AN ABSTRACT OF THE THESIS OF

Richard Orin Wheeler for the Ph. D. in Agricultural Economics

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The economic efficiency related to the use of the range resource is extremely important to an area such as Southeastern Oregon where most of the gross income of the area is obtained from range livestock production; important to the economic progress of the region and to the development of a desirable social structure for the citizenry.

The investigation of the decision making problem under conditions of uncertainty implies that the certainty problem can be solved for all uncertain alternatives. The certainty alternatives are admissible only when the economic efficiency criterion is met. The criterion used in this study for decision making under uncertainty involved maximizing expected utility through the use of a multivariate Bayesian statistical model. The uncertain states of nature were next year's cattle price and forage production. The optimal strategy involved the structure of the cattle inventory.

A set of equations was developed which generate the input-output coefficients and the objective function values for a linear program solution for any cattle inventory system with respect to expected calving percentage, death loss, replacement policy, cattle weights and prices, and for any number of time increments. A set of
homogeneous livestock inventory systems were defined such that a linear program model can be used to determine the optimal inventory structure under certainty conditions. The link between the resource equation system and the linear program improves the feasibility of effectively getting large volumes of budgeted ranch data into an optimizing framework.

Primary data were used to establish costs and returns and the land use structure for representative units. Despite the variance in physical structure that exist among the ranch population ranch units, there are certain consistencies for which some general results can be inferred:

(1) Under conditions of certainty with respect to price and forage productions, the optimal livestock inventory structure for the study area would tend toward the production of yearling and other steer beef. The study area ranch units tend toward the production and sale of calve beef. It is postulated that this discrepancy between what "should be" and "what is" is a result of the dependency of the ranch unit on public lands. The public land input is commensurate with an administrative definition rather than the physical production relationship. This difference is defined as a misallocation and the models developed can be used to quantify this misallocation. This condition is independent of the present public grazing fee structure.

(2) The primary data indicate that the area could absorb an increase of 20 percent in the total spring, summer, and fall range forage with substantially the present resource structure. Range
improvements have an expected marginal value product of $3.00 per animal unit month.

(3) It is meaningful through the use of the models developed to think in terms of a general population utility function for purposes of explaining the population's economic behavior with respect to cattle inventory structure and for predicting economic stimuli responses.

It is concluded that multivariate regression models in obtaining a posteriori weights for decision making under uncertainty can be formulated as an operational management tool. The combination of subjective and objective evidence into the scientific approach for decision making has wide use implications beyond firm management problems.
OPTIMUM CATTLE INVENTORY SYSTEMS UNDER CONDITIONS OF CERTAINTY AND UNCERTAINTY--SOUTHEASTERN OREGON

By

Richard Orin Wheeler

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Professor of Agricultural Economics in Charge of Major

Redacted for Privacy

Head of Department of Agricultural Economics

Redacted for Privacy

Dean of Graduate School

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Appreciation to one's wife and family is understood by all who have gone before or are presently involved in the task.
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CHAPTER I

INTRODUCTION

This chapter presents some of the physical characteristics of the study area and indicates the heavy dependency of the area on range livestock production. The strong dependency of the study area on range livestock production is argued to be a justification of the study. The objectives of the study are presented along with supporting arguments for the use of subjective probability in science and in the decision making process under conditions of uncertainty.

The land area of Lake, Harney, and Malheur Counties in Oregon is approximately 18 million acres. Of this 18 million, about 14 million acres are public domain. The majority of these public lands are administered by either the Forest Service of the United States Department of Agriculture or the Bureau of Land Management of the United States Department of the Interior. These agencies administer the majority of the public lands used for the grazing of domestic livestock. A large antelope refuge is maintained in Lake County and a bird refuge is a major function of the Malheur National Wildlife area in Harney County. Most of the lands administered by the United States Forest Service which influence the production patterns of the study area are located along the northwestern and western boundaries. They are the Malheur National Forest which is largely outside the study area, the Ochoco National Forest, the Deschutes National Forest, and the Fremont National Forest, all of which are in part within the study area.
The study area is characterized as a "high desert plateau" livestock grazing area. Approximately 50 percent of the land area receives less than 10 inches of annual precipitation. This semi-arid climate is further characterized by very warm and dry summers with cold winters. Most of the moisture occurs as snow between the months of November and March. A rainy period, with May being the wettest month, usually occurs between April 1 and July 1. It is the rainfall during this period which is crucial to the amount of range forage that will be available for livestock grazing.

Sagebrush covers most of the range areas, with cheatgrass and blue bunch grass being the dominant grasses. Alfalfa and alfalfa mixtures are cut for hay in the irrigated valleys. Along the rivers and streams are where the majority of the ranch headquarters are found. Wild hay is also cut from areas that receive flood and sub-irrigation during the spring months. River and stream bottom meadows are used for fall grazing and for winter hay production. The year-to-year variation in hay production is directly related to the source of irrigation on nonirrigation as the case may be. Ditch irrigation with good water rights is the most stable source for the production of hay.

Small grains harvested for sale are uncommon. For instance, in 1964 less than 30 farms in Harney County reported sales of wheat. Less than 1,000 total acres of wheat are produced in Harney County. However, the excluded area of Malheur County involves a large irrigated area that produces many different specialty crops. The Snake River provides a considerable amount of irrigation water for this area. In fact, about 70 to 75 percent of the dollar value of
agricultural production in Malheur County is produced in this small area. The amount of range livestock production in this excluded area is negligible. However, the alfalfa production along with other sources of feed for feed-lot operations do have a direct economic influence on the surrounding range area. This effect is not analyzed in this study.

As a result of the physical limitations noted above, the production alternatives for the study area are few in number. About the only present feasible agricultural use for most of the land is the grazing of livestock. The economic impact of federal land policy as related to the improvement or non-improvement of range lands for domestic livestock use and the value of the grazing privileges to the holder of the privilege have been a matter of controversy for a long time. An earlier study by Nielsen (1965) sheds a great deal of light on the economic evaluation of the investment in federal range improvements as related to time of use and the resulting value to individual ranchers. This study should supplement and will make use of many of the results of that study.

**Justification for the Study**

Justification for the expenditure of research funds for a study such as this has no single criterion. The sale of livestock other than poultry and dairy products for the study area amounted to over 20 million dollars in 1964 (U.S.D.C., B.C., 1966). An estimated one-fourth of a billion dollars invested value for private land and buildings would indicate that some economic research pertaining to management and production alternatives for these resources is in order.
An agricultural area such as Harney County where 92 percent of the farm income is derived from livestock production would indicate that the well being of the local citizens is directly related to how well the range resource and associated resources are managed. Society in general is interested in the expected return from public expenditures for such items as range improvements in this area as compared with the possible allocation in other areas.

Since the production of range livestock is the major use of the resources of this area, the composition of the livestock inventory as related to the resource base is a fundamental consideration of this investigation. Some approaches to the proper resource balance may be too complicated for direct practical application by the ranch managers but some rules of thumb may have meaningful value. Results that indicate that the only realistic recommendation is the use of rules of thumb will not be considered in this analysis as unfavorable recommendations for management. This is in agreement with the position taken in an article by Buamol and Quandt (1964, p. 23) who stated, "It is easy to jump to the conclusion that the widespread use of rules of thumb is good evidence of sloppy workmanship on the part of management." They then argue that on the contrary rules of thumb are among the more efficient pieces of equipment of optimal decision making by the entrepreneur.

If, in fact, a study pertaining to the management of the study area ranch resources is justified, then it is a matter of selecting the relevant area of investigation. Assuming that the feasible agricultural use is limited to range beef production, then it narrows
the economic problem initially is one of inward inspection of firm resource allocation as related to range livestock production. After determining the value of the resources through optimization within the ranch firm, it is then meaningful to compare this value with alternatives such as recreational use of the public lands.

The output of the ranch firm is not a single product but a multiple product. The firm produces different meat products and consequently receives different prices for these various products. The livestock inventory composition dictates what this multiple output will be and also determines the time and amount of resources necessary for production. Since the ranch entrepreneur faces an uncertainty condition with respect to product prices, as well as uncertainty related to the amount of resources that will be available for the production process, the management of the inventory composition becomes the focal point of the management problem and is one of the central issues of this thesis. This requires investigation of those forces which influence price and the availability of resources which will in turn influence the optimum inventory composition. As a result of allowing the uncertainty condition to enter the decision-making process, this will require the definition of optimum that may well be different than the optimum under the assumption of certainty.

There is a general consensus among livestock producers and extension personnel that the percent of calves that can be expected to mature as related to the number of breeding cows is the important consideration in determining the optimum composition of the livestock inventory. This is based on the rather simple rationale that if the
probability of getting a live calf is very low, then if a calf does live, let it grow to a very heavy weight before selling the animal. An economist who would dispose of the issue by simply suggesting that it is a mere question of equating the expected marginal revenue with the expected marginal cost completely disregards the practical complications of establishing either quantity. The willingness of agricultural economists to isolate and investigate problems that may be limited initially in scope to applied problems as opposed to broader economic problems seems to have been well accepted by society if one uses the amount of research resources that society has been willing to provide for agricultural economic research compared to funds offered for nonagricultural economic research.

A small addition to the present state of the arts with respect to the use of decision making under uncertainty and the use of Bayesian statistics to an applied economic problem would add to the justification of this effort. It is hoped that the probability of this being the case is greater than zero.

**Purpose and Objectives**

One of the objectives and purposes of this thesis is to fulfill the general objective of the research agreement between the Oregon Agricultural Experiment Station and the Farm Production Economics Division, Economic Research Service, United States Department of Agriculture. The objective as stated in the agreement was "to determine the adjustments in the organization and management of cattle ranches that are needed for profitable livestock ranching operations in Oregon under existing range
and ranch resource conditions and with the introduction of various range improvement practices."

An explicit objective of the study is to formulate an optimal livestock inventory system for the study area as a function of various calving percentages, livestock weights, cost structure, culling practices, availability of the feed input by time period, death rates, labor availability, and product prices. The development of the certainty model will allow fulfilling the objective of development of an uncertainty decision theory model. An objective of the uncertainty problem is the use of a multivariate prediction model using the Bayesian criterion for a decision-making model. For decision making under uncertainty, it is necessary to obtain a posteriori probabilities on the unknown state of nature "product price". Uncertainty with respect to the factor "range forage" is also proposed as a part of the decision problem.

Although the direct results of the study are intended to fulfill the objectives for the study area of Malheur, Harney and Lake Counties in Oregon, the procedures are intended to be kept sufficiently broad to allow their use for any ranch study area in the western United States.

This thesis is also an attempt to apply some recent developments in probability and statistical theory from a Bayesian viewpoint to a farm management-production economics problem. Although some of the results may rest on a theoretical basis that is closely related to a particular probability philosophy, the intent is not one of justifying old or developing new statistical techniques but rather the applica-
tion of a particular consequence of Bayes's theorem to a practical economic problem.

The reader who objects to the following statement by I. J. Good (1965, p. 12) will find the chapter pertaining to decision making under uncertainty less than satisfying. Good states:

"Bayes's theorem itself is a trivial consequence of the product axiom of probability, and it is not a belief in this theorem that makes a person a Bayesian. Rather it is a readiness to incorporate intuitive probability into statistical theory and practice, and into the philosophy of science and of the behavior of humans, animals and automata, and in an understanding of all forms of communication and everything."

This presentation does not propose to argue that there should be a different statistical theory between one that is correct for economic problems and another for "pure" science problems. However, without a great deal of explanation it seems almost self-evident that problems associated with estimating the probability of a price of a commodity at some future date and one of estimating the probable quantities of certain elements available in the soil present at least initially, some practical conceptual differences. This is true whether one allows or does not allow a role for subjective probability.

There is a noticeable lack of stated use of subjective probability in economic research. In fact, there seems at times to be almost an obsession with the researchers to emphasize and point out the use of "objective estimates" where "objective" is usually used to convey the meaning that standard classical statistical approach has been used for hypothesis testing or establishing a confidence interval and that the researcher has minimized weighting the results with prior
opinions about the outcome. An interesting statement by M. S. Bartlett (Savage, 1962, p. 85) on this matter of the degree of prior knowledge with respect to significance tests was "if you have rather vague alternatives you can justify classical tests of significance". The problem facing an entrepreneur is that he must select among many possible distinct outcomes and he needs a weighting system to apply in order to help him in selecting an action. As Dubins and Savage (1965) point out, the question is how to play, not whether. Again, it is not the purpose of this thesis to explore the philosophy associated with probability theory but rather to attempt to add to the knowledge about the range livestock industry and hope that some meaningful practical results can be presented for consumption by the entrepreneurs of cattle ranch operations. However, when one deviates somewhat from what has become rather "acceptable" procedures of problem identification and solution, then it is legitimate for others to make some demands with respect to why anyone should follow the course that has been proposed. Therefore, some additional positioning will be presented here before actually entering the specific problem area.

When is an event surprising? A layman will find an event surprising if he has never experienced a similar phenomenon. A scientist should find an event surprising only if it appears to violate his body of theory. A layman may place a probability on the happening of a future event on the basis of the number of actual experiences while the scientist should use both experience and theory. What is experience? Webster's Collegiate Dictionary (1951, p. 291)
gives as a meaning, "the sum total of the conscious events which would compose an individual life". Such a sum total would be difficult and a very long list even for a very young scientist. The Bayesian will argue that experience is evidence and that it is not unscientific to bring to bear all the evidence even if it is not possible to enumerate all the experience elements that one can muster in order to place a degree of probability on a future event or for explaining the occurrence of an event. The position taken in this study is that if two rational people differ on the probability for a state of nature given they have both observed an occurrence of a particular event then they are assumed to have different prior evidence (experience). If two rational people agree on the probability of a state of nature given, they have both observed the same event but offer different monetary odds on the state of nature, then they are assumed to have different tastes. If they pool their prior evidence, then they will be able to agree upon the probability of a state of nature but no such common monetary odds necessarily follow. According to Jeffreys (1939) the utility aspect was not specified by Bayes in that he made no distinction for utility but rather dealt with strict probability in monetary terms and would say that a one in 100 chance of receiving 100 units of money is as valuable as a certainty of receiving one unit. Jeffreys notes that he overlooks the distinction that Laplace called "moral" expectation as distinct from "mathematical" expectation: The problems associated with estimating an individual's marginal utility of money along with the similar problem of intra-personal comparisons of utility are at the moment still far from being resolved.
CHAPTER II

THE ECONOMIC OPTIMUM UNDER CERTAINTY

This chapter reviews the classical economic theory of optimum firm resource allocation under conditions of certainty. The comparison of the problem of analysis when using linear programming and the classical production function marginal analysis is reviewed. The practical problem of estimation is also noted.

In this section, the cattle ranch is assumed to be operating under conditions of certainty and in a market system that is perfectly competitive for all firm products and factors, except for a portion of the forage input which is administered by a public agency. The quantity of the input "public range" will have an upper bound as determined outside the firm. As a matter of fact, the quantity and time of use of this feed resource to the individual firm is determined as a matter of public land policy.

Determining the Optimum from Classical Production Functions

Profit maximization as the criterion for allocation of resources within the firm implies that there can be defined one action which dominates all other actions. This unique condition can be stated unequivocally as indicated by the rather fundamental result of the necessary condition for firm equilibrium. The entrepreneur desires to maximize profit subject to the constraints of the production function.

\[ \pi = \sum_{i=1}^{r+s} p_i q_i + \lambda F(q_1, q_2, ..., q_r, q_{r+1}, ..., q_s) \] (2-1)
Letting \( r \) represent profit from \( r \) products and \( s \) inputs subject to
the implicit production function \( F(q_1, q_2, \ldots, q_r, q_{r+1}, \ldots, q_s) \) then
expression (2-1) represents the profit function. The objective is
the maximization of the profit equation subject to the implicit
production function constraint. The production function must possess
the property of continuous first- and second-order partial deriva-
tives which are different from zero for all its solutions. The
maximization of the function involving the Lagrange multiplier is a
mathematical problem and not an economic issue. However, the form of
the solution is of interest from an economic interpretation viewpoint.
It is obvious from the profit function that the partial derivative
will have a form which involves a single \( p_i \) plus some part of the
production function and the Lagrange multiplier. One can then
select any two of the \( r+s \) partial derivatives and move the second
set of terms to the right of the equality and divide one by the other
yielding a general relationship such as (2-2).

\[
\frac{p_i}{p_j} = \frac{\partial q_j}{\partial q_i} \quad (i, j=1, 2, \ldots, r+s) \tag{2-2}
\]

If \( p_k \) is negative, this indicates that \( q_k \) is an input while if positive
this would indicate that \( q_k \) was an output. This expression yields the
fundamental relationship referred to in economics as the profit
maximization through the use of marginal analysis. If \( q_i \) is an input
and \( q_j \) an output then the basic statement is, "the value of the
marginal product of an input must be equated to that factor price".
In fact, this is true for an input with respect to all \( s \) outputs. If
both \( p_i \) and \( p_j \) are negative, the ratio yields the conclusion that
the rates of substitution of any two inputs must be equal to their
respective factor price ratio.

The above statements of basic economic theory involve only the
production function constraint. A more common assumed constraint is
to maximize revenue subject to some input level, say to a capital
constraint or to minimize cost subject to some output requirement
level. Equation (2-3) is an example of the type of relationship
which is used for maximizing revenue subject to a capital constraint.

\[
R = \sum_{i=1}^{s} p_i q_i + \lambda [c^0 - f(q_1, q_2 \ldots q_s)]
\]  

(2-3)

where: \( R \) = gross revenue

\( p \) = product price

\( q_i \) = quantity of product \( i \)

\( c \) = capital input

\( c^0 = f(q_1, q_2 \ldots q_s) \) a product transformation curve

for any \( c^0 \).

The "dual" of the equation (2-3) would be to minimize \( c = f(q_1, q_2 \ldots q_s) \) subject to a given level of revenue. So long as the
product transformation curve is concave from below the constrained-
revenue-maximization and a constrained-input-minimization yield
identical optimum points.

Determining the Optimum from Linear Programming

The reason for the above discussion of rather basic economic
theory is for purposes of analyzing a statement by Henderson and
Quandt (1958, p. 76) in their reputable textbook of microeconomic theory. They stated "the concept of the marginal productivity of an input is meaningless within the linear programming framework". Because of the tremendous volume of published research that involves linear programming and the references to the marginal value product concept that are commonly made, the above statement would indicate that there may be an inconsistency between the use of marginal value product in a linear programming sense as compared to using it in the classical economic theory approach. Further, a great deal of the analysis in this thesis involves results from the linear programming models in which the term marginal value product will be used; it seemed appropriate that the issue be resolved. Henderson and Quandt do note that it is not possible to change a single constraining input in a linear programming framework without changing the others proportionately to arrive at the new optimum balance. Since the classical production functions in (2-1) and (2-3) above had some very strict assumptions as did Carlson (1956, p. 15) when he stated, "the purely technical maximization problem may be said to be solved by the very definition of our production function". However, as Dorfman, Samuelson, and Solow (1958, p. 203) noted:

"the numerous restraints and the inequality signs that clutter up a programming problem are absent from the conventional formulation, not because they are inapplicable, but because it is assumed that they have already been handled. Perhaps economists would not have gotten into the habit of making this assumption so glibly if they had realized what, and how much, they were assuming."

However, these statements as such are not sufficient to resolve the point in question. It is true that linear programming as a
solution process does not seek to determine directly the optimal quantity of each factor and product, but rather, the optimal level of each activity. It seems axiomatic that if a unique maximum profit exists and if two methods are used at determining a unique point, then the conclusions about the location of this point must come out the same or one has a contradiction in which one system must not be yielding a maximum. There are two basic theorems that have been proven and which allow one to take a positive position on the results yielded by a linear program: These are (1) a feasible program is an optimal feasible program if and only if it contains a list of included activities such that no excluded activity is more profitable than its equivalent combination in terms of those included activities, and (2) a feasible program is an optimal feasible program in the sense of minimum cost if and only if it contains a list of included activities such that no excluded activity costs less per unit of operation than its equivalent combination in terms of those included activities. As Dorfman, Samuelson, and Solow (1958, p. 165) noted "these two theorems together are the linear programming analog of the 'equate your marginal productivities' dictum in the orthodox marginal analysis."

In the strict sense of marginal productivity used in the classical models referred to by Henderson and Quandt (1958), it is possible to determine the exact rate of change as well as the direction of a single factor change by evaluating the derivative at a point and one can, in fact, plot a marginal product curve. However, if one were using linear programming it would be necessary to know how to change the input-output coefficients for each succeeding single factor input in order to plot a total classical product curve.
and this would require knowledge about the production function to obtain the value of the next input-output coefficient, a rather circular requirement. However, when one is faced with the condition of inward inspection for the profit maximization to an individual firm the most powerful and perhaps the only feasible approach is through a form of programming. This is especially true when the firm can produce many products from a large number of available resources.

The statement by Henderson and Quandt is more from a definitional type argument than from an economic interpretation viewpoint as was pointed out previously by statements and theorems presented by Dorfman, Samuelson, and Solow.

For analysis here, the marginal value product will have the usual interpretation of the value added to the total product from a unit change in a particular input. In the linear program, this would result from allowing the variable resources to enter in sufficient amounts to utilize the additional unit of the constraining input but only if there is no change in the variables from non-basic to basic. In conventional programming interpretation these are the shadow prices associated with their respective disposal activity.

Aside from the problem of analysis is the practical problem of estimation. As noted by Nielsen (1965), the form in which data are usually available and the problems associated with attempting to derive a classical production function through such techniques as regression analysis using cross-sectional data generally leaves programming in a superior position. The specification error alone
in the estimation problem is a serious complication to the statistical estimation of the classical production functions.
CHAPTER III

DATA FOR THE MODELS

This chapter presents detailed information of the sampled area and population as well as the sample frame, purposes for which the primary data will be used, and the procedure for determination of the sample size for the various size strata. Instructions to the enumerators are outlined.

The purpose for which the information is needed for the objective of this thesis is not primarily to estimate industry production functions but rather to concentrate on a select group of alternatives that are available to a particular type of firm in the study area and determine the maximum profit action under conditions of certainty and to later investigate uncertainty at the firm decision-making level. In order to be of value in attempting to make some reasonable extrapolation from the hypothetical firm to the study population, the use of primary data from cross-sectional estimates of input-output relationships will be used whenever judgment dictates their use. Otherwise, secondary sources will be used and their sources indicated.

Primary Data

The study area from which the primary data for this thesis were obtained is shown in Figure 1 (page 19). A portion of Malheur County was excluded. The excluded area has diverse agricultural production patterns with no common characteristics of the general study area and very little cattle ranching. The excluded area produces specialty crops such as potatoes, dairy products, and contains some small
Figure 1. Location of the Primary Study Area of Lake, Harney, and Malheur Counties in Oregon.
livestock feed-lots. Since the sample frame was a complete listing of all agricultural operations with more than 30 head of cows and heifers, the inclusion of this area would have resulted in a large group of potential observations which would not fit the established definition of a cattle ranch.

Sampled Population

In order to qualify as a cattle ranch operation, the ranch owner must have received more than 50 percent of his income from the sale of range beef. A further requirement for an observation to qualify was that the ranch headquarters must be located within the study area. A ranch headquarters was defined in terms of the location in which the cattle were kept while on winter feed.

A complete listing of all grazing permittees was obtained from the Bureau of Land Management and the United States Forest Service. All names that appeared in these listings that did not appear on two other lists of cattle operators obtained from the Oregon Cattle-men's Association and the county assessor's records, were then checked with the county agent in each study county. A master list of cross-checked names were numbered for the sample frame. This list contained 709 operator names who qualified as determined by location, size, and judgment of the local county agents. Judgment of the local county agent assumed that he was familiar enough with the county to exclude an operator that headquartered outside the county or if a particular operation was a dairy farm, feed-lot operation or some other type that was not a range beef ranch. If there was doubt on the part of the county agent, the name remained as a potential observation.
Data Uses

The data obtained from the survey of ranch operators were to be used for establishing input-output relationships, management practices, resource structures, and general operational patterns for the study area. Further, data were to furnish similar information for Oregon State University's contribution to a regional range livestock study. As part of the regional study, consideration had to be given to size of ranch in terms of cattle numbers. This was not a primary consideration for this thesis since composition of the livestock inventory was of major interest. However, the lists from which the sample frame was developed were not reliable for determining the inventory composition but seemed highly reliable for determining size by number of cows. For this reason, it seemed reasonable to stratify the population and sample to achieve the objective for the regional study of determining three ranch unit sizes based on number of cows. This would allow maximum use of the data for the regional objective and not in any apparent way detract from the objectives of this thesis.

Selection of Sample

The selection of the sample was conditioned upon the consideration for establishing three sizes in terms of total cows. The criterion for establishing the number of cattle on a small ranch as opposed to a medium, and a medium to large, was based primarily on previous research experience in ranch organization and the ability to obtain relevant information from a survey of ranch operations.
First, ranches of a size greater than 750 head of cows become so large and complex in their operation that a special survey schedule would be required; also, the amount of time to obtain the information would amount to almost a case study of the operation. There are approximately 35 ranches in the study area that have more than 750 cows. Because of this number of very large ranch operations within the study area, the feasibility of an "economy of size study" involving these ranches is being given serious consideration as a follow-up study by the United States Department of Agriculture. For this reason, the upper size boundary for drawing the sample was the 750-cow limit.

After establishing the upper bound for the large ranch, the second step was to establish the small size ranch and, third, establish the medium size. About 80 percent of the ranches in the area are in the size of 50 to 750 cows with about 50 percent of this group in the 50 to 150 size class (see Figure 2 for relative frequency of sizes). The mid-point is 100 head in this interval and this seemed to be a reasonable size to use as the small ranch operation. About one-third of the ranches are in the 200 to 400 head size class. The mid-point is 300 head which is a reasonable medium size ranch. A 50-head increment was left between the largest and smallest of each succeeding size class. This left the 450 to 750 head size group with a mid-point of 600 head to be defined as a large ranch.

Past experience with about 155 ranch observations in the Northern Great Plains of North Dakota, South Dakota, Wyoming, and
Figure 2. Relative Frequency of Ranch Sizes by Number of Cows and Heifers—Southeastern Study Area.
Montana, as well as about 255 ranch observations of the northwestern states of Montana, Idaho, Washington, and Oregon, indicated that at least 15 ranch operations for each size class are necessary as a minimum number to establish meaningful practices and operational characteristics.

