Assimilation of Ship-Mounted ADCP Data for Barotropic Tides: Application to the Ross Sea

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ABSTRACT

The application of a generalized inverse approach for assimilating vessel-mounted acoustic Doppler current profiler (VM-ADCP) data into numerical solutions of barotropic tides is described. The derived estimates of tidal currents can be used to detide the VM-ADCP data and expose underlying mean circulation. The methodology is illustrated with data assimilation models of tidal currents in the Ross Sea. The prior solution, obtained by solving the nonlinear shallow-water equations by time stepping with a linear bottom friction parameterization and elevation of open boundary conditions obtained from a circum-Antarctic tide model, provides reasonably good fit to most available moored current meter data. Two inverse solutions were obtained: one assimilating moored current meter records and the other assimilating three cruises of VM-ADCP data. Fitting either the mooring time series or the VM-ADCP records leads to only small changes relative to the prior solution currents, except over the shelf break where short length scale, energetic diurnal topographic vorticity waves are present. It is shown that the dynamics embedded in the representer functions provides reasonable tidal corrections even with no prior information about forcing at open boundaries.

1. Introduction

Removing tidal signals from current meter data is an essential processing step in exploring mean and low-frequency (e.g., wind forced) ocean variability. For time series measurements from moored current meters, simple harmonic analyses efficiently remove tides. This detiding procedure takes advantage of the approximate temporal invariance of the amplitude and phase coefficients of the various tidal components at a specific site. In the last decade, however, a large amount of current data have been obtained from vessel-mounted acoustic Doppler current profilers (VM-ADCPs), which provide quasi-continuous profiles of upper-ocean currents measured along the ship tracks. The more complex spatial sampling provided by VM-ADCP data relative to moorings provides a valuable complementary view of ocean processes. However, simple frequency-domain analyses are no longer appropriate for detiding since the tidal amplitude and phase coefficients now vary, sometimes quite rapidly, along the cruise track. The exception to this problem is when the survey is organized to sample for a sufficiently long time at each chosen location (see, e.g., Geyer and Signell 1990).

Two straightforward approaches to detiding VM-ADCP records are described by Foreman and Freeland (1991). The first uses a purely hydrodynamic approach based on numerical modeling of the tides. The second is purely empirical, being based on least squares fitting of the data to a model (see, e.g., Candela et al. 1992). In this second approach, temporal variations are described by sines and cosines at tidal frequencies, and spatial variations of major tidal constituents are described by polynomials or other prescribed basis functions.

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A purely model-based approach is limited by many potential sources of error, such as insufficient model resolution, simplifications in the assumed dynamics (e.g., ignoring effects of stratification and poor representation of frictional energy loss), and inaccuracies in open boundary conditions. Poorly resolved or inaccurate bathymetry poses a particularly serious problem for modeling tidal currents (Foreman and Freeland 1991; Erofeeva et al. 2003). Given uncertainties in available bathymetry over much of the ocean, obtaining accurate tidal currents from a purely numerical calculation remains a challenge. A purely empirical approach also has severe limitations. Such methods can be successfully applied for small domains where spatial variations of tidal velocity fields are weak, for example, the western Mediterranean Sea (García-Gorriz et al. 2003). However, to allow for the complex patterns of current variations frequently encountered in shallow seas, higher-order polynomials or a large number of other spatial basis functions are required. A poor choice of complex basis functions can result in an ill-conditioned least squares problem, causing spurious oscillations in areas where data coverage is poor and making accurate separation of two or more tidal constituents of comparable amplitude difficult (Foreman and Freeland 1991). More fundamentally, this purely empirical approach completely ignores the physics of tidal currents, resulting in tidal velocity and height fields that are not even approximately dynamically or kinematically self-consistent.

Data assimilation methods are a promising approach for accurate mapping of tidal currents, since data can be optimally combined with dynamical equations to address the limitations of both approaches mentioned above. Griffin and Thompson (1996) and Dowd and Thompson (1996) used a “strong constraint” approach to assimilate ship-mounted ADCP data into a tidal model by adjusting open-ocean boundary conditions. With this approach the dynamical equations are satisfied exactly in the interior of the domain. The strong-constraint assumption is reasonable for the small continental shelf domain considered in these studies, where uncertainties in open boundary conditions are generally the largest source of error in a numerical model. For larger domains, and in areas with more extreme topographic variation, a weak-constraint assimilation approach is more appropriate. Bogden and O’Donnell (1998) used a weak-constraint generalized inverse (GI) approach (e.g., Bennett 1992; Egbert et al. 1994) to assimilate VM-ADCP data from the Long Island Sound. The data assimilation was performed in the time domain, using the shallow-water equations (SWEs) and data from repeated (up to 6 times daily for 10 days) sampling of a single transect across the sound. A series of short assimilation experiments, each of 1–2 day duration, demonstrated the value of this approach for removing tides, to allow for the improved resolution of subtidal flows.

