AN ABSTRACT OF THE THESIS OF

Chiang Kao for the degree of Doctor of Philosophy
in Forest Management presented on August 1, 1979

Title: A Study of Optimal Timing and Intensity of Silvicultural Practices-- Commercial and Precommercial Thinning, Fertilization and Regeneration Effort

Abstract approved: Signature redacted for privacy.
Dr. J. Douglas Brodie

One traditional problem in forest management is to find the optimal stand level management regime. Four important silvicultural practices including precommercial thinning, commercial thinning, fertilization and regeneration harvest are considered jointly in this study. The partial analyses, i.e. considering some of the silvicultural practices, are also discussed.

The inability to account for diameter acceleration in the two-dimensional dynamic programming technique is overcome by using a three-dimensional dynamic programming network with biometric relationships from DFIT. The continuous growth is fitted into a discrete dynamic programming network by using the "neighborhood" concept. The descriptors used are stand age, number of trees and basal area. The effect of the size of the state space of dynamic programming is discussed and a basal area interval between

four to 20 square feet is suggested when the tree interval used is 15.

Commercial thinning is considered every ten years and captures anticipated merchantable mortality. Precommercial thinning is considered at age ten. Different intensities of precommercial thinning can be considered jointly with other silvicultural practices. Three levels of fertilization, i.e. 400 pounds, 200 pounds and zero pounds of nitrogen per acre, are applied every ten years after commercial thinning. An extra dimension representing different levels of fertilization is eliminated by computations and using the neighborhood concept. The forward recursive relation of dynamic programming finds the best management regime for different rotations as the solution progresses.

Precommercial thinning accelerates diameter growth and will affect later commercial thinning entries. Commercial thinning lengthens rotation and fertilization increases site capacity and raises optimal stocking level.

The solution technique developed also finds the optimal management for different initial stand conditions. Plantation is solved by assuming that it is equivalent to a stand precommercially thinned at age two.

The impact of individual silvicultural practices and their interactive effects are derived. Under the revenue and cost assumptions used, it is found that fertilization has the highest economic impact, commercial thinning is the second and precommercial thinning is the last, when silvicultural practices are considered individually. The highest total effect of two silvicultural practices is precommercial thinning and commercial thinning. Commercial thinning and fertilization is the second and precommercial thinning and fertilization is the last. Precommercial thinning and commercial thinning has the highest interactive effect, commercial thinning and fertilization is the second and precommercial thinning and fertilization is the last which is negative. The interactive effect of precommercial thinning, commercial thinning and fertilization is positive, that is to say, when these practices are applied together, the total effect is larger than the sum of individual effects.

The techniques developed and discussed give practical answers to questions of stand level optimization with complex cost, revenue and growth model silvicultural interactions.

A Study of Optimal Timing and Intensity of Silvicultural Practices-- Commercial and Precommercial Thinning, Fertilization and Regeneration Effort

by

Chiang Kao

A THESIS

submitted to

Oregon State University

in partial fulfillment of
the requirement for the
degree of

Doctor of Philosophy

June 1980

APPROVED:

Signature redacted for privacy.

Assistant Professor of Forest Management
in charge of major

Signature redacted for privacy.

Head of Department of Forest Management

Signature redacted for privacy.

Dean of Graduate School

Date thesis is presented August 1, 1979

Typed by Chiang Kao for Chiang Kao

ACKNOWLEDGeMENT

Thanks are first dedicated to my parents for encouraging me to study abroad and continuous encouragement through letters.

Special appreciation is extended to Professor J. Douglas Brodie for his patient assistance in the organization and preparation of this thesis. His guidance and help in the past four years have made my study at Oregon State University fruitful and satisfactory.

Thanks are also extended to Professors John Beuter and Darius Adams in Forest Management, David Butler and Jeffrey Arthur in Statistics, and William Bregar in Computer Science for serving as my committee.

TABLE OF CONTENTS

TABLE OF CONTENTS
(continued)

TABLE OF CONTENTS
(continued)

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS
(continued)

LIST OF TABLES

LIST OF TABLES
(continued)

LIST OF TABLES
(continued)

LIST OF TABLES
(continued)

A STUDY OF OPTIMAL TIMING AND INTENSITY OF SILVICULTURAL PRACTICES--COMMERCIAL AND PRECOMMERCIAL THINNING, FERTILIZATION AND REGENERATION EFFORT

I. INTRODUCTION

The basic question of forest management for timber production can be stated simply: What treatment should be applied to each stand each period to best meet the objectives established for the forest? The question can be addressed at two levels-- the stand level and the forest level (Hann and Brodie, 1979). For individual stands the manager interested in practicing even-aged management needs to know the optimal: (1) planting density, (2) thinning scheme, (3) rotation length, (4) fertilization scheme, and (5) species mix. At the forest level the even-aged forest manager is faced with determining, for the desired number of planning periods, the optimal schedule of stand treatments, or conversion strategy and conversion period length. A forest is composed of forest stands, and the forest level questions can be better answered only after the alternative stand level treatments are known. Without the knowledge of stand level alternatives the study of forest level questions may be unrealistic and incomplete. The main purpose of this study is to investigate techniques for finding and studying the optimal management regime for an even-aged stand.

Recent increase in the real value of stumpage in the Pacific Northwest has lead to rapid attempts to implement a wide array of silvicultural practices that were regarded only as experimental a decade ago. Much attention has been given in recent years to theoretical optimization approaches at the stand level using a variety of methods including complete enumeration of limited alternatives, simple algebra, linear programming, nonlinear programming, dynamic programming, inventory theory and control theory. Most of these approaches analyse only one silvicultural practice at a time using partial analysis. If all silvicultural practices can be simultaneously considered in stand optimization then both the competitive and complementary interactions of stand level practices could be examined.

Stand growth and yield models are fairly complex systems and the timber management system over-lays this complex biological system. A way to construct a consistent system of analysis capable of optimizing the growth and yield system with its large number of silvicultural decisions has long been sought.

OBJECTIVE

The primary focus of this research is to develop applied analytic optimization techniques for key silvicultural operations in the Pacific Northwest including commercial thinning, precommercial thinning, regeneration harvest and fertilization. The techniques developed can be applied to other regions and other species. The species used in this study to demonstrate and evaluate the economic impacts of silvicultural activities is Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco).

The objectives of this study are:

(1) To adapt existing models of Douglas-fir growth and yield, commercial and precommercial thinning, fertilization and regeneration survival, so that optimization analysis will be feasible.

(2) To develop an optimization model and document its operation and application under alternate specifications of stand level problems.

(3) To analyse and present the impacts of these silvicultural activities applied as independent elements in a management regime and derive their independent impacts on rotation length, stocking and investment efficiency.

(4) To the extent it is computationally feasible to analyse the interactions between management activity impacts and derive their joint impacts on rotation length, stocking and investment efficiency.

(5) To provide management guidelines for the techniques and levels of these activities for various sites.

(6) To find the best planting density and the subsequent management regime.

(7) To differentiate between various possible criteria.

The main criterion for evaluating a management regime will be to maximize the present net worth and soil expectation. The methodology selected provides intermediate solutions for sensitivity analysis and is readily adaptable to alternate objectives such as volume maximization.

JUSTIFICATION

Demands for timber products in the United States have been projected to increase steadily (USDA Forest Service, 1974). Yet the supply of timber products potentially available from U. S. Forests show limited increases. Substantial increases in timber prices appear necessary to balance potential timber demands with available timber supplies. Work by Adams, Haynes and Darr (1977) indicates the relatively stable stumpage prices of the early 1960's have quadrupled in real terms in recent years with projections indicating future increases.

Intensified forest management offers an important means of increasing timber supplies which can mitigate the above problems. Sizable increases in timber growth and future harvest (both quantitative and qualitative) could be achieved by increased investments to expand forestry practices.

Much attention has been given to implementation of intensive management practices. The intensification of individual management inputs interacts with the intensification of other inputs and management decisions such as stocking levels, harvest schedule and rotation age. Because of the short period during which implementation of these activities has been considered, much of the analysis is partial and incomplete. Results of analyses have often not been generalizable due to reliance on simulation (Reukema and Bruce, 1977) rather than optimization techniques. This study originated from optimization techniques developed for commercial thinning analysis (Brodie, Adams and Kao, 1978), and is based on and expands the study by Brodie and Kao (1979). The goal of this research is to provide a flexible means for economic analysis of impacts and interactions of silvicultural inputs.

The management practices considered are:

Commercial Thinning

This practice is essentially a series of reductions made in stand density to maximize the net value of products removed during the whole rotation. Among the factors determining this net value are the quantity, quality and size of the products as well as the costs of harvesting. If growth is measured in terms of total volume of all parts of the trees of a given stand, it is often found that artificial changes in stand density over fairly wide ranges do not affect total volume production. Even if the total production of wood remains essentially unaltered, thinning can be used to increase the yield of "merchantable" volume (Smith, 1962). The fast diameter growth induced by thinning increases the value of stem wood. Other advantages that can be gained from thinning are: salvage of anticipated losses of merchantable volume, control of growing stock during the rotation and improvement of product quality. There could be an economic incentive for thinning even if it has no impact on total yield and quality if the present net worth contribution of early growing stock removals is greater than their contribution if left to grow for later harvest.

A thinning optimization study developed by Brodie, Adams and Kao (1978) noted that growth models that do not explicitly treat diameter growth as variable with stocking, will understate the economic desirability of thinning. If market values increase and logging costs decline with age alone, then incentives for early thinning consist primarily of light mortality capture harvests. With the development of the DFIT model (Bruce, Demars and Reukema, 1977), it became possible to describe stand dynamics including diameter growth, within a dynamic programming optimization framework.

The commercial thinning part of this research is based on the work of Brodie and Kao (1979), to find the best stocking level at each period while considering diameter growth acceleration impacts on unit prices, costs and economic returns.

Precommercial Thinning

Bruce, Demars and Reukema (1977) and the Douglas-fir Supply Study (USDA Forest Service, 1969) indicate that precommercial thinning removal of small, unsalable trees from a timber stand results in more rapid juvenile growth and qualifies stands for commercial thinning earlier than stands not receiving this treatment. The study by Brodie and Kao (1979) showed that if diameter growth acceleration is considered, the optimal thinning regime may involve a large removal of volume at the first thinning which occurred at age 30. In most cases (under different price functions) the best regime involved cutting half or more of the merchantable trees in a fully stocked stand at age 30. The stumpage price per unit at age 30 was less than the cost of thinning. So, actually, the first thinning was a special kind of precommercial thinning, since the sale return from thinning is less than the cost but the trees are salable. The above results suggest that precommercial thinning is desirable and this study investigates the circumstances and extent of application.

Fertilization

Few forest soils provide an optimal supply of nutrient elements essential for the growth of trees. Sometimes marked deficiencies may exist because of improper land management in the past or merely because of inherently low natural fertility of the site. One method of correcting such deficiencies is by chemical fertilization.

It is often possible to produce significant increases in forest growth by the application of nitrogen fertilizers. There is also evidence that fertilization might be used to increase the amount and frequency of seed production not only of seed orchards but also of individual trees being used as sources of natural regeneration (Smith, 1962). Purchase, transport and application of fertilizer is a significant expense with a deferred return. One purpose of this study is to examine methods of analysing and optimizing timing, frequency and amount of fertilization in Douglas-fir management using existing data sources (Turnbull and Peterson, 1976).

Final Harvest and Regeneration

Maximizing physical yield, cash flow, or present net worth on either a single or infinite series basis are all used as optimizing criteria in forestry. The distinctions between these criteria are reviewed in Davis (1966) and Duerr (1960). The analytic techniques of this study are adaptable to any of these criteria, however in general, optimization is done under the soil expectation maximization criterion as this is the general and accepted economic criterion. Under this criterion, levels of regeneration cost and related stocking success affect the subsequent optimal management regime.

STUDY PROCEDURE

The primary objective of this study is to find an optimal management plan which considers the key silvicultural practices. Certain assumptions will be needed to assure feasibility in modeling the system.

Assumptions

(1) Alternative rate of return.

The alternative rate of return is assumed known. Higher alternative rates of return tend to lead to shorter rotation and heavier thinning. In the certral part of the study, three percent is used as the alternative rate of return to approximate the historical real rate of return on long-term investment (Yohe and Karnosky, 1969).

(2) Timber price.

The market prices of timber per unit are assumed known. The price-diameter functional relationship used in this study is from Sessions (1979).

(3) Harvest cost.

The costs of both thinning and final harvest are assumed known. Cost is a function of both mean diameter and volume harvested. Cost functions derived from harvest simulations are available from Sessions (1979).

(4) Regeneration costs.

The costs of regeneration include the disposal of logging slash, treatment of the forest floor and competing vegetation, planting stock, and planting labor and supervision. The expected survival associated with these costs and the costs themselves is assumed known.

(5) Method of thinning.

The merchantable mortality trees will be removed at each commercial thinning. The merchantable live trees to be thinned are chosen such that the ratio of quadratic mean diameter of trees cut to quadratic mean diameter of the merchantable stand before commercial thinning is 1.00. This implies that the submerchantable part is left intact. The submerchantable trees will die eventually before they reach merchantable size. Further elaboration and stand development functions on which these assumptions are based are provided by Bruce, Demars and Reukema (1977) and are further discussed in Chapter IV.

(6) Fertilization.

Fertilizer application is modelled at each period immediately after the commercial thinning. This eliminates the necessity of an additional state descriptor for fertilization. Chapter IV will explain this procedure. Regimes with and without fertilization can be compared.

(7) Merchantable mortality in under- or over-stocked stand.

The merchantable mortality at each period is assumed to be proportional to the normality of the stand. If "p" percent of merchantable trees will die in a fully stocked stand, then it is assumed that for an under- or over-stocked stand also "p" percent of the merchantable trees of the stand will die in the next period.

(8) Mortality lost.

At each thinning the periodic merchantable mortality is removed. If no thinning occurs in a stage then the periodic merchantable mortality is lost as a result of decay. It can not be captured in the next thinning.

With these assumptions the study objectives are accomplished by executing the following tasks.

Tasks

A brief description and outline of the tasks accomplished in this dissertation follows.

(1) Derivation of growth function.

The skeleton of the growth function is from DFIT. Volume per acre of a natural stand is a function of site and age. One year volume growth of the stand is the derivative of volume over age. Volume growth is adjusted by the existing number of trees and basal area of the stand.

(2) Fertilization adjustment.

The growth response of fertilization is based on the work by Turnbull and Peterson (1976). The fertilization response model uses site, number of trees, basal area, and age as predictors of stand response. At each stage (stand age) and each state (number of trees and associated basal area) on a certain site this information is known, hence the growth can be adjusted by the fertilization effects.

(3) Deriving of mortality function.

Both submerchantable and merchantable mortality have to be calculated. Submerchantable mortality is only calculated once for the normal stand at different ages. Only the merchantable part is considered for thinning. From the number of merchantable trees the number of merchantable mortality trees is calculated as explained in assumption seven.

The number of submerchantable trees and basal area of submerchantable trees in a natural stand are calculated from equations in DFIT. The volume of submerchantable trees is calculated by using the volume/basal area ratio and adjusted by a tarif function.

The number of merchantable mortality trees is calculated as mentioned in assumption seven. The volume of merchantable mortality trees is the difference between gross growth and net growth. The calculations of net growth and gross growth will be described in the GROWTH section in Chapter IV. The basal area of merchantable mortality trees is derived from the volum/basal area ratio.

With this information the quadratic mean diameter of merchantable mortality trees, which is used to calculate the timber prices and logging costs, can be calculated. The merchantable live trees data is readily derived from net volum growth.

(4) Precommercial thinning adjustment.

DFIT has several equations for calculating precommercial thinning effects at different precommercial thinning ages. The basic idea is: The site index is adjusted due to assumed superior height growth after precommercial thinning. Since volume growth is a function of site, this adjustment will affect the subsequent volum growth and the associated basal area and mean diameter.

(5) Developing the thinning algorithm.

The state-descriptor variables of this study are: stand age, number of merchantable trees and basal area. The amount of fertilization applied could be another descriptor, but with some transformations, the extra descriptor is handled by substituting a series of computations. At each stage (stand age) and each state (number of trees) there are many thinning alternatives. The extreme cases are cut nothing and cut all of the trees in the stand. If nothing is cut then no cost is incurred, but the merchantable mortality is lost. If all trees are cut then this will be a final harvest. Between these two extremes are many cutting alternatives removing some number of trees so the remaining number of trees is a multiple of a discrete dynamic programming interval. Actually, the remaining number of trees need not be a multiple of a certain number. As long as the state of the remaining number of trees is finite, dynamic programming will work. However, computations are more easily interpreted and explained if the remaining trees is a multiple of a certain number. If that "discrete interval" is too large then only a limited number of thinning alternatives can be considered. If it is too small then the "method of thinning" assumption mentioned above may not be fulfilled, also the computer memory needed and computation time may be unacceptably large. This study found that 15 to 50 trees is a reasonable number for this discrete interval.

The basal area corresponding to the number of trees is a continuous variable. As a requirement of discrete state dynamic programming, basal areas over a range are grouped together and use the middle value of the range to represent them. The smaller the range used, the more precise the model will be. The error of comparing the same number-of-trees with different but similar basal area will be smaller, if the range of comparison is smaller. The finer the grid, the longer the computation time will be. So there is a trade-off. Within the required accuracy, a relatively small range is required. Due to the computer core capacity and the limited amount of data-packing attempted, four square feet is the smallest interval that the program developed can use with a number of trees interval of 15. (Computer used is CDC Cyber 73-16)

(6) Computer runs and interpretation

Using site index 140, the following computer runs are considered: Commercial thinning and final harvest; precommercial, commercial thinning and final harvest; commercial thinning, fertilization and final harvest; and precommercial, commercial thinning, fertilization and final harvest. The last case is also run for different site indices to demonstrate the combined silvicultural effects on different sites. The following cases are computed by using a hand calculator: Final harvest only; precommercial thinning and final harvest; fertilization and final harvest; and precommercial thinning, fertilization and final harvest. The management regimes are compared and the economic impact is discussed.

II. STAND LEVEL OPTIMIZATION

A forest is composed of forest stands. Stand level optimization forms a basis for forest level optimization. Both Williams (1977) and Nazareth (1973) independently demonstrated an approach for joint stand level and forest level optimization. The first step is to find the optimal stand level treatment, as well as the efficient treatment schedule for achieving non-optimal stand conditions. The forest level solution is formulated as an extremely large decomposable linear programming harvest scheduling problem. The first decomposition utilizes the optimal stand level treatments as activities. Due to the constraints, this solution can be enhanced in subsequent decomposition by inclusion of additional non-optimal efficient stand level treatments that provide greater levels of thinning and harvest in periods where the constraints are most binding.

The analysis presented in this study provides both the optimal stand level regime and all efficient regimes that are non-optimal. All of these alternate stand level treatments could theoretically at least be modified for inclusion in the forest level optimization discussed above. This would constitute a major undertaking and the research presented here is restricted to the stand level questions only.

In solving stand level problems two broad techniques have been utilized-- simulation and optimization. Simulation provides descriptive stand outputs when management regime is provided by the analyst. The advantage of simulation is that it can be applied to any kind of existing stand development model. The primary disadvantage is that only a "good" answer can be found. To determine a best or near best solution would require a great number of simulations and careful analysis. This approach can be expensive in terms of both time and computer costs. Optimization techniques provide a defined optimal regime when an objective criterion and cost and revenue assumptions are provided by the analyst. Usually an optimization technique is more difficult to implement than a simulation method, yet it finds an optimal solution in one set of calculations instead of an exhaustive simulation search. The results from an optimization often give more information. For example, the linear programming technique tells the shadow prices and dynamic programming techniques give many intermediate efficient solutions. In the case of forest stand optimization, efficient regimes for rotations shorter and longer than the global optimum are provided. Often a sensitivity test can be undertaken without additional computation to see the effect of deviating from the best strategy.

PREVIOUS OPTIMIZATION STUDIES

Optimization methods for solving stand level questions have long been used in forestry. Early methods of determining rotation length, such as maximum mean annual increment (MAI) and soil expectations were all optimization methods. These early methods were distinguished by the numerous assumptions made in order to solve them and by the amount of tedious calculations required. First applications of the electronic computer were aimed at easing the latter problems. More recently, mathematical programming techniques have made it easier to answer more complex problems and, therefore, reduce the need for numerous assumptions. A good example of this evolutionary process has been the development of optimization tools for jointly answering thinning scheme and rotation length questions.

Early approaches and some recent practitioners have used marginal analysis and brute force trial and error methods, or some form of complete enumeration to choose among management alternatives. Chappelle and Nelson (1964) presented an early solution to the joint optimization of thinning scheme and rotation length in loblolly pine. They used two simple volume growth equations, one in cubic feet and the other board feet, and marginal analysis to determine optimal volume stocking levels which would maximize net profit per annum for each product. Then, given an initial stocking level, the optimal stocking level, and the volume growth model, they determined the amount of volume, thinning would remove in each cutting period for a fixed rotation length. Using this information and cost and revenue data, the optimal rotation length was determined using the classical soil expectation method. Other research works used marginal analysis including applications by Duerr (1960), Duerr and Christiansen (1973) and recent applications of marginal analysis in the Douglas-fir Region by Buongiorno and Teeguarden (1973). Hardi (1977) presented thinning regimes derived from a complex biometric model. The optimization procedure was complete enumeration of a highly constrained set of alternatives.

Adams and Ek (1974) used a nonlinear programming technique to solve the two problems in the management of uneven-aged forests: (1) determination of the optimal sustainable distribution of trees by diameter class, i.e. stand structure, for a given stocking level, and (2) the optimal cutting schedule for the conversion of an irregular stand to a target structure. The solution procedure used was a modification of the gradient projection method. This problem is analagous to the even-aged situation of starting with a specified stand, growing and thinning it over a number of time periods until a target diameter distribution of zero trees in each diameter class is reached in the last time period. Their method necessitated a relatively simple stand model that explicitly predicted the net change in number of trees in a diameter class. Their method was also limited as to the number of time periods over which the method could be applied. Naslund (1969) formulated a theoretical nonlinear programming model to determine simultaneously the optimal rotation and thinning regime but presented no solution. Schreuder (1971) presented a model to solve the same problem in two forms. If time is assumed to be continuous, the problem can be formulated in the calculus of variation form. In this form no closed form solution was obtained. If the model is recast in dynamic programming form, a numerical solution can be obtained. Kao and Brodie (1979a) set up a simple "approach toward normality" growth model from Bulletin 201 (McArdle, Meyer and Bruce, 1961) and used a modified flexible polyhedron method to find a continuous time, continuous state solution.

Anderson (1976) generalized the optimal control approach to the timber management problem to include the opportunity cost of forested land. The generalized steady-state control solution was shown to be identical to the Faustmann rotation model. Anderson presented theoretical derivations only and failed to present any numerical solutions. Pelz (1977) used inventory theory to determine optimal growing stock levels and was able to duplicate the results of Chappelle and Nelson (1964).

In mathematical programming applications, dynamic programming is also quite widely used. Amidon and Akin (1968) demonstrated how dynamic programming could be used to obtain the same solutions as Chappelle and Nelson (1964). Dynamic programming is a method used for solving a wide array of scheduling and resource allocation problems. A broad sub-set of dynamic programming problems can be represented by a network of nodes. The number of "state" descriptors needed to define each node defines the dimensions of the network. The objective of dynamic programming is to find, within defined limits, the optimal, path through the network. In stand optimization the objective is to find the optimal stocking level at each stage (stand age). Solution to the problem of finding an optimal path can be accomplished by using either the forward or the backward recursive method. Amidon and Akin (1968) used two descriptors (volume and age) and a backward recursive method. Brodie, Adams and Kao (1978) and Kao and Brodie (1979b), used two descriptors (volume and age) but a forward recursive method. Risvand (1969) also used three descriptors (volume, diameter and age) and a forward recursive method. A recent study by Brodie and Kao (1979) also used three descriptors (number of trees, basal area and age) and a forward recursive method to solve the stand optimization problem.

CURRENT OPTIMIZATION WORK ASSOCIATED WITH THIS STUDY

One shortcoming that Brodie, Adams and Kao (1978) identified with their approach was that the stand model did not account for diameter growth acceleration due to thinning. If quality premiums are important, this shortcoming can lead to suboptimal solutions.

The most recent work in the application of dynamic programming to stand problems has tried to eliminate this shortcoming through the use of a much more complex stand model that incorporates quadratic mean stand diameter growth acceleration into it. Because of the added complexity of the stand model, a three descriptor dynamic programming network was used. After a careful examination of the stand model, Brodie and Kao (1979) discovered that the model could be initialized, for a specified site index, if the three values of stand age, basal area and number of trees were known. These, therefore, formed the three state descriptors of each node. Analysis results using the three descriptor framework are the optimal number of trees and basal area to maintain in each time interval. To obtain these values Brodie and Kao used the forward recursion solution method. Because quadratic mean stand diameter can be computed from number of trees and stand basal area, stumpage prices and logging costs could be more realistically introduced into the analysis as functions of quadratic mean stand diameter.

Brodie and Kao (1979) used a simple price-diameter function and a cost function for logging derived from Bureau of Land Management Schedule 20 (1977). Sessions (1979) used a complex yarding simulator to derive costs and a price function based on the distribution of log-grades related to mean stand diameter. He integrated this work into the dynamic programming framework developed by Brodie and Kao (1979) to study the interactions of logging method, terrain and optimal regime. Both of the studies considered only two silvicultural activities-- commercial thinning and final harvest. In this study two other silvicultural activities-- precommercial thinning and fertilization, are added. Combined with the dynamic programming framework developed by Brodie and Kao (1979) and price and cost functions derived by Sessions (1979), a more complete model is developed.

NECESSARY DATA

To achieve the optimal management of an even-aged pure stand several data elements are needed: (1) a growth model with silvicultural responses, (2) prices of products and (3) costs of producing products.

(1) Growth model with silvicultural responses.

DFIT (Douglas-fir Interim Tables) is a computer program developed by the Pacific Northwest Forest and Range

Experiment Station (Bruce, Demars and Reukema, 1977) which simulates stand growth and tabulates the results of the simulation. The main component of DFIT is a set of equations describing the development of natural stands. The function list contains approximately 33 relationships and the variable list 24 elements and 19 subscripts. Natural stands and plantations can be simulated with details of merchantable and unmerchantable: diameters, volumes, mortality, height, basal area and number of trees. Commercial and precommercial thinning, fertilization and growth adjusting activities such as genetic improvements are among the silvicultural activities that can be simulated and projected.

The fertilizer adjustment in DFIT consists of raising the site sidex and a volume growth adjustment through multipliers. Only one level of fertilization-- 200 pounds of nitrogen per acre is considered. In 1969 the Regional Forest Nutrition Research Project was initiated with the primary objective of providing resource managers with more accurate data on the effect of fertilizing and thinning young-growth Douglas-fir and Western hemlock. A note by Turnbull and Peterson (1976) presented the growth responses of fertilizing 400 pounds, 200 pounds and zero pounds of nitrogen per acre as functions of site index, stand age, basal area and number of trees per acre. These data are more realistic and these functional fertilizer responses were adapted to conform to the DFIT model for use in this study.

(2) Prices of products.

Sessions (1979) using data from Bulletin 201 (McArdle, Meyer and Bruce, 1961), Columbia River Scaling Rule (Dilworth, 1973) and the current 1978 pond value of logs in Western Oregon derived a relation between the age of a natural stand and the value of the stand at that age. From the arithmetic mean diameter of a natural stand at a certain age and its corresponding value, a relation between price per thousand cubic feet and arithmetic mean stand diameter is derived.

(3) Costs of products.

Sessions (1979) also derived stump-to-truck harvesting costs as a function of merchantable volume per acre removed by using his yarding simulator. The harvesting cost depends on arithmetic volume per acre. A haul cost dependent on volume harvested only was adjusted to make up the total logging cost.

III. APPLICATION OF DYNAMIC PROGRAMMING TO STAND LEVEL PROBLEMS

Among those methodologies mentioned in Chapter II, dynamic programming is a simple and flexible technique for solving optimization problems. It can overcome deficiencies in marginal analysis such as the inability to easily account for precommercial opportunities and the interdependence of harvest costs and volume removals. It does not need the restrictive assumptions of linear programming. It embodies the flexible functional forms of nonlinear programming without the difficulties in specification and solution method. Approaches such as the continuous state control theoretic formulation use complex mathematics for which solutions can be quite difficult. Dynamic programming offers an efficient method of generating and evaluating the immense number of alternatives that exist within the feasible thinning-rotation set. It has provided empirical solutions and because it operates with implicit first order conditions, theoretical derivations are also possible. Hence dynamic programming was chosen as the optimization method for this study.

DYNAMIC PROGRAMMING

Dynamic programming is a mathematical technique often useful for making a sequence of interrelated decisions. It provides a systematic procedure for determining the combination of decisions that maximizes overall effectiveness (Hillier and Lieberman, 1974; Nemhauser, 1967). The basic features which characterize dynamic programming problems are presented below, using the stand optimization problem for example and discussion:

(1) The problem can be divided into stages, with a policy decision required at each stage.

In the stand optimization study, stages are times for precommercial thinning and commercial thinnings. The policy decision at each stage is to decide the intensity of precommercial thinning or commercial thinnings. The decision in one stage will affect the decision in the next stage. That is to say dynamic programming problems require making a sequence of interrelated decisions.

(2) Each stage has a number of states associated with it.

The states associated with each stage in the stand optimization problem are stocking levels which are two-dimensional, represented by number of trees in the stand and the corresponding basal area. The states are the various possible conditions in which the system might be at that stage. The number of states may be either finite or infinite. In stand optimization the state space is finite because the largest number of trees that can grow in a stand at a certain stage is finite.

(3) The effect of the policy decision at each stage is to transform the current state into a state associated with the next stage.

In the stand optimization problem, given the state at current stage is "n" trees with "g" square feet of basal area, if the policy decision is to cut down to "n'" trees with "g'" square feet of basal area, then the state at next stage will be "n'" trees (assuming no mortality trees) with "g"" square feet of basal area, where g"-g' is the amount of basal area growth during the two consecutive stages.

(4) Given the current state, an optimal policy for the remaining stages is independent of the policy adopted in previous stages.

Given the state of the stand, the optimal management regime from this point onward is independent of how the stand state is arrived at in the current state. For dynamic programming problems in general, knowledge of the current state of the system conveys all the information about its previous behavior necessary for determining the optimal policy henceforth. This property is sometimes referred to as the principle of optimality (Bellman and Dreyfus, 1962).

(5) A recursive relationship must be specified that identifies the optimal policy for each state at stage n, given the optimal policy for each state at either stage (n-1) for a forward recursion or stage (n+1) for a backward

recursion. The distinction between forward and backward recursion is explained later.

TWO DESCRIPTOR DYNAMIC PROGRAMMING

Amidon and Akin (1968) used a backward recursion to find the best thinning regime for each given rotation. At each state (stocking level) only two actions are considered, either thin heavily or thin lightly. Figure I is a subnetwork, or grid, extracted from the larger network (Amidon and Akin, 1968). For a given rotation age, working backward the best path, or thinning regime, can be found.

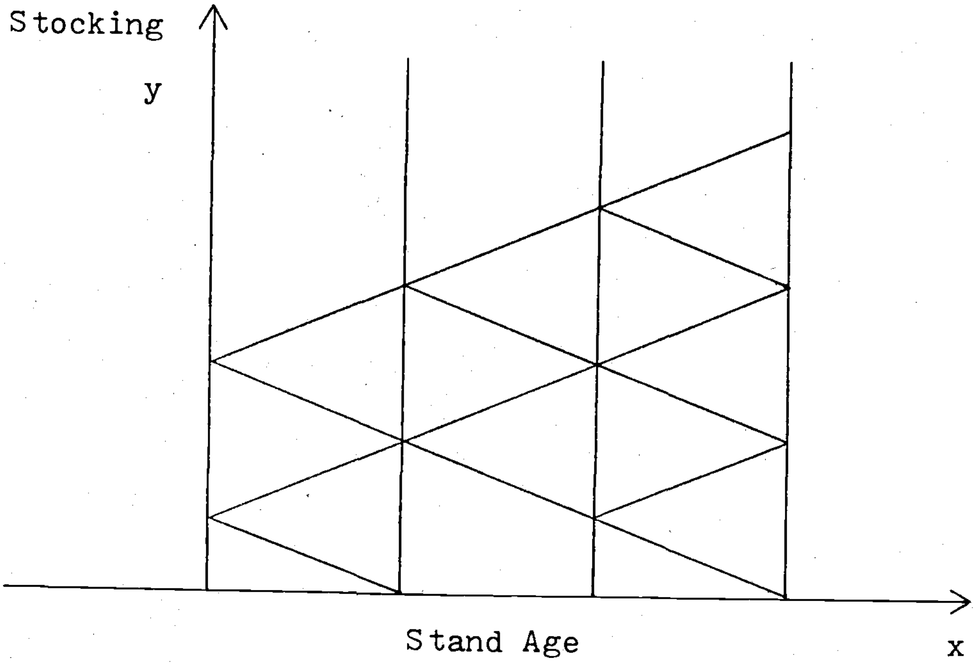


Figure I. Network of dynamic programming model by Amidon and Akin.

Let $D(x,y)$= decrement in growing stock value from $(x+1,y+1)$ to (x,y)

$I(x,y)$= increment in growing stock value from $(x+1,y-1)$ to (x,y).

$T(x,y)$= total value from the optimal schedule up to (x,y).

where x and y denote network axes as in Figure I. Then the application of the principle of optimality is expressed by the recursive function:

$$T(x,y)= \max \{D(x,y)+T(x+1,y+1);\ I(x,y)+T(x+1,y-1)\}$$

$T(n,y)$ is initial condition, i.e. final harvest value for the rotation corresponding to n. For different n the corresponding $T(n,y)$ can be calculated. Best rotation n^* is chosen such that $T(n^*,y)= \max_n \{T(n,y)\}$

Brodie, Adams and Kao (1978) used a forward solution to find the optimal stocking levels and rotation simultaneously. At each state all actions resulting in a state having less stocking level can be considered. Figure II shows the network of this dynamic programming model. $T(1,y^*)$ is the initial condition. Using the forward recursive function:

$$T(x+1,y)= \max_{y'} \{T(x,y')+P(y',y)\}$$

where y' is any node of stocking level in the current stage which can reach stocking level y in the next stage, and $P(y',y)$ is the revenue function showing the revenue obtained

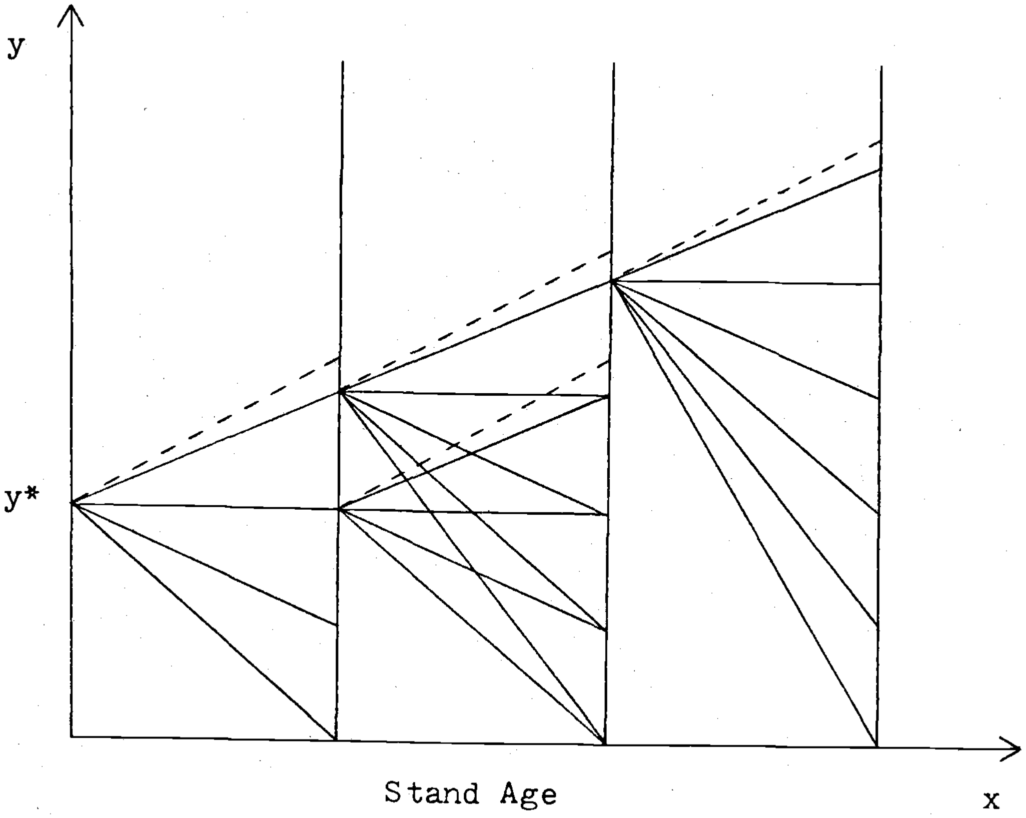


Figure II. Network of dynamic programming model by Brodie, Adams and Kao.

from state y' to y. T(n,0) will be the final harvest value for the rotation corresponding to n. The calculation goes from one stage to the nest stage until T(n*+1,0) < T(n*,0). And n* will be the best rotation.

Because the growth function is continuous while the stocking levels are discrete, a small "rounding thinning" of less than the stocking level interval is necessary to assure a discrete stocking level. If no thinning is considered at a certain stage then the stocking level, i.e. the growth will be underestimated. In Figure II the broken lines show the actual growth, and the node right below the broken line is the rounding stocking.

Both of the studies by Amidon and Akin, and Brodie, Adams and Kao considered a fixed thinning entry. The former used five years and the latter used ten years. Kao and Brodie (1979b) wrote a flexible program which considered variable thinning entry. The thinnings occur at any age if it is optimal. The age is an integer, so actually it is a quasi-continuous time dynamic programming model. They also developed the "neighborhood" concept, which the later work by Brodie and Kao (1979) is based on, to overcome the rounding stocking problem. The neighborhood concept will be discussed in the followong section.

THREE DESCRIPTOR DYNAMIC PROGRAMMING

All of the above mentioned dynamic programming studies used only two descriptors, i.e. stand age and stocking (volume). Risvand (1969) proposed a three descriptor dynamic programming model which also considered diameter growth. His network structure is similar to the structure of Amidon and Akin. At each state there are a fixed number of thinning alternatives considered. The amount of thinning is proportional to the existing stocking level, hence the stocking state is continuous but finite. Figure III shows the network of Risvand. Different paths need not go to the same stocking level.

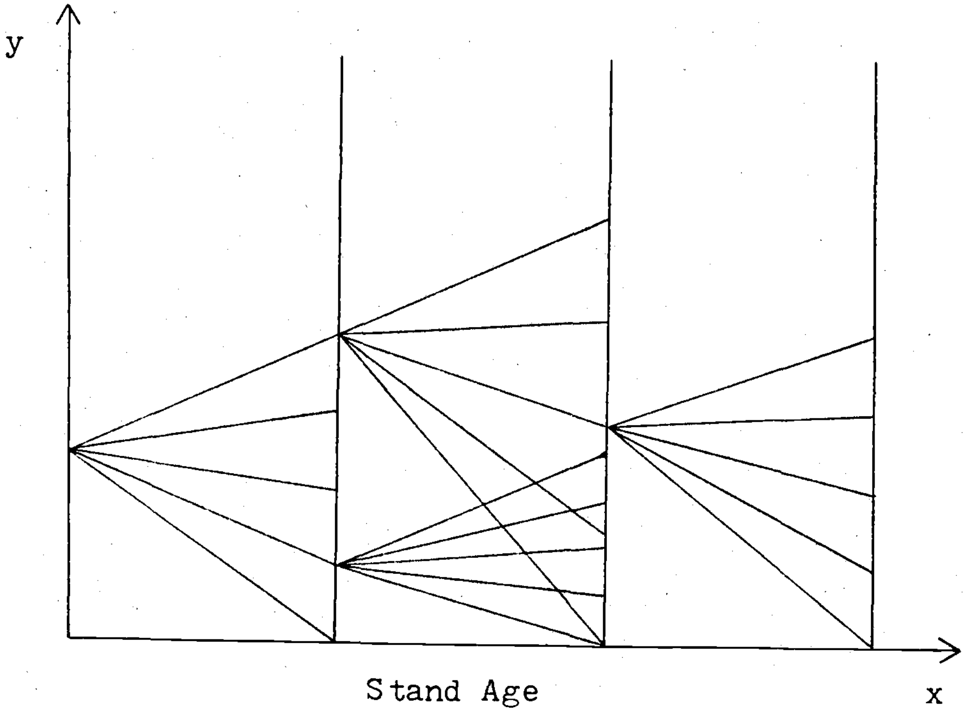


Figure III. Network of dynamic programming model by Risvand.