Whether this number (15 per size class) was sufficient from a precision viewpoint of data for costs and returns was calculated from a procedure given by Cochran (1963, p. 96). Past experience indicated no significant difference in costs of obtaining observations among size classes. Precision was the primary consideration. If the number of observations necessary to meet the precision requirement were too high for the budgeted funds, then compromises would have to be made between precision and cost. Therefore, \( c_h \) was ignored in the initial calculation of \( n \) in the following expression:

\[
\begin{align*}
    n &= \left[ \frac{\sum_{h=1}^{3} W_h S_h \sqrt{c_h}}{\sum_{h=1}^{3} W_h S_h / \sqrt{c_h}} \right] ^{3/2} \\
    &= \frac{3}{V + (1/N) \sum_{h=1}^{3} W_h S_h^2} \\
    &= \text{(3-4)}
\end{align*}
\]

where:
- \( n \) = total sample size for precision indicated by \( V \)
- \( N \) = total elements in all strata
- \( N_h \) = total elements in stratum \( h \)
- \( W_h = N_h / N \)
- \( s^2 = \sum_{h=1}^{N} (y_{hi} - \bar{Y}_h)^2 / Nh-1 \) the true stratum variance
- \( c_h = \) cost per unit sampled
- \( V = \) specified variance
- \( \bar{Y}_h = \sum_{i=1}^{Nh} y_{hi} / Nh \)
Expression (3-4) yields a value of 51 for n. The assumption made prior to the survey concluded that the S for the large, medium, and small ranches would be $4,500, $3,000, and $1,500, respectively. The dollar values are ranch operating costs. These estimates are based on previous ranch survey data for similar size classes. The range of costs for the various ranch sizes are directly related to the fact that the size range is wider for the larger ranch. This does not mean that the cost per unit of output is more variable for the larger ranch. Further, it should be noted that $S^2_h$ refers to the variance of stratum costs while V refers to the variance of the estimate. The precision selected as acceptable level for this study was $V = $250.

The Neyman allocation as presented by Cochran was then used to estimate the size of sample for each stratum. This is expressed as

$$n_h = \frac{n W_h S_h}{\sum W_h S_h}$$

(3-5)

This yields a sample size of 15 for the small ranch size, 20 for the medium, and 16 for the large ranch size. All these values were sufficiently large for the other data requirements.

Past experience had indicated that about 25 percent of the total ranch operators enumerated yielded a schedule that was fragmented or so poor in information that the data were rejected when edited by the researcher. Therefore, a 65-observation survey was initiated. Due to time and money limitations, only 62 schedules were obtained of which seven were totally unusable. The rejected observations resulted
from obvious false information as recognized by the enumerator during the interview. However, the enumerator continued the interview per instructions that he was to complete any interview in which the respondent was not abusive. In essence, 55 questionnaires of usable quality were obtained with respect to most informational sections of the survey schedule.

A list of names for each size class was given to the enumerator. These names had been obtained by a random selection from the master list of all operators. For each stratum a supplement list of ranch operations were randomly selected from the remaining operators in that stratum. The enumerator was to use this supplemental list of names when he could not obtain a questionnaire completion from one on the primary list. The enumerator made one call-back if the individual ranch operator was not at home. He would then substitute the first name of the remaining names on the supplemental list if the second attempt to locate the primary name resulted in failure or an individual refused to be enumerated.

The use of stratified sampling is useful in determining substitutes for enumeration. Experience in this area indicates that enumerators will usually substitute medium to large ranches if allowed complete freedom in selection of alternates. Small ranch operators show somewhat less enthusiasm for enumeration than the medium to large operators and the medium and large operators usually have their information on such items as costs in more available form. The enumerator soon becomes aware of this and will be inclined toward seeking out this size if built-in controls are not made. The most
effective way of avoiding this biased substitution is to assign the
alternates for each primary observation. This was done for this
survey.

A rather standardized survey schedule was used. The format
was similar to many other survey schedules that have been used and
proven to yield good structural information in previous range live-
stock studies. A complete listing of ranch resources was sought
along with sections for management practices and operating costs and
returns.
CHAPTER IV

THE ECONOMIC MODEL UNDER CERTAINTY

The purpose of this chapter is to structure a general model that can be used to establish the optimal cattle inventory system assuming decision making under certainty. A general linear program model is proposed to determine the optimal system. The general procedure for obtaining input-output coefficients along with the method for generating coefficients for the objective function is presented. A single equation regression model is used with the primary data in determining the variable cost by class of livestock. The model presented determines the optimal cattle inventory composition among any number of assumed structures.

The basic economic model for the decision making under certainty for this study is one which will select that livestock inventory composition which will maximize profit. This is a problem of selecting an optimum livestock inventory system. As was noted earlier, the resources available in the study area limit the feasible production alternatives to that of livestock.

Previous studies relating to livestock systems define a set of alternatives and would pick that one which shows the most profit when compared to the others (Middlemiss, 1965). For instance, a study of three cattle systems for the study area of Baker, Grant, and Umatilla Counties in Oregon defined the alternatives as cow-calf—all calves are sold in the fall; cow-yearling—offspring sold as yearlings; cow feed-lot—calves are placed in feed-lot and fed out to 800 to 1,100 pounds in weight (Wallace, Castle, 1956). The feed-lot alternative was the most profitable for that area with the other two alternatives not significantly different. However,
the feed-lot alternative was a relevant alternative for that area but is assumed not to be meaningful for the study area of south-eastern Oregon. The straight comparison of various assumed types of systems as opposed to optimizing the combinations of systems was the most common approach to livestock systems analysis (Laughlin, 1965; Hunter, 1963). However, Barr and Plaxico (1961) did program various alternatives which allowed the selection of more than one system. The processes investigated by these researchers were limited to different dates of calving (a labor-spreading technique) and buy-sell systems for steers within the program period with certain assumptions with respect to increases in capacity of the firm through range improvements. Another rather common approach used in research of this nature is to investigate the feasibility of adding enterprises such as a cow-calf enterprise to a "typical" grain farm. This may be a result of the desire of the researcher to find a use for the surplus resources that appeared in a previous study or a current study of optimal solutions for a grain operation. The utility associated with the additional enterprise income might certainly be less than the discomfort associated with its addition. For how would it look to his peers if a cattle rancher was to raise a few hogs!

The Input-Output Coefficients

The following set of equations were developed to generate an annual unit resource requirement for any given livestock system. This system assumes a replacement policy for livestock sold to be
replaced from within the ranch and a reasonable calving percentage (for instance \( p = 0 \) would indicate no calves are born and the solution would be indeterminate). A system resource unit requirement is defined as the resource requirement to produce the joint product of cow beef along with whatever system is being defined. For instance, a livestock inventory system might be one in which half the steer calves are sold and the remaining kept and sold as yearlings and a policy of selling all heifer calves not kept for replacement. However, the ranch firm might keep twice the number of heifer calves needed for replacement of the culled cows. As a result of this policy the firm's product sales will be in a form of joint production in which the product is made up of cull cows, steer calves, steer yearlings, heifer calves, and heifer yearlings. To produce this bundle of products requires a system resource unit. This can be for any specified resource but the objective is to get the resource in terms of system unit equivalents. Therefore, the system resource unit is defined in terms of animal unit equivalents (the problems associated with determining animal unit equivalents as well as those used in this study are taken up in pages 36-38).

\[
AUMS_{ij} = \frac{1}{p} \left[ \frac{c_{ij} + aH_{ij}}{1 + a} + cB_{ij} + (a+b)Y_{ij}^H \right] +
\]

\[
.5gY_{ij}^S + \left[ \frac{.5ph - ah - bh}{p} \right] Y_{ij}^H +
\]

\[
.5kgS_{ij}^S + \left[ \frac{.5phw - ahw - bhw}{p} \right] S_{ij}^H + K_{ij}
\]

(4-5)
where: \(0 < p \leq 1\)

\[
\frac{a+b}{p} < 0.5
\]

\(i\) = resource being considered

\(j\) = time increment assumed

\(p\) = expected percent calf crop

\(a\) = percent of breeding cow herd to be culled

\(b\) = percent of heifer calves as a percent of total cow herd carried over in excess of those needed for actual replacement

\(c\) = number of bulls per breeding cow

\(g\) = percent of steer calf carryover to yearling steers

\(h\) = percent of heifer calf carryover above those kept for cow inventory replacement

\(k\) = percent steer yearlings carryover to steers coming 2

\(w\) = percent heifer yearlings not for replacement carried over to heifers coming 2

\(C\) = breeding cow over 2 years in age

\(H\) = breeding heifer coming 2

\(B\) = breeding bull

\(Y^H\) = yearling heifer

\(Y^S\) = yearling steer

\(S^S\) = steer coming 2

\(S^H\) = heifer coming 2 not kept for replacement consideration

\(K\) = calf of either sex

The above equation may seem quite formidable at first glance but if two researchers can agree on the animal unit equivalent for
any particular resource this accounting equation solution is trivial. It is obvious that as \( p \) decreases, the system requires more resources per system unit. This is because as calving rates decrease the amount of breeding cows and bulls must increase to produce a system unit product.

Unfortunately, equation (4-5) does not fill the requirement for evaluating the entire year's operation. The equation needs to be restructured at the time in which it is assumed that the livestock sales are made and the inventory livestock classes shift internally from one livestock class to the next category.

As an example of the numerical result for equation (4-5), assume the time periods being analyzed are one-fourth a year in length. The first period starts January 1. Assume a 90 percent calf crop with a breeding cow replacement rate of 10 percent, but that the rancher selects his breeding replacement on the basis of having two heifers from which to select for each brood cow replacement. Therefore, \( a = .1 \) and \( b = .1 \). Further, a ratio of 20 cows per bull is used; i.e., \( c = .05 \). Assume further that 50 percent of the heifer calves not kept for replacement are carried over and sold as yearling heifers \( (h = .5, w = 0) \) and that 75 percent of the steer calves are carried over to steer yearlings and 10 percent of the yearling steers are carried over to steers coming 2 years of age \( (g = .75, k = .1) \). The assumed values for the animal unit feed equivalent requirements are given in Table I. Using the third period as an example, the animal unit system requirement for feed is the result given in (4-5a).
TABLE I. ANIMAL UNIT MONTHS OF FEED REQUIREMENTS IN THREE-MONTH INCREMENTS.

<table>
<thead>
<tr>
<th>Livestock Class</th>
<th>Notation</th>
<th>January March</th>
<th>April June</th>
<th>July September</th>
<th>October December</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>j=1</td>
<td>j=2</td>
<td>j=3</td>
<td>j=4</td>
</tr>
<tr>
<td>Breeding cow</td>
<td>C Fj</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Breeding bull</td>
<td>B Fj</td>
<td>4.5</td>
<td>4.5</td>
<td>3.6</td>
<td>3.6</td>
</tr>
<tr>
<td>Breeding heifer</td>
<td>H Fj</td>
<td>3.6</td>
<td>3.6</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Yearling heifer</td>
<td>Y H Fj</td>
<td>1.5</td>
<td>1.8</td>
<td>2.1</td>
<td>2.4</td>
</tr>
<tr>
<td>Yearling steer</td>
<td>Y S Fj</td>
<td>1.5</td>
<td>1.8</td>
<td>2.1</td>
<td>2.4</td>
</tr>
<tr>
<td>Steer coming 2</td>
<td>S S Fj</td>
<td>2.4</td>
<td>2.7</td>
<td>3.0</td>
<td>3.3</td>
</tr>
<tr>
<td>Heifers coming 2</td>
<td>S H Fj</td>
<td>2.4</td>
<td>2.7</td>
<td>3.0</td>
<td>3.3</td>
</tr>
<tr>
<td>(for sale only)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calf (either sex)</td>
<td>K Fj</td>
<td>0</td>
<td>0</td>
<td>.9</td>
<td>1.2</td>
</tr>
</tbody>
</table>
\[ AUMS_{F3} = \frac{1}{.90} \left[ \frac{3.0 + (.1)(3.0)}{1 + .1} + (.05)(3.6) + (.1+.1)(2.1) \right] \]

\[ + (.5)(.75)(2.1) + \left[ \frac{(.5)(.90)(.5) - (.1)(.5) - (.1)(.5)}{.90} \right] (2.1) \]

\[ + (.5)(.1)(.75)(3.0) + \]

\[ + \left[ \frac{(.5)(.90)(.5)(0) - (.1)(.5)(0) - (.1)(.5)(0)}{.90} \right] (3.0) \]

\[ + .9 \]

\[ = 6.1 \]

(4-5a)

As noted earlier, equation (4-5) is not sufficiently general to handle the complete inventory cycle. There comes a time in the accounting in which an animal such as a breeding heifer (H) must be considered as a breeding cow over two years in age (C), when a decision is made in which certain heifer calves (K^H) are considered for breeding heifer replacement (Y^H), etc. If the sale of livestock is made at the same time as the replacement selections are made, then equation (4-6) will handle the remaining portion of the accounting period.

\[ AUMS_{ij} = \frac{1}{p} \left[ \frac{C_{ij}}{1 + a} + aY^H_{ij} + cB_{ij} + (a+b)K^H_{ij} \right] \]

\[ + .5gK^S_{ij} + \left[ \frac{.5ph - ah - bh}{p} \right] K^H_{ij} + .5gY^S_{ij} \]

\[ + \left[ \frac{.5phw - ahw - bhw}{p} \right] Y^H_{ij} \]
where:  \( j \) = the period when inventory structure changes

\( K_H \) = heifer calf

\( K_S \) = steer calf

All other notions as in (4-5)

If the livestock inventory composition is assumed to be the same as indicated previously for estimating \( AUMS_{F3} \), the value of \( AUMS_{F4} \) can be calculated as follows:

\[
AUMS_{F4} = \frac{1}{.90} \left[ \frac{3.0 + (.1)(2.4)}{1 + .1} + (.05)(3.6) + (.1 + .1)(1.2) \right]
\]

\[
+ (.5)(.75)(1.2) + \left[ \frac{(.5)(.90)(.5) - (.1)(.5) - (.1)(.5)}{.90} \right] (1.2)
\]

\[
+ (.5)(.75)(.1)(2.5)
\]

\[
+ \left[ \frac{(.5)(.90)(.5)(0) - (.1)(.5)(0) - (.1)(.5)(0)}{.90} \right] (2.4)
\]

\[= 4.4 \quad (4-6a) \]

Unfortunately, equation (4-6) is not sufficiently general to handle the last period of the inventory cycle if it is assumed that the sale of the livestock is not made at the time of the selection of breeding and selling replacements. If the sale of the livestock takes place after the replacement selection, equation (4-7) is applicable for this final period.
\[ \text{AUMS}_{ij} = \frac{1}{p} \left[ \frac{C_{ij} + aY_{ij}}{1 + a} + cb_{ij} + (a+b) K_{ij}^H \right] \]
\[ + \frac{.5gK_{ij}^S}{p} + \left[ \frac{.5ph - ah - bh}{p} \right] Y_{ij}^H + .5kgY_{ij}^S \]
\[ + \left[ \frac{.5phw - ahw - bhw}{p} \right] f_{ij}^H \]
\[ + \left[ \frac{.5phw - ahw - bhw}{p} \right] f_{ij}^S \]
\[ (4-7) \]

where: 0 < f < 1

f = fraction of final period in which cattle to be sold are held after selection of replacements

All other notation is the same as in equation (4-6)

Equation (4-6) is really a special case of equation (4-7) since in (4-6) f = 0. The above examples are for a four-period cycle in which only period four is affected by inventory structure change. The year could have been broken into as many accounting periods as desired. However, the data in Table I would have to be calculated for the same incremental time periods. This is one of the most important characteristics associated with the development of this structure. For purposes of analysis in this section, the time periods will be as presented in Table I. Further refinement at this stage does not appear to be relevance for this study. Also, the sale period and the inventory replacement decision will be assumed to occur simultaneously unless otherwise indicated.

The value of the accounting equations may not be apparent to an individual who has not struggled with obtaining animal unit feed
requirements for various livestock inventory compositions for various times of the year. These equations can be readily set up for computation on a computer and various relative values assigned to the data presented in Table I. The output of the above equation is then used as the input \( a_{ij} \) for a standard linear programming model.

The problems involved with the use of equations (4-5), (4-6), and (4-7) for obtaining input coefficients for a linear program is not in the computation but with the values assigned in Table I. The use of animal unit equivalents among classes of livestock is a common technique employed in most livestock studies. However, agreement on the relative weights is more uncommon than common. It is true that if all weights in Table I were doubled this would not alter the optimum system solution in the linear program since it would merely be a proportional increase for all systems being compared. However, if in fact, the feed requirement relationship between yearling steers and steers two years old is not in the 2.1 to 3.0 relationship as shown in Table I, then the solution of the linear program will not necessarily yield the optimum livestock system.

No single standard has been established and generally accepted in economic research for determining animal unit equivalents. Most researchers agree it has some relationship to the relative amounts of total digestible nutrients required by the particular animal class either to maintain normal growth in the case of young animals or to maintain the "normal" condition required for the purpose for
which the mature animal is being kept. As Barmettler (1965) pointed out before the western range committee, there still remains wide differences among researchers. Most researchers attempt to tie total digestible nutrients requirements along with the nutrient source to the rather well-known Morrison's (1957) "Feeds and Feeding" tables. In a study by a group of 14 professional agricultural economists of the United States Department of Agriculture (1958, p. 7), the United States Department of the Interior, and three western universities, the following weights were suggested:

"In determining total and seasonal feed requirements, the following standards of animal-unit equivalents were adopted: Mature cow, 1.0; long yearlings, 0.8; weaned calf, 0.6; unweaned calf, 0.4; pregnant heifer, 1.0; bull, 1.25; five mature sheep, 1.0; and a mature horse, 1.5. These animal-unit standards were based on feeding standards. The animal-unit weights were modified in a ranching area when sample data and secondary data indicated a difference existed from the standard."

Barr, Schultz, Plaxico, and Nielsen (1960) used a breeding cow of 1,000 pounds as an animal-unit equivalent. They would then rate a 1,300 pound bull at 1.3, a 600 pound yearling as .6, etc. Figures in Table I agree somewhat with these values except that the system used allows more refinement for different seasons of the year. For instance, bulls are normally kept separate during the winter and early spring months and fed at a high nutrient rate to flesh them up for the breeding season. This fact is not reflected in studies using a flat year around rate. Young heifers that are going to calve for the first time are normally fed at a higher nutrient rate during the winter than older cows. The other categories follow
reasonably close the weight relationship indicated by Barr, Schultz, and Plaxico.

The derivation of the animal-unit labor requirements have a less objective base than the feed requirement equivalents. These figures appear in Table II. They are used in equations (4-5), (4-6), and (4-7) for calculation of the labor \( a_{ij} \) input for the linear programming model. Those animal classes which require the most attention during the winter months reflect a higher labor requirement. Further, young heifers require much more labor during the calving season. The reason that there is an increase in labor during the summer months is because the haying season labor is reflective of winter feed requirements of the various livestock classes. Winter feeding is the important labor user during the winter months.

The capital input for the various classes of livestock is presented in Table III. These values can be used in equations (4-5), (4-6), and (4-7) to obtain the capital requirement for a given livestock system unit for any time period. The values obtained from equations (4-5), (4-6), (4-7) are the basic capital \( a_{ij} \) input requirement for the linear programming model. The values appearing in this table will vary with cattle prices. Ranchers rarely sell their basic breeding herd. Quantities of sales sufficient to determine the market value of breeding cattle seldom take place; however, the relationship to current cattle prices appears to be the most realistic alternative.
## TABLE II. ANIMAL-UNIT MONTHS LABOR REQUIREMENT FOR THREE-MONTH INCREMENTS.

<table>
<thead>
<tr>
<th>Livestock Class</th>
<th>Notation</th>
<th>AUMS by Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>January March</td>
</tr>
<tr>
<td></td>
<td></td>
<td>j=1</td>
</tr>
<tr>
<td>Breeding cow</td>
<td>C&lt;sub&gt;Lj&lt;/sub&gt;</td>
<td>2.5</td>
</tr>
<tr>
<td>Breeding bull</td>
<td>B&lt;sub&gt;Lj&lt;/sub&gt;</td>
<td>4.5</td>
</tr>
<tr>
<td>Breeding heifer</td>
<td>H&lt;sub&gt;Lj&lt;/sub&gt;</td>
<td>2.7</td>
</tr>
<tr>
<td>Yearling heifer</td>
<td>Y&lt;sup&gt;H&lt;/sup&gt;&lt;sub&gt;Lj&lt;/sub&gt;</td>
<td>2.0</td>
</tr>
<tr>
<td>Yearling steer</td>
<td>Y&lt;sup&gt;S&lt;/sup&gt;&lt;sub&gt;Lj&lt;/sub&gt;</td>
<td>2.0</td>
</tr>
<tr>
<td>Steer coming 2</td>
<td>S&lt;sub&gt;Lj&lt;/sub&gt;</td>
<td>2.0</td>
</tr>
<tr>
<td>Heifer coming 2</td>
<td>S&lt;sup&gt;H&lt;/sup&gt;&lt;sub&gt;Lj&lt;/sub&gt;</td>
<td>2.0</td>
</tr>
<tr>
<td>(not for replacement)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calf (either sex)</td>
<td>K&lt;sub&gt;Lj&lt;/sub&gt;</td>
<td>0</td>
</tr>
<tr>
<td>Livestock Class</td>
<td>Notation</td>
<td>Average Investment in Livestock</td>
</tr>
<tr>
<td>----------------------</td>
<td>-----------</td>
<td>---------------------------------</td>
</tr>
<tr>
<td></td>
<td></td>
<td>January</td>
</tr>
<tr>
<td>Breeding cow</td>
<td>C&lt;sub&gt;cj&lt;/sub&gt;</td>
<td>j=1</td>
</tr>
<tr>
<td>Breeding bull</td>
<td>B&lt;sub&gt;cj&lt;/sub&gt;</td>
<td>j=2</td>
</tr>
<tr>
<td>Breeding heifer</td>
<td>H&lt;sub&gt;cj&lt;/sub&gt;</td>
<td>j=3</td>
</tr>
<tr>
<td>Yearling heifer</td>
<td>y&lt;sub&gt;H&lt;/sub&gt;&lt;sup&gt;cj&lt;/sup&gt;</td>
<td>j=4</td>
</tr>
<tr>
<td>Yearling steer</td>
<td>y&lt;sub&gt;S&lt;/sub&gt;&lt;sup&gt;cj&lt;/sup&gt;</td>
<td>j=1</td>
</tr>
<tr>
<td>Steer coming 2</td>
<td>s&lt;sub&gt;S&lt;/sub&gt;&lt;sup&gt;cj&lt;/sup&gt;</td>
<td>j=2</td>
</tr>
<tr>
<td>Heifer coming 2</td>
<td>h&lt;sub&gt;S&lt;/sub&gt;&lt;sup&gt;cj&lt;/sup&gt;</td>
<td>j=3</td>
</tr>
<tr>
<td>(not for replacement)</td>
<td></td>
<td>j=4</td>
</tr>
<tr>
<td>Steer calf</td>
<td>k&lt;sub&gt;S&lt;/sub&gt;&lt;sup&gt;cj&lt;/sup&gt;</td>
<td>j=1</td>
</tr>
<tr>
<td>Heifer calf</td>
<td>k&lt;sub&gt;H&lt;/sub&gt;&lt;sup&gt;cj&lt;/sup&gt;</td>
<td>j=2</td>
</tr>
</tbody>
</table>
Table III presents only the investment in the livestock inventory. The primary data could not be disentangled sufficiently to reflect a reasonable basis for allocating such items as machinery and equipment, at least on a basis that would allow stratification on the basis of livestock class. If the influence of investment in items such as machinery influence livestock systems, the analysis here will not reflect this condition.

**Gross Revenue Per System Unit**

In the previous section, the input-output coefficients commonly referred to as the $a_{ij}$'s were developed. The equation system along with the data presented in Tables I, II, and III establishes the number of inputs of feed (which is a function of the land resource), labor, and capital required to produce a unit of output. The output unit is defined in terms of a system unit output. A system unit output is simply a joint product of beef sales involving livestock types sold under a particular assumed system. There are technically as many possible systems as there are ways of assigning values to the coefficients in equations (4-5), (4-6), and (4-7)—an infinite number of ways.

Another problem is to establish a technique of evaluating the profit per system unit which can be used as the coefficients for any given livestock system. These coefficients are then the coefficients of the objective function. The objective function is then maximized subject to the fixed resource constraints of labor, capital, and feed.
The following equation (4-8) was developed to obtain the gross revenue for a livestock system unit.

\[ GRLSU = \frac{1}{p} \left[ a(1-dc) \right] \left[ C_{pd} W_{ch} P_{ch} + C_{pw} W_{cw} P_{cw} \right] \\
+ (0.5)(1-g)(1-d_{sk})(W_{sk} P_{sk}) \\
+ \left( \frac{5p-a-b}{p} \right)(1-h)(1-d_{hk})(W_{hk} P_{hk}) \\
+ (0.5)(1-k)(1-d_{ys})(W_{ys} P_{ys}) \\
+ h(\frac{5p-a-b}{p})(1-w)(1-d_{yh})(W_{yh} P_{yh}) \\
+ 0.5k(1-d_{st})(W_{st} P_{st}) \\
+ \left[ \frac{5phw-ahw-bhw}{p} \right] \left[ 1-d_{ht} \right] \left[ W_{ht} P_{ht} \right] \\
+ \left( \frac{5p-a-b}{p} \right)(1-d_{yh}) \left[ W_{yh} P_{yh} \right] \quad (4-8) \]

where: p, a, b, c, g, h, k, w are the same as in equation (4-5)

\[ C_{pd} + C_{pw} = 1 \]
\[ C_{pd} = \text{percent of cull cows that are dry cows} \]
\[ C_{pw} = \text{percent of cull cows that are wet cows} \]
\[ d = \text{death loss associated with the livestock class} \]
\[ W_{cd} P_{cd} = \text{weight of dry cow} \times \text{price per pound for dry cows} \]
\[ W_{cw} P_{cw} = \text{weight of wet cow} \times \text{price per pound for wet cows} \]
\[ W_{sk} P_{sk} = \text{weight of steer calf} \times \text{price per pound for steer calves} \]
\[ W_{hk} = \text{weight of heifer} \times \text{price per pound for heifer calves} \]

\[ W_{yy} = \text{weight of yearling heifer} \times \text{price per pound for yearling heifer} \]

\[ W_{st} = \text{weight of steer two years and over} \times \text{price per pound for steer 2's} \]

\[ W_{ht} = \text{weight of heifer two years and over} \times \text{price per pound for heifer 2's} \]

This equation can be evaluated for any relationship assumed in calculating the resource requirements in equations (4-5), (4-6), and (4-7).

The gross revenue figure is in the same unit system as the resource calculation. This is a necessary requirement for its use in the linear programming objective function.