In this paper we discuss the application of a frequency-domain GI approach for assimilation of VM-ADCP data into a numerical model for barotropic tides. This study differs from the work of Bogden and O’Donnell (1998) in two principal ways. First, by modeling the tides in the frequency domain we can use data from multiple surveys collected over an extended period of time, and we can also incorporate data from other sources including current moorings, tide gauges, and altimetry. Second, we apply the reduced basis scheme described in Egbert and Erofeeva (2002, hereinafter EE) to solve the GI problem efficiently for an arbitrary distribution of data within the model domain. In the study by Bogden and O’Donnell (1998), the survey was organized so that after binning to the resolution of the numerical model, data were available at only a small number of locations, making direct application of the representer approach (Bennett 1992) practical. This approach would not be practical for the sort of extensive ADCP surveys considered in this paper.

Whether it is formulated in the frequency or time domains, the GI approach results in a solution that is an expansion in a set of special spatiotemporal basis functions [the so-called representers; Bennett (1992)]. In this sense the GI approach is superficially similar to the empirical scheme of Candela et al. (1992). However, as discussed by Egbert and Bennett (1996) and Bogden and O’Donnell (1998), the basis functions for GI are derived directly from the dynamics and bathymetry of the area, and are physically consistent within assumed levels of errors in dynamics, forcing, and boundary conditions. Because the dynamics are embedded in the basis functions, the GI approach also allows extrapolation of tidal current estimates beyond the area of data coverage, provided that the error levels in the assumed dynamics are not too great.

Erofeeva et al. (2003) discuss application of the GI approach to mapping complex barotropic tidal current variations on the Oregon continental shelf by assimilating high-frequency coastal radar data into an SWE model. Kurapov et al. (2003) extended this work, applying GI to assimilate HF radar data into a linear primitive equations model for internal tides on the central Oregon shelf. Here we apply the GI method to assimilate VM-ADCP data into a barotropic tidal model for the Ross Sea. In this study, the Ross Sea is simply a convenient test environment for the application of GI methodology to improving tidal models with
data acquired from VM-ADCP. Limited data suggest that the tides are approximately barotropic, VM-ADCP data cover a significant area, and expected tidal currents vary a great deal within the region covered by ship surveys (Padman et al. 2003). The use of these methods applied to a larger set of available tidal data to produce an optimal map of Ross Sea tides will be reported elsewhere.

2. Data assimilation approach

We assume depth-integrated shallow-water dynamics as described in EE:

\[
\frac{\partial U}{\partial t} + f \mathbf{z} \times U + F + \mathbf{U} \cdot \nabla U + A_H \nabla^2 U = -g \cdot \nabla (\zeta - \zeta_{EO} - \zeta_{SL})
\]

(1)

where \(\zeta\) is the elevation of the sea surface; \(\mathbf{U}\) is the volume transport vector, equal to velocity times water depth \(H\); \(f\) is the Coriolis parameter; \(\mathbf{g}\) is the acceleration of gravity; and \(\mathbf{z}\) is oriented to the local vertical. Dissipative terms include a parameterization of bottom friction \(F\) (quadratic or linear) and horizontal viscosity with a constant eddy coefficient \(A_H\). The equilibrium tide \(\zeta_{EO}\) is derived from the astronomical tide-generating force with allowance for the earth’s body tide, and \(\zeta_{SL}\) accounts for tidal loading and self-attraction, and is computed from a global tidal model as in EE. Boundary conditions for (1) are no flow across (and no slip along) the coast, and specification of either elevations or the normal component of volume transports on (and free slip along) any open boundaries.

The SWE (1) are discretized on a C grid and solved by time stepping with a periodic forcing followed by harmonic analysis (e.g., Egbert et al. 1994), or in the frequency domain by directly factoring the coefficient matrix for a linearized version of the SWE. The model numerics and inversion approach are described in detail in EE. Here we discuss only details specifically relevant to inversion of VM-ADCP data.

We allow for \(N_c\) constituents and use the superscript \(l\) to denote complex harmonic constants for individual tidal constituents. Observations of depth-averaged current velocities at time \(t\) and location \(\mathbf{x}\) can be written as

\[
\mathbf{u}(\mathbf{x}, t) = \text{Re} \left[ \sum_{l=1}^{N_c} \alpha_l(t) \mathbf{u}(\mathbf{x}) \right] + \mathbf{e}(\mathbf{x}, t),
\]

(2)

where \(\mathbf{u}(\mathbf{x}) = [u_l(\mathbf{x}), \nu_l(\mathbf{x})], l = 1, N_c\) are tidal velocity harmonic constants at location \(\mathbf{x}\), \(\mathbf{e}\) represents error (for purposes of estimating the tides this includes all non-tidal oceanography as well as actual measurement noise), and \(\alpha_l(t) = [1 + f_l(t)] \exp[i(\omega_l(t-t_0) + V_l(t_0) + p_l(t))]\) gives the slowly modulated periodic (“nodal”: ~18.6 yr period) time variations for constituent \(l\). In this expression \(\omega_l\) is the tidal frequency, \(V_l(t_0)\) is the Greenwich phase, and \(f_l, p_l\) are nodal corrections for amplitude and phase.

