AN ABSTRACT OF THE DISSERTATION OF

Chris Dizon for the degree of Doctor of Philosophy in Electrical and Computer Engineering presented on March 11, 2022.

Title: Sizing and Performance of the Pendulum Power Take-Off Wave Energy Converter: Modularity and Chaotic Behavior

Abstract approved: ____________________________________________

Ted Brekken

Multiple wave energy converter (WEC) archetypes exist with varying power take-off (PTO) designs in the attempt to maximize ocean energy harnessed and converted into useful energy. The pendulum PTO is popular for its simple, yet robust functionality due to its internally located components and simple operation. Additionally, this PTO does not require direct contact with the ocean waves, giving it the convenient ability to act as a modular power system for ocean observing applications at sea. This is studied by examining the feasibility of a modular horizontal pendulum wave energy converter to power National Data Buoy Center’s Self-Contained Ocean Observations Payload (SCOOP) off the coast of Washington State, U.S. The effect on power output was studied when the pendulum’s radius arm, mass, and PTO damping were varied. Results using Matlab toolbox WEC-Sim revealed positive correlation between radius arm length and mass to power output, where power maximized for optimal damping values. Seasonal trends in power were not significant, where a 20 kg pendulum mass was needed to meet the SCOOP base power requirement of 5 W throughout the year. Further investigation of the potential of the pendulum PTO studies the systems display of chaotic behavior and its affect on power output in a regular sea state. Chaotic behavior in the system was verified by it exhibiting sensitivity to initial conditions, topological transitivity, and dense periodic orbits. When varying wave height and period, PTO damping, and pendulum mass, design, and radius arm, power output is consistently generally greater when the system is exhibiting stable versus chaotic behavior. Exploration of chaotic behavior present in the pendulum PTO continued through simulations in irregular
sea states. Verification was provided by the checking of asymptotic orbits, positive Lyapunov exponents, and chaotic attractors in Poincaré maps. Although positive Lyapunov exponents and asymptotic orbits were present, the lack of a chaotic attractor concludes that chaotic behavior was not present in a realistic sea environment.
Sizing and Performance of the Pendulum Power Take-Off Wave Energy Converter: Modularity and Chaotic Behavior

by

Chris Dizon

A DISSERTATION

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APPROVED:

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Major Professor, representing Electrical and Computer Engineering

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Head of the School of Electrical Engineering and Computer Sciences

__________________________________________
Dean of the Graduate School

I understand that my dissertation will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my dissertation to any reader upon request.

__________________________________________
Chris Dizon, Author
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### Figure 3.4
Bifurcation diagram of pendulum PTO WEC system simulated with a constant wave height of 4 m, PTO damping 15 Nm/(rad/s), pendulum design 1 200 kg mass, and radius arm of 0.26 m. The period changes in 0.1 steps from 6 to 16 seconds. Areas of fixed points (single dot), periodic orbits (2 dots), and chaos (multiple dots) are evident.

### Figure 3.5
Time plot of pendulum position for two initial conditions, 0° and 0.001°. Sensitivity to initial conditions is shown where the two paths begin to largely diverge noted by the black oval.

### Figure 3.6
Phase space plot of pendulum PTO WEC system throughout simulation; Top right, 0 - 300 s; Bottom left, 0 - 600 s; Bottom right, 0 - 1200 s. Topological mixing is evident by the entire phase space covered. Also illustrated is the presence of dense periodic orbits.

### Figure 3.7
Poincaré map of the phase space plot of Figure 3.6 when sampled at the period of the wave, 8.2 s. This results in a strange attractor and proves that there is an underlying pattern.

### Figure 3.8
Poincaré map of a random system where no pattern is noticed.

### Figure 3.9
Poincaré map of strange attractor, zoomed in on the black rectangle of Figure 3.7 to show fractal nature of the attractor.

### Figure 3.10
Poincaré map of strange attractor, zoomed in on the black rectangle of Figure 3.9 to show fractal nature of the attractor.

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General Introduction

Marine renewable energy (MRE) has been studied extensively due to its potential as another viable energy source. A primary challenge to MRE is its nascent state, leading to questions such as the best design topology, control methods, power take-off (PTO) systems, mooring configurations, and understanding how these interact with one another and the systems they are powering. Additionally, any of these may change depending on the operating conditions. This dissertation focuses on two areas of MRE – modular applications and PTO dynamics – presented throughout three manuscripts.

Due to MRE’s consistent, higher year-round power and the resources predictability, the Department of Energy’s (DOE) Water Power Technology Office (WPTO) has been leading the Powering The Blue Economy Initiative (PBE) to identify at-sea or remote coastal community applications where MRE could provide a different means to meet power requirements. One of these applications is ocean observation. Ocean buoys monitoring meteorological data like those used by the National Data Buoy Center (NDBC) typically rely on solar power and Lithium-Ion batteries for their energy needs. These require routine maintenance, increasing costs due to service and vessel rentals. The inclusion of a wave energy converter (WEC), a type of MRE technology, could be the sole energy provider for the buoy since this resource has a higher energy density compared to solar. It could also be used in addition to solar to provide extra power. Access to higher power opens up the possibility of longer operations, remote operations, and better quality sensors, ultimately lowering operating and maintenance costs. A WEC usually requires a mooring system to secure it, however the pendulum PTO system does not have the strict requirement of direct interaction with the ocean to produce energy. With these advantages, this PTO was used for a WEC that operates in a box structure situated on a buoy. This gives it a modular ability, as it can be removed and used from application to application. Manuscript 1 studies the feasibility of this in depth by coupling a horizontal pendulum WEC to NDBC’s coastal weather buoys and studying the effect on power output when physical aspects of the
PTO are varied. It is determined if the WEC, subject to weight and size constraints, can meet the buoys power requirements.

To further investigate the potential of the pendulum PTO, its dynamics are studied in Manuscripts 2 and 3, specifically its display of chaotic behavior. When the PTO exhibits such dynamics, it behaves in a seemingly random way, but is in fact deterministic. Understanding the relationship between this behavior, when as occurs if operating conditions of the environment or pendulum are changed, and power output could lead to application of control schemes used in chaos theory. This would allow the PTO to maximize the energy harnessed. Manuscript 2 studies this in a regular, single frequency sea state to understand the basic concept of chaotic behavior in the PTO. Manuscript 3 studies this relationship in a realistic sea state, where it is evaluated in an irregular wave environment. With multiple frequencies present, the chaotic behavior may still be present, diminish, or lead to more complex behavior. This is essential to understand how the dynamics of the PTO would behave in a real world application.

These manuscripts serve to better understand the potential of the pendulum PTO WEC, allowing for future applications in PBE, grid-use, or control strategies.
Manuscript 1 – Modular Horizontal Pendulum Wave Energy Converter: Exploring Feasibility to Power Ocean Observation Applications in the U.S. Pacific Northwest

Chris Dizon, Robert J. Cavagnaro, Bryson Robertson, Ted K. Brekken
2.1 Abstract

Marine renewable energy as a power source for ocean observation applications has the potential to allow longer deployment operations due to the consistent, higher, and denser energy available from this resource. This additionally could encourage deployments in remote locations where maintenance is costly or resource availability is low if dependent on solar power. More importantly, gaps in spatial data could be filled. This paper examines the feasibility of a modular horizontal pendulum wave energy converter to power National Data Buoy Center’s Self-Contained Ocean Observations Payload (SCOOP) off the coast of Washington State, U.S. The effect on power output was studied when the pendulum’s radius arm, mass, and power take-off damping were varied. Results using Matlab toolbox WEC-Sim revealed positive correlation between radius arm length and mass to power output, where power maximized for optimal damping values. Seasonal trends in power were not significant, where a 20 kg pendulum mass was needed to meet the SCOOP base power requirement of 5 W throughout the year.

2.2 Introduction

The desire to explore how marine applications could benefit from marine renewable energy (MRE) has increased in recent years due to the advantages this energy source could provide compared to traditional methods (e.g. battery or solar), including consistent, higher year-round power and the predictable nature of the resource. The Department of Energy’s (DOE) Water Power Technology Office (WPTO) has been leading this front under the Powering The Blue Economy Initiative to identify at-sea or remote coastal community applications where MRE could provide a different means to meet power requirements while providing these additional benefits [1]. Both the National Renewable Energy Laboratory (NREL) and the Pacific Northwest National Laboratory (PNNL) formulated the results in the report, providing background information and a value proposition for MRE integration with eight unique markets. Subsequently, both national laboratories conducted a large outreach effort to various stakeholders within the ocean observation sector of the blue economy, with a main takeaway being power limitations are a consistent issue across many applications [2][3].

While MRE could prove to be an answer to these gaps, the question still remains on how to integrate mechanisms harvesting energy from the ocean and the end-use application. A core challenge to realizing MRE-powered blue economy applications is the nascent state of MRE
technology compared to other renewable sources, with many knowledge gaps such as the optimal design topology, control methods, or power take-off (PTO) systems. This is evident by the numerous operating principles a MRE design could function by. Those specific to the focus of this paper, wave energy converters (WEC), include oscillating water columns (e.g. Ocean Energy), attenuators (e.g. Pelamis), point absorbers (e.g. PowerBuoy), oscillating surge converters (e.g. Oyster), and rotating mass (e.g. Wello Penguin) operation types [4]. Additionally, all these utilize different PTOs in the form of hydraulics, air turbines, hydro turbines, or direct mechanical or electrical drive systems to convert the energy from the waves into electricity. Although there are various ways to design a WEC, the common goal remains the same; to determine which topology produces the most power given environmental conditions. The large focus of this goal has been developing such devices that can supply power to the electric grid, however as the motivation behind the DOE report, there is large potential for marine applications to benefit from being powered by MRE. This effort in research has just begun, noted by multiple marine energy companies receiving grants on co-development of marine energy at smaller scales in 2020 from the WPTO [5], looking to further investigate the questions of coupling MRE to marine applications. The next essential step, then, is exploring specific WEC designs and their ability to meet applications power requirements.

This paper examines coupling a modular horizontal pendulum wave energy converter (HPWEC) to an application in the blue economy and its feasibility to power this application. Detailed analysis will cover the effect PTO damping, pendulum mass, and length of the pendulum radius arm values have on its power output capabilities. Performance is benchmarked against the requirements of a specific marine application: powering coastal weather buoys measuring ocean and atmospheric properties.

2.3 Application Case Study

A detailed application case study analyzing the feasibility of powering weather buoy systems using a modular HPWEC, (i.e., the WEC is directly connected/integrated onto the buoy as a distinct device) is presented. The following subsections describe aspects of the case study in detail including the power range, site location, design of the weather buoy, community responses, and additional features considered for simulation such as mooring and data range.
Figure 2.1: Average power requirements of ocean observation sensors and platforms. Abbreviations: ASV (autonomous surface vehicle), AUV (autonomous underwater vehicle), ROV (remotely operated vehicle). The SCOOP requirement of 5.0 to 16.5 W is within range of similar oceanographic sensors [2].

2.3.1 Power Range

The power range of interest is based on the requirements of the Self-Contained Ocean Observations Payload (SCOOP) system found on the National Oceanic and Atmospheric Administration (NOAA) National Data Buoy Center’s (NDBC) weather buoys. SCOOP and similar meteorological and oceanographic sensor buoys host a variety of sensors reporting values such as wave height and period, wind speed and direction, sea temperature, and air pressure which are used to understand and predict conditions and changes in the weather, climate, and ocean environment [6]. Typical power consumption ranges between a baseline of 5 W and peak of 16.5 W during periodic data transmission bursts [3], which are the basis for evaluating simulation power outputs in the results section. It is noted that this also falls within the average power requirements shown in Fig. 2.1 for oceanographic sensors and communications equipment [2].
2.3.2 Location of Interest

The study will be applied to NDBC Station 46041 located 45 nautical miles northwest of Aberdeen, WA, U.S. in a water depth of 128 m [6]. This site was chosen for two desirable characteristics. First, the weather buoy at this station hosts the instrumentation payload modeled in our case study. Compared to more power-intensive ocean observing platforms or applications (Fig. 2.1), these power requirements are generally in the lower range, providing a low bound to determining what power needs the modular HPWEC can meet. Second, the location of the buoy is in the Pacific Northwest of the United States which is known to have a large wave energy
resource [7][8] making it an ideal location for the integrated system to be operating in.

An additional note to mention is operation in remote coastal areas, a theme recorded from ocean application users in a survey showing responses on possibilities if more power were available (Fig. 2.2) [2]. The location of Station 46041 is situated along the coast of Washington State where infrastructure is limited equating to increased costs to maintain and operate systems due to difficulty of access. Typical maintenance intervals of these buoys are 12 to 18 months to keep sensors operational [3]. Currently, the buoy is powered by solar panels [2][9], however operations in the ocean environment leads to salt accumulation which can impact their lifetime and efficiency [10]. The dependence on solar has the potential to limit the amount of power provided which additionally has an affect on the location and duration of the operation, leading to decrease use in remote areas and gaps in spatial data such as locations located closer to the poles where the solar resource availability is limited [3].

Combining a HPWEC with the buoy could lead to a longer operating period, the main desire noted by ocean application users (Fig. 2.2), as well as removing the need for routine annual check-ins and opening up the possibility of different operating locations. As to how the design of the HPWEC accomplishes this is described later in its respective section.