Assuming the same livestock system that was used in the previous example for the resource requirements \((p = .9, a = .1, b = .1, h = .5, w = 0, g = .75, k = .1)\) and assuming further no death loss, a 20 percent dry cow cull, and a set of livestock weights and prices. Equation (4-8) yields the following gross return for the assumed livestock system unit:

\[
\text{GRLSU} = \frac{1}{.90} \left[ .1(1-0) \right] \left[ (.2)(1000)(.15) + (.8)(850)(.12) \right]
\]

\[ + (.5)(1 - .75)(1-0)(400)(.25) \]

\[ + \left[ \frac{(.5)(.9) - .1 - .1}{.9} \right] (1 - .5)(1-0)(380)(.24) \]

\[ + (.5)(.75)(1 - .1)(1-0)(700)(.23) \]

\[ + (.5) \left[ \frac{(.5)(.9) - .1 - .1}{.9} \right] (1-0)(1-0)(650)(.22) \]

\[ + (.5)(.75)(.1)(1-0)(1100)(.20) \]
The $135.91 is the gross revenue for the livestock system previously cited. This joint product in dollar terms is made up of $12.40 in cull cow sales, $12.50 in steer calf sales, $12.67 in heifer calf sales, $54.34 in yearling steer sales, $35.75 in yearling heifer sales, and $8.75 in steers which are two years or older.

The above livestock weights and prices are an example only and not necessarily correct for the study area. (Average weights for the study area are presented in Table VIII.) The livestock weights used in the linear program will be those obtained from the ranch survey. The prices will be derived from secondary sources such as census data.

Death losses are reflected in the revenue equation (4-8) but not in equation (4-5), (4-6), or (4-7). The death loss was ignored in the resource equation for simplicity with acknowledged reduction in accuracy. The use of (1-%) death loss could be included with each component in equations (4-5), (4-6), and (4-7) but will be assumed excluded for the analysis that follows unless specifically noted.
Variable Costs Per System Unit

The operating expenses by class of livestock are most difficult to obtain. Operating costs obtained from survey data vary considerably among observations in which there is no necessary relationship to the structure of the inventory. For instance, the variation in expenditure for gasoline among observations will in part be accounted for by differences in the location of the ranch relative to product and factor markets. This has little if any relation that could be tied directly to the inventory structure. However, some reasonable means had to be established for estimating variable costs per livestock type. It should be noted that the cost figure required from the data is not a cost per unit of output. The cost per unit of output will be computed using equation (4-5). The computed value will then be in terms of the variable cost per livestock unit of output. This result will then be in the same unit terms as the gross revenue obtained from equation (4-8), as it must be for use in the linear programming objective function.

From the survey schedule, a complete breakdown of the livestock inventory by class of livestock was obtained. An itemized list of operating expenses was also obtained. These operating expenses included seed and plants, supplies, fertilizer and lime, water, machine hire, freight, gasoline, veterinary, taxes, repairs and maintenance, automobile upkeep (ranch share), soil and water conservation expenditures (non-capital expenditure type), insurance, payroll taxes, accounting fees, and other miscellaneous expenses that are deductible from the federal income tax.
Several approaches were attempted in an effort to obtain the best relative operating expense per class of livestock. As was noted earlier in the section pertaining to establishing the $a_{ij}$ from the resource equations (4-5), (4-6), and (4-7), the relative weights among livestock classes are more important for the analysis than the absolute correctness of any single item. The relative weight among classes of livestock determines when a particular system is going to be selected over another not the absolute level of costs. However, any mis-estimation of these operating expenses will distort the true shadow prices obtained from the linear program solution and these values have important economic interpretations. For this reason, the absolute correctness as well as the relative correctness are considered important.

As was noted earlier, the information available for selecting a ranch observation for enumeration did not contain the breakdown of the livestock inventory. Further, various size operations were needed as observations for purposes other than this thesis. If a sufficient number of operations had been obtained which had only a cow-yearling operation, one might argue that it is reasonable to compare it with a cow-calf operation to determine how much additional expenses were involved with each yearling. Unfortunately, the sample did not yield a sufficient number of operations of unmixed inventory compositions to attempt this technique.

It might appear that it would be plausible to classify some operating expenses that are related to one or predominately related
to a single livestock class. The apparent justification and rationalization that would be necessary for this approach outweighed the gain.

The approach that was selected was to use a regression of the total variable operating expenses as obtained from the survey against the several individual classes of livestock. Observations of ranch operations with two-year old steers were not in sufficient number to enter that class as an independent variable. The following regression results were obtained:

\[
TOVC = 1707.77 + 14.82X_1 + 38.18X_2 + 15.67X_3 + 198.26X_4
\]  

(4-9)

where:  
\( TOVC \) = total operating variable costs  
\( X_1 \) = number of breeding cows over 2  
\( X_2 \) = number of breeding heifers 2's  
\( X_3 \) = number of yearlings  
\( X_4 \) = number of bulls  
\( R^2 = .76 \)  
\( n = 43 \)  
\( F > 115 \)

As indicated by the F value, analysis of variance for the complete model indicates a rejection of the hypothesis that the mean costs are the same. The constant term also turned out to be significantly different from zero. The procedure for handling the constant term is to take the average number of breeding cows \((257.8)\) in the sample and divide this into the constant term \(1,707.77\) yielding \$6.62 and add this to the \$14.82 giving \$21.44. The \$21.44 was then assumed as the total variable operating cost per breeding cow. The other coefficients were not changed. This rationalization
may weaken the analysis somewhat but the addition of the constant to the basic breeding cow herd, which is for any livestock system the fundamental unit, seemed the best alternative. The few observations that were excluded produced predominately steers coming 2 and these were then compared to the operating costs of the included observations. A two-year old steer or a heifer not kept for replacement was estimated to add $20.00 to the total operating variable cost. The other livestock classes were assumed to have an operating variable expense of the regression coefficient obtained from the primary data.

The following is offered as an example of establishing the variable cost per unit system with the structure assumed in the previous example. Equations (4-5) and (4-9) are used to generate this quantity.

Variable cost per assumed livestock system unit.

Variable Operating Cost per System Unit Equals

\[
\frac{1}{.9} \left( \frac{\$21.44 + (.1)(\$38.18)}{1 + .1} + (.05)(\$198.26) + (.1-.1)(\$15.67) \right) \\
+ (.5)(.75)(\$15.67) + \left[ \frac{(.5)(.9)(.5) - (.1)(.5) - (.1)(.5)}{.9} \right] (\$15.67) \\
+ (.5)(.1)(.75)(\$20.00) \\
+ \left[ \frac{(.5)(.9)(.5)(0) - (.1)(.5)(0) - (.1)(.5)(0)}{.9} \right] (\$20.00) \\
\approx \$48.81
\]

Thus, the structure assumed for the numerical structure assumed in the earlier example will have an operating variable cost per unit of output of $48.81.
The Objective Function

It is now possible to define the coefficient $C_j$ in the linear programming objective function, $Z = C_1X_1 + \ldots + C_nX_n$. The value of $C_j$ using the livestock inventory system in the above example is $135.91 - 48.81 = 87.10$. The $87.10$ is the profit per livestock system unit. The objective is to maximize the function $Z$ subject to the constraints of labor, capital and feed. The $a_{ij}$'s for labor, capital and feed were established in previous examples for the assumed livestock inventory system and can be calculated for any other assumed livestock system structure.

General Constraints

The constraints, commonly referred to in linear programming as the right-hand side, are for purposes of analysis in this study as important in terms of time availability as they are in amount. For instance, if one were interested in selecting from a set of alternative livestock systems for a particular study area, one might make the assumption that the amount of feed available in one period is twice that in another while the labor and capital available are constant for all periods. Regardless of how little or how many resources are assumed available, as long as the proportion remains the same, the result of the linear program will always select the same most favorable activity or activities. This optimum level will reveal those livestock systems which are the most profitable under the assumed constraint proportions regardless of the magnitude of the constraint.
This chapter presents three linear program models. The first model demonstrates that a set of seven homogeneous cattle inventory systems can be defined such that a unique optimal system can be structured from the program results. The second model was structured to simultaneously determine the optimal cattle inventory system when the forage input is within complete control of the firm and when it is an administered public land input. Any difference that results is defined as a misallocation. The third model uses the data from the study area and is an application of the first two models. Prices received for livestock as well as beef production and resource structure for the southeastern Oregon study area are presented. A regression equation is used to isolate the carrying capacity of various feed sources. The optimal cattle inventory structure for various calving percentage is determined for the study area. The amount of misallocation as a result of the public land administered input is isolated. This chapter concludes the investigation of the optimal cattle inventory structure for decision making under certainty.

Up to this point, the techniques developed have been for purposes of permitting complete generalization for handling an infinite number of different livestock systems. The question of relevance for this chapter is which one of these infinite livestock systems is the optimum under conditions of certainty. It is the purpose of this section to present an approach for establishing the single optimum livestock inventory composition.

For purposes of this study, a livestock inventory system is defined in terms of the alternatives available from production of all animals from birth and with replacements from within the basic structure. The exception is for breeding bulls which are assumed
to be purchased and sold with the difference being an operating expense. In other words, a buy-sell livestock activity is assumed outside the relevant alternatives. However, this would be an easy alternative to enter in the linear program. The reason for exclusion is that the primary data indicated this is not a common practice among most ranch operations of the study population. The major reason for this not being a common practice results from the practical condition that when there exists a good forage year all operators will tend to have a surplus of the same feed and a physical lack of local cattle supply to utilize the excess. Shipments of replacement cattle should take place if the local price is sufficient to cover the transportation cost of the shipments. However, disease and the speculative nature of the quality of replacement stock was given as a reason by several ranch operators for not expanding the herd size during periods of feed surplus.

Model I

To establish an exhaustive set of livestock inventory systems, a set of homogeneous livestock inventory compositions are defined. A homogeneous livestock inventory system is defined as a livestock inventory in which any class of livestock, except those kept for breeding purposes, where there is a sale of a single animal in that class then all animals in that class are sold. For instance, a cow-calf operation can be defined as an operation in which all calves except those kept for breeding purposes are sold (for example $K_1$ as presented in Table IV). A cow-calf-yearling steer operation is not
<table>
<thead>
<tr>
<th>NOTATION</th>
<th>g^a</th>
<th>h^a</th>
<th>k^a</th>
<th>w^a</th>
</tr>
</thead>
<tbody>
<tr>
<td>K_1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>K_2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>K_3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>K_4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>K_5</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
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</tr>
<tr>
<td>K_9</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

^aThis notation is defined on page 31. In general, g refers to steer calves carried over to yearlings, h refers to heifer calves carried over to yearlings, k refers to yearling steers carried over to steers coming 2, w refers to heifer yearlings not for replacement carried over to heifers coming 2.
defined. One cannot sell all the calves and have yearling steers. However, a cow-heifer calf-yearling steer operation is defined (in Table IV this is $K_2$) since one can sell all categories listed and continue the operation completely from within the structure. The list of all homogeneous livestock inventory systems is given in Table IV.

The set of homogeneous livestock inventory systems listed in Table IV make up the basic linear programming activity set. Table V is presented as an example of a simplified linear programming model using the set of homogenous livestock inventory systems along with the equation solutions in generalized form that can be developed for determining relevant coefficients.

The $P_0$ would be the total amounts of the various resources available by time of year. For instance, if $X_1 = X_2 = X_3 = X_4$, this would indicate that the same amount of feed in animal unit month equivalents is available in each period. No transfer activities are presented in this model but will be an important part of Model III. Transfer activities allow the transfer of a resource from one period to another. For instance, it is perfectly reasonable to think in terms of storing range feed from one program accounting period to a later period. In fact, as Nielsen (1965) has pointed out, the delay in time of use of certain range resources can result in a higher total range production than if used earlier. This is the result of a biological phenomenon of allowing the edible plants to gain sufficient early season vigor to sustain a high total season production.
### TABLE V. LINEAR PROGRAMMING MODEL I - THE LINEAR PROGRAM FOR HOMOGENEOUS LIVESTOCK SYSTEMS.

\[
\text{MAX}(z) = (\text{GRLS}_{K1} - \text{VCLS}_{K1}) K_1 + (\text{GRLS}_{K2} - \text{VCLS}_{K2}) K_2 + (\text{GRLS}_{K3} - \text{VCLS}_{K3}) K_3 + \ldots + (\text{GRLS}_{K9} - \text{VCLS}_{K9}) K_9
\]

<table>
<thead>
<tr>
<th>Resource</th>
<th>Unit</th>
<th>( P_0 )</th>
<th>( K_1 )</th>
<th>( K_2 )</th>
<th>( K_3 )</th>
<th>( K_4 )</th>
<th>( K_5 )</th>
<th>( K_6 )</th>
<th>( K_7 )</th>
<th>( K_8 )</th>
<th>( K_9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Winter feed (Jan-Mar)</td>
<td>AUMs</td>
<td>( X_1 )</td>
<td>AUMs(_F1)</td>
<td>AUMs(_L1)</td>
<td>AUMs(_F1)</td>
<td>AUMs(_L1)</td>
<td>AUMs(_L1)</td>
<td>AUMs(_L1)</td>
<td>AUMs(_F1)</td>
<td>AUMs(_L1)</td>
<td>AUMs(_F1)</td>
</tr>
<tr>
<td>2 Winter labor (Jan-Mar)</td>
<td>AUMs</td>
<td>( Y_1 )</td>
<td>AUMs(_L1)</td>
<td>AUMs(_L1)</td>
<td>AUMs(_L1)</td>
<td>AUMs(_L1)</td>
<td>AUMs(_L1)</td>
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<td>AUMs(_L1)</td>
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<td>AUMs(_L1)</td>
</tr>
<tr>
<td>3 Capital (Jan-Mar)</td>
<td>Dol.</td>
<td>( Z_1 )</td>
<td>AUMs(_C1)</td>
<td>AUMs(_C1)</td>
<td>AUMs(_C1)</td>
<td>AUMs(_C1)</td>
<td>AUMs(_C1)</td>
<td>AUMs(_C1)</td>
<td>AUMs(_C1)</td>
<td>AUMs(_C1)</td>
<td>AUMs(_C1)</td>
</tr>
<tr>
<td>4 Spring feed (Apr-June)</td>
<td>AUMs</td>
<td>( X_2 )</td>
<td>AUMs(_F2)</td>
<td>AUMs(_F2)</td>
<td>AUMs(_F2)</td>
<td>AUMs(_F2)</td>
<td>AUMs(_F2)</td>
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<td>AUMs(_F2)</td>
<td>AUMs(_F2)</td>
</tr>
<tr>
<td>5 Spring labor (Apr-June)</td>
<td>AUMs</td>
<td>( Y_2 )</td>
<td>AUMs(_L2)</td>
<td>AUMs(_L2)</td>
<td>AUMs(_L2)</td>
<td>AUMs(_L2)</td>
<td>AUMs(_L2)</td>
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<td>AUMs(_L2)</td>
<td>AUMs(_L2)</td>
</tr>
<tr>
<td>6 Capital (Apr-June)</td>
<td>Dol.</td>
<td>( Z_2 )</td>
<td>AUMs(_C2)</td>
<td>AUMs(_C2)</td>
<td>AUMs(_C2)</td>
<td>AUMs(_C2)</td>
<td>AUMs(_C2)</td>
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<td>AUMs(_C2)</td>
<td>AUMs(_C2)</td>
</tr>
<tr>
<td>7 Summer feed (July-Sept)</td>
<td>AUMs</td>
<td>( X_3 )</td>
<td>AUMs(_F3)</td>
<td>AUMs(_F3)</td>
<td>AUMs(_F3)</td>
<td>AUMs(_F3)</td>
<td>AUMs(_F3)</td>
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<td>AUMs(_F3)</td>
<td>AUMs(_F3)</td>
<td>AUMs(_F3)</td>
</tr>
<tr>
<td>8 Summer labor (July-Sept)</td>
<td>AUMs</td>
<td>( Y_3 )</td>
<td>AUMs(_L3)</td>
<td>AUMs(_L3)</td>
<td>AUMs(_L3)</td>
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<td>AUMs(_L3)</td>
<td>AUMs(_L3)</td>
</tr>
<tr>
<td>9 Capital (July-Sept)</td>
<td>Dol.</td>
<td>( Z_3 )</td>
<td>AUMs(_C3)</td>
<td>AUMs(_C3)</td>
<td>AUMs(_C3)</td>
<td>AUMs(_C3)</td>
<td>AUMs(_C3)</td>
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<td>AUMs(_C3)</td>
<td>AUMs(_C3)</td>
<td>AUMs(_C3)</td>
</tr>
<tr>
<td>10 Fall feed (Oct-Dec)</td>
<td>AUMs</td>
<td>( X_4 )</td>
<td>AUMs(_F4)</td>
<td>AUMs(_F4)</td>
<td>AUMs(_F4)</td>
<td>AUMs(_F4)</td>
<td>AUMs(_F4)</td>
<td>AUMs(_F4)</td>
<td>AUMs(_F4)</td>
<td>AUMs(_F4)</td>
<td>AUMs(_F4)</td>
</tr>
<tr>
<td>11 Fall labor (Oct-Dec)</td>
<td>AUMs</td>
<td>( Y_4 )</td>
<td>AUMs(_L4)</td>
<td>AUMs(_L4)</td>
<td>AUMs(_L4)</td>
<td>AUMs(_L4)</td>
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<td>AUMs(_L4)</td>
<td>AUMs(_L4)</td>
</tr>
<tr>
<td>12 Capital (Oct-Dec)</td>
<td>Dol.</td>
<td>( Z_4 )</td>
<td>AUMs(_C4)</td>
<td>AUMs(_C4)</td>
<td>AUMs(_C4)</td>
<td>AUMs(_C4)</td>
<td>AUMs(_C4)</td>
<td>AUMs(_C4)</td>
<td>AUMs(_C4)</td>
<td>AUMs(_C4)</td>
<td>AUMs(_C4)</td>
</tr>
</tbody>
</table>
The $a_{ij}$ is derived from equations (4-5) and (4-6), or (4-5) and (4-7) using the information from Table IV for establishing the values of $g$, $h$, $k$, and $w$. The coefficients of the objective function are similarly obtained by using equation (4-8) for the gross revenue per livestock system unit (CRLSU) and obtaining the variable cost per livestock system unit (VCLSU) as shown in the above section pertaining to variable costs.

The linear program solution may pick one activity as an optimum. This would indicate a homogeneous livestock inventory as the optimum system. If the optimum solution involves several activities then interpolation from the relative amounts of the linear programming model to the values assigned $g$, $h$, $k$, and $w$ of the basic equation must be solved. It must be noted that $g$, $h$, $k$, and $w$ must be equal to or between zero and one. As the results of Model III will indicate, the output may well result in an optimum heterogeneous livestock inventory system.

It is often of interest in linear programming to investigate a single or several changes in one or more of the coefficients of the objective function. This is, of course, equivalent to the dual problem of changing the coefficients of the constraints or as commonly referred to as a change in the right-hand side. This procedure is usually referred to as parametric linear programming. The implication is one of tracing out changes in optimum solutions with incremental changes in specific resources, factor prices or product prices. In the technique developed for the livestock inventory system, one might for instance want to trace out the
result of incrementing cattle prices among classes, relative weights of different cattle classes or relative costs among classes of cattle. All of these would in effect be a form of parametric linear programming. One could simulate a vast quantity of different optimum livestock inventories directly on a computerized hook-up between equations (4-5), (4-6), (4-7), and (4-8) and Table V.

Before turning to Model II, it should be noted that activities eight and nine are combinations of other activities in the first seven and as a consequence, it is only necessary to investigate seven homogeneous livestock inventory systems. As can be noted from Table IV, activity eight is a combination of two and three. Activity nine is a combination of four and seven. Therefore, activities eight and nine are redundant and disregarded in the following analysis.

Model II--The Problem of Administered Inputs

As was noted earlier, the range livestock industry in the western states is highly dependent upon the use of federal range permits for obtaining range forage for livestock production. In 1960, ranchers paid to graze 4.4 million head of cattle on lands administered by the United States Forest Service and the Bureau of Land Management. Federal range lands provided 24 million animal unit months of feed in 1960 (U.S.D.A., 1965). These 24 million animal unit months are in terms of administered animal unit months. An animal unit month is charged for any class of cattle over six months of age for one month of grazing. A calf six months of age or less is
allowed to graze free of charge and does not count in the allotment to the individual rancher.

For the western states, with the possible exception of agricultural policy with respect to grain production, no other agricultural policy question has caused more controversy than the management of public lands. To state that the emotional tone by livestock producers, public land administrators, experiment station personnel, politicians, and the public in general runs high, understates the degree of temperament.

The probability of misallocation as a result of government administrative dictum is sufficiently high to attract the scrutiny of many economists. As Gardner (1959, p. 229) has noted:

"in most cases, markets do not exist and values are difficult, if not impossible to determine. One thing seems certain . . . the non-price methods of allocating resources among users have not been amenable to rigorous economic analysis."

Gardner then developed the argument that since government charges less for a grazing feed than comparable private rates that this condition will result in misallocation. In the discussion of Gardner's paper, Taylor (1959, p. 246) states

"since grazing services are not sold on a basis of 'competetive' bidding, Gardner takes these things to be prima facie evidence of misallocation. It seems to me that the classical allocative model is hardly appropriate for this problem. The problem seems to be more one of income distribution than of allocative efficiency. The government rations a certain amount of grazing. Obviously, there can be no supply curve as such and the fact that the government fails to extract its full monopoly price has no necessary relation to efficiency in terms of marginal economic quantities, and unless permit holders can be shown to be less efficient than non-permit holders,
it is difficult, if not impossible, to perceive misallocation. Such rationing at prices below the marginal value productivity increases the income of permit holders relative to non-permit holders, but this is not necessarily an allocation problem. And even here, we would expect permit advantages to be eventually capitalized into land values so that the income advantage would disappear with time."

The question of grazing fees levels is not of issue in this thesis. However, the key statement by Taylor that unless permit holders can be shown to be less efficient than nonpermit holders, then misallocation is difficult to show is of course of concern in the analysis to follow.

The point of issue here is with respect to the effect on resource allocation. The question is, will a firm allocate resources the same when it is constrained with an animal unit feed base in which in one instance is assumed to be owned with the freedom to be used in the production process at a rate controlled through biological relationships as would be the case in private control use and in another instance in which the rate of use is modified by a government administrative decision? There is some intuitive appeal to the notion that some differences will occur. How much and how to evaluate these differences in a meaningful way is the problem to be investigated. For instance, assume a given parcel of land will furnish an adequate input range forage for 600 breeding cows for one month. If the ranch operator is assumed to own the land, the linear programming model developed earlier would permit this forage to be used by yearling steers at a rate that would allow 1,000 head to be grazed for one month. If the Bureau of Land Management
administers this land parcel, the ranch firm will through regulation be allowed to graze only 600 head of yearlings for one month. This results from the administrative agency definition that all cattle over six months in age are considered as a full animal unit. All calves six months or under are allowed to graze without charge against the allotment.

To analyze the influence of the administered forage input the linear programming model presented in Table V was expanded into Model II. A simplified version of a linear program that will be used in the next section is presented in Table VI. Table VII presents the animal unit months of feed required for the calculation of $\text{AUMs}_{F_1}$ and $\text{AUMsP}_{F_1}$ from equation (4-5), (4-6) or (4-7). The $\text{NINCK}_{1}$ is the net income to the fixed factors for the homogeneous livestock system $K_{1}$. Activities one through seven are the homogeneous livestock systems which use the feed forage at the rate given in Table I. The activities eight through 21 have the same total resource base available but .8 of the feed during the second and third periods is obtained from a public grazing permit. The other .2 of the feed resource in periods three and four is produced on private lands.

In effect, linear program II could be handled as two distinct programs for a single or for two independent firms. The first seven activities would yield the same result as program I for firm 1 and the final 14 activities would be the results for firm 2. The differences, if any, between the two must be the results of administering the resource "range forage". This difference will be defined as a
TABLE VI. LINEAR PROGRAM FOR HOMOGENEOUS LIVESTOCK SYSTEMS FOR PRIVATE LAND ($x_1$, ..., $x_7$) AND FOR PRIVATE AND PUBLIC LANDS ($x_8$, ..., $x_{21}$).

<table>
<thead>
<tr>
<th>Resource</th>
<th>Unit</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$x_5$</th>
<th>$x_6$</th>
<th>$x_7$</th>
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<th>$x_9$</th>
<th>$x_{10}$</th>
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<th>$x_{13}$</th>
<th>$x_{14}$</th>
<th>$x_{15}$</th>
<th>$x_{16}$</th>
<th>$x_{17}$</th>
<th>$x_{18}$</th>
<th>$x_{19}$</th>
<th>$x_{20}$</th>
<th>$x_{21}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winter feed (Jan-Mar)</td>
<td>$x_1$</td>
<td>$x_2$</td>
<td>$x_3$</td>
<td>$x_4$</td>
<td>$x_5$</td>
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<tr>
<td>Winter labor</td>
<td>$x_1$</td>
<td>$x_2$</td>
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<td>$x_5$</td>
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<td>$x_{19}$</td>
<td>$x_{20}$</td>
<td>$x_{21}$</td>
<td></td>
</tr>
<tr>
<td>Capital</td>
<td>$x_1$</td>
<td>$x_2$</td>
<td>$x_3$</td>
<td>$x_4$</td>
<td>$x_5$</td>
<td>$x_6$</td>
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<tr>
<td>Spring feed</td>
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<td>Spring labor</td>
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<td>$x_{19}$</td>
<td>$x_{20}$</td>
<td>$x_{21}$</td>
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</tr>
<tr>
<td>Summer feed</td>
<td>$x_1$</td>
<td>$x_2$</td>
<td>$x_3$</td>
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<td>$x_5$</td>
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<td>Summer labor</td>
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<tr>
<td>Fall feed</td>
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</tbody>
</table>
TABLE VII. ANIMAL UNIT MONTHS OF FEED EQUIVALENTS FOR GRAZING UNDER PUBLIC PERMIT--
SIX-MONTH PERIOD OF PUBLIC USE AND SIX MONTHS FULL DEPENDENCY ON PRIVATE
NUTRIENT SOURCES.

<table>
<thead>
<tr>
<th>Livestock Class</th>
<th>Notation</th>
<th>Jan - March</th>
<th>April - June</th>
<th>July - Sept</th>
<th>Oct - Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Private</td>
<td>Public</td>
<td>Private</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>AUMsF₁</td>
<td>AUMsP₁F₂</td>
<td>AUMsP₁F₃</td>
<td>AUMsF₄</td>
<td></td>
</tr>
<tr>
<td>Breeding cow</td>
<td>C₉F₂</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Breeding bull</td>
<td>B₉F₁</td>
<td>4.5</td>
<td>3.0</td>
<td>3.0</td>
<td>3.6</td>
</tr>
<tr>
<td>Breeding heifer</td>
<td>H₉F₂</td>
<td>3.6</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Yearling heifer</td>
<td>Y₉F₂</td>
<td>1.5</td>
<td>3.0</td>
<td>3.0</td>
<td>2.4</td>
</tr>
<tr>
<td>Yearling steer</td>
<td>Y₉F₁</td>
<td>1.5</td>
<td>3.0</td>
<td>3.0</td>
<td>2.4</td>
</tr>
<tr>
<td>Steer coming 2's</td>
<td>S₇F₁</td>
<td>2.4</td>
<td>3.0</td>
<td>3.0</td>
<td>3.3</td>
</tr>
<tr>
<td>Heifer coming 2's</td>
<td>S₇F₁</td>
<td>2.4</td>
<td>3.0</td>
<td>3.0</td>
<td>3.3</td>
</tr>
<tr>
<td>Calf (either sex)</td>
<td>K₉F₁</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.2</td>
</tr>
</tbody>
</table>
misallocation. These differences do not result from the charge for grazing privileges on public lands being different than similar private grazing rates. These differences can be anticipated for the most part to favor those livestock systems which are predominantly either breeding cows or older steers and heifers. That differences are almost certain to occur is not the issue here but rather how much difference and how to isolate the difference.

