On the implementation of open boundary conditions for a general circulation model: The three-dimensional case

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Abstract. This article discusses the results of an experimental survey on the performance of a select group of open boundary conditions (OBCs) for three-dimensional, primitive equation models. The focus is on passive conditions, where the model response at the open boundaries is dictated by the interior dynamics. The performance of the OBCs is tested in a series of numerical experiments conducted in a rotating channel which includes variable bottom topography and density stratification. The experiments are selected to enhance nonlinear phenomena that comprises advection and the propagation of dispersive waves. The first two experiments study coastal upwelling with constant and time-varying wind forcing. The third experiment analyses the barotropic and baroclinic response of a coastal ocean due to the passage of a traveling storm. The open boundary condition with the best overall performance is the flow relaxation scheme on sea surface elevation and transport, a radiation condition for internal velocities, and a combined scheme of flow advection and relaxation for the temperature field. A modified gravity wave radiation scheme provides reasonable responses when combined with radiation conditions for internal velocities and an advection equation for temperature if the forcing is not changing direction rapidly at the open boundary. Schemes using the method of characteristics or traditional wave advection schemes fail when combined with radiation conditions for the baroclinic mode.

1. Introduction

The objective of this article is to discuss the performance of several open boundary conditions (OBCs) for a general circulation model. Previously, we did a comparative study of the barotropic mode [Palma and Matano, 1998], and concluded that the schemes with the best overall performance were those proposed by Flather [1976] and Martinsen and Engedahl [1987]. In this article we analyze the three-dimensional (3-D) case. The experimental strategy follows that of Palma and Matano [1998]; that is, we selected a suite of benchmark experiments whose solution can be determined without the use of OBC and compared their results with experiments using OBCs.

Although there are several numerical studies using OBCs in 3-D models most of them have been focused on particular aspects of the oceanic circulation and not on the performance of an open boundary scheme per se. Miller and Bennet [1988] analyzed the compatibility conditions that arise in limited area, quasi-geostrophic models and concluded that for large-scale simulations, incompatible boundary conditions are unavoidable and that they may cause instability problems in timescales comparable to those which characterize the boundary conditions. Stevens [1990] proposed and tested a set of OBCs using a regional version of the Geophysical Fluid Dynamics Laboratory (GFDL) general circulation model. At the open boundaries he calculated the time evolution of the barotropic stream function and internal velocities from a linearized version of the momentum equations; temperature and salinity were calculated from a modified radiation condition. These conditions were later used in regional simulations [Stevens, 1991; Stevens and Johnson, 1997]. Oey and Chen [1992] implemented a suite of OBCs for a numerical simulation of the North Atlantic seas that combined a condition proposed by Flather [1976] for surface elevation and barotropic velocities, with a radiation condition for baroclinic velocities. Stordal et al. [1994] used the flow relaxation scheme (FRS) proposed by Martinsen and Engedahl [1987], to study the response of a stratified shelf to the passage of a travelling storm. Engedahl

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described a method for handling OBCs in diagnostic simulations, which required the knowledge of the depth mean current at the open boundary. The method was then used to set up a diagnostic and prognostic simulation of the Nordic Seas. Kourefalou et al. [1996] modeled the discharge of a river plume on the South Atlantic Bight using a modified version of the Blumberg and Khanta [1985] radiation condition. Barnier et al. [1998] proposed a group of OBCs for the semi-spectral primitive equation model (SPEM), [Haidvogel et al., 1991]. They used the radiation condition with allowance for phase speed oblique to the boundary [Raymond and Kuo, 1984] combined with relaxation to climatology. The resulting scheme was then employed to set up a regional model of the South Atlantic using active boundary forcing.

In spite of the widespread use of OBCs in primitive equation models, there are no comparative studies on the performance of the diverse schemes proposed in the literature. Previous comparative studies focused on linear and nonlinear 2-D equations [Chapman, 1985; Røed and Cooper, 1987; Blumberg and Matano, 1998]. The purpose of this article is to fill part of that gap by evaluating the performance of a suite of OBCs in a series of controlled experiments. The focus of this study will be on passive OBCs, i.e., conditions to be applied when the interior circulation determines the boundary values of the prognostic variables. Toward this objective the selected schemes are tested in an idealized ocean basin which includes variable bottom topography and vertical density stratification. The experiments emphasize flow conditions dominated by advection or wave propagation, and they represent situations of physical interest in coastal oceanography. The selected cases include (1) the baroclinic response of a coastal ocean to upwelling favorable winds, (2) the adjustment of a coastal zone to a change in wind conditions, and (3) the transient response of a shelf region to the passage of a traveling storm.

This article has been organized as follows. Section 2 presents background information about the numerical model used and the selected OBCs schemes. The results of the different experiments are presented and discussed in section 3. Finally, section 4 summarizes the results and presents the conclusions.

2. Model Setup

The model to be used in our experiments is the Princeton ocean model (POM), a primitive equation, sigma coordinate, finite difference model. The prognostic variables of this model are the sea surface elevation, the three components of velocity, temperature, salinity, and turbulent kinetic energy and length scale. The computation is split into an external barotropic mode, which solves the time evolution of the free surface elevation, and depth-averaged velocities and an internal baroclinic mode that solves the vertical velocity shear. Vertical mixing in the model is calculated through an embedded turbulence closure scheme [Mellor and Yamada, 1982], while horizontal mixing is parameterized following Smagorinsky [1963]. A detailed description of the model is given by Blumberg and Mellor [1987].

