The response of lightweight equipment in structures supported on resilient-friction-base isolators (R-FBI) subjected to harmonic ground motion and various earthquake ground motions is examined. The equipment-structure base system is modeled as a three degree-of-freedom discrete system (SDOF subsystems). An efficient semi-analytical numerical solution procedure for the determination of equipment response is presented. Parametric studies to examine the effects of subsystem frequency (isolator, structure, equipment), subsystem damping, mass ratio, friction coefficient and frequency content of the ground motion on the response of the equipment are performed. The equipment response on a fixed-base structure subjected to ground motion is also calculated. Friction type isolation devices can induce high frequency effects in the isolated structure due to the stick-slip action. These effects on equipment response are examined. The results show that the high frequency effect in the structure generated from a friction-type base isolator doesn't, in general, cause amplifications in the response. The R-FBI system appears to be an effective aseismic base isolator for protecting both the structure and sensitive internal equipment.
RESPONSE OF EQUIPMENT IN RESILIENT-FRICTION BASE ISOLATED STRUCTURES SUBJECTED TO GROUND MOTION

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TABLE OF CONTENTS

CHAPTER 1. INTRODUCTION

CHAPTER 2. MODELING AND GOVERNING EQUATIONS

CHAPTER 3. SOLUTION METHODOLOGY

LAPLACE TRANSFORM APPROACH

Harmonic Ground Motion
Earthquake Ground Motion

RUNGE-KUTTA APPROACH

CHAPTER 4. NUMERICAL STUDIES AND DISCUSSIONS

HARMONIC GROUND MOTION

Effect of Friction Coefficient
Effect of Damping
Effect of Frequency Content of Ground Motion
Effect of Mass Ratio

EARTHQUAKE GROUND MOTION

Effect of Friction Coefficient
Effect of Damping
Effect of Mass Ratio

CHAPTER 5. CONCLUSIONS
HARMONIC GROUND MOTION

EARTHQUAKE GROUND MOTION

CHAPTER 6. REFERENCES
**LIST OF FIGURES**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The resilient-friction base isolator (R-FBI)</td>
<td>42</td>
</tr>
<tr>
<td>2</td>
<td>Structure supported on R-FBI</td>
<td>43</td>
</tr>
<tr>
<td>3</td>
<td>Equipment-structure supported on R-FBI</td>
<td>43</td>
</tr>
<tr>
<td>4</td>
<td>Effect of friction coefficient on equipment response versus period ratio (\frac{m_e}{m_S} = 0.01)</td>
<td>44</td>
</tr>
<tr>
<td>5</td>
<td>Fourier spectra of acceleration response of structure (\mu = 0.01)</td>
<td>45</td>
</tr>
<tr>
<td>6</td>
<td>Fourier spectra of acceleration response of structure (\mu = 0.02)</td>
<td>46</td>
</tr>
<tr>
<td>7</td>
<td>Fourier spectra of acceleration response of structure (\mu = 0.04)</td>
<td>47</td>
</tr>
<tr>
<td>8</td>
<td>Fourier spectra of acceleration response of structure (\mu = 0.06)</td>
<td>48</td>
</tr>
<tr>
<td>9</td>
<td>Fourier spectra of acceleration response of structure (\mu = 0.10)</td>
<td>49</td>
</tr>
<tr>
<td>10</td>
<td>Effect of damping on equipment response versus period ratio</td>
<td>50</td>
</tr>
<tr>
<td>11</td>
<td>Effect of frequency content of ground motion on equipment response (T_e = 1.0)</td>
<td>51</td>
</tr>
<tr>
<td>12</td>
<td>Effect of frequency content of ground motion on equipment response (T_e = 0.04)</td>
<td>52</td>
</tr>
<tr>
<td>13</td>
<td>Effect of mass ratio on equipment response versus period ratio (\frac{m_e}{m_S} = 0.0001)</td>
<td>53</td>
</tr>
<tr>
<td>14</td>
<td>Effect of friction coefficient on equipment response versus period ratio (\text{El Centro S00E})</td>
<td>55</td>
</tr>
<tr>
<td>15</td>
<td>Effect of friction coefficient on equipment response versus period ratio (\text{Taft S69E})</td>
<td>56</td>
</tr>
<tr>
<td>16</td>
<td>Effect of friction coefficient on equipment response versus period ratio (\text{Pacoima S16E})</td>
<td>57</td>
</tr>
<tr>
<td>17</td>
<td>Fourier spectra of acceleration response of structure (F-B) for (\text{El Centro S00E earthquake})</td>
<td>58</td>
</tr>
<tr>
<td>18</td>
<td>Fourier spectra of acceleration response of structure (\mu = 0.01) for (\text{El Centro S00E earthquake})</td>
<td>59</td>
</tr>
</tbody>
</table>
19 Fourier spectra of acceleration response of structure ($\mu = 0.02$) for El Centro S00E earthquake

20 Fourier spectra of acceleration response of structure ($\mu = 0.04$) for El Centro S00E earthquake

21 Fourier spectra of acceleration response of structure ($\mu = 0.06$) for El Centro S00E earthquake

22 Fourier spectra of acceleration response of structure ($\mu = 0.10$) for El Centro S00E earthquake

23 Effect of equipment damping on response versus period ratio (El Centro S00E)

24 Effect of equipment damping on response versus period ratio (Taft S69E)

25 Effect of equipment damping on response versus period ratio (Pacoima S16E)

26 Effect of structure damping on response versus period ratio (El Centro S00E)

27 Effect of structure damping on response versus period ratio (Taft S69E)

28 Effect of structure damping on response versus period ratio (Pacoima S16E)

29 Effect of mass ratio on equipment response (El Centro S00E)

30 Effect of mass ratio on equipment response (Taft S69E)

31 Effect of mass ratio on equipment response (Pacoima S16E)
CHAPTER 1. INTRODUCTION

The design of earthquake resistant structures has become more and more important in recent years. The traditional technique for designing the structures to withstand strong earthquake excitations is to strengthen the structural members. Through these techniques, however, the structures are very stiff, and therefore their fundamental frequencies may fall into the range of high energy carrying frequencies of the ground motions. This will lead to large acceleration responses in the structures and their internal equipment. An alternative approach, aseismic base isolation, is to isolate structures from the ground motion and hence to reduce the seismic response of the structure. This new design methodology appears to have considerable potential in protecting the structures from the damaging effects of earthquake ground motions.

A number of base isolation systems have been developed. In general, base isolation systems can be classified in two categories: laminated rubber bearing type (LRB, NZ) [1-6] and friction type (R-FBI, and others). The LRB system is made of alternating layers of rubber and steel with the rubber being vulcanized to the steel plates. The LRB is rather flexible in the horizontal direction but quite stiff in the vertical direction. The NZ isolator is essentially the same as the LRB system with an additional lead core to provide energy dissipation and to reduce the relative displacement of the base. In all of the laminated rubber bearing systems with or without a lead core, the rubber carries both the lateral load and vertical load. The above laminated rubber bearing isolation systems have been extensively tested and implemented in such structures as nuclear power plants [7-11] as well as a historic unreinforced masonry building [12].
In this study, attention is focused on a particular friction type system, the R-FBI (resilient-friction-base isolator, proposed in 1983). The R-FBI is a relatively new system [13-17] which decouples the vertical and lateral load carrying functions. It is composed of a set of plates with a teflon sheet bounded to one side, a rubber core through the center of the plates (with or without a central steel rod), and cover plates (see figure 1). The rubber core distributes the lateral displacement across the height of the isolator and carries no vertical load. The interfacial friction serves as a structural fuse as well as an energy absorber. The plates will not slide for low level excitations. For higher level excitations, the plates slide and the central rubber core deforms generating the elastic force which tends to push the system back to its original position. The advantages of the R-FBI are its ease of fabrication as well as the fact that the vertical and lateral load carrying functions are decoupled. Preliminary laboratory tests of R-FBI bearings as well as computer experiments have demonstrated the R-FBI's potential as an effective system for isolating structures from ground motions [18-21]. An experimental program investigating the seismic response of a five-story 1/3 scaled steel structure supported on a R-FBI system has also been performed at the Earthquake Engineering Research Center, University of California, Berkeley.

In aseismic design, it is necessary in the event of severe ground motions to protect not only the primary structure but also lightweight components (equipment) that are attached to the structure. Such nonstructural components may not only serve a critical function, such as a piping system in a nuclear power plant, but may also be of substantial cost as is the case with sensitive scientific equipment.

Conventional methodologies for the analysis/design of lightweight equipment in structures subjected to seismic excitation are based either on ad-hoc or floor spectrum (cascaded system) methods [22-25]. Since the ad-hoc approaches are not based on a rational analysis of the problem, their reliability cannot be determined. Standard floor
spectrum techniques are based on a determination of structural response with the equipment removed. The motion of the equipment support locations is then known and is considered as the equipment excitation. Since the equipment and the structure are decoupled, interaction effects between the structure and equipment are ignored. An alternative approach for the determination of equipment response is a numerical integration of the combined system equations. If several design alternatives were to be investigated, this approach would be prohibitive from a computational standpoint. In addition, standard numerical schemes may mask significant response due to the ill-conditioned nature of the combined system property matrices involved.

