The Application of Internal-Wave Dissipation Models to a Region of Strong Mixing

Hemantha Wijesekera, Laurie Padman, Tom Dillon, Murray Levine, and Clayton Paulson
College of Oceanography, Oregon State University, Corvallis, Oregon

Robert Pinkel
Scripps Institution of Oceanography, La Jolla, California

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ABSTRACT

Several models now exist for predicting the dissipation rate of turbulent kinetic energy, $\alpha$, in the oceanic thermocline as a function of the large-scale properties of the internal gravity wave field. These models are based on the transfer of energy toward smaller vertical scales by wave-wave interactions, and their predictions are typically evaluated for a canonical internal wave field as described by Garrett and Munk. Much of the total oceanic dissipation may occur, however, in regions where the wave field deviates in some way from the canonical form. In this paper simultaneous measurements of the internal wave field and $\alpha$ from a drifting ice camp in the eastern Arctic Ocean are used to evaluate the efficacy of existing models in a region with an anomalous wave field and energetic mixing. By explicitly retaining the vertical wavenumber bandwidth parameter, $\beta_\alpha$, models can still provide reasonable estimates of the dissipation rate. The amount of data required to estimate $\beta_\alpha$ is, however, substantially greater than for cases where the canonical vertical wavenumber spectrum can be assumed.

1. Introduction

Microstructure measurements over the last decade indicate that mixing in the stratified ocean is closely related to the energy density (and spectral shape) of the internal gravity wave field. A large number of experiments in the ocean, atmosphere, and laboratory have identified Kelvin–Helmholtz instability (where the Richardson number becomes less than 0.25; Miles 1961) and convective instability (where parcel velocities exceed the phase speed of the wave motion; Orlanski and Bryan 1969), as likely mechanisms by which internal wave energy is dissipated. Oceanic mixing typically occurs, however, on time and space scales that are difficult to resolve well. The dye streak photography of Woods (1968) is one of the few studies where the inception of oceanic mixing has been observed. Nevertheless, because of the importance of small-scale turbulence to the evolution of the larger-scale oceanic hydrography and circulation, attempts have been made to quantitatively relate mixing rates to more easily observable variables. The common assumption to the most popular of these models is that nonlinear wave–wave interactions transfer energy and momentum from larger scales to smaller scales, and hence toward turbulence via irreversible wave instability mechanisms. These interactions can be classified as either resonant or nonresonant. A wave dissipation model described by McComas and Müller (1981; hereafter MM) computes the wave energy flux toward higher vertical wavenumbers using resonant interaction theory, whereas a model described by Henyey et al. (1986, hereafter HWF) computes the energy flux using an eikonal approach to study nonlinear, scale-separated interactions. Both studies predict dissipation rates based on the “universal” internal wave spectrum described by Garrett and Munk (1975; hereafter GM). Other theories based on strong interactions with buoyant turbulence, and direct numerical modeling techniques are not yet at a stage to be directly applicable to the oceanic internal wave field (Müller et al. 1986). Another approach discussed by Munk (1981) and Desaubies and Smith (1982) suggests that dissipation rates might be modeled using the statistics of internal wave Richardson number in a defined internal wave field.

Gregg (1989; hereafter G89) demonstrated that, at least in the midlatitude, deep-ocean pycnocline, mean dissipation rates correlate quite well with internal wave shear at vertical scales greater than 10 m. Gregg commented that the observed relationship between dissipation rate and wave shear agreed with the derivations of MM and HWF, and suggested that a first-order understanding of the dynamical link between waves and turbulence had therefore been achieved. These claims have already inspired significant controversy (for ex-
ample, see Gargett 1990). Nevertheless, if Gregg’s premise is even approximately true, the significant oceanographic problem of estimating vertical viscosities and diffusivities for large-scale modeling applications shifts from adequately sampling the processes responsible for mixing, to defining the global internal wave climate.

An intermediate but fundamental goal must also be to determine the range of validity of existing models. For example, it has been suggested that most oceanic mixing takes place near basin boundaries or over rough topography, with subsequent isopycnal advection accounting for most of the apparent diapycnal diffusion in the interior (Munk 1966). Observations suggest that the wave field near the boundary is frequently significantly different from GM. Models must therefore be sufficiently flexible to account for this deviation. A probable outcome is that a more complex description of the wave field would be required in such regions to attain a model accuracy comparable to that which is presently suggested for GM wave fields (about a factor of 2; G89).

A valuable dataset for exploring the relationship between internal waves and dissipation was obtained during the Coordinated Eastern Arctic Experiment (CEAREX). Padman and Dillon (1991) found that mixing was very energetic in this region, and they noted that both diurnal tidal motion and high-frequency wave packets were present when the mixing rate was greatest. The diurnal tide is described in more detail by Padman et al. (1992), and the wave packets are discussed by Czipott et al. (1991). The nonlinear wave packets appear to be related to the cross-slope diurnal tidal currents; however, the generation mechanism is not yet completely understood. Each of these processes is inconsistent with the hypotheses of the GM model. The CEAREX measurements are therefore well suited to testing the viability of models in non-GM environments, while also allowing us to explore possible modifications of empirical models to extend their application to such anomalous but potentially important mixing regions.

In this paper we review the dissipation models and scaling arguments (section 2), describe the CEAREX observations (section 3), then compare the model predictions and data (section 4). In section 5 we consider some of the practical difficulties in estimating the parameters for each of these models. The implications of this analysis are discussed in section 6.

### 2. Wave dissipation models

In this section we follow the approach used by G89 to describe the existing wave dissipation models that appear at present to offer the most hope for practical estimation of dissipation rates from relatively simple measurement programs.

#### a. The model of McComas and Müller (1981)

Based on the assumption of weak resonant interactions among internal waves, MM proposed a model for the dynamic balance of the oceanic internal wave field. McComas and Müller assumed that wave energy is generated at low vertical wavenumbers and dissipated at high vertical wavenumbers. They demonstrated the existence of an inertial range between the energy-containing scale and the wave dissipation scale. In this range, a constant flux of energy (independent of vertical wavenumber $\beta$) is transferred from generation to dissipation scales by weak resonant interactions. At frequencies close to the inertial frequency, $f$, the flux is provided by the parametric subharmonic instability (PSI) mechanism, in which a low-wavenumber wave with frequency $2f < \omega < 4f$ decays into two waves of half the original frequency ($f < \omega < 2f$) with higher vertical wavenumbers. At high frequencies the flux is provided by the induced diffusion (ID) mechanism, that is, scattering of a high-frequency, high-wavenumber wave by a low-frequency, low-wavenumber wave into another high-frequency, high-wavenumber wave. The energy fluxes under PSI and ID mechanisms are given by

$$Q_{\text{PSI}} = E \tau_{\text{PSI}}^{-1}$$  \hspace{1cm} (1a)

and

$$Q_{\text{ID}} = E \tau_{\text{ID}}^{-1},$$  \hspace{1cm} (1b)

where $E$ is the canonical, dimensional energy density of the GM wave field, and $\tau_{\text{PSI}}$ and $\tau_{\text{ID}}$ are the characteristic time scales of PSI and ID mechanisms, respectively. For a GM internal wave spectrum, these time scales are given approximately by

$$\tau_{\text{PSI}} = \frac{32\sqrt{10}}{27\pi} \beta^{-2} f^{-1} N^2 E^{-1}$$  \hspace{1cm} (2a)

and

$$\tau_{\text{ID}} = \beta^{-2} f^{-1} N^2 E^{-1},$$  \hspace{1cm} (2b)

where $\beta$ and $N$ are the vertical wavenumber bandwidth and the buoyancy frequency, respectively. In order to achieve the steady-state solution, the total internal wave energy flux through the spectrum should balance the small-scale dissipation rate. Following G89, and assuming that the total flux is given approximately by the sum of fluxes for the two mechanisms, we obtain

$$\epsilon_{\text{MM}} = Q_{\text{ID}} + Q_{\text{PSI}}$$

$$= \left( \frac{27\pi}{32\sqrt{10}} + 1 \right) \beta^{-2} f^{-1} N^{-2} E^2.$$  \hspace{1cm} (3)

Equation (3) can be rewritten, using the following parameterizations from the GM model (Munk 1981):

$$B_{\ast} = j_{\ast} \pi \beta^{-1} N N_0^{-1}$$  \hspace{1cm} (4)
and

$$E = b^2N_0^2E_{GM},$$

(5)

where $N_0$ is the reference buoyancy frequency (3 cph), $b$ is the scale depth of the thermocline (1300 m), $E$ is the dimensional energy density in an $N$-scaled, canonical GM wave field with dimensionless energy $E_{GM}$, and $j_*$ is the vertical-mode scale number. Then

$$\epsilon_{MM} = \left( \frac{27\pi}{32\sqrt{10}} + 1 \right) \pi^2 b^{-2}N_0^{-2}j_*^{-2}E^2$$

