AN ABSTRACT OF THE THESIS OF

Chiang Kao for the degree of Doctor of Philosophy
in Forest Management presented on August 1, 1979

Title: A Study of Optimal Timing and Intensity of Silvicultural Practices--Commercial and Precommercial Thinning, Fertilization and Regeneration Effort

Abstract approved: [Signature redacted for privacy.]

Dr. J. Douglas Brodie

One traditional problem in forest management is to find the optimal stand level management regime. Four important silvicultural practices including precommercial thinning, commercial thinning, fertilization and regeneration harvest are considered jointly in this study. The partial analyses, i.e. considering some of the silvicultural practices, are also discussed.

The inability to account for diameter acceleration in the two-dimensional dynamic programming technique is overcome by using a three-dimensional dynamic programming network with biometric relationships from DFIT. The continuous growth is fitted into a discrete dynamic programming network by using the "neighborhood" concept. The descriptors used are stand age, number of trees and basal area. The effect of the size of the state space of dynamic programming is discussed and a basal area interval between
four to 20 square feet is suggested when the tree interval used is 15.

Commercial thinning is considered every ten years and captures anticipated merchantable mortality. Precommercial thinning is considered at age ten. Different intensities of precommercial thinning can be considered jointly with other silvicultural practices. Three levels of fertilization, i.e. 400 pounds, 200 pounds and zero pounds of nitrogen per acre, are applied every ten years after commercial thinning. An extra dimension representing different levels of fertilization is eliminated by computations and using the neighborhood concept. The forward recursive relation of dynamic programming finds the best management regime for different rotations as the solution progresses.

Precommercial thinning accelerates diameter growth and will affect later commercial thinning entries. Commercial thinning lengthens rotation and fertilization increases site capacity and raises optimal stocking level.

The solution technique developed also finds the optimal management for different initial stand conditions. Plantation is solved by assuming that it is equivalent to a stand precommercially thinned at age two.

The impact of individual silvicultural practices and their interactive effects are derived. Under the revenue and cost assumptions used, it is found that fertilization has the highest economic impact, commercial thinning is the...
second and precommercial thinning is the last, when silvicultural practices are considered individually. The highest total effect of two silvicultural practices is precommercial thinning and commercial thinning. Commercial thinning and fertilization is the second and precommercial thinning and fertilization is the last. Precommercial thinning and commercial thinning has the highest interactive effect, commercial thinning and fertilization is the second and precommercial thinning and fertilization is the last which is negative. The interactive effect of precommercial thinning, commercial thinning and fertilization is positive, that is to say, when these practices are applied together, the total effect is larger than the sum of individual effects.

The techniques developed and discussed give practical answers to questions of stand level optimization with complex cost, revenue and growth model silvicultural interactions.
A Study of Optimal Timing and Intensity of Silvicultural Practices-- Commercial and Precommercial Thinning, Fertilization and Regeneration Effort

by

Chiang Kao

A THESIS

submitted to

Oregon State University

in partial fulfillment of the requirement for the degree of

Doctor of Philosophy

June 1980
APPROVED:

Signature redacted for privacy.

Assistant Professor of Forest Management
in charge of major

Signature redacted for privacy.

Head of Department of Forest Management

Signature redacted for privacy.

Dean of Graduate School

Date thesis is presented  August 1, 1979

Typed by Chiang Kao for  Chiang Kao
Thanks are first dedicated to my parents for encouraging me to study abroad and continuous encouragement through letters.

Special appreciation is extended to Professor J. Douglas Brodie for his patient assistance in the organization and preparation of this thesis. His guidance and help in the past four years have made my study at Oregon State University fruitful and satisfactory.

Thanks are also extended to Professors John Beuter and Darius Adams in Forest Management, David Butler and Jeffrey Arthur in Statistics, and William Bregar in Computer Science for serving as my committee.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Introduction</td>
</tr>
<tr>
<td></td>
<td>Objective</td>
</tr>
<tr>
<td></td>
<td>Justification</td>
</tr>
<tr>
<td></td>
<td>Commercial Thinning</td>
</tr>
<tr>
<td></td>
<td>Precommercial Thinning</td>
</tr>
<tr>
<td></td>
<td>Fertilization</td>
</tr>
<tr>
<td></td>
<td>Final Harvest and Regeneration</td>
</tr>
<tr>
<td></td>
<td>Study Procedure</td>
</tr>
<tr>
<td></td>
<td>Assumptions</td>
</tr>
<tr>
<td></td>
<td>Tasks</td>
</tr>
<tr>
<td>II</td>
<td>Stand Level Optimization</td>
</tr>
<tr>
<td></td>
<td>Previous Optimization Studies</td>
</tr>
<tr>
<td></td>
<td>Current Optimization Work Associated With This Study</td>
</tr>
<tr>
<td></td>
<td>Necessary Data</td>
</tr>
<tr>
<td>III</td>
<td>Application of Dynamic Programming to Stand Level Problems</td>
</tr>
<tr>
<td></td>
<td>Dynamic Programming</td>
</tr>
<tr>
<td></td>
<td>Two Descriptor Dynamic Programming</td>
</tr>
<tr>
<td></td>
<td>Three Descriptor Dynamic Programming</td>
</tr>
<tr>
<td></td>
<td>Precommercial Thinning and Fertilization Considerations</td>
</tr>
<tr>
<td></td>
<td>Computational Efficiency</td>
</tr>
<tr>
<td></td>
<td>Backward Solutions</td>
</tr>
<tr>
<td></td>
<td>Forward Solutions</td>
</tr>
<tr>
<td></td>
<td>Summary</td>
</tr>
<tr>
<td>IV</td>
<td>The Structure and Operation of DFIIT and DOPT</td>
</tr>
<tr>
<td></td>
<td>Precommercial Thinning (PRECOM)</td>
</tr>
<tr>
<td></td>
<td>Submerchantable Mortality (SUBMORT)</td>
</tr>
<tr>
<td></td>
<td>Growth (GROWTH)</td>
</tr>
<tr>
<td></td>
<td>Logging Cost (REVNOW)</td>
</tr>
<tr>
<td></td>
<td>Increment (INCRMNT)</td>
</tr>
<tr>
<td></td>
<td>Main Program (DOPT)</td>
</tr>
<tr>
<td></td>
<td>Input Data</td>
</tr>
<tr>
<td></td>
<td>Effect of Fertilization</td>
</tr>
<tr>
<td></td>
<td>Dynamic Programming</td>
</tr>
<tr>
<td></td>
<td>Output Information</td>
</tr>
<tr>
<td>V</td>
<td>The Operation of the DOPT Algorithm Under Alternative State Space Specification</td>
</tr>
<tr>
<td></td>
<td>Tree Interval State</td>
</tr>
<tr>
<td></td>
<td>Basal Area Interval State</td>
</tr>
</tbody>
</table>
TABLE OF CONTENTS  
(continued)

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Artifact Effects</strong></td>
<td>95</td>
</tr>
<tr>
<td>VI</td>
<td></td>
</tr>
<tr>
<td>Computations of Silvicultural Applications</td>
<td></td>
</tr>
<tr>
<td>Using the DOPT Model</td>
<td></td>
</tr>
<tr>
<td>Final Harvest Only</td>
<td>99</td>
</tr>
<tr>
<td>Precommercial Thinning and Final Harvest</td>
<td>101</td>
</tr>
<tr>
<td>Commercial Thinning and Final Harvest</td>
<td>104</td>
</tr>
<tr>
<td>Fertilization and Final Harvest</td>
<td>108</td>
</tr>
<tr>
<td>Precommercial, Commercial Thinning and Final Harvest</td>
<td>110</td>
</tr>
<tr>
<td>Precommercial Thinning, Fertilization and Final Harvest Without Commercial Thinning</td>
<td>114</td>
</tr>
<tr>
<td>Commercial Thinning, Fertilization and Final Harvest</td>
<td>116</td>
</tr>
<tr>
<td>Precommercial, Commercial Thinning, Fertilization and Final Harvest</td>
<td>119</td>
</tr>
<tr>
<td>Plantation</td>
<td>129</td>
</tr>
<tr>
<td>Non-natural Stand</td>
<td>129</td>
</tr>
<tr>
<td>Interactions Between Silvicultural Practices</td>
<td>132</td>
</tr>
<tr>
<td>VII</td>
<td></td>
</tr>
<tr>
<td>Summary, Further Applications and Discussion</td>
<td></td>
</tr>
<tr>
<td>Summary</td>
<td>142</td>
</tr>
<tr>
<td>Further Applications</td>
<td>146</td>
</tr>
<tr>
<td>Discussion</td>
<td>150</td>
</tr>
<tr>
<td>Changes in Logging Cost</td>
<td>150</td>
</tr>
<tr>
<td>Changes in Price Function</td>
<td>151</td>
</tr>
<tr>
<td>Changes in Fertilization Cost</td>
<td>152</td>
</tr>
<tr>
<td>Changes in Regeneration and Precommercial Thinning Cost</td>
<td>153</td>
</tr>
<tr>
<td>Changes in Alternative Rate of Return</td>
<td>153</td>
</tr>
<tr>
<td>Bibliography</td>
<td>155</td>
</tr>
<tr>
<td>Appendices</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td></td>
</tr>
<tr>
<td>Variable List</td>
<td>160</td>
</tr>
<tr>
<td>II</td>
<td></td>
</tr>
<tr>
<td>Program for Comparing Growths From the DFIT and the Growth Model by Turnbull and Peterson</td>
<td>164</td>
</tr>
</tbody>
</table>
TABLE OF CONTENTS
(continued)

<table>
<thead>
<tr>
<th>Appendices</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>III</td>
<td>167</td>
</tr>
<tr>
<td>IV</td>
<td>184</td>
</tr>
<tr>
<td>V</td>
<td>202</td>
</tr>
<tr>
<td>VI</td>
<td>209</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>III</th>
<th>DOPT Program Listing</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV</td>
<td>Flowchart of DOPT Main Program and Subroutines</td>
</tr>
<tr>
<td>V</td>
<td>Number of Alternatives for Reaching Each State When Tree Intervals Used Are 20, 25, 30, 40, 50 and 100</td>
</tr>
<tr>
<td>VI</td>
<td>Final Stand Conditions and Management Regimes for Tree Interval 15 and Basal Area Interval 10, 16, 20, 30 and 40 on Site 140</td>
</tr>
</tbody>
</table>
# LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>29</td>
</tr>
<tr>
<td>II</td>
<td>31</td>
</tr>
<tr>
<td>III</td>
<td>33</td>
</tr>
<tr>
<td>IV</td>
<td>34</td>
</tr>
<tr>
<td>V</td>
<td>36</td>
</tr>
<tr>
<td>VI</td>
<td>41</td>
</tr>
<tr>
<td>VII</td>
<td>42</td>
</tr>
<tr>
<td>VIII</td>
<td>50</td>
</tr>
<tr>
<td>IX</td>
<td>51</td>
</tr>
<tr>
<td>X</td>
<td>51</td>
</tr>
<tr>
<td>XI</td>
<td>53</td>
</tr>
<tr>
<td>XII</td>
<td>53</td>
</tr>
<tr>
<td>XIII</td>
<td>65</td>
</tr>
<tr>
<td>XIV</td>
<td>73</td>
</tr>
</tbody>
</table>
Relative soil expectations and computation efficiency for different basal area interval compared with basal area of four square feet.
<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>45</td>
</tr>
<tr>
<td>II</td>
<td>47</td>
</tr>
<tr>
<td>III</td>
<td>48</td>
</tr>
<tr>
<td>IV</td>
<td>67</td>
</tr>
<tr>
<td>V</td>
<td>69</td>
</tr>
<tr>
<td>VI</td>
<td>72</td>
</tr>
<tr>
<td>VII</td>
<td>80</td>
</tr>
<tr>
<td>VIII</td>
<td>87</td>
</tr>
<tr>
<td>IX-XV</td>
<td>89-91</td>
</tr>
<tr>
<td>XVI</td>
<td>91</td>
</tr>
</tbody>
</table>

**LIST OF TABLES**

I
Number of alternatives which reach a certain state at a stage when complete enumeration is considered.

II
Number of feasible nodes for a certain state at a certain stage when alternatives in four square foot neighborhood are grouped together using the best to represent the state.

III
Number of alternatives leading to a certain state at a certain stage from previous stage when alternatives in four square foot neighborhood are grouped together using the best to represent the state.

IV
Large yarder power curve coefficients for stump to truck harvesting costs as a function of merchantable volume per acre removed for a 600-foot skyline.

V
Pond value as a function of arithmetic mean stand diameter.

VI
Variance of diameter distribution for different site-age combinations.

VII
Comparison of growth models by Turnbull and Peterson and the DFIT on site 140 with different growing stock.

VIII
Stand table for Douglas-fir site index 140 at age 40.

IX-XV
Contingency table for a normal stand compared with a stand with 50, 30, 20, 15, 10, 5, and 1 tree respectively.

XVI
Mean diameters of stand with different number of trees distributed proportionally to a normal stand.
<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>XVII</td>
<td>Number of thinning alternatives can be considered for different tree intervals when rotation is 80 years.</td>
<td>92</td>
</tr>
<tr>
<td>XVIII</td>
<td>Soil expectations and computation efficiency for different basal area interval compared with basal area of four square feet.</td>
<td>94</td>
</tr>
<tr>
<td>XIX</td>
<td>Summary of final harvest, MAI and present worth data for a stand considering final harvest only, site 140.</td>
<td>100</td>
</tr>
<tr>
<td>XX</td>
<td>Summary of final harvest, MAI and present worth data for a stand considering precommercial thinning and final harvest, site 140.</td>
<td>102</td>
</tr>
<tr>
<td>XXI</td>
<td>Summary of final harvest, MAI and present worth data for a stand considering commercial thinning and final harvest, site 140.</td>
<td>105</td>
</tr>
<tr>
<td>XXII</td>
<td>Thinning schedule and stand conditions when commercial thinning and final harvest are permitted, site 140.</td>
<td>106</td>
</tr>
<tr>
<td>XXIII</td>
<td>Summary of final harvest, MAI and present worth data for a stand considering fertilization and final harvest, site 140.</td>
<td>109</td>
</tr>
<tr>
<td>XXIV</td>
<td>Summary of final harvest, MAI and present worth data for a stand considering precommercial, commercial thinning and final harvest, site 140.</td>
<td>111</td>
</tr>
<tr>
<td>XXV</td>
<td>Thinning schedule and stand conditions when precommercial, commercial thinning and final harvest are permitted, site 140.</td>
<td>112</td>
</tr>
</tbody>
</table>
## LIST OF TABLES
(continued)

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>XXVI</td>
<td>Summary of final harvest, MAI and present worth data for a stand considering pre-commercial thinning, fertilization and final harvest, site 140.</td>
<td>115</td>
</tr>
<tr>
<td>XXVII</td>
<td>Summary of final harvest, MAI and present worth data for a stand considering commercial thinning, fertilization and final harvest, site 140.</td>
<td>117</td>
</tr>
<tr>
<td>XXVIII</td>
<td>Thinning schedule and stand conditions when commercial thinning, fertilization and final harvest are permitted, site 140.</td>
<td>118</td>
</tr>
<tr>
<td>XXIX</td>
<td>Summary of final stand conditions for different rotations for site 110, with precommercial, commercial thinning, fertilization and final harvest.</td>
<td>120</td>
</tr>
<tr>
<td>XXX</td>
<td>Management regime and stand conditions for site 110, with precommercial, commercial thinning, fertilization and final harvest.</td>
<td>121</td>
</tr>
<tr>
<td>XXXI</td>
<td>Summary of final stand conditions for different rotations for site 140, with precommercial, commercial thinning, fertilization and final harvest.</td>
<td>122</td>
</tr>
<tr>
<td>XXXII</td>
<td>Management regime and stand conditions for site 140, with precommercial, commercial thinning, fertilization and final harvest.</td>
<td>123</td>
</tr>
<tr>
<td>XXXIII</td>
<td>Summary of final stand conditions for different rotations for site 170, with precommercial, commercial thinning, fertilization and final harvest.</td>
<td>124</td>
</tr>
<tr>
<td>XXXIV</td>
<td>Management regime and stand conditions for site 170, with precommercial, commercial thinning, fertilization and final harvest.</td>
<td>125</td>
</tr>
<tr>
<td>Table</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>-------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>XXXV</td>
<td>Summary of final stand conditions for different rotations for site 200, with precommercial, commercial thinning, fertilization and final harvest.</td>
<td>126</td>
</tr>
<tr>
<td>XXXVI</td>
<td>Management regime and stand conditions for site 200, with precommercial, commercial thinning, fertilization and final harvest.</td>
<td>127</td>
</tr>
<tr>
<td>XXXVII</td>
<td>Summary of final stand conditions for different rotations for site 140 plantation.</td>
<td>130</td>
</tr>
<tr>
<td>XXXVIII</td>
<td>Management regime and stand conditions for site 140 plantation.</td>
<td>131</td>
</tr>
<tr>
<td>XXXIX</td>
<td>Summary of final stand conditions for different rotations for site 140 of a nonnormal stand with 80 percent of normal basal area and number of trees at age 34.</td>
<td>133</td>
</tr>
<tr>
<td>XL</td>
<td>Management regime and stand conditions for site 140 of a nonnormal stand with 80 percent of normal basal area and number of trees at age 34.</td>
<td>134</td>
</tr>
<tr>
<td>XLI</td>
<td>Summary of soil expectations for different silvicultural activity combinations, site 140.</td>
<td>135</td>
</tr>
<tr>
<td>XLII</td>
<td>Summary of some total and interactive effects on soil expectation, site 140.</td>
<td>141</td>
</tr>
<tr>
<td>XLIII-XLVIII</td>
<td>Number of alternatives of reaching each state when tree intervals used are 20, 25, 30, 40, 50 and 100.</td>
<td>203-208</td>
</tr>
<tr>
<td>XLIX-LVIII</td>
<td>Final stand conditions and management regimes for tree interval 15 and basal area interval 10, 16, 20, 30 and 40 on site 140.</td>
<td>210-219</td>
</tr>
</tbody>
</table>
I. INTRODUCTION

The basic question of forest management for timber production can be stated simply: What treatment should be applied to each stand each period to best meet the objectives established for the forest? The question can be addressed at two levels-- the stand level and the forest level (Hann and Brodie, 1979). For individual stands the manager interested in practicing even-aged management needs to know the optimal: (1) planting density, (2) thinning scheme, (3) rotation length, (4) fertilization scheme, and (5) species mix. At the forest level the even-aged forest manager is faced with determining, for the desired number of planning periods, the optimal schedule of stand treatments, or conversion strategy and conversion period length. A forest is composed of forest stands, and the forest level questions can be better answered only after the alternative stand level treatments are known. Without the knowledge of stand level alternatives the study of forest level questions may be unrealistic and incomplete. The main purpose of this study is to investigate techniques for finding and studying the optimal management regime for an even-aged stand.
Recent increase in the real value of stumpage in the Pacific Northwest has lead to rapid attempts to implement a wide array of silvicultural practices that were regarded only as experimental a decade ago. Much attention has been given in recent years to theoretical optimization approaches at the stand level using a variety of methods including complete enumeration of limited alternatives, simple algebra, linear programming, nonlinear programming, dynamic programming, inventory theory and control theory. Most of these approaches analyse only one silvicultural practice at a time using partial analysis. If all silvicultural practices can be simultaneously considered in stand optimization then both the competitive and complementary interactions of stand level practices could be examined.

Stand growth and yield models are fairly complex systems and the timber management system over-lays this complex biological system. A way to construct a consistent system of analysis capable of optimizing the growth and yield system with its large number of silvicultural decisions has long been sought.

OBJECTIVE

The primary focus of this research is to develop applied analytic optimization techniques for key silvicultural operations in the Pacific Northwest including commercial thinning, precommercial thinning, regeneration harvest
and fertilization. The techniques developed can be applied to other regions and other species. The species used in this study to demonstrate and evaluate the economic impacts of silvicultural activities is Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco).

The objectives of this study are:

1. To adapt existing models of Douglas-fir growth and yield, commercial and precommercial thinning, fertilization and regeneration survival, so that optimization analysis will be feasible.

2. To develop an optimization model and document its operation and application under alternate specifications of stand level problems.

3. To analyse and present the impacts of these silvicultural activities applied as independent elements in a management regime and derive their independent impacts on rotation length, stocking and investment efficiency.

4. To the extent it is computationally feasible to analyse the interactions between management activity impacts and derive their joint impacts on rotation length, stocking and investment efficiency.

5. To provide management guidelines for the techniques and levels of these activities for various sites.

6. To find the best planting density and the subsequent management regime.

7. To differentiate between various possible criteria.
The main criterion for evaluating a management regime will be to maximize the present net worth and soil expectation. The methodology selected provides intermediate solutions for sensitivity analysis and is readily adaptable to alternate objectives such as volume maximization.

JUSTIFICATION

Demands for timber products in the United States have been projected to increase steadily (USDA Forest Service, 1974). Yet the supply of timber products potentially available from U. S. Forests show limited increases. Substantial increases in timber prices appear necessary to balance potential timber demands with available timber supplies. Work by Adams, Haynes and Darr (1977) indicates the relatively stable stumpage prices of the early 1960's have quadrupled in real terms in recent years with projections indicating future increases.

Intensified forest management offers an important means of increasing timber supplies which can mitigate the above problems. Sizable increases in timber growth and future harvest (both quantitative and qualitative) could be achieved by increased investments to expand forestry practices.

Much attention has been given to implementation of intensive management practices. The intensification of individual management inputs interacts with the intensi-
lication of other inputs and management decisions such as stocking levels, harvest schedule and rotation age. Because of the short period during which implementation of these activities has been considered, much of the analysis is partial and incomplete. Results of analyses have often not been generalizable due to reliance on simulation (Reukema and Bruce, 1977) rather than optimization techniques. This study originated from optimization techniques developed for commercial thinning analysis (Brodie, Adams and Kao, 1978), and is based on and expands the study by Brodie and Kao (1979). The goal of this research is to provide a flexible means for economic analysis of impacts and interactions of silvicultural inputs.

The management practices considered are:

Commercial Thinning

This practice is essentially a series of reductions made in stand density to maximize the net value of products removed during the whole rotation. Among the factors determining this net value are the quantity, quality and size of the products as well as the costs of harvesting. If growth is measured in terms of total volume of all parts of the trees of a given stand, it is often found that artificial changes in stand density over fairly wide ranges do not affect total volume production. Even if the total production of wood remains essentially unaltered, thinning
can be used to increase the yield of "merchantable" volume (Smith, 1962). The fast diameter growth induced by thinning increases the value of stem wood. Other advantages that can be gained from thinning are: salvage of anticipated losses of merchantable volume, control of growing stock during the rotation and improvement of product quality. There could be an economic incentive for thinning even if it has no impact on total yield and quality if the present net worth contribution of early growing stock removals is greater than their contribution if left to grow for later harvest.

A thinning optimization study developed by Brodie, Adams and Kao (1978) noted that growth models that do not explicitly treat diameter growth as variable with stocking, will understate the economic desirability of thinning. If market values increase and logging costs decline with age alone, then incentives for early thinning consist primarily of light mortality capture harvests. With the development of the DFIT model (Bruce, Demars and Reukema, 1977), it became possible to describe stand dynamics including diameter growth, within a dynamic programming optimization framework.

The commercial thinning part of this research is based on the work of Brodie and Kao (1979), to find the best stocking level at each period while considering diameter growth acceleration impacts on unit prices, costs and
economic returns.

Precommercial Thinning

Bruce, Demars and Reukema (1977) and the Douglas-fir Supply Study (USDA Forest Service, 1969) indicate that precommercial thinning removal of small, unsalable trees from a timber stand results in more rapid juvenile growth and qualifies stands for commercial thinning earlier than stands not receiving this treatment. The study by Brodie and Kao (1979) showed that if diameter growth acceleration is considered, the optimal thinning regime may involve a large removal of volume at the first thinning which occurred at age 30. In most cases (under different price functions) the best regime involved cutting half or more of the merchantable trees in a fully stocked stand at age 30. The stumpage price per unit at age 30 was less than the cost of thinning. So, actually, the first thinning was a special kind of precommercial thinning, since the sale return from thinning is less than the cost but the trees are salable. The above results suggest that precommercial thinning is desirable and this study investigates the circumstances and extent of application.

Fertilization

Few forest soils provide an optimal supply of nutrient elements essential for the growth of trees. Sometimes
marked deficiencies may exist because of improper land management in the past or merely because of inherently low natural fertility of the site. One method of correcting such deficiencies is by chemical fertilization.

It is often possible to produce significant increases in forest growth by the application of nitrogen fertilizers. There is also evidence that fertilization might be used to increase the amount and frequency of seed production not only of seed orchards but also of individual trees being used as sources of natural regeneration (Smith, 1962). Purchase, transport and application of fertilizer is a significant expense with a deferred return. One purpose of this study is to examine methods of analysing and optimizing timing, frequency and amount of fertilization in Douglas-fir management using existing data sources (Turnbull and Peterson, 1976).

Final Harvest and Regeneration

Maximizing physical yield, cash flow, or present net worth on either a single or infinite series basis are all used as optimizing criteria in forestry. The distinctions between these criteria are reviewed in Davis (1966) and Duerr (1960). The analytic techniques of this study are adaptable to any of these criteria, however in general, optimization is done under the soil expectation maximization criterion as this is the general and accepted economic
criterion. Under this criterion, levels of regeneration cost and related stocking success affect the subsequent optimal management regime.

**STUDY PROCEDURE**

The primary objective of this study is to find an optimal management plan which considers the key silvicultural practices. Certain assumptions will be needed to assure feasibility in modeling the system.

**Assumptions**

(1) **Alternative rate of return.**

The alternative rate of return is assumed known. Higher alternative rates of return tend to lead to shorter rotation and heavier thinning. In the central part of the study, three percent is used as the alternative rate of return to approximate the historical real rate of return on long-term investment (Yohe and Karnosky, 1969).

(2) **Timber price.**

The market prices of timber per unit are assumed known. The price-diameter functional relationship used in this study is from Sessions (1979).

(3) **Harvest cost.**

The costs of both thinning and final harvest are assumed known. Cost is a function of both mean diameter and volume harvested. Cost functions derived from
harvest simulations are available from Sessions (1979).

(4) Regeneration costs.

The costs of regeneration include the disposal of logging slash, treatment of the forest floor and competing vegetation, planting stock, and planting labor and supervision. The expected survival associated with these costs and the costs themselves is assumed known.

(5) Method of thinning.

The merchantable mortality trees will be removed at each commercial thinning. The merchantable live trees to be thinned are chosen such that the ratio of quadratic mean diameter of trees cut to quadratic mean diameter of the merchantable stand before commercial thinning is 1.00. This implies that the submerchantable part is left intact. The submerchantable trees will die eventually before they reach merchantable size. Further elaboration and stand development functions on which these assumptions are based are provided by Bruce, Demars and Reukema (1977) and are further discussed in Chapter IV.

(6) Fertilization.

Fertilizer application is modelled at each period immediately after the commercial thinning. This eliminates the necessity of an additional state descriptor for fertilization. Chapter IV will explain this procedure. Regimes with and without fertilization can be compared.
(7) Merchantable mortality in under- or over-stocked stand.

The merchantable mortality at each period is assumed to be proportional to the normality of the stand. If "p" percent of merchantable trees will die in a fully stocked stand, then it is assumed that for an under- or over-stocked stand also "p" percent of the merchantable trees of the stand will die in the next period.

(8) Mortality lost.

At each thinning the periodic merchantable mortality is removed. If no thinning occurs in a stage then the periodic merchantable mortality is lost as a result of decay. It cannot be captured in the next thinning.

With these assumptions the study objectives are accomplished by executing the following tasks.

Tasks

A brief description and outline of the tasks accomplished in this dissertation follows.

(1) Derivation of growth function.

The skeleton of the growth function is from DFIT. Volume per acre of a natural stand is a function of site and age. One year volume growth of the stand is the derivative of volume over age. Volume growth is adjusted by the existing number of trees and basal area of the stand.

(2) Fertilization adjustment.

The growth response of fertilization is based on the
work by Turnbull and Peterson (1976). The fertilization response model uses site, number of trees, basal area, and age as predictors of stand response. At each stage (stand age) and each state (number of trees and associated basal area) on a certain site this information is known, hence the growth can be adjusted by the fertilization effects.

(3) Deriving of mortality function.

Both submerchantable and merchantable mortality have to be calculated. Submerchantable mortality is only calculated once for the normal stand at different ages. Only the merchantable part is considered for thinning. From the number of merchantable trees the number of merchantable mortality trees is calculated as explained in assumption seven.

The number of submerchantable trees and basal area of submerchantable trees in a natural stand are calculated from equations in DFIT. The volume of submerchantable trees is calculated by using the volume/basal area ratio and adjusted by a tariff function.

The number of merchantable mortality trees is calculated as mentioned in assumption seven. The volume of merchantable mortality trees is the difference between gross growth and net growth. The calculations of net growth and gross growth will be described in the GROWTH section in Chapter IV. The basal area of merchantable mortality trees is derived from the volum/basal area ratio.
With this information the quadratic mean diameter of merchantable mortality trees, which is used to calculate the timber prices and logging costs, can be calculated. The merchantable live trees data is readily derived from net volume growth.

(4) Precommercial thinning adjustment.

DFIT has several equations for calculating precommercial thinning effects at different precommercial thinning ages. The basic idea is: The site index is adjusted due to assumed superior height growth after precommercial thinning. Since volume growth is a function of site, this adjustment will affect the subsequent volume growth and the associated basal area and mean diameter.

(5) Developing the thinning algorithm.

The state-descriptor variables of this study are: stand age, number of merchantable trees and basal area. The amount of fertilization applied could be another descriptor, but with some transformations, the extra descriptor is handled by substituting a series of computations. At each stage (stand age) and each state (number of trees) there are many thinning alternatives. The extreme cases are cut nothing and cut all of the trees in the stand. If nothing is cut then no cost is incurred, but the merchantable mortality is lost. If all trees are cut then this will be a final harvest. Between these two extremes are many cutting alternatives removing some number of trees.
so the remaining number of trees is a multiple of a discrete dynamic programming interval. Actually, the remaining number of trees need not be a multiple of a certain number. As long as the state of the remaining number of trees is finite, dynamic programming will work. However, computations are more easily interpreted and explained if the remaining trees is a multiple of a certain number. If that "discrete interval" is too large then only a limited number of thinning alternatives can be considered. If it is too small then the "method of thinning" assumption mentioned above may not be fulfilled, also the computer memory needed and computation time may be unacceptably large. This study found that 15 to 50 trees is a reasonable number for this discrete interval.

The basal area corresponding to the number of trees is a continuous variable. As a requirement of discrete state dynamic programming, basal areas over a range are grouped together and use the middle value of the range to represent them. The smaller the range used, the more precise the model will be. The error of comparing the same number-of-trees with different but similar basal area will be smaller, if the range of comparison is smaller. The finer the grid, the longer the computation time will be. So there is a trade-off. Within the required accuracy, a relatively small range is required. Due to the computer core capacity and the limited amount of data-packing
attempted, four square feet is the smallest interval that the program developed can use with a number of trees interval of 15. (Computer used is CDC Cyber 73-16)

(6) Computer runs and interpretation

Using site index 140, the following computer runs are considered: Commercial thinning and final harvest; precommercial, commercial thinning and final harvest; commercial thinning, fertilization and final harvest; and precommercial, commercial thinning, fertilization and final harvest. The last case is also run for different site indices to demonstrate the combined silvicultural effects on different sites. The following cases are computed by using a hand calculator: Final harvest only; precommercial thinning and final harvest; fertilization and final harvest; and precommercial thinning, fertilization and final harvest. The management regimes are compared and the economic impact is discussed.
II. STAND LEVEL OPTIMIZATION

A forest is composed of forest stands. Stand level optimization forms a basis for forest level optimization. Both Williams (1977) and Nazareth (1973) independently demonstrated an approach for joint stand level and forest level optimization. The first step is to find the optimal stand level treatment, as well as the efficient treatment schedule for achieving non-optimal stand conditions. The forest level solution is formulated as an extremely large decomposable linear programming harvest scheduling problem. The first decomposition utilizes the optimal stand level treatments as activities. Due to the constraints, this solution can be enhanced in subsequent decomposition by inclusion of additional non-optimal efficient stand level treatments that provide greater levels of thinning and harvest in periods where the constraints are most binding.

