Relationship of Specific Gravity to Moduli of Rupture and Elasticity in Commercial Hardboard[1]

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Thirty-six types of commercial hardboard were tested in static bending. Test results were analyzed statistically and predicting equations were derived. Significant correlations existed for almost every type of hardboard tested, although no one equation could give reliable results for all types of hardboard when correcting for modulus of rupture or elasticity for specific gravity differences.

PRODUCERS AND USERS OF HARDBOARD have been handicapped by a scarcity of reliable information about relationships of mechanical properties to physical characteristics of hardboard. Considerable information on relationships between manufacturing variables and board strength undoubtedly has been developed within plant laboratories; few data, however, have been published.

Currently accepted practice is to use modulus of rupture as an index of hardboard properties, although some other physical property might be satisfactory for this purpose. One perplexing problem that faces the hardboard producer or researcher is the difficulty in compensating for differences in specific gravity when comparing strength properties of hardboards with differing values for modulus of rupture.

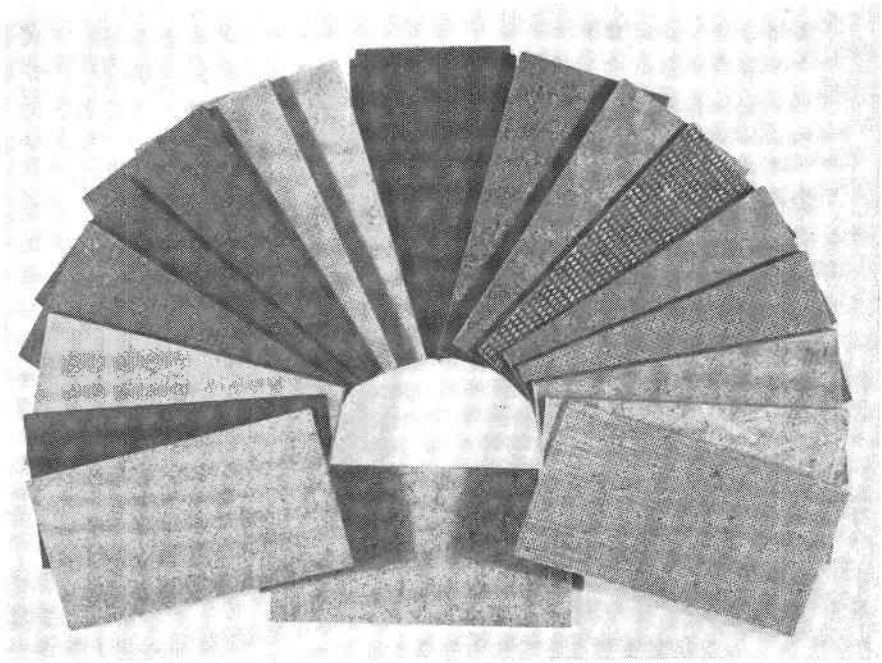
While Wilcox (3, 4, 5)[1], and Stillinger and Coggan (1) reported findings of the effect of various process variables and moisture relationships on flexural properties of laboratory and commercial hardboard, little or no information was obtained relating to the relationship of specific gravity to flexural properties of hardboard.

The U. S. Forest Products Laboratory, at Madison, Wis., undertook a comprehensive study of variables encountered in the manufacture of fiberboard.[2] Part of the study dealt with the relationship between specific gravity and modulus of rupture. The following conclusion was reached. "For practical purposes, the modulus of rupture may be assumed to be directly related to the square of the specific gravity" (2). The result was based on laboratory-size specimens manufactured from wood-fiber pulp with no additives. The range in specific gravity was from 0.15 to 1.2.

In the course of testing hardboard at the Oregon Forest Products Research Center, the practice of adjusting specific gravity to a common base came into question. The relationship of modulus of rupture to specific gravity in commercial hardboards did not appear to be the same for all kinds of boards; furthermore, relationships varied considerably among boards of one class or thickness, and among similar boards from one producer. Preliminary work suggested reliable corrections for specific gravity could be made only when appropriate factors were available for each board type from which test specimens were selected. Lack of such factors, and lack of previous work over the broad scope of current commercial hardboard, prompted the present investigation.

Definitions: Throughout this report, certain descriptive words have been used that may cause some confusion. For the purposes of the report, they are:

- Class: There were two classes of hardboard; untreated and treated.
- Board type: Hardboard of one class, one thickness, and from one producer.
- Board or sheet: A 4- by 4-foot or 4- by 8-foot piece of hardboard.
- Test specimen: A comparatively small piece of hardboard cut from a board or sheet.
- Machine direction: Orientation of the board with respect to its line of flow in the pressing operation during manufacture.



Some of the 36 hardboard types studied at the Oregon Forest Research Center to determine the relationship between strength properties and specific gravity.

[1] Submitted for publication April, 1957.

[1] Numbers refer to the Literature Cited at the end of this paper.