For VM-ADCP data there are generally only a few observations (often only one) at any fixed location \(\mathbf{x}\), making it impossible to estimate \(\mathbf{u}(\mathbf{x}), l = 1, N_c\) at a point by direct harmonic analysis as is routinely and easily done with current mooring time series. However, the dynamics impose spatial structure on the tidal current fields, allowing data from multiple locations to be combined in estimates of \(\mathbf{u}(\mathbf{x})\). In general, the data \(\mathbf{d} = \{d_x, k = 1, \ldots, K\}\) are measurements of components of the tidal current at times \(t_k, k = 1, \ldots, K\), and locations \(\mathbf{x}_k\) [i.e., \(\mathbf{u}(\mathbf{x}_k, t_k)\) or \(\mathbf{v}(\mathbf{x}_k, t_k)\)]. Let \(\mathbf{w} = (w^1, \ldots, w^K)\), where \(\mathbf{w} = (\mathbf{\zeta}, \mathbf{u})\) denotes the barotropic tidal elevations and depth-averaged currents. The relation between the tidal state vector \(\mathbf{w}\) and the data, given by (2), is denoted formally as \(\mathbf{d} = \mathbf{Lw} + \mathbf{e}\), where \(\mathbf{L} = (L_1, \ldots, L_K)\) is shorthand for the \(K\) measurement functionals. With the GI approach we compromise between satisfying the SWE and fitting the data by minimizing the quadratic penalty functional

\[
J(\mathbf{d}, \mathbf{w}) = \lambda (\mathbf{Lw} - \mathbf{d})^\dagger \mathbf{C}_e^{-1} (\mathbf{Lw} - \mathbf{d}) + (\mathbf{Sw} - \mathbf{f}_0)^\dagger \mathbf{C}_d^{-1} (\mathbf{Sw} - \mathbf{f}_0).
\]

(3)

In (3), \(\mathbf{Sw} = \mathbf{f}_0\) represents the system of Eqs. (1), with forcing \(\mathbf{f}_0\) derived from \(\zeta_{EO}, \zeta_{SL}\), and the prior open boundary conditions; the superscript \(\dagger\) denotes the complex conjugate transpose; and \(\mathbf{C}_e\) and \(\mathbf{C}_d\) are the covariance matrices for the data and dynamical errors, which express our a priori beliefs about the magnitude and correlation structure of errors in the data and in the assumed dynamical equations. The “damping factor” \(\lambda\) is explained below. For \(\mathbf{C}_d\) we use the general form discussed in Egbert et al. (1994), with spatially heterogeneous error variances, and a simple constant decorrelation length scale. Errors in the momentum balance equation result primarily from the crude parameterization of dissipative effects, the approximate treatment of tidal loading and self-attraction, and errors in the bathymetry. Variances of these errors can be estimated very roughly as a function of position using velocity and elevation amplitudes derived from the prior tidal model, as described by Egbert et al. (1994). The continuity equation is taken to be exact, and errors in the open boundary conditions are assumed to be independent of errors in the dynamics. Boundary condition er-
ror amplitudes are determined by posterior error analysis (including comparison to tide gauges and current moorings) of the larger-scale model providing the open boundary conditions. Note also that, as in EE, errors in the dynamical equations for each constituent are assumed to be uncorrelated and their amplitudes are proportional to the forcing amplitudes. Correlations between constituents of a fixed species (diurnal or semidiurnal) could be introduced as in Egbert et al. (1994). This would enforce greater smoothness in frequency of corrections to tidal admittances, but complicates calculations considerably and is not essential to the conclusions of this study. Further specific details on error covariances for our example application in the Ross Sea are given in section 3c.

For simplicity (and because we lack more specific information) we assume a simple diagonal form for the data error covariance $C_e$. The damping factor (or trade-off parameter) $\lambda$ in the data misfit term in (3) allows for the possibility that data or dynamical covariances are incorrectly specified. Increasing $\lambda$ corresponds to increasing data error variances, or decreasing dynamical error variance, by a constant factor. Trial solutions with different values of $\lambda$ are readily computed, allowing us to tune the inverse solution and check the model sensitivity to prior assumptions.

With these simplifying assumptions it can be shown (Bennett 1992, Egbert et al. 1994) that the tidal field $w^l_0$ for constituent $l$ that minimizes (3) can be written as the sum of a prior solution $w^l_0 = S^{-1}r^l_0$ (i.e., the exact solution of the SWE) and a linear combination of $K$ basis functions $r^l_k$, called representers. That is,

$$w^l_0 = w^l_0 + \sum_{k=1}^{K} \beta^l_k r^l_k.$$  (4)

The representers for constituent $l$ are given by

$$r^l_k = S^{-1}C_gS^gS^{-1}l_k,$$  (5)

where $l_k$ is the averaging kernel for the data functional $L_k$ (an impulsive forcing of the momentum equations at the observation location $x_k$). To calculate the representers for each constituent we use the frequency-domain scheme described in EE.

Note that the time domain data we consider here are real numbers, which are linear combinations of the real and imaginary parts of harmonic constants summed over all $N_c$ constituents. The actual representer for each observation is constructed from the real and imaginary parts of the frequency domain representers $r^l_k$, incorporating all $N_c$ constituents (Egbert et al. 1994; EE). The $N_c \times K$ complex representer coefficients $\beta^l_k$, $k = 1, K$, $l = 1, N_c$ for the representation in (4) are not independent, but are determined from $K$ data by solving a $K \times K$ system of linear equations with real coefficients. Further details of this calculation, which is straightforward but tedious, are given by Egbert et al. (1994) and EE.

