

AN ABSTRACT OF THE THESIS OF

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Title OPTIMUM TRANSIENT RESPONSE OF A KAPLAN TURBINE
GENERATOR SYSTEM DETERMINED FROM MATHEMATICAL
MODELS

Abstract approved 
(Major professor)

This thesis proposes a criteria for the evaluation of those variable parameters that influence the response of a hydroelectric generator driven by a hydraulic turbine of the Kaplan type. It is considered a requirement that a unit must be stable when supplying an isolated load and it is assumed that this requirement assures stability of a system of such units when interconnected. A mathematical model of a typical unit supplying an isolated load is presented. The characteristic equation of this linearized system is analyzed with the aid of a digital computer to determine relative stability of the unit for various parameter settings. A model of a Kaplan-turbine-driven unit connected to a typical system is shown. This model is solved on the digital computer to determine the response of the unit when subjected to a disturbance.

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MODELS

by

GLENN ROGERS MELOY

A THESIS

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OPTIMUM TRANSIENT RESPONSE OF A KAPLAN TURBINE GENERATOR SYSTEM DETERMINED FROM MATHEMATICAL MODELS

I. INTRODUCTION AND STATEMENT OF THE PROBLEM

Introduction

Most power systems show a predominance of a particular type of power generation. Thus, some power systems can be considered as being characterized by steam or thermal generation and others are predominantly hydroelectric in nature. The predominance of a particular type of generation is dictated by economics and the available natural resources of the area. In areas such as the Pacific Northwest, an abundant supply of water power has resulted in predominantly hydroelectric generation.

The Northwest Power Pool (NWPP) totals approximately 17000 megawatts of which only about 1400 megawatts consists of steam generation (2, p. 765). The NWPP has unique distinction among hydroelectric power systems in that its hydraulic turbines are predominantly of the Kaplan type which have adjustable runner blades controlled by the governor. Thus the transient analysis of other hydro systems is not generally applicable to the NWPP.

Any discussion of optimal response of a hydroelectric generating

unit will ultimately center about adjustments which can be made on the unit speed governor. This is because the existing water passages and machine dimensions cannot be changed. The speed governor of a Kaplan generating unit connected to a large system functions essentially as a proportional load control device. In addition it maintains maximum turbine efficiency for the operating head and gate position. This is accomplished by means of a cam which automatically drives the runner blades to the blade angle corresponding to maximum turbine efficiency at that head for the actual gate position. Thus a change in frequency will result in a corresponding change in turbine wicket gate position with a resulting change in turbine runner blade position.

The criteria now used for adjusting the turbine governors is essentially the same as that used when the output of one machine actually represented a substantial portion of the generating capability of the entire power system. These criteria are no longer valid. The growth of the system has created a considerable problem which must now be dealt with if stable system operation is to prevail. The temptation is to consider the system as infinite when adjusting an individual governor. This, coupled with a universal desire for increased speed of response, has a tendency to result in too little compensation in the governor adjustments. The destabilizing effect on the system is not immediately apparent due to the relatively small capacity of a single unit as compared to that of the system. The eventual result of this

policy, however, as more units are adjusted in this manner, is a gradual deterioration of the system stability.

Two criteria for adjusting an individual governor have found rather general acceptance. One criterion is to provide a well-damped frequency characteristic when the unit is "off-line" to facilitate either manual or automatic synchronizing of the machine to the system. The "on-line" criterion is to provide fast response to load changes. These two criteria are generally incompatible with each other and their use has resulted in a widespread application of various means of altering the governor parameters once synchronization has been accomplished. This has occasionally resulted in the practice of removing the governor compensating dashpot from service altogether after the unit is connected to the system.

Virtually since its inception the Northwest interconnected system has had occasional periods of frequency instability (2, p. 765). It has only been in relatively recent years however that these oscillatory periods have caused serious concern. Several trial-and-error tests on the system have resulted in agreements to operate with the compensating dashpots in service on all units not directly controlled by load-frequency control equipment. A five percent permanent speed droop on all units not directly participating in power system frequency control must also be maintained (2, p. 765). These corrective measures have resulted in more stable system operation and have also shown the

need for an analytical approach to determine the governor parameter settings necessary to obtain the best response.

A number of investigators have carried on extensive studies and have proposed various criteria for optimizing the response of water turbines (1, p. 9-12; 4, p. 137; 5, p. 65; 9, p. 7; 10, p. 191). Although none of these studies are directly applicable in their entirety to Kaplan-type turbines, portions of them are applicable and can serve as a guide to a study of a linear model of this type of system.

Statement of the Problem

Develop a criteria for defining the optimal response of a Kaplan-type turbine-generator system. Make use of linearization techniques to construct the necessary linear mathematical models of the system and apply the criteria to determine the response of a Kaplan-type hydroelectric unit.

II. RESPONSE CRITERIA AND METHOD OF ANALYSIS

Response Criteria

The "optimum" response of a hydroelectric unit can be defined as that response which provides maximum initial recovery rate or speed of response with minimum overshoot and maximum rate of decay of the transient components. From the point of view of the user of large Kaplan-driven hydroelectric units this "optimum" response is subject to the following constraints:

1. The "optimum" response must be consistent with the user's type of equipment. In virtually all cases of practical interest at the present time in the United States this involves a "temporary-droop" type of governor. The method of analysis presented here is applicable with appropriate changes in the governor equation to generating units utilizing the so-called "derivative stabilized" governors which have recently appeared on the market.
2. The unit physical characteristics are fixed. (water starting time, machine starting time)

Method of Analysis

Speed of Response

It has been shown by Hovey (7, p. 587) that assuming a unit connected to an infinite system and an idealized servo linkage, the

governor equation can be written as:

$$(1) \quad -\delta T_r DG = -\Delta n + \sigma G$$

In this case a step input to the speed changer causes the wicket gates to move to their final position in accordance with an exponential curve having a time constant equal to $\frac{\delta T_r}{\sigma}$. Thus it can be seen that for fast response the product of the temporary droop in per unit and the dashpot time constant in seconds should be as small as possible.

In a practical governor the temporary droop can be varied from zero to about 0.65 per-unit while the dashpot time constant can be varied from approximately zero to about forty seconds. Normal operating ranges are not less than 0.2 per-unit temporary droop and a dashpot time constant of not less than two seconds. It is evident that the dashpot time constant exerts a much greater relative influence on the speed of response of the unit than does the temporary droop. Thus, in practical applications maximum speed of response is obtained by utilizing the minimum dashpot time constant commensurate with stability.

Minimum Overshoot

Fast response of the turbine governor is required to minimize the frequency excursion following a disturbance. In a modern turbine governor the amount of compensation (output of the temporary-droop dashpot) is limited by allowing oil to bypass the small dashpot

piston. This allows the governor to respond at its maximum rate for the larger disturbances and inherently provides minimum frequency excursion following a disturbance.

Maximum Rate of Transient Decay

The determination of the maximum rate of decay of the transient components is a stability analysis. The root-locus method (3, p. 192-217) used to examine system stability makes it possible to obtain a measure of the transient characteristics of a system that is independent of the system disturbance. The method is based on the changes that take place in the roots of the characteristic equation of the system as the system parameters are varied. In this particular system the coefficients of the characteristic equation will always be real. The roots will either be real or appear as conjugate pairs of complex numbers. The real roots, α_i , will contribute terms in the solution of the form $A_i e^{\alpha_i t}$. A complex conjugate pair of roots would give rise to a term in the solution of the form $A_i e^{\alpha_i t} \sin(\beta_i + \theta_i)$. The α_i and β_i are determined by the roots of the characteristic equation and convey information about the transient nature of the system. Any roots which lie in the right half of the complex plane represent terms in the solution which grow with time and thus represent an unstable system. Roots which lie on the left side of the complex plane represent terms which decay with time. The larger the

absolute value of a negative α_i the faster the term will die out. When time, in seconds is equal to $1/\alpha$ the term will have decayed to approximately 37 percent of its initial value and when time is equal to $4/\alpha$ the term will have decayed to less than two percent of its original value and can be neglected. To improve the transient response it is necessary to move the roots of the characteristic equation further to the left when plotted on the complex plane. The distance of the complex roots from the real axis indicates the frequency of the oscillatory terms in radians per second.