Recent research has focused on the development of rational techniques which include interaction effects for the determination of equipment response in structures subjected to earthquake excitation. Several methodologies, based on either a deterministic or a random vibration analysis, have been proposed [26-39]. These studies have shown that floor spectrum approaches grossly overestimate the equipment response when the system is tuned (one or more natural frequencies of the subsystems are equal or close to each other). The new methodologies are computationally efficient since they are based on the dynamic properties of the subsystems and a response spectrum associated with the ground motion. A computationally intensive numerical integration of the combined system is specifically avoided.

Some analytical studies have been performed to determine the response of a single-degree-of-freedom (SDOF) equipment item attached to a multi-degree-of-freedom (MDOF) structure which in turn is supported on a high damping rubber or the New Zealand isolator and modeled as an equivalent linear system (i.e. a simple spring-dashpot model) [40-42]. An experimental investigation of the response of SDOF oscillators attached to various floors of a MDOF building supported on elastomeric bearings with and without a lead plug has also been performed [40]. The studies indicate that elastomeric
bearings without a lead plug may significantly protect internal equipment as well as the superstructure. Elastomeric bearings with a lead plug appear to be less beneficial in protecting the lightweight equipment attached to the base-isolated structure. A study on the response of equipment in structures supported on a frictional isolation system subjected to earthquake ground motion was performed by Ikonomou [43]. The isolation system consisted of a laminated rubber sliding bearing. A comparative study of the floor response spectra for a multi-story building with various base isolation systems was performed by Fan and Ahmadi [44]. Although interaction between the primary and secondary system is neglected in the floor response spectra study, interaction is included in later studies by Fan and Ahmadi [45]. Recently an analytical and experimental study of equipment response in structures supported on sliding bearings and helical springs [46] was performed. In addition, a comparative study of response of a SDOF secondary system in a base-isolated non-uniform elastic shear beam structure was also performed [47-48].

Although the R-FBI appears to be a very viable system for protecting the superstructure from strong ground motion, the slip-stick friction action can induce high frequency effects [21]. These high frequency effects may adversely affect relatively lightweight nonstructural components that are attached to the superstructure (i.e. sensitive scientific equipment in buildings or a coolant piping system in a nuclear power plant), if the resulting high frequency structural motions are of significant duration, intensity, and energy content. Therefore, such effects should be carefully studied.

In this report, a discrete, SDOF subsystem shear-type model supported on R-FBI subjected to harmonic ground motion and various earthquake ground motions is studied. The governing equations and the criteria for transition between different phases of motion are described. There are two phases of motion to be considered: a non-sliding phase and a sliding phase. Closed form expressions are developed for the responses in each phase.
The transition between phases is determined numerically. This results in an efficient numerical scheme to determine the response of the system. A standard numerical integration scheme using Runge-Kutta method is also applied to check some of the results.

The peak acceleration response of the equipment is evaluated and the result is compared to the fixed-base case. Parametric studies to examine the effects of subsystem frequency (isolator, structure, equipment), subsystem damping, mass ratio, friction coefficient and frequency content of the ground motion on the response of the equipment are also carried out.
CHAPTER 2. MODELING AND GOVERNING EQUATIONS

Two shear-type models subjected to ground motion are investigated in this report. The first model is a single-degree-of-freedom (SDF) structure (primary subsystem) resting on the R-FBI (figure 2). The second model is a SDF equipment (secondary subsystem) attached to a SDF structure (primary subsystem) that is supported on R-FBI (figure 3). There are two types of ground excitations in this study: harmonic ground motion and earthquake ground motion. Three earthquake excitations used in this report are El Centro 1940 SOOE, Taft 1952 S69E and Pacoima Dam 1971 S16E.

By applying Newton's Second Law to the equipment-structure-base system shown in figure 3, the equations of motion governing the responses of the superstructure (primary-secondary system) and the base are [20]

\[ m\ddot{x} + c\dot{x} + kx = -(\ddot{s} + \ddot{x}_g)m_r \]  
\[ \ddot{s} + 2\zeta_b \omega_b \dot{s} + \omega_b^2 s + \mu g \text{sgn}(\dot{s}) + \sum_{i=1}^{2} \alpha_i \ddot{x}_i = -\ddot{x}_g \]

where \( s \) is the relative displacement between the base of the structure and the ground, \( x = [x_i] \) where \( x_i \) is the relative displacement of the structure and the equipment with respect to the base, \( \ddot{x}_g \) is the horizontal ground acceleration, \( \mu \) is the friction coefficient, \( g \) is the acceleration of gravity, and \( m = [m_{ij}], c = [c_{ij}], k = [k_{ij}], i, j = 1, 2 \), represent the mass matrix, damping matrix, and stiffness matrix of the superstructure, respectively. In these equations, \( r \) is a unit vector, \( \text{sgn}(\dot{s}) \) is a sign function which is equal to +1 when \( \dot{s} \) is positive, and -1 when \( \dot{s} \) is negative. The mass ratio \( \alpha_i \) is defined as
\[
\alpha_1 = \frac{m_s}{M}, \quad \alpha_2 = \frac{m_e}{M}, \quad M = m_e + m_s + m_b
\]  

where \(m_e, m_s, m_b\) represent the mass of equipment, structure, and base, respectively.

The natural circular frequency of the base, and its effective damping ratio are defined as

\[
\omega_b^2 = \frac{K}{M}, \quad \zeta_b = \frac{C}{2M\omega_b}
\]

where \(C\) and \(K\) are the damping and the horizontal stiffness of the isolator.

Equations (1) and (2) are a set of coupled nonlinear differential equations. There are two phases of motion: a non-sliding phase and a sliding phase. In each phase the system is linear. The transition from sliding to non-sliding is determined numerically. In the sliding phase, equations (1) and (2) govern the motion of the superstructure and its isolator. It should be noted that since the sign function \(\text{sgn}(\dot{s})\) in equation (2) doesn't change sign in this phase, the coupled set of equations (1) and (2) are linear. When the system is not sliding,

\[
\dot{s} = \ddot{s} = 0
\]

and the motion is governed by equation (1) with \(\dot{s} = 0\) only.

In a non-sliding phase the frictional force is greater than the total inertia force generated from the superstructure-base mat system and therefore the friction plates are sticking to each other. The criteria for transition from a non-sliding phase to a sliding phase therefore depends on the relative values of the friction force and the inertia force of the superstructure-base mat system. Therefore, the non-sliding phase continues as long as
As soon as the condition,

\[ |\ddot{x} + \omega_b^2 s + \sum_{i=1}^{2} \alpha_i \dot{x}_i| < \mu g \]  

(6)

is satisfied, sliding occurs and the direction of sliding at this moment of transition is given by

\[ \text{sgn}(s) = - \frac{\ddot{x} + \omega_b^2 s + \sum_{i=1}^{2} \alpha_i \dot{x}_i}{\mu g} \]  

(8)

The sliding of the system in one direction may be terminated by a transition to a stick condition, or it may continue sliding in the reverse direction. During the sliding phase, whenever \( s \) becomes zero, the criterion,

\[ |\ddot{s}| \geq 2\mu g \frac{M}{m_b} \]  

(9)

is checked to determine the subsequent behavior. If equation (9) holds, the sliding continues with \( \text{sgn}(\ddot{s}) \) taking the sign of \( \ddot{s} \) at the moment of transition; otherwise, the stick condition occurs and equations (1) and (5) for a non-sliding phase control the motion [49].

The precise evaluation of the time of phase transitions is very important to the accuracy of the response. A successive-time-step-cutting procedure is used to accurately determine each of these transitions. If the difference between the left and right hand side of equation (7) is less than \( 10^{-10}\text{cm/sec}^2 \), then equation (7) is assumed to be satisfied.
Similarly, if $\dot{s} < 10^{-10}\text{cm/sec}$ then it is assumed that $\dot{s} = 0$, and equation (9) is checked for the subsequent behavior. In this manner, the time of transition is accurately determined. It is noticed that the phase change or reversal of sliding may occur more than once during the relatively small time step used for digitization of earthquake excitations (0.02 sec). This is due to the rapidly varying nature of the excitation that is characteristic of earthquake ground motion. For this type of excitation, the search for phase transitions must be performed extremely carefully. The rapid phase changes that can occur under earthquake ground motions do not occur when the system is subjected to relatively smooth harmonic ground excitations.
CHAPTER 3. SOLUTION METHODOLOGY

In this section, a solution technique using a Laplace transform approach is described in detail. A standard step-by-step numerical integration scheme, the Runge-Kutta method, is also presented. In general, a small time step for the standard integration scheme is required to insure accuracy of the results. Such a requirement is computationally intensive and may lead to accumulation of numerical errors. However, with the use of the Laplace transform approach to solve the linear equations in each phase (sliding and non-sliding) the numerical difficulties discussed above can be eliminated. The responses calculated by the Laplace transform approach are exact in each phase and no numerical approximation is involved. The criteria for locating the change in phase, however, must be performed numerically. This approach is shown to be an efficient and reliable way of evaluating the responses of the models discussed.