The analysis presented in this study provides both the optimal stand level regime and all efficient regimes that are non-optimal. All of these alternate stand level treatments could theoretically at least be modified for inclusion in the forest level optimization discussed above. This would constitute a major undertaking and the research presented here is restricted to the stand level questions only.
In solving stand level problems two broad techniques have been utilized--simulation and optimization. Simulation provides descriptive stand outputs when management regime is provided by the analyst. The advantage of simulation is that it can be applied to any kind of existing stand development model. The primary disadvantage is that only a "good" answer can be found. To determine a best or near best solution would require a great number of simulations and careful analysis. This approach can be expensive in terms of both time and computer costs. Optimization techniques provide a defined optimal regime when an objective criterion and cost and revenue assumptions are provided by the analyst. Usually an optimization technique is more difficult to implement than a simulation method, yet it finds an optimal solution in one set of calculations instead of an exhaustive simulation search. The results from an optimization often give more information. For example, the linear programming technique tells the shadow prices and dynamic programming techniques give many intermediate efficient solutions. In the case of forest stand optimization, efficient regimes for rotations shorter and longer than the global optimum are provided. Often a sensitivity test can be undertaken without additional computation to see the effect of deviating from the best strategy.
Optimization methods for solving stand level questions have long been used in forestry. Early methods of determining rotation length, such as maximum mean annual increment (MAI) and soil expectations were all optimization methods. These early methods were distinguished by the numerous assumptions made in order to solve them and by the amount of tedious calculations required. First applications of the electronic computer were aimed at easing the latter problems. More recently, mathematical programming techniques have made it easier to answer more complex problems and, therefore, reduce the need for numerous assumptions. A good example of this evolutionary process has been the development of optimization tools for jointly answering thinning scheme and rotation length questions.

Early approaches and some recent practitioners have used marginal analysis and brute force trial and error methods, or some form of complete enumeration to choose among management alternatives. Chappelle and Nelson (1964) presented an early solution to the joint optimization of thinning scheme and rotation length in loblolly pine. They used two simple volume growth equations, one in cubic feet and the other board feet, and marginal analysis to determine optimal volume stocking levels which would maximize net profit per annum for each product. Then, given an
initial stocking level, the optimal stocking level, and the volume growth model, they determined the amount of volume, thinning would remove in each cutting period for a fixed rotation length. Using this information and cost and revenue data, the optimal rotation length was determined using the classical soil expectation method. Other research works used marginal analysis including applications by Duerr (1960), Duerr and Christiansen (1973) and recent applications of marginal analysis in the Douglas-fir Region by Buongiorno and Teeguarden (1973). Hardi (1977) presented thinning regimes derived from a complex biometric model. The optimization procedure was complete enumeration of a highly constrained set of alternatives.

Adams and Ek (1974) used a nonlinear programming technique to solve the two problems in the management of uneven-aged forests: (1) determination of the optimal sustainable distribution of trees by diameter class, i.e. stand structure, for a given stocking level, and (2) the optimal cutting schedule for the conversion of an irregular stand to a target structure. The solution procedure used was a modification of the gradient projection method. This problem is analogous to the even-aged situation of starting with a specified stand, growing and thinning it over a number of time periods until a target diameter distribution of zero trees in each diameter class is reached in the last time period. Their method necessitated a relatively simple
stand model that explicitly predicted the net change in number of trees in a diameter class. Their method was also limited as to the number of time periods over which the method could be applied. Naslund (1969) formulated a theoretical nonlinear programming model to determine simultaneously the optimal rotation and thinning regime but presented no solution. Schreuder (1971) presented a model to solve the same problem in two forms. If time is assumed to be continuous, the problem can be formulated in the calculus of variation form. In this form no closed form solution was obtained. If the model is recast in dynamic programming form, a numerical solution can be obtained. Kao and Brodie (1979a) set up a simple "approach toward normality" growth model from Bulletin 201 (McArdle, Meyer and Bruce, 1961) and used a modified flexible polyhedron method to find a continuous time, continuous state solution.

Anderson (1976) generalized the optimal control approach to the timber management problem to include the opportunity cost of forested land. The generalized steady-state control solution was shown to be identical to the Faustmann rotation model. Anderson presented theoretical derivations only and failed to present any numerical solutions. Pelz (1977) used inventory theory to determine optimal growing stock levels and was able to duplicate the results of Chappelle and Nelson (1964).

In mathematical programming applications, dynamic
programming is also quite widely used. Amidon and Akin (1968) demonstrated how dynamic programming could be used to obtain the same solutions as Chappelle and Nelson (1964). Dynamic programming is a method used for solving a wide array of scheduling and resource allocation problems. A broad sub-set of dynamic programming problems can be represented by a network of nodes. The number of "state" descriptors needed to define each node defines the dimensions of the network. The objective of dynamic programming is to find, within defined limits, the optimal, path through the network. In stand optimization the objective is to find the optimal stocking level at each stage (stand age). Solution to the problem of finding an optimal path can be accomplished by using either the forward or the backward recursive method. Amidon and Akin (1968) used two descriptors (volume and age) and a backward recursive method. Brodie, Adams and Kao (1978) and Kao and Brodie (1979b), used two descriptors (volume and age) but a forward recursive method. Risvand (1969) also used three descriptors (volume, diameter and age) and a forward recursive method. A recent study by Brodie and Kao (1979) also used three descriptors (number of trees, basal area and age) and a forward recursive method to solve the stand optimization problem.
CURRENT OPTIMIZATION WORK ASSOCIATED WITH THIS STUDY

One shortcoming that Brodie, Adams and Kao (1978) identified with their approach was that the stand model did not account for diameter growth acceleration due to thinning. If quality premiums are important, this shortcoming can lead to suboptimal solutions.

The most recent work in the application of dynamic programming to stand problems has tried to eliminate this shortcoming through the use of a much more complex stand model that incorporates quadratic mean stand diameter growth acceleration into it. Because of the added complexity of the stand model, a three descriptor dynamic programming network was used. After a careful examination of the stand model, Brodie and Kao (1979) discovered that the model could be initialized, for a specified site index, if the three values of stand age, basal area and number of trees were known. These, therefore, formed the three state descriptors of each node. Analysis results using the three descriptor framework are the optimal number of trees and basal area to maintain in each time interval. To obtain these values Brodie and Kao used the forward recursion solution method. Because quadratic mean stand diameter can be computed from number of trees and stand basal area, stumpage prices and logging costs could be more realistically introduced into the analysis as functions of
quadratic mean stand diameter.

Brodie and Kao (1979) used a simple price-diameter function and a cost function for logging derived from Bureau of Land Management Schedule 20 (1977). Sessions (1979) used a complex yarding simulator to derive costs and a price function based on the distribution of log-grades related to mean stand diameter. He integrated this work into the dynamic programming framework developed by Brodie and Kao (1979) to study the interactions of logging method, terrain and optimal regime. Both of the studies considered only two silvicultural activities--commercial thinning and final harvest. In this study two other silvicultural activities--precommercial thinning and fertilization, are added. Combined with the dynamic programming framework developed by Brodie and Kao (1979) and price and cost functions derived by Sessions (1979), a more complete model is developed.

NECESSARY DATA

To achieve the optimal management of an even-aged pure stand several data elements are needed: (1) a growth model with silvicultural responses, (2) prices of products and (3) costs of producing products.

(1) Growth model with silvicultural responses.

DFIT (Douglas-fir Interim Tables) is a computer program developed by the Pacific Northwest Forest and Range
Experiment Station (Bruce, Demars and Reukema, 1977) which simulates stand growth and tabulates the results of the simulation. The main component of DFIT is a set of equations describing the development of natural stands. The function list contains approximately 33 relationships and the variable list 24 elements and 19 subscripts. Natural stands and plantations can be simulated with details of merchantable and unmerchantable: diameters, volumes, mortality, height, basal area and number of trees. Commercial and precommercial thinning, fertilization and growth adjusting activities such as genetic improvements are among the silvicultural activities that can be simulated and projected.

The fertilizer adjustment in DFIT consists of raising the site index and a volume growth adjustment through multipliers. Only one level of fertilization-- 200 pounds of nitrogen per acre is considered. In 1969 the Regional Forest Nutrition Research Project was initiated with the primary objective of providing resource managers with more accurate data on the effect of fertilizing and thinning young-growth Douglas-fir and Western hemlock. A note by Turnbull and Peterson (1976) presented the growth responses of fertilizing 400 pounds, 200 pounds and zero pounds of nitrogen per acre as functions of site index, stand age, basal area and number of trees per acre. These data are more realistic and these functional fertilizer responses
were adapted to conform to the DFIT model for use in this study.

(2) Prices of products.

Sessions (1979) using data from Bulletin 201 (McArdle, Meyer and Bruce, 1961), Columbia River Scaling Rule (Dilworth, 1973) and the current 1978 pond value of logs in Western Oregon derived a relation between the age of a natural stand and the value of the stand at that age. From the arithmetic mean diameter of a natural stand at a certain age and its corresponding value, a relation between price per thousand cubic feet and arithmetic mean stand diameter is derived.

(3) Costs of products.

Sessions (1979) also derived stump-to-truck harvesting costs as a function of merchantable volume per acre removed by using his yarding simulator. The harvesting cost depends on arithmetic volume per acre. A haul cost dependent on volume harvested only was adjusted to make up the total logging cost.
III. APPLICATION OF DYNAMIC PROGRAMMING TO STAND LEVEL PROBLEMS

Among those methodologies mentioned in Chapter II, dynamic programming is a simple and flexible technique for solving optimization problems. It can overcome deficiencies in marginal analysis such as the inability to easily account for precommercial opportunities and the interdependence of harvest costs and volume removals. It does not need the restrictive assumptions of linear programming. It embodies the flexible functional forms of nonlinear programming without the difficulties in specification and solution method. Approaches such as the continuous state control theoretic formulation use complex mathematics for which solutions can be quite difficult. Dynamic programming offers an efficient method of generating and evaluating the immense number of alternatives that exist within the feasible thinning-rotation set. It has provided empirical solutions and because it operates with implicit first order conditions, theoretical derivations are also possible. Hence dynamic programming was chosen as the optimization method for this study.

DYNAMIC PROGRAMMING

Dynamic programming is a mathematical technique often useful for making a sequence of interrelated decisions. It
provides a systematic procedure for determining the combination of decisions that maximizes overall effectiveness (Hillier and Lieberman, 1974; Nemhauser, 1967). The basic features which characterize dynamic programming problems are presented below, using the stand optimization problem for example and discussion:

(1) The problem can be divided into stages, with a policy decision required at each stage.

In the stand optimization study, stages are times for precommercial thinning and commercial thinnings. The policy decision at each stage is to decide the intensity of precommercial thinning or commercial thinnings. The decision in one stage will affect the decision in the next stage. That is to say dynamic programming problems require making a sequence of interrelated decisions.

(2) Each stage has a number of states associated with it.

The states associated with each stage in the stand optimization problem are stocking levels which are two-dimensional, represented by number of trees in the stand and the corresponding basal area. The states are the various possible conditions in which the system might be at that stage. The number of states may be either finite or infinite. In stand optimization the state space is finite because the largest number of trees that can grow in a stand at a certain stage is finite.
(3) The effect of the policy decision at each stage is to transform the current state into a state associated with the next stage.

In the stand optimization problem, given the state at current stage is "n" trees with "g" square feet of basal area, if the policy decision is to cut down to "n'" trees with "g'" square feet of basal area, then the state at next stage will be "n'" trees (assuming no mortality trees) with "g'" square feet of basal area, where g"-g' is the amount of basal area growth during the two consecutive stages.

(4) Given the current state, an optimal policy for the remaining stages is independent of the policy adopted in previous stages.

Given the state of the stand, the optimal management regime from this point onward is independent of how the stand state is arrived at in the current state. For dynamic programming problems in general, knowledge of the current state of the system conveys all the information about its previous behavior necessary for determining the optimal policy henceforth. This property is sometimes referred to as the principle of optimality (Bellman and Dreyfus, 1962).

(5) A recursive relationship must be specified that identifies the optimal policy for each state at stage n, given the optimal policy for each state at either stage (n-1) for a forward recursion or stage (n+1) for a backward
recursion. The distinction between forward and backward recursion is explained later.

TWO DESCRIPTOR DYNAMIC PROGRAMMING

Amidon and Akin (1968) used a backward recursion to find the best thinning regime for each given rotation. At each state (stocking level) only two actions are considered, either thin heavily or thin lightly. Figure I is a sub-network, or grid, extracted from the larger network (Amidon and Akin, 1968). For a given rotation age, working backward the best path, or thinning regime, can be found.

Figure I. Network of dynamic programming model by Amidon and Akin.
Let \( D(x,y) \) = decrement in growing stock value from \((x+1,y+1)\) to \((x,y)\).

\( I(x,y) \) = increment in growing stock value from \((x+1,y-1)\) to \((x,y)\).

\( T(x,y) \) = total value from the optimal schedule up to \((x,y)\).

where \( x \) and \( y \) denote network axes as in Figure I. Then the application of the principle of optimality is expressed by the recursive function:

\[
T(x,y) = \max \{ D(x,y) + T(x+1,y+1); I(x,y) + T(x+1,y-1) \}
\]

\( T(n,y) \) is initial condition, i.e. final harvest value for the rotation corresponding to \( n \). For different \( n \) the corresponding \( T(n,y) \) can be calculated. Best rotation \( n^* \) is chosen such that

\[
T(n^*,y) = \max_n \{ T(n,y) \}
\]

Brodie, Adams and Kao (1978) used a forward solution to find the optimal stocking levels and rotation simultaneously. At each state all actions resulting in a state having less stocking level can be considered. Figure II shows the network of this dynamic programming model. \( T(1,y^*) \) is the initial condition. Using the forward recursive function:

\[
T(x+1,y) = \max_{y'} \{ T(x,y') + P(y',y) \}
\]

where \( y' \) is any node of stocking level in the current stage which can reach stocking level \( y \) in the next stage, and \( P(y',y) \) is the revenue function showing the revenue obtained
Figure II. Network of dynamic programming model by Brodie, Adams and Kao.

from state \( y' \) to \( y \). \( T(n,0) \) will be the final harvest value for the rotation corresponding to \( n \). The calculation goes from one stage to the next stage until \( T(n^{*}+1,0) < T(n^{*},0) \). And \( n^{*} \) will be the best rotation.

Because the growth function is continuous while the stocking levels are discrete, a small "rounding thinning" of less than the stocking level interval is necessary to assure a discrete stocking level. If no thinning is considered at a certain stage then the stocking level, i.e. the growth will be underestimated. In Figure II the broken
lines show the actual growth, and the node right below the broken line is the rounding stocking.

Both of the studies by Amidon and Akin, and Brodie, Adams and Kao considered a fixed thinning entry. The former used five years and the latter used ten years. Kao and Brodie (1979b) wrote a flexible program which considered variable thinning entry. The thinnings occur at any age if it is optimal. The age is an integer, so actually it is a quasi-continuous time dynamic programming model. They also developed the "neighborhood" concept, which the later work by Brodie and Kao (1979) is based on, to overcome the rounding stocking problem. The neighborhood concept will be discussed in the following section.

THREE DESCRIPTOR DYNAMIC PROGRAMMING

All of the above mentioned dynamic programming studies used only two descriptors, i.e. stand age and stocking (volume). Risvand (1969) proposed a three descriptor dynamic programming model which also considered diameter growth. His network structure is similar to the structure of Amidon and Akin. At each state there are a fixed number of thinning alternatives considered. The amount of thinning is proportional to the existing stocking level, hence the stocking state is continuous but finite. Figure III shows the network of Risvand. Different paths need not go to the same stocking level.
Brodie and Kao (1979) modified the Dfit model (Bruce, Demars and Reukema; 1977) and used basal area, number of trees and age to describe stand dynamics which can handle diameter growth acceleration impacts resulting from thinning. The initial condition for the start of the solution is generated from the natural stand options in DPIT or can be set at alternate levels of number of trees and corresponding basal area. The recursion is solved forward from this single node. A graphic representation of the network is demonstrated in Figure IV.
Figure IV. Graphical representation of the three dimensional DOPT network.
Schreuder (1968) has noted that the large number of alternatives has been a barrier to realistic structuring and solution of forestry problems in a dynamic programming framework. A related problem discussed by Brodie, Adams and Kao (1978) is the rounding error associated with discrete state descriptor nodes and continuous growth models. A modification of the node definition in dynamic programming was used to more adequately represent a continuous production surface with a limited number of nodes and eliminate the rounding error problems.

Traditionally discrete dynamic programming nodes are defined in terms of precise values of the discrete state descriptors which may require a large number of nodes to adequately represent a continuous production surface. Additionally in some applications a rounding error is introduced as values computed from continuous functions are forced to be rounded to discrete node intervals. Brodie and Kao treated the network nodes as ''neighborhood storage locations'' at which exact continuous values of the descriptors for the optimal policy to the current stage are stored. Figure V helps to clarify this point. It represents a section of the number of trees (N) and basal area (G) grid for a stage (age) in the solution. Growth from optimal policy nodes of the previous stage when combined with discrete thinnings create candidate stand types in the ''neighborhood'' of the N=75, g=48 node where the neighbor-
Figure V. A representation of the DOPT network for a stage showing candidate stand types for the optimal policy at neighborhood storage location N75, G48.
hood is defined as all candidate stands with basal area between 46 and 50 cubic feet and number of trees between 60 and 75 per acre. Optimization takes place over these candidates and the result is saved at node (75,48). Growth for subsequent stages is calculated using the actual continuous N and G of the optimal policy for this node rather than the "neighborhood" value of the node itself. Values of number of trees that are not discrete multiples of the tree interval represent "no thinning arcs" with mortality. Except for the provision for these arcs, the solution is discrete in number of residual trees and time but continuous in the merchantable basal area variable. Brodie and Kao discovered no indication of systematic bias in those modifications. Alternatives are eliminated by the process that might prove to be optimal through interaction with later stages in the recursion if they were retained as exact nodes; however, this effect is not thought to be serious if the state intervals are small. The "neighborhood" approach with continuous descriptors mitigates the necessity of choosing wide state intervals for reasons of available storage and computational efficiency. A principle benefit of this approach in the thinning problem is that it allows for merchantable mortality adjustment at nodes where no thinning is optimal.

A DFIT natural stand was generated for the base year of the study by Brodie and Kao which was 30 in their
examples presented. The associated merchantable number of
trees and basal area define the single starting condition
node. The unmerchantable portion of the stand is also
generated and it is carried along creating a small retard-
ing influence on merchantable basal area growth that
declines over time. The present net worth of costs incur-
red prior to age 30 is the optimal value function value for
the starting condition node. The thinning interval is ten
years and the stand is grown in ten annual cycles of the
DFIT model.

Thinnings are removed from the diameter classes
proportional to frequency in the classes or in other words
the ratio of mean diameter of trees removed to all merchant-
able trees in the stand is maintained at 1.00. The dynamic
programming problem solved by forward recursion is as
follows:

Define the optimal value function as \( T(N,G,t) = \) the
value of the present net worth (PNW) "path" from regenera-
tion to stand age \( t \), number of trees \( N \) and basal area \( G \).
The forward recursive function is

\[
T(N,G,t) = \max \left\{ T(N',G',t-10) + P(N',G',N,G) \right\}
\]

where \( P(N',G',N,G) \) is the discounted value from node \((N',G')\)
to node \((N,G)\). \([N',G']\) is the set of all feasible number
of trees and basal area at age \( t-10 \) from which the current
neighborhood level of \( N \) and \( G \) can be reached. \( T(0,0,t) \) is
the discounted value for final harvest at age \( t \). The
recursion continues for a specified number of stages or can be automatically terminated as soon as $T(0,0,t)$ declines. The soil expectation maximization rotation $t^*$ is found by choosing rotation $t$ such that

$$Se(t^*) = \max_t \left\{ \frac{T(0,0,t)(1+r)^t}{(1+r)^{t-1}} \right\}$$

where $r$ is the alternative rate of return.

Most of the examples show heavy thinning in the first stage. DFIT optima are highly sensitive to number of trees in the young stand and this suggests as Reukema and Bruce (1977) note that precommercial thinning might be desirable. The revenues from the cutting in the first stage in the examples are negative and it is only the diameter acceleration impact that makes them optimal. Higher levels of small-diameter material logging costs eliminate early thinnings from optimal regimes. Negative current revenue thinnings can be constrained from optimal solutions for managers who are assumed to be averse to silvicultural investment of this sort.

**PRECOMMERCIAL THINNING AND FERTILIZATION CONSIDERATIONS**

As mentioned in Chapter I, most of the studies on forest stand optimization have concentrated on partial analysis which considers only one or two silvicultural practices. The interaction of silvicultural inputs are neglected. Using a three descriptor dynamic programming model, two other important silvicultural activities, i.e.
precommercial thinning and fertilization, can be considered. This makes the resulting optimal management regime an integrated best management regime.

The addition of precommercial thinning does not cause any difficulty. Precommercial thinning is only considered once, so it is just another stage prior to the stage of the first commercial thinning. The timing and intensity of precommercial thinning can be considered simultaneously, i.e. in the precommercial thinning stage the state space is a two-dimensional instead of one-dimensional state (here we combine the number of trees and basal area together as one dimension of stocking) which is comprised of intensity of thinning only. Figure VI illustrates how the two-dimensional space in the precommercial thinning stage is used without causing any dimension problem. Each intensity and timing of precommercial thinning represents a different initial condition. Hence the only work added is to calculate the growth of several initial conditions instead of only one. In DFIT it is assumed that trees left after precommercial thinning will not die except for those captured in commercial thinning. This is why the number of trees left after precommercial thinning will stay at the same level to the next stage as shown in Figure VI. This assumption does not seem to be biologically reasonable and clearly DFIT was not intended to be used with precommercial thinning alone. Some precommercial-thinning-alone results
Figure VI. Timing and intensity of precommercial thinning (PCT) is considered simultaneously as in one stage.

are presented later for comparative purposes. DFIT also represents a plantation as a stand precommercially thinned at age two. In this study the traditional timing for precommercial thinning is followed, i.e. precommercially thin at age ten. Only the intensity of precommercial thinning is considered.

Turnbull and Peterson (1976) have presented the basic preliminary models of fertilization response in Douglas-fir. These models use site, number of trees, basal area and stand age as descriptors of stand response. Since site is
handled outside of the dynamic programming model, the remaining three descriptors just fit into our three descriptor dynamic programming model.

The fertilization alternatives considered are 400 pounds, 200 pounds and zero pounds of nitrogen per acre. If a backward recursion were used one extra descriptor to represent amount of fertilization would be needed. Using the forward recursive technique this problem is eliminated. Figure VII is a two-dimensional diagram showing how the necessity of another descriptor is eliminated. The

Figure VII. Fertilization alternatives are handled through extra computations instead of an extra dimension. Broken lines and solid lines are paths for fertilization and without fertilization, respectively.
discounted value of a node with stocking \( y \) at age \( x+1 \) is chosen such that

\[
T(x+1,y) = \max \left\{ T(x,y') + P(y',y,I) - FC \right\},
\]

where \( P(y',y,I) \) is the revenue derived by going from a node with stocking \( y' \) to a node with stocking \( y \) in next stage. \( I \) is equal to one or zero depending on whether fertilization is selected or not. \( FC \) is the fertilization cost.

For example, \( T(x+1,1) = \max \{ T(x,1) + P(1,1,1) - FC; T(x,1) + P(1,1,0) \} \). Fertilization results in a higher amount of growth. In the above example, if the extra unit of stocking resulting from fertilization has a value higher than the cost of fertilization then the "do fertilize" alternative will be chosen. If the value of the extra growth does not pay the cost of fertilization then the "do not fertilize" alternative will be chosen. Node \((x+1,3)\) can be reached only by fertilization hence it carries a value of \( T(x,1) - FC \). Figure VII shows only two fertilization alternatives, the same concept can be applied to any number of fertilization alternatives. This simple approach can be applied to any treatment whose growth impact is equal to or less than the thinning interval.

**COMPUTATIONAL EFFICIENCY**

The principle merit of dynamic programming is that it offers an efficient method of generating and evaluating the immense number of alternatives that exist within the
feasible thinning-rotation set within an acceptable precision. Using the stand optimization problem as an example, with precommercial thinning at age ten to 400 trees, commercial thinning every ten years starting from age 30 with three levels of fertilization considered, and a thinning interval of 15 trees, Table I shows the number of feasible alternatives of getting to a certain stocking level (number of trees) at a certain stage (stand age) when the first commercial thinning is restricted to removals of less than 50 percent of the stocking. For example, the number of alternatives for getting to a stocking level of 270 trees at the second stage is 30 (Table I), and the number of alternatives of clearcutting the stand at stage five is 2,072,385. For an eighty year rotation there are 39,785,661 (around 40 million) alternatives. The assumption here is that each discrete 15 tree thinning alternative creates a unique basal area.

If all those 40 million alternatives are examined and if all the intermediate stages and states are recorded then all information of stage and state combinations before age 80 are known. The growth function and their price and cost functions have to be calculated 40 million times. This is the result of considering discrete tree interval and continuous basal area. If all alternatives in the neighborhood of four-square-foot basal-area intervals are grouped together, i.e. discrete tree interval and discrete basal
<table>
<thead>
<tr>
<th>Stage</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>10</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
</tr>
<tr>
<td>#Trees</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>390</td>
<td>375</td>
<td>360</td>
<td>345</td>
<td>330</td>
<td>315</td>
</tr>
<tr>
<td></td>
<td>250</td>
<td>240</td>
<td>225</td>
<td>210</td>
<td>195</td>
<td>180</td>
<td>165</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>135</td>
<td>120</td>
<td>105</td>
<td>90</td>
<td>75</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>30</td>
<td>15</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE I.** NUMBER OF ALTERNATIVES WHICH REACH A CERTAIN STATE AT A CERTAIN STAGE WHEN COMPLETE ENUMERATION IS CONSIDERED.

<table>
<thead>
<tr>
<th>#Trees</th>
<th>Number of Alternatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>1  1  3  9  27  81  243</td>
</tr>
<tr>
<td>390</td>
<td>0  1  6  27 108 405 1458</td>
</tr>
<tr>
<td>375</td>
<td>0  1  9  54 270 1215 5103</td>
</tr>
<tr>
<td>360</td>
<td>0  1 12  90 540 2835 13608</td>
</tr>
<tr>
<td>345</td>
<td>0  1 15 135 945 5670 30618</td>
</tr>
<tr>
<td>330</td>
<td>0  1 18 189 1512 10206 61236</td>
</tr>
<tr>
<td>315</td>
<td>0  1 21 252 2268 17010 112266</td>
</tr>
<tr>
<td>300</td>
<td>0  1 24 324 3240 26730 192456</td>
</tr>
<tr>
<td>285</td>
<td>0  1 27 405 4455 40095 312714</td>
</tr>
<tr>
<td>270</td>
<td>0  1 30 495 5940 57915 486486</td>
</tr>
<tr>
<td>255</td>
<td>0  1 33 594 7722 81081 729729</td>
</tr>
<tr>
<td>240</td>
<td>0  1 36 702 9828 110565 1061424</td>
</tr>
<tr>
<td>225</td>
<td>0  1 39 819 12285 147420 1503684</td>
</tr>
<tr>
<td>210</td>
<td>0  1 42 945 15120 192780 2082024</td>
</tr>
<tr>
<td>195</td>
<td>0  0 42 1071 18333 247779 2825361</td>
</tr>
<tr>
<td>180</td>
<td>0  0 42 1197 21924 313551 3766014</td>
</tr>
<tr>
<td>165</td>
<td>0  0 42 1323 25893 391230 4939704</td>
</tr>
<tr>
<td>150</td>
<td>0  0 42 1449 30240 481950 6385554</td>
</tr>
<tr>
<td>135</td>
<td>0  0 42 1575 34965 586845 8146089</td>
</tr>
<tr>
<td>120</td>
<td>0  0 42 1701 40068 707049 10267236</td>
</tr>
<tr>
<td>105</td>
<td>0  0 42 1827 45549 843696 12798324</td>
</tr>
<tr>
<td>90</td>
<td>0  0 42 1953 51408 997920 15792084</td>
</tr>
<tr>
<td>75</td>
<td>0  0 42 2079 57645 1170855 19304649</td>
</tr>
<tr>
<td>60</td>
<td>0  0 42 2205 64260 1363959 23396526</td>
</tr>
<tr>
<td>45</td>
<td>0  0 42 2331 71253 1577394 28128708</td>
</tr>
<tr>
<td>30</td>
<td>0  0 42 2457 78624 1813266 33568506</td>
</tr>
<tr>
<td>15</td>
<td>0  0 42 2583 86373 2072385 39785661</td>
</tr>
<tr>
<td>0</td>
<td>0  0 42 2583 86373 2072385 39785661</td>
</tr>
</tbody>
</table>
area interval, then the number of computations needed will be greatly reduced. For each tree stocking level, the alternatives with basal area larger than 286 square feet are grouped together as a big neighborhood. Table II shows the number of feasible nodes at different tree stocking levels. For example, 42 alternatives at age 30, tree stocking level of 210 trees (Table I) are categorized to 16 alternatives according to their basal area (Table II). From Table II Table III can be derived which shows the number of alternatives leading to a certain stage-state combination from the previous stage. The number of alternatives of clearcutting the stand at stage five is 2160 compared with 2,072,385 in Table I. For an eighty year rotation there are 2448 alternatives compared with 39,785,661 alternatives in Table I. The total number of function calculations is, from Table II, \[14+(257-1)+(517-1)+(721-1)+(817-1)+(846-1)]*3= 9501. The one subtracted in the parenthesis is because the zero stocking does not require growth calculation. The multiplier three is to indicate three levels of fertilization. Compared with the continuous basal area—39,785,661 calculations, only 0.00008 of the effort is needed for the four square foot basal area interval. The number of alternatives, 2448, is less than the number of calculations needed, 9501, because some of the resulting states are grouped together and only one alternative is used to represent that state.
TABLE II. NUMBER OF FEASIBLE NODES FOR A CERTAIN STATE AT A CERTAIN STAGE WHEN ALTERNATIVES IN A FOUR SQUARE FOOT NEIGHBORHOOD ARE GROUPED TOGETHER USING THE BEST TO REPRESENT THE STATE.

<table>
<thead>
<tr>
<th>Stage</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>10</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
</tr>
<tr>
<td>#Trees</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Number of Feasible Nodes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>6</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>390</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>8</td>
<td>8</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>375</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>11</td>
<td>11</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>360</td>
<td>0</td>
<td>1</td>
<td>7</td>
<td>12</td>
<td>13</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>345</td>
<td>0</td>
<td>1</td>
<td>9</td>
<td>15</td>
<td>16</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>330</td>
<td>0</td>
<td>1</td>
<td>10</td>
<td>16</td>
<td>18</td>
<td>16</td>
<td>15</td>
</tr>
<tr>
<td>315</td>
<td>0</td>
<td>1</td>
<td>11</td>
<td>17</td>
<td>21</td>
<td>19</td>
<td>17</td>
</tr>
<tr>
<td>300</td>
<td>0</td>
<td>1</td>
<td>11</td>
<td>18</td>
<td>23</td>
<td>21</td>
<td>20</td>
</tr>
<tr>
<td>285</td>
<td>0</td>
<td>1</td>
<td>12</td>
<td>20</td>
<td>25</td>
<td>24</td>
<td>23</td>
</tr>
<tr>
<td>270</td>
<td>0</td>
<td>1</td>
<td>13</td>
<td>21</td>
<td>28</td>
<td>26</td>
<td>25</td>
</tr>
<tr>
<td>255</td>
<td>0</td>
<td>1</td>
<td>14</td>
<td>23</td>
<td>30</td>
<td>29</td>
<td>28</td>
</tr>
<tr>
<td>240</td>
<td>0</td>
<td>1</td>
<td>15</td>
<td>24</td>
<td>33</td>
<td>31</td>
<td>30</td>
</tr>
<tr>
<td>225</td>
<td>0</td>
<td>1</td>
<td>16</td>
<td>26</td>
<td>34</td>
<td>34</td>
<td>33</td>
</tr>
<tr>
<td>210</td>
<td>0</td>
<td>1</td>
<td>16</td>
<td>26</td>
<td>35</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>195</td>
<td>0</td>
<td>0</td>
<td>15</td>
<td>26</td>
<td>36</td>
<td>39</td>
<td>38</td>
</tr>
<tr>
<td>180</td>
<td>0</td>
<td>0</td>
<td>14</td>
<td>26</td>
<td>36</td>
<td>41</td>
<td>41</td>
</tr>
<tr>
<td>165</td>
<td>0</td>
<td>0</td>
<td>13</td>
<td>26</td>
<td>36</td>
<td>44</td>
<td>43</td>
</tr>
<tr>
<td>150</td>
<td>0</td>
<td>0</td>
<td>12</td>
<td>25</td>
<td>37</td>
<td>46</td>
<td>46</td>
</tr>
<tr>
<td>135</td>
<td>0</td>
<td>0</td>
<td>11</td>
<td>25</td>
<td>37</td>
<td>46</td>
<td>49</td>
</tr>
<tr>
<td>120</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>25</td>
<td>36</td>
<td>47</td>
<td>51</td>
</tr>
<tr>
<td>105</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td>24</td>
<td>36</td>
<td>46</td>
<td>52</td>
</tr>
<tr>
<td>90</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td>22</td>
<td>34</td>
<td>45</td>
<td>50</td>
</tr>
<tr>
<td>75</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>20</td>
<td>33</td>
<td>43</td>
<td>48</td>
</tr>
<tr>
<td>60</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>18</td>
<td>30</td>
<td>42</td>
<td>49</td>
</tr>
<tr>
<td>45</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>16</td>
<td>28</td>
<td>40</td>
<td>47</td>
</tr>
<tr>
<td>30</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>12</td>
<td>23</td>
<td>34</td>
<td>39</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>9</td>
<td>17</td>
<td>24</td>
<td>28</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Total   | 1 | 14| 257| 517| 721| 817| 846
TABLE III. NUMBER OF ALTERNATIVES LEADING TO A CERTAIN STATE AT A CERTAIN STAGE FROM THE PREVIOUS STAGE WHEN ALTERNATIVES IN FOUR SQUARE FOOT NEIGHBORHOODS ARE GROUPED TOGETHER USING THE BEST TO REPRESENT THE STATE.

