FIELD EMISSION CURRENT FROM SILICON AS A FUNCTION OF APPLIED POTENTIAL

by

RICHARD LEE PERRY

A THESIS

submitted to

OREGON STATE COLLEGE

in partial fulfillment of
the requirements for the
degree of

DOCTOR OF PHILOSOPHY

June 1961
APPROVED:

Redacted for Privacy

Professor of Physics
In Charge of Major

Redacted for Privacy

Chairman of Department of Physics

Redacted for Privacy

Chairman of School Graduate Committee

Redacted for Privacy

Dean of Graduate School

Date thesis is presented May 2, 1961

Typed by Ruth Perry
ACKNOWLEDGMENTS

The author takes this opportunity to express his appreciation to many people for their aid during this investigation. Dr. E. A. Yunker, Chairman of the Oregon State College Physics Department, and Dr. W. P. Dyke, Director of Linfield Research Institute (LRI), made the arrangements for this research to be done at LRI. Dr. Dyke obtained the government contract from the U. S. Navy through the Bureau of Ships to support this work. Dr. J. J. Brady, the author's major professor, made several helpful suggestions. Valuable discussions were held with Dr. F. G. Allen and Dr. J. A. Becker of Bell Telephone Laboratories, Dr. W. R. Savage of Texas Instruments, and Dr. E. G. Brock when he was at LRI.

The aid and interest of the entire LRI staff are greatly appreciated. In particular, Drannan Hamby, Robert Carter, Charles Baltzell, and James Barbour helped with the tip fabrication and glass blowing. Dr. P. C. Bettler and Thomas Morisky helped with some of the early data taking. Dr. W. W. Dolan has permitted his unpublished notes on $\beta$ calculations to be included in this manuscript.

Last, but not least, the encouragement and understanding of the author's wife, Ruth, are greatly appreciated. She also typed the thesis after making several helpful suggestions on its composition.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>THEORY</td>
<td>12</td>
</tr>
<tr>
<td>APPARATUS</td>
<td>36</td>
</tr>
<tr>
<td>PROCEDURE</td>
<td>65</td>
</tr>
<tr>
<td>1. Electrolytic Shaping of Germanium and Silicon Field Emitters</td>
<td>65</td>
</tr>
<tr>
<td>2. Tube Evacuation Process</td>
<td>79</td>
</tr>
<tr>
<td>3. Cleaning of Silicon Field Emitter Tips in High Vacuum</td>
<td>84</td>
</tr>
<tr>
<td>4. Marbles for Atoms Model of Silicon Field Emitter</td>
<td>98</td>
</tr>
<tr>
<td>5. Determination of Tip Characteristics</td>
<td>102</td>
</tr>
<tr>
<td>EXPERIMENTAL RESULTS</td>
<td>108</td>
</tr>
<tr>
<td>COMPARISON WITH THEORY</td>
<td>144</td>
</tr>
<tr>
<td>CONCLUSIONS</td>
<td>153</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>157</td>
</tr>
<tr>
<td>APPENDIX A. Fabrication of Field Emission Microscope Tubes</td>
<td>164</td>
</tr>
<tr>
<td>APPENDIX B. Major Items of Equipment and Supplies</td>
<td>179</td>
</tr>
<tr>
<td>APPENDIX C. β Calculations</td>
<td>186</td>
</tr>
</tbody>
</table>
**LIST OF FIGURES**

1. Potential energy diagram for a metal 13
2. Potential energy diagram for n-type semiconductor 13
3. Potential energy diagram showing field penetration 19
4. Potential energy diagram with surface states 27
5. Theoretical plot of field emission from SiC 32
6. Sketch of one of the first tubes used in this study 37
7. Photograph of one of the first tubes used in this study 38
8. Sketch of the last tube used in this study 39
9. Photograph of the last tube used in this study 40
10. Schematic diagram of vacuum system 43
11. Photograph of vacuum system 44
12. Photograph of vacuum system control panel 45
13. Circuit diagram of oven and control circuits 51
14. Sketch of vacuum cart framework 52
15. Photograph of outgassing cart 54
16. Typical circuit for operation of tube 58
17. Diagrams of circuits used in calibrating instruments 62
18. Micrographs of non-symmetrical tips 66
19. Germanium sample before and midway through fabrication 68
20. Germanium sample after tip fabrication completed 69
21. Micrographs of fairly well-shaped germanium tips 70
22. Typical set-up for fabrication of semiconductor tips  
23. Field emission pattern from a clean silicon tip  
24. Field emission pattern from a contaminated silicon tip  
25. Hairpin filament support for semiconductor tips  
26. Device for mounting semiconductors on filaments  
27. Clamp holder and bombarding apparatus  
28. Helical filament support  
29. Yoke-type emitter structure  
30. Marble model of tip with diamond crystal structure  
31. Diagram for determining the emitting area  
32. Pattern showing a grain boundary trace  
33. Orthographic projection for grain boundary pattern  
34. Patterns of migration and build-up  
35. Plots of data at stages of build-up  
36. Micrographs of two built-up silicon tips  
37. Patterns of smooth and built-up tip  
38. Plot of data from built-up tip  
39. Plot of typical "non-linear" data  
40. Plot of typical "linear" data  
41. Plot of greatly "non-linear" data  
42. Plot of only slightly "non-linear" data from same tube  
43. Data from tip for two different temperatures  
44. Patterns (built-up) for heated and cold tip
45. Recording of variation of current with temperature

46. Micrographs of three very well shaped tips

47. Micrographs of four non-symmetrical tips

48. Micrographs of two very non-symmetrical tips

49. Plot of current densities and fields from nine smooth tips

50. Field patterns from the nine smooth tips

51. Plot of current densities and fields for tip Si1L33

52. Plot of current densities and fields for tip Si1L41

53. Plot of current densities and fields for tip Si1L43

54. Potential energy diagram calculated for tip Si1L41

55. Shape of metal evaporating filament

56. Equipotential surfaces for β calculations

57. Equipotential surfaces for β calculations

58. Equipotential surfaces for β calculations

59. Equipotential surfaces for β calculations

60. Equipotential surfaces for β calculations

61. Equipotential surfaces for β calculations
FIELD EMISSION CURRENT FROM SILICON AS A
FUNCTION OF APPLIED POTENTIAL

INTRODUCTION

A complete investigation of semiconductors by field emission
techniques can yield a great deal of information about the basic
properties of semiconductors. For instance, the experimental
determination of the field emission characteristic, i.e., the plot of
logarithm of the current density versus the reciprocal of the electric
field, can provide quantitative information on the electron affinity,
charge carrier density, and other properties of a variety of
semiconductor materials.

More detailed information on the dependence of these
parameters on crystallographic direction can also be derived from
the intensity distribution of the emission pattern provided by the
field emission projection microscope. The field emission
microscope (FEM), first developed in spherical symmetry by
Müller (47, p. 541-550), is a powerful tool for the detailed and
quantitative study of surface phenomena. It is a spherical projection
microscope in which electrons drawn radially from a sharp field
emission tip strike a phosphor screen after travelling a nearly radial
path between the tip and the anode. The resulting emission pattern on
the screen provides a highly magnified ($10^5$ to $10^6$ times) electrical image of the tip. Pattern detail arises from local variations in the electric field and work function at the tip surface. For a smooth clean tip the pattern is regular or symmetrical corresponding to the crystal structure of the tip material while the pattern from a tip with a thick layer of adsorbate on the surface is usually a random variation of emission intensity. The resolution, which is determined by the horizontal component of the initial energy spread of the field emitted electrons, is of the order of 30 Angstroms for a fairly sharp tip (25, p. 131). In other words, the parts of the field emission pattern from two adjacent emitting regions on a smooth field emitter can be distinguished from each other if the two regions are about 30 Angstroms apart.

Uses of the FEM include the study of crystal structure and its defects, and the study of phase transformations in metals and alloys in addition to the measurement of electron emission parameters such as work function (or electron affinity) mentioned above. Using as a reference the field emitted current and the pattern characteristic of a smooth and uncontaminated emitter surface, examination of the currents and patterns obtained under other conditions yields information on the presence and nature of surface contaminants, their
preferential adsorption sites and their binding strength to the substrate, their surface migration and desorption characteristics, and the effect of their presence on surface states and charge carrier distribution. The effect of very high electric fields on chemical reactions and surface damage caused by ion sputtering may both be studied with the FEM.

In some recent work (8, p. 1452-1459) the versatility of the FEM has been extended by using a very low duty cycle pulsed electric field (e.g., 30 one microsecond pulses per second) instead of a lower continuous field. This minimizes the effect of the applied electric field (which is required for field emission) on the surface event under study, and allows the use of a more intense field for which certain lattice steps only 2.2 Angstroms high on the tip surface become visible. This method has been used to study transport phenomena in heated single crystals, deriving values of the activation energy for surface migration and of the surface tension in the solid phase for tungsten. The same techniques might be used to study transport phenomena and to measure the surface tension and the activation energy for surface migration of semiconductor materials.

In addition to the total magnitude and spatial distribution of the field emitted current, the initial energy distribution of the emitted
electrons may also be measured, providing useful additional information on basic semiconductor properties. For example, this study should show whether all of the electrons are emitted from the conduction band or if some of them come from the valence band or from states located in the forbidden energy gap. Is there a variation in the distribution of energies of the emitted electrons with respect to the crystallographic direction? If there is, it could indicate that the energy levels of the top of the valence band and bottom of the conduction band vary with the crystallographic direction which has been suggested by other studies of semiconductors (42, p. 1163-1165). The measurement of the energy distribution of the field emitted electrons can also be coupled with the measurement of tip heating or cooling during emission to provide information on the average energy level of the charge carriers and their energy exchange with the crystal lattice. There is a strong possibility that at least some of these properties depend on the temperature of the sample. A better knowledge of the physical properties mentioned above could result in the improvement of present semiconductor devices or the development of new ones.

Before these studies can be undertaken, however, a number of technical problems must be solved. In order to be used as a
cathode in the field emission projection microscope tube, the semiconductor must be shaped into a needle with a tip radius of the order of $10^{-5}$ to $10^{-4}$ cm, mounted suitably in the center of the tube, and cleaned in vacuum so that no contaminants exist on the surface of the tip.

The purpose of this study has been to try to solve these problems and then take experimental data over an extended range of current and voltage. The experimental data itself would be a valuable contribution to the field of semiconductor physics for the following reason. All of the theories of field emission from semiconductors proposed so far have been so compromised by the extreme complexity of the problem that they probably are not valid for semiconductors like germanium and silicon. Experimental data could provide a guide for the theoreticians to follow in developing a suitable theory for field emission from semiconductors provided it is reduced to the fundamental quantities of current density and electric fields.

The field emission of electrons from a cold metallic cathode was first reported by Wood (64, p. 1-10) in 1897, but no further work on this was done until Lilienfeld (44) published his observations.
in 1922. Millikan and Lauritsen (45) developed the empirical relationship

\[ I = C \exp\left(-\frac{B}{V}\right) \]  

(1)

where \( I \) and \( V \) are current and voltage, respectively, and \( C \) and \( B \) are constants. This relationship is still used when the observed current in amperes and voltage in volts are plotted as \( \ln I \) vs \( 1/V \) or, as used later in this report, as \( \log_{10} I \) vs \( 10^4/V \), to check the observed current to be sure that it is field current. i.e., if a straight line is obtained, the current is field current.

Classical theory (59, p. 63-106) fails completely to adequately describe a mechanism for field emission. When the method of wave mechanics was applied to this problem in 1928 by Fowler and Nordheim (34, p. 173-181) and then corrected for the effect of image force by Nordheim (50, p. 626-639), a mechanism for field emission from metals was developed which fits the experimental results quite well. Two recent good review articles have summarized the comparisons between theory and experiment for metals (25, p. 89-185; 37, p. 176-231).
The history of field emission from semiconductors is very sketchy because of the experimental and theoretical difficulties of the problem. Busch and Fischer (15, p. 532-539) recently published a brief review article summarizing this history. They state that the first attempt at a theoretical calculation of field emission from semiconductors was made in 1942 by Prof. Busch who drew upon it for an explanation of the voltage dependence of the resistivity of silicon carbide, and that the first measurements of field emission from a semiconductor were made on silicon carbide by Donath in 1942 and Janussen in 1943 as thesis experiments. Janussen obtained a straight line over several orders of magnitude when his data were plotted as logarithm of the current versus the reciprocal of the voltage, except for very low voltages where the data points were far below the emission straight line. These studies were all done at the Confederated Technical High School in Zürich, Switzerland, and were either unpublished or received only local circulation.

In 1947 Morgulis (46) published his theory in which he took into account one of the fundamental peculiarities distinguishing semiconductors from metals, namely, the ability of an external electric field to penetrate into a semiconductor. This causes a distortion of the energy bands of the cathode, which decreases the
work function and increases the concentration of the conduction electrons at the cathode surface. However, he limited himself to the case of not too high fields, i.e., for $F \leq 5 \times 10^6$ electron volts/cm, when the electron gas in the conduction band remains non-degenerate. In general, the fields required for field emission are considerably higher than this, i.e., by an order of magnitude. Also, he did not take into account the possibility of the deformation of the energy bands of the semiconductor as an effect of the electron surface states. Because of these simplifying assumptions, the validity of this theory for any practical case is very doubtful.

Apker and Taft (6) studied field emission from single crystal needles of the photoconductors CdS and CdSe. They found very little emission until the emitter was illuminated and then emission came from the lighted side. Brock and Taylor (13) made a study of field emission from both germanium and silicon. They fitted their data to a straight line when plotted as $\log_{10} I$ vs $10^4/V$ in the narrow range that their data covered. Allen (4, p. 1-26) studied field emission from germanium and fitted his data to a straight line when plotted as above. He tried to calculate electric field $F$ (ev/cm) and current density $J$ (amps/cm$^2$) and plot them as $\ln \frac{J}{F^2}$ vs $\frac{1}{F}$. For metals, this gives a straight line and its slope is dependent on the work
function $\Phi$. Allen found values of $\Phi$ from 1.8 to 2.1 electron volts. The actual value is $\Phi = 4.78$ electron volts (20, p. 177).

All of the above data taken by these various investigators have been from semiconductor field emission tips which did not have clean surfaces and therefore are of doubtful value.

In 1955 Stratton (62, p. 746-757) published a theory of field emission from semiconductors in which he analyzed the problem in a more general form and eliminated some of the limitations made by Morgulis. Stratton's theory will be discussed in some detail in the section on theory because in spite of the many necessary approximations and assumptions made it is a good approach to the problem and the general shape of his example shows some resemblance to some of the data which have been taken.

This present investigation, the results of which will be described in this report, was initiated in the fall of 1956 and pursued through the fall of 1960. The study began primarily with the information obtained earlier by Allen (4, p. 1-26) and by Brock and Taylor (13). It was conducted at Linfield Research Institute where a portion of the facilities, equipment, and techniques necessary for an investigation of this type could be readily obtained.
In the meantime other laboratories have been working on this problem of field emission from semiconductors. At Bell Telephone Laboratories, L. A. D'Asaro (18) did some work on field emission from silicon, but F. G. Allen (5; 2, p. 1568-1571; 3, p. 979-985) has been very actively pursuing his study of field emission from silicon with the goal of investigating the behavior of adsorbed doping agents and contaminants on the silicon surface. Another project on field emission from silicon to study surface phenomena was recently initiated at Texas Instruments, Inc., but their early results were disappointing (57).

A number of investigations of field emission from semiconductors have been initiated in Russia, some of which have already been published. Apparently the majority of these were triggered by the article by Zernov and Elinson (65, p. 5-22) which was not only a short review of field emission, but also a call for more activity in this area of research. Elinson and Vasiliev wrote a book (29, p. 1-272) on field emission summarizing the work done through 1958, including a great deal of Russian work. Vasiliev (63) published a theory of field emission from semiconductors which will be discussed in the next section. Elinson (27) extended a part of Stratton's theory in an attempt to fit some data which he had observed. Unfortunately
his final result is not correct dimensionally, probably due to a misprint, so it will not be discussed further. Elinson and Vasiliev obtained a linear plot of \( \ln I \) vs \( 10^4/V \) for field emission from lanthanum hexaboride (30, p. 945-953) and a non-linear plot from germanium (28). Gofman and co-workers (36) reported that their results from field emitters made from tungsten carbide were non-linear, while Zubenko and co-workers (66) reported that plots of data from both tungsten carbide and germanium were linear. Klimin (43) obtained a straight line dependence for both germanium and cadmium sulfide. In almost all of these cases, however, the field emission patterns indicated that the emitting surfaces were not clean.
THEORY

Fowler and Nordheim (34, p. 173-181) considered a potential energy diagram with a triangular potential energy barrier (Figure 1) at the surface of a metal in vacuum due to the field \( F \) (in electron volts/cm). \( \phi \) is the work function in electron volts. The potential energy in electron volts is given by \( V = 0 \), \( x < 0 \) for the metal, and \( V = \phi - Fx \), \( x > 0 \) for the vacuum. The metal to vacuum interface is at \( x = 0 \). The equations to be solved are

\[
\frac{d^2 \psi}{dx^2} + K^2 (E_x - \phi + Fx) \psi = 0 \quad x > 0
\]

\[
\frac{d^2 \psi}{dx^2} + K^2 E_x \psi = 0 \quad x < 0
\]

where

\[
K^2 = \frac{2m}{\hbar^2},
\]

\( m \) is the mass of the electron and \( \hbar = \hbar/2\pi \) where \( \hbar \) is Planck’s constant.

The conditions to be satisfied are that \( \psi \) and \( \frac{d\psi}{dx} \) are continuous at \( x = 0 \) and that for \( x > 0 \), \( \psi \) represents a stream of
Figure 1. Potential energy diagram at the surface of a metal with an external electric field applied perpendicular to the surface. The effect of the image force is ignored in the triangular potential barrier, but is included in the dashed barrier.

Figure 2. Potential energy diagram at the surface of an n-type semiconductor with an external field applied perpendicular to the surface. Field penetration, surface states, and image force are not considered.
electrons moving toward the right only. After some approximations and for

\[
\frac{2}{3} \frac{K}{F} (\phi - E_x)^{3/2} \gg 1
\]  

(5)

it is found that the transmission coefficient is

\[
D(E_x) \sim \frac{4 \sqrt{E_x(\phi - E_x)}}{\phi} \exp \left[ -\frac{4}{3} \frac{K}{F} (\phi - E_x)^{3/2} \right].
\]  

(6)

The implied upper limit on \( F \) (Equation 5) does not restrict the applicability of the results since the current densities for higher fields would disrupt the material. They found that the number of electrons \( N(E_x) \) incident on a unit area of surface per unit time with energy \( E_x \) normal to the surface was

\[
N(E_x) = \frac{4 \pi m k T}{h^3} \int_{0}^{\infty} \frac{dy}{\exp \left( \frac{E_x - E_F}{kT} + y \right) + 1}
\]  

(7)

where \( E_F \) is the Fermi Energy, \( T \) is the absolute temperature, and \( k \) is the Boltzmann constant. Nordheim (50, p. 626-639) later
corrected the potential energy diagram to include the effect of the image force of the electron (dotted line barrier in Figure 1). This correction changes the transmission coefficient so that the exponent is multiplied by $f\left(\frac{q\sqrt{F}}{\phi - E_x}\right)$ where $f$ is an elliptic function and $q$ is the charge on an electron. Also the constant in front of the exponential is altered. Since the emitted current per unit area is given by

\[
J = q \int_{0}^{\infty} N_x(E_x) D(E_x) \, dE_x
\]  

then the value of $J$ as corrected by several workers (61, p. 436-441; 24, p. 799-800) is

\[
J = 1.54 \times 10^{-6} \frac{F^2}{\phi} \exp \left[ -6.83 \times 10^7 \frac{\phi^{3/2}}{F} f(y) \right]
\]  

where

\[
y = 3.79 \times 10^{-4} \frac{\sqrt{F}}{\phi}
\]  

Corrected values of $f(y)$ are available in tabular form (14), but for metals usually $f(y)$ is nearly constant over the range of fields used.
The current density $J$ is in amperes/cm$^2$, the electric field $F$ (actually potential energy gradient) is in electron volts/cm, and the work function $\phi$ is in electron volts. For metals $\phi$ is the height of the potential barrier at the surface above the Fermi level.