(6a)

or

$$\epsilon_{MM} = \left( \frac{27\pi}{32\sqrt{10}} + 1 \right) \pi^2 b^{-2}N_0^{-2}j_*^{-2}E_{GM}^2.$$  

(6b)

A summary of the GM midlatitude parameters is given in Table A1 [see MM and Müller et al. (1986) for further details]. Predictions of dissipation rate in a wave field with measured energy density, $E_{meas}$, will be made by replacing $E$ by $E_{meas}$ in (6a).

b. The model of Henyey et al. (1986)

Henyey et al. (1986) formulated an analytical model for wave dissipation using an eikonal approach, in which it was assumed that nonlinear energy transfer is dominated by scale-separated interactions, and that the large-scale background flow is unaffected by these interactions. In this model, wave action flux carries waves toward higher-frequency and higher vertical wavenumber and, finally, toward microscales via critical-layer processes. This is a completely different dynamical process than induced diffusion in the resonant interaction model (MM). In order to obtain quantitative estimates of the dissipation rate in an internal wave field, HWF assumed a GM shear spectrum that was white below a standard cutoff vertical wavenumber $\beta_c$ ($\approx (2\pi/10)$ m$^{-1}$). The spectrum was proportional to $\beta^{-1}$ from $\beta_c$ to $\beta_c$, where $\beta_c$ was regarded as the largest possible vertical wavenumber before wave breaking. Finally, it was assumed that a critical Richardson number, defined as

$$Ri_c = (3j_*bE_{GM}\beta_c)^{-1},$$

(7)

was constant at $\beta = \beta_c$. For this specific wave field, the dissipation rate predicted by the model of HWF is then

$$\epsilon_{HWF} = \left( 3\pi Ri_c^{-1/2}/f \right) \left[ 4\pi^{-1}j_*bNE_{GM}f \right] 2\left[ \frac{\beta_c}{\beta_c} \right]^2$$

$$\times \left[ 1 + \ln \left( \frac{\beta_c}{\beta_c} \right) \right]^{1/2} \left[ \frac{1 - r}{1 + r} \right]^{1/2} \cosh^{-1} \left( \frac{fN}{f} \right),$$

(8a)

where $r$ is the ratio between upscale (decreasing $\beta$) and downscale (increasing $\beta$) energy fluxes at $\beta_c$. Using values of $\beta_c/\beta_c = 2$, $Ri_c^{-1} = 0.5$, and $r = 0.4$ (see HWF for further details), (8a) reduces to

$$\epsilon_{HWF} = \left[ \frac{1.67}{\pi} \right] b^2N_0^2f \cosh^{-1} \left( \frac{N}{f} \right)^2 E_{GM}^2.$$  

(8b)

Using (5), (8b) can be expressed in terms of the dimensional energy $E$:

$$\epsilon_{HWF} = \left[ \frac{1.67}{\pi} \right] b^{-2}N_0^{-2}f \cosh^{-1} \left( \frac{N}{f} \right)^2 j_*^{-2}E^2.$$  

(8c)

HWF commented that some of the assumptions on which their analytical model was based were either clearly inaccurate or not justifiable. Nevertheless, they found good agreement between this model and predictions from a more complete eikonal Monte Carlo simulation on the same test wave field, and therefore proposed that (8) was a reasonable dissipation estimate for a GM wave field. As for the MM model, predictions of $\epsilon$ in a wave field with measured energy density $E_{meas}$ will be made by replacing $E$ by $E_{meas}$ in (8c).

c. Scaling of turbulent dissipation by Gregg (1989)

Gregg (1989) introduced a scaling formula for estimating $\epsilon$ in the midlatitude thermocline using measurements of the vertical shear of horizontal velocity. His analysis was based on five experiments involving concurrent measurements of $\epsilon$ and finite-differenced shear, $S_\delta = |\Delta U/\Delta z|$, where $U$ is the horizontal velocity vector measured with expendable current profilers (XCPs). Gregg concluded that $\epsilon$ depended on $N$, the local buoyancy frequency, and $\langle S_{10}^4 \rangle$, the fourth moment of shear at all vertical wavenumbers less than $(2\pi/10)$ m$^{-1}$,

$$\epsilon_{G89} = \frac{7 \times 10^{-10}}{N_0^2} \left( \frac{S_{10}^4}{S_{GM}^4} \right), \quad \text{[m}^2 \text{s}^{-3}].$$

(9)

Angle brackets denote some averaging process (e.g., time average or ensemble average of profiles). In (9), $S_{GM}^4$ is the fourth moment of shear from all wavenumbers less than $2\pi/10$ m$^{-1}$, evaluated for a GM wave field with the canonical energy density (Table A1) and the local buoyancy frequency. The value of $S_{GM}^4$ is obtained from the GM spectrum with the use of a continuous spectral representation of the vertical variability of horizontal currents: as Gargett (1990) noted, if the modal description of Munk (1981) is used, $S_{GM}^4$ is reduced by a factor of $(\pi/2)^2$. Gregg estimated $\langle S_{10}^4 \rangle$ from measurements of $S_\delta$ with $\Delta z = 10$ m by assuming that the total shear spectrum was flat to $\beta_c = (2\pi/10)$ m$^{-1}$, then proportional to $\beta^{-1}$ at higher wavenumbers (Gargett et al. 1981). With this assumption, $\langle S_{10}^4 \rangle = R^2 \langle S_\delta^4 \rangle$.

(10)

The coefficient $R^2$ corrects for shear that is not resolved by the finite differencing of velocity over a fixed vertical interval. For $\Delta z = 10$ m and using the above model
shear spectrum, $R = 2.11$ (Gregg and Sanford 1988). Gregg further assumed that

$$\frac{\langle S^2_{\text{GM}} \rangle}{S^2_{\text{GM}}} = \frac{E_{\text{GW}}}{E_{\text{GM}}},$$

where $E_{\text{GW}}$ is the dimensionless energy density, given in terms of the measured energy density $E_{\text{meas}}$ by

$$E_{\text{meas}} = b^2 N_0 N_0 E_{\text{GW}}.$$  

For GM wave fields in which the two orthogonal components of shear are each normally distributed with zero mean and the same variance, $\langle S^2_{\text{GM}} \rangle = 2 \langle S^2_{\text{GM}} \rangle$. That is, (9) and (11) imply that G89's dependence on the fourth moment of shear is equivalent to the $E^2$ dependence of the MM and HWF models.

The constant $7 \times 10^{-10} \text{ m}^2 \text{ s}^{-3}$ in (9) was obtained from a comparison of $\epsilon$ and $\langle S^2_{\text{GW}} \rangle$ in the PATCHEX experiment. Gregg commented that $\epsilon_{\text{GM}} \approx 2 \epsilon_{\text{HWF}}$ and $\epsilon_{\text{GM}} \approx \epsilon_{\text{MM}}/3$ for the diffusively stable midlatitude data which he considered, provided the GM canonical values of $j_0 = (53)$ and $b = (1300)$ m were used. As Gargett (1990) noted, however, Gregg's conversion of the MM and HWF models (Eqs. (6) and (8), respectively) to models written as functions of shear was incorrect because those models were based on the description of the vertical structure of the wave fields in terms of discrete modes. When the HWF and MM models are multiplied by $(\pi/2)^2 \approx 2.5$ to be compatible with G89's spectral description of the shear, $\epsilon_{\text{GW}} \approx \epsilon_{\text{HWF}}$, and $\epsilon_{\text{GM}} \approx \epsilon_{\text{MM}}/3$. The $f$ dependence of $\epsilon_{\text{HWF}}$ in (8b) was also investigated by Gregg, however, his test of $f$ scaling was inconclusive.

d. Scaling of turbulent dissipation by Gargett and Holloway (1984) and Gargett (1990)

Considering the steady-state kinetic energy equation without the classical separation of the velocity field into "turbulent" and "mean" (including internal waves) components, Gargett and Holloway (1984, hereafter GH) suggested that

$$\epsilon_{\text{GH}} \approx -u w \frac{\partial u}{\partial z} = C \left[ u^2 w^2 \left( \frac{\partial u}{\partial z} \right)^2 \right]^{1/2},$$

where $C$ is a nondimensional triple correlation coefficient, $u^2$ and $w^2$ are, respectively, the mean square horizontal and vertical velocities associated with the internal wave field, and $\langle (\partial u/\partial z)^2 \rangle$ is the integral wave shear variance up to that vertical wavenumber at which $\mathcal{R}i$ reaches $\mathcal{R}i_\ast$ (Eq. (7)). For example, for a GM wave field, at $\beta = \beta_\ast$, $\mathcal{R}i_\ast$ is approximately unity. Assuming that $C$ does not vary with $N$, GH predicted, using WKB scaling appropriate for single-Fourier components in which $u^2 \sim N$, $w^2 \sim N^{-1}$, and $\langle (\partial u/\partial z)^2 \rangle \sim N^2$,$$

