

SUSTAINABLE YIELDS FOR ECOSYSTEMS. A MATHEMATICAL VIABILITY APPROACH

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Abstract

We define the notion of sustainable yields for ecosystem, with particular emphasis on long-run consistency between ecological and economic conflicting objectives. We provide a way to compute sustainable yields by means of a viability analysis of generic ecosystem models with harvesting. We apply our approach to a Lotka–Volterra model of the anchovy–hake couple in the Peruvian upwelling ecosystem between the years 1971 and 1981. Our analysis suggests that, during the anchovy collapse, the fishery could theoretically have been viably managed to produce catches above the expected levels while ensuring biological conservation. Control theory and viability theory methods have allowed us to introduce ecosystem considerations, such as multispecies and multiobjectives, and have contributed to integrate the long term dynamics, which is generally not considered in conventional fishery management.

Keywords

control theory; state constraints; viability; predator–prey; ecosystem management; sustainable yields.

INTRODUCTION

In fisheries, yields are usually defined species by species. For instance, the maximum sustainable yield approach (MSY, see [1]) relies upon a single variable stock description; what is more, computations are made at equilibrium. The ICES precautionary approach does not assume equilibrium (it projects abundances one year ahead), but it relies upon age-class monospecific dynamical models [2].

On the other hand, more and more emphasis is put on multispecies models [3] and on ecosystem management. For instance, the World Summit on Sustainable Development encouraged the application of the ecosystem approach by 2010 [4].

At last, sustainability is a major goal of international agreements and guidelines to fisheries management [5, 2].

Our interest is in providing conceptual insight as what could be *sustainable yields for ecosystems*. In this, we follow the vein of [6] where the notion of *Ecologically Sustainable Yield* (ESY) is introduced.

However, our emphasis is on providing formal definition and practical methods to design and compute such yields. For this purpose, our approach is not based on equilibrium calculus, nor on intertemporal discounted utility maximization but on the so-called viability theory, as follows.

On the one hand, the ecosystem is described by a dynamical model controlled by harvesting. On the other hand, building upon [7], constraints are imposed: catches are expected to be above given production thresholds, and biomasses above safety biological thresholds. Sustainability is the property that such constraints can be maintained for all time by appropriate harvesting strategy.

Such problems of dynamic control under constraints refer to viability [8] or invariance [9] frameworks, as well as to reachability of target sets or tubes for nonlinear discrete time dynamics in [10].

We consider sustainable management issues formulated within such framework as in [7, 11, 12, 13, 14, 15, 16].

A viable state is an initial condition for the ecosystem dynamical system such that proper harvesting rules may drive the system on a sustainable path by maintaining catches and biomasses above their respective thresholds. We provide a way to characterize production thresholds (yields) such that the initial conditions are a viable state. These yields are sustainable in the sense that they can be indefinitely maintained, while making possible that the ecosystem remains in an ecologically viable zone.

The paper is organized as follows. In the following Section, we introduce generic harvested nonlinear ecosystem models, and we present how preservation and production constraints are modelled. Thanks to an explicit description of viable states, we are able to characterize sustainable yields. These latter are not defined species by species, but depend on the whole ecosystem dynamics and on all conservation thresholds. In the last Section, an illustration in ecosystem management and numerical applications are given for the hake–anchovy couple in the Peruvian upwelling ecosystem between the years 1971 and 1981.

ECOSYSTEM SUSTAINABLE YIELDS

We introduce generic harvested nonlinear ecosystem models. Then, we present preservation and production constraints. We provide a rather explicit description of viable states which allow us to define sustainable yields, compatible with biological conservation constraints.

Ecosystem dynamical model

For simplicity, we consider two species. However, the following Proposition 1 may be easily extended to n -dimensional systems as long as each species is harvested by a specific device: one species, one harvesting effort.

Each species is described by its biomass: the two-dimensional state vector (y, z) represents the biomasses of both species. The two-dimensional control (v, w) comprises the harvesting effort for each species, respectively. The catches are thus vy and wz (measured in biomass).¹

The discrete-time control system we consider is

$$\begin{cases} y(t+1) &= y(t)R_y(y(t), z(t), v(t)) \\ z(t+1) &= z(t)R_z(y(t), z(t), w(t)) \end{cases}, \quad (1)$$

where t stand for time (typically, periods are years), and where $R_y : \mathbb{R}^3 \rightarrow \mathbb{R}$ and $R_z : \mathbb{R}^3 \rightarrow \mathbb{R}$ are two functions representing growth factors (the growth rates being $R_y - 1$ and $R_z - 1$).