<table>
<thead>
<tr>
<th>Stage Age #Trees</th>
<th>0 1 2 3 4 5 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>1 1 3 6 15 18 12</td>
</tr>
<tr>
<td>390</td>
<td>0 1 6 18 39 42 30</td>
</tr>
<tr>
<td>375</td>
<td>0 1 9 33 72 75 54</td>
</tr>
<tr>
<td>360</td>
<td>0 1 12 54 108 114 87</td>
</tr>
<tr>
<td>345</td>
<td>0 1 15 81 153 162 129</td>
</tr>
<tr>
<td>330</td>
<td>0 1 18 111 201 216 117</td>
</tr>
<tr>
<td>315</td>
<td>0 1 21 144 252 279 234</td>
</tr>
<tr>
<td>300</td>
<td>0 1 24 177 306 348 279</td>
</tr>
<tr>
<td>285</td>
<td>0 1 27 213 366 423 369</td>
</tr>
<tr>
<td>270</td>
<td>0 1 30 252 429 507 447</td>
</tr>
<tr>
<td>255</td>
<td>0 1 33 294 498 597 534</td>
</tr>
<tr>
<td>240</td>
<td>0 1 36 339 570 696 627</td>
</tr>
<tr>
<td>225</td>
<td>0 1 39 387 648 798 729</td>
</tr>
<tr>
<td>210</td>
<td>0 1 42 435 726 903 837</td>
</tr>
<tr>
<td>195</td>
<td>0 0 42 480 804 1011 954</td>
</tr>
<tr>
<td>180</td>
<td>0 0 42 522 882 1119 1077</td>
</tr>
<tr>
<td>165</td>
<td>0 0 42 561 960 1227 1209</td>
</tr>
<tr>
<td>150</td>
<td>0 0 42 597 1035 1338 1347</td>
</tr>
<tr>
<td>135</td>
<td>0 0 42 630 1110 1449 1485</td>
</tr>
<tr>
<td>120</td>
<td>0 0 42 660 1185 1557 1626</td>
</tr>
<tr>
<td>105</td>
<td>0 0 42 687 1257 1665 1764</td>
</tr>
<tr>
<td>90</td>
<td>0 0 42 711 1323 1767 1899</td>
</tr>
<tr>
<td>75</td>
<td>0 0 42 729 1383 1866 2028</td>
</tr>
<tr>
<td>60</td>
<td>0 0 42 741 1437 1956 2154</td>
</tr>
<tr>
<td>45</td>
<td>0 0 42 753 1485 2040 2274</td>
</tr>
<tr>
<td>30</td>
<td>0 0 42 762 1521 2109 2376</td>
</tr>
<tr>
<td>15</td>
<td>0 0 42 768 1548 2160 2448</td>
</tr>
<tr>
<td>0</td>
<td>0 0 42 768 1548 2160 2448</td>
</tr>
</tbody>
</table>
If the size of basal area interval or tree interval is increased, less computations will be needed for more alternatives are grouped together in a neighborhood.

**BACKWARD SOLUTIONS**

Traditional dynamic programming problems are solved by the backward solution method using a backward recursive relation. The solution procedure is to move backward stage by stage-- each time finding the optimal policy for each state of that stage-- until the optimal policy is found at the beginning of the network. Using the notation of Amidon and Akin (1968), the recursive relation is

\[ T(x,y) = \max \{D(x,y) + T(x+1,y+1); I(x,y) + T(x+1,y-1)\} \]

A generalized notation will be

\[ f^*(s) = \max \{C(s,x_n) + f^*(x_{n+1})\} \]

Finding the optimal policy when starting in state \( s \) at stage \( n \) requires finding the maximizing value of \( x_n \). \( C(s,x_n) \) represents the cost incurred from state \( s \) in the current stage to state \( x_n \) in the next stage. Figure VIII is a graphical interpretation of the backward recursive relation. For a generalized dynamic programming problem with \( s \) states at each stage, if every state can be reached from every state, the number of computations needed will be \( s^{n-1} \) (Figure IX), for a problem with \( n \) stages and using the complete enumeration method. If the backward solution
method is used, the number of computations needed will be \((n-2)s^2 + s\) for \(n \geq 2\), and \(s\) for \(n=1\) (Figure X). As \(n\) and \(s\) get large, the efficiency of using the backward solution method is obvious.

The most important advantage of the backward solution in stand level optimization, is that the intermediate solutions represent optimal regimes from every current state, to the end of the fixed rotation. Should a stand end up off of the optimal path at some stage, then the optimal thinning regime to the end of the rotation is available as an intermediate solution. A principle disadvantage of the backward recursion is that a separate solution is required for each rotation length. The
Figure IX. Number of computations needed when all paths are considered.

Figure X. Number of computations needed when backward recursive method is used.
"neighborhood" concept discussed in this study introduces an additional problem in the backward recursion. The stands are grown from continuous basal area and (in the case of "no thinning") continuous tree states starting from a given stand state. The backward recursion would have to begin with the discrete basal area and number of tree node variables and would therefore not track the exact values of a continuous growth model and given initial stand condition precisely.

**FORWARD SOLUTIONS**

The basic idea of the forward solution method is to compare the value of every node in the current stage which can reach a certain node in the next stage. The maximal value will be chosen as the value of that certain node in the next stage and the node in the current stage resulting in the maximal value will be chosen as a path. The functional relationship is

\[ f^*_{n+1}(s) = \max_{x_n} \{ C(x_n, s) + f^*_n(x_n) \} \]

Every notation has the same meaning as mentioned before. A graphical interpretation is shown in Figure XI. For a problem with n stages and s states, the number of computations needed is also \((n-2)s^2+s\) for \(n \geq 2\) and \(s\) for \(n=1\) (Figure XII). The number of computations needed for forward solution and backward solution methods are the same.
Figure XI. Forward recursive relation.

Figure XII. Number of computations needed when forward recursive method is used.
One advantage of the forward solution method in stand optimization is it tells the best path (thinning regime) for every rotation (stage) in one set of calculations. Another advantage of the forward solution method is it handles continuous state problems in discrete state dynamic programming. Kao and Brodie (1979b) first introduced the technique of handling continuous growth in a discrete volume cutting interval. Brodie and Kao (1979) introduced the neighborhood concept to handle the continuous state problem in a three dimensional dynamic programming problem. The disadvantage of the forward recursion is that the intermediate are optimal paths from the beginning of the network to the current stage and represent management decisions already made. Should a stand end up in a non-optimal path state then the optimal path from that state to the end of the rotation can only be found by restarting the recursion from the current stand state.

SUMMARY

Dynamic programming is a powerful and flexible stand level optimization tool, for:

(1) It is computationally simple.

(2) It handles growth, cost and revenue functions of many forms.

(3) It is an efficient method of selecting among an immense number of alternatives.
(4) It considers different silvicultural activities and makes interrelated policy decisions.

Using dynamic programming, four important management activities: commercial thinning, precommercial thinning, regeneration harvest and fertilization can be combined together to get an optimal management regime, accounting for the interactions between the management activities, stocking levels and rotation. The interaction of the four silvicultural activities and size dependent costs and revenues can be examined simultaneously and analytically for particular situations and not just by partial analysis or a single empirical test as in the past.
IV. THE STRUCTURE AND OPERATION OF DFIT AND DOPT

The DFIT (Douglas-fir Interim Tables) program (Bruce, Demars and Reukema, 1977) simulates stand growth and tabulates the results of the simulation. The basic components of DFIT are: (1) equations describing the development of natural stands, (2) thinning guides (Reukema, 1975) based on Reineke's stand density index (Reineke, 1933), (3) equations describing the total cubic-foot increment of thinned stands and plantations, (4) equations predicting the amount and timing of mortality in natural stands, (5) a method for describing stand components at intervals without the direct use of stand and stock tables, and (6) many assumptions about management practices and their effects on stand development. The assumptions are based on study of current available results of Douglas-fir thinning studies. They have been balanced against growth of natural stands and have been checked for compatibility and consistency (Reukema and Bruce, 1977). It is claimed that DFIT was made as flexible as possible subject to the constraint of providing a timely interim tool. Within limits DFIT allows the user to test and observe probable results of different management regimes.

DOPT (DFIT OPTimization) is a stand optimization computer program which uses the stand growth model from DFIT and dynamic programming to find the best management
regime. DFIT is a simulator which provides descriptive stand outputs when management regime is provided, and DOPT is an optimizer which provides a regime when a criterion and revenue and cost assumptions are provided. As mentioned earlier, a simulator gives a set of results for each regime supplied. These sets of results are compared to gradually improve on the criterion sought, e.g. maximum present net worth from managing a forest stand. Such an approach may find a "good" answer but only on very simple systems, does it find the true maximum. An optimizer gives the answer one looks for in one set of calculations. In most cases a set of optimization calculations is more difficult than a set of simulation calculations. But the total time used for an exhaustive simulation search is usually larger than that used in optimization. The results from an optimization often give more information than a simulator does.

In the following sections the modules of the DOPT computer program are discussed. (The DOPT program listing is in Appendix III.) Most of them are based on DFIT. The modules include: (1) precommercial thinning (PRECOM), (2) submerchantable mortality (SUBMORT), (3) growth (GROWTH), (4) logging cost (REvNOW), and (5) increment (INCRMNT). A "*" is used to indicate equations taken directly from DFIT (Bruce, Demars and Reukema, 1977) and the DFIT program listing. The meaning of notations used in the following
equations are listed in Appendix I.

**PRECOMMERCIAL THINNING (PRECOM)**

From DFIT the number of trees per acre in a precommercially thinned stand, given diameter at first possible commercial thinning is

\[ (*) \log N = 3.9591 - 1.5006 \log D_F \]  \hspace{1cm} (1)

When number of trees after precommercial thinning is given, the corresponding \( D_F \) will be

\[ \log D_F = (3.9591 - \log N) / 1.5006 \]  \hspace{1cm} (2)

The basal area per acre of merchantable trees, given average diameter of merchantable trees at first possible commercial thinning, is

\[ (*) \log G_M = 1.6958 + 0.4994 \log D_F \]  \hspace{1cm} (3)

After precommercial thinning, the site index is adjusted for superior height growth:

\[ (*) S_P = S*(1+(210-S)^2/90000) \]

\[ \text{IF}(S > 210) \text{ } S_P = S \]  \hspace{1cm} (4)

This equation increases site index for the growth simulation between precommercial thinning and first commercial thinning. Improved vigor of the stand is assumed to last indefinitely. Relationships for transforming total age to breast height age and for calculating the breast height age at which a stand of given site will reach a given diameter
Then the age is adjusted by the age of precommercial thinning:

\[
(*) \text{IF}(A_p \geq 15 \text{ AND } A_p \leq 20) \quad A_F = A_T(0.68 + 0.016A_p) \\
\quad \text{IF}(A_p \geq 2 \text{ AND } A_p \leq 14) \quad A_F = A_T(0.8244 + 0.004333 \times (A_p/10) + 0.0158 \times (A_p/10)^2 + 0.01667 \times (A_p/10)^3)
\]

The model does not permit precommercial thinning after age 20. Using the above equations, the time for the first possible commercial thinning can be calculated when time for precommercial thinning and number of trees left after precommercial thinning are given. If the stand will not reach the prescribed diameter for the remaining trees at the time of first commercial thinning, the DOPT program stops here and prints out an error message.

If less than 400 trees per acre are left, the adjusted basal area per acre is

\[
(*) G_S = 180.22 - 5D_F
\]

If \( G_M \) calculated from equation (3) is no less than \( G_S \), the stand is simulated to grow year by year, so \( G_S \) increases, until \( G_M \) is less than \( G_S \). The growth of the stand will be discussed in the GROWTH section.
Usually the age for the first possible commercial thinning calculated from the above equations is noninteger value, hence an adjustment is added, by adding the growth of the remaining fractional year, to round the age of the stand up to the nearest integer age. If the rounded age is smaller than the age considered for the first commercial thinning, a growth function is used to grow the stand to the age of the first commercial thinning specified. A flowchart is shown in Appendix IV.

**SUBMERCHANTABLE MORTALITY (SUBMORT)**

Given the mean diameter of a natural stand of current age and site ($\bar{D}_G$) and the minimum merchantable diameter ($D_m$), the number of submerchantable trees per acre ($N_s$) and their corresponding basal area ($G_s$) in a natural stand can be calculated by using the equations:

\[
(*) \log N_s = 3.8622 + 3.1994 \log D_m - 4.70 \log \bar{D}_G \quad (8)
\]

\[
(*) \log G_s = 1.4034 + 4.9394 \log D_m - 4.44 \log \bar{D}_G \quad (9)
\]

Differences between estimates at successive ages give submerchantable mortality between those ages. The following equations give the total number of trees and their corresponding basal area per acre in a natural stand. Given these values and an estimate of submerchantable mortality, the merchantable mortality can be estimated.

\[
(*) \log N = 3.9108 + 5.2306/A_B^{0.25} - 1.5803 \log S \quad (10)
\]
The merchantable mortality is calculated in the GROWTH subroutine and will be discussed later. Here only the nonmerchantable mortality derivation is considered. First, the minimum merchantable diameter is defined as
\[(*) \quad D_m = (\overline{D}_{30}/0.875) \times 0.75 \tag{12} \]
where \(\overline{D}_{30}\) is the mean diameter of a natural stand at age 30, or
\[(*) \quad \log \overline{D} = 0.1097 - 3.4857/A_B^{0.25} - 1.0531 \log S \tag{13} \]
The mean diameter of submerchantable trees at age 30 \((D_L)\) is defined as
\[(*) \quad D_L = 0.698 \times \overline{D}_{30} \tag{14} \]
The periodic submerchantable mortality of number of trees and basal area can be calculated by using equations (8) and (9). The mortality of volume takes more effort to derive. In a natural stand the volume/basal area ratio can be expressed as a function of dominant and codominant height \((\overline{H}_D)\), and the height is a function of age and site.
\[(*) \quad \log \overline{H}_D = 0.1567 - 15.673/A_T + \log S \tag{15} \]
\[(*) \quad \log V/G = -0.0282 + 0.7917 \log \overline{H}_D \tag{16} \]
The mortality of volume is the mortality of basal area multiplied by \(V/G\) and adjusted by the Washington Department of Natural Resources tarif. A tarif diameter function, given diameter, is

\[(*) \quad \log G = 1.8669 - 1.7408/A_B^{0.25} + 0.5259 \log S \tag{11} \]
\begin{align*}
(*) \ & F = 0.004978* \frac{\overline{D}_G^2}{(0.005454* (\overline{D}_G^2 + 16) * (1.0378 + \
1.4967 * 0.0134) - 0.1745)} \\
\text{Washington Department of Natural Resources tariff is} \\
(*) \ & T = F*V/G \\
\text{This relation is approximate, not exact, and is applied} \\
generally to the smallest stand components so the error in volume is minimized. Volume of larger stand components are estimated from volume of total stand and volume of small components. From equation (18) we formulate the following equation:} \\
F(D_L) * \frac{V_S'}{G_S'} = F(\overline{D}_G) * \frac{V}{G} \\
\text{rearranging:} \\
V_S' = G_S' * \frac{(F(\overline{D}_G) / F(D_L)) * V}{G} \\
\text{Where } V_S' \text{ and } G_S' \text{ are mortalities between successive periods. After the first calculation, } D_L \text{ in equation (20) is substituted by } D_L' = ((G_S' / N_S') / 3.1416)^{0.5} * 12, \text{ and } F(\overline{D}_G) * V/G \text{ is substituted by } (F(\overline{D}_G) * V/G + F(\overline{D}_G) * (V/G)')/2 \text{ where } \overline{D}_G' \text{ is the mean diameter at the age when mortality is to be calculated and } (V/G)' \text{ is the volume/basal area ratio at that age. Hence} \\
V_S = G_S' * \frac{(F(\overline{D}_G) * V/G + F(\overline{D}_G') * (V/G)') * 0.5}{F(D_L')} \\
\text{In DOPT both yearly and ten-year mortality are calculated. If a stand is not normal, the stand condition is}
\end{align*}
compared with a normal stand and the mortalities are adjusted linearly. A flowchart of the subroutine SUBMORT is shown in Appendix IV.

**GROWTH (GROWTH)**

In a natural stand, the volume per acre can be expressed as a function of site and stand age:

\[(*) \log V = 1.9628 - 12.4083/A_T - 1.7408/A_B^{0.25} + 1.3176 \log S \]  \tag{22}

One-year volume growth of the stand is just the derivative of equation (22) over age:

\[(*) \log \frac{dV}{dA_T} = \log V + \log 2.3026 + \log \left( \frac{12.4083}{A_T} + 0.4352/A_B^{1.25} \right) \]  \tag{23}

Equation (23) was compared with Staebler's (1955) and Curtis' (1967) estimates of gross growth for ages 30 to 100 and adjusted by \( \log dV_A \) to approximate gross growth ratios. Equations (23) and (24) are combined to equation (25):

\[(*) \log dV_A = \log (1.12 + 0.0105A_T - 0.00005A_T^2) \]

\[\text{IF}(A_T \geq 105) \log dV_A = 0.22304 \]  \tag{24}

\[(*) \log \frac{dV'}{dA_T} = \log \frac{dV}{dA_T} + \log dV_A \]  \tag{25}

Using the same argument, height growth is the derivative of the function of tree height, equation (15), over age:

\[(*) \log dH/dA_T = 1.7141 + \log S - 15.673/A_T - 2\log A_T \]  \tag{26}
Given total number of trees, the top limit of basal area in natural stand can be expressed as

\[(*) \log G_L = 3.3446 - 0.3328 \log N \quad (27)\]

The volume growth calculated from equation (25) is first adjusted by an adjustment factor which is a function of age of first commercial thinning \((A_{CT})\).

\[(*) ADJ1 = (405 - A_{CT})/400 \quad (28)\]

Then the volume growth is adjusted by a competition growth reduction multiplier which is a function of the average basal area of living merchantable trees \((G_M)\) to the top limit of basal area in a natural stand:

\[(*) G_Z = G_M/G_L \quad (29)\]

\[(*) ADJ2 = 1 -16*(G_Z - 0.5)^4 \quad (30)\]

Figure XIII shows the shape of \(ADJ2\) over \(G_Z\). When \(G_Z\) is small, it implies \(G_M\) is small, hence the growth will be small compared with the growth from a normal stand. When \(G_Z\) is zero, i.e. no stocking in the stand, the growth is also zero. When \(G_Z\) is large, it means the stand is overstocked, hence the growth is smaller than the growth of a natural stand. If \(G_Z\) is one, i.e. the stand has reached the top limit that a stand can be, then the growth is zero. This symmetrical function gives an approximation of the plateau of maximum yield hypothesized for stands of different density.

Given number of trees \((N)\), basal area \((G)\), volume \((V)\),
Figure XIII. The relation between growth adjustment ADJ2 and stocking level $G_Z$. 
age and site index of a stand, the gross growth can be calculated by following the above procedures. The net growth can be calculated by using equation (23), instead of equation (25), i.e. the growth without adjustment of mortality. The difference between gross growth and net growth is merchantable mortality. A flowchart which shows how growth is calculated is in Appendix IV.

**LOGGING COST (REVNOM)**

Sessions (1979) derived stump-to-truck harvesting costs as a function of merchantable volume removed per acre by using simulation. In DOPT the cost of a 600-foot skyline was chosen and divided by 1.5 to approximate the cost of using a tractor. This ratio was chosen because a study by Aulerich, Johnson and Froehlich (1974) showed that skyline costs are 1.5 to 1.66 times those for tractor logging. Not many studies have discussed logging cost in as much detail as Sessions did. DOPT will operate with any computable logging cost functions for which DFIT will provide the necessary stand variables. This study was conducted at cost levels 2/3 those derived by Sessions and demonstrates the impact on regime of these lower cost levels. Comprehensive sensitivity to logging cost is dealt with in Sessions (1979). Table IV lists larger yarder power curve coefficients for stump to truck harvesting costs as a function of merchantable volume per acre removed.
for a 600-foot skyline.

TABLE IV. LARGER YARDER POWER CURVE COEFFICIENTS FOR STUMP TO TRUCK HARVESTING COSTS AS A FUNCTION OF MERCHANTABLE VOLUME REMOVED PER ACRE FOR A 600-FOOT SKYLINE.

\[
\text{COST} (\$/\text{MCF}) = A \cdot V^B
\]

<table>
<thead>
<tr>
<th>Mean Diameter</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.87</td>
<td>8479.6</td>
<td>-0.3626</td>
</tr>
<tr>
<td>12.31</td>
<td>6839.5</td>
<td>-0.3492</td>
</tr>
<tr>
<td>13.66</td>
<td>6276.4</td>
<td>-0.3498</td>
</tr>
<tr>
<td>15.03</td>
<td>5848.5</td>
<td>-0.3548</td>
</tr>
<tr>
<td>16.19</td>
<td>4980.1</td>
<td>-0.3475</td>
</tr>
<tr>
<td>17.26</td>
<td>3765.6</td>
<td>-0.3238</td>
</tr>
<tr>
<td>18.31</td>
<td>4215.0</td>
<td>-0.3402</td>
</tr>
<tr>
<td>20.25</td>
<td>3377.5</td>
<td>-0.3207</td>
</tr>
<tr>
<td>21.90</td>
<td>4209.5</td>
<td>-0.3488</td>
</tr>
</tbody>
</table>

When mean diameter is smaller than 10.87 inches the volume removed in each cut is also a factor which determines the harvest cost. How the cost is calculated when diameter is smaller than 10.87 is shown in the flowchart of REVNOW and DOPT program listings in Appendix IV and III. For any given mean diameter, a linear interpolation is used to calculate harvest cost. For example, if mean diameter cut is 20 inches, then the cost will be $4215.0 \times V^{-0.3402}$. 
Revenue from either thinning or regeneration harvest of the stand in the current period is

\[ \text{Revenue} = (\text{Volume Cut}) \times ((\text{Pond Value}) - (\text{Stump To Truck Cost}) - (\text{Haul Cost})) \]

Where pond value is defined as: The market price of logs delivered to a wet site, log pond, or tidewater (Pearce and Stenzel, 1972), i.e. it is the value a mill would pay for logs received at the mill gate. Sessions (1979) used data from Bulletin 201 (McArdle, Meyer and Bruce, 1961), the Columbia River Scaling Rule (Dilworth, 1973) and the current market value of logs to derive a relation between arithmetic mean stand diameter and pond value. Table V shows the relation.

A least squares linear regression line was fitted to the data with an \( R^2 = 0.99 \) providing the following relationship:

\[
\text{Pond Value ($/MCF)} = 9.91 + 70.81 \times (\text{Mean Stand Dbh})
\]

When quadratic mean diameter is larger than 22 inches, there is no additional price premium attached. Haul cost is the cost of transporting logs from the forest stand to mill. A constant $150/MCF was used in this study although it should be recognized that smaller logs cost more to transport due to a lower board foot to cubic foot ratio.
A flowchart which helps to understand how the revenue is calculated is in Appendix IV.

**TABLE V. POND VALUE AS A FUNCTION OF ARITHMETIC MEAN STAND DIAMETER.**

<table>
<thead>
<tr>
<th>Mean Stand Dbh (Inch)</th>
<th>Age</th>
<th>BF/CF Ratio</th>
<th>Pond Value ($/MCF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.60</td>
<td>40</td>
<td>1.93</td>
<td>413.96</td>
</tr>
<tr>
<td>10.40</td>
<td>60</td>
<td>3.46</td>
<td>782.16</td>
</tr>
<tr>
<td>13.70</td>
<td>80</td>
<td>4.15</td>
<td>1002.50</td>
</tr>
<tr>
<td>16.20</td>
<td>100</td>
<td>4.58</td>
<td>1203.83</td>
</tr>
<tr>
<td>18.40</td>
<td>120</td>
<td>4.94</td>
<td>1351.88</td>
</tr>
<tr>
<td>20.20</td>
<td>140</td>
<td>5.11</td>
<td>1417.42</td>
</tr>
<tr>
<td>22.00</td>
<td>160</td>
<td>5.29</td>
<td>1509.37</td>
</tr>
</tbody>
</table>

When the above procedures are used, two transformations must be made. First, the volume used in the growth model is total cubic volume and the volume used in calculating harvest cost is cubic volume to a four-inch top (CV4). The following equations calculate the cubic volume to a four-inch top from total cubic volume (V).

\[
(*) \quad \overline{H}_M = \overline{H}_D \cdot (3040 - N)/3000 \\
\text{IF}(\overline{H}_M \text{ GT } \overline{H}_D) \quad \overline{H}_M = \overline{H}_D \\
(*) \quad CV4 = V \cdot (0.8758 + 0.001049 \cdot \overline{H}_M - 0.000002824 \cdot \overline{H}_M^+ + 0.3221\overline{D}_G - 45.647/\overline{D}_G^3)
\]
Where $\bar{H}_M$ is the stand mean height, $\bar{H}_D$ is average height of dominants and codominants, and $N$ is total number of trees.

Second, the diameter used in the growth model is quadratic mean diameter ($\bar{D}_G$), and the diameter used in calculating harvest cost is arithmetic mean diameter ($\bar{D}_N$). Sessions (1979) showed that

$$\bar{D}_N^2 = \bar{D}_G^2 - \text{Var}(\bar{D}_N) \quad (34)$$

where $\text{Var}(\bar{D}_N)$ is the variance of $\bar{D}_N$.

In a stand table, trees are grouped into diameter classes.

Let $N =$ total number of trees in a stand.

$N_i =$ number of trees in the $i$th diameter class.

$D_{ij} =$ diameter of the $j$th tree in the $i$th diameter class.

$\bar{D}_i =$ mean of the $i$th diameter class.

$\bar{D} =$ mean of all trees in the stand.

Then, $\text{Var}(D_{ij}) = \sum_{i} \sum_{j} (D_{ij} - \bar{D})^2 / N$

$$= \sum_{i} \sum_{j} ((D_{ij} - \bar{D}_i) - (\bar{D}_i - \bar{D}))^2 / N$$

$$= \sum_{i} \sum_{j} (D_{ij} - \bar{D}_i)^2 / N + \sum_{i} \sum_{j} (\bar{D}_i - \bar{D})^2 / N +$$

$$2 \sum_{i} \sum_{j} (D_{ij} - \bar{D}_i)(\bar{D}_i - \bar{D}) / N$$

$$= \sum_{i} \sum_{j} (N_i * (D_{ij} - \bar{D}_i)^2 / N_i) / N + \sum_{i} \sum_{j} (\bar{D}_i - \bar{D})^2 / N +$$
\[ 2 \sum_i (\bar{D}_i - \bar{D}) \sum_j (D_{ij} - \bar{D}_i) / N \]
\[ = \sum_i N_i \times \text{Var}(D_{ij} \text{ within the } i\text{th class}) / N + \]
\[ \sum_i N_i \times (\bar{D}_i - \bar{D})^2 / N + 0 \]

(35)

It is reasonable to assume that the diameter of the trees in every diameter class is uniformly distributed. Hence

\[ \bar{D}_i = (L_i + U_i) / 2 \]  

\[ \text{Var}(D_{ij} \text{ within the } i\text{th class}) = (U_i - L_i)^2 / 12 \]  

(36)  

(37)

Where \( U_i \) and \( L_i \) are the upper and lower bound of the \( i\text{th} \) diameter class. Equations (36) and (37) are the definitions for uniform distributions (Mood, Graybill and Boes, 1974). In our case, \( U_i - L_i = 2 \) for every \( i \), so \( \text{Var}(\text{Diameter of the } i\text{th class}) = 1/3 \) for every class. Therefore,

\[ \text{Var}(D_{ij}) = \sum_i (N_i / 3) / N + \sum_i N_i \times (\bar{D}_i - \bar{D})^2 / N \]
\[ = 1/3 + \sum_i N_i \times (\bar{D}_i - \bar{D})^2 / N \]  

(38)

Where \( \sum_i N_i \times (\bar{D}_i - \bar{D})^2 / N \) is the variance among diameter classes. That is to say the variance of \( D_{ij} \) is the sum of variance within diameter class and variance among diameter classes. The above derivation is similar to the study of Sessions (1979). The difference is that Sessions assumed that there was no variance within each diameter class, i.e.
Var(D_{ij}) = \sum N_i (\bar{D}_{i} - \bar{D})^2 / N \text{ compared with equation (38).}

Table VI shows the variances calculated by using equation (38) and data from Bulletin 201. Figure XIV is a diagram showing the same thing.

TABLE VI. VARIANCE OF DIAMETER DISTRIBUTION FOR DIFFERENT SITE-AGE COMBINATION.

<table>
<thead>
<tr>
<th>Age</th>
<th>Site</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
<th>160</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>-</td>
<td>4.42</td>
<td>8.26</td>
<td>8.70</td>
<td>10.66</td>
<td>13.19</td>
<td>15.49</td>
<td></td>
<td></td>
</tr>
<tr>
<td>140</td>
<td>-</td>
<td>6.18</td>
<td>12.17</td>
<td>18.12</td>
<td>23.69</td>
<td>29.92</td>
<td>35.29</td>
<td>40.44</td>
<td></td>
</tr>
<tr>
<td>170</td>
<td>2.28</td>
<td>8.86</td>
<td>17.83</td>
<td>27.07</td>
<td>36.33</td>
<td>44.89</td>
<td>50.22</td>
<td>60.91</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>3.77</td>
<td>14.10</td>
<td>26.96</td>
<td>41.08</td>
<td>55.31</td>
<td>66.45</td>
<td>75.44</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

A least squares linear regression line is fitted with $R^2 = 0.9950$, and

\[
\text{Variance} = 4.0725 - 0.065722s + 0.00001508A_Ts^2
\]  

(39)

Each time a revenue calculation is requested, first, the total cubic volume is changed to cubic volume to a four-inch top, second, the quadratic mean diameter is changed to the arithmetic mean diameter, and finally, the harvest cost and pond value are calculated to get the revenue.
Figure XIV. Variances of diameter distribution for different ages and sites.
INCREMENT (INCRMNT)

Sometimes a stand may be disturbed by climatic factors and biological factors and may not be a normal stand any more. What DOPT will do with this kind of stand is to simulate the stand to the next age of multiple ten based on the current stand condition. Then use dynamic programming to find the best management regime from the simulated initial condition. How to simulate the stand based on the stand condition is done in the subroutine INCRMNT.

The current age of the stand is tested to see if it is a multiple of ten and larger than 30. If it is not, then the next age of multiple ten is the target age that the stand will be projected to. The current number of trees and basal area are compared with those of a normal stand. And NRATIO (current number of trees/number of trees from natural stand) and GRATIO (current basal area/basal area from natural stand) are calculated. The merchantable mortalities between the current age and the next age of multiple ten are adjusted by the NRATIO and GRATIO. The merchantable part is the difference between the total and submerchantable part. The following simulations of growth are the same as those discussed in the GROWTH section. A flowchart showing the structure of this subroutine is in Appendix IV.
MAIN PROGRAM (DOPT)

The responsibilities of the main program are to: (1) input data, (2) calculate the effect of fertilization, (3) use the dynamic programming technique to find the best alternative, and (4) output information. The main program also calls those subroutines mentioned above to fulfill these responsibilities.

Input Data

The data needed for DOPT are INTVR, INTHR, TBASE, SITE, R, TESTN, TESTP, TESTF, REGENC, R1, PCTAGE, NPCT, PCTCOST, STREE, and SBA. Where

INTVR is tree interval considered for cutting. Twelve is the smallest number that can be used and 15 to 50 is suggested.

INTHR is the basal area interval. Four square feet is the smallest value that can be used, and four to 20 is suggested.

TBASE is either the age to be considered for first commercial thinning for a normal stand, or the current age of a non-normal stand. In the current version of DOPT this age must be 30 or larger.

SITE is the 100-year based site index of the stand considered.

R is the alternative rate of return (3% will be 0.03).
TESTN = \{ 
\begin{align*} 
0 & \quad \text{if the stand is a normal stand.} \\
1 & \quad \text{if the stand is not a normal stand.} 
\end{align*} 
\}

TESTP = \{ 
\begin{align*} 
0 & \quad \text{if the stand is not considered for} \\
& \quad \text{precommercial thinning.} \\
1 & \quad \text{if the stand is considered for} \\
& \quad \text{precommercial thinning.} 
\end{align*} 
\}

TESTF = \{ 
\begin{align*} 
0 & \quad \text{if the stand is not considered for} \\
& \quad \text{fertilization.} \\
1 & \quad \text{if the stand is considered for} \\
& \quad \text{fertilization.} 
\end{align*} 
\}

REGENC is regeneration cost ($).

R1 is inflation rate (5% will be 0.05).

PCTAGE is the age considered for precommercial thinning.

NPCT is the number of precommercial thinning alternatives to be considered. The first alternative is to leave STREE trees. The second is to leave \((\text{STREE} - \text{INTVR})\) trees, \((\text{STREE} - 2*\text{INTVR})\) for the third, and so on. When the number of remaining trees is too few, resulting in a first possible commercial thinning age larger than TBASE, the program DOPT will stop considering those heavier precommercial thinning alternatives.

