THE DETERMINATION OF TRANSFORMER LEAKAGE REACTANCE BY USING AN IMPULSE DRIVING FUNCTION

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLE

TABLE OF SYMBOLS

Symbol	Description
A	Real part of a complex number.
a	Ratio of primary to secondary turns.
a	Dimention of air gap.
C	Capacitance.
E	Constant electromotive force.
E	Effective value of electromotive force.
e	Instantaneous electromotive force.
F	Magnetomotive force.
f	Frequency of a periodic function.
I	Effective value of current.
I	Effective value of exciting current.
i	Instantaneous current.
i'_{φ}	Instantaneous value of exciting current.
K	Constant of circuit.
L	Self-inductance.
L_{eq}	Equivalent inductance.
L	Leakage inductance.
L_{sc}	Short- circuit inductance.
M	Mutual inductance.
N	Number of turns.

Symbol	
P	Power (average).
p	Instantaneous power.
q	Electric charge.
R	Resistance.
R_{sc}	Short-circuit resistance.
U	Difference of magnetic potential.
V	Constant voltage: difference of potential.
X_{eq}	Equivalent reactance.
X	Leakage reactance.
X_{sc}	Short-circuit reactance.
Z_{eq}	Equivalent impedance.
Z_{sc}	Short-circuit impedance.
λ	Instantaneous flux linkage.
μ	Permeability.
π	Ratio of circumference to diameter of circle. (3.14159..)
ϕ	Magnetic flux.
φ	Instantaneous flux
φ_{ρ}	Instantaneous leakage flux.
ω	Angular frequency.
$\approx$	Approximately equal to.

THE DETERMINATION OF TRANSFORMER LEAKAGE REACTANCE BY USING AN IMPULSE DRIVING FUNCTION

INTRODUCTION

Electrical power system transformers are generally represented by their equivalent circuits for the purpose of system analysis. The equivalent circuit is a graphical representation of the transformer in terms of electrical circuit elements. The circuit configuration can be changed from a very accurate representation to a simplified approximation depending upon the degree of accuracy desired by the calculations.

In power system applications the leakage reactance of a transformer is the most important element in the equivalent circuit. This reactance causes a voltage drop and reactive power consumption in the transformer.

The magnetic leakage in the transformer has been studied and found that, in spite of the magnetic nonlinearity of the iron, the leakage reactance is constant and linear over the normal operating range. The component leakage fluxes are nearly directly proportional to the currents producing them, since the paths of the component fields are in air for a considerable portion of their lengths.

The impulse test was used in this investigation to determine the leakage reactance of the transformer.

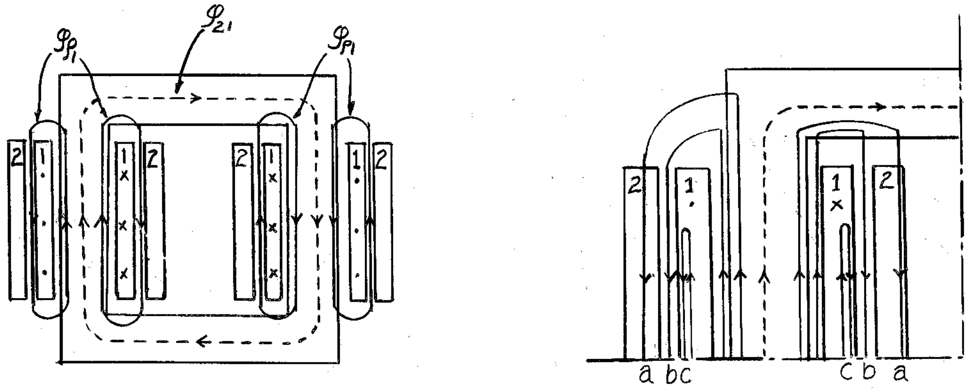
The impulse response characteristics for the equivalent circuit of a transformer was studied and the oscillatory terminal voltage was determined experimental.. The oscillation of the transformer terminal potential is due to the energy relation between the electrostatic and electromagnetic fields just after the application of an impulse of current.

The leakage reactance was calculated from the oscillograms of terminal voltage.

MAGNETIC LEAKAGE IN TRANSFORMERS

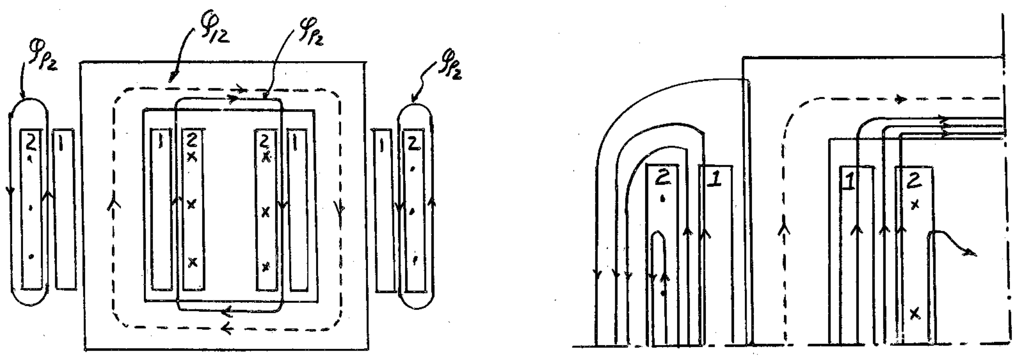
Theory of the electrical characteristics of transformers must account for the following imperfections which occur in iron-cored transformers; winding resistance, magnetic leakage, exciting current, and hysteresis and eddy current losses in the core. Of course it is necessary to take into account the equivalent capacitance of the windings when high frequences are involved. It is very important to take into account the magnetic leakage and the magnetic properties of the core, when determining the relations between the flux linkages and currents.

As an example (6, p 313-324), consider the roughly drawn field map of Figure 1 which shows in cross section, the core and coil of a core type transformer with concentric windings. In Figure 1 there is current only in the inner winding 1, as indicated by the dots and crosses representing the heads and tails of arrows pointing the direction of the current i_1. The principal components of the flux produced by i_1 are shown in Figure 1(a). Most of the flux is confined to the core and therefore links all the turns of both windings, as shown by the broken lines in Figure 1. Additional flux, not entirely confined to the core, is shown by the light unbroken flux lines. Figure 1(a) shows the approximate path of this air-borne flux, and Figure 1(b) shows a roughly drawn map of the magnetic



(a) (b)

Figure 1. Magnetic field due to current in the inner winding of a core-type transformer. The principal fluxes are shown in (a). (b) shows the general character of the field in the upper, left-hand quadrant of (a).



(a) (b)

Figure 2. Magnetic field due to current in the outer winding of Figure 1.

field in the upper, left-hand quadrant of Figure 1(a). An examination of Figure 1(b) shows that some of the air-borne flux as lines (a) and (b), link half the turns of winding 1, while another portion, such as line (c), links only a fraction of the turns of winding 1. Figure 1(b) shows the partial flux linkages produced by fields (a) and (c), whose paths lie among the turns of the windings. Thus the distinction between leakage and mutual fields in an actual transformer is less evident than in devices having compact coils.

It is convenient to simplify the picture of the magnetic field in a transformer by introducing the concept of equivalent flux in the following manner. A small contribution to the flux linkages λ_{11} is produced by fields, such as that shown as line (c), Figure 1(b). This field links only a fraction of the turns of the winding. Let φ_{11} be an equivalent flux considered as linking all N_1 turns of winding 1 and producing a flux linkage $N_1 \varphi_{11}$ equal to the actual flux linkage λ_{11}. Then

$$\varphi_{11} \equiv \frac{\lambda_{11}}{N_1} \qquad \text{Eq. (1)}$$

The flux φ_{11} is the average flux linkage with winding 1 per turn. A portion of this flux also links winding 2. Let λ_{21} be the flux linkage with winding 2 produced by the current in winding 1. A small contribution to the flux linkage λ_{21} is made by a field such as that shown by lines (a), Figure 1 (b), that links only a fraction of the turns of winding 2. Let φ_{21} be an equivalent flux considered as linking all N_2 turns of winding 2 and producing a flux linkage $N_2 \varphi_{21}$ equal to the actual flux linkage λ_{21}. Then

$$\varphi_{21} \equiv \frac{\lambda_{21}}{N_2} \qquad \text{Eq. (2)}$$

The flux φ_{21} is the average mutual flux linkage with winding 2 per turn.

The difference between the average flux φ_{11} linking winding 1 and the average mutual flux φ_{21} which also links winding 2 is the leakage flux $\varphi_{\ell 1}$ of winding 1 with respect to winding 2. That is

$$\varphi_{\ell 1} = \varphi_{11} - \varphi_{21} \qquad \text{Eq. (3)}$$

$$\equiv \frac{\lambda_{11}}{N_1} - \frac{\lambda_{21}}{N_2} \qquad \text{Eq. (4)}$$

Thus the actual flux distribution of Figure 1(b) is equivalent to the simplified picture of Figure 1(a), in which the magnetic field produced by i_1 is represented by an equivalent mutual flux φ_{21}, shown by the broken lines in Figure 1 (a), linking all the turns of both windings and a leakage flux $\varphi_{\ell 1}$ linking all the turns of winding 1 but none of the turns of winding 2, as shown by the light unbroken lines in Figure 1(a). The magnetic field of an actual transformer then may be visualized in terms of equivalent linkage.

Similarly, if there is a current i_2 in winding 2 while winding 1 is open circuited, the flux distribution is approximately as shown in Figure 2(b). The average flux φ_{22} linking winding 2 is

$$\varphi_{22} = \frac{\lambda_{22}}{N_2} \qquad \text{Eq. (5)}$$

$$\varphi_{12} = \frac{\lambda_{12}}{N_1} \qquad \text{Eq. (6)}$$

Then

$$\varphi_{\ell 2} = \varphi_{22} - \varphi_{12} \qquad \text{Eq. (7)}$$

$$= \frac{\lambda_{22}}{N_2} - \frac{\lambda_{12}}{N_2} \qquad \text{Eq. (8)}$$

The equivalent fluxes φ_{12} and $\varphi_{\ell 2}$ are shown in Figure 2(a).