This model is generic in that no explicit assumptions are made on how the growth factors R_y and R_z indeed depend upon both biomasses (y, z) .

Preservation and production sustainability

Let us be given

- on the one hand, *minimal biomass thresholds* $B_y^b \geq 0$, $B_z^b \geq 0$, one for each species,
- on the other hand, *minimal catch thresholds* $C_y^b \geq 0$, $C_z^b \geq 0$, one for each species.

¹In fact, any expression of the form $c(y, v)$, instead of vy , would fit for the catches in the following Proposition 1 as soon as $v \mapsto c(y, v)$ is strictly increasing and goes from 0 to $+\infty$ when v goes from 0 to $+\infty$. The same holds for $d(z, w)$ instead of wz .

A state (y_0, z_0) is said to be a *viable state* if there exist appropriate controls $(v(t), w(t))$, $t = t_0, t_0 + 1, \dots$ such that the state path $(y(t), z(t))$, $t = t_0, t_0 + 1, \dots$, starting from $(y(t_0), z(t_0)) = (y_0, z_0)$ and generated by the dynamics (1), satisfies the following goals:

- preservation (minimal biomass thresholds)

$$\text{biomasses: } y(t) \geq B_y^b, \quad z(t) \geq B_z^b, \quad \forall t = t_0, t_0 + 1, \dots \quad (2)$$

- and production requirements (minimal catch thresholds)

$$\text{catches: } v(t)y(t) \geq C_y^b, \quad w(t)z(t) \geq C_z^b, \quad \forall t = t_0, t_0 + 1, \dots \quad (3)$$

The set of all viable states is called the *viability kernel* [8].

Hence, characterizing viable states makes it possible to test whether or not minimal biomasses and catches can be guaranteed for all time.

The following Proposition 1 gives a rather explicit description of the viable states, under some conditions on the minimal thresholds.

We shall say that *growth factors are nice* if the function $R_y : \mathbb{R}^3 \rightarrow \mathbb{R}$ is continuously decreasing² in the control v and satisfies $\lim_{v \rightarrow +\infty} R_y(y, z, v) \leq 0$, and if $R_z : \mathbb{R}^3 \rightarrow \mathbb{R}$ is continuously decreasing in the control variable w , and satisfies $\lim_{w \rightarrow +\infty} R_z(y, z, w) \leq 0$.

Proposition 1 *Assume that the growth factors are nice. If the thresholds $B_y^b \geq 0$, $B_z^b \geq 0$, and $C_y^b \geq 0$, $C_z^b \geq 0$ are such that the following growth factors are greater than one*

$$R_y(B_y^b, B_z^b, \frac{C_y^b}{B_y^b}) \geq 1 \text{ and } R_z(B_y^b, B_z^b, \frac{C_z^b}{B_z^b}) \geq 1, \quad (4)$$

then viable states are (y, z) such that

$$y \geq B_y^b, \quad z \geq B_z^b, \quad yR_y(y, z, \frac{C_y^b}{y}) \geq B_y^b, \quad zR_z(y, z, \frac{C_z^b}{z}) \geq B_z^b. \quad (5)$$

Let us comment the assumptions of Proposition 1. That the growth factors are decreasing with respect to the harvesting effort is a natural assumption. Conditions (4) mean that, at the point (B_y^b, B_z^b) and applying efforts $u^b = \frac{C_y^b}{B_y^b}$, $v^b = \frac{C_z^b}{B_z^b}$,

²In all that follows, a mapping $\varphi : \mathbb{R} \rightarrow \mathbb{R}$ is said to be increasing if $x \geq x' \Rightarrow \varphi(x) \geq \varphi(x')$. The reverse holds for decreasing. Thus, with this definition, a constant mapping is both increasing and decreasing.

the growth factors are greater than one, hence both populations grow; hence, it could be thought that computing viable states is useless since everything looks fine. However, if all is fine at the point (B_y^b, B_z^b) , it is not obvious that this also goes for a larger domain. Indeed, the ecosystem dynamics f given by (1) has no monotonicity properties that would allow to extend a result valid for a point to a whole domain. What is more, if continuous-time viability results mostly relies upon assumptions at the frontier of the constraints set, this is no longer true for discrete-time viability.

We shall explicitly draw a viability kernel in the next section, for a discrete-time Lotka–Volterra model for the hake–anchovy couple in the Peruvian upwelling ecosystem.

