Establishing suitability of an ocean model for a poleward undercurrent study.

Expository paper submitted in partial fulfillment for the degree of

Master of Science

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"All models are wrong, but some are useful." – George E. P. Box

Summary

Modifications of an ocean model are described, as the objective for which the model was used changed to study the kinematics and dynamics of an eastern-boundary poleward undercurrent.
1 Introduction.

Ocean models are essential yet very complex oceanographic tools. It is safe to anticipate that for the foreseeable future, specialists will spend considerable time producing and analyzing numerical solutions. However, the complexity behind these models makes it difficult, if not impossible, to completely understand the meaning and origin of the numerical solutions they provide. Some degree of understanding and trusting the solutions is necessary if rigorous progress is to be made. Thus, heuristic and empirical approaches are used—at least should be used—to establish the appropriateness of any given model. Analogies between different models and between a model and observations—or more generally a model and reality—are helpful tools when trying to establish the appropriateness of a particular model. However, this approach should not come at the expense of a sound numerical representation.

This paper describes the modifications—and their rationale—done to the numerical model used in Rivas & Samelson (2011; from now on Rivas & Samelson) as the focus of research shifted to the model’s poleward undercurrent (PUC) for the author’s PhD research, which is expected to be published in two papers (Duran & Samelson, in preparation). The effect of the modifications are assessed by reproducing some of the results from the Rivas & Samelson paper.

Ocean models as oceanographic tools and the means to establish their credibility is briefly introduced in section 2. Differences between the two model configurations are described in section 3.1, while the differences in some aspects of the solutions are shown in section 3.2 (cross-referencing the corresponding figures in Rivas & Samelson will be necessary), followed by discussion and conclusions in section 4. An appendix (A) includes a complementary analysis.

2 Ocean models as oceanographic tools— and their validation?

Navier-Stokes and the associated coupled equations that comprise the governing equations for ocean dynamics, are complicated enough that even solving them numerically presents considerable challenges (e.g. chapter 10 of Samelson 2011). However, ocean models are undoubtedly the tool of choice for a vast number of oceanographic problems, often being the only viable alternative in the quest for solutions. It is my own personal appreciation from both physical oceanography seminars at Oregon State University and from reading scientific literature, that on one hand there may be a high degree of skepticism in ocean models, and on the other hand some researchers seem to imply that "validating" a model—whatever that may mean in any particular case—is sufficient to establish credibility, even when basic numerical constraints raise legitimate and serious concerns in the model’s ability to simulate the object of study. The following few paragraphs—comprised entirely by quoting leading experts as appropriately referenced—introduce the problem.

What are ocean models?

Numerical ocean models are computational tools used to understand and predict aspects of the ocean. They are a repository for our best ocean theories, and they provide an essential means to probe a mathematical representation of this very rich and complex geophysical system (Griffies, 2005). Numerical models are powerful tools to perform investigative experiments and to extrapolate observations into the future via prediction. For example, the modeler is able to test ideas by altering sub-grid scale parameters, changing boundary forcing or domain geometry, refining the grid mesh sizes to allow the flow to become more nonlinear, simplifying the dynamical degrees of freedom via approximations, identifying dominant balances by measuring terms within the model equations, and projecting scenarios for future ocean climate change due to alterations in atmospheric radiative forcing. Furthermore, the suite of solutions available numerically is far larger than those available analytically.
Hence, when combined with analytical methods of highly idealized configurations, laboratory experiments of small-scale phenomena, and in situ, remote, and paleo ocean observations, numerical models round out the suite of tools for ocean climate science. Indeed, numerical models have become the main repository for observations and theories of ocean climate (Griffies, 2004). During the past decade, large-scale models have become the experimental tool of choice for many oceanographers and climate scientists. The reason for this state of affairs is largely due to improved understanding of both the ocean and ocean models, as well as increased computer power allowing for increasingly realistic representations of ocean fluid dynamics. Without computer models, our ability to develop a robust and testable intellectual basis for ocean and climate dynamics would be severely handicapped (Griffies, 2005).

Can ocean models be validated?