A programming model similar to Model II allows one to make systematic parametric changes in the basic inventory equations, generate the new set of program coefficients, solve the linear program, and be able to trace out the changes all from a single computer program output. For instance, changes in the calving percentage "p" in the basic equations will be investigated for the study area. This is a form of indirect parametric programming. However, parametric programming typically refers to systematic changes in the objective function or the right-hand side values while the change in "p" in the equation system will involve changes in the objective function coefficients and the input-output coefficients. These differences are important largely in the definitive sense but not in an operations research sense.

The linear program to be developed in this section is to be formulated such that the resource base reflects a typical ranch resource situation for the range area of southeastern Oregon.
Model III--The Study Area Optimum Livestock System

The total operating variable costs for the various livestock classes for this model are those determined by the regression equation (4-9). Survey data indicates that the ranch operators normally keep three heifer calves for every two needed for actual replacement. The normal replacement rate was 15 percent. Therefore, the value of .15 was assigned to "a" in the relevant resource, cost, and revenue equations and the value of .075 for "b".

Calving percentages at five levels were investigated. This percentage is assumed to be the probability of a cow in the livestock inventory on January 1 having a calf that lives to maturity. Death rates were obtained for cows but the number of calves born alive and later died were not obtained. The surveyed ranches had negligible death losses among other classes of livestock. As a consequence of the survey data a probability of zero is assigned to death losses for all classes except for breeding cows. The revenue function is used to reflect the 2.5 percent death loss in the breeding herd. An interesting relationship was noted between the calving percentage and the number of cows per breeding bull. The survey data indicated a large variation in calving percentage from a low of 37 percent to a high of 96 percent. Also, the data revealed that the number of cows per breeding bull varied from a low of 12 to a high of 60. As a result of this phenomenon and the probable direct relationship of calving percentage "p" and the number of bulls per cow "c" in the resource requirement equations, a least-squares
estimate was made to establish this relationship and is presented in equation (5-10).

\[ p = 0.58174 + 5.23796c \quad 0 \leq c \leq 0.07 \] (5-10)

The correlation coefficient for the above relationship was approximately 0.86. The probability of observing the above relationship when in fact no such relationship exists was estimated to be less than one percent. For this reason the relationship estimated by equation (5-10) was used as a part of the computer format for calculating resource requirements for the linear programming model. The data indicated that the average calving percentage for the study area would be approximately 83 percent with about one bull per 21 cows.

Prices received for the various livestock classes are assumed to be known with certainty. One of the major objectives is to determine the optimum livestock inventory system. For this purpose, relative prices among the various classes are the only significant relationship, not the level of cattle prices. The reason being that the programming model selects among the activities in which relative prices have a direct impact on the most profitable activity.

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1There are some disturbing consequences if expression (5-10) is not viewed within a narrow perspective. Perhaps a more refined specification would have been more meaningful. This specification would involve a functional relationship in which \( p \) would asymptotically approach the value of zero at the lower limit with an upper limit of one. The structure presented was easily entered in the computer program that had been structured to obtain resource requirements and this "naive" expression was accepted as adequate for the analysis.
The accuracy with respect to the level of prices is important for evaluating the marginal value product for such items as summer range, winter feed, labor and capital but not in the selection of the optimum inventory structure.

For purposes of analysis all beef cattle prices that are relevant to the study area are assumed to move together. This will also be the assumption for the next chapter pertaining to decision making under uncertainty. To aid in the justification of this assumption the correlation coefficient was calculated between various livestock classes. The correlation coefficient between the index price of feeder steers and feeder calves for the period 1947 through 1965 was calculated to be .95. Between feeder steers and commercial slaughter cow index prices, the correlation coefficient value was .97 and between feeder calves and commercial slaughter cows the value was .92. This strong relationship can be noted from Figure 3.

The assumption that all relevant cattle prices move together will permit the selection of a unique optimum livestock system. If this were not the case, the system would require an infinite number of relative prices to be analyzed. The alternative if prices did not move together in a direct relationship would be the selection of a few relative price situations and optimizing with respect to these and this would have entered a degree of arbitrariness and would have considerably weakened the analysis.

The question might be raised whether or not the market system is working sufficiently well to relate prices from the national markets to the local Oregon markets. David W. Norman (1964)
Figure 3. Indexed Prices of Selected Livestock Classes at Kansas City and Chicago Markets Over Time Period 1947-1965.\(^a\)

calculated the correlation coefficient between reported Oregon and United States slaughter cattle prices and obtained a value of .9999.

As can be noted from Figure 3, the various classes of cattle prices have been relatively stable for the period from 1953 through 1965 compared to the longer period of 1947 through 1965. For this reason, the average prices for 1953 to 1965 by class were calculated and are presented in Table VIII. These prices will be used for the assumed product price in this chapter. The relationship between steer calves and heifer calves and the relationship between the larger steer and heifer stockers were calculated using the relative price differences established for use in a grazing fee study conducted by the United States Department of Agriculture (1965).

Study Area Input-Output Coefficients

Table IX is presented as an example of the use of the equation system developed to yield values that are relevant for the study area. The first three homogenous livestock systems; \( K_1 \), \( K_2 \), and \( K_3 \) are sufficient to illustrate the output of the equation system developed earlier for the input to the linear programming model.

The decrease of gross revenue per system unit as calving percentage goes up, at first glance, may seem paradoxical. However, for livestock system \( K_1 \) the equation system requires that a complete system unit be produced. This means that the system must furnish a sufficient number of breeding cows to produce a steer calf and a part of a heifer calf for sale as well as the breeding cow replacement. As the calving percentage goes down this requires a greater number of
TABLE VIII. PRICES PER HUNDREDWEIGHT FOR CATTLE, BY CLASS, 1953-1965 AVERAGE AT SELECTED MARKETS, ADJUSTED TO OREGON MARKETS, AND AVERAGE WEIGHTS BY CLASS FOR SOUTHEASTERN OREGON.a

<table>
<thead>
<tr>
<th>Class of Cattle</th>
<th>1953-1965 Average</th>
<th>Adjusted for Oregon b</th>
<th>Average Weight for Southeastern Oregon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dollars</td>
<td>Dollars</td>
<td>Pounds</td>
</tr>
<tr>
<td>Cows (Chicago market):</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commercial, utility</td>
<td>14.83</td>
<td>13.85</td>
<td>1,077 (dry)</td>
</tr>
<tr>
<td>Canner, cutter</td>
<td>12.84</td>
<td>11.99</td>
<td>926 (wet)</td>
</tr>
<tr>
<td>Feeders and stockers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Kansas City market):</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steers (over 800 lbs)</td>
<td>21.09</td>
<td>19.70</td>
<td>967</td>
</tr>
<tr>
<td>Heifers (over 700 lbs)c</td>
<td>18.13</td>
<td>16.93</td>
<td>791</td>
</tr>
<tr>
<td>Yearling steers (500 to 800 lbs)</td>
<td>21.79</td>
<td>20.35</td>
<td>647</td>
</tr>
<tr>
<td>Yearling heifers (500 to 700 lbs)c</td>
<td>18.73</td>
<td>17.49</td>
<td>606</td>
</tr>
<tr>
<td>Calves steer (300 to 500 lbs)</td>
<td>25.05</td>
<td>23.40</td>
<td>394</td>
</tr>
<tr>
<td>Calves heifer (300 to 500 lbs)c</td>
<td>21.95</td>
<td>20.50</td>
<td>374</td>
</tr>
</tbody>
</table>

aSimple average of monthly prices LMS Statistical Bulletin No. 337, August 1966, and simple average of selling weights obtained from survey data.

bThe average Oregon commerical slaughter cattle price divided by U. S. commercial slaughter cattle price yields an adjustment weight of .934. Assumed to be the same differential for stocker and feeder. (Source of slaughter prices from Oregon State Bulletin 594.)

cDetermined by interpolation from price information in ERS 248, U. S. Department of Agriculture bulletin.
TABLE IX. GROSS REVENUE, TOTAL OPERATING VARIABLE COSTS, PROFIT, AND TOTAL FEED REQUIREMENTS—ALL PER LIVESTOCK SYSTEM UNIT, FOR FIVE LEVELS OF CALVING PERCENTAGES, FOR THREE OF THE SEVEN HOMOGENEOUS LIVESTOCK SYSTEMS.a

<table>
<thead>
<tr>
<th>Pure Livestock System</th>
<th>Unit</th>
<th>Calving Percentage</th>
<th>.75</th>
<th>.80</th>
<th>.85</th>
<th>.90</th>
<th>.95</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gross revenue:</strong> b</td>
<td>K1</td>
<td>Dol.</td>
<td>96.14</td>
<td>95.40</td>
<td>94.76</td>
<td>94.18</td>
<td>93.67</td>
</tr>
<tr>
<td></td>
<td>K2</td>
<td>Dol.</td>
<td>115.87</td>
<td>115.14</td>
<td>114.50</td>
<td>113.92</td>
<td>113.41</td>
</tr>
<tr>
<td></td>
<td>K3</td>
<td>Dol.</td>
<td>102.00</td>
<td>101.82</td>
<td>101.66</td>
<td>101.51</td>
<td>101.39</td>
</tr>
<tr>
<td><strong>Total variable operating costs:</strong> c</td>
<td>K1</td>
<td>Dol.</td>
<td>44.65</td>
<td>44.23</td>
<td>43.85</td>
<td>43.52</td>
<td>43.22</td>
</tr>
<tr>
<td></td>
<td>K2</td>
<td>Dol.</td>
<td>52.50</td>
<td>52.08</td>
<td>51.70</td>
<td>51.37</td>
<td>51.07</td>
</tr>
<tr>
<td></td>
<td>K3</td>
<td>Dol.</td>
<td>47.79</td>
<td>47.66</td>
<td>47.55</td>
<td>47.44</td>
<td>47.35</td>
</tr>
<tr>
<td><strong>Profit:</strong> d</td>
<td>K1</td>
<td>Dol.</td>
<td>51.49</td>
<td>51.17</td>
<td>50.91</td>
<td>50.66</td>
<td>50.45</td>
</tr>
<tr>
<td></td>
<td>K2</td>
<td>Dol.</td>
<td>63.37</td>
<td>63.06</td>
<td>62.80</td>
<td>62.55</td>
<td>62.34</td>
</tr>
<tr>
<td></td>
<td>K3</td>
<td>Dol.</td>
<td>54.21</td>
<td>54.16</td>
<td>54.11</td>
<td>54.07</td>
<td>54.04</td>
</tr>
<tr>
<td><strong>Total AUMs feed:</strong> e</td>
<td>Private</td>
<td>AUMs</td>
<td>19.68</td>
<td>18.70</td>
<td>17.83</td>
<td>17.06</td>
<td>16.38</td>
</tr>
<tr>
<td></td>
<td>K1</td>
<td>AUMs</td>
<td>22.98</td>
<td>22.00</td>
<td>21.13</td>
<td>20.36</td>
<td>19.68</td>
</tr>
<tr>
<td></td>
<td>K3</td>
<td>AUMs</td>
<td>21.00</td>
<td>20.14</td>
<td>19.38</td>
<td>18.71</td>
<td>18.11</td>
</tr>
</tbody>
</table>

(continued)
TABLE IX. (continued) GROSS REVENUE, TOTAL OPERATING VARIABLE COSTS, PROFIT, AND TOTAL FEED REQUIREMENTS—ALL PER LIVESTOCK SYSTEM UNIT, FOR FIVE LEVELS OF CALVING PERCENTAGES, FOR THREE OF THE SEVEN HOMOGENEOUS LIVESTOCK SYSTEM.

<table>
<thead>
<tr>
<th>Pure Livestock System Unit</th>
<th>.75</th>
<th>.80</th>
<th>.85</th>
<th>.90</th>
<th>.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public K₁ AUMs</td>
<td>19.21</td>
<td>18.18</td>
<td>17.27</td>
<td>16.46</td>
<td>15.74</td>
</tr>
<tr>
<td>Public K₂ AUMs</td>
<td>23.56</td>
<td>22.53</td>
<td>21.62</td>
<td>20.81</td>
<td>20.08</td>
</tr>
<tr>
<td>Public K₃ AUMs</td>
<td>20.95</td>
<td>20.08</td>
<td>19.32</td>
<td>18.65</td>
<td>18.02</td>
</tr>
</tbody>
</table>

\( a \) This table uses Table IV for the livestock systems and Table VIII for the livestock prices and weights; also assumes \( a = .15, b = .075, c = (p - .58174)/5.23796 \).

\( b \) Calculated using equation (4-8).

\( c \) Calculated using equations (4-5) and (4-9).

\( d \) (Gross revenue - total variable costs) = profit per system unit.

\( e \) Uses equations (4-5) and (4-6) with Table I for private forage requirements; equations (4-5) and (4-6) with Table VII for public forage requirements.
breeding cows to maintain this output and as a consequence, more beef
cow sales results from the breeding herd cull. This cow beef sale
increases the gross revenue as the calving percentage goes down. In
fact, for the illustrated systems, the profit per system unit
decreased as the calving percentage increased. This result was a
consequence of operating expenses increasing more rapidly than gross
revenue as calving percentage decreases. As would be expected, the
amount of feed resource, and this would also be true for labor and
capital, to produce the system unit varies inversely to the calving
percentage. The profit per unit of feed, labor, and capital varies
directly with the calving percentage as would be expected. The
values for the feed unit requirements presented are for the total
year. The linear program model developed uses each fixed resource
with respect to four distinct time periods. Each parametric change
with respect to calving percentage requires the complete evaluation
of the linear program. Not only do the objective function values
change as a result of calving percentage changes, but the input-output
coefficients also change. The calculations for the objective function
and the resource coefficients are easily programmed for computer
computation. The calving percentage intervals to be investigated
can be made arbitrarily small. Although this may appear to be an
example of a system which can generate upon itself an inexhaustible
supply of data, with the use of proper controls, this is not the
case but is, instead, a very efficient way of investigating a rather
complex inventory system. The system developed here and presented
in the linear program examples in Tables V and VI allows one to
investigate any resource base relationship as opposed to specification of actual resource amounts. Mere relative amounts of resources available at various time periods are sufficient to evaluate the most profitable livestock inventory composition. The major advantage of the resource accounting equations, when compared say to a standard partial budgeting process, are their simplicity in determining the resource requirements necessary to maintain the assumed inventory in terms of total resource requirements and for as small a specified time interval required for proper analysis.

Specific Resource Constraints for the Study Area

The resource requirements calculated by the equations developed for establishing the input-output coefficients per livestock inventory system units are in terms of animal unit month equivalents for feed and labor and in terms of dollars for the capital requirement. Determining the amount of each resource available in the same unit equivalent for each time period is the central topic of this section.

Feed Constraint

The survey information obtained included data on the amount of acreage per ranch by land use type. In addition to the amount of each land type, the time and length of use of the land by the livestock was obtained. To establish the amount of feed produced by each type of land the data was fitted to expression (5-11).
$Y = 38.113045 + .106104X_1 + .444000X_2 + .063955X_3$

$\text{.026004} \text{ (.066562) (.067889)}$

$+ .017092X_4 + .086778X_5 + .005469X_6 + .086109X_7$

$\text{.004558} \text{ (.044524) (.005702) (.005140)}$

where: $R^2 = .93$

$Y =$ number of AUs in ranch

$X_1 =$ acres of irrigated hayland

$X_2 =$ acres of irrigated pasture

$X_3 =$ acres of dryland hay

$X_4 =$ acres of private unimproved range

$X_5 =$ acres of improved range

$X_6 =$ feed supplement purchased

$X_7 =$ AUMs of public range permit

(numbers in parentheses are standard error estimates for the respective parameter)

It should be pointed out that the parameter estimates on $X_3$ and $X_6$ are from a probability viewpoint very doubtful of being different than zero. These two sources of feed account for only slightly more than two percent of the total animal unit feed. A partial explanation for the large error relative to the coefficient on $X_3$ may result from the large number of zero elements in the observation for that variable. The only explanation offered for retaining a value for $X_6$ is that a minimum amount of feed must be purchased. Feed supplement purchases such as minerals and salt can be purchased and stored to take advantage of income tax provisions and therefore may appear or not appear on the farm records for any given operating.
year. For purposes of analysis these two feed sources were included in the feed base with the constant term under the category of "feed supplement and miscellaneous sources".

It is interesting to note that the coefficient on $X_5$ would "a priori" be assigned the value of 0.083333. The number of animal unit months times 12 should give the increase in animal units if no error is involved and the ranch operator is using the entire public permit.

To facilitate the use of the information from equation (5-11) and the time of use patterns obtained from the survey some of the land types were combined. For instance, the irrigated pasture and the improved private range were generally used together during the grazing period. Dryland hay was combined with the feed supplement purchases along with the animal units equivalents accounted for by the constant term and are listed as "feed supplement and miscellaneous sources".

Figure 4 is presented to supplement the equation system of the linear programming model presented in Appendix A. The quantities presented in Figure 4 are the right-hand sides for the feed resource of the linear program models. The arrows indicate the feasible delay in time of use for any given feed source. This condition is reflected in the linear program through the use of transfer activities. The transfer of an animal unit month equivalent from one period to another is assumed to be one to one. As indicative by Nielsen (1965) this may not be the case for early spring use of range lands since delay in use may result in a greater total
Figure 4. Flow Chart of Animal Unit Month Equivalents of Feed, By Source of Feed, Time of Use, for Four Time Periods Involving a Representative Southeastern Oregon Cattle Ranch.
capacity than would be the case if grazing is allowed over the entire region for the whole season. Further evidence pertaining to this matter was indicated by the experiments at the Squaw Butte range experiment station. Raleigh and Wallace (1961, 1962) suggest that grazing on sagebrush-grass range during the spring green up removes too much of the early herbage that is necessary for the shrubs and the grasses to regain their healthy vigor. These authors suggest that this condition emphasizes the need for crested wheat-grass to take the pressure off native ranges during the early spring months. In an earlier publication, the evidence indicated that the sagebrush-bunchgrass ranges that are typical of southeastern Oregon are fully mature by July and thereafter decline rather steadily in nutritive value (Gates, Goetze; 1964). To account for all of these differences in the linear program would require a significant increase in the number of time periods along with the establishment of values other than one to one in the transfer activities. If one justified the differences in time of use then it seems that there is sufficient evidence to indicate that differences also exist for the value of certain feeds to vary among cattle classes. For instance, there is evidence that indicates that if weaner calves are fed only meadow hay during the winter months that they will merely maintain themselves (Raleigh, Wallace; 1961). Also that some supplement late in the summer months was found to improve yearling growth. However, as the supplement ration was increased to the yearlings the animals foraged less and this offset the supplement gain. With full recognition that this evidence is important to a range livestock operation,
it is assumed that the model using the structure presented in Figure 4 is sufficiently broad to implicitly encompass these conditions. For instance for the linear program feed constraints, as shown in Figure 4, there is considerably more of the feed resource "feed supplement" available during the late fall and winter months as would be the case for increased feeding of feed concentrates to the younger stock. Further, there appears to be sufficient flexibility through a large number of transfer activities to gain the proper time balance without refining the relationships to reflect the gain or loss in specific time of use with respect to a specific source of feed.

Labor and Capital Constraints

The amount of labor required and the contribution of family labor for the programmed operation was estimated from the survey data and with additional information from a study conducted by the University of Nevada (Lloyd, Reuben, Hecht; 1959). The majority of the surveyed operations which were within 50 animal units of the program model size employed two full-time hired men plus some additional hired labor during the summer "haying period".

Hired labor for most cattle ranching operations is not a completely divisible input. A common comment by ranch operators is that the ranch labor requirements would not demand the full use of the last man hired. In order to have the labor available during certain work peaks such as during the calving season, the ranch operators carry a surplus labor input for extended slack periods.
If one wanted to investigate a program solution in which the labor resource becomes available in rather large incremental values, this would require the use of an algorithm of the type employed in integer programming. For the analysis here, this additional complication was avoided; the ranch size established from the feed resource seemed to fit the two full-time hired men, with some additional labor available either from temporary hired labor or from additional family labor during the spring and summer months.

The labor constraint was investigated at the assumed minimum that would be available from the family and full-time hired labor and for the lowest and highest calving percentage. This was done to investigate the influence of this factor on the livestock inventory system. This same right-hand side change was made for the capital constraint for the lowest and highest calving percentage investigated. The magnitude of these changes and the program results are discussed in the following section pertaining to the solutions obtained from the linear programming model.

**Results of Model III**

Although a sensitivity analysis was not performed on the linear program model, the results indicate that there would be no change in the optimal variables in the basis for the range of capital and labor constraints that appear to prevail in the study area. This agrees with the survey questionnaire section related to obtaining the ranchers' opinions of limiting resources. Additional capital and labor usually did not appear as a reason given by the ranch
owner for the present size limitation, or structure of the ranch organization. Generally, the answer related to lack of feed or water. However, enumerator comments on the schedule indicated that some ranchers believed they could carry more cattle but really did not have a "desire" to expand their operations. Whether this indicated a backward bending supply supply curve for entrepreneur labor and management or a reluctance to risk capital is not evident from the data. Optimum program results indicate that additional capital, when capital is the constraining resource, has a marginal value product of two percent for a three-month period, or approximately eight percent on an annual basis.

The optimum livestock inventory composite for the study area is presented in Table X. The feed source is the limiting resource with respect to size. Interpreting the linear program solution in terms of the optimum composition of the various livestock types requires inversion of the system units back through the equation structure. For instance, the optimum programmed solution indicates approximately 184 units of system $K_1$ and 47 units of $K_5$. This is for the 80 percent calving rate and the ranch base that depends both on private and public lands. From Table IV, $K_1$ is a cow-calf homogeneous activity and $K_5$ is a cow-heifer calf-steer coming 2 activity. In other words, the optimum structure calls for selling heifer calves, steer calves, two-year old steers and the normal cow sales. Table IV indicates that $g = h = k = w = 0$ for the 184 units of $K_1$ while $g = k = 1$ and $h = w = 0$ for the 47 units of $K_5$. 
TABLE X. OPTIMUM LIVESTOCK INVENTORY SYSTEM FOR FIVE CALVING PERCENTAGES, FOR REPRESENTATIVE SOUTHEASTERN OREGON CATTLE RANCH WHEN FEED IS THE LIMITING RESOURCE, ONE RANCH OPERATES USING PRIVATE FEED SOURCE, THE OTHER USES PRIVATE AND PUBLIC ADMINISTERED LANDS.

<table>
<thead>
<tr>
<th>Type of Livestock</th>
<th>Calving Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.75</td>
</tr>
<tr>
<td>Breeding cow (over 2 years)</td>
<td>222</td>
</tr>
<tr>
<td>Breeding heifer (2 years old)</td>
<td>39</td>
</tr>
<tr>
<td>Bulls</td>
<td>9</td>
</tr>
<tr>
<td>Steers (2 years and over)</td>
<td>0</td>
</tr>
<tr>
<td>Beef heifers (2 years &amp; over)</td>
<td>0</td>
</tr>
<tr>
<td>Yearling steers</td>
<td>101</td>
</tr>
<tr>
<td>Yearling heifers</td>
<td>60</td>
</tr>
<tr>
<td>Calves</td>
<td>201</td>
</tr>
</tbody>
</table>

\(^a\)Assumes a breeding cow replacement rate of 15 percent \([a = .15 \text{ in equation (4-5)}]\) with the selection made on the basis of three yearling heifers for every two required for actual replacement \([b = .075 \text{ in equation (4-5)}]\). Assumes a July 1 composition in which the 2.5 percent breeding herd death loss has already occurred. All values rounded to nearest whole number.
From the above information, equation (4-5) can be used to determine the structure of the inventory. For instance, to produce 184 units of $K_1$ and 47 units of $K_5$, which is equal to a total of 231 total combination units, requires a total of 231 divided by $p = .80$ or approximately 289 total cows and heifers in the inventory on January 1. After a death loss of 2.5 percent or approximately seven head, the inventory of breeding cows after death losses is 281. Of the 281 breeding cows, the 15 percent young cows (2-year olds) requirement is approximately 42 as indicated in Table X.

Again using equation (4-5) it is obvious that many of the coefficients become zero for the 184 system units since $g = h = k = w = 0$, but for the 47 system units the $(47)(.5)(1)$ times yearling steers ($Y^S$) and the $(47)(.5)(1)(1)$ times steers coming 2 ($S^S$) come into the solution and were not a part of 184 units of system $K_1$. The proper interpretation with respect to active inventory composition for the output of the linear programming model requires that a return path be made through the resources structure equations. There are two approaches which lead directly to the proper animal composition of the inventory structure and both can easily be constructed for direct computation by a computer. It should again be noted that a system unit assures that all the breeding stock and replacements necessary to produce the net output are a part of the inventory. This is true for any reasonable calving percentage, death loss of any livestock type, bull-cow ratio, and breeding stock replacement policy. Examples of an unreasonable condition would be 100 percent death loss or a zero calving percentage or replacing more than 50
percent of the breeding herd each year. The reasons for the limited breeding herd replacement is simply the requirement that the system must be self-contained (no purchases except breeding bulls) and one can expect no more than 50 percent of the calf crop being heifer calves. The restriction on calving percentage being greater than zero is merely to exclude the problem associated with division by zero.

The most direct route in the transformation of the system's optimum result to physical inventory terms is to take each optimum system unit output quantity, calculate the physical animal requirements for each homogeneous system, and sum the component parts. This approach involves more computation than the alternative of doing some structuring before the actual computations but may be more readily programmed for a computer where the number of necessary iterations become less significant.

The alternative procedure would take the sum of all the units of the optimum program and calculate the necessary breeding cows, bulls, and yearling heifers kept for replacement. The total optimal units would be used as a multiplier for all remaining livestock types of equation (4-5). However, the relative weight to the total of each homogeneous class would have to be calculated and used as a multiplier. For instance if in the above example, the optimal program called for 60 units of $K_2$ along with the 184 units of $K_1$ and 47 units of $K_5$, the homogeneous system $K_2$ (see Table IV) requires $g = 1$ and $h = k = w = 0$. The total optimal system now calls for 291 units of
production. To use 291 as the common multiplier requires that g and k equal some other value than one or zero. The value of g is the percent of steer calves carried over to steer yearlings. Both K₂ and K₅ require this carryover. The value of g is then equal to 
\[(47 + 60) ÷ 291; \approx 0.37.\]
The value of k is then equal to 
\[47 ÷ (47 + 60); \approx 0.44.\]
This would result in the number of steer calves carried over as 
\[
(291)(0.5)(0.37); \approx 54
\]
head of steer yearlings would be in the optimum inventory. The number of steers coming 2 is then 
\[
(291)(0.5)(0.37)(0.44); \approx 24
\]
as was required previously.

