors.
Table C-1. Estimated Experimental Uncertainty.

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<th>$\Delta T_W$</th>
<th>$\Delta h_{T_W}$</th>
<th>$\frac{\partial h}{\partial T_W}$</th>
<th>$\Delta h_{\sigma}$</th>
<th>$\Sigma \Delta h$</th>
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</tbody>
</table>

* Units of $h$ and $\Delta h$ are Btu/hr ft$^2$ °F. Units of $\Delta T$ are °F, and those for $\frac{\partial h}{\partial T}$ are Btu/hr ft$^2$ °F$^2$.

** Corrected assuming $h_{\text{non-porous}} = h_0$. 

86
Uncertainty in the value of $T_\infty$ was due to fluctuations of the reading during a particular run. While the thermocouples were not inherently noisy transducers, nonetheless there were fluctuations of $\pm 20$ to $70$ microvolts ($\pm 0.6$ to $2.4^\circ F$) due to convection currents within the environment. The estimate of uncertainty in $T_\infty$ given in Table C-1 is one-half the fluctuations described above. It is assumed that errors due to calibration of the thermocouples were negligible compared to the uncertainties reported in Table C-1.

Uncertainties in $h$ due to possible errors in $T_\infty$ were considered negligible. $T_\infty$ measurements were quite stable, showing only very small (0-6 microvolts) fluctuations for every run.

b. **Errors In Flow Measurement**

As discussed under EXPERIMENTAL EQUIPMENT, calibration of the orifice meter was accurate to $\pm 1/2\%$. An error of this magnitude would not be discernible on Figure V-4. As shown in Appendix F, possible leakage in the system is also negligible.

2. **UNCERTAINTIES IN THE VALUE OF EMISSIVITY**

As discussed under EXPERIMENTAL EQUIPMENT, the emissivity of the porous surface was taken as 0.3, the value obtained by a previous investigator [20] using the same type of material from the same manufacturer. It is recognized, however, that some slight difference in fabrication may have caused a slightly different surface condition of the stainless steel wire mesh. A range of
0.25-0.35 is assumed to be broad enough to include the true value of emissivity.

To determine the error in heat transfer coefficient possible because of the uncertainty in the value of emissivity described above, \( h \) was calculated using three values of emissivity: 0.25, 0.30, and 0.35. The range in \( h \) is given numerically in Table C-1 as \( \Delta h_o \) and shown graphically in Figures V-4a, b, c, d by horizontal brackets facing right (see legend).

3. **UNCERTAINTY IN CORRECTION FOR IMPERMEABLE AREA**

The two methods of correcting for the outer, non-porous area of the rotating disk were discussed at some length under RESULTS. Certainly the assumption that \( h_{\text{non-porous}} = h_{\text{porous}} \) provides an indisputable lower limit for the correction. The other assumption used to calculate a second estimate of the correction, viz., \( h_{\text{non-porous}} = h_o \) (\( h \) for no injection) could possibly be argued inadequate as an upper limit. For example, \( h_o \) for a rotating cylinder might give a better upper limit. Or some arbitrary multiple of \( h_o \) for a rotating disk might be used, or \( h_o \) for turbulent flow over a disk might be used.

The author feels, however, that the value used is adequate and is readily calculated from the data at hand. Quantitative estimates for the other corrections suggested above would be difficult to determine and more difficult to justify. Therefore,
it is assumed that the two corrections calculated for the non-porous area of the disk provide adequate upper and lower limits.

4. **SUM OF UNCERTAINTIES**

The individual uncertainties described in the sections 1 and 2 above contribute to the total uncertainty in the heat transfer coefficient. To determine the total uncertainty, the individual uncertainties must be combined in a statistically appropriate way.

According to Davies [17], if $y$ is a function of several variables $x_1, x_2 \ldots$, then the variance of $y$ (i.e., the square of the standard deviation) is given by:

$$ s_y^2 = \left( \frac{\partial y}{\partial x_1} \right)^2 s_{x_1}^2 + \left( \frac{\partial y}{\partial x_2} \right)^2 s_{x_2}^2 + \ldots \quad [C-1] $$

This equation assumes no interaction among the variances of $x_1, x_2 \ldots$ (covariance = 0).