[2] Fiberboard is a term applicable to a sheet material manufactured of refined or partly refined vegetable fiber. These board products are manufactured in densities from 2 to about 90 pounds per cubic foot. Hardboard has a density range of 50 to 70 pounds per cubic foot. Density values must be considered as approximate, since properties vary with base materials, binders, impregnants, and methods of manufacture.

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Objective: The objective of the investigation was to determine relationships between specific gravity and flexural properties of commercial hardboard; specifically, modulus of rupture, modulus of elasticity, maximum work, and work to 0.2-inch deflection. It was assumed the ultimate goal would be to devise an acceptable method to correct for differences in specific gravity among various types of commercial hardboard when comparing strength properties. For this reason, the study included a range of three thicknesses, two classes, and 11 producers of commercial hardboard in the United States and Canada.

Scope: Thirty-six different types of commercial hardboard were included in the study. Eleven producers (or sources of raw material), five basically different processes, three thicknesses, and two classes (treated and untreated) were represented in the study.

Results of static bending tests were analyzed statistically to determine relationships of specific gravity to modulus of rupture, modulus of elasticity, maximum work, and work to 0.2-inch deflection. Results of the statistical analysis for maximum work and work to 0.2-inch deflection were inconclusive and therefore have not been included in this report.

Table 1.—RELATIONSHIPS OF SPECIFIC GRAVITY TO MODULI OF RUPTURE AND ELASTICITY IN COMMERCIAL HARDBOARDS

Board	Specific gravity range		Predicting equation[1]	
	Low	High	Mod. of rupture	Mod. of elasticity
Untreated, ⅛-inch				
1	0.919	1.040	10,502 x — 5,475	1,260 x — 632
2	0.930	1.069	6,247 x — 862	727 x — 183
3	0.817	1.044	11,889 x — 5,801	806 x — 232
4	0.838	1.022	9,106 x — 3,357	921 x — 272
5	0.872	1.058	9,335 x — 3,259	491 x — 208
6	1.000	1.111	14,056 x — 7,638	1,634 x — 705
7	1.074	1.242	14,833 x — 12,315	1,343 x — 836
8	1.007	1.165	9,213 x — 5,563	680 x — 118
9	0.991	1.136	7,264 x — 3,298	307 x + 112
10	0.870	1.045	12,104 x — 6,126	1,069 x — 471
Untreated, 3/16-inch				
11	1.003	1.069	22,451 x — 16,376	1,634 x — 705
Untreated, ¼-inch				
12	0.904	1.044	15,726 x — 9,778	998 x — 372
13	0.955	1.080	9,249 x — 4,266	1,031 x — 542
14	0.865	1.064	13,211 x — 6,705	803 x — 181
15	0.840	1.031	12,902 x — 6,024	786 x — 212
16	0.891	1.090	16,995 x — 10,095	1,013 x — 379
17	0.748	1.069	13,190 x — 6,146	1,281 x — 483
18	0.742	1.002	11,576 x — 5,279	1,373 x — 549
19	0.823	1.072	14,378 x — 8,426	2,044 x — 1,262
20	0.981	1.188	14,651 x — 10,148	1,406 x — 925
21	1.083	1.159	8,379 x — 3,990	232 x + 286
22	0.847	1.006	10,355 x — 5,566	1,036 x — 458
Treated, ⅛-inch				
23	0.983	1.112	20,328 x — 12,159	1,476 x — 645
24	0.816	1.077	15,586 x — 7,431	1,293 x — 509
25	0.913	1.074	16,050 x — 7,317	404 x + 361
26	0.997	1.069	16,679 x — 9,053	1,594 x — 517
27	0.953	1.139	12,240 x — 4,491	1,006 x — 263
Treated, 3/16-inch				
28	1.060	1.122	3,037 x + 5,072	571 x + 508
Treated, ¼-inch				
29	1.029	1.118	21,574 x — 14,115	1,047 x — 221
30	0.893	1,049	13,701 x — 6,065	959 x — 302
31	0.889	1.083	10,931 x — 2,189	913 x — 162
32	0.921	1.038	13,571 x — 5,780	1,005 x — 257
Utility, ¼-inch				
33	0.742	0.969	9,223 x — 3,774	683 x — 188
34	0.782	0.855	15,518 x — 8,423	1,354 x — 522
35	0.757	0.925	12,182 x — 5,785	969 x — 337
36	0.796	0.916	11,080 x — 5,667	801 x — 247

[1] Where x is specific gravity, and modulus of rupture or elasticity is equal to the entire expression for each hardboard.

Procedure

Each of the 36 types of hardboard was represented by five 4- by 4-foot sheets selected at random by the producers' personnel. The sheets were cut into 12-inch squares and numbered in a uniform pattern from 1 to 16. By means of a table of random numbers, 12 of the 16 squares were selected for experimental purposes. The 3- by 12-inch test specimens were cut with their longitudinal axes parallel to the machine direction from six of the squares, and perpendicular to the machine direction in the other six squares. Thus, for a given board type, there were 12 specimens for each of the five 4- by 4-foot sheets, or a total of 60. Since there were 36 types of hardboard, the total number of specimens was 2,160.