Stratton (62, p. 746-757) introduced his theory with a very brief discussion of the Fowler-Nordheim theory and then extended it to include semiconductors as well as metals. For his first extension Stratton points out that since the dielectric constant cannot be neglected (the dielectric constant $\varepsilon$ for silicon is $\varepsilon = 11.7$) the image force correction must be multiplied by $\frac{\varepsilon - 1}{\varepsilon + 1}$, so that the argument of $f$ must be multiplied by $\left(\frac{\varepsilon - 1}{\varepsilon + 1}\right)^{1/2}$.

Next Stratton discusses the problem of field penetration into the semiconductor. When a field is applied to the surface of a metal or a dielectric, it induces a surface charge which may be calculated by electrostatic theory. The distribution of this surface charge for the case of a metal has been examined by Fan (32, p. 388-394), who found that the lower edge of the conduction band dips down as the surface is approached from the interior, or that there is a decrease in potential energy. This means that there is an increase in the Fermi energy (positive for metals) near the surface since the Fermi level remains constant. This increase of the Fermi energy is
required to take care of the additional electronic charge induced. The maximum change $\Delta R$ in the Fermi energy is calculated by equating the field on both sides of the surface barrier and this turns out to be $F = \Delta R/\lambda$ where $\lambda$ is the depth of penetration of the surface charge which is in the form of an exponential decay. In a metal $\lambda$ is about $3 \times 10^{-9} \text{ cm}$ as indicated by theory, so $\Delta R \ll \phi$ even for the highest fields and may be neglected. $\phi$ is the height of the surface barrier above the Fermi level, i.e., $\phi$ is the work function.

The situation is different in the case of semiconductors because of the relatively low density $n$ of carriers. Morgulis pointed this out and he derived the potential drop when the field is so low that the carriers in the barrier are always non-degenerate. This is the case when the bottom of the conduction band will not drop below the Fermi level and Maxwellian statistics will apply to the electrons in the conduction band. Stratton, however, was also interested in the case of stronger fields which cause the bottom of the conduction band to drop below the Fermi level and the carriers become degenerate, i.e., they obey Fermi-Dirac statistics.

Figure 2 shows a potential energy diagram for an n-type semiconductor with an electric field $F$ applied perpendicular to its
surface with the image force and field penetration neglected. Figure 3a takes the image force and field penetration for the non-degenerate case into account and Figure 3b takes the image force and field penetration for the degenerate case into account. If $U$ is the decrease in potential energy in electron volts of any point a distance $x$ into the semiconductor from the surface, then the boundary conditions are

\[
U_{x=0} = \Delta R ; \quad U(x) \to 0 \quad \text{as} \quad x \to \infty ;
\]

\[
\left( \frac{dU}{dx} \right)_{x=0} = \frac{F}{\varepsilon} ; \quad \frac{dU}{dx} \to 0 \quad \text{as} \quad x \to \infty .
\]

(11)

Note that in this particular part of the discussion, Stratton reversed the direction of $x$ so that positive $x$ is from the surface into the semiconductor. This simplification does not affect the final results. When field emission is discussed, positive $x$ is from the surface into the vacuum. If Poisson's equation is integrated once, boundary conditions applied, and if classical statistics hold right up to the surface, i.e., when the conditions are

\[
- (\Delta R + f') \gg kT
\]

(12)
Figure 3. Potential energy diagram at the surface of an n-type semiconductor (without surface states) with an external field applied perpendicular to the surface. (a) weak penetration of the external field: non-degenerate case; (b) strong penetration of the external field: degenerate case.
where $\hat{\sigma}$ is the Fermi energy as shown in Figure 3a, it will reduce to

$$\Delta R = 2kT \sinh^{-1} \left( \frac{F x_o}{2 \varepsilon k T} \right)$$  \hspace{1cm} (13)$$

where

$$x_o = \left( \frac{\varepsilon k T}{8\pi nq} \right)^{1/2} = x_o \left( \frac{2(2\pi m k T/h^2)^{3/2}}{n} \right)^{1/2}$$  \hspace{1cm} (14)$$

$q$ is the electronic charge, and $n$ is the density of electrons in the conduction band. This is the case that Morgulis considered and the condition given above is equivalent to $F \ll \sqrt{\varepsilon} \cdot 1.5 \times 10^6$ ev/cm at room temperature.

Since the fields required for field emission are usually much higher than this, the situation involving much greater field penetration must be considered. The condition for the case when the bottom of the conduction band dips well below the Fermi level near the surface is

$$\hat{\sigma} + \Delta R \gg kT \cdot$$  \hspace{1cm} (15)$$
After integrating Poisson's equation in this case, with several approximations and neglecting the small terms due to the ionic charge, it is found that

\[ J' + \Delta R = \nu F^{4/5} \]  \hspace{1cm} (16)

where

\[ \nu = \left( \frac{15 \sqrt{\pi}}{16} \right)^{2/5} kT \left( \frac{X_0}{e kT} \right)^{4/5} \]  \hspace{1cm} (17)

The constant \( \nu \) is really independent of temperature since \( X_0 \sim T^{-1/4} \). The condition for this case is essentially \( F \gg \sqrt{\epsilon} \ 1.2 \times 10^6 \text{ ev/cm} \) at room temperature. When this condition on \( F \) applies, the Fermi energy, \( J' \), near the surface is positive and greater than \( kT \), because of field penetration, while farther in from the surface the Fermi energy is negative (Figure 3b).

Stratton then turns from his discussion of field penetration to the topic of field emission from a semiconductor. First he considers a semiconductor that would have no surface barrier due to the surface states and finds that two extreme situations may result,
i. e., there may be only slight field penetration or there may be strong field penetration causing degeneracy at the surface. In the first case Maxwellian statistics can be applied to the current carriers right up to the surface of the semiconductor from the inside. If \( N_x(E_x) \) is the number of electrons incident on a unit surface from the inside per unit time with energy \( E_x \) normal to the surface, then

\[
N_x(E_x) = \frac{qn}{2\pi m} \left( \frac{kT}{2\pi m} \right)^{1/2} \exp \left( \frac{-E_x}{kT} \right)
\]

(18)

and this can be substituted into Equation (8). Then for \( D(E_x) \) he uses either Equation (6) or the equation as corrected by Nordheim for the image force. Either way the resulting exponential can be simplified if

\[
2K_0 \theta^{1/2} / F \gg \frac{1}{kT}
\]

(19)

where \( \theta \) is the potential energy of the electrons at the bottom of the conduction band, but the degree of inequality need not be large, since the quantities occur as exponents. This condition means that
emission by tunnelling, i.e., field emission, is much more important than thermionic emission. This condition and the condition in Equation (5) together limit $F$ to the finite range

$$2 K \theta^{1/2} k T \ll F \ll 2 K \theta^{3/2}/3,$$

or at room temperature

$$2.6 \times 10^6 \theta^{1/2} \ll F \ll 3.4 \times 10^7 \theta^{3/2}$$

where $\theta$ is in electron volts and $F$ is electron volts/cm. If the image force is neglected, the result is

$$J = n q \frac{2 k T}{(2 m \theta)^{1/2}} \exp \left[ -\frac{4 K}{3 F} \theta^{3/2} \right]$$

but if the image force is included, then the current density (amps/cm$^2$) is

$$J = n q \left( \frac{k T}{2 \pi m} \right)^{1/2} \exp \left[ -\frac{4 K}{3 F} \theta^{3/2} f \left( \frac{\pi \theta}{\theta + 1} \right)^{1/2} \frac{q \sqrt{F}}{\theta} \right].$$

This result should not be applied to real semiconductors because the penetration by the field was neglected. If the field penetration does not lower the conduction band enough to invalidate the Maxwellian statistics near the surface, then the only correction required is that
the value of \( n \) must be multiplied by \( \exp(\Delta R/kT) \) where \( \Delta R \) is given by Equation (13).

If the second situation applies, i.e., the field penetration is sufficient to cause degeneracy of the electrons near the surface, as seems very probable because of the condition given immediately after Equation (17) that \( F \gg \sqrt{\varepsilon} \text{ } 1.2 \times 10^6 \text{ ev/cm} \), then \( n_x(E_x) \) will have a value appropriate to a metal. In this case the calculation for the field emission current is similar to the Fowler-Nordheim calculation, but the work function is \( \theta - (\Delta R + \varepsilon) = \theta - \nu F^{4/5} \)
and the Fermi energy is \( \Delta R + \varepsilon = \nu F^{4/5} \). Then following the method of Fowler and Nordheim,

\[
J = \frac{q}{8\pi h} \frac{F^2}{\theta - \nu F^{4/5}} \exp \left[ -\frac{4}{3} \frac{K(\theta - \nu F^{4/5})^{3/2}}{F} \right] \left[ f \left( \frac{\epsilon - 1}{\epsilon + 1} \right)^{1/2} \frac{q \sqrt{F}}{\theta - \nu F^{4/5}} \right] \]

\[
x \left[ 1 - \left( 1 + \frac{2K\nu F^{4/5}(\theta - \nu F^{4/5})^{1/2}}{F} \right) \exp \left( \frac{2K\nu F^{4/5}(\theta - \nu F^{4/5})^{1/2}}{F} \right) \right] \]

where \( F \ll \frac{2}{3} K (\theta - \nu F^{4/5})^{3/2} \)

\[
f \left[ \frac{\epsilon - 1}{\epsilon + 1} \right]^{1/2} \frac{q \sqrt{F}}{\theta - \nu F^{4/5}} \]

\[
\varepsilon \times \left[ 1 - \left( 1 + \frac{2K\nu F^{4/5}(\theta - \nu F^{4/5})^{1/2}}{F} \right) \exp \left( \frac{2K\nu F^{4/5}(\theta - \nu F^{4/5})^{1/2}}{F} \right) \right] \]

(22)
If \( \Delta R + \hat{\mathbf{j}} = \nu F^{4/5} \ll \theta \) these reduce to

\[
J \approx \frac{q}{8\pi h} \frac{F^2}{\theta} \exp \left[ -\frac{4}{3} \frac{K\theta^{3/2}}{F} f \left( \frac{\epsilon - 1}{\epsilon + 1} \right)^{1/2} \frac{q\sqrt{F}}{\theta} \right]
\]

\[
x \left[ \exp \left( \frac{2K\nu\theta^{1/2}}{F^{1/5}} \right) - \left( 1 + \frac{2K\nu\theta^{1/2}}{F^{1/5}} \right) \right] \equiv \left( \frac{2K\nu\theta^{1/2}}{F^{1/5}} \right) \left( \epsilon + 1 \right)^{1/2} \frac{q\sqrt{F}}{\theta} \right)
\]

where \( F \ll \frac{2}{3} K\theta^{3/2} f \left( \frac{\epsilon - 1}{\epsilon + 1} \right)^{1/2} \frac{q\sqrt{F}}{\theta} \). This theory, unfortunately, does not analyze the intermediate conditions of the semiconductor between complete absence of degeneration and intense degeneration of the electrons in the conduction band.

Next Stratton considers the case of field emission from a semiconductor with a surface barrier due to surface states. Many of the contact phenomena of semiconductors can be explained by assuming that states having energies within the forbidden energy band exist on the surface. These states cause an internal potential barrier at the free surface of the semiconductor. If the density of surface states is high enough, the internal potential barrier may be almost unchanged by putting the surface in contact with a metal.
Instead of putting the semiconductor in contact with metal, Stratton considers it in a high vacuum with a high electric field applied to it. In the potential energy diagram of Figure 4, $D$ represents donor-type states with energy $E_d$ below the conduction band while $A$ represents the acceptor states with an energy $E_a$ below the conduction band. The $D$ states are neutral when occupied by electrons and the $A$ states are neutral when occupied by holes (lack of electrons). This model with

$$\exp\left(\frac{E_d}{kT}\right) \ll \exp\left(\frac{E_g}{kT}\right) \text{ and } \exp\left(\frac{E_g - E_a}{kT}\right) \ll \exp\left(\frac{E_g}{kT}\right)$$

is the model that has been used by Brattain and Bardeen (12, p. 1-41) to describe the surface properties of germanium. These expressions indicate that the donor surface states are very close to the conduction band and the acceptor states are very close to the valence band. These conditions are assumed and arbitrary. Stratton simplified this case considerably by assuming that there are enough surface states to provide all the charge induced by the external field, or in other words, that field penetration does not occur.
Figure 4. Potential energy diagram at the surface of an n-type semiconductor with donor and acceptor surface states at D and A respectively. The external electric field reduced the height of the internal potential barrier from its zero field value of $\phi_0$ by an amount $V$ to the value $(\phi_0 - V)$. 
A number of new symbols are required in order to discuss this portion of the theory. With $F = \text{the applied electric field in electron volts per cm}$, let

\[
\begin{align*}
\phi_o & = \text{the height of the internal barrier in electron volts when } F = 0; \\
(\phi_o - V) & = \text{the height of the internal barrier in electron volts when } F \neq 0; \\
N_d & = \text{the number of donor surface states per cm}^2; \\
N_a & = \text{the number of acceptor surface states per cm}^2; \\
n_{do} & = \text{the number of electrons on donor surface states when } F = 0; \\
p_{ao} & = \text{the number of holes on acceptor surface states when } F = 0; \\
n_d & = \text{the number of electrons on donor surface states when } F \neq 0; \\
p_a & = \text{the number of holes on acceptor surface states when } F \neq 0; \\
E_o/\epsilon & = \text{the field in electron volts per cm at the top of the barrier inside the semiconductor when } F = 0 \text{ and} \\
E/\epsilon & = \text{the field in electron volts per cm at the top of the barrier inside the semiconductor when } F \neq 0.
\end{align*}
\]
The charge on the surface states is given by Gauss's theorem as

\[
\frac{F + E}{4\pi q^2} = -(N_d - n_d) + (N_a - p_a)
\]  

(25)

and reduces to

\[
\frac{E_n}{4\pi q^2} = -(N_d - n_{do}) + (N_a - p_{ao})
\]  

(26)

when \( F = 0 \). Stratton says that it can be shown that if the forbidden band gap is large with respect to the energy difference between the bottom of the conduction band at the surface and the Fermi level, i.e., if the acceptor states at the surface lie far below the Fermi level as indicated in the following expression

\[
\exp \left( \frac{E g}{2kT} \right) \gg \exp \left[ \frac{(\Phi - \phi)}{kT} \right],
\]  

(27)

then the acceptor states will carry electrons, or \( p_{ao} = p_a = 0 \), and this is assumed in the derivation. Then after another assumption and some algebraic juggling, Stratton obtains the relation between \( E \) and \( V \)
\[
\frac{a + a'}{E_0} (F + E) = \left[ 1 + \frac{1-a}{a} \exp \left( -\frac{V}{kT} \right) \right]^{-1} + a',
\]  

(28)

where \( a = \frac{n_0}{N_d} \), and \( a' = (N_a - N_d)/N_d \). He indicates that there is another relation between \( E \) and \( V \) which depends on the model used for the barrier, but he does not give this relationship. Unfortunately a second relationship between \( E \) and \( V \) is necessary before any practical calculations can be made, since this is an equation involving two unknown quantities.

Then Stratton returns to Equation (8) to calculate the emitted current with

\[
N_x(E_x) = nq \left( \frac{kT}{2\pi m} \right)^{1/2} \exp \left( \frac{V - \phi_0}{kT} \right) = J_0 \exp \left( -\frac{\phi_0 - V}{kT} \right)
\]  

(29)

and with \( D(E_x) \) as corrected for the image force by Nordheim.

The result obtained is

\[
J = J_0 \exp \left( -\frac{\phi_0 - V}{kT} \right) \exp \left[ -\frac{4K}{3F} \theta^{3/2} \left( \frac{\epsilon - 1}{\epsilon + 1} \right)^{1/2} \frac{q \sqrt{F}}{\theta} \right].
\]  

(30)
This equation is identical to Equation (21) except for the multiplier
\[ \exp \left( - \frac{\Phi_0 - V}{kT} \right). \]

Finally, Stratton assumes values of the constants which he says might be appropriate for the semiconductor SiC, and he plots curves of \( \log_{10} J \) vs \( 10^8/F \) for the different cases he treated. In Figure 5, curves I and II for the case of field emission from a semiconductor neglecting both strong field penetration and surface states, with curve I for the case neglecting the image force and curve II with the image correction, are very similar to those for a metal, since the factor \( F^2 \) in front of the exponential in the equation for field emission from a metal is relatively unimportant. One important difference, however, is that the field currents from semiconductors are strongly temperature dependent because of the factor \( n \). It has already been pointed out that neither curve I nor curve II should be applied to a real semiconductor since variations in the lower edge of the conduction band near the surface must be considered.

Curve III is for the case of strong field penetration with the image force correction when no surface states exist and all fields will cause the level of the conduction band near the surface to drop
Figure 5. Field emission currents from silicon carbide according to Stratton, plotted as the logarithm of the current density versus the reciprocal of the field strength. I, no image force, field penetration, or surface states; II, image force, but no field penetration or surface states; III, image force and strong field penetration, but no surface states; IV, image force and surface states.
below the Fermi level. This causes much greater field emission than that obtained from case II, because the electron gas near the surface becomes degenerate, thus providing a greatly increased density of electrons available for emission.

Stratton says that curve IV corresponds to the case in which an internal surface barrier is present because of the surface levels. If the density of surface states is high enough, the internal barrier is large enough to greatly reduce the current for low fields, so curve IV falls far below curve II for weak fields. As the applied field is increased, the internal barrier is lowered and the current increases much more rapidly until curve IV intersects curve II at \( P \). This corresponds to the complete breakdown of the internal surface barrier, i.e., \( V = \phi_0 \) which occurs in this example at \( F_0 \approx 2.5 \times 10^7 \) ev/cm. For fields higher than this critical value \( F_0 \), field penetration will occur. The emitted current will be given by an equation like Equation (22) for case III except that the field used to determine \( \Delta R \) is in this case the applied field \( F \) reduced by the field \( F_0 \) which is required to reduce the internal barrier to zero, or to almost completely fill the surface states. If \( F - F_0 \) is small enough (\( < 1.5 \times 10^6 \) ev/cm in this example), \( \Delta R \) is given by equation (13) and the current is given by
Equation (21) with $n$ increased by the factor $\exp\left(\frac{\Delta R}{kT}\right)$, and this corresponds to portion PO of curve IV. If $F - F_0$ is large enough ($> 3.2 \times 10^6$ ev/cm in this example), Equation (22) or (23) must be used with $F$ replaced by $F - F_0$ in the last factor only since the other factors correspond to the penetration coefficient and this corresponds to the portion RS of curve IV. Finally, Stratton drew in a broken line to "suggest the connection between the two regions of approximation". He says that the dotted line indicates a slight lowering of the current below the value calculated from IV in the range $V \rightarrow \phi_0$ because the electron density in the barrier is appreciable in this range but was neglected in deriving Equation (30). The high value of the critical field $F_0$ is due to the high value of the surface states density. Lower values of the surface states density would shift the point $P$ down curve II while the portion of the curve corresponding to RS would, according to Stratton, approach curve III more and more closely.