2.1. OBCs for the External Mode

The OBC schemes to be used for the barotropic mode follows the recommendations of our previous study [Blumberg and Matano, 1998]; that is, the characteristic scheme proposed by Røed and Cooper [1987], the radiation condition proposed by Flather [1976], and the flow relaxation scheme (FRS) proposed by Martinsen and Engedahl [1987]. The characteristic method is a modified version of a method first proposed by Hartree [1953] and later modified to include rotation and wind forcing by Røed and Cooper [1987]. It combines the characteristic equations of the problem with a linearized version of the primitive equations to yield an equation for normal velocity at the open boundaries. The Flather condition combines a Sommerfeld-type radiation condition with a one-dimensional version of the continuity equation to yield an equation for the normal velocity at the open boundary. In the FRS the prognostic variables are restored to a reference state within specified regions. The method, originally designed for nested models, was implemented by Martinsen and Engedahl [1987] as a pure OBC. In dynamical situations with nonzero steady state solutions or strong forcing at the open boundaries, the best performance of the Flather's and Martinsen's schemes were supplemented with a local solution. Table 1 summarizes the general characteristics of all the schemes and defines the acronyms used hereafter. A detailed description of their numerical implementation is given by in Blumberg and Matano [1998].

2.2. OBCs for the Internal Mode

For the baroclinic velocities we test two variations of Sommerfeld's one-dimensional radiation condition. In the first case we calculate the baroclinic velocities at the open boundary using

\[
\frac{\partial \phi}{\partial t} + C_i \frac{\partial \phi}{\partial x} = -\frac{\phi}{P_f},
\]

where \( \phi \) stands for \( u \) and \( v \), \( C_i \) is the (fixed) baroclinic internal wave phase speed, \( P_f \) is a relaxation time and the plus (minus) sign applies to the right (left) open boundary. This scheme was originally proposed by Blumberg and Khanta [1985] for use on a barotropic simulation and later modified by Oey and Mellor [1993] and Kourefalou et al. [1996] to handle three-dimensional baroclinic situations. To solve (1) at the open boundaries we employ an implicit and upstream method for the evaluation of the partial derivatives. The internal wave phase speed \( C_i \) is fixed at \( C_i = \sqrt{gH \times 10^{-3}} \), where \( g \) is gravity and \( H \) is the local water depth. Earlier results derived using this condition show a relative...
Table 1. Description of the OBCs Tested With the Numerical Experiments

<table>
<thead>
<tr>
<th>Open Boundary Conditions</th>
<th>Analytic Form</th>
<th>References</th>
</tr>
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<tbody>
<tr>
<td><strong>External Mode</strong></td>
<td></td>
<td></td>
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<tr>
<td>MRO (Martinsen plus Reed local solution)</td>
<td>$\phi_e = \alpha \phi_0(t) + (1 - \alpha) \phi^*$</td>
<td>Martinsen and Engedahl [1987]</td>
</tr>
<tr>
<td>PRO (Flather radiation plus Røed local solution)</td>
<td>$U - U_0(t) = \pm \frac{1}{H} [\eta - \eta_0(t)]$</td>
<td>Flather [1976] and Palma and Matano [1998]</td>
</tr>
<tr>
<td>BKI (Blumberg and Khanta Implicit)</td>
<td>$\phi^<em><em>e \pm C_0 \phi</em></em> = - \frac{\phi_0}{H}$; with $C_0 = \sqrt{\gamma H}$</td>
<td>Blumberg and Khanta [1985]</td>
</tr>
<tr>
<td>HOC (Hedstrom O'Brien characteristic)</td>
<td>$(UD)_x = \mp 0.5 C_0(UD \pm \eta \eta_0)_x + F_x$</td>
<td>Reed and Cooper [1987] and Palma and Matano [1998]</td>
</tr>
<tr>
<td><strong>Internal Mode</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BKI (Blumberg and Khanta Implicit)</td>
<td>$\phi^<em><em>e \pm C_0 \phi</em></em> = - \frac{\phi_0}{H}$; with $C_0 = \sqrt{10^{-3} g H}$</td>
<td>Koutrakolou et al. [1996]</td>
</tr>
<tr>
<td>ORI (Orlanski radiation implicit)</td>
<td>$\phi^<em><em>e \pm C_0 \phi</em></em> = 0$ with $C_0 = \pm \frac{\phi_0}{H}$</td>
<td>Orlanski [1976] and Chapman [1985]</td>
</tr>
<tr>
<td><strong>Tracers</strong></td>
<td></td>
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</tr>
<tr>
<td>FRS (flow relaxation scheme)</td>
<td>$T = \alpha T_{trim} + (1 - \alpha) T_{b}$; $\alpha = 1 - \tanh[0.5(x - x_B)]$</td>
<td>Martinsen and Engedahl [1987] and Engedahl [1995]</td>
</tr>
<tr>
<td>ADV (advection scheme)</td>
<td>$T_x \pm \zeta(T + u) T_x = 0$</td>
<td>Stevens [1990] and Mellor [1996]</td>
</tr>
<tr>
<td><strong>Composite Schemes</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MOA</td>
<td>MRO + ORI + [FRS (inflow) + ADV (outflow)]</td>
<td>this paper</td>
</tr>
<tr>
<td>FOA</td>
<td>FRO + ORI + ADV ($C_T = 0$)</td>
<td>Oey and Chen [1992]</td>
</tr>
<tr>
<td>FOO</td>
<td>FRO + ORI + [ADV + ORI; $u \neq 0$, $C_T \neq 0$]</td>
<td>this paper</td>
</tr>
<tr>
<td>BBA</td>
<td>BKI + BKI + ADV ($C_T = 0$)</td>
<td>Koutrakolou et al. [1996]</td>
</tr>
<tr>
<td>HOA</td>
<td>HOC + ORI + ADV ($C_T = 0$)</td>
<td>this paper</td>
</tr>
</tbody>
</table>