Brodie and Kao (1979) modified the Dfit model (Bruce, Demars and Reukema, 1977) and used basal area, number of trees and age to describe stand dynamics which can handle diameter growth acceleration impacts resulting from thinning. The initial condition for the start of the solution is generated from the natural stand options in DFIT or can be set at alternate levels of number of trees and corresponding basal area. The recursion is solved forward from this single node. A graphic representation of the network is demonstrated in Figure IV.

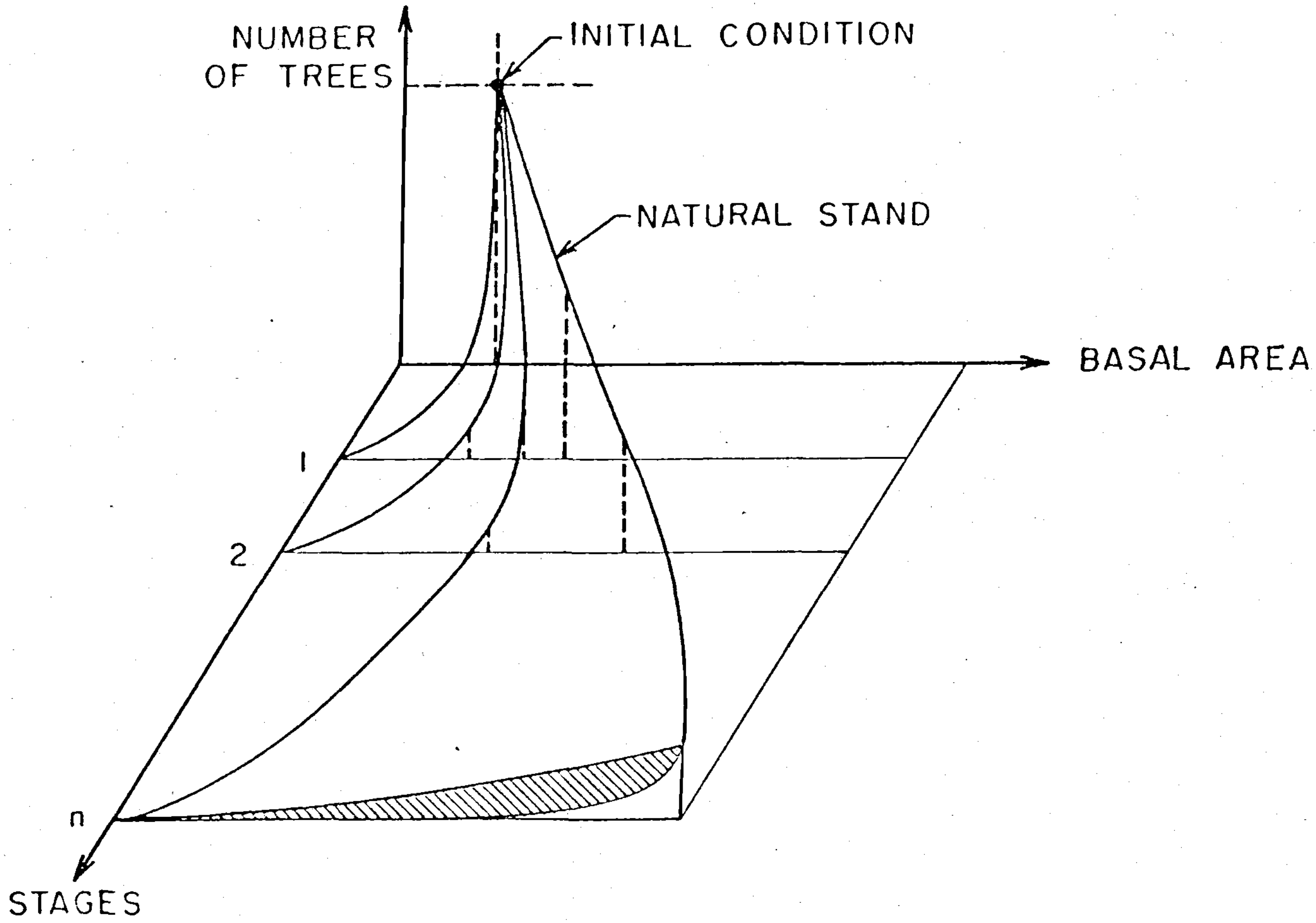


Figure IV. Graphical representation of the three dimensional DOPT network.

Schreuder (1968) has noted that the large number of alternatives has been a barrier to realistic structuring and solution of forestry problems in a dynamic programming framework. A related problem discussed by Brodie, Adams and Kao (1978) is the rounding error associated with discrete state descriptor nodes and continuous growth models. A modification of the node definition in dynamic programming was used to more adequately represent a continuous production surface with a limited number of nodes and eliminate the rounding error problems.

Traditionally discrete dynamic programming nodes are defined in terms of precise values of the discrete state descriptors which may require a large number of nodes to adequately represent a continuous production surface. Additionally in some applications a rounding error is introduced as values computed from continuous functions are forced to be rounded to discrete node intervals. Brodie and Kao treated the network nodes as "neighborhood storage locations" at which exact continuous values of the descriptors for the optimal policy to the current stage are stored. Figure V helps to clarify this point. It represents a section of the number of trees (N) and basal area (G) grid for a stage (age) in the solution. Growth from optimal policy nodes of the previous stage when combined with discrete thinnings create candidate stand types in the "neighborhood" of the N=75, g=48 node where the neighbor-

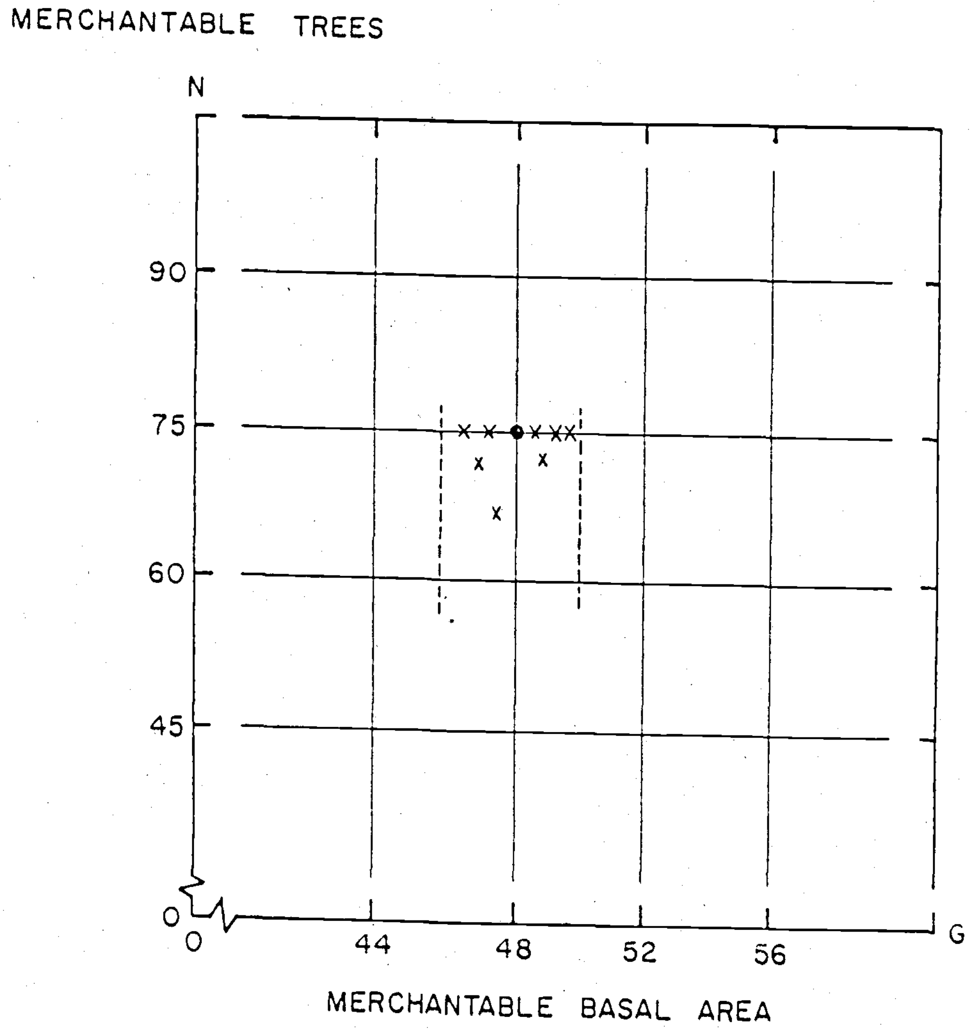


Figure V. A representation of the DOPT network for a stage showing candidate stand types for the optimal policy at neighborhood storage location N_{75},G_{48}.

hood is defined as all candidate stands with basal area between 46 and 50 cubic feet and number of trees between 60 and 75 per acre. Optimization takes place over these candidates and the result is saved at node (75,48). Growth for subsequent stages is calculated using the actual continuous N and G of the optimal policy for this node rather than the "neighborhood" value of the node itself. Values of number of trees that are not discrete multiples of the tree interval represent "no thinning arcs" with mortality. Except for the provision for these arcs, the solution is discrete in number of residual trees and time but continuous in the merchantable basal area variable. Brodie and Kao discovered no indication of systematic bias in those modifications. Alternatives are eliminated by the process that might prove to be optimal through interaction with later stages in the recursion if they were retained as exact nodes; however, this effect is not thought to be serious if the state intervals are small. The "neighborhood" approach with continuous descriptors mitigates the necessity of choosing wide state intervals for reasons of available storage and computational efficiency. A principle benefit of this approach in the thinning problem is that it allows for merchantable mortality adjustment at nodes where no thinning is optimal.

A DFIT natural stand was generated for the base year of the study by Brodie and Kao which was 30 in their examples presented. The associated merchantable number of trees and basal area define the single starting condition node. The unmerchantable portion of the stand is also generated and it is carried along creating a small retarding influence on merchantable basal area growth that declines over time. The present net worth of costs incurred prior to age 30 is the optimal value function value for the starting condition node. The thinning interval is ten years and the stand is grown in ten annual cycles of the DFIT model.

Thinnings are removed from the diameter classes proportional to frequency in the classes or in other words the ratio of mean diameter of trees removed to all merchantable trees in the stand is maintained at 1.00. The dynamic programming problem solved by forward recursion is as follows:

Define the optimal value function as T(N,G,t)= the value of the present net worth (PNW) "path" from regeneration to stand age t, number of trees N and basal area G. The forward recursive function is

$$T(N,G,t) = \max_{\{[N',G']\}} \left\{ T(N',G',t-10) + P(N',G',N,G) \right\}$$

where P(N',G',N,G) is the discounted value from node (N',G') to node (N,G). $\{[N',G']\}$ is the set of all feasible number of trees and basal area at age t-10 from which the current neighborhood level of N and G can be reached. T(0,0,t) is the discounted value for final harvest at age t. The recursion continues for a specified number of stages or can be automatically terminated as soon as T(0,0,t) declines. The soil expectation maximization rotation t* is found by choosing rotation t such that

$$Se(t^*) = \max_{t} \left\{ T(0,0,t)*(1+r)^{t}/[(1+r)^{t}-1] \right\}$$

where r is the alternative rate of return.

Most of the examples show heavy thinning in the first stage. DFIT optima are highly sensitive to number of trees in the young stand and this suggests as Reukema and Bruce (1977) note that precommercial thinning might be desirable. The revenues from the cutting in the first stage in the examples are negative and it is only the diameter acceleration impact that makes them optimal. Higher levels of small-diameter material logging costs eliminate early thinnings from optimal regimes. Negative current revenue thinnings can be constrained from optimal solutions for managers who are assumed to be averse to silvicultural investment of this sort.

PRECOMMERCIAL THINNING AND FERTILIZATION CONSIDERATIONS

As mentioned in Chapter I, most of the studies on forest stand optimization have concentrated on partial analysis which considers only one or two silvicultural practices. The interaction of silvicultural inputs are neglected. Using a three descriptor dynamic programming model, two other important silvicultural activities, i.e.

precommercial thinning and fertilization, can be considered. This makes the resulting optimal management regime an integrated best management regime.

The addition of precommercial thinning does not cause any difficulty. Precommercial thinning is only considered once, so it is just another stage prior to the stage of the first commercial thinning. The timing and intensity of precommercial thinning can be considered simultaneously, i.e. in the precommercial thinning stage the state space is a two-dimensional instead of one-dimensional state (here we combine the number of trees and basal area together as one dimension of stocking) which is comprised of intensity of thinning only. Figure VI illustrates how the two-dimensional space in the precommercial thinning stage is used without causing any dimension problem. Each intensity and timing of precommercial thinning represents a different initial condition. Hence the only work added is to calculate the growth of several initial conditions instead of only one. In DFIT it is assumed that trees left after precommercial thinning will not die except for those captured in commercial thinning. This is why the number of trees left after precommercial thinning will stay at the same level to the next stage as shown in Figure VI. This assumption does not seem to be biologically reasonable and clearly DFIT was not intended to be used with precommercial thinning alone. Some precommercial-thinning-alone results

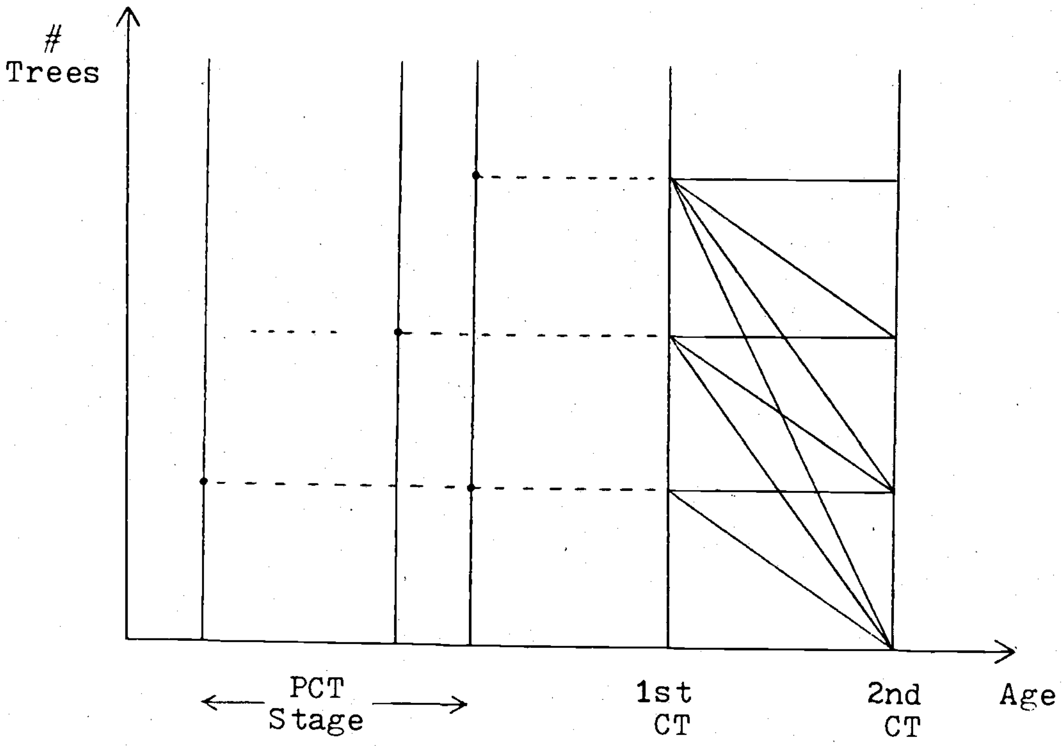


Figure VI. Timing and intensity of precommercial thinning (PCT) is considered simultaneously as in one stage.

are presented later for comparative purposes. DFIT also represents a plantation as a stand precommercially thinned at age two. In this study the traditional timing for precommercial thinning is followed, i.e. precommercially thin at age ten. Only the intensity of precommercial thinning is considered.

Turnbull and Peterson (1976) have presented the basic preliminary models of fertilization response in Douglas-fir. These models use site, number of trees, basal area and stand age as descriptors of stand response. Since site is handled outside of the dynamic programming model, the remaining three descriptors just fit into our three descriptor dynamic programming model.

The fertilization alternatives considered are 400 pounds, 200 pounds and zero pounds of nitrogen per acre. If a backward recursion were used one extra descriptor to represent amount of fertilization would be needed. Using the forward recursive technique this problem is eliminated. Figure VII is a two-dimensional diagram showing how the necessity of another descriptor is eliminated. The

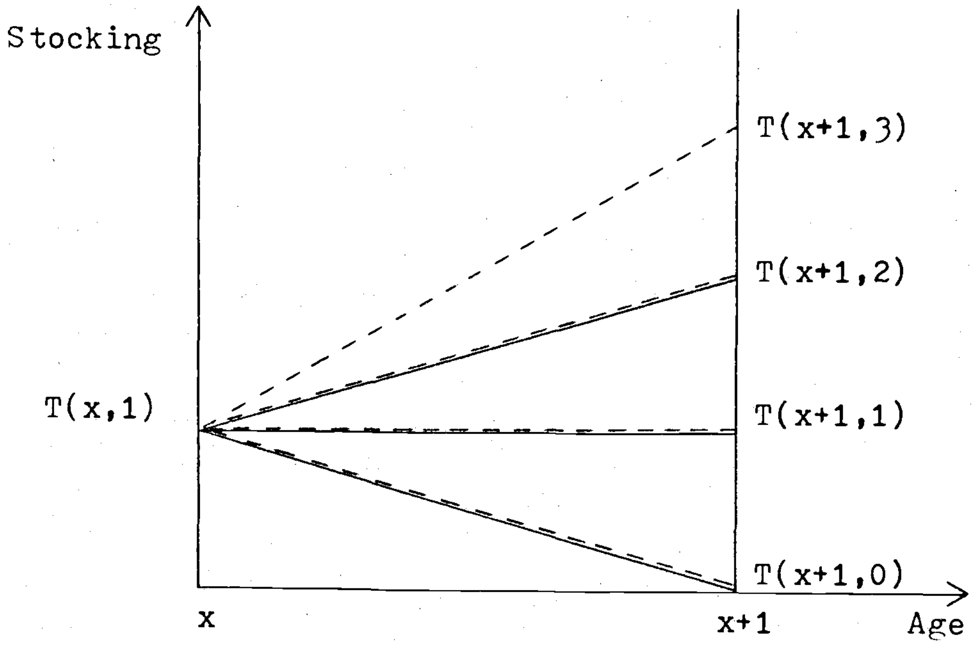


Figure VII. Fertilization alternatives are handled through extra computations instead of an extra dimension. Broken lines and solid lines are paths for fertilization and without fertilization, respectively.

discounted value of a node with stocking y at age x+1 is chosen such that

$$T(x+1,y)=\max_{\{[y',I]\}}\{T(x,y')+P(y',y,I)-FC\}$$

where P(y',y,I) is the revenue derived by going from a node with stocking y' to a node with stocking y in next stage. I is equal to one or zero depending on whether fertilization is selected or not. FC is the fertilization cost. For example, $T(x+1,1)=\max\{T(x,1)+P(1,1,1)-FC;\ T(x,1)+P(1,1,0)\}$. Fertilization results in a higher amount of growth. In the above example, if the extra unit of stocking resulting from fertilization has a value higher than the cost of fertilization then the "do fertilize" alternative will be chosen. If the value of the extra growth does not pay the cost of fertilization then the "do not fertilize" alternative will be chosen. Node (x+1,3) can be reached only by fertilization hence it carries a value of T(x,1)-FC. Figure VII shows only two fertilization alternatives, the same concept can be applied to any number of fertilization alternatives. This simple approach can be applied to any treatment whose growth impact is equal to or less than the thinning interval.

COMPUTATIONAL EFFICIENCY

The principle merit of dynamic programming is that it offers an efficient method of generating and evaluating the immense number of alternatives that exist within the feasible thinning-rotation set within an acceptable precision. Using the stand optimization problem as an example, with precommercial thinning at age ten to 400 trees, commercial thinning every ten years starting from age 30 with three levels of fertilization considered, and a thinning interval of 15 trees, Table I shows the number of feasible alternatives of getting to a certain stocking level (number of trees) at a certain stage (stand age) when the first commercial thinning is restricted to removals of less than 50 percent of the stocking. For example, the number of alternatives for getting to a stocking level of 270 trees at the second stage is 30 (Table I), and the number of alternatives of clearcutting the stand at stage five is 2,072,385. For an eighty year rotation there are 39,785,661 (around 40 million) alternatives. The assumption here is that each discrete 15 tree thinning alternative creates a unique basal area.

If all those 40 million alternatives are examined and if all the intermediate stages and states are recorded then all information of stage and state combinations before age 80 are known. The growth function and their price and cost functions have to be calculated 40 million times. This is the result of considering discrete tree interval and continuous basal area. If all alternatives in the nieghborhood of four-square-foot basal-area intervals are grouped together, i.e. discrete tree interval and discrete basal

TABLE I. NUMBER OF ALTERNATIVES WHICH REACH A CERTAIN STATE AT A CERTAIN STAGE WHEN COMPLETE ENUMERATION IS CONSIDERED.

Stage	0	1	2	3	4	5	6
Age	10	30	40	50	60	70	80
#Trees	Number of Alternatives						
400	1	1	3	9	27	81	243
390	0	1	6	27	108	405	1458
375	0	1	9	54	270	1215	5103
360	0	1	12	90	540	2835	13608
345	0	1	15	135	945	5670	30618
330	0	1	18	189	1512	10206	61236
315	0	1	21	252	2268	17010	112266
300	0	1	24	324	3240	26730	192456
285	0	1	27	405	4455	40095	312714
270	0	1	30	495	5940	57915	486486
255	0	1	33	594	7722	81081	729729
240	0	1	36	702	9828	110565	1061424
225	0	1	39	819	12285	147420	1503684
210	0	1	42	945	15120	192780	2082024
195	0	0	42	1071	18333	247779	2825361
180	0	0	42	1197	21924	313551	3766014
165	0	0	42	1323	25893	391230	4939704
150	0	0	42	1449	30240	481950	6385554
135	0	0	42	1575	34965	586845	8146089
120	0	0	42	1701	40068	707049	10267236
105	0	0	42	1827	45549	843696	12798324
90	0	0	42	1953	51408	997920	15792084
75	0	0	42	2079	57645	1170855	19304649
60	0	0	42	2205	64260	1363959	23396526
45	0	0	42	2331	71253	1577394	28128708
30	0	0	42	2457	78624	1813266	33568506
15	0	0	42	2583	86373	2072385	39785661
0	0	0	42	2583	86373	2072385	39785661

area interval, then the number of computations needed will be greatly reduced. For each tree stocking level, the alternatives with basal area larger than 286 square feet are grouped together as a big neighborhood. Table II shows the number of feasible nodes at different tree stocking levels, For example, 42 alternatives at age 30, tree stocking level of 210 trees (Table I) are categorized to 16 alternatives according to their basal area (Table II). From Table II Table III can be derived which shows the number of alternatives leading to a certain stage-state combination from the previous stage. The number of alternatives of clearcutting the stand at stage five is 2160 compared with 2,072,385 in Table I. For an eighty year rotation there are 2448 alternatives compared with 39,785,661 alternatives in Table I. The total number of function calculations is, from Table II, [14+(257-1)+(517-1)+(721-1)+(817-1)+(846-1)]*3= 9501. The one subtracted in the parenthesis is because the zero stocking does not require growth calculation. The multiplier three is to indicate three levels of fertilization. Compared with the continuous basal area-- 39,785,661 calculations, only 0.00008 of the effort is needed for the four square foot basal area interval. The number of alternatives, 2448, is less than the number of calculations needed, 9501, because some of the resulting states are grouped together and only one alternative is used to represent that state.

TABLE II. NUMBER OF FEASIBLE NODES FOR A CERTAIN STATE AT A CERTAIN STAGE WHEN ALTERNATIVES IN A FOUR SQUARE FOOT NEIGHBORHOOD ARE GROUPED TOGETHER USING THE BEST TO REPRESENT THE STATE.

Stage	0	1	2	3	4	5	6
Age	10	30	40	50	60	70	80
#Trees	Number of Feasible Nodes						
400	1	1	2	5	6	4	3
390	0	1	4	8	8	6	5
375	0	1	5	11	11	8	7
360	0	1	7	12	13	11	10
345	0	1	9	15	16	14	12
330	0	1	10	16	18	16	15
315	0	1	11	17	21	19	17
300	0	1	11	18	23	21	20
285	0	1	12	20	25	24	23
270	0	1	13	21	28	26	25
255	0	1	14	23	30	29	28
240	0	1	15	24	33	31	30
225	0	1	16	26	34	34	33
210	0	1	16	26	35	36	36
195	0	0	15	26	36	39	38
180	0	0	14	26	36	41	41
165	0	0	13	26	36	44	43
150	0	0	12	25	37	46	46
135	0	0	11	25	37	46	49
120	0	0	10	25	36	47	51
105	0	0	9	24	36	46	52
90	0	0	8	22	34	45	50
75	0	0	6	20	33	43	48
60	0	0	4	18	30	42	49
45	0	0	4	16	28	40	47
30	0	0	3	12	23	34	39
15	0	0	2	9	17	24	28
0	0	0	1	1	1	1	1
Total	1	14	257	517	721	817	846

TABLE III. NUMBER OF ALTERNATIVES LEADING TO A CERTAIN STATE AT A CERTAIN STAGE FROM THE PREVIOUS STAGE WHEN ALTERNATIVES IN FOUR SQUARE FOOT NEIGHBORHOODS ARE GROUPED TOGETHER USING THE BEST TO REPRESENT THE STATE.

Stage	0	1	2	3	4	5	6
Age	10	30	40	50	60	70	80
#Trees	Number of Alternatives						
400	1	1	3	6	15	18	12
390	0	1	6	18	39	42	30
375	0	1	9	33	72	75	54
360	0	1	12	54	108	114	87
345	0	1	15	81	153	162	129
330	0	1	18	111	201	216	117
315	0	1	21	144	252	279	234
300	0	1	24	177	306	348	279
285	0	1	27	213	366	423	369
270	0	1	30	252	429	507	447
255	0	1	33	294	498	597	534
240	0	1	36	339	570	696	627
225	0	1	39	387	648	798	729
210	0	1	42	435	726	903	837
195	0	0	42	480	804	1011	954
180	0	0	42	522	882	1119	1077
165	0	0	42	561	960	1227	1209
150	0	0	42	597	1035	1338	1347
135	0	0	42	630	1110	1449	1485
120	0	0	42	660	1185	1557	1626
105	0	0	42	687	1257	1665	1764
90	0	0	42	711	1323	1767	1899
75	0	0	42	729	1383	1866	2028
60	0	0	42	741	1437	1956	2154
45	0	0	42	753	1485	2040	2274
30	0	0	42	762	1521	2109	2376
15	0	0	42	768	1548	2160	2448
0	0	0	42	768	1548	2160	2448

If the size of basal area interval or tree interval is increased, less computations will be needed for more alternatives are grouped together in a neighborhood.

BACKWARD SOLUTIONS

Traditional dynamic programming problems are solved by the backward solution method using a backward recursive relation. The solution procedure is to move backward stage by stage-- each time finding the optimal policy for each state of that stage-- untill the optimal policy is found at the beginning of the network. Using the notation of Amidon and Akin (1968), the recursive relation is

$$T(x,y)= \max \left\{D(x,y)+T(x+1,y+1);\ I(x,y)+T(x+1,y-1)\right\}$$

A generalized notation will be

$$f_n^*(s)= \max_{x_n} \left\{C(s,x_n)+f_{n+1}^*(x_n)\right\}$$

Finding the optimal policy when starting in state s at stage n requires finding the maximizing value of x_n. $C(s,x_n)$ represents the cost incurred from state s in the current stage to state x_n in the next stage. Figure VIII is a graphical interpretation of the backward recursive relation. For a generalized dynamic programming problem with s states at each stage, if every state can be reached from every state, the number of computations needed will be s^{n-1} (Figure IX), for a problem with n stages and using the complete enumeration method. If the backward solution

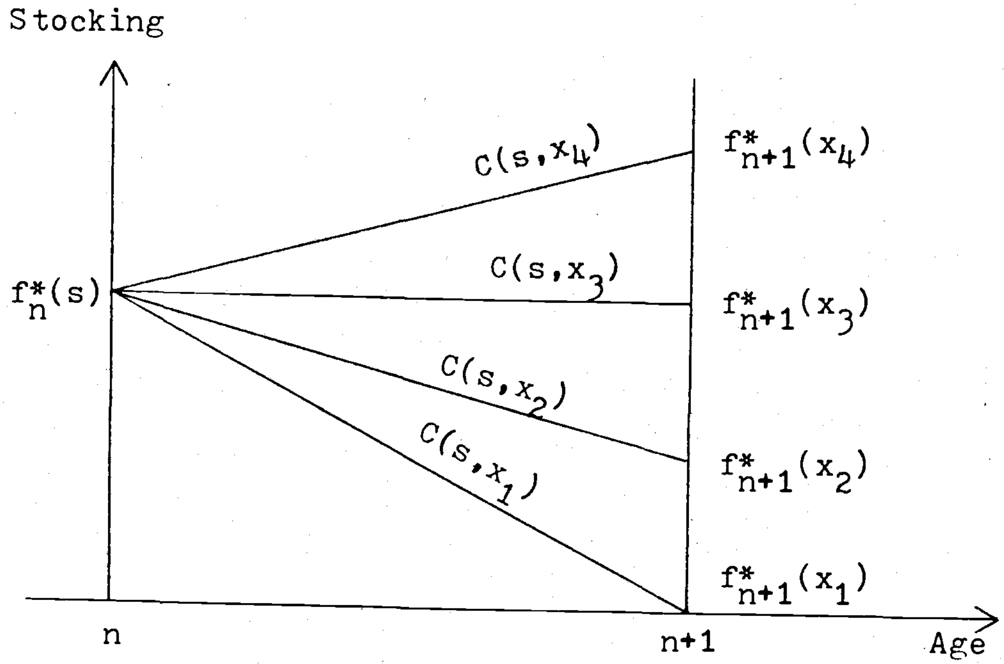


Figure VIII. Backward recursive relation.

method is used, the number of computations needed will be $(n-2)*s^2+s$ for $n \geq 2$, and s for n=1 (Figure X). As n and s get large, the efficiency of using the backward solution method is obvious.

The most important advantage of the backward solution in stand level optimization, is that the intermediate solutions represent optimal regimes from every current state, to the end of the fixed rotation. Should a stand end up off of the optimal path at some stage, then the optimal thinning regime to the end of the rotation is available as an intermediate solution. A principle disadvantage of the backward recursion is that a separate solution is required for each rotation length. The

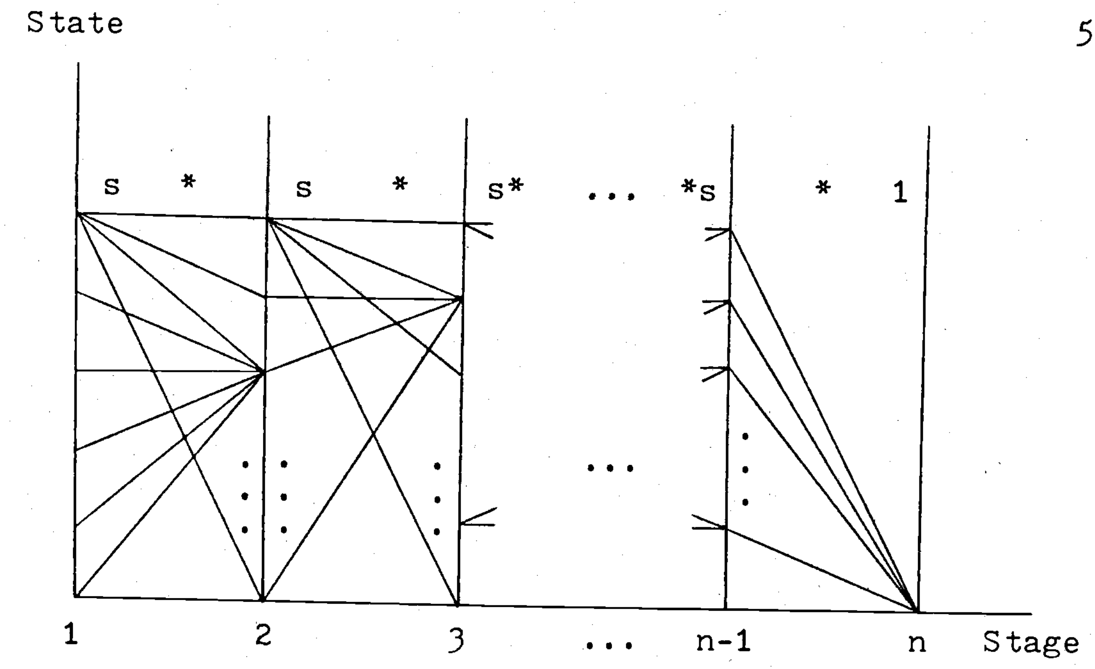


Figure IX. Number of computations needed when all paths are considered.

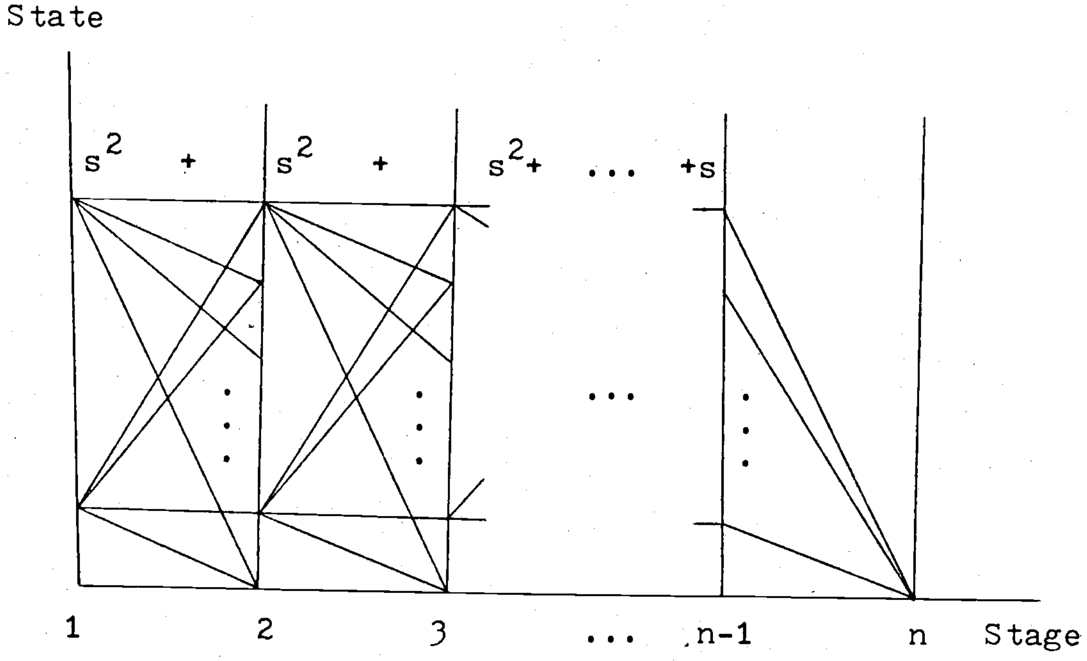


Figure X. Number of computations needed when backward recursive method is used.

"neighborhood" concept discussed in this study introduces an additional problem in the backward recursion. The stands are grown from continuous basal area and (in the case of "no thinning") continuous tree states starting from a given stand state. The backward recursion would have to begin with the discrete basal area and number of tree node variables and would therefore not track the exact values of a continuous growth model and given initial stand condition precisely.

FORWARD SOLUTIONS

The basic idea of the forward solution method is to compare the value of every node in the current stage which can reach a certain node in the next stage. The maximal value will be chosen as the value of that certain node in the next stage and the node in the current stage resulting in the maximal value will be chosen as a path. The functional relationship is

$$f^*_{n+1}(s) = \max_{x_n} \left\{ C(x_n, s) + f^*_n(x_n) \right\}$$

Every notation has the same meaning as mentioned before. A graphical interpretation is shown in Figure XI. For a problem with n stages and s states, the number of computations needed is also $(n-2)*s^2+s$ for $n \geq 2$ and s for $n=1$ (Figure XII). The number of computations needed for forward solution and backward solution methods are the same.

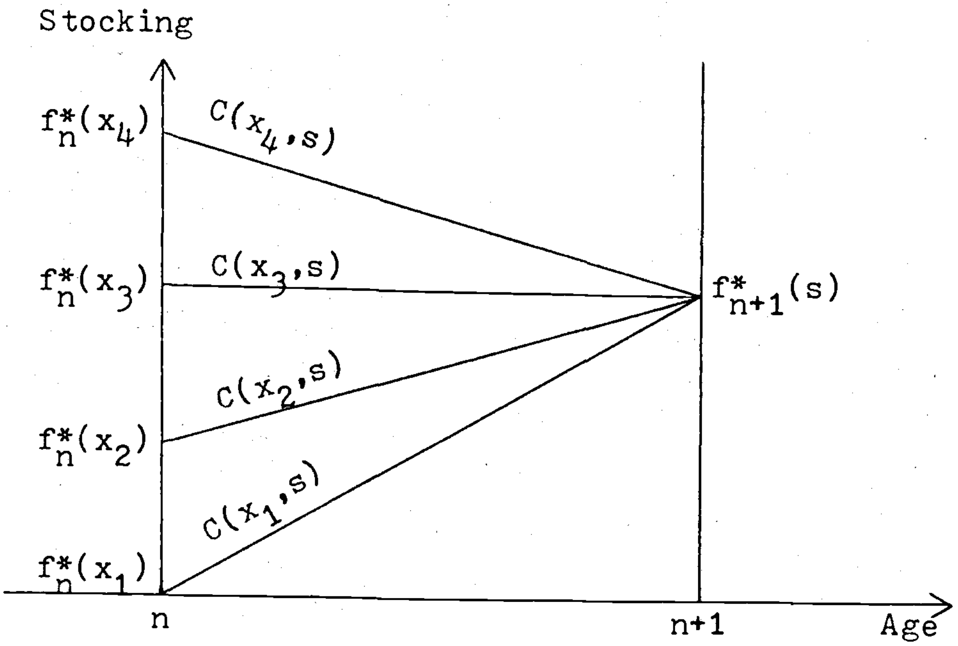


Figure XI. Forward recursive relation.