The test specimens were stored in a heated building with no temperature or humidity control for three months prior to testing to simulate conditions practical for manufacturers who want to test their own products. Examination of coupons cut from the test specimens at time of test indicated virtually all specimens had reached a state of moisture equilibrium. All test specimens were measured to the nearest 0.01 inch in width with a modified gage comparator of the type described in ASTM Standard D 1037–52T, and to the nearest 0.001 inch in thickness with an Ames dial mounted over a plane table.

Loads were applied and measured with a Baldwin Tate-Emery air-cell weighing system attached to the movable crosshead of an electro-mechanical testing machine. Loads were applied in the centers of specimens over a span of 8.1 inches and were read to the nearest 0.5 pound. A head speed of 0.5 inch per minute was used, and load-deflection curves were drawn for each test specimen.

After each test specimen was broken in bending, a 1- by 3-inch coupon was cut near the area of failure and weighed immediately. After the coupons were weighed, they were placed in an oven and dried to constant weight at 212° ± 3° F. Initial and final weights of the coupons and their volumes, determined by the mercury-immersion method, furnished data for calculation of moisture content at time of test and specific gravity based on oven-dry weight and oven-dry volume.

The relationships between specific gravity and modulus of rupture, modulus of elasticity, work to 0.2-inch deflection, and work to maximum load were determined statistically by regression and correlation studies.

Many comparisons could be made from the collected data. The method adopted in the present study for comparison purposes was to determine the respective correlation and regression coefficients of specific gravity on modulus of rupture or modulus of elasticity, and then statistically compare these regressions and correlations for significant differences. If no significant differences existed, then a common regression or correlation value could be used; conversely, significant differences meant that the flexural properties of a board type needed to be determined by their respective regression or correlation coefficients when accurate predictions were desired. The resulting physical properties then were compared visually or statistically.

Persons interested in a more complete explanation of statistical methods followed than space allows here may obtain a supplement to this report from the Forest Products Research Center on request.

Results and Discussion

Relationships of specific gravity to maximum work, or to work to 0.2-inch deflection, were such that reliable equations could not be developed for many hardboards tested. The statistical analysis showed relationships

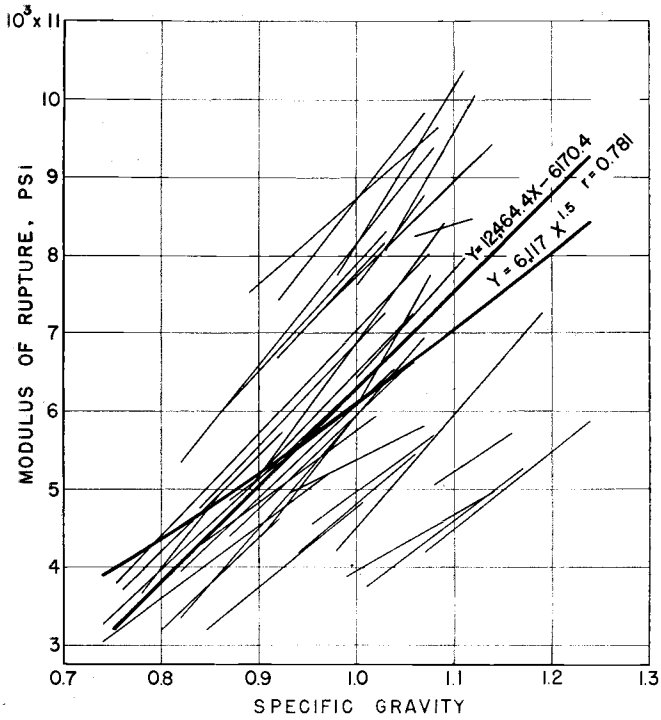

Fig. 1.—Relationship of specific gravity and modulus of rupture for 36 hardboard types as determined by analyses of regression.