The assumption is made that if each unit is stable when supplying an isolated load whose damping characteristics result in zero system damping then the system of interconnected units will also be stable.

III. DERIVATION OF THE MATHEMATICAL MODELS

General

A mathematical model is defined as an equation which mathematically relates the relevant parameters of a physical system. In order for the model to be useful it must exist in a form suitable for the intended purpose. For this analysis two forms will be useful. One form consists of a model of a Kaplan unit feeding a reasonable approximation of the existing power system. This model will be solved with a digital computer by the step-by-step method to predict the actual response of the unit to an arbitrary disturbance. The other form will be a linear model of the unit supplying an isolated load. The characteristic equation of this model will be solved with the digital computer. The roots of the characteristic equation for various parameter settings will be plotted on the complex plane to determine the relative transient stability of the unit.

The equations which follow are in terms of per unit deviations about a given operating point. Per unit bases are rated head, rated horsepower, rated KVA, rated speed and frequency, rated servomotor stroke and rated blade angle in degrees.

Differential Equation of Governor

The differential equation describing the dynamic performance of the temporary droop type governor which is representative of virtually all domestic governor manufacturers can be obtained by noting that the gate velocity is proportional to the sum of the speed error signal, the output of the compensating dashpot and the permanent speed-droop signal (8, p. 10). The transformed differential equation relating wicket gate servo-motor velocity to speed error is as follows:

$$(2) \quad -T_g \dot{G}S = -n_r + n_g + \sigma G + \frac{\delta T_r G S}{T_r S + 1}$$

The block diagram illustrative of the governor action is shown in Figure 1. Limits are shown in Figure 1a for the gate velocity and for the output of the temporary droop dashpot. In the actual governor the gate velocity is limited by stop nuts on the oil distributing valve. The output of the compensating dashpot is limited by allowing oil to bypass the small dashpot piston for a given displacement. This form of governor model will be used in the step-by-step solution.

Figure 1b is a linear representation of Figure 1a and is in suitable form for inclusion as a linear representation of the dynamic performance of the governor.

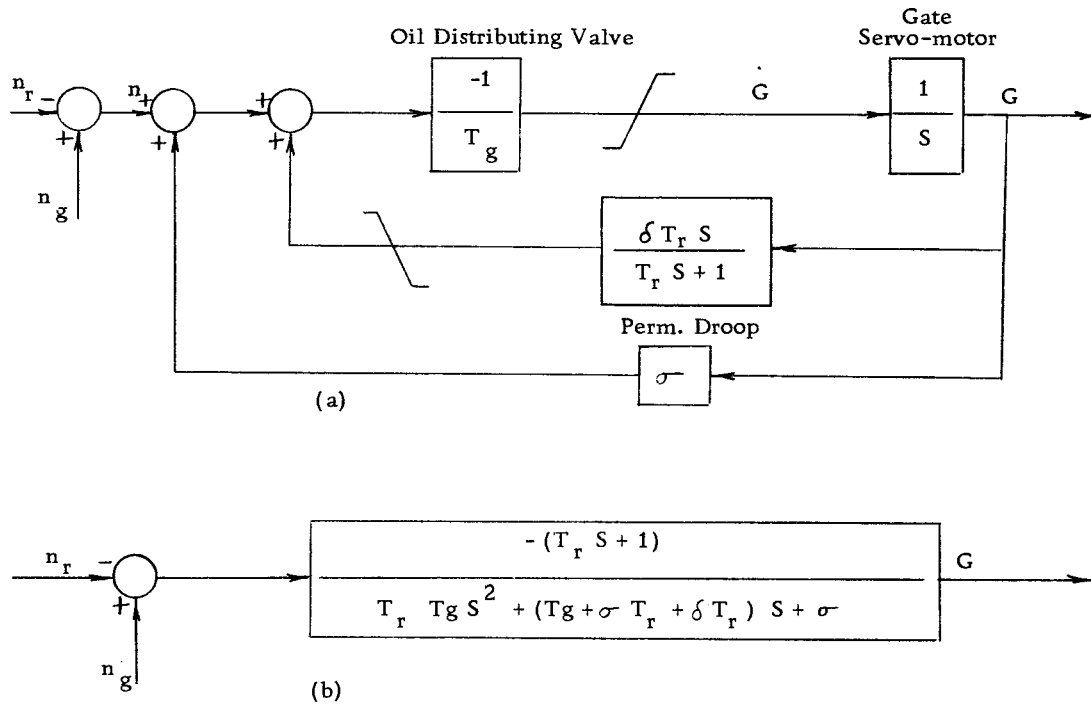


Figure 1. (a) Complete governor block diagram.

(b) Reduced block diagram of governor.

Differential Equation of Turbine

The change in turbine output as a result of a change in gate position can be determined by a modification of the slope constant method proposed by Paynter (6, p. 369).

$$(3) \quad q = \frac{\partial q}{\partial G} G + \frac{\partial q}{\partial h} h + \frac{\partial q}{\partial n_g} n_g = a_{11} G + a_{12} h + a_{13} n_g$$

$$(4) \quad m = \frac{\partial m}{\partial G} G + \frac{\partial m}{\partial h} h + \frac{\partial m}{\partial n_g} n_g = a_{21} G + a_{22} h + a_{23} n_g$$

The rate of change of per-unit flow is proportional to head (5, p. 68).

The transformed differential equation is as follows:

$$(5) \quad T_w q S = -h$$

where

$$(6) \quad T_w = \frac{\Sigma LV}{gH}$$

combine (5) and (3) and solve for h

$$(7) \quad h = \frac{-a_{11} T_w GS - a_{13} T_w n_g S}{a_{12} T_w S + 1}$$

substitute (7) in (4) and collect terms

$$(8) \quad m = \frac{[(a_{12} a_{21} - a_{11} a_{22}) T_w S + a_{21}] G - [(a_{13} a_{22} - a_{12} a_{23}) T_w S - a_{23}] n_g}{a_{12} T_w S + 1}$$

The block diagram descriptive of equation (8) is shown in Figure 2.

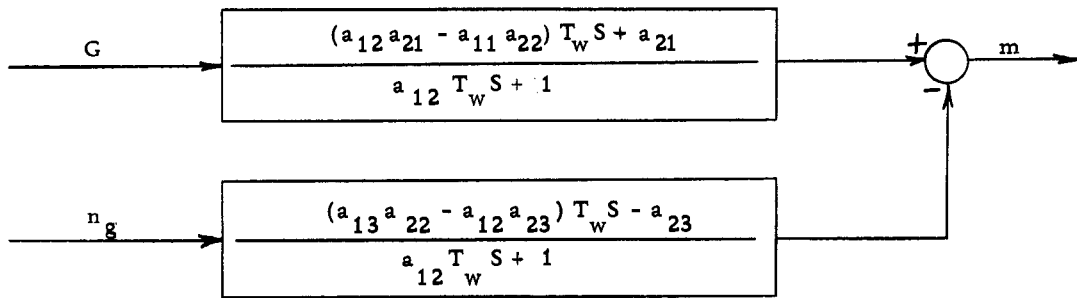


Figure 2. Block diagram of turbine using variable slope constants.

Thus it can be seen that the output of the turbine is composed of two components. One component represents the torque derived from the flow of water through the turbine. The other component represents the self-damping of the turbine as a function of speed. For a typical Kaplan turbine the constants a_{13} and a_{23} are negligibly small. For

this study the range of speed variation will be small. Thus the damping term can be neglected with negligible error.

The rest of the a_{ij} variables can be considered as constants for small changes about the operating point if the blades remain "on-cam". In actuality the blades are controlled by the gates and thus lag behind the gates in attaining their "on-cam" position. This momentarily decreases the turbine output by an amount Δe which is a function of how far "off-cam" the blades actually are. This effect can be approximated by a single-order lag function whose initial effect is to subtract from the turbine output to compensate for this blade effect. The block diagram of a linear model to accomplish this effect is shown in Figure 3.