As defined in Chapter V, differences between the product that results from an optimum livestock inventory program when the feed source is related to a basic ranch unit which is completely under control of the private firm and when the feed source is administered through a public land agency is defined as misallocation. Tables XI and XII are presented as a supplement to Table X. The differences in product among calving percentages and between the control of the feed base are evident. The greatest structure change takes place between the 75 percent and the 80 percent calving percentage. This change is basically within the ranch firm which uses the public permit as a feed source.

An interesting aspect relating to the optimum livestock inventory composition is the general consensus of opinion of the personnel at the Squaw Butte Range Experiment Station. Much of the range research that is directly related to the study area is carried on at this station. There seemed to be a general agreement among the
TABLE XI. POUNDS OF BEEF BY TYPE OF ANIMAL PRODUCED FOR SALE BY OPTIMUM LIVESTOCK SYSTEM, FOR SOUTHEASTERN OREGON REPRESENTATIVE RANCH OPERATION, FOR FIVE CALVING PERCENTAGES, WITH COMPARISON OF PRIVATE FEED SOURCE AND A FEED SOURCE PARTIALLY ADMINISTERED BY A PUBLIC LAND AGENCY.\(^a\)

<table>
<thead>
<tr>
<th>Type of Livestock Sales</th>
<th>.75</th>
<th>.80</th>
<th>.85</th>
<th>.90</th>
<th>.95</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lbs.</td>
<td>lbs.</td>
<td>lbs.</td>
<td>lbs.</td>
<td>lbs.</td>
</tr>
<tr>
<td>Dry breeding cows</td>
<td>14,001</td>
<td>13,641</td>
<td>13,641</td>
<td>15,078</td>
<td>13,283</td>
</tr>
<tr>
<td>Wet breeding cows</td>
<td>24,076</td>
<td>23,458</td>
<td>23,458</td>
<td>25,928</td>
<td>22,841</td>
</tr>
<tr>
<td>Steers (2 years &amp; over)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>23,208</td>
<td>0</td>
</tr>
<tr>
<td>Beef heifers (2 years &amp; over)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Yearling steers</td>
<td>65,347</td>
<td>64,053</td>
<td>67,935</td>
<td>70,523</td>
<td>73,758</td>
</tr>
<tr>
<td>Yearling heifers</td>
<td>12,726</td>
<td>12,726</td>
<td>13,938</td>
<td>12,120</td>
<td>12,120</td>
</tr>
<tr>
<td>Steer calves</td>
<td>0</td>
<td>0</td>
<td>36,248</td>
<td>38,218</td>
<td>40,188</td>
</tr>
<tr>
<td>Heifer calves</td>
<td>14,960</td>
<td>14,960</td>
<td>17,204</td>
<td>19,448</td>
<td>22,814</td>
</tr>
<tr>
<td>TOTAL BEEF</td>
<td>131,110</td>
<td>128,838</td>
<td>134,864</td>
<td>133,100</td>
<td>142,946</td>
</tr>
</tbody>
</table>

\(^a\)See Table X for optimum livestock inventory by type of livestock. Sales from that inventory using weights from Table VIII are assumed in the calculations. Note that bull sales are not considered as a result of earlier assumptions.
TABLE XII. PERCENT OF TOTAL DOLLARS OF BEEF PRODUCED, BY LIVESTOCK OF OPTIMUM LIVESTOCK SYSTEM, FOR SOUTHEASTERN OREGON REPRESENTATIVE RANCH OPERATION, FOR FIVE CALVING PERCENTAGES, WITH COMPARISON OF PRIVATE FEED SOURCE AND A FEED SOURCE PARTIALLY ADMINISTERED BY A PUBLIC LAND AGENCY.\(^a\)

<table>
<thead>
<tr>
<th>Type of Livestock Sale Revenue</th>
<th>Calving Percentage</th>
<th>.75</th>
<th>.80</th>
<th>.85</th>
<th>.90</th>
<th>.95</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Private</td>
<td>Public</td>
<td>Private</td>
<td>Public</td>
<td>Private</td>
<td>Public</td>
</tr>
<tr>
<td>Cows</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>19</td>
<td>21</td>
</tr>
<tr>
<td>Steers (2 years &amp; over)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>19</td>
<td>0</td>
</tr>
<tr>
<td>Beef heifers (2 years &amp; over)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Yearling steers</td>
<td>57</td>
<td>57</td>
<td>58</td>
<td>0</td>
<td>57</td>
<td>16</td>
</tr>
<tr>
<td>Yearling heifers</td>
<td>10</td>
<td>10</td>
<td>9</td>
<td>10</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Steer calves</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>35</td>
<td>0</td>
<td>35</td>
</tr>
<tr>
<td>Heifer calves</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>16</td>
<td>16</td>
<td>18</td>
</tr>
</tbody>
</table>

\(^a\)See Table XI for pounds of beef and Table VIII for prices.
scientists that there existed a contradiction in the inventory system which they would recommend and the inventory structure that appears to prevail in the study area. The general opinion was that the ranchers in the area should be producing more older steer beef and less marketing of calf beef. This is exactly the recommendation that would be forthcoming from the analysis here if one assumed the ranch feed base control is entirely within the control of the entrepreneur. However, the results of the models developed in this study would indicate a general inventory complex that would favor the marketing of steer calves as the optimum program. This is directly related to the high dependency of the area on the public lands for a livestock feed source. The survey data indicated that the "typical" marketing pattern would be the sale of "weaner" calves in the fall.

Therefore, the results are consistent. They indicate what "should be" as defined by the experiment station scientists and are supported by the analysis for optimum resource allocation when the control of the resources are completely within the firm. The model suggests what "should be" from the viewpoint of maximizing profit by the entrepreneur when part of the resources are administered by a public agency. The ranch population appears to be following the optimum pattern and the model directs attention toward a factor which appears significantly to control "what is".

The ranch operation which has less than a 85 percent calf crop would have a higher return to the fixed factors if the feed base was controlled entirely by the firm as opposed to the mixture of
private and public. From the 90 percent calving percentage and above, the advantage shifts to the ranch operation which has a public land feed base. This result is not surprising and can be satisfactorily rationalized. However, there exist few areas in the range livestock regions of the entire west where a 90 percent calf crop can be expected. Also, the areas where public lands are a predominate range source of livestock feed are usually at the lower end of the expected calving percentage when compared to the entire western range region.

Analyzing the computer results of the linear program indicate at the lower calving percentage of .75 that the ranking of the magnitude of changes necessary in the objective function to bring about the entering of a specific activity into the basis places the cow-calf (K₁) livestock system in a weak fourth position. The same activity (K₁) is a strong third position for the combination of public and private lands. The influence of the public land policy for evaluating the nutrient requirements are the only explanation that can account for these differences. The return to fixed factors is lower for the ranch firm using the public land administered inputs as can be noted directly from the pounds of beef produced for sale in Table XI. The direct implication is simply that if the calving percentage is sufficiently low, the production advantage of larger and older animals outweighs the influence of the administered input. At this low level of calving percentage, the rancher who uses public lands then bears a "cost" for the administered input. This "cost" is a misallocation as the term was previously defined.
The marginal value product associated with an additional animal unit month of grazing for the 75 percent calving level for the public unit has a value of $2.63 while that for the private unit is approximately $2.75. It should again be noted that in all previous analysis the level of product and factor prices were not a factor in determining the optimum structure. The only relevant condition was that the weights of the costs and returns among the various livestock types were in the right proportion. However, the marginal value product magnitude is related to the level of the assumed product prices and cost structures. Further, the representative ranch did not have a single period in which the feed supply limited the other periods. The feed systems developed for the representative ranch were taken from the primary data and the system developed is shown in Figure 4. The lack of a single period of the year being the limiting resource was not a planned structure but a consequence of the numerous transfer alternatives. Although several ranchers noted periodic limiting feed conditions, a general observation for the study area would indicate that the ranch operations in general are sufficiently flexible to have a well overall balanced operation. However, as can be noted in Figure 4, the public permit is not a flexible feed source. If this large block of dependency during the period of April through September were reduced, this would seriously upset the balanced feed source. The systematic reduction of this public permit was not investigated but the problem is structured such that this would be a relatively simple operation. However, it would require additional computer runs of the linear program model.
At the 80 percent calving percentage level the optimum inventory structure of the ranch operation using the public grazing permit changes significantly from the optimum at the 75 percent level. The combination of homogeneous livestock systems, $K_1$ and $K_5$, enter the basis with $K_2$ leaving the basis. Of the non-basic activities for the privately owned operation, $K_1$ requires a larger improvement in objective function value than either $K_4$ or $K_5$ in order for it to enter the basis. The evidence indicates that the disparity widens between the optimum inventory structures as the calving percentage increased from the low level of 75 percent toward the average of 84 percent of the sampled ranches.

The 85 percent calving percentage optimal is the inventory structure which should closely resemble the average ranch firm situation. Further, the actual resource balance of public and private feed source associated with the other physical constraints would be representative for the study area. The marginal value product associated with an additional grazing unit would be approximately $3.00 for the study ranch. Additional capital when forced to the position of a constraining resource showed a simple rate of return of about 2 percent for a three-month period.

At the 85 percent calving percentage this study indicates that for various livestock systems, the ranch using only private resources would place the systems in the following order: $K_2, K_5, K_1, K_4, K_3, K_7, K_6$. For the ranch dependent on public grazing permits, the order is $K_1, K_2, K_5, K_3, K_4, K_6, K_7$. 
The labor budgeted, as determined from the primary data, did not constrain output. As a consequence, the marginal value product for labor is zero in the programming framework. This should not be interpreted to mean that all ranches have surplus labor. The ranch operators, especially among the larger units, in many instances noted that the quantity of dependable ranch labor supplied at the present wage offerings was becoming increasingly difficult to locate. For this reason, many ranch operators kept surplus labor when viewed from a year around requirement basis. This allowed for labor availability during peak work load periods and showed up in the data as an overall surplus. A right-hand linear program side change sufficient to bring about a change in the basis was not investigated for the labor constraint.
CHAPTER VI

THE UNCERTAINTY PROBLEM

This chapter takes the results of the certainty model of the previous chapter and investigates the use of a multivariable Bayesian statistical decision theory model for an optimal cattle inventory system. A set of equations is developed which express the money loss or gain involved with the decision of holding over any class of cattle as a function of next season's price. The uncertainty decision theory model is extended to include uncertainty with respect to forage production as well as price. A function to transform money losses and gains to utility gains and losses is developed. A technique for establishing a posteriori probability statements from a multivariate single equation regression model is proposed as the scheme for weighting the regret function. The criterion for decision making is the selection of the cattle inventory structure which will maximize expected utility using the a posteriori probabilities from the regression model. The model developed allows the use of separate subjective probability distributions for an analyst and the decision maker. The independence of the subjective and objective probability statements allows the scientific reproducibility of the experiment. The optimal livestock inventory is dependent upon the individual utility function, however it is argued that from a prediction viewpoint some general population statements can be made. For instance if an individual ranch operator is observed carrying over yearling heifers for beef production, the individual must accept unfair bets or have a strong a priori subjective probability distribution favoring the increase in cattle prices or both; a testable hypothesis.

In the previous chapter, the input-output coefficients (the $a_{ij}$'s); the constraints, (the $b$'s) and the value of the objective function (the $c_i$'s) were assumed to hold with probability one. The problem is solved for a single time period. The results are in terms of a static-condition. The condition to be investigated in this chapter is one of relaxing the assumption of certainty and investigate
the problem of decision-making under uncertainty. The problem situation is related to a representative cattle ranch resource structure.

There have been programming methods developed related to decision making under uncertainty. These methods are commonly referred to as linear programming under uncertainty or stochastic programming. The most common approach is to get the problem reduced to an ordinary linear programming problem and then solve by the simplex method. For instance, consider the linear programming problem in matrix form.

Maximize \( z = c^T x \) subject to

\[
A x < b \quad \text{and} \quad x \geq 0.
\]

An expectation operator could then be employed such as \( E(z) = \sum_{j=1}^{n} E(c_j)X_j = (c^T X) \) to indicate that a statistical technique has been used to estimate the coefficients of the objective function. One might conjecture that the optimum resource arrangement is quite stable as indicated through the use of sensitivity analysis when compared to the variance of the estimator.

No procedure is presently available for solving the general chance-constrained programming problems as developed by G. A. Charnes and W. W. Cooper (1959). This type of programming appears to have the most potential for problems in which scheduling of production where a given set of probable orders over \( n \) time periods is the issue.
The use of these techniques did not appear well adapted for the problem of decision making under certainty as related to the livestock inventory problem. For instance, the hypothesis that the prices of the various cattle types were independent was rejected. The assumption was then made that the various cattle prices move together and when the general price of cattle is known then the individual cattle prices can be calculated from this index. From the viewpoint of the individual firm this means that changes in the price of cattle over time merely changes the objective function in a parallel fashion and may increase the amount of use of certain resources such as borrowed capital along with proportional increases in the other resources but this will not alter the optimum inventory structure determined by the initial program. Further, the condition that year-to-year decisions must be made within the framework of an optimum livestock composition which includes the necessary breeding replacement as dictated by growing animal structure could not be realistically formulated for a simplex solution when an additional time period was added. This would suggest that a dynamic programming approach might be used since this technique is often useful for making a sequence of optimal interrelated decisions. One of the consequences common to dynamic programming problem solutions is that given some current state an optimal policy for the remaining stages is independent of the policy adopted in previous stages. The solution plots out the optimal decision pattern for the remaining periods for specified conditions. The problem to be solved in this section
deals only with the next time period in which the individual decision maker is allowed to use all relevant evidence pertaining to price. This evidence may be objectively listed or subjective data. The objective is to evaluate the next move. It was not obvious how this problem could be structured for solution by the present dynamic programming algorithm. For this reason, the analysis will proceed in a more pedestrian approach to decision making under uncertainty.

Components of the Uncertainty Decision Problems

The models developed for analysis herein will use terminology similar to that used in a book on decision theory by Chernoff and Moses (1959). These authors break down the basic factors involved with the typical statistical problem of decision making as (1) a set of possible actions, (2) a set of states of nature, (3) a set of strategies, (4) a probability distribution for component (3) above, (5) a regret function, and (6) a risk function (this is essentially the expected value of the regret function).

The synthesized cattle ranch firm will use the optimum composition as determined in the previous chapter. The system selected is the one presented in Table X for a calving percentage of 85 percent and feed base of private and public lands. The essence of the uncertainty situation is that it is marketing time and a decision must be made with respect to what, if any, modification is to be made in the normal optimal marketing pattern. The previous optimum was determined by the linear programming for a single period.
Uncertainty with Respect to Price

The entrepreneur has the alternative to sell the 97 steer calves or 60 heifer calves or hold any number of them for marketing in period (t). The entrepreneur can sell the normal 30 yearling steers or hold any number of them until period (t). Of the normal 44 cow sales, 15 are dry cows and assumed sold along with nine others which are assumed diseased or too old for consideration of holding. This leaves 20 breeding cows for decision purposes. The imposed marketing constraints are based largely on the condition that the entrepreneur may hold the present optimum livestock inventory established in Chapter V or market a portion of the normal marketing but is not allowed to reduce the level of the inventory. The longer run decisions related to expanding the breeding herd through holding more than the normal replacements of heifer calves is not a part of this analysis. The inferior position of holding heifers beyond yearling beef sales when compared to steers, as determined by the linear programming model in the previous chapter, resulted in the exclusion of this alternative.

The following set of equations were developed for the calculation of the money values for the loss function. The term "loss function" in the context used at this point is to define the quantity of money that accrues to the firm as a result of a particular consequence of an action. The derivation and explanation of the relationships defined by equations (6-12), (6-13), (6-14), and (6-15)
are expressed in detail in the relationships (6-16), (6-17), (6-18), and (6-19).

\begin{align*}
\text{UPV}_{ys} &= -31.87 - 5.17 P_{BC}^{t-1} + 7.38 P_{BC}^t \quad (6-12) \\
\text{UPV}_{yh} &= -31.87 - 4.30 P_{BC}^{t-1} + 5.94 P_{BC}^t \quad (6-13) \\
\text{UPV}_{st} &= -48.80 - 7.38 P_{BC}^{t-1} + 10.68 P_{BC}^t \quad (6-14) \\
\text{UPV}_{cc} &= -69.84 - 7.19 P_{BC}^{t-1} + 9.84 P_{BC}^t \quad (6-15)
\end{align*}

where: \text{UPV}_{ys} = \text{undiscounted present value of steer yearling in (t) when the price in t-1 is } P_{t-1} \\
\text{UPV}_{yh} = \text{undiscounted present value of heifer yearling in (t) when the price in t-1 is } P_{t-1} \\
\text{UPV}_{st} = \text{undiscounted present value of steer coming 2 in (t) when the price in t-1 is } P_{t-1} \\
\text{UPV}_{cc} = \text{undiscounted present value of cow-calf in (t) when the price in t-1 is } P_{t-1} \\
P_{BC}^{t-1} = \text{price of beef cattle in time } t-1 \\
P_{BC}^t = \text{price of beef cattle in time } t

\textbf{Calculation of Undiscounted Present Value of Various Cattle Classes}

No additional charges are made for labor or capital in evaluating the carryover of additional livestock above the inventory level. Labor charges could have been added to the constant term (the sign would be negative since increases in cost must decrease the left-hand side of the equation). Also, a charge for capital could be added.
However, the major analysis pertaining to the central thesis of uncertainty in this section is with respect to prices received. The forage requirements by type of livestock, cattle prices, cattle weights, as well as the operating expenses are those developed in the previous chapter.

It is assumed that an animal unit month of feed during the fall and winter period costs $6 per unit. A steer calf held over to a yearling steer requires 1.2 AUMs of fall feed and 1.5 AUMs of winter feed. The variable operating expense for a yearling steer was determined by equation (4-9) as $15.67. Therefore, the $15.67 plus the $16.20 for purchase of feed yields $31.87. The $31.87 is the cost of producing a yearling steer starting from a steer calf in the fall. This assumes no charge for spring and summer feed. The $31.87 appears as a constant in the undiscounted present value of the yearling steer equation. The other constants were calculated in the same fashion. The $31.87 in (6-16), (6-17), and (6-18) are used as the starting base for determining the coefficients for equations (6-12), (6-13), and (6-14).

Undiscounted present value of yearling steers in time \( (t-1) \) =

\[
\text{UPV'}_{ys} = -31.87 - 3.94P_{\text{steer calf}} + 6.47P_{\text{yearling steer}}
\]

(6-16)

Undiscounted present value of yearling heifer in time \( (t-1) \) =

\[
\text{UPV'}_{yh} = -31.87 - 3.74P_{\text{heifer calf}} + 6.06P_{\text{yearling heifer}}
\]

(6-17)

Undiscounted present value of a steer coming 2 in time \( (t-1) \) =

\[
\text{UPV'}_{st} = -48.80 - 7.38P_{\text{yearling steer}} + 10.68P_{\text{steer 2}}
\]

(6-18)
The coefficients on the price variables in equations (6-16), (6-17), and (6-18) are the weights of the various cattle types. The prices are expressed in dollars per hundred weight. This yields an undiscounted present dollar value.

The price predicting equation (6-26) yields values that are in terms of the general indexed beef cattle price. The general indexed beef cattle price was obtained from the United States Department of Agriculture statistical reports. As a consequence of accepting the hypothesis that beef cattle prices move together, the above equations (and the one developed later also) are adjusted such that the price in time t-1 and t can be expressed in terms of the general beef cattle price.

Dividing the average price received at Kansas City for feeder steer yearlings from 1953 through 1965 by the general beef cattle price for the same period yields a factor of 1.14086. This factor can be used to interpolate from the general beef cattle price in Oregon. For instance, if the general price of beef cattle in Oregon was $20 per hundred weight then the price of feeder steers in Oregon would be 1.14086 x $20 or approximately $22.82 per hundred weight.

As shown in Table VIII the price of feeder steers in Oregon relative to steer calves in Oregon is the proportion of $23.40 to $20.35 or 1.14988. Therefore, the indexed price of steer calves in Oregon would be (1.14988 x 1.14086) times the general indexed price of cattle in Oregon. Using the above example of prices, this would mean that the steer calf price would be approximately $26.37 per hundred weight when the general price of cattle is $20.
Through the use of the above adjustment factors it is now possible to express equations (6-16), (6-17), and (6-18) in terms of a coefficient times a common price. In effect the coefficients are now in terms of livestock involved. For instance, in equation (6-12) the coefficient 5.17 is pounds of Oregon steer calves in terms of beef cattle equivalents. This resulted from $1.14988 \times 1.14086 \times 3.94 = 5.17$. The remaining coefficients in equations (6-16), (6-17), and (6-18) were adjusted similarly to obtain equations (6-12), (6-13), and (6-14).

In order to develop an equation for expressing the present value of producing a calf in the next period, the problem is slightly more complicated. This results from the fact that the product involved is not a single product but rather a joint product of a breeding cow and the expected offspring. The resource requirement and revenue accounting equations developed in Chapter IV are not applicable since the full complement of replacement livestock is not a requirement for these marginal carryovers.

The following conditions are assumed to prevail with respect to the carryover of an extra breeding cow. These conditions could be looked upon as stochastic phenomenon and investigated on that basis. The expected calving percentage drops to 80 percent. The sale weights of cows normally culled are reduced by 10 percent. Death losses double for this set of animals from 2.5 to 5 percent. The ratio of cows to bulls is fixed at 20 to 1. The variable operating expenses for this combination of beef cattle is $69.84.
This includes fall and winter feed for the breeding cow and bull plus the normal operating expense.

Undiscounted present value of cow-calf in time $t-1 =$

$$UPV_{cc} = -$69.84 - 9.26P^{wet\ cow}_{t-1} + (.40)(3.94)P^{steer\ calf}_{t} + (.40)(3.74)P^{heifer}_{t} + (.375)(10.77 - 1.08)P^{dry\ cow}_{t} + (.575)(9.26 - .93)P^{wet\ cow}_{t} \tag{6-19}$$

The conversion of Oregon cattle prices to the general beef price followed the same general procedures used in expressions (6-16), (6-17), and (6-18). The condition that all cattle prices move together is especially useful in this case. The combination of various prices that would need to be considered if the prices were independent would seriously complicate the problem.

Uncertainty with Respect to Future Price

One of the first issues in a decision-making problem under uncertainty is to establish a model which relates actions to consequences. Consequences for a Bayesian model are losses that will ensue from actions taken partially on the basis of prior beliefs.

The action-space to be explored in this section of the analysis relates the decision of a ranch manager in holding various cattle types above the optimum inventory level to the price in the next period. An example of losses in money terms per type of cattle is shown in Figure 5. The values plotted indicate money consequences
Figure 5. The Undiscounted Present Value of One Head, By Livestock Type, in Time t, Assuming the Present General Price of Beef Cattle is $23 per CWT.
per head of holding a particular animal as a function of a given present price \( P_{t-1} \) and the future price \( P_t \). The decision maker is assumed to know the price in time \((t-1)\) with probability one. The uncertainty is with respect to cattle prices in \((t)\). The uncertain prices in \((t)\) result in an uncertainty position with respect to the present value of the various cattle types. The following sections will explore the statistical model with a portion of the theoretical framework necessary to generate the posterior probabilities for the undiscounted present values of the various livestock types.

The Posterior Probability Distribution of Cattle Prices in Time \((t)\)

The decision maker for this applied problem is assumed to have an analyst (an extension agent or an experiment station scientist) with whom he agrees with respect to one of the two subjective probability distributions which are involved in determining the a posteriori probability distribution of cattle prices in \((t)\). The term "decision maker" will be used to characterize the individual that bears directly the loss for the action taken. It is perfectly reasonable to assume that the analyst and the decision maker are the same person. However, as was indicated earlier in Chapter I, an assumption that two rational people will agree on the probability of a state of nature given they have the same evidence must hold. It must hold for the distribution provided by the analyst to the decision maker. Further the condition expressed in (20) must hold.
In words, the two sets of evidence used by the decision maker must be stochastically independent. The condition implies that the occurrence of the set of circumstances that make up the evidence of \( E_1 \) may not effect the probability of occurrence of the evidence \( E_2 \).

Expression (6-20) is a frequency concept that is not contingent on prior degrees of belief. The use of the symbol \( \pi \) will be used as a functional notation which will indicate that the probability distribution does involve prior degrees of belief. This is substantially the same notation as used by Lindley in his recent work related to Bayesian statistics (1965a, 1965b).

The relationship \( P_{i1}(E_1|\theta) \) is commonly called the likelihood of \( \theta \) on \( E_1 \). The classical procedure of asking what is the probability of observing this phenomenon given that the state of nature is \( \theta \) is the usual line of reasoning used for probability statements related to hypothesis testing. The admission that the probability of \( \theta \) is a meaningful and necessary probability statement starts one down the path of investigating Bayesian statistics. Statement (6-21)

\[
P_{i1}(\theta|E_1) \propto P_{i1}(E_1|\theta)P_i(\theta) \tag{6-21}
\]

is a posteriori probability statement about \( \theta \). In words this reads "the a posteriori probability is proportional to the product of the likelihood and the prior probability". The symbol \( \propto \) is used to indicate that the expression on the right side differs only by a
constant of proportionality which does not involve \( \theta \). This statement results from a set of probability axioms and its validity has nothing to do with degrees of belief about \( \theta \).

The key requirement for this section of the thesis and perhaps of the entire uncertainty area results from work by Lindley who showed how one might express (6-21) above as shown in (6-22) below and to interpret \( \theta \) as next year's cattle price

\[
\pi(\theta | E_1) \propto P(E_1 | \theta) \pi(\theta)
\]

(a future observation) where \( \theta \) is the dependent variable of a least squares multivariate estimating system and not a parameter. This will then allow the analyst to present the decision maker with a weight to associate with various cattle prices.