2.3.3 Coastal Weather Buoy Modelling

A buoy like the one used for NDBC Station 46041 can be divided into two main components: the hull and the instrument payload. The hull is an AXYS 3 m discus buoy (3 m buoy) [9][11] whose dimensions and weight of each separate structure were available after discussions with AXYS technologies (A. Velasco, personal communication, February 15 2021). Information specific to the SCOOP payload was found through NDBC public presentations and research

<table>
<thead>
<tr>
<th>Type</th>
<th>Mass [kg]</th>
<th>COG</th>
<th>Diameter [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 m buoy</td>
<td>1500</td>
<td>[0.0,-0.03]</td>
<td>3.4</td>
</tr>
<tr>
<td>SCOOP</td>
<td>90</td>
<td>[0.0,1.7]</td>
<td>0.3</td>
</tr>
<tr>
<td>HPWEC housing</td>
<td>10</td>
<td>[-0.48,-0.64,0.50]</td>
<td>0.6</td>
</tr>
<tr>
<td>Pendulum</td>
<td>14-100</td>
<td>[-0.37,-0.54,0.52]</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 2.1: Mass Property and Dimension Values for System Components
Figure 2.3: NDBC buoy with SCOOP payload [11].

articles [11][12]. This led to knowledge of the overall waterline, dimensions, and weight of the integrated AXYS 3 m-discus SCOOP buoy. Physical modeling was done in SolidWorks [13] to obtain mass properties such as moment of inertia (MOI) and center of gravity (COG) after applying the information from the aforementioned sources. To acquire the hydrodynamic response of the buoy in various sea states, it was then simulated in Ansys AQWA [14] using the aforementioned information to generate the hydrodynamic files of the coastal buoy. This software simulates the system through a range of frequencies and outputs responses for each frequency such as the added mass, radiation damping, Froude-Krylov, Diffraction, and response amplitude operators. Both solid modeling and hydrodynamic output files are used during power simulations described in Section 2.7. General dimensions and estimated mass properties of the 3 m buoy and the SCOOP payload are reported in Table 2.1 and Fig. 2.3.
2.3.4 Mooring

Mooring systems used by NDBC can consist of a combination of chain, wire, nylon, and other polypropylene materials depending on the depth and buoy dimensions [15][16]. Specifically, single-point surface moorings have been used by NDBC for their 3 m buoys and an all-chain mooring setup for depths up to 90 m and semi-taut for depths between 60 to 600 m [16]. To stay within the main focus of the paper, chain is used to represent the entirety of a simplified mooring system. During simulation, the library MoorDyn was used to include mooring into the simulation [17]. Values needed to run this were vessel and fixed locations of the mooring, its material, diameter, axial stiffness, weight, and unstretched length. The vessel and fixed locations are known from the AXYS schematics and the depth of the water, and the type of mooring has been chosen as chain. The axial stiffness of the chain can be calculated using elongation properties provided by the manufacturer [18]:

\[
E = (5.40 - 4d) \times 10^{10} \tag{2.1}
\]

\[
A = \frac{2\pi d^2}{4} \tag{2.2}
\]

where \(E\) [N/m²] is the elastic modulus, \(A\) [m²] is the chain area, and \(d\) [m] is the chain diameter. The area is multiplied by two to account for the cross sectional area of two bars. The axial stiffness, \(EA\) is \(E\) multiplied by \(A\). A 10 to 45 mm diameter chain is used in depths 2 to 150 m with a mooring scope (ratio of chain length to water depth) between 1.5 to 4.0 [19]. For JEYCO Studless Chain Grade 3 25 mm diameter chain, \(EA = 5.2 \times 10^7\) N and the weight of this chain is 12.63 kg/m [18]. For a depth of 128 m with a mooring scope of 2, the unstretched length is then 256 m.

2.3.5 Data Range

To understand seasonal variation of the power output of the HPWEC, simulations were performed for each season of the year, defined as follows: Fall - September, October, November; Winter - December, January, February; Spring - March, April, May; Summer - June, July, August. NDBC’s yearly data from each buoy station are publicly available. The year 2019 was chosen due to it having the most recent data with the least amount of missing values.
2.4 HPWEC Modelling

2.4.1 Overview of Design

The pendulum-PTO design is straightforward and has been shown to operate well in both small and large sea states [20]. The basic components and assembly of this design are shown in Fig. 2.4. When developed appropriately, such designs require a minimum amount of mechanical parts, possibly eliminating the need for gearboxes or complex control techniques [21]. The concept is that rotation of a pendulum attached to a gear-and-pinion system drives an electrical generator. To charge a battery, an AC-DC converter is required, where a properly designed system will have a combined approximate efficiency of 90%. Additionally, the round trip efficiency of a lithium-ion battery is around 86% [22], leading to a total efficiency of 77.4%.

The HPWEC design offers the advantage of a completely enclosed system that is not subject to the ocean’s harsh environment, therefore increasing longevity whereas other PTO methods’ lifespans are cut short due to corrosion or features used to mitigate corrosion failing early. An example of such would be the linear generator point absorber, where principal components such as the stator, translator, and heaving buoy are generally exposed to salty conditions. While various sealing approaches can be used to protect sensitive components on such a point absorber, a dynamic seal separating a dry housing from a rotating or oscillating component exposed to the elements is likely required. Reducing friction and heating in dynamic seals becomes an engineering challenge with failure creating the potential to shorten the system’s lifespan. This again speaks well to the fact that the pendulum’s design depends on an internal moving system. Also, due to the fact that these components are within an enclosed system, all major assembly takes place onshore as opposed to offshore making deployment and recovery easier. When using a modular version of it, this gives the user the ability to produce power for their application simply by loading the device onto their pre-existing platform where it operates with the motion to the waves. Electrical integration may be as simple as connecting a single power cord to the end-user system. The modular nature of the device also provides the advantage of not being limited to one application or requiring an additional separate mooring system as it can operate directly on the platform. This additionally allows convenient maintenance of the WEC as it is its own distinct device not integrated into the system, such as the design proposed by Triton Systems [5][23]. This would allow removal of the device for maintenance on the vessel or on land as opposed to working on the buoy or removing the buoy itself, increasing cost. This makes the pendulum
design ideal for remote locations, where maintenance is costly and potentially dangerous.

Since the advantage of the pendulum-PTO compared to other PTO designs is its robustness and longevity, this would pair well with systems deployed for long periods of time (years) and in remote locations where maintenance and routine check-ups are not practical options such as the case study location. Referencing the results of stakeholder surveys (Fig. 2.2) again, the theme with the highest amount of relevant comments was “Longer Deployments” which aligns well with the HPWECS design and also with the possibility of operating in “Remote Sites, and Harsh Conditions” [2]. Additionally, most rechargeable systems utilize solar PV panels, but in the Pacific Northwest, solar (compared to wave energy) is not amply available year-round nor is it the most concentrated energy source, requiring more panels and therefore space to equate to the energy available in waves [7]. With more power available, better quality sensors that require higher power, but less calibration and maintenance, may become viable as well. The above items represent an opportunity to study the feasibility of utilizing a HPWEC to power such systems.

2.4.2 Modular System

This subsection details the basic shape of the modular HPWEC for the application case study. The design is based on a simple empty cylinder shape that houses the pendulum-PTO. The housing radius is 0.3 m and height 0.18 m with a thickness of 0.01 m. The pendulum design considered was a half moon shape of radius 0.2 m and height 0.15 m. The radius arm length is
the distance from the HPWEC central axis to the pendulum COG which will be varied in simulation to study the effect it has on power output. The modular HPWEC is shown in Fig. 2.5. Dimensions were based on designing the smallest pendulum size with the ability to be physically manufactured within the mass ranges considered (14 to 100 kg) and to be as close as possible to the center of a 3 m buoy. Additionally, dimension and location decisions were chosen to mitigate as much as possible the effects of the pendulum’s motion on the buoy’s original hydrodynamics.

Hypothetical volume and mass limits for modular devices on NDBC 3 m buoys were obtained from NOAA of 0.06 m$^3$ and 14 kg, respectively. The volume of the studied modular HPWEC is 0.02 m$^3$ and HPWEC weights used during simulations that met the SCOOP power requirement will be compared to the 14 kg limit to examine if this restriction can be met. The model of the integrated system is shown in Fig. 2.6. General dimensions of the HPWEC housing and pendulum are in Table 2.1, noting that the values are taken from its location on the buoy with a radius arm length of 0.18 m.
Figure 2.6: Isometric transparent view of the integrated system; pendulum highlighted in blue, HPWEC housing rigidly connected to 3 m buoy.
2.5 Resource Assessment: Quantifying the Wave Climate

To generate power outputs from the integrated system, the sea state it will be simulated through needs to be calculated. A resource assessment is used to determine the distribution of significant wave height ($H_s$) and peak period ($T_p$) during year 2019 at NDBC Station 46041. These will help define the most frequently occurring sea states and necessary parameters for developing an irregular sea state realization using the Bretschneider spectrum to be used in power simulations.

2.5.1 Bivariate Histograms

When representing an irregular sea state, the parameters needed ($H_s$ and $T_p$) are based on a bivariate histogram; a representation of the distribution of sea state parameters. To determine what sea states NDBC Station 46041 operates in, bivariate histograms were made for each of

![NDBC Buoy 46041 Bivariate Histogram - Fall](image)

Figure 2.7: Bivariate Histogram of NDBC Station 46041, Fall; bin numbers indicate total occurrences (observed/hr) of the sea state over the year; most frequent lies in bin $H_s$ 1.0 to 1.5 m and $T_p$ 10 to 11 s, a value of 593 hrs/yr; color indicates percent of total energy in the sea state.
the 4 seasons using historical yearly data from the Station’s site page [6]. The data file has a record of hourly samples. The most frequently occurring sea state values were used to develop the irregular sea state as opposed to sea states containing the highest annual energy contribution so that power outputs during analyses represent the power that is most frequently generated at this location. The numbers in the bins on Fig. 2.7 indicate occurrences of the sea state whereas color indicates the respective percentage of annual energy. As an example, Fig. 2.7 is the bivariate histogram for the Fall season with the most occurring sea state taking on $H_s$ and $T_p$ values between 1.0 to 1.5 m and 10 to 11 s, respectively (noted by the 593 in the bin) whereas the sea state with the most energy had bin values of $H_s=2.5$ to 3.0 m and $T_p=12$ to 13 s. Table 2.2 summarizes the bivariate histogram bins with the most occurring sea states for each season. Note that the absence of data in the 15 s wave period bin is an artifact of the non-linear frequency discretization utilized by NDBC, which is heavily focused on resolving the high frequency tail of the spectrum rather than the low frequency components. The NDBC post-processing results in no wave periods records between 14.9 s and 16.0 s.

### 2.5.2 Building the Irregular Sea State

Many empirical spectra have been developed to characterize sea states. The Bretschneider spectrum was chosen based on the fact that it does not have the hard requirement of needing a fully developed sea and that it is formulated using two parameters, $H_s$ and $w_m (2\pi f_p=\frac{2\pi}{T_p})$ giving a more accurate representation of the true sea state as opposed to one parameter spectra [20][24]. The span of frequencies, $w$, used to generate the spectrum contain the lowest and highest frequency seen in each season to ensure all observed data from NDBC Station 46041 during 2019

<table>
<thead>
<tr>
<th>season</th>
<th>$H_s$[m]</th>
<th>$T_p$[s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fall</td>
<td>1.0–1.5</td>
<td>10–11</td>
</tr>
<tr>
<td>Winter</td>
<td>2.5–3.0</td>
<td>12–13</td>
</tr>
<tr>
<td>Spring</td>
<td>2.0–2.5</td>
<td>12–13</td>
</tr>
<tr>
<td>Summer</td>
<td>0.5–1.0</td>
<td>7–8</td>
</tr>
</tbody>
</table>

Table 2.2: Most Occurring $H_s$ and $T_p$ for each season
Figure 2.8: Bretschneider Spectrum for NDBC Station 46041 for each season; frequency range to ensure all frequencies from bivariate histograms included was used in the spectrum calculation. The spectrum is realized using [24]:

\[
S(w) = \frac{5}{16} H_s^2 \frac{w_m^4}{w_s} e^{-\frac{5}{4} \frac{w_m^2}{w_s}}
\] (2.3)

\[
w_m = 2\pi f_p = \frac{2\pi}{T_p}
\] (2.4)

Fig. 2.8 shows the Bretschneider spectrum when inputting (2.3) with the higher range values of Table 2.2 for each season. The wave surface elevation can then be obtained from:

\[
\bar{\eta} = \frac{\sum_{i=1}^{n} \sqrt{2S(w_i)} \delta w}{n}
\] (2.5)

\[
\eta = \bar{\eta} + \sum_{i=1}^{n} \sqrt{S(w_i)} \delta w \ast \cos(wt + \varepsilon)
\] (2.6)

where \(n\) are the total frequencies considered, \(\delta w\) is the frequency step, and \(\varepsilon\) is a random phase. The wave surface elevation for the Fall season is shown in Fig. 2.9. The seasonal surface
2.6 Simplified Pendulum Equation of Motion

In this section we demonstrate the derivation of the equation of motion (EOM) of a horizontal pendulum in only the pitch degree of freedom (Fig. 2.10), for simplicity of illustration, and demonstration of the principal concept of motion. Formulation of the full equation of motion for all six DOF is complex (See Appendix A.1), and therefore, we solve for the motion in that case numerically using the toolbox WEC-Sim described in Section 2.7.1. The EOM in pitch is found using Lagrangian mechanics of the difference between the integrated system’s total kinetic and potential energy [25]:

\[ L = K_{\text{total}} - P_{\text{total}} \]  

(2.7)
where $L$ is the Lagrangian, and $K_{total}$ and $P_{total}$ are the total kinetic and potential energy, respectively. For the system in pitch, $K_{total}$ is:

$$K_{total} = \frac{1}{2} m_p r^2 \dot{\psi}^2 + \frac{1}{2} m_p (r \cos \psi)^2 \dot{\theta}^2$$  \hspace{1cm} (2.8)

For the system in pitch, $P_{total}$ is:

$$P_{total} = g m_p r \cos \psi \cos \theta$$  \hspace{1cm} (2.9)

Figure 2.10: Free Body Diagram of Integrated System to define the Lagrangian and pendulum EOM; $\theta$ denotes pitch, $\psi$ denotes yaw; radius arm length $r$ denotes length between HPWEC COG and pendulum COG; $\tau_{PTO}$ denotes torque from PTO
Combining (2.8) and (2.9) into (2.7) leads to the Lagrangian in pitch as:

\[
L = \frac{1}{2} m_p r^2 \dot{\psi}^2 + \frac{1}{2} m_p (r \cos \psi)^2 \dot{\theta}^2 - m_p g r \cos \psi \cos \theta
\]  

(2.10)

where \( m_p \) [kg] is the mass of the pendulum, \( g \) is gravitational acceleration (9.81 m/s\(^2\)), \( r \) [m] is the radius arm length, \( \theta \) [rad] and \( \dot{\theta} \) [rad/s] is the angular position and velocity about the y-axis (pitch), \( \psi \) [rad] and \( \dot{\psi} \) [rad/s] is the angular position and velocity about the z-axis (yaw).

The Euler-Lagrange equation is used to obtain the EOM for the angular response of the pendulum:

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\psi}} \right) = \frac{\partial L}{\partial \psi}
\]

(2.11)

\[
\ddot{\psi} = \frac{m_p g r \sin \psi \cos \theta - m_p r^2 \dot{\theta}^2 \cos \psi \sin \psi}{m_p r^2}
\]

(2.12)

where \( \ddot{\psi} \) [rad/s\(^2\)] is the angular acceleration of the pendulum. After applying the non-conservative torque from the damping of the PTO:

\[
\ddot{\psi} = \frac{m_p g r \sin \psi \cos \theta - m_p r^2 \dot{\theta}^2 \cos \psi \sin \psi - c \dot{\psi}}{m_p r^2}
\]

(2.13)

where \( c \) [Nm/(rad/s)] represents PTO damping. Motor viscous damping is neglected as its value is small compared to the PTO damping values considered, explained in detail later in Section 2.7 numerical modelling. Power from the PTO is then found by:

\[
P = \tau_{pto} \dot{\psi} = c \dot{\psi}^2
\]

(2.14)

where \( \tau_{pto} \) [Nm] is the torque from the PTO.