Verification and validation of numerical models of natural systems is impossible. Models can be confirmed by the demonstration of agreement between observation and prediction, but confirmation is inherently partial. Models can only be evaluated in relative terms, and their predictive value is always open to question (Oreskes et al., 1994). Usage of terms like ‘validated’ or ‘verified’ may be misleading and create a false sense of truth, particularly in practical policy applications (Sterman et al., 1994). Model output should not be viewed as an accurate prediction of the future state of the system. Short timeframe model output can and should be used to evaluate models and suggest avenues for future study. Model output can also generate "what if" scenarios that can help to evaluate alternative courses of action (or inaction), including worst-case and best-case outcomes. Scientists should eschew long-range deterministic predictions, which are likely to be erroneous and may damage the credibility of the communities that generate them (Oreskes, 2003).

After reading these quotes from leading experts, a natural question is: How then can we use numerical models and trust the results in our quest for rigorous scientific answers to our research questions? The approach here outlined is to pursue a model that could conceivably simulate a phenomena of interest. The results of the model can then be used to guide observational and dynamical studies which can then confirm or reject the model’s results.

3 An example: Establishing suitability of an ocean model for a poleward undercurrent study.

Poleward undercurrents are ubiquitous large-scale circulation patterns present in eastern boundary current systems. Observations show they are regular features off the U.S. west coast, in the Peru-Chile system, in the Canary and Portugal systems (possibly reaching as far north as England or even Norway), in the Benguela current system and even off the west coast of India. However, the dynamics of PUCs have remained elusive despite many studies now spanning several decades. A study of the dynamics and kinematics of the PUC off the U.S. coast from 41 to 48N (Fig. 1) using a numerical ocean model are the objectives of the author's doctoral dissertation. There are a number of desirable, even necessary, properties that qualify an oceanic numerical model as appropriate for the study of a relatively narrow current (order 20 km) with an alongshore reach of thousands of kilometers, often adjacent to the continental slope, at times possibly over the shelf, and interacting non-linearly with both the coastal and offshore regimes. This motivated the modification of the numerical model used in Rivas & Samelson. The original model was shown to be appropriate for the questions addressed in the Rivas & Samelson paper and it was noted in their solution that their model had a convincing representation of a PUC. New questions were asked and the objectives for which the model was to be used changed accordingly.
Current capabilities of ocean models suggest the time is ripe for further numerical studies of the PUC (Samelson et al. 2008). However, as the model was adapted for a PUC-study, it became clear that successfully simulating flow over the slope is still challenging for ocean models, regardless of the vertical coordinate it uses. Therefore an underlying assumption was necessary during the modifications: Given that PUCs are robust features of all eastern boundary currents, appearing consistently in both observations and models (see references in Duran & Samelson), that therefore even a semi-idealized model is a helpful (even realistic) tool for researching PUCs. Emphasis was placed on having a model that could be expected to reproduce the dynamics of a PUC rather than a model that would exactly simulate any particular eastern boundary system.

3.1 Modifications to a model and their rationale.

In this section, the modifications made to the Rivas & Samelson model are described, as well as the author’s rationale for what constitutes an appropriate model for a numerical study focusing on the dynamics of a PUC.

Bathymetric features have a prominent role in some of the theories suggesting pathways for poleward flow over an eastern-boundary slope; for example by topographic lee-wave drag that generates a mean flow (Samelson & Allen 1986), or by interaction between eddies and bathymetry (e.g. Salmon et al. 1976, Merryfield et al. 2001). lentz (2008) suggests the former is unlikely while the latter is of the right magnitude and should be investigated further. Even if neither are the main driver, either of these methods could enhance poleward flow over the slope. In this regard, terrain-following coordinates are a natural choice for the vertical coordinate (alternatively a hybrid vertical coordinate with sigma coordinate near the bottom could also be considered).

The terrain-following coordinate approach has proved useful over many years for shelf circulation modeling, and thus it was used by Rivas & Samelson. As the main subject changed to the PUC, this choice was retained to allow direct connection with the Rivas & Samelson simulations and analysis, to build upon that prior work. As progress was made Lagrangian and Eulerian approaches allowed Duran & Samelson to assess and evaluate the ability of this approach to represent slope flows adequately.