Further, the analyst prefers to use his past data and present data (an experiment in Chernoff and Moses' terminology) in a way which takes full advantage of his disciplines theoretical foundations in forming the uncertainty model. The oversimplification of the model as related to price uncertainty has probably been one of the two major reasons for the lack of extended use of decision models under uncertainty by the economics profession. The other dilemma is the specification of the regret function. However, as a result of the recent work by Lindley (1965a), it is now meaningful, with some restrictive assumptions to use regression analysis for the economic model for predicting price and using these results in the form of expression (6-22) above.
If one assumes that the prior knowledge of the \( \beta \)'s is so diffuse that the prior density is sensibly constant over the effective range of the likelihood function such that the prior distribution of \( \beta_1, \beta_2, \ldots, \beta_p \), \( \text{ln}^2 \) are uniform and independent, then Lindley has shown that the posterior distribution \( \pi(P_t | E_1, E_3) \) of

\[
P_t = \frac{X \hat{\beta}}{t} + e_t
\]

is such that

\[
\frac{P_t - \frac{X \hat{\beta}}{t}}{(s^2[l + \frac{1}{t} + X_t(X'X)^{-1}X'_t])^{1/2}}
\]

has a \( t \) distribution with \( (t-1-P) \) degrees of freedom. Where \( E_1 = \{P_1, X_1; P_2, X_2; \ldots; P_{t-1}, X_{t-1}\} = \{PX\} \). That is \( P_i \) are observed prices and the \( X_i \)'s are vectors of independent variables associated with the respective prices. \( E_3 \) is the data that becomes available at decision-making time (in terms of Chernoff and Moses this could be an experiment in the attempt to find what state of nature will prevail). When the true state of nature (\( P_t \)) becomes available, which is after the decision but before the time of the next decision period, this information is used to calculate \( P_{t+1} \). The important result is that the analyst obtains \( \pi(P_t | E_1, E_3) \) and then presents a weighting system to the decision maker. These weights are the analyst's a posteriori probabilities for various cattle prices. Under the above assumptions, this effectively means that the analyst assumes all prices (within a reasonable range) are a priori equally likely.
Since the assumption is made that the decision maker can in fact define a posteriori probability distribution $\pi(P_t | E_2)$ which is independent of the data of the analyst, the posteriori distribution relationship (6-25) can be used for weighting the prices for decision purposes.

$$\pi(P_t | E_1, E_2, E_3) \propto \pi_1(P_t | E_1, E_3) \pi_2(P_t | E_2)$$ (6-25)

It should be noted at this point that the subjective probability distribution assumed for the future price ($P_t$) is not related to the magnitude of the loss or gain in either the mind of the analyst or the decision maker. The ability of a decision maker to separate losses from the subjective probability scale is seriously open to question and needs further investigation of the implications to decision models. One major implication of violating the independence is the entering of "wishful thinking" into the subjective probability distribution. This may in effect help to explain and in fact predict human behavior but is assumed not a consequence to be dealt with in this study.

**Cattle Price Prediction Model**

The model used for predicting cattle prices is a single equation regression model. It was necessary to formulate this model such that the data are available at decision period time. It should be noted that the objective is solely to predict and not to investigate the economic demand and supply structure of the beef livestock.
sector. It is not essential that any other analyst would use the relationship established for predicting price but it is necessary to make the same underlying assumptions if the posterior probability statement presented in (6-25) above is of interest.

The prediction model is presented in expression (6-26) and is of the form suggested in (6-23) above. The raw data with its source indicated are presented in Table XIII.

$$\hat{P}_t = f(X_1, X_2, X_3, X_4, X_5)$$

$$= \hat{B}_0 + \hat{B}_1 X_1(t-1) + \hat{B}_2 X_2(t-1) + \hat{B}_3 X_3(t-1) + \hat{B}_4 X_4(t-1)$$

$$+ \hat{B}_5 X_5(t-1)$$  \hspace{1cm} (6-26)

where:  

- $X_1(t-1) = \text{per capita number of cows and heifers in } t-1$
- $X_2(t-1) = \text{per capita number of calves not kept for milk in } t-1$
- $X_3(t-1) = \text{change in cows and heifers 1 to 2 years not kept for milk from } t-2 \text{ to } t-1$
- $X_4(t-2) = \text{non-indexed price of good dressed steers in Chicago in } t-2$
- $X_5(t-1) = \text{per capital income in year } t-1$
- $P_t = \text{indexed price of beef cattle in } t$
### TABLE XIII. DATA USED IN DEVELOPMENT OF PREDICTION EQUATION FOR BEEF CATTLE PRICES.\(^a\)

<table>
<thead>
<tr>
<th>Year</th>
<th>U.S. Civilian Food Source Consumer Income</th>
<th>Total Consumer Income</th>
<th>Consumer Price Index</th>
<th>No. Cows &amp; Heifers Over 2 Years Old</th>
<th>No. Calves Not Kept For Milk</th>
<th>Change in Cows &amp; Heifers Not Kept for Milk</th>
<th>Non-Indexed Price Good Steers-Chic.</th>
<th>Real Income Per Capita</th>
<th>Indexed Price of Beef Cattle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No.-M1.</td>
<td>Dol.-Bill. 1957-59=100</td>
<td>Per Capita</td>
<td>Per Capita</td>
<td>Per Capita</td>
<td>Per Cwt.</td>
<td>dol.</td>
<td>Per Cwt.</td>
<td></td>
</tr>
<tr>
<td>1946</td>
<td>138.4</td>
<td>160.6</td>
<td>68.0</td>
<td>.30818</td>
<td>.09196</td>
<td>+.00185</td>
<td>27.88</td>
<td>1695.4</td>
<td>--</td>
</tr>
<tr>
<td>1947</td>
<td>142.6</td>
<td>170.1</td>
<td>77.8</td>
<td>.29684</td>
<td>.08979</td>
<td>+.00102</td>
<td>23.22</td>
<td>1533.2</td>
<td>23.65</td>
</tr>
<tr>
<td>1948</td>
<td>145.2</td>
<td>189.3</td>
<td>83.8</td>
<td>.27979</td>
<td>.08296</td>
<td>+.00408</td>
<td>35.40</td>
<td>1555.8</td>
<td>26.49</td>
</tr>
<tr>
<td>1949</td>
<td>147.6</td>
<td>189.7</td>
<td>83.0</td>
<td>.26952</td>
<td>.08152</td>
<td>-.00032</td>
<td>45.06</td>
<td>1548.5</td>
<td>23.86</td>
</tr>
<tr>
<td>1950</td>
<td>150.2</td>
<td>207.7</td>
<td>83.8</td>
<td>.27028</td>
<td>.08333</td>
<td>-.00613</td>
<td>37.66</td>
<td>1650.1</td>
<td>27.08</td>
</tr>
<tr>
<td>1951</td>
<td>151.1</td>
<td>227.5</td>
<td>90.5</td>
<td>.27858</td>
<td>.09477</td>
<td>-.01424</td>
<td>43.45</td>
<td>1663.7</td>
<td>31.71</td>
</tr>
<tr>
<td>1952</td>
<td>153.4</td>
<td>238.7</td>
<td>92.5</td>
<td>.28633</td>
<td>.10319</td>
<td>-.02077</td>
<td>54.26</td>
<td>1682.2</td>
<td>26.27</td>
</tr>
<tr>
<td>1953</td>
<td>156.0</td>
<td>252.5</td>
<td>93.2</td>
<td>.30026</td>
<td>.11179</td>
<td>-.01918</td>
<td>49.20</td>
<td>1736.7</td>
<td>17.49</td>
</tr>
<tr>
<td>1954</td>
<td>159.1</td>
<td>256.9</td>
<td>93.6</td>
<td>.30764</td>
<td>.11300</td>
<td>-.01000</td>
<td>36.02</td>
<td>1725.1</td>
<td>17.09</td>
</tr>
<tr>
<td>1955</td>
<td>162.3</td>
<td>274.4</td>
<td>93.3</td>
<td>.30266</td>
<td>.11386</td>
<td>-.00467</td>
<td>36.56</td>
<td>1812.1</td>
<td>16.72</td>
</tr>
<tr>
<td>1956</td>
<td>165.3</td>
<td>292.9</td>
<td>94.7</td>
<td>.29209</td>
<td>.11415</td>
<td>+.00360</td>
<td>35.05</td>
<td>1871.1</td>
<td>15.73</td>
</tr>
<tr>
<td>1957</td>
<td>168.4</td>
<td>308.8</td>
<td>98.0</td>
<td>.27826</td>
<td>.10929</td>
<td>+.00663</td>
<td>33.24</td>
<td>1872.1</td>
<td>17.55</td>
</tr>
<tr>
<td>1958</td>
<td>171.5</td>
<td>317.9</td>
<td>100.7</td>
<td>.26490</td>
<td>.10656</td>
<td>+.00228</td>
<td>36.22</td>
<td>1840.8</td>
<td>21.75</td>
</tr>
<tr>
<td>1959</td>
<td>174.5</td>
<td>337.1</td>
<td>101.5</td>
<td>.25928</td>
<td>.11121</td>
<td>-.00917</td>
<td>42.15</td>
<td>1903.2</td>
<td>22.27</td>
</tr>
<tr>
<td>1960</td>
<td>178.2</td>
<td>350.0</td>
<td>103.1</td>
<td>.25741</td>
<td>.11462</td>
<td>-.00960</td>
<td>42.81</td>
<td>1905.0</td>
<td>19.79</td>
</tr>
<tr>
<td>1961</td>
<td>181.2</td>
<td>364.4</td>
<td>104.2</td>
<td>.25642</td>
<td>.11270</td>
<td>-.00436</td>
<td>41.57</td>
<td>1930.0</td>
<td>19.39</td>
</tr>
<tr>
<td>1962</td>
<td>183.8</td>
<td>385.3</td>
<td>105.4</td>
<td>.25828</td>
<td>.11997</td>
<td>-.00798</td>
<td>39.39</td>
<td>1988.9</td>
<td>20.21</td>
</tr>
<tr>
<td>1963</td>
<td>186.7</td>
<td>404.6</td>
<td>106.7</td>
<td>.26057</td>
<td>.12496</td>
<td>-.01200</td>
<td>42.81</td>
<td>2031.0</td>
<td>18.65</td>
</tr>
<tr>
<td>1964</td>
<td>189.4</td>
<td>436.6</td>
<td>108.1</td>
<td>.26346</td>
<td>.12975</td>
<td>-.01192</td>
<td>39.28</td>
<td>2132.4</td>
<td>16.65</td>
</tr>
<tr>
<td>1965</td>
<td>194.6</td>
<td>469.1</td>
<td>109.9</td>
<td>.25887</td>
<td>.13004</td>
<td>-.00596</td>
<td>37.59</td>
<td>2193.4</td>
<td>18.13</td>
</tr>
<tr>
<td>1966</td>
<td>196.8</td>
<td>499.9</td>
<td>113.3</td>
<td>.25022</td>
<td>.13174</td>
<td>+.00082</td>
<td>39.22</td>
<td>2242.0</td>
<td>17.94</td>
</tr>
</tbody>
</table>

\(^a\) Sources for this data and for obtaining data for current year to be used for predicting are: (1) Agricultural Statistics, U.S. Department of Agriculture; (2) Livestock and Meat Statistics, ERS and SRS, U.S. Department of Agriculture; and (3) Economic Indicators, Joint Economic Committee, Council of Economic Advisors, U.S. Department of Commerce.
The prediction equation was first fitted to data from Table XIII for the period of 1946 to 1963. The predicted values, starting in 1963 are shown in Table XIV along with some other output information. The prediction equation uses the information obtained during the preceding period to obtain an estimate of the coefficients to use in estimating the unknown state of nature price for the current decision period. An increasing $R^2$ and a decreasing standard error of the estimate as new observations are added is an encouraging sign that the model is an expression of a reasonable relationship.

If one assumes that the decision maker assigns an equal probability to all prices in 1967 (commonly referred to in decision theory jargon as complete ignorance), then the probabilities presented in Column 3 of Table XV can be assumed to be the joint probability of the decision maker and the analyst. This results from the density of equal probabilities for $\pi(P_{1967}|E_2)$ and the independence assumption of expression (6-26) above.

The calculation of $\sum_{i=1}^{16} P_{i3} M_{ij}$ from Table XV, where $P_{i3}$ is the probability of the next period's price as related to holding over a

$$d = \frac{\sum_{t=2}^{N} (\epsilon_t - \epsilon_{t-1})^2}{\sum_{t=1}^{N} \epsilon_t^2}$$

large such that the hypothesis of independent random disturbances is not rejected. The table of values used for the test were obtained from an article "Testing for Serial Correlation in Least Square Regression", Part II, Biometrika, vol. 38, 1951, pp. 159-178.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{B}_0$</td>
<td>71.95</td>
<td>71.01</td>
<td>71.41</td>
<td>69.34</td>
<td>69.51</td>
<td>--</td>
</tr>
<tr>
<td>$\hat{B}_1$</td>
<td>-74.71</td>
<td>-69.93</td>
<td>-68.82</td>
<td>-66.14</td>
<td>-66.38</td>
<td>--</td>
</tr>
<tr>
<td>$\hat{B}_2$</td>
<td>-430.15</td>
<td>-430.48</td>
<td>-428.11</td>
<td>-440.92</td>
<td>-440.05</td>
<td>--</td>
</tr>
<tr>
<td>$\hat{B}_3$</td>
<td>-198.20</td>
<td>-184.63</td>
<td>-175.45</td>
<td>-187.18</td>
<td>-187.02</td>
<td>--</td>
</tr>
<tr>
<td>$\hat{B}_4$</td>
<td>.281</td>
<td>.265</td>
<td>.259</td>
<td>.267</td>
<td>.267</td>
<td>--</td>
</tr>
<tr>
<td>$\hat{B}_5$</td>
<td>.01417</td>
<td>.01362</td>
<td>.01294</td>
<td>.01459</td>
<td>.01448</td>
<td>--</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.907</td>
<td>.908</td>
<td>.912</td>
<td>.913</td>
<td>.915</td>
<td>--</td>
</tr>
<tr>
<td>SP$_t$$X$</td>
<td>1.69</td>
<td>1.63</td>
<td>1.58</td>
<td>1.53</td>
<td>1.47</td>
<td>--</td>
</tr>
<tr>
<td>$\hat{P}_t$</td>
<td>--</td>
<td>19.74</td>
<td>17.52</td>
<td>17.25</td>
<td>17.98</td>
<td>16.78</td>
</tr>
<tr>
<td>$P_t$</td>
<td>20.21</td>
<td>18.65</td>
<td>16.65</td>
<td>18.12</td>
<td>17.94</td>
<td>--</td>
</tr>
</tbody>
</table>

aData are in Table XIII. The prediction equation uses the data from year $t-1$ for prediction in $t$, as the coefficients for 1967 to predict price for 1968 will not be available until the fall of 1967.
TABLE XV. THE PROBABILITY OF GENERAL BEEF CATTLE PRICES FOR 1967 WITH ASSOCIATED MONEY LOSSES OR GAINS OF HOLDING ONE ANIMAL OF VARIOUS TYPES OF CATTLE OVER THE NORMAL HOLDOVER LEVEL.

| 1967 General Beef Price | t Statistic with 14 Degrees of Freedom | $P_{1967} |E_1, E_3| | Gain or Loss Per Head Carryover Using 1966 Price for Price in t-l and Midpoint of Price Interval for Time t = 1967 |
|-------------------------|---------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| Under 10                | Under 3.5                             | .0008                                         | -60.00                                       | -56.98                                        | -87.20                                        | -111.83                                       |
| 10.01 - 11.00           | -3.50                                 | .0010                                         | -47.13                                       | -46.64                                        | -69.06                                        | -95.51                                        |
| 11.01 - 12.00           | -3.00                                 | .0030                                         | -39.75                                       | -40.70                                        | -58.38                                        | -85.67                                        |
| 12.01 - 13.00           | -2.40                                 | .0106                                         | -32.37                                       | -34.13                                        | -47.70                                        | -75.83                                        |
| 14.01 - 15.00           | -1.30                                 | .0606                                         | -17.61                                       | -22.25                                        | -26.34                                        | -56.15                                        |
| 15.01 - 16.00           | - .72                                | .1345                                         | -10.23                                       | -16.31                                        | -15.66                                        | -46.31                                        |
| 16.01 - 17.00           | - .16                                | .1959                                         | - 2.85                                       | -10.37                                        | - 4.98                                        | -36.47                                        |
| 17.01 - 18.00           | + .41                                | .2182                                         | + 4.53                                       | - 4.43                                        | + 5.70                                        | -26.63                                        |
| 18.01 - 19.00           | + .97                                | .1696                                         | +11.91                                       | + 1.51                                        | +16.38                                        | -16.79                                        |
| 19.01 - 20.00           | +1.50                                | .0966                                         | +19.29                                       | + 7.45                                        | +27.06                                        | - 6.95                                        |
| 20.01 - 21.00           | +2.10                                | .0507                                         | +26.67                                       | +13.39                                        | +37.74                                        | + 2.89                                        |
| 21.01 - 22.00           | +2.60                                | .0167                                         | +34.05                                       | +19.33                                        | +48.42                                        | +12.73                                        |
| 22.01 - 23.00           | +3.20                                | .0073                                         | +41.43                                       | +25.27                                        | +59.10                                        | +22.57                                        |
| 23.01 - 24.00           | +3.80                                | .0022                                         | +48.81                                       | +31.21                                        | +69.78                                        | +32.41                                        |
| Over 24                 | Over +3.8                            | .0010                                         | +60.00                                       | +41.56                                        | +87.80                                        | +48.17                                        |
breeding cow shown in Column 3 and $M_{ij}$ is the $i^{th}$ row and the $j^{th}$ column of monetary gain or loss depending on the sign, presents the monetary value of the action of carrying over the animal of the respective column. For instance, the \textit{a posteriori} weighted monetary value of carrying over a steer calf is \( \sum_{i=1}^{16} P_{i3} M_{i4} = \$2.92 \). The probabilities presented in Column 3 would be the \textit{a posteriori} weights turned over to the rancher by the analyst. The continuous \( t \) distribution has been transformed to a discrete expression of probabilities. The consequence of this is a degree of error which increases as the width of the interval increases. This may seem somewhat unimportant but for the above example the weighted average of a positive amount of $2.92 per head carryover would indicate that one should carry over all steer calves assuming of course that utility gains and losses are a direct linear combination of money gains and losses. However, the expected price of beef cattle in 1968 is $16.78 per CWT and this figure along with the present price in 1967 of $17.94 using equation (5-12) results in a loss of $.78 per head. The discrete intervals can be made arbitrarily small thus resulting in a degree of error specification for any tolerable level.

\textbf{The Transformation of Money Gains to Utility Gains}

Probably no single concept in economics has been subjected to more controversy than the theory involved with measurability of utility. To review even the major facets would require too much detail for purposes of this presentation and in fact would probably
be vacuous from the standpoint of contributing to the problem at hand. However, to completely ignore the issue by assuming utility losses are equivalent to money losses would be an act of complete disrespect for a large body of economic theory.

The problem associated with the specification of a utility function and the measurement problems involved will probably only be resolved through some very close scientific cross-discipline endeavors. Statements such as "experience with statistical problems shows that 'good' procedures are insensitive to small changes in the loss function, especially when considerable data are available" and "one can measure the random loss by taking expectation just so long as these losses are themselves measured in terms of utility function" help very little in resolving the problem (Mood, Graybill, 1963).

One study that met the problem of utility measurement squarely was a study of "Managerial Processes of Midwestern Farmers" (Johnson, et al., 1961). Another was a study by Grayson (1960) "Decisions Under Uncertainty--Drilling Decisions by Oil and Gas Operations". Halter presents an attempt in fitting of a utility function from empirical evidence taken from the agricultural sector. The approach is based primarily on the Von Neumann and Morgenstern approach of establishing numerical estimates of utility. The Halter study indicated that it was meaningful to suggest that groups of farmers who engage in risky types of enterprises can be classified on the basis of the relative marginal utility of gains and marginal disutility of losses. It is unfortunate that the format for the
survey to obtain similar data were not included as a part of this study. As a result of this lack of empirical data from the study population as related to attitudes in taking risks, the following development will largely be developed on a set of assumed conditions. The general utility function presented is for example purposes.

For purposes of analysis, the total utility function translates dollars (the independent variable) into what is sometimes called "utils" (the dependent variable). The function has the property that for low money amounts the marginal curve is increasing then decreases for increasing money amounts. The function presented here never reaches a maximum, i.e., the first derivative is greater than zero over the entire money domain. The function used demonstrates increasing, constant, and decreasing marginal utility for money but does not reach a maximum total utility. The function developed is substantially of the form suggested by R. G. D. Allen (1956). The function violates the condition that a utility function must be bounded. The fact that the general function is unbounded does not present problems over the relevant range that is considered for the applied problem. This results largely from the condition that the opportunity to weigh very large quantities of money gains is considered not relevant for the applied problem. Further, the applied problem range of money gains are sufficiently small such that the decision maker does not change into a different socio-economic class. In other words, the monetary gains and losses are sufficiently small such that the decision maker can evaluate them
from his present position without considering how he would act if he should suddenly "strike it rich" and in turn require that he act as expected for a "rich" man.

It is assumed that all money positions lower than zero are not defined. This means that the decision maker is always in a positive money position before the decision and after the consequence of the action. This means that he will never enter a situation which will totally wipe out all his money holdings. This seems to be a realistic assumption for the study population since the decision maker must hold over almost all livestock and the lowest price considered actually must prevail before he would end up with a negative money level. However, this point can be considered immaterial from a utility viewpoint. The index is arbitrary—the zero point on the utility scale is arbitrary.

For the increasing portion of the utility function, the general form presented in (6-27) was developed.

\[
U(m) = \frac{1}{2}m^2 - \frac{1}{\alpha}m^\phi
\]

\[
0 \leq m^{\phi-2} \leq \frac{\alpha}{\phi(\phi-1)}
\]

\[
\phi > 2
\]

\[
m = \frac{\text{dollars}}{100}
\]

The individual decision maker is assumed to know at what level of money holdings his marginal utility of money is constant or some method such as the Von Neumann-Morgenstern technique of deriving a utility index can be used to establish this point. This means that
(6-27) is defined only for values in which the second derivative is positive and up to the point where the second derivative is zero. For the function defined in expression (6-27) above, the upper limit must be where \( \frac{d^2U(m)}{dm^2} = 0 \). This means that \( 1 - \frac{\phi(\phi-1)m^{\phi-2}}{\alpha} \) must be set to zero. If for instance, it has been determined that the individual at a certain income level suddenly shifts from accepting lotteries that have unfair odds, i.e., risk preference condition, to an expression of preference toward a lottery only when the odds are in his favor, i.e., a risk aversion position, then it is assumed that he has changed from a risk taker to a risk hedger. This is the inflection point of the total utility function for money and has been defined by Siegel (1957) as the level of aspiration for money. Assuming that this point of inflection is in the neighborhood of $10,000 on the money scale, then \( 1 - \frac{\phi(\phi-1)(100)^{\phi-2}}{\alpha} = 0 \). Two values of \( \phi (\phi=3,\phi=4) \) are shown in Figure 6. Once the value of m and the general form is established that appears to best represent the individual's degree of preferences over the relevant range, a unique value of \( \alpha \) can be determined.

As can be noted in Figure 6 if the expression given in (6-27) was evaluated beyond the limits, the function soon reaches a maximum and declines very rapidly. Since human satisfaction is assumed never to be totally satiable over the relevant decision-making range and that money can be translated directly into satisfaction, the utility function is an increasing function of money, another branch is added to expression (6-27) and the total specification is presented in (6-28) below.
Figure 6. The Total Utility Function for Money. The Relevant Portion is for Increasing Marginal Utility of Money for Dollar Values between $0 and $10,000.

\[
U(m) = \frac{1}{2}m^2 - \frac{1}{120,000}m^4
\]

\[
U(m) = \frac{1}{2}m^2 - \frac{1}{600}m^3
\]
\[ U(m) = \frac{1}{2} m^2 - \frac{1}{\alpha} m^{\phi} \quad 0 < \frac{m^{\phi-2}}{\phi(\phi-1)} \]

\[ = \left( \frac{1}{2} m^2 - \frac{1}{\alpha} m^\phi \right) + \lambda \ln \left( \frac{m-m_0}{100} + B \right) \quad m > m_0 \]

\[ m = \frac{\alpha}{\phi(\phi-1)} \]

\[ = 0 \quad \text{Otherwise} \quad (6-28) \]

The value assigned to B will control the rate of decrease of the function. The value of \( \lambda \) can be used to force the second branch of the function to a rate of change that is equal or slightly less than the lower branch at their point of contact. For instance, assuming \( \phi = 3 \), and \( \alpha = 600 \), then \( \frac{d^2U(m)}{dm^2} = 0 \) when \( m = 100 \) and \( \frac{dU(m)}{dm} = 50 \) when \( m = 100 \). Assuming the risk aversion scaling index indicated that \( \beta = 1 \) was a reasonably good expression for the decision maker in the area of risk aversion, then the specification of the additional increment that the second branch must add to total utility can be uniquely defined by the value of \( \lambda \). For example, from the above assumed condition, \( \lambda \ln \left( \frac{m-m_0}{100} + 1 \right) \leq 50 \) when \( m = 101 \). This results from the above where \( \frac{dU(m)}{dm} = 50 \) since the new branch could not increase at a per unit rate of greater than 50 or increasing marginal utility would still be the case. Therefore, \( \lambda \ln \left( \frac{101-100}{100} + 1 \right) = \lambda \ln \left( 1.01 \right) \leq 50 \). This results in a value for \( \lambda \) to be approximately 5025.1 when the equality holds. These assumed values are shown for the utility function shown in Figure 7, and are used for the utility function for the decision making problem that follows.
Figure 7. The Total Utility Function Assumed for Analysis Involving Uncertain Outcome.\textsuperscript{a}

\textsuperscript{a}This function is plotted from expression (6-27) with $\phi = 3$, $\alpha = 600$, $\lambda = 5025$ and $\beta = 1$. 
Utility is relevant to this inventory decision model as a result of the earlier assumed conditions. These assumptions make it analogous to a single gamble decision problem. The issue is not the expectation of money but rather the expectation of utility. It is assumed throughout the analysis that any gamble in which the expected utility places the individual decision maker at a higher utility level will be an accepted action.

Since a strategy is a function on the possible data to the set of possible actions, the enumeration of all strategies is not a feasible approach for the decision problem under investigation. However, a systematic approach for determining an optimum decision in terms of maximizing expected utility based on a posteriori probabilities for a future price which is estimated from a single equation regression model must be developed if the results are to be meaningful from an applied viewpoint. As outlined earlier, the actions available for the livestock inventory involve not only a specific type of cattle sale, such as a steer calf, but the sale of any number within each class up to the total number available for sale. This extremely large action space is a stimulus for developing a systematic way of obtaining a decision rule.

If it is possible to exclude a particular class of cattle carry-over as outside the feasible strategy set, such as heifer yearlings, this significantly reduces the strategy space and scales down the
number of alternatives for decision purposes. From Figure 5, the strategy action set of the holding over a cull cow for purposes of obtaining a calf in the next period is inferior to holding either a steer calf to become a yearling steer or a yearling steer to become a steer coming 2 regardless of what the price turns out to be in the next period. Because the money gain is always less or the money loss always more regardless of the uncertain price outcome, the strategy set which calls for its action of holding one or more cows is inferior in the utility scale when compared to holding a steer calf or yearling steer. The criterion for selection of the "best" strategy is that combination of actions which yields the highest expected utility. Therefore, it is necessary that the set of strategies for holding steer calves and steer yearlings must be totally exhausted before the consideration of holding cows is admissible action as a part of the admissible strategy set.