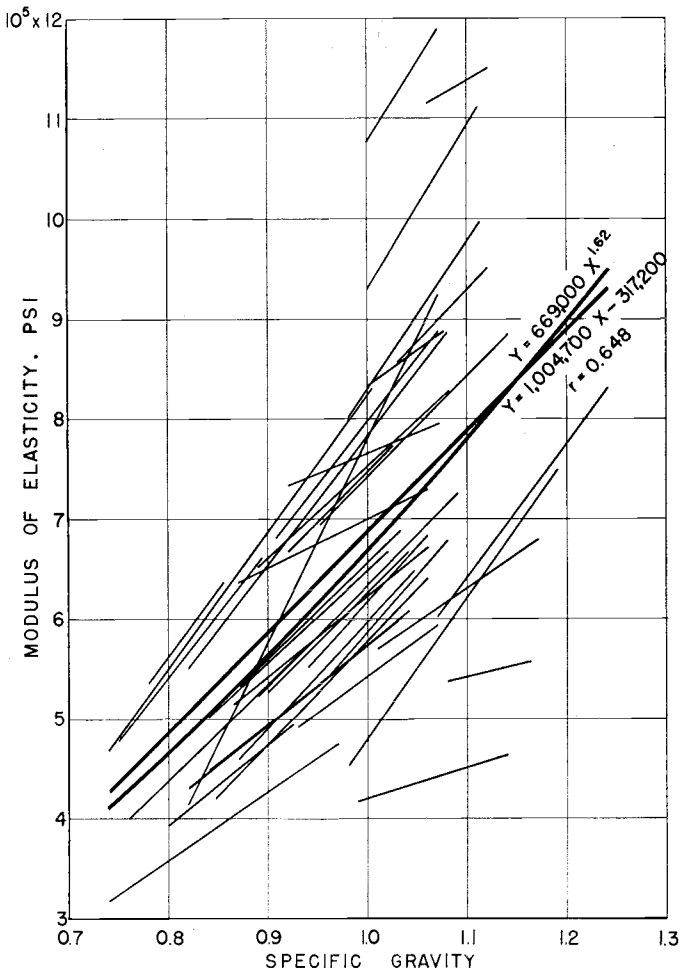

Fig. 2.—Relationship of specific gravity and modulus of elasticity for 36 hardboard types as determined by analyses of regression.

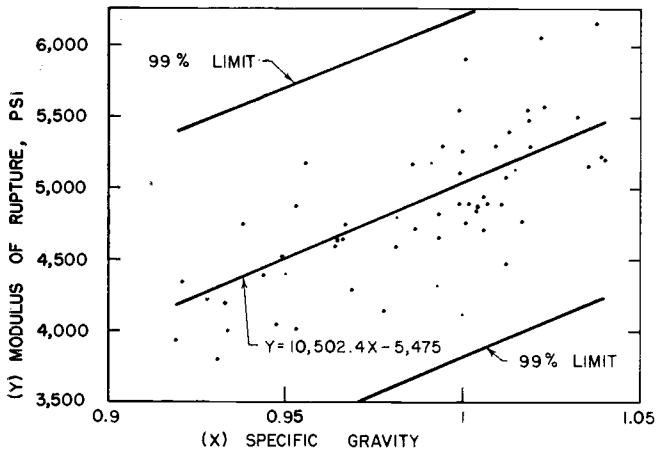

Fig. 3—Scatter diagram of modulus of rupture for 60 specimens of hardboard type 1, with formula and confidence limits.

of specific gravity to moduli of rupture and elasticity, however, and allowed the derivation of fairly reliable equations.

Derived equations, and ranges in specific gravity of hardboard for which the equations were calculated, are listed in Table 1. Curves for each equation for moduli of rupture and elasticity are shown in Figs. 1 and 2 for each of the 36 hardboards tested. General equations for all 36 types were calculated by two variations of statistical methods. Of the general equations, the two that produced straight lines when plotted in Figs. 1 and 2 appeared to fit the data more closely than did the two plotted curves.

The desirability of developing individual equations for different board types is apparent from inspection of the wide spread in equations shown in Table 1. A quick indication of relationships for a particular board can be had by plotting points for specimens of several specific gravities and fitting a curve to the points.

When such information is lacking for a given hardboard, a general formula such as shown in Figs. 1 and 2 will provide a means of partially correcting for differences in specific gravity.

The curve plotted in Fig. 3 from the formula developed for one board studied (board 1 in Table 1) can provide an example of a method to follow for other boards. Note in Fig. 3 that, at 0.95 specific gravity, the formula curve shows an average modulus of rupture of 4,500 pounds per square inch. A specimen of similar hardboard, but with specific gravity of 1.02 could then be expected to have a modulus of rupture somewhere in the neighborhood of 5,250 pounds per square inch. The scatter of observations indicates any single specimen, however, may vary considerably from the average.

The interpretation of results was based solely on data obtained over the measured range in specific gravity for each board type included in the study, and should not be construed to hold for the entire specific gravity range of 0.75—1.25 for hardboard. This limitation does not invalidate comparisons here, but indicates that judgment must be used in evaluating differences in specific gravity and properties of hardboards other than those included in the study.

Because of roughly parallel alignment of fibers during board formation, some board types exhibit directional properties. These directional properties sometimes have a significant effect on modulus of rupture and modulus of elasticity, and whenever this effect occurs, accuracy of predictions can be increased by deriving an equation for each machine direction. Significant directional properties were disregarded in this study, however.

Conclusions

It is recognized that hardboards not included in the study may or may not have similar specific gravity—strength relationships, but the following conclusions are drawn from results of the investigation applied to most hardboard made in the United States in 1953:

1) Significant correlations were found to exist between specific gravity and modulus of rupture and between specific gravity and modulus of elasticity in nearly all board types studied.
2) No one single equation gave accurate corrections for moduli of rupture or elasticity when compensating for differences in specific gravity for all types of commercial hardboards studied.