To translate this formula into the present problem, several substitutions and assumptions will be made. First, $h$ is substituted for $y$, $T_w$ for $x_1$, $T_\infty$ for $x_2$ and $c$ for $x_3$. Second, recognizing that standard deviation or standard error describes the tendency of a measurement to be distributed about the mean, the estimated error of a given variable is substituted for a multiple of its standard deviation, i.e.

$$ \Delta T_{est} = N \, s_T $$

Finally, it is assumed that the errors estimated in each of the
previous sections enclosed limits at equal levels of significance, i.e. if

\[ \Delta T_{est} = N_1 s_T \]

and if

\[ \Delta \sigma_{est} = N_2 s_\sigma \]

then it is assumed that \( N_1 = N_2 \). Making the above substitutions, equation [C-1] becomes

\[ \Delta h^2 = \left( \frac{\partial h}{\partial T_W} \right)^2 \Delta T_W^2 + \left( \frac{\partial h}{\partial T_\infty} \right)^2 \Delta T_\infty^2 + \left( \frac{\partial h}{\partial \sigma} \right)^2 \Delta \sigma^2 \]  

[C-2]

If we replace \( \frac{\partial h}{\partial T} \) \( \Delta T \) with \( \Delta h_T \), as described in section 1 above, and \( \frac{\partial h}{\partial \sigma} \) \( \Delta \sigma \) with \( \Delta h_\sigma \), then

\[ \Delta h^2 = \Delta h_T^2 + \Delta h_T^2 + \Delta h_\sigma^2 \]  

[C-3]

Equation [C-3] embodies the method used to combine individual errors in order to estimate the total uncertainty in heat transfer coefficient presented in this work. While the individual uncertainties are indicated in Figures V-4a, b, c, d with brackets facing right or left, the total uncertainty is indicated by a vertical line. It is also presented in Table C-1, both in terms of \( \Delta h_{total} \) and as a per cent of \( h \).

It is recognized that the total uncertainty in \( h \) may be larger than the values tabulated due to factors not quantitatively accounted for. For example, if the heat transfer coefficient for the non-porous area of the disk is different from that calculated, the actual
value of $h_{\text{porous}}$ could fall beyond the limits indicated in Figures V-4a, b, c, d. Nonetheless, the author submits these intervals, with a high degree of confidence, as estimates of a range of values into which the true value of $h$ falls.
EXTRAPOLATING $h$ TO FIND $h_0$. 
Many of the calculations described in EXPERIMENTAL RESULTS required values for $h_0$, the heat transfer coefficient for zero injection. Since the apparatus described herein did not permit the direct determination of heat transfer with zero injection, it was necessary to find $h_0$ by extrapolation.

Extrapolation of experimental data is, of course, subject to error. Experimental error causes some degree of uncertainty within the range of the variables studies. Furthermore, when one extrapolates outside of the range of experimentation, the uncertainty may become magnified. For this reason, the method used to extrapolate should be chosen carefully so as to minimize the effect of experimental uncertainty on values obtained in this way.

Two methods of extrapolation were attempted for finding $h_0$. The first isolated the data for a given rotational rate, plotted $h$ as a function of injection rate, then extrapolated a line through those points to zero injection. Data for this method are compiled in Table D-1, corrected for minor deviations from the nominal rotational speed, and plotted in Figure D-1. It is apparent from the figure that this method is unsatisfactory because, for any one rate of rotation, there are so few data points that the uncertainty causes a very high uncertainty in the extrapolated value at zero injection.

The second method of extrapolation overcomes many of the ob-
Table D-1. Values of $h_o$, $\theta_T$, and $R_T$.