λ_{11} - The flux linkage of winding 1 produced by the current in winding 1.

φ_{11} - An equivalent flux considered as linking all N_1 turns of winding 1 and producing a flux linkage $N_1 \varphi_{11}$.

λ_{21} - The flux linkage with winding 2 produced by the current in winding 1.

λ_{12} - The flux linkage with winding 1 produced by the current in winding 2.

φ_{21} - An equivalent flux considered as linking all N_2 and producing a flux linkage $N_2 \lambda_{21}$ equal to the actual flux linkage λ_{21}.

λ_{22} - The flux linkage of winding 2 produced by the current in winding 2.

φ_{12} - An average mutual flux linking with winding 1.

$\varphi_{\ell 2}$ - The leakage flux of winding 2 with respect to winding 1.

The above illustration was made when current flows only in the inner winding or in the outer winding. In this manner it is easy to visualize the flux linkages in the iron core. The flux distribution in a transformer depends not only on the geometrical arrangement of its core and windings, but also on the instantaneous magnitudes and directions of the currents.

When there are currents in both windings, the resultant magnetic field depends on the instantaneous values of both currents. Therefore, determining the distribution of the magnetic field is more difficult than when there is current in only one winding. For this reason it is convenient to resolve the resultant fields into the components produced by each current alone.

The resultant fluxes φ_1 and φ_2 can be expressed as

$$\varphi_1 = \varphi_{\ell 1} + \varphi_{21} \qquad \text{Eq. (9)}$$

$$\varphi_2 = \varphi_{\ell 2} + \varphi_{12} \qquad \text{Eq. (10)}$$

$\varphi_{\ell 1}$ and $\varphi_{\ell 2}$ are the component leakage fluxes produced by each current.

φ_{21} is the component mutual flux produced by i_1.

φ_{12} is the component mutual flux produced by i_2

The resultant magnetic field at any instant can be regarded as the result of superposing these component fields with proper consideration for their instantaneous magnitudes and directions. For the present, consider a transformer having a core of magnetic material whose permeability is constant.

The component fluxes in equation 9 and 10 can also be combined in another way which is particularly convenient for analysis of iron core transformer.

When there are currents in both windings, the resultant flux linking can be expressed as the sum of the components due to each current acting by itself. These components are;

(1) The leakage flux due to the current in the winding,

(2) The component mutual flux due to the current in the winding,

(3) The component mutual flux due to the current in the other winding.

If φ is resultant mutual flux,

$$\varphi = \varphi_{21} + \varphi_{12} \qquad \text{Eq. (11)}$$

Hence equation 9 and 10 can be written as

$$\varphi_1 = \varphi_{\ell 1} + \varphi \qquad \text{Eq. (12)}$$

$$\varphi_2 = \varphi_{\ell 2} + \varphi \qquad \text{Eq. (13)}$$

That is, the resultant flux linking each winding can be expressed as the sum of the leakage flux due to the current in the winding alone plus the resultant mutual flux due to the combined magnetomotive forces of both primary and secondary currents acting simultaneously.

The mutual and leakage fluxes in equation 9 and 10 are merely components, and cannot be identified with any of the lines of force in a map of the resultant magnetic field, except when current is in only one winding as in Figure 1 and 2.

When there are instantaneous currents in both windings, most of the air-borne flux links whichever winding has the greater instantaneous magnetomotive force. Therefore the magnetic field that links one winding without linking the other does not represent the component leakage flux of this winding with respect to the other. (6, p. 324).

Most of the mutual flux in an iron core transformer is confined to the core. Because of the magnetic nonlinearity of the iron, the mutual flux is not proportional to the magnetomotive force producing it. The leakage flux, however is in air for a considerable portion of the length of its path. Hence the reluctance of the iron portion of the leakage flux path is small compared with the reluctance of the paths in air. (6, p. 317).

Therefore, in spite of the magnetic nonlinearity of the iron, the leakage flux is nearly directly proportional to the current producing it. This important property of the leakage flux greatly simplifies the analytical treatment of iron core transformers.

A graphical method of solution is very useful as a means for determing the flux. For series connected air and iron core paths, (6, p. 70-73)

$$F = U_a + U_i \qquad \text{Eq. (14)}$$

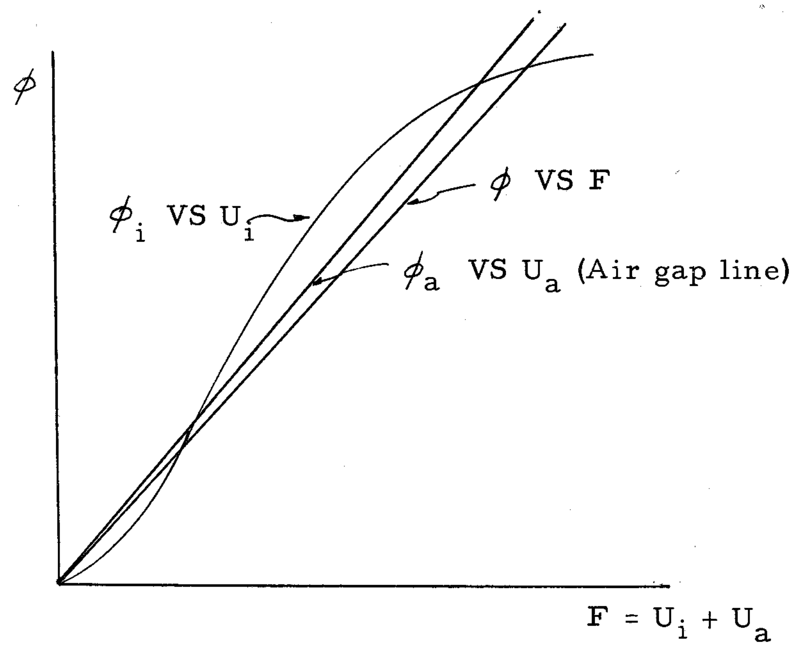


Figure 3. Construction for graphical solution for flux in an iron core and with an air gap.

Where the subscripts (a) and (i) refer to the air and the iron respectively. Equation 14 is satisfied for a graphical solution because the leakage flux comes out of the iron core, through the air and then goes back to the iron as shown in the Figures 1 and 2.

The curve for the air gap and iron combined is then obtained by plotting arbitrary values of flux as a function of the corresponding sum of U_i and U_a or F.

F - Magnetomotive force (Total).

U_a - Magnetizing force in the air gap.

U_i - Magnetizing force in the iron core.

According to these illustrations it will be assumed that leakage flux in a transformer is linear and proportional to the current producing it.

LEAKAGE REACTANCE IN TRANSFORMERS

When the resultant flux is expressed as the sum of the components in Equation (12) and (13), the transformer equations become

$$V_1 = R_1 i_1 + N_1 \frac{d\varphi_{\ell 1}}{dt} + N_1 \frac{d\varphi}{dt} \qquad \text{Eq. (15)}$$

$$V_2 = -R_2 i_2 - N_2 \frac{d\varphi_{\ell 2}}{dt} + N_2 \frac{d\varphi}{dt} \qquad \text{Eq. (16)}$$

Each terminal voltage can hence be expressed as the sum of a resistance drop, a voltage induced by leakage flux and voltage induced by the resultant mutual flux. The component leakage fluxes $\varphi_{\ell 1}$ and $\varphi_{\ell 2}$ induce voltages in only the winding with which each is associated but the resultant mutual flux φ links both windings and induce in them voltages whose ratio nearly equals the turns ratio.

In spite of the magnetic nonlinearity of the iron core, the components of leakage flux are nearly directly proportional to the currents producing them, since the paths of these fields are in air for a considerable portion of their lengths. Hence the components of the self-inductance of each winding due to the leakage flux is very nearly constant. Thus it is convenient to introduce inductance parameters to account for the voltage induced by the leakage flux.

The component of self inductance of winding 1 due to the leakage flux $\varphi_{\ell 1}$ with respect to winding 2 is defined as the leakage inductance of winding 1 with respect to winding 2. Thus the leakage inductance $L_{\ell 1}$ of winding 1 with respect to winding 2 is the leakage flux linkages per unit current, or

$$L_{\ell 1} = \frac{N_1 \varphi_{\ell 1}}{i_1} \qquad \text{Eq. (17)}$$

Similarly, the leakage inductance $L_{\ell 2}$ of winding 2 with respect to winding 1 is

$$L_{\ell 2} = \frac{N_2 \varphi_{\ell 2}}{i_2} \qquad \text{Eq. (18)}$$

The leakage inductance is a property of one winding with respect to another. If the transformer has more than two independent windings the leakage inductance of the primary with respect to the secondary generally differs from the leakage inductance of the primary to the third or tertialy winding. (6, p. 325).

If the voltage induced by the primary and secondary leakage fluxes are expressed in terms of the leakage inductance, Equation (15) and Equation (16) become

$$V_1 = R_1 i_1 + L\rho_1 \frac{di_1}{dt} + N_1 \frac{d\varphi}{dt}$$

$$= R_1 i_1 + L\rho_1 \frac{di_1}{dt} + e_1 \qquad \text{Eq. (19)}$$

$$N_2 \frac{d\varphi}{dt} = R_2 i_2 + L\rho_2 \frac{di_2}{dt} + V_2$$

$$e_2 = R_2 i_2 + L\rho_2 \frac{di_2}{dt} + V_2 \qquad \text{Eq. (20)}$$

Where e_1 and e_2 are the voltage induced by the resultant mutual flux φ. The resistances and leakage inductances in Equation (19) and (20) are essentially constant parameters. The only effect of the nonlinear magnetic properties of the core on the Equation (19) and (20) are in the relation between the resultant mutual flux φ and the magnetomotive force required to produce it.