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RELATIONSHIP OF SPECIFIC GRAVITY TO MODULI OF RUPTURE AND ELASTICITY IN COMMERCIAL HARDBOARD*

STATISTICAL SUPPLEMENT

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PREFACE

Some details of statistical treatment not included in the report printed in Forest Products Journal, June 1958, are presented here for those interested. For background, test procedures, and analysis of results refer to the Journal article.

CONTENTS

RELATIONSHIP OF SPECIFIC GRAVITY TO MODULI OF RUPTURE AND ELASTICITY IN COMMERCIAL HARDBOARD

STATISTICAL SUPPLEMENT

A. D. Hofstrand

INTRODUCTION

Statistics is a scientific tool to aid a research worker in organizing and digesting large amounts of data. While the mathematical background of statistics is complicated and can be best understood only after extensive study of mathematical theory, the research worker needs only a working knowledge of statistical processes to estimate the reliability of data he purposes to evaluate.

Linear regression and correlation is but one of many statistical tools available to the experimenter. Procedures in analysis of data, while perhaps tedious, are relatively simple and do not require extensive knowledge of statistical mathematics. Purpose of this supplement is to acquaint the research worker with typical procedures used to undertake regression and correlation analyses. Basis for the discussion is a study of 36 types of commercial hardboard. Boards were tested at the Forest Products Research Center and results were reported in the Forest Products Journal, Vol. 8, No. 6, June 1958.

STATISTICAL METHODS

Regression

A linear relationship between an independent variable, X in this presentation, and its dependent variable Y, can be shown by locating a straight line through plotted data points. It must be determined, after

examining the data, that no definite curvilinear trends are apparent. If such a trend is not present, the curve best fitting data will be a straight line, with the general equation $Y = a + bX$. This equation yields the simplest and probaby the most nearly correct relationship of X to Y in many groups of research data.

This best-fitting line is called the line of regression of Y on X. Its position and inclination are determined from original data by the method of least squares. The resulting regression is a line associated with all points (X-Y coordinates) in such a way that the sum of the squares of the Y deviations of points about the line is a minimum. The regression line passes through the intersections of the means $\bar{x}$ and $\bar{y}$. Its equation is:

$$\bar{Y} = a + bX$$

where "a" is the $\bar{Y}$ intercept and "b", the regression coefficient, is the slope or rise in $\bar{Y}$ per unit change in the independent variable X.

Correlation

In contrast to the regression coefficient, which measures rate of change of the means of Y with respect to X (geometrically, the slope of the line), the correlation coefficient "r" measures the "degree", or closeness, of the linear relationship. Briefly, the correlation coefficient is a measure of the variability (dispersion) of data about the regression line. The correlation coefficient ranges from -1 to +1. Zero coefficient indicates a horizontal line, and a coefficient near -1 or +1 indicates a close linear relationship. The plus sign means that Y increases with increasing X, whereas the minus sign indicates that Y decreases with increasing X. When the regression coefficient "b" equals zero, the correlation coefficient "r" also equals zero, and the regression is a horizontal line parallel to the X axis. In this instance, any value of X will predict the general mean of Y and no correlation exists. Should "b" equal other than zero, the best estimate of Y will depend upon X, and a correlation exists.

Significance of regression and correlation coefficients

Although it is possible to establish a regression in data for which no true relationship exists, there are methods to test the validity of any regression. The F test, used in this study, is one such measure. In brief, this test determines whether the regression coefficient "b" is significantly different from zero. When the probability of being wrong was less than one per cent, the relationship was termed significant at the 99 per cent level; when between one and five per cent, significant at the 95 per cent level; and when greater than five per cent, non-

significant. Since the slope of a regression line cannot be greater than zero until the degree is, also, a significant correlation coefficient will result in a significant regression coefficient. The converse also holds true; a significant regression results in a significant correlation.

Existence of a significant correlation does not, however, mean that such correlation can serve profitably for prediction purposes. Generally, when working with wood and wood-base materials, a correlation coefficient between $\pm$ 0.8 is deemed unreliable for prediction purposes on individual tests. This statement should be amended somewhat, because the aim of the research worker can determine whether or not a 0.80 correlation level should be used. For example, let $r = 0.80$ and be significant at the five per cent level. Therefore, upon assuming the hypothesis $r = 0$, the probability is 0.05 or less that the results are caused by sampling errors, rather than a real influence of X on Y. The square of the correlation coefficient, r^2, gives the amount of the variation in Y accounted for by the variation in X. Again, with $r = 0.80$, $r^2 = 0.64$, which, multiplied by 100, gives a "coefficient of determination." For the example, 64 per cent of the variability in the dependent variable is accounted for by variability of the independent variable, and 36 per cent is unexplained. It can be seen that as the correlation coefficient becomes smaller than $\pm$ 0.80, r^2 rapidly diminishes, and the unexplained portion of variation in the dependent variable increases. Therefore, the prognostic value of test result is diminished when correlations between $\pm$ 0.80 are obtained.