PCTCOST is the cost of precommercial thinning.

If TESTN=1 or TESTP=0, the value of PCTAGE, NPCT and PCTCOST are not used.
STREE is either the total number of trees of a non-normal stand or the first alternative considered for precommercial thinning. When TESTN=0 and TESTP=0 this value is not used.

SBA is the basal area of a non-normal stand. This value is used only when TESTN=1.

The data are read in through free format, so zero values must be applied.

Effect of Fertilization

In 1969 the Regional Forest Nutrition Research Project was initiated with the primary objective of providing resource managers with more accurate data on the effect of fertilizing and thinning young-growth Douglas-fir and Western hemlock forests. A note by Turnbull and Peterson (1976) presented the results of fertilizing 87 installations of six plots each in Douglas-fir natural stands in Western Washington and Oregon. In each installation two of the 1/10 acre plots were not treated, two received 200 pounds nitrogen/acre, and two 400 pounds nitrogen/acre, in the form of urea.

The primary objective was to provide an estimate of mean response to fertilizer according to age, site, and other relevant variables. Response is measured as increase in growth rate due to fertilizer. In simple equation form,
The installation chosen vary in site (site classes I, II, III, and IV, or site indices 200, 170, 140, and 110 using 100-year basis) and breast height age (10, 20, 30, 40, and 50). The following equations show the volume and basal area growth per acre per year with different amounts of fertilization:

\[
\Delta V = (0.10493 A_B^{-0.2269} S_{50}^{1.07647} G^{0.56805} N^{0.10875}) \\
+ (-0.27823 - 0.46375 S_{50} \log(1+PN) + 78.9023 \log(1+PN)) \\
\log(1+PN)
\]

(40)

\[
\Delta G = (12.70144 - 0.22684 A_B - 0.02066 S_{50} + 0.04469 G - 1.64715 G / A_B + 21.74006 G / A_B^2) \\
+ (40.40841 \log(1+PN) / G + 0.00056 \log(1+PN))
\]

(41)

where \(\Delta V\) = periodic annual gross increment of volume, cu. ft./acre/year.

\(\Delta G\) = periodic annual gross increment of basal area,
sq. ft./acre/year.

\[ A_B = \text{breast height age.} \]

\[ S_{50} = 50\text{-year based site index.} \]

\[ G = \text{initial basal area, sq. ft./acre.} \]

\[ N = \text{number of stems/acre.} \]

\[ \text{PN = pounds of nitrogen/acre.} \]

When \( \text{PN}=0 \), \( \Delta V \) and \( \Delta G \) is just the usual growth of a stand. Table VII compares the growths calculated from the growth model of DFIT and the growth model of Turnbull and Peterson (T&P) under different stocking levels for site index 140. A program from which Table VII was derived is in Appendix II. From Table VII, in young and understocked stands, and old and overstocked stands the difference of the growth between these two models is larger. In the first commercial thinning we restrict the thinning to no more than 50 percent of the stocking. And from the DOPT computer runs, under different conditions, none of them has stocking more than 80 percent of a normal stand at every age. Hence we are not in the region of inconsistency.

To make the two models comparable, the percent fertilization effect from the model by Turnbull and Peterson is adjusted by the growth of DFIT model, i.e.

\[ \text{GR(Fertilization)} = \frac{\text{GR(Fertilization)*GR(DFIT)}}{\text{GR(T&P)}} \]  

(42)
<table>
<thead>
<tr>
<th>Normality</th>
<th>( A_B (A_T) )</th>
<th>( \Delta G )</th>
<th>( \Delta V )</th>
<th>( \Delta G )</th>
<th>( \Delta V )</th>
<th>( \Delta G )</th>
<th>( \Delta V )</th>
<th>( \Delta G )</th>
<th>( \Delta V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>DFIT</td>
<td>46.8</td>
<td>2420</td>
<td>22.2</td>
<td>1771</td>
<td>16.2</td>
<td>1478</td>
<td>14.0</td>
<td>1300</td>
</tr>
<tr>
<td></td>
<td>T&amp;P</td>
<td>77.2</td>
<td>3091</td>
<td>62.1</td>
<td>3310</td>
<td>52.2</td>
<td>3207</td>
<td>42.6</td>
<td>3082</td>
</tr>
<tr>
<td>1.0</td>
<td>DFIT</td>
<td>54.7</td>
<td>2483</td>
<td>39.1</td>
<td>2201</td>
<td>32.5</td>
<td>1968</td>
<td>28.2</td>
<td>1769</td>
</tr>
<tr>
<td></td>
<td>T&amp;P</td>
<td>73.4</td>
<td>2732</td>
<td>57.1</td>
<td>2926</td>
<td>45.1</td>
<td>2835</td>
<td>33.4</td>
<td>2724</td>
</tr>
<tr>
<td>0.8</td>
<td>DFIT</td>
<td>60.2</td>
<td>2478</td>
<td>45.5</td>
<td>2264</td>
<td>37.8</td>
<td>2031</td>
<td>32.3</td>
<td>1821</td>
</tr>
<tr>
<td></td>
<td>T&amp;P</td>
<td>69.6</td>
<td>2349</td>
<td>52.1</td>
<td>2515</td>
<td>38.0</td>
<td>2437</td>
<td>24.2</td>
<td>2342</td>
</tr>
<tr>
<td>0.6</td>
<td>DFIT</td>
<td>64.1</td>
<td>2445</td>
<td>50.0</td>
<td>2256</td>
<td>41.2</td>
<td>2024</td>
<td>35.1</td>
<td>1814</td>
</tr>
<tr>
<td></td>
<td>T&amp;P</td>
<td>65.7</td>
<td>1933</td>
<td>47.1</td>
<td>2070</td>
<td>30.9</td>
<td>2006</td>
<td>15.0</td>
<td>1928</td>
</tr>
<tr>
<td>0.4</td>
<td>DFIT</td>
<td>62.4</td>
<td>2229</td>
<td>50.7</td>
<td>2115</td>
<td>41.3</td>
<td>1884</td>
<td>34.6</td>
<td>1673</td>
</tr>
<tr>
<td></td>
<td>T&amp;P</td>
<td>61.9</td>
<td>1470</td>
<td>42.0</td>
<td>1574</td>
<td>23.9</td>
<td>1525</td>
<td>5.8</td>
<td>1465</td>
</tr>
<tr>
<td>0.2</td>
<td>DFIT</td>
<td>44.4</td>
<td>1518</td>
<td>37.0</td>
<td>1467</td>
<td>28.8</td>
<td>1257</td>
<td>23.2</td>
<td>1082</td>
</tr>
<tr>
<td></td>
<td>T&amp;P</td>
<td>58.1</td>
<td>919</td>
<td>37.0</td>
<td>985</td>
<td>16.8</td>
<td>954</td>
<td>-3.4</td>
<td>916</td>
</tr>
</tbody>
</table>

TABLE VII. COMPARISON OF TOTAL CUBIC FOOT GROWTHS FROM GROWTH MODELS BY TURNBULL AND PETERSON AND THE DFIT MODEL ON SITE 140 WITH DIFFERENT GROWING STOCK.
where \( GR(\text{Fertilization}) \) is the growth with fertilization.

\( GR(\text{DFIT}) \) is the growth from DFIT model without fertilization.

\( GR(\text{T&P}) \) is the growth from Turnbull and Peterson growth model without fertilization.

Since data collected from the fertilization experiment is restricted to breast height age younger than 50, or 58.6 total age for site index 140, only stands with age younger than or equal to 60 total age are considered for fertilization in the DOPT program.

The site index used in DFIT is 100-year based. To make the two models comparable, an equation derived by King (1966) is used to change 100-year based site index to 50-year based site index:

\[
S_{50} = 21.5 - 0.18127A_T + 0.72144S_{100}
\]

(43)

where \( A_T \) is total age, and 30 is used here to transform 100-year based site index to 50-year based site index.

The assumed cost for 200 pounds of applied nitrogen fertilization is $45/acre and $85/acre for 400 pounds. These include all transport and application costs required for fertilization.

**Dynamic Programming**

In each stage, every combination of number of trees and basal area is checked to see if it is feasible, i.e. if it can be reached from a node (a combination of trees and
basal area) in the previous stage. If it is feasible then there is nothing to be done. If it is feasible, then three levels of fertilization are considered, if it is requested to do so, to grow the stand to three different states. Every state can be considered for different levels of thinning. The resulting states are categorized to appropriate nodes. The alternative which has the highest cumulative present net worth will occupy that node. The information associated with each node (node value) will be the quadratic mean diameter, level of fertilization, merchantable volume, merchantable basal area, total volume removed, merchantable number of trees and cumulative present net worth of the treatments and harvests to that age. The optimal stand condition (optimal policy) in the previous state, from which the current state is reached, is also stored at each node. This is the forward recursive method. The information carried by each node permits the tracing of all elements of a regime once the recursion is completed.

Output Information

First, the input data are echoed.
Second, the node-value information associated with each state at each stage and state are printed out.
Third, the number of nonmerchantable trees at every stage is printed out.
Finally, there is a table showing the final stand
conditions, present net worth and soil expectations of different rotations.

At each stage, the node with zero trees carries the information of the best management regime for a rotation length of this age. From this node we can use information item (8)-- the optimal path from the previous stage, to trace back and find the management regime. The DOPT program stops when present net worth declines or when 14 stages have been calculated. The soil expectation associated with each rotation is also calculated. The largest value gives the soil expectation rotation.

A flowchart of program DOPT is shown in Appendix IV. The program listing is in Appendix III.
V. THE OPERATION OF THE DOPT ALGORITHM UNDER ALTERNATE STATE SPACE SPECIFICATION

When diameter acceleration is considered in a stand optimization problem, in addition to volume growth, another descriptor is needed to represent the diameter growth. In this model number of trees and basal area are used as descriptors. Basal area indicates the volume growth, and basal area and number of trees together indicate the diameter growth. In dynamic programming both stage and state space have to be defined. Here stage is defined as age of the stand at commercial thinning and precommercial thinning. State is defined as the number of trees in the stand and its corresponding basal area. For a fully stocked young Douglas-fir medium site stand at age 30, there are around 865 trees. At age 100 there are still around 184 trees. If every single tree is considered as alternative for thinning then the computations needed will be tremendous. One way to solve this problem is to consider a certain number of trees, e.g. K, as an alternative for thinning. That is, to thin K, 2K, 3K, ..., or nK trees. This can reduce the state space, hence the corresponding large number of computations needed. The larger the K is, the larger the reduction in computations. But the trade-off is that as K increases, the number of alternatives considered decreases. Hence an optimal range of
tree interval is required that considers a fairly large number of thinning alternatives yet not an excessive number of computations.

Once the tree interval $K$ is fixed, the optimal basal area interval must also be defined. If the basal area interval is too small, then many states (number of tree-basal area conditions) will be infeasible, i.e. some basal area will not have any number-of-trees associated. For example, if 20 alternatives can reach the 210-tree stocking level, when there are 50 basal area intervals then at least 30 (=50-20) basal area intervals will be empty (or infeasible). The more the basal area intervals, i.e. the smaller the size of basal area interval, the more empty nodes there will be. If the interval is too large then we expect the error due to choosing a wrong alternative in the neighborhood will be larger. This artifact effect (Brodie and Kao, 1979) will be discussed later in this Chapter.

The following sections discuss what is the reasonable tree interval and basal area interval for a fixed tree interval.

**TREE INTERVAL STATE**

As mentioned earlier, if the tree interval is too large, then only a limited number of thinning alternatives can be considered. If the tree interval is too small, then the "method of thinning" assumption, i.e. the ratio of
diameter cut to diameter of the merchantable stand before cut (assumed equal to one), may not be fulfilled. Another problem will be the computer capacity. A reasonable size of tree interval offers a reasonable number of thinning alternatives, meets the computer capacity and yet gives a result of reasonable precision.

One assumption used in this study is that the diameter distribution of a stand before and after thinning have the same normal distribution when d/D ratio is one. For different sizes of tree interval this assumption may not hold. Two kinds of test can be used to check the effect of tree interval size. One is to test whether the diameter distribution of the remaining trees in the stand is normally distributed with certain mean and variance. Another is to test whether the diameter distribution of the trees in the stand before cut is the same as the diameter distribution of the trees in the stand after cut. If small components are normally distributed, the aggregate will also be normally distributed--a variation of the central-limit theorem, i.e. when the number of trees increases, the diameter distribution is more likely to be normally distributed. Hence only the normality of the smallest residual has to be tested. Only the second test is explained here. One example is used to show how the test is applied.

From Bulletin 201 (McArdle, Meyer and Bruce, 1961), a
fully stocked Douglas-fir stand with site index 140 at age 40 has 585 trees. Its diameter distribution is shown in Table VIII which has a mean of 6.5752 inches and variance 5.8572 square inches. A chi-square test (Mood, Graybill and Boes, 1974) shows the stand diameter distribution is normally distributed with mean 6.5752 and variance 5.8572. The chi-square value is calculated by using the formula

\[ \chi^2 = \sum_{i=1}^{n} \frac{(X_i - \text{Expected } X_i)^2}{\text{Expected } X_i}. \]

Here we have six diameter classes. Since two degrees of freedom are used in estimating mean and variance, hence there are 6-2=4 degrees of freedom left. The calculated chi-square value is 8.96 which is smaller than the chi-square value from a chi-square distribution with four degrees of freedom at the 95 percent significance level. \((\chi^2(4, 0.95) = 9.49)\). Hence we accept the hypothesis that this diameter distribution is normal.

**TABLE VIII. STAND TABLE FOR DOUGLAS-FIR SITE INDEX 140 AT AGE 40.**

<table>
<thead>
<tr>
<th>Diameter Class</th>
<th>2-3</th>
<th>4-5</th>
<th>6-7</th>
<th>8-9</th>
<th>10-11</th>
<th>12-13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Trees</td>
<td>52</td>
<td>159</td>
<td>175</td>
<td>129</td>
<td>54</td>
<td>16</td>
</tr>
</tbody>
</table>

The second test is to test whether the diameter distribution of trees before thinning and after thinning are the same no matter what the diameter distribution is. A contingency table (Davies and Goldsmith, 1972; Daniel, 1976) is set up for each thinning (tree) interval with the
normal stand to find if they have the same distribution. Tables IX, X, XI, XII, XIII, XIV, and XV are contingency tables for different thinning, or tree, intervals. The second row in each table shows the remaining number of trees in each diameter class when thinned proportionally to the initial diameter distribution. The chi-square values are calculated by using the formula

\[ \chi^2 = \sum_{i=1}^{2} \sum_{j=1}^{6} \frac{(X_{ij} - \text{Expected } X_{ij})^2}{\text{Expected } X_{ij}} \]

Where (Expected \( X_{ij} \)) = (\( X_{i.} \) / Total) * \( X_{.j} \). The calculated chi-square value is compared with the chi-square value at 95 percent significance level with degrees of freedom of (number of rows - 1)*(number of columns - 1), or (2-1)*(6-1)=5 in this case, which is 11.1. None of the calculated chi-square values is larger than 11.1, so we believe that the remaining trees of every cutting interval has the same distribution as the normal stand no matter what distribution the normal stand is. We have already tested that the normal stand is normally distributed, hence the remaining trees of different cutting, or tree, interval are also normally distributed.

The above test indicates that for every cutting, or tree, interval, the smallest residual has the same distribution (normally distributed with same mean and variance).
### TABLE IX. CONTINGENCY TABLE FOR A NORMAL STAND COMPARED WITH A STAND WITH 50 TREES.

<table>
<thead>
<tr>
<th>Normal Stand</th>
<th>52</th>
<th>159</th>
<th>175</th>
<th>129</th>
<th>54</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval 50</td>
<td>4</td>
<td>14</td>
<td>15</td>
<td>11</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>56</td>
<td>173</td>
<td>190</td>
<td>140</td>
<td>59</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>635</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\chi^2 = 0.18$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### TABLE X. CONTINGENCY TABLE FOR A NORMAL STAND COMPARED WITH A STAND WITH 30 TREES.

<table>
<thead>
<tr>
<th>Normal Stand</th>
<th>52</th>
<th>159</th>
<th>175</th>
<th>129</th>
<th>54</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval 30</td>
<td>3</td>
<td>8</td>
<td>9</td>
<td>6</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>55</td>
<td>167</td>
<td>184</td>
<td>135</td>
<td>57</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>615</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\chi^2 = 0.15$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### TABLE XI. CONTINGENCY TABLE FOR A NORMAL STAND COMPARED WITH A STAND WITH 20 TREES.

<table>
<thead>
<tr>
<th>Normal Stand</th>
<th>52</th>
<th>159</th>
<th>175</th>
<th>129</th>
<th>54</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval 20</td>
<td>2</td>
<td>5</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>54</td>
<td>164</td>
<td>181</td>
<td>133</td>
<td>56</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>605</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\chi^2 = 0.46$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TABLE XII. CONTINGENCY TABLE FOR A NORMAL STAND COMPARED WITH A STAND WITH 15 TREES.

<table>
<thead>
<tr>
<th>Normal Stand</th>
<th>52</th>
<th>159</th>
<th>175</th>
<th>129</th>
<th>54</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval</td>
<td>15</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>53</td>
<td>163</td>
<td>180</td>
<td>132</td>
<td>55</td>
<td>17</td>
</tr>
<tr>
<td>Total</td>
<td>600</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \chi^2 = 1.07 \]

### TABLE XIII. CONTINGENCY TABLE FOR A NORMAL STAND COMPARED WITH A STAND WITH 10 TREES.

<table>
<thead>
<tr>
<th>Normal Stand</th>
<th>52</th>
<th>159</th>
<th>175</th>
<th>129</th>
<th>54</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval</td>
<td>10</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>53</td>
<td>162</td>
<td>178</td>
<td>131</td>
<td>55</td>
<td>16</td>
</tr>
<tr>
<td>Total</td>
<td>595</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \chi^2 = 0.34 \]

### TABLE XIV. CONTINGENCY TABLE FOR A NORMAL STAND COMPARED WITH A STAND WITH 5 TREES.

<table>
<thead>
<tr>
<th>Normal Stand</th>
<th>52</th>
<th>159</th>
<th>175</th>
<th>129</th>
<th>54</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval</td>
<td>5</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>52</td>
<td>160</td>
<td>177</td>
<td>130</td>
<td>55</td>
<td>16</td>
</tr>
<tr>
<td>Total</td>
<td>590</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \chi^2 = 1.47 \]
Table XVI shows that for small cutting, or tree, intervals the mean diameter is unstable. The mean diameter stabilizes after cutting, or tree, interval 15.

Statistically speaking, every cutting, or tree, interval results in the same distribution. If the cutting, or tree, interval is small, every time only the diameter classes with higher proportion of trees can be considered for cutting. If this situation continues for several cuttings, the diameter distribution will become uniform.

In this example if we want every diameter class to have a chance of being considered for cutting then the cutting interval must be no less than 15.
The above discussion is for a medium site (site index 140) Douglas-fir stand at age 40. Sessions (1979) showed that the diameter distribution of a fully stocked stand at other ages is also normally distributed. From the low chi-square values calculated from the above contingency tables we believe that the remaining trees for different cutting, or tree, interval at different ages are also normally distributed. This leads us to the result of 15 as the smallest tree interval which would result in reasonable solutions.

<table>
<thead>
<tr>
<th>Tree Interval</th>
<th>Number of Alternatives</th>
<th>Compared with Tree Interval 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>39,785,661</td>
<td>1.0000</td>
</tr>
<tr>
<td>20</td>
<td>10,175,382</td>
<td>0.2558</td>
</tr>
<tr>
<td>25</td>
<td>3,655,206</td>
<td>0.0919</td>
</tr>
<tr>
<td>30</td>
<td>2,020,788</td>
<td>0.0508</td>
</tr>
<tr>
<td>40</td>
<td>472,878</td>
<td>0.0119</td>
</tr>
<tr>
<td>50</td>
<td>187,353</td>
<td>0.0047</td>
</tr>
<tr>
<td>100</td>
<td>13,392</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

The number of thinning alternatives considered for different tree interval can be found by setting up a
table like Table I (see Appendix V). If a stand is pre-commercially thinned to 400 trees, and commercially thinned and fertilized every ten years starting from age 30 to final harvest at age 80, for different tree intervals, the number of alternatives is summarized in Table XVII.

Choice of the largest tree interval is rather arbitrary. When a rough approximation of the result is wanted, a larger tree interval not only gives the desired result but also saves computation time. The size of tree interval really depends on the objective of the research. However, from Table XVII, 50 seems to be the largest tree interval that can be reasonably considered.

**BASAL AREA INTERVAL STATE**

The size of basal area interval is dependent on the size of tree interval chosen. When tree interval is large, only a few thinning alternatives will be categorized into the same neighborhood, i.e. the error resulting from comparing different alternatives in the same neighborhood will be smaller. Hence when tree interval is large, a small basal area interval will not increase the desired precision. When tree interval is small, many thinning alternatives will be categorized into the same neighborhood, so a reduced basal area interval will increase the desired precision. In general, if the same basal area interval is used, the smaller the tree interval used, the better the
model will be, because more alternatives are considered. In this study 15 is chosen as tree interval, hence the discussion of basal area interval is based on a tree interval of 15.

A sensitivity analysis is used to see the effect of basal area interval. For fixed tree interval size, different basal area intervals are used, through dynamic programming, to find the best thinning regime and the corresponding soil expectation value.

### TABLE XVIII. SOIL EXPECTATIONS AND COMPUTATION EFFICIENCY FOR DIFFERENT BASAL AREA INTERVALS COMPARED WITH BASAL AREA INTERVAL OF FOUR SQ. FT.

<table>
<thead>
<tr>
<th>Basal Area Interval</th>
<th>Soil Expectation ($)</th>
<th>%</th>
<th>Execution Time (Sec.)</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1780.17</td>
<td>100.00</td>
<td>89.43</td>
<td>100.00</td>
</tr>
<tr>
<td>10</td>
<td>1756.22</td>
<td>98.65</td>
<td>41.86</td>
<td>46.79</td>
</tr>
<tr>
<td>16</td>
<td>1760.20</td>
<td>98.88</td>
<td>28.47</td>
<td>31.83</td>
</tr>
<tr>
<td>20</td>
<td>1750.66</td>
<td>98.34</td>
<td>23.30</td>
<td>26.05</td>
</tr>
<tr>
<td>30</td>
<td>1689.66</td>
<td>94.92</td>
<td>18.62</td>
<td>20.82</td>
</tr>
<tr>
<td>40</td>
<td>1713.12</td>
<td>96.23</td>
<td>16.37</td>
<td>18.30</td>
</tr>
</tbody>
</table>

Table XVIII shows the soil expectation values for different basal area intervals when tree interval is 15, the relative soil expectation values compared with basal area interval four, execution time of using optimization technique, and the relative execution time compared with
basal area interval four. Figure XV is a corresponding diagram of the results in Table XVIII. The management regimes for different basal area intervals are shown in Appendix VI.

An intuitive judgement is that small basal area would result in larger soil expectations because of fewer comparisons. But due to the artifact effect of dynamic programming, this is not the case. The artifact effect will be discussed in detail in the next section. From either Table XVIII or Figure XV we find the soil expectation values are stable in the range of four to 20 square feet of basal area. Hence four to 20 is considered as a reasonable range of basal area interval when tree interval is 15. The basal area interval used in this study is four square feet. Figure XV also shows that the execution time drops negative exponentially when basal area interval increases. Within certain precision, larger basal area interval saves a lot of computing time.

ARTIFACT EFFECTS

The neighborhood concept categorizes some alternatives with slightly different basal area and some with slightly different number of trees into the same node (state) and the alternative with the highest cumulative present net worth is chosen to represent the state. If the alternative have the same number of trees and same basal area, as in a
Figure XV. Relative soil expectations and computation efficiency for different basal area intervals compared with basal area of four square feet.
traditional discrete state dynamic programming problem, then there will not be any problem. The alternative chosen to represent the node is the best alternative. When basal area and number of trees are not the same, the alternatives with slightly lower cumulative present net worth might have a higher value contribution in subsequent stages, i.e. the thinning regime chosen may not be the true optimal solution. The difference between the present net worths from the strategy chosen and the true solution will be very small.

From Table XVIII we find that when basal area interval increases from ten to 16 square feet and from 30 to 40, the soil expectation increases. This is an artifact effect. When ten square feet is used, alternatives with basal areas 71 and 74 square feet are categorized to a node of 70 square feet (covers from 65 to 75). Suppose the alternative with 71 square feet would contribute more to present net worth in later stages than the alternative with 74 square feet, but has less cumulative present net worth in the current stage. Then the 74 square foot alternative will be chosen to represent the node of 70 square feet even though it will have less future value. If 16 square feet is used as the interval size, the alternative with 71 square feet will be categorized to the node of 64 square feet (covers from 56 to 72) and the alternative with 74 square feet will be categorized to node of 80 square feet (covers from 72 to 88). In this case the 71 square foot
alternative may be chosen to represent the node of 64 square feet. With higher future values, the 71 square foot alternative will override the 74 square foot alternative which represents the node of 80 square feet in the future. This explains why a smaller basal area interval may not have a higher soil expectation.

Comparing the management regimes for basal area of ten square feet and 16 square feet in Appendix VI we find that at age 30 the same state with basal area 82.0 was chosen in both cases. At age 40 the state with basal area 85.2 was chosen in the case of ten square foot interval and 73.0 was chosen in the case of 16 square foot interval. When the ten square foot interval is used, 85.2 and 73.0 will be categorized to the node of 90 (covers from 85 to 95) and 70 (covers from 65 to 75) respectively. But when the 16 square foot interval is used both 85.2 and 73.0 will be categorized to the node of 80 (covers from 72 to 88). This may happen in every categorization and the consequent artifact effect follows.
VI. COMPUTATIONS OF SILVICULTURAL APPLICATIONS USING THE DOPT MODEL

In this Chapter best rotation and stocking levels under different silvicultural considerations are studied. The interaction between silvicultural practices can be derived by comparing the information from different silvicultural considerations. Other impacts of silvicultural activities incorporated in the analysis are also discussed. Throughout the discussion, the discount rate used is three percent to represent the real rate and zero for the inflation rate, i.e. no ballooning of soil expectation due to inflation, current real dollars are considered. The tree interval used is 15 and four square feet for basal area interval. Regeneration cost is $200 per acre and $80 per acre for precommercial thinning at age ten is used. Site index will be 140 unless otherwise noted. The cost, price and other data were discussed in Chapter IV.

FINAL HARVEST ONLY

When only final harvest is considered as a silvicultural practice, the rotation is found by looking at maximal soil expectation value. Table XIX shows the statistics of a Douglas-fir stand of site index 140 at final harvest. The soil expectation culminates at age 70 and maximum mean annual increment (MAI) culminates ten years earlier. The numbers shown in Table XIX are calculated from DFIT for a
TABLE XIX. SUMMARY OF FINAL HARVEST, MAI AND PRESENT WORTH DATA FOR A STAND CONSIDERING FINAL HARVEST ONLY, SITE 140.

<table>
<thead>
<tr>
<th>Rotation Age (years)</th>
<th>Harvest Diameter (inches)</th>
<th>Total Number of Trees</th>
<th>Harvest Basal area (sq. ft.)</th>
<th>Harvest Volume (cu. ft.)</th>
<th>MAI (cu. ft.)</th>
<th>Present Net Worth ($/acre)</th>
<th>Soil Expectation ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>6.4</td>
<td>894</td>
<td>125.5</td>
<td>3156.2</td>
<td>105.21</td>
<td>-628.24</td>
<td>-1068.41</td>
</tr>
<tr>
<td>40</td>
<td>8.0</td>
<td>536</td>
<td>162.6</td>
<td>4981.3</td>
<td>124.53</td>
<td>-216.64</td>
<td>-312.41</td>
</tr>
<tr>
<td>50</td>
<td>9.7</td>
<td>381</td>
<td>184.4</td>
<td>6520.9</td>
<td>130.42</td>
<td>98.82</td>
<td>128.02</td>
</tr>
<tr>
<td>60</td>
<td>11.4</td>
<td>296</td>
<td>202.2</td>
<td>7840.8</td>
<td>130.68</td>
<td>309.17</td>
<td>372.37</td>
</tr>
<tr>
<td>70</td>
<td>12.9</td>
<td>244</td>
<td>216.6</td>
<td>8986.6</td>
<td>128.38</td>
<td>385.60</td>
<td>441.34</td>
</tr>
<tr>
<td>80</td>
<td>14.3</td>
<td>208</td>
<td>228.9</td>
<td>9993.1</td>
<td>124.91</td>
<td>397.11</td>
<td>438.30</td>
</tr>
</tbody>
</table>
fully stocked natural origin stand. This result tells us that for an undisturbed forest stand, when no special silvicultural practices are considered, the stand should be clear cut at age 70 and regenerated thereafter to achieve the greatest economic income. This result is used as a basis for calculating the economic impacts of different silvicultural activities.

PRECOMMERCIAL THINNING AND FINAL HARVEST

Precommercial thinning removes small and unsalable trees and leaves promising trees which result in more rapid juvenile growth. Table XX is the result of a stand precommercially thinned to 400 trees at age ten and final harvested at different ages. As promised by precommercial thinning, juvenile growth is rapid. Compared with a natural stand (Table XIX), at age 30 the quadratic mean diameter is 6.4 inches without precommercial thinning and 8.5 with precommercial thinning. At age 40 it is 8.0 inches compared with 9.9. After age 40 the precommercially thinned stand grows much slower than the natural stand does. The reason is that in DFIT it is assumed that the trees left after precommercial thinning will not die except those that could be captured in the next commercial thinning. Here the number of trees left after precommercial thinning is 400 and no commercial thinning are considered. When 400 trees crowd together, the growth is retarded. At
TABLE XX. SUMMARY OF FINAL HARVEST, MAI AND PRESENT WORTH DATA FOR A STAND CONSIDERING PRECOMMERCIAL THINNING AND FINAL HARVEST, SITE 140.

<table>
<thead>
<tr>
<th>Rotation Age (years)</th>
<th>Harvest Diameter (inches)</th>
<th>Total Number of Trees</th>
<th>Harvest Basal Area (sq. ft.)</th>
<th>Harvest Volume (cu. ft.)</th>
<th>MAI (cu. ft.)</th>
<th>Present Net Worth ($/acre)</th>
<th>Soil Net Worth ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>8.5</td>
<td>400</td>
<td>156.2</td>
<td>4041.8</td>
<td>134.73</td>
<td>-192.63</td>
<td>-327.59</td>
</tr>
<tr>
<td>40</td>
<td>9.9</td>
<td>400</td>
<td>213.1</td>
<td>6505.6</td>
<td>162.64</td>
<td>206.92</td>
<td>298.40</td>
</tr>
<tr>
<td>50</td>
<td>10.6</td>
<td>400</td>
<td>243.3</td>
<td>8570.5</td>
<td>171.41</td>
<td>361.40</td>
<td>468.20</td>
</tr>
<tr>
<td>60</td>
<td>10.9</td>
<td>400</td>
<td>261.3</td>
<td>10120.6</td>
<td>168.68</td>
<td>322.46</td>
<td>388.38</td>
</tr>
</tbody>
</table>
age 50 a natural stand has 381 trees (Table XIX) but the precommercially thinned stand still has 400 trees. The best rotation for a precommercially thinned (only) stand is 50 which has the highest soil expectation $468.20. The rotation here is shorter than the rotation of the natural stand because the retarded growth due to overstocking makes waiting for the higher price of larger trees unprofitable. The soil expectation value for the precommercially thinned stand is slightly larger than that of a natural undisturbed stand (Tables XIX and XX). Without other silvicultural considerations precommercial thinning only contributes $468.20-441.34=$26.86 to soil expectation. Yet precommercial thinning is a good silvicultural practice for producing small size logs. The maximum MAI is 171.41 cubic feet for rotation 50 compared with 130.68 for rotation 60 of the natural stand.

Due to the limitations of the growth model and to make the results comparable with the results from other silvicultural considerations, different intensities of precommercial thinnings are not considered in this section. If fewer than 400 trees are left after precommercial thinning, the time for the first possible commercial thinning will be delayed. When only precommercial thinning is considered we believe heavier precommercial thinning will make precommercial thinning more profitable.
COMMERCIAL THINNING AND FINAL HARVEST

The merits of commercial thinning are capture of those trees which will die in the following years because of suppression by stronger competitors and the subsequent faster diameter growth due to the greater growing space and lesser competition created. From the DOPT program the best thinning regime for different rotations and the stand conditions can be obtained in one computer run. Commercial thinning is considered every ten years starting at age 30. Table XXI shows the soil expectations and final stand conditions for different rotations. Table XXII shows the corresponding thinning regime and stand conditions at different ages for the best rotation, which is 80 in this case. In order to accelerate diameter growth, many trees are removed in the first commercial thinning. A constraint on maximum removals in the first commercial thinning of 50 percent of the merchantable stocking level is set due to consideration of blow-down, sunscald and thinning shock, etc. which are not included in the growth model. In this example the first commercial thinning did not remove more than 50 percent of the stocking. At every stage "thin" is better than "do not thin". This is because when "do not thin" is taken, the large amounts of mortality will be lost. Hence, mortality capture makes a heavy contribution to the desirability of thinning in this example. At age 50
TABLE XXI. SUMMARY OF FINAL HARVEST, MAI AND PRESENT WORTH DATA FOR A STAND CONSIDERING COMMERCIAL THINNING AND FINAL HARVEST, SITE 140.