LAPLACE TRANSFORM APPROACH

The procedure to analyze the equipment-structure-base isolator system is developed here. The structure-base isolator system can be done in a similar way. The idea of this approach is to apply the Laplace transform technique in two phases as discussed in the previous section. The formulations for the harmonic ground motion and the earthquake ground motion are derived separately in this section. Since the earthquake excitation is a known piecewise linear function and the system is linear in each phase, a closed form solution for the response in each digitized interval of ground motion can be obtained.
Harmonic Ground Motion

A. Non-sliding Phase

The governing equations for non-sliding phase are equations (1) and (5). The harmonic ground motion is assumed to be of the form \( \ddot{x}_g = a \sin(2\pi t/T_g) \), where \( a \) is a constant, \( T_g \) is the period of the ground motion. Introducing the excitation into equation (1) and changing the time variables lead to

\[
\mathbf{m}\ddot{x}(\tau) + \mathbf{c}\dot{x}(\tau) + \mathbf{k}x(\tau) = -m_r a \sin(\omega_g (t^*_i + \tau))
\]  

(10)

where \( \omega_g = 2\pi/T_g \), \( 0 \leq \tau \leq (t_{i+1} - t_i) \), \( i = 1, 3, 5, \ldots, t_1 = 0, t_i \) is the moment of transition from sliding to non-sliding and \( \tau \) is the time variable (\( \tau = 0 \) at moment of transition). Taking the Laplace transform of both sides of equation (10), one obtains

\[
p^2\overline{x} + p\overline{c} + k\overline{x} - (c + pm)x^* - mx^* = -m_r a\frac{\omega_g \cos(\omega_g t) + p\sin(\omega_g t)}{p^2 + \omega_g^2}
\]  

(11a)

where

\[
\overline{x} = \begin{bmatrix} \overline{x}_1 \\ \overline{x}_2 \end{bmatrix}, \quad x^* = \begin{bmatrix} x_{1i} \\ x_{2i} \end{bmatrix}, \quad \dot{x}^* = \begin{bmatrix} \dot{x}_{1i} \\ \dot{x}_{2i} \end{bmatrix}
\]  

(11b)

\( \overline{x} \) is the Laplace transform of \( x \) and \( p \) is the Laplace variable, \( x^* \) and \( \dot{x}^* \) are the initial displacement and the initial velocity at the beginning of this phase. Expanding equation (11a) yields
Solving equations (12) and (13) for the transform response $x_1$ and $x_2$ leads to

$$\bar{x}_1 = \frac{A(m_e p^2 + c_{22} p + k_{22}) - B(c_{12} p + k_{12})}{(p^2 + \omega_g^2)[(m_s p + c_{11}) (m_e p^2 + c_{22} p + k_{22}) - (c_{12} p + k_{12}) (c_{21} p + k_{21})]}$$  \hspace{1cm} (14)$$

$$\bar{x}_2 = \frac{B(m_s p + c_{11}) - A(c_{21} p + k_{21})}{(p^2 + \omega_g^2)[(m_s p + c_{11}) (m_e p^2 + c_{22} p + k_{22}) - (c_{12} p + k_{12}) (c_{21} p + k_{21})]}$$  \hspace{1cm} (15)$$

where

$$A = [(m_s p + c_{11}) x_{11} + m_s \dot{x}_{11} + c_{12} x_{21}](p^2 + \omega_g^2) - a m_s [\omega_g \cos(\omega_g t_1) + p \sin(\omega_g t_1)]$$  \hspace{1cm} (16)$$

$$B = [(m_e p + c_{22}) x_{21} + m_e \dot{x}_{21} + c_{21} x_{11}](p^2 + \omega_g^2) - a m_e [\omega_g \cos(\omega_g t_1) + p \sin(\omega_g t_1)]$$  \hspace{1cm} (17)$$

The equations (14) and (15) give the solution in the Laplace domain. The solution in the time domain is obtained by taking the inverse Laplace transform of equations (14)
and (15). Using the residue theorem one can write

\[ x_1 = \sum \text{Res} [\bar{x}_1(p)e^{pl}] = \sum_k R_{1k} \]  

(18)

\[ x_2 = \sum \text{Res} [\bar{x}_2(p)e^{pl}] = \sum_k R_{2k} \]  

(19)

In order to evaluate the residues in equations (18) and (19), one must obtain the zeroes of the denominators in equations (14) and (15) - a sixth degree polynomial in the transform variable \( p \). The roots of the polynomial are determined numerically using the IMSL library (specifically routine DZPLRC). After finding the zeroes of the denominator, the residues are calculated by the following formulas

\[ R_{1k} = \lim_{p \to p_k} \frac{A(m_p^2 + c_{22}p + k_{22}) - B(c_{12}p + k_{12})}{(p - p_1)(p - p_2)(p - p_3)(p - p_4)(p - p_5)(p - p_6)} e^{pl} \]  

(20)

\[ R_{2k} = \lim_{p \to p_k} \frac{B(m_p^2 + c_{11}p + k_{11}) - A(c_{21}p + k_{21})}{(p - p_1)(p - p_2)(p - p_3)(p - p_4)(p - p_5)(p - p_6)} e^{pl} \]  

(21)

where \( p_k (k = 1, 2, ..., 6) \) are the roots of the sixth degree polynomial. Then the response is determined by equations (18) and (19).

It should be noted that the responses obtained from the above calculations are the relative displacements of the structure and the equipment. The relative acceleration responses of the structure and the equipment are

\[ \ddot{x}_1 = \sum \text{Res} [p^2\bar{x}_1(p)e^{pl}] = \sum_k R_{1k}^* \]  

(22)
\[
\ddot{x}_2 = \sum \text{Res} \left[ p^2 \ddot{x}_2(p)e^{pt} \right] = \sum_k R_{2k}^*
\] (23)

B. Sliding Phase

The governing equations for the sliding phase are equations (1) and (2).

Introducing the excitation and changing the time variables \( t \rightarrow (t_i + \tau) \) results in

\[
\ddot{s} + 2\zeta_b \omega_b \dot{s} + \omega_b^2 s + \mu g \cdot \text{sgn}(s) + \sum_{j=1}^{2} \alpha_j \ddot{x}_j = -a \sin(\omega_g (t_i + \tau))
\] (24)

\[
m\ddot{x} + c\dot{x} + k x = -m r \left[ \ddot{s} + a \sin(\omega_g (t_i + \tau)) \right]
\] (25)

where \( 0 \leq \tau \leq (t_{i+1} - t_i) \), \( i = 2, 4, 6, \ldots \), \( t_i \) is the moment of transition from non-sliding to sliding and \( \tau \) is a time variable (\( \tau = 0 \) at moment of transition). Taking the Laplace transform of both sides of the above equations and simplifying leads to

\[
(p^2 + 2\zeta_b \omega_b p + \omega_b^2) \ddot{s} + \alpha_1 p^2 \ddot{x}_1 + \alpha_2 p^2 \ddot{x}_2 = (p + 2\zeta_b \omega_b) s_i + \ddot{s}_i + \alpha_1 (p \ddot{x}_1_i + \ddot{x}_1_i)
\]

\[
+ \alpha_2 (p \ddot{x}_2_i + \ddot{x}_2_i) + \frac{F}{p} - a \left[ \frac{\omega_g \cos(\omega_g t_i) + p \sin(\omega_g t_i)}{p^2 + \omega_g^2} \right]
\] (26)

\[
m_s p^2 \ddot{\ddot{s}} + (m_s p^2 + c_{11} p + k_{11}) \ddot{x}_1 + (c_{12} p + k_{12}) \ddot{x}_2 = m_s (p \ddot{s}_i + \ddot{s}_i) + (m_s p + c_{11}) x_1_i + m_s \ddot{x}_1_i + c_{12} x_2_i
\]

\[- a m_s \left[ \frac{\omega_g \cos(\omega_g t_i) + p \sin(\omega_g t_i)}{p^2 + \omega_g^2} \right]
\] (27)
\[ m_e p^2 \ddot{s} + (c_{21} p + k_{21}) \ddot{x}_1 + (m_e p^2 + c_{22}) \ddot{x}_2 = m_e (p \ddot{s} + \dddot{s}) + (m_e p + c_{22}) x_2 + m_e \dddot{x}_2 + c_{21} \dot{x}_1 \]

\[ -a m_e \left[ \frac{\omega_g \cos(\omega t) + p \sin(\omega_g t)}{p^2 + \omega_g^2} \right] \]

where \( F = -\mu g \mathcal{L}[s] \) and \( \mathcal{L}[s] \) is the Laplace transform of \( s \). Note that \( \mathcal{L}[s] \) doesn't change sign in this phase. The initial displacement and the initial velocity of the base are \( s_i \) and \( \dot{s}_i \), respectively. Solving equations (26), (27), and (28) leads to

\[ \ddot{s} = \frac{N_1}{D} \]

\[ \ddot{x}_1 = \frac{N_2}{D} \]

\[ \ddot{x}_2 = \frac{N_3}{D} \]

where

\[ D = p(p^2 + \omega_g^2)[(p^2 + 2\zeta_b \omega_b p + \omega_b^2)(m_s p^2 + c_{11} p + k_{11})(m_e p^2 + c_{22} p + k_{22})] \]

\[ + \alpha_2 m_s p^4 (c_{21} p + k_{21}) + \alpha_1 m_e p^4 (c_{12} p + k_{12}) \]

\[ - \alpha_2 m_e p^4 (m_s p^2 + c_{11} p + k_{11}) - \alpha_1 m_s p^4 (m_e p^2 + c_{22} p + k_{22}) \]

\[ - (c_{12} p + k_{12})(c_{21} p + k_{21})(p^2 + 2\zeta_b \omega_b p + \omega_b^2) \]