Examination of Equation (19) shows that it applies to a circuit in which the primary terminal voltage V_1 is impressed on the primary resistance and leakage inductance in series with the counter electromotive force e_1 induced in the primary by the resultant mutual flux, as shown in the following figure.

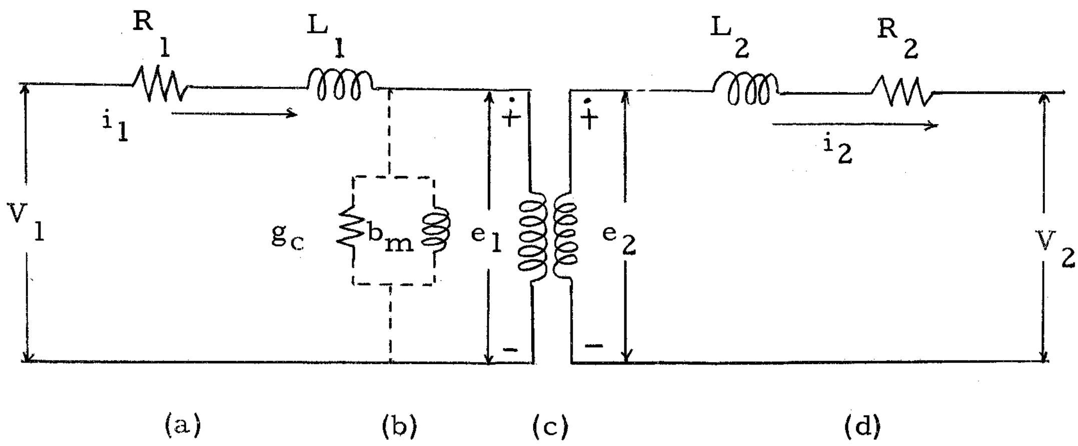


Figure 4. An equivalent circuit representing an iron core transformer. The excitation characteristics are represented at (b). The turns ratio of the ideal transformer (c) equals N_1/N_2

The primary and secondary voltage e_1 and e_2 induced by the resultant mutual flux is directly proportional to the number of turns on each winding, as are the terminal voltage of an ideal transformer.

Since the exciting current depends on the resultant mutual flux the exciting current i'_φ can be accounted for by connecting an iron core reactor (6, p. 195) in parallel with the voltage e_1 induced by the resultant mutual flux as in part (b) in Figure 4. The core loss and excitation characteristics of this reactor are assumed to be those of the actual transformer. The reactor winding resistance is also assumed zero.

The voltage equations for the primary and secondary windings, Equation (15) and (16) can be written in phasor form, as

$$\overset{o}{V}_1 = (R_1 + jX\rho_1)\overset{o}{I}_1 + \overset{o}{E}_1 \qquad \text{Eq. (21)}$$

$$\overset{o}{E}_2 = (R_2 + jX\rho_2)\overset{o}{I}_2 + \overset{o}{V}_2 \qquad \text{Eq. (22)}$$

Where

$$X\rho_1 = wL\rho_1 \qquad \text{Eq. (23)}$$

$$X\rho_2 = wL\rho_2 \qquad \text{Eq. (24)}$$

$\overset{o}{V}_1$ and $\overset{o}{V}_2$ are the phasors representing the terminal voltage, $\overset{o}{E}_1$ and $\overset{o}{E}_2$ are the phasors representing the voltage induced by the resultant mutual flux,

$\overset{o}{I}_1$ and $\overset{o}{I}_2$ are the phasors representing the currents,

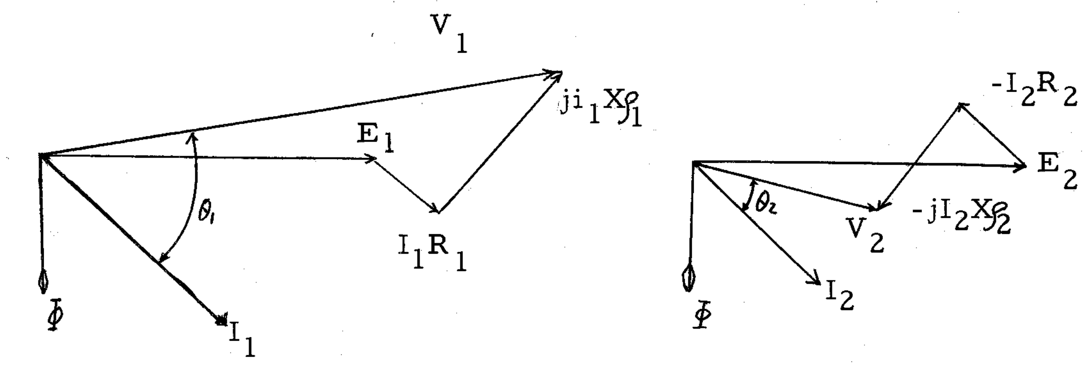
R_1 and R_2 are the effective resistance of the winding,

X_1 and X_2 are the leakage reactance.

Then the phasor diagrams of Equation (21) and (22) will be as in Figure 5.

The voltage E_1 and E_2 are induced by the resultant mutual flux and effective values are related to the maximum voltage of this flux. (6, p. 168) The magnitudes of E_1 and E_2 are proportional to the number of turns in the winding.

In Figure (4), the effects of the magnetic non-linearity of the iron core are separated from the linear leakage reactance by the



(a) (b)

Figure 5. Phasor diagrams for a transformer. The primary voltage and currents are shown in (a) and secondary voltage and currents are in (b).

shunt reactor representing the excitation characteristics. Except for this one consideration the rion core transformer has essentially the properties of a linear circuit, since the winding resistances and leakage inductances are very nearly constant In many problems involving the characteristics of a transformer as a circuit element, the core loss may be neglected and the resultant core flux may be assumed to be instantaneously proportional to themagnetizing current. That is, for a given operating condition, it may be assumed that the self and mutual inductances do not vary cyclically, whence the classical theory of magnetically coupled circuits may be applied to the iron core transformer.

In spite of the magnetic nonlinearity of the core, the characteristics of a transformer depends ultimately on an essentially linear magnetic leakage and on nonlinear exciting current which, however, is often so small that its peculiarities may be neglected.

Hence, at the beginning of the analysis, the transformer is considered essentially as a linear circuit element. (8, p 329)

PULSED LINEAR NETWORKS ANALYSIS

According to the preceding chapter, the leakage reactance of a transformer is a linear element and it is possible to determine the equivalent circuit of a transformer as a linear network. The impulse-response characteristics of a series or parallel network that contains resistance, inductance, and capacitance was investigated to obtain mathmetical expressions for the currents and voltages in the linear R-L-C network.

The application of an impulse current source to a circuit having capacitance, resistance and inductance demonstrates the following facts.

If there is an impulse current in the inductance, as in Figure 6 (a), the voltage V_{ab} must be L_{ab} times the rate of change of this current. Since $i(t) = 0$ for $t = 0$ and $i_{ab} = 0$ for $t>0$, the voltage V_{ab} must be zero for $t<0$ and for $t>0$. And $t = 0$, however, the rate of change of current is infinite; hence the voltage across the inductance is infinite at $t = 0$ for an infinitesimal time interval. A discontinuous current in an inductance requires an impulse of voltage across it.

On the other hand the impulse current through a capacitance can change discontinuously without a discontinuity in the voltage across it.

This means there is a charge on the capacitor after the impulse disappears. Then the voltage and current equations for the capacitance are

$$V_{ab} = 0 \qquad t<0 \qquad \text{Eq. (27)}$$

$$V_{ab} = V \qquad t>0 \qquad \text{Eq. (28)}$$

$$i_{ab} = C_{ab} \frac{dV_{ab}}{dt} \qquad \text{Eq. (29)}$$

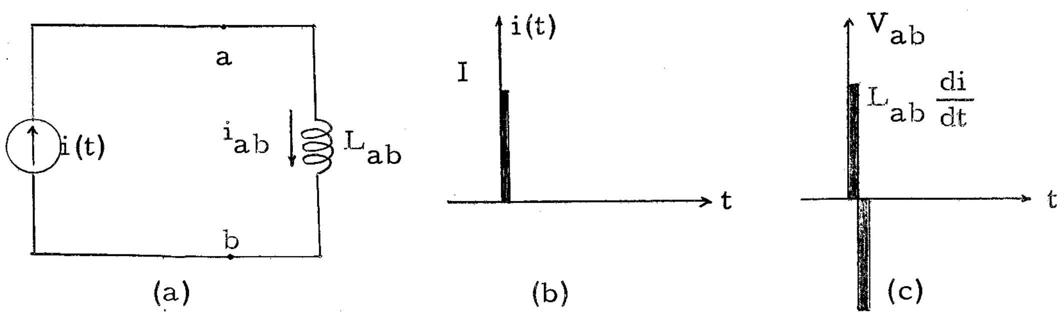


Figure 6 Impulse current source applied to an inductance.

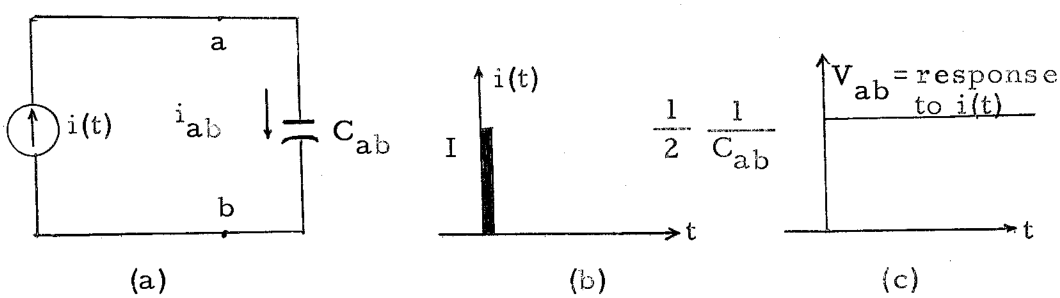


Figure 7. Impulse current source applied to a capacitance.