\epsilon_{\text{GH}} \sim N^{1.0}.$$  

while for a Garrett–Munk (Munk 1981) internal wave spectrum in which $u^2 \sim N$, $w^2 \sim N^0$, and $\langle (\partial u/\partial z)^2 \rangle \sim N^2$,$$

\epsilon_{\text{GH}} \sim N^{1.5}.$$  

The prediction in (14b) was also based on the assumption that $E_{\text{GW}} = E_{\text{GM}}$ is constant. Relaxing this assumption, Gargett (1990, hereafter GA) argued that

$$\epsilon_{\text{GA}} \sim E_{\text{GW}} N^{1.5}.$$  

\[\epsilon_{\text{GA}} \sim E_{\text{GW}} N^{1.5}.\]

e. Mixing models based on Richardson number statistics

Other models exist that predict internal wave dissipation rate from the probability of wave breaking (Munk 1981; Desaubies and Smith 1982, hereafter DS). When applied to the present dataset, Munk's prediction was almost one order of magnitude greater than $\epsilon_{\text{meas}}$ at most depths. Since G89 found a similar result for the open-ocean cases that he studied, Munk's model is not considered further. Desaubies and Smith demonstrated that, while the mean Richardson number may be well-above critical [i.e., $\langle \mathcal{R}i \rangle \gt 0.25$], the variability in the internal wave field can occasionally drive the local Richardson number, $\mathcal{R}i$, to below the critical value. The probability that $\mathcal{R}i$ is less than 0.25 was derived within a framework of the GM model as a function only of the rms strain,

$$\lambda_{\text{rms}} = \langle (\partial \eta/\partial z)^2 \rangle^{1/2},$$

where $\eta$ is the vertical displacement of an isopycnal from its mean depth. Desaubies and Smith showed, using the Desaubies and Gregg (1981) analytic approximation to GM, that $\lambda_{\text{rms}}$ was given by

$$\lambda^2_{\text{rms}} = \frac{1}{3} S^2 N^{-2}.$$  

The total shear variance $S^2$ is just twice the single component shear variance ($\sigma^2$ in DS). Following the philosophy of G89, in which the dissipation rate was expressed in terms of variables on scales greater than 10 m, we evaluated the rms strain over a 10-m vertical differencing scale $\lambda_{10, \text{rms}}$. This value was obtained by first tracing isopycnal pairs with mean vertical separations of 10 m through the Rapid Sampling Vertical Profiler (RSVP) time series. For each profile and isopycnal pair we evaluated the local strain as

$$\lambda_{10} = \frac{\eta_i - \eta_{i+1}}{10},$$

where $\eta_i$ and $\eta_{i+1}$ are the vertical displacements, from their mean depths, of two isopycnals with a mean separation of 10 m. The rms 10-m strain is then simply the square root of the variance of $\lambda_{10}$, evaluated over the time and density (or depth) ranges of interest. For comparison with the other internal wave dissipation
models, we filtered $\lambda_{10}$ to exclude subinertial (such as diurnal) variability prior to evaluating $\lambda_{10, \text{rms}}$.

3. Observations

During CEAREX in March/April 1989, microscale dissipation rate, horizontal velocities, and isopycnal displacements were measured beneath a drifting ice camp near the Yermak Plateau in the eastern Arctic Ocean. Oceanographic variability along the camp's drift track, and the instrumentation used to measure the velocity microstructure and vertical hydrographic profiles was discussed in detail in Padman and Dillon (1991) and is summarized below.

a. Microscale dissipation rates

The microstructure observations were obtained with the Rapid Sampling Vertical Profiler (Caldwell et al. 1985). Approximately 1500 profiles were obtained over a typical depth range of 0 to 340 m, with about 2-3 profiles per hour. Energetic mixing events in the pycnocline, with maximum turbulent kinetic energy (TKE) dissipation rates ($\epsilon$) of about $10^{-6}$ m$^2$ s$^{-3}$, were observed near the plateau (Fig. 3c and Plate 1d in Padman and Dillon 1991). The measured, depth-averaged dissipation rate in the pycnocline, $\langle \epsilon_{\text{mean}} \rangle$, varied substantially during the 25 days ($90 < t < 114.3$) of microstructure measurements (Fig. 1a). The time dependence of pycnocline dissipation rate was calculated by averaging $\epsilon_{\text{mean}}$ between isopycnals having mean depths of 90 and 300 m. The smallest values of $\langle \epsilon_{\text{mean}} \rangle$ were found as the ice camp was drifting in the deep, and relatively featureless, Nansen Basin at the start of the experiment, while the largest values were found over the upper slope adjacent to the Yermak Plateau. Padman and Dillon also observed a significant increase in the amplitudes of high-frequency isopycnal displacements during $107 < t < 110$, when turbulence was most energetic and dissipation rates were largest. They proposed that the most energetic mixing events were associated with the presence of these internal wave packets (see also Czipott et al. 1991). In the present study, we divide the period from $t = 87.0$ to $t = 114.3$ into four different intervals, considering the intensity of $\langle \epsilon_{\text{mean}} \rangle$, the position of the ice camp relative to bottom topography (Fig. 1b), and the strength of the diurnal tidal currents (Padman et al. 1992). During the first period (Pd. 1: $87 < t < 93$), while the ice camp was drifting in deep water, the turbulent mixing was significantly weaker than during the other periods. During the third period (Pd. 3: $103 < t < 110$) when the ice camp was over the upper slope of the Yermak Plateau, the dissipation rate was significantly larger than in the second (Pd. 2: $96.8 < t < 103$) and fourth (Pd. 4: $110 < t < 114.3$) periods. We started the second period at $t = 96.8$ because velocity shear estimates were only available from this time.

b. The internal wave field

Horizontal velocity was measured with moored S-4 current meters at depths of 100, 150, 200, and 250 m throughout the experiment. A seven-element horizontal array was also deployed at 100 m to measure the horizontal coherence of internal waves and to determine the horizontal phase propagation velocity of specific wave packets (see Czipott et al. 1991). Measurements of currents at higher vertical resolution than was possible with the moorings were obtained from 307-kHz and 161-kHz acoustic Doppler current profilers (ADCPs). The 307-kHz unit was manufactured by RD Instruments, while the 161-kHz unit was produced by the Scripps Institution of Oceanography and utilized repeat-sequence coding for improved precision (Pinkel and Smith 1992). The combination of finite acoustic pulse length and return gate width in the ADCPs resulted in a trapezoidal filter being applied to the vertical profile of horizontal velocity. A totally independent estimate was obtained approximately every 16 m for the 307-kHz ADCP and 24 m for the 161-kHz ADCP (see Appendix for details). Since the filters are tapered, however, estimates were nearly independent every 12

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1 We have used decimal year-day for time $t$ in this paper. The integer part of the time is the day of the year since 1 January 1989, with 1 January as day 1. The time of day (UTC) is given as a fraction of the day.
m for the 307 kHz ADCP and 18 m for the 161 kHz ADCP. The two ADCPs were deployed at $t \approx 94.0$ (161 kHz) and $t \approx 96.8$ (307 kHz). No ADCP data is available during Pd. 1.

1) ENERGY DENSITY AND SHEAR VARIANCE

The dominant signal in isopycnal displacement over the plateau slope (Pds. 2, 3, and 4) is diurnal and is associated with the large-amplitude, cross-slope barotropic tidal currents (Padman and Dillon 1991; Padman et al. 1992). Sherman and Pinkel (1991) found that vertical advection of internal waves can strongly influence the shear spectrum observed at a fixed (Eulerian) sensor. These influences are likely to be even more significant where the vertical gradient of mean buoyancy frequency $\delta \langle N \rangle / \delta z$ is large. Sherman and Pinkel proposed that more useful information regarding the time and space scales of wave shear and strain could be obtained by tracking these parameters on isopycnals. The method is denoted “semi-Lagrangian,” since we simply track isopycnals vertically, rather than tracking particles in three dimensions. We have attempted where possible in the present analysis to estimate dissipation rates and wave field parameters in approximately isopycnal-following (semi-Lagrangian) coordinates. This is achieved by tracking specific isopycnals through the RSVP hydrographic data time series and monitoring dissipation rate, horizontal velocity, and vertical shear along these isopycnals. The method is imperfectly applied because 1) the RSVP sampling rate does not fully resolve the internal wave frequency spectrum, and 2) the RSVP data is collected from a site displaced several hundreds of meters horizontally from the ADCP. Nevertheless, in regions where most of the isopycnal displacement is due to tides with long vertical and horizontal scales relative to the free internal waves, we expect that semi-Lagrangian statistics will provide a more useful description of the internal wave field.