The equations for a simple pendulum assume a massless string, the pendulum as a point mass, small oscillations, and no friction, allowing predictions on period and angular frequency to be made based on knowledge of radius arm length [26]. Given that (2.13) is a second-order nonlinear differential equation for one DOF, the same cannot be said, more so with inclusion of
all six DOFs (see Appendix equation (A.43)). Changing parameters and observing results from numerical simulations are used to draw conclusions about the relationship the design described in Section 2.3 and 2.4 has between power output and varying PTO damping, radius arm length, and pendulum mass values.

2.7 Numerical Modelling Results and Discussions

The integrated system was simulated using an iterative process of altering the pendulum radius arm length, pendulum mass, and PTO damping, then checking the effect on power output to examine the effects these parameters have. For clarity, the following definitions are reminded of: *radius arm* as the distance from the origin of the axis of rotation (center of HPWEC) to the COG of the pendulum; *mass* as the mass of the pendulum; and *damping* as the damping value of the PTO system. Reported power output is the mean power after 20 minute *WEC-Sim* simulations multiplied by 77.4\% for total efficiencies as described in Section 2.4.1. Mass values were based on the size of the HPWEC housing and pendulum within NOAA’s hypothetical size constraints (as explained in Section 2.4.2). This led to a range starting at NOAA’s weight limitation of 14 kg to the heaviest weight manufacturing could allow for the size of the pendulum, 100 kg. Damping values were chosen based on preliminary simulations to obtain a rough range of values where the SCOOP base power requirements of 5 W was met and where global maximums in power were observed. This led to a range from 1.5 Nm/(rad/s) to 35 Nm/(rad/s). As mentioned in Section 2.6, viscous damping is neglected due to its small value compared to the considered damping values. General values of viscous damping are around 1x10^{-6} Nm/(rad/s) [27] whereas the lowest value of PTO damping to be used in simulation is 1.5 Nm/(rad/s). Radius arm values were chosen based on the closest and farthest the pendulum can be to the axis of rotation while staying within the HPWEC cylinder housing.

After describing the software model used to calculate power, results from simulations are then presented. The first parameter investigated is the effect changing the length of the radius arm has on power performance. This reveals a radius arm value that maximizes power output for the conditions considered in this case study. This value will be used for the remaining simulations, allowing only a change in two variables instead of three. Patterns in mass and damping values are further analyzed in Section 2.7.3 for each season. Additionally, this section will show which mass and damping values meet the SCOOP power requirements and if they are within imposed weight constraints. Section 2.7.4 is a brief note on energy storage, followed by a comparison
between the pitch motion of the 3 m buoy with and without the addition of the HPWEC, an important consideration to study the effects on the hydrodynamics of the 3 m buoy and what this could mean for accuracy of sensor readings.

2.7.1 WEC-Sim Model

To calculate power (2.14) is used, and as briefly mentioned in Section 2.6, $\psi$ in (2.14) is found from the six DOF EOM of the system using WEC-Sim (Wave Energy Converter SIMulator), an open-source code developed by NREL and Sandia National Laboratories (SNL) for simulating WECs using Matlab/Simulink’s Simscape Multibody, a multi-body dynamics solver. WEC-Sim allows users to upload custom geometry files, hydrodynamic output files, and setup various PTO and mooring configurations to simulate their design through regular or irregular sea states [28]. The WEC-Sim model used to represent the 3 m buoy and HPWEC modular system are described next.

![Figure 2.11: WEC-Sim Simulink model of 3 m SCOOP buoy with modular HPWEC; describes how devices of user defined system interact with each other](image-url)
The SolidWorks files made for each component in the system were saved as stereolithography files to be used in WEC-Sim as visualization. It was also used to determine COG values at the varying radius arm lengths. MOI values were determined from the assumption of the inertia of a point mass yielding a value equal to the mass times radius arm squared ($mr^2$). The hydrodynamic files from Ansys AQWA allow WEC-Sim to know the system’s response for certain frequencies that the user chooses to simulate the system through, then uses that information to solve the governing WEC equations of motion in the 6 Cartesian degrees-of-freedom (DOF).

WEC-Sim’s wecSimInputFile.m initialization script has multiple classes that allow users to define components in their system. Examples of relevant inputs users can make are briefly described. The bodyClass script defines each body (device) in the system and allocates the appropriate hydrodynamic and visual file to it. Users can define the COG (if a non-hydro body), mass, and MOI to the body with their own inputs. The script wavesClass allows users to define the sea state they want to simulate their device in. This was used to import the irregular wave surface elevation for each season derived from Section 2.5 with a ramp up time of 100 s. The script constraintClass defines the location and orientation of constraints determining how bodies move in relation to one another or to the global coordinate system such as being fixed or moving in all 6-DOF. The script ptoClass defines the PTOs stiffness, damping, location, and orientation. The script mooringClass with lines.txt specific to MoorDyn defines the stiffness properties, locations, and connection points of the mooring system.

The WEC-Sim Simulink Simscape Multibody model describes how devices of the user’s system interact with each other and works with the initialization script to define properties of the blocks in the model. Fig. 2.11 is the Simulink model for the integrated system of this case study. Shown is every component of the integrated system having its own body block (3 m buoy, HPWEC cylinder housing, pendulum) defined by their respective COG, MOI, mass, hydrodynamic (if a hydro body) and visual files. The constraint blocks describe how the follower (F) is connected to the base (B). For example, the 3 m buoy is connected to the seabed with a 6DOF block meaning it is free to translate or rotate in any of the 6-DOFs with respect to the seabed. The HPWEC cylinder housing has a fixed constraint on the 3 m buoy to resemble it being rigidly connected to the buoy, and the pendulum has a rotational PTO which allows it to spin around the HPWEC’s central axis defined as location in the ptoClass. Inside the PTO block are values for stiffness and damping that the user can change to influence power output and motion response. Finally, the force of the mooring is represented using the MoorDyn library as calculated in Section 2.3.4. Various radius arm, mass, and damping values were run through this model and
Figure 2.12: PWEC Power outputs for the Summer sea state when the radius arm length is 0.09 m; observed lower power results for decrease in length.

Figure 2.13: HPWEC Power outputs for the Summer sea state when the radius arm length is 0.18 m; observed higher power results for increase in length.

the initialization script in calculating and observing the effects on power and motion response, described next.

2.7.2 Radius Arm Choice

To study the effects radius arm length have on power output, the value is changed between lengths of 0.09 m to 0.18 m for the Summer sea state. Summer was chosen as it has the lowest energy sea state and is expected to produce the lowest power output out of all seasons. Results are shown in Fig. 2.12 and Fig. 2.13 for \( r = 0.09 \) m and \( r = 0.18 \) m, respectively.

The trend of increase in power was consistent with increase in radius arm length and was evident across the Fall, Spring, and Winter sea states. To simplify the remaining simulations, the longest radius arm allowed within the size of the HPWEC, 0.18 m, is used for the remaining simulations as it allowed for the highest production of power. It is important to note that the pendulum’s ever changing COG has a direct effect on the hydrodynamics of the 3 m buoy in its response to the sea state. Drastic behavior of this is evident by the ‘NaN’ (not a number) value in Fig. 2.12 caused by singularities encountered when solving the governing equations. This could be a result of non-linear hydrodynamics not included in simulations and is an important topic for
future work.

2.7.3 Mass and Damping Effect on Seasonal Power Outputs

Now that the radius arm value that yields the highest power output across all sea states has been determined this variable can be fixed, simplifying the remaining simulations to explore the effect of damping and mass values on power output for all seasons. This is shown in Fig. 2.14, Fig. 2.15, Fig. 2.16, and Fig. 2.17 for Fall, Winter, Spring, and Summer seasons respectively.

Power outputs consistently increased with higher pendulum mass values and a constant damping value with the exception of certain damping values at 50 kg. Power maximized for certain ranges of damping with a constant mass, decreasing once the damping was too high as the direct negative impact it had on speed ultimately lowered the power. These two observations can lead to general ranges of damping desired for mass ranges. For example, during the Fall season (Fig. 2.14) power was generally maximized for a damping range of 10.5 to 21.0 Nm/(rad/s) for heavier masses (50 kg and above) and 1.5 to 10.5 Nm/(rad/s) for lighter masses (50 kg and below). Power outputs were seen to decrease when below or above these damping ranges for the corresponding mass range, again, due to the dependent relationship of speed and damping and their trade-off on power output. Interestingly, while a power increase was evident with mass increase, the magnitude of power did not significantly change throughout the four seasons. For example, the power values in the lowest energy sea state, Summer, are similar to that of the highest energy sea state, Winter. This aligns well with [19] where tank tests were conducted on a pendulum WEC, concluding that larger waves resulted in higher mean power outputs only a portion of the time. This is true when observing Fig. 2.15 and Fig. 2.17 where Winter generally outperforms Summer, but not significantly in magnitude for each mass and damping combination. Although seasonal power trends are typically common among WEC devices, the absence of outstanding variability could prove to be an advantage to users when choosing sensors to operate within their rated power conditions.

Finally, it is evident power outputs in every season other than Winter and Spring do not produce the base SCOOP power requirement of 5 W within the presently-assumed mass constraint of 14 kg. Therefore the minimum mass value to meet the base power requirement yearly is examined. A minimum mass of 20 kg is capable of meeting the base requirement of 5 W for all seasons of the year. The damping value may change per season to capture more energy or stay at 3.5 Nm/(rad/s) to meet the base requirement.
Figure 2.14: HPWEC Power outputs for the Fall irregular sea state: 5 W base requirement met for masses 20 kg and over and varying damping ranges

Figure 2.15: HPWEC Power outputs for the Winter irregular sea state: 5 W base requirement met for masses 14 kg and over and varying damping ranges

Figure 2.16: HPWEC Power outputs for the Spring irregular sea state: 5 W base requirement met for masses 14 kg and over and varying damping ranges

Figure 2.17: HPWEC Power outputs for the Summer irregular sea state: 5 W base requirement met for masses 20 kg and over and varying damping ranges
Another important observation is the effect of inertia on the system’s power performance. This study showed that an increase in inertia from heavier masses led to a higher power output, however in this particular case study the problem was restricted in ways such as the assumption of a point mass for MOI and limitations in mass and radius length values due to NOAA size constraints and physical manufacturing capabilities. It is key to continue further simulations in the future that have a broader focus with less restrictions on using differing pendulum designs and MOIs to better understand the inertia effect on the dynamics of the integrated system and its response to the sea state as well as the effects on power output.

2.7.4 Energy Storage

The current power system for the SCOOP is a 10.8 V 1.34 kWh battery [29]. The range of energy ripple produced by the HPWEC was determined by integrating the power time series after removing the average power, representing the energy leftover to be stored. To establish the upper storage capacity needed, the time series was taken from the season that produced the highest power using the earlier defined yearly mass value of 20 kg. This corresponds to the Winter season with a 3.5 Nm/(rad/s) damping value to produce the highest power. The power, power ripple, and energy ripple time series of this scenario is shown in Fig. 2.18. The battery capacity needed to meet the energy range is:

\[
Batt_{cap} = \frac{E_{\text{max}}}{3600 \text{ s/hr} \times V}
\]  

(2.15)

where \( Batt_{cap} \) [Ah] is the capacity of the battery, \( E_{\text{max}} \) [J] is the maximum energy to be stored, and \( V \) [V] is the voltage of the SCOOP power system (10.8 V). This results in a battery capacity of 0.03 Ah. To get units of Watt-hours, the battery capacity is multiplied by the voltage of the SCOOP power system, yielding a value of roughly 0.5 Wh. The existing system of 1.34 kWh is sufficient then to buffer the output from the HPWEC. It is also noted that while the yearly mass value of 20 kg does not produce the peak power requirement of 16.5 W for any season, the accumulation from energy storage would allow supply of peak power when needed for periodic data transmission bursts.
2.7.5 Comparing Pitch Motion between 3 m Buoy with and without the HPWEC

With the addition of the HPWEC on the 3 m buoy, the response to oncoming waves will differ from without the HPWEC. As a result, the SCOOP payload on the buoy will have different sensor readings of environmental conditions, possibly leading to false representations of the true conditions. To study the effect of the HPWEC on the 3 m buoy's response to sea states, the root mean square error (RMSE) values for the pitch motion of the 3 m buoy with and without the HPWEC are shown in Table 2.3 for the lowest and largest sea states (Summer and Winter). Additionally, Table 2.3 shows the difference in RMSE when the mass of the pendulum is 20 kg versus 100 kg. Each scenario used the damping value that maximized power from the results in Section 2.7.3 for Summer and Winter, and a radius arm of 0.18 m. Fig. 2.19, Fig. 2.20, Fig. 2.21, and Fig. 2.22 depict the time series of pitch motion for an arbitrary time range for these cases.
Figure 2.19: Pitch motion of 3 m buoy with and without the HPWEC for the Summer sea state with a pendulum mass of 20 kg, damping of 3.5 Nm/(rad/s), and 0.18 m radius arm; RMSE = 0.2828 over entire simulation time; arbitrary time range shown.

Figure 2.20: Pitch motion of 3 m buoy with and without the HPWEC for the Summer sea state with a pendulum mass of 100 kg, damping of 17.5 Nm/(rad/s), and 0.18 m radius arm; RMSE = 0.3190 over entire simulation time; arbitrary time range shown.

Figure 2.21: Pitch motion of 3 m buoy with and without the HPWEC for the Winter sea state with a pendulum mass of 20 kg, damping of 3.5 Nm/(rad/s), and 0.18 m radius arm; RMSE = 0.3248 over entire simulation time; arbitrary time range shown.