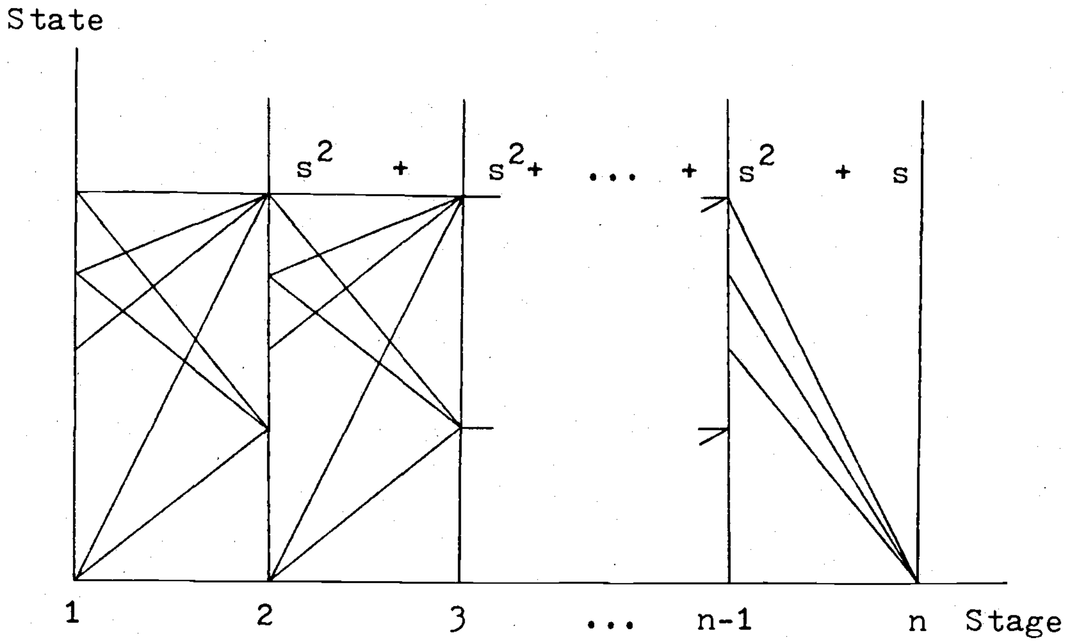


Figure XII. Number of computations needed when forward recursive method is used.

One advantage of the forward solution method in stand optimization is it tells the best path (thinning regime) for every rotation (stage) in one set of calculations. Another advantage of the forward solution method is it handles continuous state problems in discrete state dynamic programming. Kao and Brodie (1979b) first introduced the technique of handling continuous growth in a discrete volume cutting interval. Brodie and Kao (1979) introduced the neighborhood concept to handle the continuous state problem in a three dimensional dynamic programming problem. The disadvantage of the forward recursion is that the intermediate are optimal paths from the beginning of the network to the current stage and represent management decisions already made. Should a stand end up in a non-optimal path state then the optimal path from that state to the end of the rotation can only be found by restarting the recursion from the current stand state.

SUMMARY

Dynamic programming is a powerful and flexible stand level optimization tool, for:

(1) It is computationally simple.

(2) It handles growth, cost and revenue functions of many forms.

(3) It is an efficient method of selecting among an immense number of alternatives.

(4) It considers different silvicultural activities and makes interrelated policy decisions.

Using dynamic programming, four important management activities: commercial thinning, precommercial thinning, regeneration harvest and fertilization can be combined together to get an optimal management regime, accounting for the interactions between the management activities, stocking levels and rotation. The interaction of the four silvicultural activities and size dependent costs and revenues can be examined simultaneously and analytically for particular situations and not just by partial analysis or a single empirical test as in the past.

IV. THE STRUCTURE AND OPERATION OF DFIT AND DOPT

The DFIT (Douglas-fir Interim Tables) program (Bruce, Demars and Reukema, 1977) simulates stand growth and tabulates the results of the simulation. The basic components of DFIT are: (1) equations describing the development of natural stands, (2) thinning guides (Reukema, 1975) based on Reineke's stand density index (Reineke, 1933), (3) equations describing the total cubic-foot increment of thinned stands and plantations, (4) equations predicting the amount and timing of mortality in natural stands, (5) a method for describing stand components at intervals without the direct use of stand and stock tables, and (6) many assumptions about management practices and their effects on stand development. The assumptions are based on study of current available results of Douglas-fir thinning studies. They have been balanced against growth of natural stands and have been checked for compatibility and consistency (Reukema and Bruce, 1977). It is claimed that DFIT was made as flexible as possible subject to the constraint of providing a timely interim tool. Within limits DFIT allows the user to test and observe probable results of different management regimes.

DOPT (DFIT OPTimization) is a stand optimization computer program which uses the stand growth model from DFIT and dynamic programming to find the best management regime. DFIT is a simulator which provides descriptive stand outputs when management regime is provided, and DOPT is an optimizer which provides a regime when a criterion and revenue and cost assumptions are provided. As mentioned earlier, a simulator gives a set of results for each regime supplied. These sets of results are compared to gradually improve on the criterion sought, e.g. maximum present net worth from managing a forest stand. Such an approach may find a "good" answer but only on very simple systems, does it find the true maximum. An optimizer gives the answer one looks for in one set of calculations. In most cases a set of optimization calculations is more difficult than a set of simulation calculations. But the total time used for an exhaustive simulation search is usually larger than that used in optimization. The results from an optimization often give more information than a simulator does.

In the following sections the modules of the DOPT computer program are discussed. (The DOPT program listing is in Appendix III.) Most of them are based on DFIT. The modules include: (1) precommercial thinning (PRECOM), (2) submerchantable mortality (SUBMORT), (3) growth (GROWTH), (4) logging cost (REVNOW), and (5) increment (INCRMNT). A "*" is used to indicate equations taken directly from DFIT (Bruce, Demars and Reukema, 1977) and the DFIT program listing. The meaning of notations used in the following equations are listed in Appendix I.

PRECOMMERCIAL THINNING (PRECOM)

From DFIT the number of trees per acre in a precommercially thinned stand, given diameter at first possible commercial thinning is

$$(*)\ \log N = 3.9591 - 1.5006 \log \bar{D}_F \qquad (1)$$

When number of trees after precommercial thinning is given, the corresponding $\bar{D}_F$ will be

$$\log \bar{D}_F = (3.9591 - \log N\ )/1.5006 \qquad (2)$$

The basal area per acre of merchantable trees, given average diameter of merchantable trees at first possible commercial thinning, is

$$(*)\ \log G_M = 1.6958 + 0.4994 \log \bar{D}_F \qquad (3)$$

After precommercial thinning, the site index is adjusted for superior height growth:

$$(*)\ S_P = S*(1+(210-S)^2/90000)$$

$$\text{IF(S GT 210) } S_P = S \qquad (4)$$

This equation increases site index for the growth simulation between precommercial thinning and first commercial thinning. Improved vigor of the stand is assumed to last indefinitly. Relationships for transforming total age to breast height age and for calculating the breast height age at which a stand of given site will reach a given diameter

$\overline{D}_F$ are:

$$(*)\ A_B = (3.4857/(0.1097 - \log(0.875*\overline{D}_F) + 1.0531 \log S))^4$$

$$(*)\ A_T = A_B + 13.22 - 0.033*S \qquad (5)$$

Then the age is adjusted by the age of precommercial thinning:

$$(*)\ \text{IF}(A_P \text{ GE } 15 \text{ AND } A_P \text{ LE } 20)\ A_F = A_T(0.68+0.016*A_P)$$

$$\text{IF}(A_P \text{ GE } 2 \text{ AND } A_P \text{ LE } 14)\ A_F = A_T(0.8244+0.004333*(A_P/10)+0.015*(A_P/10)^2+0.01667*(A_P/10)^3) \qquad (6)$$

The model does not permit precommercial thinning after age 20. Using the above equations, the time for the first possible commercial thinning can be calculated when time for precommercial thinning and number of trees left after precommercial thinning are given. If the stand will not reach the prescribed diameter for the remaining trees at the time of first commercial thinning, the DOPT program stops here and prints out an error message.

If less than 400 trees per acre are left, the adjusted basal area per acre is

$$(*)\ G_S = 180.22-5*\overline{D}_F \qquad (7)$$

If G_M calculated from equation (3) is no less than G_S, the stand is simulated to grow year by year, so G_S increases, until G_M is less than G_S. The growth of the stand will be discussed in the GROWTH section.

Usually the age for the first possible commercial thinning calculated from the above equations is noninteger value, hence an adjustment is added, by adding the growth of the remaining fractional year, to round the age of the stand up to the nearest integer age. If the rounded age is smaller than the age considered for the first commercial thinning, a growth function is used to grow the stand to the age of the first commercial thinning specified. A flowchart is shown in Appendix IV.

SUBMERCHANTABLE MORTALITY (SUBMORT)

Given the mean diameter of a natural stand of current age and site ($\bar{D}_G$) and the minimum merchantable diameter (D_M), the number of submerchantable trees per acre (N_s) and their corresponding basal area (G_s) in a natural stand can be calculated by using the equations:

$$(*)\ \log N_s = 3.8622+3.1994 \log D_m-4.70 \log \bar{D}_G \qquad (8)$$

$$(*)\ \log G_s = 1.4034+4.9394 \log D_m-4.44 \log \bar{D}_G \qquad (9)$$

Differences between estimates at successive ages give submerchantable mortality between those ages. The following equations give the total number of trees and their corresponding basal area per acre in a natural stand. Given these values and an estimate of submerchantable mortality, the merchantable mortality can be estimated.

$$(*)\ \log N = 3.9108+5.2306/A_B^{0.25}-1.5803 \log S \qquad (10)$$

$$(*)\ \log G = 1.8669 - 1.7408/A_B^{0.25} + 0.5259 \log S \qquad (11)$$

The merchantable mortality is calculated in the GROWTH subroutine and will be discussed later. Here only the nonmerchantable mortality derivation is considered. First, the minimum merchantable diameter is defined as

$$(*)\ D_m = (\bar{D}_{30}/0.875)*0.75 \qquad (12)$$

where $\bar{D}_{30}$ is the mean diameter of a natural stand at age 30, or

$$(*)\ \log \bar{D} = 0.1097 - 3.4857/A_B^{0.25} + 1.0531 \log S \qquad (13)$$

The mean diameter of submerchantable trees at age 30 (D_L) is defined as

$$(*)\ D_L = 0.698*\bar{D}_{30} \qquad (14)$$

The periodic submerchantable mortality of number of trees and basal area can be calculated by using equations (8) and (9). The mortality of volume takes more effort to derive. In a natural stand the volume/basal area ratio can be expressed as a function of dominant and codominant height ($\bar{H}_D$), and the height is a function of age and site.

$$(*)\ \log \bar{H}_D = 0.1567 - 15.673/A_T + \log S \qquad (15)$$

$$(*)\ \log V/G = -0.0282 + 0.7917 \log \bar{H}_D \qquad (16)$$

The mortality of volume is the mortality of basal area multiplied by V/G and adjusted by the Washington Department of Natural Resources tarif. A tarif diameter function, given diameter, is

$$(*)\ F = 0.004978*\bar{D}_G^2/(0.005454*(\bar{D}_G^2+16)*(1.0378+$$

$$1.4967*0.0134^{\bar{D}_G/10})-0.1745) \qquad (17)$$

Washington Department of Natural Resources tarif is

$$(*)\ T = F*V/G \qquad (18)$$

This relation is approximate, not exact, and is applied generally to the smallest stand components so the error in volume is minimized. Volume of larger stand components are estimated from volume of total stand and volume of small components. From equation (18) we formulate the following equation:

$$F(D_L)*V_s'/G_s' = F(\bar{D}_G)*V/G \qquad (19)$$

rearranging:

$$V_s' = G_s'*(F(\bar{D}_G)/F(D_L))*V/G \qquad (20)$$

Where V_s' and G_s' are mortalities between successive periods. After the first calculation, D_L in equation (20) is substituted by $D_L' = ((G_s'/N_s')/3.1416)^{0.5}*12$, and $F(\bar{D}_G)*V/G$ is substituted by $(F(\bar{D}_G)*V/G+F(\bar{D}_G')*(V/G)')/2$ where $\bar{D}_G'$ is the mean diameter at the age when mortality is to be calculated and $(V/G)'$ is the volume/basal area ratio at that age. Hence

$$V_s = G_s*(F(\bar{D}_G)*V/G+F(\bar{D}_G')*(V/G)')*0.5/F(D_L') \qquad (21)$$

In DOPT both yearly and ten-year mortality are calculated. If a stand is not normal, the stand condition is compared with a normal stand and the mortalities are adjusted linearly. A flowchart of the subroutine SUBMORT is shown in Appendix IV.

GROWTH (GROWTH)

In a natural stand, the volume per acre can be expressed as a function of site and stand age:

$$(*)\ \log V = 1.9628-12.4083/A_T-1.7408/A_B^{0.25}+ 1.3176 \log S \qquad (22)$$

One-year volume growth of the stand is just the derivative of equation (22) over age:

$$(*)\ \log dV/dA_T = \log V + \log 2.3026 + \log (12.4083/ A_T^2 + 0.4352/A_B^{1.25} \qquad (23)$$

Equation (23) was compared with Staebler's (1955) and Curtis' (1967) estimates of gross growth for ages 30 to 100 and adjusted by $\log dV_A$ to approximate gross growth ratios. Equations (23) and (24) are combined to equation (25):

$$(*)\ \log dV_A = \log (1.12 + 0.0105*A_T - 0.00005*A_T^2$$

$$\text{IF}(A_T \text{ GE } 105)\ \log dV_A = 0.22304 \qquad (24)$$

$$(*)\ \log dV'/dA_T = \log dV/dA_T + \log dV_A \qquad (25)$$

Using the same argument, height growth is the derivative of the function of tree height, equation (15), over age:

$$(*)\ \log d\bar{H}/dA_T = 1.7141 + \log S - 15.673/A_T- 2*\log A_T \qquad (26)$$

Given total number of trees, the top limit of basal area in natural stand can be expressed as

(*) $\log G_L = 3.3446 - 0.3328*\log N$ (27)

The volume growth calculated from equation (25) is first adjusted by an adjustment factor which is a function of age of first commercial thinning (A_{CT}).

(*) $ADJ1 = (405 - A_{CT})/400$ (28)

Then the volume growth is adjusted by a competition growth reduction multiplier which is a function of the average basal area of living merchantable trees (G_M) to the top limit of basal area in a natural stand:

(*) $G_Z = G_M/G_L$ (29)

(*) $ADJ2 = 1 -16*(G_Z - 0.5)^4$ (30)

Figure XIII shows the shape of ADJ2 over G_Z. When G_Z is small, it implies G_M is small, hemce the growth will be small compared with the growth from a normal stand. When G_Z is zero, i.e. no stocking in the stand, the growth is also zero. When G_Z is large, it means the stand is overstocked, hence the growth is smaller than the growth of a natural stand. If G_Z is one, i.e. the stand has reached the top limit that a stand can be, then the growth is zero. This symmetrical function gives an approximation of the plateau of maximum yield hypothesized for stands of different density.

Given number of trees (N), basal area (G), volume (V),

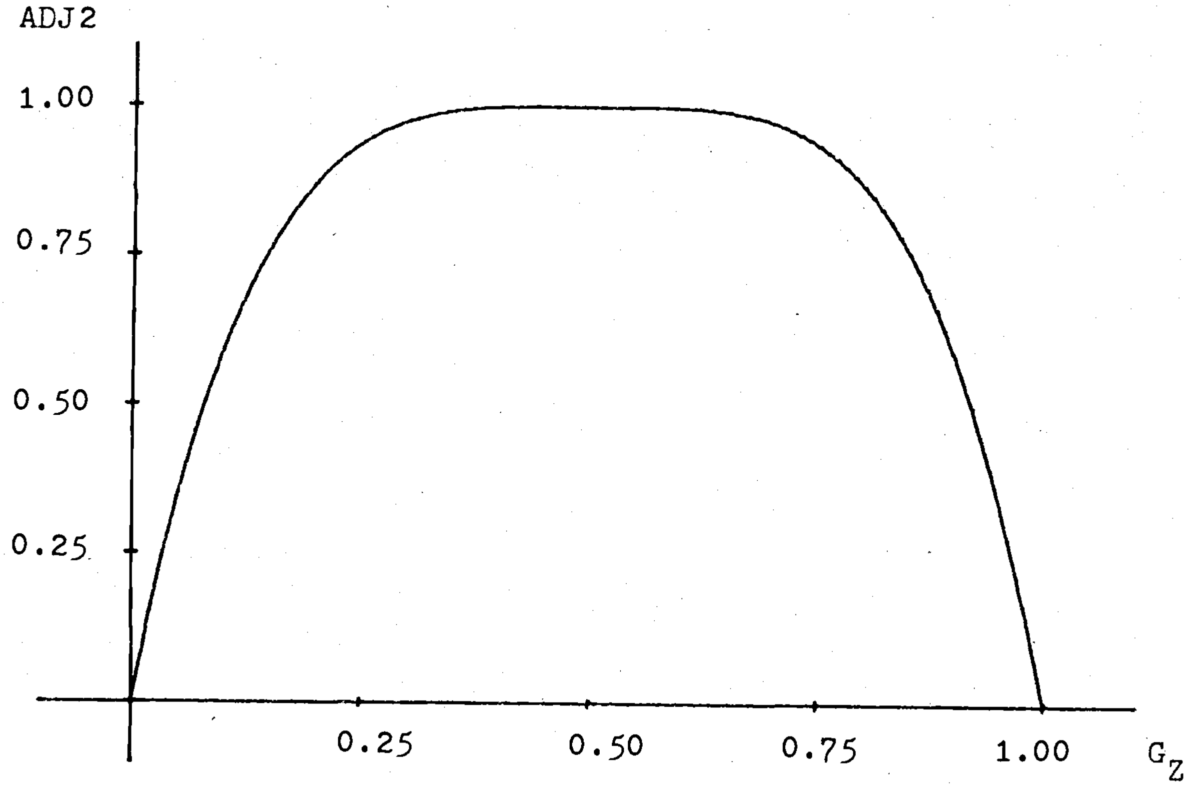


Figure XIII. The relation between growth adjustment ADJ2 and stocking level G_Z.

age and site index of a stand, the gross growth can be calculated by following the above procedures. The net growth can be calculated by using equation (23), instead of equation (25), i.e. the growth without adjustment of mortality. The difference between gross growth and net growth is merchantable mortality. A flowchart which shows how growth is calculated is in Appendix IV.

LOGGING COST (REVNOW)

Sessions (1979) derived stump-to-truck harvesting costs as a function of merchantable volume removed per acre by using simulation. In DOPT the cost of a 600-foot skyline was chosen and divided by 1.5 to approximate the cost of using a tractor. This ratio was chosen because a study by Aulerich, Johnson and Froehlich (1974) showed that skyline costs are 1.5 to 1.66 times those for tractor logging. Not many studies have discussed logging cost in as much detail as Sessions did. DOPT will operate with any computable logging cost functions for which DFIT will provide the necessary stand variables. This study was conducted at cost levels 2/3 those derived by Sessions and demonstrates the impact on regime of these lower cost levels. Comprehensive sensitivity to logging cost is dealt with in Sessions (1979). Table IV lists larger yarder power curve coefficients for stump to truck harvesting costs as a function of merchantable volume per acre removed for a 600-foot skyline.

TABLE IV. LARGER YARDER POWER CURVE COEFFICIENTS FOR STUMP TO TRUCK HARVESTING COSTS AS A FUNCTION OF MERCHANTABLE VOLUME REMOVED PER ACRE FOR A 600-FOOT SKYLINE.

$$\text{COST}(\$/\text{MCF}) = A*V^{B}$$

Mean Diameter	A	B
10.87	8479.6	-0.3626
12.31	6839.5	-0.3492
13.66	6276.4	-0.3498
15.03	5848.5	-0.3548
16.19	4980.1	-0.3475
17.26	3765.6	-0.3238
18.31	4215.0	-0.3402
20.25	3377.5	-0.3207
21.90	4209.5	-0.3488

When mean diameter is smaller than 10.87 inches the volume removed in each cut is also a factor which determines the harvest cost. How the cost is calculated when diameter is smaller than 10.87 is shown in the flowchart of REVNOW and DOPT program listings in Appendix IV and III. For any given mean diameter, a linear interpolation is used to calculate harvest cost. For example, if mean diameter cut is 20 inches, then the cost will be $4215.0*V^{-0.3402}$-

$(20-18.31)*((4215.0*V^{-0.3402}-3375.5*V^{-0.3207})/(20.25-18.31))$.

Revenue from either thinning or regeneration harvest of the stand in the current period is

Revenue = (Volume Cut)*((Pond Value)-(Stump To Truck Cost)-(Haul Cost))

Where pond value is defined as: The market price of logs delivered to a wet site, log pond, or tidewater (Pearce and Stenzel, 1972), i.e. it is the value a mill would pay for logs received at the mill gate. Sessions (1979) used data from Bulletin 201 (McArdle, Meyer and Bruce, 1961), the Columbia River Scaling Rule (Dilworth, 1973) and the current market value of logs to derive a relation between arithmetic mean stand diameter and pond value. Table V shows the relation.

A least squares linear regression line was fitted to the data with an $R^2 = 0.99$ providing the following relationship:

$$\text{Pond Value } (\$/\text{MCF}) = 9.91 + 70.81 * (\text{Mean Stand Dbh}) \qquad (31)$$

When quadratic mean diameter is larger than 22 inches, there is no additional price premium attached. Haul cost is the cost of transporting logs from the forest stand to mill. A constant $150/MCF was used in this study although it should be recognized that smaller logs cost more to transport due to a lower board foot to cubic foot ratio.

A flowchart which helps to understand how the revenue is calculated is in Appendix IV.

TABLE V. POND VALUE AS A FUNCTION OF ARITHMETIC MEAN STAND DIAMETER.

Mean Stand Dbh (Inch)	Age	BF/CF Ratio	Pond Value ($/MCF)
6.60	40	1.93	413.96
10.40	60	3.46	782.16
13.70	80	4.15	1002.50
16.20	100	4.58	1203.83
18.40	120	4.94	1351.88
20.20	140	5.11	1417.42
22.00	160	5.29	1509.37

When the above procedures are used, two transformations must be made. First, the volume used in the growth model is total cubic volume and the volume used in calculating harvest cost is cubic volume to a four-inch top (CV4). The following equations calculate the cubic volume to a four-inch top from total cubic volume (V).

$$(*)\ \overline{H}_M = \overline{H}_D*(3040 - N)/3000$$

$$\mathrm{IF}(\overline{H}_M\ \mathrm{GT}\ \overline{H}_D)\ \overline{H}_M = \overline{H}_D$$

$$(*)\ CV4 = V*(0.8758+0.001049*\overline{H}_M-0.000002824*\overline{H}_M+$$

$$0.3221/\overline{D}_G-45.647/\overline{D}_G^3 \qquad (33)$$

Where $\bar{H}_M$ is the stand mean height, $\bar{H}_D$ is average height of dominants and codominants, and N is total number of trees.

Second, the diameter used in the growth model is quadratic mean diameter ($\bar{D}_G$), and the diameter used in calculating harvest cost is arithmetic mean diameter ($\bar{D}_N$). Sessions (1979) showed that

$$\bar{D}_N^2 = \bar{D}_G^2 - \mathrm{Var}(\bar{D}_N) \tag{34}$$

where $\mathrm{Var}(\bar{D}_N)$ is the variance of $\bar{D}_N$.

In a stand table, trees are grouped into diameter classes.

Let N = total number of trees in a stand.

N_i = number of trees in the ith diameter class.

D_{ij} = diameter of the jth tree in the ith diameter class.

$\bar{D}_i$ = mean of the ith diameter class.

$\bar{D}$ = mean of all trees in the stand.

Then,
$$\begin{aligned}\mathrm{Var}(D_{ij}) &= \sum_i \sum_j (D_{ij}-\bar{D})^2/N \\ &= \sum_i \sum_j ((D_{ij}-\bar{D}_i)-(\bar{D}_i-\bar{D}))^2/N \\ &= \sum_i \sum_j (D_{ij}-\bar{D}_i)^2/N + \sum_i \sum_j (\bar{D}_i-\bar{D})^2/N + \\ &\quad 2\sum_i \sum_j (D_{ij}-\bar{D}_i)(\bar{D}_i-\bar{D})/N \\ &= \sum_i \sum_j (N_i*(D_{ij}-\bar{D}_i)^2/N_i)/N + \sum_i \sum_j (\bar{D}_i-\bar{D})^2/N +\end{aligned}$$

$$2 \sum_i (\overline{D}_i - \overline{D}) \sum_j (D_{ij} - \overline{D}_i)/N$$

$$= \sum_i N_i * \mathrm{Var}(D_{ij} \text{ within the ith class})/N + \sum_i N_i * (\overline{D}_i - \overline{D})^2/N + 0 \qquad (35)$$

It is reasonable to assume that the diameter of the trees in every diameter class is uniformly distributed. Hence

$$\overline{D}_i = (L_i + U_i)/2 \qquad (36)$$

$$\mathrm{Var}(D_{ij} \text{ within the ith class}) = (U_i - L_i)^2/12 \qquad (37)$$

Where U_i and L_i are the upper and lower bound of the ith diameter class. Equations (36) and (37) are the definitions for uniform distributions (Mood, Graybill and Boes, 1974). In our case, $U_i - L_i = 2$ for every i, so Var(Diameter of the ith class) = 1/3 for every class. Therefore,

$$\mathrm{Var}(D_{ij}) = \sum_i (N_i/3)/N + \sum_i N_i * (\overline{D}_i - \overline{D})^2/N$$

$$= 1/3 + \sum_i N_i * (\overline{D}_i - \overline{D})^2/N \qquad (38)$$

Where $\sum_i N_i * (\overline{D}_i - \overline{D})^2/N$ is the variance among diameter classes. That is to say the variance of D_{ij} is the sum of variance within diameter class and variance among diameter classes. The above derivation is similar to the study of Sessions (1979). The difference is that Sessions assumed that there was no variance within each diameter class, i.e.

$\text{Var}(D_{ij}) = \sum_i N_i*(\bar{D}_i-\bar{D})^2/N$ compared with equation (38). Table VI shows the variances calculated by using equation (38) and data from Bulletin 201. Figure XIV is a diagram showing the same thing.

TABLE VI. VARIANCE OF DIAMETER DISTRIBUTION FOR DIFFERENT SITE-AGE COMBINATION.

	Age							
Site	20	40	60	80	100	120	140	160
80	-	-	4.42	8.26	8.70	10.66	13.19	15.49
110	-	3.75	7.85	11.59	14.88	19.19	22.85	26.32
140	-	6.18	12.17	18.12	23.69	29.92	35.29	40.44
170	2.28	8.86	17.83	27.07	36.33	44.89	50.22	60.91
200	3.77	14.10	26.96	41.08	55.31	66.45	75.44	-

A least squares linear regression line is fitted with $R^2 = 0.9950$, and

$$\text{Variance} = 4.0725-0.065722*s+0.00001508*A_T*S^2 \qquad (39)$$

Each time a revenue calculation is requested, first, the total cubic volume is changed to cubic volume to a four-inch top, second, the quadratic mean diameter is changed to the arithmetic mean diameter, and finally, the harvest cost and pond value are calculated to get the revenue.

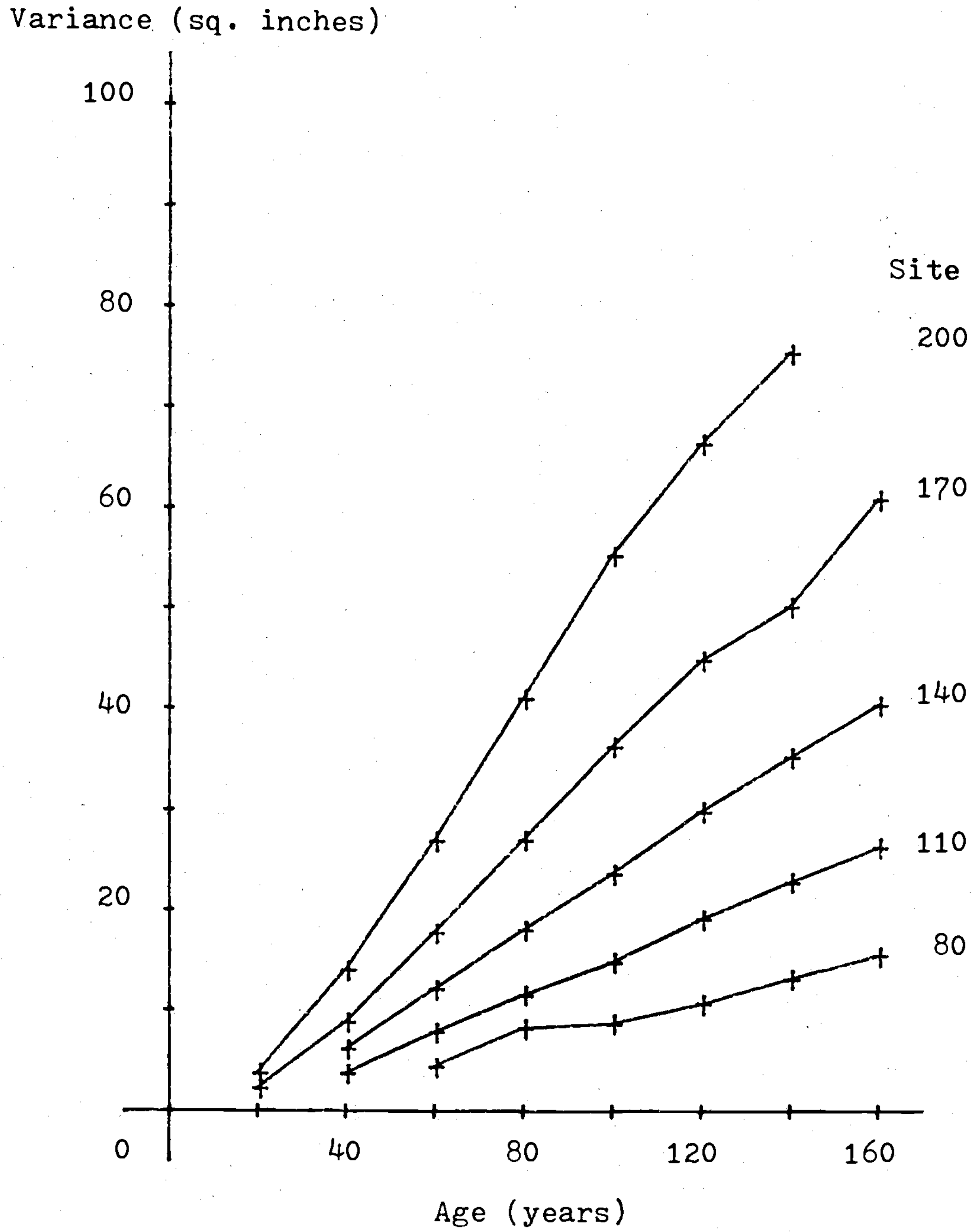


Figure XIV. Variances of diameter distribution for different ages and sites.

INCREMENT (INCRMNT)

Sometimes a stand may be disturbed by climatic factors and biological factors and may not be a normal stand any more. What DOPT will do with this kind of stand is to simulate the stand to the next age of multiple ten based on the current stand condition. Then use dynamic programming to find the best management regime from the simulated initial condition. How to simulate the stand based on the stand condition is done in the subroutine INCRMNT.

The current age of the stand is tested to see if it is a multiple of ten and larger than 30. If it is not, then the next age of multiple ten is the target age that the stand will be projected to. The current number of trees and basal area are compared with those of a normal stand. And NRATIO (current number of trees/number of trees from natural stand) and GRATIO (current basal area/basal area from natural stand) are calculated. The merchantable mortalities between the current age and the next age of multiple ten are adjusted by the NRATIO and GRATIO. The merchantable part is the difference between the total and submerchantable part. The following simulations of growth are the same as those discussed in the GROWTH section. A flowchart showing the structure of this subroutine is in Appendix IV.

MAIN PROGRAM (DOPT)

The responsibilities of the main program are to: (1) input data, (2) calculate the effect of fertilization, (3) use the dynamic programming technique to find the best alternative, and (4) output information. The main program also calls those subroutines mentioned above to fulfill these responsibilities.

Input Data

The data needed for DOPT are INTVR, INTHR, TBASE, SITE, R, TESTN, TESTP, TESTF, REGENC, R1, PCTAGE, NPCT, PCTCOST, STREE, and SBA. Where

INTVR is tree interval considered for cutting. Twelve is the smallest number that can be used and 15 to 50 is suggested.

INTHR is the basal area interval. Four square feet is the smallest value that can be used, and four to 20 is suggested.

TBASE is either the age to be considered for first commercial thinning for a normal stand, or the current age of a non-normal stand. In the current version of DOPT this age must be 30 or larger.

SITE is the 100-year based site index of the stand considered.

R is the alternative rate of return (3% will be 0.03).

$$\text{TESTN} = \begin{cases} 0 & \text{if the stand is a normal stand.} \\ 1 & \text{if the stand is not a normal stand.} \end{cases}$$

$$\text{TESTP} = \begin{cases} 0 & \text{if the stand is not considered for precommercial thinning.} \\ 1 & \text{if the stand is considered for precommercial thinning.} \end{cases}$$

$$\text{TESTF} = \begin{cases} 0 & \text{if the stand is not considered for fertilization.} \\ 1 & \text{if the stand is considered for fertilization.} \end{cases}$$

REGENC is regeneration cost ($).

R1 is inflation rate (5% will be 0.05).

PCTAGE is the age considered for precommercial thinning.

NPCT is the number of precommercial thinning alternatives to be considered. The first alternative is to leave STREE trees. The second is to leave (STREE - INTVR) trees, (STREE - 2*INTVR) for the third, and so on. When the number of remaining trees is too few, resulting in a first possible commercial thinning age larger than TBASE, the program DOPT will stop considering those heavier precommercial thinning alternatives.

PCTCOST is the cost of precommercial thinning.

If TESTN=1 or TESTP=0, the value of PCTAGE, NPCT and PCTCOST are not used.

STREE is either the total number of trees of a non-normal stand or the first alternative considered for precommercial thinning. When TESTN=0 and TESTP=0 this value is not used.

SBA is the basal area of a non-normal stand. This value is used only when TESTN=1.

The data are read in through free format, so zero values must be applied.

Effect of Fertilization

In 1969 the Regional Forest Nutrition Research Project was initiated with the primary objective of providing resource managers with more accurate data on the effect of fertilizing and thinning young-growth Douglas-fir and Western hemlock forests. A note by Turnbull and Peterson (1976) presented the results of fertilizing 87 installations of six plots each in Douglas-fir natural stands in Western Washington and Oregon. In each installation two of the 1/10 acre plots were not treated, two received 200 pounds nitrogen/acre, and two 400 pounds nitrogen/acre, in the form of urea.

The primary objective was to provide an estimate of mean response to fertilizer according to age, site, and other relevant variables. Response is measured as increase in growth rate due to fertilizer. In simple equation form,

$$\begin{bmatrix}\text{Treated stand}\\ \text{growth rate}\end{bmatrix} = \begin{bmatrix}\text{Growth rate as in}\\ \text{untreated stand}\end{bmatrix} + \begin{bmatrix}\text{Increase in growth rate}\\ \text{due to fertilizer}\end{bmatrix}$$

and

$$\begin{bmatrix}\text{Treatment}\\ \text{Response}\end{bmatrix} = \begin{bmatrix}\text{Treated stand actual}\\ \text{growth rate in plot}\end{bmatrix} - \begin{bmatrix}\text{Estimated growth rate}\\ \text{as in untreated stand}\end{bmatrix}$$

The installation chosen vary in site (site classes I, II, III, and IV, or site indices 200, 170, 140, and 110 using 100-year basis) and breast height age (10, 20, 30, 40, and 50). The following equations show the volume and basal area growth per acre per year with different amounts of fertilization:

$$\Delta V = (0.10493*A_B^{-0.2269}*S_{50}^{1.07647}*G^{0.56805}*N^{0.10875}) +(-0.27823-0.46375*S_{50}*\log(1+PN)+78.9023*\log(1+PN)) \quad (40)$$

$$\Delta G = (12.70144-0.22684*A_B-0.02066*S_{50}+0.04469*G-1.64715*G/A_B+21.74006*G/A_B^2)+(40.40841*\log(1+PN)/G+0.00056*\log(1+PN)) \quad (41)$$

where ΔV = periodic annual gross increment of volume, cu. ft./acre/year.

ΔG = periodic annual gross increment of basal area, sq. ft./acre/year.

A_B = breast height age.

S_{50} = 50-year based site index.

G = initial basal area, sq. ft./acre.

N = number of stems/acre.

PN = pounds of nitrogen/acre.

When PN=0, ΔV and ΔG is just the usual growth of a stand. Table VII compares the growths calculated from the growth model of DFIT and the growth model of Turnbull and Peterson (T&P) under different stocking levels for site index 140. A program from which Table VII was derived is in Appendix II. From Table VII, in young and understocked stands, and old and overstocked stands the difference of the growth between these two models is larger. In the first commercial thinning we restrict the thinning to no more than 50 percent of the stocking. And from the DOPT computer runs, under different conditions, none of them has stocking more than 80 percent of a normal stand at every age. Hence we are not in the region of inconsistency.

To make the two models comparable, the percent fertilization effect from the model by Turnbull and Peterson is adjusted by the growth of DFIT model, i.e.

$$GR(Fertilization) = GR(Fertilization)*GR(DFIT)/GR(T\&P) \qquad (42)$$

TABLE VII. COMPARISON OF TOTAL CUBIC FOOT GROWTHS FROM GROWTH MODELS BY TURNBULL AND PETERSON AND THE DFIT MODEL ON SITE 140 WITH DIFFERENT GROWING STOCK.

	A_B (A_T)	21.4 (30)		31.4 (40)		41.4 (50)		51.4 (60)	
Normality	10-year Growth	ΔG	ΔV	ΔG	ΔV	ΔG	ΔV	ΔG	ΔV
1.2	DFIT	46.8	2420	22.2	1771	16.2	1478	14.0	1300
	T&P	77.2	3091	62.1	3310	52.2	3207	42.6	3082
1.0	DFIT	54.7	2483	39.1	2201	32.5	1968	28.2	1769
	T&P	73.4	2732	57.1	2926	45.1	2835	33.4	2724
0.8	DFIT	60.2	2478	45.5	2264	37.8	2031	32.3	1821
	T&P	69.6	2349	52.1	2515	38.0	2437	24.2	2342
0.6	DFIT	64.1	2445	50.0	2256	41.2	2024	35.1	1814
	T&P	65.7	1933	47.1	2070	30.9	2006	15.0	1928
0.4	DFIT	62.4	2229	50.7	2115	41.3	1884	34.6	1673
	T&P	61.9	1470	42.0	1574	23.9	1525	5.8	1465
0.2	DFIT	44.4	1518	37.0	1467	28.8	1257	23.2	1082
	T&P	58.1	919	37.0	985	16.8	954	-3.4	916

where GR(Fertilization) is the growth with fertilization.

GR(DFIT) is the growth from DFIT model without fertilization.

GR(T&P) is the growth from Turnbull and Peterson growth model without fertilization.

Since data collected from the fertilization experiment is restricted to breast height age younger than 50, or 58.6 total age for site index 140, only stands with age younger than or equal to 60 total age are considered for fertilization in the DOPT program.

The site index used in DFIT is 100-year based. To make the two models comparable, an equation derived by King (1966) is used to change 100-year based site index to 50-year based site index:

$$S_{50} = 21.5-0.18127*A_T+0.72144*S_{100} \tag{43}$$

where A_T is total age, and 30 is used here to transform 100-year based site index to 50-year based site index.

The assumed cost for 200 pounds of applied nitrogen fertilization is $45/acre and $85/acre for 400 pounds. These include all transport and application costs required for fertilization.