The semi-Lagrangian approach could not be applied to data from Pd. 1 since the hydrographic sampling rate was irregular and the only current measurements were from the vertical mooring with 50-m resolution. During this period, however, wave energy density in both the diurnal and free internal wave frequency bands was low. The validity of using Eulerian internal wave band velocity variance from Pd. 1 in comparisons with semi-Lagrangian variance is tested in Table 1 for Pds. 2, 3, and 4. We consider only the upper pycnocline, where the change in $N$ due to vertical advection is small. The Eulerian estimates were averaged between 100-m and 180-m depth, and the semi-Lagrangian estimates were averaged over isopycnals with mean depths between 100 and 180 m. The difference in velocity variance between the two methods was less than 20% for all three periods. Since the vertical advection during Pd. 1 was small at all depths, we expect that the Eulerian velocity variance in this period is a reasonable estimate of the semi-Lagrangian value. A vertical averaging interval of 30 m, centered around the depth of each current meter, was used to calculate the time-averaged dissipation rate and the buoyancy frequency from RSVP profiles in this period. For the latter periods when the ADCP data was available, dissipation rate, buoyancy frequency, energy density, and shear variance were all calculated along 14 isopycnals spaced approximately evenly in depth between densities of $\sigma_t = 27.5$ ($\sim 90$ m) and 27.91 ($\sim 300$ m). The mean vertical spacing is comparable to the decorrelation scale of the ADCP trapezoidal filters (see Appendix). Dissipation rate and buoyancy frequency were averaged vertically over $\pm 5$ m about the local isopycnal position, then time averaged for each period.

For determining energy density and shear variance, we first filtered the time series to exclude frequencies that are not described by the model physics. The excluded components (predominantly subinertial) may, however, contribute to the dissipation rate, either directly or through nonlinear interactions with the free internal gravity wave field. Figure 2 shows the total velocity spectra for the least energetic (Pd. 1) and most energetic (Pd. 3) periods. The GM canonical spectrum based on the local values of $f$ and $N$ is provided as a reference. For $\omega \gg f$, the wave field is reasonably well described by the $\omega^{-2}$ slope of the GM frequency spectrum, with the energy density in Pd. 1 being about one-

![Fig. 2. Total velocity spectra (clockwise + anticlockwise) at 150-m fixed depth for Pd. 1 (dashed line) and Pd. 3 (solid line). Velocity estimates were obtained from the S-4 current meter mooring. The GM canonical spectrum based on local $f$ and $N$ is superimposed for comparison. The midlatitude GM spectral level is lower by a factor of 2 compared with the GM spectrum based on local $f$. Note that the semidiurnal frequency (sd) and inertial frequency ($f$) are almost identical at 83°N. The diurnal frequency is indicated by d.](image-url)
half of the GM level, and about three times greater than GM in Pd. 3. Nevertheless, most of the total internal wave energy is provided by near-inertial waves, which in the present case for Pd. 3 are less amplified than the high-frequency continuum. Note that the semidiurnal tides are indistinguishable from near-inertial, free internal waves, however, the dominant tidal line, M2, is subinertial ($\omega_{M2} \approx 0.97f$) in the absence of significant background relative vorticity. Some fraction of these forced semidiurnal tidal motions are included in our estimates of velocity (and shear) variance, but their role in the wave energy flux toward dissipation scales is uncertain.

A different view of the frequency partition of velocity and shear is provided by area-preserving spectra (Fig. 3). Velocity variance is dominated by the diurnal tidal motion. Padman et al. (1992) showed that the diurnal tide was almost entirely barotropic, and is therefore not expected to contribute to internal wave dissipation via instability processes, which rely on the presence of vertical shear. The shear spectrum peaks near $f$ as expected, but there is significant energy both throughout the internal wave continuum and at subinertial frequencies.

With these spectra in mind, the internal wave energy density $E_{\text{meas}}$ was estimated as

$$E_{\text{meas}} = \frac{1}{2} [\langle \eta^2 \rangle N^2 + \langle U^2 \rangle + \langle V^2 \rangle],$$

(19a)

where

$$\langle \eta^2 \rangle = \int_{f}^{\omega_{\text{Nyq}}} \Phi_\eta(\omega) d\omega$$

(19b)

and

$$\langle U^2, V^2 \rangle = \int_{f}^{\omega_{\text{Nyq}}} [\Phi_u(\omega), \Phi_v(\omega)] d\omega.$$

(19c)

In (19), the power spectral density of RSPV isopycral displacements is $\Phi_\eta(\omega)$, while $\Phi_u(\omega)$ and $\Phi_v(\omega)$ are the power spectral densities of orthogonal, horizontal velocity components, $f \approx 1.45 \times 10^{-4}$ s$^{-1}$, and $\omega_{\text{Nyq}}$ is the Nyquist frequency (about 1 cycle per hour for RSPV data). The vertical velocity variance is negligible compared with the horizontal components and has therefore been ignored in (19a). The local dimensionless energy $E_{\text{IW}}$ was estimated from $E_{\text{meas}}$ using (12). For estimating $\epsilon_{\text{MM}}$ and $\epsilon_{\text{HWF}}$, $E_{\text{GM}}$ in Eqs. (6) and (8) was replaced with $E_{\text{IW}}$.

Fig. 3. Area-preserving frequency spectra of (a) horizontal velocity, and (b) vertical shear of horizontal velocity. Spectra were calculated along five isopycnals with mean depths between 110 and 170 m, where the mean buoyancy frequency was nearly constant (see Fig. 7), then depth averaged. Both velocity and vertical shear were obtained from the 307-kHz ADCP, with shear being estimated from finite-differenced velocity over a two-bin separation (8 m).

Estimates of $S_{10}$ are needed for the G89 model. Several difficulties arise in determining the most appropriate technique for estimating shear from the ADCPs in a form that is compatible with the analysis of XCP data by G89. First, the G89 XCP profiles include shear at all frequencies, not just in the free internal gravity wave band, $f < \omega < N$ (Fig. 3b). Nevertheless, the theoretical models (MM and HWF) assume that all the shear variance is described by a GM spectrum in which no subinertial shear is present. The coefficient, $R$ [see Eq. (10)], applied by G89 to correct 10-m finite-differenced shear variance to total shear variance is also based on the GM spectrum: if a significant fraction of the total shear is due to subinertial, or non-GM, shear, then it is incorrect to scale total measured shear variance by this value of $R$. It has also been suggested to us by M. Gregg (personal communication 1991) that, given the difficulty of using ADCP shear measurements in his model based on XCP shears, we should use Eulerian rather than semi-Lagrangian statistics in this comparison. Given the large diurnal displacements and large mean vertical gradients of $N$, $E_{\text{meas}}$, and $\epsilon_{\text{meas}}$ below 180 m (see section 3a), however, we believe it is most appropriate to use semi-Lagrangian statistics wherever possible. As demonstrated in Table 1, Eulerian and semi-Lagrangian internal wave band velocity variances are approximately the same in the region of nearly uniform $N$ (100–180 m; Table 1). Table 2 presents a similar comparison for the semi-Lagrangian and Eulerian second and fourth moments of shear, for both total shear and shear at frequencies between $f$ and $N$ only. The filtered ($f < \omega < N$) fourth moment of shear is typically about 50% of the total fourth moment in
both reference frames. With the G89 model and using a constant value of $R$, independent of $\omega$, this implies that the predicted dissipation rate based on total shear would be twice as large as for internal wave band shear only. If it is desired to retain the subinertial shear for comparison with the XCP data analyzed by G89, a more conservative approach would be to assume that the subinertial shear is fully resolved, that is, it is dominated by very low wavenumbers, unlike the white shear spectrum to $\beta_*$ as assumed for internal gravity waves.