Figure 2.22: Pitch motion of 3 m buoy with and without the HPWEC for the Winter sea state with a pendulum mass of 100 kg, damping of 14 Nm/(rad/s), and 0.18 m radius arm; RMSE = 0.3485 over entire simulation time; arbitrary time range shown.
<table>
<thead>
<tr>
<th>Season</th>
<th>Mass [kg]</th>
<th>Damping [Nm/(rad/s)]</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summer</td>
<td>20</td>
<td>3.5</td>
<td>0.2828</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>17.5</td>
<td>0.3190</td>
</tr>
<tr>
<td>Winter</td>
<td>20</td>
<td>3.5</td>
<td>0.3248</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>14</td>
<td>0.3485</td>
</tr>
</tbody>
</table>

Table 2.3: RMSE values for Winter and Summer sea states with 20 kg and 100 kg pendulum masses

As expected, the pitch response of the 3 m buoy has changed when the HPWEC is fixed onto its platform. The effect is slightly more dramatic in the Winter versus Summer seen by the increase in RMSE values compared between the two sea states due to the more energetic sea state present in Winter. The use of a heavier mass also results in a larger RMSE due to the different COG and its influence as it changes location according to the pendulums motion, thus changing the COG of the entire integrated system. Future work will include more detailed analysis on the impact the HPWEC has on the buoy and accuracy of sensor measurements.

2.8 Conclusion

MRE as the main power source for marine applications has received positive feedback by users in this sector [2]. Compared to solar, battery, wind, or diesel generators, it has been identified as an advantageous alternative [1][3]. To further these studies, this paper investigated the feasibility of a specific WEC design powering one of these applications. The HPWEC design was chosen for its robustness and was applied to NDBC SCOOP weather buoys as a low bound for determining power requirements that this design can meet. This paper analyzed the effects pendulum mass, radius arm length, and damping of the PTO had on the seasonal power output of the HPWEC. Increases in mass and radius arm equated to higher power output, a consistent trend seen for every season, while ranges of damping that maximized power were dependent on the mass value and sea state.

The most notable finding related to power performance was that in order to meet the SCOOP base power requirement of 5 W for the entire year, the mass of the pendulum (and therefore the modular HPWEC) was over the considered mass constraint of 14 kg, calling for a minimum value of 20 kg. Although the 20 kg pendulum mass is required to meet the base SCOOP power of 5 W,
another scenario to consider would be using both the HPWEC and solar panels already installed on the 3 m SCOOP buoys to supply power to the payload. This way, solar could be the main power source in the lower sea state seasons with the additional production by the HPWEC. The HPWEC could then be the main source in the higher sea state seasons. This would effectively add supplementary power to the current solar panel power source of the 3 m SCOOP buoy while maintaining the possibility for longer deployments and remote area operation. With a combined system, the weight of the HPWEC could decrease as it is not the sole supplier of power, thus meeting the mass constraint.

It was confirmed that the addition of the HPWEC on the 3 m buoy changed the buoy’s motion when compared to operation without it as a result of the changing COG associated with the rotational movement of the pendulum. The RMSE between the pitch response of the 3 m buoy with and without the HPWEC showed a clear change in buoy behavior, increasing with larger sea states and mass. This will have effects on the sensor data output and is important to further study to ensure the addition of a modular device such as the HPWEC still permits the buoy to record accurate data readings. Also, the possible non-linear hydrodynamic behavior of the 3 m buoy caused by the changing COG is another feature to study as this can affect the power output and buoy motion as well. Furthermore, the findings in this paper hold for a system with a certain range of considered values for one pendulum design due to restrictions on size and weight. To better understand pendulum influence and the associated inertial properties on the 3 m buoys hydrodynamic response, a variety of designs should be studied. Future work will focus on a better understanding of this behavior leading to more accurate results of power output and buoy motion, possibly reducing or increasing the range of masses and size constraints considered. This will further aid understanding the feasibility of using HPWECs to realize the benefits they may provide.

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Manuscript 2 – Relationship Between Power Output and Chaotic Behavior of a Pendulum Power Take-Off Wave Energy Converter

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IFAC Design and Control of Ocean Wave Energy Converters
2022
3.1 Abstract

Multiple wave energy converter archetypes exist with varying power take-off (PTO) designs in the attempt to maximize ocean energy harnessed and converted into useful energy. The pendulum PTO is popular for its simple, yet robust functionality due to its internally located components and simple operation. The pendulum system also exhibits chaotic behavior and its affect on power output is studied. When varying wave height and period, PTO damping, and pendulum mass, design, and radius arm, power output is consistently generally greater when the system is exhibiting stable versus chaotic behavior.

3.2 Introduction

The pendulum system is well-known in chaos theory. It exhibits chaotic behavior under certain operating conditions, generating seemingly random and unusual results. This behavior has been studied extensively for pendulum systems [1][2][3]. The pendulum is also a power take-off (PTO) system used in the field of marine renewable energy for wave energy converters (WEC), popular for its simple, yet robust functionality due to its internally located components and straightforward operation. In an effort to maximize the energy harnessed from the ocean, many features of a WEC are studied including their effect on power output when environmental conditions, or design and control factors are varied. In this paper, the wave height and period, PTO damping, and pendulum mass, design, and radius arm are varied to study how this affects the pendulum’s chaotic versus stable behavior, and ultimately the relationship between this behavior and power output.

3.3 Pendulum WEC Modelling

3.3.1 Design Overview

The hull of the pendulum WEC used throughout this paper is based on the 3 meter buoys used by the National Data Buoy Center (NDBC). These buoys are used to monitor the weather and climate of the ocean environment and have a service life of up to 20 years [4]. This gives valid reasons to use this design as the base hull of a WEC that can survive the harsh conditions of the ocean. Dimensions and mass properties of the buoy were obtained through conversations with
Figure 3.1: Isometric view of 3 m buoy based pendulum PTO WEC hull with pendulum design 1 highlighted in red

AXYS technologies [5] and modelled in Solidworks [6]. It was then modelled through Ansys AQWA [7] to obtain its hydrodynamic response. Two pendulums were modelled in Solidworks by the author to study the effects of moment of inertia on the WEC system when pendulum shape, radius arm, and mass are varied. The two components, the hull and pendulum, make the WEC system. This system with pendulum design 1 is shown in Figure 3.1. Readers are to refer to Appendix B.1 for depictions of both pendulum designs.

3.3.2 Mooring

Although the mooring used with NDBC buoys is a single-point system [8], a catenary system was used here, reasons described later at the end of Section 3.4, preliminary results. The library MoorDyn [9] was used to emulate mooring effects, where various inputs such as the moorings connection points, material, axial stiffness, unstretched length, weight, and diameter were required. In depths of 2 – 150 m, chain diameters ranging from 10 – 45 mm are commonly used, as well as mooring scopes within values of 1.5 – 4.0 [10]. The chain’s axial stiffness can be
calculated using its mass properties [11]:

\[
E = (5.40 - 4d) \times 10^{10}
\]

(3.1)

\[
A = \frac{2\pi d^2}{4}
\]

(3.2)

where the elastic modulus is \( E \) [N/m\(^2\)], the chain area is \( A \) [m\(^2\)], and the chain diameter is \( d \) [m]. The area is multiplied by two for the cross sectional area of two bars. Simulations were performed for various diameters, weights, and mooring scopes until a stable response was achieved. This occurred with 3, 18 mm diameter chains, each with weights of 6.55 kg/m and \( EA = 2.9 \times 10^7 \) N [11]. The unstretched length was 192 m when a mooring scope of 1.5 and a depth of 128 m was used. The choice of depth is dependent on the buoy used for sea state ranges, explained next.

### 3.3.3 Simulation Overview

**WEC-Sim (Wave Energy Converter SIMulator)** is an open-source code developed by the National Renewable Energy Laboratory and Sandia National Laboratories for simulating WECs [12]. This toolbox was used to study the effect on power output the pendulum’s chaotic or stable behavior had when design and environmental features were changed. The hull and pendulum designs, mooring, mass properties, and hydrodynamic responses were uploaded into **WEC-Sim** as inputs to custom geometry files and hydrodynamic output files. Additional necessary information to run the simulation included information on the PTO, constraints, and waves.

Geometry files provide visualization of the system during simulation. The hydrodynamic files from Ansys AQWA allow **WEC-Sim** to know the system’s response for certain frequencies that the user chooses to simulate the system through, then uses that information to solve the governing WEC equations of motion in the 6 Cartesian degrees-of-freedom (DOF). Body class inputs include mass, moment of inertia, and center of gravity of the body. These were obtained from the mass properties tool in Solidworks. Constraints describe how the users system can move in regards to the seabed. These can include all six degrees or a select few as deemed appropriate by the user. The WEC system was simulated in all DOF except yaw, reasons noted later at the end of Section 3.4. As for the PTO, in Figure 3.2 the pendulum is connected to the hull via a rotational PTO, meaning that it will rotate around the hull in the DOF chosen by the
user, in this case, yaw. Power, $P \text{ [W]}$, is calculated as torque times speed squared

$$P = c \omega^2$$  \hspace{1cm} (3.3)

where $c \text{ [Nm/(rad/s)]}$ is the PTO damping and $\omega \text{ [rad/s]}$ is the relative velocity between the two bodies the PTO is connecting.

*WEC-Sim* is additionally used to provide the dynamic response of the pendulum throughout its operation, specifically the pendulums position and velocity throughout the simulation. This information is needed to characterize chaotic or stable behavior. For wave inputs, simulations have been restricted to regular sea states to illustrate and obtain a basic understanding of the concept of chaos and its relationship on power output. Sea state ranges were based on frequent sea state occurrences in the Pacific Northwest, chosen due to its high wave energy resource [13]. These ranges were obtained by creating a bivariate histogram of the annual peak period and significant wave height data obtained by NDBC buoy 46041, whose depth range was 128 m [14].
3.4 Preliminary Results: Identifying Chaotic Behavior in a Pendulum WEC

Chaos theory is a branch of mathematics that focuses on nonlinear dynamic systems and complex patterned behavior. Chaotic systems display seemingly random behavior if their inputs or initial conditions perturb the system towards non-convergent solutions. Identifying chaotic data is done by proving three properties, which also helps set it apart from random data. A chaotic system will have: 1) sensitivity to initial conditions, 2) topological mixing, and 3) dense periodic orbits [15]. This all means that, even if a chaotic system is stable and predictable for some amount of time, it could suddenly veer into unpredictable behavior; there is no certainty of its actual state. The chaotic behavior of the WEC system described in Section 2.3 will be proven by displaying all three properties through a preliminary simulation study of the WEC system. Visual aids will be used such as bifurcation diagrams, time plots, phase plots, and Poincaré maps.

3.4.1 Bifurcation Diagram: Determining Regions of Possible Chaos

In order to focus in on a state of the WEC system that results in chaotic behavior of the pendulum, regions of possible chaos first need to be determined. A bifurcation diagram is used for this purpose. This diagram plots the final value(s) of an interested variable that the system settles to, either resulting in a single value, oscillating between values, or taking on many values (chaos). For the pendulum, the variable of interest is the position every period. This requires the extra step of obtaining the Poincaré map of the phase space plot, or in other words, sampling the data at some fixed rate. This is chosen as the wave period used in the simulation. If the system is in a single periodic orbit, every sample should reveal the same value. If it is in a double periodic orbit, there will be two values. If the system has multiple points it converges to, it is stated to be displaying chaotic behavior. An example of this sampling is shown in Figure 3.3. The blue lines trace out the phase space plot of the pendulum where the red dot indicates the position and velocity of the pendulum at the time of sampling over several simulations. The Poincaré map would be illustrated as only the red dots, absent of the blue lines. As shown in the top left, a periodic orbit should be in the same spot at every sampling point, resulting in a single red dot. A double period would have 2 points at each sampling, a quadruple period, 4. The bottom right shows the Poincaré map for a pendulum displaying chaotic behavior. As expected, there are many red dots, showing that the system never converges to any orbit(s) [16].
Figure 3.3: Phase space plots of various behaviors of a forced, damped pendulum with the x-axis as position and y-axis as velocity. The value obtained when sampled at a fixed rate reveals the Poincaré map, shown by red dots. Top left exhibits a single periodic orbit (one red dot), top right two orbits (two red dots), bottom left four orbits (four red dots), and bottom right multiple orbits indicating chaos (multiple red dots) [16].

The process described above is applied to the WEC system where it is simulated over a range of constant values: wave height of 4 m, PTO damping 15 Nm/(rad/s), pendulum design 1, 200 kg mass, and a radius arm of 0.26 m. The changing variable is the period of the waves, ranging from 101 values between 6 and 16 seconds in 0.1 steps. The system is simulated over 1200 seconds for each period, where the last 300 seconds are used to see where the pendulum position settles to after one period. The result is shown in Figure 3.4. Areas of chaos dominate the periods between 6 - 7.5 seconds and 8.1 - 9.8 seconds. After 9.8 seconds and in between 7.5 - 8.1 seconds the pendulum is in a single or two period orbit.

Now that areas of chaos have been identified, one state of the system is chosen to examine further to ensure that all three chaotic properties are satisfied. For this, a wave height of 4 m, period of 8.2 s, PTO damping 15 Nm/(rad/s), pendulum design 1, 200 kg mass, and a radius arm of 0.26 m is used to demonstrate.
Figure 3.4: Bifurcation diagram of pendulum PTO WEC system simulated with a constant wave height of 4 m, PTO damping 15 Nm/(rad/s), pendulum design 1 200 kg mass, and radius arm of 0.26 m. The period changes in 0.1 steps from 6 to 16 seconds. Areas of fixed points (single dot), periodic orbits (2 dots), and chaos (multiple dots) are evident.

3.4.2 Sensitivity to Initial Conditions

Due to sensitivity and limited information, chaotic systems can only be accurately predicted up to a certain time horizon. To evaluate this, Figure 3.5 shows the time plot of pendulum initial start positions $0^\circ$ vs $0.001^\circ$ where the black oval encircles the region of the paths initial divergence. This was repeated for a smaller change in initial conditions, positions $0^\circ$ vs $0.000001^\circ$ to further check sensitivity where the initial divergence was found to occur in the same time scale. If the initial divergence rather took 1000 times longer to appear, the growth would be too small to be considered as chaos. To verify the arbitrary choice of initial condition was not coincidental, the process was repeated at $18^\circ$, $18.001^\circ$, and $18.000001^\circ$, achieving similar results.
Figure 3.5: Time plot of pendulum position for two initial conditions, $0^\circ$ and $0.001^\circ$. Sensitivity to initial conditions is shown where the two paths begin to largely diverge noted by the black oval.

3.4.3 Topological Mixing and Dense Periodic Orbits

Topological Mixing and dense periodic orbits are often discussed together. With topological mixing, the states of a system from close-valued initial conditions can suddenly diverge far apart from each other and eventually overlap with points in another phase space region. This stretching of points is only possible when the periodic orbits are significantly different, though.