Vasiliev (63) based his theory on a new equation for the transparency of the potential barrier which was derived by Murphy and Good (49, p. 1464-1466). He sets up the problem in the same way that Stratton did, using some of Stratton's results. Vasiliev obtained very complicated equations for the field emission current
density for the case of a non-degenerate electron gas in a semiconductor and the case of a degenerate electron gas due to strong field penetration. These include an integral which is tabulated (51, p. 116-120), and a t function which is also tabulated (37, p. 187).

Unfortunately, Vasiliev gave his results in atomic (Hartree) units, which makes the current density, electric field, and all energies into dimensionless quantities. Since he gave no intermediate steps, it is not possible to check his final results for misprints or mistakes. Two misprints were found in other parts of his paper. He did not give an example or a "typical plot" such as that given by Stratton.
APPARATUS

The primary tool used in this investigation was the field emission projection microscope, which has proved in other studies to be a valuable and fruitful instrument for the study of surfaces, surface defects, and reactions at surfaces. Figure 6 shows a sketch and Figure 7 a photograph of one of the first field emission projection microscope tubes used in this study, and Figure 8 a sketch and Figure 9 a photograph of the last tube used in this study. The field emission cathode is fabricated from the material to be studied and it is placed in the center of the tube. This cathode must be in the shape of a very sharp, smooth and symmetrical needle so that sufficiently high electric fields can be applied to its surface to obtain measurable field emission currents. The radius of curvature of the end of the tip is of the order of one micron or less. In this study the field cathode was made from a single crystal of germanium or silicon. In the first tubes used, the anode consisted of a Willemite phosphor screen with a metal backing, usually of aluminum or gold, which was deposited opposite the field cathode onto the inside surface of a 500 milliliter spherical Pyrex flask. Contact was made to the anode by a button seal through the glass to a platinum and silver ring painted on the inside surface of the glass.
Figure 6. Sketch of a modified field emission projection microscope used early in this study.
Figure 7. Modified field emission projection microscope used early in this study. The small bulb contains the getter.
Figure 8. A sketch of the last field emission projection microscope used in this study.
Figure 9. A photograph of the last tube used in this study. The silicon field emitter is mounted in the molybdenum spring clamps in the center.
before the gold or aluminum anode was evaporated onto the screen and glass. Many of these tubes had a ring of tungsten wire in the center around the field emitter tip which was used to provide a source for electron bombardment of the tip or to serve as a grid or as a second anode. The tube had a side tubulation through which it could be exhausted by the vacuum system and a getter bulb was usually located close to the tube in the line to the vacuum system. In some of the later tubes, the metal backed phosphor anode was replaced by an anode consisting of a conducting glass layer on the inside surface of the tube onto which the Willemite phosphor screen was deposited. The last two tubes used in this study were made from "boron free" EE-2 glass which was supplied as small oscilloscope tube blanks with three inch faces. Both of these tubes had Willemite phosphor deposited onto a layer of conducting glass on the inside of the tube face. In addition, an aluminum backing was evaporated onto the phosphor screen of the second tube to provide a conducting path on both sides of the screen. In all cases the contact to the anode was made by a button seal through the glass to a platinum and silver ring painted on the inside surface of the glass. The details of the tube construction will be given in Appendix A.
The evacuation of the tubes in the initial phase of this study was done on any of the several vacuum systems that were available. The tubes were sealed off from these systems when a pressure of the order of $10^{-10}$ millimeters of mercury was reached, and then various cleaning processes were tried. It soon became quite apparent that the existing vacuum systems were not as versatile as needed to attempt the numerous cleaning processes desired. Furthermore, they were not always available to this study as a result of scheduling difficulties. Therefore, when the funds became available under a new support contract, a new vacuum system was designed and built especially for this study. Provision was made for handling pure gases and allowing these gases into the experimental tubes during the evacuation and attempted cleaning processes. A schematic diagram of the vacuum system is shown in Figure 10 as originally built, and Figures 11 and 12 show the vacuum system and its control panel. The roughing pump was a rotary gas ballast mechanical pump (App. B, 1)\(^1\), and the diffusion pump was a three-stage mercury pump (App. B, 2) with a 70 liter per second pumping speed. As a safety measure a pressure switch (App. B, 3)

---

\(^1\)The numbers in parentheses following major items of equipment and supplies refer to Appendix B where the items are described in more detail.
Figure 10. Schematic diagram of the high vacuum system built especially for this study.
Figure 11. Ultra-high vacuum system built especially for this study. Four flasks of pure gases are sealed to the gas transfer system at the upper left and seven field emission microscope tubes are sealed to the high vacuum side of the system.
Figure 12. Control panel of the vacuum system. The ionization gauge circuit is at the top and the oven controls are below it.
was installed in the water line from the diffusion pump to the drain so that if the flow stopped the switch would open and break the circuit to the diffusion pump heater (App. B, 4). The cold traps were two non-demountable liquid air traps (App. B, 5) in the gas handling system, and one demountable liquid air trap (App. B, 6) which was inserted later in the foreline, while the cold trap on the high vacuum side of the mercury pump was an integral part of the pump. A stainless steel, three liter capacity, Dewar flask (App. B, 7) was used for the large high vacuum cold trap and one liter capacity Pyrex thermos flasks (App. B, 8) were used for the smaller traps. The fore pressure and the pressure in the gas line were measured with Veeco thermocouple gauge tubes (App. B, 9) and control circuits (App. B, 10). The high vacuum was measured by a Veeco non-burnout ion gauge (App. B, 11) and control circuit (App. B, 12) which measures from $2 \times 10^{-10}$ mm Hg to $1 \times 10^{-3}$ mm Hg. The gas was allowed into the high vacuum part of the circuit through a small gas leak which consisted of a drawn down Pyrex tubing with a capillary opening adjusted so that a pressure of one micron could be maintained on one side by a mechanical forepump when the other side of the opening was exposed to the atmosphere. A pressure of $2 \times 10^{-10}$ mm Hg or less was obtained
on the high vacuum side of the leak when the pressure was a few microns\(^1\) on the other side. This leak adjustment was a trial and error process using a spark coil (App. B, 13) to enlarge the opening or a gas torch to constrict the opening. The leak was connected through one of the small cold traps to a one liter expansion bulb and through gas lines of Pyrex and valves to the flasks of pure gases. The valves used were Veeco high vacuum valves (App. B, 14) with teflon gaskets. The pure gases, e.g., argon (App. B, 15), were obtained in one-liter spherical Pyrex flasks equipped with a small break seal. These were sealed to the system with a small glass capsule containing iron filings in the line directly above the break seal. The capsule could be raised with a magnet and then released, thus falling on the break seal and breaking it. This admitted gas into the tubulation to the first valve. A small wad of fibreglass was enclosed near the valve to prevent glass chips from the broken seal from entering the valve and fouling the teflon gasket. A ground glass ball and socket valve which could be opened or closed by an electromagnet acting on iron filings in a bulb attached to the valve was placed between the diffusion pump and the high vacuum part of the system. If a gas were allowed into the gas lines to a

\(^1\)One micron = \(10^{-3}\) mm Hg.
pressure above 1000 microns, then a pressure of the order of $10^{-7}$ mm Hg could be maintained in the high vacuum side of the system with the valve open, but if the valve were closed, the pressure would rise to above $10^{-3}$ mm Hg in less than a half hour. All of the glass used in the vacuum system was Pyrex, except for the Nonex envelope of the ionization gauge.

There were three ovens: a main oven to go over the high vacuum part of the system and two trap ovens, one for the large high trap and the second for the small gas trap and expansion bulb. The main oven was raised off and lowered onto the high vacuum part of the system by cables over overhead pulleys and a hand winch. The trap ovens each had two doors, one at the side and the other at the bottom. Eight 635 watt Chromalox tubular heating elements (App. B, 16) were used for the main oven and two 500 watt Chromalox tubular elements (App. B, 17) were used for each trap oven. PV supertemp blocks (App. B, 18), 3 feet x 1 foot x 2 inches were used for the oven insulation. A small fan was made to improve the circulation of the main oven, and this was driven by a 115 volt electric motor (App. B, 19). The air was forced over the main oven heating elements by a duct of stainless steel between the elements and the interior of the oven. The heating elements were controlled
by West Gardsman Model J on-off controllers (App. B, 20), one for each oven, which were calibrated for six feet of 14 gauge Chromel-Alumel thermocouples, two of which were covered with fish-spine insulation (App. B, 21) and one with double-bore insulation (App. B, 22). During installation of these thermocouples, the double-bore insulation was used where thermocouple leads were straight and the fish-spine insulation used where the thermocouple leads had to be bent or coiled. Manuflex timers with ten-hour dials (App. B, 23) were used to time the Gardsman controllers while the Gardsman controllers operated mercury relays (App. B, 24) which controlled the power to the ovens. Switches were later installed so that the timers could be by-passed, thus permitting the ovens to remain on longer than ten hours. A safety circuit was built in later when it was found on other systems that occasionally the mercury relays would stick shut, thus allowing the oven temperature to go far above the annealing temperature of the glass on the system, causing the tubes to collapse or melt. This circuit consisted of a large power relay (App. B, 25) with a separate thermocouple switch (App. B, 26), normally closed, in each oven in series with it. If any one of these three thermocouple switches opened, the circuit for this master relay would open, causing the master relay to open and
remove all power from the oven circuitry. A circuit diagram of the oven and safety control circuits is shown in Figure 13. The gas lines were wrapped with two circuits of heating tape (App. B, 27) and then asbestos tape. The gas lines could then be heated safely to about 250°C by connecting a variac to each circuit. The temperatures at four different points were monitored by iron-constantan thermocouples and a Sim-ply-trol meter (App. B, 28).

The vacuum system was built onto a portable cart whose framework was made of 1" angle iron, a sketch of which is shown in Figure 14. One end of this cart was reserved for the controls for the pumps, gauges, and ovens as well as the gas manifold for the bottles of pure gases. The cart was mounted on four small wheels so that it could be moved readily.

An "outgassing cart" was constructed to operate in conjunction with the vacuum system. This was a cart containing several power supplies which were used primarily for various types of outgassing procedures during a typical vacuum run. A dc power supply, which was capable of providing from 0 to 2 amperes at voltages from 0 to 2000 volts, was built into the cart, primarily to supply the voltages necessary for electron bombardment procedures. In addition, there were two powerstats (App. B, 29) providing from
Figure 13. Diagram of the oven and oven safety control circuits. The three switches at the lower left are thermocouple switches which open if their temperatures exceed a predetermined value.
Figure 14. Framework of the portable cart for the vacuum system.
0 to 140 volts ac and four powerstats connecting through step-down transformers such that two provided from 0 to 6 volts ac, one from 0 to 10 volts ac and one from 0 to 24 volts ac. Switches, meters, and the associated circuitry were included so that voltages and currents from the different supplies could be read (except for the voltages from the two 0 to 140 volt variacs which were installed later). The four low voltage ac supplies were each built into individual units which could be lifted out of the cart and used elsewhere. Figure 15 is a photograph of the top of this outgassing cart, showing the various controls.

During the operation of a new tip in a microscope tube, a source of high voltage is necessary for field emission and a source of low voltage ac is necessary for the heating of the sample. In most cases the high voltage was supplied from a Spellman 30 kv reversible polarity power supply (App. B, 30) which utilized an rf circuit to obtain the high voltage. The low voltage ac used to heat the tip in the vacuum tube was supplied from a variac, usually one of the 140 volt variacs on the outgassing cart. It was found that passing current through the semiconductor to heat it was a tricky operation because of the negative temperature coefficient of resistance of the semiconductor. When the semiconductor was heated the resistance
Figure 15. Outgassing cart.
would decrease, the current would increase, and the tip would get hotter and quickly melt. This difficulty was overcome by inserting a resistor (actually, a Chromalox strip heater, rated at 500 watts, 115 volts, giving a resistance of about 26-1/2 ohms) in the heating circuit which counteracted this effect. After this scheme was tested, it was found most convenient to build the circuit into the control section of the vacuum system cart. The circuit included, in addition to the large wattage resistor, a 100 ohm rheostat which protected the tip from melting when the heating was first started and could then be turned out of the circuit once the tip had been initially heated. This additional safeguard was necessary because a high resistance oxide layer would form on the semiconductor under the metal contacts when the tip was sealed into the tube and the tube evacuated and baked out. This high resistance had to be broken down before the tip could be heated, but when it did break down the total resistance through metal contacts and semiconductor would usually drop suddenly from the order of 10,000 ohms to the order of 10 ohms or less. In addition, a one to one isolation transformer (App. B, 31) insulated for 30 kv was installed between this circuitry and the tip in the tube so that a high negative voltage could be applied to the tip at the same time it was being heated. The temperature of the silicon
or germanium sample was measured by a micro-optical pyrometer (App. B, 32) and corrected for the emissivity, which was measured by Allen (1).

After a tip was cleaned, measurements were made of the current obtained from the tube for each voltage applied across it. Either an electrostatic voltmeter (App. B, 33) or a kilovoltmeter (App. B, 34) was used to measure the voltage applied to the tube from the 30 kv Spellman supply. A string of high resistors totalling approximately 240 megohms was placed between the Spellman supply and the tube to protect the tip in the event of any voltage surges or accidents. (The voltmeters measured the voltage at the tube.) The current through the tube was measured by a Keithley micro-micro-ammeter (App. B, 35) whose output was recorded with a Varian chart recorder (App. B, 36). The field emission pattern from the tip was photographed with a 35 mm camera (App. B, 37) using either fast Tri-X film or fine grain Adox film. Occasionally a tip would be sharp enough that it would operate at low enough voltage that a VR board could be used in place of the Spellman power supply. This VR board consisted of a bank of 120 OB2 tubes in series. A voltage from a war surplus RA-38 radar power supply capable of providing 15 kv was applied through a resistance of
2 x 10^5 ohms and a 0-35 ma dc meter to the top of this bank such that 10 milliamps of dc current would flow through the bank to another 0-35 ma dc meter to ground. Each tube was shunted with a 1 megohm resistor to provide a continuous path to ground even if the voltage from the RA-38 were not high enough to cause the tubes to fire. With the tubes stacked in this manner, the voltage regulation was constant (within one percent) and voltages could be obtained in steps of about 108 volts between the tubes up to a maximum of about 13 kv.

Using this equipment, it was simple to take data just by writing the voltage read from the voltmeter or the VR tube number onto the Varian chart at the appropriate current level measured. Also, any notation of photographs of patterns could be made on this chart at the time it was made. The chart could then be read at a more convenient time for the data reduction. A diagram of a typical circuit including the heating circuit is shown in Figure 16.

Since some of the silicon tips yielded non-linear plots of the data (when it was plotted as log of the current versus reciprocal voltage), the question of the accuracy of the equipment was raised. This led to a series of calibrations which showed that most of the equipment was quite accurate and only a few corrections were
Figure 16. A typical circuit used for testing silicon tips in field emission microscope tubes.
necessary. The first calibrations were made, using a resistor string of 0.1 percent, 1 megohm resistors (App. B, 38) and a Präz.-Mikroamperemeter (App. B, 39) on the 750 microampere range which has an accuracy of 0.5 percent of full scale. These, of course, also had to be checked for their accuracy. The resistors were checked by applying a set voltage from the VR board to five resistors in series at a time and measuring the current from each set of five, averaging and checking the deviations. It was found that each set of five resistors deviated from the average value less than the 0.1 percent allowed by the stated tolerance so it could therefore be assumed that the whole string of twenty resistors was within 0.1 percent of the sum of the average values which was assumed to be exactly 20 megohms. Another string of 1.0 percent resistors was checked against this string and the current through each string was the same, as near as could be read, so it could be assumed again that the average value of each resistor was one megohm and the total of both strings was then 40 megohms within 0.1 percent. The accuracy of the Präz.-Mikroamperemeter was checked by measuring the current through a secondary standard resistor of 100 ohms (App. B, 40) and the meter while a Leeds and Northrup type K-3 Universal potentiometer (App. B, 41) was used to measure the
voltage across the 100 ohm standard resistor. Five readings of this voltage were taken on the potentiometer for each setting of the current through the resistor and then averaged. The current through the resistor (and the meter) was set so that the meter could be read accurately, i.e., by placing the meter needle directly over a scale division using the scale mirror to eliminate parallax. This was repeated for each scale division of the "Práz. -meter." The exact current through the meter for each scale division of the 750 μA range was then calculated. The electrostatic voltmeter and the kilovoltmeter were then calibrated against the resistor string and "Práz. -meter" in series. Multiplication of the current through the "Práz. -meter" by the resistance in the resistor string (either 20 or 40 megohms) gave the voltage across them to less than 0.5 percent. The electrostatic voltmeter was found to be accurate to within less than one percent and so its readings were usually not corrected, while the kilovoltmeter was found to be off by over three percent on the 30 kv range so its readings always had to be corrected. A rough check of the accuracy of the Keithley 410 micro-microammeter and the Varian chart recorder showed accuracies in general within four percent. Since the current is plotted on the log scale, these errors are negligible and no further attempt was made to obtain greater
accuracy of current measurements. Figure 17 shows circuit diagrams used for these calibrations.

Some measurements were obtained using pulsed voltages at higher levels. The pulser was a homemade hard tube pulser with a pulse length which was varied from 30 to 80 microseconds in length for these measurements. Since only a few pulsed measurements were obtained, because, in part, the pulser was unreliable, no further description of this pulser will be given. A Tektronix oscilloscope (App. B, 42) with a dual channel input (App. B, 43) was used to measure both the current through the field emission tube and the pulsed voltage across the tube using a capacity divider. The calibrations of these were checked by operating a tungsten field emitter on the same equipment, and since the pulsed data plotted on an extension of the straight line obtained for the dc data (as they should for a metal tip), these calibrations were presumed to be correct.

A thermoelectric probe, such as that described by Dunlap (23, p. 189-190), was built to test the surfaces of the semiconductors to determine whether these surfaces were n-type or p-type. This consisted of two molybdenum points machined so that they could be fitted to a pencil soldering iron (App. B, 44). One of these was
Figure 17. Circuits used to check accuracy of instruments:
(a) value of $R_1$ and the calibration of voltmeters, (b) calibration of the "Praz. -meter", and (c) accuracy of the micro-microammeter chart recorder.

A "Praz. -meter" $R_1 = 1$ megohm $\pm$ 0.1% resistors
B Micro-microammeter and chart recorder $R_2 = 100$ ohm sec. std. resistor $R_3 = 50,000$ ohms $R_4 = 500$ ohms
C 6 volt battery $R_5 = 1$ megohm $\pm$ 1.0%
D 67-1/2 volt battery $R_6 = 0.25$ megohm $R_7 = $ load
screwed onto the heating element of the soldering iron and contact was made from this through a center zeroing 25-0-25 dc micro-ammeter (App. B, 45) to the other molybdenum point. If both the hot point and the cold point are placed on the surface of a piece of semiconductor a small dc current will flow in one direction or the other, depending on whether this surface is n-type or p-type.