$\phi^*$ represents any of the three external mode dependent variables, $U$, $V$, or $\eta$; $\phi^*$ represents any of the two internal mode velocities $u$ and $v$; and $T$ represents temperature. The upper (lower) sign in $\pm$ and $\mp$ corresponds to the OBC at the right (left) side of the model. Other variables are $D = H + \eta$, with $H$ the local water depth, $C_0$ the baroclinic mode wave phase speed, $C_T$ the shallow water wave speed, $C_T$ a computed tracer phase speed, $P_T$ a friction timescale, $U_0(t)$, $\eta_0(t)$, $\phi_0(t)$, and $T_{trim}$ prescribed boundary values, $x$ the zonal coordinate, $x_B$ the position of the open boundary, and $\alpha$ the relaxation parameter. Also, $T = \partial(\cdot)/\partial t$; and $T_x = \partial(\cdot)/\partial x$.

$C_1$ is computed using an implicit numerical scheme of the form

$$C_1 = \frac{\phi_{B+1}^{n} - \phi_{B+1}^{n-1}}{\phi_{B+1}^{n+1} + \phi_{B+1}^{n-1} - 2\phi_{B+1}^{n}}$$

where $B$ is a boundary node, $n$ is a time level index, and the lower (upper) sign corresponds to the right (left) boundary (Figure 1).

The second scheme, ORI, is based on Orlanski’s [1976] OBC. $C_1$ is computed using a simplified version of the advection equation, i.e.,

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = 0$$

where $u$ and $\partial(\cdot)/\partial x$ are the component of velocity and...
the derivative normal to the boundary, respectively. We implement (3) at the open boundaries using an upstream in space and forward in time numerical scheme in the form [Mellor, 1996]

\[ T_{B+1}^{n+1} = T_B^n + r_1(T_B^n - T_{B-1}^n) + r_2(T_e^{n+1} - T_B^n) \]  

where

\[ r_1 = 0.5(\Delta t/\Delta x)(u_B^2 + |u_B^2|) \]

\[ r_2 = 0.5(\Delta t/\Delta x)(u_B^2 - |u_B^2|) \]

and \( \Delta t \) is the internal time step, \( \Delta x \) is the grid size, \( u_B \) is the velocity normal to the boundary, and \( T_e \) is a prescribed external temperature at the same boundary (Figure 1). Note that the prescribed data are only used during inflow.

The second scheme uses a modification suggested by Stevens [1990] that consists of adding a correction term to the advection velocity which allows the outward propagation of internal waves. The modified equation for temperature is

\[ \frac{\partial T}{\partial t} + (u + C_T) \frac{\partial T}{\partial x} = 0 \]  

where \( C_T \) is computed using scheme (2) replacing \( \phi \) for \( T \). Equation (5) is discretized using a modified version of the leap-frog Orlanski scheme [Orlanski, 1976; Chapman, 1985].

For inflow boundaries we use the FRS which consists in relaxing temperature and salinity to a priori prescribed value within a specified region along the open
boundary. If $T_i$ is the prognostic temperature calculated with the full set of equations in the interior of the model domain, then in the FRS zone the variable is relaxed by

$$T = \alpha T_e + (1 - \alpha) T_i$$

(6)

where $T_e$ is the external prescribed temperature, $T_i$ is the time-integrated unrelaxed temperature, and $T$ is the new updated value of temperature in the FRS zone. The relaxation parameter is taken to be

$$\alpha(i) = 1 - \tanh[0.5(i - 1)] \quad i = 1, 10,$$

(7)

where index $i$ is counted from the western open boundary and a similar expression is valid for the eastern relaxation zone.

2.3. Composite Schemes

Of all the possible combinations among OBCs for the external and internal mode we will restrict our discussion to the following stable schemes (Table 1). MOA is Martinsen’s scheme with the local solution for the barotropic mode, Orlanski’s implicit OBC for the baroclinic velocities and FRS (inflow) plus an advection scheme (outflow) for temperature (3). FOA is Flather’s scheme with the local solution for the barotropic mode, Orlanski’s OBC for the baroclinic velocities and an advection scheme for temperature. FOO is the same as FOA but using the modified advection scheme for temperature (5). BBA is Blumberg’s OBC (1) for the barotropic mode and baroclinic velocities and an advection scheme for temperature [Kourafalou et al., 1996]. HOA is the characteristic method for the barotropic mode, Orlanski’s scheme for the baroclinic velocities and the advection OBC for temperature.

3. Numerical Experiments

The performance of the OBCs schemes described in section 2 have been evaluated in a series of numerical experiments in a rectangular basin on an $f$ plane, with variable bottom topography and vertical density stratification. A shelf and continental slope is simulated using a hyperbolic tangent profile, with a minimum depth of 20 m at the coast, a maximum of 2000 m offshore, and uniform in the along-shore direction. The vertical structure of the model includes 15 sigma levels with high resolution at the upper and bottom layers of the ocean. The initial state of the model simulates a vertically stratified ocean at rest whose temperature structure and buoyancy frequency are shown in Figure 2. The horizontal eddy viscosity of the model is computed from a Smagorinsky-type formulation. The three experiments that are selected for discussion consider the oceanic adjustment to (1) constant wind forcing, (2) variable wind stress forcing, and (3) the passage of a storm.

The oceanic adjustment to upwelling favorable winds permits the evaluation of the different OBCs schemes in a transient and strongly nonlinear regime. Variable wind forcing allows the evaluation of OBCs schemes in flow conditions characterized by rapid transitions between inflow and outflow situations. The experiments on the traveling storm focuses on flow regimes dominated by advection and coastal trapped wave propagation.