Dynamic Programming

In each stage, every combination of number of trees and basal area is checked to see if it is feasible, i.e. if it can be reached from a node (a combination of trees and basal area) in the previous stage. If it is feasible then there is nothing to be done. If it is feasible, then three levels of fertilization are considered, if it is requested to do so, to grow the stand to three different states. Every state can be considered for different levels of thinning. The resulting states are categorized to appropriate nodes. The alternative which has the highest cumulative present net worth will occupy that node. The information associated with each node (node value) will be the quadratic mean diameter, level of fertilization, merchantable volume, merchantable basal area, total volume removed, merchantable number of trees and cumulative present net worth of the treatments and harvests to that age. The optimal stand condition (optimal policy) in the previous state, from which the current state is reached, is also stored at each node. This is the forward recursive method. The information carried by each node permits the tracing of all elements of a regime once the recursion is completed.

Output Information

First, the input data are echoed.

Second, the node-value information associated with each state at each stage and state are printed out.

Third, the number of nonmerchantable trees at every stage is printed out.

Finally, there is a table showing the final stand conditions, present net worth and soil expectations of different rotations.

At each stage, the node with zero trees carries the information of the best management regime for a rotation length of this age. From this node we can use information item (8)-- the optimal path from the previous stage, to trace back and find the management regime. The DOPT program stops when present net worth declines or when 14 stages have been calculated. The soil expectation associated with each rotation is also calculated. The largest value gives the soil expectation rotation.

A flowchart of program DOPT is shown in Appendix IV. The program listing is in Appendix III.

V. THE OPERATION OF THE DOPT ALGORITHM UNDER ALTERNATE STATE SPACE SPECIFICATION

When diameter acceleration is considered in a stand optimization problem, in addition to volume growth, another descriptor is needed to represent the diameter growth. In this model number of trees and basal area are used as descriptors. Basal area indicates the volume growth, and basal area and number of trees together indicate the diameter growth. In dynamic programming both stage and state space have to be defined. Here stage is defined as age of the stand at commercial thinnings and precommercial thinning. State is defined as the number of trees in the stand and its corresponding basal area. For a fully stocked young Douglas-fir medium site stand at age 30, there are around 865 trees. At age 100 there are still around 184 trees. If every single tree is considered as alternative for thinning then the computations needed will be tremendous. One way to solve this problem is to consider a certain number of trees, e.g. K, as an alternative for thinning. That is, to thin K, 2K, 3K, ..., or nK trees. This can reduce the state space, hence the corresponding large number of computations needed. The larger the K is, the larger the reduction in computations. But the trade-off is that as K increases, the number of alternatives considered decreases. Hence an optimal range of tree interval is required that considers a fairly large number of thinning alternatives yet not an excessive number of computations.

Once the tree interval K is fixed, the optimal basal area interval must also be defined. If the basal area interval is too small, then many states (number of tree-basal area conditions) will be infeasible, i.e. some basal area will not have any number-of-trees associated. For example, if 20 alternatives can reach the 210-tree stocking level, when there are 50 basal area intervals then at least 30 (=50-20) basal area intervals will be empty (or infeasible). The more the basal area intervals, i.e. the smaller the size of basal area interval, the more empty nodes there will be. If the interval is too large then we expect the error due to choosing a wrong alternative in the neigborhood will be larger. This artifact effect (Brodie and Kao, 1979) will be discussed later in this Chapter.

The following sections discuss what is the reasonable tree interval and basal area interval for a fixed tree interval.

TREE INTERVAL STATE

As mentioned earlier, if the tree interval is too large, then only a limited number of thinning alternatives can be considered. If the tree interval is too small, then the "method of thinning" assumption, i.e. the ratio of diameter cut to diameter of the merchantable stand before cut (assumed equal to one), may not be fulfilled. Another problem will be the computer capacity. A reasonable size of tree interval offers a reasonable number of thinning alternatives, meets the computer capacity and yet gives a result of reasonable precision.

One assumption used in this study is that the diameter distribution of a stand before and after thinning have the same normal distribution when d/D ratio is one. For different sizes of tree interval this assumption may not hold. Two kinds of test can be used to check the effect of tree interval size. One is to test whether the diameter distribution of the remaining trees in the stand is normally distributed with certain mean and variance. Another is to test whether the diameter distribution of the trees in the stand before cut is the same as the diameter distribution of the trees in the stand after cut. If small components are normally distributed, the aggregate will also be normally distributed-- a variation of the central-limit theorem, i.e. when the number of trees increases, the diameter distribution is more likely to be normally distributed. Hence only the normality of the smallest residual has to be tested. Only the second test is explained here. One example is used to show how the test is applied.

From Bulletin 201 (McArdle, Meyer and Bruce, 1961), a fully stocked Douglas-fir stand with site index 140 at age 40 has 585 trees. Its diameter distribution is shown in Table VIII which has a mean of 6.5752 inches and variance 5.8572 square inches. A chi-square test (Mood, Graybill and Boes, 1974) shows the stand diameter distribution is normally distributed with mean 6.5752 and variance 5.8572. The chi-square value is calculated by using the formula $^{2} = \sum_{i=1}^{n} (X_i\text{-Expected } X_i)^2/(\text{Expected } X_i)$. Here we have six diameter classes. Since two degrees of freedom are used in estimating mean and variance, hence there are 6-2=4 degrees of freedom left. The calculated chi-square value is 8.96 which is smaller than the chi-square value from a chi-square distribution with four degrees of freedom at the 95 percent significance level. ($\chi^2(4,95\%)=9.49$). Hence we accept the hypothesis that this diameter distribution is normal.

TABLE VIII. STAND TABLE FOR DOUGLAS-FIR SITE INDEX 140 AT AGE 40.

Diameter Class	2-3	4-5	6-7	8-9	10-11	12-13
Number of Trees	52	159	175	129	54	16

The second test is to test whether the diameter distribution of trees before thinning and after thinning are the same no matter what the diameter distribution is. A contingency table (Davies and Goldsmith, 1972; Daniel, 1976) is set up for each thinning (tree) interval with the normal stand to find if they have the same distribution. Tables IX, X, XI, XII, XIII, XIV, and XV are contingency tables for different thinning, or tree, intervals. The second row in each table shows the remaining number of trees in each diameter class when thinned proportionally to the initial diameter distribution. The chi-square values are calculated by using the formula

$$\chi^2 = \sum_{i=1}^{2} \sum_{j=1}^{6} (X_{ij}\text{-Expected } X_{ij})^2/(\text{Expected } X_{ij})$$

Where $(\text{Expected } X_{ij})=(X_{i.}/\text{Total})*X_{.j}$. The calculated chi-square value is compared with the chi-square value at 95 percent significance level with degrees of freedom of (number of rows - 1)*(number of columns - 1), or (2-1)*(6-1)=5 in this case, which is 11.1. None of the calculated chi-square values is larger than 11.1, so we believe that the remaining trees of every cutting interval has the same distribution as the normal stand no matter what distribution the normal stand is. We have already tested that the normal stand is normally distributed, hence the remaining trees of different cutting, or tree, interval are also normally distributed.

The above test indicates that for every cutting, or tree, interval, the smallest residual has the same distribution (normally distributed with same mean and variance).

TABLE IX. CONTINGENCY TABLE FOR A NORMAL STAND COMPARED WITH A STAND WITH 50 TREES.

Normal Stand	52	159	175	129	54	16	585	$X_{i.}$
Interval 50	4	14	15	11	5	1	50	
	56	173	190	140	59	17	635	Total
				$X_{.j}$			$\chi^2 = 0.18$	

TABLE X. CONTINGENCY TABLE FOR A NORMAL STAND COMPARED WITH A STAND WITH 30 TREES.

Normal Stand	52	159	175	129	54	16	585	$X_{i.}$
Interval 30	3	8	9	6	3	1	30	
	55	167	184	135	57	17	615	Total
				$X_{.j}$			$\chi^2 = 0.15$	

TABLE XI. CONTINGENCY TABLE FOR A NORMAL STAND COMPARED WITH A STAND WITH 20 TREES.

Normal Stand	52	159	175	129	54	16	585	$X_{i.}$
Interval 20	2	5	6	4	2	1	20	
	54	164	181	133	56	17	605	Total
				$X_{.j}$			$\chi^2 = 0.46$	

TABLE XII. CONTINGENCY TABLE FOR A NORMAL STAND COMPARED WITH A STAND WITH 15 TREES.

Normal Stand	52	159	175	129	54	16	585	$X_{i.}$
Interval 15	1	4	5	3	1	1	15	
	53	163	180	132	55	17	600	Total
				$X_{.j}$			$\chi^2 = 1.07$	

TABLE XIII. CONTINGENCY TABLE FOR A NORMAL STAND COMPARED WITH A STAND WITH 10 TREES.

Normal Stand	52	159	175	129	54	16	585	$X_{i.}$
Interval 10	1	3	3	2	1	0	10	
	53	162	178	131	55	16	595	Total
				$X_{.j}$			$\chi^2 = 0.34$	

TABLE XIV. CONTINGENCY TABLE FOR A NORMAL STAND COMPARED WITH A STAND WITH 5 TREES.

Normal Stand	52	159	175	129	54	16	585	$X_{i.}$
Interval 5	0	1	2	1	1	0	5	
	52	160	177	130	55	16	590	Total
				$X_{.j}$			$\chi^2 = 1.47$	

TABLE XV. CONTINGENCY TABLE FOR A NORMAL STAND COMPARED WITH A STAND WITH 1 TREE.

Normal Stand	52	159	175	129	54	16	585	$X_{i.}$
Interval 1	0	0	1	0	0	0	1	
	52	159	176	129	54	16	586	Total
				$X_{.j}$			$\chi^2 = 2.33$	

Table XVI shows that for small cutting, or tree, intervals the mean diameter is unstable. The mean diameter stablizes after cutting, or tree, interval 15.

TABLE XVI. MEAN DIAMETER OF STAND WITH DIFFERENT NUMBER OF TREES DISTRIBUTED PROPORTIONALLY TO A NORMAL STAND.

Tree Interval	1	5	10	15	20	30	50	585
Mean Diameter	6.50	7.30	6.30	6.77	6.70	6.57	6.58	6.5752

Statistically speaking, every cutting, or tree, interval results in the same distribution. If the cutting, or tree, interval is small, every time only the diameter classes with higher proportion of trees can be considered for cutting. If this situation continues for several cuttings, the diameter distribution will become uniform.

In this example if we want every diameter class to have a chance of being considered for cutting then the cutting interval must be no less than 15.

The above discussion is for a medium site (site index 140) Douglas-fir stand at age 40. Sessions (1979) showed that the diameter distribution of a fully stocked stand at other ages is also normally distributed. From the low chi-square values calculated from the above contingency tables we believe that the remaining trees for different cutting, or tree, interval at different ages are also normally distributed. This leads us to the result of 15 as the smallest tree interval which would result in reasonable solutions.

TABLE XVII. NUMBER OF THINNING ALTERNATIVES THAT CAN BE CONSIDERED FOR DIFFERENT TREE INTERVALS WHEN ROTATION IS 80 YEARS.

Tree Interval	Number of Alternatives	Compared with Tree Interval 15
15	39,785,661	1.0000
20	10,175,382	0.2558
25	3,655,206	0.0919
30	2,020,788	0.0508
40	472,878	0.0119
50	187,353	0.0047
100	13,392	0.0003

The number of thinning alternatives considered for different tree interval can be found by setting up a table like Table I (see Appendix V). If a stand is precommercially thinned to 400 trees, and commercially thinned and fertilized every ten years starting from age 30 to final harvest at age 80, for different tree intervals, the number of alternatives is summarized in Table XVII.

Choice of the largest tree interval is rather arbitrary. When a rough approximation of the result is wanted, a larger tree interval not only gives the desired result but also saves computation time. The size of tree interval really depends on the objective of the research. However, from Table XVII, 50 seems to be the largest tree interval that can be reasonably considered.

BASAL AREA INTERVAL STATE

The size of basal area interval is dependent on the size of tree interval chosen. When tree interval is large, only a few thinning alternatives will be categorized into the same neighborhood, i.e. the error resulting from comparing different alternatives in the same neighborhood will be smaller. Hence when tree interval is large, a small basal area interval will not increase the desired precision. When tree interval is small, many thinning alternatives will be categorized into the same neighborhood, so a reduced basal area interval will increase the desired precision. In general, if the same basal area interval is used, the smaller the tree interval used, the better the model will be, because more alternatives are considered. In this study 15 is chosen as tree interval, hence the discussion of basal area interval is based on a tree interval of 15.

A sensitivity analysis is used to see the effect of basal area interval. For fixed tree interval size, different basal area intervals are used, through dynamic programming, to find the best thinning regime and the corresponding soil expectation value.

TABLE XVIII. SOIL EXPECTATIONS AND COMPUTATION EFFICIENCY FOR DIFFERENT BASAL AREA INTERVALS COMPARED WITH BASAL AREA INTERVAL OF FOUR SQ. FT.

Basal Area Interval	Soil Expectation ($)	%	Execution Time (Sec.)	%
4	1780.17	100.00	89.43	100.00
10	1756.22	98.65	41.86	46.79
16	1760.20	98.88	28.47	31.83
20	1750.66	98.34	23.30	26.05
30	1689.66	94.92	18.62	20.82
40	1713.12	96.23	16.37	18.30

Table XVIII shows the soil expectation values for different basal area intervals when tree interval is 15, the relative soil expectation values compared with basal area interval four, execution time of using optimization technique, and the relative execution time compared with basal area interval four. Figure XV is a corresponding diagram of the results in Table XVIII. The management regimes for different basal area intervals are shown in Appendix VI.

An intuitive judgement is that small basal area would result in larger soil expectations because of fewer comparisons. But due to the artifact effect of dynamic programming, this is not the case. The artifact effect will be discussed in detail in the next section. From either Table XVIII or Figure XV we find the soil expectation values are stable in the range of four to 20 square feet of basal area. Hence four to 20 is considered as a reasonable range of basal area interval when tree interval is 15. The basal area interval used in this study is four square feet. Figure XV also shows that the execution time drops negative exponentially when basal area interval increases. Within certain precision, larger basal area interval saves a lot of computing time.

ARTIFACT EFFECTS

The neighborhood concept categorizes some alternatives with slightly different basal area and some with slightly different number of trees into the same node (state) and the alternative with the highest cumulative present net worth is chosen to represent the state. If the alternative have the same number of trees and same basal area, as in a

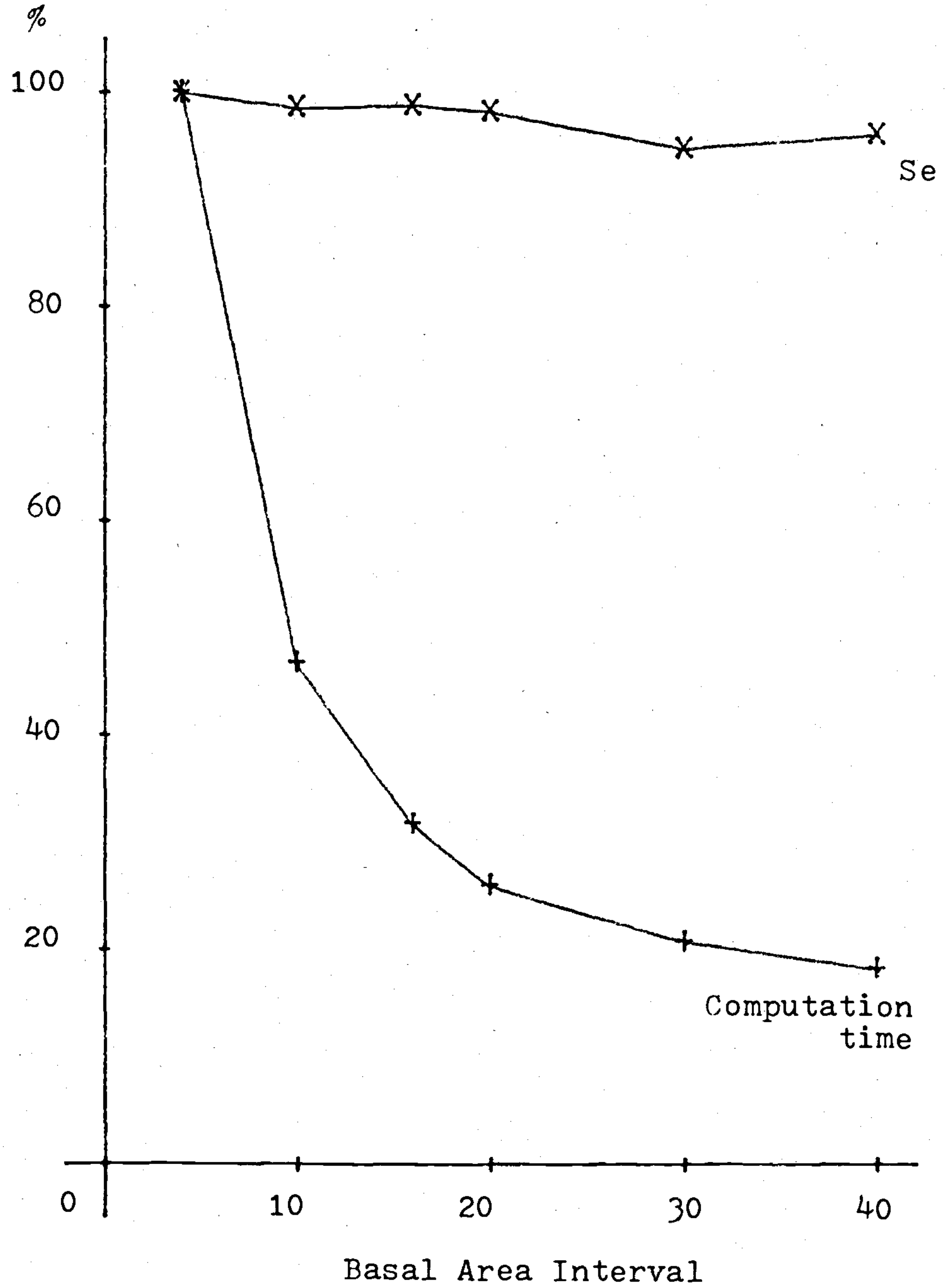


Figure XV. Relative soil expectations and computation efficiency for different basal area intervals compared with basal area of four square feet.

traditional discrete state dynamic programming problem, then there will not be any problem. The alternative chosen to represent the node is the best alternative. When basal area and number of trees are not the same, the alternatives with slightly lower cumulative present net worth might have a higher value contribution in subsequent stages, i.e. the thinning regime chosen may not be the true optimal solution. The difference between the present net worths from the strategy chosen and the true solution will be very small.

From Table XVIII we find that when basal area interval increases from ten to 16 square feet and from 30 to 40, the soil expectation increases. This is an artifact effect. When ten square feet is used, alternatives with basal areas 71 and 74 square feet are categorized to a node of 70 square feet (covers from 65 to 75). Suppose the alternative with 71 square feet would contribute more to present net worth in later stages than the alternative with 74 square feet, but has less cumulative present net worth in the current stage. Then the 74 square foot alternative will be chosen to represent the node of 70 square feet even though it will have less future value. If 16 square feet is used as the interval size, the alternative with 71 square feet will be categorized to the node of 64 square feet (covers from 56 to 72) and the alternative with 74 square feet will be categorized to node of 80 square feet (covers from 72 to 88). In this case the 71 square foot alternative may be chosen to represent the node of 64 square feet. With higher future values, the 71 square foot alternative will override the 74 square foot alternative which represents the node of 80 square feet in the future. This explains why a smaller basal area interval may not have a higher soil expectation.

Comparing the management regimes for basal area of ten square feet and 16 square feet in Appendix VI we find that at age 30 the same state with basal area 82.0 was chosen in both cases. At age 40 the state with basal area 85.2 was chosen in the case of ten square foot interval and 73.0 was chosen in the case of 16 square foot interval. When the ten square foot interval is used, 85.2 and 73.0 will be categorized to the node of 90 (covers from 85 to 95) and 70 (covers from 65 to 75) respectively. But when the 16 square foot interval is used both 85.2 qnd 73.0 will be categorized to the node of 80 (covers from 72 to 88). This may happen in every categorization and the consequent artifact effect follows.

VI. COMPUTATIONS OF SILVICULTURAL APPLICATIONS USING THE DOPT MODEL

In this Chapter best rotation and stocking levels under different silvicultural considerations are studied. The interaction between silvicultural practices can be derived by comparing the information from different silvicultural considerations. Other impacts of silvicultural activities incorporated in the analysis are also discussed. Throughout the discussion, the discount rate used is three percent to represent the real rate and zero for the inflation rate, i.e. no ballooning of soil expectation due to inflation, current real dollars are considered. The tree interval used is 15 and four square feet for basal area interval. Regeneration cost is $200 per acre and $80 per acre for precommercial thinning at age ten is used. Site index will be 140 unless otherwise noted. The cost, price and other data were discussed in Chapter IV.

FINAL HARVEST ONLY

When only final harvest is considered as a silvicultural practice, the rotation is found by looking at maximal soil expectation value. Table XIX shows the statistics of a Douglas-fir stand of site index 140 at final harvest. The soil expectation culminates at age 70 and maximum mean annual increment (MAI) culminates ten years earlier. The numbers shown in Table XIX are calculated from DFIT for a

TABLE XIX. SUMMARY OF FINAL HARVEST, MAI AND PRESENT WORTH DATA FOR A STAND CONSIDERING FINAL HARVEST ONLY, SITE 140.

Rotation Age (years)	Harvest Diameter (inches)	Total Number of Trees	Harvest Basal area (sq. ft.)	Harvest Volume (cu. ft.)	MAI (cu. ft.)	Present Net Worth ($/acre)	Soil Expectation ($/acre)
30	6.4	894	125.5	3156.2	105.21	-628.24	-1068.41
40	8.0	536	162.6	4981.3	124.53	-216.64	- 312.41
50	9.7	381	184.4	6520.9	130.42	98.82	128.02
60	11.4	296	202.2	7840.8	130.68	309.17	372.37
70	12.9	244	216.6	8986.6	128.38	385.60	441.34
80	14.3	208	228.9	9993.1	124.91	397.11	438.30

fully stocked natural origin stand. This result tells us that for an undisturbed forest stand, when no special silvicultural practices are considered, the stand should be clear cut at age 70 and regenerated thereafter to achieve the greatest economic income. This result is used as a basis for calculating the economic impacts of different silvicultural activities.

PRECOMMERCIAL THINNING AND FINAL HARVEST

Precommercial thinning removes small and unsalable trees and leaves promising trees which result in more rapid juvenile growth. Table XX is the result of a stand precommercially thinned to 400 trees at age ten and final harvested at different ages. As promised by precommercial thinning, juvenile growth is rapid. Compared with a natural stand (Table XIX), at age 30 the quadratic mean diameter is 6.4 inches without precommercial thinning and 8.5 with precommercial thinning. At age 40 it is 8.0 inches compared with 9.9. After age 40 the precommercially thinned stand grows much slower than the natural stand does. The reason is that in DFIT it is assumed that the trees left after precommercial thinning will not die except those that could be captured in the next commercial thinning. Here the number of trees left after precommercial thinning is 400 and no commercial thinning are considered. When 400 trees crowd together, the growth is retarded. At

TABLE XX. SUMMARY OF FINAL HARVEST, MAI AND PRESENT WORTH DATA FOR A STAND CONSIDERING PRECOMMERCIAL THINNING AND FINAL HARVEST, SITE 140.

Rotation Age (years)	Harvest Diameter (inches)	Total Number of Trees	Harvest Basal Area (sq. ft.)	Harvest Volume (cu. ft.)	MAI (cu. ft.)	Present Net Worth ($/acre)	Soil Expectation ($/acre)
30	8.5	400	156.2	4041.8	134.73	-192.63	-327.59
40	9.9	400	213.1	6505.6	162.64	206.92	298.40
50	10.6	400	243.3	8570.5	171.41	361.40	468.20
60	10.9	400	261.3	10120.6	168.68	322.46	388.38

age 50 a natural stand has 381 trees (Table XIX) but the precommercially thinned stand still has 400 trees. The best rotation for a precommercially thinned (only) stand is 50 which has the highest soil expectation $468.20. The rotation here is shorter than the rotation of the natural stand because the retarded growth due to overstocking makes waiting for the higher price of larger trees unprofitable. The soil expectation value for the precommercially thinned stand is slightly larger than that of a natural undisturbed stand (Tables XIX and XX). Without other silvicultural considerations precommercial thinning only contributes 468.20-441.34=$26.86 to soil expectation. Yet precommercial thinning is a good silvicultural practice for producing small size logs. The maximum MAI is 171.41 cubic feet for rotation 50 compared with 130.68 for rotation 60 of the natural stand.

Due to the limitations of the growth model and to make the results comparable with the results from other silvicultural considerations, different intensities of precommercial thinnings are not considered in this section. If fewer than 400 trees are left after precommercial thinning, the time for the first possible commercial thinning will be delayed. When only precommercial thinning is considered we believe heavier precommercial thinning will make precommercial thinning more profitable.

COMMERCIAL THINNING AND FINAL HARVEST

The merits of commercial thinning are capture of those trees which will die in the following years because of suppression by stronger competitors and the subsequent faster diameter growth due to the greater growing space and lesser competition created. From the DOPT program the best thinning regime for different rotations and the stand conditions can be obtained in one computer run. Commercial thinning is considered every ten years starting at age 30. Table XXI shows the soil expectations and final stand conditions for different rotations. Table XXII shows the corresponding thinning regime and stand conditions at different ages for the best rotation, which is 80 in this case. In order to accelerate diameter growth, many trees are removed in the first commercial thinning. A constraint on maximum removals in the first commercial thinning of 50 percent of the merchantable stocking level is set due to consideration of blow-down, sunscald and thinning shock, etc. which are not included in the growth model. In this example the first commercial thinning did not remove more than 50 percent of the stocking. At every stage "thin" is better than "do not thin". This is because when "do not thin" is taken, the large amounts of mortality will be lost. Hence, mortality capture makes a heavy contribution to the desirability of thinning in this example. At age 50

TABLE XXI. SUMMARY OF FINAL HARVEST, MAI AND PRESENT WORTH DATA FOR A STAND CONSIDERING COMMERCIAL THINNING AND FINAL HARVEST, SITE 140.

Rotation Age (years)	Harvest Diameter (inches)	Harvest Basal Area (sq. ft.)	Harvest Volume (cu. ft.)	MAI (cu. ft.)	Present Net Worth ($/acre)	Soil Expectation ($/acre)
40	8.80	122.09	4494.00	142.61	-270.36	-389.88
50	9.73	184.80	7238.81	144.78	14.06	18.22
60	11.38	202.18	8538.16	142.30	247.66	298.29
70	18.21	112.97	5322.97	160.70	394.09	451.06
80	20.97	123.60	5984.46	159.10	468.84	517.47
90	23.82	81.58	4112.19	152.19	465.70	500.72

TABLE XXII. THINNING SCHEDULE AND STAND CONDITIONS WHEN COMMERCIAL THINNING AND FINAL HARVEST ARE PERMITTED, SITE 140.

Stand Age (years)	Mean Diameter (inches)	Mer-chantable Trees	Nonmer-chantable Trees	Basal Area (sq. ft.)	Volume Left (cu. ft.)	Volume Cut (cu. ft.)	Cumulative PNW ($/acre)
30	6.4	360	334	80.8	2030.4	1125.8	-367.2
40	8.7	150	67	62.3	1910.3	2674.5	-374.9
50	12.1	105	23	83.7	2958.9	997.9	-280.2
60	15.2	75	11	95.0	3687.0	1128.0	-209.1
70	18.2	60	6	108.5	4505.8	817.1	-153.4
80	21.0	0	4	0.0	0.0	5984.5	468.6

Total Harvest= 12727.8 cu. ft.

MAI= 159.10 cu. ft.

forty-five trees were removed, at age 60 thirty trees were removed and at age 70 fifteen trees were removed (Table XXII). Compared with a normal stand we find that in the current stand 36 merchantable trees died between ages 40-50, 21 trees died between ages 50-60, and 12 died between ages 60-70. So we cut nine live merchantable trees (=45-36) at age 50, 9 (=30-21) at age 60, and 3 (=15-12) at age 70, which are the fewest trees we have to cut in each thinning, to capture this merchantable mortality. Our model structure is not explicit as to whether this mortality capture is comprised of anticipated mortality or harvest of dead trees but we would assume that as a regime progresses, the capture would be largely anticipated mortality.

An increased price premium is not assumed for trees larger than 22 inches. Hence a thinning regime ending with a mean diameter around 22 is usually chosen as the best regime under this assumption. In this example 23.82 inches is the mean diameter of the final stand of present net worth rotation and 20.97 for soil expectation rotation. If a price premium limit is not included, the rotation could be longer and the final mean diameter could be larger. By the same reasoning, if price premium is limited to a smaller diameter, the rotation will be shorter. Compared with an unthinned stand, the commercially thinned stand has faster diameter growth and a larger amount of total volume harvested. The rotation is lengthened by ten years, from 70 for the unthinned stand to 80 for the thinned stand. Thinning produces a longer rotation because the marginal value growth percent criterion can be met for longer in the thinned stand.

Another point that should be mentioned is that at each commercial thinning only merchantable trees are considered for cutting, the nonmerchantable trees are left intact. The fourth column of Table XXII shows the number of live nonmerchantable trees in the stand at each age.

FERTILIZATION AND FINAL HARVEST

The optimal rotation with fertilization only is found in a rather straightforward manner. At each ten-year interval, three levels of fertilization are considered. Every fertilization combination was calculated by using a hand calculator. The strategy which results in the highest present net worth and the corresponding soil expectation for a given rotation are recorded. The rotation which has the highest soil expectation is the best rotation with fertilization. Table XXIII shows the results for site 140 with zero, 200 and 400 pounds of nitrogen fertilizer considered for application at ten-year intervals beginning at age 30. The best rotation occurs at age 70, and the strategy is to fertilize 400 pounds at age 30, 200 pounds at ages 40, 50 and 60 and clear cut the stand at age 70. The total volume gained by fertilization is 10740.4-8986.6=

TABLE XXIII. SUMMARY OF FINAL HARVEST, MAI AND PRESENT WORTH DATA FOR A STAND CONSIDERING FERTILIZATION AND FINAL HARVEST, SITE 140.

Rotation Age (years)	Harvest Diameter (inches)	Total Number of Trees	Harvest Basal Area (sq. ft.)	Harvest Volume (cu. ft.)	MAI (cu. ft.)	Present Net Worth ($/acre)	Soil Expectation ($/acre)
30	6.4	894	125.5	3156.2	105.21	-628.24	-1068.41
40	8.4	536	178.5	5469.2	136.75	-150.49	- 217.02
50	10.4	381	209.7	7398.5	147.97	250.14	324.06
60	12.3	296	236.2	9158.7	152.65	492.56	593.26
70	14.1	244	258.9	10740.4	153.43	603.24	690.44
80	15.5	208	268.0	11697.9	146.22	571.22	630.47

The best regime is to fertilize 400 pounds at age 30, 200 pounds at ages 40, 50, and 60, and clear cut the stand at age 70.

1753.4 cubic feet or about 22 cubic feet per year (Tables XIX and XXIII) under the soil expectation criterion. The effect of fertilization delayed MAI culmination ten years, i.e. from 130.68 at rotation age 60 for an undisturbed stand to 153.43 at rotation age 70 for a fertilized stand. Fertilization keeps the growth at a higher rate for a longer period.

PRECOMMERCIAL, COMMERCIAL THINNING AND FINAL HARVEST

The heavy thinning in the first commercial thinning of a natural stand and its negative contribution to the present net worth (Table XXII, $-367.2 is less than regeneration cost of $-200.0) suggest that precommercial thinning would be beneficial. Table XXIV gives the final stand conditions and soil expectations of different rotations which are precommercially thinned to 400 trees at age ten and commercially thinned every ten years after age 30. The soil expectations are much higher than those in Tables XIX, XX and XXI. Precommercial thinning accelerates juvenile diameter growth and commercial thinnings keep the subsequent diameter growth at a high rate. At age 30 compared with a stand considering only precommercial thinning, both stands have mean diameter of 8.5 inches which is higher than a stand without precommercial thinning. After age 30, diameter growth is much faster than a natural stand (Table XIX), and a precommercially thinned stand without

TABLE XXIV. SUMMARY OF FINAL HARVEST, MAI AND PRESENT WORTH FOR A STAND CONSIDERING PRECOMMERCIAL, COMMERCIAL THINNING AND FINAL HARVEST, SITE 140.

Rotation Age (years)	Harvest Diameter (inches)	Harvest Basal Area (sq. ft.)	Harvest Volume (cu. ft.)	MAI (cu. ft.)	Present Net Worth ($/acre)	Soil Expectation ($/acre)
40	11.42	149.50	4564.58	162.11	189.09	272.68
50	12.91	177.18	6240.13	175.50	558.70	723.81
60	16.83	162.15	6281.29	174.72	781.89	941.74
70	18.34	137.58	5705.00	172.57	931.74	1066.42
80	20.93	107.49	4690.30	166.84	1004.57	1108.76
90	21.91	157.05	7130.30	168.53	1024.92	1101.98
100	24.23	48.02	2250.63	156.36	1004.22	1059.34

TABLE XXV. THINNING SCHEDULE AND STAND CONDITIONS WHEN PRECOMMERCIAL, COMMERCIAL THINNING AND FINAL HARVEST ARE PERMITTED, SITE 140.

Stand Age (years)	Mean Diameter (inches)	Merchantable Trees	Basal Area (sq. ft.)	Volume Left (cu. ft.)	Volume Cut (cu. ft.)	Cumulative PNW ($/acre)
30	8.5	240	93.7	2425.1	1616.7	-317.2
40	11.1	195	130.4	3979.9	918.4	-160.8
50	12.9	75	68.1	2400.0	3840.1	248.6
60	16.0	75	104.1	4033.5	0.0	248.6
70	18.3	45	82.6	3423.0	2282.0	506.7
80	20.9	0	0.0	0.0	4690.3	1004.6

subsequent commercial thinnings. One could expect the rotation of the stand with precommercial thinning and commercial thinnings to be no shorter than a stand with commercial thinnings only. Because one more thinning (precommercial thinning) should keep the marginal growth percent higher. Table XXIV shows that the maximum present net worth rotation is 90 years and maximum soil expectation rotation is 80 years compared with 80 years and 80 years respectively for a stand considering commercial thinning operations only (Table XXI)-- an answer as was expected. The soil expectation rotation is ten years earlier than the present net worth rotation. The reason is because the cost of waiting out the rotation is higher than the price premium and growth in that extra ten years (Duerr, 1960).

Table XXV is the stand condition at different ages for the soil expectation rotation. At age 60 "do not thin" is the best alternative. In the precommercially thinned option of the DFIT model, all trees left after precommercial thinning will survive and hence there are the same number of trees at ages 50 and 60. "Do not thin" implies no cost is incurred nor is income earned, so the cumulative present net worth is the same for ages 50 and 60. The MAI increases from 159.10, without precommercial thinning, to 166.84, with precommercial thinning (Tables XXII and XXV).

PRECOMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST WITHOUT COMMERCIAL THINNING

When precommercial thinning and fertilization are considered jointly, first the stand is precommercially thinned to 400 trees at age ten. Then starting from age 30, at every ten-year interval, three different levels of fertilization are considered. Every fertilization combination is calculated by using a hand calculator. The strategy which results in the highest present net worth and the corresponding soil expectation are recorded. The rotation which has the highest soil expectation is the best rotation. In the site index 140 Douglas-fir stand it occurs at age 50 which is also the age that MAI culminates. The corresponding best alternative is to leave 400 trees at age ten, fertilize 200 pounds of nitrogen at ages 30 and 40, and clear cut the stand at age 50. The rotation is relatively short. The reason is the same as for a precommercially thinned stand without fertilization-- the retarded growth due to too many trees crowding together makes the waiting unprofitable. Here both soil expectation and total volume harvested are higher than a precommercially thinned stand without considering fertilization (Table XX). The extra contribution, 667.31-468.20=$199.11, is from fertilization.

TABLE XXVI. SUMMARY OF FINAL HARVEST, MAI AND PRESENT WORTH DATA FOR A STAND CONSIDERING PRECOMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST, SITE 140.

Rotation Age (years)	Mean Diameter (inches)	Merchantable Trees	Harvest Basal Area (sq. ft.)	Harvest Volume (cu. ft.)	MAI (cu. ft.)	Present Net Worth ($/acre)	Soil Expectation ($/acre)
30	8.5	400	156.2	4041.8	134.73	-192.63	-327.59
40	10.3	400	230.2	7028.6	175.72	327.88	472.83
50	11.1	400	267.3	9414.2	188.28	515.09	667.31
60	11.4	400	282.1	10928.8	182.15	460.09	554.15

The best regime is to fertilize 200 pounds of nitrogen at ages 30 and 40 and clear cut the stand at age 50.

COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST

Table XXVII shows the final stand conditions for different rotations considering commercial thinning and fertilization. The best rotation is 70 years. The stand condition at different ages for the best rotation is shown in Table XXVIII. The fast diameter growth due to fertilization shortens the rotation by ten years compared with a commercially thinned stand without fertilization (Table XXI). For a fertilized stand we would expect that it would have a higher marginal value growth percent than a stand not fertilized. Hence a fertilized stand should have a longer rotation. But here we have an opposite result. This is because the price function is flattened for diameters larger than 22 inches. When a stand is fertilized, it grows faster and maximal present net worth rotation occurs when mean diameter reaches 22 inches. Table XXVII shows that the maximum soil expectation rotation is 70 which has a mean diameter of 22.17 inches. And the maximum present net worth rotation is 80 with a mean diameter of 24.89 inches-- already exceeding the 22 inches limit.

TABLE XXVII. SUMMARY OF FINAL HARVEST, MAI AND PRESENT WORTH DATA FOR A STAND CONSIDERING COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST, SITE 140.

Rotation Age (years)	Harvest Diameter (inches)	Harvest Basal Area (sq. ft.)	Harvest Volume (cu. ft.)	MAI (cu. ft.)	Present Net Worth ($/acre)	Soil Expectation ($/acre)
40	8.32	176.66	6203.42	155.09	-146.24	-210.89
50	13.11	161.15	6515.99	182.83	263.51	341.38
60	17.15	153.94	6762.21	194.46	670.26	807.29
70	22.17	100.47	4795.09	195.17	942.83	1079.12
80	24.89	87.08	4263.44	186.84	936.29	1033.47

TABLE XXVIII. THINNING SCHEDULE AND STAND CONDITIONS WHEN COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST ARE PERMITTED, SITE 140.

Stand Age (years)	Amount of Fertilization (pounds)	Mean Diameter (inches)	Mer-chantable Trees	Basal Area (sq. ft.)	Volume Left (cu. ft.)	Volume Cut (cu. ft.)	Cumulative PNW ($/acre)
30	200	6.4	345	77.4	1945.8	1210.4	-373.7
40	200	9.3	165	78.3	2402.7	2642.8	-335.3
50	400	13.4	120	116.8	4125.5	1052.4	-227.0
60	400	17.2	45	72.2	2801.1	3961.1	238.5
70	-	22.2	0	0.0	0.0	4795.1	942.8

Total Harvest= 13661.8 cu. ft.

MAI= 195.17 cu. ft.

PRECOMMERCIAL, COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST

In this section all four silvicultural practices are considered in stand optimization. The effect of precommercial thinning, commercial thinning, fertilization and final harvest for site indices 110, 140, 170 and 200 are shown in Tables XXIX to XXXVI. Table XXXI is the final stand conditions for different rotations for site index 140. Best soil expectation rotation is 70 years. The corresponding management regime and stand conditions are shown in Table XXXII. Compared with Tables XXIV and XXV, which are the results of considering precommercial thinning and commercial thinning, the rotation is shortened by ten years due to the effect of fertilization. In the section "Commercial Thinning, Fertilization and Final Harvest" it was discussed that fertilization shortened rotation also by ten years when precommercial thinning is not included.