As in EE we use a reduced basis approach to reduce computations. A subset of representers is calculated for each constituent, using $N \leq K$ of the data locations spread throughout the sampled part of the model domain. Then approximating (4) as a sum over the calculated subset of representers, the coefficients $\beta^l = (\beta^l_1, \ldots, \beta^l_N)^T$, $l = 1, N_c$ that minimize the penalty functional (3) are computed. Again, details (and a justification for this approach) are discussed in Egbert et al. (1994) and in EE.

3. Application of the GI approach to the Ross Sea

a. Physical setting

The Ross Sea (Fig. 1) is a large embayment in the Pacific sector of the Southern Ocean. Much of the area consists of a continental shelf with a typical depth of $\sim 400–600$ m; the southern portion of the shelf area is overlaid by the floating Ross Ice Shelf. Tides in the Ross Sea are dominated by diurnal components (Padman et al. 2003). Limited data suggest that most of the kinetic energy is barotropic; that is, there is little depth variation in tidal current amplitude and phase except for within a frictional boundary layer a few tens of meters thick. This layer is typically only a small fraction of the water column. Thus, tidal currents extracted from middepth and upper-ocean current meters may be regarded as fairly representative of the barotropic tide (the current associated with the surface pressure gradient). This lack of significant baroclinic energy makes the Ross Sea an excellent test environment for the data assimilation methods reported herein.

A further virtue of the Ross Sea for the present study is the extreme variability of anticipated diurnal tidal kinetic energy over the domain. As Padman et al. (2003) showed, quite small diurnal currents are expected for the deep water north of the shelf break and also over the fairly flat continental shelf south of the break. Along the shelf break, however, significant tidal kinetic energy is contained in diurnal, topographic vorticity waves (DTVWs) that propagate along the topographic waveguide of the continental slope. From previous modeling studies and analyses of recently acquired data from the shelfbreak region, the major axis of the dominant diurnal $K_1$ and $O_1$ ellipses can exceed $70 \text{ cm s}^{-1}$ over the upper slope. The cross-slope decay scale and along-slope wavelength of DTVWs scale with the slope width (Middleton et al. 1987; Padman et al. 1992), so that variability on scales of order 10–100 km is expected.
For modeling, we consider a domain from 63° to 86°S and from 159° to 215°E (Fig. 1a). The northern limit is chosen for consistency with our studies that incorporate assimilation of TOPEX/Poseidon altimetry data (Padman et al. 2003). Bathymetry is based on Earth Topography-5 Minute (ETOPO-5; National Geophysical Data Center 1992), but significantly updated as described in Padman et al. (2002) and Padman et al.
Bathymetry under the Ross Ice Shelf is replaced with “water column thickness,” that is, the distance from the ice base to the seabed. This approach is valid for barotropic models of domains containing land-fast, glacial ice shelves, since the horizontal momentum transport associated with a given free-surface displacement field is forced into a thinner water column by the laterally immobile but free-floating ice.

For the present model, no effort is made to account for the additional friction associated with the tidal flow against the ice shelf base. Furthermore, sea ice is not included in this model. The effect of sea ice on ocean tidal velocities is expected to be small (Kowalik and Proshutinsky 1994).

b. Current data

We have two types of current data: moored current meters and VM-ADCP surveys. In our inverse modeling we did several experiments, using one type of data for assimilation and the other for validation. Current meters were deployed in the Ross Sea as part of various projects in the 1970s and 1990s; see Padman et al. (2003). The data are generally hourly time series of 7–12-month duration sampled at two to four depths in a total of 21 locations, shown in Fig. 1. All moorings are concentrated within the smaller domain outlined with a red frame in Fig. 1a and zoomed in Fig. 1b.

To derive harmonic constants from the current mooring time series we applied conventional harmonic analysis for each depth separately, focusing on the records measured well above the bottom and well below the surface. Analyses of time series in shorter segments (see Erofeeva et al. 2003 for further details) revealed little seasonal or long-term variability in tidal currents. For most sites there was also very little vertical shear in the tidal band except in the frictional boundary layer extending a few tens of meters above the seabed. Such sampling obviously does not allow for precise estimation of the barotropic component. Phase errors due to insufficient sampling with depth may be as much as 10°–40° (Erofeeva et al. 2003). Harmonic constants for the northern current meters were thus derived using the middle water column record on the assumption that the correction to depth-averaged currents for the reduced and rotated currents in the boundary layer would be small. Semidiurnal constituents are all very small in the Ross Sea (Padman et al. 2003), so we focus on the three most energetic diurnal constituents: K1, O1, and P1. To summarize model and data comparisons for the $N_m$ current moorings we consider root-mean-square (rms) misfit in the diurnal band, synthesized from harmonic constants:

$$\text{rms}_\text{RI} = \left( \frac{1}{4N_m} \sum_{k=1}^{N_k} \sum_{l=1}^{N_l} \left( \text{Re}(\hat{u}_k^l - u_{MK}^l)^2 + \text{Im}(\hat{u}_k^l - u_{MK}^l)^2 \right)^{1/2} \right)^{-1/2}$$

where $\hat{u}_k^l = (\hat{u}_k^l, \hat{v}_k^l)$ and $u_{MK}^l = (u_{MK}^l, v_{MK}^l)$ are vectors of harmonic constants for velocity components for the $l$th tidal constituent at location $x_s$, derived from the current meter time series and model, respectively. The sum in (6) is over $N_m$ current meter measurement sites and $N_c = 3$ diurnal constituents. We also calculate

$$\text{rms}_\text{AMP} = \left( \frac{1}{2N_m} \sum_{k=1}^{N_k} \sum_{l=1}^{N_l} \left( |\hat{u}_k^l| - |u_{MK}^l| \right)^2 \right)^{1/2}$$

for comparison of amplitudes only, that is, ignoring errors in phase.