3.1. Constant Wind Forcing

In this experiment we simulate the spin-up of a coastal ocean to a constant, upwelling favorable (easterly) wind stress of magnitude 0.1 Pa. The numerical domain consists of a channel that is 600 km long and 200 km wide. The along-shore grid spacing is 10 km, while the cross-shore spacing varies from 2 km at the coast, to 10 km farther offshore. Since there are no analytical solutions to the fully nonlinear problem in a semi-enclosed basin, we used as benchmark the results obtained in an experiment using cyclic conditions (see discussion by Palma and Matano, 1998). A detailed analysis of the spin-up process is given by Allen et al. [1995] and references therein; therefore only a brief description is included here. Figure 3 shows cross-sectional plots of velocity and temperature (at the middle of the domain) at day 10. After the impulsive start the surface wind stress forces an offshore Ekman drift in the upper layer.
Figure 3. Benchmark results of the wind driven spin-up experiment using cyclic open boundary conditions. (a) Cross section of the temperature field at the middle of the computational domain and for day 10 of the simulation. (b) Cross section of alongshore velocity. (c) Cross section of offshore velocity. Shaded areas indicate negative values.

Figure 4. Time series of (a) area-averaged kinetic energy and (b) SSH at a coastal node from the constant wind experiment. The solid line is the result from the benchmark experiment (cyclic OBC), marked lines are results obtained using the OBC scheme indicated on top of each curve.

which is compensated by an onshore flow below the surface. The return flow is initially spread out over the water column, and it gradually becomes concentrated in a bottom boundary layer (Figure 3c). As the onshore flow advects warm waters offshore, colder waters move upward into the shelf (Figure 3a). The tilting of the density surfaces by the onshore flow accelerates a coastal jet that is vertically and horizontally sheared (Figure 3b). The jet has a width of ~20 km, and its axis slowly drifts offshore. The results obtained using the cyclic condition are consistent with existing descriptions of coastal upwelling [e.g., Brink, 1983; Allen et al., 1995].

As a general overview on the performance of the different OBCs schemes during the spin-up process, Figure 4 shows the time evolution of the basin averaged kinetic energy and sea surface height (SSH) for a coastal node. The small oscillations superimposed on the curves are related to inertial oscillations. Of all the tested schemes, only MOA, FOO, and FOA produce a similar, albeit slower, response than the bench-
Figure 5. Snapshots of SSH contours and surface velocity vectors after 10 days from the constant wind experiment. (top) The result from the benchmark experiment (CYC OBC); other panels are results obtained using the OBC scheme indicated on top of each panel. Contour interval (CI) is 4 cm; shaded areas indicate negative values. Every third velocity vector is plotted.

mark experiment. BBA drifts toward a steady state with lower energy levels than the benchmark case. This scheme shows large SSH oscillations caused by the partial clamping of the open boundary. Since the HOA scheme showed a steady emptying of the basin after a few days of integration time, we included an integral constraint (IC) on mean sea level to force volume conservation (e.g., Palma and Matano, 1998). Although this device stopped the loss of volume, it also contaminated the interior solution with spurious oscillations associated to boundary reflections of coastal trapped waves, which generates an increase of the mean kinetic energy level (Figure 4a). Experiments replacing HOC with a gravity wave implicit scheme (GWI) and an Orlanski-type condition (ORI) for the barotropic component of the flow show the same tendency toward emptying and filling of the basin. For those particular schemes the integral constraint does not alleviate the problem. This particular behavior of HOC, GWI, and ORI seems to be related to the strong coupling between barotropic and baroclinic components of the flow during the upwelling event, since the schemes performed well
in barotropic experiments [Palma and Matano, 1998].

For the discussion of the internal fields we will first show horizontal (x-y) and vertical cross sections (y-z) of the prognostic variables. Figure 5 compares the SSH distribution and surface velocity vectors of the different schemes after 10 days of model simulation. Figure 5 (top left) shows the results of the benchmark (CYC). In this experiment the SSH contours run parallel to the shore where there is a marked sea level set down. The core of the jet is located at ~30 km from the coast and decreases rapidly farther offshore. The schemes with the best performance were MOA and FOO. We also did experiments using the FOO version without the local wind solution but the results were poor since the lack of wind-driven outflow at the western boundary clamps the solution and causes spurious reflections. The characteristic scheme (HOA+IC) has a uniform tilting of the contour lines and a strong setup on the northern (closed) boundary. BBA shows strong reflections in the region close to the western (outflowing) boundary.

The sea surface temperature (SST) fields for the different experiments are shown in Figure 6. Figure 6 (top left) shows the benchmark (CYC) results after 10 days of model simulation. The other panels depict the root mean square error corresponding to the different open boundary schemes. The schemes with the best performance are MOA and FOO, while FOA, HOA, and BBA do not allow the proper upwelling of cold water over
a large proportion of the continental shelf. The root mean square error in BBA extends up to the center of the model domain, rendering this OBC unsuitable for long-term simulations. Although it appears that all the OBC schemes fail to reproduce the flow at the eastern (inflowing) boundary, this is not necessarily the case. The fact is that the simulation using cyclic conditions makes no distinction between outflowing and inflowing boundaries. In the test experiments, however, the OBCs schemes restore temperature to its climatological value at the inflowing boundary. Therefore the differences between the cyclic case and the experiments using OBCs at the eastern boundary are more related to the limitations of the benchmark case than to the deficiencies of the OBC schemes.