The shape of a field emitter must be known accurately before the current-voltage data can be reduced to the more basic quantities of current density and electric field. An RCA electron microscope (App. B, 46) with a magnification of about 3000X had been modified previously so that metallic field emitters could be examined in all profiles provided the emitter and its filament support structure could be inserted into a hole 1/8 inch in diameter. A new holder was made so that the silicon and germanium tips could be mounted on it, inserted into the electron microscope, and micrographed provided that they were kept less than 1/8 inch in diameter (the length was not critical). In general, except at the very beginning of the study, the electron microscope was not used until after the silicon tips had been tested and removed from the field emission microscope tubes in order to avoid contamination from oil vapors and other materials in the electron microscope. Experience gained
previously from metallic field emitters indicates that these contaminants would cause the cleaning process to be even more difficult than it already is. A light microscope (App. B, 47) with a 100X magnification was used to check the gross geometry of these tips periodically during the tip fabrication process.
PROCEDURE

1. Electrolytic shaping of germanium and silicon field emitters.

This investigation required the use of smooth, symmetrical needle shaped semiconductor field emitters with a tip radius of the order of one micron or less. Therefore the initial phase of this study dealt with the development of techniques capable of meeting these requirements. Both germanium and silicon bars were supplied by Fred Allen of Bell Telephone Laboratories; E. G. Brock, now at IBM Research Laboratories, furnished some germanium cylinders. In addition, both germanium and silicon bars were purchased from Anchor Metal Company (App. B, 48) and bars and ingots of silicon were purchased from Thermosen, Inc. (App. B, 49).

The first attempts to produce germanium field emitters by electrolytic etch, using either a solution of 3 parts HF to 1 part HNO₃ or the CP-4 solution (consisting of 25 ml HNO₃ conc., 15 ml HC₂H₃O₂ glacial, 15 ml HF conc., and several drops of bromine), resulted in the undesirably non-symmetrical tips shown in Figure 18. The next method tried was to mount the crystals in a drill press and rotate them at a high speed while they were being electrolytically etched. Three symmetrical tips were made this way in three tries,
Figure 18. Micrographs (approximately 3000X) of the first five germanium tips produced in this study.
but since the fumes from the electrolyte were starting to tarnish the drill chuck, it became imperative to make a device to accomplish the same purpose, and a pin-vise was mounted on the shaft of a sewing machine motor. Unfortunately, not a single symmetrical tip was obtained using this homemade device. Therefore this method was discarded and experiments were started to find a combination of etching solutions and voltages which might produce symmetrical tips. Since then it was found possible to round and polish the blanks electrolytically without mechanical grinding (see Figures 19 and 20) and better results were obtained with a number of solutions which yielded much more symmetrical tips as shown in Figure 21-a. For germanium this was accomplished by slowly dipping the blanks into and removing them from an electrolyte composed of 95 percent concentrated HNO₃ and five percent concentrated HF by volume. The germanium blanks were held in platinum tipped tweezers (App. B, 50) and the solution was contained in a small polyethylene beaker (App. B, 51) placed on a lab jack. The dipping process was accomplished by raising and lowering the solution to and from the tip by means of a lab jack, thus permitting the germanium blank to be held motionless and observed through a binocular microscope (App. B, 52). A potential of 70 to 90 volts dc from a homemade dc
Figure 19. Sample XGe1G34 (A) before being reduced and polished electrolytically and (B) when partially reduced. Note that the rectangular bar is being rounded electrolytically.
Figure 20. Sample XGe1G34 (a) after the tip has been formed electrolytically and composite micrographs (b) and (c) of the tip 40° apart. The magnification of the micrographs is about 3000 times.
Figure 21. Micrograph profiles 90° apart for three germanium tips (A) after shaping with a nitric acid and hydrofluoric acid electrolyte, and (B) after finishing with a nitric acid and phosphoric acid electrolyte. The field of view is about 15 microns for each micrograph.
power supply was maintained between the germanium which was made anodic and a platinum cathode in the form of a small wire loop which could be raised up around the blank if necessary by means of a micro-manipulator. A floodlight was held on the work, not only for illumination, but also for improvement of the fabrication results. Apparently the light generates pairs of electrons and holes in the semiconductor, increasing its conductivity and leading to a more uniform distribution of current density in the blank. The reaction was accompanied by copious gas evolution and had to be carried out under a hood. Figure 22 is a photograph of a typical setup for electrolytic fabrication of semiconductor tips.

Because the resistivity of the blank was much higher than that of the solution, the current density on the blank was believed to be non-uniform, being highest near the surface of the solution and becoming progressively lower near the end of the blank. This should result in a speeding up of the rate of material removed near the surface. Experimentally it was observed that if the blank was not dipped into and drawn from the solution but left immersed in one position, it soon became necked in near the surface and was eventually cut through, the lower portion dropping away. By slowly dipping the blank into and removing it from the solution it was
Figure 22. Typical setup for the fabrication of semiconductor tips.
possible to shape the blank until the lower portion was cylindrical in shape with a diameter of approximately 0.001 inch. The most reasonable explanation for the rounding action is that the removal of material is a process controlled by diffusion and convection, in which oxidizing and complexing reagents are prevented free access to the surface by evolved oxygen. The most easily accessible portion of the blank would be the protruding corners and edges, which would therefore be more rapidly attacked.

The surface of the semiconductor was found to be electro-polished after this shaping treatment, which is to say that surface irregularities had been reduced to dimensions well below one micron. Electrochemical polishing is generally associated with the presence of a coherent oxide layer on the surface of the work. Such layers are observable under certain conditions and have been observed and studied in the case of the semiconductors germanium (31, p. 1262-1269) and silicon (58, p. 230-236). It is felt that the presence of such a layer is necessary to effect the non-preferential removal of atoms from a surface, which is essential in a polish attack. In order to explain electrochemical removal of material from a surface while maintaining polishing conditions, it is necessary to postulate the movement of ions through the covering oxide layer and also the
dissolution of the oxide at the oxide-solution interface. In the present case the literature supports the view that under polishing conditions germanium is covered with a coherent layer of GeO₂ and silicon with a layer of SiO₂, both of which are only sparingly soluble in concentrated HNO₃ but quite soluble in a solution which is a mixture of HNO₃ and HF. The increased solubility in this case is probably a result of the surface complexing action of the fluoride ion. This explanation would indicate that in the steady state polishing condition there is an oxide layer present on the surface of the work and material is being transported through the layer under the influence of the applied field to the oxide-solution interface to form oxide at the same rate that the oxide is being dissolved. It is believed that the oxide thickness is dependent on the voltage across the polishing cell and a certain thickness must be maintained to obtain polishing (58, p. 230-236).

In HNO₃-HF solutions the HNO₃ is thought to act as the oxidizing agent which reacts to form and maintain a coherent layer of dioxide on the surface of the semiconductor. It is reported that the dioxide GeO₂ dissolves in basic but is insoluble in acid electrolytes in the absence of a complexing agent such as the fluoride ion (38, p. 268). The effectiveness of the fluoride ion in such
dissolution processes has been related to a complexing tendency which results from unshared electron pairs and a relatively small ionic radius with consequent high charge density (52, p. 1531). According to Clay and Thomas, in their work on the dissolution of hydrous aluminum oxide (16, p. 2384-2390), the fluoride ion is thought to attach itself to a metal atom in the oxide lattice by means of a coordinate link involving an unshared electron pair in the fluorine. This high negative charge density of the metal-fluorine bond shifts the electrons of the metal-to-oxygen bond closer to the oxygen. The metal-to-oxygen bond is thus weakened, the oxygen becoming more negative and in consequence having increased affinity for protons over that of the fluorine atom whose proton affinity is weakened by its linkage to the metal. In this situation protons could then be transferred from the solution to the oxygen to form water. The water so formed could remain with the metal as a hydrated complex and hence further contribute to the solubility.

The opposite of the non-preferential polishing attack, that is, preferential removal of atoms from a surface according to lattice free energy considerations, is commonly referred to as etching. Etching exposes surface grain boundaries and other surface imperfections; high energy crystal planes (i.e., planes in which
the surface atoms are in shallow potential wells) are preferentially attacked; and the resulting surface is not micropolished. This type of attack is also possible electrolytically, e.g., in the case of tungsten a certain critical voltage must be applied in a given solution to achieve polishing; below this critical voltage etching is obtained.

This information is important in two respects: a point which is both micropolished and symmetrical is desired for field emission studies. Etch attack would be detrimental to the attainment of either of these desired features. Failure to maintain polishing conditions at the very tip of the emitter during the final shaping process could lead to (1) roughness and (2) non-symmetry due to the anisotropy of the crystal. The difficulty involved in obtaining the desired conditions is believed to be associated with the above mentioned difference in current distribution along an immersed blank due to the high resistivity of the semiconductor.

After the shaping process the points were rinsed consecutively in distilled water and methanol. According to the literature the surface of the emitter will be covered with a layer a few angstroms thick, the composition and nature of which will vary with the solutions with which it has come into contact (31, p. 1262-1269; 7).
Another shaping technique applicable to the fabrication of germanium emitters has been found in which the shaping and polishing of the emitter is performed with an electrolyte consisting of four parts by volume $H_3PO_3$ (phosphorous acid) and one part by volume of $HNO_3$. An immediate advantage of this electrolyte is in the complete elimination of the use of fluoride solutions which is desirable from the standpoint of laboratory safety. Figure 21-b shows the same three germanium tips as in Figure 21-a after they had been tailored by very briefly dipping the tips into a droplet of $HNO_3 - H_3PO_4$ (phosphoric acid) solution (This is not quite as effective as the $HNO_3 - H_3PO_3$ solution.) held in a small platinum loop. The tailoring voltage was about 35 volts with the germanium positive.

The use of $HNO_3 - H_3PO_3$ acid mixtures as a shaping medium for germanium was suggested by the work of Clay and Thomas (16, p. 2384-2390), who found that the rate of dissolution of hydrous alumina in various acids depended on the nature of the anions of the acids. They decided that the rate determining factor in the dissolution was chemical in nature and involved the formation of a complex at the solid-liquid interface. The acids found most effective in accelerating the dissolution process were HF, $H_3PO_4$, and
$\text{H}_3\text{PO}_3$. They concluded that if the anion can form coordinate covalent bonds with the metal, even small amounts of it in acid solution would catalyze the dissolution process.

Unfortunately the $\text{HNO}_3 - \text{H}_3\text{PO}_3$ solutions were not effective in the case of silicon. The most effective electrolyte found for shaping and polishing silicon field emitter tips was a solution of about three parts by volume concentrated $\text{HNO}_3$ and one part by volume concentrated $\text{HF}$. The potential difference maintained between the silicon which is made anodic and the platinum cathode ranged from about 75 volts to as much as 150 volts. These voltages and solutions depended to a great extent on both the length and resistivity of the silicon blank. The electropolishing process discussed above is believed to apply to the case of silicon as well as germanium.

Finally, the process is also quite dependent on the skill and judgment of the technician. If the technician has not worked with these materials for a period of several weeks, he usually must practice on several blanks before he succeeds in fabricating a usable field emitter tip of germanium or silicon.
2. Tube Evacuation Process

Previous experience with metallic field emitters has shown that a high vacuum environment is essential for reliable field emission. In general, pressures higher than \(10^{-8}\) mm Hg cannot be tolerated, while pressures lower than \(10^{-10}\) mm Hg are very desirable. Experience gained with silicon field emitters during this study indicates that the silicon tips can be operated satisfactorily at \(10^{-9}\) mm Hg or less for extended periods, but only briefly up to about \(10^{-7}\) mm Hg.

The evacuation process must not only remove as much of the air as possible from inside the tube, but also the gases which have been adsorbed onto the glass and metal surfaces inside the tube. In addition, some gases diffuse into some of the materials used in the tube construction and these too must be driven out. If the high vacuum part of the vacuum system, i.e., everything beyond the diffusion pump, is not baked out during the evacuation process, these trapped gases will be slowly liberated causing a "virtual leak" and the pressure cannot be reduced below about \(10^{-7}\) mm Hg in a reasonable length of time. For this reason, the evacuation process must include several bakeout and outgassing cycles. The bakeout
drives the trapped gases out of the phosphor screen and glass envelope while outgassing the metal parts drives the gases out of them. In each case some of the liberated gases are exhausted by the pumps while the rest are readsorbed and must be released again by another bakeout and outgassing cycle. This process must be repeated until the desired vacuum is obtained.

A typical vacuum run followed in this study, particularly with the EE-2 glass tubes, would not be started until the tube to be evacuated had already been evacuated to a pressure less than $10^{-9}$ mm Hg during which it had been baked out at least once at a temperature over 450°C. This insured that the insides of the tube and the vacuum system were clean. Then as soon as the silicon tip had been fabricated, the diffusion pump was turned off. As soon as the diffusion pump cooled off, nitrogen gas was slowly passed through two auxiliary cold traps, a container of a desiccant, and into the vacuum system through the valve in the diffusion pump foreline. All of the cold traps on the vacuum system as well as the two auxiliary cold traps were immersed in liquid nitrogen. This elaborate procedure was deemed necessary to prevent water vapor, oil vapors, carbon dioxide, and other contaminants from reaching the high vacuum part of the system. There was evidence that even some
oxygen could be trapped out. As soon as the pressure in the system reached atmospheric pressure, the tube was cracked off the system, mounted on a glass lathe, and opened. The forming gas to the glass lathe was passed through liquid nitrogen to remove water vapor. The two wire press seal and clamps were cleaned by a phosphoric acid etch, distilled water rinse, and a final rinse in alcohol (to remove the water). The silicon tip was then inserted into the clamps, the press seal sealed back into the tube, the tube back onto the system, and the pumps turned back on as quickly as possible. In the meantime, the flow of dried nitrogen through the system had been reversed, i.e., it was admitted to the system through the opening where the tube had been cracked off in an effort to force any contaminants which had entered the system to move to the fore side of the diffusion pump. In addition, the second EE-2 glass tube had a provision for allowing the dried nitrogen to enter above the point at which the seal on was made so that any contaminants which gained admittance during the seal on would be swept away from the tube.

After the diffusion pump had been pumping long enough to reduce the pressure to less than $10^{-5}$ mm Hg (usually within an hour or two), the main oven was lowered onto the system and the bakeout started at a temperature of 200°C. Shortly after the main
oven reached 200°C, the liquid nitrogen was removed from the high trap and the gas trap. As soon as these traps reached room temperature, the small ovens were closed and set to bake at 175°C. The first bakeout was usually about 18 hours long, i.e., a bakeout overnight. As soon as the trap ovens had cooled to about 150°C, they were opened, and the traps were allowed to cool to near room temperature. Then the traps were immersed in liquid nitrogen. An hour or two later the main oven was cooled to less than 150°C, then lifted from the system, and the ionization gauge was turned on. The getter, ion gauge, and all other metal parts except the clamp supports for the emitter were then outgassed by resistive heating. The emitter and its clamp supports were not outgassed until the last cycle in order to keep the total heating time of the emitter to a minimum. This was permissible since the clamps had previously been outgassed in an auxiliary vacuum system. After a half hour or so of outgassing, another cycle was started with another bakeout, usually with the ovens at the same temperatures given above. Occasionally the main oven was baked at 300°C and the trap ovens at 275°C, but higher temperatures were avoided since some earlier experiences hinted that bakeouts at the higher temperatures may cause the contaminants to become more strongly bound to the
semiconductor tip. Other workers (40, p. 1546-1554) have found that heat treating silicon samples at 450°C causes considerable changes in their electrical properties. A pressure of less than $10^{-9}$ mm Hg was usually obtained after only three cycles, two of which included overnight bakeouts.

The high trap was found in most of the vacuum runs to be the most difficult part of the vacuum system to clean up, presumably because of the low temperature bakeouts. Generally a pressure of about $10^{-7}$ mm Hg was obtained after the first bakeout. At first leaks in the tubes were suspected, but usually none were found. Later it was found that if the ground glass ball and socket valve were closed (thereby isolating the tubes and ionization gauge from the high trap and diffusion pump), the gauge would usually reduce the pressure about an order of magnitude by ion pumping. The pressure immediately jumped back to its former value when the valve was opened again, thus indicating that the "virtual leak" was in the high trap even when it was cooled by liquid nitrogen. Another bakeout was started immediately. This test saved a considerable amount of time which would otherwise have been spent on unnecessary leak chasing.
Cleaning of the field emitter surface is another requirement for any thorough study of field emission. In this investigation the problem was very difficult because some of the oxides of germanium and silicon melt at temperatures higher than the melting points of these two elements, thus precluding the simple high temperature vacuum heating method of cleaning used for field emission tips made of refractory materials such as tungsten. It was during this phase of the investigation that the emphasis was shifted from germanium to silicon due to the greater promise shown by the latter.

The field emission projection microscope is a very powerful tool for the study of the surface of a material since magnifications of the order of $10^5$ times of any directional variations of the current emitted from the surface can be observed on the phosphor screen of the tube when a large positive potential is applied to the screen. These directional variations of the emitted current can be caused by variations of the physical or electrical properties with crystallographic direction of the clean field emitter tip or they can be caused by a layer of adsorbate on the surface of the tip; the
former is a regular or symmetrical variation corresponding to the crystal structure while the latter is usually a random variation. The field emission pattern from a clean silicon tip is shown in Figure 23 and from a contaminated tip in Figure 24.

Several techniques were investigated in the attempt to clean the semiconductor field emitter surface during and following the evacuation process. These included high temperature heating in vacuum, in hydrogen, and even in chlorine, electron bombardment, bombardment of the tip surface by argon ions, the application of a strong electric field while heating the tip to moderately high temperatures, and field desorption (48, p. 618-624), i.e., the process in which surface atoms are pulled off and evaporated upon application of a high electric field of polarity opposite that required for electron emission. Repeated high temperature flash heating in vacuum has proved to be the most successful method for cleaning silicon field emitter tips. The emitter tip is heated by conduction from the base of the emitter, which, in turn, is heated resistively. The conical emitter shank must have the proper angle for good results, since too small an angle causes inadequate heat conduction, while too large a cone angle results in an undesirably large operating
Figure 23. A field emission pattern from the clean silicon tip Si1L43 with the features corresponding to the major crystallographic planes identified.

Figure 24. A field emission pattern from a contaminated silicon tip.
voltage through excessive "dulling" of the emitter tip (i.e., increase in tip radius caused by surface migration) during processing.

The design of the field cathode and its supporting structure was revised a number of times during the effort to obtain clean field emitter surfaces consistently. The first structure, which was a simple two-wire clip as shown in Figure 19, too often yielded unsatisfactory mechanical, electrical and thermal contact between the semiconductor and the wire supports. In order to observe the emitters in the electron microscope, the over-all diameter of the sample and, if possible, of its support structure had to be limited to less than 1/8 inch. With this requirement in mind, a technique for fusing a semiconductor blank onto a small etched down tungsten wire filament loop was developed (see Figure 25). The shift of emphasis from germanium to silicon took place during the development and testing of this type of mounting. The wire filament was lowered onto the end of the semiconductor sample and heated to well above the melting point of the semiconductor in a vacuum of a few microns. As soon as the end of the sample melted and flowed around the wire, the heating current was shut off, thus permitting the sample to solidify around the wire. Better results were obtained
Figure 25. Illustration of emitter blank mounted by flash heating technique. (A) Tungsten filament loop and a germanium blank mounted on a similar filament, (B) reduced germanium blank on filament, (C) and (D) light micrographs of point after shaping.
for silicon bars when the ends to be mounted were first reduced electrolytically to about 10 mils in diameter. Flash heating of the wire also improved the results since a smaller portion of the sample was heated significantly. The mounting apparatus is shown in Figure 26 and was designed to fit an available vacuum system. This method proved very convenient and easy to use, and gave good control of the emitter temperature at the beginning of the experimental tests. However, the fusing process disturbed the crystal structure of the semiconductor near the boundary of the fused region and probably introduced dislocations and fracture zones. Subsequent heating and cooling of the emitter during the experimental tests aggravated this condition and led to the loss of the emitter needle under the action of the high electrostatic field forces. To eliminate this difficulty, a clamp type holder as shown in Figure 27 was designed. With this unit the emitter may be effectively heated by passing a current through the emitter material itself since its resistivity at room temperature is much greater than that of the clamp material. Very uniform heating of the emitter was obtained since the heat originated in the semiconductor itself. A new difficulty introduced with the use of this clamp arose from the non-uniform and negative thermal coefficient of resistance encountered in most semiconductor
Figure 26. Device for mounting germanium and silicon samples on a "hairpin" filament in vacuum. The two small flasks contain samples that have just been mounted.
Figure 27. (A) Silicon field emitter mounted in clamp holder. Its legs are insulated from each other to permit heating by passing current through the emitter material. (B) The emitter and holder are inserted into a grid structure used for ion bombardment.
materials, which made it difficult to control accurately the temperature of the emitter during experimental tests. This was overcome by the insertion of a high wattage but low resistance resistor into the heating circuit. This clamp emitter holder was fairly successful since several field emission patterns and some data were obtained with it. The most serious attempts to adapt the argon ion bombardment cleaning method of Farnsworth and coworkers (33, p. 1150-1161; 19, p. 1197) were made with this clamp holder (see Figure 27). The argon bombardment may have helped clean some of the silicon tips, but it was not sufficient by itself and also some were cleaned without its use. Another emitter holder (Figure 28) was developed concurrently with the clamp in an effort to avoid the bulk as well as the temperature control difficulty associated with the clamp. A thread was ground on the shank of the emitter needle and this was screwed into a helically wound support filament. The unit was heated by resistive heating of the filament wire which was etched to a smaller diameter in the helical region than the support legs in order to concentrate the heating near the emitter. Very good control of the emitter temperature was obtained in some cases and good data were obtained from a tip with this type of support. Unfortunately, however, the mechanical and thermal
Figure 28. A helical filament mount for silicon field emitters.