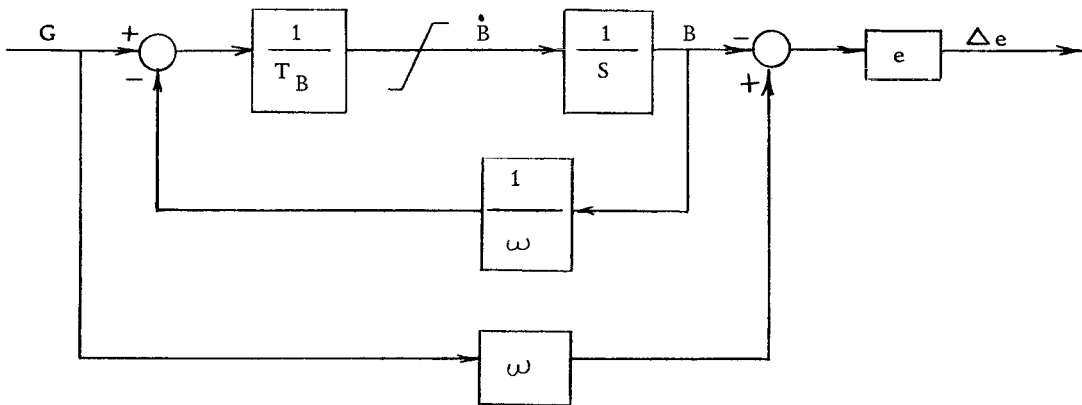


Figure 3. Block diagram of a linear model to include blade effect.

The slope constants ω and e are determined from the turbine test data at the desired operating point. The linear representation, obtained from the equations represented by Figure 3 is as follows:

$$(9) \quad \frac{\Delta e}{G} = \frac{e \omega_t^2 B S}{\omega_t B S + 1}$$

The step-by-step solution permits the use of tables derived from the turbine test data. The associated block diagram is shown in Figure 4.

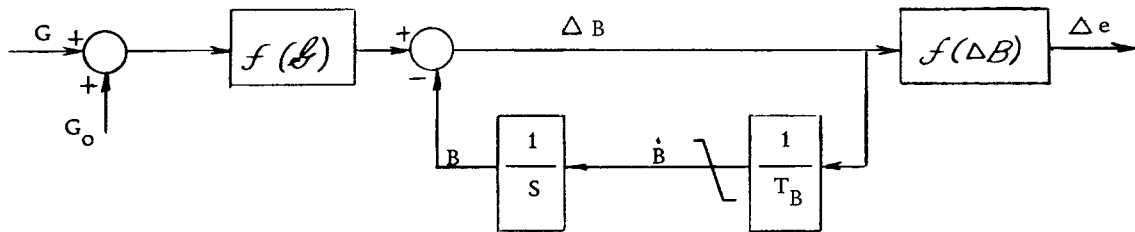


Figure 4. Block diagram to accomplish blade effect for step-by-step solution.

The function $f(B)$ relates the "on-cam" position of the blades to the actual gate position. The function $f(\Delta B)$ is determined from the turbine efficiency curve at various blade angles.

Representation of the System

The representation of the system used by Concordia and Kirchmayer (4, p. 133) is used in this study with only minor modifications. The acceleration of the machine, in per unit, is proportional to the accelerating torque.

$$(10) \quad T_m \cdot Dn = m - T_e$$

The block diagram of the system is shown in Figure 5.

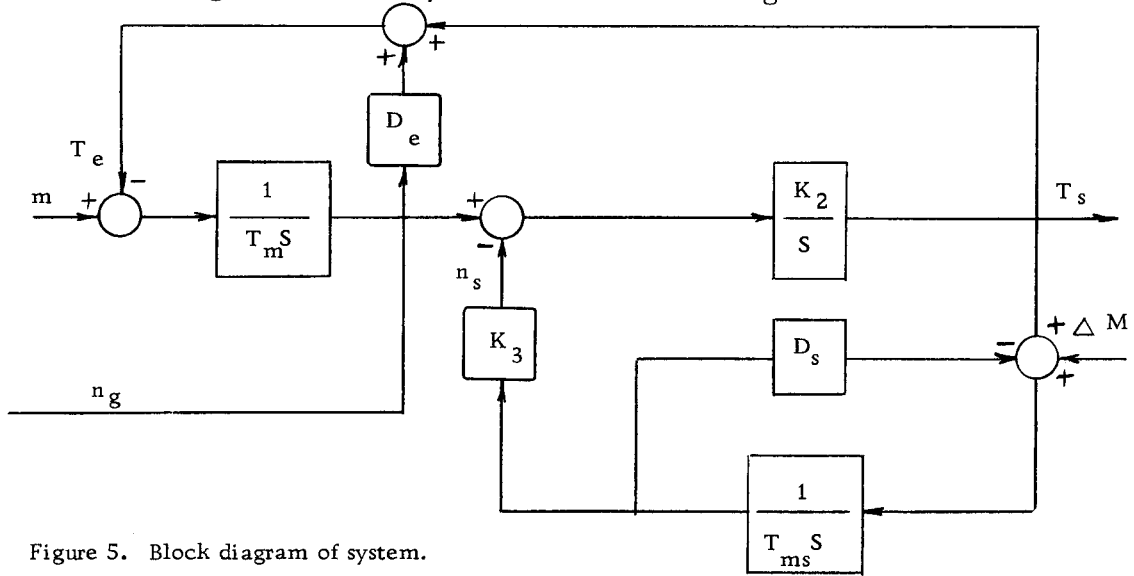


Figure 5. Block diagram of system.

Mathematical Model for Step-by-Step Solution

The system of equations describing the transient performance of a power generating unit driven by a Kaplan turbine prime mover determined from Figures 1a, 2, 4, and 5 is as follows:

$$(11) \quad n = n_g - n_r$$

$$(12) \quad \dot{y} = \frac{\delta T_r \dot{G} - y}{T_r} \quad -K_4 \leq y \leq K_4$$

$$(13) \quad \dot{G} = \frac{n + \sigma G + y}{-T_g} \quad \frac{-1}{t_g} \leq \dot{G} \leq \frac{1}{t_g}$$

$$(14) \quad (a_{12}a_{21} - a_{11}a_{22}) T_w G + a_{21} G = a_{12} T_w \dot{P}_w + P_w$$

$$(15) \quad \mathcal{G} = G + G_o$$

$$(16) \quad B = \text{TLU of } \mathcal{G}$$

$$(17) \quad ek = B' - B$$

$$(18) \quad \dot{B} = \frac{e}{t_B} \qquad \frac{-1}{t_{BC}} \leq \dot{B} \leq \frac{1}{t_{BO}}$$

$$(19) \quad \Delta e = \text{TLU of } ek$$

$$(20) \quad m = P_w - \Delta e$$

$$(21) \quad \dot{n}_g = \frac{m - T_e - D_m n_g}{T_m}$$

$$(22) \quad \dot{T}_s = K_2(n_g - n_s)$$

$$(23) \quad T_e = D_e(n_g - n_s) + T_s - \Delta T_s$$

$$(24) \quad \dot{n}_s = \frac{K_3 \Delta M + K_3 \Delta T_s - D_s n_s}{T_{ms}}$$

The block diagram for this system of equations is shown in Appendix A.