Currently, all vertical coordinate options for ocean models come with strengths and weaknesses that need to be considered according to the object of study. While originally it was circumstantial that a terrain-following vertical was used, in retrospect it seems (at least to the author) that it was likely the best option:

Isopycnic coordinates are simply unable to resolve mixed layers near bottom and surface boundaries, making it a bad choice for the PUC. Quoting Griffies (2004), the following is one of the three main disadvantages of z-models:

"Representation of the solid-earth boundary is generally cumbersome, as it is discretized with rectangular steps instead of piecewise linear fits to the topography. Rectangular steps impose a distinction between side and bottom. There are means to overcome aspects of this awkward representation by using partial or shaved bottom cells. These methods have been successfully implemented in many popular z-models. However, there remains the issue of parameterizing the bottom boundary layer flow, which is quite important in the ocean. In general, even with more faithful representation of bottom topography, the z-coordinate framework is cumbersome for representing bottom intensified flows and transport."

In contrast, and again quoting Griffies (2004), one of the main advantages of \( \sigma \)-models is that:

"They provide a smooth representation of the solid earth boundary, and so provides for a natural representation of bottom boundary layer physics and bottom intensified currents."

Thus—in the absence of quantifying the differences arising from the choice of vertical coordinate in a study focusing on a PUC— it seems that a terrain-following coordinate is a good first choice. However, it comes with
McCreary's linear model (1981) of a PUC suggests that vertical mixing is of importance, thus relatively high vertical resolution is a natural consideration. The vertical resolution from Rivas & Samelson was increased from 31 to 52 \( \sigma \)-levels. To offset the computational cost related to the increase in vertical resolution, the western boundary was moved from -132°W to -129°W (the latter a typical western limit for modeling studies off Oregon, e.g. Springer et al. 2009) so that the horizontal grid consisted of 160 \( \times \) 296 points instead of 250 \( \times \) 296.

Cross-shore scales for the PUC off the U.S. West coast have been reported between 10 to 50 km (Gay & Chereskin 2009 and references therein), so clearly a numerical model used to study the PUC should be able to resolve this scale. Too fine a resolution, however, has been associated with increased variance at grid scales, which stresses the numerical model and often causes numerical divergence rather than convergence (Griffies, 2004; Schulman et al. 2013). A fine resolution also increases the computational cost, thus the roughly 2.6-km resolution of the Rivas & Samelson model was retained as a good compromise.

Because the pressure gradient error typical of \( \sigma \)-coordinate models depends on several factors (bathymetry slope, depth, stratification, algorithm used to compute the pressure gradient, horizontal and vertical resolution), sensitivity tests are the correct way to determine if the smoothing reduces the error to acceptable levels. As pointed out by Miller (2007), the Beckman and Haidvogel r-parameter value of 0.2—often invoked in the literature as if it was a sufficient condition for an acceptable error—was found an unreliable measure of the smoothing required. (It seems that a value of 0.2 was originally suggested as an upper bound estimate.) Our tests showed that without forcing and a horizontally-constant, vertically-stable stratification, poleward flow of about 0.1 m/s formed over the slope (depth-averaged between 150- and 500-m) while using the Rivas & Samelson bathymetry that had been smoothed to a 0.2 r-parameter (Fig. 2). After increasing the vertical resolution without changing the horizontal resolution, the need for even further smoothing can be expected. Consequentially, significant smoothing of the bathymetry was applied—preferentially over the slope and seamounts (Fig. 3). A poleward undercurrent has typical velocities of 0.1 m/s, it was (somewhat arbitrarily) decided that a tolerable spurious velocity should be no greater than 0.01 m/s after three days of unforced integration with closed boundaries, the maximum r-parameter for these tolerable levels was 0.069 (Fig. 4). This step was particularly important given the intent to study the dynamics of the model PUC.

Given the importance of the bathymetry to a number of dynamical matters (e.g. propagation and decay of coastally trapped waves, vertical structure of velocity and density fields, cross-shore exchange), the problem now being solved is different from the problem solved in Rivas & Samelson (Fig. 4). It should also be expected that the degree of idealization in the bathymetry would cause the solution to diverge from observations (Schulman et al. 2013). As the problem changed from simulating the currents that would give the trajectories of upwelled parcels to researching the dynamics of the model PUC, the emphasis of the model shifted from simulating reality—and therefore showing skill in reproducing observations—to setting up a model that would allow for credible reproduction of the physics involved in a model PUC. The trade-off was the need to delegate reproducing actual observations to secondary importance. The underlying assumption is that ocean models, in particular version 3.6 of ROMS-Rutgers, have enough positive analogs with reality—as known from a multitude of studies—that the solution would be of interest even in a semi-idealized case, provided that negative analogs were under control. In particular, PUCs are a phenomena spontaneously appearing in all eastern boundary currents suggesting that the physical mechanism forcing them is robust without too much consideration to the exact characteristics of the slope.