Confidence limits

To use results established by a study of correlation, one must decide upon the confidence to be placed in the best computed value of Y. Here, two types of confidence limits are presented. In one type, regression of the sample is no more than an estimate of true regression of the entire population. Among samples, estimates of both the vertical positions, "a", and the slope, "b", in the equation $Y = a + bX$, vary about the two true, but unknown, values for the population. A confidence interval with any percentage confidence coefficient can be established about the regression line for a sample. Ninety-five and 99 per cent confidence limits are those generally accepted. For the 99 per cent interval, there is a 99 per cent probability that the interval includes the true average of Y, and for the 95 per cent interval, there is a 95 per cent probability that the interval includes the true average of Y.

In the second type, confidence limits were established for predicting probable behavior of individual specimens as contrasted to random samples. Again, the custom is to choose 95 and 99 per cent limits. Because of the rotational nature of uncertainty, these limits are preferable to limits based upon the mean

and a chosen number of standard deviations as a reduction factor, as most generally is done.*

METHODS OF CALCULATION

The brief introduction to regression and correlation was to acquaint the research worker with general background. Detailed mathematical manipulations, while not difficult, are somewhat tedious and any short cuts for arriving successfully at a final answer are welcomed. With this idea in mind, much of the following discussions are of procedures followed by personnel at the Forest Products Research Center to evaluate physical properties of 36 different hardboard types as reported in the June, 1958, issue of Forest Products Journal.

Analysis of data is best done with a calculating machine, although the computations could be completed by longhand. Use of a machine eliminates much labor, since it is often possible to conduct more than one statistical operation at a time. For instance, it is possible with some calculators to obtain the squares and the product of two variables in one operation.

One of the important functions of regression analysis is to derive regression coefficients and, indirectly therefrom, useful predicting equations. Predicting equations provide a way to establish one variable from a second variable. In the study of various hardboard types, specific gravity was one variable, X, and either modulus of rupture or modulus of elasticity was the second variable, Y. Having an equation including these two variables, and knowing one, it is possible to predict what the value of the second variable should be. The predicted value will be an average, but it is possible also that the true value is not the average, but ranges above or below the predicted average.

To aid the research worker in following computations illustrated here, a set of data forms for the hardboard study is included. These forms have been filled out with values computed from data given in Tables 1 and 2. Data are actual test values of specific gravity and modulus of rupture for 5 boards each of 2 hardboard

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* An explanation of this statistical phenomenon is beyond the scope of this report. The reader will be able to find an explanation in most statistic books.

Table 1. Summary of Data for 60 Test Specimens for Illustrated Problem. Board Type W.

Specimen	BOARD									
	A		B		C		D		E	
	x*	y**	x	y	x	y	x	y	x	y
		Psi		Psi		Psi		Psi		Psi
Parallel to machine direction										
1	0.930	5,190	0.902	4,240	0.870	3,920	0.914	4,620	0.935	4,650
2	1.021	5,740	0.958	5,250	0.963	4,940	0.968	5,590	0.991	5,680
3	1.026	6,330	0.989	5,490	0.981	5,410	0.977	5,650	1.026	6,130
4	1.016	6,220	1.026	6,010	0.987	5,570	1.040	6,110	1.020	6,110
5	1.029	6,010	0.981	5,330	0.978	5,840	0.972	5,510	1.014	5,710
6	0.982	5,600	0.973	5,570	0.960	5,490	0.939	5,510	0.967	5,510
Sub-total	6.004	35,090	5.829	31,890	5.739	31,170	5.810	32,990	5.953	33,790
Perpendicular to machine direction										
1	0.974	5,790	0.959	5,360	0.941	5,040	0.942	5,330	0.970	6,160
2	1.020	6,890	1.004	6,540	1.004	6,010	1.032	6,760	1.014	6,460
3	1.027	6,560	1.012	6,180	1.033	6,140	0.986	5,970	1.045	7,120
4	1.020	6,910	1.018	6,140	1.018	5,950	1.014	6,290	1.037	6,660
5	0.943	5,510	0.928	5,080	0.913	5,220	0.965	5,460	0.941	5,510
6	0.963	5,950	0.895	5,000	0.922	5,250	0.917	5,200	0.947	6,060
Sub-total	5.947	37,610	5.816	34,300	5.831	33,610	5.856	35,010	5.954	37,970
ΣX, ΣY	11.951	72,700	11.645	66,190	11.570	64,780	11.666	68,000	11.907	71,760

*x = Specific gravity; volume at test, oven-dry weight.
**y = Modulus of rupture, pounds per square inch.

Table 2. Summary of Data for 60 Test Specimens for Illustrated Problem. Board Type Z.