So far, our discussion has been focused on circulation patterns immediate to the open boundaries. To illustrate how the different schemes affect the circulation in the middle of the basin, Figures 7 and 8 show cross sections (y-z) of the prognostic variables. Figure 7 shows the velocity structure of the different experiments superimposed on temperature fields. The scheme with the poorest performance was BBA, which shows a relative weak upwelling regime associated to relative small vertical velocities over the shelf break. Figure 8 shows a comparison of along-shelf velocity fields across the same section. None of the schemes is capable of reproducing the vertical and horizontal structure of the coastal jet of the benchmark experiment. The most obvious difference is that the coastal jet in the cyclic case has a more marked barotropic structure than in any of the other experiments. This might be caused by the fact that reflections associated to the different OBC schemes tend to create an artificial along-shore pressure gradient which arrests the wind driven jet everywhere but at the uppermost layers where mixing effects are the strongest. An obvious example is BBA which, as shown in Figures 5 and 6, is the most reflective of all the tested schemes and, according to Figure 8, has also the more shallow and weakest of the upwelling jets.

Of all the schemes tested in this experiment, MOA and FOO show the best response, although the amount of upwelling produced is not as strong as in the benchmark case. In this regard it should be remembered that although the tested OBCs distinguish between inflow and outflow conditions, the cyclic case does not and al-
Figure 8. Cross-sectional plots of alongshore velocity after 10 days from the constant wind experiment. (top left) The result from the benchmark experiment (CYC OBC); (other panels) Results obtained using the OBC scheme indicated on top of each panel. CI is 5 cm/s. The FOA scheme has a very similar response to FOO.

allows cold water to circulate endlessly along the model domain. For this reason, a similar field pattern should be expected but with warmer temperatures and slower velocities in the cases using the proposed OBCs compared with the benchmark experiment.

3.2. Variable Wind Forcing

The objective of this experiment is to test the OBCs in flows characterized by periodic reversals between inflow and outflow conditions. To simulate these conditions, the model is forced by a time varying, along-shore wind stress of the form

$$\tau_x = \tau_0 \sin \left( \frac{2\pi}{P} \right)$$

(8)

where $\tau_0 = 1$ dyn/cm$^2$ and $P = 5$ days is the period of wind forcing. The wind is eastward (upwelling favorable) during the first half cycle and westward (downwelling favorable) during the second half. The benchmark experiment is conducted using the numerical domain and boundary conditions (CYC) described in the previous experiment. Philander and Yoon [1982] have discussed at length the oceanic adjustment process to this particular forcing; therefore only aspects relevant to the OBCs performance will be included in our discussion.

After a rapid adjustment during the first cycle the mechanical energy of the system converges slowly toward a periodic state. Results from the final sixth cycle were used to describe the numerical experiments. Figure 9 shows the time evolution of the SSH and along-shore velocity in a near-shore transect. Although according to the linear, inviscid theory the along-shore velocity should lag the forcing by an amount of $\sim 3$ hours; in our case this lag is slightly greater (4.8 hours) because of enhanced bottom friction near the coast. Figure 10 shows the offshore structure of the along-shore velocity and temperature anomaly fields for a section in the middle of the domain and at two different times. The offshore scale is the radius of deformation ($\sim 20$ km), but the structure of the coastal jet is confined closer to the coast than in the experiment using constant wind stress forcing (Figure 10, top left). This is related to the fact that the strength of the surface Ekman transport, responsible for the offshore drifting of the coastal jet, is weakened by the periodic change in wind direction.
Figure 9. Time zonal plots of sea surface elevation (CI = 1 cm) and alongshore velocity (CI = 5 cm/s) along the transect $y = 2$ km for the variable wind experiment obtained with the cyclic OBC. Dashed lines indicate negative values; the thick solid line is the zero contour line.

An interesting feature of the response is the presence of adjacent along-shore currents in opposite directions (Figure 10, bottom left). Philander and Yoon [1982] gave the explanation of these phenomena. When the wind starts to blow eastward a westward current is still present. The wind effect is strongest near the coast, and a westward current appears there while the flow offshore is still eastward. The sequence is not merely reversed when the wind blows westward because of nonlinear effects. Momentum from the western current close to the coast is advected offshore in the surface Ekman layers so that the surface flow is westward everywhere at this time (Figure 10, top left). A similar situation occurs in the temperature field. The surface temperature close to shore was already mixed with warmer water carried by onshore flow during the westward wind; consequently, the maximum temperature anomaly (negative for upwelling) is not attained at the maximum upwelling favorable wind but at the end of the spell of the eastward wind. As the surface heat flux is zero, the overall effect is a net decrease of temperature at the coast and over the shelf break (Figure 11). The asymmetry in the response between eastward and westward wind forcing is also evident in the mechanical energy plot (Figure 12, top left). The energy curve presents a higher peak during the upwelling phase when the coastal jet is strengthened by the upward bend of the isotherms.

As a first glimpse on the behavior of the different OBCs schemes, Figure 12 shows the time series of the average barotropic kinetic energy for each experiment. The dashed line represents the benchmark experiment, while the solid line corresponds to the particular OBC scheme. Of all the tested schemes, only results using MOA and FOO are similar to the benchmark case. The response using the FOA scheme is close to the benchmark case until day 25 when there appears a slow increase of the energy level. The characteristic method (HOA), boosted by an integral constraint in surface elevation, produces a partial clamping of the boundary, and the retained energy stays inside the domain oscillating around an increased mean level. A similar response is obtained with BBA ($P_f = 3$ days), which shows strong oscillations immediately after the initial transients (gravity and shelf waves) have reached the eastern boundary. A closer look at the time evolution of the SSH field at a transect over the shelf (Figure 13) shows the boundary reflections produced by HOA, BBA, and FOO. In the case of HOA, the reflections (mainly at the western boundary) created an along-shore pressure gradient and a resultant elevation field that is out of phase with the wind (compare with CYC). The failure of BBA seems more pronounced; the incorrect cross-shore slope of the surface prevents a proper geostrophic balance and obstructs the advective flow at both boundaries generating strong wave reflection. The trapped energy oscillates at the basin normal mode frequency superimposed to the forcing frequency, and the resulting field presents improper lows and highs in the interior out of phase with the wind. The failure is more noticeable when there is a reversal of the flow. Test
experiments changing the relaxation time of the BBA scheme show a better response as $P_f$ increases, but an incorrect alongshore gradient is still present even for $P_f \rightarrow \infty$. For the FOA scheme the growth of a small alongshore gradient at the western boundary is clearly seen after the fifth cycle. This improper response is reflected in the steady increase in the basin-averaged energy level shown in Figure 12.