(32)
\[ N_1 = \overline{A}^*[(m_sp^2 + c_{11}p + k_{11})(m_e p^2 + c_{22}p + k_{22}) - (c_{21}p + k_{12})(c_{12}p + k_{12})] \\
+ \overline{B}^* p^3[\alpha_2 (c_{21}p + k_{21}) - \alpha_1 (m_e p^2 + c_{22}p + k_{22})] \\
+ \overline{C}^* p^3[\alpha_1 (c_{12}p + k_{12}) - \alpha_2 (m_s p^2 + c_{11}p + k_{11})] \]  

(33)

\[ N_2 = \overline{A}^* p^2[m_e (c_{12}p + k_{12}) - m_s (m_e p^2 + c_{22}p + k_{22})] \\
+ \overline{B}^* p[(p^2 + 2\zeta_b \omega_b p + \omega_b^2)(m_e p^2 + c_{22}p + k_{22}) - \alpha_2 m_e p^4] \\
+ \overline{C}^* p[\alpha_2 m_s p^4 - (p^2 + 2\zeta_b \omega_b p + \omega_b^2)(c_{12}p + k_{12})] \]  

(34)

\[ N_3 = \overline{A}^* p^2[m_s (c_{21}p + k_{21}) - m_e (m_s p^2 + c_{11}p + k_{11})] \\
+ \overline{B}^* p[\alpha_1 m_e p^4 - (p^2 + 2\zeta_b \omega_b p + \omega_b^2)(c_{21}p + k_{21})] \\
+ \overline{C}^* p[(p^2 + 2\zeta_b \omega_b p + \omega_b^2)(m_s p^2 + c_{11}p + k_{11}) - \alpha_1 m_s p^4] \]  

(35)

and

\[ \overline{A}^* = [(p + 2\zeta_b \omega_b)\dot{s}_i + \dot{s}_i + \alpha_1 (\dot{p}_x \dot{x}_{1i} + \dot{x}_{1i}) + \alpha_2 (\dot{p}_x \dot{x}_{2i} + \dot{x}_{2i})] p(p^2 + \omega_g^2) \\
+ F(p^2 + \omega_g^2) - a_p[\omega_g \cos(\omega_g t) + p \sin(\omega_g t)] \]  

(36)

\[ \overline{B}^* = [m_s (p \dot{s}_i + \dot{s}_i) + (m_sp + c_{11})x_{1i} + m_s \dot{x}_{1i} + c_{12} \dot{x}_{2i}](p^2 + \omega_g^2) \\
- a_m s[\omega_g \cos(\omega_g t) + p \sin(\omega_g t)] \]  

(37)

\[ \overline{C}^* = [m_e (p \dot{s}_i + \dot{s}_i) + (m_e p + c_{22})x_{2i} + m_e \dot{x}_{2i} + c_{21} \dot{x}_{1i}](p^2 + \omega_g^2) \\
- a_m e[\omega_g \cos(\omega_g t) + p \sin(\omega_g t)] \]  

(38)
The equations (29), (30), and (31) give the solution in the Laplace domain. Again, the residue theorem is applied to calculate the inverse Laplace transform. The procedure is exactly the same as described previously for the non-sliding phase.

Earthquake Ground Motion

A. Non-sliding Phase

The governing equations for non-sliding phase are equations (1) and (5). The earthquake excitation is assumed to be of the form

\[ \ddot{x}_{gi} = \dot{x}_{gi} + a\tau \] (39)

where \(\ddot{x}_{g\tau}\) is the earthquake acceleration at time \(\tau\), \(0 \leq \tau \leq t_{i+1} - t_i\), \(\dot{x}_{gi}\) is the earthquake acceleration at time \(t_i\) and \(a\) is the slope between two consecutive digitized points of the earthquake motion at times \(t_i\) and \(t_{i+1}\). Introducing the excitation into equation (1) and changing the time variables, one obtains

\[
m\ddot{x}(\tau) + c\dot{x}(\tau) + kx(\tau) = -mr(\ddot{x}_{gi} + a\tau) \] (40)

Taking the Laplace transform of both sides leads to

\[
p^2m\ddot{x} + pec\ddot{x} + k\ddot{x} - (c + pm)x^* - m\ddot{x}^* = -mr\left(\frac{\ddot{x}_{gi}}{p} + \frac{a}{p^2}\right) \] (41a)

where
\[ \bar{x} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}, \quad x^* = \begin{bmatrix} x_{1i} \\ x_{2i} \end{bmatrix}, \quad \dot{x}^* = \begin{bmatrix} \dot{x}_{1i} \\ \dot{x}_{2i} \end{bmatrix} \] (41b)

\( \bar{x} \) is the Laplace transform of \( x \) and \( p \) is the Laplace variable, \( x^* \) and \( \dot{x}^* \) are the initial displacement and the initial velocity at time \( t_i \). Expanding equation (41a) yields

\[ (m_s p^2 + c_{11} p + k_{11})\bar{x}_1 + (c_{12} p + k_{12})\bar{x}_2 = [(m_s p + c_{11})x_{1i} + m_s \dot{x}_{1i} + c_{12} x_{2i}] \]

\[ -\frac{m_s (\ddot{x}_1 p + a)}{p^2} \]

(42)

\[ (c_{21} p + k_{21})\bar{x}_1 + (m_p p^2 + c_{22} p + k_{22})\bar{x}_2 = [(m_p p + c_{22})x_{2i} + m_p \dot{x}_{2i} + c_{21} x_{1i}] \]

\[ -\frac{m_p (\ddot{x}_2 p + a)}{p^2} \]

(43)

Solving equations (42) and (43) for the transform response leads to

\[ \bar{x}_1 = \frac{A(m_p p^2 + c_{22} p + k_{22}) - B(c_{12} p + k_{12})}{p^2 [(m_s p^2 + c_{11} p + k_{11})(m_p p^2 + c_{22} p + k_{22}) - (c_{12} p + k_{12})(c_{21} p + k_{21})]} \] (44)

\[ \bar{x}_2 = \frac{B(m_p p^2 + c_{11} p + k_{11}) - A(c_{21} p + k_{21})}{p^2 [(m_s p^2 + c_{11} p + k_{11})(m_p p^2 + c_{22} p + k_{22}) - (c_{12} p + k_{12})(c_{21} p + k_{21})]} \] (45)

where

\[ A = [(m_s p + c_{11})x_{1i} + m_s \dot{x}_{1i} + c_{12} x_{2i}] p^2 - m_s (\ddot{x}_1 p + a) \] (46)
\[ B = \left( m_e p + c_{22} \right) x_{2i} + m_e \dot{x}_{2i} + c_{21} x_{11} \left[ p^2 - m_e (x_{gi} p + a) \right] \]  

(47)

The responses in Laplace domain are given by equations (44) and (45). In order to obtain the responses in the time domain, one must take the inverse Laplace transform of equations (44) and (45). Using the residue theorem one can write

\[ x_1 = \sum_k \text{Res} [\bar{x}_1(p)e^{pt}] = \sum_k R_{1k} \]  

(48)

\[ x_2 = \sum_k \text{Res} [\bar{x}_2(p)e^{pt}] = \sum_k R_{2k} \]  

(49)

In order to evaluate the residues in equations (48) and (49), one must obtain the zeroes of the denominators in equations (44) and (45) - a sixth degree polynomial in the transform variable \( p \). There are two zero roots. The remaining roots of the polynomial are determined numerically using the routine from IMSL library. It is noted that there only five residues need to be evaluated (four for the distinct roots and one for repeated root). Therefore the summation in equations (48), (49) is from \( k=1 \) to 5. After the zeroes of the denominator are obtained, the residues are calculated as follows:

a) for the distinct roots \( p_k \) (\( k=1,2,3,4 \))

\[ R_{1k} = \lim_{p \to p_k} \left( p - p_k \right) \frac{A(m_e p^2 + c_{22} p + k_{22}) - B(c_{12} p + k_{12})}{m_e m_p^2 (p - p_1)(p - p_2)(p - p_3)(p - p_4)} e^{pt} \]  

(50)

\[ R_{2k} = \lim_{p \to p_k} \left( p - p_k \right) \frac{B(m_e p^2 + c_{11} p + k_{11}) - A(c_{21} p + k_{21})}{m_e m_p^2 (p - p_1)(p - p_2)(p - p_3)(p - p_4)} e^{pt} \]  

(51)
b) for the repeated root

\[ R_{15} = \lim_{p \to p_5} \frac{d}{dp} \left[ \frac{(p - p_5)^2}{\bar{x}_1(p)e^{p \tau}} \right] \]  
\[ R_{25} = \lim_{p \to p_5} \frac{d}{dp} \left[ \frac{(p - p_5)^2}{\bar{x}_2(p)e^{p \tau}} \right] \]