Characteristic Equation for Root-Locus Analysis

The block diagram of a unit supplying an isolated load is shown in Appendix B and can be obtained from Figures 1b and 2 and equations 9 and 10. The characteristic equation is obtained from the equations represented in Appendix B by assuming n_r equal to zero and solving for the ratio of n_g to T_e . The characteristic equation is as follows:

$$(25) \quad \text{C. E.} = A_1 S^5 + A_2 S^4 + A_3 S^3 + A_4 S^2 + A_5 S + A_6$$

where

$$A_1 = a_1 2 T_g T_m T_r T_w$$

$$A_2 = T_g T_m T_r \left(1 + \frac{a_{12} T_w}{T_B}\right) + a_{12} T_m T_w (T_g + \sigma T_r + \delta T_r) + D_m a_{12} T_g T_r T_w$$

$$A_3 = D_m T_g T_r \left(1 + \frac{a_{12} T_w}{T_B}\right) + a_{12} D_m T_w (T_g + \sigma T_r + \delta T_r) + \frac{T_m T_r T_g}{T_B}$$

$$+ T_m (T_g + \sigma T_r + \delta T_r) \left(1 + \frac{a_{12} T_w}{T_B}\right) + T_m \sigma a_{12} T_w$$

$$+ T_r (K_1 T_w - a_{12} T_w e\omega)$$

$$A_4 = \frac{T_m}{T_B} (T_g + \sigma T_r + \delta T_r) + T_m \sigma \left(1 + \frac{a_{12} T_w}{T_B}\right) + \frac{D_m T_r T_g}{T_B}$$

$$+ D_m (T_g + \sigma T_r + \delta T_r) \left(1 + \frac{a_{12} T_w}{T_B}\right) + D_m \sigma a_{12} T_w$$

$$+ a_{21} T_r + \frac{K_1 T_w T_r}{T_B} - e\omega T_r + K_1 T_w - a_{12} T_w e\omega$$

$$A_5 = \frac{T_m \sigma}{T_B} + \frac{D_m}{T_B} (T_g + \sigma T_r + \delta T_r) + \sigma D_m \left(1 + \frac{a_{12} T_w}{T_B}\right) + \frac{a_{21} T_r}{T_B}$$

$$+ a_{21} + \frac{K_1 T_w}{T_B} - e\omega$$

$$A_6 = \frac{a_{21}}{T_B} + \frac{D_m \sigma}{T_B}$$

$$T_B = \omega t_B$$

$$K_1 = a_{12} a_{21} - a_{11} a_{22}$$

IV. COMPUTER PROGRAMS

General

Programs for both models were developed for the IBM 1920 computer system. Use was made of existing programs as subroutines where possible. Both programs will ultimately become a part of the users library.

Root-Locus Program

A flow chart, as shown in Appendix C, is intended to show in a very general way the make-up of this program. This program calculates the coefficients of the characteristic equation from the input data and then uses program number 7.0.040 from the 1620 General Program Library to calculate the roots of the auxiliary equation by Bairstow's method. Various parameters are automatically stepped through a range of values and the roots printed out. These roots must then be manually plotted on the complex plane to obtain the root-locus plot.

Step-by-Step Solution

A flow chart of the salient feature of this program is shown in Appendix D. This program has the option of either printing all the

variables in tabular form or plotting not more than five of the variables by a standard plot subroutine. The program is a step-by-step solution of the differential equations of the machine connected to the system and takes into account the limits of gate and blade velocity and compensating dashpot output. A table-look-up routine of the blade-gate relationship and the off-cam efficiency relationship is used.

V. APPLICATION OF CRITERIA

A typical Kaplan-turbine generating unit was analyzed by the root-locus method for various settings of the governor parameters T_g , T_r , σ and δ .

Changes in the parameter T_g did not materially affect the root-locus plot. This parameter is a measure of the sensitivity of the governor and its adjustment can be left to the judgement of station operators with little affect on system performance. A representative value for T_g is 0.15.

Variations in the permanent droop parameter had a significant effect on the root-locus plot. Increased values of permanent droop made all roots more negative which would increase the relative stability and speed of response. This can be predicted from Equation 1 and thus a root-locus plot is not shown. The typical hydro governor can be adjusted to a maximum of five percent permanent speed droop and thus it appears beneficial to set all permissible units at the maximum.

If values of 0.15 for T_g and 0.05 for σ are selected the problem is reduced to a two dimensional problem. Root-locus plots for various values of T_r and δ are shown in Appendix E. From this plot it can be seen that there are two roots which are relatively close to the imaginary axis, $\alpha_1 \pm j\beta_1$, and α_2 . Increasing the temporary droop to about 0.3 makes α_1 , and α_2 both more negative with

beneficial results on the transient response. Higher values of δ make α_2 more negative but α_1 , becomes less negative thus 0.3 is selected as the optimal value of δ . Increasing the dashpot time constant makes α_1 , more negative but makes α_2 less negative. Equation 1 prescribes that the minimum T_r consistent with stability be used. At best the selection of T_r is an arbitrary one depending on the stability margin desired. A dashpot time constant of four seconds would appear appropriate in this case. A more sophisticated means of selecting T_r is not justified as actual operating experience has shown that it is subject to rather wide fluctuations due to temperature changes.

Using these values of temporary droop and dashpot time constant the speed of response of the unit to a change in load setting is shown in Appendix F.

VI. CONCLUSIONS

The determination of the optimal response of a Kaplan turbine is essentially a stability analysis. To be successful this analysis must be an analytical one as the effects of varying the individual turbine governor adjustments is well obscured by the stabilizing effect of the rest of the system until a major percentage of the governors in the system have been changed correspondingly.

The root-locus method can be used to define the limits of stability. Once absolute stability is obtained the degree of relative stability is one of choice in which the stability margin is obtained at the expense of fast return to equilibrium conditions.

The adjustment of the governor sensitivity within the range provided on modern governors has very little effect on the system performance and can be adjusted to provide minimum regulating effort for the particular machine.

When possible the permanent speed droop should be set at its maximum of five percent contributing to increased stability and speed of response.

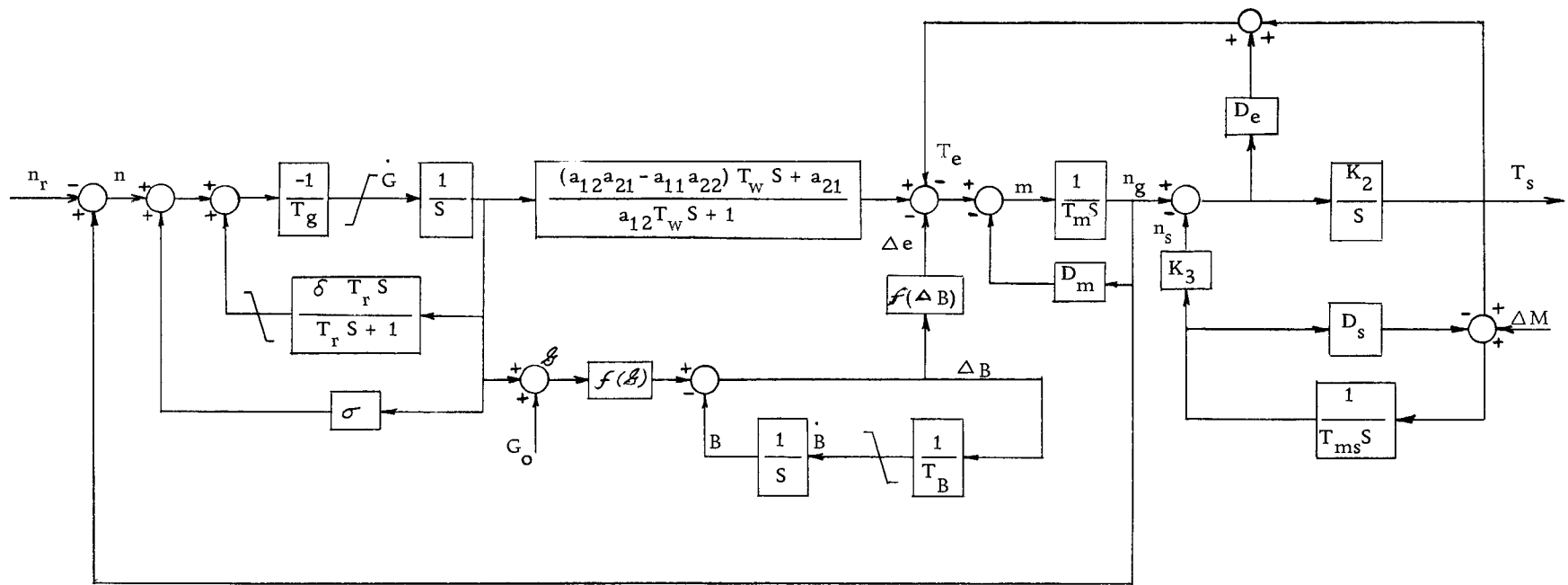
The temporary droop and dashpot time constant should be adjusted, consistent with stability, so that their product is a minimum. In practice this will result in the minimum permissible dashpot time

constant as it dominates the product. . A considerable margin of stability should be provided in practice to compensate for rather wide fluctuations in this parameter after it is adjusted.