Simulating geophysical fluid dynamics over the continental slope is still very challenging for ocean models. Preliminary developments in numerical methods like the discontinuous Galerkin method are encouraging, if this method indeed proves suitable for general ocean dynamics, then the pressure gradient error would vanish regardless of the vertical coordinate used (Robert Higdon, personal communication).
Although climatological open-boundary conditions have been shown to support a PUC-like circulation (e.g. Rivas & Samelson, Coelho et. al. 1999), daily boundary conditions allow for higher frequency variability to be included; they were also noted to improve the integrity of the solution (not shown). The monthly boundary conditions used in Rivas & Samelson required a sponge layer that was no longer necessary after changing to daily instantaneous boundary conditions from the data assimilative NCOM-CCS at roughly 9-km resolution. Coastally trapped waves are important drivers of meridional flow over the shelf and slope; they can also setup large-scale alongshore pressure gradients – an important part of the dynamics in the linear PUC model by McCreary (1981).

An important focus of the dynamical analysis is the vorticity balance. One previously suggested candidate for the PUC dynamics is a simple Sverdrup response to regional wind-stress curl. Although the daily wind-stress forcing used in Rivas & Samelson showed good skill when compared to observations at a point, the roughly 50-km resolution and an interpolation problem resulting in unusual spatial derivative fields (see appendix A for a detailed description), encouraged the decision to use a different wind product with higher spatial resolution and an interpolation scheme with continuous derivatives. Wind from the 12-km data-assimilative NAM model was bicubically interpolated to the ROMS grid.

There is some evidence from numerical models that heat flux may help trap poleward flow at depth so that it is an undercurrent during the summer rather than poleward flow reaching the surface as in the winter (Mateos et. al. 2013). Other than the ability to stratify during the summer, heat fluxes are expected to be of secondary importance and therefore the same climatological heat-flux forcing from Rivas & Samelson was considered acceptable.

### 3.2 Differences in the solutions and a possible explanation.

Cumulative wind-stress from the NAM forcing crosses zero around the beginning of September while the wind from Rivas & Samelson crosses zero around mid-July (Fig. 5). Upwelling-favourable winds are considerably weaker in the modified model as can be seen by the magnitude of the minimum (about -1 Pa per day compared to about -5), surprisingly both minimums happen near the beginning of October despite the month and a half lag in the zero crossing.

The Rivas & Samelson model is able to better capture direction and magnitude of velocities at the NH10 buoy (Fig. 6). This is likely related to the bathymetry or the wind forcing or a combination of the two. Remote forcing coming through the southern boundary of the Duran & Samelson model may also be a cause.

Sea surface temperature is overall warmer in the modified model (Fig. 10). While some of the SST variability during August is captured a bit better, the modified model is too warm June through July and again in October (Fig. 7). This may be due to the different wind-forcings, and how deep they mix the surface layer.

Sea level anomaly of the modified model is somewhat convincing near the coast, but degrades offshore completely missing a maximum between 46 and 47 °N and 127-128 °W (Fig. 8). It does slightly better with a maximum near the lower left corner. Overall the original model seems to do better. The higher-frequency open boundary conditions may be causing the problem. Near the coast, SLA in the modified model does much better when compared to observations (Fig. 7), a performance comparable to the original model.

Bottom and surface velocities near the upwelling region (Fig. 10) are comparable in both models, perhaps the most striking difference is a more coherent poleward undercurrent signal (near the 200-m isobath) in the modified model. The PUC also seems to be slightly stronger.

Meridional and zonal velocity cross-sections at roughly 44.6 °N are similar in both models (Fig. 11), the most notable differences being that poleward flow is confined to an undercurrent in the modified model instead of reaching the surface as in the original model, and that onshore flow extends considerably deeper in the modified model while offshore Ekman transport seems comparable in magnitude and location.
4 Discussion and Conclusions

The objective for Rivas & Samelson was determining the likely pathways for upwelled water parcels off the Oregon Coast. Modifying a model with a convincing PUC representation became necessary when the object of study changed to studying the dynamics of the model PUC. Desirable properties in a model for the study of PUC dynamics include: high-frequency boundary conditions, appropriate vertical and horizontal resolutions, relatively high-resolution wind, a realistic bathymetry (in this case as allowed by terrain-following coordinates), and an acceptable pressure gradient error.