Substituting equations (44), (45) into (52), (53) and simplifying the expressions, one obtains

\[ R_{15} = g \frac{(m_{s_{22}} - m_{s_{12}}) + (m_{c_{12}} - m_{c_{22}})}{k_{11}k_{22} - k_{12}k_{21}} + \frac{\alpha(m_{k_{22}} - m_{k_{12}})(c_{11k_{22}} + c_{22k_{11}} - c_{12k_{21}} + c_{21k_{12}})}{(k_{11}k_{22} - k_{12}k_{21})^2} + \frac{\alpha(m_{e_{12}} - m_{e_{22}})}{k_{11}k_{22} - k_{12}k_{21}} \]  
\[ R_{25} = g \frac{(m_{s_{21}} - m_{s_{11}}) + (m_{c_{11}} - m_{c_{21}})}{k_{11}k_{22} - k_{12}k_{21}} + \frac{\alpha(m_{k_{21}} - m_{k_{11}})(c_{11k_{22}} + c_{22k_{11}} - c_{12k_{21}} + c_{21k_{12}})}{(k_{11}k_{22} - k_{12}k_{21})^2} + \frac{\alpha(m_{e_{11}} - m_{e_{21}})}{k_{11}k_{22} - k_{12}k_{21}} \]

The responses obtained from equations (48) and (49) are the relative displacements of the structure and the equipment. The relative velocity responses of the structure and equipment are calculated by

\[ \dot{x}_1 = \sum \text{Res} \left[ p\bar{x}_1(p)e^{p \tau} \right] = \sum_{k} R_{1k}^* \]
\[ \dot{x}_2 = \sum \text{Res} [p \bar{x}_2(p)e^{pt}] = \sum_k R_{2k}^* \] (57)

From equations (56) and (57), it is observed that there are only five distinct roots in the denominator. The residues are given by the following formulas

\[ R_{1k}^* = \lim_{p \to p_k} (p - p_k) \frac{A(m_e p^2 + c_{22} p + k_{22}) - B(c_{12} p + k_{12})}{m_s m_e (p - p_1)(p - p_2)(p - p_3)(p - p_4)(p - p_5)} e^{pt} \] (58)

\[ R_{2k}^* = \lim_{p \to p_k} (p - p_k) \frac{B(m_s p^2 + c_{11} p + k_{11}) - A(c_{21} p + k_{21})}{m_s m_e (p - p_1)(p - p_2)(p - p_3)(p - p_4)(p - p_5)} e^{pt} \] (59)

where \( k = 1, 2, 3, 4, 5 \). The relative acceleration responses of the structure and equipment are given by

\[ \ddot{x}_1 = \sum \text{Res} [p^2 \bar{x}_1(p)e^{pt}] = \sum_k R_{1k}^{**} \] (60)

\[ \ddot{x}_2 = \sum \text{Res} [p^2 \bar{x}_2(p)e^{pt}] = \sum_k R_{2k}^{**} \] (61)

Since there are only four distinct roots in the denominator of equations (60) and (61), only four residues need to be calculated;

\[ R_{1k}^{**} = \lim_{p \to p_k} (p - p_k) \frac{A(m_e p^2 + c_{22} p + k_{22}) - B(c_{12} p + k_{12})}{m_s m_e (p - p_1)(p - p_2)(p - p_3)(p - p_4)} e^{pt} \] (62)
\[
R_{2k}^{**} = \lim_{p \to p_k} \frac{B(m_s p^2 + c_{11} p + k_{11}) - A(c_{21} p + k_{21})}{m_s m_e (p - p_1)(p - p_2)(p - p_3)(p - p_4)} e^{pt} \tag{63}
\]

where \( k = 1, 2, 3, 4 \). The summation in equations (60) and (61) is from \( k = 1 \) to 4.

B. Sliding Phase

The motion of the sliding phase is governed by equations (1) and (2). Introducing the earthquake excitation and changing the time variables leads to

\[
\ddot{s} + 2\zeta_b \omega_b s + \omega_b^2 s + \mu g \text{sgn}(s) + \sum_{j=1}^{2} \alpha_j \dot{x}_j = - (\ddot{x}_{gi} + a\tau) \tag{64}
\]

\[
m\ddot{x} + c\dot{x} + kx = -mr (\ddot{x}_{gi} + a\tau) \tag{65}
\]

where \( 0 \leq \tau \leq (t_i + 1 - t_i) \). Taking the Laplace transform of both sides of the above equations and simplifying gives

\[
(p^2 + 2\zeta_b \omega_b p + \omega_b^2)\ddot{s} + \alpha_1 p^2 \ddot{x}_1 + \alpha_2 p^2 \ddot{x}_2 = (p + 2\zeta_b \omega_b)\dot{s}_i + \dot{s}_i + \alpha_1 (p\dot{x}_{1i} + \dot{x}_{1i})
\]

\[
+ \alpha_2 (p\dot{x}_{2i} + \dot{x}_{2i}) + \frac{F}{p} - \frac{\ddot{x}_{gi} p + a}{p^2} \tag{66}
\]

\[
m_s p^2 \ddot{s} + (m_s p^2 + c_{11} p + k_{11})\ddot{x}_1 + (c_{12} p + k_{12})\ddot{x}_2 = m_s (p\ddot{s}_i + \ddot{s}_i) + (m_s p + c_{11})\dot{x}_i + m_s \ddot{x}_{1i} + c_{12} \ddot{x}_{2i}
\]

\[
m_s (\ddot{x}_{gi} p + a) - \frac{m_s \ddot{x}_{gi} p + a}{p^2} \tag{67}
\]
\[
\begin{align*}
\frac{m_e P^2 \bar{s} + (c_{21}^p + k_{21}^p) \bar{x}_1 + (m_e P^2 + c_{22}^p + k_{22}^p) \bar{x}_2}{(x_{21}^p)^2 + (x_{21}^p)^2 + c_{22}^p + k_{22}^p) x_{21}^p + m_e E_2^e + c_{21}^p x_{11}^p} = m_e (p s_i + \dot{s}_i) + (m_e P^2 + c_{22}^p) x_{21}^p + m_e x_{21}^p + c_{21}^p x_{11}^p
\end{align*}
\]
\[
(68)
\]

where \( F = -\mu g \hat{\text{sgn}}(\dot{s}) \) and \( \bar{s} \) is the Laplace transform of \( s \). It should be noted that \( \hat{\text{sgn}}(\dot{s}) \) doesn't change sign in the sliding phase. The initial displacement and the initial velocity of the base at time \( t_i \) are \( s_i \) and \( \dot{s}_i \), respectively. Solving equations (66), (67), and (68) leads to

\[
\bar{s} = \frac{N_1}{D}
\]
\[
(69)
\]
\[
\bar{x}_1 = \frac{N_2}{D}
\]
\[
(70)
\]
\[
\bar{x}_2 = \frac{N_3}{D}
\]
\[
(71)
\]

where

\[
D = P^2 \left[ (P^2 + 2\zeta_b \omega_b P + \omega_b^2)(m_s P^2 + c_{11}^p + k_{11}) (m_e P^2 + c_{22}^p + k_{22})
\right.
\]
\[
+ \alpha_2 m_s P^4 (c_{21}^p + k_{21}) + \alpha_1 m_e P^4 (c_{12}^p + k_{12})
\]
\[
- \alpha_2 m_e P^4 (m_s P^2 + c_{11}^p + k_{11}) - \alpha_1 m_s P^4 (m_e P^2 + c_{22}^p + k_{22})
\]
\[
- (c_{12}^p + k_{12})(c_{21}^p + k_{21})(P^2 + 2\zeta_b \omega_b P + \omega_b^2)
\]
\[
(72)
\]
\[ N_1 = \overline{A}^* \left[ (m_e p^2 + c_{11} p + k_{11})(m_e p^2 + c_{22} p + k_{22}) - (c_{21} p + k_{21})(c_{12} p + k_{12}) \right] \\
+ \overline{B}^* p^2 \left[ \alpha_2 (c_{21} p + k_{21}) - \alpha_1 (m_e p^2 + c_{22} p + k_{22}) \right] \\
+ \overline{C}^* p^2 \left[ \alpha_1 (c_{12} p + k_{12}) - \alpha_2 (m_s p^2 + c_{11} p + k_{11}) \right] \tag{73} \]

\[ N_2 = \overline{A}^* p^2 \left[ m_e (c_{12} p + k_{12}) - m_s (m_e p^2 + c_{22} p + k_{22}) \right] \\
+ \overline{B}^* \left[ (p^2 + 2\zeta_b \omega_b p + \omega_b^2)(m_e p^2 + c_{22} p + k_{22}) - \alpha_2 m_e p^4 \right] \\
+ \overline{C}^* \left[ \alpha_2 m_s p^4 - (p^2 + 2\zeta_b \omega_b p + \omega_b^2)(c_{12} p + k_{12}) \right] \tag{74} \]