In light of the above results, the discussion of the baroclinic response will be restricted to the MOA, FOA, and FOO schemes. Figure 14 shows a plot of the time evolution of the average baroclinic kinetic energy. MOA has the best performance of all the tested schemes, having only minor differences with the benchmark case.

FOA shows a response close to the benchmark case but with larger amplitude of the energy oscillations. The FOO scheme shows a marked increase of the kinetic energy after 15 days of simulation, which contrasts with the good performance shown in the graphic of the barotropic kinetic energy (Figure 12). To understand the behavior of the different OBCs, we analyzed the three-dimensional structure of the model prognostic fields. Figure 15 compares cross-sectional plots of alongshore velocity at a transect located 100 km from the eastern boundary and at the end of the spell of the eastward (upwelling favorable) wind for the benchmark and FOA schemes. Although the results using MOA and FOO (not shown) are in good agreement with the

**Figure 10.** Cross-sectional plots of alongshore velocity (left, CI = 2 cm/s) and temperature anomaly (right, CI = 0.1°) for the variable wind experiment obtained with the CYC OBC. The flow is westward in shaded areas. The temperature anomaly is computed with respect to the reference state and is negative in shaded areas. (top) The instant of maximum westward (upwelling favorable) wind stress. (bottom) The instant of maximum eastward wind stress.

**Figure 11.** The time evolution of surface temperature for the variable wind experiment obtained with the CYC OBC. (left) A coastal node; (right) a node over the shelf break.
benchmark experiment, the FOA scheme shows a countercurrent ~40 km offshore and extending vertically until 250 m depth. This countercurrent decays in the along-shore direction and is almost negligible at 150 km from the eastern boundary. Through geostrophic balance the countercurrent affects the temperature fields (Figure 16) which show an intense downward trend 50 km offshore that balances the countercurrent at this location. The existence of a countercurrent in upwelling favorable flows is symptomatic of alongshore pressure gradients [e.g. Philander and Yoon, 1982]. Since in our particular model setting, there are no alongshore variations in topography or forcing, these gradients are most likely to be related to spurious reflections at the open boundaries.

Finally, we examine the response of FOO in more detail since it performed well during the first 15 days of simulation (Figure 14). Figure 17 shows a cross-sectional plot of alongshore velocity at y = 450 km and t = 27.5 days, which focus on the northern boundary wall. FOO (Figure 17, right) presents an intense flow trapped against the wall which is absent in the benchmark experiment. This flow is generated by the passage of a baroclinic Kelvin wave travelling westward. Note that the wave produces an alongshore gradient, which in turn generates a countercurrent [Philander and Yoon, 1982]. The Kelvin wave at the northern wall is not present in the experiments using the MOA and FOA schemes.

The differences between FOO and FOA reflect differences in the implementation of the OBC for temperature. FOO is an implicit, leap-frog scheme which includes a wave speed correction to the advection velocity. FOA uses an upstream scheme and has no wave
Figure 13. Time zonal plots of sea surface elevation along the transect $y = 2$ km for the variable wind experiment. (top left) The result from the benchmark experiment; (other panels) the results obtained using the OBCs indicated on top of each panel. CI is 2 cm; dashed lines indicate negative values.
speed correction. To highlight the differences related to the numerical schemes, we set to zero the wave speed of FOO and conduct an experiment using variable wind forcing. The results showed a marked improvement over those using FOA, indicating that for our particular dynamical scenario the Orlanski implicit scheme performs better than the upstream method. From the results of this experiment we concluded that the poor performance of FOO shown in Figures 14 and 17 relates to inconsistencies between the imposed wave speed and the periodic changes in the advection velocity at the open boundary. There are times during the experiment when the flow velocity is zero at the open boundary and the modeled temperature equation is transformed into a pure radiation condition, a scheme which has a reported tendency to numerical instabilities [Stevens, 1990; Barnier et al., 1998]. The spurious reflections generated numerically at the eastern boundary travelled along the boundary and fed the northern wall Kelvin wave shown in Figure 17.

Figure 14. Time series of area averaged baroclinic kinetic energy for the variable wind experiment. The dashed line is the result from the benchmark experiment (CYC); solid line is result obtained using the OBC scheme indicated on top of each panel.

Figure 15. Cross section of alongshore velocity for the variable wind experiment. (left) The result from the benchmark experiment; (right) the result obtained using the FOA scheme. Flow is westward in shaded areas. The figure corresponds to the instant of maximum upwelling favorable wind at a cross section 100 km from the eastern boundary.
3.3. The Passage of a Storm

The last experiment of this study evaluates the performance of the OBC schemes in a dynamical scenario which combines flow advection and wave propagation. The simulation consists in the passage of a midlatitude cyclone over a shelf region [e.g., Gjevik, 1991]. In our previous study [Palma and Matano, 1998] we investigated the performance of barotropic OBCs to this particular forcing. Following Roed and Cooper [1987], the wind stress is generated by a cyclone of Gaussian shape and maximum amplitude of 3 Pa, translating at 8 m/s from the northwest to the southeast. The storm has a radius of 100 km and enters the computational domain at $x = 250$ km, $y = 500$ km, leaving 24 hours later at $x = 750$ km, $y = 0$. Since there is no known analytical solution to this problem, the benchmark experiment for this test is conducted in a closed basin with the same meridional extension, bottom topography, and stratification as the cases with OBCs but with a zonal extension of 10,000 km. The discussion will concentrate on the central 1000 km of the basin, and the time integration was stopped before reflections from the closed boundaries could reach the domain of interest.