It is obvious that, from the definition of resistance, the ratio $V_{ab}/i_{ab} = R_{ab}$ is independent of the wave form of the current. But the voltage-current ratio for an inductance L_{ab} or a capacitance C_{ab}, depends on the wave form of i_{ab} and hence will be a function of time. For a current impulse source, current is a function of time, but the variation of the current with time will not be affected by the nature or the magnitude of the circuit elements connected to the terminals of the source and hence i(t) will be independent of V_{ab}.

To obtain the response of combinations of circuit elements to a current impulse source, inhomogeneous differential equations must be solved. The equivalent circuit of a transformer as in Figure 8 was studied to obtain the impulse response characteristics.

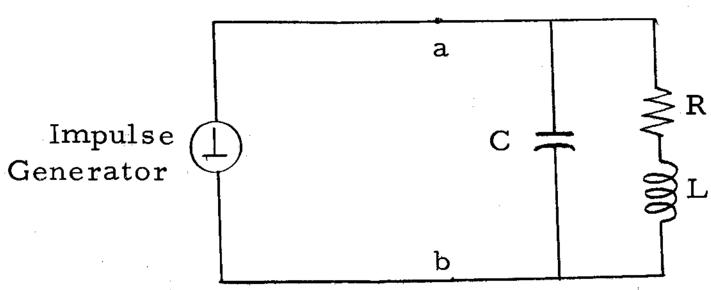


Figure 8 Parallel RLC network with a current impulse generator.

When the parallel network of Figure 8 was connected to a current impulse generator, the impulse left the capacitor charged and zero current in the inductor at $t = 0^+$. After the impulse disappeared, the energy stored in the capacitance discharged through the resistance and inductance. The energy relation between the capacitance and inductance results in oscillations with the amplitude of the oscillations dimishing exponentially due to the resistance of the circuit elements. The complicating factor arises from the fact that the roots of the auxiliary equation, which largely determine the form of the response wave, are dependent upon the relative values of resistance, inductance and capacitance in the network. The type of circuit response is determined by the relative values of the

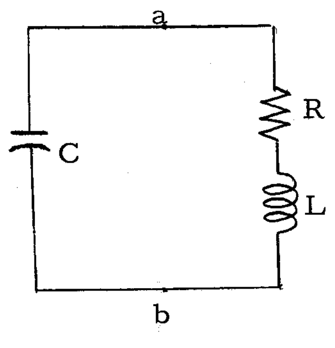


Figure 9 Deenergizing circuit for a series RLC network circuit

parameters in the circuit. The value of resistance usually determines whether the response will be overdamped, underdamped or oscillatory or critically damped. Each type has physical significance and typical response equations, but in this thesis only the oscillatory case is of interest and used in the investigation.

The application of Kirchhoff's law to the circuit of Figure 9 gives the equation, (4, p. 141)

$$V_{ab}(t) = A_1 \epsilon^{s_1 t} + A_2 \epsilon^{s_2 t} \qquad \text{Eq. (30)}$$

Three types of response conditions can be defined, depending on the nature of the characteristic roots. The underdamped or oscillatory case is investigated in the following discussion.

For the oscillatory case the circuit values should be such that,

$$(R/2L)^2 < 1/LC$$

The general form of the solution in the underdamped case can be written as follows, (4, p. 145)

$$V_{ab}(t) = 2\mathrm{Re}\,(A_1 \epsilon^{s_1 t})$$

if $K = 2A_1$

$$V_{ab}(t) = \mathrm{Re}\,(K \epsilon^{s_1 t}) \qquad \text{Eq. (31)}$$

Where

$$A_1 = \frac{I_o/C - s_2V_o}{s_1 - s_2}$$

$$A_2 = \frac{s_1V_o - I_0/C}{s_1 - s_2}$$

$$s_1 = -\alpha + jw_d \qquad \text{and} \qquad s_2 = -\alpha - jw_d$$

In Equation (31) the value of the complex number $K = 2A_1$ will depend on the initial conditions, as well as the circuit elements. At any rate K can be written in the exponential form $K = K\epsilon^{j\gamma}$, where K and γ will be evaluated from the following equations,

$$V_{ab}(0) = V_0 \text{ and } (dV_{ab}/dt)_0 = \frac{1}{C} I_0$$

By convention K is always a positive number, and the value of γ, depending on the initial conditions, may be anywhere between zero and 2π. In contrast s_1 is a complex number whose value is determined by the circuit elements only and is independent of the initial conditions.

$$S_1 = -R/2L + j\sqrt{\omega_o^2 - \alpha^2} = -\alpha + j\omega_d \qquad \text{Eq. (32)}$$

or

$$S_1 = \sqrt{\alpha^2 + \omega_d^2} \,\angle \tan^{-1}\frac{\omega_d}{-\alpha} \qquad \text{Eq. (33)}$$

Where

$$\omega_0 = 1/\sqrt{LC} \qquad \alpha = R/2L$$

Using the exponential form of K in Equation (31),

$$V_{ab} = \mathrm{Re}\left[K\epsilon^{j\gamma}\epsilon^{(-\alpha+j\omega_d)t}\right]$$

$$= \mathrm{Re}\left[K\epsilon^{-\alpha t}\epsilon^{j(\omega_d t+\gamma)}\right]$$

$$= K\epsilon^{-\alpha t}\cos(\omega_d t + \gamma) \qquad \text{Eq. (34)}$$

The complex number $s_1 = -\alpha + j\omega_d$ is called the complex radian frequency of the function $\epsilon^{s_1 t}$. Complex frequencies are characteristics of damped oscillations. The real part of the complex frequency is the attenuation factor of the natural mode of a source-free circuit, and its imaginary part, ω_d is the radian frequency of the oscillatory term of the mode.

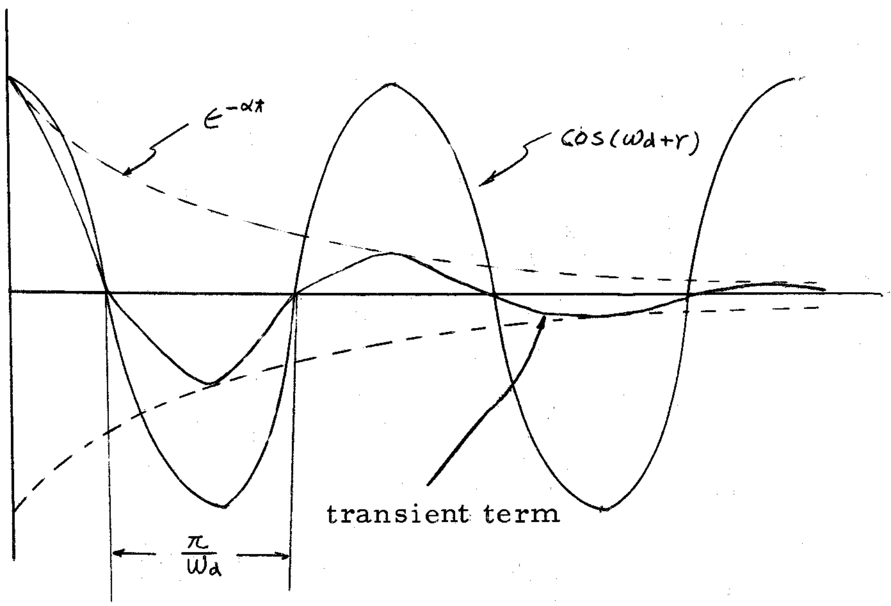


Figure 10. Construction of a wave form in the under-damped case.

The oscillation of the capacitor potential after the impulse disappears can be represented by Equation (34) and by the wave form in Figure 10. The amplitude of the oscillation decreases exponentially and finally approaches the steady state value of zero. The energy exchange between inductance and capacitance is the same as the energy transformation of the electromagnetic to the electrostatic field. And each time, the energy transfer occurs part of the energy is dissipated in the resistance. This is why the energy in the circuit is reduced to zero. The frequency of oscillation will depend upon the value of the parameters in the circuit. This is indicated by the term "w_d" in Equation (32).

The current behavior can be visualized by considering the slope of the capacitor voltage, the slope being proportional to the current. Each time the capacitor voltage is a maximum or a minimum, the instantaneous current in the network is zero, and each time the slope of the capacitor voltage is a maximum, point of inflection or zero value, the instantaneous current is a maximum.

IMPULSE TEST

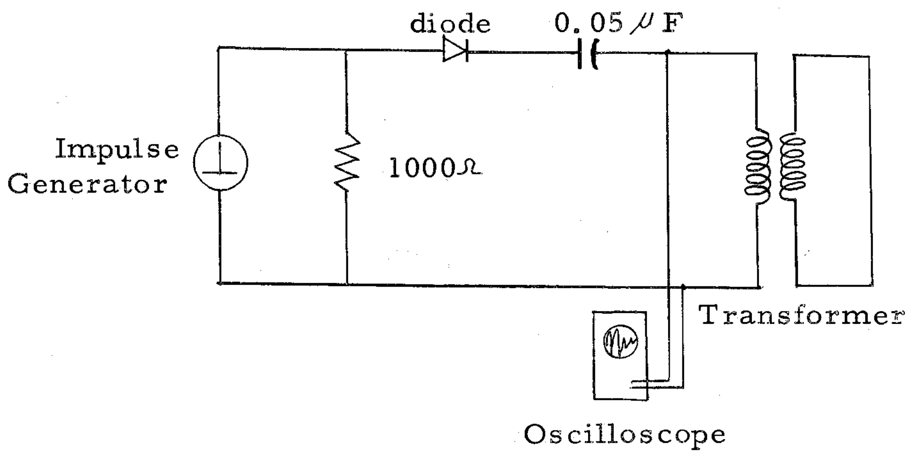
One RLC network was analyzed in the previous section in terms of the potential across the capacitance. The application of this linear network theory to the equivalent transformer circuit was used to find the response after an impulse is applied to the network. The test circuit was arranged as in Figure 11, and the impulse current source connected to the primary or secondary of the transformer. The connections of the test transformer are shown in Table III of this section.