To prove topological mixing, it is demonstrated that the phase space plot of the pendulum will fill the entire area after a number of iterations [15]. This is shown throughout Figure 3.6, where phase space plots of the WEC system are plotted throughout time. Figure 3.6 also proves that the system has no periodic behavior, and an orbit does not repeat itself exactly the same, forever cycling around instability. This is an important reminder that one does not truly know what is going to happen in a chaotic system. It may appear stable for some time, then diverge suddenly to unexpected behavior. However, it is not random, as proven next.
Figure 3.6: Phase space plot of pendulum PTO WEC system throughout simulation; Top right, 0 - 300 s; Bottom left, 0 - 600 s; Bottom right, 0 - 1200 s. Topological mixing is evident by the entire phase space covered. Also illustrated is the presence of dense periodic orbits.

The presence of topological mixing and dense periodic orbits means that there is some underlying structure to the chaotic behavior of the pendulum. To view this, the phase space plot of Figure 3.6 over the entire time range is sampled at the period of the excitation wave, obtaining a section of the motion, creating the Poincaré map (as explained by the red dots in Figure 3.3). The Poincaré map of Figure 3.6 is shown in Figure 3.7. One can easily see that the apparently random looking behavior of Figure 3.6 has a pattern. If the system were instead truly random, the poincaré map would yield a result such as in Figure 3.8. Additionally, Figure 3.7 is a strange attractor, meaning it is fractal. By zooming into the image, one can see that the pattern repeats itself at small detail. Figure 3.9 is the black rectangular portion of Figure 3.7, and similarly with Figure 3.10 to Figure 3.9.
Figure 3.7: Poincaré map of the phase space plot of Figure 3.6 when sampled at the period of the wave, 8.2 s. This results in a strange attractor and proves that there is an underlying pattern.

Figure 3.8: Poincaré map of a random system where no pattern is noticed.

Figure 3.9: Poincaré map of strange attractor, zoomed in on the black rectangle of Figure 3.7 to show fractal nature of the attractor.

Figure 3.10: Poincaré map of strange attractor, zoomed in on the black rectangle of Figure 3.9 to show fractal nature of the attractor.
3.4.4 Application to Numerical Modelling

The previous subsections detailed how the WEC system has chaotic behavior by proving the three properties needed to be classified as chaotic. While performing these preliminary simulations, it was noted that when the WEC system was allowed to move in all six DOFs, the behavior of the WEC resulted in a random Poincaré map similar to that of Figure 3.8. However, when the DOF that the pendulum was allowed to move around (yaw) was restricted for the motion of the WEC system, the Poincaré map generated images similar to Figure 3.7. This could be due to the fact that if the WEC system is allowed to rotate in yaw, the reference for the pendulum and its chaotic behavior is constantly being changed. For these reasons noted, the catenary mooring system was chosen over the commonly used single point mooring by NDBC on their 3 m buoys in an effort to reduce the yaw motion. It was found that this was not restrictive enough, however, and therefore the constraint blocks in WEC-Sim were chosen to only allow WEC movement in all DOFs except yaw.

3.5 Numerical Modelling and Results

Now that the simulation conditions to produce and tools to identify chaotic behavior have been discussed, the relationship between this behavior and output power of the WEC system are studied. This process involves changing features of the WEC itself (pendulum mass, design, radius arm, and PTO damping) or of the environment (wave height and period) and creating a bifurcation diagram as was done in the previous section. In addition to this, the power output from the WEC is plotted where a polynomial trendline was applied to the power data points in regions where stable behavior dominated. Areas of outliers were left out of the creation of the trendline, creating low power due to no movement in the WEC system at either the beginning or the end of simulations, likely due to small excitation by e.g. short wave heights, long periods, or too large a damping value.

Stable areas are evident by a light green shade in the power plot whereas chaotic regions are shaded with light blue. Vertical lines between the two plots additionally help to relate stable and chaotic areas. Figure 3.11, Figure 3.12, and Figure 3.13 detail various configurations of chaotic behavior and power output when period, height, or damping are changed. Similar simulations were run where pendulum mass and design were changed (see Appendix B.2). All simulations had a chaos-check process as was done throughout Section 3.4 to ensure chaotic behavior was
present. The concluding find shows that the trendline for each scenario reveals a consistent pattern of stable behavior generally outputting higher power.

3.6 Conclusion

Identifying conditions when a WEC generates maximum power is essential for effective operation. The chaotic behavior of a pendulum was studied when utilized as a PTO system in a WEC subject to changing variables affecting its motion response. Stable behavior proved to yield a higher output of power and is therefore a desirable response. To achieve such a response in chaotic regions, control techniques used in chaos theory can be applied to the system. Familiar techniques include Ott, Gebogi, and York or resonant parametric perturbation [17][18]. Control of chaotic regions to stability is a future step in understanding the potential of this type of PTO in a WEC and control strategy. Furthermore, now that the relationship between chaotic behavior and power has been identified, simulations in irregular sea states can follow to study any changes in the pendulums chaotic behavior.
Figure 3.11: Bifurcation diagram and power plot of the WEC system. The changing variable is the period which ranges from 6 s to 16 s. Blue shaded regions in the power plot indicate chaos, green shaded regions indicate stability, also noted by the black vertical lines relating the two visuals. The red trendline applied to stable data points shows how chaotic behavior generally underperforms in power output when compared to stable behavior.
Figure 3.12: Bifurcation diagram and power plot of the WEC system. The changing variable is the height which ranges from 0.5 m to 5 m. Blue shaded regions in the power plot indicate chaos, green shaded regions indicate stability, also noted by the black vertical lines relating the two visuals. The red trendline applied to stable data points shows how chaotic behavior generally underperforms in power output when compared to stable behavior.
Figure 3.13: Bifurcation diagram and power plot of the WEC system. The changing variable is the damping which ranges from 0 Nm/(rad/s) to 100 Nm/(rad/s). Blue shaded regions in the power plot indicate chaos, green shaded regions indicate stability, also noted by the black vertical lines relating the two visuals. The red trendline applied to stable data points shows how chaotic behavior generally underperforms in power output when compared to stable behavior.
References


Manuscript 3 – Investigation of Chaotic Behavior in a Pendulum PTO WEC Operating in an Irregular Sea State

Chris Dizon, Ted Brekken, Eduardo Cotilla-Sanchez
4.1 Abstract

The study of chaotic behavior in power take-off (PTO) systems for wave energy converters (WEC) is limited and evaluated in regular, single frequency sea states. The pendulum PTO displays chaotic behavior and is investigated in this paper to address the question of whether this behavior is present when simulated in a realistic setting via an irregular sea state. This would allow for control strategies used in chaos theory to be applied to the system to maximize power output. Chaos was defined in this work as data that was asymptotically periodic, had a positive Lyapunov exponent, and contained a chaotic attractor. Visual inspection of data through tables and time plots, numerical calculation of the exponent, and modified use of traditional Poincaré maps were applied to verify the chaotic hypothesis. Although the first two criteria were met, multiple simulation runs of the system with differing parameters lacked a chaotic attractor viewed by Poincaré map sample points filling in the area of the phase space rather than remaining close in pattern overtime. It was concluded that chaotic behavior as defined was not present in the pendulum PTO when evaluated in irregular sea states.

4.2 Introduction

Understanding the dynamics of the power take-off (PTO) system in a wave energy converter (WEC) is essential to maximize energy captured from the ocean. Systems such as pendulums, Chua's circuits, hydraulics, permanent magnet motors, and vibro-impact dynamics (multiple impact interactions in the form of jumps in state space) exhibit chaotic behavior, where certain operating conditions cause it to behave in a seemingly random way. Huang et al. (2019), Guo et al. (2021), and Dizon et al. (2022) [1],[2],[3] examine coupling a WEC with one of the previously mentioned chaotic systems taking on the role of the PTOs to observe the relationship between the two. Dizon et al. (2022) [3] for example, concluded that when using the pendulum system as a PTO, greater power output was generally observed when the PTO exhibited stable versus chaotic behavior. Although an important finding in Dizon et al. (2022) [3] as it becomes the goal for a control scheme, Huang et al. (2019), Guo et al. (2021), and Dizon et al. (2022)[1],[2],[3] focus on the system performing in an unrealistic environment with regular sinusoidal waves. Additionally, most literature on pendulum systems in the research field of chaos theory have only focused on the system when input with a single frequency [4],[5]. Before moving forward to control or other applications, it is first necessary to verify if chaotic behavior is still present.
when these PTOs are operating in a realistic sea state where multiple frequencies are present. In this paper, the pendulum PTO WEC in Dizon et al. (2022)[3] is evaluated in irregular waves to study this hypothesis. The Lyapunov exponent is used as a quantitative measurement to identify chaotic behavior in addition to visual aids such as time plots and Poincaré maps to confirm other chaotic properties. As in Dizon et al. (2022)[3], sea states were based on frequent sea state occurrences in the Pacific Northwest.

4.3 Chaos Criteria

Chaos theory stems from the branch of nonlinear deterministic systems. It is of interest because these systems described by equations that appear simple sometimes do not converge to fixed or periodic solutions. Instead, when input with certain parameter values, they can be perturbed to solutions that do not converge. As mentioned previously, a PTO following a pendulum model will be used throughout the paper. A pendulum system when forced and damped can be chaotic. It is typically defined as a variation of (4.1) when subject to one degree-of-freedom (DOF):

\[ \ddot{\theta} = -c\dot{\theta} - \sin\theta + p\sin t \]  \hspace{1cm} (4.1)

where \( \ddot{\theta} \) is the pendulum angular acceleration, \( \dot{\theta} \) is angular velocity, \( c \) is a damping term, and \( p \) is the force amplitude. For a WEC with such a PTO, this equation in 6 DOF under the influence of multiple effects such as the PTO damping, mooring, and multiple wave frequencies leads to a more complex equation with many non-linearities (see Appendix equation (A.43)). Although it is more intriguing when a simpler system as (4.1) displays such uncertain behavior, a main focus when studying WECs is to understand how systems interact with one another in an irregular sea state. Therefore, the pendulum WEC must be studied for chaotic behavior in this realistic setting.

4.3.1 Definitions

The most common definition of chaotic systems include that it is sensitive to initial conditions [6], often measured by the Lyapunov exponent, \( \lambda \). For example, [7] states that an orbit is chaotic if it is not periodic (asymptotically periodic) and it has at least one positive Lyapunov exponent. This allows for a more quantitative measurement of chaos as opposed to relying on visual aids.
as in Dizon et al. (2022)[3]. These criteria when considered alone fail to address that chaotic systems trajectories are also bounded to regions in phase space, creating patterned points or chaotic attractors [8]. A chaotic attractor is an attractor that is made up of the chaotic orbit and attracts a set of initial values that has a nonzero area in the plane. An attractor is what orbits continually return to starting from any basin of the attractor (initial values whose orbits converge to the attractor). For the pendulum, attractors are revealed by sampling the phase space at the forcing frequency to create Poincarè maps. As noted above, this pattern should remain the same for longer simulations and for any initial value in the basin of the attractor.

Thus, the criteria used to identify chaos in this paper includes confirming chaotic attractors in phase space via Poincarè maps, asymptotically periodic orbits, and obtaining Lyapunov exponents greater than 0. Other common visual aids such as time plots, Fourier transforms, or bifurcation diagrams may be used to assist in explanations. Additionally, instead of relying only on tables to identify asymptotically periodic data, each position-velocity Poincarè sample will be compared to one another. It will be defined as a repeatable sample if the difference between the two samples is less than a tolerance of $\varepsilon = 0.00001$:

$$|\text{norm}(\phi, \omega)_{\text{poin}_1} - \text{norm}(\phi, \omega)_{\text{poin}_n}| < \varepsilon$$

where $\phi$ is the pendulum position [rad], $\omega$ is the pendulum velocity [rad/s], and $\text{poin}_1$ is the fixed Poincarè sample point that is being compared to each $\text{poin}_n$ (sample points other than $\text{poin}_1$). For a fixed point or periodic orbit, the amount of repeatable samples should be less than the original amount of samples. For a chaotic orbit, the amount of repeatable samples should be equal to the original amount of samples.

### 4.3.2 Lyapunov Exponent Calculation

The Lyapunov exponent is defined as [6][9][10]:

$$\lambda_t = \ln \left( \frac{|\delta Z(t)|}{|\delta Z(0)|} \right)$$

$$\delta Z(t) = \sqrt{(\phi_2(t) - \phi_1(t))^2 + (\omega_2(t) - \omega_1(t))^2}$$

$$\delta Z(0) = \sqrt{(\phi_2(0) - \phi_1(0))^2 + (\omega_2(0) - \omega_1(0))^2}$$
where $\delta Z(t)$ is the Euclidean difference between two trajectories through time whose initial conditions are close to one another, $\delta Z(0)$ is the difference between the initial conditions at time zero, $\phi$ is the pendulum position [rad] for each initial condition, and $\omega$ is the pendulum velocity [rad/s] for each initial condition. Equation (4.3) yields a time plot of the logarithmic difference between trajectories where $\lambda$ can be measured as the slope. Care should be taken when using this method as the measurement for slope should only be fitted until the paths converge or separate. Different meanings of the value of $\lambda$ reveal behavior on the sensitivity of the system to small changes. When $\lambda < 0$, the orbits are stable and attract to a point or periodic orbit; for $\lambda = 0$, the orbits are in steady state and maintain the same distance away from each other; for $\lambda > 0$, the orbits are unstable and move away from each other exponentially. To ensure that the Lyapunov exponent tells us that orbits separate exponentially, the slopes from (4.3) should be similar with any chosen orbit from the same attractor basin. Therefore, calculations for the Lyapunov exponent are the overall average from four initial conditions:

1. $0^\circ$ and $0.0001^\circ$
2. $18^\circ$ and $18.0001^\circ$
3. $172^\circ$ and $172.0001^\circ$
4. $243^\circ$ and $243.0001^\circ$

These were verified previously to ensure the basin requirement was met.

### 4.4 Regular Waves: Demonstration of Chaos Criteria

To better understand identification of the chaos criteria defined in the previous section for the pendulum PTO WEC, we demonstrate first with the familiar case of regular, single frequency waves before moving on to irregular waves using the model in Dizon et al. (2022)[3].

#### 4.4.1 Single Periodic Orbit

In Dizon et al. (2022)[3], a single periodic orbit occurred for a wave height of 4 m, period of 6 seconds, and damping of 32 Nm/(rad/s). This is used here. For a single periodic orbit, one would expect repeating values in a table of pendulum position-velocity sampled at the wave frequency, a single point on the Poincarè map, and $\lambda < 0$. An example of the time plot for two trajectories
is shown in Figure 4.1. It can be seen that the two trajectories appear to be identical, as expected. Figure 4.2 is the logarithmic difference of the two trajectories where the top figure represents the fitted slope applied until the trajectories converge. This was repeated for each of the four initial condition cases and after averaging out the slope values, a result of $\lambda = -0.1002$ was calculated, yielding a negative Lyapunov exponent. The overall average and each respective initial condition

Table 4.1: Table of pendulum position and velocity for a period one orbit. The amount of iterations until values or sequences of values repeat indicates the period. In this case, the period eventually settles to a period of one.