The VM-ADCP data were obtained from three cruises in December–January 1997, December–January 1998, and February–April 2003, referred to subsequently as cruise 1, cruise 2, and cruise 3. All ship tracks were located within the framed area of Fig. 1a. The VM-ADCP data cover a range of about 30-m depth to between 200 and 300 m, depending on acoustic conditions. We use an average velocity for the depth range of 100–200 m. Inspection of current profiles indicates that the surface layer (shallower than 100 m) frequently contains significant semidiurnal energy associated with wind-forced, near-inertial oscillations. The lower depth limit of 200 m is chosen so that almost all profiles contain data; that is, the depth averaging, is consistent for all sections of all three cruises. Current shears within the 100–200-m averaging range are usually small, which is consistent with our belief that the regional currents are dominated by the barotropic component.

Total rms currents for each of the three cruises are shown in the first row of Table 1. On average, cruise 3 currents exceed those of cruises 1 and 2 by more than a factor of 2. Most of the cruise 3 track is in the northern shelf break area where currents associated with DTVWs are strong, while the cruise 1 and 2 tracks are mostly located over the southern shelf away from shelf-edge enhancement of diurnal tidal currents. All VM-ADCP data were depth averaged over 100–200 m, which avoids the highly variable surface layer and the thinner portion of the water column where data loss is frequent. Data records for each cruise were prepro-
Table 1. The model–data rms misfits (cm s\(^{-1}\)). The misfits are shown for each of the three VM-ADCP cruises, for the group of 19 southern moored current meters, and for each of the 2 northern current meters (JB and JC). For the VM-ADCP cruises rms misfit between the predicted tidal currents (sum of \(K_1\), \(O_1\), and \(P_1\) constituents) and measured VM-ADCP values, averaged over two velocity components, is shown. For moored current meters, rms_{RI} and rms_{AMP} are computed as in (6) and (7) with \(u\) shown in Fig. 1a. Model computations were done for the full domain for the smaller area shown in Fig. 1b. However, all main to verify our models, we focus on modeling results for two moorings nearer the shelf break.

For the errors in the dynamical equations we assume a diagonal covariance. For data errors we assume a diagonal covariance. The VM-ADCP data can contain significant nontidal power in the tidal frequency band. In particular, near-inertial motions (roughly semidiurnal at high latitudes) are common in the upper ocean in the Ross Sea. We estimate the nontidal variance in the VM-ADCP data using the current mooring time series. The estimated nontidal rms values for current moorings are 6.0 (JB), 9.2 (JC), and 3.6–10.6 cm s\(^{-1}\) for southern current moorings. We take the value of 9 cm s\(^{-1}\) from the most energetic current meter (JC) as our initial guess for VM-ADCP data error. For the current moorings a larger value for \(r_0\) was tested, but the assimilation was found to perform slightly better in this domain with this smaller value for \(r_0\). This finding is consistent with our assumption that errors are predominantly in bathymetry (assumed to be 5%), and dissipation (100% of the amplitude of the appropriate terms in the prior). These amplitudes were computed for each constituent using the prior solution, so the assumed dynamical errors are roughly proportional to forcing amplitudes. Constant error variance was assumed for the open boundary conditions (with amplitudes 2% of typical values of tidal forcing fields, whether height or normal-flow velocity).

Table 1. Note that these rms values are for the three harmonic constants derived at each depth and then averaged over the seven moorings provide an estimate of present-day variation. The estimated spatially variable amplitudes, with a constant decorrelation length scale of \(r_0 = 10\) km. A range of larger values for \(r_0\) was tested, but the assimilation was found to perform slightly better in this domain with this smaller value for \(r_0\). This finding is consistent with our assumption that errors are predominantly in bathymetry (assumed to be 5%), and dissipation (100% of the amplitude of the appropriate terms in the prior). These amplitudes were computed for each constituent using the prior solution, so the assumed dynamical errors are roughly proportional to forcing amplitudes. Constant error variance was assumed for the open boundary conditions (with amplitudes 2% of typical values of tidal forcing fields, whether height or normal-flow velocity).

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the amplitude of this error. These values are 0.5, 0.4, and 0.2 cm s\(^{-1}\) for \(K_1\), \(O_1\), and \(P_1\), respectively. The estimated current meter data error in the diurnal band, summed over all three constituents, does not exceed 0.8 cm s\(^{-1}\).

d. Representers

As outlined in section 2, our data assimilation approach is based on explicitly calculating representers for the data functionals (5), which depend on the assumed SWE dynamics and the dynamical and data error covariances. The representers, which do not depend on the actual values of the observations, are analogous to the spatial basis functions used for tidal estimation with the Candela et al. (1992) approach. It is instructive to plot examples.