In the transformer, the distributed capacitance is very important when there are high frequencies. Only the primary equivalent capacitance of the transformer is considered, because the distributed capacitance configuration is difficult to determine. The effects of the secondary distributed capacitance are neglected because the secondary winding is short-circuited.

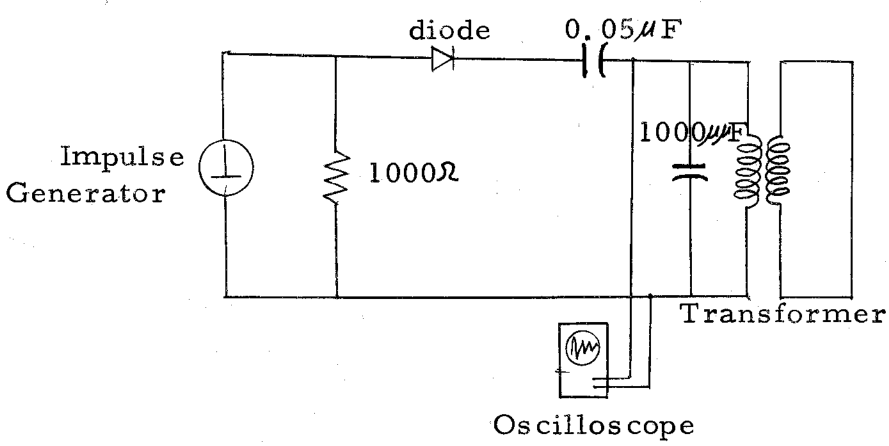
When an impulse current is applied to the equivalent circuit of a transformer, the inherent primary capacitance of the transformer is charged and after the impulse disappears this capacitance will discharge through the resistance and inductance element of the equivalent circuit. It is noted from the linear circuit analysis, that the voltage across the inductance will be zero when an impulse current disappears.

The response voltage will oscillate if the parameters of the circuit are such that $(R/2L)^2 < 1/LC$. Thus the capacitance value should be small to cause oscillations in the circuit.

The equivalent capacitance C_0 is considered to be parallel with



(a)



(b)

Figure 11. Impulse-test circuit. (b) shows the connection of a capacitance across the transformer input.

the low frequency transformer equivalent impedance. In Figure 12, the leakage inductance of the equivalent circuit of the transformer, is constand and linear and the resistance is assumed very small.

If the resistance is neglected, the resonant frequency of this parallel circuit is

$$F_1 = \frac{1}{2\pi\sqrt{LC_0}} \qquad \text{Eq. (35)}$$

In Equation 35, there are two unknown elements, thus to find the leakage inductance L, a known capacitance C_1 is connected in parallel with the equivalent capacitance C_0. This is indicated in Fibure 11(b). The resonant frequency equation can then be written as follows, provided that R is negligible,

$$F_2 = \frac{1}{2\pi\sqrt{LC_2}} \qquad \text{Eq. (36)}$$

Where C_2 is equal to $C_1 + C_0$

Substituting the C_2 equality in Equation 36 yields the following equation in terms of the known value of C_1

$$2\pi F_2 = \frac{1}{\sqrt{L\ (C_1 + C_0)}}$$

$$L(C_1 + C_0) = \frac{1}{4\pi^2 F_2{}^2}$$

Solving Equation 35 for C_0 and substituting in the above equation yields the following equation for the determination of L.

$$L = \frac{1}{4\pi^2 C_1} \left(\frac{F_1^{\,2} - F_2^{\,2}}{F_1^{\,2} F_1^{\,2}} \right) \qquad \text{Eq. (37)}$$

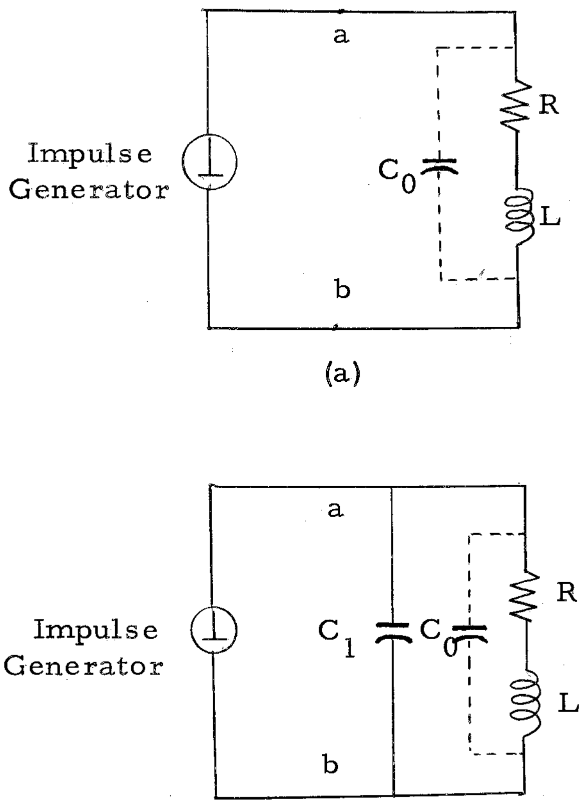


Figure 12. Simplified equivalent circuit of figure 11 (a) and (b).

The leakage inductance can be calculated from Equation 37. The capacitance C_1 is known and the frequency F_1 and F_2 can be obtained from the response voltage of the transformer as shown in Figures 13 to 18.

From the oscillograms, the frequency will be,

$$F = \frac{1}{T}$$

If the capacitance value of the equivalent parallel circuit of Figure 12 is changed, the frequency of the oscillation will be changed. Thus the frequency change will depend upon the capacitance value of the circuit. For more accurate calculations the value of the capacitors used in the test circuit were measured with an impedance bridge (General Electric, Type 650-A. serial no. 1181). The 0.05μF and 1000$\mu\mu$F capacitors were corrected to 0.0508μF and 1015$\mu\mu$F each. The corrected values were used in the calculations to obtain the leakage inductance.

The response voltages of the transformer were photographed and are shown in Figures 13 to 18. The resistance was assumed to be very small because the oscillatory response amplitude decreased exponentially at a slow rate.

It is obvious that the resonant frequency is independent of the magnitude of the charge on the capacitance and is dependent on the

R, L, and C of the circuit. In other words, the resonant frequency and the damping constant are both independent of the initial conditions.

From Equation 34 it is noted that the oscillatory wave form is a cosine term and the radian frequence is a function of $\epsilon^{S_1 t}$. The complex number S_1 of Equation 32 is called the complex frequency of the function .

The maximum and the minimum values of the wave form occur at the position of the maxima and minima of the cosine function.

The calculated values of leakage inductance and reactance from the impulse tests are shown in Table I. The comparison of the transformer impulse and short circuit tests are shown in Table II. The leakage reactance obtained from both tests are the equivalent leakage reactance of the primary and secondary referred to the input winding.

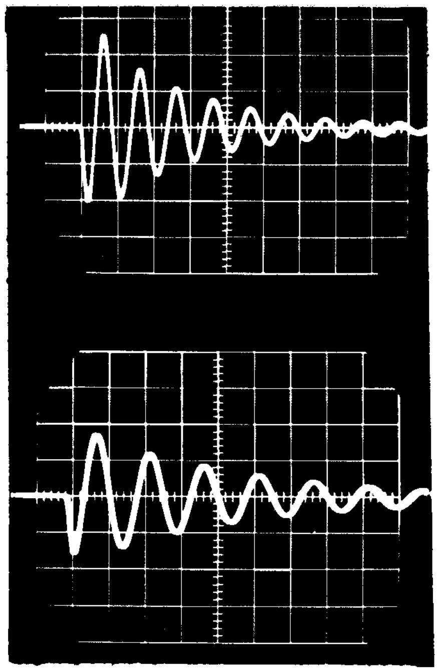


Fig. 13. Response voltage of the transformer Fig. 9. Test connection 1, Table III.

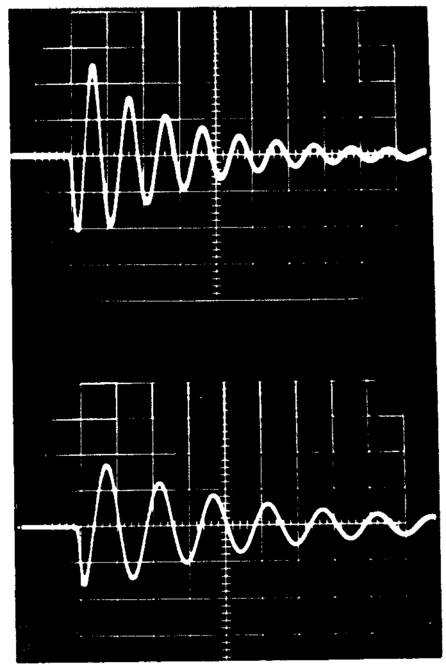


Fig. 14. Response voltage of the transformer Fig. 9. Test connection 2, Table III.

Top view (circuit A) Figure 9.

Bottom view (circuit B) Figure 9.

Horizontal scale- 2 microseconds/div.

Vertical scale - 5 volts/div.