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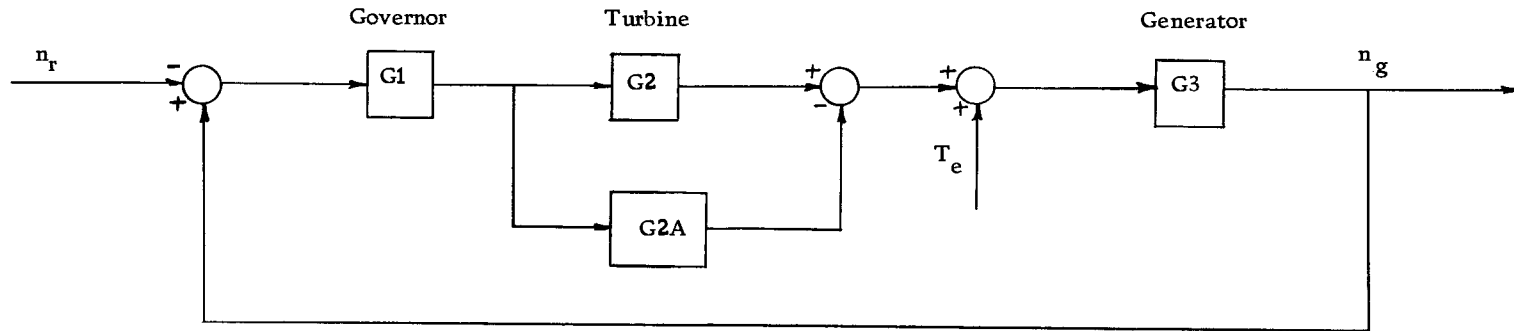
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APPENDICES



APPENDIX A

Block diagram of system for
step-by-step solution



where

$$G1 = \frac{-(T_r S + 1)}{T_r T_g S^2 + (T_g + \sigma T_r + \delta T_r) S + \sigma}$$