Figure 29. A shadow photograph of the yoke type emitter structure of silicon. The total width is 100 mils, thickness 40 mils, and length less than 300 mils.
contact between the helix and emitter was not always reliable, apparently due in part to the formation at high temperatures of a lower melting eutectic at the points of contact, which caused in some cases a break in the helix.

All of the emitter assemblies described above were of such a design that the metal-semiconductor contact was the hottest region of the semiconductor during the heating process. Becker (9) suggested that perhaps this very hot contact was the source of much of the contamination which was observed to arrive at the tip during the heating with field cleaning process. An emitter assembly described by D'Asaro (18) and used by Allen (5) had the advantage that the metal-semiconductor contact would be considerably cooler than the region near the emitter tip, but it had previously been avoided because it was much too large to fit into the electron microscope and because it was very difficult to machine this structure without breaking it. However, machining techniques (using rubber bonded cut-off wheels (App. B, 53)) and modifications were successfully made so that this yoke or bridge type structure as shown in Figure 29 could be made, the tip fabricated and tested, and then inserted into the electron microscope. Two shapes have been used, i.e., for one the maximum dimensions were 0.100" x 0.040"
x 0.300" while for the other they were about 0.060" x 0.060" x 0.300". It has been found that symmetrical tips were more easily obtained from the samples with the square cross-section. The hottest region in this structure during the heating process was in the yoke or bridge region which was thinned down electrolytically when the tip was formed by electropolishing. This places the source of heat near the tip, thus allowing it to reach a higher temperature than could be reached previously. Good temperature control could be maintained by using a resistor of the proper specifications in series in the heating circuit as mentioned above. This semiconductor bridge structure was mounted in thin molybdenum spring clamps, which not only permitted easy mounting and dismounting, but also dissipated heat rapidly enough to keep the metal-semiconductor contact relatively cool. These features all contributed to the successful use of the flash heating technique for cleaning the silicon field emitter tip. It should be re-emphasized that the cone angle of the emitter shank was very important since too wide an angle would have permitted excessive dulling of the tip while too narrow an angle would have prevented the tip from reaching a high enough temperature.
A few silicon field emitters were successfully cleaned by applying an electric field to the tip while it was heated to temperatures of about 1100\(^\circ\)K to 1400\(^\circ\)K. Generally the field was strong enough for a field emission pattern to be observed and the temperature of the tip was carefully controlled so that the spots of pattern caused by contaminants on the tip would suddenly disappear one by one, thus indicating that that contaminant had suddenly been pulled off the tip. Unfortunately the final result was usually either that the end of the tip was pulled off, leaving a tip too blunt for field emission at reasonable voltages, or that the tip was so misshapen that reliable field calculations would be next to impossible.

However, four of the silicon tips which yielded data that were successfully reduced to electric fields and current densities were cleaned by this method.

The flash heating technique for cleaning the silicon tip consists of flashing a region (which is large enough to be observed with a micro-optical pyrometer) near the tip to at least 1550\(^\circ\)K and, if possible, to above 1625\(^\circ\)K for one to three seconds. The sample is allowed to cool for four to ten seconds after the flash and then the field emission pattern may be observed to determine the progress of the cleaning before the next flash. The maximum temperature at the
tip is limited by the length and cone angle of the emitter shank and by the melting point (1688°C) of the silicon itself at the thinnest (and therefore hottest) region of the yoke. At some time during the cleaning process, the hottest region of the yoke has usually reached the melting temperature, allowing a small spot to melt and solidify again. Since silicon is one of the few substances which expand on freezing, a small bump or projection is formed such as the one on the inside of the yoke almost directly behind the tip in Figure 29. The usual number of flashes required to clean the tip has varied from about 200 flashes to about 2000 flashes. However, one tip was cleaned in only 24 flashes while another was not cleaned after 10,000 flashes. The flashing technique appears to have reduced the migration effects resulting in dulling the emitter tip while the repeated thermal shock associated with the flashing seems to aid the cleaning process.

Occasionally data were taken from a tip which still had a spot or two of contamination contributing to the emission, but in these cases the contribution appeared to be negligible when compared to the total current and field emission pattern. This alternative was chosen over that of further dulling the tip and risking its total loss.
4. Marbles for Atoms Model of Silicon Field Emitter

Each material that has been used as a field emitter has a characteristic pattern when observed with a field emission projection tube. This pattern, a familiar feature in field emission research, corresponds to the crystal structure of the emitter material and its state of cleanliness. With metal emitters it is invariably observed that the most atomically dense or smoothest crystal planes emit the fewest electrons, while the more irregular surfaces emit profusely, thus producing detail, i.e., bright and dark regions in the pattern. This detail is due to the two facts that, first, some surfaces are rougher than others, thus causing an enhancement of the electric field, and second, the work function varies with crystallographic direction. The latter is probably the predominant factor in producing pattern contrast. The work function is composed of two component parts including a contribution from the bulk material which is the same in all crystal directions and a contribution from a dipole layer which depends in part on the atomic structure of the surface. It can be shown that a rough surface has less dipole moment and therefore a lower work function. Therefore a rough surface is expected to emit more electrons since it has both a higher local electric field
and a lower work function. Conversely, a field emission pattern from a clean surface yields information on the work function of the various crystallographic directions. In addition, a pattern from a contaminated surface supplies information about the contaminant.

In order to interpret the field emission pattern from silicon, the crystal detail of silicon must be examined. A "marbles for atoms" model of a field emitter tip with the diamond crystal structure, which is the structure of silicon crystals, has been built. This model was designed to have a radius of 20 unit cells, which corresponds to a silicon tip with a radius of $1.08 \times 10^{-6}$ cm, or about 1/50 the actual radius of the tip commonly used in the field emission microscope tube. It was limited to only 1/4 of a hemisphere instead of a complete hemisphere because of size and weight considerations; all sets of major planes, however, were represented. The model was built "inside out" on a framework designed to fit the outside surface of the model. When finished, a backing made of liquid metal (App. B, 54), plastic wood (App. B, 55), and plywood was built onto it and then it was lifted out of the framework. This method provided a more precise determination of the locations of the marbles on the surface. The open structure of the tetrahedral bond did not permit simple stacking of marbles to make
the model. Instead, "chains of marbles" first had to be made and these were then stacked. Approximately 8000 marbles were glued together with Duco cement.

The marbles representing the surface atoms are in the same positions they would have occupied if the surface had not interrupted the volume where it did, i.e., they are in their normal volume equilibrium positions. As a result some of the marbles, particularly on the \( \{100\} \) planes, have only two filled bonds to neighboring marbles, leaving two unfilled bonds. Since this is an unstable condition, these marbles should be displaced in such a way that a third bond is filled. Very low energy electron diffraction studies (33, p. 1155-1161) indicate that a displacement does occur for some of the surface atoms of germanium and silicon. No attempt was made to displace any of the marbles from their normal volume equilibrium positions.

Two views of the model are shown in Figure 30. The open network-like structure of the tetrahedral bond (similar to the carbon bond in diamond) is very apparent and may account for some of the difficulty in obtaining a clean emitter surface, since some of the contaminants may easily diffuse into the bulk of the material. Also the relatively rough surface may account for a tighter binding of
Figure 30. Two views of a marble model of a quarter section of a field emission tip with the diamond crystal structure.
contaminants than is the case for other materials such as tungsten.
Inspection of the model shows the smoothest and most extended
crystal planes to be the \{111\} and \{100\} planes. Next in order are
the \{110\} and the \{211\} or \{311\} planes. One would expect these
to show up as dark areas in the emission pattern. This is borne
out in the pattern in Figure 23 with the \{100\} and \{111\} planes
being most prominent and the others well defined.

5. Determination of Tip Characteristics

The size and shape of a field emitter must be determined as
accurately as possible so that the current and voltage data may be
reduced to the more fundamental quantities of current density and
electric field. Those silicon tips which yielded good current and
voltage data along with good patterns were carefully removed from
the experimental tubes and inserted into the electron microscope.
The shadow profile, magnified about 3000 times, of each of these
tips was observed closely while the tip was rotated at least 180°
about its axis. Then micrographs, i.e., exposures on Kodak 2" x 2"
glass lantern slide medium plates, were made of from three to five
profiles including the narrowest and widest profiles for each of these
tips and the focus settings of the electron microscope were recorded for each micrograph. The electron microscope was calibrated by making micrographs of a carbon grating replica (App. B, 56) with 28,800 lines per inch over the range of focus settings. The ratio of the spacing between lines on the micrograph to the spacing between lines on the replica gives the magnification for the focus setting at which each micrograph was taken.

The radius of the profile tip on each micrograph was measured with a small pocket comparator (App. B, 57) using either a "diameters" reticle or a "radii" reticle, depending on the size of the tip. This radius was then divided by the magnification corresponding to the focus setting at which the micrograph was taken to find the actual radius of the tip. A radius was calculated for each micrograph profile of a tip and if the tip was fairly symmetrical these profile radii were averaged to obtain an average tip radius, \( r \). If the tip was not fairly symmetrical, the radii were not averaged, but the radius for each profile was used to find the tip characteristics discussed below corresponding to that profile and then these were averaged.

The average current density at a particular voltage was obtained by dividing the current emitted at that voltage by the emitting
area of the tip. An estimate of the emitted area was made by assuming that it consisted of a zone of a sphere in which one of the two parallel planes intersecting the sphere to define the zone is just tangent to the very end of the tip. In this case, with the area $A = 2\pi rh$, $h$ is found by the relationship $h = r - r \cos \omega = r (1 - \cos \omega)$, where $\omega$ is the angle between the axis of the tip and the effective edge of the emitting region (see Figure 31-a). The location of the edge of the emitting area, and therefore the value of $\omega$, was obtained by drawing a circle on a photograph of the emission pattern concentric with the center of the pattern. The circle was drawn so that it included the major portion of the pattern and excluded some of the fringes of the pattern, with allowances made for the non-emitting areas inside the circle as shown in Figure 31-b. The value of $\omega$ was estimated from the circle by obtaining the angle from the axis to one of the crystallographic directions of the pattern that the circle crossed. The angles between the major crystallographic directions in the cubic system have been tabulated (11, p. 393-394). Although the uncertainty in the value of emitting area is appreciable, its effect is small because the logarithm of the current density is the value plotted. Furthermore, the values obtained for the emitting area of
Figure 31. A sketch (a) and pattern (b) from the tip Si1L41 used in the determination of the emitting area of this tip.
the silicon tips were approximately those expected on the basis of past experience with tungsten tips.

The electric field at the surface of the tip at a particular voltage was obtained by multiplying the voltage applied across the tube by the geometrical factor, $\beta$, which is defined as the ratio of the electric field, $F$, to the applied voltage, $V$, or $F = \beta V$. The difficult problem is, of course, the determination of $\beta$. Since the electric field is concentrated at the tip, the shape of the tip is far more important than the shape of the anode in the determination of $\beta$. A number of methods for approximating the $\beta$ factor have been devised and these have been reviewed briefly by Dyke and Dolan (25, p. 98-102). The most accurate method for obtaining the $\beta$ factor is to use the equipotential surfaces surrounding a charged isolated sphere-on-orthogonal-cone to approximate the emitter shape. Using this model, Dyke and co-workers (26, p. 573-576) calculated expressions for $\beta$ which require the evaluation of some Legendre polynomials. Values of some of these polynomials were laboriously obtained from a series which in some cases was carried to 120 terms (26, p. 576).

Dolan (21) has extended these calculations in some unpublished work by drawing sketches of several equipotential surfaces for the
sphere-on-orthogonal-cone model which correspond to some of the typical field emitter shapes. A set of values was calculated for each of these surfaces which can be used with the actual tip radius and cathode to anode spacing in two equations to calculate $\beta$ for the tip. The simplest way to fit the emitter shape to one of these surfaces is to project the micrograph with a slide projector, focusing on the sketches. Then choose the sketch (or sketches) which best matches the projected tip shape and use the values for that sketch to calculate $\beta$. This greatly simplifies the calculation for $\beta$ using the sphere-on-orthogonal-cone model. The equations, calculated values, and the sketches of equipotential surfaces are given in Appendix C.
EXPERIMENTAL RESULTS

The first experimental result obtained during this investigation from a field emission microscope tube containing a silicon tip was also one of the most unusual results. This was a field emission pattern from a fairly clean silicon tip with a grain boundary intersecting the surface of the tip. Since this tip was mounted in a two wire clip similar to the one shown in Figure 19, it could not be heated adequately by heating the clip resistively because of poor thermal contact so an attempt was made to clean it by electron bombardment. During the bombardment the emitter temperature was monitored with the optical pyrometer. The temperature suddenly increased and the emitter started to melt before the bombarding power could be turned off. Immediately after the bombardment, voltage was applied to the tube by a Van de Graaff generator (App. B, 58) which can supply a maximum current of about 10 microamperes, thus providing a good current limited power supply which helps to protect the emitters from vacuum arcs. The field emission pattern was observed on one side of the screen with some spots of contamination at the center of the screen. A photograph (which was overexposed) of this pattern is shown in Figure 32. The pattern suddenly disappeared after a few minutes, presumably
Figure 32. Field emission pattern from the silicon tip XSi1G3 with a grain boundary trace.

Figure 33. Orthographic projection for the tip yielding the pattern above. Several crystallographic directions are identified along the dashed line which corresponds to the trace when the right side of the pattern is fitted to the projection.
because of a vacuum arc or fracture of the emitter by the high
electrostatic forces caused by the field, and did not return with
further heat treatment. Many further attempts were made to clean
both silicon and germanium tips by this method, but this was the only
tip cleaned by electron bombardment, apparently by accident. The
grain boundary pattern was discussed briefly in a paper (54) read at
a professional meeting. In the discussion following the paper,
Dr. William Shockley made the comment that the probability of
obtaining a grain boundary pattern was about one in $10^4$ when the
size of the tip and the usual density of grain boundaries in silicon
are taken into consideration.

Analysis of the grain boundary pattern has shown that the
relative orientation of the two grains was about eight degrees and
that the boundary trace corresponded to the intersection of a $(12\bar{1})$
plane with the emitting surface. The analysis was accomplished by
making an orthographic projection of the crystal planes and then
rotating and reducing its size to match the pattern. Several
crystallographic directions along the boundary trace were identified,
differences were taken between these directions in order to obtain
vectors in the boundary plane, and finally cross products of pairs of
these vectors were taken to obtain the direction normal to the plane,
which identified the plane. Figure 33 shows the results of this analysis.

Surface migration was one of the first phenomena observed when clean field emission projection microscope patterns were obtained from silicon tips since heating with the field applied was the first successful cleaning method used with the exception of the one tip discussed above. When no electric field is applied at the surface of a field emitter which is heated to high temperatures, surface migration is caused by surface tension forces which produce a rounding and smoothing of the emitter tip. The surface atoms tend to move from regions of high chemical potential to regions of lower chemical potential, i.e., from regions of large curvature to regions of small curvature. This tendency leads to subsequent dulling of the tip by a transport of material from the apex toward the shank. Surface tension forces still predominate at low values of the applied field, causing the heated emitter tip to recede and dull with time.

When a sufficiently high electric field is applied, the electrostatic field forces predominate and reverse the direction of the migration. By analogy with the low field case one might expect the heated emitter to grow in length, but simple extension does not occur because of the very low probability of nucleating new atom
layers in certain crystallographic plane orientations corresponding to relatively smooth surfaces. This low probability inhibits crystal growth in the corresponding directions, and as a result the heated field emitter does not remain smoothly rounded when subjected to high electrostatic fields, but builds up into a polyhedral shape in which the tip surface eventually consists of large plane facets separated by sharp edges and corners. A recent publication (10, p. 85-93) discusses this phenomenon in more detail and includes further references on the subject.

Surface migration and build-up will occur when a silicon tip is heated to about 1000°K or above while the electric field is applied strongly enough to observe the field emission pattern. A series of patterns illustrating the migration and build-up process are shown in Figure 34. Note that the initial changes seem to occur on the edges of the planes, while the final result is the filling in or build-up of the (110) planes into ridges. Care must be taken to reduce the applied voltage as the (110) planes build up since the field emitted current at a constant voltage increases considerably due to an enhancement of the electric field at the built-up ridges. This behavior is illustrated in the plot shown in Figure 35. Figure 36 shows enlarged micrographs of two built-up silicon tips.
Figure 34. A sequence of patterns from silicon tip Si1L24 showing "migration" and "build-up" which occur when both temperature and field are high. Initial changes occur on the edges of the planes; the final result is the build-up of the \{110\} planes into ridges.
Figure 35. Plots of data and corresponding emission patterns at different stages of build-up.
Figure 36. Enlarged micrographs of two built-up silicon tips: (A) Si1L36 and (B) Si1L40. Note the large steps between the planes.
Two silicon field emitter tips were successfully operated in such a way that just one (110) plane was built up into a single ridge on each tip. The results obtained from one of these tips are shown in Figures 37 and 38. Figure 37 shows both the original smooth pattern and the built-up pattern and Figure 38 shows a plot of $\log_{10} I$ versus $10^4/V$ for the built up case as well as electron micrographs of the built-up tip. The micrographs show that the end of the tip had several large planes or facets and the region of emission was probably a ridge between two of these. This tip was cleaned suddenly by heating with the electric field applied, which probably pulled off the end of the tip leaving the small emitting projection. The maximum current obtained from this built-up tip was 16.5 microamperes and the pattern showed that practically all of it was from the one built-up region, thus giving a beam with fairly narrow cone angle, which would greatly decrease the difficulty of refocusing the beam into a small spot and would therefore be advantageous in electron optical applications.

The field emission characteristics of several silicon and germanium cold cathodes have been studied with the field emission projection microscope. Two distinct forms, illustrated in Figures 39 and 40 and designated below as "type 1" and "type 2"
Figure 37. Field emission patterns from silicon tip Si1L34 which yielded the data shown in Figure 38. (A) Pattern corresponding to a smooth tip, and (B) pattern corresponding to the built-up tip. Note that nearly all of the current (16-1/2 microamperes) is in the narrow beam.
Figure 38. A plot of dc data taken from the silicon tip Si1L34 when in the built-up form. Two profiles of the tip are shown in the inset.
Figure 39. Typical "non-linear" data, designated as "type 1".
Figure 40. Typical "linear" data, designated as "type 2".
respectively have been found for the relationship between the field emitted current, I, and the applied voltage, V.