<table>
<thead>
<tr>
<th>Rotation Age (years)</th>
<th>Harvest Diameter (inches)</th>
<th>Harvest Basal Area (sq. ft.)</th>
<th>Harvest Volume (cu. ft.)</th>
<th>MAI (cu. ft.)</th>
<th>Present Net Worth ($/acre)</th>
<th>Soil Expectation ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>8.80</td>
<td>122.09</td>
<td>4494.00</td>
<td>142.61</td>
<td>-270.36</td>
<td>-389.88</td>
</tr>
<tr>
<td>50</td>
<td>9.73</td>
<td>184.80</td>
<td>7238.81</td>
<td>144.78</td>
<td>14.06</td>
<td>18.22</td>
</tr>
<tr>
<td>60</td>
<td>11.38</td>
<td>202.18</td>
<td>8538.16</td>
<td>142.30</td>
<td>247.66</td>
<td>298.29</td>
</tr>
<tr>
<td>70</td>
<td>18.21</td>
<td>112.97</td>
<td>5322.97</td>
<td>160.70</td>
<td>394.09</td>
<td>451.06</td>
</tr>
<tr>
<td>80</td>
<td>20.97</td>
<td>123.60</td>
<td>5984.46</td>
<td>159.10</td>
<td>468.84</td>
<td>517.47</td>
</tr>
<tr>
<td>90</td>
<td>23.82</td>
<td>81.58</td>
<td>4112.19</td>
<td>152.19</td>
<td>465.70</td>
<td>500.72</td>
</tr>
</tbody>
</table>
TABLE XXII. THINNING SCHEDULE AND STAND CONDITIONS WHEN COMMERCIAL THINNING AND FINAL HARVEST ARE PERMITTED, SITE 140.

<table>
<thead>
<tr>
<th>Stand Age (years)</th>
<th>Mean Diameter (inches)</th>
<th>Merchantable Trees</th>
<th>Nonmerchantable Trees</th>
<th>Basal Area (sq. ft.)</th>
<th>Volume Left (cu. ft.)</th>
<th>Volume Cut (cu. ft.)</th>
<th>Cumulative PNW ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>6.4</td>
<td>360</td>
<td>334</td>
<td>80.8</td>
<td>2030.4</td>
<td>1125.8</td>
<td>-367.2</td>
</tr>
<tr>
<td>40</td>
<td>8.7</td>
<td>150</td>
<td>67</td>
<td>62.3</td>
<td>1910.3</td>
<td>2674.5</td>
<td>-374.9</td>
</tr>
<tr>
<td>50</td>
<td>12.1</td>
<td>105</td>
<td>23</td>
<td>83.7</td>
<td>2958.9</td>
<td>997.9</td>
<td>-280.2</td>
</tr>
<tr>
<td>60</td>
<td>15.2</td>
<td>75</td>
<td>11</td>
<td>95.0</td>
<td>3687.0</td>
<td>1128.0</td>
<td>-209.1</td>
</tr>
<tr>
<td>70</td>
<td>18.2</td>
<td>60</td>
<td>6</td>
<td>108.5</td>
<td>4505.8</td>
<td>817.1</td>
<td>-153.4</td>
</tr>
<tr>
<td>80</td>
<td>21.0</td>
<td>0</td>
<td>4</td>
<td>0.0</td>
<td>0.0</td>
<td>5984.5</td>
<td>468.6</td>
</tr>
</tbody>
</table>

Total Harvest = 12727.8 cu. ft.

MAI = 159.10 cu. ft.
forty-five trees were removed, at age 60 thirty trees were removed and at age 70 fifteen trees were removed (Table XXII). Compared with a normal stand we find that in the current stand 36 merchantable trees died between ages 40-50, 21 trees died between ages 50-60, and 12 died between ages 60-70. So we cut nine live merchantable trees (=45-36) at age 50, 9 (=30-21) at age 60, and 3 (=15-12) at age 70, which are the fewest trees we have to cut in each thinning, to capture this merchantable mortality. Our model structure is not explicit as to whether this mortality capture is comprised of anticipated mortality or harvest of dead trees but we would assume that as a regime progresses, the capture would be largely anticipated mortality. An increased price premium is not assumed for trees larger than 22 inches. Hence a thinning regime ending with a mean diameter around 22 is usually chosen as the best regime under this assumption. In this example 23.82 inches is the mean diameter of the final stand of present net worth rotation and 20.97 for soil expectation rotation. If a price premium limit is not included, the rotation could be longer and the final mean diameter could be larger. By the same reasoning, if price premium is limited to a smaller diameter, the rotation will be shorter. Compared with an unthinned stand, the commercially thinned stand has faster diameter growth and a larger amount of total volume harvested. The rotation is lengthened by ten years, from 70
for the unthinned stand to 80 for the thinned stand. Thinning produces a longer rotation because the marginal value growth percent criterion can be met for longer in the thinned stand.

Another point that should be mentioned is that at each commercial thinning only merchantable trees are considered for cutting, the nonmerchantable trees are left intact. The fourth column of Table XXII shows the number of live nonmerchantable trees in the stand at each age.

FERTILIZATION AND FINAL HARVEST

The optimal rotation with fertilization only is found in a rather straightforward manner. At each ten-year interval, three levels of fertilization are considered. Every fertilization combination was calculated by using a hand calculator. The strategy which results in the highest present net worth and the corresponding soil expectation for a given rotation are recorded. The rotation which has the highest soil expectation is the best rotation with fertilization. Table XXIII shows the results for site 140 with zero, 200 and 400 pounds of nitrogen fertilizer considered for application at ten-year intervals beginning at age 30. The best rotation occurs at age 70, and the strategy is to fertilize 400 pounds at age 30, 200 pounds at ages 40, 50 and 60 and clear cut the stand at age 70. The total volume gained by fertilization is 10740.4 - 8986.6
The best regime is to fertilize 400 pounds at age 30, 200 pounds at ages 40, 50, and 60, and clear cut the stand at age 70.
1753.4 cubic feet or about 22 cubic feet per year (Tables XIX and XXIII) under the soil expectation criterion. The effect of fertilization delayed MAI culmination ten years, i.e. from 130.68 at rotation age 60 for an undisturbed stand to 153.43 at rotation age 70 for a fertilized stand. Fertilization keeps the growth at a higher rate for a longer period.

PRECOMMERCIAL, COMMERCIAL THINNING AND FINAL HARVEST

The heavy thinning in the first commercial thinning of a natural stand and its negative contribution to the present net worth (Table XXII, $-367.2 is less than regeneration cost of $-200.0) suggest that precommercial thinning would be beneficial. Table XXIV gives the final stand conditions and soil expectations of different rotations which are precommercially thinned to 400 trees at age ten and commercially thinned every ten years after age 30. The soil expectations are much higher than those in Tables XIX, XX and XXI. Precommercial thinning accelerates juvenile diameter growth and commercial thinnings keep the subsequent diameter growth at a high rate. At age 30 compared with a stand considering only precommercial thinning, both stands have mean diameter of 8.5 inches which is higher than a stand without precommercial thinning. After age 30, diameter growth is much faster than a natural stand (Table XIX), and a precommercially thinned stand without
TABLE XXIV. SUMMARY OF FINAL HARVEST, MAI AND PRESENT WORTH FOR A STAND CONSIDERING PRECOMMERCIAL, COMMERCIAL THINNING AND FINAL HARVEST, SITE 140.

<table>
<thead>
<tr>
<th>Rotation Age (years)</th>
<th>Harvest Diameter (inches)</th>
<th>Harvest Basal Area (sq. ft.)</th>
<th>Harvest Volume (cu. ft.)</th>
<th>MAI (cu. ft.)</th>
<th>Present Net Worth ($/acre)</th>
<th>Soil Expectation ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>11.42</td>
<td>149.50</td>
<td>4564.58</td>
<td>162.11</td>
<td>189.09</td>
<td>272.68</td>
</tr>
<tr>
<td>50</td>
<td>12.91</td>
<td>177.18</td>
<td>6240.13</td>
<td>175.50</td>
<td>558.70</td>
<td>723.81</td>
</tr>
<tr>
<td>60</td>
<td>16.83</td>
<td>162.15</td>
<td>6281.29</td>
<td>174.72</td>
<td>781.89</td>
<td>941.74</td>
</tr>
<tr>
<td>70</td>
<td>18.34</td>
<td>137.58</td>
<td>5705.00</td>
<td>172.57</td>
<td>931.74</td>
<td>1066.42</td>
</tr>
<tr>
<td>80</td>
<td>20.93</td>
<td>107.49</td>
<td>4690.30</td>
<td>166.84</td>
<td>1004.57</td>
<td>1108.76</td>
</tr>
<tr>
<td>90</td>
<td>21.91</td>
<td>157.05</td>
<td>7130.30</td>
<td>168.53</td>
<td>1024.92</td>
<td>1101.98</td>
</tr>
<tr>
<td>100</td>
<td>24.23</td>
<td>48.02</td>
<td>2250.63</td>
<td>156.36</td>
<td>1004.22</td>
<td>1059.34</td>
</tr>
</tbody>
</table>
TABLE XXV. THINNING SCHEDULE AND STAND CONDITIONS WHEN PRECOMMERCIAL, COMMERCIAL THINNING AND FINAL HARVEST ARE PERMITTED, SITE 140.

<table>
<thead>
<tr>
<th>Stand Age (years)</th>
<th>Mean Diameter (inches)</th>
<th>Merchantable Trees</th>
<th>Basal Area (sq. ft.)</th>
<th>Volume Left (cu. ft.)</th>
<th>Volume Cut (cu. ft.)</th>
<th>Cumulative PNW ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>8.5</td>
<td>240</td>
<td>93.7</td>
<td>2425.1</td>
<td>1616.7</td>
<td>-317.2</td>
</tr>
<tr>
<td>40</td>
<td>11.1</td>
<td>195</td>
<td>130.4</td>
<td>3979.9</td>
<td>918.4</td>
<td>-160.8</td>
</tr>
<tr>
<td>50</td>
<td>12.9</td>
<td>75</td>
<td>68.1</td>
<td>2400.0</td>
<td>3840.1</td>
<td>248.6</td>
</tr>
<tr>
<td>60</td>
<td>16.0</td>
<td>75</td>
<td>104.1</td>
<td>4033.5</td>
<td>0.0</td>
<td>248.6</td>
</tr>
<tr>
<td>70</td>
<td>18.3</td>
<td>45</td>
<td>82.6</td>
<td>3423.0</td>
<td>2282.0</td>
<td>506.7</td>
</tr>
<tr>
<td>80</td>
<td>20.9</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>4690.3</td>
<td>1004.6</td>
</tr>
</tbody>
</table>
subsequent commercial thinnings. One could expect the rotation of the stand with precommercial thinning and commercial thinnings to be no shorter than a stand with commercial thinnings only. Because one more thinning (precommercial thinning) should keep the marginal growth percent higher. Table XXIV shows that the maximum present net worth rotation is 90 years and maximum soil expectation rotation is 80 years compared with 80 years and 80 years respectively for a stand considering commercial thinning operations only (Table XXI)-- an answer as was expected. The soil expectation rotation is ten years earlier than the present net worth rotation. The reason is because the cost of waiting out the rotation is higher than the price premium and growth in that extra ten years (Duerr, 1960).

Table XXV is the stand condition at different ages for the soil expectation rotation. At age 60 "do not thin" is the best alternative. In the precommercially thinned option of the DFIT model, all trees left after precommercial thinning will survive and hence there are the same number of trees at ages 50 and 60. "Do not thin" implies no cost is incurred nor is income earned, so the cumulative present net worth is the same for ages 50 and 60. The MAI increases from 159.10, without precommercial thinning, to 166.84, with precommercial thinning (Tables XXII and XXV).
When precommercial thinning and fertilization are considered jointly, first the stand is precommercially thinned to 400 trees at age ten. Then starting from age 30, at every ten-year interval, three different levels of fertilization are considered. Every fertilization combination is calculated by using a hand calculator. The strategy which results in the highest present net worth and the corresponding soil expectation are recorded. The rotation which has the highest soil expectation is the best rotation.

In the site index 140 Douglas-fir stand it occurs at age 50 which is also the age that MAI culminates. The corresponding best alternative is to leave 400 trees at age ten, fertilize 200 pounds of nitrogen at ages 30 and 40, and clear cut the stand at age 50. The rotation is relatively short. The reason is the same as for a precommercially thinned stand without fertilization--the retarded growth due to too many trees crowding together makes the waiting unprofitable. Here both soil expectation and total volume harvested are higher than a precommercially thinned stand without considering fertilization (Table XX). The extra contribution, $667.31 - 468.20 = $199.11$, is from fertilization.
TABLE XXVI. SUMMARY OF FINAL HARVEST, MAI AND PRESENT WORTH DATA FOR A STAND CONSIDERING PRECOMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST, SITE 140.

<table>
<thead>
<tr>
<th>Rotation Age (years)</th>
<th>Mean Diameter (inches)</th>
<th>Merchantable Trees</th>
<th>Harvest Basal Area (sq. ft.)</th>
<th>Harvest Volume (cu. ft.)</th>
<th>MAI (cu. ft.)</th>
<th>Present Net Worth ($/acre)</th>
<th>Soil Net Worth Expectation ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>8.5</td>
<td>400</td>
<td>156.2</td>
<td>4041.8</td>
<td>134.73</td>
<td>-192.63</td>
<td>-327.59</td>
</tr>
<tr>
<td>40</td>
<td>10.3</td>
<td>400</td>
<td>230.2</td>
<td>7028.6</td>
<td>175.72</td>
<td>327.88</td>
<td>472.83</td>
</tr>
<tr>
<td>50</td>
<td>11.1</td>
<td>400</td>
<td>267.3</td>
<td>9414.2</td>
<td>188.28</td>
<td>515.09</td>
<td>667.31</td>
</tr>
<tr>
<td>60</td>
<td>11.4</td>
<td>400</td>
<td>282.1</td>
<td>10928.8</td>
<td>182.15</td>
<td>460.09</td>
<td>554.15</td>
</tr>
</tbody>
</table>

The best regime is to fertilize 200 pounds of nitrogen at ages 30 and 40 and clear cut the stand at age 50.
COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST

Table XXVII shows the final stand conditions for different rotations considering commercial thinning and fertilization. The best rotation is 70 years. The stand condition at different ages for the best rotation is shown in Table XXVIII. The fast diameter growth due to fertilization shortens the rotation by ten years compared with a commercially thinned stand without fertilization (Table XXI). For a fertilized stand we would expect that it would have a higher marginal value growth percent than a stand not fertilized. Hence a fertilized stand should have a longer rotation. But here we have an opposite result. This is because the price function is flattened for diameters larger than 22 inches. When a stand is fertilized, it grows faster and maximal present net worth rotation occurs when mean diameter reaches 22 inches. Table XXVII shows that the maximum soil expectation rotation is 70 which has a mean diameter of 22.17 inches. And the maximum present net worth rotation is 80 with a mean diameter of 24.89 inches-- already exceeding the 22 inches limit.
TABLE XXVII. SUMMARY OF FINAL HARVEST, MAI AND PRESENT WORTH DATA FOR A STAND CONSIDERING COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST, SITE 140.

<table>
<thead>
<tr>
<th>Rotation Age (years)</th>
<th>Harvest Diameter (inches)</th>
<th>Harvest Basal Area (sq. ft.)</th>
<th>Harvest Volume (cu. ft.)</th>
<th>MAI (cu. ft.)</th>
<th>Present Net Worth ($/acre)</th>
<th>Soil Expectation ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>8.32</td>
<td>176.66</td>
<td>6203.42</td>
<td>155.09</td>
<td>-146.24</td>
<td>-210.89</td>
</tr>
<tr>
<td>50</td>
<td>13.11</td>
<td>161.15</td>
<td>6515.99</td>
<td>182.83</td>
<td>263.51</td>
<td>341.38</td>
</tr>
<tr>
<td>60</td>
<td>17.15</td>
<td>153.94</td>
<td>6762.21</td>
<td>194.46</td>
<td>670.26</td>
<td>807.29</td>
</tr>
<tr>
<td>70</td>
<td>22.17</td>
<td>100.47</td>
<td>4795.09</td>
<td>195.17</td>
<td>942.83</td>
<td>1079.12</td>
</tr>
<tr>
<td>80</td>
<td>24.89</td>
<td>87.08</td>
<td>4263.44</td>
<td>186.84</td>
<td>936.29</td>
<td>1033.47</td>
</tr>
</tbody>
</table>
TABLE XXVIII. THINNING SCHEDULE AND STAND CONDITIONS WHEN COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST ARE PERMITTED, SITE 140.

<table>
<thead>
<tr>
<th>Stand Age (years)</th>
<th>Amount of Fertilization (pounds)</th>
<th>Mean Diameter (inches)</th>
<th>Merchantable Trees</th>
<th>Basal Area (sq. ft.)</th>
<th>Volume Left (cu. ft.)</th>
<th>Volume Cut (cu. ft.)</th>
<th>Cumulative PNW ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>200</td>
<td>6.4</td>
<td>345</td>
<td>77.4</td>
<td>1945.8</td>
<td>1210.4</td>
<td>-373.7</td>
</tr>
<tr>
<td>40</td>
<td>200</td>
<td>9.3</td>
<td>165</td>
<td>78.3</td>
<td>2402.7</td>
<td>2642.8</td>
<td>-335.3</td>
</tr>
<tr>
<td>50</td>
<td>400</td>
<td>13.4</td>
<td>120</td>
<td>116.8</td>
<td>4125.5</td>
<td>1052.4</td>
<td>-227.0</td>
</tr>
<tr>
<td>60</td>
<td>400</td>
<td>17.2</td>
<td>45</td>
<td>72.2</td>
<td>2801.1</td>
<td>3961.1</td>
<td>238.5</td>
</tr>
<tr>
<td>70</td>
<td>-</td>
<td>22.2</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>4795.1</td>
<td>942.8</td>
</tr>
</tbody>
</table>

Total Harvest = 13661.8 cu. ft.
MAI = 195.17 cu. ft.
PRECOMMERCIAL, COMMERCIAL THINNING, FERTILIZATION
AND FINAL HARVEST

In this section all four silvicultural practices are considered in stand optimization. The effect of precommercial thinning, commercial thinning, fertilization and final harvest for site indices 110, 140, 170 and 200 are shown in Tables XXIX to XXXVI. Table XXXI is the final stand conditions for different rotations for site index 140. Best soil expectation rotation is 70 years. The corresponding management regime and stand conditions are shown in Table XXXII. Compared with Tables XXIV and XXV, which are the results of considering precommercial thinning and commercial thinning, the rotation is shortened by ten years due to the effect of fertilization. In the section "Commercial Thinning, Fertilization and Final Harvest" it was discussed that fertilization shortened rotation also by ten years when precommercial thinning is not included.

Table XXX tells the best management regime for a stand with site index 110: Leave 340 trees in precommercial thinning at age ten; cut 130 trees at age 40, 150 at age 50, skip one period and clear cut at age 70; fertilize 400 pounds of nitrogen at ages 40, 50 and 60. Tables XXXII, XXXIV and XXXVI can be interpreted in the same way to get the best management regime for site indices 140, 170 and 200. The optimal thinning regime of site 200 might not be
<table>
<thead>
<tr>
<th>Rotation Age (years)</th>
<th>Harvest Diameter (inches)</th>
<th>Harvest Basal Area (sq. ft.)</th>
<th>Harvest Volume (cu. ft.)</th>
<th>MAI</th>
<th>Present Net Worth ($/acre)</th>
<th>Soil Expectation ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>11.93</td>
<td>197.98</td>
<td>5760.81</td>
<td>137.11</td>
<td>291.08</td>
<td>377.10</td>
</tr>
<tr>
<td>60</td>
<td>17.23</td>
<td>121.45</td>
<td>3887.11</td>
<td>148.43</td>
<td>584.22</td>
<td>703.65</td>
</tr>
<tr>
<td>70</td>
<td>21.90</td>
<td>156.90</td>
<td>5375.11</td>
<td>153.79</td>
<td>872.11</td>
<td>998.18</td>
</tr>
<tr>
<td>80</td>
<td>22.38</td>
<td>122.91</td>
<td>4431.17</td>
<td>148.71</td>
<td>870.96</td>
<td>961.30</td>
</tr>
</tbody>
</table>
Precommercially thin to 340 trees at age ten.
<table>
<thead>
<tr>
<th>Rotation Age (years)</th>
<th>Harvest Diameter (inches)</th>
<th>Harvest Basal Area (sq. ft.)</th>
<th>Harvest Volume (cu. ft.)</th>
<th>MAI (cu. ft.)</th>
<th>Present Net Worth ($/acre)</th>
<th>Soil Expectation ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>12.29</td>
<td>173.07</td>
<td>5284.15</td>
<td>185.15</td>
<td>376.05</td>
<td>542.29</td>
</tr>
<tr>
<td>50</td>
<td>16.92</td>
<td>140.51</td>
<td>4948.57</td>
<td>201.80</td>
<td>936.49</td>
<td>1213.24</td>
</tr>
<tr>
<td>60</td>
<td>20.86</td>
<td>178.08</td>
<td>6898.71</td>
<td>204.88</td>
<td>1379.41</td>
<td>1661.40</td>
</tr>
<tr>
<td>70</td>
<td>22.62</td>
<td>167.47</td>
<td>6944.26</td>
<td>213.83</td>
<td>1555.34</td>
<td>1780.17</td>
</tr>
<tr>
<td>80</td>
<td>25.72</td>
<td>54.10</td>
<td>2360.71</td>
<td>194.91</td>
<td>1536.94</td>
<td>1696.36</td>
</tr>
</tbody>
</table>

TABLE XXXI. SUMMARY OF FINAL STAND CONDITIONS FOR DIFFERENT ROTATIONS FOR SITE 140 WITH PRECOMMERICAL, COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST.
TABLE XXXII. MANAGEMENT REGIME AND STAND CONDITIONS FOR SITE 140 WITH PRECOMMERCIAL, COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST.

<table>
<thead>
<tr>
<th>Stand Age (years)</th>
<th>Amount of Fertilization (pounds)</th>
<th>Mean Diameter (inches)</th>
<th>Merchandiseable Trees</th>
<th>Basal Area (sq. ft.)</th>
<th>Volume Left (cu. ft.)</th>
<th>Volume Cut (cu. ft.)</th>
<th>Cumulative PNW ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>400</td>
<td>8.5</td>
<td>210</td>
<td>82.0</td>
<td>2121.9</td>
<td>1919.8</td>
<td>-309.4</td>
</tr>
<tr>
<td>40</td>
<td>400</td>
<td>12.3</td>
<td>105</td>
<td>86.5</td>
<td>2642.1</td>
<td>2642.1</td>
<td>-34.4</td>
</tr>
<tr>
<td>50</td>
<td>400</td>
<td>16.4</td>
<td>105</td>
<td>154.2</td>
<td>5431.9</td>
<td>0.0</td>
<td>-60.5</td>
</tr>
<tr>
<td>60</td>
<td>400</td>
<td>19.1</td>
<td>60</td>
<td>119.1</td>
<td>4615.4</td>
<td>3461.6</td>
<td>494.8</td>
</tr>
<tr>
<td>70</td>
<td>-</td>
<td>22.6</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>6944.3</td>
<td>1555.3</td>
</tr>
</tbody>
</table>

Precommercially thin to 400 trees at age ten.
TABLE XXXIII. SUMMARY OF FINAL STAND CONDITIONS FOR DIFFERENT ROTATIONS FOR SITE 170 WITH PRECOMMERCIAL, COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST.

<table>
<thead>
<tr>
<th>Rotation Age (years)</th>
<th>Harvest Diameter (inches)</th>
<th>Harvest Basal Area (sq. ft.)</th>
<th>Harvest Volume (cu. ft.)</th>
<th>MAI</th>
<th>Present Net Worth ($/acre)</th>
<th>Soil Expectation ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>12.81</td>
<td>214.81</td>
<td>7648.01</td>
<td>214.72</td>
<td>1045.27</td>
<td>1507.36</td>
</tr>
<tr>
<td>50</td>
<td>17.14</td>
<td>144.28</td>
<td>5925.62</td>
<td>232.93</td>
<td>1680.46</td>
<td>2177.06</td>
</tr>
<tr>
<td>60</td>
<td>21.71</td>
<td>154.18</td>
<td>6965.14</td>
<td>236.91</td>
<td>2197.13</td>
<td>2646.29</td>
</tr>
<tr>
<td>70</td>
<td>23.18</td>
<td>175.83</td>
<td>8502.30</td>
<td>245.27</td>
<td>2338.23</td>
<td>2676.24</td>
</tr>
<tr>
<td>80</td>
<td>26.61</td>
<td>57.94</td>
<td>2948.62</td>
<td>224.01</td>
<td>2325.44</td>
<td>2566.65</td>
</tr>
</tbody>
</table>
TABLE XXXIV. MANAGEMENT REGIME AND STAND CONDITIONS FOR SITE 170 WITH PRECOMMERCIAL, COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST.

<table>
<thead>
<tr>
<th>Stand Age (years)</th>
<th>Amount of Fertilization (pounds)</th>
<th>Mean Diameter (inches)</th>
<th>Merchantable Trees</th>
<th>Basal Area (sq. ft.)</th>
<th>Volume Left (cu. ft.)</th>
<th>Volume Cut (cu. ft.)</th>
<th>Cumulative PNW ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>200</td>
<td>10.3</td>
<td>240</td>
<td>139.9</td>
<td>4104.9</td>
<td>940.7</td>
<td>-130.0</td>
</tr>
<tr>
<td>40</td>
<td>200</td>
<td>12.8</td>
<td>90</td>
<td>80.6</td>
<td>2868.0</td>
<td>4780.0</td>
<td>547.0</td>
</tr>
<tr>
<td>50</td>
<td>400</td>
<td>17.1</td>
<td>90</td>
<td>143.0</td>
<td>5875.2</td>
<td>0.0</td>
<td>533.2</td>
</tr>
<tr>
<td>60</td>
<td>400</td>
<td>20.0</td>
<td>60</td>
<td>130.4</td>
<td>5892.1</td>
<td>2946.1</td>
<td>1030.5</td>
</tr>
<tr>
<td>70</td>
<td>-</td>
<td>23.2</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>8502.3</td>
<td>2338.2</td>
</tr>
</tbody>
</table>

Precommercially thin to 295 trees at age ten.
TABLE XXXV. SUMMARY OF FINAL STAND CONDITIONS FOR DIFFERENT ROTATIONS FOR SITE 200 WITH PRECOMMERCIAL, COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST.

<table>
<thead>
<tr>
<th>Rotation Age (years)</th>
<th>Harvest Diameter (inches)</th>
<th>Harvest Basal Area (sq. ft.)</th>
<th>Harvest Volume (cu. ft.)</th>
<th>MAI (cu. ft.)</th>
<th>Present Net Worth ($/acre)</th>
<th>Soil Expectation ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>14.49</td>
<td>251.89</td>
<td>10199.62</td>
<td>254.99</td>
<td>1785.22</td>
<td>2574.43</td>
</tr>
<tr>
<td>50</td>
<td>18.93</td>
<td>146.55</td>
<td>6845.75</td>
<td>171.36</td>
<td>2572.88</td>
<td>3333.21</td>
</tr>
<tr>
<td>60</td>
<td>21.95</td>
<td>197.11</td>
<td>10127.41</td>
<td>280.83</td>
<td>3123.01</td>
<td>3761.45</td>
</tr>
<tr>
<td>70</td>
<td>26.35</td>
<td>113.57</td>
<td>6245.97</td>
<td>272.07</td>
<td>3198.54</td>
<td>3660.91</td>
</tr>
<tr>
<td>80</td>
<td>29.62</td>
<td>143.55</td>
<td>8307.56</td>
<td>263.83</td>
<td>3197.83</td>
<td>3529.53</td>
</tr>
</tbody>
</table>
TABLE XXXVI. MANAGEMENT REGIME AND STAND CONDITIONS FOR SITE 200 WITH PRECOMMERCIAL, COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST.

<table>
<thead>
<tr>
<th>Stand Age (years)</th>
<th>Amount of Fertilization (pounds)</th>
<th>Mean Diameter (inches)</th>
<th>Merchantable Trees</th>
<th>Basal Area (sq. ft.)</th>
<th>Volume Left (cu. ft.)</th>
<th>Volume Cut (cu. ft.)</th>
<th>Cumulative PNW ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>200</td>
<td>12.5</td>
<td>220</td>
<td>186.9</td>
<td>6155.8</td>
<td>0.0</td>
<td>- 259.5</td>
</tr>
<tr>
<td>40</td>
<td>200</td>
<td>14.5</td>
<td>75</td>
<td>85.9</td>
<td>3477.1</td>
<td>6722.5</td>
<td>1030.6</td>
</tr>
<tr>
<td>50</td>
<td>200</td>
<td>18.9</td>
<td>75</td>
<td>146.6</td>
<td>6845.7</td>
<td>0.0</td>
<td>1016.8</td>
</tr>
<tr>
<td>60</td>
<td>200</td>
<td>22.0</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>10127.4</td>
<td>3123.0</td>
</tr>
</tbody>
</table>

Precommercially thin to 220 trees at age ten.
acceptable to a silviculturist because there is a heavy precommercial thinning at age ten (down to 220 trees), then two thirds of the stocking is harvested at age 40, and the stand is clear cut at age 60. Though trees grow faster on site 200, thinning shock may still exist. One could use some additional constraints to restrict the amount of thinning at age 40. The first commercial thinning for site 110 is delayed to age 40 because at age 30 the mean diameter of the merchantable trees has not reached the minimum merchantable size of eight inches. We could consider commercial thinning as soon as the mean diameter of the merchantable trees reaches merchantable size. But DOPT is constructed to consider every silvicultural activity at an age of multiple ten.

From the information in the above tables we can conclude: First, higher site has shorter rotation because the trees grow faster and reach 22 inches sooner. Second, higher site needs less fertilizer as most people expected. The reason is that higher site already has relatively more nutrients in the soil, the nature of diminishing rate of return of production factors makes a larger amount of fertilizer less valuable. Third, for a stand inherited with "reasonable" fertility, e.g. site 170, fertilizer has less effect in the fast growing young stage and a more prominent effect in the slow growing older stages. Fourth, on higher site more trees are removed in precommercial
thinning and fewer trees are cut in the first commercial thinning. Site index 110 does not follow this pattern because the structure of the DFIT growth model restricts the first commercial thinning to occur later.

**PLANTATION**

DFIT represents a plantation via the device of precommercial thinning at age two. After precommercial thinning all procedures are the same as usual. Tables XXXVII and XXXVIII show the results of the plantation option on site 140.

Table XXXVIII shows that the best management regime is to plant 385 trees, fertilize 400 pounds of nitrogen every ten years from age 30, and cut 175, 135, 0, 45, and 30 trees in sequence. The rotation is 70 years. The advantage of the plantation assumption over the natural regeneration assumption in soil expectation is 1874.22-1780.17=$94.05 (Tables XXXI and XXXVIII), according to the model which assumes equal establishment cost. If the natural regeneration assumption was enough cheaper then it could actually have an advantage over the plantation under our assumptions.

**NON-NATURAL STAND**

For a non-natural stand at any age, DOPT first calculates the merchantable and non-merchantable parts by comparing with the natural stand. Then the growth model of
<table>
<thead>
<tr>
<th>Rotation Age (years)</th>
<th>Harvest Diameter (inches)</th>
<th>Harvest Basal Area (sq. ft.)</th>
<th>Harvest Volume (cu. ft.)</th>
<th>MAI (cu. ft.)</th>
<th>Present Net Worth ($/acre)</th>
<th>Soil Expectation ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>12.54</td>
<td>180.22</td>
<td>5502.29</td>
<td>182.16</td>
<td>469.99</td>
<td>677.76</td>
</tr>
<tr>
<td>50</td>
<td>17.69</td>
<td>128.07</td>
<td>4510.43</td>
<td>196.63</td>
<td>1038.07</td>
<td>1344.84</td>
</tr>
<tr>
<td>60</td>
<td>21.92</td>
<td>157.24</td>
<td>6091.45</td>
<td>205.25</td>
<td>1517.93</td>
<td>1828.25</td>
</tr>
<tr>
<td>70</td>
<td>26.39</td>
<td>113.98</td>
<td>4726.09</td>
<td>204.49</td>
<td>1637.51</td>
<td>1874.22</td>
</tr>
<tr>
<td>80</td>
<td>28.52</td>
<td>66.54</td>
<td>2903.35</td>
<td>192.59</td>
<td>1611.05</td>
<td>1778.16</td>
</tr>
</tbody>
</table>
TABLE XXXVIII. MANAGEMENT REGIME AND STAND CONDITIONS FOR SITE 140 PLANTATION.