An example of a \(K_1\) representer for an observation of the north velocity component (\(v\)) at a data site in the southern part of the domain, near the Ross Ice Shelf front, is presented in Figs. 2a and 2b. Currents associated with this single representer are plotted as vectors separately for in-phase (Fig. 2a) and quadrature (Fig. 2b) components. Here, and subsequently, “in phase”
refers to the standard Greenwich phase; that is, the in-phase plots give the tidal currents when the phase of the tidal potential is zero at the Greenwich meridian. The quadrature component gives the current $\frac{1}{4}$ of a tidal period earlier.

The spatial structure of this single representer is very complex, particularly over the shelf break. As expected, it mimics the structure of tidal current speed presented by Padman et al. (2003). This is not surprising, given that the representer is formed using SWE physics similar to that used by Padman et al. Its relevance to the present study is that, even though we chose a data site from the less energetic southern domain, the representer amplitude is still much larger over the shelf break than at the observation site. Thus, because representers are consistent with the SWE physics, they can be used to extrapolate data in a way that could not be achieved with an ad hoc set of basis functions. Furthermore, the spatial complexity, which is tightly coupled to bathymetric features, could not be easily reproduced with polynomials or other simple sets of basis functions.

To solve for the representer coefficients, required to reconstruct the tidal currents via (4), the representer matrix $\mathbf{R}$ (Egbert et al. 1994) is formed. The matrix $\mathbf{R}$ may be shown (Bennett 1985) to be the covariance of the deviation of observations from the prior model at the measurement sites. The eigenvector decomposition of the representer matrix can be viewed as a rotation of the data vectors and the measurement functionals into a coordinate system where $\mathbf{R}$ becomes diagonal (Egbert et al. 1994). In this system the minimizer to (3) is expressed as a sum of the prior model and a series of “array modes” (Bennett 1985). A very good approximation to the full solution of (4) can usually be formed using a small number of array modes associated with the largest eigenvalues of $\mathbf{R}$ (e.g., Parker 1994). The first and second representer array modes are plotted in Figs. 2c,d and 2e,f, respectively. As in the case of the representer for a point measurement, both modes have significant spatial variations along the shelf break where the biggest changes due to the data assimilation should be expected due to the dominance of DTVWs in the total tidal kinetic energy (Padman et al. 2003).

4. Results

a. Prior model

Experiments with various friction parameterizations revealed that the magnitude of diurnal tidal currents in the Ross Sea was more accurately reproduced with linear rather than quadratic bottom friction (Padman et al. 2003). Thus, to get a prior model we solved the nonlinear SWE by time stepping, with the linear bottom friction parameterization of EE. Open boundary conditions were elevations taken from the Circum-Antarctic Tidal Simulation version 02.01 (CATS02.01) described by Padman et al. (2002). The prior model was compared to the current meter and VM-ADCP data. Total rms misfits are presented in the second row of Table 1. For the VM-ADCP records, the misfit represents all ocean velocity that is not explained by the three fitted diurnal constituents, $K_1$, $O_1$, and $P_1$. For the moorings, we use (6) and (7) to calculate only the misfit for the combination of these three diurnal constituents. The purely hydrodynamic prior model reduces the rms current for all three VM-ADCP records. The decrease in rms current is especially significant for cruise 3, indicating that the complex tidal currents in this area are at least approximately reproduced by the dynamics-only model. Diurnal-band misfit to the southern current moorings is reduced from 2.4 to 1.6 cm s$^{-1}$, indicating that the prior model is quite good in this area. The prior does not reduce the rms $H_1$ misfit (6) to the more energetic northern current meters (JB and JC); however, the rms$_{AMP}$ misfit (7) is small for both moorings, indicating that the prior and the current meter harmonic constants disagree mostly in phase rather than amplitude.

b. Inverse models

We conducted three inversion experiments using the GI approach and the calculation scheme described in EE. For all experiments we fit $K_1$, $O_1$, and $P_1$. Separation of $K_1$ and $P_1$ by harmonic analysis requires 6 months of data; thus, the ambiguity between these constituents may be a problem here, where each cruise is approximately 2 months long. In the inversion, the division between $K_1$ and $P_1$ of the nearly diurnal signal is controlled by the assumed dynamical error covariance. We have estimated dynamical error amplitudes using a prior model. Hence for each constituent the amplitude of dynamical errors, and thus of the inverse solution, scales with the tidal forcing. Thus the covariance enforces smoothness of at least the amplitude of the tidal admittance. Using a more complex covariance that allows for interconstituent correlation of dynamical errors would also enforce smoothness of the phase and, thus, might be expected to improve separation of nearby constituents such as $K_1$ and $P_1$. This refinement to our procedure is an issue for future study.

For the first experiment we inverted only harmonic constants from the moored current meters (referred to as the CM case). This allows us to address the extent to which it is possible to fit the current meter data using the assumed dynamics and error covariance. Both the northern and southern current meter harmonic con-
The best choice of parameters was assessed by varying the damping parameter $\lambda$. As $\lambda$ is reduced, data are fit better and the model fit to the SWE is relaxed. A value of $\lambda = 1$ corresponds to our prior assumptions about data and dynamical errors: a value of $\lambda = 0.1$ corresponds to reducing data error variances by a factor of 10, or equivalently increasing dynamical errors by the same factor. Reducing $\lambda$ improves the model fit to the JB and JC harmonic constants, but the southern current mooring rms misfits change only slightly. We conclude that the southern current meter dataset is already well represented by the prior, and the fit cannot be improved with any plausible adjustments to our prior error assumptions.