Table XXX tells the best management regime for a stand with site index 110: Leave 340 trees in precommercial thinning at age ten; cut 130 trees at age 40, 150 at age 50, skip one period and clear cut at age 70; fertilize 400 pounds of nitrogen at ages 40, 50 and 60. Tables XXXII, XXXIV and XXXVI can be interpreted in the same way to get the best management regime for site indices 140, 170 and 200. The optimal thinning regime of site 200 might not be

TABLE XXIX. SUMMARY OF FINAL STAND CONDITIONS FOR DIFFERENT ROTATIONS FOR SITE 110 WITH PRECOMMERCIAL, COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST.

Rotation Age (years)	Harvest Diameter (inches)	Harvest Basal Area (sq. ft.)	Harvest Volume (cu. ft.)	MAI (cu. ft.)	Present Net Worth ($/acre)	Soil Expectation ($/acre)
50	11.93	197.98	5760.81	137.11	291.08	377.10
60	17.23	121.45	3887.11	148.43	584.22	703.65
70	21.90	156.90	5375.11	153.79	872.11	998.18
80	22.38	122.91	4431.17	148.71	870.96	961.30

TABLE XXX. MANAGEMENT REGIME AND STAND CONDITIONS FOR SITE 110 WITH PRECOMMERCIAL, COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST.

Stand Age (years)	Amount of Fertilization (pounds)	Mean Diameter (inches)	Mer-chantable Trees	Basal Area (sq. ft.)	Volume Left (cu. ft.)	Volume Cut (cu. ft.)	Cumulative PNW ($/acre)
40	400	9.2	210	96.9	2704.2	1674.1	-251.8
50	400	12.5	60	51.1	1486.5	3716.2	90.3
60	400	17.9	60	104.6	3349.2	0.0	70.9
70	-	21.9	0	0.0	0.0	5375.1	872.1

Precommercially thin to 340 trees at age ten.

TABLE XXXI. SUMMARY OF FINAL STAND CONDITIONS FOR DIFFERENT ROTATIONS FOR SITE 140 WITH PRECOMMERCIAL, COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST.

Rotation Age (years)	Harvest Diameter (inches)	Harvest Basal Area (sq. ft.)	Harvest Volume (cu. ft.)	MAI (cu. ft.)	Present Net Worth ($/acre)	Soil Expectation ($/acre)
40	12.29	173.07	5284.15	185.15	376.05	542.29
50	16.92	140.51	4948.57	201.80	936.49	1213.24
60	20.86	178.08	6898.71	204.88	1379.41	1661.40
70	22.62	167.47	6944.26	213.83	1555.34	1780.17
80	25.72	54.10	2360.71	194.91	1536.94	1696.36

TABLE XXXII. MANAGEMENT REGIME AND STAND CONDITIONS FOR SITE 140 WITH PRECOMMERCIAL, COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST.

Stand Age (years)	Amount of Fertilization (pounds)	Mean Diameter (inches)	Merchantable Trees	Basal Area (sq. ft.)	Volume Left (cu. ft.)	Volume Cut (cu. ft.)	Cumulative PNW ($/acre)
30	400	8.5	210	82.0	2121.9	1919.8	-309.4
40	400	12.3	105	86.5	2642.1	2642.1	- 34.4
50	400	16.4	105	154.2	5431.9	0.0	- 60.5
60	400	19.1	60	119.1	4615.4	3461.6	494.8
70	-	22.6	0	0.0	0.0	6944.3	1555.3

Precommercially thin to 400 trees at age ten.

TABLE XXXIII. SUMMARY OF FINAL STAND CONDITIONS FOR DIFFERENT ROTATIONS FOR SITE 170 WITH PRECOMMERCIAL, COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST.

Rotation Age (years)	Harvest Diameter (inches)	Harvest Basal Area (sq. ft.)	Harvest Volume (cu. ft.)	MAI (cu. ft.)	Present Net Worth ($/acre)	Soil Expectation ($/acre)
40	12.81	214.81	7648.01	214.72	1045.27	1507.36
50	17.14	144.28	5925.62	232.93	1680.46	2177.06
60	21.71	154.18	6965.14	236.91	2197.13	2646.29
70	23.18	175.83	8502.30	245.27	2338.23	2676.24
80	26.61	57.94	2948.62	224.01	2325.44	2566.65

TABLE XXXIV. MANAGEMENT REGIME AND STAND CONDITIONS FOR SITE 170 WITH PRECOMMERCIAL, COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST.

Stand Age (years)	Amount of Fertilization (pounds)	Mean Diameter (inches)	Mer-chantable Trees	Basal Area (sq. ft.)	Volume Left (cu. ft.)	Volume Cut (cu. ft.)	Cumulative PNW ($/acre)
30	200	10.3	240	139.9	4104.9	940.7	-130.0
40	200	12.8	90	80.6	2868.0	4780.0	547.0
50	400	17.1	90	143.0	5875.2	0.0	533.2
60	400	20.0	60	130.4	5892.1	2946.1	1030.5
70	-	23.2	0	0.0	0.0	8502.3	2338.2

Precommercially thin to 295 trees at age ten.

TABLE XXXV. SUMMARY OF FINAL STAND CONDITIONS FOR DIFFERENT ROTATIONS FOR SITE 200 WITH PRECOMMERCIAL, COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST.

Rotation Age (years)	Harvest Diameter (inches)	Harvest Basal Area (sq. ft.)	Harvest Volume (cu. ft.)	MAI (cu. ft.)	Present Net Worth ($/acre)	Soil Expectation ($/acre)
40	14.49	251.89	10199.62	254.99	1785.22	2574.43
50	18.93	146.55	6845.75	171.36	2572.88	3333.21
60	21.95	197.11	10127.41	280.83	3123.01	3761.45
70	26.35	113.57	6245.97	272.07	3198.54	3660.91
80	29.62	143.55	8307.56	263.83	3197.83	3529.53

TABLE XXXVI. MANAGEMENT REGIME AND STAND CONDITIONS FOR SITE 200 WITH PRECOMMERCIAL, COMMERCIAL THINNING, FERTILIZATION AND FINAL HARVEST.

Stand Age (years)	Amount of Fertilization (pounds)	Mean Diameter (inches)	Mer-chantable Trees	Basal Area (sq. ft.)	Volume Left (cu. ft.)	Volume Cut (cu. ft.)	Cumulative PNW ($/acre)
30	200	12.5	220	186.9	6155.8	0.0	- 259.5
40	200	14.5	75	85.9	3477.1	6722.5	1030.6
50	200	18.9	75	146.6	6845.7	0.0	1016.8
60	200	22.0	0	0.0	0.0	10127.4	3123.0

Precommercially thin to 220 trees at age ten.

acceptable to a silviculturist because there is a heavy precommercial thinning at age ten (down to 220 trees), then two thirds of the stocking is harvested at age 40, and the stand is clear cut at age 60. Though trees grow faster on site 200, thinning shock may still exist. One could use some additional constraints to restrict the amount of thinning at age 40. The first commercial thinning for site 110 is delayed to age 40 because at age 30 the mean diameter of the merchantable trees has not reached the minimum merchantable size of eight inches. We could consider commercial thinning as soon as the mean diameter of the merchantable trees reaches merchantable size. But DOPT is constructed to consider every silvicultural activity at an age of multiple ten.

From the information in the above tables we can conclude: First, higher site has shorter rotation because the trees grow faster and reach 22 inches sooner. Second, higher site needs less fertilizer as most people expected. The reason is that higher site already has relatively more nutrients in the soil, the nature of diminishing rate of return of production factors makes a larger amount of fertilizer less valuable. Third, for a stand inherited with "reasonable" fertility, e.g. site 170, fertilizer has less effect in the fast growing young stage and a more prominent effect in the slow growing older stages. Fourth, on higher site more trees are removed in precommercial thinning and fewer trees are cut in the first commercial thinning. Site index 110 does not follow this pattern because the structure of the DFIT growth model restricts the first commercial thinning to occur later.

PLANTATION

DFIT represents a plantation via the device of precommercial thinning at age two. After precommercial thinning all procedures are the same as usual. Tables XXXVII and XXXVIII show the results of the plantation option on site 140.

Table XXXVIII shows that the best management regime is to plant 385 trees, fertilize 400 pounds of nitrogen every ten years from age 30, and cut 175, 135, 0, 45 and 30 trees in sequence. The rotation is 70 years. The advantage of the plantation assumption over the natural regeneration assumption in soil expectation is 1874.22-1780.17=$94.05 (Tables XXXI and XXXVIII), according to the model which assumes equal establishment cost. If the natural regeneration assumption was enough cheaper then it could actually have an advantage over the plantation under our assumptions.

NON-NATURAL STAND

For a non-natural stand at any age, DOPT first calculates the merchantable and non-merchantable parts by comparing with the natural stand. Then the growth model of

TABLE XXXVII. SUMMARY OF FINAL STAND CONDITIONS FOR DIFFERENT ROTATIONS FOR SITE 140 PLANTATION.

Rotation Age (years)	Harvest Diameter (inches)	Harvest Basal Area (sq. ft.)	Harvest Volume (cu. ft.)	MAI (cu. ft.)	Present Net Worth ($/acre)	Soil Expectation ($/acre)
40	12.54	180.22	5502.29	182.16	469.99	677.76
50	17.69	128.07	4510.43	196.63	1038.07	1344.84
60	21.92	157.24	6091.45	205.25	1517.93	1828.25
70	26.39	113.98	4726.09	204.49	1637.51	1874.22
80	28.52	66.54	2903.35	192.59	1611.05	1778.16

TABLE XXXVIII. MANAGEMENT REGIME AND STAND CONDITIONS FOR SITE 140 PLANTATION.

Stand Age (years)	Amount of Fertilization (pounds)	Mean Diameter (inches)	Mer-chantable Trees	Basal Area (sq. ft.)	Volume Left (cu. ft.)	Volume Cut (cu. ft.)	Cumulative PNW ($/acre)
30	400	8.9	210	90.5	2341.5	1784.0	- 287.2
40	400	12.5	75	64.4	1965.1	3537.2	147.7
50	400	17.7	75	128.1	4510.4	0.0	121.7
60	400	21.2	30	73.4	2844.9	4267.3	931.2
70	-	26.4	0	0.0	0.0	4726.1	1637.5

Plant 385 trees.

DFIT is used to simulate the stand to the next age of multiple ten (no less than 30). Then commercial thinning, fertilization and final harvest are considered as usual. Tables XXXIX and XL are the results of a slightly understocked stand with 80 percent normal stocking of both number of trees and basal area, at age 34 with site index 140.

The first commercial thinning and fertilization are considered at age 40 because the initial age of the stand is 34 which is larger than 30. From Tables XXVI, XXVIII, XXXII and XL we find that with relatively more trees in the stand, the larger amount of fertilization (400 pounds/acre) does not have the expected effect because of the limited growing space. Hence the lesser amount of fertilizer is applied when there are many trees in the stand. Rotation is longer due to the increased length of time required to reach 22 inches diameter.

INTERACTIONS BETWEEN SILVICULTURAL PRACTICES

Table XLI summarizes the soil expectations for different silvicultural activity combinations for site 140. Where

PCT = precommercial thinning.
CT = commercial thinning.
FERT = fertilization.
Se = soil expectation.

TABLE XXXIX. SUMMARY OF FINAL STAND CONDITIONS FOR DIFFERENT ROTATIONS FOR SITE 140 OF A NON-NORMAL STAND WITH 80 PERCENT OF NORMAL BASAL AREA AND NUMBER OF TREES AT AGE 34.

Rotation Age (years)	Harvest Diameter (inches)	Harvest Basal Area (sq. ft.)	Harvest Volume (cu. ft.)	MAI (cu. ft.)	Present Net Worth ($/acre)	Soil Expectation ($/acre)
50	10.66	183.28	7252.76	145.06	153.67	199.08
60	12.82	212.21	8968.86	149.48	442.13	532.51
70	20.31	112.47	5323.50	166.08	679.25	777.44
80	22.62	107.81	5235.17	162.07	757.52	825.06
90	22.20	106.24	5307.31	154.68	767.22	824.91
100	24.71	44.67	2296.02	143.86	744.07	784.92

TABLE XL. MANAGEMENT REGIME AND STAND CONDITIONS FOR SITE 140 OF A NON-NORMAL STAND WITH 80 PERCENT OF NORMAL BASAL AREA AND NUMBER OF TREES AT AGE 34.

Stand Age (years)	Amount of Fertilization (pounds)	Mean Diameter (inches)	Merchantable Trees	Basal Area (sq. ft.)	Volume Left (cu. ft.)	Volume Cut (cu. ft.)	Cumulative PNW ($/acre)
40	200	8.3	300	111.5	3438.7	996.6	-224.5
50	400	11.1	105	70.3	2480.3	3735.4	- 92.1
60	400	15.2	60	75.8	2940.2	1923.6	57.0
70	-	19.6	45	94.5	3923.6	1075.0	151.2
80	-	22.6	0	0.0	0.0	5235.2	747.5

TABLE XLI. SUMMARY OF SOIL EXPECTATIONS FOR DIFFERENT SILVICULTURAL ACTIVITY COMBINATIONS, SITE 140.

Number	Activities Considered	Maximum Se
1	Final Harvest Only	441.34
2	PCT	468.20
3	CT	517.47
4	FERT	690.44
5	PCT and CT	1108.76
6	PCT and FERT	667.31
7	CT and FERT	1079.12
8	PCT, CT and FERT	1780.17

Let $C(A)$ = the contribution of activity A on soil expectation.

$C(A \cup B)$ = the total contribution of activities A and B.

$C(A \mid B)$ = the contribution of activity A when B is included.

$C(A \cup B \mid C)$ = the total contribution of activities A and B when activity C is included.

$C(A \cap B)$ = the interactive contribution of activities A and B.

From Table XLI we can calculate the independent contribution of the three silvicultural activities to soil expectation:

C(PCT) = (2)-(1) = 468.20-441.34 = \$26.86

C(CT) = (3)-(1) = 517.47-441.34 = \$76.13

C(FERT) = (4)-(1) = 690.44-441.34 = \$249.10

Among C(PCT), C(CT) and C(FERT), C(FERT) has the highest value, hence if only one additional silvicultural practice could be considered, fertilization should be considered first. Intuitively we would expect commercial thinning to have the highest contribution on soil expectation. But due to the high harvesting cost, the relatively cheap practice of fertilization takes the first place. From Tables XX, XXII and XXIII we find the maximum MAI of PCT is 171.41 cubic feet, 159.10 for CT and 153.43 for FERT. The trees harvested from a precommercially thinned stand have relatively small diameter and because of the high harvesting cost, fertilization has the highest independent contribution to soil expectation.

From Table XLI the contribution of one activity given that another activity is included can be calculated:

$C(PCT \mid CT)^{1/}$ = (5)-(3) = 1108.76-517.47 = 591.29

1/ Logically it is not reasonable to talk about the contribution of PCT given CT because PCT occurs earlier than CT. Here it is just a notation for further calculation of interactive effect.

$C(PCT \mid FERT)$[2/] = (6)-(4) = 667.31-690.44 = -23.13

$C(CT \mid PCT)$ = (5)-(2) = 1108.76-468.20 = 640.56

$C(CT \mid FERT)$[2/] = (7)-(4) = 1079.12-690.44 = 388.68

$C(FERT \mid PCT)$ = (6)-(2) = 667.32-468.20 = 199.11

$C(FERT \mid CT)$[2/] = (7)-(3) = 1079.12-517.47 = 561.65

The contribution of PCT when FERT is considered is negative. Since C(PCT | FERT) is smaller than C(PCT), this implies that the inclusion of another activity makes the contribution of PCT negative. The total contribution of every two activities can also be calculated from Table XLI:

$C(PCT \cup CT)$ = (5)-(1) = 1108.76-441.34 = 667.42

$C(PCT \cup FERT)$ = (6)-(1) = 667.31-441.34 = 225.97

$C(CT \cup FERT)$ = (7)-(1) = 1079.12-441.34 = 637.78

Among the above three contributions, C(PCT ∪ CT) has the highest value. This shows that if only two silvicultural practices can be considered, PCT and CT should be chosen. When only one activity is considered FERT has the highest contribution, adding in another activity makes PCT and CT the highest contributors. This is because PCT and CT have the highest interaction effect. Comparing C(PCT) with C(PCT | CT) we find that the contribution of PCT is increased from 26.86 to 591.29 due to a positive interac-

---

[2/] See footnote 1.

tion effect between PCT and CT. Using the notation mentioned above, the interaction effect between PCT and CT is:

$$C(PCT \cap CT) = C(PCT \mid CT)-(PCT) = 591.29-26.86 = 564.43$$

Using the same argument, $C(PCT \cap CT)$ can also be obtained by comparing $C(CT)$ with $C(CT \mid PCT)$, and the same answer is derived:

$$C(PCT \cap CT) = (CT \mid PCT)-(CT) = 640.56-76.13 = 564.43$$

Another way to look at this problem is that the total effect of PCT and CT should be the sum of the individual contribution and their interactive contribution:

$$C(PCT \cup CT) = C(PCT)+C(CT)+C(PCT \cap CT)$$

This implies

$$C(PCT \cap CT) = C(PCT \cup CT)-C(PCT)-C(CT) = 564.43$$

The same answer as we derived before. Using either of the two argument we derive

$$C(PCT \cap FERT) = -49.99$$

$$C(CT \cap FERT) = 312.55$$

The interaction effect between PCT and FERT is negative, while the interaction effects for PCT and CT, and CT and FERT are positive. That is to say, when PCT and FERT are considered together their individual effects are reduced by their negative interaction. For CT, the addition of any other activity reinforces the contribution of CT. PCT and CT have the largest interaction effect.

When two activities are considered, the effect of adding another activity can be calculated as:

$C(PCT \cup CT \mid FERT) = (8)-(4) = 1780.17-690.44 = 1089.73$

$C(PCT \cup FERT \mid CT) = (8)-(3) = 1780.17-517.47 = 1262.70$

$C(CT \cup FERT \mid PCT) = (8)-(2) = 1780.17-468.20 = 1311.97$

Since $C(PCT \cup CT \mid FERT)$ is larger than $C(PCT \cup CT)$, $C(PCT \cup FERT \mid CT)$ is larger than $C(PCT \cup FERT)$ and $C(CT \cup FERT \mid PCT)$ is larger than $C(CT \cup FERT)$, we conclude that given any two activities already considered, the inclusion of another activity increases the total contribution of the two given activities. Hence, whenever possible, as many activities should be considered because the more activities considered the more "extra" contributions can be gained under the cost and revenue assumptions of these examples. Higher cost levels might well cause a negative contribution from some activities.

Finally, the interaction effect of PCT, CT and FERT can also be derived in two ways. The first method is by using the simple intuition: Total effect of PCT, CT and FERT should be the sum of the individual effect, interactive effect of every two activities and the interactive effect of the three activities:

$$\begin{aligned} C(PCT \cup CT \cup FERT) = {} & C(PCT)+C(CT)+C(FERT)+C(PCT \cap CT)+ \\ & C(PCT \cap FERT)+C(CT \cap FERT)+ \\ & C(PCT \cap CT \cap FERT) \end{aligned}$$

First, $C(PCT \cup CT \cup FERT)$ is calculated from Table XLI:

$C(PCT \cup CT \cup FERT) = (8)-(1) = 1780.17-441.34 = 1338.83$

Then, $C(PCT \cap CT \cap FERT)$ is derived:

$C(PCT \cap CT \cap FERT)$

$= C(PCT \cup CT \cup FERT) - C(PCT) - C(CT) - C(FERT) - C(PCT \cap CT) - C(PCT \cap FERT) - C(CT \cap FERT)$

$= 1338.83 - 26.86 - 76.13 - 249.10 - 564.43 - (-49.99) - 312.55$

$= 159.75$

The interactive effect of PCT, CT and FERT is positive. When these three activities are considered together, an extra contribution of $159.75 on soil expectation is gained because the positive interactive effect among these three activities.

The second way of deriving $C(PCT \cap CT \cap FERT)$ is by using mathematical relations:

First, $C((CT \cup FERT) \cap PCT)$

$= C(CT \cup FERT \mid PCT) - C(CT \cup FERT)$

$= 1311.97 - 637.78$

$= 674.19$

Second, $C((CT \cup FERT) \cap PCT)$

$= C((CT + FERT + CT \cap FERT) \cap PCT)$

$= C(CT \cap PCT) + C(FERT \cap PCT) + C(CT \cap FERT \cap PCT)$

Then, $C(CT \cap FERT \cap PCT)$

$= C((CT \cup FERT) \cap PCT) - C(CT \cap PCT) - C(FERT \cap PCT)$

$= 674.19 - 564.43 - (-49.99)$

$= 159.75$

The same result can also be derived by looking at either $C((PCT \cup FERT) \cap CT)$ or $C((PCT \cup CT) \cap FERT)$. Table XLII summarizes the results developed above for interactive and total contributions.

TABLE XLII. SUMMARY OF SOME TOTAL AND INTERACTIVE EFFECTS ON SOIL EXPECTATION, SITE 140.

Combinations	Contribution ($)
PCT	26.86
CT	76.13
FERT	249.10
PCT $\cup$ CT	667.42
PCT $\cup$ FERT	225.97
CT $\cup$ FERT	637.78
PCT $\cap$ CT	564.43
PCT $\cap$ FERT	- 49.99
CT $\cap$ FERT	312.55
PCT $\cup$ CT $\cup$ FERT	1338.83
PCT $\cap$ CT $\cap$ FERT	159.75

VII. SUMMARY, FURTHER APPLICATIONS AND DISCUSSION

SUMMARY

Solution of stand level optimization problems requires a suitable growth model with responses to all silvicultural activities considered. Appropriate cost and revenue responses to stand management alternatives are also required. Finally, an efficient optimization procedure for handling the potential numerous combinations of treatments is required. The study presented has utilized the DFIT model with modifications, current cost and revenue data from Western Oregon and dynamic programming to accomplish the task.

Dynamic programming is a powerful technique for solving optimization problems. The continuous growth of the stand can be discretized by using the "neighborhood" concept to fit into the discrete state dynamic programming model. To find the optimal solution, the number of alternatives examined is greatly reduced by applying the principle of optimality of dynamic programming. Dynamic programming is flexible accepting any form of price, cost and growth functions. The choice of a growth function depends on the dimensions of the dynamic programming framework to be used. When only volume or basal area is involved, a two-dimensional dynamic programming network can solve the problem. If both stocking (volume or basal area) and diameter are involved then a three-dimensional dynamic programming network is needed. The structure of the dynamic programming framework is not affected by the complexities of price, cost and growth functions. This is a major advantage over most other optimization techniques. Analysts are not biased towards selection of simple functional forms. The following characteristics make dynamic programming a useful optimization method: (1) computationally simple, (2) efficient, (3) compatible to a wide array of functional forms, and (4) flexible in use.

The growth model used is DFIT (Bruce, Demars and Reukema, 1977). It has functional relations to explain the results of precommercial, commercial thinning, fertilization and submerchantable mortalities. Some of the functions in DFIT can be changed without upsetting internal relations in DFIT. For example the fertilization part of this research is derived from the experimental results of Turnbull and Peterson (1976) instead of the functional equations in DFIT. The cost function used in this research is derived by Sessions (1979) who used a simulation technique. The price function is also derived from Bulletin 201 (McArdle, Meyer and Bruce, 1961) by Sessions.

Incorporation of diameter growth acceleraction impacts of thinning into dynamic programming analysis is necessary to account for reduction in logging cost and increases in price received as the size of material harvested increases.

The effect of fertilization and its interaction with other silvicultural practices and diameter dependent costs and revenues must also be considered. DOPT is a general model that considers single silvicultural practices and some combinations of silvicultural practices. DOPT can consider (1) finding the best thinning regime for a natural stand at ten-year intervals, (2) finding the best precommercial thinning intensity at year ten and the best subsequent thinning regime for a natural stand starting from age 30, (3) finding the best regime for commercial thinning and fertilization for a natural stand at ten-year intervals, (4) finding the best precommercial thinning intensity and the subsequent commercial thinning and fertilization regime for a natural stand, (5) finding the best thinning regime for a non-natural stand, and (6) finding the best regime for thinning and fertilization for a non-natural stand. The "best" is this instance means maximum soil expectation.

In this study we have come up with a method for testing complex silvicultural responses and interactions. The impact of silvicultural inputs can be studied either independently or in combination taking account of simultaneous density interactions over time and rotation length alternatives.

Precommercial thinning-- it is generally assumed that early stocking control accelerates diameter growth and reduces competition and will affect later commercial thinning entries in value and volume. This study comfirms these assumptions and indicates the impact of precommercial thinning on later commercial thinning and rotation. Deficiencies due to DFIT limitations make a precise analysis of the impact of precommercial thinning without considering the subsequent commercial thinning difficult. But the process presented would work with a better specified precommercial thinning response model to better provide these results.

Commercial thinning-- this has been described and discussed in an earlier paper (Brodie and Kao, 1979). The consideration of diameter acceleration makes thinnings more attractive. If diameter acceleration is not considered, the purpose of thinning is only to capture the mortality. The fact that commercial thinning lengthens rotation by maintaining the efficiency of the growing stock over longer periods of time is also generally assumed and is confirmed in this study.

Fertilization-- the usual assumptions about fertilization were contingent on uncertain knowledge of physical response. Does fertilization substitute for growing stock or site capacity? Using some of the first good empirical response data to fertilization of Douglas-fir, this study indicates that fertilization increases site capacity and raises optimal stocking levels.

Initial density-- the general assumptions are that lower stocking has faster growth and longer rotation. This model can be used to study the exact impact. In general, a lesser stocked stand will reach a given diameter sooner and the rotation impact may be hidden by the quality premium termination effect. The methodology presented confirms previous tneory and can be used to study initial stocking impacts.

In general, silvicultural impacts when aggregated have positive interaction so that the sum of the treatments effect is greater than the independent effects summed independently. The combinatorial approach to treatments outlined in this study may help to justify intensive silviculture in marginal situations where independent effects are insufficient.

FURTHER APPLICATIONS

DOPT considers first commercial thinning at the first age of multiple ten that is older than the age of the first possible commercial thinning. Let A_F be the age of the first possible commercial thinning, DOPT considers thinning starting from the age of the next nearest multiple of ten. One could consider thinnings at ages A_F, A_F+10, A_F+20, and so on; (A_F+1), $(A_F+1)+10$, $(A_F+1)+20$, and so on; ...; and (A_F+9), $(A_F+9)+10$, $(A_F+9)+20$, and so on. From these ten alternatives the one with the maximum soil expectation could be chosen. The current result from DOPT is one of the above ten alternatives. To make the model simple we did not consider the others. We also believe the soil expectations from those ten alternatives will not differ by much. But DOPT could be modified to analyse any of the alternatives.

The problem of finding the best management regime for maximizing MAI can also be solved by DOPT by changing the objective function from maximizing soil expectation to maximizing MAI, i.e. from $\sum_i (R(V_i,D_i)-C(V_i,D_i))*(1+r)^{n-1}/((1+r)^n-1)$ to $\sum_i V_i/n$. Where i is the year harvest occurred, V_i is the volume harvested at age i, $R(V_i,D_i)$ is the revenue and $C(V_i,D_i)$ is the cost associated with harvesting volume V_i with mean diameter D_i at year i, r is the discount rate, and n is the rotation age. A recent study by Ritters (1979) showed that this could be done when only commercial thinnings are considered. The inclusion of precommercial thinning and fertilization should not cause any difficulties.

A note by Kao and Brodie (1979b) showed that the determination of optimal thinning entry interval can also be solved by using dynamic programming. DOPT might also be modified to solve this question. From the study by Kao and Brodie it is found that, under their price function, changing from ten-year entry interval to variable entry interval the soil expectation only increases 1.11 percent but the computation time needed is quadrupled. The duration of the effect of fertilizer also causes some problem. The duration of the effect of fertilizer is assumed to be ten years. If variable entry interval is considered, more than one fertilization may occur in ten years. How the growth response would be is not clear. The principle of optimality would be violated, since continuing effects of fertilization would not be taken into account in the stage by stage optimization. With a fertilization effect of ten years and a stage interval of five years, fertilization applications that were cost effective over the ten year period but not the five year period would be rejected. Another problem is mortality capture. It is assumed that thinning captures the mortality in the last ten years. If the thinning interval is less than ten years then there is no problem, the mortality after the last thinning can all be captured. When the thinning is longer than ten years then we need some more assumptions to make the solution feasible. This research concentrated on studying the interactions among silvicultural practices, hence the above mentioned details were not considered.

In each thinning it is assumed that the mean diameter of the trees removed (d) is equal to the mean diameter of the stand before cut (D), i.e. d/D=1.00. In a silvicultural sense only mechanical thinning achieves this restriction. Other thinning methods like thinning from above (or crown thinning), thinning from below (or low thinning), and selection thinning are not considered. When other thinning methods are also considered, i.e. d/D need not be 1.00, the current DOPT will not be able to solve this problem. One solution method would be to use another descriptor to represent the d/D ratio. The d/D ratios chosen to represent the state must be finite to fit into the finite state dynamic programming model. At each thinning, different combinations of trees removed would have different d/D ratio values. Using the "neighborhood" concept, some d/D ratios can be grouped together and the alternative which has the highest cumulative present net worth would be chosen to represent the node. All situations would be the same as discussed in this study except one more descriptor and more computations would be needed.

In a natural stand the number of trees per acre decreases as the stand gets older. One way to increase the precision of the solution yet use the same computer memory is to narrow the tree interval size as the solution progresses from stage to stage (Sessions, 1979). One technique is to define the network size by the number of trees in an unthinned natural stand and determine the interval by some constant divisor of this number. For example, for a site index 140 Douglas-fir stand, there are around 900 trees at age 30, 600 trees at age 40, 420 at 50, 300 at 60, and 270 at 70. If network size is defined to be 30, then the interval size at age 30 is 30 trees (=900/30), at 40 is 20 trees (=600/30), at 50 is 14 (=420/30), at 60 is 10 (= 300/30) and at 70 is 9 (=270/30).

Some forest managers are interested in optimal thinning regimes, where all thinnings are constrained to have positive money income. Noncommercial thinnings are not considered even if they result in higher value growth in later stages. The result of this additional constraint will be a thinning regime which has a present net worth less than or equal to the unconstrained problem. With some modifications, DOPT can find the conditional optimal solution with no difficulty (Sessions, 1979).

DISCUSSION

Changes in Logging and Haul Costs

Some sensitivity tests and break-even tests can also be done. Increases in logging cost or haul cost will change thinning regime. As Sessions (1979) indicated when haul cost increases, among all candidates which can reach a certain node, the candidate with the largest volume harvested is least affected in terms of present net worth. The following equation explains the reason:

$$\begin{aligned}\text{Revenue} &= V^*(\text{Pond Value} - \text{Harvest Cost} - \text{Haul Cost}) \\ &= V^*(\text{Pond Value} - \text{Harvest Cost}) - V^*(\text{Haul Cost})\end{aligned}$$

where V is volume harvested in thinning. When haul cost is reduced, the candidate with the largest V will gain the most.

If harvest cost is changed linearly, i.e. K*(Harvest Cost), the optimal regime will also be changed. (Sessions, 1979). The result was that early thinnings were skipped. That is when harvest cost is increased linearly, it is not worth harvesting small amounts of volume.

Changes in Price Function

The pond value is expressed as a linear function of mean diameter of trees harvested:

Pond Value = K + K'*D

hence

Revenue = V*(K + K'*D - Harvest Cost - Haul Cost)

A change in the intercept of the price function K will be the same as a change on haul cost. Increasing the intercept K has the same effect as reducing haul cost. Increasing the slope of the function usually lengthens rotation because larger diameter material is more valuable relative to small diameter material. But there are still two possibilities: First, fewer trees will be removed in early stages because trees in the early stages are relatively less valuable due to small diameter. Second, more trees will be removed in early stages to stimulate the concentration of growth on fewer, larger trees. Sessions (1979) changed the slope from 70.81 to 100 and the results came out to be the second case. Although Sessions only considered commercial thinnings, the results of considering commercial, precommercial thinning and fertilization should be similar.

Changes in Fertilization Cost

When fertilization cost is changed the management regime may also be changed. Because the best alternative among all candidates that can reach a certain node in the next stage will be different when fertilization cost is changed. Generalizing, we can assume that increasing the cost of fertilization makes fertilization less favorable. When fertilization cost is high enough then "do not fertilize" will be chosen as the optimal management regime for the cost of fertilization does not pay for the return. Between "fertilize" and "do not fertilize" there is a fertilization cost which makes no difference between "fertilize" and "do not fertilize", i.e. the optimal management regime of considering fertilization has the same soil expectation as that of optimal management regime without fertilization. The "break-even" point can be found by a "trial and error" method.

The consideration of only three levels of fertilization may also have artifact effect. When data are available, the consideration of more levels of fertilization will decrease the artifact effect.

Changes in Regeneration and Precommercial Thinning Costs

Let R_i be the revenue earned and C_i be the cost incurred at age i, r be the discount rate, n the rotation, REGENC the regeneration cost, and PCTCOST the precommercial thinning cost, the soil expectation Se is:

$$Se = \sum_i((R_i-C_i)*(1+r)^{n-i}-REGENC*(1+r)^n-PCTCOST*(1+r)^{n-10})/((1+r)^n-1)$$

When REGENC is increased the rotation will be lengthened to spread the cost over a longer time interval so the cost is reduced relatively. The management regime will be changed accordingly when the rotation is changed. Changing PCTCOST can be shown to have the same effect as REGENC does:

$$\begin{aligned} PCTCOST*(1+r)^{n-10} &= (PCTCOST*(1+r)^{-10})*(1+r)^n \\ &= PCTCOST'*(1+r)^n \end{aligned}$$

where $PCTCOST*(1+r)^{-10}$ is a constant because it is a function of two constants PCTCOST and R. Hence increasing PCTCOST will lengthen the rotation.

Changes in Alternative Rate of Return

Raising the discounting rate shortens the rotation almost invariably as future returns become less valuable than present returns. As discussed in Chapter VI thinnings keep the marginal value growth percent high, hence a thinned stand will have a longer rotation than an unthinned stand. But due to the price premium constraint, i.e. trees larger than a certain diameter have a price no better than trees with that specific diameter, a thinned stand grows faster and reaches that specific diameter faster, so a shorter rotation may result from this premium limit. If the discounting rate is raised high enough to shorten the rotation to the age that mean diameter of the stand does not reach that specific diameter, then the effect of lengthening rotation of thinning can be checked. Fertilization may have the same effect on rotation as thinning does. When the discounting rate is raised, this effect can also be checked.

The strength of the model presented are first of all that the solution is general and reflects the diameter growth acceleration interactions of thinning. When fertilization is included the interaction among precommercial thinning, commercial thinning and fertilization can be studied. The technique will accept price and cost functions of any form, provided they are comprised of state-descriptor variables or their transformations. The model is also transferable to other species for which DFIT type models are available.

BIBLIOGRAPHY

Adams, D. M. and A. R. Ek. 1974. Optimizing the management of uneven-aged forest stands. Canadian Journal of Forest Research. Dec. 1974. p274-287.

Adams, D. M., R. W. Haynes and D. R. Darr. 1977. Alternative techniques for analysis of price effects in national forest flow. U. S. Forest Service General Technical Report PNW-60. 18p.

Amidon, E. L. and G. S. Akin. 1968. Dynamic programming to determine optimal level of growing stock. Forest Science 14:287-291.

Anderson, F. J. 1976. Control theory and the optimum timber rotation. Forest Science 22:241-246.

Aulerich, D. E., K. N. Johnson and H. Froehlich. 1974. Tractors or skylines: what's best for thinning young-growth Douglas-fir? Forest Industries 101(12):42-45.

Bellman, R. and S. E. Dreyfus. 1962. Applied dynamic programming. Princeton Univ. Press. 363p.

Brodie, J. D., D. M. Adams and C. Kao. 1978. Analysis of economic impacts on thinning and rotation for Douglas-fir using dynamic programming. Forest Science 24: 513-522.

Brodie, J. D. and C. Kao. 1979. Optimization of thinning regime in Douglas-fir using three descriptor dynamic programming to account for diameter growth acceleration. Unpublished paper. Dept. of Forest Management. Oregon State Univ.

Bruce, D., D. J. Demars and D. L. Reukema. 1977. Douglas-fir yield simulator--DFIT-user's guide. U. S. Forest Service General Technical Report PNW-57. 26p.

Buongiorno, J. and D. Teeguarden. 1973. An economic model for selecting Douglas-fir reforestation projects. Hilgardia 42(3):35-120.

Chappelle, D. E. and T. C. Nelson. 1964. Estimation of optimal stocking levels and rotation ages of loblolly pine. Forest Science 10:471-502.

Curtis, R. O. 1967. A method of estimation of gross yield of Douglas-fir. Forest Science Monograph 13. 24p.

Daniel, W. W. 1978. Applied nonparametric statistics. Houghton Mifflin Co. 503p.

Darr, D. R. 1973. Stumpage price: does it vary by tree diameter. Forest Products Journal. 23(Oct):58-60.

Davis, K. P. 1966. Forest management: regulation and valuation. Second edition. McGraw Hill Book Co. 519p.

Davies, O. L. and P. L. Goldsmith. 1972. Statistical methods in research and production. Hafner Publishing Co. 478p.

Dilworth, J. R. 1970. Log scaling and timber cruising. Oregon State Univ. Bookstore. 466p.

Duerr, W. A. 1960. Fundamentals of forestry economics. McGraw Hill Book Co. 579p.

Duerr, W. A. and N. B. Christiansen. 1973. Exercises in the managerial economics of forestry. Oregon State Univ. Bookstore. 325p.

Hann, D. W. and J. D. Brodie. 1979. Even-aged management: Basic managerial questions and available or potential techniques for answering them. Unpublished report. Dept. of Forest Management, Oregon State Univ.

Hardie, I. W. 1977. Optimal management plans for loblolly pine plantation in the mid-Atlantic region. Univ. of Marylan. Agricultural Experiment Station Report MP-906. 107p.

Hillier, F. S. and G. J. Lieberman. 1974. Operations research. Second edition. Holden-Day Inc. 800p.

Johnson, K. N. and Scheurman, H. L. 1977. Techniques for prescribing optimal timber harvest and investment under different objectives-- discussion and synthesis. Forest Science Monograph 18. 31p.

Kao, C. and J. D. Brodie. 1979a. A modified flexible polyhedron approach to solution of the continuous entry interval continuous stocking level thinning and rotation problem. Unpublished report. Dept. of Forest Management, Oregon State Univ.

Kao, C. and J. D. Brodie. 1979b. A note on determination of optimal thinning entry interval using dynamic programming. Unpublished report. Dept. of Forest Management, Oregon State Univ.

Kao, C. and J. D. Brodie. 1979c. Goal programming as a tool for reconsiliation of economic, even-flow and regulation objectives in forest harvest scheduling. Unpublished report. Dept. of Forest Management, Oregon State Univ.

King, J. E. 1966. Site index curves for Douglas-fir in the Pacific Northwest. Weyerhaeuser Forestry Paper No. 8. Weyerhaeuser Co. 49p.

Lembersky, M. R. 1976. Maximum average annual volume for managed stand. Forest Science 22:69-81.

Lembersky, M. R. and K. N. Johnson. 1975. Optimal policies for managed stand: an infinite horizon Markov decision process approach. Forest Science 21:109-122.

McArdle, R. E., W. H. Meyer and D. Bruce. 1961. The yield of Douglas-fir in the Pacific Northwest. Revised Technical Bulletin No. 201. U. S. Dept. of Agriculture. Washington D. C. 74p.

Mood, A. M., F. A. Graybill and D. C. Boes. 1974. Introduction to the theory of statistics. Third edition. McGraw Hill Book Co. 546p.