Simulating flow over the continental slope with realistic (and smooth) bathymetry is still challenging for current ocean models. As much progress as has been made, numerical limitations may still make it necessary to surrender, say, a realistic bathymetry, in order to reduce an error – e.g., a pressure gradient that is solely due to the numerics. Consequently, the modified model that seems adequate to study the dynamics of a PUC, is to a certain degree, an idealization of the U.S. West coast. Even though the forcing is realistic, the way the ocean responds to that forcing can be expected to be different from a more realistic model. The alternative would be to use a z-coordinate model, but it remains to be seen if the general problems these models have with bottom intensified currents and bottom boundary layers can be satisfactorily minimized given an appropriate horizontal resolution. A definitive answer would require a quantitative comparison, which in turn requires a generalized coordinate ocean model. This approach allows most numerical aspects to remain unchanged while different vertical coordinates are probed.

PUCs develop in all eastern boundary systems, the only exemption possibly being the west coast of Australia, which seems to be an eastern boundary system dominated by unique dynamics. A convincing PUC representation develops in the semi-idealized, modified model considered in this paper as adequate for PUC dynamics study. It is therefore hoped – despite showing an overall slight degradation in the ability to reproduce observations— that the model PUC is representative of the observed ocean phenomenon. As with any modeling study, further research will be needed to confirm or reject the conclusions drawn from the model PUC.
5 References


A Appendix

A.1 Analyzing the wind from Rivas & Samelson

A.1.1 Summary

It is shown that the curl of the wind stress that was used to force the R & S simulation does not match general analytic properties of the curl of a vector field that has been bilinearly interpolated from a coarser to a finer grid. An example of the errors is shown.

A.1.2 Analysis

Consider the square formed by the centers of a coarse-grid cell (Fig. 12; data is positioned at the center of the coarse-grid cell and a square is formed between four centers). Normalize the independent variables (longitude and latitude in this case) so that this square becomes the unit square. The formula for bilinear interpolation at any point \((x, y)\) within the unit square is:

\[
f(x, y) = f(0, 0)(1 - x)(1 - y) + f(1, 0)x(1 - y) + f(0, 1)(1 - x)y + f(1, 1)xy
\]

Suppose we have a finer grid within this square. A centered difference in the zonal direction on the finer, staggered grid is given by

\[
\frac{\partial f}{\partial x} \approx \frac{f(x + \delta, y) - f(x, y)}{\delta} = f(1, 0) - f(0, 0) - [f(1, 0) + f(0, 1) - f(0, 0) - f(1, 1)] y
\]

where \(\delta\) is the distance between cells in the finer grid. In the meridional direction a similar expression:

\[
\frac{\partial f}{\partial y} \approx \frac{f(x, y + \delta) - f(x, y)}{\delta} = f(0, 1) - f(0, 0) - [f(1, 0) + f(0, 1) - f(0, 0) - f(1, 1)] x
\]

Note that the derivatives are exact in the sense that they do not depend on the size \(\delta\) of the finer grid. For example:

\[
\lim_{\delta \to 0} \frac{f(x, y + \delta) - f(x, y)}{\delta} = \frac{f(x, y + \delta) - f(x, y)}{\delta}
\]

(Although clearly not exact in the sense that \(f(x, y)\) is an approximation.)

The curl of a vector-valued function \(\text{curl}(\mathbf{F}) = \mathbf{k} \cdot \nabla \times (f, g, 0)\) is thus given by

\[
\text{curl}(\mathbf{F}) = \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} = b_y - m_y y - [b_x - m_x x]
\]

where (according to above) we have defined:

---

1 Possibly a rectangle in which case each independent variable has its own normalization factor.
\[ b_x = f(0,1) - f(0,0), \]
\[ m_x = [f(1,0) + f(0,1) - f(0,0) - f(1,1)] \]
\[ b_y = g(1,0) - g(0,0) \text{ and } \]
\[ m_y = [g(1,0) + g(0,1) - g(0,0) - g(1,1)] \]

Clearly \( \text{curl}(F) \) is continuous within the unit square.