<table>
<thead>
<tr>
<th>Re nominal</th>
<th>Re actual</th>
<th>$n$</th>
<th>$\left( \frac{Re_{nom}}{Re_{act}} \right)^{\frac{1}{3}}$</th>
<th>$h$ actual</th>
<th>$h$ corrected to $Re_{nom}$ from Fig V-4c</th>
<th>$h_{o}$</th>
<th>$\theta_T$</th>
<th>$R_T$</th>
<th>$\frac{\rho \times}{\rho_{oc}}$</th>
<th>$R_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>19,000</td>
<td>20,300</td>
<td>0.441</td>
<td>0.967</td>
<td>2.04</td>
<td>1.97</td>
<td>3.90</td>
<td>0.51</td>
<td>3.22</td>
<td>2.94</td>
<td></td>
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<td>19,500</td>
<td>20,300</td>
<td>0.580</td>
<td>0.988</td>
<td>1.78</td>
<td>1.76</td>
<td>4.56</td>
<td>0.45</td>
<td>4.72</td>
<td>4.21</td>
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<tr>
<td>18,600</td>
<td>19,045</td>
<td>0.724</td>
<td>1.01</td>
<td>0.87</td>
<td>0.87</td>
<td>0.224</td>
<td>17.2</td>
<td>15.1</td>
<td></td>
<td></td>
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<tr>
<td>26,000</td>
<td>29,900</td>
<td>0.362</td>
<td>0.932</td>
<td>3.38</td>
<td>3.15</td>
<td>4.65</td>
<td>0.69</td>
<td>1.60</td>
<td>1.50</td>
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<tr>
<td>27,600</td>
<td>29,900</td>
<td>0.438</td>
<td>0.971</td>
<td>2.92</td>
<td>2.83</td>
<td>0.62</td>
<td>2.22</td>
<td>2.04</td>
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<td>0.990</td>
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<td>2.48</td>
<td>0.54</td>
<td>3.34</td>
<td>3.30</td>
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<tr>
<td>25,900</td>
<td>25,900</td>
<td>0.734</td>
<td>1.00</td>
<td>2.32</td>
<td>2.32</td>
<td>0.51</td>
<td>4.53</td>
<td>4.02</td>
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<td></td>
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<tr>
<td>26,000</td>
<td>25,900</td>
<td>0.734</td>
<td>1.00</td>
<td>2.24</td>
<td>2.24</td>
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<td>4.14</td>
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<td>25,600</td>
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<td>1.01</td>
<td>1.06</td>
<td>1.08</td>
<td>0.246</td>
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<td>25,600</td>
<td>1.044</td>
<td>1.01</td>
<td>0.95</td>
<td>0.96</td>
<td>0.210</td>
<td>15.7</td>
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<td>0.970</td>
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<td>0.61</td>
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<td>38,400</td>
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<td>1.01</td>
<td>2.78</td>
<td>2.81</td>
<td>0.50</td>
<td>3.72</td>
<td>3.29</td>
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<td>38,400</td>
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<td>2.67</td>
<td>0.48</td>
<td>3.91</td>
<td>3.45</td>
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<td>37,500</td>
<td>1.043</td>
<td>1.02</td>
<td>1.58</td>
<td>1.61</td>
<td>0.29</td>
<td>9.2</td>
<td>8.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>51,000</td>
<td>54,600</td>
<td>0.350</td>
<td>0.967</td>
<td>4.25</td>
<td>4.11</td>
<td>6.38</td>
<td>0.65</td>
<td>1.22</td>
<td>1.12</td>
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<td>54,600</td>
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<td>0.350</td>
<td>0.967</td>
<td>4.35</td>
<td>4.20</td>
<td>0.66</td>
<td>1.19</td>
<td>1.10</td>
<td></td>
<td></td>
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<tr>
<td>50,200</td>
<td>50,200</td>
<td>0.728</td>
<td>1.01</td>
<td>3.55</td>
<td>3.58</td>
<td>0.56</td>
<td>2.92</td>
<td>2.52</td>
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<tr>
<td>49,700</td>
<td>49,700</td>
<td>1.043</td>
<td>1.01</td>
<td>2.24</td>
<td>2.26</td>
<td>0.354</td>
<td>6.62</td>
<td>5.50</td>
<td></td>
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<tr>
<td>49,600</td>
<td>49,600</td>
<td>1.042</td>
<td>1.01</td>
<td>2.59</td>
<td>2.62</td>
<td>0.41</td>
<td>5.70</td>
<td>4.94</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: $h$ at $Re_{nom} = h_{actual} \left( \frac{Re_{nom}}{Re_{act}} \right)^{\frac{1}{3}}$. 