Ecosystem sustainable yields

Considering that minimal biomass conservation thresholds $B_y^b \geq 0, B_z^b \geq 0$ are given first (for prominent biological issues), we shall now examine conditions for the existence of minimal catch thresholds $C_y^b \geq 0, C_z^b \geq 0$ susceptible to be sustainably maintained.

Proposition 2 *Assume that growth factors are nice. Assume that the growth factors at the conservation thresholds without harvest are greater than one:*

$$R_y(B_y^b, B_z^b, 0) \geq 1 \text{ and } R_z(B_y^b, B_z^b, 0) \geq 1 . \quad (6)$$

Define equilibrium efforts as the largest nonnegative v^b, w^b such that

$$R_y(B_y^b, B_z^b, v^b) = 1 \text{ and } R_z(B_y^b, B_z^b, w^b) = 1 , \quad (7)$$

and define catches

$$C_y^{b,*} := B_y^b v^b \text{ and } C_z^{b,*} := B_z^b w^b . \quad (8)$$

Consider (y_0, z_0) such that $y_0 \geq B_y^b$ and $z_0 \geq B_z^b$ and satisfying

$$y_0 R_y(y_0, z_0, 0) \geq B_y^b \text{ and } z_0 R_z(y_0, z_0, 0) \geq B_z^b . \quad (9)$$

We define

$$\begin{cases} C_y^{b,*}(y_0, z_0) & := \max\{C_y \in [0, C_y^{b,*}] \mid y_0 R_y(y_0, z_0, \frac{C_y}{y_0}) \geq B_y^b\} \\ C_z^{b,*}(y_0, z_0) & := \max\{C_z \in [0, C_z^{b,*}] \mid z_0 R_z(y_0, z_0, \frac{C_z}{z_0}) \geq B_z^b\} , \end{cases} \quad (10)$$

and consider catches C_y^b and C_z^b such that $0 \leq C_y^b \leq C_y^{b,}(y_0, z_0)$ and $0 \leq C_z^b \leq C_z^{b,*}(y_0, z_0)$.*

Then, starting from the initial point $(y(t_0), z(t_0)) = (y_0, z_0)$, there exists appropriate harvesting paths which can provide, for all time, at least the sustainable yields C_y^b and C_z^b . Indeed, such levels of production can be guaranteed by appropriate viable controls respecting minimal biomass conservation thresholds.

From the practical point of view, the upper quantities $C_y^{b,*}(y_0, z_0)$ and $C_z^{b,*}(y_0, z_0)$ in (10) cannot be seen as catches targets, but rather as *crisis limits*. Indeed, the closer to them, the more risky. since the initial point is close to the viability kernel frontier.

Notice that the yield $C_y^{b,*}(y_0, z_0)$ depends, first, on both species biomasses (y_0, z_0) , second, on both conservation thresholds B_y^b and B_z^b , third, on the ecosystem model by the growth factor R_y ; the same holds for $C_z^{b,*}(y_0, z_0)$. Thus, the yields in (10) are designed jointly on the basis of the whole ecosystem model and of all the conservation thresholds: they are *ecosystem sustainable yields*.

This observation may have practical consequences. Indeed, the catches guaranteed for one species depend not only on the biological threshold of the same species, but on the other species. For instance, in the Peruvian upwelling ecosystem, it is customary to set the biological threshold of the anchovy considering El Niño event, but without explicitly considering the interactions with other species. Our analysis stresses the point that thresholds have to be designed globally to guarantee sustainability for the whole ecosystem.

NUMERICAL APPLICATION TO THE HAKE–ANCHOVY COUPLE IN THE PERUVIAN UPWELLING ECOSYSTEM (1971–1981)

We provide a viability analysis of the hake–anchovy Peruvian fisheries between the years 1971 and 1981. For this, we shall consider a discrete-time Lotka–Volterra model for the couple anchovy (prey y) and hake (predator z), then provide an explicit description of viable states. The emphasis is not on developing a biological model, but rather on decision-making using such a model.

Viable states and ecosystem sustainable yields for a Lotka–Volterra system

Consider the following discrete–time Lotka–Volterra system of equations with density–dependence in the prey

$$\begin{cases} y(t+1) &= Ry(t) - \frac{R}{\kappa}y^2(t) - \alpha y(t)z(t) - v(t)y(t), \\ z(t+1) &= Lz(t) + \beta y(t)z(t) - w(t)z(t), \end{cases} \quad (11)$$

where $R > 1$, $0 < L < 1$, $\alpha > 0$, $\beta > 0$ and $\kappa = \frac{R}{R-1}K$, with $K > 0$ the carrying capacity for prey.