With these difficulties in mind, we evaluated the variance of velocity shear by integrating the spectral estimates of measured ADCP velocity shear $\Phi_\nu(\omega)$ between $f$ and $N$, where the time series of shear was obtained from 8-m and 12-m finite-difference estimates for the 307-kHz ADCP and the 161-kHz ADCP, respectively. Then the total variance of internal wave shear at wavenumbers less than $(2\pi/10) \text{ m}^{-1}$ was approximated by

$$S_\nu^2 \approx R \int_f^N \Phi_\nu(\omega) d\omega,$$  \hspace{1cm} (20)

where $\Phi_\nu(\omega)$ and $\Phi_\nu(\omega)$ are the frequency spectral densities of the measured, finite-difference shears, $\Delta u/\Delta z$ and $\Delta v/\Delta z$. In G89, $R$ was used to correct for the attenuation of total shear variance due to applying a 10-m finite-difference filter to the XCP velocity profiles ($R = 2.11$ for the GM spectrum used by G89). In the present study, $R$ is calculated to correct both for the finite differencing ($\Delta z = 8$ m and 12 m for the 307-kHz and 161-kHz ADCP, respectively), and for the ADCP trapezoidal filter applied to the velocity estimates prior to finite differencing (see Appendix). For the ADCP and with the same assumed vertical wavenumber spectrum used by G89 (section 2c), $R = 2.54$ for the 307-kHz unit and 3.18 for the 161-kHz unit.

The G89 model is actually based on measurements of $S_\nu^2$; however, in the present experiment both components of shear are sufficiently close to being normally distributed that the approximation,

$$\langle S_\nu^2 \rangle = 2 \langle S_\nu^4 \rangle^2$$  \hspace{1cm} (21)

is valid to within 10% (Table 2).

2) INTERNAL WAVE COHERENCE

The HWF and MM models both require that the shape of the vertical wavenumber spectrum be parameterized by $\beta_*$, or $j_*$ using Eq. (4). These variables describe the spatial coherence of the wave field: increasing values of each implies that the spatial correlation scales in the wave field become smaller. Either horizontal or vertical coherences can be used for the free internal wave band since the two are related via the internal wave dispersion relationship. For a model spectrum, such as GM, measured coherence can be used to estimate $j_*$ (for example, see Levine 1990). A detailed review of the coherence structure during CEAREX is beyond the scope of the present paper;

<table>
<thead>
<tr>
<th>Period</th>
<th>$\langle u^2 \rangle_L$</th>
<th>$\langle v^2 \rangle_L$</th>
<th>$\langle u^2 \rangle_L + \langle v^2 \rangle_L$</th>
<th>$\langle u^2 \rangle_E$</th>
<th>$\langle v^2 \rangle_E$</th>
<th>$\langle u^2 \rangle_E + \langle v^2 \rangle_E$</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
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<td>39.0</td>
<td>71.0</td>
</tr>
<tr>
<td>4</td>
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<td>28.4</td>
<td>17.0</td>
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<td>29.0</td>
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</table>

Table 2. Comparison between semi-Langrangian and Eulerian estimates of $\langle S_\nu^2 \rangle$ and $\langle S_\nu^4 \rangle$, where

$$\langle S_\nu^2 \rangle = R \left[ \left( \frac{\Delta u}{\Delta z} \right)^2 + \left( \frac{\Delta v}{\Delta z} \right)^2 \right]$$

and

$$\langle S_\nu^4 \rangle = R \left[ \left( \frac{\Delta u}{\Delta z} \right)^2 + \left( \frac{\Delta v}{\Delta z} \right)^2 \right]^2.$$
however, we present herein a brief analysis as background to generating predictions from the MM and HWF models.

Two independent techniques were applied to the CEAREX data. The first involved comparing the horizontal coherences of horizontal velocity from the seven-element current meter array at 100-m depth, which is usually in the upper pycnocline but is occasionally in the lower surface mixed layer. Horizontal coherence is used to estimate the wavenumber bandwidth $\beta_s$ of the internal wave field (Levine 1990). Then, assuming a GM parameterization, $j_s$ can be determined as a function of $\beta_s$. In the present data, $j_s$ was a function of both the period of data being analyzed and wave frequency. The optimum $j_s$ values are given in Table 3; the range represents the variation with frequency for each period. The most coherent waves (smallest $j_s$) were found over the upper slope of the plateau when the total energy density was greatest (Pd. 3). Within this period, $j_s$ was also independent of frequency, unlike the other periods.

An independent check on the temporal variability of $j_s$ was obtained from the vertical coherence of isopycnal displacements. Figure 4 shows the measured correlations (for all variability in the frequency band $f < \omega < N$) for Pds. 2, 3, and 4 as functions of mean vertical separation of isopycnals. Provided the cutoff wavenumber $\beta_0$ of the GM wave field is much greater than $\beta_{s}$, Desaubies and Smith (1982) showed that the correlation function $\rho$ is given by

$$\rho(\delta) = e^{-\delta \beta_0},$$

where $\delta$ is the mean isopycnal separation. Estimates of $j_s$ from (22) and (4) range from $j_s \approx 4$ during Pd. 3 to $j_s \approx 10$ and $j_s \approx 8$ for Pd. 2 and Pd. 4, respectively. While these values are different from the estimates obtained from horizontal coherence, they indicate the same time dependence, with highest coherence (lowest $j_s$) during the most energetic period. As with horizontal currents, the coherence of isopycnal depth is also a function of frequency (Fig. 5). For example, the coherence of the 0.1-0.3 cycles per hour band during Pd. 3, is extremely high, and would be represented by $j_s \approx 2-3$.

Table 3. Variances of horizontal velocity ($f < \omega < N$) for clockwise ($\langle U_{\omega} \rangle$) and counterclockwise ($\langle U_{-\omega} \rangle$) spectra from moored S-4 current meters; total velocity variance ($\langle U_{\omega}^2 + U_{-\omega}^2 \rangle$); mean buoyancy frequency squared (N$^2$); isopycnal displacement variance (\$\eta^2\$); ratio of potential to horizontal kinetic energy [$PE/KE = N^2/\langle U_{\omega}^2 + U_{-\omega}^2 \rangle$]; and $j_s$ estimated from the horizontal coherence of horizontal velocity and the vertical correlation of isopycnal displacements.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>$\langle U_{\omega}^2 \rangle$</th>
<th>$\langle U_{-\omega}^2 \rangle$</th>
<th>$\langle U_{\omega}^2 + U_{-\omega}^2 \rangle$</th>
<th>$N^2$</th>
<th>$\eta^2$ (m$^2$)</th>
<th>PE/KE</th>
<th>$j_s$ (hor)</th>
<th>$j_s$ (vert)</th>
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</thead>
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<tr>
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<td>—</td>
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<tr>
<td>200</td>
<td>8.4</td>
<td>1.5</td>
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<td>—</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>4.6</td>
<td>1.5</td>
<td>6.1</td>
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<td>—</td>
<td>—</td>
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<td></td>
</tr>
<tr>
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<td></td>
<td>6</td>
<td>10</td>
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<tr>
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<td>6.9</td>
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<td></td>
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<tr>
<td>250</td>
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<td>13.3</td>
<td>33.6</td>
<td>0.08</td>
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<td>0.24</td>
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</tr>
<tr>
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<td>Period 4: $110 &lt; t &lt; 114.3$</td>
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<td>3-12</td>
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<td>0.08</td>
<td>39</td>
<td>0.06</td>
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</tbody>
</table>
two variables are not tightly coupled, preventing an accurate determination of the optimum coefficient, \( n^2 \), for a model based on \( \epsilon \propto S^2 \). Figure 6d compares \( N^2 \) as a function of the estimated variance of 10-m shear \( S^2 \). Lines of constant slope indicate different 10-m variance Richardson numbers which, in this dataset, are generally between 0.5 and 2. This value is consistent with both the GM canonical value (Munk 1981) and the value assumed by HWF.

The depth dependence of the parameters required by the various models is shown in Fig. 7. The average values are based on semi-Lagrangian estimates for Pds. 2, 3, and 4, plotted at the mean depth of each reference isopycnal. Each parameter is approximately constant in the upper pycnocline, where \( N \) is fairly uniform, then decays with depth as \( N \) decreases. We shall discuss these profiles in more detail in the following section.

4. Comparison between observations and models

Predicted and observed values of \( \epsilon \), computed as functions of depth for all four periods, are shown in Fig. 8. The nondimensionalized, measured wave energy density, \( E_{IW} \) [Eq. (12)], and a period-dependent \( j_a \) were used to compute MM [Eq. (6)] and HWF [Eq. (8)] dissipation rates. A value of \( j_a = 4 \) was used for Pd. 1 and Pd. 3, and \( j_a = 6 \) for Pd. 2 and Pd. 4. Each value of \( j_a \) is a compromise between estimates based on the horizontal coherence of velocity and the vertical coherence of isopycnal displacements (Table 3). The dissipation rate predicted by G89 was obtained from the estimated fourth moment of shear at wavenumbers less than \( (2\pi/10) \text{ m}^{-1} \). For Pds. 2, 3, and 4, \( \langle S^4 \rangle \) was obtained from measured \( S^4 \); for Pd. 1, shear was estimated from \( E_{\text{meas}} \) and the canonical GM values for \( j_a, b, \) and \( \beta_e \).