Since the strategy set of holding over steer calves and steer yearlings dominate the set for carrying over a cow for all outcomes, the selection of the superior strategy did not involve probable prices or expected utility. In other words, if the strategy that calls for the action of holding over only steer calves and steer yearlings and selling the others is not admissible, then any strategy that involves holding over cows above the normal inventory level is not admissible. Expected utility is derived from weighting the decision maker's utility function for money with respect to the probability of obtaining that utility level. The probability weights
are the \textit{a posteriori} probabilities of equation (6-25). Since the probability distributions of the analyst and the decision maker are independent, the result is invariant to the order in which the values are combined. In other words, if the analyst has access to the decision maker's estimated utility function, the analyst can formulate the information in a fashion in which the decision maker need only apply his subjective probability distribution for purposes of decision making. This has many advantages one of which is that the analyst will most likely have access to computer facilities for involved computations. Another advantage is the analyst will be able to make statements of the type, the decision maker will not take the following action unless the decision maker has \textit{a posteriori} probability distribution which indicates that next year's price will be greater than some value \( E \) with probability \( P \).

A further reduction of the number of admissible strategies can be obtained through the use of relevant regions of the utility function. Starting from any given money level a decision maker that accepts a fair bet has an expected monetary return which places him at the same income level that existed prior to the bet. However, as noted earlier, expected money gains and losses are not the issue. Expected utility is the relevant issue in the decision to place a bet. If, for example, the decision maker is assumed to be in a present money position of \( \text{OM}_1 \) in Figure 8, his present utility is \( \text{OU}_1^0 \). If the decision maker places a fair bet indicated by the chord \( ab \) in Figure 8, his expected utility is \( \text{OU}_1^1 \) which is greater than \( \text{OU}_1^0 \).
Figure 8. The Convex Set of Utilities Available to a Decision Maker for Fair Bets for Positive Initial Money Position between 0 and \( \Omega M^1 \).
In fact, he would maximize his utility through placing the fair bet indicated by the boundary line and obtaining an expected utility level of $OU_1^2$.

The shaded area in Figure 8 represents a convex set which contains all chords relevant to the placing of bets with the associated utility function. A convex set has the property that if two points are in the set then a straight line connecting the two points is also in that set.

Without attempting to evaluate the probabilities one can note from Figure 8 that a decision maker with a present money position of $OM_1$ would accept bets which are unfair. An unfair bet from the decision maker's viewpoint is one in which the money gains weighted by the probability of the gain is less than the money losses weighted by the probability of the loss. In other words, the expected money level of the decision maker engaging in an unfair bet is lower than his present money position. The term "unfair" is not associated with utility, only money. However, assuming again the decision maker is at a present money position of $OM_1$, he should to be consistent engage in any lottery so long as the odds were not so extremely unfair as to move the expected money value down to or below the point $C$ on the chord. Therefore, the decision maker in a present money position of either $OM_1$ or $OM_2$ can a priori be expected to accept all more than fair and exactly fair bets. However, unfair bets will have to be evaluated.
An analogous argument holds for the concave downward portion of the utility function. For this area, all unfair and fair bets can \textit{a priori} be expected to be rejected. The more than fair lottery must be evaluated to determine if the decision maker should accept the bet.

An important technical point relating to preference and utility should be noted. It is not proper to say that lottery A is preferred to lottery B because lottery A has a higher utility than lottery B. The correct statement is that lottery A has a higher utility than lottery B because lottery A is preferred to lottery B (Luce, Baiffa, 1958). This condition results from the fact that the utility index is generated from the selection of lotteries. The reason for attempting to specify as nearly as possible the parameters of the utility function (expression 6-28) from a set of responses to selected lotteries is for the purpose of predicting what decision the ranch operator will make when faced with more complicated choices. The summarization of preferences and rules of consistency permit, through the use of the utility framework, the determination of the decision strategy which will be selected over an alternative strategy. This is in essence the result which allows one to generalize from the convex set of Figure 8.

One additional point needs to be made with respect to the implication of both the establishment and use of the utility function proposed by expression (6-28) and the general form presented in Figure 8. This is with reference to the region between the point
of inflection and the maximum average utility. The implication of an earlier argument relating to the establishment of the inflection point (second derivative of the utility function is set to zero and the second branch established for expression (6-28) indicated that this point might be established empirically at the money level in which the decision maker suddenly changed from accepting fair bets to rejecting fair bets. This is basically the argument of Siegel (1957, p. 255) as expressed by his statement:

"The level of aspiration of an individual is a point in the positive region of his utility scale of an achievement variable; it is at the least upper bound of that chord (connecting two goals) which has maximum slope; i.e., the level of aspiration is associated with the higher of the two goals between which the rate of change of the utility function is a maximum".

However, one must be careful in the selection of the domain of the money bet (achievement variable in Siegel's terminology). As can be noted for Figure 8, a fair bet can be designed such that it is accepted at a present money level between the point of maximum marginal and maximum average utility. If one is interested in establishing an expression for a utility function these are some of the more relevant focal points. If one uses a set of lotteries, the rational individual will not accept fair bets between the points of marginal maximum and average maximum so long as the range of losses and gains does not involve losses below the marginal maximum point. This point is crucial and sometimes ignored for some designs of lottery systems developed for generating individual utility surfaces. A lottery system which is designed such that large money
losses are placed at very low odds but are a part of the lottery are likely to indicate an inflection point which may be closer in actuality to the maximum average point than to the true inflection point.

Expression (6-28) was designed such that the second branch of the function is located at the inflection point. As a result of this condition, the maximum of the average utility function can be isolated by considering only the second branch. This means that the average marginal utility can be expressed as, \( 3333m^{-1} + 5025m^{-1} \ln m - 5025n^{-1} \ln 100 \). This assumed the inflection point was at the point where \( m = 100 \). The maximum average point is then found directly through the use of derivatives to be at the point approximately where \( \ln m = 4.94187 \) or \( m = 140 \).

Through the use of these two points (\( m = 100 \) and \( m = 140 \)) and the general relationship expressed by Figure 8, it may be possible to exclude a large set of strategies as not admissible. For example, if the present money position of the decision maker is assumed to be at or above 140 and one assumes the decision maker's subjective probability distribution for next year's price is uniform, then all strategies sets that call for holding over any livestock above the normal carryover with the exception of steer calves and steer yearlings will be inadmissible. The exception set is that set of strategies which call for the sale of all livestock normally sold except the steer calf carryover to steer yearlings, and perhaps some steer yearling carryover. The strategy set that calls for
carrying over a steer calf and steer yearling and zero amounts for all other classes is the only strategy set which has an equivalent lottery value that is more than fair to the lottery holder. This result is derived from the summation of probability weights of next year's price with their respective gains and losses for the steer calf and steer yearling livestock class indicated in Column 4 and Column 6 of Table XV. These calculations result in an expected lottery gain of $2.92 per head for steer calves and $3.64 for steer yearlings carried over to steer coming 2. This result indicates that only the set of strategies which involve steer calves and steer yearlings need be investigated.

The number of steer calves and steer yearlings carried over must still be determined. It is not obvious what number must be carried over in order to maximize utility. No method of solving other than numerical iteration was found for determining the optimum number within the livestock type once a particular type has been found to be admissible. Table XVI indicates that total utility for the carryover of 10 steer calves is 5035. This is 11 points higher than the initial point of 5024 total units at the initial position of m=140. The carryover of steer calves to steer yearlings results in a slightly higher utility gain than the carryover of steer yearlings to steers coming 2. This results from the utility position of the decision maker such that the higher gains of the steer yearling to steer coming 2 is discounted at a higher rate because of the diminishing marginal utility. The losses of the steer yearling carryover
are also more than for steer calves; therefore, the losses in utility terms are also greater for the steer yearling carryover.

It should be noted that the utility index must implicitly contain a factor for discounting the year's delay in income or the final utility figure must be discounted. It is assumed throughout the examination of specific values that the income in money terms is not discounted for time but the utility function implicitly contains the time discount. The gain in utility from 10 to 20 steer calf carryover is nine units which is less than the gain from 0 to 10 animals as could be predicted since marginal utility is decreasing as is average utility for all gains. From Table XVI, it appears without examining all values that utility is maximized only if all steer calves and steer yearlings are carried over. As noted earlier, only through examining more points between 10 steer yearlings plus all steer calves and 30 steer yearlings plus all steer calves can one be absolutely sure that a maximum has not been reached and that total utility is actually decreasing before reaching the sale of all steer calves and steer yearlings. This is not a very serious problem if the system is programmed for a computer. For practical purposes a few calculated points with Figure 8 should usually yield a good approximation of the utility level achievement.

A point noted earlier from Figure 8 was that one could exclude a priori all strategies that involved any inventory holdover except steer calves and steer yearlings. A single example of a strategy that called for carrying over 10 heifer calves is presented in
This is $\pi(P_t|E_1,E_2,E_3)$ assuming that $\pi(P_t|E_3)$ is a uniform distribution and $\pi(P_t|E_1,E_2)$ as in Table XV, Column 3.

Values for money gains and losses are calculated from values in Table VIII. Utility values assume an initial money position of maximum average utility for money ($14,000) and use expression (6-28) with $\phi=3$, $\alpha=600$, $\lambda=5025.1$, and $\beta=1$.
Table XVII. The expected total utility after the carryover of 10 heifer calves is 5003 which is 21 utility units lower than holding over zero heifer calves. The Bayes strategy for the above assumed condition must be that strategy that calls for the action of holding over above the normal replacement all steer calves, all steer yearlings, holding over no heifer calves nor breeding cows above the normal inventory level. For it is admissible strategy which will maximize the utility for the \textit{a priori} probability of the decision maker with a uniform subjective probability distribution with respect to next year's general cattle price. The set of admissible strategies contains that set of all combinations of actions which require the carryover of steer calves and steer yearlings and no others. The action elements of the strategy for only steer calve carryover contains the number of steer calves carried over. This may be any number between and including zero to 97 with all other livestock types at the zero level.

**Implications of Modified Prior Subjective Distributions and Initial Money Position**

As the decision maker modifies his \textit{a priori} probability distribution, some of the admissible strategies must be evaluated to determine which strategy is the Bayes strategy. As noted earlier, for problems that involve an infinite number of possible values (experiments in Chernoff and Moses' terminology), as is the case for the independent variable used for estimating the dependent variable—the enumeration of all strategies is not meaningful. If, as a result
TABLE XVII.  A POSTERIORI PROBABILITY FOR CATTLE PRICES IN 1967
ASSUMING VARIOUS A PRIORI DISTRIBUTIONS FOR THE
DECISION MAKER.

| General Price of Beef Cattle | $P(P_{1967} | E_1, E_3) \times \pi_2(P_t | E_2) = \pi_2(P_t | E_3) = \pi_2(P_t | E_3) = (60, 40)^a | (40, 60)^a | (30, 70)^a |
|-----------------------------|------------------------------------------------|------------------------------------------------|------------------------------------------------|
| Under $10                  | .0008                                          | .0010                                          | .0006                                          | .0004                                          |
| 10.01 - 11.00              | .0010                                          | .0013                                          | .0008                                          | .0006                                          |
| 11.01 - 12.00              | .0030                                          | .0037                                          | .0023                                          | .0017                                          |
| 12.01 - 13.00              | .0106                                          | .0130                                          | .0083                                          | .0060                                          |
| 13.01 - 14.00              | .0313                                          | .0385                                          | .0244                                          | .0179                                          |
| 14.01 - 15.00              | .0606                                          | .0746                                          | .0473                                          | .0346                                          |
| 15.01 - 16.00              | .1345                                          | .1655                                          | .1050                                          | .0769                                          |
| 16.01 - 17.00              | .1959                                          | .2411                                          | .1529                                          | .1120                                          |
| 17.01 - 18.00              | .2182                                          | .1790                                          | .2555                                          | .2910                                          |
| 18.01 - 19.00              | .1696                                          | .1391                                          | .1986                                          | .2262                                          |
| 19.01 - 20.00              | .0966                                          | .0792                                          | .1131                                          | .1288                                          |
| 20.01 - 21.00              | .0507                                          | .0416                                          | .0594                                          | .0676                                          |
| 21.01 - 22.00              | .0167                                          | .0137                                          | .0196                                          | .0223                                          |
| 22.01 - 23.00              | .0073                                          | .0060                                          | .0085                                          | .0097                                          |
| 23.01 - 24.00              | .0022                                          | .0019                                          | .0026                                          | .0030                                          |
| Over 24                    | .0010                                          | .0008                                          | .0011                                          | .0013                                          |

^a^The quantity (a,b) is defined to mean that the decision maker's evidence indicates that the price for the next period will be uniform below the predicted interval by $P_t(a)$ and uniform above $P_t(b)$. The resulting probability in the respective column is then $\pi(P_t | E_1, E_2, E_3)$. 


of the assumed initial position a means can be developed to allow the decision maker, as before, to exclude large sets of strategies, this will improve the efficiency and perhaps make the application more meaningful.

Assuming as in the previous example that the ranch operator is at the present time either at the money position of maximum average utility or beyond, then if the decision maker has a "pessimistic" prior distribution with respect to next year's price, the admissible strategies are the same or reduced. A "pessimistic" prior distribution is defined to be when the decision maker is attaching a higher probability to the lower prices relative to the distribution furnished by the analyst. Column 3 of Table XVII is pessimistic since the decision maker is attaching relatively larger weights to the lower prices than the analyst has assigned. The converse is true for the assumed conditions in Column 4 and 5 of Table XVII.

The decision maker may have evidence in the form which allows him to express his probability distribution in smaller increments than for instance a 60 - 40 dichotomy with a uniform distribution over each part. From the viewpoint of the ranch operator, an extremely refined distribution probably is not very realistic. The ranch operator might have sufficient data such that the 60 - 40 could be expanded to a 20 - 40 - 30 - 10 specification. This would still define the prior distribution to have 60 percent weight for the lower price interval except it would further specify that the
very low price intervals (the bottom quartile) would have half the weight of the second quartile and the price intervals above the projected (third quartile) are more likely than the price intervals in the fourth quartile. A more sophisticated continuous or discrete distribution is acceptable from an analysis standpoint, but a decision maker at the ranch operation level would probably not realistically specify a highly complex or refined distribution.

The implication of an increasing a priori probability for lower prices when the decision maker is assumed to be in the area of decreasing average utility is simply that the number of admissible strategies cannot be greater than the number when the a priori distribution was uniform. A strategy that is not admissible when the probability is favorable toward high prices will not suddenly become admissible as the odds become less favorable. For the example used previously, the Bayes strategy calls for the carry-over of approximately 35 steer calves when the ranch operator has an a posteriori probability distribution that is in the (60-40) range shown in Column 3 of Table XVII. This reduces the carryover from 97 steer calves and 30 steer yearlings to 35 steer calves; a sizable reduction. Further calculation at less favorable odds could be used until the Bayes strategy indicated a zero carryover for all livestock classes. If a general utility index of the study population of the farm suggested by Halter was available, a survey of ranch operations to obtain their subjective probabilities with respect to next year's price could then be used to predict the size above normal of the livestock carryover (Halter, Beringer, 1960).
If the decision maker has an "optimistic" outlook for next year's price as indicated in Columns 4 and 5 of Table XVII, the minimum carryover will be at least all the steer calves and steer yearlings. This condition is assured since at less favorable odds the actions must be taken to maximize utility. The only question remaining relates to the problem of determining if additional carryover of other livestock types must be a part of the strategy that will be the Bayes strategy. Since the initial position of the decision maker is assumed to be in the diminishing average utility section of the utility function (the concave downward section of the curve), if a single animal unit such as a heifer calf or a breeding cow cannot be a part of the admissible strategies then the entire set of strategies that involve all combinations of holding over this type of animal can be excluded. For the assumed utility functions and the cost and returns calculated for the carryover of a breeding cow or a heifer calf, the optimistic odds of Column 5 of Table XVII are not sufficiently favorable for a carryover of either a breeding cow or a heifer calf.

The above analysis suggests that if in general the study population is represented by the assumed utility function, then that part of the population which is presently in an initial position of diminishing marginal utility would not be observed holding over breeding cows or heifer calves above the normal breeding herd.

One of the major reasons that a technique could not be found for solving the problem more efficiently was for the same reason
which has held up the general advancement of quadratic programming. If the objective function (maximizing utility) related to a function that was a concave upward function throughout or a concave downward function throughout or if it could be completely separable, then the problem might have been formulated such that the general convex programming algorithm could be used and would have made the problem more manageable. Unfortunately this was not the case. However, with some of the general relationships that can be determined from a utility function of the form presented in Figure 9, the number of admissible strategies may still remain within a manageable proportion.

If the Bayes strategy called for the carryover of any livestock type when the initial money position of the decision maker was in the region beyond or at the maximum average utility, then the admissible strategy set for the same decision maker at all lower defined initial money positions will contain none of the strategies which call for a lower animal carryover. In fact, the Bayes strategy for any decision maker at an initial money position of diminishing marginal utility will set the lower limit of the carryover for the admissible strategies for the same decision maker in the region of increasing marginal utility. In other words, one need not investigate, with one exception, any livestock carryover strategy which was found admissible in the previous example. The only additional strategies that need be investigated for the increasing marginal utility section are the strategies that
Figure 9. The Convex Set of Admissible Strategies Generated by the Increasing Marginal Utility Section of a Utility Function for Money.
call for a larger carryover. The exception will arise if the losses extend to the negative money scale for which the specification of a particular utility function is not defined.

The decision maker at the initial position of say \( m = 50 \) must have preferred a lottery which had an expected money gain of zero with losses and gains extending from \( m = 0 \) to \( m = 140 \) then any equivalent probability lottery which involved losses between 0 and 50 and gains between 50 and 140. This result can be read directly from Figure 8. The importance of this result as related to the livestock inventory carryover is in the reduction of the strategies that need to be investigated.

The reduction of the number of strategies to be investigated is not the only important implication that can be drawn from the general convex set generated by the utility function over the range of increasing marginal utility. One can predict the combined \textit{a posteriori} probability distribution of the analyst and decision maker necessary to alter a given strategy. From the assumption of independence of the probability distributions of the analyst and the decision maker, it is possible for the analyst to specify the level for which the decision maker must view the probability of next year's price before a strategy will be rejected. This can be shown through the use of Figure 9.

If the present position of the decision maker is at a money level of \( OM_2 \), then a fair bet would be of the form \( p(0) + (1-p)(OM_3) = OM_2 \). The decision maker must accept this lottery since the
utility function properly represents a previous lottery system. He can improve his utility position from \( U(M_2) \) to \( U(M_L) \). The question here is how unfavorable must the odds become or rather how pessimistic must the decision maker's probability distribution be such that one can predict that the decision maker will not take the actions involved with a given strategy. The point of indifference between accepting a lottery and not accepting must be for a value of \( p \) such that \( pU(\$0) + (1-p)U(OM_2) = U(OM_L) \). In the case of the lottery in which the expected utility was \( U(M_L) \), the value of \( p \) was such that \( p = \frac{oc-ob}{oc} \). However, the value \( p = \frac{oc-oa}{oc} \) is the largest value of \( p \) for which the decision maker will accept the risk of the strategy involved. Therefore, the prediction can be made that the decision maker will take the specified decision route so long as the \textit{a posteriori} probability is not greater than \( p = \frac{oc-oa}{oc} \).

Another important implication is that clearly the statistical expectation may indicate a strategy expectation value below the present value of the monetary position of the decision maker but one can still make the statement that the decision maker "should" accept the lottery so long as the expected money loss is less than say \( OM_2 - OM_1 \). An analyst could predict which lotteries will be accepted or rejected. This result is not surprising from a theory viewpoint since elementary economic theory would lead directly to the conclusion, as has been pointed out previously, that money losses are not synonymous to utility losses. However, the implication that one might place a numerical range on expected money losses
and gains has definite shades of cardinal measurement. In fact, this is precisely the essence of the Von Neumann-Morgenstern utility measure. It differs from the classical cardinal measurement because it does not suggest how much better off or worse off the individual will be nor does its validity rest on whether we can sum over individuals. It allows the comparison of lotteries which in turn have monetary payoffs. This in turn permits the holder of the information to design related lottery systems and rank their preference without consulting the decision maker directly.

The major reason for the above explanation is an attempt to suggest that the use of the uncertainty analysis has meaning for an analyst even if it may appear too complex for application by a ranch decision maker. The amount of calculations necessary may require an iteration procedure which is practical only if the analyst gets the information processed on a computer. The processing of farm record data by extension personnel for agricultural firms has been an increasing service. This service could be expanded to include a degree of uncertainty analysis.

The Extension of Uncertainty to More than a Single Factor

There is no theoretical constraint to keep one from extending the uncertainty analysis to cover several factors other than price. The major concern from an applied viewpoint is the computational complexity. This becomes a highly practical constraint for a ranch decision maker.
The previous analysis assumed that the uncertainty involved in the decision process was restricted to the next period's price. The objective was to determine the strategy that involved the actions of holding over a specific number of various types of cattle which in turn would maximize the expected utility of the decision maker. The availability of feed during the grazing period is a function of the moisture received. This is supported by research at the Oregon State University Experiment Station and expressed by the statements:

"Precipitation fluctuation is the most important element that causes yearly fluctuations in range forage production. Therefore, precipitation can be used as an index of range herbage production." (Sneva, Ryder, 1962a, p. 3)

This introduces another uncertainty factor into the decision process—range forage production.

From the decision maker's viewpoint the additional cost of holding over livestock when there exists a shortage of range forage or the loss of forage when there is insufficient number of livestock to consume the supply is a relevant uncertainty issue. It has been shown that one can express empirically the relationship of a precipitation index to a range forage yield index (Sneva, Ryder, 1962b). There are factors other than the total crop year precipitation that influence forage production but for practical applied problems these other factors may not be traceable. Using the weather data from the reporting station in the study area would indicate that for the study area a crop year precipitation index as used by Sneva and Hyder has a range of 45 percent to 173 percent for the years 1937 through 1963 (U.S.D.I., 1964). The low
extreme occurred in 1949 and the high extreme in 1940. A crop year precipitation is defined as the precipitation received during the period September 1 to June 30 with the crop year defined as the calendar year of June 30.

It is probably true that weather affects cattle prices. Cattle prices do not affect weather. It is also probably true that the level of livestock production in the study area does not affect the general cattle price level. It is not clear how reflective weather conditions of the study area are to general weather patterns. However, the assumption that weather conditions of the study area have no defineable influence on the general cattle price level appears to be reasonably acceptable. It is probably true that the forage supply may have some localized price effect but it is assumed that the magnitude of this factor is negligible with respect to local cattle prices. This assumption allows the decision making probability distributions to have independent components with respect to price and forage conditions. In other words, 

$$\pi(P_t \mid E_1, E_2, E_3)P(r_t)$$

can be used as the expression for the subjective probability of price P in time t and the probability of a forage condition of index level r in time t. There is no claim made here that a subjective probability distribution $$\pi(r_t)$$ for a forage index level is meaningful. The assumption that there is a non-stochastic relationship between the amount of crop season precipitation and the forage index simplifies the analysis. The assumption would also suggest that one need only look at past precipi-
itation records for the study area in order to specify a reasonable probability distribution for the forage index.

The survey data indicated that for the representative ranch unit structure approximately one-fifth of the total livestock feed source would be private range which would be directly affected by the precipitation level. Approximately one-third of the feed source is derived from public permit. A public permit for a specified number of livestock does not insure that the feed will be available. Therefore, the public permit is an uncertain source of feed and it too can be tied directly to crop year precipitation.

As can be noted in Table XVIII, the impact of holding over additional livestock with the consequence of rather large variation in the production of the necessary feed supply is a significant factor to be considered by the decision maker. The problem of specifying a dollar loss function weighted by the joint probability distribution of subjective prices and the forage level could be handled without a great deal of effort in computer terms with the use of the procedure developed earlier. However, the problem of finding the strategy which will be a Bayes strategy is more complex if the solution requires iteration as it must if a utility function of the form presented in this thesis is accepted. No method other than the iteration procedure used earlier was found for establishing the Bayes strategy among the feasible strategies when the money losses can extend over a utility function that has both concave from above and concave from below regions to be investigated.
TABLE XVIII. FORAGE INDEX LEVEL WITH ASSOCIATED PROBABILITY FOR THAT LEVEL AND THE SURPLUS ANIMAL UNIT MONTHS FOR A REPRESENTATIVE CATTLE RANCH IN SOUTHEASTERN OREGON.

<table>
<thead>
<tr>
<th>Forage Index Level</th>
<th>Study Area Surplus for Representative Ranch</th>
<th>Percent of Total Ranch Feed Involved</th>
<th>Study Area Probability of Forage Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index a</td>
<td>AUMs</td>
<td>Percent</td>
<td>Percent</td>
</tr>
<tr>
<td>Under 50</td>
<td>-1440</td>
<td>29</td>
<td>4</td>
</tr>
<tr>
<td>50 - 60</td>
<td>-1296</td>
<td>26</td>
<td>5</td>
</tr>
<tr>
<td>60 - 70</td>
<td>-1008</td>
<td>20</td>
<td>6</td>
</tr>
<tr>
<td>70 - 80</td>
<td>-720</td>
<td>15</td>
<td>7</td>
</tr>
<tr>
<td>80 - 90</td>
<td>-432</td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td>90 - 100</td>
<td>-144</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>100 - 110</td>
<td>+144</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>110 - 120</td>
<td>+432</td>
<td>9</td>
<td>15</td>
</tr>
<tr>
<td>120 - 130</td>
<td>+720</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>130 - 140</td>
<td>+1008</td>
<td>20</td>
<td>4</td>
</tr>
<tr>
<td>140 - 150</td>
<td>+1296</td>
<td>26</td>
<td>4</td>
</tr>
<tr>
<td>Over 150</td>
<td>+1440</td>
<td>29</td>
<td>3</td>
</tr>
</tbody>
</table>

See reference (Sneva, Hyder, 1962b) for definition of index.
For the above reason additional examples of a Bayes strategy is not presented for the joint distribution since it would be applicable only for the assumed utility function and the problem of a generalized system would amount to precisely the same issue that was involved earlier.
CHAPTER VII

IMPLICATIONS OF THE RESULTS OF THE STUDY

Throughout this thesis exist side issues which appear as a natural extension of the analysis and were explored as the analysis flowed from developing a model which could be used for determining and evaluating an optimal livestock inventory system under conditions of certainty through the investigation of decision making under uncertainty. In addition to these side issues, it was necessary to develop a system which would allow the determination and evaluation of an optimal unique livestock inventory system.

An important result was the development of a set of equations which can be used to systematically account for the amount of feed, labor, and capital necessary to maintain any given cattle inventory structure and transform the resource requirements into a structural form which can be directly used in conjunction with an optimizing procedure technique such as linear programming. At the present time most of the western states are developing budgeted primary data related to range livestock production for the regional W-79 project. This will result in a total of approximately 150 different ranch budgeted firms of various sizes, resource structures, and livestock inventories.