<table>
<thead>
<tr>
<th>Simulation Time [s]</th>
<th>Poincarè Sample</th>
<th>Position [rad]</th>
<th>Velocity [rad/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>204.01</td>
<td>35</td>
<td>-1.72591</td>
<td>0.67745</td>
</tr>
<tr>
<td>210.01</td>
<td>36</td>
<td>-1.72592</td>
<td>0.67744</td>
</tr>
<tr>
<td>216.01</td>
<td>37</td>
<td>-1.72591</td>
<td>0.67745</td>
</tr>
<tr>
<td>222.01</td>
<td>38</td>
<td>-1.72591</td>
<td>0.67745</td>
</tr>
<tr>
<td>228.01</td>
<td>39</td>
<td>-1.72591</td>
<td>0.67745</td>
</tr>
<tr>
<td>234.01</td>
<td>40</td>
<td>-1.72591</td>
<td>0.67745</td>
</tr>
</tbody>
</table>

Figure 4.1: Unwrapped pendulum position trajectories starting from two close initial conditions; the two trajectories are identical, as expected.
Figure 4.2: Plots of Lyapunov Exponent obtained after applying (4.3) to the pendulum data. A negative value of $\lambda = -0.1002$ is obtained, which indicates converging trajectories, as expected for stable behavior.

Figure 4.3: Poincarè map of a single periodic orbit. Due to simulation ramp up time, the pendulum does not take on the orbit immediately, but settles to it over time, indicated by the encircled dot.
case values were similar in magnitude, thus, it can be confirmed that this Lyapunov exponent is well-defined for this fixed point attractor.

When sampled at the wave frequency, the position and velocity should return to the same spot. In Table 4.1 it is possible to see that the pendulum eventually settles to a single periodic orbit, with a decimal precision of five places. The total position-velocity Poincarè samples was 201. These samples were compared to one another by the process described in (4.2) yielding 168 samples that were repeatable. The Poincarè map (Figure 4.3) should display one dot, however due to the simulation ramp time of the wave the pendulum does not settle to its period one orbit immediately. The final period one orbit the system settles to is indicated by the circled dot.

4.4.2 Chaotic Orbit

In Dizon et al. (2022)[3], a single periodic orbit occurred for a wave height of 4 m, period of 6 seconds, and damping of 12 Nm/(rad/s). This is used here. For a chaotic orbit, one would expect no repeating values in a table of pendulum position-velocity sampled at the wave frequency and $\lambda > 0$. Furthermore, points on the Poincarè map should form a chaotic attractor. An example of the time plot for two trajectories is shown in Figure 4.4. It can be seen that the two trajectories follow the same path for roughly 100 sec, then diverge to independent trajectories. Figure 4.5 is the logarithmic difference of the two trajectories where the top figure represents the fitted slope applied until the trajectories diverge. This was repeated for each of the four initial condition cases and after averaging out the slope values, a result of $\lambda = 0.1328$ was calculated, yielding a positive Lyapunov exponent.

Table 4.2: Table of pendulum position and velocity for a chaotic orbit. The amount of iterations until values or sequences of values repeat indicates the period. In this case, there are none up to a 6 periodic orbit.

<table>
<thead>
<tr>
<th>Simulation Time [s]</th>
<th>Poincarè Sample</th>
<th>Position [rad]</th>
<th>Velocity [rad/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>204.01</td>
<td>35</td>
<td>-0.95610</td>
<td>1.818596</td>
</tr>
<tr>
<td>210.01</td>
<td>36</td>
<td>-2.27792</td>
<td>0.028166</td>
</tr>
<tr>
<td>216.01</td>
<td>37</td>
<td>0.937867</td>
<td>2.077382</td>
</tr>
<tr>
<td>222.01</td>
<td>38</td>
<td>2.115762</td>
<td>-1.26201</td>
</tr>
<tr>
<td>228.01</td>
<td>39</td>
<td>-2.70280</td>
<td>0.041113</td>
</tr>
<tr>
<td>234.01</td>
<td>40</td>
<td>0.075607</td>
<td>1.541454</td>
</tr>
</tbody>
</table>
Figure 4.4: Unwrapped pendulum position trajectories starting from two close initial conditions; the two trajectories diverge and separate, as expected for chaotic behavior.

Figure 4.5: Plots of Lyapunov Exponent obtained after applying (4.3) to the pendulum data. A positive value of $\lambda = 0.1328$ is obtained, which indicates divergent trajectories, as expected for chaotic behavior.
When sampled at the wave frequency, the position should never return to the same spot. In Table 4.2 the pendulum position is observed for 6 iterations and never repeats. This means up to a period 6 orbit could exist if one were to examine the next 6 iterations and see if they repeat the original 6. The total position-velocity Poincarè samples was 201. These samples were compared to one another by the process described in (4.2) yielding 0 samples that were repeatable, implying non-periodicity. The Poincarè map in Figure 4.6 displays a chaotic attractor. It was confirmed earlier that the orbit is chaotic which makes up the attractor. Property of points returning arbitrarily close to previous points is shown in Figure 4.7a by running the simulation longer. Property attracting initial conditions that form a non-zero plane area is achieved by showing that the same attractor shows up from a different initial condition (Figure 4.7b) (confirmed for other close initial conditions that create a square in the plane). It is verified that all three represent the same chaotic attractor.

4.5 Irregular Waves: Application and Modification of Traditional Tools to Identify Chaos

It has been demonstrated using regular, single frequency waves how the dynamics of the pendulum PTO WEC will be evaluated in determining if it meets the criteria established earlier for

Figure 4.6: Poincarè map of the chaotic orbit for a 3600 sec simulation time from 0°
4.5.1 Asymptotically Periodic and Lyapunov Exponent

As stated in Section 4.4.2, a chaotic orbit will have no repeating values in a table of pendulum position sampled at the wave frequency and $\lambda > 0$. Lastly, points on the Poincarè map should form a chaotic attractor. An example of the time plot for two trajectories is shown in Figure 4.8. It can be seen that the two trajectories diverge to become independent trajectories, as well as for each of the four initial condition cases when plotted. Figure 4.9 are the logarithmic difference of the two trajectories where the top figure represents the fitted slope applied until the trajectories
Figure 4.8: Unwrapped pendulum position trajectories starting from two close initial conditions; the two trajectories diverge and separate.

Figure 4.9: Plots of Lyapunov Exponent obtained after applying (4.3) to the pendulum data. A positive value is obtained as $\lambda = 0.1926$
diverge. This was repeated for each of the four initial condition cases and after averaging out the slope values, a positive exponent result of \( \lambda = 0.1926 \) was calculated.

For identifying a chaotic orbit, the challenge is that in an irregular wave environment multiple frequencies are present. Thus, there is no obvious frequency to sample the data by. Analyzing the data in irregular waves will require a modified approach than demonstrated in Section 4.4. To determine non-periodicity, the process described in (4.2) will be applied at position-velocity data sampled by the frequencies most present in the irregular wave to create multiple Poincarè sample sets per frequency. These were identified as frequencies between 0.11 Hz – 0.5 Hz, chosen after transforming the wave to the frequency domain via the Fourier transform. The total position-velocity Poincarè samples was 201 for each frequency case, and each case returned 0 repeatable elements. Thus, no elements repeated equating to asymptotically periodic behavior. This result along with the positive Lyapunov exponent satisfy the criteria of a chaotic orbit. The presence of a chaotic attractor is investigated next.

4.5.2 Chaotic Attractor Via Modified Poincarè Maps: Time Ranged Maps and Frequency Summations

The creation of a chaotic attractor requires sampling the position-velocity data for a pendulum. As noted in Section 4.5.1, there are an abundance of frequencies present in the irregular wave, ranging between 0.11 Hz and 0.50 Hz. Additionally, the most prevalent frequency of the wave can change throughout the wave signal during certain time ranges. To study this, the Fourier transform is used again to identify the most prevalent frequency during sectioned time ranges throughout the simulation. A 3D Poincarè map is then created in which an individual Poincarè map is made for each time range and its respective sampling frequency. For example, Figure 4.10 shows the Fourier transform of the 1200 sec simulation in 300 sec ranges where the color bar on the right helps visualize which frequencies are most present. This can be used to determine the sampling frequency for each time range by using the frequency with the maximum amplitude content. The Poincarè map in each of these time ranges is seen in Figure 4.11. Figures 4.12-4.13 repeat the process in 600 sec intervals. Simulations were run for 100 sec and 400 sec time ranges, as well as the additional cases stated at the beginning of Section 4.5. A clear pattern was yet to show for any. However, is what is normally considered a “pattern” necessarily required to develop in the Poincarè maps?
Figure 4.10: FFT for time ranges of 300 sec intervals throughout 1200 sec simulation; used to identify the most energetic frequency in each time range to be used as the sampling frequency for the respective Poincarè map.

Figure 4.11: Resulting Poincarè maps for each 300 sec interval; no chaotic attractor is immediately observed.
Figure 4.12: FFT for time ranges of 600 sec intervals throughout 1200 sec simulation; used to identify the most energetic frequency in each time range to be used as the sampling frequency for the respective Poincarè map.

Figure 4.13: Resulting Poincarè maps for each 600 sec interval; no chaotic attractor is immediately observed.
By the definition in Section 4.3.1, a chaotic attractor is an attractor made up of an orbit that is not periodic, has a positive Lyapunov exponent, and returns arbitrarily close to the same points for any length of simulation time and any initial condition in the basin. The first two requirements have already been satisfied. To further investigate the presence of an attractor, another modification of the traditional Poincaré map is made. The entire data is sampled at not one, but the top three most dominant frequencies – 0.11, 0.15, and 0.17 Hz – then summed together to make one Poincaré map. Figure 4.14 details the Poincaré maps for different simulation times. As before, the scattering of dots should return arbitrarily close, but they are quite different unlike the case of Figure 4.6-4.7a. Figure 4.15 details the Poincaré maps with different initial conditions. If it were a chaotic attractor, the two maps would overlay one another and one would not be able to tell which dots were for which condition. However, the two are easily distinguishable, unlike the case of Figure 4.6 and 4.7b.

Although the orbit is considered chaotic as proven by its non-periodic behavior and positive Lyapunov exponent, the properties of the orbit creating a chaotic attractor are not met. Instead, as the simulation time is lengthened, the phase space area fills up entirely and there is not a non-zero phase plane area that attracts to an attractor. This process was repeated for the additional cases stated at the beginning of Section 4.5 yielding similar results.

Figure 4.14: Poincaré map of the same initial condition simulated for 1200 sec and 46800 sec. The points do not remain bounded to the same scattering pattern as simulation time is increased, but instead fill the phase space.
4.6 Conclusions

This paper investigated the presence of chaotic behavior of a pendulum PTO WEC when operating in an irregular sea state. This allowed for studying if chaotic behavior was exhibited by the system in a realistic environmental setting as opposed to regular, single frequency waves. If found to do so, control strategies such as OGY or constant perturbation could be applied with the goal of moving the system towards stability as determined by Dizon et al. (2022)[3]. Under varying environmental, initial conditions, and PTO damping, chaotic orbits were confirmed by meeting the criteria of being asymptotically periodic and having a positive Lyapunov exponent. Although modifications were made to tailor traditional tools used in analyzing chaotic systems, the presence of a chaotic attractor was not found. The main reasons for this were points in the Poincarè map did not return arbitrarily close to each other as simulation time was lengthened, but rather began to fill in the phase space. By the definitions of chaos in Section 4.3.1, chaotic behavior in the pendulum PTO WEC was not present when evaluated in an irregular sea state. With these findings, the route of controlling the system via control strategies commonly used in chaos theory application is not recommended to be pursued. There are multiple aspects on
the interaction between the PTO and WEC to study in determining how efficiently to operate them. Evaluation in irregular sea states is inevitable and, as seen here, dynamics of systems can drastically change.

References


General Conclusion

MRE has the potential to play a significant role as an alternative to powering of applications as opposed to traditional sources. It’s advantages of providing a high energy density make it appealing to uses that are conscious of energy demand and costs. This dissertation studied the pendulum PTO WEC, evaluating its ability to be used as a modular energy source and what conclusions could be drawn of its dynamic chaotic behavior.

It was studied in Manuscript 1 that the HPWEC as a modular component to a NDBC ocean observing buoy was not able to meet the yearly base power requirements of the SCOOP (5 W) without exceeding set weight constraints (14 kg) by 6 kg. While the setup of the HPWEC over varying masses and PTO damping values failed to meet concrete conditions, the findings of the study pave a path for future uses of this setup. The ability of the HPWEC to "plug-and-play" removes the need for mooring configurations, permanent use to a single application, and ease of maintenance without having to remove the entire system it is operating on. The possible advantages of using such a modular device to applications in remote areas or for multiple purposes has been shown.