**Note:** We will refer to the square or rectangle formed by four centers of the coarse grid simply as a square (Fig. 12). This square is where the linear interpolation is done by using the data at the four corners. The corners of the square are the centers of the coarse grid – where the coarse data is positioned.

At the boundary of the squares however, a discontinuity in the curl can be expected: A general expression for bilinear interpolation within an arbitrary square (e.g. Fig. 12, square 1) is given by:

\[
\begin{align*}
    f(x,y) &\approx \frac{1}{(x_2-x_1)(y_2-y_1)} \left[ f(x_1,y_1)(x_2-x)(y_2-y) + f(x_2,y_1)(x-x_1)(y_2-y) + \\
    &\quad f(x_1,y_2)(x_2-x)(y-y_1) + f(x_2,y_2)(x-x_1)(y-y_1) \right] \\
\end{align*}
\]

Computing finite differences as above, we see that \( \text{curl}(F_1) = \hat{k} \cdot \nabla \times (f, g, 0) \) within square 1 is given by

\[
    \text{curl}(F_1) = \frac{1}{(x_2-x_1)(y_2-y_1)} \left[ B_y + M_y y - (B_x + M_x x) \right]
\]

where

\[
    M_y = g(x_1, y_1) - g(x_1, y_2) - g(x_2, y_1) + g(x_2, y_2) \\
    B_y = [g(x_2, y_1) - g(x_1, y_1)]y_2 + [g(x_1, y_2) - g(x_2, y_2)] y_1 \\
    M_x = f(x_1, y_1) - f(x_1, y_2) - f(x_2, y_1) + f(x_2, y_2) \\
    B_x = [f(x_1, y_2) - f(x_1, y_1)]x_2 + [f(x_2, y_1) - f(x_2, y_2)] x_1
\]

The curl within square 2 is given by:

\[
    \text{curl}(F_2) = \frac{1}{(x_2-x_1)(y_3-y_2)} \left[ \tilde{B}_y + \tilde{M}_y y - (\tilde{B}_x + \tilde{M}_x x) \right]
\]

where

\[
    \tilde{M}_y = g(x_1, y_2) - g(x_1, y_3) - g(x_2, y_2) + g(x_2, y_3) \\
    \tilde{B}_y = [g(x_2, y_2) - g(x_1, y_2)]y_3 + [g(x_1, y_3) - g(x_2, y_3)] y_2 \\
    \tilde{M}_x = f(x_1, y_2) - f(x_1, y_3) - f(x_2, y_2) + f(x_2, y_3) \\
    \tilde{B}_x = [f(x_1, y_2) - f(x_1, y_1)]x_2 + [f(x_2, y_1) - f(x_2, y_2)] x_1
\]

For the curl to be continuous between squares 1 and 2, \( \text{curl}(F_1(x, y_2)) = \text{curl}(F_2(x, y_2)) \) needs to hold.
If we redefine \( M_x, B_x, M_y, B_y \) to include a factor \( \frac{1}{(x_2-x_1)(y_2-y_1)} \) and \( \widehat{M}_x, \widehat{B}_x, \widehat{M}_y, \widehat{B}_y \) to include a factor \( \frac{1}{(x_2-x_1)(y_2-y_1)} \), then \( \text{curl} (F_1(x,y)) = \text{curl} (F_2(x,y)) \) at the boundary of two meridionally contiguous squares (e.g. squares 1 and 2 in Fig. 12), when

\[
\begin{align*}
\widehat{B}_y + \widehat{M}_y y_2 - B_y - M_y y_2 &= \widehat{B}_x + \widehat{M}_x x - B_x - M_x x \\
\widehat{B}_y - B_y - \widehat{B}_x + B_x + [\widehat{M}_y - M_y] y_2 &= [\widehat{M}_x - M_x] x \end{align*}
\]

(1)

Condition (1) cannot be satisfied \( \forall x \in [x_1, x_2] \) (it could be satisfied for at most one point). A similar conclusion can be reached with two zonally contiguous squares. Thus, the curl is not continuous across the boundary of the squares — except possibly at one point.