Over the shelf area the traveling storm generates continental shelf waves that propagate with the coast to their right. Since the shelf is relatively shallow and well mixed, the oceanic response is mostly barotropic. Figure 18 illustrates the spatial and temporal characteristics of the model response in the benchmark experiment. Figure 18a shows a snapshot of the barotropic velocity field after 50 hours of model simulation. A periodic structure, with alongshore wavelength of approximately 550 km is clearly visible. Figure 18b shows the time evolution of surface current speed and direction for
Figure 18. (a) Snapshot of depth averaged velocity at day 4 for the traveling storm experiment (benchmark case). Maximum velocity vector is 0.15 m/s. (b) Time series of surface current speed and direction at a coastal node located at the center of the domain. (c) Dispersion diagram for the two lowest barotropic shelf wave modes corresponding to the hyperbolic tangent depth profile.

A station placed near the coast which is characterized by oscillations with a period of ~25 hours. Using the computer code of Brink and Chapman [1987], we evaluated the dispersion relation of the lowest barotropic shelf waves for the hyperbolic tangent bathymetry. Figure 18c shows the period of the waves as a function of wavelength. For a mode 1 wave 550 km long the estimated period is ~30 hours. This result compares very well with our experiment, the differences could be related to internal friction and stratification [Huthnance, 1978].

The oceanic response at the shelf slope, a region characterized by large depth and density gradients, is affected by the interaction between baroclinic and barotropic motions. The vertical current shear is depicted in Figure 19, plots of the horizontal velocity field at 10, 100, and 1000 m below the surface. Maximum velocities at 10 m depth are 1.0 m/s with a periodic structure of horizontal wavelength of ~500 km. Following Gill [1982], the estimated wavelength of the wake pattern expected for storms traveling faster than the first baroclinic wave mode is $\lambda = 2\pi U/f$, where $U$ is
the speed of the storm. For our model, $U = 8 \text{ m/s}$, and $f = 1.028 \times 10^{-4}$, which renders $\lambda = 500 \text{ km}$, in good agreement with the model result. At 100 m depth the maximum current speed reaches $0.35 \text{ m/s}$ and occurs in the slope region where the storm has passed. The response in this area is characterized by near inertial oscillating current vectors (Figure 20), intense horizontal gradients, and high current shear (e.g., Figure 19). Overall, the results obtained with the extended domain are consistent with previous models of storm driven events on coastal regions [Gjevik, 1991; Storløkke et al., 1994].
The experimental results can be summarized as follows: The simulations using MOA, FOA, and FOO are in close agreement with the benchmark case. The experiments using HOA and BBA show reflections and spurious sea level anomalies that are particularly accentuated at the western boundary. As an example of the performance of the different schemes, Figure 21 shows a snapshot of the SSH distribution at day 4, which corresponds to the adjustment period after the passage of a mode 1 shelf wave. There are no significant differences between the benchmark case and the simulations using MOA, FOA, and FOO (the latter two not shown). The characteristic method (HOA) shows a SSH distribution that resembles the pattern of the extended case but with an increased sea level. The most serious disagreements are found between the benchmark case and the experiments using BBA which not only shows a general increase of sea level, like HOA, but also a marked gradient of SSH over the western portion of the continental shelf. The analysis of the barotropic velocities (not shown) indicates that these SSH anomalies are associated to a westward jet that flows up to 150 km offshore. Although the HOA experiments show differences with the benchmark along the western boundary, the performance of this particular scheme for the barotropic mode was not as poor as that of BBA.

To illustrate the performance of the different schemes for the baroclinic fields, Figure 22 compares the horizontal distribution of the velocity fields at 100 m depth, while Figure 23 shows the depth structure of the alongshore velocity near the western boundary. MOA, FOA, and FOO reproduce correctly the velocity pattern of the benchmark case. HOA shows an artificial inflow at the western boundary between 200 and 300 km off-
shore. Similar discrepancies at the western boundary occur at other levels. Since the differences between HOA and the benchmark case are relatively small for the barotropic velocities, it is possible that the observed differences relate to the boundary conditions for the internal mode. The cause can be traced to the structure of the forced mode that remains after the passage of the storm. As shown in Figure 19, the upper level structure of this mode in the northwestern portion of the basin has a velocity component, which is tangential to the open boundary. Since the OBCs for the internal velocities consider only propagation perpendicular to the boundary, the OBC scheme creates artificial circulation patterns which are then propagated through the vertical by the barotropic OBC. A possible alternative to handle this situation could be the use of a radiation condition with allowance for oblique phase speed propagation [Raymond and Kuo, 1984]. Reed and Cooper [1987] reported a bad performance of this scheme for a similar barotropic experiment, but recently, Barnier et al. [1998] suggested some modifications which may improved the scheme performance.

The poor performance of BBA for the SSH fields is also present in the barotropic velocity fields and the baroclinic velocities (Figures 22 and 23). The largest differences occur in the coastal regions where BBA shows a partial clamping at the western boundary (Figure 22) and the development of a strong barotropic structure (Figure 23). The performance of BBA does not improve when the relaxation time is increased; in fact, it worsens. Experiments in which the relaxation time was increased to 10, 24 hours, and infinity show a more major clamping than for the shown example. The major problem faced by this particular scheme seems to be associated with the fact that the path of the storm impinges upon the open western boundary. Experiments in which the storm is displaced eastward show considerable improvement on the discussed case. This result is consistent with the barotropic experiments reported in our previous study [Palma and Matano, 1998].