<table>
<thead>
<tr>
<th>Stand Age (years)</th>
<th>Amount of Fertilization (pounds)</th>
<th>Mean Diameter (inches)</th>
<th>Merchantable Trees</th>
<th>Basal Area (sq. ft.)</th>
<th>Volume Left (cu. ft.)</th>
<th>Volume Cut (cu. ft.)</th>
<th>Cumulative PNW ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>400</td>
<td>8.9</td>
<td>210</td>
<td>90.5</td>
<td>2341.5</td>
<td>1784.0</td>
<td>287.2</td>
</tr>
<tr>
<td>40</td>
<td>400</td>
<td>12.5</td>
<td>75</td>
<td>64.4</td>
<td>1965.1</td>
<td>3537.2</td>
<td>147.7</td>
</tr>
<tr>
<td>50</td>
<td>400</td>
<td>17.7</td>
<td>75</td>
<td>128.1</td>
<td>4510.4</td>
<td>0.0</td>
<td>121.7</td>
</tr>
<tr>
<td>60</td>
<td>400</td>
<td>21.2</td>
<td>30</td>
<td>73.4</td>
<td>2844.9</td>
<td>4267.3</td>
<td>931.2</td>
</tr>
<tr>
<td>70</td>
<td>-</td>
<td>26.4</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>4726.1</td>
<td>1637.5</td>
</tr>
</tbody>
</table>

Plant 385 trees.
DFIT is used to simulate the stand to the next age of multiple ten (no less than 30). Then commercial thinning, fertilization and final harvest are considered as usual. Tables XXXIX and XL are the results of a slightly understocked stand with 80 percent normal stocking of both number of trees and basal area, at age 34 with site index 140.

The first commercial thinning and fertilization are considered at age 40 because the initial age of the stand is 34 which is larger than 30. From Tables XXVI, XXVIII, XXXII and XL we find that with relatively more trees in the stand, the larger amount of fertilization (400 pounds/acre) does not have the expected effect because of the limited growing space. Hence the lesser amount of fertilizer is applied when there are many trees in the stand. Rotation is longer due to the increased length of time required to reach 22 inches diameter.

INTERACTIONS BETWEEN SILVICULTURAL PRACTICES

Table XLI summarizes the soil expectations for different silvicultural activity combinations for site 140. Where

\[
\begin{align*}
PCT & = \text{precommercial thinning.} \\
CT & = \text{commercial thinning.} \\
FERT & = \text{fertilization.} \\
Se & = \text{soil expectation.}
\end{align*}
\]
TABLE XXXIX. SUMMARY OF FINAL STAND CONDITIONS FOR DIFFERENT ROTATIONS FOR SITE 140 OF A NON-NORMAL STAND WITH 80 PERCENT OF NORMAL BASAL AREA AND NUMBER OF TREES AT AGE 34.

<table>
<thead>
<tr>
<th>Rotation Age (years)</th>
<th>Harvest Diameter (inches)</th>
<th>Harvest Basal Area (sq. ft.)</th>
<th>Harvest Volume (cu. ft.)</th>
<th>MAI (cu. ft.)</th>
<th>Present Net Worth ($/acre)</th>
<th>Soil Expectation ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>10.66</td>
<td>183.28</td>
<td>7252.76</td>
<td>145.06</td>
<td>153.67</td>
<td>199.08</td>
</tr>
<tr>
<td>60</td>
<td>12.82</td>
<td>212.21</td>
<td>8968.86</td>
<td>149.48</td>
<td>442.13</td>
<td>532.51</td>
</tr>
<tr>
<td>70</td>
<td>20.31</td>
<td>112.47</td>
<td>5323.50</td>
<td>166.08</td>
<td>679.25</td>
<td>777.44</td>
</tr>
<tr>
<td>80</td>
<td>22.62</td>
<td>107.81</td>
<td>5235.17</td>
<td>162.07</td>
<td>757.52</td>
<td>825.06</td>
</tr>
<tr>
<td>90</td>
<td>22.20</td>
<td>106.24</td>
<td>5307.31</td>
<td>154.68</td>
<td>767.22</td>
<td>824.91</td>
</tr>
<tr>
<td>100</td>
<td>24.71</td>
<td>44.67</td>
<td>2296.02</td>
<td>143.86</td>
<td>744.07</td>
<td>784.92</td>
</tr>
</tbody>
</table>
**TABLE XL.** MANAGEMENT REGIME AND STAND CONDITIONS FOR SITE 140 OF A NON-NORMAL STAND WITH 80 PERCENT OF NORMAL BASAL AREA AND NUMBER OF TREES AT AGE 34.

<table>
<thead>
<tr>
<th>Stand Age (years)</th>
<th>Amount of Fertilization (pounds)</th>
<th>Mean Diameter (inches)</th>
<th>Merchantable Trees</th>
<th>Basal Area (sq. ft.)</th>
<th>Volume Left (cu. ft.)</th>
<th>Volume Cut (cu. ft.)</th>
<th>Cumulative PNW ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>200</td>
<td>8.3</td>
<td>300</td>
<td>111.5</td>
<td>3438.7</td>
<td>996.6</td>
<td>-224.5</td>
</tr>
<tr>
<td>50</td>
<td>400</td>
<td>11.1</td>
<td>105</td>
<td>70.3</td>
<td>2480.3</td>
<td>3735.4</td>
<td>-92.1</td>
</tr>
<tr>
<td>60</td>
<td>400</td>
<td>15.2</td>
<td>60</td>
<td>75.8</td>
<td>2940.2</td>
<td>1923.6</td>
<td>57.0</td>
</tr>
<tr>
<td>70</td>
<td>-</td>
<td>19.6</td>
<td>45</td>
<td>94.5</td>
<td>3923.6</td>
<td>1075.0</td>
<td>151.2</td>
</tr>
<tr>
<td>80</td>
<td>-</td>
<td>22.6</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>5235.2</td>
<td>747.5</td>
</tr>
</tbody>
</table>
TABLE XLI. SUMMARY OF SOIL EXPECTATIONS FOR DIFFERENT SILVICULTURAL ACTIVITY COMBINATIONS, SITE 140.

<table>
<thead>
<tr>
<th>Number</th>
<th>Activities Considered</th>
<th>Maximum Se</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Final Harvest Only</td>
<td>441.34</td>
</tr>
<tr>
<td>2</td>
<td>PCT</td>
<td>468.20</td>
</tr>
<tr>
<td>3</td>
<td>CT</td>
<td>517.47</td>
</tr>
<tr>
<td>4</td>
<td>FERT</td>
<td>690.44</td>
</tr>
<tr>
<td>5</td>
<td>PCT and CT</td>
<td>1108.76</td>
</tr>
<tr>
<td>6</td>
<td>PCT and FERT</td>
<td>667.31</td>
</tr>
<tr>
<td>7</td>
<td>CT and FERT</td>
<td>1079.12</td>
</tr>
<tr>
<td>8</td>
<td>PCT, CT and FERT</td>
<td>1780.17</td>
</tr>
</tbody>
</table>

Let \( C(A) \) = the contribution of activity A on soil expectation.

\( C(A \cup B) \) = the total contribution of activities A and B.

\( C(A \mid B) \) = the contribution of activity A when B is included.

\( C(A \cup B \mid C) \) = the total contribution of activities A and B when activity C is included.

\( C(A \cap B) \) = the interactive contribution of activities A and B.
From Table XLI we can calculate the independent contribution of the three silvicultural activities to soil expectation:

\[
\begin{align*}
C(\text{PCT}) &= (2)-(1) = 468.20-441.34 = 26.86 \\
C(\text{CT}) &= (3)-(1) = 517.47-441.34 = 76.13 \\
C(\text{FERT}) &= (4)-(1) = 690.44-441.34 = 249.10
\end{align*}
\]

Among \(C(\text{PCT})\), \(C(\text{CT})\) and \(C(\text{FERT})\), \(C(\text{FERT})\) has the highest value, hence if only one additional silvicultural practice could be considered, fertilization should be considered first. Intuitively we would expect commercial thinning to have the highest contribution on soil expectation. But due to the high harvesting cost, the relatively cheap practice of fertilization takes the first place.

From Tables XX, XXII and XXIII we find the maximum MAI of PCT is 171.41 cubic feet, 159.10 for CT and 153.43 for FERT. The trees harvested from a precommercially thinned stand have relatively small diameter and because of the high harvesting cost, fertilization has the highest independent contribution to soil expectation.

From Table XLI the contribution of one activity given that another activity is included can be calculated:

\[
\frac{1}{C(\text{PCT} | \text{CT})} = (5)-(3) = 1108.76-517.47 = 591.29
\]

1/ Logically it is not reasonable to talk about the contribution of PCT given CT because PCT occurs earlier than CT. Here it is just a notation for further calculation of interactive effect.
The contribution of PCT when FERT is considered is negative. Since C(PCT | FERT) is smaller than C(PCT), this implies that the inclusion of another activity makes the contribution of PCT negative. The total contribution of every two activities can also be calculated from Table XLI:

\[ C(\text{PCT} \cup \text{CT}) = (5)-(1) = 1108.76-441.34 = 667.42 \]
\[ C(\text{PCT} \cup \text{FERT}) = (6)-(1) = 667.31-441.34 = 225.97 \]
\[ C(\text{CT} \cup \text{FERT}) = (7)-(1) = 1079.12-441.34 = 637.78 \]

Among the above three contributions, C(\text{PCT} \cup \text{CT}) has the highest value. This shows that if only two silvicultural practices can be considered, PCT and CT should be chosen. When only one activity is considered FERT has the highest contribution, adding in another activity makes PCT and CT the highest contributors. This is because PCT and CT have the highest interaction effect. Comparing C(PCT) with C(PCT | CT) we find that the contribution of PCT is increased from 26.86 to 591.29 due to a positive interaction. 

\[ C(\text{PCT} | \text{FERT})^2 = (6)-(4) = 667.31-690.44 = -23.13 \]
\[ C(\text{CT} | \text{PCT})^2 = (5)-(2) = 1108.76-468.20 = 640.56 \]
\[ C(\text{CT} | \text{FERT})^2 = (7)-(4) = 1079.12-690.44 = 388.68 \]
\[ C(\text{FERT} | \text{PCT})^2 = (6)-(2) = 667.32-468.20 = 199.11 \]
\[ C(\text{FERT} | \text{CT})^2 = (7)-(3) = 1079.12-517.47 = 561.65 \]
tion effect between PCT and CT. Using the notation men-
tioned above, the interaction effect between PCT and CT is:
\[ C(PCT \cap CT) = C(PCT | CT) - (PCT) = 591.29 - 26.86 = 564.43 \]

Using the same argument, \( C(PCT \cap CT) \) can also be ob-
tained by comparing \( C(CT) \) with \( C(CT | PCT) \), and the same
answer is derived:
\[ C(PCT \cap CT) = (CT | PCT) - (CT) = 640.56 - 76.13 = 564.43 \]

Another way to look at this problem is that the total
effect of PCT and CT should be the sum of the individual
contribution and their interactive contribution:
\[ C(PCT \cup CT) = C(PCT) + C(CT) + C(PCT \cap CT) \]

This implies
\[ C(PCT \cap CT) = C(PCT \cup CT) - C(PCT) - C(CT) = 564.43 \]
The same answer as we derived before. Using either of the
two argument we derive
\[ C(PCT \cap FERT) = -49.99 \]
\[ C(CT \cap FERT) = 312.55 \]

The interaction effect between PCT and FERT is
negative, while the interaction effects for PCT and CT, and
CT and FERT are positive. That is to say, when PCT and
FERT are considered together their individual effects are
reduced by their negative interaction. For CT, the
addition of any other activity reinforces the contribution
of CT. PCT and CT have the largest interaction effect.

When two activities are considered, the effect of
adding another activity can be calculated as:
\[
C(PCT \cup CT \cup FERT) = (8)-(4) = 1780.17-690.44 = 1089.73
\]
\[
C(PCT \cup FERT \mid CT) = (8)-(3) = 1780.17-517.47 = 1262.70
\]
\[
C(CT \cup FERT \mid PCT) = (8)-(2) = 1780.17-468.20 = 1311.97
\]
Since \(C(PCT \cup CT \mid FERT)\) is larger than \(C(PCT \cup CT)\), \(C(PCT \cup FERT \mid CT)\) is larger than \(C(PCT \cup FERT)\) and \(C(CT \cup FERT \mid PCT)\) is larger than \(C(CT \cup FERT)\), we conclude that given any two activities already considered, the inclusion of another activity increases the total contribution of the two given activities. Hence, whenever possible, as many activities should be considered because the more activities considered the more "extra" contributions can be gained under the cost and revenue assumptions of these examples. Higher cost levels might well cause a negative contribution from some activities.

Finally, the interaction effect of PCT, CT and FERT can also be derived in two ways. The first method is by using the simple intuition: Total effect of PCT, CT and FERT should be the sum of the individual effect, interactive effect of every two activities and the interactive effect of the three activities:

\[
C(PCT \cup CT \cup FERT) = C(PCT) + C(CT) + C(FERT) + C(PCT \cap CT) + C(PCT \cap FERT) + C(CT \cap FERT) + C(PCT \cap CT \cap FERT)
\]
First, \(C(PCT \cup CT \cup FERT)\) is calculated from Table XLI:
\[
C(PCT \cup CT \cup FERT) = (8)-(1) = 1780.17-441.34 = 1338.83
\]
Then, \(C(PCT \cap CT \cap FERT)\) is derived:
C(PCT ∩ CT ∩ FERT)

\[
= C(PCT \cup CT \cup FERT) - C(PCT) - C(CT) - C(FERT) - C(PCT \cap CT) - C(PCT \cap FERT) - C(CT \cap FERT) \\
= 1338.83 - 26.86 - 76.13 - 249.10 - 564.43 - (-49.99) - 312.55 \\
= 159.75
\]

The interactive effect of PCT, CT and FERT is positive. When these three activities are considered together, an extra contribution of $159.75 on soil expectation is gained because the positive interactive effect among these three activities.

The second way of deriving \(C(PCT \cap CT \cap FERT)\) is by using mathematical relations:

First, \(C((CT \cup FERT) \cap PCT)\)

\[
= C(CT \cup FERT | PCT) - C(CT \cup FERT) \\
= 1311.97 - 637.78 \\
= 674.19
\]

Second, \(C((CT \cup FERT) \cap PCT)\)

\[
= C((CT + FERT + CT \cap FERT) \cap PCT) \\
= C(CT \cap PCT) + C(FERT \cap PCT) + C(CT \cap FERT \cap PCT)
\]

Then, \(C(CT \cap FERT \cap PCT)\)

\[
= C((CT \cup FERT) \cap PCT) - C(CT \cap PCT) - C(FERT \cap PCT) \\
= 674.19 - 564.43 - (-49.99) \\
= 159.75
\]

The same result can also be derived by looking at either \(C((PCT \cup FERT) \cap CT)\) or \(C((PCT \cup CT) \cap FERT)\). Table XLII summarizes the results developed above for interactive
and total contributions.

**TABLE XLII. SUMMARY OF SOME TOTAL AND INTERACTIVE EFFECTS ON SOIL EXPECTATION, SITE 140.**

<table>
<thead>
<tr>
<th>Combinations</th>
<th>Contribution ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCT</td>
<td>26.86</td>
</tr>
<tr>
<td>CT</td>
<td>76.13</td>
</tr>
<tr>
<td>FERT</td>
<td>249.10</td>
</tr>
<tr>
<td>PCT ∪ CT</td>
<td>667.42</td>
</tr>
<tr>
<td>PCT ∪ FERT</td>
<td>225.97</td>
</tr>
<tr>
<td>CT ∪ FERT</td>
<td>637.78</td>
</tr>
<tr>
<td>PCT ∩ CT</td>
<td>564.43</td>
</tr>
<tr>
<td>PCT ∩ FERT</td>
<td>- 49.99</td>
</tr>
<tr>
<td>CT ∩ FERT</td>
<td>312.55</td>
</tr>
<tr>
<td>PCT ∪ CT ∪ FERT</td>
<td>1338.83</td>
</tr>
<tr>
<td>PCT ∩ CT ∩ FERT</td>
<td>159.75</td>
</tr>
</tbody>
</table>
VII. SUMMARY, FURTHER APPLICATIONS AND DISCUSSION

SUMMARY

Solution of stand level optimization problems requires a suitable growth model with responses to all silvicultural activities considered. Appropriate cost and revenue responses to stand management alternatives are also required. Finally, an efficient optimization procedure for handling the potential numerous combinations of treatments is required. The study presented has utilized the DFIT model with modifications, current cost and revenue data from Western Oregon and dynamic programming to accomplish the task.

Dynamic programming is a powerful technique for solving optimization problems. The continuous growth of the stand can be discretized by using the "neighborhood" concept to fit into the discrete state dynamic programming model. To find the optimal solution, the number of alternatives examined is greatly reduced by applying the principle of optimality of dynamic programming. Dynamic programming is flexible accepting any form of price, cost and growth functions. The choice of a growth function depends on the dimensions of the dynamic programming framework to be used. When only volume or basal area is involved, a two-dimensional dynamic programming network can solve the problem. If both stocking (volume or basal
area) and diameter are involved then a three-dimensional dynamic programming network is needed. The structure of the dynamic programming framework is not affected by the complexities of price, cost and growth functions. This is a major advantage over most other optimization techniques. Analysts are not biased towards selection of simple functional forms. The following characteristics make dynamic programming a useful optimization method: (1) computationally simple, (2) efficient, (3) compatible to a wide array of functional forms, and (4) flexible in use.

The growth model used is DFIT (Bruce, Demars and Reukema, 1977). It has functional relations to explain the results of precommercial, commercial thinning, fertilization and submerchantable mortalities. Some of the functions in DFIT can be changed without upsetting internal relations in DFIT. For example the fertilization part of this research is derived from the experimental results of Turnbull and Peterson (1976) instead of the functional equations in DFIT. The cost function used in this research is derived by Sessions (1979) who used a simulation technique. The price function is also derived from Bulletin 201 (McArdle, Meyer and Bruce, 1961) by Sessions.

Incorporation of diameter growth acceleration impacts of thinning into dynamic programming analysis is necessary to account for reduction in logging cost and increases in price received as the size of material harvested increases.
The effect of fertilization and its interaction with other silvicultural practices and diameter dependent costs and revenues must also be considered. DOPT is a general model that considers single silvicultural practices and some combinations of silvicultural practices. DOPT can consider (1) finding the best thinning regime for a natural stand at ten-year intervals, (2) finding the best precommercial thinning intensity at year ten and the best subsequent thinning regime for a natural stand starting from age 30, (3) finding the best regime for commercial thinning and fertilization for a natural stand at ten-year intervals, (4) finding the best precommercial thinning intensity and the subsequent commercial thinning and fertilization regime for a natural stand, (5) finding the best thinning regime for a non-natural stand, and (6) finding the best regime for thinning and fertilization for a non-natural stand.

The "best" is this instance means maximum soil expectation.

In this study we have come up with a method for testing complex silvicultural responses and interactions. The impact of silvicultural inputs can be studied either independently or in combination taking account of simultaneous density interactions over time and rotation length alternatives.

Precommercial thinning-- it is generally assumed that early stocking control accelerates diameter growth and reduces competition and will affect later commercial
thinning entries in value and volume. This study confirms these assumptions and indicates the impact of precommercial thinning on later commercial thinning and rotation. Deficiencies due to DFIT limitations make a precise analysis of the impact of precommercial thinning without considering the subsequent commercial thinning difficult. But the process presented would work with a better specified precommercial thinning response model to better provide these results.

Commercial thinning-- this has been described and discussed in an earlier paper (Brodie and Kao, 1979). The consideration of diameter acceleration makes thinnings more attractive. If diameter acceleration is not considered, the purpose of thinning is only to capture the mortality. The fact that commercial thinning lengthens rotation by maintaining the efficiency of the growing stock over longer periods of time is also generally assumed and is confirmed in this study.

Fertilization-- the usual assumptions about fertilization were contingent on uncertain knowledge of physical response. Does fertilization substitute for growing stock or site capacity? Using some of the first good empirical response data to fertilization of Douglas-fir, this study indicates that fertilization increases site capacity and raises optimal stocking levels.

Initial density-- the general assumptions are that
lower stocking has faster growth and longer rotation. This model can be used to study the exact impact. In general, a lesser stocked stand will reach a given diameter sooner and the rotation impact may be hidden by the quality premium termination effect. The methodology presented confirms previous theory and can be used to study initial stocking impacts.

In general, silvicultural impacts when aggregated have positive interaction so that the sum of the treatments effect is greater than the independent effects summed independently. The combinatorial approach to treatments outlined in this study may help to justify intensive silviculture in marginal situations where independent effects are insufficient.

FURTHER APPLICATIONS

DOPT considers first commercial thinning at the first age of multiple ten that is older than the age of the first possible commercial thinning. Let \( A_P \) be the age of the first possible commercial thinning, \( A_P \) considers thinning starting from the age of the next nearest multiple of ten. One could consider thinnings at ages \( A_P \), \( A_P+10 \), \( A_P+20 \), and so on; \( (A_P+1) \), \( (A_P+1)+10 \), \( (A_P+1)+20 \), and so on; \(...\); and \( (A_P+9) \), \( (A_P+9)+10 \), \( (A_P+9)+20 \), and so on. From these ten alternatives the one with the maximum soil expectation could be chosen. The current result from DOPT
is one of the above ten alternatives. To make the model simple we did not consider the others. We also believe the soil expectations from those ten alternatives will not differ by much. But DOPT could be modified to analyse any of the alternatives.

The problem of finding the best management regime for maximizing MAI can also be solved by DOPT by changing the objective function from maximizing soil expectation to maximizing MAI, i.e. from \( \sum_1^i (R(V_i, D_i) - C(V_i, D_i)) \times (1+r)^{n-1} \) to \( \frac{\sum_i V_i}{n} \). Where \( i \) is the year harvest occurred, \( V_i \) is the volume harvested at age \( i \), \( R(V_i, D_i) \) is the revenue and \( C(V_i, D_i) \) is the cost associated with harvesting volume \( V_i \) with mean diameter \( D_i \) at year \( i \), \( r \) is the discount rate, and \( n \) is the rotation age. A recent study by Ritters (1979) showed that this could be done when only commercial thinnings are considered. The inclusion of precommercial thinning and fertilization should not cause any difficulties.

A note by Kao and Brodie (1979b) showed that the determination of optimal thinning entry interval can also be solved by using dynamic programming. DOPT might also be modified to solve this question. From the study by Kao and Brodie it is found that, under their price function, changing from ten-year entry interval to variable entry interval the soil expectation only increases 1.11 percent
but the computation time needed is quadrupled. The duration of the effect of fertilizer also causes some problem. The duration of the effect of fertilizer is assumed to be ten years. If variable entry interval is considered, more than one fertilization may occur in ten years. How the growth response would be is not clear. The principle of optimality would be violated, since continuing effects of fertilization would not be taken into account in the stage by stage optimization. With a fertilization effect of ten years and a stage interval of five years, fertilization applications that were cost effective over the ten year period but not the five year period would be rejected.

Another problem is mortality capture. It is assumed that thinning captures the mortality in the last ten years. If the thinning interval is less than ten years then there is no problem, the mortality after the last thinning can all be captured. When the thinning is longer than ten years then we need some more assumptions to make the solution feasible. This research concentrated on studying the interactions among silvicultural practices, hence the above mentioned details were not considered.

In each thinning it is assumed that the mean diameter of the trees removed (d) is equal to the mean diameter of the stand before cut (D), i.e. d/D=1.00. In a silvicultural sense only mechanical thinning achieves this restriction. Other thinning methods like thinning from above (or
crown thinning), thinning from below (or low thinning), and selection thinning are not considered. When other thinning methods are also considered, i.e. d/D need not be 1.00, the current DOPT will not be able to solve this problem. One solution method would be to use another descriptor to represent the d/D ratio. The d/D ratios chosen to represent the state must be finite to fit into the finite state dynamic programming model. At each thinning, different combinations of trees removed would have different d/D ratio values. Using the "neighborhood" concept, some d/D ratios can be grouped together and the alternative which has the highest cumulative present net worth would be chosen to represent the node. All situations would be the same as discussed in this study except one more descriptor and more computations would be needed.

In a natural stand the number of trees per acre decreases as the stand gets older. One way to increase the precision of the solution yet use the same computer memory is to narrow the tree interval size as the solution progresses from stage to stage (Sessions, 1979). One technique is to define the network size by the number of trees in an unthinned natural stand and determine the interval by some constant divisor of this number. For example, for a site index 140 Douglas-fir stand, there are around 900 trees at age 30, 600 trees at age 40, 420 at 50, 300 at 60, and 270 at 70. If network size is defined to be 30, then
the interval size at age 30 is 30 trees (=900/30), at 40 is 20 trees (=600/30), at 50 is 14 (=420/30), at 60 is 10 (=300/30) and at 70 is 9 (=270/30).

Some forest managers are interested in optimal thinning regimes, where all thinnings are constrained to have positive money income. Noncommercial thinnings are not considered even if they result in higher value growth in later stages. The result of this additional constraint will be a thinning regime which has a present net worth less than or equal to the unconstrained problem. With some modifications, DOPT can find the conditional optimal solution with no difficulty (Sessions, 1979).

DISCUSSION

Changes in Logging and Haul Costs

Some sensitivity tests and break-even tests can also be done. Increases in logging cost or haul cost will change thinning regime. As Sessions (1979) indicated when haul cost increases, among all candidates which can reach a certain node, the candidate with the largest volume harvested is least affected in terms of present net worth. The following equation explains the reason:

\[
\text{Revenue} = V*(\text{Pond Value} - \text{Harvest Cost} - \text{Haul Cost}) \\
= V*(\text{Pond Value} - \text{Harvest Cost}) - V*(\text{Haul Cost})
\]

where \( V \) is volume harvested in thinning. When haul cost is reduced, the candidate with the largest \( V \) will gain the
most.

If harvest cost is changed linearly, i.e. \( K \times (\text{Harvest Cost}) \), the optimal regime will also be changed. (Sessions, 1979). The result was that early thinnings were skipped. That is when harvest cost is increased linearly, it is not worth harvesting small amounts of volume.

Changes in Price Function

The pond value is expressed as a linear function of mean diameter of trees harvested:

\[
Pond \text{ Value} = K + K' \times D
\]

hence

\[
Revenue = V \times (K + K' \times D - \text{Harvest Cost} - \text{Haul Cost})
\]

A change in the intercept of the price function \( K \) will be the same as a change on haul cost. Increasing the intercept \( K \) has the same effect as reducing haul cost. Increasing the slope of the function usually lengthens rotation because larger diameter material is more valuable relative to small diameter material. But there are still two possibilities: First, fewer trees will be removed in early stages because trees in the early stages are relatively less valuable due to small diameter. Second, more trees will be removed in early stages to stimulate the concentration of growth on fewer, larger trees. Sessions (1979) changed the slope from 70.81 to 100 and the results came out to be the second case. Although Sessions only
considered commercial thinnings, the results of considering commercial, precommercial thinning and fertilization should be similar.

Changes in Fertilization Cost

When fertilization cost is changed the management regime may also be changed. Because the best alternative among all candidates that can reach a certain node in the next stage will be different when fertilization cost is changed. Generalizing, we can assume that increasing the cost of fertilization makes fertilization less favorable. When fertilization cost is high enough then "do not fertilize" will be chosen as the optimal management regime for the cost of fertilization does not pay for the return. Between "fertilize" and "do not fertilize" there is a fertilization cost which makes no difference between "fertilize" and "do not fertilize", i.e. the optimal management regime of considering fertilization has the same soil expectation as that of optimal management regime without fertilization. The "break-even" point can be found by a "trial and error" method.

The consideration of only three levels of fertilization may also have artifact effect. When data are available, the consideration of more levels of fertilization will decrease the artifact effect.
Changes in Regeneration and Precommercial Thinning Costs

Let $R_i$ be the revenue earned and $C_i$ be the cost incurred at age $i$, $r$ be the discount rate, $n$ the rotation, REGENC the regeneration cost, and PCTCOST the precommercial thinning cost, the soil expectation $S_e$ is:

$$S_e = \frac{\sum_i ((R_i - C_i)(1+r)^{n-i} - \text{REGENC}(1+r)^n - \text{PCTCOST}*(1+r)^{n-10})}{((1+r)^n - 1)}$$

When REGENC is increased the rotation will be lengthened to spread the cost over a longer time interval so the cost is reduced relatively. The management regime will be changed accordingly when the rotation is changed. Changing PCTCOST can be shown to have the same effect as REGENC does:

$$\text{PCTCOST}*(1+r)^{n-10} = (\text{PCTCOST}*(1+r)^{-10})*(1+r)^n$$

$$= \text{PCTCOST}^* (1+r)^n$$

where $\text{PCTCOST}^* (1+r)^{-10}$ is a constant because it is a function of two constants PCTCOST and R. Hence increasing PCTCOST will lengthen the rotation.

Changes in Alternative Rate of Return

Raising the discounting rate shortens the rotation almost invariably as future returns become less valuable than present returns. As discussed in Chapter VI thinnings keep the marginal value growth percent high, hence a thinned stand will have a longer rotation than an unthinned
stand. But due to the price premium constraint, i.e. trees larger than a certain diameter have a price no better than trees with that specific diameter, a thinned stand grows faster and reaches that specific diameter faster, so a shorter rotation may result from this premium limit. If the discounting rate is raised high enough to shorten the rotation to the age that mean diameter of the stand does not reach that specific diameter, then the effect of lengthening rotation of thinning can be checked. Fertilization may have the same effect on rotation as thinning does. When the discounting rate is raised, this effect can also be checked.

The strength of the model presented are first of all that the solution is general and reflects the diameter growth acceleration interactions of thinning. When fertilization is included the interaction among precommercial thinning, commercial thinning and fertilization can be studied. The technique will accept price and cost functions of any form, provided they are comprised of state-descriptor variables or their transformations. The model is also transferable to other species for which DFIT type models are available.