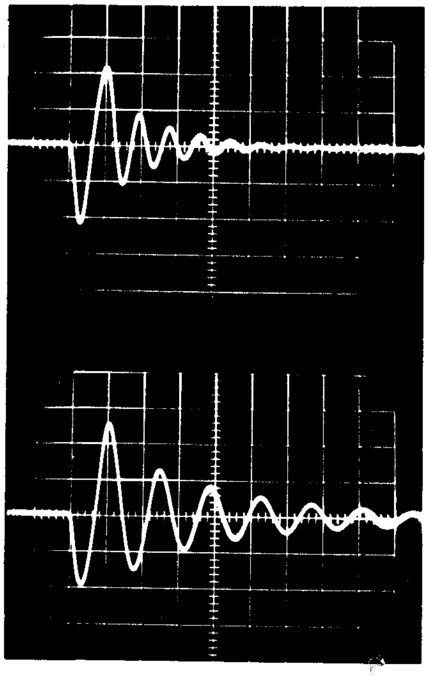


Fig. 15. Response voltage of the transformer Fig. 9. Test connection 3, Table III.

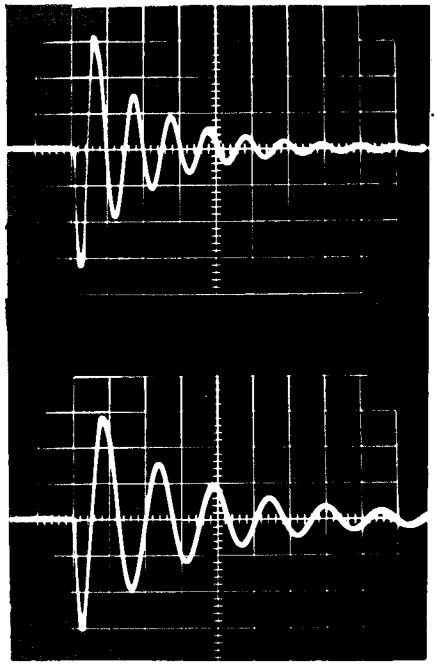


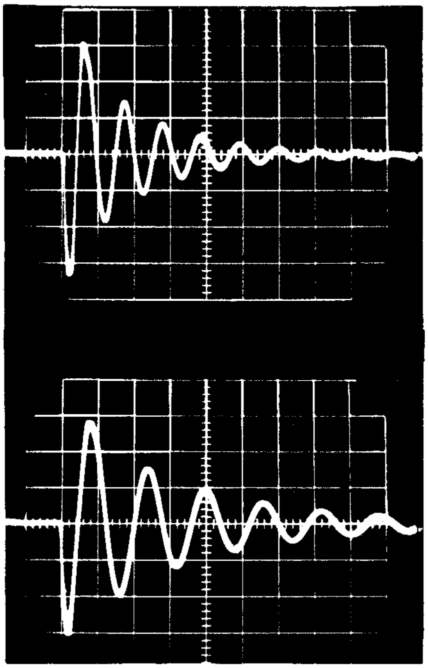
Fig. 16. Response voltage of the transformer Fig. 9. Test connection 4, Table III.

Top view (circuit A) Figure 9.

Bottom view (circuit B) Figure 9.

Horizontal scale - 4 microseconds/div

Vertical scale - 5 volts/div



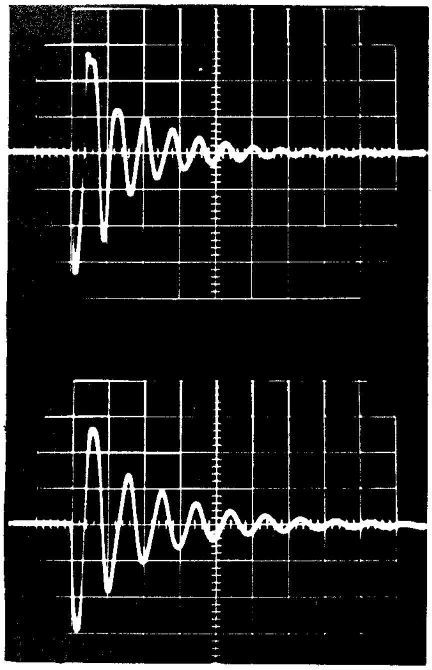


Fig. 17. Response voltage of the transformer Fig. 9. Test connection 5, Table III.

Fig. 18. Response voltage of the transformer Fig. 9. Test connection 6. Table III.

Top view (circuit A) Figure 9.

Bottom view (circuit B) Figure 9.

Horizontal scale - 4 microseconds/div.

Vertical scale - 5 volts/div.

TABLE I

CALCULATED VALUE OF REACTANCE AND INDUCTANCE FROM THE IMPULSE TEST

Test No.	Impulse Generator		Calculated value	
	Frequency (cps)	Pulse width (μ sec)	Inductance milli-henry	Reactance ohms
1	10,000	0.5	0.125	0.04715
2	10,000	0.5	0.125	0.04715
3	30,000	0.5	0.050	0.01885
4	10,000	1.0	0.540	0.20330
5	10,000	1.0	0.580	0.21850
6	10,000	1.0	0.160	0.06030

TABLE II

COMPARISON OF TRANSFORMER OF LEAKAGE INDUCTANCE AND REACTANCE

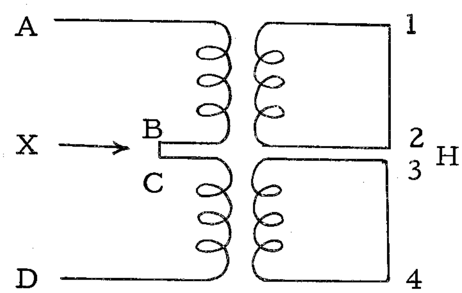
Test No.	Short circuit test		Impulse test	
	Inductance milli-henry	Reactance ohms/60cps	Inductance milli-henry	Reactance ohms
1	0.1301	0.0490	0.125	0.04715
2	0.1288	0.0485	0.125	0.04715
3	0.0548	0.0200	0.050	0.01885
4	0.577	0.2190	0.540	0.20330
5	0.594	0.2236	0.580	0.21850
6	0.1832	0.0693	0.160	0.06030

TABLE III

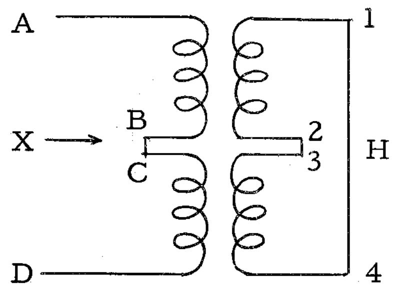
CONNECTION OF TRANSFORMER USED FOR IMPULSE AND SHORT CIRCUIT TEST

(Transformer: 5KVA. 120/240V. 240/48OV. 1.8% impedance 60cps).

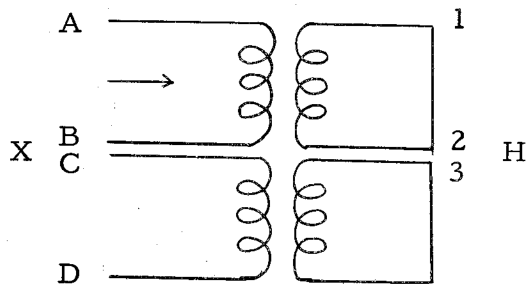
Test No. 1.



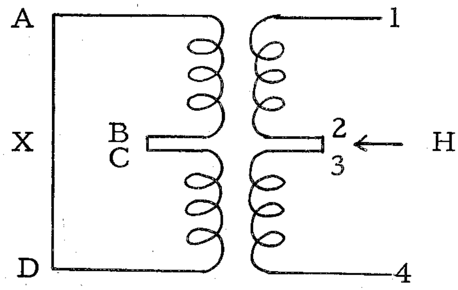
Test No. 2.



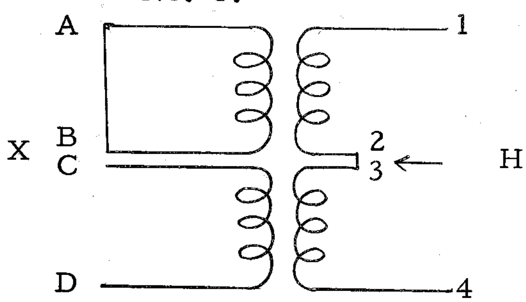
Test No. 3.



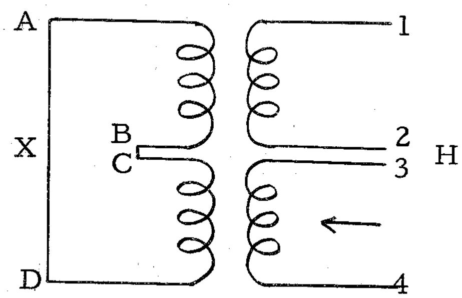
Test No. 4.



Test No. 5.



Test No. 6.



1. Arrow shows input of the short-circuit test.
2. X - Low voltage side 120/24OV.
3. H - High voltage side 240/48OV.

CONCLUSION

The leakage reactance of power system transformers is a very important parameter for the users and designers of the transformers. The characteristics of a transformer depend to a considerable extent on magnetic leakage, thus the accurate determination of the leakage reactance is necessary.

Ordinarily, the leakage reactance is determined by the short circuit method. The leakage reactance of the transformer was found to be linear and constant. In this thesis, an impulse driving function was applied to determine the leakage reactance of the transformer. The impulse response characteristics of the linear circuit were studied and applied to the equivalent transformer circuit.

The leakage reactance calculated from the impulse test data on the transformer is the equivalent leakage reactance of the input winding with the other winding short circuited.

As a control, the short-circuit test was made using a voltmeter, ammeter and wattmeter. The calculated values of leakage inductance and reactance by using an impulse driving function are approximately 4% to 7% lower than those obtained by the short circuit test (See Appendix A for the short circuit test results).