Specimen	BOARD									
	A		B		C		D		E	
	x*	y**	x	y	x	y	x	y	x	y
		Psi		Psi		Psi		Psi		Psi
Parallel to machine direction										
1	0.942	5,070	0.938	5,310	0.905	5,360	0.961	5,770	0.921	5,020
2	0.867	4,260	0.887	4,710	0.913	5,460	1.006	6,280	0.875	4,470
3	0.978	5,420	0.994	5,940	0.857	4,230	0.939	5,460	0.978	5,810
4	1.014	6,040	1.013	6,040	0.897	4,550	1.033	6,740	1.040	6,150
5	0.989	6,370	0.963	5,990	0.944	5,500	0.960	5,300	0.980	6,080
6	0.919	4,860	0.950	5,280	0.817	4,430	0.835	4,190	0.976	5,460
Subtotal	5.709	32,020	5.745	33,270	5.333	29,530	5.734	33,740	5.770	32,990
Perpendicular to machine direction										
1	0.906	4,890	0.965	5,810	0.879	4,380	1.009	6,420	0.921	5,190
2	0.919	5,260	0.935	5,580	0.903	5,190	0.951	5,490	0.891	4,490
3	0.934	4,710	0.948	5,540	0.931	5,510	0.979	6,250	0.912	4,960
4	1.017	6,070	1.044	7,230	0.927	5,420	1.026	6,690	1.025	5,730
5	0.950	5,140	0.978	5,710	0.906	6,040	0.968	6,090	0.963	5,280
6	0.932	4,710	0.905	4,890	0.903	5,000	0.917	5,610	0.944	5,020
Subtotal	5.658	30,780	5.775	34,760	5.449	31,540	5.850	36,550	5.656	30,670
ΣX, ΣY	11.367	62,800	11.520	68,030	10.782	61,070	11.584	70,290	11.426	63,660

* x = Specific gravity; volume at test, weight oven-dry.

** y = Modulus of rupture, pounds per square inch.

types (W and Z). There are 12 test values for each hardboard panel, 6 each parallel to and perpendicular to machine direction of the board. Thus, there are 60 test values for each hardboard type.

Calculation of regression coefficients

Calculation of regression coefficients requires computation of sums of squares for each variable being studied, and sums of the products for each pair of variables (XY). In this instance, the variables are specific gravity, X, and either modulus of rupture or modulus of elasticity as (Y). For ease of understanding and following calculations, specific gravity and modulus of rupture are included in the illustrated problem.

From data given in Table 1, sums of squares for each variable can be calculated from the following equation:

$$(1) \quad ss_x = \Sigma(X)^2 - \frac{(\Sigma X_\perp)^2}{N} + \frac{(\Sigma X_\parallel)^2}{N}$$

where ss_x = sums of squares of variable X.

ΣX^2 = sum of each X variable squared

$\Sigma X_\perp$ = sum of X variables, perpendicular direction

$\Sigma X_\parallel$ = sum of X variable, parallel direction

N = observations in each direction

These values from data given in Table 1 and the solution to equation (1) are presented in Form A.

For example, to determine the sum of squares for the specific gravity variable X for board A, the values for board A from Form A are substituted into equation (1). Thus, the specific gravity sums of squares for board A would be:

$$ss_x = 11.916261 - \frac{(6.004)^2 + (5.947)^2}{6}$$

$$ss_x = 11.916261 - \frac{(36.048016 + 35.366809)}{6}$$

$$ss_x = 0.013790$$

In like manner, sums of squares for the other four boards can be calculated. Sums of squares for modulus of rupture are found also in this way, substituting appropriate values of Y in equation 1. Once the sums of squares are calculated, they are tabulated as in Form B. Sums of squares for specific gravity are in column 3, while columns 4 and 5 list sums of products of variables XY and sums of squares for modulus of rupture. Notice that the total sum of squares for specific gravity in Form A and Form B, column 3, are identical. In other words, column 5, Form A is the same as column 3, Form B.

Sum of products XY for a board is found by adding the products of XY for each test specimen, then subtracting the product of the sum of all 12 values multiplied by the sum of all 12 Y values and divided by the number of observations, 12.

After the sums of squares for specific gravity X, sum of the products XY, and modulus of rupture Y are tabulated in Form B, the calculation of regression coefficients is fairly simple.

To calculate the regression coefficient for board A, refer to the values for board A in Form B and divide that in column 4 by that in column 3. In other words, the regression coefficient for board A is equal to 172.02/0.013790, or 12,474.257. Regression coefficients are determined likewise for the other 4 boards. To find the regression coefficient of the hardboard <u>type</u>, divide the sum of column 4 by the sum of column 3, or

$$\bar{b} = \frac{1130.98}{0.093436} = 12,104.328$$

FORM A

VARIABLE: Specific gravity and _____

BOARD	$\sum^{12} X^2$	$\sum^{2} T^{2*}$	$\frac{\sum T^2}{6}$	$\sum X^2 - \frac{\sum T^2}{6}$ (s.s. x)
	1	2	3	4
A	11.916261	71.414825	11.902471	0.013790
B	11.321629	67.803097	11.300516	.021113
C	11.179446	66.936682	11.156114	.023332
D	11.359948	68.048836	11.341473	.018475
E	11.831447	70.888325	11.814721	.016726
SUM	57.608731	345.091765	57.515295	0.093436