APPENDICES
APPENDIX I

VARIABLE LIST
<table>
<thead>
<tr>
<th>SYMBOLS</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Stand age (years)</td>
</tr>
<tr>
<td>ADJ</td>
<td>Adjustment</td>
</tr>
<tr>
<td>BF</td>
<td>Board foot</td>
</tr>
<tr>
<td>CF</td>
<td>Cubic foot</td>
</tr>
<tr>
<td>CT</td>
<td>Commercial thinning</td>
</tr>
<tr>
<td>CV4</td>
<td>Cubic volume to four-inch top, no stump (CF)</td>
</tr>
<tr>
<td>d/D</td>
<td>Ratio diameter cut to diameter before thinning</td>
</tr>
<tr>
<td>D</td>
<td>Diameter (inches)</td>
</tr>
<tr>
<td>D̄</td>
<td>Average diameter (inches)</td>
</tr>
<tr>
<td>Dbh</td>
<td>Diameter at breast height</td>
</tr>
<tr>
<td>F</td>
<td>Diameter function</td>
</tr>
<tr>
<td>FERT</td>
<td>Fertilization</td>
</tr>
<tr>
<td>G</td>
<td>Basal area per acre (square foot)</td>
</tr>
<tr>
<td>GR</td>
<td>Growth</td>
</tr>
<tr>
<td>GRATIO</td>
<td>Current basal area/basal area from natural stand</td>
</tr>
<tr>
<td>H</td>
<td>Mean height (foot)</td>
</tr>
<tr>
<td>INTHR</td>
<td>Basal area interval (square foot)</td>
</tr>
<tr>
<td>INTVR</td>
<td>Tree interval</td>
</tr>
<tr>
<td>MAI</td>
<td>Mean annual increment (CF)</td>
</tr>
<tr>
<td>N</td>
<td>Number of trees per acre</td>
</tr>
<tr>
<td>n</td>
<td>Rotation age</td>
</tr>
<tr>
<td>NPCT</td>
<td>Number of PCT alternatives to be considered</td>
</tr>
<tr>
<td>NRATIO</td>
<td>Current number of trees/number of trees from natural stand</td>
</tr>
<tr>
<td>PCT</td>
<td>Precommercial thinning</td>
</tr>
</tbody>
</table>
SYMBOLS (continued)

PCTCOST  Cost of PCT
PN       Pounds of nitrogen
PNW      Present net worth
R        Alternative rate of return
R1       Inflation rate
REGENC   Regeneration cost
S        Site index
SITE     Site index
Se       Soil expectation
SBA      Basal area of a non-normal stand
STREE    Total number of trees of a non-normal stand
T        Department of Natural Resource tariff
TBASE    Base age, age to be considered for first CT
TESTF    Indicator variable of fertilization
TESTN    Indicator variable of natural stand
TESTP    Indicator variable of precommercial thinning
V        Total cubic-foot volume per acre
Var      Variance
V/G      Volume to basal area ratio
### Subscripts

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>Adjusted or adjuster</td>
</tr>
<tr>
<td>(B)</td>
<td>Breast height</td>
</tr>
<tr>
<td>(C)</td>
<td>Commercial thinning</td>
</tr>
<tr>
<td>(CT)</td>
<td>Commercial thinning</td>
</tr>
<tr>
<td>(D)</td>
<td>Dominant and codominant</td>
</tr>
<tr>
<td>(F)</td>
<td>First possible</td>
</tr>
<tr>
<td>(G)</td>
<td>Basal area</td>
</tr>
<tr>
<td>(i)</td>
<td>Diameter class (i)</td>
</tr>
<tr>
<td>(K)</td>
<td>Limit</td>
</tr>
<tr>
<td>(L)</td>
<td>Limit</td>
</tr>
<tr>
<td>(m)</td>
<td>Merchantable limit</td>
</tr>
<tr>
<td>(M)</td>
<td>Merchantable</td>
</tr>
<tr>
<td>(N)</td>
<td>Nitrogen fertilizer</td>
</tr>
<tr>
<td>(P)</td>
<td>Precommercial thinning</td>
</tr>
<tr>
<td>(s)</td>
<td>Submerchantable</td>
</tr>
<tr>
<td>(T)</td>
<td>Total</td>
</tr>
<tr>
<td>(Z)</td>
<td>Ratio</td>
</tr>
<tr>
<td>(50)</td>
<td>Fifty year basis</td>
</tr>
<tr>
<td>(100)</td>
<td>One hundred year basis</td>
</tr>
</tbody>
</table>
APPENDIX II

PROGRAM FOR COMPARING GROWTHS FROM THE DFIT AND
THE GROWTH MODEL BY TURNBULL AND PETERSON
PROGRAM COMPARE(INPUT,OUTPUT)
WRITE *,"READ IN TREES, BA, BHAGE, AND SITE"
READ *, TREE, BA, BHAGE, SITE
IF(TREE.LE.0.) STOP
B30=30. - 13.22 + 0.033 * SITE
D30=10**((0.1097 - 3.4857/B30**0.25 + 1.0531*ALOG10(SITE))
DM=0.75*D30/0.875
AGE=BHAGE+13.22-0.033*SITE
DNATURE=10**((0.1097 - 3.4857/BHAGE**0.25 + 1.0531*ALOG10(SITE))
TNATURE=10**((3.9108+5.2306/BHAGE**0.25 - 1.5803*ALOG10(SITE))
GNATURE=10**((1.8669-1.7408/BHAGE**0.25 + 0.5259*ALOG10(SITE))
TREE1=TREE+TNONMER
BA1=BA+BNNMER
HT=10**((0.1567 - 15.673/AGE + ALOG10(SITE))
VG=10**(-0.0282 + 0.7917*ALOG10(HT))
V1=BA1*VG
ADJ1=(405. - 30.)/400.
GLIMIT=10**((3.3446 - 0.3328*ALOG10(TREE))
DAGE1=10**((0.1097 - 3.4857/BHAGE**0.25 + 1.0531*ALOG10(SITE))
VGRDW=0.
SITE50=21.5 - 0.18127*AGE + 0.72114*SITE
VGRDW1=10**((0.10493*BHAGE**(-0.2269) + SITE50**1.07647*BA1)**
$0.56805*TREE1**0.10875)**10.
$GRW=(12.70144 + 0.2266*AGE - 0.02066*SITE50 - 0.04469*$$BA1-1.64715*BA1/BHAGE + 1.74006*BA1/BHAGE**2)**10.
AGE=AGE-0.5
DO 20 I=1,10
AGE=AGE+1.
BHAGE=AGE-13.22 + 0.033 * SITE
DVOL=10**((ALOG10(2.3026) + ALOG10(12.4083/AGE + 2) + 0.4352/BHAGE**
$1.25) + ALOG10(1.12 + 0.0105*AGE - 0.00005*AGE**2) + 1.9628 - 12.4083/AGE
$-1.7408/BHAGE**0.25 + 1.3176*ALOG10(SITE))
DVOL=DVOL+ADJ1
THPVOL=V1 + DVOL + VGRDW
HT=10**((0.1567 - 15.673/(AGE+0.5) + ALOG10(SITE))
VG=10**(-0.0282 + 0.7917*ALOG10(HT))
G1=THPVOL/VG
DAGE=DAGE1
DAGE1=10**((0.1097 - 3.4857/BHAGE + 0.5)**0.25 + 1.0531*ALOG10(SITE))
GMRT=10**((1.4034 + 4.9394*ALOG10(DM))** (10**(-4.44*ALOG10(DAGE))
$-10**(-4.44*ALOG10(DAGE1)))
G=G1-GMRT
CR=G/GLIMIT
ADJ2=1.16*(CR-0.5)**4
DVOL=DVOL+ADJ2
VGROW=VGROW+DVOL
CONTINUE
G1=(VI+VGROW)/VG
G6GROW=G1-BA1
WRITE *,G6GROW,G6GROW1,VGROW,VGROW1
GOTO 10
END
APPENDIX III

DOPT PROGRAM LISTING
PROGRAM DOPT(INPUT,OUTPUT,TAPE60=INPUT,TAPE61=OUTPUT)
DIMENSION VAL(39,72,2),VMN(39,72,2),TRUEA(39,72,2),TRUEB(39,72,2)
DIMENSION VCUT(39,72),DMEAN(39,72),OPLVR(39,72),OPLHR(39,72)
DIMENSION HRVSTD(16),HRVSTB(16),HRVSTP(16),HRVSTP(16),HRVSTC(16)
DIMENSION NN(100),TNORM(16),GHORM(16),VNORM(16),Z(16)
COMMON TAGE,TBASE,SITE,TRATIO,GRATIO,VGRATIO
COMMON PMORTN(25),PHORMG(25),PMORTV(25),YMORTN(200),YMORTG(200)
COMMON TNOMMER(25),GNOMMER(25),VNOMMER(25)
INTEGER OPLV,R,OPOLVR,OPOLHR
N=1
MAXVR=39
MAXHR=72
READ(60,*) INTVR,INTHR,TBSE,SITE,R,TESTN,TESTP,TESTF,
$REGENC,R1,PCTAGE,NPCT,PCTCOST,STREE,SBA
WRITE(61,3) TBASE,SITE,R,R1,REGENC
SITE=SITE*SITE**2
REGENC=REGENC+PCTCOST/(1.+R)**PCTAGE
DO 5 I=1,18
5 PMORTN(I)=PMORTG(I)=PMORTV(I)=TNOMMER(I)=GNOMMER(I)=VNOMMER(I)=0.
K=(I-1)*10
DO 5 J=1,10
7 YMORTN(K+J)=YMORTG(K+J)=0.
TRATIO=GRATIO=1.
WRITE(61,6) PCTAGE,PCTCOST,SITE1,NPCT,STREE
$ALTERNATIVES WOULD LIKE TO BE CONSIDERED IS",I5,/," MAXIMUM NUMBER OF TREES LEFT AFTER PCT IS",F9.2)
7 TMERCH=STREE-IN*INTVR
SITE=SITEOLD
CALL PRECOM(PCTAGE,SITE,TBASE,TMERCH,GHERCH,VMERCH,ERROR)
IF(ERROR.EQ.0.) GOTO 9
WRITE(61,8) IN
8 FORMAT(\\"**SOME PCT ALTERNATIVES RESULT IN TOO")
" FEW TREES FOR CT AT SPECIFIED CT AGE" /" **THE NUMBER OF"
" PCT ALTERNATIVES CONSIDERED IS",15)
IF(EVER.EQ.0.) STOP
GOTO 17
EVER=1.
IN=IN+1
GOTO 16
IF(TESTN.EQ.0.) GOTO 15
******FOR NON-NATURAL STAND GROW TO AGE OF MULTIPLES OF 10
IF(TBASE.GE.30..AND.10.*IFIX(TBASE/10.).EQ.TBASE) GOTO 15
WRITE(61,12) STREE,SBA
12 FORMAT(/" THIS IS NOT A NORMAL STAND, AND THE INITIAL NUMBER"
" OF TREES IS",F7.2," AND BA IS",F7.2," SQUARE FEET")
CALL INCRuft(STREE,SBA,TBASE,SITE)
TAGE=TBASE
WRITE(61,14) TBASE,STREE,SBA
14 FORMAT(/" STAND GROWS TO AGE",F6.1" WITH",F8.2," TREES AND BA",
" SQ FT")
******CALCULATE NATURAL STAND AT BASE AGE
15 BHAPE=TAGE-13.22+0.033*SITE
DNATURE=10**((0.1097-3.4857/BHAGE**0.25+1.0531*ALOG10(SITE))
TNATURE=10**((3.9108+5.2306/BHAGE**0.25-1.5803*ALOG10(SITE))
GNATURE=10**((1.8669-1.7408/BHAGE**0.25+0.5259*ALOG10(SITE))
HT=10**((0.1567-15.673/TAGE+ALOG10(SITE))
VG=VGRATIO=10**((-0.0282+0.7917*ALOG10(HT))
VNATURE=GNATURE*VGRATIO
TRATIO=STREE/TNATURE
GRATIO=SBA/GRATIO
IF(TESTN.EQ.0.) TRATIO=GRATIO=1.
******CALCULATE MERCHANTABLE PART AND MORTALITY LATER PART
CALL SUBMort(DNATURE)
L=TBASE/10.-2.
GMERCH=GNATURE-GNONM(ER(L)/GRATIO
TMERCH=TNATURE-TNONM(L)/TRATIO
UMERCH=VNATURE-VNONM(L)/GRATIO
16 I=GMERCH*TRATIO/INTVR+1.999999
IF(I.GT.MAXVR) I=MAXVR
J=(GMERCH*GRATIO+INTHR/2.)/INTVR+1.999999
TRUET(I,J,1)=GMERCH*TRATIO
TRUEB(I,J,1)=GMERCH*GRATIO
VME(I,J,1)=VMERCH*GRATIO
VAL(I,J,1)=REGENC
IF(IN.LT.NPCT.AND.TESTP.EQ.1.) GOTO 7
17 NS=GMERCH/(2.*INTVR)+1.999999
IF((NS-1.)*INTVR.GT.TRUET(I,J,1)) NS=TRUET(I,J,1)/INTVR+1.999999
IF(TES1tP.EQ.1.) NS=400./(2.*INTVR)+1.99999

*****CALCULATE NATURAL STAND AND MERCHANTABLE TREES.
DO 18 I=L,14
   AB=30+10*(I-1)
   B=AB-13.22+0.033*SITE
   TNORM(I)=10**(3.9108+5.2306/B**0.25-1.5803*ALOG10(SITE))
   GNORM(I)=10**(1.8669-1.7408/B**0.25+0.5259*ALOG10(SITE))
   VNORM(I)=10**(1.9628-12.4083/AB-1.7408/B**25+1.3176*ALOG10(SITE))
18
Z(I)=TNORM(I)-THONMER(I)/TRATIO
SITE50=21.5-0.18127*30.+0.72114*SITE

DO 20 I=1,MAXVR
   DO 20 J=1,MAXHR
      VAL(I,J,1)=VLM(I,J,1)-TRUEBA(I,J,1)=TRUEET(I,J,1)=-999999.9
      OPOLVI(I,J)=OPOLHR(I,J)=VCUT(I,J)=DMEAN(I,J)=-999999.9
      IF(N.EQ.1) GOTO 25
      NS=1
      HT=10**(0.1567-15.673/(TAGE-10.)+ALOG10(SITE))
      VG=10**(-0.0282+0.7917*ALOG10(HT))
      KJ=TAGE/10.
      BHAGE=(TAGE-10.)-13.22+0.033*SITEOLO
      GBOUND=0.
      IQUANT=3
      IF(N.EQ.1.OR.TAGE.GT.70..OR.TESTP.EQ.0.) IQUANT=1
      DO 55 I=2,MAXVR
      IF(V(AV(I,J,1)).LE.-999999.9) 6010 55
      *$***PULL OUT NEXT CARD WHEN MORE INFORMATIONS ARE NEEDED
      IF(TES1tP.EQ.1..AND.I.GT.(20/N+7)*15/INTVR+2) 6010 55
      DO 50 J=1,MAXHR
      IF(VAL(I,J,1).LE.-999999.9) GOTO 50
      ****IN THE FIRST STAGE THIN FIRST THEN GROW
      IF(N.EQ.1) GOTO 28
      VOL=VLM(I,J,1)+VNONMER(N+L-1)
      CALL GROWTH(TRUEET(I,J,1),VOL,VMER,VMORT1,GMER,GMORT1,N,TESTP)
      TOTAL=TRUEET(I,J,1)+THONMER(N+L-2)
      TOTALG=TRUEBA(I,J,1)+GNONMER(N+L-2)
      VMORT2=VMORT1
      GMORT2=GMORT1
      VFERT=VOL+10.*(0.10493*BHAGE**(-0.2269)+SITE50**1.07647)*
      *TOTALG**0.56805+TOTAL**0.10875)
      VLM(I,J,1)=VMER
      TRUEBA(I,J,1)=GMER
      PNORM=TRUEET(I,J,1)/Z(N+L-2)
      TMORT1=(Z(N+L-2)-Z(N+L-1)))*PNORM
      IF(TES1tP.EQ.1.) TMORT1=0.
      TRUEET(I,J,1)=TRUEET(I,J,1)-TMORT1
      ****CALCULATE FERTILIZATION EFFECTS
DO 45 IFERT=1,1QUANT
   IF(IFERT.EQ.1) GOTO 30
   VFERT1=VFERT+10.*(-0.27823-0.46375*SITE50+ALOG10(1.+IFERT-1)*200.)+78.90236+ALOG10(1.+IFERT-1)*200.)
   ADJST=VFERT1/VFERT
   VLM(I,J,1)=VMER*ADJST
   TRUEBA(I,J,1)=GMER*ADJST
   VMORT1=VMORT2*ADJST
   GMORT1=GMORT2*ADJST
30 DIAM=SQRT(TRUEBA(I,J,1)/(0.005454154*TRUET(I,J,1)))
   NT=TRUET(I,J,1)/INTVR+1.99999
   ****WITH I TREES THERE ARE I+1 KINDS OF THINNING.
   ****THIN PROPORTIONALLY TO THE DISTRIBUTION OF DIAMETER
   ****SO THE MEAN DIAMETER IS UNCHANGED.
   DO 40 K=NS,NT
      IF(K.LT.NT) GOTO 32
      TMPBA=TRUEBA(I,J,1)
      TMPVLM=VLM(I,J,1)
      CUT=REV=0.
   GOTO 35
32 TMPBA=TRUEBA(I,J,1)*(K-1)*INTVR/TRUET(I,J,1)
   TMPVLM=VLM(I,J,1)*(K-1)*INTVR/TRUET(I,J,1)
   CUT=VLM(I,J,1)-TMPVLM+VMORT1
   DIAM1=SQRT((TRUEBA(I,J,1)-TMPBA+GMORT1)/(0.005454154*(TRUET(I,J,1)-$(K-1)*INTVR+TMORT1))
   ****CALCULATE CUBIC VOLUME REMOVED TO A 4-INCH TOP
   HD=10**(.1567-15.673/TAGE+ALOG10(SITE))
   N1=(TAGE-20.)/10.
   ALLTREE=TRUET(I,J,1)+TNONMER(N1)
   HM=HD*(3040.-ALLTREE)/3000.
   IF(HM.GT.HD) HM=HD
   VOL4=CUT*0.0725-0.065722*SITE*0.00001508*TAGE**2
   IF(DIAM1.GT.22.) DIAM1=22.
   TRUED=SQRT(DIAM1**2-VAR)
   REV=REVNDM(VOL4,TRUED)
35 FERTCST=(-50.+52.5*IFERT-2.5*IFERT**2)/(1.+R)**(TAGE-10.)
   TMPVAL=REV*(1.+R)**TAGE/(1.+R)**TAGE-FERTCST+VAL(I,J,1)
   KK=IFIX((TMPBA+INTHR/2.)/INTHR)+1
   IF(KK.GT.MAXHR) KK=MAXHR
   IF(TMPVAL.LE.VAL(K,KK,2)) GOTO 40
   DMEAN(K,KK)=IFERT+1000.+DIAH
   IF(TESTF.EQ.0.) DMEAN(K,KK)=DIAM
   VAL(K,KK,2)=TMPVAL
OPOLVR(K,KK)=(I-1)*INTVR
OPOLHR(K,KK)=(J-1)*INTHR
TRUEBA(K,KK,2)=TMPBA
IF(TMTPBA.GT.GBOUND) GBOUND=TMTPBA
TRUE(K,KK,2)=(K-1)*INTVR
IF(K.EQ.NT) TRUET(K,KK,2)=TRUET(I,J,1)
IF(K.EQ.1) HRVSTB(KJ)=TRUEBA(I,J,1)
VLM(K,KK,2)=TMPVLM
VCUT(K,KK)=CUT

40 CONTINUE
45 CONTINUE
50 CONTINUE
55 CONTINUE

******OUTPUT

WRITE(61,60) TAGE

***("IAGE="",F5.1"" 1=MEAN DIAMETER (INCH)/
**" (FIRST DIGIT INDICATES FERTILIZATION: 1=0 PB, 2=200 PB, 3=400 PB
**" 2=VOLUME (CUBIC FEET)"
**" 3=VOLUME CUT IN THIS STAGE (CUBIC FEET)"/" 4=TRUE BASAL ARE
**" 5=TRUE NUMBER OF TREES"/" 6=CUMULATIVE VALUE FR
**" 7=WHRNE IT COMES FROM( TREES,BA)"

WRITE(61,61) TNONMER(N+L-1)

DO 62 I=1,72

62 NN(I)=(I-1)*INTHR
IS=-11
DO 75 J=1,6
IS=IS+12
IE=IS+11
WRITE(61,64) TAGE,(NN(I),I=IS,IE)

64 FORMAT("AGE="",F5.1"" TREES*BA ",12(13,7X))
NI=Z(N+L-1)*TRATIO/INTVR+1.999999

******CHANGE NEXT CARD TO NI=400./INTVR+1.999999 WHEN NECESSARY
IF(TECSTP.EQ.1.) NI=(20/N+7.)*15./INTVR+1.999999
DO 70 I=1,NI
II=(I-1)*INTVR
WRITE(61,65) II,(DMEAN(I,J),J=IS,IE)

65 FORMAT(3X,13,12(3X,F7.1))
WRITE(61,66) (VLM(I,J,2),J=IS,IE)
WRITE(61,66) (VCUT(I,J),J=IS,IE)
WRITE(61,66) (TRUEBA(I,J,2),J=IS,IE)
WRITE(61,66) (TRUEET(I,J,2),J=IS,IE)
WRITE(61,66) (VAL(I,J,2),J=IS,IE)

66 FORMAT(3X,12(3X,F7.1))
WRITE(61,68) (OPOLVR(I,J),OPOLHR(I,J),J=IS,IE)
FORM(7X,12("","I3","","I3",""),1X))
CONTINUE
IF(IE.GE.IFIX((GBOUND+INTHR/2.)/INTHR)+1) GOTO 80
WRITE(61,72)
FORMAT("1")
CONTINUE
HRVSTD(KJ)=DMEAN(1,1)-100.*IFIX(DMEAN(1,1)/100.)
HRVSTP(KJ)=VAL(1,1,2)
HRVSTS(KJ)=VAL(1,1,2)*(1.+R)**(TAGE/((1.+R)**TAGE-1.))
HRVSTC(KJ)=VCUT(1,1)
IF(VAL(1,1,2).LE.VAL(1,1,1).OR.N.GE.14) GOTO 95
N=N+1
TAGE=TAGE+10.
DO 90 I=1,MAXVR
   DO 90 J=1,MAXHR
      VAL(I,J,1)=VAL(I,J,2)
      VLM(I,J,1)=VLM(I,J,2)
      TRUEET(I,J,1)=TRUEET(I,J,2)
      TRUEBA(I,J,1)=TRUEBA(I,J,2)
   GOTO 90
90 TRUEBA(I,J,1)=TRUEBA(I,J,2)
GOTO 19
WRITE(61,96)
FORMAT("1ROTATION AGE HARVEST VOLUME DIAMETER HARVEST BA
$ PNW SE")
KI=TBASE/10.+1.
DO 90 L=KI,KJ
   LL=L*10
   WRITE(61,99) LL,HRVSTC(L),HRVSTD(L),HRVSTB(L),HRVSTP(L),HRVSTS(L)
STOP
END
SUBROUTINE PRECOM(PCTA, SITE, TBASE, TMERCH, GMERCH, VMERCH, ERROR)
ERROR=0.
DMERCH=10**((3.9591-ALOG10(DMERCH))/1.5006)
GMERCH=10**((1.6958+0.4994*ALOG10(DMERCH))
ADJ=(405.-PCTA)/400.
SITE1=SITE+(1.+((210.-SITE)/300.))**2)

*****CALCULATE THE AGE WHEN DM REACHES 8 INCHES
BHAGE=(3.4857/((1.097-ALOG10(1.875*DMERCH)+1.0531*ALOG10(SITE1)))**4
TAGE=BHAGE+13.22-0.033*SITE
TAGE=TAGE* (0.824+0.00433*PCTA/10. +0.015*(PCTA/10.)**2+
$0.01667*(PCTA/10.)**3)
IF(TAGE.LE.TBASE) GOTO 20
ERROR=1.
RETURN

20 SITE=SITE1
HT=10**((0.1567-15.673/TAGE+ALOG10(SITE))
VG=10**(-0.0282+0.7917*ALOG10(HT))
VMERCH=GMERCH*VG
GP=180.22-5.*DMERCH
IF(GP.GE.GMERCH) GOTO 25
TAGE=TAGE+0.5
BHAGE=TAGE+13.22+0.033*SITE
DVA=10*(ALOG10(2.3026)+ALOG10(12.4083/TAGE**2+0.4352/BHAGE**1.7408/BHAGE**0.25+1.3176*ALOG10(SITE))
VMERCH=VMERCH+DVA*ADJ
TAGE=TAGE+0.
HT=10**((0.1567-15.673/TAGE+ALOG10(SITE))
VG=10**(-0.0282+0.7917*ALOG10(HT))
GP=VMERCH/VG
IF(GP.LT.GMERCH) GOTO 25
GMERCH=GP

*****ROUND THE AGE UP TO AN INTEGER NUMBER
28 IAGE=TAGE+1.
FRACT=IMAGE-TAGE
HT=10**((0.1567-15.673/TAGE+ALOG10(SITE))
VG=10**(-0.0282+0.7917*ALOG10(HT))
VMERCH=GMERCH*VG
AGE=TAGE+FRACT/2.
BHAGE=AGE+13.22+0.033*SITE
DVA=10*(ALOG10(2.3026)+ALOG10(12.4083/AGE**2+0.4352/BHAGE**1.7408/BHAGE**0.25+1.3176*ALOG10(SITE))
DVA=DVA*ADJ*FRACT
VMERCH=VMERCH+DVA
HT = 10**(0.1567 - 15.673/IAGE + ALOG10(SITE))
VG = 10**((-0.0282 + 0.7917*Aalog10(HT))
GMERCH = VMERCH/VG

******GROW STAND TO THE AGE OF FIRST CT
TAGE = IAGE
ADJ = (405. - TAGE)/400.
GLIMIT = 10**(3.3446 - 0.3328*Alog10(TMVRCH))

30 IF (TAGE .GE. TBASE) RETURN
AGE = TAGE + 0.5
BHAGE = AGE - 13.22 + 0.033*SITE
DVA = 10**(ALOG10(2.3026) + ALOG10(12.4083/AGE)**2 + 0.4352/BHAGE**1.25
* + ALOG10(1.12 + 0.0105*AGE - 0.00005*AGE**2) + 1.9628 - 12.4083/AGE -
$1.7408/BHAGE**0.25 + 1.3176*ALOG10(SITE))
DVA = DVA*ADJ
TMPV = VMERCH + DVA
HT = 10**(0.1567 - 15.673/(TAGE + 1.) + ALOG10(SITE))
VG = 10**((-0.0282 + 0.7917*Alog10(HT))
G = TMPV/VG
CR = G/GLIMIT
ADJ = 1. - 16.*(CR - 0.5)**4
VMERCH = VMERCH + DVA*ADJ
GMERCH = VMERCH/VG
TAGE = TAGE + 1.
GOTO 30.

END
SUBROUTINE INCMNT(STREE,SBA,TBASE,SITE)

TBASE1 = TBASE
IF(10.*IFIX(TBASE/10.).NE.TBASE) TBASE=10.*IFIX(TBASE/10.)+1.
IF(TBASE.LT.30.) TBASE=30.
BHA6E1 = TBASE-13.22+0.033*SITE
TNATURE=10**(3.9108+5.2306/BHA6E1**0.25-1.5803*ALOG10(SITE))
GNATURE=10**(1.8669-1.7408/BHA6E1**0.25+0.5259*ALOG10(SITE))
GRATIO=SBA/GNATURE
TRATIO=STREE/TNATURE

D1=10**(1.097-3.4857/BHA6E1**0.25+1.0531*ALOG10(SITE))
BHA6E1 = TBASE-13.22+0.033*SITE
D2=10**(0.1097-3.4857/BHA6E1**0.25+1.0531*ALOG10(SITE))
DM=D1*0.75/0.875

TNONMER=10**(3.8622+3.1994*ALOG10(DM)-4.7*ALOG10(D1))
TNONMER=TNONMER*TRATIO

T1=10**(3.8622+3.1994*ALOG10(DM)-4.7*ALOG10(D1))
T2=10**(3.8622+3.1994*ALOG10(DM)-4.7*ALOG10(D2))
TMORT=TNONMER-TNOMER*(T2/T1)

GNONMER=10**(1.4034+4.9394*ALOG10(DM)-4.44*ALOG10(D1))
GNONMER=GNONMER*GRATIO
ADJI=(405.-TBASE)/400.
TREE=STREE-TNONMER

GLIMIT=10**(3.3446-0.3328*ALOG10(TREE))
HT=10**(0.1567-15.673/TBASE1+ALOG10(SITE))
VG=10**(-0.0282+0.7917*ALOG10(H1))
V=SBA*VG
VGROW=0.

10 TBASE1=TBASE1+0.5
BHA6E1 = TBASE1-13.22+0.033*SITE
DVOL=10**(ALOG10(2.3026)+ALOG10(12.4083/TBASE1**2+0.4352
D/BHA6E1**1.25)+
$1.9628-12.4083/TBASE1-1.7408/BHA6E1**0.25+1.3176*ALOG10(SITE))
DVOL=DVOL+ADJI
TBASE1=TBASE1+0.5
HT=10**(0.1567-15.673/TBASE1+ALOG10(SITE))
VG=10**(-0.0282+0.7917*ALOG10(H1))
TMPV=V+DVOL+VGROW
G=TMPV/VG
GMERCH=G-GNONMER
CR=GMERCH/GLIMIT
ADJ2=1.-16*(CR-0.5)**4
DVOL=DVOL+ADJ2
VGROW=VGROW+DVOL
IF(TBASE1.LT.TBASE) GOTO 10
STREE=STREE-THMORT
SBA=(V+VGROW)/VG
RETURN
END
SUBROUTINE SUBMORT(DD)
COMMON TAGE, TBASE, SITE, TRATIO, GRATIO, VGRATIO
COMMON PMORTN(25), PMORTG(25), PMORTV(25), YMORTN(200), YMORTG(200)
COMMON TNONMER(25), VNOMN MER(25), VNOMMER(25)

***** CALCULATE PERIODIC MORTALITY STARTS FROM THE AGE OF FIRST THINNING
B30 = 30. - 13.22 + 0.033*SITE
D30 = 10**((0.1097 - 3.4857/BHAGE**0.25 + 1.0531*ALOG10(SITE))
DM = (D30/0.875) * 0.75
DL = 0.698*D30
L = TBASE/10. - 2.
TNONMER(L) = YMORTN(L) = 10**((3.8622 + 3.1994*ALOG10(DM) - 4.7*ALOG10(DD))
*TRATIO
GNONMER(L) = YMORTG(L) = 10**((1.4034 + 4.9394*ALOG10(DM) - 4.44*ALOG10(DD))
*GRATIO
VNOMMER(L) = GNOMMER(L) * VGRATIO * TARIF(DD) / TARIF(DL)
DO 10 JJ = L, 18
AGE = 30 + JJ*10.
BHAGE = AGE - 13.22 + 0.033*SITE
D = 10**((0.1097 - 3.4857/BHAGE**0.25 + 1.0531*ALOG10(SITE))
TNONMER(JJ+1) = 10**((3.8622 + 3.1994*ALOG10(DM) - 4.7*ALOG10(DD)) * TRATIO
GNONMER(JJ+1) = 10**((1.4034 + 4.9394*ALOG10(DM) - 4.44*ALOG10(DD)) * GRATIO
DMORTL = SQRT((GNOMMER(JJ+1)/TNOMMER(JJ+1))/0.005454154)
HT = 10**((0.1567 - 15.673/AGE + ALOG10(SITE))
VG = 10**((-0.0282 + 0.7917*ALOG10(HT))
TAVE = (VG*TARIF(D)+VGRATIO*TARIF(DD))/2.
VNOMMER(JJ+1) = TAVE/TARIF(DMORTL)*GNOMMER(JJ+1)
PMORTN(JJ) = (TNOMMER(JJ) - TNOMMER(JJ+1))
PMORTG(JJ) = (GNOMMER(JJ) - GNOMMER(JJ+1))
DMORT = SQRT((PMORTG(JJ)/PMORTN(JJ))/0.005454154)
PMORTV(JJ) = TAVE/TARIF(DMORT)*PMORTG(JJ)
10 CONTINUE

***** CALCULATE YEARLY MORTALITY STARTS FROM THE AGE OF FIRST THINNING
LL = (L-1)*10+1
DO 20 II = LL, 180
AGE = 30 + II
BHAGE = AGE - 13.22 + 0.033*SITE
D = 10**((0.1097 - 3.4857/BHAGE**0.25 + 1.0531*ALOG10(SITE))
YMORTN(II+1) = 10**((3.8622 + 3.1994*ALOG10(DM) - 4.7*ALOG10(DD))
YMORTG(II+1) = 10**((1.4034 + 4.9394*ALOG10(DM) - 4.44*ALOG10(DD))
YMORTN(II) = (YMORTN(II) - YMORTN(II+1)) * TRATIO
YMORTG(II) = (YMORTG(II) - YMORTG(II+1)) * GRATIO
20 CONTINUE
RETURN
END
SUBROUTINE GROWTH(TREE, V, VMER, VMDRT1, GMER, GMORT1, M, TESTP)
COMMON TAGE, TBASE, SITE, TRATIO, GRATIO, VGRATIO
COMMON PMORTN(25), PMORTG(25), PMORTV(25), YMORTN(200), YMORTG(200)
COMMON TNONMER(25), GNONMER(25), VNONMER(25)
VGROW=VGROW1=0.