\[ N_3 = \overline{A}^* p^2 \left[ m_s (c_{21} p + k_{21}) - m_e (m_s p^2 + c_{11} p + k_{11}) \right] \\
+ \overline{B}^* \left[ \alpha_1 m_e p^4 - (p^2 + 2\zeta_b \omega_b p + \omega_b^2)(c_{21} p + k_{21}) \right] \\
+ \overline{C}^* \left[ (p^2 + 2\zeta_b \omega_b p + \omega_b^2)(m_s p^2 + c_{11} p + k_{11}) - \alpha_1 m_s p^4 \right] \tag{75} \]

and

\[ \overline{A}^* = \left[ (p + 2\zeta_b \omega_b) s_i + \dot{s}_i + \alpha_1 (p x_{11} + \dot{x}_{11}) + \alpha_2 (p x_{21} + \dot{x}_{21}) \right] p^2 + F p - (\ddot{x}_{gi} p + a) \tag{76} \]

\[ \overline{B}^* = \left[ m_s (p s_i + \dot{s}_i) + (m_s p + c_{11}) x_{11} + m_s \ddot{x}_{11} + c_{12} x_{21} \right] p^2 - m_s (\ddot{x}_{gi} p + a) \tag{77} \]

\[ \overline{C}^* = \left[ m_e (p s_i + \dot{s}_i) + (m_e p + c_{22}) x_{21} + m_e \ddot{x}_{21} + c_{21} x_{11} \right] p^2 - m_e (\ddot{x}_{gi} p + a) \tag{78} \]

Applying the residue theorem to equations (69), (70) and (71), the relative displacements of the base, structure and equipment are given by
The zeroes of the eighth degree polynomial [equation (72)] must be calculated to obtain the residues in equations (79), (80) and (81). There are six distinct roots and one repeated root and therefore seven residues need to be calculated. The residues are obtained by the following formulas:

a) for the distinct roots \( p_k \) (\( k=1,2,3,4,5,6 \))

\[
R_{sk} = \lim_{p \to p_k} \frac{s(p) \exp(p \cdot t)}{m_s m_e (1 - \alpha_1 - \alpha_2) p^2 (p - p_1)(p - p_2)(p - p_3)(p - p_4)(p - p_5)(p - p_6)}
\]

b) for the repeated root

\[
R_{s7} = \lim_{p \to p_7} \frac{d}{dp} \left[ (p - p_7)^2 s(p) \exp(p \cdot t) \right]
\]
Expanding and simplifying the above equations results in

\[
R_{s7} = \frac{\omega_b(F - \ddot{x}_g) + 2a\ddot{\tau}b}{\omega_b^3} - \frac{a}{\omega_b^2} \tau
\]  

\[
R_{17} = \frac{\ddot{x}_g(m_{s11}k_{12} - m_{s22}k_{12}) + a(m_{s11} - m_{s22})}{k_{11}k_{22} - k_{12}k_{21}} + \frac{\alpha(m_{e12} - m_{e22})}{k_{11}k_{22} - k_{12}k_{21}} \tau
\]  

\[
R_{27} = \frac{\ddot{x}_g(m_{s21} - m_{s11}) + a(m_{s21} - m_{e11})}{k_{11}k_{22} - k_{12}k_{21}} + \frac{\alpha(m_{s21} - m_{e11})}{k_{11}k_{22} - k_{12}k_{21}} \tau
\]

It should be noted that equations (89) and (90) for determining the structure and equipment responses are exactly the same as equations (54) and (55) in the non-sliding phase. Finally, in order to calculate the relative velocity and acceleration response, the same procedure used for the non-sliding phase is applied to the sliding phase.
RUNGE-KUTTA APPROACH

In order to verify the accuracy of the previously developed solution approaches, the Runge-Kutta numerical integration scheme is used. The governing equations, however, need to be modified [49].

A. Non-Sliding Phase

Multiplying equation (1) by \( m^{-1} \), the result may be written as

\[
\ddot{x} = -m^{-1} c \dot{x} - m^{-1} k x - r \ddot{x}_g
\]

Applying matrix multiplication yields

\[
\ddot{x}_1 = -\frac{1}{m_s} (c_{11} \dot{x}_1 + c_{12} \dot{x}_2 + k_{11} x_1 + k_{12} x_2) - \ddot{x}_g
\]

\[
\ddot{x}_2 = -\frac{1}{m_e} (c_{21} \dot{x}_1 + c_{22} \dot{x}_2 + k_{21} x_1 + k_{22} x_2) - \ddot{x}_g
\]

Equations (92) and (93) are the explicit form used in the Runge-Kutta numerical integration procedure. Equations (92) and (93) are then transferred into a set of first order equations in a standard manner. The calculations are accomplished by calling IMSL subroutine DIVPRK.

B. Sliding Phase

Multiplying equation (1) by \( m^{-1} \) and expanding the matrix equations leads to
\[ \ddot{x}_1 = -\frac{1}{m_s} (c_{11} \dot{x}_1 + c_{12} \dot{x}_2 + k_{11} x_1 + k_{12} x_2) - (\ddot{s} + \ddot{x}_g) \] (94)

\[ \ddot{x}_2 = -\frac{1}{m_e} (c_{21} \dot{x}_1 + c_{22} \dot{x}_2 + k_{21} x_1 + k_{22} x_2) - (\ddot{s} + \ddot{x}_g) \] (95)

Substituting equations (94) and (95) into equation (2) yields

\[ \ddot{s} = -\ddot{x}_g + \frac{1}{m_b} (2m_s \omega_s \xi_s \dot{x}_1 + m_s \omega_s^2 x_1) - \frac{1}{\alpha_b} (2\xi_b \omega_b \ddot{s} + \omega_b^2 s + \mu g \hat{\text{sgn}}(s)) \] (96)

where \( \alpha_b = m_b/M \). Substituting equation (96) into equations (94) and (95), it follows that

\[ \ddot{x}_1 = -\frac{1}{m_s} (c_{11} \dot{x}_1 + c_{12} \dot{x}_2 + k_{11} x_1 + k_{12} x_2) - \frac{1}{m_b} (2m_s \omega_s \xi_s \dot{x}_1 + m_s \omega_s^2 x_1) \]

\[ + \frac{1}{\alpha_b} (2\xi_b \omega_b \ddot{s} + \omega_b^2 s + \mu g \hat{\text{sgn}}(s)) \] (97)

\[ \ddot{x}_2 = -\frac{1}{m_c} (c_{21} \dot{x}_1 + c_{22} \dot{x}_2 + k_{21} x_1 + k_{22} x_2) - \frac{1}{m_b} (2m_s \omega_s \xi_s \dot{x}_1 + m_s \omega_s^2 x_1) \]

\[ + \frac{1}{\alpha_b} (2\xi_b \omega_b \ddot{s} + \omega_b^2 s + \mu g \hat{\text{sgn}}(s)) \] (98)

Equations (96) (97) and (98) are then transferred into a series of first order equations in a standard manner. Again, IMSL routine DIVPRK is used.
CHAPTER 4. NUMERICAL STUDIES AND DISCUSSIONS

In this section, parametric studies to examine the effects of friction coefficient, subsystem frequency (isolator, structure, equipment), subsystem damping, mass ratio and frequency content of the ground motion on the response of the equipment are performed and the results are discussed. The peak absolute acceleration response, \((\ddot{x}_g + \ddot{s} + \ddot{x}_2)_{\text{max}}\), of the equipment in structures supported on R-FBI (figure 3) subjected to a sinusoidal horizontal ground motion and various earthquake ground motions are calculated by the approach developed in section 3. In order to examine the effects of the slip-stick friction action on the equipment response, the absolute acceleration time history of the structure supported on R-FBI (figure 2) is also calculated. The Fourier spectra of this time history is then determined and the equipment is tuned to peaks in this spectra.

Throughout this work, in order to assure the accuracy of the results, the time step of the calculations is chosen such that

\[
\Delta t \leq \min \left[ \frac{T_e}{20}, \frac{T_s}{20}, \frac{T_g}{20}, 0.02 \right]
\]

(99)

where \(T_e\), \(T_s\), and \(T_g\) are the natural periods of equipment subsystem, structure subsystem, and ground motion, respectively. For harmonic ground excitation, in general, a computation time of 9 seconds (10 cycles of ground motion) is sufficient to obtain the maximum equipment response. However, there are some instances when this computation time is insufficient; in these instances the duration of computation is extended until the maximum equipment response is achieved. Furthermore, the amplitude of the harmonic ground motion is 0.5g. The earthquake ground motions considered are the first 30 seconds of the Pacoima Dam S16E (peak acceleration 1.17g), El Centro S00E (peak
acceleration 0.35g), and Taft S69E (peak acceleration 0.18g). The maximum equipment responses are normalized with respect to the peak ground acceleration of the particular earthquake.