54
Figure D-1. Extrapolation of Heat Transfer Coefficient to Zero Injection

- Calculated value, based on \( Nu_0 = 0.302 \)
- \( Re = 19,000 \)
- \( Re = 26,000 \)
- \( Re = 39,000 \)
- \( Re = 51,000 \)
jections to the first. This method used all of the data for all rotational rates and all injection rates, as plotted on Figure V-4c. Since there are many more data points, one has greater confidence in the inferred relationship, and thus in the extrapolated value of $Nu_0$ and $h_0$.

Using standard statistical techniques (see [5] for example), a confidence interval was determined for the intercept of the line inferred by the linear regression technique. At the 98% confidence level,

$$\log_{10} Nu_0 = -0.51962 \pm 0.0247$$

or

$$Nu_0 = 0.302 \pm 0.017$$

Thus, with 98% confidence, $Nu_0$ and $h_0$ are estimated within a 6% interval.

The conversion of $Nu_0$ to $h_0$ is obvious, since $Nu = \frac{h(v/\omega)^{1/2}}{k}$. Interestingly, the ratio $k/\sqrt{v}$ is very nearly independent of temperature (the variation from 90°F to 150°F is less than 1/2%) for air, so one can choose either environment temperature or film temperature to evaluate fluid properties when calculating $h_0$ from $Nu_0$. The data reduction program used film temperature ($T_{film} = 0.5 (T_w + T_\infty)$).

It should be noted that the determination of $Nu_0$ is actually an iterative procedure. Each data point on Figure V-4c required a value for $h_0$ in calculating the correction for the non-porous
portion of the rotating disk. Because the non-porous area is only about 11% of the total area, however, the extrapolated value of $N_u_o$ was rather insensitive to the value used in the correction. For example, a 10% change in the value of $N_u_o$ used for the correction caused a change of only 1.2% in the extrapolated value of $N_u_o$.

The last four columns of Table D-1 present the values of $h_o$ for each nominal Reynolds number and $\theta_T$ for each run (corrected to nominal Re), and $R_T$ and $\frac{\rho_w}{\rho_\infty} R_T$. The significance of these data is discussed in EXPERIMENTAL RESULTS. They are plotted in Figures V-5a and b.
APPENDIX E