There will be a 2% or 3% error in the short circuit test result, because of instrument and reading errors. If it is assumed that the values obtained by the short circuit test are accurate, then the leakage reactance obtained by using an impulse driving function are accurate within the range of the oscilloscope accuracy.

The impulse driving function method is very simple and seems to be applicable to most power system transformers in determining the leakage reactance. The equivalent circuit of a transformer can be simplified to that of a linear parallel R-L-C network, and the desired oscillatory response can be obtained by changing the parallel capacitance added to the transformer input terminals.

The impulse driving function method is very simple and gives significant results in determining the leakage reactance of a transformer.

BIBLIOGRAPHY

1. Allibone, T. E. D. B. MeKenzi and F. R. Perry. The effects of impulse voltage on transformer winding. Journal of the Institution on Electrical Engineers 80:117-135. 1937.

2. Bewely, L. V. Transient oscillations in distributed circuits with special reference to transformer rindings. American Institute of ~~of~~ Electrical Engineers Transactions 50:1215-1220. 1931.

3. Blume, L. F. Transformer engineering. New York, Wiley, 1956. 494 p.

4. Brenner, Egon and Javid, Mansour. Analysis of electric circuits. New York, McGraw-Hill, 1959. 703 p.

5. Ernest, Frank. Pulse linear network. New York, McGraw Hill, 1945. 267 p.

6. Golding, E. W. Electrical measurements and measuring instruments. London, Pitman and Sons, 1949. 698 p.

7. Holcom, J. E. Impulse test on low-voltage windings of distribution transformers. American Institute of Electrical Engineers 74:1152-1154. 1955.

8. Massachusetts Institute of Technology. Dept. of Electrical Engineering. Magnetic circuits and transformers. New York, Wiley, 1958. 718 p.

INDEX TO THE APPENDIX

INDEX

APPENDIX A

SHORT CIRCUIT TEST

The short-circuit impedance of a transformer is the impedance measured at the terminals of one winding when the other winding is short circuited. The short circuit test data were taken for a comparison with the impulse data in the determination of the equivalent circuit of the transformer. The test transformer is schematically shown in Figure A-1 and has the following ratings.

Capacity	Primary	Secondary	Impedance	Frequency
5KVA	120V 240V	240V 480V	1.8%	60 cps

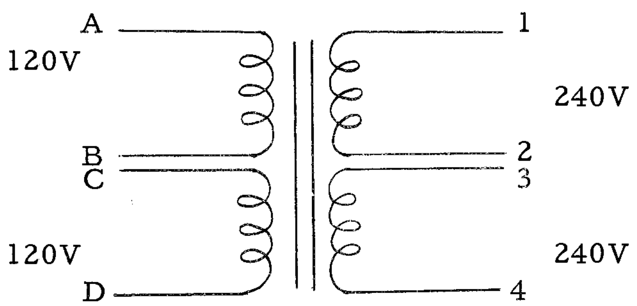


Figure A-1 Circuit of transformer used for test.

Two VARIACS were used to control the voltage and current for the tests. The test circuit is shown in Figure A-2. Six (6) tests were made for the determination of the equivalent impedance, from which resistance, reactance and inductance values were calculated. If the secondary winding of the transformer is short circuited the equivalent circuit of the transformer is shown in Figure A-3.

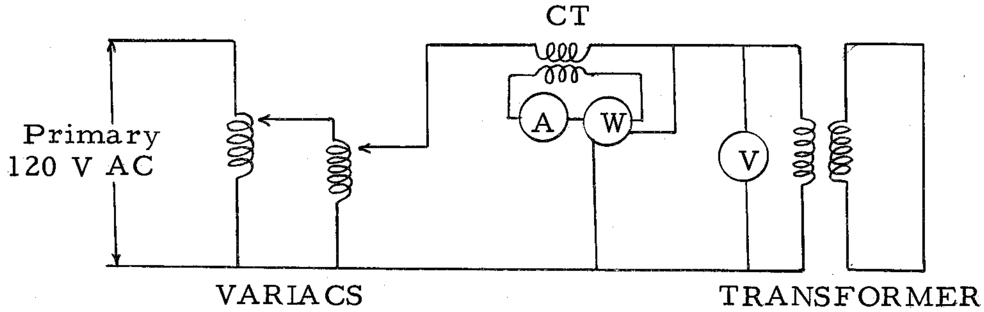


Figure A-2. Short circuit test arrangement.

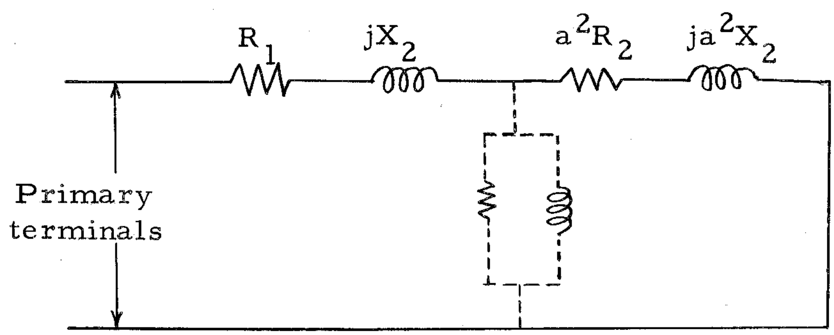


Figure A-3. Equivalent circuit of a short circuited transformer referred to the primary.

If the exciting current is neglected, Figure A-3 shows that the impedance at the primary terminals equals the equivalent impedance of the transformer referred to the primary. The short circuit test was made by the application of current values equal to 50%, 75%, 100% and 120 percent of the rated current.

The applied voltage under short circuit conditions is called the impedance voltage and when the current is equal to the rated value, this potential is called the impedance voltage. The full-load impedance voltage in this test is 1.8 percent of the transformer rated voltage. Thus the core flux during the short circuit test is only a few percent of its value for rated voltage operation.

In spite of this fact, however, the leakage reactance, being practically independent of saturation, has essentially the same value for short circuit as for normal operating conditions.

When the primary current is equal to the rated value and the exciting current is negligible, the secondary current is also equal to its rated value. To produce this rated secondary current at short circuit, the voltage induced in the secondary must equal the secondary leakage impedance of voltage drop. In this transformer this voltage is about 0.9 percent of the rated voltage, that is, the core flux is about 0.009 of its value at no load with rated voltage. If magnetic saturation is neglected, the exciting current on short circuit therefore is about 0.009 of its normal value. Since the exciting current at rated voltage usually is about 5 percent of rated current, the exciting component of the primary current when the secondary is short circuited is only about 0.009x5 or 0.045 percent of the primary current and is quite insignificant, if linearity is assumed. Also, since the core loss varies approximately as the square of the flux, the core loss at short circuit is approximately $(0.009)^2$ or 0.000081 of the normal voltage core loss. Since the normal voltage core loss usually is less than the combined primary and secondary copper losses at rated current, the core loss at short circuit should be less than 0.000081 of the copper loss and this is entirely negligible. Hence, on short circuit, the power input P, very nearly equals the equivalent resistance loss, I^2R_{eq}.

If the corrected readings of the voltmeter and wattmeter in the test are V, I and P, the components of the short circuit impedance are,

$$Z_{sc} = \frac{V}{I} \approx Z_{eq} \qquad \text{Eq. (A-1)}$$

$$R_{sc} = \frac{P}{I_2} \approx R_{eq} \qquad \text{Eq. (A-2)}$$

$$X_{sc} = \sqrt{Z^2_{sc} - R^2_{sc}} \approx X_{eq} \qquad \text{Eq. (A-3)}$$

$$L_{sc} = \frac{X_{sc}}{\omega} = \frac{X_{eq}}{2\pi f} \approx L_{eq} \qquad \text{Eq. (A-4)}$$

Z_{sc} - Short-circuit impedance
R_{sc} - Short-circuit resistance
X_{sc} - Short-circuit reactance
L_{sc} - Short-circuit inductance

Z_{eq} - Equivalent impedance
R_{eq} - Equivalent resistance
X_{eq} - Equivalent reactance
L_{eq} - Equivalent inductance

In these tests, either the low voltage or the high voltage winding was used as the primary. The calculated equivalent impedance is then the total impedance value referred to the excited winding. If desired, this impedance value may be referred to the other winding base by a factor equal to the turns ratio squared. Thus if $Z_{eq}X$ is the equivalent impedance referred to or measured on the low voltage side, the equivalent impedance $Z_{eq}H$ referred to the high voltage side is,

$$Z_{eq}H = a^2 Z_{eq}X \qquad \text{Eq. (A-5)}$$

Where a is the turns ratio $\frac{N_H}{N_X}$

Figure A-4 shows the equivalent circuit as it would be determined using Equations A-1, A-2, and A-3.

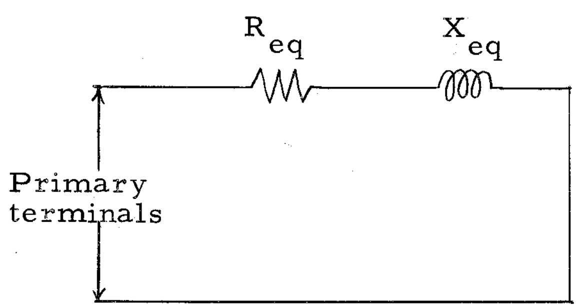


Figure A-4 Equivalent circuit of a short circuited transformer.

Figure A-4 shows that the exciting admittance of Figure A-3 is neglected and is the approximate equivalent of the transformer when short circuited.

Table IV shows the test and calculated results.

The short circuit reactance is assumed to be due to magnetic leakage flux except for the effects of distributed capicitance and skin effect. (2, p. 1215-1220). In this test the effect of distributed capacitance is neglected. Thus, the short circuit resistance and inductance are substantially constant.