HARMONIC GROUND MOTION

Effect of Friction Coefficient

In the following, the effects of friction coefficient of the base isolator on the response of the equipment are investigated. The results are shown in figure 4. In the figure, $\mu$ is the friction coefficient, which varies from 0.01 to 0.1. The natural periods of the structure ($T_s$), base ($T_b$), and ground motion ($T_g$) are 0.3 sec, 4.0 sec, and 0.9 sec, respectively. The damping ratios and masses of the equipment, structure, and base are $\zeta_e = 0.02$, $\zeta_s = 0.02$, $\zeta_b = 0.08$; and $m_e = 0.01$ kg, $m_s = 1.0$ kg, $m_b = 1.0$ kg, respectively. One notices that the largest amplification occurs when the equipment natural period is tuned to the period of the ground motion. Amplification also occurs when the equipment and structure natural periods are the same (conventional tuning). In most instances the R-FBI reduces the response of the equipment, and the smaller the friction coefficient the greater the reduction. There are several exceptions, however, when the response of the equipment in the isolated structure exceeds the equipment response in the fixed base structure. This occurs at $\mu = 0.1$ and $T_e/T_s = 0.5 - 0.8, 1.0$; $\mu = 0.06$ and $T_e/T_s = 0.6 - 0.8$; as well as $\mu = 0.04$ and $T_e/T_s = 07$. For these particular systems, the stick-slip friction action imparts significant energy into the structure which in turn is transmitted into
the equipment. There are also amplifications in the high period region $T_e/T_s \geq 12.0$. At $T_e/T_s = 13.33$ the equipment is tuned to the natural period of the isolator which yields a resonance effect. Interaction between the equipment and the base-isolator contributes to the amplifications in equipment response in this region.

The amplifications in equipment response in the isolated structure due to the stick-slip friction action can perhaps best be explained by examining the Fourier spectra of the structure-R-FBI system alone (figure 2). The properties of the structure-R-FBI system were obtained by removing the equipment for those systems where amplification in the equipment response due to stick-slip effects occurred in the equipment-structure-R-FBI system. The Fourier spectra of the structure acceleration response is presented in figures 5 through 9. The Fourier spectra are generated by first calculating the acceleration time histories of the structure supported on the R-FBI and the applying the Fourier transform technique. The structure response reaches a quasi-steady periodic state for computation times greater than 18 seconds. After this time 12 seconds of structure acceleration response are used as the sample for the IMSL Fast Fourier Transform routine FCOST. The energy containing frequencies of the Fourier spectra of the structure response are roughly located at $(2n -1)f_g$, $n = 1, 2, 3, \ldots$, where $f_g = 1/T_g$ [49]. The lowest frequency where a peak in the Fourier transform occurs is the frequency of the ground excitation and the friction force generates peaks at higher frequencies. The three frequencies with the largest magnitude in the Fourier spectra contain the most energy and therefore dominate the acceleration response of the system. For frequencies higher than 10 Hz, the Fourier amplitudes decrease rapidly. The magnitudes of the peaks in the Fourier spectra increase as the friction coefficient increases. The three frequencies with dominant peaks are approximately 1.125, 3.375, and 5.625 Hz. When the frequency of the lightweight equipment is tuned (equal or close) to one of these frequencies amplifications in equipment
response may occur due to a resonant effect. For example, an equipment frequency of 5.625 Hz, it is equivalent to the case of $T_e/T_s = 0.6$ in the previous analysis and hence amplifications in equipment response in the isolated structure occur. The peak in the equipment response due to the stick-slip friction action that occurs at $T_e/T_s = 0.7$ is due to the interaction between two of the dominant frequencies (3.375 and 5.625 Hz) in the Fourier spectra of the structural response.

**Effect of Damping**

The effect of equipment damping on the response of the equipment in structures supported on R-FBI subjected to harmonic ground motion are shown in figure 10. The damping ratios of the structure and base are 0.02 and 0.08, and the damping ratio of the equipment varies from small damping 0.02 to relatively large damping 0.1. Base friction coefficients of 0.01 and 0.1 representing the two extremes of small and relatively large friction, are considered. In addition, the natural period of the structure, base, and ground motion are 0.3 sec, 4.0 sec and 0.9 sec respectively while the natural period of the equipment varies. The masses of the equipment, structure and base are 0.01 kg, 1.0 kg and 1.0 kg, respectively.

It is observed that the acceleration response of the equipment is reduced as the damping ratio increases for both friction coefficients. The reduction in the peaks where resonant amplifications occur is more prominent. This indicates that damping in the equipment is an effective mechanism for reducing peaks in equipment response resulting from either the stick-slip friction action or tuning. In the low period region ($T_e/T_s \leq 0.4$), however, equipment damping does not noticeably affect the response.
Effect of Frequency Content of Ground Motion

The effect of frequency content of the ground motion on the response of the equipment are shown in figures 11 and 12. In figures 11 and 12, the equipment natural periods of 0.04 sec and 1.0 sec representing relatively high and low frequency equipment, respectively, were used. The remaining properties of the equipment, structure, and base were those described in the previous section.

Figure 11 shows that there are two peaks in the fixed base response. The first peak occurs at $T_g/T_s = 1.0$ (structure natural frequency tuned to the ground motion) and the second peak is approximately at $T_g/T_s = 3.3$ (equipment natural frequency tuned to the ground motion). It is also seen that the isolated response increases as the period of the ground motion increases. At $T_g/T_s = 3.3$ the response reaches its maximum value and for $T_g/T_s \geq 9$ the isolated response exceeds the fixed base response. In this region, the frequency of the ground motion approaches that of the base isolator, thus causing amplifications in response. The frequency of the ground motion is identical to the base isolator for $T_g/T_s = 13.33$.

Figure 12 shows two peaks for the equipment response in the fixed base system when the ground motion is tuned to the equipment and structure natural periods, i.e. $T_g/T_s = 0.1333$ and $T_g/T_s = 1.0$, respectively. The isolated response increases slightly until it reaches a peak at $T_g/T_s = 0.7$. After this peak, increasing the period of the ground motion does not cause a large change in equipment response.
Effect of Mass Ratio

Figure 4 and 13 shows the effect of the equipment to structure mass ratio on the acceleration response of the equipment. In figure 13, three friction coefficients (0.01, 0.04, 0.1) and the fixed base case are considered. The natural period and damping ratio of each subsystem are the same as those used in the section on the effect of friction coefficient. The masses of the equipment, structure and base are 0.0001 kg, 1.0 kg and 1.0 kg, respectively, which implies $m_e/m_s = 0.0001$. In figure 4 the mass ratio $m_e/m_s$ is 0.01. From figure 4 and figure 13, it is observed that, for most systems, there is no noticeable change in the equipment response as the mass ratio is reduced. When the equipment natural frequency is equal to the structure natural frequency ($T_e/T_s = 1.0$), there are amplifications in the equipment response as the mass ratio decreases (see table 1). However, when the system is isolated, there is no noticeable change in the equipment response as the mass ratio decreases. Significant amplifications in equipment response occur as the mass ratio decreases for $T_e/T_s = 0.7$, a system where a peak in the equipment response results from the stick-slip friction action.

EARTHQUAKE GROUND MOTION

Effect of Friction Coefficient

The effects of friction coefficient of the base isolator on the response of the equipment are shown in figures 14 through figure 16. In these figures, $\mu$ is the friction coefficient, which varies from 0.01 to 0.1. The natural period of the structure ($T_s$) and base ($T_b$) are 0.3 and 4.0. The damping ratios and masses of the equipment, structure and
base are $\zeta_e = 0.02$, $\zeta_s = 0.02$, $\zeta_b = 0.08$; and $m_e = 0.01$ kg, $m_s = 1.0$ kg, $m_b = 1.0$ kg, respectively. In this study, the maximum acceleration response of the equipment supported on the fixed-base structure is also calculated for comparison. In order to explain the amplifications in equipment response due to the stick-slip friction action of the base isolator, the Fourier spectra of the acceleration response of the structure supported on R-FBI subjected to the El Centro 1940 S00E earthquake ground motion are examined. The properties of the structure-R-FBI system are the same as the original system with the equipment removed. The different friction coefficients are given in figures 17 through figure 22.

From figures 14, 15 and 16, it is observed that the acceleration response of the equipment has a sharp peak at $T_e/T_s = 1.0$ for the fixed-base case. One notices from the corresponding Fourier spectra (figure 17) that the dominant peak is located at a frequency of 3.3 Hz which corresponds to the natural frequency of the structure. This indicates that significant energy is transmitted into the structure at this particular frequency. When the natural frequency of the equipment is tuned to the natural frequency of the structure ($T_e/T_s = 1.0$), large amplification in equipment response occurs due to the resonant effect.

Figures 14, 15 and 16 show that for a wide range of frequencies (including the tuned case $T_e/T_s = 1.0$), the R-FBI effectively reduces the response of the equipment as compared to the fixed base system. It is noticed that the smaller the friction coefficient the greater the reduction in equipment response. This effect can be seen from the Fourier spectra of the structure response (figures 18-22). As the friction coefficient decreases, the amplitude of the Fourier spectra and hence the amount of energy imparted into the equipment decreases as well. For $T_e/T_s > 10.0$, there is relatively little change in the response of the equipment in the isolated system as compared to the fixed base case.
In order to illustrate the magnitude of the reduction in equipment response, consider a typical friction coefficient (μ = 0.04) and the three ground motions Taft S69E, El Centro S00E, and Pacoima S16E. In the high frequency region (T_e/T_s ≤ 0.3), the normalized responses are reduced by factors of 3, 5, and 11 respectively. At tuning (T_e/T_s = 1.0), the normalized responses are reduced by factors of 3, 6, and 18, respectively. This comparison indicates that the R-FBI is extremely effective in reducing the equipment response when the system is subjected to strong ground motions such as the Pacoima S16E record.