The time-averaged MM prediction \( \epsilon_{MM} \) is higher than \( \epsilon_{\text{meas}} \) by a factor of 2–5 during Pd. 2 and Pd. 4 (Figs. 8b and 8d) and is lower by a factor of 2–10 during Pd. 1 (Fig. 8a) (although note that \( \epsilon_{\text{meas}} \) is close to the noise floor of \( 10^{-9} \text{ m}^2 \text{s}^{-3} \) during this period). During the most energetic period, \( \epsilon_{MM} \) is very close to the observed values (Fig. 8c). In the last three periods, the depth dependence of \( \epsilon_{MM} \) is consistent with the measured profiles. As G89 noted, the MM and HWF models have essentially the same parameter dependencies and differ only by the constant and the \( \cosh^{-1}(N/f) \) term in the HWF model. The latter term is approximately constant within realistic ranges for \( N \) and \( f \), hence we find the same relationship that G89 found, that is, \( \epsilon_{MM} \approx \epsilon_{HWF} \). Compared with the measured dissipation rates, \( \epsilon_{HWF} \) is low by a factor of about 10 for Pd. 1 and by a factor of 2–8 for the other periods.

The Richardson number estimated from observations, \( R_{10} = N^2/S^2 \), Fig. 6d, is comparable to the value used in the HWF simplified analytical model, where \( R_i = 2.0 \) for a wavenumber shear spectrum integrated to \( \beta_e \). The validity of the other assumptions (such as
\( \beta_i / \beta_e = 2 \) and \( r = 0.4 \) used in the HWF model cannot, however, be determined from this data.

The G89 model based on internal wave band (\( f < \omega < N \)) shear underestimates the CEAREX thermocline observations by approximately a factor of 10 for Pd. 2 and Pd. 3. During Pd. 4, the G89 prediction is lower than \( \epsilon_{meas} \) by a factor of 10–30 in the upper 175 m while it is within a factor of 2 below 175-m depth. During Pd. 4, unlike other periods, the predicted vertical structure of \( \epsilon_{G89} \) is not comparable to the structure of \( \epsilon_{meas} \), even though \( \epsilon_{MM} \) and \( \epsilon_{HWF} \) follow the measured profile. The deep estimates in Pd. 4 are the points in Fig. 6a that have substantially more measured shear variance than is expected from the measured energy density.

The GH and GA scaling arguments are tested in Fig. 9 for all four periods. In each panel, the profile mean of each scaled property is unity. There is some suggestion in Pd. 1 that the \( N^{1.0} \) scaling (GH) is better than the \( N^{1.5} \) scaling; however, we caution that the dissipation rates in this period are near the noise floor. In the other periods there is no obvious preferred scaling, even when the dependence of \( E_{IW} \) is included (GA).

This ambiguity is due to the small relative changes of \( E_{meas} \) and \( N \) within each profile, and to the significant cross correlation between \( E_{meas} \) and \( N \) (see Fig. 7). Tests of the \( E_{IW} N^2 \) dependence of the HWF and MM models against the GA dependence on \( E_{IW} N^{1.5} \) (Fig. 10) are similarly inconclusive.

Finally, for completeness we consider the empirical strain-based model discussed by Padman et al. (1991). This model is a hybrid of the \( \langle S^2 \rangle \) dependence of G89 and the strain dependence of Desaubies and Smith (1982; DS). Using (17), we assume that the strain variance is proportional to the shear variance, that is, \( S^2 \propto \lambda_{rms}^2 \). Note, however, that the constant of proportionality depends on the shape of the wave-field frequency spectrum, since \( r_m (= S^2/\lambda_{rms}^2) \) is frequency dependent. For example, if most of the energy is near \( f_i \), where shear is large but strain is small, \( r_m \) is large. On the other hand, if higher frequencies are relatively energetic, \( r_m \) will be small. Figure 2 shows that the near-
The 10-m rms internal wave strain, \( \lambda_{10, \text{rms}} \), \( = \langle \lambda_{10}^2 \rangle^{1/2} \), was evaluated for each of the last three periods. For each period, three density ranges were considered, based on the profile of \( N \) in Fig. 7: the upper thermocline, constant-\( N \) region, 100 to 170 m (U); a transition region, 170 to 220 m (T); and the lower pycnocline, 220 to 270 m (L). Only the variation of strain in the internal wave frequency band, \( f < \omega < N \), was considered for compatibility with our analyses of other models. For the nine resulting variance estimates, \( \lambda_{10, \text{rms}} \) varied between 0.32 and 0.53, significantly higher than in the canonical GM ocean. Predicted dissipation rates based on (23) are compared with \( \langle \epsilon_{\text{meas}} \rangle \) for the same time and density ranges in Table 4. For comparison, the HWF, MM and G89 predictions are shown for the same averaging time and density ranges. The mean predictive ability of the strain-based model is very good, suggesting either that strain is itself dynamically relevant, or that \( \langle \lambda_{10}^2 \rangle \) is a good indicator of the total shear variance.

5. Difficulties in estimating model parameters

The G89 model requires that we obtain reliable estimates of both \( \langle \epsilon \rangle \) and \( \langle S_{10}^4 \rangle \). Gibson (1991; and related papers) has discussed the need for adequate statistics to provide a stable estimate of \( \langle \epsilon \rangle \), given the inherent intermittency of oceanic dissipation rates. In the present dataset, however, we found that the expected value of \( \langle \epsilon \rangle \) based on lognormal statistics for \( \epsilon \) was very close to the arithmetic mean for each period and depth range. The details of the distribution of dissipation rates will be discussed in a future manuscript.

Potentially greater errors arise from our method of obtaining \( \langle S_{10}^4 \rangle \) from ADCP profiles. The ADCPs provide much better temporal resolution of the velocity shear than the XCP profiles used by G89; however, the vertical resolution of the velocity profiles is significantly worse, resulting in a larger correction coefficient \( R \) [Eq. (10)]. For the HWF and MM models, the calculation of \( \langle E \rangle \) is relatively straightforward, however, the determination of \( j_\lambda \) (or \( \beta_\lambda \)) is not. As we have seen above, \( j_\lambda \) is a function of both wave frequency and time in the CEAREX region. Furthermore, esti-
mates of \( j_* \) from the horizontal coherence of horizontal velocity and the vertical coherence of isopycnal displacement agree only qualitatively (Table 3). We therefore address the difficulties in estimating both \( \langle S^{r}_{10} \rangle \) and \( j_* \) below.

**a. Evaluating \( \langle S^{r}_{10} \rangle \) for the G89 model**

As we found during our analysis of ADCP data, without prior knowledge of the wave spectrum it is impossible to determine the fourth moment of total

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**Table 4. Model predictions for the four periods.** The three depth ranges refer to isopycnals with mean depths as follows: (U) the upper thermocline (100 < \( z \) < 170); (T) the transition region (170 < \( z \) < 220); and (L) the lower thermocline (220 < \( z \) < 270) m (see Fig. 7). The 10-m rms strain \( \lambda_{10,\text{rms}} \) is evaluated after band passing the measured 10-m strain [Eq. (18)] between \( f \) and \( N \). The predictions of each model have been scaled by \( \langle \epsilon_{\text{meas}} \rangle \). The scaled MM and HWF models in Pd. 1 may be underestimated because \( \langle \epsilon_{\text{meas}} \rangle \) is near the noise floor of about \( 10^{-8} \) m$^2$s$^{-3}$ in this period. Mean values are arithmetic means of the nine bin averages for the last three time periods.