Naslund, B. 1969. Optimal rotation and thinning. Forest Science 15:446-451.

Nautiyal, J. C. and P. H. Pearse. 1967. Optimizing the conversion to sustained yield-- a programming solution. Forest Science 13:131-139.

Nazareth, L. 1973. A resource allocation model for land management and a solution strategy that combines Dantzig-Wolfe decomposition and dynamic programming. Univ. of California, Berkeley. Coll. of Eng. Operations Res. Center. ORC 73-20. 100p.

Nemhauser, G. L. 1966. Introduction to dynamic programming. John Wiley and Sons. Inc. 256p.

Pearce, J. K. and G. Stenzel. 1972. Logging and pulpwood production. The Ronald Press Co. 453p.

Pelz, D. P. 1977. Determination of optimal growing stock levels by inventory theory. Forest Science 23:183-189.

Porterfield, R. L. and D. Schweitzer. 1975. A quick and data-poor approach to evaluating precommercial thinning. Journal of Forestry 73:771-773.

Randall, R. M. 1977. Financial consequences of commercial thinning regimes in young-growth Douglas-fir. USFS Research Note PNW-292. 8p.

Randall, R. M. and D. Darr. 1974. Douglas-fir thinning values sensitive to price-diameter relationships. USDA Forest Service Research Note PNW-227. 7p.

Reineke, L. H. 1933. Perfecting a stand density index for even-aged forests. J. Agric. Res. 46(7):627-638.

Reukema, D. L. and D. Bruce. 1977. Effects of thinning in yield of Douglas-fir: concept and some estimates obtained by simulation. US Forest Service General Technical Report PNW-58. 36p.

Risvand, J. 1969. Economic analysis of cutting programs applying dynamic programming. In: Reading in forest economics. Universitesforlaget Oslo. p 73-79.

Ritters, K. C. 1978. Maximum mean annual increment used as a criterion in the DOPT model. Unpublished paper. Dept. of Forest Management, Oregon State Univ. 12p.

Schreuder, G. F. 1968. Optimal forest investment decisions through dynamic programming to determine optimal levels of growing stock. Yale University. School of Forestry. Bulletin No. 72. 71p.

Schreuder, G. F. 1971. The simultaneous determination of optimal thinning schedule and rotation for an even-aged forest. Forest Science 17:333-339.

Sessions, J. 1979. Effects of harvesting technology upon optimal stocking regimes of forest stands in mountainous terrain. Ph.D. Thesis. Dept. of Forest Management, Oregon State Univ. 259p.

Smith, D. M. 1962. The practice of silviculture. Seventh edition. John Wiley and Sons, Inc. 578p.

Staebler, G. R. 1955. Gross yield and mortality tables for fully stocked stands of Douglas-fir. USDA Forest Service PNW Research Paper 14. 20p.

Turnbull, K. J. and C. E. Peterson. 1976. Analysis of Douglas-fir growth response to nitrogenous fertilizer. Part I: regional trends. Institute of forest products contribution number 13. Univ. of Washington. 15p.

Turnbull, K. J. and C. E. Peterson. 1976. Analysis of response data from single installations. Institute of forest products contribution number 15. Univ. of Washington. 7p.

U. S. Department of Agriculture, Forest Service. 1969. Douglas-fir supply study. Pacific Northwest Forest and Range Station. 53p.

U. S. Department of Agriculture, Forest Service. 1974. The outlook for timber in the United States. Forest Resource Report No. 20. Washington D. C.

U. S. Department of Interior, Bureau of Land Management. 1977. Schedule 20-- Timber production cost. Portland Office. (periodically updated).

Williams, D. H. 1977. Linking timber RAM to a simple stand model with Dantzig-Wolfe decomposition-- a test case. A paper presented to Western Forest Economists' Meeting. Wemme, OR on May 2-3, 1977.

Yohe, W. P. and D. S. Karnosky. 1969. Interest rates and price level changes, 1952-1969. Federal Reserve Bank of St. Louis. Reprint Series No. 49. 38p.

APPENDICES

APPENDIX I

VARIABLE LIST

SYMBOLS

A	Stand age (years)
ADJ	Adjustment
BF	Board foot
CF	Cubic foot
CT	Commercial thinning
CV4	Cubic volume to four-inch top, no stump (CF)
d/D	Ratio diameter cut to diameter before thinning
D	Diameter (inches)
$\overline{D}$	Average diameter (inches)
Dbh	Diameter at breast height
F	Diameter function
FERT	Fertilization
G	Basal area per acre (square foot)
GR	Growth
GRATIO	Current basal area/basal area from natural stand
$\overline{H}$	Mean height (foot)
INTHR	Basal area interval (square foot)
INTVR	Tree interval
MAI	Mean annual increment (CF)
N	Number of trees per acre
n	Rotation age
NPCT	Number of PCT alternatives to be considered
NRATIO	Current number of trees/number of trees from natural stand
PCT	Precommercial thinning

SYMBOLS (continued)

PCTCOST	Cost of PCT
PN	Pounds of nitrogen
PNW	Present net worth
R	Alternative rate of return
R1	Inflation rate
REGENC	Regeneration cost
S	Site index
SITE	Site index
Se	Soil expectation
SBA	Basal area of a non-normal stand
STREE	Total number of trees of a non-normal stand
T	Department of Natural Resource tarif
TBASE	Base age, age to be considered for first CT
TESTF	Indicator variable of fertilization
TESTN	Indicator variable of natural stand
TESTP	Indicator variable of precommercial thinning
V	Total cubic-foot volume per acre
Var	Variance
V/G	Volume to basal area ratio

SUBSCRIPTS

A	Adjusted or adjuster
B	Breast height
C	Commercial thinning
CT	Commercial thinning
D	Dominant and codominant
F	First possible
G	Basal area
i	Diameter class i
K	Limit
L	Limit
m	Merchantable limit
M	Merchantable
N	Nitrogen fertilizer
P	Precommercial thinning
s	Submerchantable
T	Total
Z	Ratio
50	Fifty year basis
100	One hundred year basis

APPENDIX II

PROGRAM FOR COMPARING GROWTHS FROM THE DFIT AND THE GROWTH MODEL BY TURNBULL AND PETERSON

```
      PROGRAM COMPARE(INPUT,OUTPUT)
      WRITE *," READ IN TREES, BA, BHAGE, AND SITE"
 10   READ *, TREE,BA,BHAGE,SITE
      IF(TREE.LE.0.) STOP
      B30=30.-13.22+0.033*SITE
      D30=10**(0.1097-3.4857/B30**0.25+1.0531*ALOG10(SITE))
      DM=0.75*D30/0.875
      AGE=BHAGE+13.22-0.033*SITE
      DNATURE=10**(0.1097-3.4857/BHAGE**0.25+1.0531*ALOG10(SITE))
      TNATURE=10**(3.9108+5.2306/BHAGE**0.25-1.5803*ALOG10(SITE))
      GNATURE=10**(1.8669-1.7408/BHAGE**0.25+0.5259*ALOG10(SITE))
      TREE1=TREE+TNONMER
      BA1=BA+GNONMER
      HT=10**(0.1567-15.673/AGE+ALOG10(SITE))
      VG=10**(-0.0282+0.7917*ALOG10(HT))
      V1=BA1*VG
      ADJ1=(405.-30.)/400.
      GLIMIT=10**(3.3446-0.3328*ALOG10(TREE))
      DAGE1=10**(0.1097-3.4857/BHAGE**0.25+1.0531*ALOG10(SITE))
      VGROW=0.
      SITE50=21.5-0.18127*AGE+0.72114*SITE
      VGROW1=(0.10493*BHAGE**(-0.2269)*SITE50**1.07647*BA1**
     $0.56805*TREE1**0.10875)*10.
      GGROW1=(12.70144-0.22684*BHAGE-0.02066*SITE50+0.04469*
     $BA1-1.64715*BA1/BHAGE+21.74006*BA1/BHAGE**2)*10.
      AGE=AGE-0.5
      DO 20 I=1,10
      AGE=AGE+1.
      BHAGE=AGE-13.22+0.033*SITE
      DVOL=10**(ALOG10(2.3026)+ALOG10(12.4083/AGE**2+0.4352/BHAGE**
     $1.25)+ALOG10(1.12+0.0105*AGE-0.00005*AGE**2)+1.9628-12.4083/AGE
     $-1.7408/BHAGE**0.25+1.3176*ALOG10(SITE))
      DVOL=DVOL*ADJ1
      TMPVOL=V1+DVOL+VGROW
      HT=10**(0.1567-15.673/(AGE+0.5)+ALOG10(SITE))
      VG=10**(-0.0282+0.7917*ALOG10(HT))
      G1=TMPVOL/VG
      DAGE=DAGE1
      DAGE1=10**(0.1097-3.4857/(BHAGE+0.5)**0.25+1.0531*ALOG10(SITE))
      GMORT=10**(1.4034+4.9394*ALOG10(DM))*(10**(-4.44*ALOG10(DAGE))
     $-10**(-4.44*ALOG10(DAGE1)))
      G=G1-GMORT
      CR=G/GLIMIT
      ADJ2=1.-16.*(CR-0.5)**4
      DVOL=DVOL*ADJ2
```

```
      VGROW=VGROW+DVOL
20    CONTINUE
      G1=(V1+VGROW)/VG
      GGROW=G1-BA1
      WRITE *,GGROW,GGROW1,VGROW,VGROW1
      GOTO 10
      END
```

APPENDIX III

DOPT PROGRAM LISTING

```
      PROGRAM DOPT(INPUT,OUTPUT,TAPE60=INPUT,TAPE61=OUTPUT)
      DIMENSION VAL(39,72,2),VLM(39,72,2),TRUEBA(39,72,2),TRUET(39,72,2)
      DIMENSION VCUT(39,72),DMEAN(39,72),OPOLVR(39,72),OPOLHR(39,72)
      DIMENSION HRVSTD(16),HRVSTB(16),HRVSTP(16),HRVSTS(16),HRVSTC(16)
      DIMENSION NN(100),TNORM(16),GNORM(16),VNORM(16),Z(16)
      COMMON TAGE,TBASE,SITE,TRATIO,GRATIO,VGRATIO
      COMMON PMORTN(25),PMORTG(25),PMORTV(25),YMORTN(200),YMORTG(200)
      COMMON TNONMER(25),GNONMER(25),VNONMER(25)
      INTEGER OPOLVR,OPOLHR
      N=1
      MAXVR=39
      MAXHR=72
      READ(60,*) INTVR,INTHR,TBASE,SITE,R,TESTN,TESTP,TESTF,
     $REGENC,R1,PCTAGE,NPCT,PCTCOST,STREE,SBA
      WRITE(61,3) TBASE,SITE,R,R1,REGENC
 3    FORMAT("1AGE IS",F6.1//" SITE INDEX IS",F7.1//" INTEREST RATE IS",
     $F5.3//" PRICE INFLATION RATE IS",F6.3//" REGENERATION COST"
     $" IS $",F7.2)
      SITEOLD=SITE
      TAGE=TBASE
      DO 4 I=1,MAXVR
      DO 4 J=1,MAXHR
 4    VAL(I,J,1)=-999999.99
      IF(TESTP.NE.1.) GOTO 10
******PRECOMMERCIAL THINNING
      IN=EVER=0.
      SITE1=SITE*(1.+((210.-SITE)/300.)**2)
      REGENC=REGENC+PCTCOST/(1.+R)**PCTAGE
      DO 5 I=1,18
      PMORTN(I)=PMORTG(I)=PMORTV(I)=TNONMER(I)=GNONMER(I)=VNONMER(I)=0.
      K=(I-1)*10
      DO 5 J=1,10
 5    YMORTN(K+J)=YMORTG(K+J)=0.
      TRATIO=GRATIO=1.
      WRITE(61,6) PCTAGE,PCTCOST,SITE1,NPCT,STREE
 6    FORMAT(/" THE STAND IS PCT AT AGE",F6.1,//" THE COST OF PCT"
     $" IS $",F7.2,//" SITE INDEX WAS ADJUSTED TO",F8.2,//" PCT "
     $"ALTERNATIVES WOULD LIKE TO BE CONSIDERED IS",I5,//,
     $" MAXIMUM NUMBER OF TREES LEFT AFTER PCT IS",F9.2)
 7    TMERCH=STREE-IN*INTVR
      SITE=SITEOLD
      CALL PRECOM(PCTAGE,SITE,TBASE,TMERCH,GMERCH,VMERCH,ERROR)
      IF(ERROR.EQ.0.) GOTO 9
      WRITE(61,8)IN
 8    FORMAT(//////" **SOME PCT ALTERNATIVES RESULT IN TOO"
```

```
     $" FEW TREES FOR CT AT SPECIFIED CT AGE"//" **THE NUMBER OF"
     $" PCT ALTERNATIVES CONSIDERED IS",I5)
      IF(EVER.EQ.0.) STOP
      GOTO 17
9     EVER=1.
      IN=IN+1
      GOTO 16
10    IF(TESTN.EQ.0.) GOTO 15
******FOR NON-NATURAL STAND GROW TO AGE OF MULTIPLES OF 10
      IF(TBASE.GE.30..AND.10.*IFIX(TBASE/10.).EQ.TBASE) GOTO 15
      WRITE(61,12) STREE,SBA
12    FORMAT(/" THIS IS NOT A NORMAL STAND, AND THE INITIAL NUMBER"
     $" OF TREES IS",F7.2," AND BA IS",F7.2," SQUARE FEET")
      CALL INCRMNT(STREE,SBA,TBASE,SITE)
      TAGE=TBASE
      WRITE(61,14) TBASE,STREE,SBA
14    FORMAT(/" STAND GROWS TO AGE",F6.1" WITH",F8.2," TREES AND BA",
     $F8.2," SQ FT")
******CALCULATE NATURAL STAND AT BASE AGE
15    BHAGE=TAGE-13.22+0.033*SITE
      DNATURE=10**(0.1097-3.4857/BHAGE**0.25+1.0531*ALOG10(SITE))
      TNATURE=10**(3.9108+5.2306/BHAGE**0.25-1.5803*ALOG10(SITE))
      GNATURE=10**(1.8669-1.7408/BHAGE**0.25+0.5259*ALOG10(SITE))
      HT=10**(0.1567-15.673/TAGE+ALOG10(SITE))
      VG=VGRATIO=10**(-0.0282+0.7917*ALOG10(HT))
      VNATURE=GNATURE*VGRATIO
      TRATIO=STREE/TNATURE
      GRATIO=SBA/GNATURE
      IF(TESTN.EQ.0.) TRATIO=GRATIO=1.
******CALCULATE MERCHANTABLE PART AND MORTALITY LATER PART
      CALL SUBMORT(DNATURE)
      L=TBASE/10.-2.
      GMERCH=GNATURE-GNONMER(L)/GRATIO
      TMERCH=TNATURE-TNONMER(L)/TRATIO
      VMERCH=VNATURE-VNONMER(L)/GRATIO
16    I=TMERCH*TRATIO/INTVR+1.999999
      IF(I.GT.MAXVR) I=MAXVR
      J=(GMERCH*GRATIO+INTHR/2.)/INTHR+1.999999
      TRUET(I,J,1)=TMERCH*TRATIO
      TRUEBA(I,J,1)=GMERCH*GRATIO
      VLM(I,J,1)=VMERCH*GRATIO
      VAL(I,J,1)=-REGENC
      IF(IN.LT.NPCT.AND.TESTP.EQ.1.) GOTO 7
17    NS=TMERCH/(2.*INTVR)+1.999999
      IF((NS-1.)*INTVR.GT.TRUET(I,J,1)) NS=TRUET(I,J,1)/INTVR+1.999999
```

```
      IF(TESTP.EQ.1.) NS=400./(2.*INTVR)+1.999999
******CALCULATE NATURAL STAND AND MERCHANTABLE TREES.
      DO 18 I=L,14
      AB=30+10*(I-1)
      B=AB-13.22+0.033*SITE
      TNORM(I)=10**(3.9108+5.2306/B**0.25-1.5803*ALOG10(SITE))
      GNORM(I)=10**(1.8669-1.7408/B**0.25+.5259*ALOG10(SITE))
      VNORM(I)=10**(1.9628-12.4083/AB-1.7408/B**.25+1.3176*ALOG10(SITE))
18    Z(I)=TNORM(I)-TNONMER(I)/TRATIO
      SITE50=21.5-0.18127*30.+0.72114*SITE
19    DO 20 I=1,MAXVR
      DO 20 J=1,MAXHR
      VAL(I,J,2)=VLM(I,J,2)=TRUEBA(I,J,2)=TRUET(I,J,2)=-999999.9
20    OPOLVR(I,J)=OPOLHR(I,J)=VCUT(I,J)=DMEAN(I,J)=-999999.9
      IF(N.EQ.1) GOTO 25
      NS=1
      HT=10**(0.1567-15.673/(TAGE-10.)+ALOG10(SITE))
      VG=10**(-0.0282+0.7917*ALOG10(HT))
25    KJ=TAGE/10.
      BHAGE=(TAGE-10.)-13.22+0.033*SITEOLD
      GBOUND=0.
      IQUANT=3
      IF(N.EQ.1.OR.TAGE.GT.70..OR.TESTF.EQ.0.) IQUANT=1
      DO 55 I=2,MAXVR
******PULL OUT NEXT CARD WHEN MORE INFORMATIONS ARE NEEDED
      IF(TESTP.EQ.1..AND.I.GT.(20/N+7)*15/INTVR+2) GOTO 55
      DO 50 J=1,MAXHR
      IF(VAL(I,J,1).LE.-999999.9) GOTO 50
******IN THE FIRST STAGE THIN FIRST THEN GROW
      IF(N.EQ.1) GOTO 28
      VOL=VLM(I,J,1)+VNONMER(N+L-1)
      CALL GROWTH(TRUET(I,J,1),VOL,VMER,VMORT1,GMER,GMORT1,N,TESTP)
      TOTALT=TRUET(I,J,1)+TNONMER(N+L-2)
      TOTALG=TRUEBA(I,J,1)+GNONMER(N+L-2)
      VMORT2=VMORT1
      GMORT2=GMORT1
      VFERT=VOL+10.*(0.10493*BHAGE**(-0.2269)*SITE50**1.07647*
     $TOTALG**0.56805*TOTALT**0.10875)
      VLM(I,J,1)=VMER
      TRUEBA(I,J,1)=GMER
      PNORM=TRUET(I,J,1)/Z(N+L-2)
      TMORT1=(Z(N+L-2)-Z(N+L-1))*PNORM
      IF(TESTP.EQ.1.) TMORT1=0.
      TRUET(I,J,1)=TRUET(I,J,1)-TMORT1
******CALCULATE FERTILIZATION EFFECTS
```

```
28    DO 45 IFERT=1,IQUANT
      IF(IFERT.EQ.1) GOTO 30
      VFERT1=VFERT+10.*(-0.27823-0.46375*SITE50*ALOG10(1.+(IFERT-1)*200.
     $)+78.90236*ALOG10(1.+(IFERT-1)*200.))
      ADJST=VFERT1/VFERT
      VLM(I,J,1)=VMER*ADJST
      TRUEBA(I,J,1)=GMER*ADJST
      VMORT1=VMORT2*ADJST
      GMORT1=GMORT2*ADJST
30    DIAM=SQRT(TRUEBA(I,J,1)/(0.005454154*TRUET(I,J,1)))
      NT=TRUET(I,J,1)/INTVR+1.999999
******WITH I TREES THERE ARE I+1 KINDS OF THINNING.
******THIN PROPORTIONALLY TO THE DISTRIBUTION OF DIAMETER
******SO THE MEAN DIAMETER IS UNCHANGED.
      DO 40 K=NS,NT
      IF(K.LT.NT) GOTO 32
      TMPBA=TRUEBA(I,J,1)
      TMPVLM=VLM(I,J,1)
      CUT=REV=0.
      GOTO 35
32    TMPBA=TRUEBA(I,J,1)*(K-1)*INTVR/TRUET(I,J,1)
      TMPVLM=VLM(I,J,1)*(K-1)*INTVR/TRUET(I,J,1)
      CUT=VLM(I,J,1)-TMPVLM+VMORT1
      DIAM1=SQRT((TRUEBA(I,J,1)-TMPBA+GMORT1)/(.005454154*(TRUET(I,J,1)-
     $(K-1)*INTVR+TMORT1)))
******CALCULATE CUBIC VOLUME REMOVED TO A 4-INCH TOP
      HD=10**(.1567-15.673/TAGE+ALOG10(SITE))
      N1=(TAGE-20.)/10.
      ALLTREE=TRUET(I,J,1)+TNONMER(N1)
      HM=HD*(3040.-ALLTREE)/3000.
      IF(HM.GT.HD) HM=HD
      VOL4=CUT*(.8758+.001049*HM-.000002824*HM**2+.3221/DIAM1-
     $45.647/DIAM1**3)
      VAR=4.0725-0.065722*SITE+0.00001508*TAGE*SITE**2
      IF(DIAM1.GT.22.) DIAM1=22.
      TRUED=SQRT(DIAM1**2-VAR)
      REV=REVNOW(VOL4,TRUED)
35    FERTCST=(-50.+52.5*IFERT-2.5*IFERT**2)/((1.+R)**(TAGE-10.))
      TMPVAL=REV*(1.+R1)**TAGE/((1.+R)**TAGE)-FERTCST+VAL(I,J,1)
      KK=IFIX((TMPBA+INTHR/2.)/INTHR)+1
      IF(KK.GT.MAXHR) KK=MAXHR
      IF(TMPVAL.LE.VAL(K,KK,2)) GOTO 40
      DMEAN(K,KK)=IFERT*1000.+DIAM
      IF(TESTF.EQ.0.) DMEAN(K,KK)=DIAM
      VAL(K,KK,2)=TMPVAL
```

```
      OPOLVR(K,KK)=(I-1)*INTVR
      OPOLHR(K,KK)=(J-1)*INTHR
      TRUEBA(K,KK,2)=TMPBA
      IF(TMPBA.GT.GBOUND) GBOUND=TMPBA
      TRUET(K,KK,2)=(K-1)*INTVR
      IF(K.EQ.NT) TRUET(K,KK,2)=TRUET(I,J,1)
      IF(K.EQ.1) HRVSTB(KJ)=TRUEBA(I,J,1)
      VLM(K,KK,2)=TMPVLM
      VCUT(K,KK)=CUT
40    CONTINUE
45    CONTINUE
50    CONTINUE
55    CONTINUE
******OUTPUT
      WRITE(61,60) TAGE
60    FORMAT("1AGE=",F5.1/" 1=MEAN DIAMETER (INCH)"/
     $" (FIRST DIGIT INDICATES FERTILIZATION: 1=0 PB, 2=200 PB, 3=400 PB
     $)"/" 2=VOLUME (CUBIC FEET)"
     $    /" 3=VOLUME CUT IN THIS STAGE (CUBIC FEET)"/" 4=TRUE BASAL ARE
     $A (SQUARE FEET)"/" 5=TRUE NUMBER OF TREES"/" 6=CUMULATIVE VALUE FR
     $OM THINNING ($PNW)"/" 7=WHERE IT COMES FROM( TREES,BA)")
      WRITE(61,61) TNONMER(N+L-1)
61    FORMAT(" NONMERCHANTABLE TREES=",F5.1//)
      DO 62 I=1,72
62    NN(I)=(I-1)*INTHR
      IS=-11
      DO 75 IJ=1,6
      IS=IS+12
      IE=IS+11
      WRITE(61,64) TAGE,(NN(I),I=IS,IE)
64    FORMAT(" AGE=",F5.1/" TREES*BA ",12(I3,7X))
      NI=Z(N+L-1)*TRATIO/INTVR+1.999999
******CHANGE NEXT CARD TO NI=400./INTVR+1.999999 WHEN NECESSARY
      IF(TESTP.EQ.1.) NI=(20/N+7.)*15./INTVR+1.999999
      DO 70 I=1,NI
      II=(I-1)*INTVR
      WRITE(61,65) II,(DMEAN(I,J),J=IS,IE)
65    FORMAT(3X,I3,12(3X,F7.1))
      WRITE(61,66) (VLM(I,J,2),J=IS,IE)
      WRITE(61,66) (VCUT(I,J),J=IS,IE)
      WRITE(61,66) (TRUEBA(I,J,2),J=IS,IE)
      WRITE(61,66) (TRUET(I,J,2),J=IS,IE)
      WRITE(61,66) (VAL(I,J,2),J=IS,IE)
66    FORMAT(6X,12(3X,F7.1))
      WRITE(61,68) (OPOLVR(I,J),OPOLHR(I,J),J=IS,IE)
```

```
68    FORMAT(7X,12("(",I3,",",I3,")",1X))
70    CONTINUE
      IF(IE.GE.IFIX((GBOUND+INTHR/2.)/INTHR)+1) GOTO 80
      WRITE(61,72)
72    FORMAT("1")
75    CONTINUE
80    HRVSTD(KJ)=DMEAN(1,1)-100.*IFIX(DMEAN(1,1)/100.)
      HRVSTP(KJ)=VAL(1,1,2)
      HRVSTS(KJ)=VAL(1,1,2)*(1.+R)**TAGE/((1.+R)**TAGE-1.)
      HRVSTC(KJ)=VCUT(1,1)
      IF(VAL(1,1,2).LE.VAL(1,1,1).OR.N.GE.14) GOTO 95
      N=N+1
      TAGE=TAGE+10.
      DO 90 I=1,MAXVR
      DO 90 J=1,MAXHR
      VAL(I,J,1)=VAL(I,J,2)
      VLM(I,J,1)=VLM(I,J,2)
      TRUET(I,J,1)=TRUET(I,J,2)
90    TRUEBA(I,J,1)=TRUEBA(I,J,2)
      GOTO 19
95    WRITE(61,96)
96    FORMAT(/"1ROTATION AGE   HARVEST VOLUME   DIAMETER   HARVEST BA
     $  PNW       SE")
      KI=TBASE/10.+1.
      DO 98 L=KI,KJ
      LL=L*10
98    WRITE(61,99) LL,HRVSTC(L),HRVSTD(L),HRVSTB(L),HRVSTP(L),HRVSTS(L)
99    FORMAT(1X,I7,10X,F9.2,5X,F7.2,4X,F9.2,5X,F7.2,F9.2)
      STOP
      END
```

```
      SUBROUTINE PRECOM(PCTA,SITE,TBASE,TMERCH,GMERCH,VMERCH,ERROR)
      ERROR=0.
      DMERCH=10**((3.9591-ALOG10(TMERCH))/1.5006)
      GMERCH=10**(1.6958+0.4994*ALOG10(DMERCH))
      ADJ=(405.-PCTA)/400.
      SITE1=SITE*(1.+((210.-SITE)/300.)**2)
******CALCULATE THE AGE WHEN DM REACHES 8 INCHES
      BHAGE=(3.4857/(.1097-ALOG10(.875*DMERCH)+1.0531*ALOG10(SITE1)))**4
      TAGE=BHAGE+13.22-0.033*SITE
      TAGE=TAGE*(0.8244+0.004333*PCTA/10.+0.015*(PCTA/10.)**2+
     $0.01667*(PCTA/10.)**3)
      IF(TAGE.LE.TBASE) GOTO 20
      ERROR=1.
      RETURN
20    SITE=SITE1
      HT=10**(0.1567-15.673/TAGE+ALOG10(SITE))
      VG=10**(-0.0282+0.7917*ALOG10(HT))
      VMERCH=GMERCH*VG
      GP=180.22-5.*DMERCH
      IF(GP.GE.GMERCH) GOTO 28
25    TAGE=TAGE+0.5
      BHAGE=TAGE-13.22+0.033*SITE
      DVA=10**(ALOG10(2.3026)+ALOG10(12.4083/TAGE**2+0.4352/BHAGE**1.25
     $)+ALOG10(1.12+0.0105*TAGE-0.00005*TAGE**2)+1.9628-12.4083/TAGE-
     $1.7408/BHAGE**0.25+1.3176*ALOG10(SITE))
      VMERCH=VMERCH+DVA*ADJ
      TAGE=TAGE+0.
      HT=10**(0.1567-15.673/TAGE+ALOG10(SITE))
      VG=10**(-0.0282+0.7917*ALOG10(HT))
      GP=VMERCH/VG
      IF(GP.LT.GMERCH) GOTO 25
      GMERCH=GP
******ROUND THE AGE UP TO AN INTEGER NUMBER
28    IAGE=TAGE+1.
      FRACT=IAGE-TAGE
      HT=10**(0.1567-15.673/TAGE+ALOG10(SITE))
      VG=10**(-0.0282+0.7917*ALOG10(HT))
      VMERCH=GMERCH*VG
      AGE=TAGE+FRACT/2.
      BHAGE=AGE-13.22+0.033*SITE
      DVA=10**(ALOG10(2.3026)+ALOG10(12.4083/AGE**2+0.4352/BHAGE**1.25
     $)+ALOG10(1.12+0.0105*AGE-0.00005*AGE**2)+1.9628-12.4083/AGE-1.7408
     $/BHAGE**0.25+1.3176*ALOG10(SITE))
      DVA=DVA*ADJ*FRACT
      VMERCH=VMERCH+DVA
```

```
      HT=10**(0.1567-15.673/IAGE+ALOG10(SITE))
      VG=10**(-0.0282+0.7917*ALOG10(HT))
      GMERCH=VMERCH/VG
******GROW STAND TO THE AGE OF FIRST CT
      TAGE=IAGE
      ADJ=(405.-TAGE)/400.
      GLIMIT=10**(3.3446-0.3328*ALOG10(TMERCH))
30    IF(TAGE.GE.TBASE) RETURN
      AGE=TAGE+0.5
      BHAGE=AGE-13.22+0.033*SITE
      DVA=10**(ALOG10(2.3026)+ALOG10(12.4083/AGE**2+0.4352/BHAGE**1.25
     $)+ALOG10(1.12+0.0105*AGE-0.00005*AGE**2)+1.9628-12.4083/AGE-
     $1.7408/BHAGE**0.25+1.3176*ALOG10(SITE))
      DVA=DVA*ADJ
      TMPV=VMERCH+DVA
      HT=10**(0.1567-15.673/(TAGE+1.)+ALOG10(SITE))
      VG=10**(-0.0282+0.7917*ALOG10(HT))
      G=TMPV/VG
      CR=G/GLIMIT
      ADJ1=1.-16.*(CR-0.5)**4
      VMERCH=VMERCH+DVA*ADJ1
      GMERCH=VMERCH/VG
      TAGE=TAGE+1.
      GOTO 30
      END
```

```
      SUBROUTINE INCRMNT(STREE,SBA,TBASE,SITE)
      TBASE1=TBASE
      IF(10.*IFIX(TBASE/10.).NE.TBASE) TBASE=10.*(IFIX(TBASE/10.)+1.)
      IF(TBASE.LT.30.) TBASE=30.
      BHAGE1=TBASE1-13.22+0.033*SITE
      TNATURE=10**(3.9108+5.2306/BHAGE1**0.25-1.5803*ALOG10(SITE))
      GNATURE=10**(1.8669-1.7408/BHAGE1**0.25+0.5259*ALOG10(SITE))
      GRATIO=SBA/GNATURE
      TRATIO=STREE/TNATURE
      D1=10**(.1097-3.4857/BHAGE1**0.25+1.0531*ALOG10(SITE))
      BHAGE=TBASE-13.22+0.033*SITE
      D2=10**(0.1097-3.4857/BHAGE**0.25+1.0531*ALOG10(SITE))
      DM=D1*0.75/0.875
      TNONMER=10**(3.8622+3.1994*ALOG10(DM)-4.7*ALOG10(D1))
      TNONMER=TNONMER*TRATIO
      T1=10**(3.8622+3.1994*ALOG10(DM)-4.7*ALOG10(D1))
      T2=10**(3.8622+3.1994*ALOG10(DM)-4.7*ALOG10(D2))
      TMORT=TNONMER-TNONMER*(T2/T1)
      GNONMER=10**(1.4034+4.9394*ALOG10(DM)-4.44*ALOG10(D1))
      GNONMER=GNONMER*GRATIO
      ADJ1=(405.-TBASE)/400.
      TREE=STREE-TNONMER
      GLIMIT=10**(3.3446-0.3328*ALOG10(TREE))
      HT=10**(0.1567-15.673/TBASE1+ALOG10(SITE))
      VG=10**(-0.0282+0.7917*ALOG10(HT))
      V=SBA*VG
      VGROW=0.
 10   TBASE1=TBASE1+0.5
      BHAGE=TBASE1-13.22+0.033*SITE
      DVOL=10**(ALOG10(2.3026)+ALOG10(12.4083/TBASE1**2+0.4352
     $/BHAGE**1.25)+
     $1.9628-12.4083/TBASE1-1.7408/BHAGE**0.25+1.3176*ALOG10(SITE))
      DVOL=DVOL*ADJ1
      TBASE1=TBASE1+0.5
      HT=10**(0.1567-15.673/TBASE1+ALOG10(SITE))
      VG=10**(-0.0282+0.7917*ALOG10(HT))
      TMPV=V+DVOL+VGROW
      G=TMPV/VG
      GMERCH=G-GNONMER
      CR=GMERCH/GLIMIT
      ADJ2=1.-16*(CR-0.5)**4
      DVOL=DVOL*ADJ2
      VGROW=VGROW+DVOL
      IF(TBASE1.LT.TBASE) GOTO 10
      STREE=STREE-TMORT
      SBA=(V+VGROW)/VG
      RETURN
      END
```

```
      SUBROUTINE SUBMORT(DD)
      COMMON TAGE,TBASE,SITE,TRATIO,GRATIO,VGRATIO
      COMMON PMORTN(25),PMORTG(25),PMORTV(25),YMORTN(200),YMORTG(200)
      COMMON TNONMER(25),GNONMER(25),VNONMER(25)
******CALCULATE PERIODIC MORTALITY STARTS FROM THE AGE OF FIRST THINNING
      B30=30.-13.22+0.033*SITE
      D30=10**(0.1097-3.4857/B30**0.25+1.0531*ALOG10(SITE))
      DM=(D30/0.875)*0.75
      DL=0.698*D30
      L=TBASE/10.-2.
      TNONMER(L)=YMORTN(L)=10**(3.8622+3.1994*ALOG10(DM)-4.7*ALOG10(DD))
     $*TRATIO
      GNONMER(L)=YMORTG(L)=10**(1.4034+4.9394*ALOG10(DM)-4.44*ALOG10(DD)
     $)*GRATIO
      VNONMER(L)=GNONMER(L)*VGRATIO*TARIF(DD)/TARIF(DL)
      DO 10 JJ=L,18
      AGE=30+JJ*10.
      BHAGE=AGE-13.22+0.033*SITE
      D=10**(0.1097-3.4857/BHAGE**0.25+1.0531*ALOG10(SITE))
      TNONMER(JJ+1)=10**(3.8622+3.1994*ALOG10(DM)-4.7*ALOG10(D))*TRATIO
      GNONMER(JJ+1)=10**(1.4034+4.9394*ALOG10(DM)-4.44*ALOG10(D))*GRATIO
      DMORTL=SQRT((GNONMER(JJ+1)/TNONMER(JJ+1))/0.005454154)
      HT=10**(0.1567-15.673/AGE+ALOG10(SITE))
      VG=10**(-0.0282+0.7917*ALOG10(HT))
      TAVE=(VG*TARIF(D)+VGRATIO*TARIF(DD))/2.
      VNONMER(JJ+1)=TAVE/TARIF(DMORTL)*GNONMER(JJ+1)
      PMORTN(JJ)=(TNONMER(JJ)-TNONMER(JJ+1))
      PMORTG(JJ)=(GNONMER(JJ)-GNONMER(JJ+1))
      DMORT=SQRT((PMORTG(JJ)/PMORTN(JJ))/0.005454154)
      PMORTV(JJ)=TAVE/TARIF(DMORT)*PMORTG(JJ)
10    CONTINUE
******CALCULATE YEARLY MORTALITY STARTS FROM THE AGE OF FIRST THINNING
      LL=(L-1)*10+1
      DO 20 II=LL,180
      AGE=30.+II
      BHAGE=AGE-13.22+0.033*SITE
      D=10**(0.1097-3.4857/BHAGE**0.25+1.0531*ALOG10(SITE))
      YMORTN(II+1)=10**(3.8622+3.1994*ALOG10(DM)-4.7*ALOG10(D))
      YMORTG(II+1)=10**(1.4034+4.9394*ALOG10(DM)-4.44*ALOG10(D))
      YMORTN(II)=(YMORTN(II)-YMORTN(II+1))*TRATIO
      YMORTG(II)=(YMORTG(II)-YMORTG(II+1))*GRATIO
20    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE GROWTH(TREE,V,VMER,VMORT1,GMER,GMORT1,M,TESTP)
      COMMON TAGE,TBASE,SITE,TRATIO,GRATIO,VGRATIO
      COMMON PMORTN(25),PMORTG(25),PMORTV(25),YMORTN(200),YMORTG(200)
      COMMON TNONMER(25),GNONMER(25),VNONMER(25)
      VGROW=VGROW1=0.
******ASSUME FIRST CT AT FIRST STAGE
      ADJ1=(405.-TBASE)/400.
      GLIMIT=10**(3.3446-0.3328*ALOG10(TREE))
      HT=10**(0.1567-15.673/(TAGE-10.)+ALOG10(SITE))
      TMPAGE=TAGE-9.5
      DO 10 I=1,10
      NN=TMPAGE-30.+2.
      BHAGE=TMPAGE-13.22+0.033*SITE
      DHT=10**(1.7141+ALOG10(SITE)-15.673/TMPAGE-2.*ALOG10(TMPAGE))
      HT=HT+DHT
      VG=10**(-0.0282+0.7917*ALOG10(HT))
******CALCULATE GROSS GROWTH
      DVA=1.12+0.0105*TMPAGE-0.00005*TMPAGE**2
      IF(TMPAGE.GT.105.) DVA=10**0.22304
      DVOL=10**(ALOG10(2.3026)+ALOG10(12.4083/TMPAGE**2+.4352/BHAGE**
     $1.25)+ALOG10(DVA)+1.9628-12.4083/TMPAGE-1.7408/BHAGE**0.25+
     $1.3176*ALOG10(SITE))
      DVOL=DVOL*ADJ1
      TMPVOL=VGROW+DVOL+V
      G=TMPVOL/VG
      GMERCH=G-YMORTG(NN)
      CR=GMERCH/GLIMIT
      ADJ2=1.-16.*(CR-0.5)**4
      DVOL=DVOL*ADJ2
      VGROW=VGROW+DVOL
******CALCULATE NET GROWTH
      DVOL1=10**(ALOG10(2.3026)+ALOG10(12.4083/TMPAGE**2+.4352/BHAGE**
     $1.25)+1.9628-12.4083/TMPAGE-1.7408/BHAGE**.25+1.3176*ALOG10(SITE))
      DVOL1=DVOL1*ADJ1
      TMPVOL1=VGROW1+DVOL1+V
      G1=TMPVOL1/VG
      GMERCH1=G1-YMORTG(NN)
      CR1=GMERCH1/GLIMIT
      ADJ21=1.-16.*(CR1-0.5)**4
      DVOL1=DVOL1*ADJ21
      VGROW1=VGROW1+DVOL1
      TMPAGE=TMPAGE+1.
10    CONTINUE
******CALCULATE MERCHANTABLE MORTALITY AND MERCHANTABLE LIVE TREES.
      N=(TBASE-30.)/10.+M
```

```
VMORT1=VGROW-VGROW1
VMER=V+VGROW1-VNONMER(N+1)
IF(TESTP.EQ.1.) VMER=VMER+VMORT1
GMORT1=(V+VGROW)/VG-(V+VGROW1)/VG
GMER=(V+VGROW)/VG-GMORT1-GNONMER(N+1)
IF(TESTP.EQ.0.) RETURN
VMER=V+VGROW-VNONMER(N+1)
GMER=VMER/VG
GMORT1=VMORT1=0.
RETURN
END
```

```
      FUNCTION REVNOW(VOL,D)
******CALCULATE STUMP TO TRUCK LOGGING COST
      IF(D.GT.6.05) GOTO 5
      COST=7790.9*VOL**(-0.2834)
      GOTO 150
    5 IF(D.GT.7.63) GOTO 10
      IF(VOL.GT.1000.) GOTO 7
      C1=7790.9*VOL**(-.2834)
      C2=4954.7*VOL**(-.2726)
      DELTA=(C1-C2)/(7.63-6.05)
      COST=C1-DELTA*(D-6.05)
      GOTO 150
    7 C1=7800.8*VOL**(-.2539)
      C2=7187.5*VOL**(-.2891)
      DELTA=(C1-C2)/(7.63-6.05)
      COST=C1-DELTA*(D-6.05)
      GOTO 150
   10 IF (D.GT.9.23) GO TO 20
      IF(VOL.GT.1000.) GOTO 15
      C1=4954.7*VOL**(-.2726)
      C2=3768.0*VOL**(-.2662)
      DELTA=(C1-C2)/(9.23-7.63)
      COST=C1-DELTA*(D-7.63)
      GOTO 150
   15 IF(VOL.GT.2000.) GOTO 18
      C1=7187.5*VOL**(-.2891)
      C2=6254.8*VOL**(-.3013)
      DELTA=(C1-C2)/(9.23-7.63)
      COST=C1-DELTA*(D-7.63)
      GOTO 150
   18 C1=14353.0*VOL**(-.3782)
      C2=10336.6*VOL**(-.3627)
      DELTA=(C1-C2)/(9.23-7.63)
      COST=C1-DELTA*(D-7.63)
      GOTO 150
   20 IF(D.GT.10.87) GOTO 30
      IF(VOL.GT.1000.) GOTO 25
      C1=3768.0*VOL**(-.2662)
      C2=4375.0*VOL**(-.2833)
      DELTA=(C1-2)/(10.87-9.23)
      COST=C1-DELTA*(D-9.23)
      GOTO 150
   25 IF(VOL.GT.2000.) GOTO 28
      C1=6524.8*VOL**(-.3013)
      C2=4375.0*VOL**(-.2833)
```

```
      DELTA=(C1-C2)/(10.87-9.23)
      COST=C1-DELTA*(D-9.23)
      GOTO 150
   28 C1=10336.6*VOL**(-.3627)
      C2= 8479.6*VOL**(-.3626)
      DELTA=(C1-C2)/(10.87-9.23)
      COST=C1-DELTA*(D-9.23)
      GOTO 150
   30 IF(D.GT.12.31) GOTO 40
      C1=8479.6*VOL**(-.3626)
      C2=6839.5*VOL**(-.3492)
      DELTA=(C1-C2)/(12.31-10.87)
      COST=C1-DELTA*(D-10.87)
      GOTO 150
   40 IF(D.GT.13.66) GOTO 50
      C1=6839.5*VOL**(-.3492)
      C2=6276.4*VOL**(-.3498)
      DELTA=(C1-C2)/(13.66-12.31)
      COST=C1-DELTA*(D-12.31)
      GOTO 150
   50 IF(D.GT.15.03) GOTO 60
      C1=6276.4*VOL**(-.3498)
      C2=5848.5*VOL**(-.3548)
      DELTA=(C1-C2)/(15.03-13.66)
      COST=C1-DELTA*(D-13.66)
      GOTO 150
   60 IF(D.GT.16.19) GOTO 70
      C1=5848.5*VOL**(-.3548)
      C2=4980.1*VOL**(-.3475)
      DELTA=(C1-C2)/(16.19-15.03)
      COST=C1-DELTA*(D-15.03)
      GOTO 150
   70 IF(D.GT.17.26) GOTO 80
      C1=4980.1*VOL**(-.3475)
      C2=3765.6*VOL**(-.3238)
      DELTA=(C1-C2)/(17.26-16.19)
      COST=C1-DELTA*(D-16.19)
      GOTO 150
   80 IF(D.GT.18.31) GOTO 90
      C1=3765.6*VOL**(-.3238)
      C2=4215.0*VOL**(-.3402)
      DELTA=(C1-C2)/(18.31-17.26)
      COST=C1-DELTA*(D-17.26)
      GOTO 150
   90 IF(D.GT.20.25) GOTO 100
```