Effect of Damping

The effect of equipment damping on the response of the equipment in structures supported on R-FBI subjected to various earthquake excitations is shown in figures 23, 24 and 25.

The damping ratios of the structure and base are 0.02 and 0.08, and the damping ratio of the equipment varies from light damping of 0.02 to relatively heavy damping of 0.1. Base friction coefficients of 0.01 and 0.1 representing two extremes of small and relatively large friction coefficients are considered. In addition, the natural period of the structure and base are 0.3 sec and 4.0 sec respectively while the natural period of the equipment varies. The masses of the equipment, structure and base are 0.01 kg, 1.0 kg, and 1.0 kg, respectively.

It is observed that the acceleration response of the equipment is reduced as the equipment damping ratio increases for both friction coefficients. The reduction in the resonant peaks is more prominent. This indicates that damping in the equipment is an effective mechanism for reducing peaks in equipment response resulting from either the
stick-slip friction action or tuning. In the low period (high frequency) region \( \frac{T_e}{T_s} \leq 0.2 \), however, varying the equipment damping has no effect on the equipment response in the isolated structure.

Figures 26, 27 and 28 show the effects of structural damping on the equipment response in structures subjected to various earthquake excitations. The damping ratio of the equipment is fixed at 0.02 and the damping ratio of the structure varies from 0.02 to 0.1. The remaining properties of the equipment, structure and base were those described above.

It is noticed that the acceleration response of the equipment is significantly reduced by increasing the damping ratio of the structure for \( \frac{T_e}{T_s} \leq 2.0 \). For \( \frac{T_e}{T_s} \geq 2.0 \), there is no noticeable change in the equipment response as the damping in the isolated structure varies.

**Effect of Mass Ratio**

The effect of equipment to structure mass ratio on the acceleration response of the equipment for various earthquake excitations is shown in figures 29, 30 and 31. In each figure, three equipment natural periods of 1.0 sec, 0.3 sec and 0.04 sec representing relatively low to high frequency equipment are used. The natural period of the structure and base are 0.3 sec and 4.0 sec, respectively. The damping of the equipment, structure and base are 0.02, 0.02 and 0.08, respectively. In addition, two friction coefficients 0.01 and 0.10 are considered.

It is observed that for \( T_e = 1.0 \) and \( T_e = 0.04 \) (equipment natural frequency detuned from the natural frequency of either the structure or the base), and for both small and large friction coefficients, the mass ratio has no noticeable effect on the equipment response for
the various earthquakes considered (figures 29 - 31).

For $T_e = 0.3$ (equipment natural frequency tuned to the structure natural frequency) and a small friction coefficient ($\mu = 0.01$), varying the mass ratio has no effect, in general, on the equipment response (figures 29, 30). However, figure 31 shows that the response of the equipment increases as the mass ratio decreases. This is possibly due to the high intensity ground motion associated with the Pacoima Dam record. For $T_e = 0.3$ and a relatively large friction coefficient (0.1) the equipment response increases as the mass ratio decreases. From previous research on fixed base structures, when the system is tuned, the amplification in equipment response depends directly on the mass ratio. The equipment-structure-R-FBI system would behave like a fixed base equipment-structure system if the friction coefficient in the R-FBI is infinitely large. For finite but relatively large values of the friction coefficient, the system behaves more like a fixed base system than if the friction coefficient is small. This explains the amplifications in equipment response when $\mu = 0.1$ that are absent when $\mu = 0.01$. 
CHAPTER 5. CONCLUSIONS

The dynamic response of lightweight equipment in structures supported on resilient-friction-base isolators (R-FBI) subjected to harmonic ground motion and earthquake ground motion is investigated. The model is a SDOF equipment item attached to a SDOF structure which is attached to a base mat supported on the R-FBI. The peak absolute acceleration of the equipment is evaluated. An efficient semi-analytical numerical solution procedure for the determination of equipment response is developed. Closed form expressions for the response in each phase, sliding and non-sliding, are derived. The expressions in each phase are exact, thus eliminating any numerical difficulties associated with a standard numerical integration scheme. The transition from sliding to non-sliding is determined numerically. A series of parametric studies is performed to examine the effects of friction coefficient, damping ratio, mass ratio and frequency content of ground motion on the response of the equipment. Based on the present results, the following conclusions may be drawn:

HARMONIC GROUND MOTION

1. The R-FBI is effective in reducing the acceleration response of the equipment.

2. The variations of the friction coefficient of the R-FBI affect the equipment response. In general, the smaller the friction coefficient, the greater the reduction in the response of the equipment.

3. Equipment damping effectively reduces resonant effects.
4. The response of the equipment is affected by the period of the ground motion. Amplifications in equipment response occur when the natural period of the structure, equipment, or base-isolator are tuned to the period of the ground motion.

5. When the equipment natural frequency is tuned to the structure natural frequency, amplifications in the equipment response occur as the mass ratio decreases for the fixed base case. When the system is isolated, however, there is no noticeable amplification in the equipment response as the mass ratio decreases. Significant amplifications occur in the equipment response as the mass ratio decreases in systems where there is a peak due to the stick-slip friction action.

EARTHQUAKE GROUND MOTION

1. The acceleration response of the equipment in structures subjected to earthquake excitations is, in general, effectively reduced by the use of the R-FBI. The R-FBI is extremely effective in protecting the equipment when the isolated structure is subjected to severe earthquake ground motions.

2. Varying the friction coefficient of the R-FBI affects the equipment response. In general, the smaller the friction coefficient, the greater the reduction in the response of the equipment.

3. The damping of the equipment and the structure effectively reduces resonant effects. The response of the equipment is reduced as the damping of the equipment and/or the structure increases.
4. When the equipment natural frequency is tuned to the structural frequency, amplifications in the equipment response occur as the mass ratio decreases for large friction coefficients. When the friction coefficient is small, however, there is no noticeable amplification in the equipment response as the mass ratio decreases. In addition, when the natural frequencies of the equipment and the structure are detuned, the mass ratio has no noticeable effect on the equipment response; in either isolated or fixed base structures.
Figure 1. The resilient-friction base isolator (R-FBI)
Figure 2. Structure supported on R-FBI

Figure 3. Equipment-structure supported on R-FBI
Figure 4. Effect of friction coefficient on equipment response versus period ratio ($m_e/m_s = 0.01$)
Figure 5. Fourier spectra of acceleration response of structure ($\mu = 0.01$)
Figure 6. Fourier spectra of acceleration response of structure ($\mu = 0.02$)
Figure 7. Fourier spectra of acceleration response of structure ($\mu = 0.04$)
Figure 8. Fourier spectra of acceleration response of structure ($\mu = 0.06$)
Figure 9. Fourier spectra of acceleration response of structure (μ = 0.10)
Figure 10. Effect of damping on equipment response versus period ratio
Figure 11. Effect of frequency content of ground motion on equipment response

(\(T_e = 1.0\))
Figure 12. Effect of frequency content of ground motion on equipment response

\( (T_e = 0.04) \)
Figure 13. Effect of mass ratio on equipment response versus period ratio 

\( \frac{m_c}{m_s} = 0.0001 \)
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Table 1. Effect of mass ratio on equipment response
Figure 14. Effect of friction coefficient on equipment response versus period ratio (El Centro S00E)
Figure 15. Effect of friction coefficient on equipment response versus period ratio (Taft S69E)
Figure 16. Effect of friction coefficient on equipment response versus period ratio (Pacoima S16E)
Figure 17. Fourier spectra of acceleration response of structure (F-B) for El Centro S00E earthquake
Figure 18. Fourier spectra of acceleration response of structure (μ=0.01) for El Centro S00E earthquake.
Figure 19. Fourier spectra of acceleration response of structure ($\mu=0.02$) for El Centro S00E earthquake
Figure 20. Fourier spectra of acceleration response of structure ($\mu=0.04$) for El Centro S00E earthquake
Figure 21. Fourier spectra of acceleration response of structure (μ=0.06) for El Centro S00E earthquake
Figure 22. Fourier spectra of acceleration response of structure ($\mu=0.10$) for El Centro S00E earthquake.
Figure 23. Effect of equipment damping on response versus period ratio
(El Centro S00E)
Figure 24. Effect of equipment damping on response versus period ratio (Taft S69E)
Figure 25. Effect of equipment damping on response versus period ratio (Pacoima S16E)
Figure 26. Effect of structure damping on response versus period ratio (El Centro S00E)
Figure 27. Effect of structure damping on response versus period ratio (Taft S69E)
Figure 28. Effect of structure damping on response versus period ratio (Pacoima S16E)
Figure 29. Effect of mass ratio on equipment response (El Centro S00E)
Figure 30. Effect of mass ratio on equipment response (Taft S69E)
Figure 31. Effect of mass ratio on equipment response (Pacoima S16E)
CHATER 6. REFERENCES


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