Figure 39 shows a type 1 relationship, plotted in the form \( \log_{10} I \) versus \( 10^4/V \), for a silicon emitter. In the case of pulsed emission the data points represent peak values of the emitted current and applied voltage; the pulses were usually 30, 60 or 80 microseconds in duration and were repeated at the rate of 60 pulses per second. The dc relationship for type 1 emission differs from the simple straight line characteristic of metals, and instead exhibits two well defined bends separating straight sections, with the slope of the center section being smaller than that of the other two. This behavior conforms at least qualitatively with that predicted by Stratton (62, p. 746-757). Similar data had been obtained previously from several germanium field emitters, both at this laboratory in the early part of the investigation and by Elinson and Vasiliev (28); however in both cases observation of the emission patterns showed contamination of the emitting surface, so that the behavior could not be attributed exclusively to the characteristic of the germanium. Since then, similar dc relationships have been obtained from twelve different silicon field emitters, for which concurrent observation of the emission pattern displayed on the screen of the projection
microscope indicated a clean (2, p. 1568-1570) or very nearly clean condition of the emitting surface. Four of these twelve tips were built-up while the remaining eight tips were smooth. In addition, a similar dc relationship was obtained from a whisker on the leg of a silicon field emitter. It was not possible to observe the pattern from the whisker. The end of this whisker glowed red hot when it emitted about 45 microamperes dc. As shown in Figure 39, the pulsed data obtained at higher emission densities fall on an extension of the straight line defined by the lower portion of the dc relationship.

A different behavior, "type 2", illustrated in Figure 40, was obtained from fourteen silicon field emitters with clean or nearly clean emitting surfaces. Four of these fourteen tips were built-up. For these emitters the characteristic does not show definite bends but is nearly straight with a slight upward curvature when plotted in the coordinates shown. This observed behavior of field emission from silicon has been reported elsewhere (18; 5) for a more restricted range of emitted currents. As shown in Figure 40, the ranges of the dc and pulsed data overlap and the plot of the pulsed data for type 2 behavior falls on a direct extension of the line defined by the dc data.
An attempt has been made to correlate the type of emission characteristic with the physical nature of the emitting material. It was first suggested (55) that type 1 and type 2 emission corresponded to n-type and p-type silicon respectively. However, more recent measurements have yielded instances where type 1 data were obtained from field emitters made of initially p-type silicon or where type 2 data were obtained from field emitters made of initially n-type silicon. Since then thermoelectric probe tests have been made on several field emitters following their operation in the field emission microscope. These tests indicate that, in cases where type 2 emission was obtained, very strongly p-type layers (5; 2, p. 1570; 3, p. 979) had formed on the silicon surface, thus causing the whole emitter tip to be strongly and perhaps even degenerately p-type, whereas the surface of a field emitter yielding type 1 emission, though still p-type, was much less strongly so.

This interpretation is also supported by the observed influence of the emitter cleaning procedure on the type of emission subsequently obtained. Most of the tips yielding type 2 emission were cleaned by the flash heating technique described earlier. In this case the silicon received a total of several minutes heating at 1600°K or above, thus giving surface contaminants sufficient
opportunity to diffuse into the silicon to form a strong p-layer. One contaminant is boron (3, p. 979-985), which is a constituent of the tube envelope and is likely to be deposited on the emitter surface during tube bake-out. In contrast to this, the first seven tips yielding type 1 data were cleaned by applying a high electric field while heating at a considerably lower temperature as discussed earlier. The field emission patterns and electron micrographs indicated that these tips were cleaned suddenly by stripping off a portion of the silicon surface or by removal of the end of the tip by the electric field, thus exposing relatively uncontaminated silicon which would not have such a strong p-layer. This evidence indicated that the silicon field emitters yielding type 2 data were degenerately p-type or nearly so at the tip, thus explaining the similarity of the emission characteristics to those obtained from metallic emitters, while the silicon field emitters yielding type 1 data were not degenerate at the tip and therefore, as discussed by Stratton (62, p. 746-757), the energy bands were free to bend when the high electric field was applied. A letter discussing these two types of data and the above interpretation has recently been published (53).

Further work, including the reduction of the measurable current and voltage to the more fundamental quantities of current
density and electric field, was undertaken, and all of this work lends additional support to the interpretation given above for the two types of data. A new "boron free" glass (EE-2, manufactured by the Kimble Glass Company) was used to make a field emission microscope tube with a conducting glass anode. Unfortunately it was necessary to seal this tube onto a Pyrex vacuum system, but in spite of this disadvantage five silicon field emitters contained therein yielded type I data following successful cleaning after considerable flash heat treatment. Three of these field emitters were made from 0.05 ohm-cm n-type silicon, one from 0.4 ohm-cm n-type silicon and one from 1-2 ohm-cm n-type silicon. Although these latter two yielded type I data, the deviations from a straight line were very small. In spite of the considerable precautions taken to exclude water vapor, which appears (3, p. 985) to be the carrier of the boron, a p-layer was found on all five of these silicon field emitters after their operation in this new tube. However, in all cases the layer was much thinner and apparently weaker than it had been for those tips operated in Pyrex tubes and in one case, Si1L41, n-type indications were obtained on about 50% of the surface of the sample. This tip which was smooth and one other which was built up yielded much larger deviations from straight line data than had previously
been observed as shown in Figure 41. The last tip, Si1L43, made of 0.4 ohm-cm silicon, operated in this EE-2 glass tube yielded type 1 data (Figure 42) which had only a slight deviation from a straight line, thus indicating that the large deviations observed earlier from other tips in the same tube were not malfunctions of the tube itself. Then four silicon field emitters were sealed into Pyrex tubes with no precaution taken to prevent boron doping of the silicon. Two of these tips were successfully cleaned and operated and both yielded type 2 data. Thermal probe tests showed that all four of these samples had thicker and apparently stronger p-layers. An attempt was made to operate a silicon tip in a second tube made of EE-2 glass, which had an aluminum backed phosphor screen for an anode, but a series of accidents permitted a strong p-layer to form and type 2 instead of type 1 data were obtained. Lack of time prevented another test.

A dependence of the magnitude of the field emitted current on the temperature of the tip was observed for several silicon tips. This was to be expected since raising the temperature of the emitter should make more electrons available for emission. One of these tips was operated in the built up form (to reduce to a negligible value the contribution to the total current of side crystallites of
Figure 41. A plot of "non-linear" data from a tip yielding indications of surface regions of n-type silicon after operation in a tube of EE-2 glass.
Figure 42. A plot of only slightly "non-linear" data from a later tip in the same tube of EE-2 glass, showing that the large deviation in the previous figure was not due to a defective tube.
contamination) while heated to a very faint red, i.e., to approximately 900°K. Then it was cooled to room temperature (~300°K) while a high field was still applied to prevent smoothing, and was then operated further. Figure 43 shows plots of the data for the two temperatures. Figure 44 shows field emission microscope patterns for the heated and cold tip, indicating that there was very little or no change in the geometrical surface conditions other than a very slight "thermal vibration". The enlarged micrograph of this same built-up tip shows in Figure 36-A that there are large steps between some of the planes. One of the three other silicon tips tested for this temperature dependence was a smooth tip which showed a tendency to start building up even at the low temperature when the tip was just faintly red hot (~900°K). No change in its smooth pattern was observed when it was heated at a much lower temperature and yet there was a very definite increase in the current emitted over that obtained when the tip was operated at room temperature. Figure 45 shows a chart recording of the current emitted from a third silicon tip (built-up) operated at a constant voltage of 20.0 kv, first with the tip cold, then hot, and then cold again. As in the cases above, no change could be seen in the pattern except for a very slight apparent vibration when the tip was warm.
Figure 43. Plot of data from a built up silicon field emitter which exhibited a slight dependence of the emitted current on the temperature of the tip.
Figure 44. Field emission microscope patterns of (A) the heated and (B) the cold built-up tip photographed at the data points indicated in Figure 43.
Figure 45. A chart recording of the variation of field emitted current with temperature while the voltage on the built-up silicon tip was held constant.
Seven silicon field emitters were successfully cleaned, smoothed, operated, removed from the tubes, and micrographed in the electron microscope during the last six months of the support contract. Three of these tips were very well shaped as shown in Figure 46, thus permitting good calculations of the emitter radius, r, the emitting area, A, and the geometrical factor, β. (Then the fundamental quantities of current density, J, and the electric field, $F$, can be obtained by the relationships $J = I/A$ and $F = \beta V$.)

Although the other four tips were not as symmetrical, they still were well enough shaped (see Figure 47) that fairly accurate determinations of r, A, and β could be made. Then relying on the experience gained with these seven tips, estimates and calculations were made on the two very non-symmetrical tips (see Figure 48) which had been micrographed previously, and the values obtained were in surprisingly close agreement with the other seven tips. As discussed earlier, the emitter profile radius was measured on the micrograph and then divided by the magnification factor to obtain the actual field emitter radius. The emitting area was obtained from the tip radius and an estimate of the effective half-angle included in the emission pattern. The value of β was determined from the tip radius and information obtained by matching a projection of the
Figure 46. Electron micrographs of three profiles of three very well shaped silicon field emitters yielding "non-linear" data. The field of view in each micrograph is about 15 microns.
Figure 47. Electron micrographs of four profiles of four non-symmetrical but fairly well shaped silicon tips. The field of view in each micrograph is about 15 microns.
Figure 48. Electron micrographs of four profiles of two very non-symmetrical silicon tips. The field of view in each micrograph is about 15 microns.
micrograph to sketches of profiles based on the sphere-on-
orthogonal-cone model discussed by Dyke and co-workers
(26, p. 573-576). No attempt was made to take into account such
factors as the variation of \( \beta \) with angle from the emitter apex, the
effect of the support structure on the value of \( \beta \), decrease of the
value of the field due to penetration, and submicroscopic surface
roughness due to possible slight faceting. It was assumed that these
would all be small effects and that the last one mentioned would tend
to cancel the others. These calculated values for \( A \) and \( \beta \) for even
the three best shaped tips may have an inaccuracy as high as 15 per-
cent.

A typical data run for each of these nine silicon tips is plotted
in the form \( \log_{10} J \) versus \( 10^8/F \) in Figure 49 and corresponding
patterns are shown in Figure 50. An interesting feature to be
observed in Figure 49 is the high field required for field emission
from silicon, covering the approximate range
\[
2 \times 10^7 \text{ v/cm} \leq F \leq 9 \times 10^7 \text{ v/cm}.
\]
In addition, the four plots on the
left (higher field region) in Figure 49 correspond to tips with the
best known geometries while the four plots on the right (lower field
region) correspond to tips with the more non-symmetrical or less
known geometries which, if better known, would probably shift the
Figure 49. A plot of current densities and electric fields for a typical data run from each of the silicon tips shown in the three preceding figures.
Figure 50. A typical field emission pattern from each of the tips with data plotted in Figure 49.
plots to the left into the higher field region. The current densities calculated for these nine tips all fell in the approximate range $10^{-3} \text{ amps/cm}^2 \leq J \leq 3 \times 10^3 \text{ amps/cm}^2$ for the dc case. The small amount of pulsed data obtained from three of these tips indicated that the pulsed current densities probably lie in the range $5 \times 10^2 \text{ amps/cm}^2 \leq J \leq 10^5 \text{ amps/cm}^2$ corresponding to applied electric fields in the range $4 \times 10^7 \text{ v/cm} \leq F \leq 9 \times 10^7 \text{ v/cm}$.

In general, tips made from metals such as tungsten, which has a work function of 4.5 ev, yield much higher current densities at these same values of the electric field.

The typical data runs included in Figure 49 for the three tips with the best known geometries are plotted again on a larger scale in Figures 51, 52, and 53.

A total of 163 tips were put into tubes, of which 36 were germanium and 127 were silicon. Of the 127 silicon tips tested, 27 yielded good data, and of these, nine smooth tips and four built-up tips were successfully micrographed after they yielded the data.
Figure 51. Plot of data from the silicon field emitter Si1L33 after transformation from current-voltage values to current density - electric field values.
Figure 52. Plot of data from the silicon field emitter Si1L41 after transformation from current-voltage values to current density - electric field values.
Figure 53. Plot of data from the silicon field emitter Si1L43 after transformation from current-voltage values to current density - electric field values.
Comparison of the experimental results with the available theory shows only limited agreement. Four of the plots in Figure 49 are from silicon tips which appear to be degenerately p-type and there is at present no theory which covers this case. The remaining five plots exhibit a type 1 relationship between the field emitted current density, \( J \), and the applied electric field, \( F \), and are similar to the central portion of the example (portion \( PQ \) of curve IV in Figure 5) given by Stratton to illustrate his theory. Accordingly, attempts have been made to fit this portion of Stratton's theory to these five plots.

As was discussed in the section on theory, Stratton derived three equations relating the field emitted current density to the electric field at the surface of a semiconductor. The plot of typical type 1 data from tip Si1L33 shown in Figure 51 should resemble the Curve IV of Figure 5. Equation (30) which corresponds to the low field, low current density portion of Curve IV below point \( P \) in Figure 5 should correspond to the lowest straight line of the plot in Figure 51. In this case Stratton predicted that both the internal potential barrier due to surface states and the familiar external potential barrier must be overcome by the applied electric field,
thus causing the slope of the plot of $\log_{10} J$ versus $1/F$ to be quite steep. When the internal barrier is reduced to zero by the increasing electric field, the slope of the plot should be less according to Equation (21) because then only the external potential barrier is being overcome by the field, and this should correspond to the middle straight line in Figure 51. At very high electric fields strong field penetration should occur, causing near the surface a degenerate condition giving the electron density a value appropriate to a metal. The emitted current density would therefore be very high and according to Equation (22) it would be several orders of magnitude higher than the maximum value shown in Figure 51. Stratton did not derive an equation corresponding to the highest straight line shown in Figure 51. Elinson (27), however, attempted to extend Stratton's theory to include this region by considering the influence of internal electric fields in a semiconductor on the field emission current density from that semiconductor. As mentioned in the introduction, his theory is not correct dimensionally, so it will not be considered further.

By combining the known constants, Equation (21) can be written into a "type form" which is easier to work with:
\[ J = n D \exp \left[ -G \frac{\theta^{3/2}}{F} f(y') \right] \]  

(31)

where \( D \) and \( G \) are constants, \( y' = \left( \frac{\epsilon - 1}{\epsilon + 1} \right)^{1/2} \frac{q \sqrt{F}}{\theta} \) and the other terms are the same as before. The density of electrons in the conduction band, \( n \), and the value of the potential energy of the bottom of the conduction band, \( \theta \), are unknown and must be determined from the data. The derivative with respect to \( 1/F \) of \( \log_{10} J \) was found to be

\[
\frac{d (\log_{10} J)}{d (1/F)} = - (\log_{10} e) G \frac{\theta^{3/2}}{F} \left[ f(y') - \frac{1}{2} y' \frac{df(y')}{dy'} \right]
\]

or

\[
\frac{d (\log_{10} J)}{d (1/F)} = - (\log_{10} e) G \frac{\theta^{3/2}}{F} s(y') \]

(32)

where \( s(y') \) has been tabulated (14; 37, p. 187). Since this derivative should be equal to the slope of the middle straight line in Figure 51, which can be measured, then the value of \( \theta \) for the tip Si1L33 is determined. Returning to Equation (31) and using values
for J and F taken from the midpoint of the middle straight line in Figure 51, a value for n, the electron density, can be obtained for this tip. These calculations were carried out for the other four tips yielding type I data shown in Figure 49. The correction for n discussed in the sentences following Equation (21) was not used in these calculations. Attempts to include it did not lead to reasonable values for n. Therefore the data fit more closely to Curve II than the line PQ in Figure 5.

Thermoelectric probe tests on four of these five samples showed that fairly strong p-layers had formed on the legs of three of them while the fourth, Si1L41, still had n-type regions on about 50% of the surface. It is presumed that all of the tips were p-type since the boron contamination was probably uniform over the surfaces while the volume enclosed within the tip was much smaller than that within the legs, thus allowing the boron to diffuse throughout the whole tip during the heating process. It is also presumed that this was the case for the first tip (Si1L13) micrographed although no thermoelectric probe test was made on it. Based on this information, estimates using the equation $p = n_1 \exp \left[ \left( E_i - E_F \right) / kT \right]$ were then made of the hole density, $p$, and the average work function, $\varphi$, for each of these tips except for the tip Si1L43. This tip yielded
unreasonably large values for $\theta$, the potential energy of the bottom of the conduction band, and $n$, the electron density in the conduction band. These values for the five tips yielding type 1 data shown in Figure 49 are summarized in the following table. The column headed "Orig. Prop." lists the resistivity and type of silicon as described by the supplier before the tips were fabricated from it. Figure 54 shows an energy level diagram based on these calculations for the tip Si1L41.

<table>
<thead>
<tr>
<th>Silicon Tip No.</th>
<th>Orig. Prop.</th>
<th>$\rho$ (\text{\Omega \cdot cm})</th>
<th>type</th>
<th>$\theta$ (eV)</th>
<th>$n$ (cm$^{-3}$)</th>
<th>$p$ (cm$^{-3}$)</th>
<th>$\phi$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si1L13</td>
<td>180, n</td>
<td>3.87</td>
<td>2.54 x 10$^{17}$</td>
<td>3.28 x 10$^{17}$</td>
<td>4.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Si1L33</td>
<td>50, p</td>
<td>3.86</td>
<td>6.70 x 10$^{15}$</td>
<td>2.24 x 10$^{17}$</td>
<td>4.84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Si1L39</td>
<td>0.05, n</td>
<td>3.45</td>
<td>8.62 x 10$^{17}$</td>
<td>3.72 x 10$^{18}$</td>
<td>4.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Si1L41</td>
<td>0.05, n</td>
<td>3.74</td>
<td>5.06 x 10$^{16}$</td>
<td>5.41 x 10$^{17}$</td>
<td>4.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Si1L43</td>
<td>0.4, n</td>
<td>6.01</td>
<td>5.50 x 10$^{19}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The calculations of the hole density and the average work function yield fairly reasonable results, although perhaps both of these as well as the value of the electron density may be somewhat high. The value of the work function obtained for each of these tips
Figure 54. A potential energy diagram calculated for the silicon tip Si1L41 without taking field penetration or surface states into account. The ordinate is potential energy in electron volts.
is an average value for the tip as a whole, while each of the work function values obtained by other workers (5; 19, p. 1197) was for a particular crystal face, so close agreement is not expected. These published values range from about 4.74 ev to 5.05 ev, depending on which crystal face was tested and whether it was thermally quenched or annealed after high temperature heating.

When it is recalled that the electron and hole densities are limited by the equation \( np = n_t^2 \leq 2 \times 10^{20} / \text{cm}^3 \), while the above calculations yield a product of the order \( np \approx 10^{34} / \text{cm}^3 \), a number of searching questions need to be asked:

1. Is the limitation \( np = n_t^2 \leq 2 \times 10^{20} / \text{cm}^3 \) valid in this situation?

2. Should the p-type thermal probe indications, which lead to values for \( p \) and \( \phi \), obtained under conditions of no applied electric field, be discarded here when the high electric field is applied, the conditions under which \( \theta \) and \( n \) are determined?

3. If the thermal probe indications are not to be discarded, then the holes should predominate, and the electron density should be less than \( 10^{10} / \text{cm}^3 \). Then where are the electrons coming from? Are they coming from the valence band as suggested by the calculations for tip Si1L43 (which showed only a slight
type 1 effect) or are they coming from deep within the semiconductor which is still strongly n-type (except for Si1L33 which was p-type throughout)?