<table>
<thead>
<tr>
<th>Period</th>
<th>Depth range</th>
<th>( N/N_0 )</th>
<th>( \lambda_{10,\text{rms}} )</th>
<th>( \epsilon_{\text{MM}}/\langle \epsilon_{\text{meas}} \rangle )</th>
<th>( \epsilon_{\text{HWF}}/\langle \epsilon_{\text{meas}} \rangle )</th>
<th>( \epsilon_{\text{G89}}/\langle \epsilon_{\text{meas}} \rangle )</th>
<th>( \epsilon_{j}/\langle \epsilon_{\text{meas}} \rangle )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>1.3</td>
<td>—</td>
<td>0.55$^\dagger$</td>
<td>0.07$^\dagger$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>0.92</td>
<td>—</td>
<td>0.33$^\dagger$</td>
<td>0.04$^\dagger$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>L</td>
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<td>—</td>
<td>0.09$^\dagger$</td>
<td>0.01$^\dagger$</td>
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<td>—</td>
</tr>
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<td></td>
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<td>0.56</td>
<td>0.32</td>
<td>1.8</td>
<td>0.26</td>
<td>0.50</td>
<td>1.05</td>
</tr>
<tr>
<td>Mean</td>
<td>—</td>
<td>—</td>
<td>0.42</td>
<td>2.3</td>
<td>0.31</td>
<td>0.23</td>
<td>1.01</td>
</tr>
<tr>
<td>GM</td>
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<td>0.27</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

$^\dagger$ Values may be underestimated due to the low signal-to-noise ratio for \( \langle \epsilon_{\text{meas}} \rangle \) in Pd. 1.
(dynamically relevant) shear from profiles of filtered, finite-differenced shear. The problems arise in both frequency and vertical wavenumber space. As Fig. 3b indicates, a significant fraction of the total shear in the CAREX region is subinertial, including the M2 semi-diurnal tide ($\omega_{M2} \approx 0.97/\tau$). There are two obvious difficulties introduced by this subinertial shear. From the data analyst's perspective, the problem is that the transition from the fourth moment of finite-differenced shear, $\langle S^4_{\theta_0} \rangle$, to $\langle S^4_{\theta_0} \rangle$ requires that the fully resolved vertical wavenumber spectrum is known a priori. While this may be true for $f < \omega < N$ if there is indeed some universality of the oceanic internal gravity wave field as supposed by G89, it is not obvious how to scale the finite-differenced subinertial shear. Even within the free internal wave band, the fourth moment of total shear is only well represented by $\langle S^4_{\theta_0} \rangle$ if the shear spectrum rolls off rapidly near a wavenumber of $(2\pi/10) \text{ m}^{-1}$, independent of the wave field energy density (Gargett 1990). For example, Duda and Cox (1989) present evidence for an energy-dependent value of $\beta_c$. Indeed, as Gargett (1990) noted, the cutoff wavenumber ($\beta_c$) and dissipation scale wavenumber ($\beta_d$: HWF) must vary with $\nu_{w}$ if HWF's hypothesis of a constant critical Richardson number [Eq. (7)] is valid.

In the present study, we have used only the shear between $f$ and $N$, and a scaling coefficient, $R$, consistent with the Gargett et al. (1981) model vertical wavenumber spectrum as the best estimate of $\langle S^4_{\theta_0} \rangle$. The incorporation of the subinertial shear scaled by the same value of $R$ leads to dissipation rate predictions that are about twice as large as for internal wave band shear only (Table 2). We note that the frequency content of the shear from G89's XCP sets is at best very poorly known, so that G89's use of total shear and the value of $R$ appropriate to his measurement technique requires that the shear in his datasets is predominantly due to free internal gravity waves described by the GM spectrum.

A second difficulty is that the mechanisms by which energy is transported through the wave spectrum will be different if significant subinertial or non-GM shear is present. This problem is also relevant to the HWF and MM models, each of which has assumed a GM spectrum and specific resonant or nonresonant interaction mechanisms between different wavenumber-frequency pairs. It is not obvious, without further modeling of the observed wave spectrum, how this anomalous shear would affect the relationship between $\langle \epsilon \rangle$ and $\langle S^4_{\theta_0} \rangle$.

b. Estimating $j_{\ast}$ for the HWF and MM models

The GM model wave field has a vertical wavenumber dependence whose spectral shape is independent of frequency and is dependent only on the vertical wave number bandwidth parameter $\beta_{\ast}$ and the cutoff wavenumber $\beta_c$. The vertical-mode scale number $j_{\ast}$ is sim-
It is not known whether any of these, or similar, anomalous properties of the wave field were present in the regions analyzed by G89, although the PATCHEX wave field is statistically similar to GM when viewed in semi-Lagrangian coordinates (Sherman and Pinkel 1991). It is, however, likely that the deviation of the CEAREX wave field from the GM model is more pronounced than in the midlatitude, open-ocean datasets because in the present case we appear to be very close to a topographic source of internal waves.

6. Discussion

Gargett (1990; GA) has already commented on some of the perceived weaknesses of G89's analysis. In particular, GA notes that 1) the HWF model is very sensitive to the assumption of the scale at which the transition from waves to irreversible turbulence occurs, and 2) the relationship [Eq. (11)] used by G89 to relate his shear-based model to HWF is incorrect if \( \beta \) varies with energy density. Gargett's third conclusion, that the ranges of \( E_{IW} \) and \( N \) in G89's data are too small to distinguish between plausible scaling arguments, has been shown (Figs. 9 and 10) to be also true in the present dataset. The failure of the G89 model in the CEAREX study region is not surprising, however, given the anomalous features of the wave field (section 3b). Nevertheless, the present dataset remains consistent with G89's observation that the measured dissipation rates lie between the HWF and MM predictions, provided the local value of \( j \) is used and the variable wave field coherence is retained through the models' dependences on \( j \). Furthermore, when strain rather than shear is used in a scaling model [Eq. (23)], the predicted and measured dissipation rates are very close to each other (Table 4). We therefore suggest that, in spite of Gargett's (1990) concerns about G89's analysis, it remains a useful exercise to refine empirical models of internal wave dissipation rates while improved dynamical models are being developed.

The internal wave parameters required by the models that we have discussed are, in theory, obtainable from mooring data. If the goal is to obtain long-term averages of climate-relevant diapycnal fluxes, even fairly inaccurate model predictions may be preferable to extrapolating the measured mean dissipation rates from short-duration microstructure programs.

The uncertainties in the HWF and MM predictions due to the variability in \( j \) estimates prevent an unambiguous test between the mechanisms for wave-wave interactions proposed by each model. For Pd. 3, however, the estimated \( j \) from horizontal coherences is independent of wave frequency and is comparable with \( j \) obtained from isopycnal correlations. For this period, the MM model prediction is very close to \( \epsilon_{meas} \) (Fig. 8). Is this a fortuitous agreement, or is it reasonable to expect that the modeled mechanisms will be valid in this region, which is presumably near an internal wave source? If this is a plausible scenario, then high-frequency, low-wavenumber waves would transfer

![Fig. 9. The time-averaged vertical profile of dissipation rate, plotted in nondimensional form using the two GH scaling arguments [Eq. (14a), \( \epsilon \propto N^{1.5} \); open circles], and [Eq. (14b): \( \epsilon \propto N^{1.5} \); solid squares], and the GA scaling [Eq. (15); \( \epsilon \propto E_{IW} N^{1.5} \); solid circles], for (a) Pd. 1, (b) Pd. 2, (c) Pd. 3, and (d) Pd. 4. The profile average of each scaled quantity is unity, and the profiles are plotted on a logarithmic scale from 0.5 to 2.](image-url)
their energy toward low-frequency, high-wavenumber waves through the parametric subharmonic instability (PSI) and induced diffusion (ID) mechanisms. We have previously postulated (Padman et al. 1991) that significant sources of internal waves in the pycnocline are bottom-generated 6-hour tides and high-frequency wave packets. The PSI and ID mechanisms might therefore smooth the spectrum around these source frequencies and transfer energy toward the inertial frequency.

In contrast, G89 found that the open-ocean observations were much closer to the predictions of the HWF model, in which the energy flux is consistently toward higher frequencies and wavenumbers. It is therefore plausible that the energy flux through a "steady-state" wave field occurs via scale-separated interactions with the principal wave energy source near the inertial frequency, while the PSI and ID mechanisms are more effective near high-frequency wave sources. While this is obviously speculative, it suggests that the dissipation rate might lie anywhere between the HWF and MM predictions depending on the proximity to wave sources, and the relative importance of near-inertial and high-frequency wave generation.

Somewhat surprisingly, the most effective model tested by us was the ad hoc strain-based model [Eq. (23)], which was a hybrid of the DS analysis of Richardson number statistics and G89's scaling arguments [Eq. (9)]. The relationship [Eq. (17)] used to relate strain variance to shear variance is only valid for a GM spectrum: as we have shown, the CEAREX spectrum deviates significantly from GM. For example, while most shear occurs at frequencies near \( f \), the strain spectrum in the GM model peaks at higher frequencies, and in the CEAREX data is distributed over a broad range of frequencies (Fig. 7e of Padman et al. 1991). Since the wave strain is a measure of the variation of the buoyancy gradient, an increasing ratio of strain to shear variance implies that changes in \( N \) become more important to determining the Richardson number. Perhaps close to wave sources, the strain of high-frequency waves plays a larger role in creating critical (low) Richardson number events than it does in the open ocean, where near-inertial shear is usually implicated.