In the dynamics (1), we identify $R_y(y, z, v) = R - \frac{R}{\kappa}y - \alpha z - v$ and $R_z(y, w) = L + \beta y - w$. One can prove the following corollary.

Corollary 3 *Consider the Lotka–Volterra predator–prey model (11). Whenever*

$$B_y^b \geq \frac{1-L}{\beta} \text{ and } B_z^b \leq \frac{R-1}{\alpha} - \frac{R(1-L)}{\alpha\beta\kappa}, \quad (12)$$

for any minimal catch thresholds C_y^b and C_z^b such that

$$C_y^b \leq C_y^{b,*} := B_y^b \left(R - \frac{R}{\kappa}B_y^b - \alpha B_z^b - 1 \right) \quad (13a)$$

$$C_z^b \leq C_z^{b,*} := B_z^b (L + \beta B_y^b - 1), \quad (13b)$$

viable states are given by

$$y \geq B_y^b, \quad B_z^b \leq z \leq \frac{1}{\alpha} \left[R \left(\frac{\kappa - y}{\kappa} \right) - \frac{C_y^b + B_y^b}{y} \right]. \quad (14)$$

By Proposition 2, we obtain that, for any initial point (y_0, z_0) such that

$$y_0 \geq B_y^b, \quad z_0 \geq B_z^b, \quad y_0 \left(R - \frac{R}{\kappa}y_0 - \alpha z_0 \right) \geq B_y^b, \quad (15)$$

the ecosystem sustainable yields are given by

$$\begin{cases} C_y^{b,*}(y_0, z_0) &= \min \left\{ C_y^{b,*}, y_0 \left(R - \frac{R}{\kappa}y_0 - \alpha z_0 \right) - B_y^b \right\} \\ C_z^{b,*}(y_0, z_0) &= C_z^{b,*}. \end{cases} \quad (16)$$

In other words, if viably managed, the ecosystem could produce at least $C_y^{b,*}(y_0, z_0)$ and $C_z^{b,*}$, while respecting biological thresholds B_y^b and B_z^b .

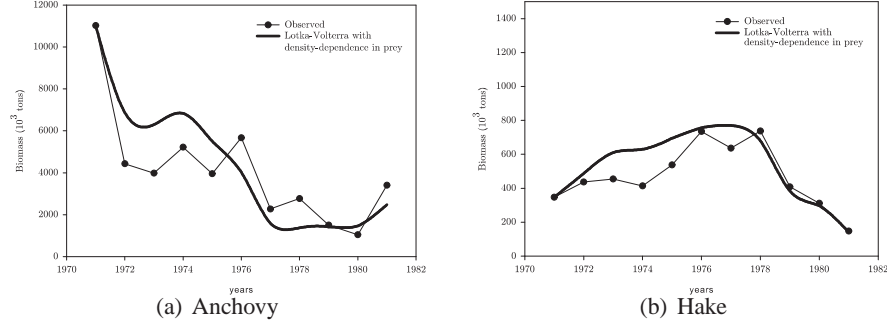


Figure 1: Comparison of observed and simulated biomasses of anchovy and hake using a Lotka–Volterra model with density-dependence in the prey (1971–1981). Model parameters are $R = 2.25 \text{ year}^{-1}$, $L = 0.945 \text{ year}^{-1}$, $\kappa = 67\,113 \times 10^3 \text{ t}$ ($K = 37\,285 \times 10^3 \text{ t}$), $\alpha = 1.220 \times 10^{-6} \text{ t}^{-1}$, $\beta = 4.845 \times 10^{-8} \text{ t}^{-1}$.

A viability analysis of the hake–anchovy Peruvian fisheries between the years 1971 and 1981

The period between the years 1971 and 1981 has been chosen because the competition between the fishery and hake was reduced due to low anchovy catches, and because of the absence of strong El Niño events. We have 11 couples of biomasses, and the same for catches. The 5 parameters of the model are estimated minimizing a weighted residual squares sum function using a conjugate gradient method, with central derivatives. Estimated parameters and comparisons of observed and simulated biomasses are shown in Figure 1.