$$G2 = \frac{(a_{12} a_{21} - a_{11} a_{22}) T_w S + a_{21}}{a_{12} T_w S + 1}$$

$$G2A = \frac{e \omega_B^2 t_B S}{\omega_B t_B S + 1}$$

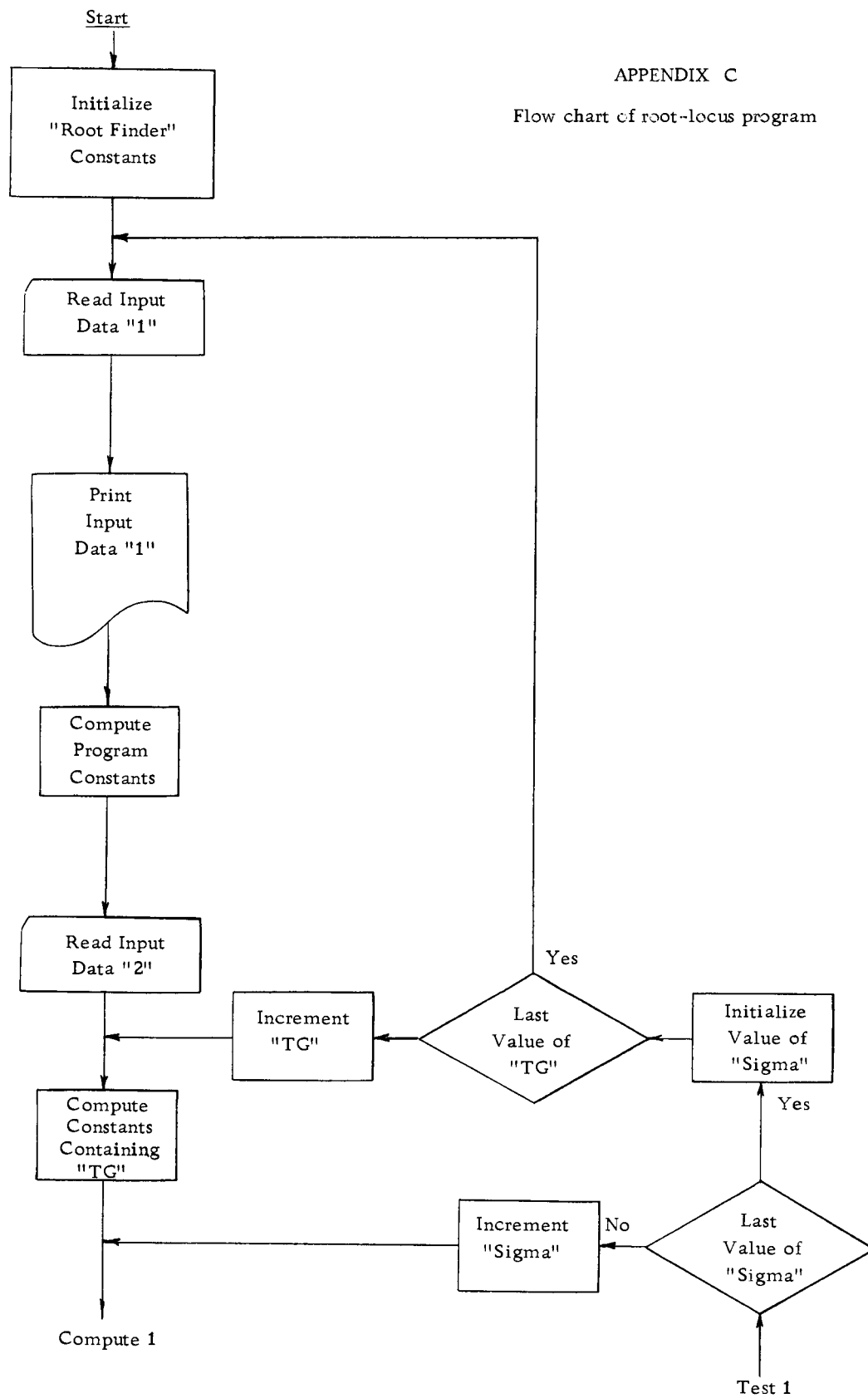
$$G3 = \frac{1}{T_m S}$$

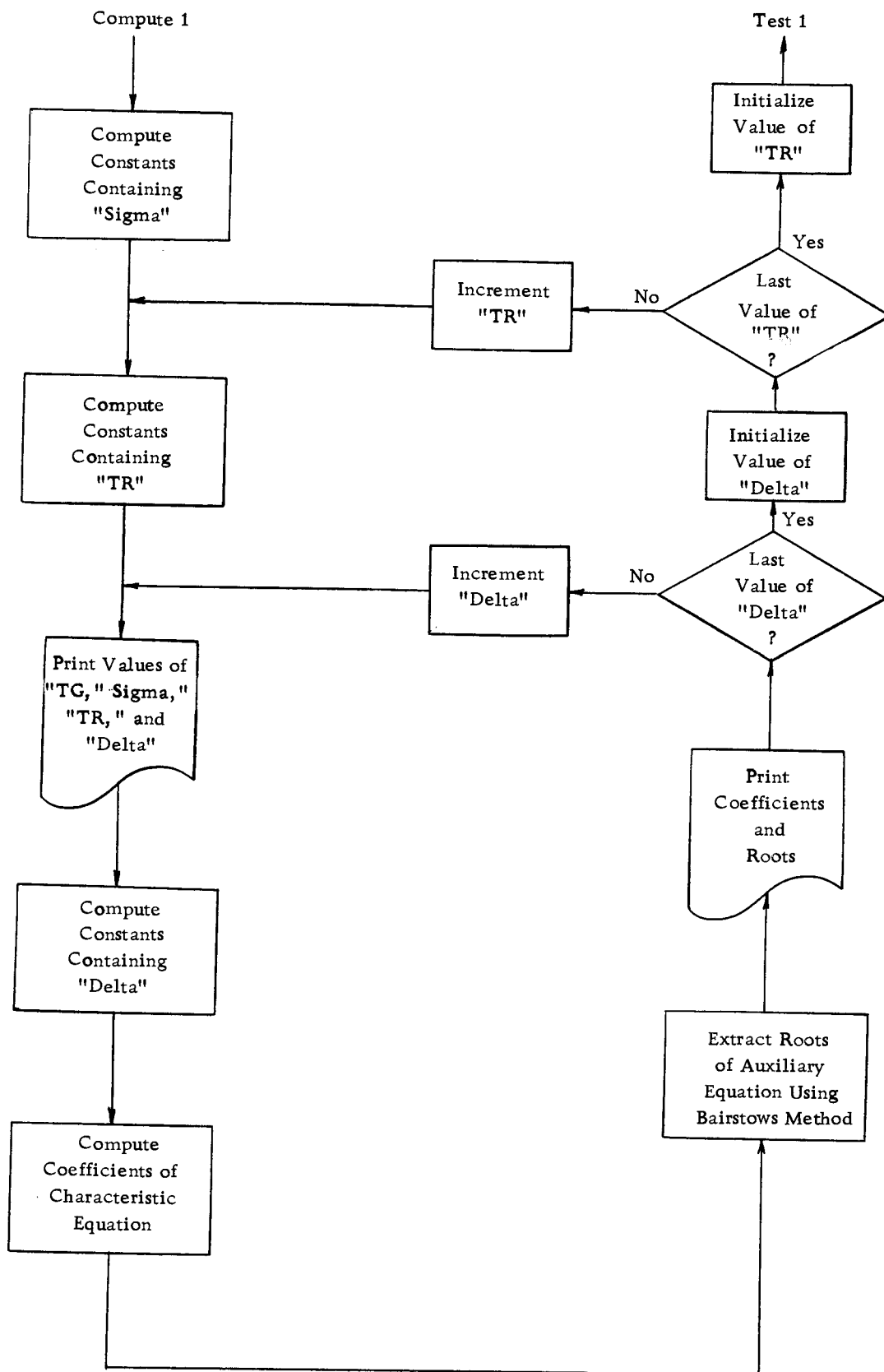
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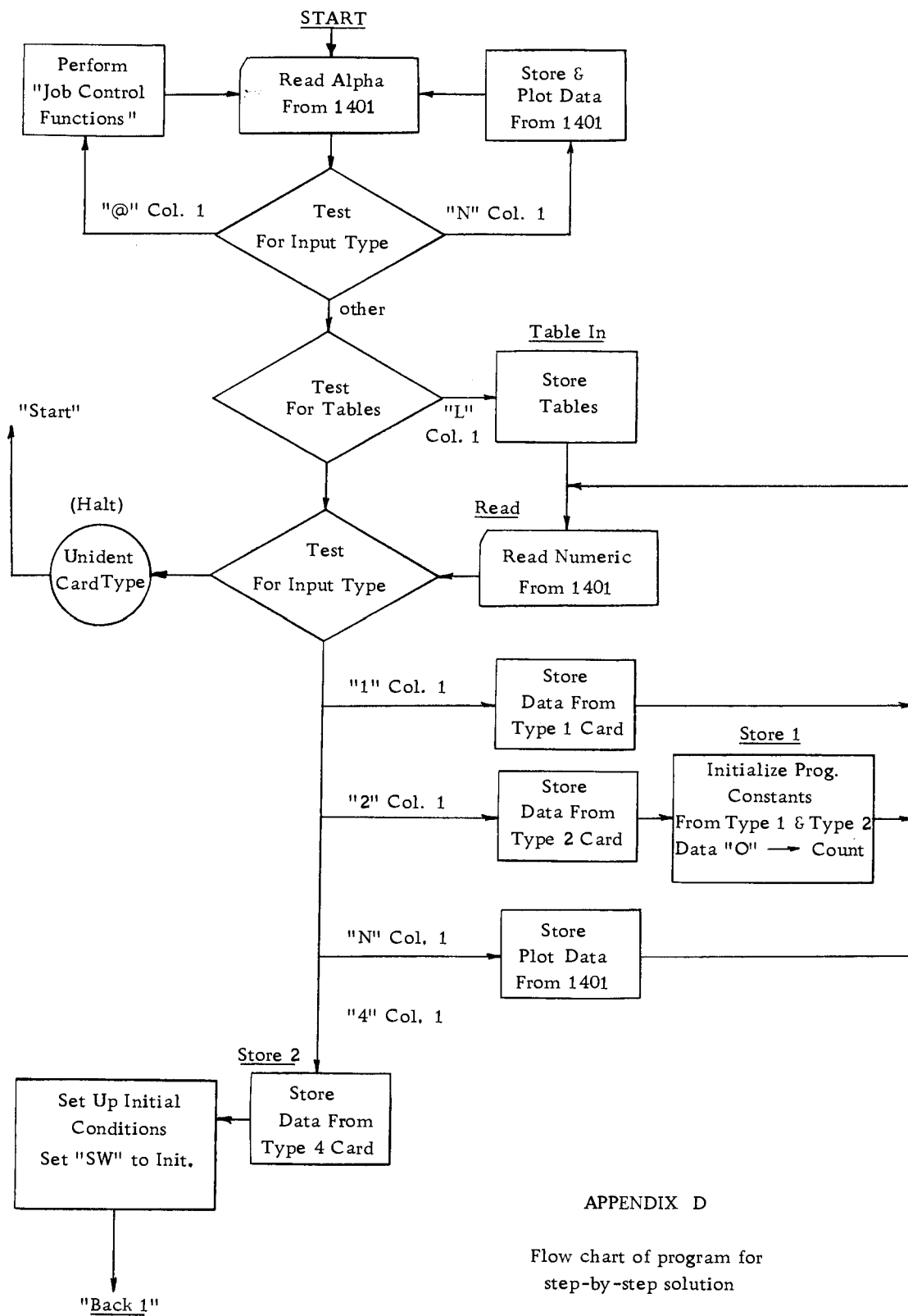
Block Diagram of Unit Supplying
an Isolated Load