From the above analysis we arrive at the following two conclusions:

After doing bilinear interpolation from a coarser to a finer grid:

- The curl of the interpolated fields – computed on the finer grid – is a linear function in both \( y \) and \( x \) (and therefore continuous) within a square.

- The curl computed on the finer grid is discontinuous at the boundary of any two squares (except possibly at one point).

**Remark:** The discontinuity can be generalized to any derivative, not only the curl.

A.1.3 Discussion.

We can now use these conclusions to analyze the wind curl from the interpolated fields that were used in the R & S simulation. The wind data used to force the model is a function of longitude, latitude and time. The original data had one data set per day with \( t = 12 \) pm. ROMS required wind data at \( t = 12 \) am, averaging two data sets (each at 12 pm) gives a data set at 12 am. The data set resulting from this averaging can now be bilinearly interpolated as assumed above. This is effectively a trilinear interpolation. However, our above analysis holds since once the averaging has been done, the problem reduces to bilinear interpolation exactly as described above.

The curl of the interpolated wind fields used in the model is not linear within a square, in fact the curl has a discontinuity within the square (Fig. 13; that the curl is in fact discontinuous was verified but is not shown here). This suggests that the interpolation of the wind fields with which ROMS was forced are not a linear interpolation as described in the Rivas & Samelson paper, which reads:

*The daily varying wind stress forcing for year 2005 was obtained from a gridded Quick Scatterometer (QuikSCAT) product. The 1/2°-resolution data were linearly interpolated to the model grid; for the model grid points closest to the coastline, the values were extrapolated by replacing them with the*
Using Matlab’s interp2 function, the original coarse data was bilinearly interpolated to the finer grid so that a comparison with the interpolated fields used to force ROMS could be made. The raw data was bilinearly interpolated both with and without time averaging, as described above. When doing the averaging, interpolation is effectively trilinear – an averaging followed by bilinear interpolation. The results for the newly interpolated winds -both the bilinear and trilinear interpolations- show what we expected from the above conclusions, while the interpolated wind data used to force the ROMS simulation does not behave as expected (Figs. 14 and 15). Within each square, the curl from the data used to force the model is approximately (although not exactly) given by two different constants instead of a linear function, this explains the triangular shapes (Fig. 16). The bilinear and trilinear interpolations give a curl that is discontinuous at the boundary of the squares but linear within (Figs. 17, 18 and 19). The only difference between the trilinear and bilinear interpolation is the magnitude of the slope as well as the magnitude of the intercepts $B_x, B_y, \tilde{B}_x, \tilde{B}_y$, as expected from the time-averaging. With linear interpolation in more than one dimension, the order in which interpolation is done does not matter. Thus, the average of two consecutive-in-time bilinear interpolations (Figs. 18 and 19) should equal the corresponding trilinear interpolation (Fig. 17)