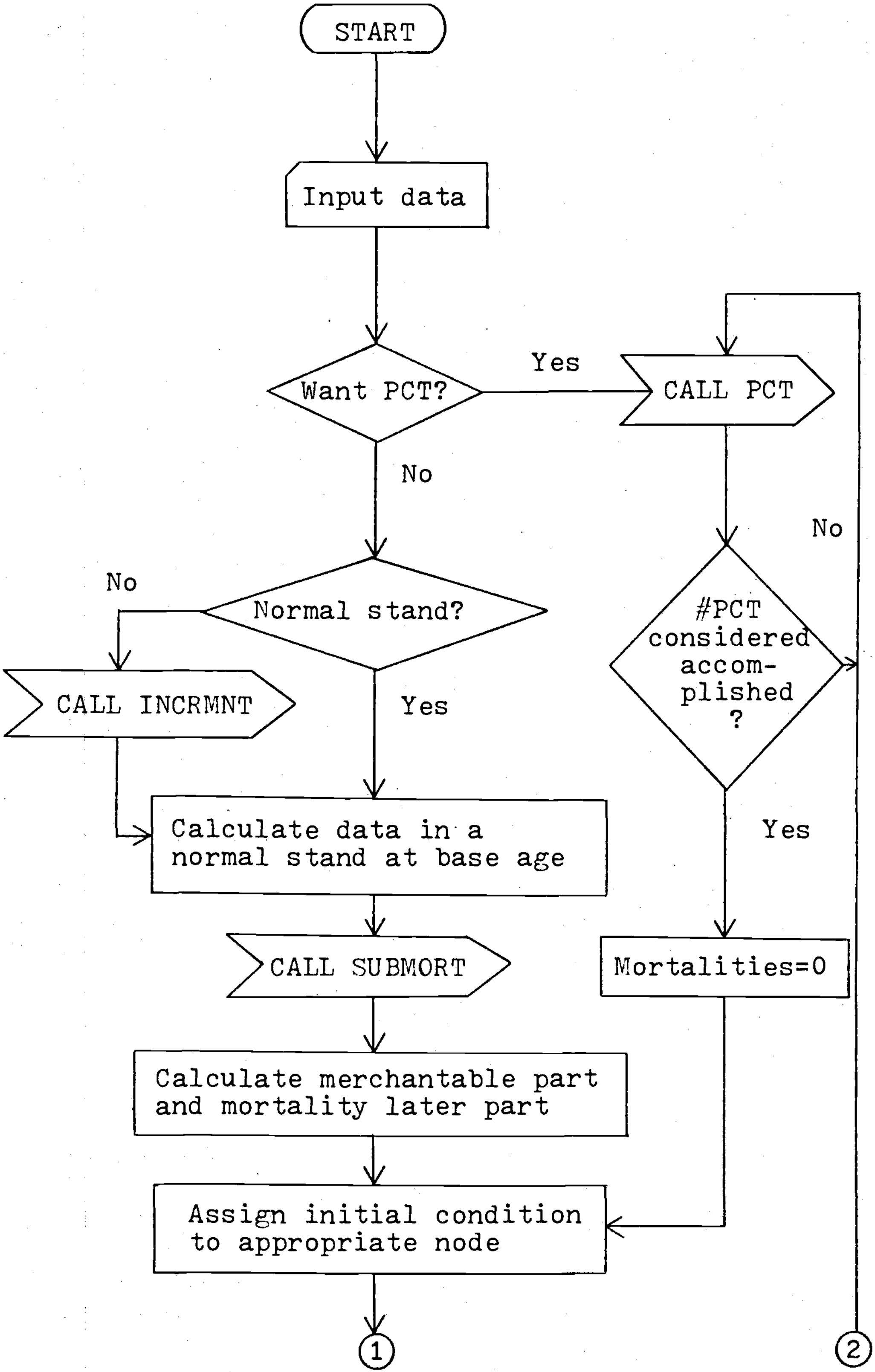
```
      C1=4215.0*VOL**(-.3402)
      C2=3377.5*VOL**(-.3207)
      DELTA=(C1-C2)/(20.25-18.31)
      COST=C1-DELTA*(D-18.31)
      GOTO 150
  100 IF(D.GT.21.90) GOTO 110
      C1=3377.5*VOL**(-.3207)
      C2=4209.5*VOL**(-.3488)
      DELTA=(C1-C2)/(21.9-20.25)
      COST=C1-DELTA*(D-20.25)
      GOTO 150
  110 COST=4209.5*VOL**(-.3488)
******CALCULATE CURRENT REVNEW AS POND VALUE LESS
******LOGGING AND HAUL COST ($/CF)
  150 PONDVAL=9.91+70.81*D
      HAUL=150.
      REVNOW=VOL*(PONDVAL-COST/1.5-HAUL)*0.001
      RETURN
      END
```

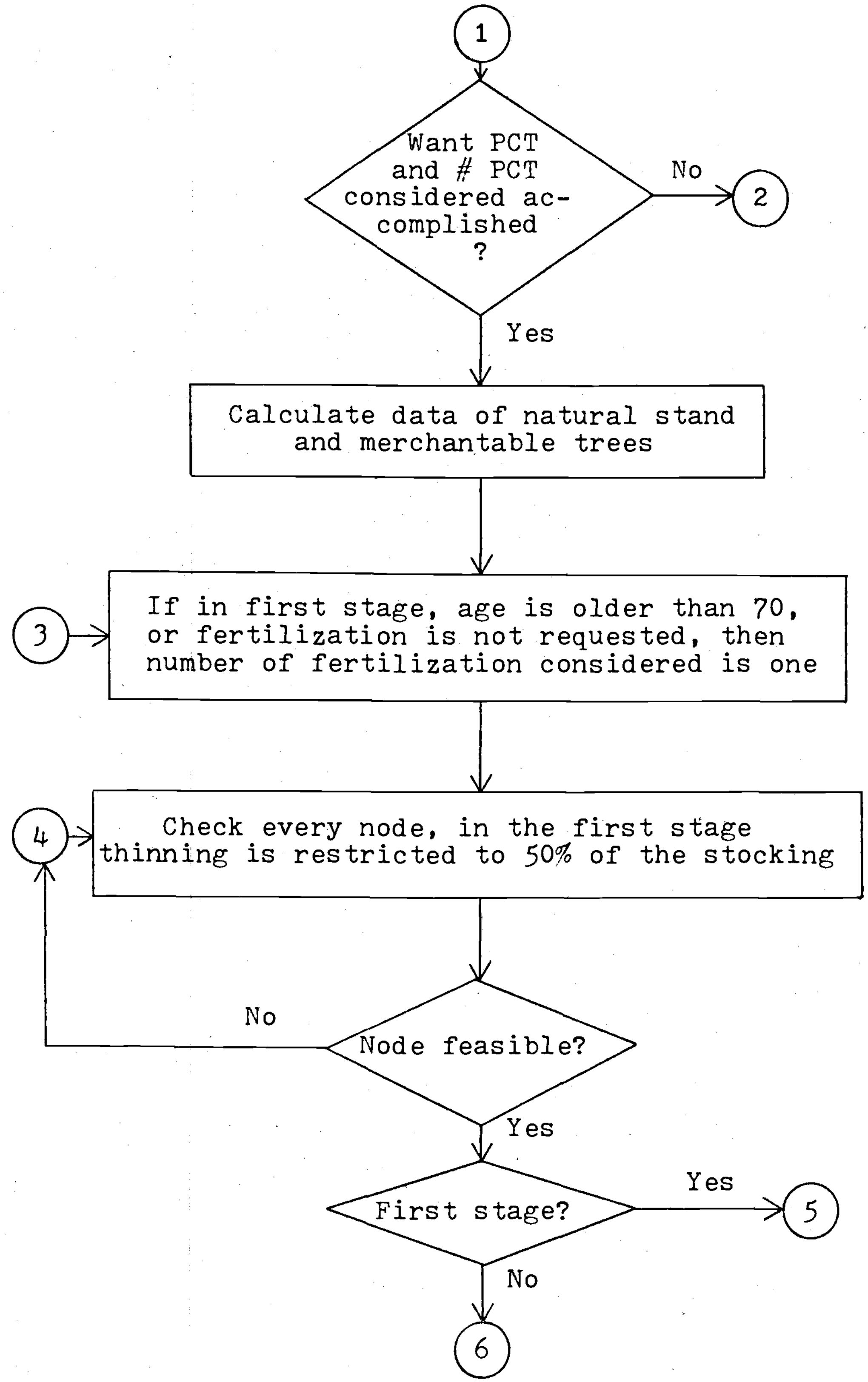
```
      FUNCTION TARIF(DIAM)
      TARIF=(0.00497819*DIAM**2)/(0.005454154*(DIAM**2+16.)*(1.0378+
     $1.4967*(0.0134**(DIAM/10.)))-0.174532)
      RETURN
      END
```

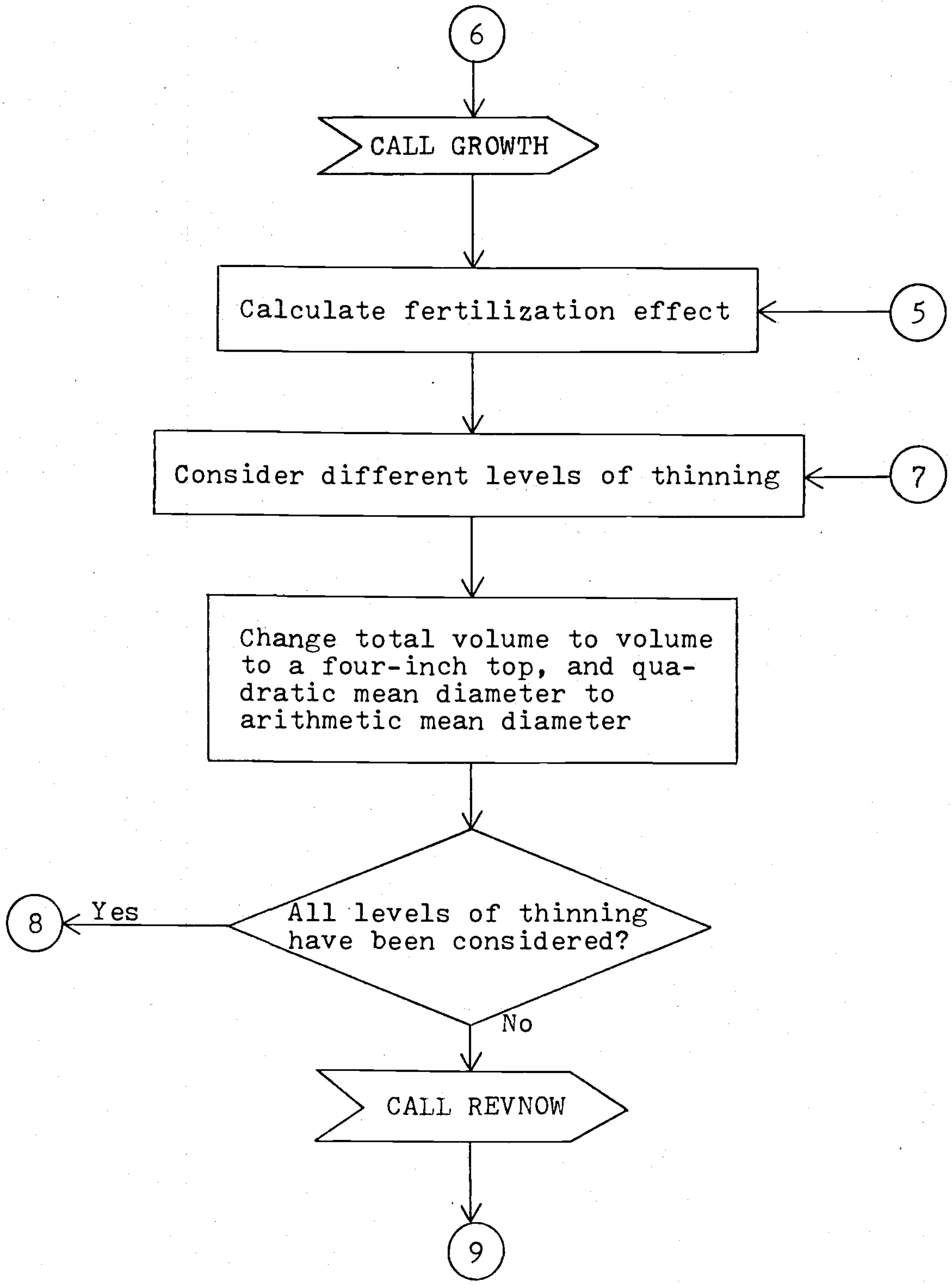
APPENDIX IV

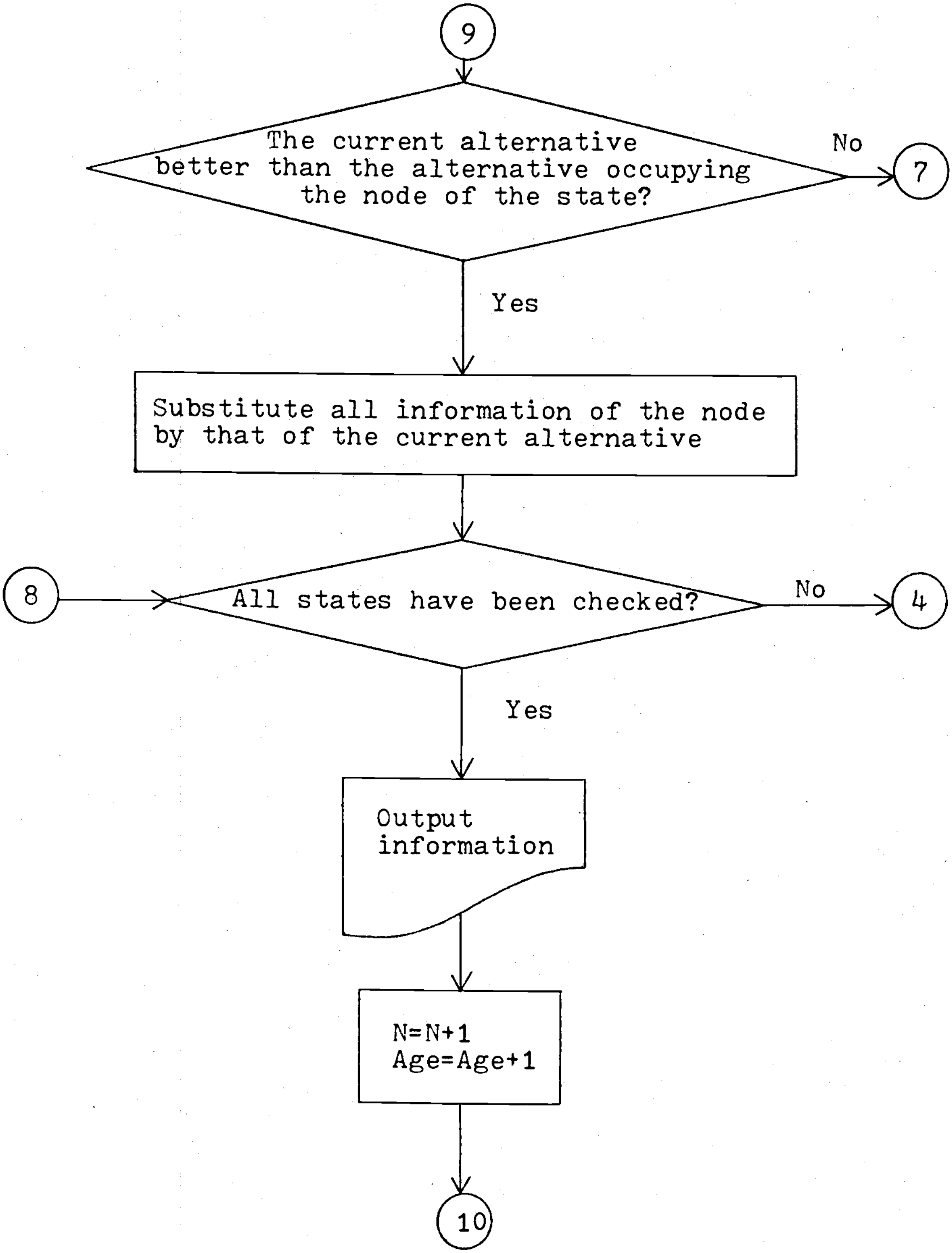
FLOWCHARTS OF DOPT MAIN PROGRAM AND SUBROUTINES

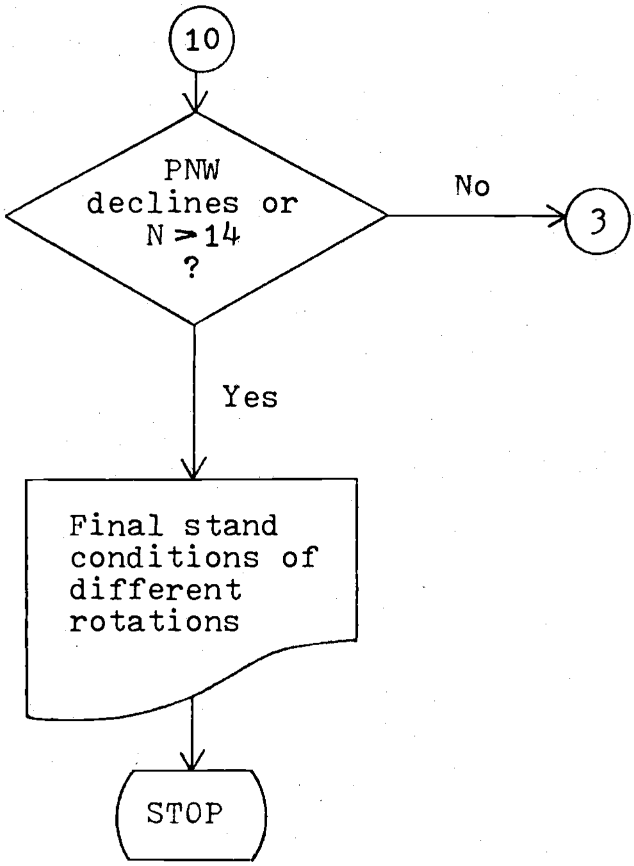
DOPT


START
Input data
Want PCT?
Yes
CALL PCT
No
No
Normal stand?
#PCT considered accom-plished ?
CALL INCRMNT
Yes
Calculate data in a normal stand at base age
Yes
CALL SUBMORT
Mortalities=0
Calculate merchantable part and mortality later part
Assign initial condition to appropriate node
1
2

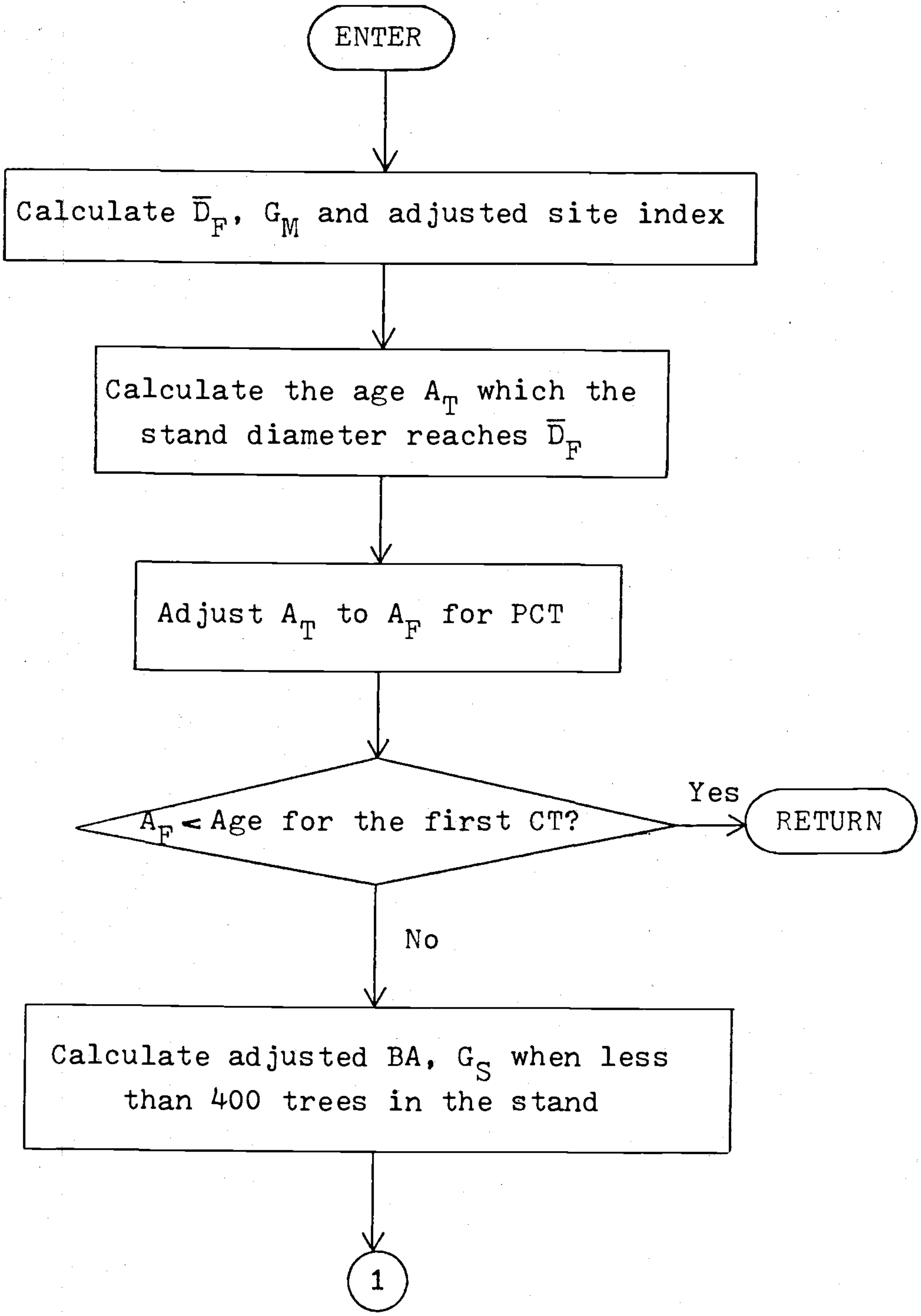

1
Want PCT and # PCT considered accomplished ?
No
2
Yes
Calculate data of natural stand and merchantable trees
3
If in first stage, age is older than 70, or fertilization is not requested, then number of fertilization considered is one
4
Check every node, in the first stage thinning is restricted to 50% of the stocking
No
Node feasible?
Yes
First stage?
Yes
5
No
6

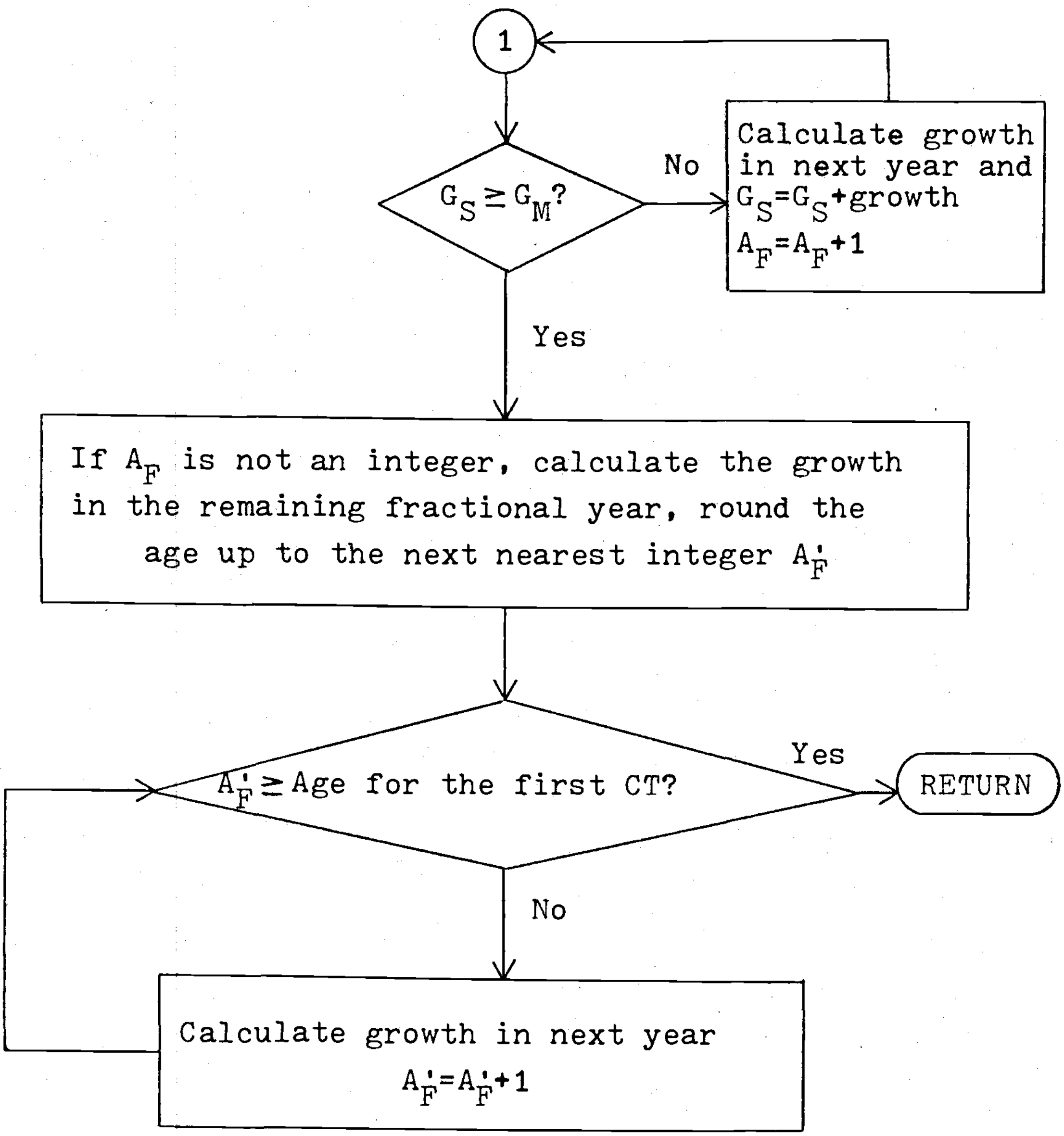

6
CALL GROWTH
Calculate fertilization effect
5
Consider different levels of thinning
7
Change total volume to volume to a four-inch top, and quadratic mean diameter to arithmetic mean diameter
All levels of thinning have been considered?
Yes
8
No
CALL REVNOW
9


9
The current alternative better than the alternative occupying the node of the state?
No
7
Yes
Substitute all information of the node by that of the current alternative
8
All states have been checked?
No
4
Yes
Output information
N=N+1
Age=Age+1
10

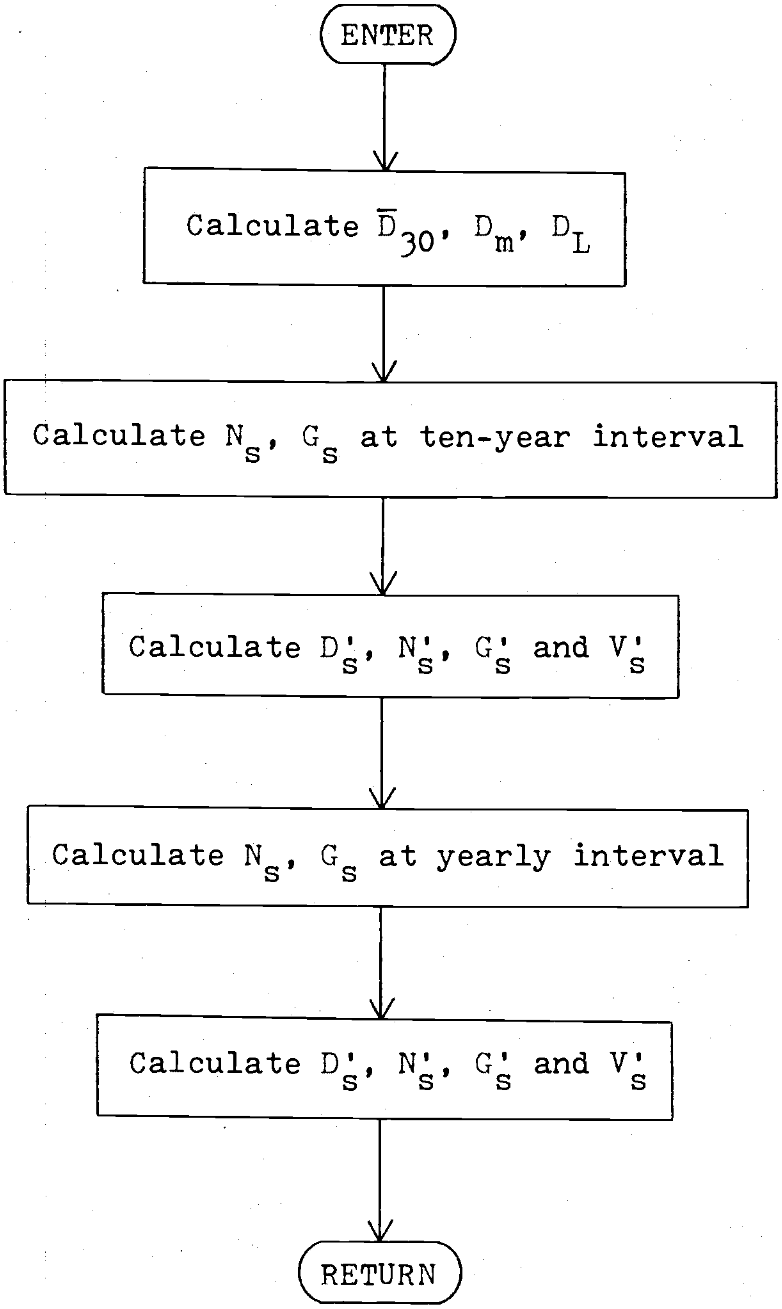

10
PNW declines or N > 14 ?
No
3
Yes
Final stand conditions of different rotations
STOP

PRECOM

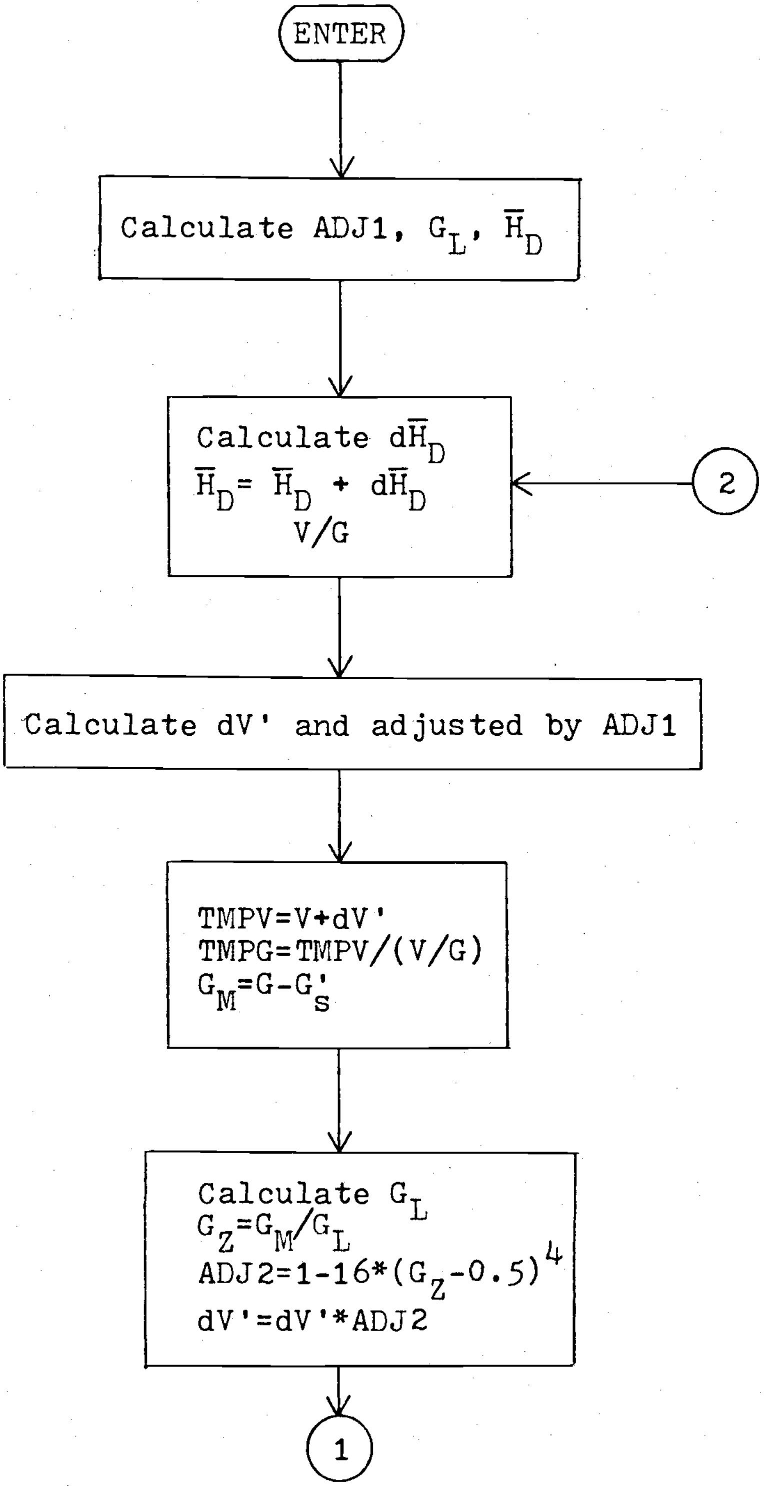

ENTER
Calculate $\overline{D}_F$, G_M and adjusted site index
Calculate the age A_T which the stand diameter reaches $\overline{D}_F$
Adjust A_T to A_F for PCT
$A_F <$ Age for the first CT?
Yes
RETURN
No
Calculate adjusted BA, G_S when less than 400 trees in the stand
1