4. What is the type of energy diagram to be expected from the deep interior which is n-type, through the p-layer, to the surface which is unknown when the high fields are applied? How is the flow of electrons to the surface to be described in a situation such as this? How do the surface states affect this situation?

The limitation \( np = n_1^2 \) is probably not valid here since it is valid only when the semiconductor sample is in equilibrium (23, p. 123) which presumably is not the case when field emission current is drawn from the sample. The other questions should be kept in mind, however, because they are still pertinent.

An attempt was made to fit Equation (30) to the low field, low current density part of the plot of data from the silicon tip Si1L33 shown in Figure 51. This is a more difficult case with several more unknown quantities which must be estimated. These include \( \phi_0 \), i.e., the height of the internal barrier when \( F = 0 \), and \( a \) and \( a' \). The latter two must be used in Equation (28) to determine \( V \) from \( E \), both of which are also unknown. Since Stratton did not give another
relationship between $V$ and $E$, it was necessary to derive another. The method of Kingston and Neustadter (41) was followed and the relationship obtained was almost identical to one given by Dousmanis and Duncan (22). Some degree of fit was obtained near the top of this region, but the calculated plots all departed from the straight line plot exhibited by the data. No further studies were made of the low field, low current density part of the data.

An attempt was also made to fit the data from the tip Si1L33 to the theory proposed by Vasiliev (63) using as a starting point the values for $\theta$, $n$, and $\theta - \gamma$ obtained by Equations (31) and (32). No fit was obtained and the apparent region of validity was found to be over a very narrow range of fields which were about an order of magnitude lower than those observed experimentally. No attempt was made to test the theory by Morgulis (46) since he limits his region of validity to fields about an order of magnitude lower than those observed experimentally.
CONCLUSIONS

The majority of the technical problems associated with this study of field emission from semiconductors has been solved. Suitable methods of tip fabrication and tip cleaning in high vacuum have been developed and yet the sample size and shape have been kept within the limits required by the electron microscope. The field emission projection microscope has yielded information on the surface cleanliness and smoothness of the silicon tips.

Two types of electrical data have been obtained from silicon field emitter tips. One of these types has been associated with a surface p-layer which is strong enough to cause degeneracy of the silicon tip, thus preventing the energy bands at the surface from shifting with changing high electric fields. This results in nearly linear data when plotted in the form of $\log_{10} I$ versus $10^4/V$. The other type has been obtained from silicon tips which apparently were not degenerately p-type and in which the energy bands were therefore free to bend with changing electric fields, resulting in non-linear data when plotted as above. The p-layer appears to be a result of boron from the Pyrex tube envelope doping the surface of the silicon. This contamination has been considerably reduced, but not eliminated, and it is still a very troublesome problem which needs further work.
A dependence of the field emitted current on the temperature of the emitter tip has been observed. Although no quantitative tests were made, the existence of this dependence verifies qualitatively some of the theories of field emission from semiconductors.

The Equation number (21) which was derived by Stratton, yields values for the electron affinity, $\theta$, and the electron density, $n$, which appear to be fairly reasonable for four of the five tips yielding type 1 data which were successfully micrographed. The attempts to apply the correction to $n$ by a factor $\exp(\Delta R/kT)$ where $\Delta R$ is the amount the energy bands are depressed at the surface by the applied electric field as recommended by Stratton for Equation (21) did not lead to reasonable values for $n$. Since the tips were all fairly strongly p-type, probably the Fermi level was so far below the bottom of the conduction band that the "tail" of the Fermi distribution function had almost no effect on the density of electrons in the conduction band when the bands shifted due to changes of the applied electric field. It should be kept in mind that Stratton's theory was derived for the case of an n-type semiconductor in which the Fermi level lies fairly close to the bottom of the conduction band. This is quite different from the experimental conditions obtained.
The calculations of the hole density and the work function yield fairly reasonable results, although perhaps both of these as well as the value of the electron density may be somewhat high. Since these results are fairly reasonable, they indicate that Stratton's approach is basically sound in spite of the several simplifying assumptions made because of the complexity of the problem.

The middle portion of each of three of the plots in Figure 49, i.e., that for Si1L13, Si1L39, and Si1L41, has two slopes. In each of these cases the best value of \( \theta \) was obtained using the slope of the "lower middle" portion of the plot. The indications are that perhaps the further downward bend of the "upper middle" portion of the plot may be due to the hot electron effect or decrease of electron mobility caused by the strong internal fields necessary to keep the surface supplied with enough electrons for the emission that is observed. This hot electron effect was discovered by Ryder and Shockley (56) and has been observed by several workers (39, p. 87-94; 35, p. 259-266; 60). No attempt has been made so far to incorporate this into the theory although Elinson (27) mentioned and then neglected it. The top portion of each of the five type 1 plots in Figure 49 (which bend upward again) appear to coincide with the increase of conduction electrons due to the internal
fields as discussed by Elinson, but no attempt has yet been made to compare the data with his theory. The bottom portion of each of the five type 1 plots in Figure 49 should correspond to Equation (30) by Stratton, but the attempt to fit data from Si1L33 to this equation yielded very poor results.

The present theories of field emission from semiconductors are incomplete and unsatisfactory, partly because of the many necessary simplifying assumptions made in the theories and partly because of inadequate understanding of semiconductor materials. The hot electron effect and emission from the valence band should also be included in the theories. More and better experimental data are needed to serve as a guide for the theoreticians to follow in deriving a suitable theory.

In conclusion, some experimental data on field emission from clean silicon tips have been successfully reduced to the basic quantities of current density and electric fields, yet much more work, both theoretical and experimental, needs to be done in order to answer the many questions which have been raised.
BIBLIOGRAPHY


APPENDIX A

Fabrication of Field Emission Microscope Tubes

The tube envelope is usually made from a 500 ml round bottom, short ring neck chemical type boiling flask of Pyrex 7740 glass (App. B, 59). The flask is washed with a solution of Alconox and water, rinsed with water, and dried, since working the glass is much easier when it is clean. A small hole is blown in the side of the flask and 12 mm Pyrex tubing is sealed to this opening to serve as the exhaust line from the tube to the pumps. If a flat glass window is to be inserted, blow a hole in the opposite side of the flask from the exhaust tubing approximately 1-3/4 inches to 2 inches in diameter and seal on a piece of 51 mm Pyrex tubing about 1/2 inch long. Then seal a two inch window to this tubing and flame anneal the bulb.

A button seal must also be sealed into the bulb to provide electrical contact to the internal phosphor screen which will be added later. To make the button seal, oxidize a tungsten Kulite lead (App. B, 60) by heating in a flame and bead it with a piece of uranium glass (App. B, 61) by putting a short piece of small diameter glass onto the lead and heating to shrink it until it makes good contact to the wire. This beaded Kulite lead is then sealed into a graded seal of uranium glass to Pyrex. Cut off the excess Pyrex, leaving only a
short piece attached to the graded seal and then seal this into the bulb, leaving a small hollow on the inside of the tube by pulling the button seal out about 1/4 inch during the sealing. Flame anneal the bulb again and then insert it into an oven and bake it at about 560°C for 10 to 15 minutes to anneal the glass sufficiently.

The emitter is mounted on a two-wire press which is inserted into the neck of the tube just before the tube is sealed onto a vacuum system. This two-wire press is made in the following manner. Seal a piece of 12 mm uranium glass tubing to 12 mm Pyrex tubing and flare the end of the Pyrex tubing until it is about 1-1/4 inch in diameter at the widest part of the flare. Two tungsten Kulite leads which have been oxidized and beaded with uranium glass are mounted in an iron block jig. This is mounted on the glass lathe along with the graded seal and the Kulite leads are inserted into the graded seal. The uranium glass part of the graded seal is then heated, pressed, and sealed to the beaded leads. When the seal is complete, flame anneal the press, remove from the lathe, insert into an oven and bake at about 560°C for 10 to 15 minutes.

The getter is usually in a bulb which is mounted in the exhaust line close to the tube when it is mounted on the vacuum system. To make the getter bulb, blow a small bulb 2 to 2-1/2 inches in diameter
from one inch Pyrex tubing. Insert a two-wire press seal which has a 10 mil tungsten filament wrapped with a 10 mil titanium wire attached between the two Kulite leads and of sufficient length that it consists of a five turn coil. This completes the glass work for the tubes made of Pyrex.

If the tube is made of EE-2 glass (App. B, 62), it is made from a small oscilloscope blank with a three inch face. This glass must be worked at a much higher temperature than Pyrex and therefore is much more difficult to use. A hydrogen-oxygen flame must be used to get a hot enough fire. The tube must be cleaned thoroughly to remove all dirt, grease, fingerprints, etc. Wash the blank with Alconox and water and rinse with water; rinse or wipe the surfaces to be blown with acetone or alcohol just before blowing. Attach an exhaust side arm of EE-2 glass tubing and insert the button seal on the other side of the tube blank. Because of different expansion properties, the button seal is made of molybdenum Kulite lead which is beaded with EE-2 glass. Most of the excess length of the solid part of the Kulite lead is cut off, leaving about 1/8 inch exposed which then is nickel plated to prevent oxidation at high temperatures. Sometimes the plating deteriorates during glass working and the tube can then be baked only in a nitrogen atmosphere
to prevent further oxidation of the molybdenum. A 1/2 mil thick platinum foil about 1/8 inch wide and 3/4 inch long is spotwelded to the lead. The beaded Kulite lead is then sealed into the EE-2 glass bulb and the platinum foil is sealed to the inside surface of the tube to make a better contact to the anode. If it fails to seal down properly, it may have to be painted down later with silver paint. The two-wire press seal is made by pressing a flared piece of EE-2 glass tubing down onto two molybdenum Kulite leads which were previously beaded with EE-2 glass.

The EE-2 glass must be flame annealed continuously during the blowing and working. At the beginning, flame the glass with a Litton hand annealing burner. When the glass reaches a red color, turn on the hydrogen fires on the lathe. Introduce oxygen into the hydrogen and increase the ratio of oxygen to hydrogen until the reduction, which is a darkening of the glass, just begins to disappear. Do not increase the ratio of oxygen to hydrogen any further since too much oxygen will then cause the glass to reboil, which is the formation of tiny white bubbles in the glass. The working temperature range between reduction and reboil is very narrow. The EE-2 glass can be sealed to Pyrex 7740 glass through a uranium glass graded seal.
After the tube envelope has been blown and annealed, it is ready to have the screen built into it. The glass envelope must be carefully washed with Alconox and water and thoroughly rinsed with water. Then rinse with acetone to speed the drying and dry for 24 hours. Apply a ring of platinum paint (App. B, p. 63) about 1/4 inch wide, using a small paint brush, at the same latitude at which the button seal is located. Do not paint closer than about 1/4 inch to the button seal because the sodium hydroxide etch which is to follow later would cause the paint to peel at the button seal. Dry the platinum ring with a stream of air and then bake the envelope at 650°C for about 30 minutes. In the case of 1720 or EE-2 glass bulbs which use the platinum foil lead from the button seal to the ring for more reliable contact, the tube must be baked in a nitrogen atmosphere when the temperature is above 475°C to prevent the platinum foil from powdering. When the tube has cooled, paint, dry, and bake a second and then repeat for a third platinum layer on this ring. Then apply and air dry a ring of gold paint (App. B, p. 63) and bake at 525°C for about 10 minutes. The layer of gold is painted on top of the platinum ring and in this case the tube may be baked in an air atmosphere since the bakeout is at a lower temperature. As
mentioned above, do not paint the gold within 1/4 inch of the button
seal, but do paint gold over the edge of the platinum.

Now place a small amount of sodium hydroxide (1-5 N) into the
hollow around the button seal on the inside of the tube and apply
24 volts ac between the button seal lead and a probe wire inserted
into the solution. This etches the button seal lead to remove the
oxide on the lead which formed during the glass blowing and, in the
case of the molybdenum lead into the EE-2 or 1720 glass envelopes,
it removes any remaining traces of the nickel plating. Etch until it
looks clean. Then wash thoroughly with Alconox and water and
rinse with water and dry for about 24 hours. Next apply a ring of
silver paint (App. B, 64) over the ring of platinum and gold and be
very generous with the silver around and on the button seal lead to
insure good contact between it and the rings. Dry with a stream of
air as before and then bake at 500°C for 5 minutes. Wash
thoroughly with Alconox and water, rinse with water and dry for
about 24 hours.

The silver paint provides the electrical contact between the
button seal and the screen, but it does not stick to the glass reliably.
The silver paint adheres well to the layer of gold and the latter
adheres to the platinum ring. The platinum paint adheres to the
glass surface very well. Neither the platinum paint nor the gold paint makes a reliable contact to the button seal, but the silver paint does.

The tube is now ready to receive the phosphor screen which will be deposited by the lacquer method. The steps in preparing the phosphor are as follows. Grind 40 grams of Willemite (App. B, 65) with 75 ml of amyl acetate (App. B, 66) in a slowly turning ball mill (App. B, 67) which is very tightly closed. The ball mill is a one quart porcelain jar with smooth agates in it. After four hours of grinding, drain the solution from the ball mill into a flask and rinse the ball mill and agates with 25 ml of amyl acetate to remove as much of the phosphor as possible. Add this rinse to the solution, then cover and let it settle for 24 hours. Exercise extreme precautions to prevent dust particles and any other contaminants from falling into the phosphor. After settling, carefully pour off 50 ml of the clear solution without disturbing the phosphor, and discard the clear solution. Add 25 ml of Varniton varnish (App. B, 68) which acts as a glue or binder for the phosphor, and mix well. The phosphor mixture is now ready to be coated onto the inside of the tube face.
Mount the tube with its face down and pour in a sufficient amount of the solution to cover the face of the tube with a thin film when the tube is rolled and tilted to distribute the phosphor evenly over the tube face. Invert the tube to allow the excess phosphor to drain out and start rotating the tube immediately with its axis approximately 30° from the vertical. This allows the excess phosphor to drain away evenly. At the same time admit a gentle stream of air into the tube to dry the phosphor. Continue rotating the tube about seven or eight minutes, until the phosphor appears to be dry. Stand the tube face up in a rack for 24 hours to complete drying. Then bake the tube and phosphor at 500°C for 20 minutes to remove the Varniton. When the tube has cooled, use a sharp instrument to make a mark around the face of the tube at least 1/4 inch in front of the ring of platinum, gold, and silver; scrape away as much phosphor as possible behind this line with the tube dry. This exposes a ring of clean glass between the phosphor on the face and the ring to which the metal backing will adhere better.

A "metal carrier" is needed on the back of the phosphor screen to provide a smooth surface on which to evaporate the metal. This will allow the metal layer to be a smooth, uniform, good conducting, and good reflecting thin layer. Otherwise the rough
phosphor screen will not permit the above desired qualities. The metal carrier is made by mixing one part collodion, one part amyl acetate, and five parts ethyl acetate (App. B, 69). The tube should be prepared for the application of the metal carrier by carefully rinsing the inside surfaces of the tube and screen with a solution of 10% (by volume) acetone, reagent grade (App. B, 69), to 90% distilled water. Repeat this rinsing two or three times until the tube appears to be free of excess phosphor particles. Mount the tube securely face down on a tilt table. Insert a cork which is pierced by a small drawn down glass tubing into the exhaust side arm. Then fill the tube to a level which is above the painted ring and the side arm with the solution of 10% acetone and 90% distilled water. Insert an eye dropper containing a few drops of metal carrier into the neck of the tube to within a few millimeters of the surface of the solution. Deposit the metal carrier onto the surface of the solution quickly so that it will spread over the surface in one single uniform and continuous sheet. Then tilt the tube slowly, allowing the solution to drain out from under the sheet through the drawn down glass tubing inserted into the exhaust side arm without puncturing the floating sheet of metal carrier. The drawn down glass tubing causes the draining to be slow to prevent tearing of the sheet. The
sheet of metal carrier is thus deposited on the back of the phosphor and painted metal ring. Then stand the tube face up in the rack and allow to dry overnight.

The next day seal off the end of the exhaust side arm and prepare the tube for vacuum pumping. Insert a roll or cylinder of nickel sheet into the neck of the tube almost to the center of the tube to prevent metal film from evaporating onto the back part of the tube. If observation of the emitter during the experiments is desired, small windows (blank regions) in the metal backing may be obtained by placing pieces of nickel sheet about 1 inch square at the desired position on the inside surface of the tube envelope to shield the small regions from the evaporating metal. These pieces of nickel sheet may be held at the desired place on the inside of the tube by small magnets taped to the outside of the tube.

The metal to be evaporated, which was usually aluminum or gold, is mounted on a filament made of 20 mil tungsten wire (App. B, 70) bent into the shape shown in Figure 55 at the right. This particular shape of the filament

Figure 55. Shape of metal evaporating filament.
eliminates the shadows caused by the side supports, since the metal which is placed in one of the two low regions will melt and flow up the center high region from which it evaporates. The filament is cleaned by electrolytic etching in sodium hydroxide and rinsed in water and then acetone. For aluminum backings, a small piece of aluminum is looped over one of the two low spots of the filament and crimped securely onto it. The aluminum is cleaned by dipping in concentrated hydrochloric acid and then is rinsed in water and acetone. Immediately after cleaning, the filament is mounted vertically on filament supports with external leads on a quick change vacuum system. The tube to receive the metal backing is then placed over the filament onto a flat table with a hole through the center for the exhaust line and heavy filament leads and pumping is started immediately. The seal between the neck of the tube and the table is made with a flat rubber gasket coated with vacuum grease. The filament must be located as near the center of the tube as possible so that the distances from it to all parts of the tube to be covered are about equal. It is heated by ac power from a variac through a 36 volt transformer (App. B, 71).

Before the filament and aluminum are outgassed, the system is pumped until the pressure is less than $5 \times 10^{-4}$ mm Hg. The
Outgassing must not be done at higher pressures than that mentioned because an oxide layer would form on the aluminum. The oxide layer must be avoided since it melts at a temperature so much higher than the melting point of aluminum that it forms a crust-like coating which the aluminum cannot penetrate. The outgassing process is therefore usually a series of flashes separated by long intervals. When the outgassing is completed, the current in the filament is adjusted (usually from outgassing at 10 or 11 amps to evaporation at about 9 amps) to cause the metal to evaporate onto the inside of the tube at a fairly slow rate to avoid sputtering. If the pressure rises to \(5 \times 10^{-4}\) mm Hg or above, the evaporation must be interrupted until the pressure drops down again, in order to prevent oxidation of the aluminum. About midway through the evaporation process, the tube should be rotated 90° about its axis, leaving the filament stationary, to insure a more uniform layer of aluminum. After the evaporation is completed, the tube is baked out at 340°C for about 1/2 hour. The temperature must be brought up to 340°C very slowly, i.e., about 90 Centigrade degrees per hour. The purpose of this bakeout is to remove the metal carrier from under the metal film and the temperature rise must be slow to prevent the metal backing from peeling off. After this bakeout the tube is ready for use.
If a conducting glass anode instead of an aluminum backed anode is desired, a different process is required following the painting of the gold ring and etching the button seal with sodium hydroxide as discussed above. After the button seal has been etched with sodium hydroxide until it looks clean, wash the tube thoroughly with Alconox and water, rinse in water, and dry for 24 hours. Then apply Aquadag (App. B, 72) evenly with a pipe cleaner to all parts of the inside and outside of the tube that are not to have a conducting surface, since glass with a conducting surface has been found to be difficult to blow or work. It will probably be necessary to thin the Aquadag with distilled water in order to make it easier to apply evenly. Also, the addition of a few drops of household ammonia to the Aquadag makes it much easier to remove the Aquadag afterwards.