In conclusion the interpolated fields used to force the model show features that are inconsistent with a linear interpolation suggesting an error in the computations done for R & S; these features can sometimes be noticed from a visual inspection (Figs. 20 and 21).
Figure 1: Model domain with smoothed bathymetry (m), grey and black contours correspond to the 200-m isobath of the smoothed and Rivas & Samelson bathymetries, respectively. The northern and southern limits of the upwelling region (inshore of the 200-m isobath between 41.5°N and 45.5°N) are also shown in grey. The location of buoy NH10 is marked by a triangle, buoy NDBC 46050 is marked with a cross, South Beach, the location of sea surface height observations, is marked with a circle and the location of buoy Re is marked with a square.
Figure 2: Depth-averaged meridional geostrophic velocity between 150 and 500m—solely due to the pressure gradient error typical of \( \sigma \)-coordinate ocean models— that resembles a poleward undercurrent in magnitude, cross-shelf and along-shelf scales. Color scale is velocity in m/s. Zero-contour in black.
Figure 3: Cross-sections showing vertical resolution and bathymetry smoothing at 42°N (top), 44.6°N (middle) and 47°N (bottom). Also shown is the PUC region used for the analysis in Duran & Samelson, bounded vertically between 150- and 500-m and horizontally between the 200-m isobath and roughly 36km offshore.
Figure 4: Comparison of bathymetry before and after smoothing, and 0.069 r-criterion.
Figure 5: Cumulative wind-stress averaged over the upwelling region (see Fig. 1), compare to fig. 2(a) of Rivas & Samelson.
Figure 6: Observed ADCP (obs) and model (mod) velocity vectors (cm s$^{-1}$, eastward along positive time axis) at (a) 10 and (b) 68 m at the NH10 location (Fig. 1) vs time (months) for the year 2005. Model velocities are displaced from the zero line, but have the same scale as the observed velocities. Compare to fig. 3 of Rivas & Samelson.
Figure 7: (a) SST at the position of NDCBC buoy 46050 and (b) adjusted sea level anomaly (relative to mean annual cycle, represented by a single fitted annual harmonic) at South Beach (see Fig. 1) vs time (months) during 2005: observed (gray lines) and modeled values (black). Compare to Fig. 4 of Rivas & Samelson.
Figure 8: Temporal means (May-October 2005) of (a) SST (°C) and (b) sea level anomaly (SLA in cm, is the difference from the annual mean) from the model. Compare to Fig. 5 of Rivas & Samelson.
Figure 9: Compare to Fig. 6 of Rivas & Samelson
Figure 10: Model mean mid-July-September near surface (left) and near-bottom (right) velocities (colorscale is magnitude in cms$^{-1}$), in and near the upwelling region (Fig. 1). At each point, these near-surface and near-bottom velocities are vertical averages over the uppermost 80m and lowermost 50m, respectively. Compare to Fig. 8 of Rivas & Samelson
Figure 11: Model mean mid-July-September (a) meridional and (b) zonal velocity (colorscale is magnitude in $\text{cm s}^{-1}$, zero-contour in black) vs longitude and depth (m) at the NH line (Fig. 1). The point near (-124.8,-200) with a white marker in (a) is the core of the model undercurrent represented by the cross-sectional maximum northward velocity. Compare to Fig. 9 of Rivas & Samelson.
Figure 12: Two contiguous interpolation squares (bottom is square 1 and top is square 2); the corners are the points where the data of the coarse grid is known (in this case the center of the coarse-grid cells, see figure 20 and compare to figure 16). Thus, the shown coordinates are where the function that needs to be interpolated is known.
Figure 13: Close-up of the curl of the interpolated wind-fields along a constant longitude as a function of latitude. The boundary of the squares are marked with vertical lines. The discontinuities are within the squares which is not as expected. All the data in the subsequent plots is for day 100 (April 10, 2005) unless otherwise noted, similar results hold for other days.
Figure 14: Curl of the interpolated wind-fields along a constant longitude as a function of latitude. Data marked as Model is the curl from the wind fields used to force the simulation, (Bi) is bilinear interpolation from the coarse data, and (Tri) is bilinear interpolation of the coarse data after averaging two consecutive data sets in time. The boundary of the squares are marked with vertical lines.
Figure 15: Curl of the interpolated wind-fields along a constant latitude as a function of longitude. Data marked as Model is the curl from the wind fields used to force the simulation, (Bi) is bilinear interpolation from the coarse data, and (Tri) is bilinear interpolation of the coarse data after averaging two consecutive data sets in time. The boundary of the squares are marked with vertical lines.
Figure 16: Curl of the interpolated wind-fields used to force the model as a function of longitude and latitude. The centers of the coarse-grid cells (which coincide with the boundary of squares) are marked with black dots.
Figure 17: Curl of the newly trilinearly interpolated wind-fields as a function of longitude and latitude; time is 12 am day 100. The centers of the coarse-grid cells are marked with black dots.
Figure 18: Curl of the newly bilinearly interpolated wind-fields as a function of longitude and latitude. Time is 12 pm day 100. The centers of the coarse-grid cells are marked with black dots.
Figure 19: Curl of the newly bilinearly interpolated wind-fields as a function of longitude and latitude time is 12 pm day 99. The centers of the coarse-grid cells are marked with black dots.
Figure 20: Left column is the interpolated wind data (meridional component in top, zonal in bottom) used to force the model. Middle column is the raw data at 50km resolution from where interpolations are done. The right column is the trilinearly interpolated data using Matlab’s averaging and interpolation routines. Some differences in the interpolated fields are highlighted with a black box in the left column (compare to right column). Data is for December 26, 2005.
Figure 21: Left column is the interpolated wind data (meridional component in top, zonal in bottom) used to force the model minus the trilinear interpolation done with Matlab – left column minus right column in figure 20. The right column is the relative error.