In this short circuit test the values read from the instruments were corrected for instrument losses. The voltmeter and wattmeter have I^2R losses (6, p. 589) and errors (6, p. 591) produced by heat generated in the instruments, or changes in temperatures. The readings of the wattmeter and ammeter are actually higher than the true values. The wattmeter correction was made in the form of; True power = Wattmeter reading - (wattmeter potential coil power loss + voltmeter power loss).

The current transformer may introduce errors. The secondary current of a current transformer is displaced in phase from the primary current, because the existance of the magnetizing and iron loss component of the primary current. This causes the angle of the secondary current to be different than the angle of the primary current relative to a common potential.

The current ratio of the transformers differs from the mere "turns ratio"(number of secondary turns/number of primary turns) by an amount that depends upon the magnitude of the exciting current of the transformer. The current ratio is, therefore not constant under all conditions of load, and the error so introduced may be of considerable importance (6, p. 651).

The wattmeter will read high for lagging power factor loads, since the effect of the inductance of the potential coil is to shift the current in this circuit so that it is more nearly in phase with the load current.

All the instrument corrections were made except for the current transformer phase angle, ratio errors and temperature.

The circuit connections for short circuit and results are in Table III and IV.

TABLE IV

THE 60 CYCLE RESULTANT VALUE OF SHORT CIRCUIT TEST, AND THE CALCULATED VALUES OF IMPEDANCE, INDUCTIVE REACTANCE AND INDUCTANCE.

(TRANSFORMER: 5KVA. 120/240V 240/48OV 1.8% impedance, 60 cps)

Test No.	*Test Value			Calculated Value			
	V volts	A amps	W watts	$Z = \frac{V}{I}$ ohms	$R = \frac{P}{I^2}$ ohms	X(L) ohms	L mh
1	3.65	20.8	72.89	0.1755	0.16850	0.0490	0.1301
2	3.68	20.8	73.75	0.1770	0.17040	0.0485	0.1288
3	1.4	20.8	27.80	0.0673	0.06425	0.0200	0.0548
4	7.25	10.4	71.70	0.6970	0.66250	0.2190	0.5770
5	10.6	10.4	107.50	1.0190	0.99000	0.2236	0.5940
6	2.46	10.4	23.95	0.2318	0.22130	0.0693	0.1832

V. A. W. - Voltmeter, Ammeter and Wattmeter.

*These three values are corrected for instrument error.

INSTRUMENTS USED IN THE TEST

1. Wattmeter:

Waston AC & DC wattmeter   Model 310
Potential circuit

Maximum volts	Res. in ohms(at 25c)	to correct for temperature.
75	1, 477. 4	1-0. 000. 26(t-25c)
150	2, 954. 8	1-0. 000. 34(t-25c)

Approx: Res. of field coils in series 0. 127 int ohms
in multiple 0. 0325 int ohms

Approx: self-inductance of field coil in series
0. 000. 21 henry

self-inductance of field coil in multiple
0. 000. 052 henry

Self-inductance of potential circuit 0. 004. 1 henry

2. Ammeter:

Westinghouse   AC Ammeter

Type PA-5   Style No. 701347
Serial No. 2253218
Frequency rage   D-C to 135c

Rating	Ohms	Millihenries	K
5	.013	.024	-0. 000. 4

3. Voltmeter

General Electric   AC voltmeter

Type P8   No. 627686

Resistance	Rating
99.6	7.5

4. DC voltmeter

Westinghouse Type PX-14

Scale 0-15 F.S. = 1MA
0-150

Style 1208822 Serial No. 2283764

5. Pulse generator

Model 79B

6. Oscilloscope

Model 504 A

APPENDIX B

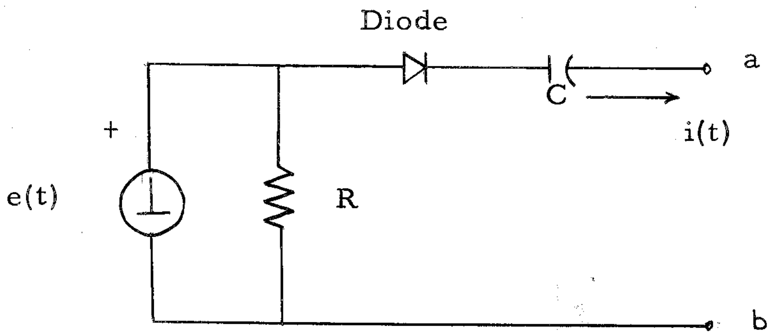


Figure B-1. Circuit used for a current impulse source in impulse test.

In the impulse test, a current source was used instead of a voltage source, because an impulse of current can leave a charge on the capacitance of the equivalent transformer circuit. An impulse function, while having a certain amplitude, lasts for almost zero time, thus it represents a theoretical mechanism for transfering energy in near zero time.

The generator used for the impulse test was a voltage source converted by the circuit in Figure B-1 to an impulse current source. To obtain an equivalent current source from a voltage source, the circuit was arranged according to the Norton theorem illustrated in Figure B-2. For the same terminal characteristics of the two circuits in Figure B-2, the following condition must be fulfilled.

$$V_{ab}(t) = V_{ab}(t) \qquad \text{Eq. (B-1)}$$

$$i'(t) = i_{ab}(t) \qquad \text{Eq. (B-2)}$$

The application of Kirchhoff's voltage law gives

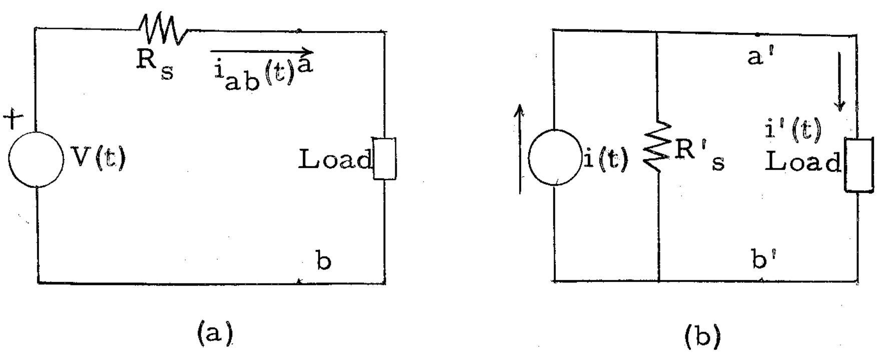


Figure B-2. The two sources shown here the same terminal characteristics. (a) Voltage source. (b) Current source.

$$i_{ab}(t) = \frac{V(t)}{R_s} - \frac{V_{ab}(t)}{R_s} \qquad \text{Eq. (B-3)}$$

$$i'(t) = i(t) - \frac{V_{a'b'}(t)}{R'_s} \qquad \text{Eq. (B-4)}$$

The conditions (Equations B-1, B-2) must hold true for an arbitrary time function V(t). Then

$$i(t) - \frac{V_{a'b'}(t)}{R'_s} = \frac{V(t)}{R_s} - \frac{V_{ab}(t)}{R_s} \qquad \text{Eq. (B-5)}$$

If the equivalence is to be valid $V_{a'b'} = V_{ab}$ and Equation B-5

$$V_{ab}(t)\left(\frac{1}{R_s} - \frac{1}{R'_s}\right) = \frac{V(t)}{R} - i(t) \qquad \text{Eq. (B-6)}$$

Now V(t) is an unspecified time function; V_{ab} is arbitrary in the sense that it will depend on the elements in the load as well as on V(t), equation B-6 states that the arbitrary time function $V(t)/(R_s) - i(t)$ must equal the arbitrary time function $V_{ab}(t)(1/R_s - 1/R'_s)$.

The conditions for equivalence of Figure B-2 therefore

$$R_s = R'_s$$

$$\frac{V(t)}{R_s} = i(t)$$

Where i(t) is the current which flows from terminal a to b in either circuit if a short circuit is connected between those terminal.

A diode is connected in series with the source to give a uni-directional current flow and limit the current that will flow back to the source. A capacitor C was connected in series with the source as an energy storing device, and the impulse current through this capacitor will increase the magnitude of the impulse current. This will provide a sufficient quantity of charge to the circuit connected to the source.

BIBLIOGRAPHY

1. Bachman, G. Bryant and James L. Dever. The $BF_3 \cdot N_2O_5$ complex. Its use as a nitrating agent. Journal of the American Chemical Society 80:5871-5873. 1958.

2. Bachman, G. Bryant, Henry Feuer, B.R. Bluestein and C.M. Vogt. The boron trifluoride-nitrogen tetroxide complex. Its composition and chemical behavior. Journal of the American Chemical Society 77:6188-6190. 1955.

3. Bachman, G. Bryant and Hokama Takeo. The $BF_3 \cdot N_2O_3$ complex. Its use in diazotization and nitration. Journal of the American Chemical Society 79:4370-4373. 1957.

4. Batey, Harry H. and Harry H. Sisler. Some inorganic reactions of nitryl chloride. Journal of the American Chemical Society 74:3408-3410. 1952.

5. Booth, Harold Simmons and Donald Ray Martin. Systems with boron trifluoride. Journal of the American Chemical Society 64:2198-2205. 1942.

6. Booth, Harold Simmons and Francis Carl Mericola. The fluorination of thionyl chloride. Journal of the American Chemical Society 62:640-642. 1940.

7. Booth, Harold Simmons and John Harper Walkup. Systems with boron trifluoride. III. Journal of the American Chemical Society 65:2334-2339. 1943.

8. Booth, Harold Simmons and Karl Stuart Willson. Thermal analysis of the system argon-boron trifluoride. Compounds with the inert gases of the atmosphere. Journal of the American Chemical Society 57:2273-2280. 1935.

9. Brown, Herbert C. and Robert R. Holmes. The heats of reaction of pyridine and nitrobenzene with boron trifluoride, trichloride and tribromide; The relative acceptor properties of the boron halides. Journal of the American Chemical Society 78:2173-2176. 1956.