4. Summary and Conclusions

In this article we analyzed the performance of a suite of OBCs for a 3-D, primitive equation, ocean model. The first experiment evaluates the oceanic adjustment to the onset of upwelling favorable winds. For this case, MOA had the best overall performance. Although the OBCs based on Fletter's condition performs reasonably well, they show divergences near the eastern (inflowing) boundary. The implementation of a corrected advection scheme for temperature using implicit leapfrog differencing (FOO) improves the overall behavior of this scheme. The remaining OBCs, HOA and BBA, perform poorly because of the generation of artificial alongshelf pressure gradients. These pressure gradients reduce the strength of the coastal jet and the amount of the water that is upwelled on the shelf area.

The second experiment considers the effects of a variable wind forcing which induces periodic reversals of the flow. In this series of experiments, MOA had the best performance as compared with the benchmark results. The experiments using HOA and BBA show a clamping of the open boundary that generates energetic oscillations of the SSH that deteriorate the interior solution. These effects are more marked during the reversals of the flow. Although the schemes based on Fletter's condition behave correctly in the barotropic mode, they
show major differences in the baroclinic structure with the benchmark case. FOA develops a spurious counter-current that is related to an alongshelf pressure gradient generated by reflections at the open boundaries. FOO showed an artificial increase of baroclinic energy after 15 days of simulation time. A closer look at the solution shows a Kelvin wave travelling at the northern (closed) boundary. This wave is presumably generated by inconsistencies between the imposed wave speed and the periodic change in advection velocity at the eastern boundary. Numerical experiments conducted with FOO imposing a zero phase speed gave results equivalent to MOA and superior to FOA, showing that for this particular dynamical scenario the ORI scheme is better suited than upstream differencing for the temperature OBC.

The neglect of alongshore variations in the first two experiments facilitates the comparison between the experiments using OBC and existing solutions. The difficulty of using experiments with alongshore variations of bottom topography is the definition of the reference case. There are no analytical solutions to this problem, and the setup of a benchmark is not trivial (the difficulty in setting up these types of experiments is the whole motivation for using OBCs). During the first stage of our numerical simulations we used a channel with alongshore variations of the bottom topography and cyclic conditions. The results were similar to those reported in this article, but since cyclic conditions are not technically correct for this benchmark experiment (since alongshore gradients differentiates the eastern from the western side of the basin), we choose a straight coastline. This particular case had the advantage of having analytical and numerical solutions to compare. Moreover, since the lack of alongshore variations precludes the existence of alongshore pressure gradients, the existence of these gradients in some of our experiments was a clear indicator of failures of the corresponding OBC’s schemes.

The third experiment investigates the combined effects of variable wind forcing and wave propagation on OBC performance. MOA, FOA, and FOO are able to radiate the outgoing energy without significant reflections. BBA and HOA are not able to handle the propagation dispersive wave packets which generate an increase in mean sea level. Integral constraints, which solved this problem in barotropic experiments [Palma and Matano, 1998], were useless in the present case.
The perturbations generated by the OBCs are not restricted to the barotropic component of the flow since a spurious baroclinic jet develops near the western boundary for the BBA scheme. This internal jet smooths out the current shear and produces an incorrect barotropic response. HOA also shows problems at the western boundary with an intense inflow between 150 and 250 km offshore that extends ~300 m depth.

The experiments and combination of OBCs schemes used in this study reflect a choice that, although guided by physical and practical considerations, is still highly arbitrary. Since the possible combinations of numerical schemes and test cases are endless, we choose those schemes and test cases that have either been used frequently in the literature or were relatively successful in our previous studies. During the execution of this study, however, we did several experiments using different OBCs combinations or experimental setups because of space constraints cannot be included in this article. The practical experience gained from these studies can be combined with the results of our previous discussion in the following general conclusions:

1. MOA is the only scheme that provides a reasonable response in all cases studied. Experiments using the FRS in all prognostic variables (not shown) also yield reasonable results although the best performance is attained using a radiation condition for the baroclinic tangential velocity and a FRS for temperature at inflow conditions. It is interesting to note that MOA has been used successfully in a simulation of the upwelling regime along the coast of Chile [Matano et al., 1998], where the model included the combined effects of alongshelf variations of bottom topography, density stratification, and wind forcing.

2. FOO could be a valuable alternative in cases where the forcing is not changing directions rapidly at the open boundary. Numerical experiments (not shown) changing the period of the wind forcing show an improvement in the response of the scheme as the period of the wind increases.

3. HOA, which provided reasonably good results in barotropic simulations [Rued and Cooper, 1987; Palma and Matano, 1998], fails when combined with radiation conditions for the internal baroclinic mode. This failure is more evident when the response has a tangential component to the open boundary (e.g., experiment 3). Numerical experiments (not shown) moving the storm track away from the open boundary improved the overall response of the scheme. In all experiments the OBC must be supplemented by integral constraints to ensure stability and volume conservation.

4. BBA performs poorly in all our test cases. The two major drawbacks of the scheme are the determination of the proper relaxation time and the correct internal wave phase speed. Although during our experiments variations of these variables did not improve the overall performance of the OBC, it should be noted that BBA had a good performance in real-time simulations that included tidal, wind, and buoyancy forcing [Kourafalou et al., 1996]. The inclusion of tidal forcing, however, changes the OBC from passive to active.

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