Place the tube in an oven with the tube face up. Insert a 12 mm Pyrex tubing into the center of the neck of the tube so that the end of the tubing extends about 1/2 inch into the tube and is aimed at the center of the face of the tube. The other end of the tubing extends outside of the oven. The oven is turned on and the temperature is raised to between 500 and 550°C. Air at just slightly over atmospheric pressure is passed through rubber tubing over the surface of anhydrous fuming stannic chloride (App. B, 73) contained
in a closed flask and then the mixture of fumes and air is passed through the tubing into the tube which is to receive the conducting glass layer. This mixture should be admitted to the tube in small puffs or bursts several times until the glass shows slight rainbow colors or rings. Occasionally move the glass tubing extending from the oven in order to distribute the fumes more evenly over the inside of the tube. Then turn off the oven and allow to cool. Great care must be exercised in handling the fuming stannic chloride since it is a toxic material. When the tube has cooled, measure the resistance from the center of the face of the tube to the button seal. If the resistance is over 1000 ohms, return the tube to the oven and deposit more fumes on the glass as described above. If the resistance is less than 1000 ohms, remove the Aquadag by scrubbing the tube with Alconox and water. Avoid scratching the conducting regions since this will damage the conducting ability of the material. This process is described in more detail in a recent technical report (17).

Apply a silver ring as described above, make a good contact to the button seal, and dry and bake out as was described above for the metal backed tubes. Rinse the tube with concentrated ammonium hydroxide (App. B, 74), rinse with water, then acetone, and dry for 24 hours. Apply the phosphor screen as was described above for the
other type of tube and remove the excess phosphor from behind the painted ring to avoid scratching the conducting layer between the ring and the screen. Rinse as before and allow to dry. The tube is ready for use.
APPENDIX B

Major Items of Equipment and Supplies

1. NRC, 4D Rotary Gas Ballast Mechanical Pump, type 4512. Carl Herrmann Associates, P. O. Box 1179, Palo Alto, Calif.


3. Contact Maker, Model N42, Type L, Class 1, adjustable 1/2 to 50 P. S. I. United Electric Controls Co., Boston, Mass.


5. 1386 Liquid Air Traps, 38 mm dia., 250 mm depth. W. M. Welch Scientific Co., 1515 Sedgewick St., Chicago, Ill.

6. 1387 Liquid Air Traps, demountable, 38 mm dia., 250 mm depth. W. M. Welch Scientific Co., 1515 Sedgewick St., Chicago, Ill.

7. Hofman D2 Stainless Steel Dewar Flask, 4-1/2" inside diameter, 14" inside depth. Hofman Laboratories, Inc., 212-18 Wright St., Newark 5, N. J.


10. Veeco GV-31 Thermocouple gauge control circuit with gauge tube and cable. Carl Herrmann Associates, P. O. Box 1179, Palo Alto, California.

12. Veeco RG21-R Ionization gauge control, range $2 \times 10^{-10}$ mm Hg to 1 micron, for rack mounting. Carl Herrmann Associates, P.O. Box 1179, Palo Alto, California.


17. No. T1-2248, 120 volt, 500 watt Chromalox Tubular Elements. Montgomery Bros., 1632 N.W. Johnson St., Portland 9, Ore.

18. PV Supertemp Blocks, 36" x 12" x 2". Fire Brick Supply Co., 2914 First Avenue South, Seattle 4, Washington.


20. West Gardsman, Model J On-Off Controller. BB-100, 0-1000°C calibrated for 6 ft. of 14 gauge (C/A), 115 volts, 60 cycle. Electro-Glass Laboratory, 12335 S.W. Faircrest St., Portland 25, Oregon.

21. Thermocouple, K14E 6' Chromel-Alumel, 14 gauge, straight, fish-spine covered, 6' long each. Electro-Glass Laboratory, 12335 S.W. Faircrest St., Portland 25, Oregon.

22. Thermocouple, K14D 6' Chromel-Alumel, 14 gauge, straight, double bore insulation, 6' long each. Electro-Glass Laboratory, 12335 S.W. Faircrest St., Portland 25, Oregon.
23. Manuflex Timers, AB25A6, 110 volts, 60 cycle, panel mounting, 10 hour dials, type 2. Electro-Glass Laboratory, 12335 S. W. Faircrest St., Portland 25, Oregon.


25. Magnetic Contactor, class 8502, type SO 8, 3 poles, 110 volt coil circuit. Square D Co., Los Angeles, California.


27. 33737 Heating Tape, flexible, heavy, 8 ft. long, 1 inch wide, 115 volts, 700 watts. Scientific Supplies Co., 600 Spokane St., Seattle 4, Washington.

28. Sim-Ply-Trol Indicating Pyrometer No. 2614, 0-750°F, 0-400°C; Thermocouples: Iron-Constantan 20 gauge, solid with 5 ft. lead. Assembly Products Co., Chesterland, Ohio.


33. Model ESH Electrostatic Voltmeter, ranges 5/15/30 kv.  
Sensitive Research Instrument Corp., Mt. Vernon, N. Y.

34. DCH1 Kilovoltmeter, 0-3/7.5/15/30 kilovolt ranges.  
Sensitive Research Instrument Corp., New Rochelle, N. Y.


36. G-10 Graphic Recorder, span - 10 mv.  Varian Associates,  
Palo Alto, California.

37. Rapid Sequence IIa, 35 mm Camera, West Germany.

38. 1.00 megohm, wire wound Precision Resistor, 1/2 watt,  
type WW3, 0.1% tolerance.  Reliance Merchandising Co.,  
2223 Arch St., Philadelphia 3, Pa.

39. Präz. -Mikroamperemeter, Model no. 134313 (±0.5%),  
ranges 150/750/1500 microamperes.  Physics Research  
Laboratories, Inc., Hempstead, N. Y.

40. 100 ohm Secondary Standard Resistor.  Leeds and Northrup Co.,  

41. Type K-3 Universal Potentiometer.  Leeds and Northrup Co.,  

42. Type 545 Oscilloscope.  Tektronix, Inc., P. O. Box 500,  
Beaverton, Oregon.

43. Type 53/54 C Dual Channel Input.  Tektronix, Inc.,  
P. O. Box 500, Beaverton, Oregon.

44. No. 776 Unger Soldering Iron and Tip.  Lou Johnson Inc.,  
1506 N. W. Irving, Portland, Oregon.

45. Simpson dc Microammeter, center zeroing, scale range  
Portland 9, Oregon.
46. Type EMT Electron Microscope, MI-12952. Radio Corporation of America, Engineering Products Dept., Camden, N. J.

47. AO Microstar Trinocular Microscope with 35 mm photomicrographic camera model 635 and model 600 Ortho-Illuminator. American Optical Co., Instrument Div., Buffalo 15, N. Y.

48. Silicon bars, 1/16" x 1/16" x 5/16", minimum resistivity 50 ohm-cm, n-type.

Germanium bars, 0.040" x 0.040" x 5/16", minimum resistivity 30 ohm-cm, n-type.

Silicon bars, 0.060" x 0.060" x 0.400", 1-2 ohm-cm, n-type.

Anchor Metal Co., Inc., 966 Meeker Ave., Brooklyn 22, N. Y.

49. Silicon bars, 0.050" x 0.050" x 0.300", ~ 50 ohm-cm, p-type.

Silicon bars, 0.040" x 0.100" x 0.300", ~ 1 ohm-cm, n-type.

Silicon bars, 0.040" x 0.100" x 0.300", ~ 50 ohm-cm, p-type.

Silicon bars, 0.040" x 0.100" x 0.300", ~ 180 ohm-cm, n-type.

Silicon ingots: 0.05 ohm-cm, n-type; 3.5 - 13.6 ohm-cm, n-type; 0.4 ohm-cm, n-type; 0.26 - 0.77 ohm-cm, p-type; 9.5 - 11.0 ohm-cm, p-type.

Thermosan, Inc., 375 Fairfield Ave., Stamford, Conn.

50. No. 507 Platinum Tipped Tweezers. Englehard Industries, Inc., 113 Astor St., Newark 5, N. J.


53. No. C240-P-RR7 Rubber Bonded Cut-off Wheels, 0.010" thick, 3" diameter, 1/4" hole; No. C240-P-RR7 Rubber Bonded Cut-off Wheels, 0.015" thick, 3" diameter, 1/4" hole.


55. Dura-Lite Wood Dough. Peterson Paint Co.,
321 N. Baker St., McMinnville, Oregon.

56. Carbon Grating Replica No. 115. Ernest F. Fullam, Inc.,
Electron and Light Microscopy, P. O. Box 444, Schenectady,
New York.

57. Pee-Gee Pocket Comparator; Radii Reticule No. 102; Diameters
Reticule No. 109. National Tool Co., Cleveland, Ohio.

58. Model SC21 Electrostatic Generator, American Electrostatic Co.,
Box 4181, Tulsa, Oklahoma.

59. 500 ml round bottom, short ring neck, chemical type boiling
flask, Pyrex 7740. Scientific Supplies Co., 600 Spokane St.,
Seattle 4, Washington.

60. Kulite lead, tungsten wire .050" x 1-1/2" with 3" Kulgrid Flex.
Kulite Tungsten Co., 723 Sip St., Union City, N. J.


62. Boron free EE-2 glass, tubing and oscilloscope blanks.
Kimble Glass Co., Toledo 1, Ohio.

63. Liquid Bright Platinum No. 05, Hanovia; Liquid Bright Gold
No. 261, Hanovia. Englehard Industries, Inc., Hanovia Liquid
Gold Div., 1 West Central Ave., East Newark, N. J.

64. Silver 4760. E. I. DuPont De Nemours and Co., Inc.,
Electrochemicals Dept., Wilmington, Delaware.

65. Willemite type 2282. Sylvania Electric Products, Inc.,
Chemical and Metallurgical Div., Towanda, Pa.

66. Amyl Acetate, Purified. J. T. Baker Chemical Company,
Phillipsburg, New Jersey.

67. Ball Mill, one quart size. Scientific Supplies Company,
600 Spokane St., Seattle 4, Wn.
68. Varniton Label Varnish V-21. The Varniton Company,
    416 N. Varney St., Burbank, California.

69. Collodion, Flexible U. S. P.; Ethyl Acetate, Reagent Grade;
    Acetone, Reagent Grade. J. T. Baker Chemical Company,
    Phillipsburg, New Jersey.

70. Tungsten wire (all sizes). Fansteel Metallurgical Corporation,
    North Chicago, Illinois.

71. Transformer, input 117 volts, output 36 volts. George Sturley,
    2830 S. W. Texas, Portland, Oregon.


73. Stannic Chloride, Anhydrous-Fuming. Fisher Scientific Co.,
    Fair Lawn, New Jersey.

74. Ammonium Hydroxide, concentrated, Reagent Grade.
    J. T. Baker Chemical Co., Phillipsburg, N. J.
APPENDIX C

\[ \beta \text{ Calculations} \]

The sphere-on-orthogonal-cone method of calculating the \( \beta \)-factor was discussed briefly on pages 106 and 107. Dolan (21) extended the calculations of Dyke and co-workers (26, p. 573-576) in some unpublished work. He has generously given permission for his notes, equations, a table of values, and sketches of equipotential surfaces for this model to be included in this thesis. These follow with only a few minor changes in wording and equation numbers. The values for his standard configuration are \( a = 1 \times 10^{-5} \) cm, \( R = 1 \) cm, and \( V_R = 1 \) volt.

Method for Finding \( \beta \) by Matching a Given Emitter to One of a Set of Standard Sphere-on-Orthogonal-Cone Equipotentials

W. W. Dolan

The process consists of correcting the \( \beta \)-factor from its value for the chosen standard curve to the actual value for the given emitter. An outline of steps, with an example, is followed by a detailed explanation below.

(1) Match profile of emitter to one of the set of standard curves (can use enlarging projector for this purpose). Note these associated quantities, which are listed with each curve:

- \( n \) : Legendre function parameter.
- \( r' \) : radius of standard curve at vertex, measured from origin at center of sphere.
- \( V' \) : potential on standard curve.
- \( \beta' \) : \( \beta \)-factor for standard curve.
You also need \( r \) and \( R \), the radius and anode distance for the actual emitter. If \( R \) is very small, it should also be measured from the same origin.

Example: \( n = 0.16, \quad r' = 2.0 \times 10^{-5} \text{ cm}, \quad V' = .1062 \text{ v}, \quad \beta' = 6.19 \times 10^3 \text{ cm}^{-1}; \quad r = 4.0 \times 10^{-5} \text{ cm}, \quad R = 5 \text{ cm}. \)

(2) Calculate the auxiliary quantity \( C = \left( \frac{r'}{r} \right)^n \).

Example (continued): \( C = \left( \frac{2}{4} \times 5 \right)^{16} = (2.5)^{16} = 1.158 \)

(3) Calculate \( \frac{1 - V'}{C - V'} \).

Example (continued): \( \frac{1 - .1062}{1.158 - .1062} = \frac{.8938}{1.052} = .8496 \)

(4) Calculate \( \beta = \beta' \left( \frac{r'}{r} \right) \left( \frac{1 - V'}{C - V'} \right) \).

Example (continued): \( \beta = (6.19 \times 10^3) \cdot (.5) \cdot (.8496) = 2.63 \times 10^3 \)

Explanation:

The two factors applied to \( \beta' \) in Step (4) can be explained as the effects on \( \beta' \) of two successive operations leading from the standard configuration to the actual. First, the entire standard configuration is scaled up (or down) to bring the standard radius \( r' \) to the actual radius \( r \). The equation for \( \beta \) in general is this:

\[
\beta = \frac{F}{V_R - V_r} = \frac{1}{r} \left( \frac{r}{R} \right)^n \left[ n + (n + 1) \left( \frac{a}{r} \right)^{2n+1} \right] \frac{VR}{V_R - V_r}
\]

where \( a \) is the radius of the sphere-on-cone. Evidently the ratios \( r/R \) and \( a/r \) are not affected by the scaling process. The first
factor on the right of Equation (33) is such as to require multiplication by \( \frac{r'}{r} \) for the indicated scaling process. As for the last factor, the equation for axial values of the potential is

\[
\frac{V_r}{V_R} = \left(\frac{r}{R}\right)^n \left[1 - \left(\frac{a}{r}\right)^{2n+1}\right].
\] (34)

Since this is not changed by linear scaling of \( a, r, R \) neither is the factor

\[
\frac{1}{1 - \left(\frac{V_r}{V_R}\right)} = \frac{V_R}{V_R - V_r}.
\]

hence the entire effect of this scaling step is to introduce the factor \( \frac{r'}{r} \) as in Step (4).

Second, the new configuration is changed to the actual by keeping \( a \) and \( r \) fixed, but changing \( R \) to its actual value. The effect of this appears in both the second and the last factors on the right side of Equation (33). Since \( R = 1 \) in the standard configuration, the first step above leaves it at the value \( r/r' \); this is now to be replaced by \( R \), i.e., multiplied by

\[
\frac{r'}{r} R,
\]

and the effect on the second factor on the right side of Equation (33) is to divide it by

\[
\left(\frac{r'}{r} R\right)^n = C.
\]

The effect on the final factor of Equation (33) is seen by reference to Equation (34) where again there must be division by \( C \), i.e., the left side of Equation (34) becomes \( V_r/CV_R \), whence the
final factor of Equation (33) becomes

\[
\frac{1}{V_R} \left( 1 - \frac{V_R}{CV_R} \right) = \frac{CV_R}{CV_R - V_R}.
\]

This requires multiplying the former value by

\[
\frac{C (V_R - V_x)}{CV_R - V_R}.
\]

or, since $V_R = 1$ and $V_x = V'$ in the standard configuration, the factor is

\[
\frac{C (1 - V')}{C - V'}.
\]

This may be combined with the factor $1/C$ of the preceding paragraph to get the factor appearing in Step (4), viz.

\[
\frac{1 - V'}{C - V'}.
\]

The table of associated quantities is given on the next page.

The sketch number refers to one of the standard curves shown in the figures following the table.
<table>
<thead>
<tr>
<th>Sketch Number</th>
<th>n</th>
<th>$r'$</th>
<th>$V_r$</th>
<th>$\beta'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.18</td>
<td>$1.6 \times 10^{-5}$</td>
<td>0.0647</td>
<td>$7.35 \times 10^3$</td>
</tr>
<tr>
<td>2</td>
<td>0.30</td>
<td>$2.0 \times 10^{-5}$</td>
<td>0.0260</td>
<td>$1.46 \times 10^3$</td>
</tr>
<tr>
<td>3</td>
<td>0.26</td>
<td>$2.0 \times 10^{-5}$</td>
<td>0.0391</td>
<td>$2.18 \times 10^3$</td>
</tr>
<tr>
<td>4</td>
<td>0.14</td>
<td>$3.0 \times 10^{-5}$</td>
<td>0.1757</td>
<td>$3.95 \times 10^3$</td>
</tr>
<tr>
<td>5</td>
<td>0.14</td>
<td>$2.0 \times 10^{-5}$</td>
<td>0.1294</td>
<td>$7.70 \times 10^3$</td>
</tr>
<tr>
<td>6</td>
<td>0.10</td>
<td>$2.0 \times 10^{-5}$</td>
<td>0.1914</td>
<td>$1.21 \times 10^4$</td>
</tr>
<tr>
<td>7</td>
<td>0.10</td>
<td>$3.0 \times 10^{-5}$</td>
<td>0.2586</td>
<td>$6.26 \times 10^3$</td>
</tr>
<tr>
<td>8</td>
<td>0.10</td>
<td>$4.0 \times 10^{-5}$</td>
<td>0.2945</td>
<td>$3.97 \times 10^3$</td>
</tr>
<tr>
<td>9</td>
<td>0.08</td>
<td>$15.0 \times 10^{-5}$</td>
<td>0.4730</td>
<td>$7.94 \times 10^2$</td>
</tr>
<tr>
<td>10</td>
<td>0.06</td>
<td>$15.0 \times 10^{-5}$</td>
<td>0.5613</td>
<td>$9.95 \times 10^2$</td>
</tr>
<tr>
<td>11</td>
<td>0.06</td>
<td>$7.0 \times 10^{-5}$</td>
<td>0.4995</td>
<td>$2.89 \times 10^3$</td>
</tr>
<tr>
<td>12</td>
<td>0.04</td>
<td>$4.0 \times 10^{-5}$</td>
<td>0.5178</td>
<td>$9.43 \times 10^3$</td>
</tr>
<tr>
<td>13</td>
<td>0.04</td>
<td>$5.0 \times 10^{-5}$</td>
<td>0.5547</td>
<td>$6.73 \times 10^3$</td>
</tr>
<tr>
<td>14</td>
<td>0.04</td>
<td>$8.0 \times 10^{-5}$</td>
<td>0.6131</td>
<td>$3.32 \times 10^3$</td>
</tr>
<tr>
<td>15</td>
<td>0.04</td>
<td>$10.0 \times 10^{-5}$</td>
<td>0.6343</td>
<td>$2.39 \times 10^3$</td>
</tr>
<tr>
<td>16</td>
<td>0.04</td>
<td>$20.0 \times 10^{-5}$</td>
<td>0.6832</td>
<td>$9.10 \times 10^2$</td>
</tr>
</tbody>
</table>
Figure 56. Standard curves representing equipotential surfaces for $\beta$ calculations.
Figure 57. Standard curves representing equipotential surfaces for $\beta$ calculations.
Figure 58. Standard curves representing equipotential surfaces for $\beta$ calculations.
Figure 59. Standard curves representing equipotential surfaces for $\beta$ calculations.
Figure 60. Standard curves representing equipotential surfaces for $\beta$ calculations.
Figure 61. Standard curves representing equipotential surfaces for $\beta$ calculations.