DATA REDUCTION PROGRAM
DATA REDUCTION PROGRAM

The following program was used to convert the raw data, recorded in millivolts, centimeters of manometer fluid, millimeters of mercury, etc., to heat transfer coefficients, Reynolds numbers, Nusselt numbers, etc. It is written in FOCAL, a conversational language provided by Digital Equipment Corporation for users of their PDP-series computers. The computer used was a PDP-9 in the College of Mines Computing Laboratory at The University of Arizona.
C-FDIAL V36
01:01 IC HEATFLX DATA REDUCTION
01:07 "DATA INPUT": "ASK" "DATE...": "LAIN!!"
01:09 ASK " 1 COOLANT FL " "LFC" 10 WALL, BEG 1 "SWALL
01:15 ASK " T INFO NV ON BEG C "SWALL", T INFO NV TRANS!
01:17 ASK " ORIFICE NO. "ON" DELTA F: ON "SWALL
01:19 ASK " UPSTREAM PRESS ON "SWALL" MINUS PRESS ON "VAMOS
01:11 ASK " ORIFICE AV "SWALL!!", RPM "SWALL!!
01:12 SET TIBIL = 550; DO 63 SET TV = 1K
01:13 SET TWALL = (TWALL + 450) / 1.8
01:14 SET TICEL = MVIV; DO 63 SET TINF = 1K
01:15 SET SSW = 6
01:16 SET TICEL = MVIV; DO 63 SET TINF = 1K
01:17 SET TICEL = MVIV; DO 63 SET TINF = 1K
01:18 SET TIFILM = 6.5 * (TWALL + TINF)
01:19 SET HI = 6.5 * TINF(1.)
01:20 CC
01:21 SET OMEGA = HI * HOM / 30.
01:22 CC
01:23 CC CALCULATE FLUID PROPERTIES
01:24 SET TK = TIFILM; DO 71; SET COV = VISV
01:25 LO 83 SET FORILK = FR3
01:26 SET TK = TWALL; DO 83; SET FORALL = FR3
01:27 SET NO = MU2/OFILM
01:28 SET TK = TIFILM; DO 9
01:29 LO 103 SET OFILM = FR
01:30 SET TK = 6.5 * (TWALL + TINF); DO 103; SET CORAV = GF
01:31 CC
01:32 SET MSILON = V; 31; SET STOPA = 0.171428
01:33 SET B = 3.97414; I SET F = 170; SET AD = 2.123724
01:34 SET AREA = HI * F: K: SET ACURR = (0.70) * (1070) - 1.
1.560 OUTPUT FORMAT
1.57 IF (SSC) 1.571, 1.61, 1.66
1.58 'T !!!! "TRANSPORT TOOLING" !!!, OR A ROTATING DISK" !!!!
1.61 'T "DATA" !!!, "MIXED" FLOW!!!
1.64 'T "S. 45, "LEAK IN ALL INJECTION" !!!
1.63 'T "S. 1000" % INJETION TEMPERATURE!!!
1.64 'T "WALL TEMPE!!!
1.65 'T "ENVIRONMENT THARM!!!
1.66 'T "RUN " % "DATE" !!!
1.67 'T "DISK REYNOLDS NUMBER" 38, 0.1
1.68 'T "NUSSLETT NUMBER" 38, 0.1 NUSSELT!!!
1.69 'T "INJECTION PARAMETERS, HW = WALL," (FLOW/HOINT)*hw), hw!!!
1.70 'T "HEAT TRANSFER COEFFICIENT, R = %S*COFF!!!
1.71 'T "NU/HQW (HOINT) (FILM PROPERTIES) XH!!!
1.72 'T "0X - 10)/C11(0 = 2.01), (10-001)/01-12), !!!!
01.73 T: "PE = .5. 3. SAVEL!!.
01.74 T: "OMEGA = .5. 3. OMEGA!!" "WALL = .5. WALL!!
01.75 T: "EXH = .5. EXHII/COND!!
01.76 T: "KIN. VISCO = .5. NO!!
01.77 T: "PHON = .5. PHON!!
01.78 T: "COND = .5. COND!!
01.79 IF (SSW) 57.1 58. 1.38
01.80 S: COND = COND = COND = GCR - E - 1.157*LEAD
01.81 S: SSW = 1 . GOTO 1.26
01.82 T: "NEGATIVE SSW; HELP!!"; GUIT
01.83 T: "COND = .5. COND!!
01.84 T: "!!!!!!!; GOTO 1.02
06.01 FUNCTION THCFL
06.10 IF (THCEL = 15) 6.12, 6.11, 6.11
06.11 SET TK = THCFL + 273.16; GOTO 6.21
06.12 IF (THCEL = 1) 6.10, 6.10, 6.14
06.13 TYPE "THCEL NO FLOW RANGE OF CALIBRATION"; T 5
06.14 IF (THCEL = 2) 6.10, 6.10, 6.16
06.15 SET TK = 19.0255*THCEL + 273.1617; GOTO 6.21
06.16 IF (THCEL = 4) 6.10, 6.10, 6.16
06.17 SET TK = 6.557*THCEL + 273.1617; GOTO 6.21
06.18 IF (THCEL = 7) 6.10, 6.10, 6.20
06.19 SET TK = 13.1818*THCEL + 273.1617; GOTO 6.21
06.20 TYPE "THCEL NO AVG RANGE OF CALIBRATION"; GUIT
06.21 RETURN
07.01 C
07.02 FUNCTION VISC
07.03 C
07.04 SET VISC = (1.153E-5) * 141.8 * 145.8 / (171E-3 < K + 116.43)
07.05 RETURN
08.01 C
08.02 FUNCTION HLU
08.03 C
08.04 SET HLU = 0.24e8 * (273.16/10) * 10*10.275
08.05 RETURN
29.410
29.420 FUNCTION CONNECTIVITY
29.430
29.440 SET Z = 5.77 * (1. + MID(1.01, Z/10))
29.450 SET TEND = 1.5 / (1.01 - 0.5) * KEND / Z
29.460 RETURN
10.010
10.020 FUNCTION CP
10.030
10.050 RETURN
10.990
11.010 FUNCTION AFLOW
11.020
11.040 S X = FACT<DELPHS+POLE+FAIL>/1000>
11.050 IF (N = 2011) 11.1011.25
11.101 IF (X = 5.111) 11.1111.12
11.111 S W = 0.99159*X + 0.984123 5010 11.5
11.121 S X = 0.101474*X + 0.10173 5010 11.5
11.201 IF (X = 4.111) 11.2111.22
11.211 S W = 0.99625*X; 5010 11.5
11.221 S X = 0.9934*X + 0.9911; 5010 11.5
11.251 IF (N = 4.111) 3.11.411.48
11.301 S W = 0.98675*X + 0.98493; 5010 11.5
11.401 S W = 0.97148*X + 0.97403; 5010 11.5
11.451 "WRONG UNIFORM N": N = "", &N, "?"; GOL1
11.50 RETURN
APPENDIX F