Assuming that the cutoff wavenumber, \( \beta_c \), is close to the value of \( (2\pi/10) \) m \(^{-1} \) assumed by DS and G89, the true rms strain [Eq. (16)] implied by these 10-m finite-differenced measurements is between 0.6 and 1, compared with \( \lambda_{rms} \approx 0.5 \) quoted by DS for a canonical GM wave field. The probability that Ri is less than 0.25 varies from 13\% for \( \lambda_{rms} \approx 0.60 \) to 40\% for \( \lambda_{rms} = 1.0 \) [Eq. (7) in DS]. Richardson number statistics are therefore very sensitive to \( \lambda_{rms} \). The numerical simulations of mixing rates by DS reflect this high sensitivity of Ri statistics to \( \lambda_{rms} \). In DS, \( \langle \epsilon \rangle \propto \lambda^n \), where \( n \approx 9 \), compared with \( n = 4 \) in our hybrid model. The limited range of \( \lambda_{10,rms} \) and the uncertainty in \( \beta_c \) prevents us from discriminating between these models. If, however, as Duda and Cox (1989) suggest, \( \beta_c \) increases with increasing \( E_{meas} \), then the true rms strain (\( \lambda_{rms} \)) will not change as rapidly as \( \lambda_{10,rms} \). The probability of shear instability [expressed as \( Pr(Ri < 0.25) \)] will therefore be smaller than expected as \( \lambda_{10,rms} \) increases, suggesting a lower value of the exponent, \( n \), than the DS simulations indicate.
7. Summary

In this paper we have explored the viability of using internal wave dissipation models to predict the turbulent kinetic energy dissipation rate in a region where the wave field deviates significantly from the GM assumptions on which the models are based. The principal conclusions are as follows.

1) Although the measured wave energy and estimated shear variance are comparable to midlatitude ocean Garrett–Munk values, \( \epsilon_{\text{meas}} \) is an order of magnitude higher than in the open ocean. This implies that the G89 model, which is based only on the measured fourth moment of shear and mean buoyancy frequency, underpredicts the dissipation rate by a factor of about 10.

2) The measured dissipation rate in the present study usually lies between the predictions of HWF and MM, when the value of \( j_a \) used in these models is allowed to vary according to measured spatial coherences in the wave field. Since this is consistent with G89’s midocean observations, it suggests that an approximate, empirical model of dissipation based on relatively simple measurements of the wave field is still viable even in regions with non-GM wave properties. The data requirement for measuring \( j_a \) is, however, much greater than was proposed by G89 for estimating \( \left< S_{10}^4 \right> \).

3) A modified G89 model based on internal wave strain rather than shear gives the correct magnitude for mean dissipation rates. This may reflect the increased importance in this region of wave strain compared to shear in creating wave instabilities. It may, however, simply indicate that the strain, which is obtained from high-resolution hydrographic profiles, is better resolved than the shear estimated from ADCP velocity profiles and an assumed vertical wavenumber spectrum.

4) Without some a priori understanding of the total wave spectrum, there are significant difficulties in obtaining the fourth moment of 10-m shear, \( \left< S_{10}^4 \right> \), required for the G89 model. Frequency dependence of the vertical wavenumber spectrum, including \( j_a \), precludes the scaling of the fourth moment of measured, finite-differenced shear to \( \left< S_{10}^4 \right> \) by the coefficient \( R \) based on the measurement technique and the GM spectrum. Furthermore, if the cutoff wavenumber (\( \beta_c \)) is not constant, then it is not clear that \( \left< S_{10}^4 \right> \) is actually the dynamically relevant quantity to use.

5) The small relative ranges of \( N \) and \( E_{\text{meas}} \), and the correlation between these variables, prevents us from discriminating between the parameter dependence of the HWF and MM models and the GH and GA scaling arguments.

Further study of the dissipation mechanisms for internal waves is required before models of dissipation rate based on low-order internal wave statistics can be applied successfully in all oceanic regimes. The strong dependence of \( \epsilon \) on rms strain in the DS stochastic model, and on rms shear in the MM and HWF models, indicates that extensive measurements of the internal wave field wave energy and coherence structure are required in order for these models to be applied successfully to wave fields that deviate significantly from GM. The GH and GA scaling arguments predict simple scaling dependencies of \( \epsilon \) on \( N \) and wave energy density, but do not predict the absolute magnitude of dissipation rates. It is probable that in regions close to internal wave sources such as the Yermak Plateau, wave anisotropy and higher-order internal wave statistics need to be considered for effective modeling of the dissipation rate, eddy viscosity, diffusivity, and associated diapycnal fluxes. Nevertheless, such local mixing “hot spots” might dominate the average diapycnal exchange processes in the world’s oceans and therefore require further theoretical and observational study.

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APPENDIX

ADCP Estimation of Shear Variance at Scales Greater than 10 m, \( S_{10}^4 \)

Estimates of vertical shear of horizontal velocity were obtained with an Acoustic Doppler Current Profiler (ADCP). The ADCP consists of four beams that point 60° downward from the horizontal (Fig. A1). The 307-kHz ADCP transmits a 20.4 ms pulse of acoustic energy while the pulse length for the 161-kHz unit is 30 ms. The Doppler-shifted frequency between transmitted and reflected pulses determines the velocity of the water relative to the ADCP. For the 307-kHz unit, the 6.2-ms range gate for the returned signal produces a vertical profile of velocity at 4-m intervals to approximately 260 m. Reliable velocity estimates were consistently

![Fig. A1](attachment:image.png)
Table A1. Parameters used in the Garrett and Munk (1975) internal wave model.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Ω</em></td>
<td>$7.3 \times 10^{-3} \text{s}^{-1}$</td>
</tr>
<tr>
<td><em>f</em></td>
<td>$2\Omega \sin(\text{latitude})$</td>
</tr>
<tr>
<td><em>N₀</em></td>
<td>$5.2 \times 10^{-3} \text{s}^{-1}$</td>
</tr>
<tr>
<td><em>b</em></td>
<td>1300 m</td>
</tr>
<tr>
<td><em>β₂</em></td>
<td>0.6 m⁻¹</td>
</tr>
<tr>
<td><em>J₀</em></td>
<td>3</td>
</tr>
<tr>
<td><em>E₀</em></td>
<td>$6.3 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

obtained above 225 m. Good velocity estimates with the 161-kHz unit were obtained above 300 m. The ADCP velocity estimates are effectively filtered by a trapezoidal filter because of the finite length of the pulse and the range gating (Fig. A1). A trapezoidal filter was derived as a function vertical wavenumber ($\beta$ m⁻¹; in order to estimate the total velocity and shear variances). The magnitude of the resulting ADCP trapezoidal filter is

$$F_T(\beta) = \frac{1}{2(\beta d)^2} [ (1 - \cos \beta d - \cos 2\beta d + \cos 3\beta d)^2 + (\sin \beta d + \sin 2\beta d - \sin 3\beta d)^2 ]^{1/2},$$

where $d$ is the bin size which is 4 m for the 307-kHz ADCP and 6 m for the 161-kHz ADCP.

The velocity shear squared over successive 10-m finite-difference intervals is defined by $S_{10}^2 = (\Delta u/\Delta z)^2 + (\Delta v/\Delta z)^2$. The first-difference filter $F_{FD}(\beta) = (\sin \beta d)^2/(\Delta z)^2$ falls below the transfer function of the differentiating filter $F_{TD}(\beta) = \beta^2$ (Gregg and Sanford 1988). Therefore, the attenuation of $S_{10}^2$ relative to the GM shear spectrum ($\Phi_{GM}$), accounting for both finite differencing and trapezoidal filtering is

$$R^{-1} = \int_{0}^{\beta_\mu} \Phi_{GM}(\beta)(\sin \beta d)^2/(\Delta z)^2 F_{TD} \, d\beta,$$

where $\Phi_{GM}$ is the GM shear spectrum: $\Phi_{GM} = C\beta^0$ for $\beta < \beta_c$; $\Phi_{GM} = C(\beta_c/\beta)^2$ for $\beta > \beta_c$, and $C$ is a constant (Gargett et al. 1981; Gregg 1989). The integration was continued to $\beta_\mu$, in order to include the contribution of high wavenumber to $S_{10}^2$. The attenuation coefficients for $\Delta z = 8$ m and $\Delta z = 12$ m with $\beta_\mu = 4\pi$ m⁻¹ are 2.54 and 3.18, respectively. The same coefficient for the finite-difference attenuation only, with $\Delta z = 10$ m and $\beta_\mu = 4\pi$ m⁻¹ is 2.11 (Gregg and Sanford 1988).

REFERENCES


Gargett, A. E., 1990: Do we really know how to scale the turbulent kinetic energy dissipation rate $\varepsilon$ due to breaking of oceanic internal waves? *J. Geophys. Res.*, 95, 15971–15974.