If the regional economic analysis is to be meaningful, it will be first necessary to develop a systematic procedure for handling this large volume of data. One of the objectives of the regional
project is to investigate the various resource bases and the proper balance of livestock under present management practices. Present and potential management practices relates to the management of the range resource by the public as well as the private sectors. Since the budgeted data will include physical input-output information related to calving percentages, death rates, replacement rates, labor requirements, livestock weights, and management practices related to time of use with respect to the forage availability, the resource accounting equations developed in this thesis appear at the present time the only feasible way of using the effort expended in collecting and compiling the data. Experience has shown that probability of a regional research committee formulating a single report from 150 budgeted firms is not significantly different from zero. However, with a format such as developed in this thesis for optimizing from a set of resource requirement equations in which the parameter estimates are available from the budgets developed at the various universities, it seems that a consensus of the procedural use of the structure developed in this thesis would be accepted.

Once the parameters of the accounting equations are formalized for all regions, the analysis of the region's total range, labor, and capital resources can be parametrically investigated. The implication being that the marginal value products for expenditures for such items as range improvements can serve as guidelines for both public and private regional land improvements. It should be noted that the implications do not enter the argument of whether budgeted
representative units can be used to give meaningful results. The point is that if one accepts that the procedure of developing budgets of representative units as an acceptable technique for economic analysis then the procedure developed in this thesis will have direct application for handling the data for the economic analysis.

Although certain activities such as a purchase activity for additional feed was not directly a part of the linear program in this thesis, the model can be easily expanded to include these activities for land purchases, leases, and changes in costs or price levels. All these points are part of the W-79 cooperative regional project objective. If, in fact, this thesis makes a contribution, it may well be that the major gain may result from the aspect of aiding in the processing and the economic analysis of the data from the regional study.

Another area of direct application of the equation linear programming system is in the area of extension farm management consulting. A local county agent can obtain a specified set of parameter values from a local ranch operator and forward these to the extension specialist for analysis through the developed system. The practical reason for forwarding to a specialist is the general availability of a computer for solution of the linear program. The optimum livestock inventory structure along with the marginal value products of the constraining resources can then be returned to the local county agent for consultation with the ranch operator. One of
the implications of this study is that for a complete consulting service the uncertainty aspect needs to be included. The derivation of the relevant utility function along with a more operational handling of the uncertainty action space problem, such as was developed for the certainty problem, needs to be developed for purposes of operational effectiveness by extension specialists.

Any problem associated with western range livestock production implies that somewhere in the analysis the subject of administering the public lands will pose some issues. The implication of some of the results of this study indicate that the misallocation of resources related to the administered input "range forage" can be isolated and quantified. This misallocation can be shown to exist outside the fees structure. The results of the study imply that the optimum inventory structure is significantly different if the input "range forage" is used at the present public lands administered rate than the structure when the input is allowed to be used at a rate which is normal for private land control. Since the study area is between one-third to two-fifths dependent upon public lands for livestock nutrients, this would imply that the optimum production patterns should tend toward the misallocation structure. The study population appears to be performing as would be expected for the administered input structure and the results of this study. The model which represents the structure for the private control of the resource indicates a structure that closely approximates the optimum suggested by the range experiment station personnel engaged in range livestock research for the study area.
A public grazing fee structure which more closely approximates a charge for the resource that is commensurate with the consumption by various livestock types could be structured to move the production patterns toward the free market optimum. If from a policy viewpoint the changes in fee structure are ruled out, the models developed here would accomplish substantially the same result as a free market allocation. At worst it would be an improvement over the present structure.

If one accepts the validity of using subjective probability in conjunction with multivariate regression analysis in predicting economic phenomenon such as future prices, it seems reasonable that such analysis could well be used for other economic estimates. The acceptability of "subjective" as opposed to "objective" estimates does involve personal philosophy. One of the dangers of subjective values entering the scientific procedures is in the area of the subjective values becoming hidden such that science becomes a private art with the result that the process loses its reproducibility. The position held for this thesis is that the subjective probability distribution must be explicitly flagged such that the entire process can be reproduced. The results will differ only by the explicit weight of the individual's subjective probability distribution. It is not necessary that the individual be able to enumerate all the experiences which are involved with establishing the subjective probability distribution but the independence of the data used in the objective estimates from the subjective must be maintained. The
sections of this thesis dealing with decision making under uncertainty implies that this independence of probability distributions can be maintained.

One of the results, which is common to most studies, is the implication that further work is needed and the analysis should be extended by relaxing some of the assumptions. The rather limited analysis relating to the applied decision making under uncertainty is in part the result of first, not having data from which one might make conjectures with respect to the general attitude of the study population with respect to risk; and second, the failure to devise a system which would isolate a unique optimum strategy by some other method than iteration over a large strategy space. To paraphrase Samuelson's (1967) comments he maintained that it may be a pain in the neck to have to work out the whole subjective probability distribution rather than giving a single point estimate but to satisfy one's scientific conscience there may well exist a payoff that is worth the extra work involved. However, from an applied research viewpoint, it is imperative that a less cumbersome approach in establishing the optimum strategy be developed for use at the firm decision making level.

It may be that a simpler form of utility function than used in this study would be adequate. If one assumes that for all practical prediction requirements the study population can be represented in the diminishing marginal utility for money section and remain in this section for certain specified marketing decisions then the
computations are significantly reduced. The implication is that the only additional information necessary to predict what actions the study population will take is to determine their present subjective probability distribution about next year's price. A feel of the "pulse" type survey to predict the actions the rancher will take as a result of his present belief in the future price.

It should be noted that a subjective probability distribution involving a multivariate regression equation can be used quite apart from actual decision making. The general multivariate structure presented in this thesis, with of course a reasonably good set of variables for predicting the purpose for which the model was developed, could for instance, be used to predict next year's gross national product. The implication being that as more evidence becomes available and is not a part of the objective regression structure that this evidence can be used for modifying the prediction weights through the use of a subjective probability distribution. The ability of a system to allow current developments to become a part of the prediction mechanism is a powerful tool for policy decisions that must be formulated and the action taken before the outcome of the uncertain event becomes reality. The use of such techniques may become "acceptable" by the scientific community as its feasibility is demonstrated in the more "glamour" areas such as tracking of space vehicles (Carroll, 1965).

The results of this study make no claim that the selection of the criteria for decision making problems under uncertainty should
be limited to the Bayesian criterion. For instance, another approach would be the minimax criterion. The objective of the minimax criterion for the decision making problem is to minimize the maximum average loss. In other words, pick that strategy for which the largest average loss is as small as possible. The major objection and the reason for not examining some of the possible outcomes of the livestock inventory decision problem is the objectionable characteristic of minimax that it can be shown that the optimal strategy can change as a result of removing or adding an undesirable strategy. It appears to this author that the presence of an undesirable strategy should not influence the choice among the other strategies. The problem of the necessary calculation of the large set of strategies is not necessarily reduced by adopting a different criterion for decision making.
CHAPTER VIII

SUMMARY AND CONCLUSIONS

The economic welfare of southeastern Oregon is highly dependent upon the management of the range resource. Since approximately 14 of the 18 million acres of the study area are public lands, the investigation of proper management of these lands was an important part of the analysis.

This study was initiated as part of a cooperative range research agreement between Oregon State University and the Economic Research Service. The objective as stated in this agreement was

"to determine the adjustments in the organization and management of cattle ranches that are needed for profitable livestock ranching operations in Oregon under existing range and ranch resource conditions and with the introduction of various range improvement practices."

In addition to this objective, the problem of decision making under uncertainty was investigated.

A random sample of ranch operators located in Malheur, Harney, and Lake counties of Oregon were enumerated. The survey data were used in establishing the physical input-output relationships along with costs, returns, and current management practices of the range livestock firms. The structure of the "representative" ranch presented for purposes of analysis in this thesis was developed from the survey data.

A set of accounting equations was developed. These equations can be used to determine the amount of resources, the variable
operating costs and gross revenue for any arbitrarily proposed range livestock inventory structure. These equations can be used to generate the resource requirements (the $a_{ij}$'s) and the objective function values ($c_j$'s) for any range livestock inventory structure such that the structure can be evaluated for optimization by linear programming. The resource structure equations reflect the resources, costs, and returns. The parameters include, calving percentages, death rates of all livestock classes, number of cows per bull, weights of all livestock classes, replacement rates of breeding herd, the number of cull cows that are wet or dry, the prices received for all livestock classes, the percent carryover for any livestock class and an evaluation for the requirements for an unlimited number of time increments within a given production period.

The system developed does not require that the physical amount of available resources (the $b_j$'s for linear programming) be specified in order to select an optimum range livestock inventory structure. If the relative amounts of feed, labor, and capital available at different times of the production period are specified, the optimum livestock system can be determined along with the marginal value product for all constraining resources. Model I presents this format. Public ranges predominately furnish the feed during certain times of the total production period. This impact at specific time intervals, during the production period can be handled by the system developed and the system allows for the direct evaluation of public expenditures for range improvements for any specified livestock inventory.
The use of the resource structure equations in conjunction with the optimizing linear programming approach for establishing an optimum livestock inventory structure is not the only significant use of the developed equation system. There will become available this year a set of about 150 budgeted ranch operations for various range livestock areas in the western states. Each land grant institution that is contributing to the Regional W-79 research project will develop ranch organization and budgeted input-output data for representative ranch operations for their respective areas. It is concluded here that if these data are to be used effectively then it is necessary that a format such as developed in this thesis be used for the economic analysis. To attempt the analysis using an approach such as partial budgeting for this large set of diverse data appears impractical. This suggests that the approach developed here for processing these data rather than the conclusions of the analysis may be the most important professional contribution of the effort involved with this thesis.

Because there exists such a large number of possible livestock inventory compositions it was necessary to develop a system which could be analyzed to find the optimum inventory composition without enumerating all possible combinations. It was shown that a set of homogeneous livestock systems (seven in number) could be defined such that the linear programming could be used to determine the optimum combination of livestock classes for any resource, cost, and return structures.
Model II was developed to demonstrate how the inventory structure could be evaluated and compared for two resource bases. One resource structure related the feed resource to a use that was at a public land administered rate. The other resource structure assumed the same feed base was available except the rate of use was completely under the private control of the ranch entrepreneur. The private control structure used the feed at a rate that was biological rather than administered. The difference in the economic optimum, as determined by comparing the activities in the basis that relate to the two structures within the same linear program model, must be a result of the difference caused by public land administered policy. This difference is defined as a "misallocation of resources".

Model III was used as an application of Model II. A representative ranch resource structure was developed from the primary data for use in Model III. The operating expenses for various cattle types were estimated by a single regression equation. The use of this approach for isolating costs through primary data was not found as an approach in any contemporary literature pertaining to range livestock cost structure estimation. Livestock weights were representative for the study area and prices received were adjusted to an Oregon base. The economic optimum livestock inventory structure from the results of Model III would lead to the conclusion that if the range resource was in the private sector that this study area should produce and market predominately cow, heifer calf, and yearling steer beef. This result agrees substantially with the
opinion of the scientists at the Squaw Butte range experiment station with respect to the general study area. However, in general, the predominant pattern in the study area is one of producing and marketing cow, heifer and steer calve beef. This is the structure that one would conclude as the economic optimal for the administered range forage resource activity set from Models II and III. Since the study area is heavily dependent upon the public land administered policy, it appears that the Models are in fact reflecting the study population. The models developed can be used to quantify this misallocation for the representative ranch unit. The objective function maximum was substantially the same for both ranch structures. The public land administered input ranch structure was two to three percent lower. This slightly lower value reflected in the lower marginal value product of an additional animal unit month of range forage for the public lands.

One concludes from the program results that an additional animal unit month of forage has a marginal value product of approximately $3 in both structure cases. This establishes a basic dollar figure for use in evaluating range improvements. The programmed representative ranch would indicate that a 10 to 20 percent increase in total spring, summer, and fall range forage could be absorbed with substantially the same resources presently available. This is concluded from the programmed model where the amount of range forage increase is a direct evaluation of the amount of change necessary to bring about a change in the basis. Assuming a six percent opportunity cost, this would indicate that a range
improvement that results in certainty of one animal unit month increase in carrying capacity should be initiated if the cost of the improvement is $50 per animal unit month or less. This, of course, implies that the entire cost of the improvement must be carried by the range livestock enterprise.

The uncertainty problem was investigated from Bayesian viewpoint. The decision making under uncertainty was investigated predominately from the uncertainty with respect to next year's cattle price. A single equation regression model was developed for the "objective" price predictor. This model was then used in the transformation to an a posteriori subjective probability weight for various cattle price levels. The results of programmed optimum would suggest that the ranch operations that were observed carrying over steer calves to yearling steers were doing so on the basis of the utility function over the monetary range presented in this thesis and an optimistic subjective evaluation of next year's price. The program results would lead to the hypothesis that the ranch observations that deviate from the optimal programmed results are acting rationally and their actions can be explained from the decision making under uncertainty viewpoint. The set of ranch operators that are near the representative structure and holding over steer calves to steer yearlings must either accept unfair bets or have an "optimistic" subjective probability distribution with respect to next year's cattle price or both—a testable hypothesis. The term "near" in the above statement has a bothersome degree of arbitrary width.
In the context used here, it is true that some ranch observations differ from the representative structures investigated. However, the range of calving percentages investigated covers over 95 percent of the cases. Other factors such as surplus labor, livestock weights and unique physical resources would not get as high an investigation coverage as did the calving percentage comparisons but here one must rely on the merit of central tendency to draw inferences with respect to the study population.

The single equation general cattle price prediction model developed in this thesis is exactly that—a predictive model. A simultaneous equation model was not attempted since it is not clear how to generate a posteriori probability distribution for an endogenous variable such as the general cattle price level. The decision making under uncertainty model requires a set of a posteriori probability weights. In words the probability statement is

"the probability of price p in time t given the evidence in the estimation of the single equation prediction model, the additional evidence that has become available in the interim and is available at time t-1 (decision time), the subjective evidence of the decision maker, is equal to \( p_t \)"

The subjective probability statement is \( \pi(p_t|E_1, E_2, E_3) \). It is shown that since it is assumed that the sets of evidence are completely independent that the statement

\[
\pi(p_t|E_1, E_2, E_3) = \pi_1(p_t|E_1, E_2) \pi_2(p_t|E_3)
\]

is valid. This implies that an experiment station scientist could turn over to the decision maker a set of a posteriori weights that are independent of the subjective distribution of the decision maker. The decision maker could then decide on the strategy on the basis of his personally generated probability distribution.
The implication of the use of a multivariate single equation model such that one can obtain a posteriori probability weights for purposes other than next year's cattle price is of general importance. The use of subjective probability is finding more general acceptance in the economic science along with other science disciplines. Samuelson's (1967) article suggests that he is really a "Bayesian". The use of Bayesian models in the area of missile trajectory and satellite tracking systems is an example of contemporary use for a scientific area in which would initially appear to an outside observer as an unlikely consumer of the approach (Carroll, 1965). The dangers involved with subjective probability entering any social science analysis should not be taken lightly. However, the conclusion here is that any system that uses subjective probability should be identified such that the system results can be reproduced by another scientist with the results differing only by the explicit subjective weights.

It is concluded that a computational algorithm for handling the maximization of utility for a large action space using the Bayesian criterion will be necessary for applied use of the uncertainty model of the type developed in this thesis. This is basically an operations research-mathematical problem and not an economic problem. The iteration procedure developed in this thesis resembles a simulation of the action space with rather large sets of strategies necessarily being investigated to insure an optimal solution. Some efficiencies were obtained through rather rigid
assumptions about the form of the utility function and the range of losses. A more general solution procedure is proposed as a need for future research.

It is suggested that the uncertainty model developed helps to explain the variance of some firms of the study population which have an inventory structure unlike that which would be optimal from the certainty model results. The uncertainty model would suggest that a ranch operator that carried over heifer calves and heifer yearlings for beef production would be in that set of operators with a highly optimistic outlook for next year's price and probably be in the increasing marginal utility for money gains section with respect to his utility function. Personal experience with ranch operators' viewpoints would suggest this would be a very small proportion. The survey data appears to support this position. It should not be concluded that the results of the uncertainty model will allow expression about what "should be" for the entire study population. This is ruled out by the lack of theoretical justification for inter-personal aggregation of utility functions. The suggested result here is that it may be meaningful to think in terms of a general population utility function structure for purposes of explaining the population's behavior and for predicting economic stimuli responses through the use of decision theory models.


Taylor, Maurice C. 1959. Discussion: Resource policies and price policies and the changing west. In: The west in a growing economy; Proceedings of the Thirty-Second Annual Meeting of the Western Farm Economics Association. Logan, Utah. p. 244-246. (Mimeographed)


APPENDIX A

EQUATIONS AND INEQUALITIES REPRESENTING THE LINEAR PROGRAM FORMAT FOR SELECTION OF THE OPTIMUM LIVESTOCK SYSTEM

The below program uses the 90 percent calving percentage parameter. The other parameters are the same as generally used throughout the thesis. The coefficients were generated using the livestock accounting equations developed for the thesis. Activities $K_1$ through $K_7$ relate to a feed base that assumes private control while $K_8$ through $K_{21}$ assumes the feed base of private and public lands that are grazed at an administered rate.

$$\text{Max } Z = \sum_{K}^{n} \text{ constraint equations}$$

4.0990 $K_1 + 4.8490 K_2 + 4.4740 K_3 + 6.4240 K_4 +$
6.0490 $K_5 + 5.0740 K_6 + 5.8240 K_7 - K_22$
K23 - K26 + K86
K22 + K31
K23 + K78

4.0990 $K_8 + 4.8490 K_9 + 4.4740 K_{10} + 6.4240 K_{11} +$
6.0490 $K_{12} + 5.0740 K_{13} + 5.8240 K_{14} + 4.0990 K_{15} +$
4.8490 $K_{16} + 4.4740 K_{17} + 6.4240 K_{18} + 6.0490 K_{19} +$
5.0740 $K_{20} + 5.8240 K_{21} - K_{24} - K_{25} -$
K27 + K87
K24 + K32
K25 + K79
K26
K27

$\leq 652.8^*$
$97^*$
$442.1^*$
$300^*$
$300^*$

304.0999 K1 + 369.0999 K2 + 332.8499 K3 + 493.3499 K4 + 464.5999 K5 + 336.5999 K6 + 401.5999 K7 \leq b


K28 \leq b

K29 \leq b

4.1740 K1 + 5.0740 K2 + 4.6240 K3 + 6.8740 K4 + 6.4240 K5 + 5.2990 K6 + 6.1990 K7 \leq K30 - K31 - K33 - K35 - K37 + K48 \leq 1046*

4.2858 K8 + 5.7858 K9 + 5.0358 K10 + 8.0358 K11 + 7.2858 K12 + 5.7858 K13 + 7.2858 K14 + K49 \leq 1046*

4.1740 K15 + 5.0740 K16 + 4.6240 K17 + 6.8740 K18 + 6.4240 K19 + 5.2990 K20 + 6.1990 K21 \leq K30 - K34 - K36 - K38 + K50 \leq 315.3*

K30 + K51 \leq 315.3*

K31 + K52 \leq 177.3*

K31 + K52 \leq 177.3*

K32 + K53 \leq 176.8*

K33 + K56 \leq 176.8*

K34 + K57 \leq 176.8*

K35 + K58 \leq 39.8*

K36 + K59 \leq 39.8*

K37 + K54 \leq 272.2*

K38 + K55 \leq 272.2*

5.8545 K1 + 6.8545 K2 + 6.3545 K3 + 8.3545 K4 + 7.8545 K5 + 6.8545 K6 + 7.8545 K7 \leq K39 \leq a
170

\[
5.8545 \ K_1 + 6.8545 \ K_2 + 6.3545 \ K_3 + 8.3545 \ K_4 + 7.8545 \ K_{12} + 6.8545 \ K_{13} + 7.8545 \ K_{14} + 5.8545 \ K_{15} + 6.8545 \ K_{16} + 6.3545 \ K_{17} + 8.3545 \ K_{18} + 7.8545 \ K_{19} + 6.8545 \ K_{20} + 7.8545 \ K_{21} - \ K_{40} \leq a
\]

\[
K_{39} \leq a
\]

\[
K_{40} \leq a
\]

\[
361.6192 \ K_1 + 434.1192 \ K_2 + 393.6192 \ K_3 + 568.6192 \ K_4 + 536.6192 \ K_5 + 397.7442 \ K_6 + 470.2442 \ K_7 - \ K_{41} \leq b
\]

\[
361.6192 \ K_8 + 434.1192 \ K_9 + 393.6192 \ K_{10} + 568.6192 \ K_{11} + 536.6192 \ K_{12} + 397.7442 \ K_{13} + 470.2442 \ K_{14} + 361.6192 \ K_{15} + 434.1192 \ K_{16} + 393.6192 \ K_{17} + 568.6192 \ K_{18} + 536.6192 \ K_{19} + 397.7442 \ K_{20} + 470.2442 \ K_{21} - \ K_{42} \leq b
\]

\[
K_{41} \leq b
\]

\[
K_{42} \leq b
\]

\[
5.0013 \ K_1 + 6.0513 \ K_2 + 5.5263 \ K_3 + 8.0763 \ K_4 + 7.5513 \ K_5 + 6.2763 \ K_6 + 7.3263 \ K_7 - \ K_{43} - K_{44} - K_{46} - K_{48} - K_{51} - K_{52} - K_{54} - K_{56} - K_{58} \leq 697.4^*
\]

\[
4.2858 \ K_8 + 5.7858 \ K_9 + 5.5263 \ K_{10} + 8.0358 \ K_{11} + 7.2858 \ K_{12} + 5.7858 \ K_{13} + 7.2858 \ K_{14} - \ K_{49} \leq 697.4^*
\]

\[
5.0013 \ K_{15} + 6.0513 \ K_{16} + 4.4263 \ K_{17} + 8.0763 \ K_{18} + 7.5513 \ K_{19} + 6.2763 \ K_{20} + 7.3263 \ K_{21} - \ K_{45} - K_{47} - K_{50} - K_{53} - K_{55} - K_{57} - K_{59} + K_{77} \leq 354.7^*
\]

\[
K_{43} + K_{76} \leq 354.7^*
\]

\[
K_{44} + K_{70} \leq 367.4^*
\]

\[
K_{45} + K_{71} \leq 367.4^*
\]

\[
K_{46} + K_{84} \leq 39.8^*
\]

\[
K_{47} + K_{85} \leq 39.8^*
\]

\[
K_{48} \leq 174.3^*
\]

\[
K_{49} \leq 174.3^*
\]

\[
K_{50} + K_{74} \leq 116.1^*
\]

\[
K_{51} + K_{75} \leq 116.1^*
\]
K52 + K72 ≤ 35.5*
K53 + K73 ≤ 35.5*
K54 + K68 ≤ 204.2*
K55 + K69 ≤ 204.2*
K56 + K80 ≤ 88.4*
K57 + K81 ≤ 88.4*
K58 + K82 ≤ 20*
K59 + K83 ≤ 20*

6.7070 K1 + 8.5570 K2 + 7.6320 K3 + 11.4820 K4 +
10.5570 K5 + 8.6320 K6 + 10.4820 K7 - K60 -
K62 ≤ a

6.7070 K8 + 8.5570 K9 + 7.6320 K10 + 11.4820 K11 +
10.5570 K12 + 8.6320 K13 + 10.4820 K14 + 6.7070 K15 +
8.5570 K16 + 7.6320 K17 + 11.4820 K18 + 10.5570 K19 +
8.6320 K20 + 10.4820 K21 - K61 - K63 ≤ a

K60 ≤ a
K61 ≤ a
K62 ≤ a
K63 ≤ a

392.0297 K1 + 472.5297 K2 + 427.7797 K3 + 618.2797 K4 +
582.5297 K5 + 432.2797 K6 + 512.7797 K7 - K64 ≤ b

392.0297 K8 + 472.5297 K9 + 427.7797 K10 + 618.2797 K11 +
582.5297 K12 + 432.2797 K13 + 512.7797 K14 + 392.0297 K15 +
472.5297 K16 + 427.7797 K17 + 618.2797 K18 + 582.5297 K19 +
432.2797 K20 + 512.7797 K21 - K65 ≤ b

K64 ≤ b
K65 ≤ b

3.7894 K1 + 4.3894 K2 + 4.0894 K3 + 5.8894 K4 +
5.5894 K5 + 4.6894 K6 + 5.2894 K7 - K66 -
K68 - K70 - K72 - K75 -
K77 - K78 - K80 - K82 -
K84 - K86 - K91 ≤ 40.8*
3.7894 K8 + 4.3894 K9 + 4.0894 K10 + 5.8894 K11 +
5.5894 K12 + 4.6894 K13 + 5.2894 K14 + 3.7894 K15 +
4.3894 K16 + 4.0894 K17 + 5.8894 K18 + 5.5894 K19 +
4.6894 K20 + 5.2894 K21 - K67 - K69 -
K71 - K73 - K74 - K76 -
K79 - K81 - K83 - K85 -
K87 - K92 ≤ 40.8*
K66 ≤ 118.2*
K67 ≤ 118.2*
K91 ≤ 41.2*
K92 ≤ 41.2*
K68 ≤ 82.7*
K69 ≤ 82.7*
K70 ≤ 220.4*
K71 ≤ 220.4*
K72 ≤ 20 *
K73 ≤ 20 *
K75 ≤ 87.1*
K74 ≤ 87.1*
K77 ≤ 266.0*
K76 ≤ 266.0*
K78 ≤ 132.6*
K79 ≤ 132.6*
K80 ≤ 44.2*
K81 ≤ 44.2*
K82 ≤ 20 *
K83 ≤ 20 *
\[2.9247 K_1 + 3.9247 K_2 + 3.4247 K_3 + 5.4247 K_4 + 4.9247 K_5 + 3.9247 K_6 + 4.9247 K_7 \leq \alpha\]
\[2.9247 K_8 + 3.9247 K_9 + 3.4247 K_{10} + 5.4247 K_{11} + 4.9247 K_{12} + 3.9247 K_{13} + 4.9247 K_{14} + 4.9247 K_{15} + 3.9247 K_{16} + 3.4247 K_{17} + 5.4247 K_{18} + 4.9247 K_{19} + 2.9247 K_{20} + 3.9247 K_{21} \leq \alpha\]
\[275.7061 K_1 + 333.2061 K_2 + 301.9561 K_3 + 447.4561 K_4 + 421.2061 K_5 + 340.7061 K_6 + 398.2061 K_7 - K_9 \leq \beta\]
\[275.7061 K_8 + 333.2061 K_9 + 301.9561 K_{10} + 447.4561 K_{11} + 421.2061 K_{12} + 340.7061 K_{13} + 398.2061 K_{14} + 275.7061 K_{15} + 333.2061 K_{16} + 301.9561 K_{17} + 447.4561 K_{18} + 421.2061 K_{19} + 340.7061 K_{20} + 398.2061 K_{21} - K_9 \leq \beta\]

*See Figure 4 for values.

*a* Labor parametrically investigated.

*b* Capital parametrically investigated.