LEAKAGE ANALYSIS
LEAKAGE ANALYSIS

Leakage in the tubing, manometers, rotary union, etc., was too low to be measured directly by the smallest orifice meter available. Therefore an indirect means of determining leak rate was used, as follows:

The porous disk was removed, the thermowell placed up through the rotating pipe was replaced with a slightly shorter one, and a rubber stopper was inserted into the upper end of the rotating pipe. Then the system was pressurized by opening the needle valve. When the valve was closed, the mercury manometer measuring total pressure in the system slowly fell, indicating some leakage of air from the "closed" system. This test was repeated many times during the development of the apparatus. Rotation of the pipe had no apparent influence on the leak rate.

When plotted on semi-logarithmic paper, total pressure in the system against time showed an exponential decay, as indicated in Figure F-1. Due to subsequent leak testing and tightening of fittings, etc., the data in the figure show a much greater slope (more leakage) than those taken just before the heat transfer runs. The latter data, however, showed such a slow decay that it was difficult to ascertain the slope accurately. In the analysis that follows, therefore, the data of Figure F-1 will be used as an upper limit to the actual pressure decay.
Figure F-1. Decay of Pressure In Sealed Piping System

Data of 7-20-71

\[ p = p_0 e^{-0.30t} \]
Consider a pressurized system consisting of a certain volume of gas with a slow leakage outward.

For such a system, the general conservation equation for total mass in the system is:

\[
\text{Accumulation} = \text{Input} - \text{Output}
\]

or

\[
\text{Accumulation} = -\text{Output}
\]

If \( V \) = total volume, \( v \) = specific volume, and \( w \) = the mass rate of output, Equation [F-1] may be written

\[
\frac{d}{dt} \left( \frac{V}{v} \right) = -w
\]

For an ideal gas,

\[
Pv = \frac{R}{M} T
\]

or

\[
\frac{1}{v} = \frac{MP}{RT}
\]

Since the volume of the system is independent of time, Equation [F-2] becomes

\[
\frac{MV}{RT} \frac{dP}{dt} = -w
\]
Now, if pressure decays exponentially with time,

\[ P = P_0 e^{-at} \]

and

\[ \frac{dP}{dt} = aP_0 e^{-at} = -aP \]  \hspace{1cm} [F-5]

where \( a \) is determined from the data, for example, of Figure F-1.

Substitution of \([F-5]\) into \([F-4]\) yields

\[ w = \frac{MV}{RT} (aP) \]  \hspace{1cm} [F-6]

As mentioned earlier, the data of Figure F-1 provide an upper limit to the actual leakage rates.