The three cruises of VM-ADCP data provide an independent test of whether current meter assimilation improves the overall skill of the model. In fact, it does not. The rms misfit for all CM cases is poorer than for the prior model, albeit only slightly for cruises 1 and 2. For cruise 3, the fit becomes significantly poorer as the current meter data are more tightly fit (decreasing $\lambda$). The best choice of $\lambda$, based on fitting the current meter data and minimizing degradation of the model fit to VM-ADCP data, is $\lambda = 0.3$–1; that is, our a priori assumptions about the error covariances were reasonably good. Other inverse models, described below, are then obtained with $\lambda = 1$.

The result, that assimilating the mooring data slightly degrades ADCP data misfits, indicates that sparsely distributed current meter data are of limited value for correcting the tidal fields over a complex area such as the Ross Sea. Thus, even though current moorings can provide very accurate tidal information through harmonic analyses of long records, their value for improving tidal models over a large area depends on mooring locations being chosen with assimilation tidal modeling in mind. The locations of all the Ross Sea moorings used herein were chosen for other reasons, such as identifying dense water outflow paths and flow across the front of the Ross Ice Shelf.

For the second experiment we inverted the VM-ADCP data from all three cruises using the same prior and covariance (we refer to this solution as VM-ADCP). In this case we calculated 406 representers in locations evenly distributed along the ADCP tracks and used the moored current meter harmonic constants only for validation. As expected, this solution fits ADCP data better, especially for cruise 3; however, all current meter rms misfits are slightly increased compared to the prior (see Table 1). The southern current meter rms misfit change is less than the assumed data error and thus is probably not significant. The JB and JC rms$_{RI}$ misfit changes are more noticeable; however, they do not exceed 1.3 cm s$^{-1}$, which is comparable to the estimated current meter data error of 0.8 cm s$^{-1}$. For the JC current meter the rms$_{SAMP}$ misfit degrades more significantly than the rms$_{RI}$ misfit, indicating that while inverse amplitudes are worse than in the prior, the inverse phases are improved around the JC mooring. The in-phase and quadrature $K_1$ currents for the VM-ADCP solution are shown in Figs. 3c and 3d. Compared to the prior, slightly stronger currents are predicted in the area of Iselin Bank and around the JB and JC current meters: the inverse model currents elsewhere are very similar to the prior. This again supports our conclusion that the prior model is reasonably accurate except over the shelf break where energetic and difficult-to-model DTWVs dominate.

For the third experiment we inverted only VM-ADCP data using no prior: we refer to this solution as NP-ADCP. Our two goals in this experiment were to 1) test how well tidal current fields could be recovered using only data without constraints imposed by a prior model and 2) demonstrate how the dynamics are embedded in the basis functions. This is representative of the case where no good prior model is available, for instance because of large uncertainties in open boundary conditions. As in the second case we assimilated only VM-ADCP data and use the current moorings for validation. The rms misfits for this inverse model are given in the last row of Table 1. As for the VM-ADCP inversion case, the rms misfit for cruise 3 is improved significantly compared to the rms currents, while misfits for cruises 1 and 2 are improved slightly. The harmonic constants for the less energetic southern current meters are fit by the NP-ADCP inverse model as well as they are by the prior model and better than by the VM-ADCP model. The rms$_{SAMP}$ misfit is also significantly improved for the more energetic northern current meters; however, the rms$_{RI}$ misfit is similar to the prior, still indicating a discrepancy in phase between the data and the model. The $K_1$ currents for the NP-ADCP inverse solution (Figs. 3e and 3f) are essentially identical to the VM-ADCP solution. Therefore, with no open boundary forcing information, fitting the VM-ADCP data reproduces the complex pattern of tidal currents including the amplitude of DTWVs along the shelf break.

c. A practical comparison of models

The prior model based on the SWE performs reasonably well, and the differences between this and the two inverse solutions are difficult to see in the regional maps (Fig. 3). Similarly, the rms misfits (Table 1) show...
only modest changes between the models. However, as we have previously noted, the largest differences between models occur over the northern shelf break where DTVWs dominate the total tidal kinetic energy. In early 2004, several moorings deployed over the outer shelf and continental slope as part of the AnSlope Project (Gordon et al. 2004) were recovered. We choose a midwater-column current meter from one mooring (Central E2; 71.9138°S, 173.2115°E, water depth ~1770 m) for comparison with our models. The measured water depth is within 2% of the depth at the same location in our model grid. The record length is ~340 days. Data were kindly provided by A. Orsi (2004, personal communication).