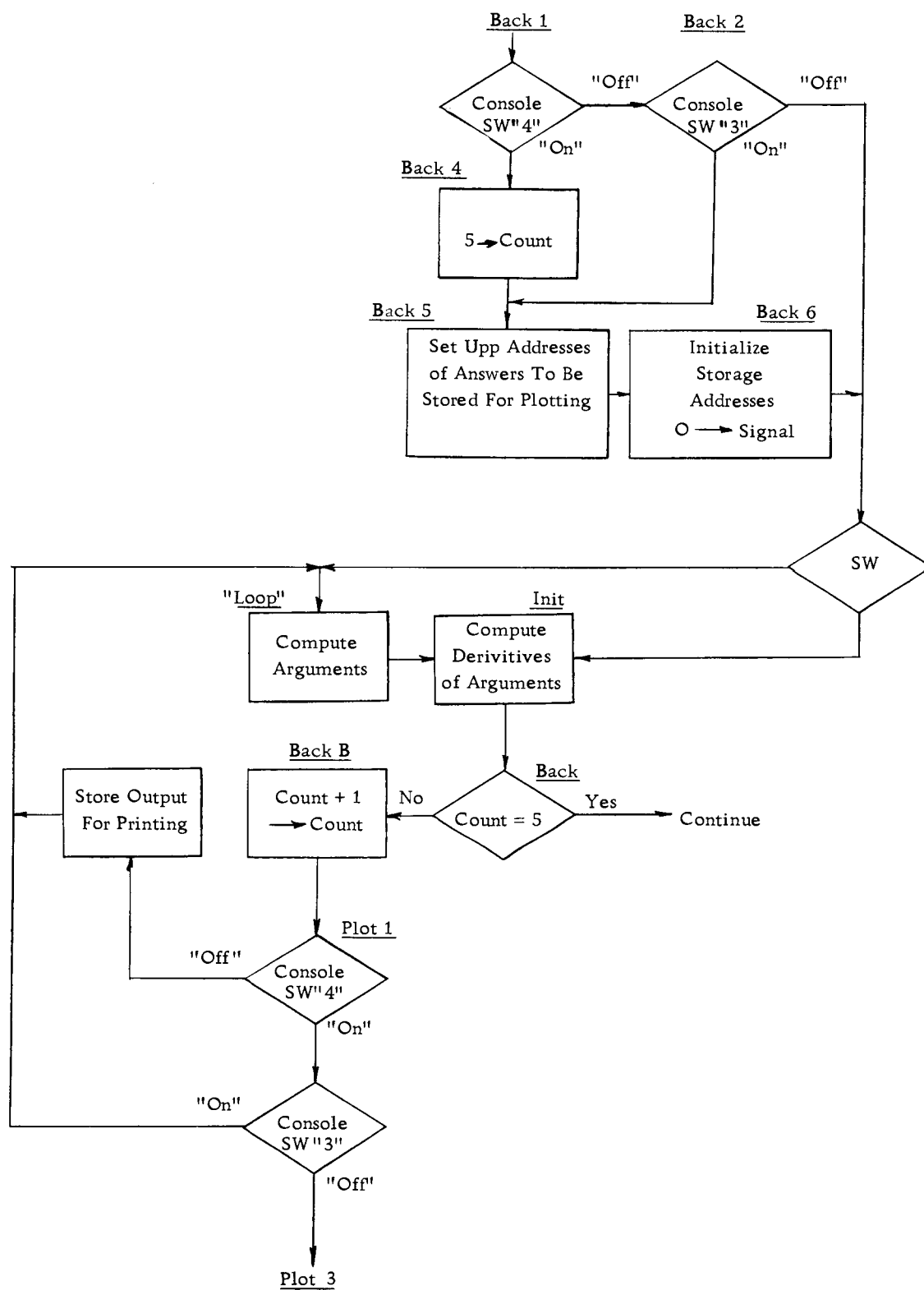
APPENDIX C

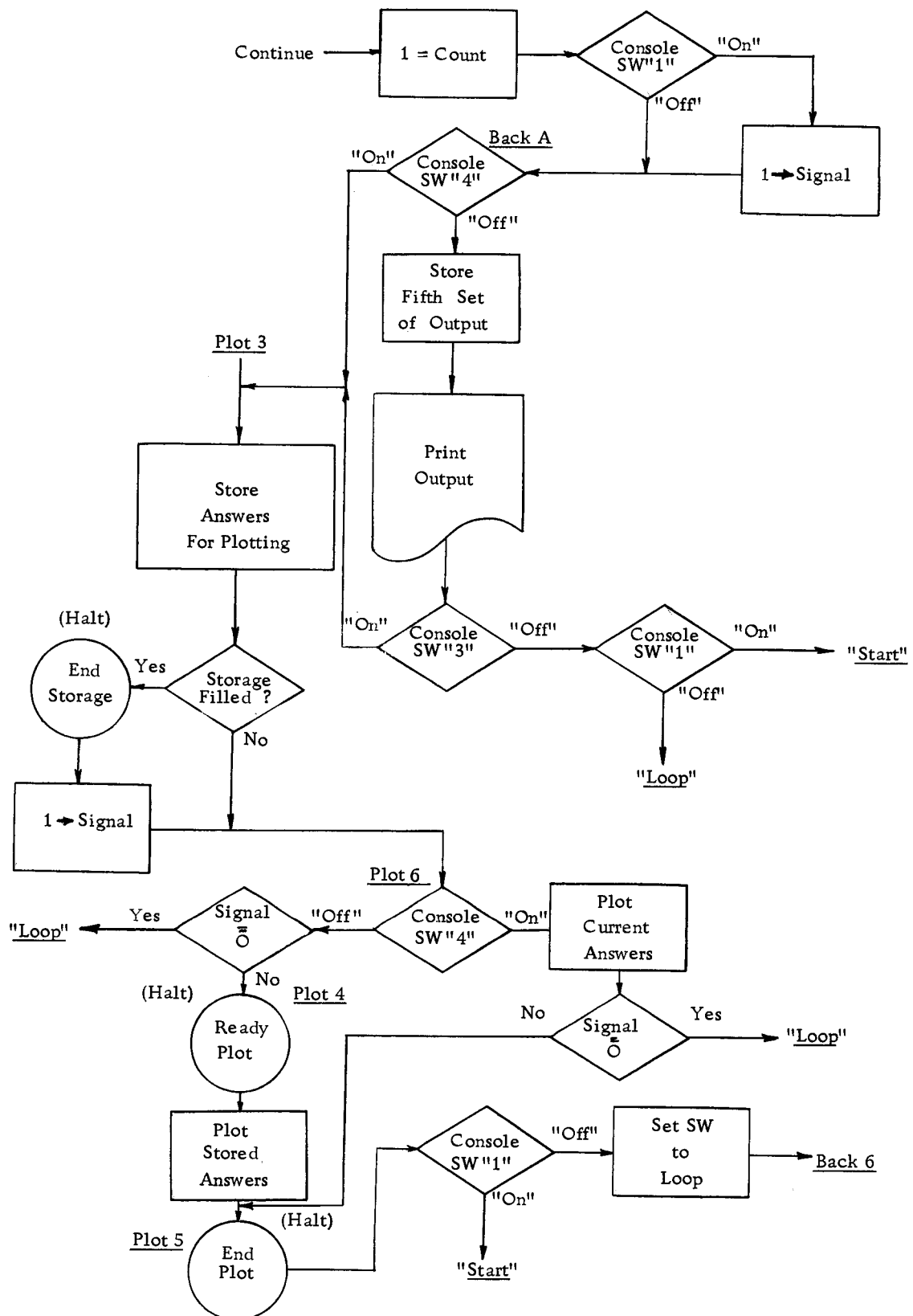
Flow chart of root-locus program









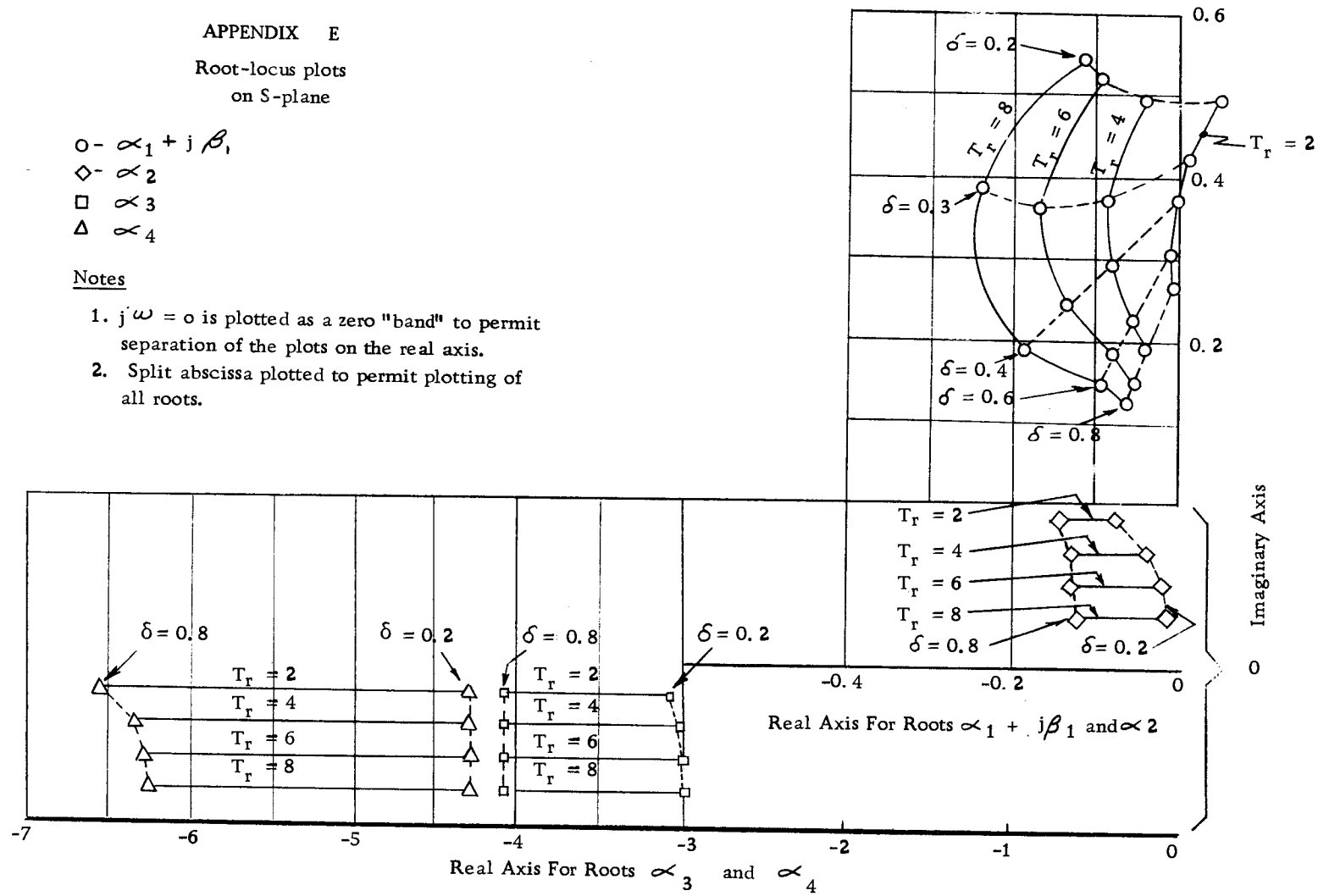


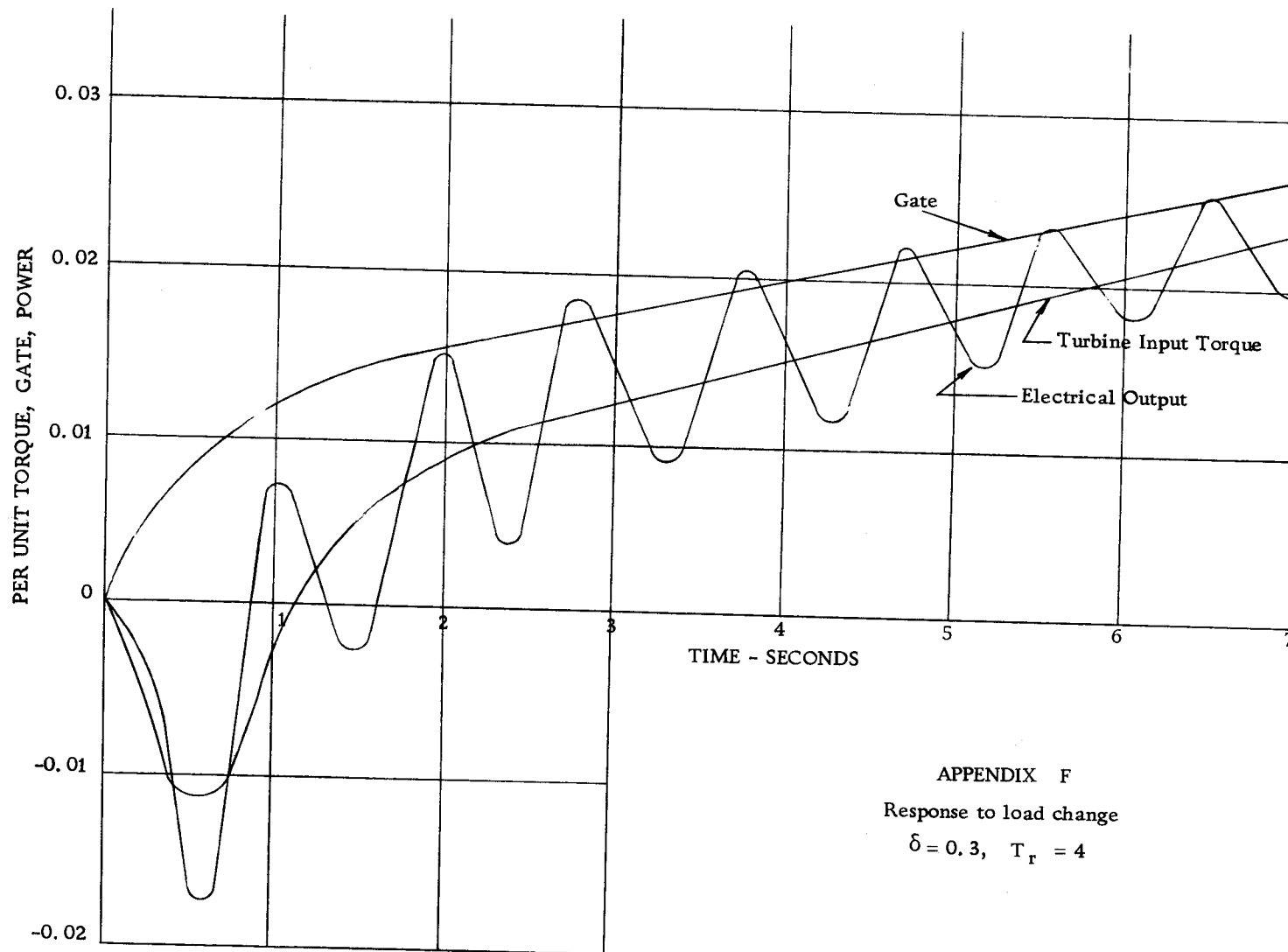
APPENDIX E
Root-locus plots
on S-plane

- - $\alpha_1 + j\beta_1$
◇ - α_2
□ - α_3
△ - α_4

Notes

1. $j\omega = 0$ is plotted as a zero "band" to permit separation of the plots on the real axis.
2. Split abscissa plotted to permit plotting of all roots.





APPENDIX F
 Response to load change
 $\delta = 0.3, T_r = 4$

APPENDIX G

List of Nomenclature

a_{ij}	Slope constants from turbine test data
B_o	Blade deviation necessary to saturate blade relay valve
B	Blade deviation starting position
B'	Blade "on-cam" position
ΔB	Blade deviation from "on-cam" position
D	Operator $\frac{d}{dt}$
D_e	Generator electrical damping
D_m	Turbine self-regulation
	$\frac{(a_{13}a_{22} - a_{12}a_{23})T_w S - a_{23}}{a_{12}T_w S + 1}$
D_s	System damping
e	Turbine efficiency deviation slope constant
ek	ΔB
Δe	Decrease in turbine efficiency due to blades being "off-cam"
G	Gate deviation from starting position
g	Acceleration of gravity
G_o	Initial gate position
\mathcal{G}	Actual gate position