***** ASSUME FIRST CT AT FIRST STAGE
ADJ1=(405.-TBASE)/400.
GLIMIT=10**(3.3446-.3328*ALOG10(TREE))
HT=10**(0.1567-15.673/(TAGE-10.)+ALOG10(SITE))
TMPAGE=TAGE-9.5
DO 10 I=1,10
NN=TMPAGE-30.+2.
BHA6E=TMPAGE-13.22+0.033*SITE
DHT=10**(1.7141+ALOG10(SITE)-15.673/TMPAGE-2.*ALOG10(TMPAGE))
HT=HT+DHT
VG=10**((-0.0282+0.7917*ALOG10(HT)))

***** CALCULATE GROSS GROWTH
DVA=1.12+0.0105*TMPAGE-0.00005*TMPAGE**2
IF (TMPAGE.GT.105.) DVA=10**0.22304
DVOL=10**(ALOG10(2.3026)+ALOG10(12.4083/TMPAGE**2+.4352/BHAGE**
*1.25)+ALOG10(DVA)+1.9628-12.4083/TMPAGE-1.7408/BHAGE**0.25+
*1.3176+ALOG10(SITE))
DVOL=DVOL*ADJ1
TMPVOL=VGROW+DVOL*V
G=TMPVOL/VG
GMERCH=G-YMORTG(NN)
CR=GMERCH/GLIMIT
ADJ2=1.-16.*(CR-0.5)**4
DVOL=DVOL*ADJ2
VGROW=VGROW+DVOL

***** CALCULATE NET GROWTH
DVOL1=10**(ALOG10(2.3026)+ALOG10(12.4083/TMPAGE**2+.4352/BHAGE**
*1.25)+1.9628-12.4083/TMPAGE-1.7408/BHAGE**0.25+1.3176+ALOG10(SITE))
DVOL1=DVOL1*ADJ1
TMPVOL1=VGROW1+DVOL1*V
G1=TMPVOL1/VG
GMERCH1=G1-YMORTG(NN)
CR1=GMERCH/GLIMIT
ADJ21=1.-16.*(CR1-0.5)**4
DVOL1=DVOL1*ADJ21
VGROW1=VGROW1+DVOL1
TMPAGE=TMPAGE+1.
10 CONTINUE

***** CALCULATE MERCHANTABLE MORTALITY AND MERCHANTABLE LIVE TREES.
N=(TBASE-30.)/10.+M
VMORT1 = VGROW - VGROW1
VMER = V + VGROW1 - VNORMER(N+1)
IF (TESTP.EQ.1.) VMER = VMER + VMORT1
GMORT1 = (V + VGROW) / VG - (V + VGROW1) / VG
GMER = (V + VGROW) / VG - GMORT1 - GNORMER(N+1)
IF (TESTP.EQ.0.) RETURN
VMER = V + VGROW - VNORMER(N+1)
GMER = VMER / VG
GMORT1 = VMORT1 = 0.
RETURN
END
FUNCTION REVNOW(VOL,D)
*****CALCULATE STUMP TO TRUCK LOGGING COST
IF (D.GT.6.05) GOTO 5
  COST=7790.9*VOL**(-0.2834)
GOTO 150
5 IF (D.GT.7.63) GOTO 10
  IF (VOL.GT.1000.) GOTO 7
  C1=7790.9*VOL**(-0.2834)
  C2=4954.7*VOL**(-0.2726)
  DELTA=(C1-C2)/(7.63-6.05)
  COST=C1-DELTA*(D-6.05)
GOTO 150
7 C1=7800.8*VOL**(-.2539)
  C2=7187.5*VOL**(-.2891)
  DELTA=(C1-C2)/(7.63-6.05)
  COST=C1-DELTA*(D-6.05)
GOTO 150
10 IF (D.GT.9.23) GOTO 20
  IF (VOL.GT.1000.) GOTO 15
  C1=4954.7*VOL**(-0.2726)
  C2=3768.0*VOL**(-0.2662)
  DELTA=(C1-C2)/(9.23-7.63)
  COST=C1-DELTA*(D-7.63)
GOTO 150
15 IF (VOL.GT.2000.) GOTO 18
  C1=7187.5*VOL**(-.2891)
  C2=6254.8*VOL**(-.3013)
  DELTA=(C1-C2)/(9.23-7.63)
  COST=C1-DELTA*(D-7.63)
GOTO 150
18 C1=14353.0*VOL**(-.3782)
  C2=10336.6*VOL**(-.3627)
  DELTA=(C1-C2)/(9.23-7.63)
  COST=C1-DELTA*(D-7.63)
GOTO 150
20 IF (D.GT.10.87) GOTO 30
  IF (VOL.GT.1000.) GOTO 25
  C1=3768.0*VOL**(-.2662)
  C2=4375.0*VOL**(-.2833)
  DELTA=(C1-2)/(10.87-9.23)
  COST=C1-DELTA*(D-9.23)
GOTO 150
25 IF (VOL.GT.2000.) GOTO 28
  C1=6524.8*VOL**(-.3013)
  C2=4375.0*VOL**(-.2833)
DELTA=(C1-C2)/(10.87-9.23)
COST=C1-DELTA*(D-9.23)
GOTO 150
28 C1=10336.6*VOL**(-.3627)
C2=8479.6*VOL**(-.3626)
DELTA=(C1-C2)/(10.87-9.23)
COST=C1-DELTA*(D-9.23)
GOTO 150
30 IF(D.GT.12.31) GOTO 40
C1=8479.6*VOL**(-.3626)
C2=6839.5*VOL**(-.3492)
DELTA=(C1-C2)/(12.31-10.87)
COST=C1-DELTA*(D-10.87)
GOTO 150
40 IF(D.GT.13.66) GOTO 50
C1=6839.5*VOL**(-.3492)
C2=6276.4*VOL**(-.3498)
DELTA=(C1-C2)/(13.66-12.31)
COST=C1-DELTA*(D-12.31)
GOTO 150
50 IF(D.GT.15.03) GOTO 60
C1=6276.4*VOL**(-.3498)
C2=5848.5*VOL**(-.3548)
DELTA=(C1-C2)/(15.03-13.66)
COST=C1-DELTA*(D-13.66)
GOTO 150
60 IF(D.GT.16.19) GOTO 70
C1=5848.5*VOL**(-.3548)
C2=4980.1*VOL**(-.3475)
DELTA=(C1-C2)/(16.19-15.03)
COST=C1-DELTA*(D-15.03)
GOTO 150
70 IF(D.GT.17.26) GOTO 80
C1=4980.1*VOL**(-.3475)
C2=3765.6*VOL**(-.3238)
DELTA=(C1-C2)/(17.26-16.19)
COST=C1-DELTA*(D-16.19)
GOTO 150
80 IF(D.GT.18.31) GOTO 90
C1=3765.6*VOL**(-.3238)
C2=4215.0*VOL**(-.3402)
DELTA=(C1-C2)/(18.31-17.26)
COST=C1-DELTA*(D-17.26)
GOTO 150
90 IF(D.GT.20.25) GOTO 100
C1=4215.0*VOL**(-.3402)
C2=3377.5*VOL**(-.3207)
DELTA=(C1-C2)/(20.25-18.31)
COST=C1-DELTA*(D-18.31)
GOTO 150

100 IF(D.GT.21.90) GOTO 110
C1=3377.5*VOL**(-.3207)
C2=4209.5*VOL**(-.3488)
DELTA=(C1-C2)/(21.9-20.25)
COST=C1-DELTA*(D-20.25)
GOTO 150

110 COST=4209.5*VOL**(-.3488)

******CALCULATE CURRENT REVENUE AS POND VALUE LESS
******LOGGING AND HAUL COST ($/CF)
150 PONDVAL=9.91+.7061*D
HAUL=150.
REVNOU=VOL*(PONDVAL-COST/1.5-HAUL)*0.001
RETURN
END
FUNCTION TARIF(DIAM)
TARIF=(0.00497819*DIAM**2)/(0.005454154*(DIAM**2+16.)*(1.0378+0.497*(0.0134*(DIAM/10.3)))-0.174532)
RETURN
END
APPENDIX IV

FLOWCHARTS OF DOPT MAIN PROGRAM AND SUBROUTINES
DOPT

START

Input data

Want PCT?

Yes

CALL PCT

No

Normal stand?

Yes

CALL INCRMNT

Calculate data in a normal stand at base age

CALL SUBMORT

Calculate merchantable part and mortality later part

Assign initial condition to appropriate node

Mortalities = 0

No

#PCT considered accomplished?

Yes

No

CALL INCRMNT

1

2
Calculate data of natural stand and merchantable trees

If in first stage, age is older than 70, or fertilization is not requested, then number of fertilization considered is one

Check every node, in the first stage thinning is restricted to 50% of the stocking

Node feasible?

First stage?
CALL GROWTH

Calculate fertilization effect

Consider different levels of thinning

Change total volume to volume to a four-inch top, and quadratic mean diameter to arithmetic mean diameter

All levels of thinning have been considered?

Yes

No

CALL REVNOW
The current alternative better than the alternative occupying the node of the state?

Substitute all information of the node by that of the current alternative

All states have been checked?

Output information

N = N + 1
Age = Age + 1
Final stand conditions of different rotations

STOP
Calculate $\overline{D}_p$, $G_M$ and adjusted site index

Calculate the age $A_T$ which the stand diameter reaches $\overline{D}_p$

Adjust $A_T$ to $A_F$ for PCT

$A_F < \text{Age for the first CT?}$

Yes → RETURN

No

Calculate adjusted BA, $G_S$ when less than 400 trees in the stand
If $A_F$ is not an integer, calculate the growth in the remaining fractional year, round the age up to the next nearest integer $A_F^t$.

Calculate growth in next year

If $A_F^t \geq \text{Age for the first CT}$?

Yes: RETURN

No: Calculate growth in next year

$A_F^t = A_F^t + 1$
Calculate $D_{30}$, $D_m$, $D_L$

Calculate $N_s$, $G_s$ at ten-year interval

Calculate $D_s'$, $N_s'$, $G_s'$ and $V_s'$

Calculate $N_s$, $G_s$ at yearly interval

Calculate $D_s'$, $N_s'$, $G_s'$ and $V_s'$

RETURN
GROWTH

ENTER

Calculate ADJ1, \( G_L, \bar{H}_D \)

Calculate \( d\bar{H}_D \)
\[ \bar{H}_D = \bar{H}_D + d\bar{H}_D \]
\[ V/G \]

Calculate \( dV' \) and adjusted by ADJ1

\[ \text{TMPV} = V + dV' \]
\[ \text{TMPG} = \text{TMPV} / (V/G) \]
\[ G_M = G - G_s' \]

Calculate \( G_L \)
\[ G_Z = G_M / G_L \]
\[ \text{ADJ2} = 1 - 16 \times (G_Z - 0.5)^4 \]
\[ dV' = dV' \times \text{ADJ2} \]
Growth1 = Growth1 + dV

Calculate dV and adjusted by ADJ1

TMPV = V + dV
TMPG = TMPV / (V/G)
G_M = G - G'

G_z = G_M / G_L
ADJ2 = 1 - 16*(G_z - 0.5)^4

dV = dV * ADJ2

Growth2 = Growth2 + dV

AGE = AGE + 1

AGE ≥ Next CT age?

No → 2

Yes → 3
Mortality = Growth1 - Growth2

Is stand PCT?

No → RETURN

Yes

Mortality = 0

RETURN
5

Calculate cost

Calculate pond value, haul cost, revenue

RETURN
INCRMNT

ENTER

Find the next age of multiple ten

Compare current stand with normal stand to get NRATIO and GRATIO

Calculate submerchantable trees which will die in the period from current age to the next age of multiple ten

Calculate live trees and the corresponding BA

Calculate growth dV' and adjusted by ADJ1

\[
\begin{align*}
\text{TMPV} &= V + dV' \\
\text{TMPG} &= \frac{\text{TMPV}}{V/G} \\
G_M &= G - G'_S
\end{align*}
\]
1

Calculate $G_L$, $G_Z$, ADJ2
$dV' = dV' \times ADJ2$

Growth = Growth + $dV'$
Age = Age + 1

Age ≥ Next age of multiple ten?

No → 2

Yes

RETURN
APPENDIX V

NUMBER OF ALTERNATIVES FOR REACHING EACH STATE WHEN
TREE INTERVALS USED ARE 20, 25, 30, 40, 50 AND 100
### TABLE XLIII. INTERVAL 20

<table>
<thead>
<tr>
<th>Stage</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>10</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
</tr>
<tr>
<td>#Trees</td>
<td>400</td>
<td>380</td>
<td>360</td>
<td>340</td>
<td>320</td>
<td>300</td>
<td>280</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>27</td>
<td>54</td>
<td>90</td>
<td></td>
<td>135</td>
<td>252</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>108</td>
<td>270</td>
<td></td>
<td>945</td>
<td>189</td>
<td>252</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>405</td>
<td></td>
<td>90</td>
<td>540</td>
<td>189</td>
<td>2268</td>
</tr>
<tr>
<td></td>
<td>81</td>
<td>405</td>
<td>1215</td>
<td>2835</td>
<td>13608</td>
<td>10206</td>
<td>17010</td>
</tr>
<tr>
<td></td>
<td>243</td>
<td>1458</td>
<td>5103</td>
<td>13608</td>
<td>30618</td>
<td>61236</td>
<td>112266</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>380</td>
<td>360</td>
<td>340</td>
<td>320</td>
<td>300</td>
<td>280</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>27</td>
<td>54</td>
<td>90</td>
<td></td>
<td>135</td>
<td>252</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>108</td>
<td>270</td>
<td></td>
<td>945</td>
<td>189</td>
<td>2268</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>405</td>
<td></td>
<td>90</td>
<td>540</td>
<td>189</td>
<td>2268</td>
</tr>
<tr>
<td></td>
<td>81</td>
<td>405</td>
<td>1215</td>
<td>2835</td>
<td>13608</td>
<td>10206</td>
<td>17010</td>
</tr>
<tr>
<td></td>
<td>243</td>
<td>1458</td>
<td>5103</td>
<td>13608</td>
<td>30618</td>
<td>61236</td>
<td>112266</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE XLIV. INTERVAL 25

<table>
<thead>
<tr>
<th>Stage</th>
<th>Age 0</th>
<th>Age 1</th>
<th>Age 2</th>
<th>Age 3</th>
<th>Age 4</th>
<th>Age 5</th>
<th>Age 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trees</td>
<td>Number of Alternatives</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>------------------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>1 1 3 9 27 81 243</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>375</td>
<td>0 1 6 27 108 405 1458</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>350</td>
<td>0 1 9 54 270 1215 5103</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>325</td>
<td>0 1 12 90 540 2835 13608</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>0 1 15 135 945 5670 30618</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>275</td>
<td>0 1 18 189 1512 10206 61236</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>0 1 21 252 2268 17010 112266</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>225</td>
<td>0 1 24 324 3240 26730 192456</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>0 1 27 405 4455 40095 312741</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>175</td>
<td>0 0 27 486 5913 57834 486243</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>0 0 27 567 7614 80676 728271</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>125</td>
<td>0 0 27 648 9558 109350 1056321</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0 0 27 729 11745 144585 1490076</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>0 0 27 810 14175 187110 2051406</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0 0 27 891 16848 237654 2764368</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>0 0 27 972 19764 296946 3655206</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0 0 27 972 19764 296946 3655206</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stage</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>-------</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Age</td>
<td>10</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
</tr>
<tr>
<td>#Trees</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>9</td>
<td>27</td>
<td>81</td>
<td>243</td>
</tr>
<tr>
<td>390</td>
<td>0</td>
<td>1</td>
<td>6</td>
<td>27</td>
<td>108</td>
<td>405</td>
<td>1458</td>
</tr>
<tr>
<td>360</td>
<td>0</td>
<td>1</td>
<td>9</td>
<td>54</td>
<td>270</td>
<td>1215</td>
<td>5103</td>
</tr>
<tr>
<td>330</td>
<td>0</td>
<td>1</td>
<td>12</td>
<td>90</td>
<td>540</td>
<td>2835</td>
<td>13608</td>
</tr>
<tr>
<td>300</td>
<td>0</td>
<td>1</td>
<td>15</td>
<td>135</td>
<td>945</td>
<td>5670</td>
<td>30618</td>
</tr>
<tr>
<td>270</td>
<td>0</td>
<td>1</td>
<td>18</td>
<td>189</td>
<td>1512</td>
<td>10206</td>
<td>61236</td>
</tr>
<tr>
<td>240</td>
<td>0</td>
<td>1</td>
<td>21</td>
<td>252</td>
<td>2268</td>
<td>17010</td>
<td>112266</td>
</tr>
<tr>
<td>210</td>
<td>0</td>
<td>1</td>
<td>24</td>
<td>324</td>
<td>3240</td>
<td>26730</td>
<td>192456</td>
</tr>
<tr>
<td>180</td>
<td>0</td>
<td>0</td>
<td>24</td>
<td>396</td>
<td>4428</td>
<td>40014</td>
<td>312498</td>
</tr>
<tr>
<td>150</td>
<td>0</td>
<td>0</td>
<td>24</td>
<td>468</td>
<td>5832</td>
<td>57510</td>
<td>485028</td>
</tr>
<tr>
<td>120</td>
<td>0</td>
<td>0</td>
<td>24</td>
<td>540</td>
<td>7452</td>
<td>79866</td>
<td>724626</td>
</tr>
<tr>
<td>90</td>
<td>0</td>
<td>0</td>
<td>24</td>
<td>612</td>
<td>9288</td>
<td>107730</td>
<td>1047816</td>
</tr>
<tr>
<td>60</td>
<td>0</td>
<td>0</td>
<td>24</td>
<td>684</td>
<td>11340</td>
<td>141750</td>
<td>1473066</td>
</tr>
<tr>
<td>30</td>
<td>0</td>
<td>0</td>
<td>24</td>
<td>756</td>
<td>13608</td>
<td>182574</td>
<td>2020788</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>24</td>
<td>756</td>
<td>13608</td>
<td>182754</td>
<td>2020788</td>
</tr>
</tbody>
</table>
## TABLE XLVI. INTERVAL 40

<table>
<thead>
<tr>
<th>Stage</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>10</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
</tr>
<tr>
<td>#Trees</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>9</td>
<td>27</td>
<td>81</td>
<td>243</td>
</tr>
<tr>
<td>360</td>
<td>0</td>
<td>1</td>
<td>6</td>
<td>27</td>
<td>108</td>
<td>405</td>
<td>1458</td>
</tr>
<tr>
<td>320</td>
<td>0</td>
<td>1</td>
<td>9</td>
<td>54</td>
<td>270</td>
<td>1215</td>
<td>5103</td>
</tr>
<tr>
<td>280</td>
<td>0</td>
<td>1</td>
<td>12</td>
<td>90</td>
<td>540</td>
<td>2835</td>
<td>13608</td>
</tr>
<tr>
<td>240</td>
<td>0</td>
<td>1</td>
<td>15</td>
<td>135</td>
<td>945</td>
<td>5670</td>
<td>30618</td>
</tr>
<tr>
<td>200</td>
<td>0</td>
<td>1</td>
<td>18</td>
<td>189</td>
<td>1512</td>
<td>10206</td>
<td>61236</td>
</tr>
<tr>
<td>160</td>
<td>0</td>
<td>0</td>
<td>18</td>
<td>243</td>
<td>2241</td>
<td>16929</td>
<td>112023</td>
</tr>
<tr>
<td>120</td>
<td>0</td>
<td>0</td>
<td>18</td>
<td>297</td>
<td>3132</td>
<td>26325</td>
<td>190998</td>
</tr>
<tr>
<td>80</td>
<td>0</td>
<td>0</td>
<td>18</td>
<td>351</td>
<td>4185</td>
<td>38880</td>
<td>307638</td>
</tr>
<tr>
<td>40</td>
<td>0</td>
<td>0</td>
<td>18</td>
<td>405</td>
<td>5400</td>
<td>55080</td>
<td>472878</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>18</td>
<td>405</td>
<td>5400</td>
<td>55080</td>
<td>472878</td>
</tr>
</tbody>
</table>
### Table XLVII. Interval 50

<table>
<thead>
<tr>
<th>Stage</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>10</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
</tr>
<tr>
<td>#Trees</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>1</td>
<td>11</td>
<td>3</td>
<td>9</td>
<td>27</td>
<td>81</td>
</tr>
<tr>
<td></td>
<td>350</td>
<td>0</td>
<td>1</td>
<td>6</td>
<td>27</td>
<td>108</td>
<td>405</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>0</td>
<td>1</td>
<td>9</td>
<td>54</td>
<td>270</td>
<td>1215</td>
</tr>
<tr>
<td></td>
<td>250</td>
<td>0</td>
<td>1</td>
<td>12</td>
<td>90</td>
<td>540</td>
<td>2835</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0</td>
<td>1</td>
<td>15</td>
<td>135</td>
<td>945</td>
<td>5670</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>0</td>
<td>0</td>
<td>15</td>
<td>180</td>
<td>1485</td>
<td>10125</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>15</td>
<td>225</td>
<td>2160</td>
<td>16605</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0</td>
<td>0</td>
<td>15</td>
<td>270</td>
<td>2970</td>
<td>25515</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>15</td>
<td>270</td>
<td>2970</td>
<td>25515</td>
</tr>
</tbody>
</table>
### TABLE XLVIII. INTERVAL 100

<table>
<thead>
<tr>
<th>Stage</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>10</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
</tr>
<tr>
<td>#Trees</td>
<td>400</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>9</td>
<td>27</td>
<td>81</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>0</td>
<td>1</td>
<td>6</td>
<td>27</td>
<td>108</td>
<td>405</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0</td>
<td>1</td>
<td>9</td>
<td>54</td>
<td>270</td>
<td>1215</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td>81</td>
<td>513</td>
<td>2754</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td>81</td>
<td>513</td>
<td>2754</td>
</tr>
</tbody>
</table>
APPENDIX VI

FINAL STAND CONDITIONS AND MANAGEMENT REGIMES FOR
TREE INTERVAL 15 AND BASAL AREA INTERVALS
10, 16, 20, 30 AND 40 ON SITE 140
<table>
<thead>
<tr>
<th>Rotation Age (years)</th>
<th>Harvest Diameter (inches)</th>
<th>Harvest Basal Area (sq. ft.)</th>
<th>Harvest Volume (cu. ft.)</th>
<th>MAI (cu. ft.)</th>
<th>Present Net Worth ($/acre)</th>
<th>Soil Expectation ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>12.29</td>
<td>173.07</td>
<td>5284.15</td>
<td>180.10</td>
<td>376.05</td>
<td>542.29</td>
</tr>
<tr>
<td>50</td>
<td>16.34</td>
<td>152.88</td>
<td>5384.52</td>
<td>198.09</td>
<td>912.87</td>
<td>1182.64</td>
</tr>
<tr>
<td>60</td>
<td>20.68</td>
<td>139.99</td>
<td>5423.02</td>
<td>203.60</td>
<td>1340.49</td>
<td>1614.53</td>
</tr>
<tr>
<td>70</td>
<td>22.42</td>
<td>164.47</td>
<td>6819.85</td>
<td>210.22</td>
<td>1534.42</td>
<td>1756.22</td>
</tr>
<tr>
<td>80</td>
<td>24.80</td>
<td>50.33</td>
<td>2196.27</td>
<td>192.41</td>
<td>1504.61</td>
<td>1660.67</td>
</tr>
<tr>
<td>Stand Age (years)</td>
<td>Amount of Fertilization (pounds)</td>
<td>Mean Diameter (inches)</td>
<td>Merch- chantable Trees</td>
<td>Basal Area (sq. ft.)</td>
<td>Volume Left (cu. ft.)</td>
<td>Volume Cut (cu. ft.)</td>
</tr>
<tr>
<td>------------------</td>
<td>----------------------------------</td>
<td>------------------------</td>
<td>------------------------</td>
<td>----------------------</td>
<td>-----------------------</td>
<td>----------------------</td>
</tr>
<tr>
<td>30</td>
<td>200</td>
<td>8.5</td>
<td>210</td>
<td>82.0</td>
<td>2121.9</td>
<td>1919.8</td>
</tr>
<tr>
<td>40</td>
<td>200</td>
<td>12.2</td>
<td>105</td>
<td>85.2</td>
<td>2600.4</td>
<td>2600.4</td>
</tr>
<tr>
<td>50</td>
<td>200</td>
<td>16.2</td>
<td>105</td>
<td>150.6</td>
<td>5303.7</td>
<td>0.0</td>
</tr>
<tr>
<td>60</td>
<td>400</td>
<td>18.8</td>
<td>60</td>
<td>116.2</td>
<td>4500.5</td>
<td>3375.4</td>
</tr>
<tr>
<td>70</td>
<td>-</td>
<td>22.4</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>6819.8</td>
</tr>
</tbody>
</table>
### TABLE LI. FINAL STAND CONDITIONS FOR BASAL AREA INTERVAL 16 SQUARE FEET.

<table>
<thead>
<tr>
<th>Rotation Age (years)</th>
<th>Harvest Diameter (inches)</th>
<th>Harvest Basal Area (sq. ft.)</th>
<th>Harvest Volume (cu. ft.)</th>
<th>MAI (cu. ft.)</th>
<th>Present Net Worth ($/acre)</th>
<th>Soil Expectation ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>12.29</td>
<td>173.07</td>
<td>5284.15</td>
<td>180.10</td>
<td>376.05</td>
<td>542.29</td>
</tr>
<tr>
<td>50</td>
<td>16.84</td>
<td>139.27</td>
<td>4904.92</td>
<td>195.93</td>
<td>921.95</td>
<td>1194.40</td>
</tr>
<tr>
<td>60</td>
<td>21.09</td>
<td>145.58</td>
<td>5639.58</td>
<td>202.31</td>
<td>1362.52</td>
<td>1641.06</td>
</tr>
<tr>
<td>70</td>
<td>24.06</td>
<td>142.13</td>
<td>5893.47</td>
<td>207.31</td>
<td>1537.89</td>
<td>1760.20</td>
</tr>
<tr>
<td>80</td>
<td>26.79</td>
<td>117.46</td>
<td>5125.69</td>
<td>196.04</td>
<td>1503.89</td>
<td>1659.88</td>
</tr>
</tbody>
</table>
**TABLE LII. MANAGEMENT REGIME FOR BASAL AREA INTERVAL 16 SQUARE FEET.**

<table>
<thead>
<tr>
<th>Stand Age (years)</th>
<th>Amount of Fertilization (pounds)</th>
<th>Mean Diameter (inches)</th>
<th>Merchantable Trees</th>
<th>Basal Area (sq. ft.)</th>
<th>Volume Left (cu. ft.)</th>
<th>Volume Cut (cu. ft.)</th>
<th>Cumulative PNW ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>200</td>
<td>8.5</td>
<td>210</td>
<td>82.0</td>
<td>2121.9</td>
<td>1919.8</td>
<td>- 309.4</td>
</tr>
<tr>
<td>40</td>
<td>200</td>
<td>12.2</td>
<td>90</td>
<td>73.0</td>
<td>2228.9</td>
<td>2971.9</td>
<td>23.2</td>
</tr>
<tr>
<td>50</td>
<td>400</td>
<td>16.7</td>
<td>90</td>
<td>136.9</td>
<td>4822.1</td>
<td>0.0</td>
<td>9.4</td>
</tr>
<tr>
<td>60</td>
<td>400</td>
<td>19.8</td>
<td>45</td>
<td>96.2</td>
<td>3726.5</td>
<td>3726.5</td>
<td>645.4</td>
</tr>
<tr>
<td>70</td>
<td>-</td>
<td>24.1</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>5893.5</td>
<td>1537.9</td>
</tr>
</tbody>
</table>
TABLE LIHI. FINAL STAND CONDITIONS FOR BASAL AREA INTERVAL 20 SQUARE FEET.

<table>
<thead>
<tr>
<th>Rotation Age (years)</th>
<th>Harvest Diameter (inches)</th>
<th>Harvest Basal Area (sq. ft.)</th>
<th>Harvest Volume (cu. ft.)</th>
<th>MAI (cu. ft.)</th>
<th>Present Net Worth ($/acre)</th>
<th>Soil Expectation ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>12.29</td>
<td>173.07</td>
<td>5284.15</td>
<td>180.10</td>
<td>376.05</td>
<td>542.29</td>
</tr>
<tr>
<td>50</td>
<td>16.66</td>
<td>136.25</td>
<td>4798.56</td>
<td>195.62</td>
<td>894.39</td>
<td>1158.70</td>
</tr>
<tr>
<td>60</td>
<td>21.06</td>
<td>145.11</td>
<td>5621.25</td>
<td>203.39</td>
<td>1351.68</td>
<td>1628.01</td>
</tr>
<tr>
<td>70</td>
<td>22.11</td>
<td>200.04</td>
<td>8294.90</td>
<td>183.71</td>
<td>1529.55</td>
<td>1750.66</td>
</tr>
<tr>
<td>80</td>
<td>25.18</td>
<td>51.86</td>
<td>2262.95</td>
<td>193.29</td>
<td>1510.99</td>
<td>1667.72</td>
</tr>
</tbody>
</table>
TABLE LIV. MANAGEMENT REGIME FOR BASAL AREA INTERVAL 20 SQUARE FEET.

<table>
<thead>
<tr>
<th>Stand Age (years)</th>
<th>Amount of Fertilization (pounds)</th>
<th>Mean Diameter (inches)</th>
<th>Merchantable Trees</th>
<th>Basal Area (sq. ft.)</th>
<th>Volume Left (cu. ft.)</th>
<th>Volume Cut (cu. ft.)</th>
<th>Cumulative PNW ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>400</td>
<td>8.5</td>
<td>210</td>
<td>82.0</td>
<td>2121.9</td>
<td>1919.8</td>
<td>- 309.4</td>
</tr>
<tr>
<td>40</td>
<td>200</td>
<td>12.3</td>
<td>135</td>
<td>111.3</td>
<td>3390.7</td>
<td>1887.2</td>
<td>- 143.8</td>
</tr>
<tr>
<td>50</td>
<td>200</td>
<td>15.5</td>
<td>75</td>
<td>97.9</td>
<td>3446.9</td>
<td>2757.5</td>
<td>262.6</td>
</tr>
<tr>
<td>60</td>
<td>400</td>
<td>19.3</td>
<td>75</td>
<td>151.8</td>
<td>5881.6</td>
<td>0.0</td>
<td>252.3</td>
</tr>
<tr>
<td>70</td>
<td>-</td>
<td>22.1</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>6294.9</td>
<td>1529.6</td>
</tr>
</tbody>
</table>


TABLE LV. FINAL STAND CONDITIONS FOR BASAL AREA INTERVAL 30 SQUARE FEET

<table>
<thead>
<tr>
<th>Rotation Age (years)</th>
<th>Harvest Diameter (inches)</th>
<th>Harvest Basal Area (sq. ft.)</th>
<th>Harvest Volume (cu. ft.)</th>
<th>MAI (cu. ft.)</th>
<th>Present Net Worth ($/acre)</th>
<th>Soil Expectation ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>12.29</td>
<td>173.07</td>
<td>5284.15</td>
<td>180.10</td>
<td>376.05</td>
<td>542.29</td>
</tr>
<tr>
<td>50</td>
<td>18.93</td>
<td>87.93</td>
<td>3096.85</td>
<td>183.37</td>
<td>849.03</td>
<td>1099.93</td>
</tr>
<tr>
<td>60</td>
<td>22.66</td>
<td>126.00</td>
<td>4881.09</td>
<td>192.91</td>
<td>1264.77</td>
<td>1523.33</td>
</tr>
<tr>
<td>70</td>
<td>21.88</td>
<td>156.68</td>
<td>6496.88</td>
<td>211.73</td>
<td>1476.26</td>
<td>1689.66</td>
</tr>
<tr>
<td>80</td>
<td>24.78</td>
<td>100.49</td>
<td>4385.04</td>
<td>199.47</td>
<td>1459.51</td>
<td>1610.89</td>
</tr>
<tr>
<td>Stand Age (years)</td>
<td>Amount of Fertilization (pounds)</td>
<td>Mean Diameter (inches)</td>
<td>Merchantable Trees</td>
<td>Basal Area (sq. ft.)</td>
<td>Volume Left (cu. ft.)</td>
<td>Volume Cut (cu. ft.)</td>
</tr>
<tr>
<td>------------------</td>
<td>---------------------------------</td>
<td>------------------------</td>
<td>--------------------</td>
<td>----------------------</td>
<td>-----------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>30</td>
<td>400</td>
<td>8.5</td>
<td>210</td>
<td>82.0</td>
<td>2121.9</td>
<td>1919.8</td>
</tr>
<tr>
<td>40</td>
<td>200</td>
<td>12.3</td>
<td>165</td>
<td>136.0</td>
<td>4151.8</td>
<td>1132.3</td>
</tr>
<tr>
<td>50</td>
<td>200</td>
<td>14.8</td>
<td>90</td>
<td>108.0</td>
<td>3803.1</td>
<td>3169.2</td>
</tr>
<tr>
<td>60</td>
<td>400</td>
<td>18.2</td>
<td>60</td>
<td>108.9</td>
<td>4205.2</td>
<td>2102.6</td>
</tr>
<tr>
<td>70</td>
<td>-</td>
<td>21.9</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>6496.9</td>
</tr>
</tbody>
</table>
TABLE LVII. FINAL STAND CONDITIONS FOR BASAL AREA INTERVAL 40 SQUARE FEET.

<table>
<thead>
<tr>
<th>Rotation Age (years)</th>
<th>Harvest Diameter (inches)</th>
<th>Harvest Basal Area (sq. ft.)</th>
<th>Harvest Volume (cu. ft.)</th>
<th>MAI (cu. ft.)</th>
<th>Present Net Worth ($/acre)</th>
<th>Soil Expectation ($/acre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>12.29</td>
<td>173.07</td>
<td>5284.15</td>
<td>180.10</td>
<td>376.05</td>
<td>542.29</td>
</tr>
<tr>
<td>50</td>
<td>17.49</td>
<td>125.15</td>
<td>4407.86</td>
<td>194.49</td>
<td>932.39</td>
<td>1207.93</td>
</tr>
<tr>
<td>60</td>
<td>21.75</td>
<td>154.84</td>
<td>5998.40</td>
<td>203.28</td>
<td>1410.38</td>
<td>1698.71</td>
</tr>
<tr>
<td>70</td>
<td>25.66</td>
<td>161.58</td>
<td>6700.23</td>
<td>205.42</td>
<td>1496.76</td>
<td>1713.12</td>
</tr>
<tr>
<td>80</td>
<td>31.62</td>
<td>81.81</td>
<td>3569.88</td>
<td>178.34</td>
<td>1485.87</td>
<td>1639.99</td>
</tr>
<tr>
<td>Stand Age (years)</td>
<td>Amount of Fertilization (pounds)</td>
<td>Mean Diameter (inches)</td>
<td>Merchantable Trees</td>
<td>Basal Area (sq. ft.)</td>
<td>Volume Left (cu. ft.)</td>
<td>Volume Cut (cu. ft.)</td>
</tr>
<tr>
<td>------------------</td>
<td>---------------------------------</td>
<td>------------------------</td>
<td>--------------------</td>
<td>----------------------</td>
<td>----------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>30</td>
<td>400</td>
<td>8.5</td>
<td>210</td>
<td>82.0</td>
<td>2121.9</td>
<td>1919.8</td>
</tr>
<tr>
<td>40</td>
<td>400</td>
<td>12.3</td>
<td>75</td>
<td>61.8</td>
<td>1887.2</td>
<td>3397.0</td>
</tr>
<tr>
<td>50</td>
<td>200</td>
<td>17.5</td>
<td>60</td>
<td>100.1</td>
<td>3526.3</td>
<td>881.6</td>
</tr>
<tr>
<td>60</td>
<td>400</td>
<td>21.6</td>
<td>45</td>
<td>114.6</td>
<td>4441.4</td>
<td>1480.5</td>
</tr>
<tr>
<td>70</td>
<td>-</td>
<td>25.7</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>6700.2</td>
</tr>
</tbody>
</table>