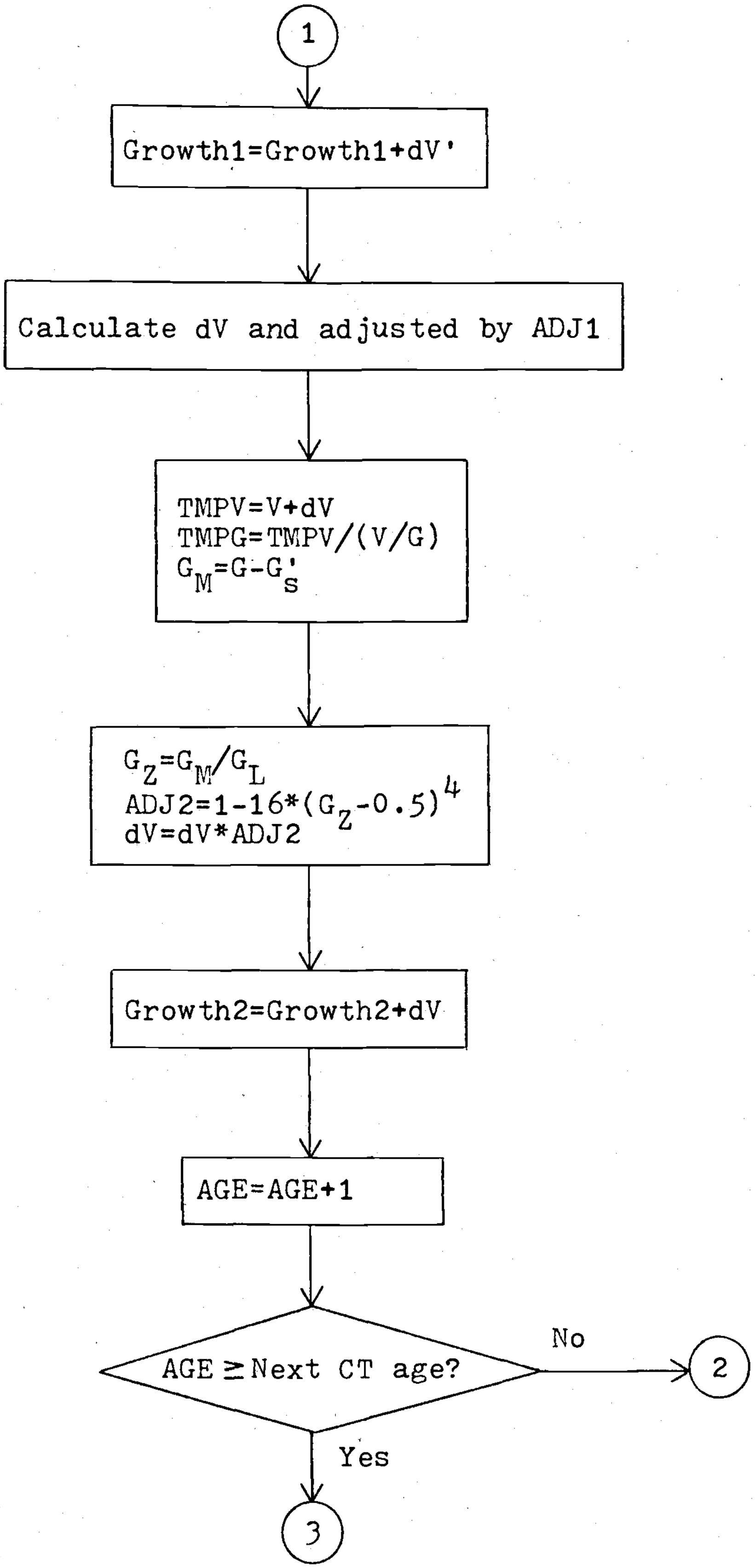

1
$G_S \geq G_M$?
No
Calculate growth in next year and
$G_S = G_S + \text{growth}$
$A_F = A_F + 1$
Yes
If A_F is not an integer, calculate the growth in the remaining fractional year, round the age up to the next nearest integer A_F'
$A_F' \geq$ Age for the first CT?
Yes
RETURN
No
Calculate growth in next year
$A_F' = A_F' + 1$

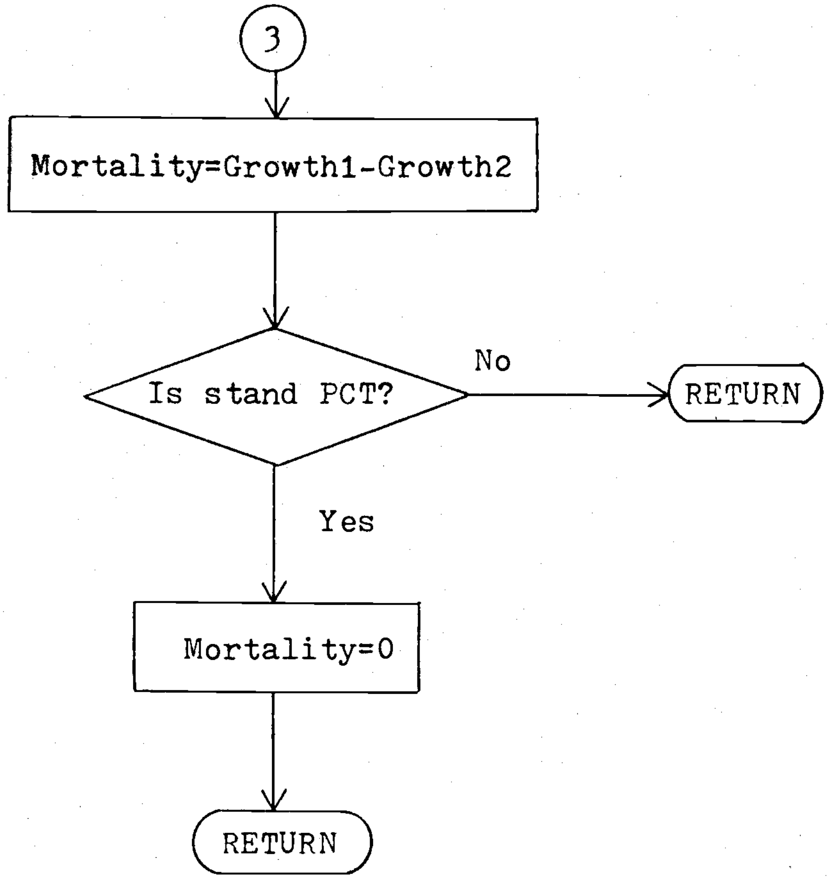
SUBMORT


ENTER
Calculate $\overline{D}_{30}$, D_m, D_L
Calculate N_s, G_s at ten-year interval
Calculate D'_s, N'_s, G'_s and V'_s
Calculate N_s, G_s at yearly interval
Calculate D'_s, N'_s, G'_s and V'_s
RETURN

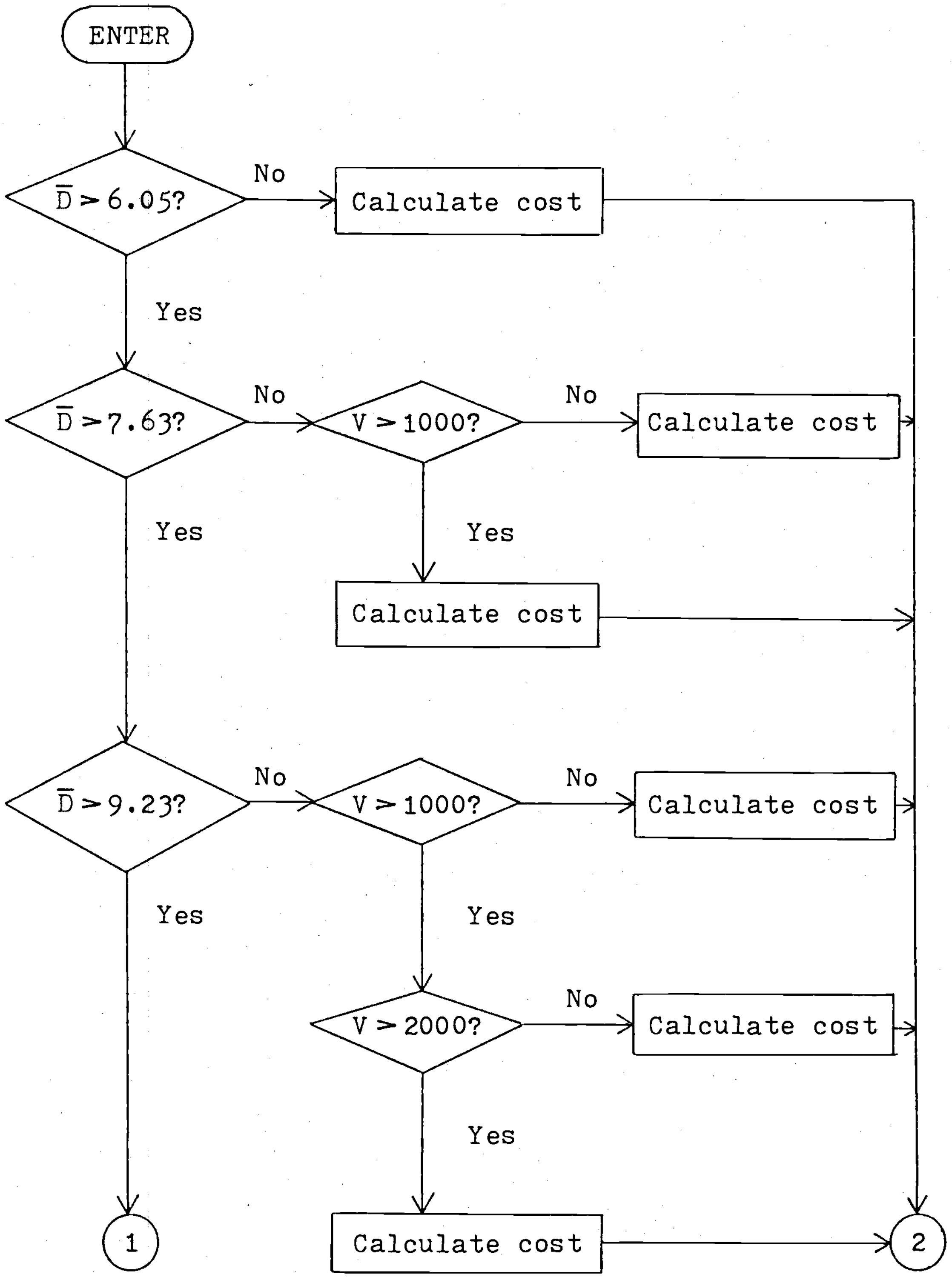
GROWTH

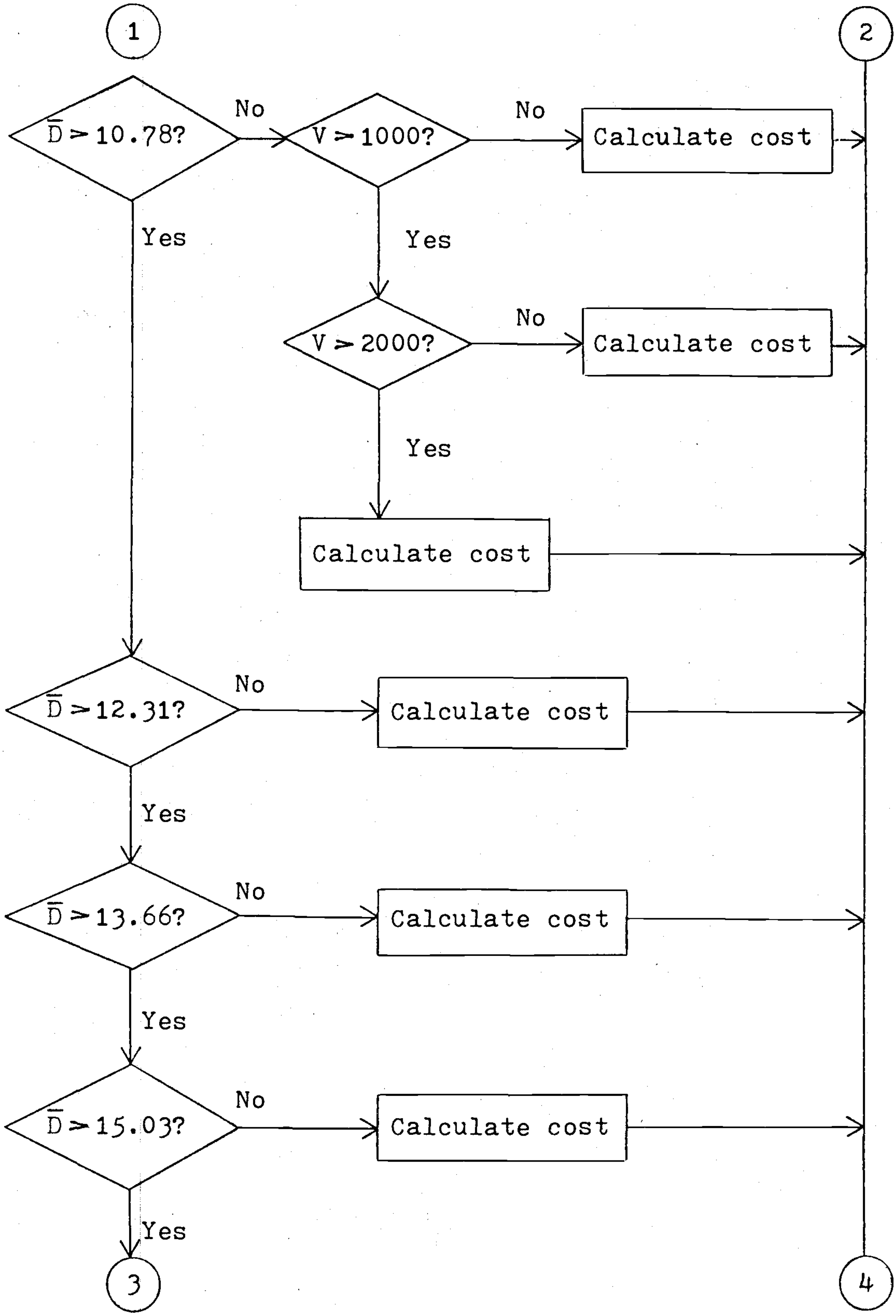

ENTER
Calculate ADJ1, G_L, $\overline{H}_D$
Calculate $d\overline{H}_D$
$\overline{H}_D = \overline{H}_D + d\overline{H}_D$
V/G
2
Calculate dV' and adjusted by ADJ1
TMPV=V+dV'
TMPG=TMPV/(V/G)
$G_M=G-G'_s$
Calculate G_L
$G_Z=G_M/G_L$
$ADJ2=1-16*(G_Z-0.5)^4$
dV'=dV'*ADJ2
1

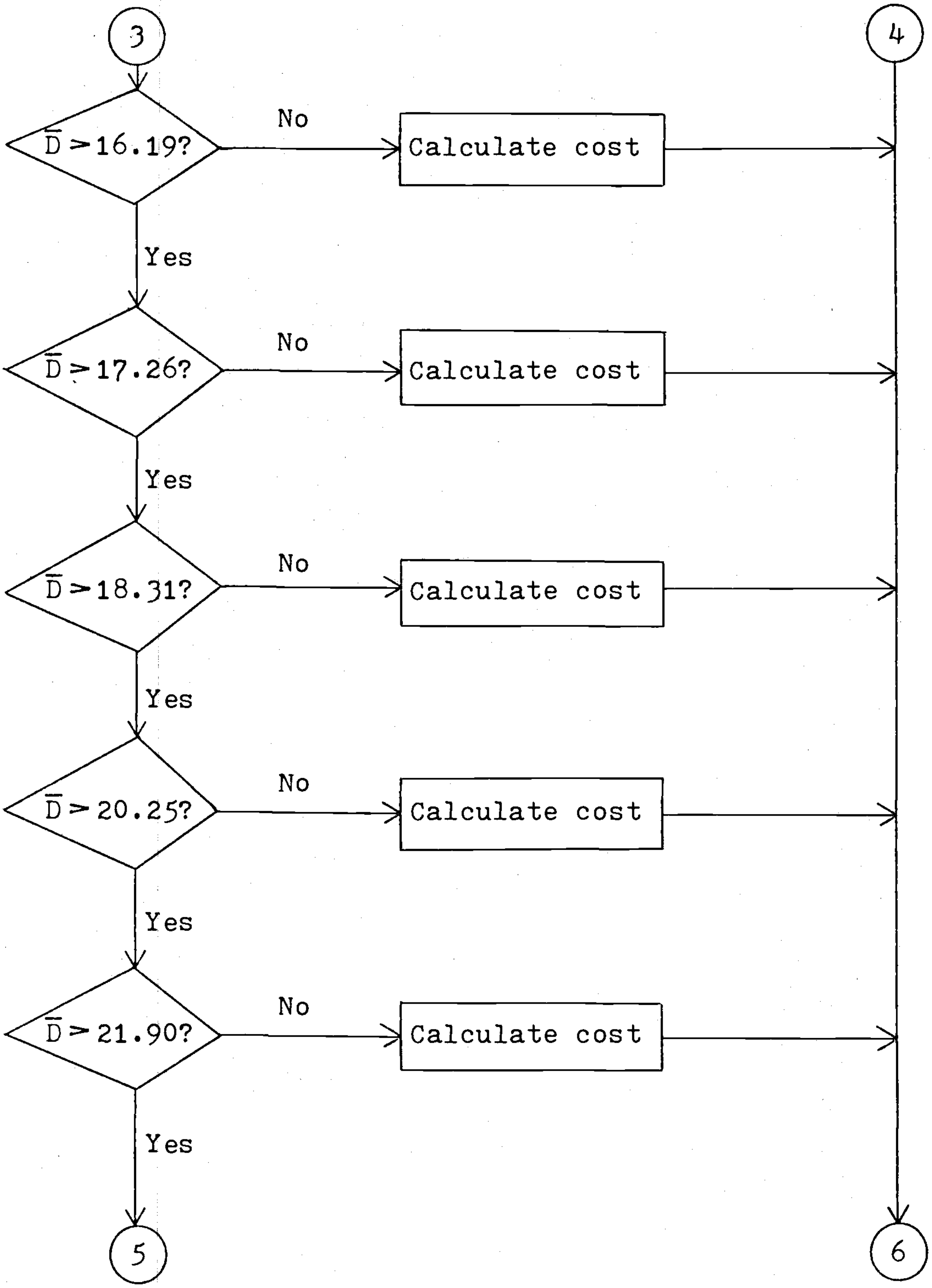

1
Growth1=Growth1+dV'
Calculate dV and adjusted by ADJ1
TMPV=V+dV
TMPG=TMPV/(V/G)
$G_M=G-G'_s$
$G_Z=G_M/G_L$
$ADJ2=1-16*(G_Z-0.5)^4$
dV=dV*ADJ2
Growth2=Growth2+dV
AGE=AGE+1
AGE ≥ Next CT age?
No
2
Yes
3

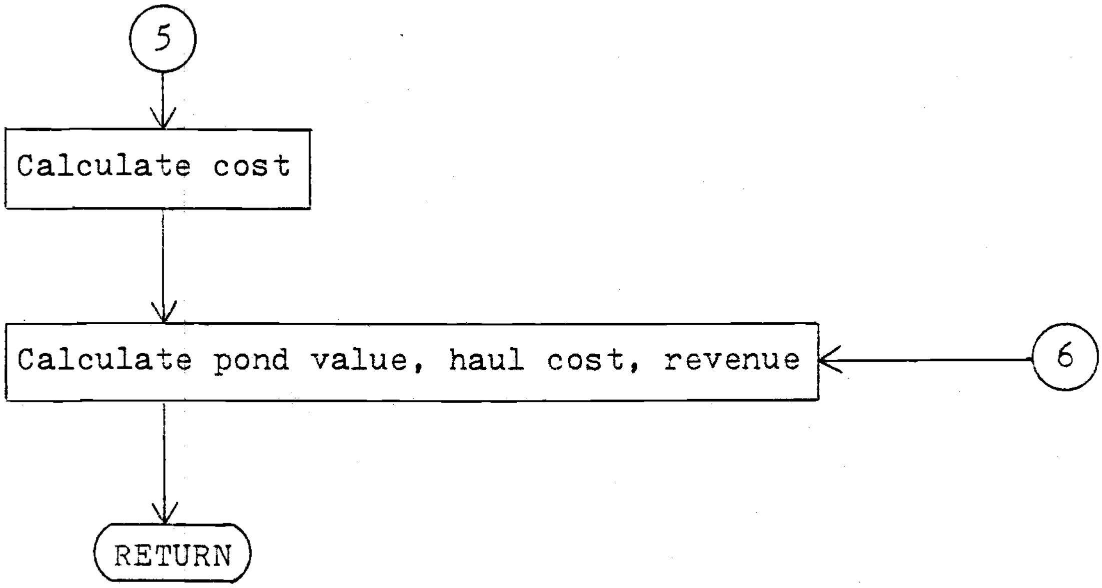

3
Mortality=Growth1-Growth2
Is stand PCT?
No
RETURN
Yes
Mortality=0
RETURN

REVNOW

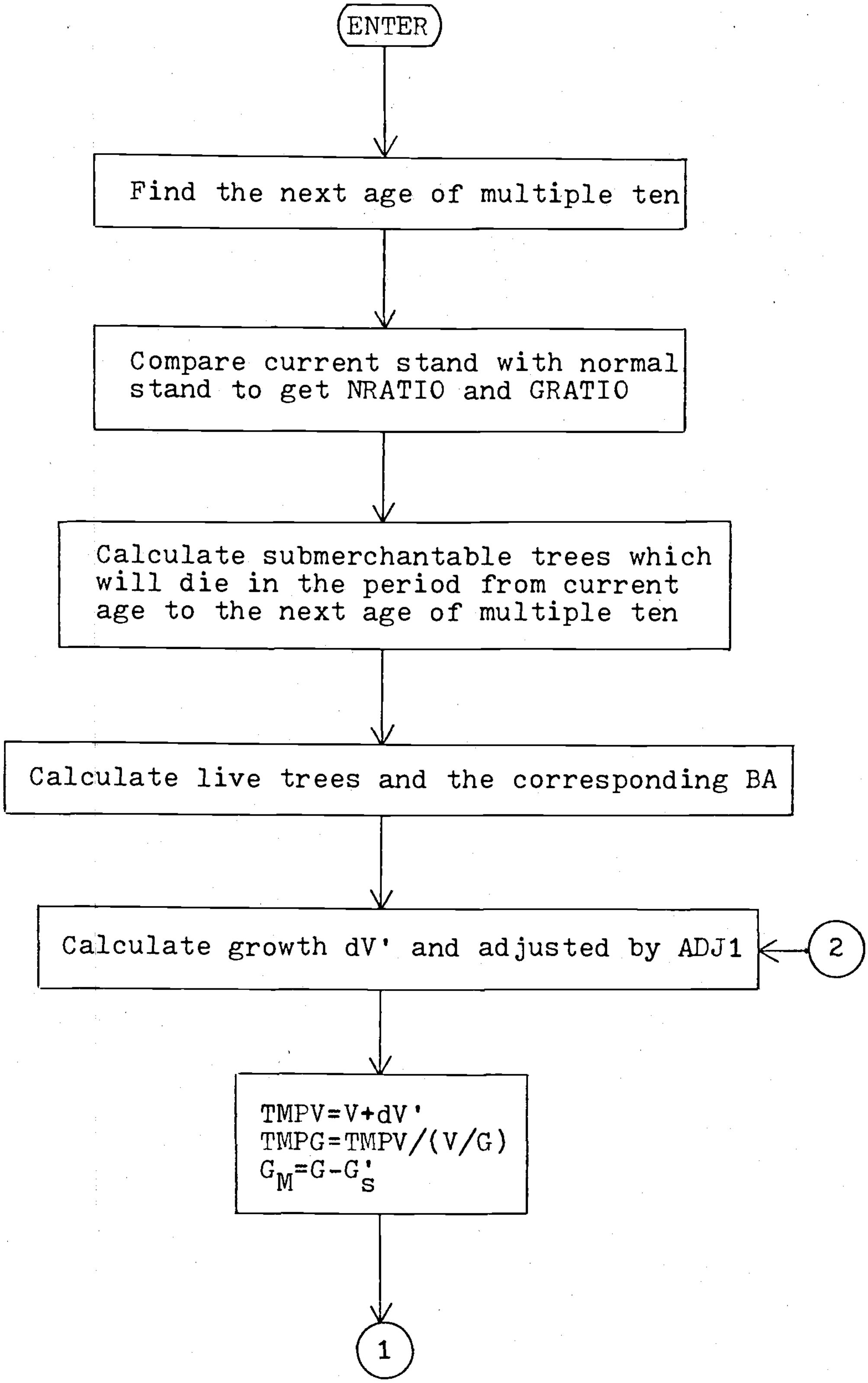

ENTER
$\overline{D} > 6.05?$
No
Yes
Calculate cost
$\overline{D} > 7.63?$
No
Yes
$V > 1000?$
No
Yes
Calculate cost
Calculate cost
$\overline{D} > 9.23?$
No
Yes
$V > 1000?$
No
Yes
Calculate cost
$V > 2000?$
No
Yes
Calculate cost
Calculate cost
1
2

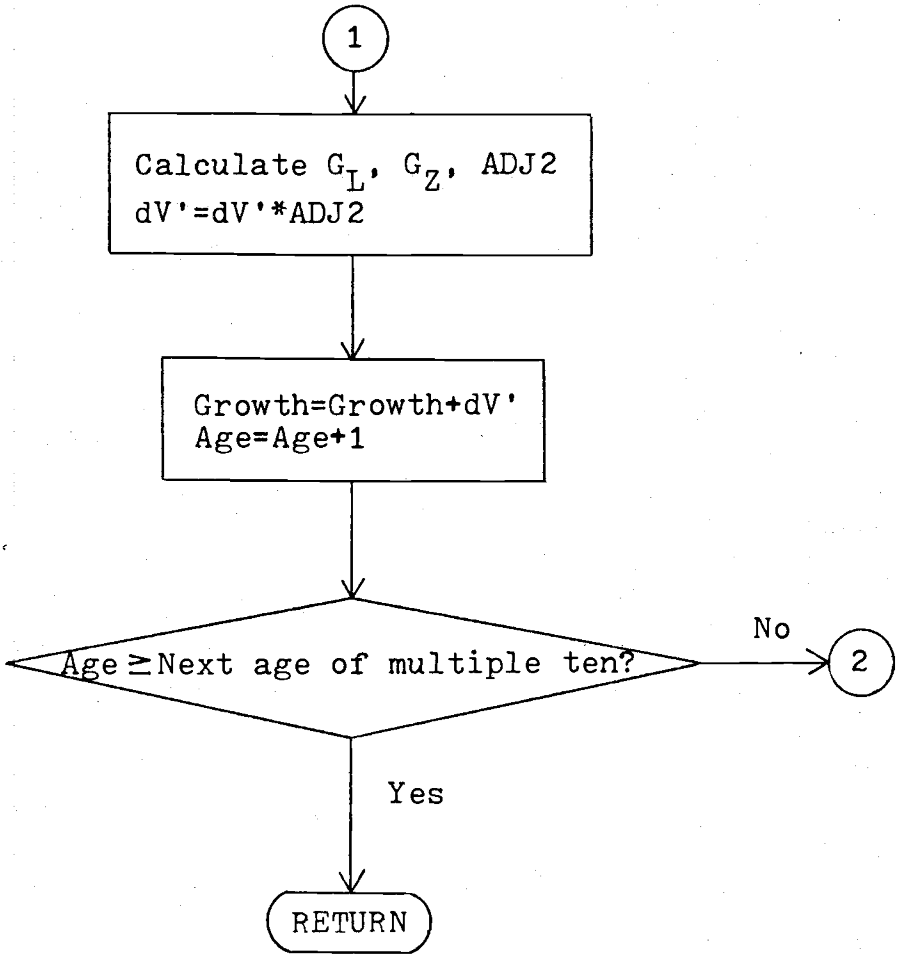

1
2
$\bar{D} > 10.78$?
No
V > 1000?
No
Calculate cost
Yes
Yes
V > 2000?
No
Calculate cost
Yes
Calculate cost
$\bar{D} > 12.31$?
No
Calculate cost
Yes
$\bar{D} > 13.66$?
No
Calculate cost
Yes
$\bar{D} > 15.03$?
No
Calculate cost
Yes
3
4


3
4
$\bar{D}>16.19?$
No
Calculate cost
Yes
$\bar{D}>17.26?$
No
Calculate cost
Yes
$\bar{D}>18.31?$
No
Calculate cost
Yes
$\bar{D}>20.25?$
No
Calculate cost
Yes
$\bar{D}>21.90?$
No
Calculate cost
Yes
5
6


5
Calculate cost
Calculate pond value, haul cost, revenue
6
RETURN

INCRMNT


ENTER
Find the next age of multiple ten
Compare current stand with normal stand to get NRATIO and GRATIO
Calculate submerchantable trees which will die in the period from current age to the next age of multiple ten
Calculate live trees and the corresponding BA
Calculate growth dV' and adjusted by ADJ1
2
TMPV=V+dV'
TMPG=TMPV/(V/G)
$G_M=G-G'_S$
1


1
Calculate G_L, G_Z, ADJ2
dV'=dV'*ADJ2
Growth=Growth+dV'
Age=Age+1
Age ≥ Next age of multiple ten?
No
2
Yes
RETURN

APPENDIX V

NUMBER OF ALTERNATIVES FOR REACHING EACH STATE WHEN TREE INTERVALS USED ARE 20, 25, 30, 40, 50 AND 100

TABLE XLIII. INTERVAL 20

Stage	0	1	2	3	4	5	6
Age	10	30	40	50	60	70	80
#Trees	Number of Alternatives						
400	1	1	3	9	27	81	243
380	0	1	6	27	108	405	1458
360	0	1	9	54	270	1215	5103
340	0	1	12	90	540	2835	13608
320	0	1	15	135	945	5670	30618
300	0	1	18	189	1512	10206	61236
280	0	1	21	252	2268	17010	112266
260	0	1	24	324	3240	26730	192456
240	0	1	27	405	4455	40095	312741
220	0	1	30	495	5940	57915	486486
200	0	1	33	594	7722	81081	729729
180	0	0	33	702	9828	110565	1061424
160	0	0	33	819	12285	147420	1503684
140	0	0	33	945	15120	192780	2082024
120	0	0	33	1044	18252	247536	2824632
100	0	0	33	1143	21681	312579	3762369
80	0	0	33	1242	25407	388800	4928769
60	0	0	33	1341	29430	477090	6360039
40	0	0	33	1440	33750	578340	8095059
20	0	0	33	1539	38367	693441	10175382
0	0	0	33	1539	38367	693441	10175382

TABLE XLIV. INTERVAL 25

Stage	0	1	2	3	4	5	6
Age	10	30	40	50	60	70	80
#Trees	Number of Alternatives						
400	1	1	3	9	27	81	243
375	0	1	6	27	108	405	1458
350	0	1	9	54	270	1215	5103
325	0	1	12	90	540	2835	13608
300	0	1	15	135	945	5670	30618
275	0	1	18	189	1512	10206	61236
250	0	1	21	252	2268	17010	112266
225	0	1	24	324	3240	26730	192456
200	0	1	27	405	4455	40095	312741
175	0	0	27	486	5913	57834	486243
150	0	0	27	567	7614	80676	728271
125	0	0	27	648	9558	109350	1056321
100	0	0	27	729	11745	144585	1490076
75	0	0	27	810	14175	187110	2051406
50	0	0	27	891	16848	237654	2764368
25	0	0	27	972	19764	296946	3655206
0	0	0	27	972	19764	296946	3655206

TABLE XLV. INTERVAL 30

Stage	0	1	2	3	4	5	6
Age	10	30	40	50	60	70	80
#Trees	Number of Alternatives						
400	1	1	3	9	27	81	243
390	0	1	6	27	108	405	1458
360	0	1	9	54	270	1215	5103
330	0	1	12	90	540	2835	13608
300	0	1	15	135	945	5670	30618
270	0	1	18	189	1512	10206	61236
240	0	1	21	252	2268	17010	112266
210	0	1	24	324	3240	26730	192456
180	0	0	24	396	4428	40014	312498
150	0	0	24	468	5832	57510	485028
120	0	0	24	540	7452	79866	724626
90	0	0	24	612	9288	107730	1047816
60	0	0	24	684	11340	141750	1473066
30	0	0	24	756	13608	182574	2020788
0	0	0	24	756	13608	182754	2020788

TABLE XLVI. INTERVAL 40

Stage	0	1	2	3	4	5	6
Age	10	30	40	50	60	70	80
#Trees	Number of Alternatives						
400	1	1	3	9	27	81	243
360	0	1	6	27	108	405	1458
320	0	1	9	54	270	1215	5103
280	0	1	12	90	540	2835	13608
240	0	1	15	135	945	5670	30618
200	0	1	18	189	1512	10206	61236
160	0	0	18	243	2241	16929	112023
120	0	0	18	297	3132	26325	190998
80	0	0	18	351	4185	38880	307638
40	0	0	18	405	5400	55080	472878
0	0	0	18	405	5400	55080	472878

TABLE XLVII. INTERVAL 50

Stage	0	1	2	3	4	5	6
Age	10	30	40	50	60	70	80
#Trees	Number of Alternatives						
400	1	11	3	9	27	81	243
350	0	1	6	27	108	405	1458
300	0	1	9	54	270	1215	5103
250	0	1	12	90	540	2835	13608
200	0	1	15	135	945	5670	30618
150	0	0	15	180	1485	10125	60993
100	0	0	15	225	2160	16605	110808
50	0	0	15	270	2970	25515	187353
0	0	0	15	270	2970	25515	187353

TABLE XLVIII. INTERVAL 100

Stage	0	1	2	3	4	5	6
Age	10	30	40	50	60	70	80
#Trees	Number of Alternatives						
400	1	1	3	9	27	81	243
300	0	1	6	27	108	405	1458
200	0	1	9	54	270	1215	5103
100	0	0	9	81	513	2754	13392
0	0	0	9	81	513	2754	13392

APPENDIX VI

FINAL STAND CONDITIONS AND MANAGEMENT REGIMES FOR TREE INTERVAL 15 AND BASAL AREA INTERVALS 10, 16, 20, 30 AND 40 ON SITE 140

TABLE XLIX. FINAL STAND CONDITIONS FOR BASAL AREA INTERVAL 10 SQUARE FEET.

Rotation Age (years)	Harvest Diameter (inches)	Harvest Basal Area (sq. ft.0	Harvest Volume (cu. ft.)	MAI (cu. ft.)	Present Net Worth ($/acre)	Soil Expectation ($/acre)
40	12.29	173.07	5284.15	180.10	376.05	542.29
50	16.34	152.88	5384.52	198.09	912.87	1182.64
60	20.68	139.99	5423.02	203.60	1340.49	1614.53
70	22.42	164.47	6819.85	210.22	1534.42	1756.22
80	24.80	50.33	2196.27	192.41	1504.61	1660.67

TABLE L. MANAGEMENT REGIME FOR BASAL AREA INTERVAL 10 SQUARE FEET.

Stand Age (years)	Amount of Fertilization (pounds)	Mean Diameter (inches)	Mer-chantable Trees	Basal Area (sq. ft.)	Volume Left (cu. ft.)	Volume Cut (cu. ft.)	Cumulative PNW ($/acre)
30	200	8.5	210	82.0	2121.9	1919.8	- 309.4
40	200	12.2	105	85.2	2600.4	2600.4	- 31.4
50	200	16.2	105	150.6	5303.7	0.0	- 45.2
60	400	18.8	60	116.2	4500.5	3375.4	493.8
70	-	22.4	0	0.0	0.0	6819.8	1534.4

TABLE LI. FINAL STAND CONDITIONS FOR BASAL AREA INTERVAL 16 SQUARE FEET.

Rotation Age (years)	Harvest Diameter (inches)	Harvest Basal Area (sq. ft.)	Harvest Volume (cu. ft.)	MAI (cu. ft.)	Present Net Worth ($/acre)	Soil Expectation ($/acre)
40	12.29	173.07	5284.15	180.10	376.05	542.29
50	16.84	139.27	4904.92	195.93	921.95	1194.40
60	21.09	145.58	5639.58	202.31	1362.52	1641.06
70	24.06	142.13	5893.47	207.31	1537.89	1760.20
80	26.79	117.46	5125.69	196.04	1503.89	1659.88

TABLE LII. MANAGEMENT REGIME FOR BASAL AREA INTERVAL 16 SQUARE FEET.

Stand Age (years)	Amount of Fertilization (pounds)	Mean Diameter (inches)	Merchantable Trees	Basal Area (sq. ft.)	Volume Left (cu. ft.)	Volume Cut (cu. ft.)	Cumulative PNW ($/acre)
30	200	8.5	210	82.0	2121.9	1919.8	- 309.4
40	200	12.2	90	73.0	2228.9	2971.9	23.2
50	400	16.7	90	136.9	4822.1	0.0	9.4
60	400	19.8	45	96.2	3726.5	3726.5	645.4
70	-	24.1	0	0.0	0.0	5893.5	1537.9

TABLE LIII. FINAL STAND CONDITIONS FOR BASAL AREA INTERVAL 20 SQUARE FEET.

Rotation Age (years)	Harvest Diameter (inches)	Harvest Basal Area (sq. ft.)	Harvest Volume (cu. ft.)	MAI (cu. ft.)	Present Net Worth ($/acre)	Soil Expectation ($/acre)
40	12.29	173.07	5284.15	180.10	376.05	542.29
50	16.66	136.25	4798.56	195.62	894.39	1158.70
60	21.06	145.11	5621.25	203.39	1351.68	1628.01
70	22.11	200.04	8294.90	183.71	1529.55	1750.66
80	25.18	51.86	2262.95	193.29	1510.99	1667.72

TABLE LIV. MANAGEMENT REGIME FOR BASAL AREA INTERVAL 20 SQUARE FEET.

Stand Age (years)	Amount of Fertilization (pounds)	Mean Diameter (inches)	Merchantable Trees	Basal Area (sq. ft.)	Volume Left (cu. ft.)	Volume Cut (cu. ft.)	Cumulative PNW ($/acre)
30	400	8.5	210	82.0	2121.9	1919.8	- 309.4
40	200	12.3	135	111.3	3390.7	1887.2	- 143.8
50	200	15.5	75	97.9	3446.9	2757.5	262.6
60	400	19.3	75	151.8	5881.6	0.0	252.3
70	-	22.1	0	0.0	0.0	6294.9	1529.6

TABLE LV. FINAL STAND CONDITIONS FOR BASAL AREA INTERVAL 30 SQUARE FEET

Rotation Age (years)	Harvest Diameter (inches)	Harvest Basal Area (sq. ft.)	Harvest Volume (cu. ft.)	MAI (cu. ft.)	Present Net Worth ($/acre)	Soil Expectation ($/acre)
40	12.29	173.07	5284.15	180.10	376.05	542.29
50	18.93	87.93	3096.85	183.37	849.03	1099.93
60	22.66	126.00	4881.09	192.91	1264.77	1523.33
70	21.88	156.68	6496.88	211.73	1476.26	1689.66
80	24.78	100.49	4385.04	199.47	1459.51	1610.89

TABLE LVI. MANAGEMENT REGIME FOR BASAL AREA INTERVAL 30 SQUARE FEET.

Stand Age (years)	Amount of Fertilization (pounds)	Mean Diameter (inches)	Mer-chantable Trees	Basal Area (sq. ft.)	Volume Left (cu. ft.)	Volume Cut (cu. ft.)	Cumulative PNW ($/acre)
30	400	8.5	210	82.0	2121.9	1919.8	- 309.4
40	200	12.3	165	136.0	4151.8	1132.3	- 246.1
50	200	14.8	90	108.0	3803.1	3169.2	189.6
60	400	18.2	60	108.9	4205.2	2102.6	494.3
70	-	21.9	0	0.0	0.0	6496.9	1476.3

TABLE LVII. FINAL STAND CONDITIONS FOR BASAL AREA INTERVAL 40 SQUARE FEET.

Rotation Age (years)	Harvest Diameter (inches)	Harvest Basal Area (sq. ft.)	Harvest Volume (cu. ft.)	MAI (cu. ft.)	Present Net Worth ($/acre)	Soil Expectation ($/acre)
40	12.29	173.07	5284.15	180.10	376.05	542.29
50	17.49	125.15	4407.86	194.49	932.39	1207.93
60	21.75	154.84	5998.40	203.28	1410.38	1698.71
70	25.66	161.58	6700.23	205.42	1496.76	1713.12
80	31.62	81.81	3569.88	178.34	1485.87	1639.99

TABLE LVIII. MANAGEMENT REGIME FOR BASAL AREA INTERVAL 40 SQUARE FEET.

Stand Age (years)	Amount of Fertilization (pounds)	Mean Diameter (inches)	Mer-chantable Trees	Basal Area (sq. ft.)	Volune Left (cu. ft.)	Volume Cut (cu. ft.)	Cumulative PNW ($/acre)
30	400	8.5	210	82.0	2121.9	1919.8	- 309.4
40	400	12.3	75	61.8	1887.2	3397.0	79.4
50	200	17.5	60	100.1	3526.3	881.6	206.8
60	400	21.6	45	114.6	4441.4	1480.5	475.6
70	-	25.7	0	0.0	0.0	6700.2	1496.8