We consider values of $B_y^b = 7\,000\,000 \text{ t}$ (anchovy) and $B_z^b = 200\,000 \text{ t}$ (hake) for minimal biomass thresholds and values of $C_y^b = 2\,000\,000 \text{ t}$ and $C_z^b = 5\,000 \text{ t}$ for minimal catch thresholds [17, 18]. Conditions (12) in Corollary 3 are satisfied with these values and the expressions in (13a)–(13b) give:

$$\begin{cases} C_y^b = 2\,000\,000 \text{ t} \leq C_y^{b,*} = 5\,399\,000 \text{ t} \\ C_z^b = 5\,000 \text{ t} \leq C_z^{b,*} = 56\,800 \text{ t} . \end{cases} \quad (17)$$

The viability kernel is depicted in Figure 2. The star point within the viability kernel is the initial point: thus, based upon this model, the fishery could have been managed – with appropriate viable controls – to produce catches above $C_y^b = 2\,000\,000 \text{ t}$ and $C_z^b = 5\,000 \text{ t}$, while ensuring biological conservation.

What is more, due to (16), catches up to $C_y^{b,*}(y_0, z_0) = 5\,399\,000 \text{ t}$ and $C_z^{b,*}(y_0, z_0) = 56\,800 \text{ t}$ were theoretically achievable in a sustainable way starting from year 1971.

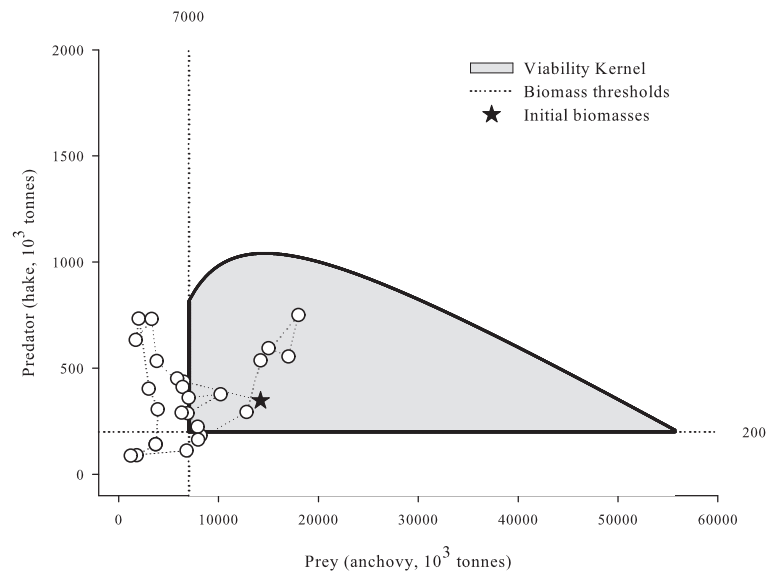


Figure 2: Viability kernel (in grey) for a Lotka–Volterra model with density-dependence in the prey (1971–1981) in the predator–prey phase space (with $B_y^b = 7\,000\,000$ t, $B_z^b = 200\,000$ t, $C_y^b = 2\,000\,000$ t, $C_z^b = 5\,000$ t). The star point within the viability kernel is the initial point (1971).

However, the catches of year 1971 were very high and the biomasses trajectories went outside of the biological thresholds for fourteen years. Using viable quotas may have prevented anchovy collapse, and possibly improved the state of hake, which has currently declined [19, 20]. Current ecosystem sustainable yields computation based on a viability analysis is beyond the scope of this paper, because it ought to rely on new models reflecting the new ecosystem functioning.

CONCLUSION

We have defined the notion of sustainable yields for ecosystem, and provided ways to compute them by means of a viability analysis of generic ecosystem models with harvesting. Our analysis stresses the point that thresholds have to be designed globally to guarantee sustainability for the whole ecosystem.

Our results have then been applied to a Lotka–Volterra model using the anchovy–hake couple in the Peruvian upwelling ecosystem. Despite simplicity³ of the models considered, our approach has provided reasonable figures and new insights: it may be a mean of designing sustainable yields from an ecosystem point of view.

Thus, control and viability theory methods have allowed us to introduce ecosystem considerations, such as multispecies and multiobjectives, and have contributed to integrate the long term dynamics, which is generally not considered in conventional fishery management.

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³In addition to hake, there are other important predators of anchovy in the Peruvian upwelling ecosystem, such as mackerel and horse mackerel, seabirds and pinnipeds, which were not considered. Also, anchovy has been an important prey of hake, but other prey species have been found in the opportunistic diet of hake [21]

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