In order to make a quantitative estimate of the leakage, the volume of the pipes, hoses, tubes, etc., was determined to be approximately 0.045 cubic feet. At \( 80^\circ \text{F} \), the right hand side of Equation \([F-6]\) reduces to

\[ w = \frac{MV}{RT} (aP) = 1.34 \times 10^{-6} \frac{P}{\text{mmHg}} \frac{1 \text{b}}{\text{min}} \]

For example, at the maximum flow rate encountered in the heat transfer runs (0.087 lb/min), \( P = 117 \text{ mmHg} \). For this case,

\[ w_{\text{leak}} = 1.57 \times 10^{-4} \frac{1 \text{b}}{\text{min}} \]

\[ \frac{w_{\text{leak}}}{w_{\text{injection}}} = \frac{1.57 \times 10^{-4}}{8.7 \times 10^{-2}} = 0.0018 \]

or leakage is less than 0.2% of the injection rate. At the minimum injection rate (0.029 lb/min), \( P = 30 \text{ mmHg} \). For this case also, the
leakage is less than 0.2% of the injection rate. Because the orifice calibration for measuring injection rate were reproducible to 0.5%, and because the leakage calculated here is an upper limit to the actual loss, it is concluded that leakage rate makes a negligible contribution to the sources of error in the heat transfer coefficients reported in this research.
APPENDIX G

NOMENCLATURE
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B, B'$</td>
<td>constants used in Equations [V-4] and [V-4']</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$B'^+!!$</td>
<td>matching parameter used in Kays' work</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$C, C'$</td>
<td>constants used in Equations [V-4] and [V-4']</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$C_f$</td>
<td>friction coefficient</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$C_p$</td>
<td>heat capacity</td>
<td>BTU/lb °F</td>
</tr>
<tr>
<td>$D$</td>
<td>diameter of disk</td>
<td>ft</td>
</tr>
<tr>
<td>$D$</td>
<td>diffusivity</td>
<td>ft²/hr</td>
</tr>
<tr>
<td>$D$</td>
<td>damping factor in mixing length expression</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$F$</td>
<td>dimensionless radial velocity</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration</td>
<td>ft/hr²</td>
</tr>
<tr>
<td>$G$</td>
<td>dimensionless angular velocity</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$Gr$</td>
<td>Grashof number, $\beta g \rho R^3 (T_w - T_\infty)/\nu^2$</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$h$</td>
<td>heat transfer coefficient</td>
<td>BTU/hr ft² °F</td>
</tr>
<tr>
<td>$H$</td>
<td>enthalpy</td>
<td>BTU/lb</td>
</tr>
<tr>
<td>$H$</td>
<td>clearance, distance from disk to stationary surface opposite</td>
<td>ft</td>
</tr>
<tr>
<td>$H$</td>
<td>dimensionless axial velocity</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity</td>
<td>BTU/hr ft °F</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Prandtl mixing length</td>
<td>ft</td>
</tr>
<tr>
<td>$M$</td>
<td>molecular weight</td>
<td>lb/lb mole</td>
</tr>
<tr>
<td>$n$</td>
<td>mass flux</td>
<td>lb/min ft²</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>ratio of error estimate to standard deviation</td>
<td></td>
</tr>
<tr>
<td>Nu</td>
<td>Nusselt number, ( \frac{h_0}{k} )</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>pressure</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>dimensionless pressure, ( \frac{p}{\mu \omega} )</td>
<td></td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number, ( \frac{c_p \mu}{k} )</td>
<td></td>
</tr>
<tr>
<td>q</td>
<td>heat flux</td>
<td></td>
</tr>
<tr>
<td>r</td>
<td>radial coordinate</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>radius of the disk</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>universal gas constant</td>
<td></td>
</tr>
<tr>
<td>( \frac{n c_p}{h} )</td>
<td>flux ratio, ( \frac{n c_p}{h} )</td>
<td></td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number</td>
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</tr>
<tr>
<td>s</td>
<td>standard deviation</td>
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</tr>
<tr>
<td>s^2</td>
<td>variance</td>
<td></td>
</tr>
<tr>
<td>Sc</td>
<td>Schmidt number, ( D/\nu )</td>
<td></td>
</tr>
<tr>
<td>t</td>
<td>time</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
<td></td>
</tr>
<tr>
<td>U</td>
<td>velocity in streamwise direction</td>
<td></td>
</tr>
<tr>
<td>u*</td>
<td>friction velocity in two-dimensional boundary layer, ( (\tau_0/\rho)^{\frac{1}{2}} )</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>velocity</td>
<td></td>
</tr>
<tr>
<td>W</td>
<td>mass flow rate</td>
<td></td>
</tr>
<tr>
<td>Wl</td>
<td>weight fraction</td>
<td></td>
</tr>
<tr>
<td>y</td>
<td>distance normal to the wall in two-dimensional boundary layer</td>
<td></td>
</tr>
</tbody>
</table>
\( y^+ \) non-dimensionalized value of \( y, \frac{y v}{U} \) 