The modeled $K_1$ current meter ellipse parameters for a north–south transect passing close to this mooring (Fig. 4) show that, while all models are qualitatively similar, they do vary significantly from each other. The $K_1$ major axis, $u_{maj}(K_1)$, over the outer shelf and upper slope for the VM-ADCP model is up to 9 cm s$^{-1}$ greater than for the prior model. Ellipse phases also differ by order $10^\circ$ (~2/3 h), an additional source of error when the models are used predictively. At the mooring location, $u_{maj}(K_1)$ from the VM-ADCP model

![Fig. 3. In-phase and quadrature $K_1$ currents for the (a), (b) prior, (c), (d) VM-ADCP, and (e), (f) NP-ADCP solutions. Locations of moored current meters (gray filled circles) are shown.](image-url)
is \( \sim 4 \) cm s\(^{-1}\) greater than for the prior model and is very close to the value obtained by harmonic analysis of the mooring time series. As a test of the predictive skill of the VM-ADCP and prior models for this site, we correlate the \( u \) and \( v \) components of the measured tidal currents [obtained using the \texttt{t\_tide} Matlab scripts (Pawlowicz et al. 2002)] with the model predictions from the prior and inverse models. For predictive purposes, the five main tidal constituents (\( M_2, S_2, N_2, K_2, \) and \( Q_1 \)) that are not evaluated by VM-ADCP assimilation are obtained from the prior model. The correlation coefficients and linear regression slopes between measured and modeled tidal currents are presented in Table 2. The correlations do not change significantly; however, the regression slopes demonstrate that the VM-ADCP inverse model performs significantly better than the prior model at predicting the tidal contribution to total currents on the central slope.

Significantly, the mooring data do not temporally overlap the assimilated VM-ADCP data records. That is, at least along the slope, the inverse model is a much better predictive tide model than the prior. This is a substantially more rigorous test of the efficacy of assimilation than simply improving the extraction of tides from the assimilated data records.

### 5. Discussion and conclusions

As in the harmonic method proposed by Candela et al. (1992), the GI approach discussed herein uses a set of basis functions to interpolate (and extrapolate) reconstructed tidal fields from VM-ADCP records. How-

**Table 2.** Comparison of linear regression statistics for \( u \) (east) and \( v \) (north) components of the tidal component of measured currents at AnSlope mooring Central E2 (71.9138°S, 173.2115°E, water depth \( \sim 1770 \) m) with the predicted values from the Ross Sea prior solution based on the SWE, and the VM-ADCP assimilation model. Shown are the correlation coefficients \( r(u) \) and \( r(v) \), and the regression line slopes \( m(u) \) and \( m(v) \).

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<tr>
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<th>Prior</th>
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<tr>
<td>( r(u) )</td>
<td>0.93</td>
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<tr>
<td>( r(v) )</td>
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<tr>
<td>( m(u) )</td>
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<td>0.97</td>
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<tr>
<td>( m(v) )</td>
<td>0.74</td>
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**Fig. 4.** Comparison of \( K_1 \) ellipse parameters (a) major axis, (b) minor axis, (c) phase, and (d) inclination, for the prior (blue line), VM-ADCP (green), and NP-ADCP (red) models, for a north–south transect along 173.25°E, passing close to AnSlope mooring Central E2. The black line in each panel shows water depth, with the scale indicated on the right-hand side. The location of the mooring is indicated in each panel by the thin vertical line.
ever, the Candela et al. method uses arbitrary basis functions (polynomials, biharmonic splines, etc.), while the GI approach uses functions derived from SWE dynamics with bathymetry appropriate for the area under consideration. These basis functions can be quite complex where tidal variability on short spatial scales is physically realistic, such as along the shelf break and when significant tidal kinetic energy is carried by topographically trapped vorticity waves. To model such complex current fields with arbitrarily chosen basis functions would be a challenge, and would likely lead to significant numerical problems during least squares fitting of the VM-ADCP data. We demonstrated the value of dynamically determined basis functions with our no-prior VM-ADCP model: this model showed that tidal signals in the ship-mounted ADCP records can be used to provide reasonable tidal corrections throughout the model domain, even with no prior information about forcing at open boundaries.

For the Ross Sea example discussed herein, the prior solution obtained by forcing the SWE with elevation open boundary conditions obtained from a circum-Antarctic tidal model already accounts for much of the measured tidal velocity variance well south and north of the shelf break. Assimilation of either the moored current meter data or the VM-ADCP data leads to only small changes in the fraction of explained tidal variance. That is, in this region a purely dynamical model would generally provide reasonable tidal corrections to depth-averaged VM-ADCP data. Along the continental slope, however, tidal currents are large and the prior and inverse models differ significantly in their predictions of tidal ellipse magnitude and phase (Fig. 4). Comparisons between the tidal components of velocity, measured at a recently recovered midslope mooring, with the prior and VM-ADCP models indicates that the moored currents are much better predicted when VMADCP data are assimilated (see Table 2). Thus, this type of data assimilation can be a valuable tool for regions where tidal currents are large and have large spatial gradients.

While we have described assimilation of ship-mounted ADCP data into a barotropic model, a similar approach could be applied for assimilation into a baroclinic model. Successful assimilation of high-frequency coastal radar surface current data into a baroclinic model for the Oregon coast using the representor method is described in Kurapov et al. (2003). This internal tide assimilation could be extended to include depth-dependent VM-ADCP measurements. We note, however, that the tidal ellipse characteristics for baroclinic tides are sensitive to background conditions such as changing stratification and mean flow. Thus, more caution will be required when combining multiple VMADCP records for baroclinic tide extraction than for the simpler case of barotropic tides.

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REFERENCES