h	Instantaneous per unit head

H	Rated head
K_1	$a_{12}a_{21} - a_{11}a_{22}$
K_2	Power constant
K_3	Constant relating per unit base of unit to per unit base of system
K_4	Limit on output of dashpot
L	Length of water passage
m	Per unit torque deviation
ΔM	Step load change on system
n	Per unit speed error
n_g	Per unit speed deviation of generator
n_r	Per unit reference speed deviation
n_s	Per unit speed deviation of system
P_w	Input torque to turbine
q	Flow
S	Complex variable
t_B	$B_o t_{BC}$
T_B	ωt_B
t_{BC}	Blade timing in the close direction
t_{BO}	Blade timing in the open direction
T_e	Output power or torque, per unit
t_g	Gate timing of governor
T_g	βt_g

T_m	Machine starting time
T_{ms}	Equivalent system inertia
T_r	Dashpot relaxation time constant
T_s	Electrical power or torque transmitted to system, per unit
T_w	Water starting time
ΔT_s	Load rejection constant used in computer program
TLU	Table look up
V	Velocity of water in section
y	Output of compensating dashpot
∂	Partial derivative
ω	Slope constant determined from turbine blade-gate relationship
σ	Permanent droop
δ	Temporary droop
β	Speed deviation necessary to "saturate" governor distributing valve