\( z \) axial coordinate 

\( \beta \) temperature coefficient of volume expansion 

\( \delta \) boundary layer thickness, \((v/\omega)^{1/2}\) 

\( \Delta \) estimated uncertainty 

\( \varepsilon \) emissivity 

\( \eta \) dimensionless distance normal to the disk, \( z/\delta \) 

\( \theta \) angular coordinate 

\( \Theta \) dimensionless temperature ratio, \( \frac{T - T_\infty}{T_W - T_\infty} \) 

\( \Theta_T \) ratio of heat transfer coefficient with injection to heat transfer coefficient without injection, \( h/h_0 \) 

\( \mu \) viscosity 

\( \nu \) kinematic viscosity 

\( \rho \) density 

\( \sigma \) Stefan-Boltzmann constant 

\( \Sigma \) summation, total 

\( \tau \) shear stress 

\( \phi \) dimensionless ratio of weight fractions, \( \frac{W_1 - W_{1\infty}}{W_1 - W_{1\infty}} \) 

\( \omega \) angular velocity 

\( \text{dimensionless} \) 

\( \text{ft} \) 

\( ^{\circ} \text{R}^{-1} \) 

\( \text{dimensionless} \) 

\( \text{dimensionless} \) 

\( \text{radians} \) 

\( \text{dimensionless} \) 

\( \text{dimensionless} \) 

\( \text{lb/ft hr} \) 

\( \text{ft}^2/\text{hr} \) 

\( \text{lb/ft}^3 \) 

\( \text{BTU/hr ft}^2 \text{ } ^{\circ} \text{R}^4 \) 

\( \text{dimensionless} \) 

\( \text{lb}_{f}/\text{ft}^2 \) 

\( \text{dimensionless} \) 

\( \text{sec}^{-1} \)
**Subscripts**

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Definition</th>
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<tr>
<td>conv</td>
<td>convection</td>
</tr>
<tr>
<td>corr</td>
<td>corrected</td>
</tr>
<tr>
<td>e</td>
<td>in external flow of two-dimensional boundary layer</td>
</tr>
<tr>
<td>o</td>
<td>zero injection</td>
</tr>
<tr>
<td>o</td>
<td>orifice meter</td>
</tr>
<tr>
<td>rad</td>
<td>radiation</td>
</tr>
<tr>
<td>w</td>
<td>evaluated at the wall or disk surface</td>
</tr>
<tr>
<td>w∞</td>
<td>evaluated on the surface of the surroundings, far from the disk</td>
</tr>
<tr>
<td>x</td>
<td>using distance along the surface for a typical length in a dimensionless group</td>
</tr>
<tr>
<td>∞</td>
<td>evaluated in the fluid, far from the disk surface</td>
</tr>
</tbody>
</table>

**Superscripts**

<table>
<thead>
<tr>
<th>Superscript</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>^</td>
<td>per lb</td>
</tr>
<tr>
<td>~</td>
<td>per mole</td>
</tr>
</tbody>
</table>