Manuscript 2 and 3 delved into the field of chaos theory and reviewed the chaotic behavior present in a pendulum PTO styled WEC. The concluding remarks of Manuscript 2 confirmed that a pendulum style PTO WEC exhibits chaotic behavior. It also identified a goal for a control system that could be applied to the pendulum PTO WEC to maximize the amount of power output by moving the system from chaotic to stable states. However as noted in Manuscript 3, chaotic systems are often evaluated in single frequency environments when real world application of WEC systems involves an environment containing many frequencies; the ocean. The findings of Manuscript 3 showed that chaotic behavior (per the definitions of 4.3.1) was not present in a pendulum PTO WEC operating in an irregular sea state and recommended not pursuing a control scheme such as those used in chaos theory as proposed at the end of Manuscript 2. However, studying all aspects of a system is necessary in understanding its full potential.
The pendulum PTO for a WEC remains an existing choice for users in the field and has been used by commercial companies for grid use as noted in Manuscript 1. Its simple and robust operation, given all parts are internally located, make it a strong candidate for the harsh conditions of the ocean. Furthermore, with the possibility of modularity and the study of other non-chaotic dynamics it may present, the understanding of its potential will only increase.
APPENDICES
Pendulum Euler-Lagrange Equation of Motion in 6 DOF

This appendix contains the derivation of the EOM of a pendulum moving in 6 DOF. It is derived for the motion of yaw, denoted as $\psi$ in the derivation, shown in Figure A.1. The resulting equation shows how non-linear and complex the EOM can become when deriving for 6 DOF (A.43) compared to 1 DOF (EQ(2.6) or EQ(4.1)), giving justification to choosing numerical modeling methods.
A.1 6 DOF Derivation

To derive the EOM of the pendulum in 6-DOF, the Lagrangian of the system will be derived in surge, sway, heave, roll, pitch, and yaw where the former three are the translational motions along the x-y-z axis and the latter three are the rotational motions along the x-y-z axis. The Euler-Lagrange equation will then be applied to the Lagrangian in yaw to obtain the EOM of the pendulum rotating about the z-axis of the buoy. Orientation of the pendulum in 3-D space is found through use of rotation matrices. Assumptions and conventions used throughout the derivation include:

1. Pendulum is represented as a point mass to use parallel-axis theorem
2. Tait-Bryan angles are used
3. Z-Y-X sequential order of rotation
4. Global coordinate system angle convention
5. X-Y-Z rotation matrix order (given item #4)

The Lagrangian for the pendulum is defined as:

\[ L = T_{\text{total}} - V_{\text{total}} \]  \hspace{1cm} (A.1)

where \( T_{\text{total}} \) is the system total kinetic energy and \( V_{\text{total}} \) is the system total potential energy. \( T_{\text{total}} \) is made up of the linear and rotational motion. The kinetic energy of linear and rotational motion is:

\[ T_{\text{linear}} = \frac{1}{2} m \vec{v}^2 \]  \hspace{1cm} (A.2)

\[ T_{\text{rotational}} = \frac{1}{2} I \vec{w}^2 \]  \hspace{1cm} (A.3)

where \( m \) is the mass of the interested body, \( \vec{v} = \vec{v}_x \hat{i} + \vec{v}_y \hat{j} + \vec{v}_z \hat{k} \) is the translational velocity of the body, \( I \) is the moment of inertia of the body, and \( \vec{w} = \vec{w}_x \hat{i} + \vec{w}_y \hat{j} + \vec{w}_z \hat{k} \) is the rotational velocity
of the body. Using assumption item #1, \( I \) can be represented as:

\[
I = mr^2 
\]

(A.4)

where \( r \) is distance from the origin of the body to the pendulum’s center of mass. (A.2) can further be distributed into the translational 3-DOFs, surge, sway and heave as:

\[
T_{\text{linear}} = \frac{1}{2} m \vec{v}_X^2 + \frac{1}{2} m \vec{v}_Y^2 + \frac{1}{2} m \vec{v}_Z^2 
\]

(A.5)

where \( \vec{v}_X, \vec{v}_Y, \) and \( \vec{v}_Z \) represent translational velocity in the surge, sway, and heave motions, respectively. (A.3) can further be broken down into the rotational 3-DOFs, roll, pitch and yaw as:

\[
T_{\text{rotational}} = \frac{1}{2} m (r_X \vec{w}_X)^2 + \frac{1}{2} m (r_Y \vec{w}_Y)^2 + \frac{1}{2} m (r_Z \vec{w}_Z)^2 
\]

(A.6)

where \( \vec{w}_X, \vec{w}_Y, \) and \( \vec{w}_Z \) represent the rotational velocity vector in the roll, pitch, and yaw motions, respectively. In 2-D, rotational motion about, e.g., the x-axis would lead to \( \vec{w}_X = \dot{\phi} \hat{i} \), where \( \dot{\phi} \) is the angular velocity around the x-axis and \( \hat{i} \) is the unit vector along the x-axis. When rotated in 3-D, this (dependent on rotation order) does not hold true because the x-axis of the new mobile frame could not be aligned with the global frame anymore. Thus in 3-D,

\[
\vec{w} \neq \dot{\phi} \hat{i} + \dot{\theta} \hat{j} + \dot{\psi} \hat{k} 
\]

(A.7)

where \( \dot{\phi} \) is the angular velocity around the x-axis, \( \hat{i} \) is the unit vector along the x-axis, \( \dot{\theta} \) is the angular velocity around the y-axis, \( \hat{j} \) is the unit vector along the y-axis, \( \dot{\psi} \) is the angular velocity around the z-axis, and \( \hat{k} \) is the unit vector along the z-axis. However, it holds in 3D that:

\[
\vec{v} = \vec{w} \times \vec{p}' 
\]

(A.8)

where \( \vec{p}' = px \hat{i} + py \hat{j} + pz \hat{k} \) are the current x-y-z coordinates of the point mass. To determine both translational and rotational velocities, the skew matrix, \( S(w) \), and rotation matrix, \( R \), are utilized along with a few definitions:

1. \( \vec{p}' = R \vec{p} \)
2. \( \vec{p}' = \vec{p} R \)
3. $RR^T = I$

4. $S(w) + S(w)^T = 0$

5. $S(w) = \dot{R}R^T$

6. $RS(w)R^T = S(Rw)$

where $I$ is the identity matrix, $\vec{p}'$ are the new x-y-z coordinates of the point mass after being rotated by $R$, $\dot{\vec{p}}'$ is the derivative of $\vec{p}$ with respect to time, and $\dot{R}$ is the derivative of $R$ with respect to time. The definitions are now proven. Def. #1, #4, and #6 are known.

Def. #2 proof. By using the chain rule:

$$\dot{\vec{p}}' = R\dot{\vec{p}} + p\dot{R}$$  \hspace{1cm} (A.9)

$$\dot{\vec{p}}' = p\dot{R}$$

as $p$ does not change with time.

Def. #3 proof.

$$RR^T = I$$  \hspace{1cm} (A.10)

$$R^{-1}RR^T = R^{-1}$$

$$R^T = R^{-1}$$

Def. #5 proof. Differentiate Def. #3 using chain rule.

$$\dot{RR^T} + R\dot{R^T} = 0$$  \hspace{1cm} (A.11)

using the definition:

$$(AB)^T = B^T A^T$$  \hspace{1cm} (A.12)

(A.11) continues as:

$$\dot{RR^T} + (\dot{R}R^T)^T = 0$$  \hspace{1cm} (A.13)
Comparing (A.13) to Def. #4 leads to Def. #5. Now that the definitions have been proven, the definition of the rotation matrices along each axis are provided:

\[
R_x = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi
\end{bmatrix}
\]

(A.14)

\[
R_y = \begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix}
\]

(A.15)

\[
R_z = \begin{bmatrix}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(A.16)

where \(\phi\) is the angular position around the x-axis, \(\theta\) is the angular position around the y-axis, and \(\psi\) is the angular position around the z-axis. The definition of \(S(w)\) is:

\[
S(w) = \begin{bmatrix}
0 & -w_z & w_y \\
w_z & 0 & -w_x \\
w_y & w_x & 0
\end{bmatrix}
\]

(A.17)

leading to:

\[
\vec{w} = \begin{bmatrix}
w_x \\
w_y \\
w_z
\end{bmatrix}
\begin{bmatrix}
\hat{i} \\
\hat{j} \\
\hat{k}
\end{bmatrix}
\]

(A.18)

With Item #3:

\[
R = R_x(\phi)R_y(\theta)R_z(\psi)
\]

(A.19)

Using Def. #5 to find the skew matrix using the full matrices of (A.14), (A.15), and (A.16) would require complex and long matrix multiplication. Def. #5 is then first calculated using variables
to take advantage of other definitions in an attempt to simplify the equation. Again, we have:

\[ S(w) = \dot{R}R^T \]  
(A.20)

\[ S(w) = \dot{R}_x R_y R_z^T + \dot{R}_y R_z R_x^T + \dot{R}_z R_x R_y^T \]  
(A.21)

Working through 1:

\[ \dot{R}_x R_y R_z^T = \dot{R}_x R_y (R_x R_y R_z)^T \]
\[ = \dot{R}_x R_y R_y^T R_y R_z^T \]
\[ = I \]
\[ = \dot{R}_x R_y^T \]
\[ = \dot{R}_x \psi R_x(\psi)^T \]
\[ = S(\dot{\phi} \hat{\theta}) \]  
(A.22)

Working through 2:

\[ \dot{R}_x \dot{R}_y R_z^T = \dot{R}_x \dot{R}_y R_z (R_x R_y R_z)^T \]
\[ = \dot{R}_x \dot{R}_y R_z^T R_y^T R_x^T \]
\[ = I \]
\[ = \dot{R}_x \dot{R}_y R_z^T \]
\[ = R_x S(\dot{\theta} \hat{j}) R_y R_y^T R_x^T \]
\[ = R_x S(\dot{\theta} \hat{j}) R_x^T \]
\[ = S(R_x \dot{\theta} \hat{j}) \]  
(A.23)
Working through 3:

\[ R_x R_y R_z R^T = R_x R_y (R_y R_z)^T \]

\[ = R_x R_y \begin{bmatrix} R_z & S(\psi \hat{k}) & R_z^T \end{bmatrix} R^T_y R^T_z \]

\[ = R_x R_y S(\psi \hat{k}) (R_y R_z)^T \]

\[ = S(R_x R_y \psi \hat{k}) \]

(A.24)

Putting (A.21), (A.22), (A.23), and (A.24) together yields:

\[ S(w) = S(\phi \hat{i}) + S(R_x \hat{j}) + S(R_y \hat{k}) \]

(A.25)

Thus:

\[ \vec{w} = \phi \hat{i} + R_x \hat{j} + R_y \hat{k} \]

(A.26)

This is expanded out to:

\[ w = \phi \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + R_x \hat{j} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + R_y \hat{k} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \]

(A.27)

Performing the matrix multiplication after combining (A.13), (A.14), (A.15), (A.17), and (A.26) yields:

\[ w = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & \sin \theta \\ 0 & \cos \phi & -\sin \phi \cos \theta \\ 0 & \sin \phi & \cos \phi \cos \theta \end{bmatrix} \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \]

(A.28)

Leading to the final equations for angular velocities about the x-y-z axis of the object in 3-D.
space:

\[ w_X = \dot{\phi} + \psi \sin \theta \]  \hspace{1cm} (A.29)  
\[ w_Y = \dot{\theta} \cos \phi - \dot{\psi} \sin \phi \cos \theta \]  \hspace{1cm} (A.30)  
\[ w_Z = \dot{\theta} \sin \phi + \dot{\psi} \cos \phi \cos \theta \]  \hspace{1cm} (A.31)

The cross product of (A.8) leads to:

\[ \vec{v} = (w_Y p_Z - w_Z p_Y) \hat{i} - (w_X p_Z - w_Z p_X) \hat{j} + (w_X p_Y - w_Y p_X) \hat{k} \]  \hspace{1cm} (A.32)

Leading to the final equations for translational velocity about the x-y-z axis of the object in 3-D space:

\[ v_X = w_Y p_Z - w_Z p_Y \]  \hspace{1cm} (A.33)  
\[ v_Y = w_X p_Z - w_Z p_X \]  \hspace{1cm} (A.34)  
\[ v_Z = w_X p_Y - w_Y p_X \]  \hspace{1cm} (A.35)

\( V_{total} \) is then equal to:

\[ V_{total} = m g h \]  \hspace{1cm} (A.36)

\[ = m g p_Z \]

and \( T_{total} \) is:

\[ T_{total} = \frac{1}{2} m \vec{v}_X^2 + \frac{1}{2} m \vec{v}_Y^2 + \frac{1}{2} m \vec{v}_Z^2 \]  
\[ + \frac{1}{2} m (r_X \vec{w}_X)^2 + \frac{1}{2} m (r_Y \vec{w}_Y)^2 + \frac{1}{2} m (r_Z \vec{w}_Z)^2 \]  \hspace{1cm} (A.37)
The total Lagrangian is now defined as:

\[
L = \frac{1}{2} m \dot{v}_X^2 + \frac{1}{2} m \dot{v}_Y^2 + \frac{1}{2} m \dot{v}_Z^2 + \frac{1}{2} m (r_X \dot{w}_X)^2 + \frac{1}{2} m (r_Y \dot{w}_Y)^2 + \frac{1}{2} m (r_Z \dot{w}_Z)^2 - mg p_Z \tag{A.38}
\]

\[
= \frac{1}{2} m (w_Y p_Z - w_Z p_Y)^2 + \frac{1}{2} m (w_X p_Z - w_Z p_X)^2 + \frac{1}{2} m (w_X p_Y - w_Y p_X)^2 \\
+ \frac{1}{2} m (r_X \dot{w}_X)^2 + \frac{1}{2} m (r_Y \dot{w}_Y)^2 + \frac{1}{2} m (r_Z \dot{w}_Z)^2 - mg p_Z \\
L = \frac{1}{2} m ((\dot{\theta} \cos \phi - \dot{\psi} \sin \phi \cos \theta) p_Z - (\dot{\theta} \sin \phi + \dot{\psi} \cos \phi \cos \theta) p_Y)^2 \\
+ \frac{1}{2} m ((\dot{\phi} + \dot{\psi} \sin \theta) p_Z - (\dot{\theta} \sin \phi + \dot{\psi} \cos \phi \cos \theta) p_X)^2 \\
+ \frac{1}{2} m ((\dot{\phi} + \dot{\psi} \sin \theta) p_Y - (\dot{\theta} \cos \phi - \dot{\psi} \sin \phi \cos \theta) p_X)^2 \\
+ \frac{1}{2} m (p_X (\dot{\phi} + \dot{\psi} \sin \theta))^2 + \frac{1}{2} m (p_Y (\dot{\theta} \cos \phi - \dot{\psi} \sin \phi \cos \theta))^2 \\
+ \frac{1}{2} m (p_Z (\dot{\theta} \sin \phi + \dot{\psi} \cos \phi \cos \theta))^2 - mg p_Z \tag{A.39}
\]

To get the EOM of the pendulum and it’s rotational velocity about the z-axis, the Euler-Lagrange equation is applied:

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{\psi}} - \frac{\partial L}{\partial \psi} = S \tag{A.40}
\]

where \(S\) are the non-conservative forces and torques. This yields:
\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{\psi}} = m (p_Y (\psi \sin \phi + \dot{\phi} \psi \cos \phi + \phi) + p_X (\psi \cos \theta \sin \phi - \dot{\theta} \cos \phi + \phi \dot{\theta} \sin \phi) \]
\[ + \dot{\phi} \psi \cos \phi \cos \theta - \psi \dot{\theta} \sin \phi \sin \theta) \]  
\[ + \psi \cos \phi \cos \theta - p_Z (\psi \cos \theta - \dot{\psi} \cos \theta \sin \phi)) (p_Y \dot{\phi} \psi \cos \phi) + p_Y \dot{\theta} \cos \phi \sin \theta + p_Z \dot{\theta} \sin \phi \sin \theta) + m (p_Y (\dot{\theta} \sin \phi) \]
\[ + \dot{\phi} \psi \cos \phi \cos \theta - \dot{\theta} \cos \phi \sin \theta) - \psi \dot{\theta} \sin \phi \sin \theta)) (p_Y \dot{\phi} \psi \cos \phi + p_X \dot{\phi} \psi \cos \phi \cos \theta - p_X \dot{\theta} \sin \phi \sin \theta) - m (p_X (\dot{\theta} \sin \phi + \psi \cos \phi \cos \theta) \]
\[ - p_Z (\psi \sin \theta + \dot{\phi}) (p_Z \dot{\phi} \psi \cos \phi \cos \theta + \dot{\theta} \cos \phi \sin \theta) + m p_X^2 \cos \theta (\psi \sin \theta + \dot{\phi}) + m p_Z^2 \cos \phi \cos \theta (\psi \sin \phi + \dot{\theta} \cos \phi \sin \theta) \]
\[ + \dot{\phi} \psi \cos \phi \cos \theta - \dot{\theta} \cos \phi \sin \theta) + m p_Z^2 \dot{\theta} \cos \phi \cos \theta (\dot{\theta} \sin \phi + \psi \cos \phi \cos \theta) \]
\[ - m p_Z^2 \dot{\theta} \cos \phi \cos \theta (\dot{\theta} \sin \phi + \psi \cos \phi \cos \theta) \dot{\theta} + m p_Z^2 \dot{\theta} \co \sin \phi (\dot{\theta} \sin \phi + \psi \cos \phi \cos \theta) \dot{\theta} \]
\[ + m p_Z^2 \dot{\theta} \cos \phi \cos \theta (\dot{\theta} \sin \phi + \psi \cos \phi \cos \theta) \dot{\theta} \]
\[ \frac{\partial L}{\partial \dot{\psi}} = 0 \]  
(A.42)
Substituting (A.41) and (A.42) into (A.40) solves for \( \psi \) as:

\[
\psi = \frac{(S + m (p_Y (\theta \sin \phi + \psi \cos \phi \cos \theta) - p_Z (\theta \cos \phi - \psi \cos \theta \sin \phi)) (p_Y \phi \cos \theta \sin \phi)}{(A.43)}
\]

\[
= \frac{\left( -p_Y \phi \cos \phi \cos \theta + p_Y \theta \cos \phi \sin \theta + p_Z \theta \sin \theta \sin \phi \right) + m (p_X (\theta \cos \phi - \psi \cos \theta \sin \phi))}{(A.43)}
\]

\[
- p_Z \phi \cos \phi \cos \theta \left( \psi \sin \phi + \theta \right) (p_Y \phi \cos \phi \sin \theta) + \left( p_Y \phi \psi \cos \phi \cos \theta \sin \phi - \psi \theta \cos \phi \sin \theta \right) + m (p_X (\theta \cos \phi - \psi \theta \cos \phi \sin \theta) + p_Z \sin \theta \sin \phi)
\]

\[
- p_Z \phi \cos \phi \cos \theta \left( \psi \sin \phi + \theta \right) (p_Y \phi \cos \phi \sin \theta) + \left( p_Y \phi \psi \cos \phi \cos \theta \sin \phi - \psi \theta \cos \phi \sin \theta \right) + m (p_X (\theta \cos \phi - \psi \theta \cos \phi \sin \theta) + p_Z \sin \theta \sin \phi)
\]

\[
+ m (p_Y \phi \psi \cos \phi \cos \theta \sin \phi - \psi \theta \cos \phi \sin \theta) + m (p_X \phi \psi \cos \phi \cos \theta \sin \phi - \psi \phi \psi \cos \phi \cos \theta \sin \phi)
\]

\[
= \frac{\left( -p_Y \phi \cos \phi \cos \theta + p_Y \theta \cos \phi \sin \theta + p_Z \theta \sin \theta \sin \phi \right) + m (p_X (\theta \cos \phi - \psi \theta \cos \phi \sin \theta) + p_Z \sin \theta \sin \phi)}{m (p_Z \sin \theta - p_X \cos \phi \cos \theta)^2 + m (p_Y \sin \theta + p_X \cos \theta \sin \phi)^2}
\]

\[
+ m (p_Y \cos \phi \cos \theta + p_Z \cos \theta \sin \phi)^2 + m (p_X \sin \theta^2 + m p_Z \cos \phi^2 \cos \theta^2 + m p_Y \cos \theta^2 \sin \phi^2)^2
\]
IFAC Additional Case Runs

This appendix contains examples of data ran from Chapter 3 section 3.5 detailing how stable power generally outperforms chaotic behavior for the cases of different pendulum design, mass, radius arm, and wave height and period, than presented in Chapter 3 section 3.5.

B.1 Pendulum Designs

This section of the appendix shows the two pendulum designs used throughout the simulations. Pendulum design 1 was shown in Chapter 3 and is shown below with pendulum design 2. Both are shown side-by-side (B.1) and inside the pendulum PTO WEC (B.2 and B.3).
Figure B.1: Solidworks model of pendulum design 1 (left) and 2 (right).

Figure B.2: Isometric view of pendulum design 1 inside the pendulum PTO WEC.

Figure B.3: Isometric view of pendulum design 2 inside the pendulum PTO WEC.
B.2 Bifurcation-Power Diagrams

Figures B.5, B.6, B.7, B.8, and B.9 illustrate the stable versus chaotic behavior of the pendulum PTO WEC as well as its power output for different values of pendulum mass, radius arm, and pendulum design than presented in Chapter 3 section 3.5. The figures depicted through section 3.5 were for simulations of pendulum design 1 \((r = 0.26m)\) and a mass of 200 kg over varying wave height, period, and PTO damping. The following figures are for a constant wave period and PTO damping, and the bifurcation changing variable is wave height, similar to that of Figure 3.12. It is studied how changing the constant value of pendulum mass (200 kg and 100 kg), radius arm \((r = 0.26m \text{ and } r = 0.45m)\), and design 1 and 2 affects dynamics and power performance. Figure 3.12 is provided again for easy access and comparison as Figure B.4.

Figure B.5 is similar to that of Figure B.4 (Figure 3.12) except that the pendulum design 1 radius arm has been changed from \(r = 0.26m\) to \(r = 0.45m\).

Figure B.6 is similar to that of Figure B.4 (Figure 3.12) except that the pendulum mass has been changed from 200 kg to 100 kg.

Figure B.7 is similar to that of Figure B.4 (Figure 3.12) except that the pendulum design 1 radius arm has been changed from \(r = 0.26m\) to \(r = 0.45m\), and the mass has been decreased from 200 kg to 100 kg.

Figure B.8 is similar to that of Figure B.4 (Figure 3.12) except that the pendulum design has been changed from 1 to 2.

Figure B.9 is similar to that of Figure B.4 (Figure 3.12) except that the pendulum design has been changed from 1 to 2 and the mass has been decreased from 200 kg to 100 kg.

It is confirmed that the conclusion of section 3.5 regarding stable behavior outperforming chaotic behavior still holds true for these differing scenarios.
Figure B.4: Bifurcation diagram and power plot of the WEC system. The changing variable is the period which ranges from 0.5 m to 6 m. This is Figure 3.12, with pendulum design 1 and pendulum mass 200 kg. Blue shaded regions in the power plot indicate chaos, green shaded regions indicate stability, also noted by the black vertical lines relating the two visuals. The red trendline applied to stable data points shows how chaotic behavior generally underperforms in power output when compared to stable behavior.
Figure B.5: Bifurcation diagram and power plot of the WEC system. The changing variable is the period which ranges from 0.5 m to 6 m. This figure is similar to Figure 3.12 except that it obtains pendulum design 1 \( r = 0.45m \) instead of \( r = 0.26m \). Blue shaded regions in the power plot indicate chaos, green shaded regions indicate stability, also noted by the black vertical lines relating the two visuals. The red trendline applied to stable data points shows how chaotic behavior generally underperforms in power output when compared to stable behavior.
Figure B.6: Bifurcation diagram and power plot of the WEC system. The changing variable is the period which ranges from 0.5 m to 6 m. This figure is similar to Figure 3.12 except that it obtains pendulum mass 100 kg instead of 200 kg. Blue shaded regions in the power plot indicate chaos, green shaded regions indicate stability, also noted by the black vertical lines relating the two visuals. The red trendline applied to stable data points shows how chaotic behavior generally underperforms in power output when compared to stable behavior.
Figure B.7: Bifurcation diagram and power plot of the WEC system. The changing variable is the period which ranges from 0.5 m to 6 m. This figure is similar to Figure 3.12 except that it obtains pendulum design 1 $r = 0.45 m$ instead of $r = 0.26 m$ and pendulum mass 100 kg instead of 200 kg. Blue shaded regions in the power plot indicate chaos, green shaded regions indicate stability, also noted by the black vertical lines relating the two visuals. The red trendline applied to stable data points shows how chaotic behavior generally underperforms in power output when compared to stable behavior.
Figure B.8: Bifurcation diagram and power plot of the WEC system. The changing variable is the period which ranges from 0.5 m to 6 m. This figure is similar to Figure 3.12 except that it obtains pendulum design 2 instead of 1. Blue shaded regions in the power plot indicate chaos, green shaded regions indicate stability, also noted by the black vertical lines relating the two visuals. The red trendline applied to stable data points shows how chaotic behavior generally underperforms in power output when compared to stable behavior.
Figure B.9: Bifurcation diagram and power plot of the WEC system. The changing variable is the period which ranges from 0.5 m to 6 m. This figure is similar to Figure 3.12 except that it obtains pendulum design 2 instead of 1 and pendulum mass 100 kg instead of 200 kg. Blue shaded regions in the power plot indicate chaos, green shaded regions indicate stability, also noted by the black vertical lines relating the two visuals. The red trendline applied to stable data points shows how chaotic behavior generally underperforms in power output when compared to stable behavior.
Additional Fourier Transforms and 3D Poincarè Maps

This appendix contains additional analysis into the frequencies present in the wave environment and the pendulum dynamics. Section C.1 shows that in regular waves the pendulum’s most prevalent frequency is identical to that of the wave. Section C.2 depicts different simulations than those shown in Chapter 4 Section 4.5.2, illustrating that even under varying conditions, a chaotic attractor with a traditional pattern (such as Figure 4.6) does not appear.

C.1 Fourier Transform of Regular Waves

C.1.1 Stable Pendulum Dynamics

The Fourier transform of a regular wave throughout an entire 1200 sec simulation is shown in Figure C.1. Additionally, the Fourier transform of the pendulum when exhibiting stable behavior is provided using the parameters used in Chapter 4 section 4.4.1. It is seen that the pendulums most prevalent frequency is that of the waves; i.e. 6 sec. Included also are the frequencies associated with the pendulums non-linearity.
Figure C.1: The Fourier transform of a regular wave and pendulum dynamics when the pendulum is exhibiting stable behavior. It is seen that the pendulum’s most prevalent frequency is the same as the waves. They both also represent the wave period, 6 sec.

C.1.2 Chaotic Pendulum Dynamics

The Fourier transform of a regular wave throughout an entire 1200 sec simulation is shown in Figure C.2. Additionally, the Fourier transform of the pendulum when exhibiting chaotic behavior is provided using the parameters used in Chapter 4 section 4.4.2. It is seen that the pendulums most prevalent frequency is still that of the waves; i.e. 6 sec. However as it is behaving chaotically, the frequencies the pendulum moves at during its simulation has greatly increased. This details how the same system can behave much more complex when changing certain parameters.
Figure C.2: The Fourier transform of a regular wave and pendulum dynamics when the pendulum is exhibiting chaotic behavior. It is seen that the pendulum’s most prevalent frequency is the same as the waves. They both also represent the wave period, 6 sec. The increase in frequency content experienced by the pendulum is due to its chaotic state.
C.2 3D Poincarè Maps

The 3D Poincarè maps detailed throughout Chapter 4 section 4.5.2 were produced from simulations using irregular wave parameters of 4 m significant wave height and 6 sec peak period with a Pierson-Moskowitz (PM) spectrum. The damping for the PTO was 32 Nm/(rad/s) and time ranges shown for determining the sampling frequency were 300 sec and 600 sec. The following two subsections show additional Poincarè maps for two sea states using the PM spectrum (sea state A: 4 m significant wave height and 6 sec peak period; sea state B: 3 m significant wave height and 8 sec peak period) with PTO damping values of 5, 12, and 32 Nm/(rad/s) and time ranges of 300, 400, and 600 sec. The concluding the findings of section 4.5.2, a lack of pattern for any combination of considered sea states, PTO damping, and time ranges is also shown here.

C.2.1 Sea State A

The figures presented in this subsection are for simulations throughout a 4 m 6 sec PM spectrum. Figure C.3 and C.4 detail the 3D Poincarè map in 400 sec intervals at damping values of 12 and 5 Nm/(rad/s), respectively. This differs from those shown in section 4.5.2 which were 300 sec and 600 sec intervals, both at 32 Nm/(rad/s) damping. Despite these differences, a traditional pattern does not show up (such as Figure 4.6), but instead displays a scattering of dots.
Figure C.3: The resulting 3D Poincarè map for the WEC system simulated in sea state A, taken at 400 sec intervals, with a PTO damping of 12 Nm/(rad/s). No traditional pattern is observed.
Figure C.4: The resulting 3D Poincaré map for the WEC system simulated in sea state A, taken at 400 sec intervals, with a PTO damping of 5 Nm/(rad/s). No traditional pattern is observed.
C.2.2 Sea State B

The figures presented in this subsection are for simulations throughout a 3 m 8 sec PM spectrum. Figure C.5, C.6, C.7, and C.8 detail the 3D Poincarè map in 300, 400, and 600 sec intervals at damping values of 5, 12, and 32 Nm/(rad/s). This differs from those shown in section 4.5.2 as the sea state has changed completely. Despite these differences, a traditional pattern does not show up (such as Figure 4.6), but instead displays a scattering of dots.

Figure C.5: The resulting 3D Poincarè map for the WEC system simulated in sea state B, taken at 300 sec intervals, with a PTO damping of 32 Nm/(rad/s). No traditional pattern is observed.
Figure C.6: The resulting 3D Poincarè map for the WEC system simulated in sea state B, taken at 600 sec intervals, with a PTO damping of 32 Nm/(rad/s). No traditional pattern is observed.
Figure C.7: The resulting 3D Poincaré map for the WEC system simulated in sea state B, taken at 400 sec intervals, with a PTO damping of 12 Nm/(rad/s). No traditional pattern is observed.
Figure C.8: The resulting 3D Poincarè map for the WEC system simulated in sea state B, taken at 400 sec intervals, with a PTO damping of 5 Nm/(rad/s). No traditional pattern is observed.