DIRECTION	$\sum^{30} X^2$	$\sum^{5} T^2$	$\frac{\sum T^2}{6}$	$\sum X^2 - \frac{\sum T^2}{6}$
PARALLEL	28.733297	172.155687	28.692614	0.040683
PERPENDICULAR	28.875434	172.936078	28.822680	.052754
SUM	57.608731	345.091765	57.515294	0.093437

BOARD TOTALS (BT)	
A	11.951
B	11.645
C	11.570
D	11.666
E	11.907
Grand total (GT)	58.739

$\sum (BT)^2 = 690.169531$ $\sum (BT)^2/12 = 57.514128$

$(GT)^2 = 3,450.270121$ $GT^2/60 = 57.504502$

BOARD ss = 0.009626

$^{*}\sum T^2 = \left[(\sum X_{\perp})^2 + (\sum X_{||})^2 \right]$

FORM B

Test of Homogeneity of Regression Coefficients Among Boards.

X : Specific gravity Y : Modulus of rupture

Board	Deg. free-dom	ss_x	Sum of prod. XY	ss_y	(4 ÷ 3) Regression coefficient b	(4 x 6) Regression s.s.
1	2	3	4	5	6	7
A	10	0.013790	172.02	2,619,600	12,474.257	2,145,821.69
B	10	.021113	269.24	3,056,700	12,752.333	3,433,438.14
C	10	.023332	259.68	3,543,200	11,129.779	2,890,181.01
D	10	.018475	221.74	3,057,000	12,002.165	2,661,360.07
E	10	.016726	208.30	2,988,200	12,453.665	2,594,098.42
Sum	50	0.093436	1,130.98	16,136,700	- - - - *	13,724,899.33

* Regression coefficient of hardboard type, or weighted average of b's, $\bar{b}$ = 12,104.328 (sum of column 4 divided by sum of column 3).

Analysis of Variance of the Regression Value.

Sources of Variation	d.f.	Sum of squares	Mean squares	F
Regression due to $\bar{b}$	1	13,689,752.99	13,689,752.99	255.43**
Among b's	4	35,146.45	8,786.61	0.16
Residual	45	2,411,800.67	53,595.57	---
Sum (same as above)	50	16,136,700.00	-------	---

** Significantly different from zero at 99 % confidence level.

Homogeneity of regression coefficient

When the regression coefficient for the hardboard type has been calculated, the next step is to test for homogeneity of the regression coefficient. To do so, refer to the lower half of Form B. This section of the form is the analysis of variance of the regression value.

Again, the calculation of values for this section is relatively simple. But first, an explanation of terms used in this section is in order. Under the column headed "sources of variation", "regression due to $\bar{b}$" is the explained amount of the source of variation because of regression; regression "among b's" is the explained variation between the calculated regression coefficients, while "residual" is unexplained variation due to variables other than specific gravity.

"Degrees of freedom (d.f.) are the number of observations which are free to vary after certain restrictions are imposed. In testing the reliability of a statistic, the degrees of freedom are one less than the number of observations. Since the calculated statistic is a fixed value, all the observations cannot fluctuate freely and independently of one another. All observations but one may have any value regardless of size of their average. However, after all the observations but one are determined, the last one is automatically fixed. It is fixed because it must be such a number that all the numbers will average the statistic."* Thus, the degrees of freedom for regression due to $\bar{b}$ is 1, since only two variables X and Y are used to establish the regression. If one variable is fixed and cannot change, there is only one variable that can move about freely, hence only 1 degree of freedom. In like manner, degrees of freedom are computed for regression among "b's". Degrees of freedom for residual values amount to the sum of all degrees of freedom, less those already allocated.

Sums of squares for analysis of variance are computed from the totals of columns 3, 4, 5 and 7 of Form B. To compute sums of squares for regression due to $\bar{b}$, take the sum of column 4, square this value and divide by the sum of column 3. The sum of squares for regression among "b's" can be found by subtracting the sum of squares for regression due to $\bar{b}$ from the sum of regression sums of squares (column 7 minus sums of squares regression due to $\bar{b}$). Residual sums of squares can be found by subtracting the sum of column 7 from the sum of column 5.

---

* Pearson, Frank A. and Bennett, Kenneth R., Statistical Methods, John Wiley and Sons, Inc., New York, New York. November 1947.

Mean squares are calculated by dividing the different sums of squares by their respective degrees of freedom. Each mean square is in turn divided by the mean square of the residual. The result is an "F" value. Checking this calculated F value against F tables found in most statistic books determines whether or not the regression coefficient is significant. Should the regression among "b's" be not significant, the regression coefficients for individual boards (A, B, C, D, or E) can be combined, and a single regression coefficient can be calculated and used to establish the predicting equation.

Calculation of predicting equations

When predicting equations of the form $Y = a + bX$ are derived for small samples, the method of "least squares" generally is used. The values of "a" and "b" are calculated by solving two simultaneous equations. Their solution requires previous calculation of the following quantities:

ΣY, ΣX, ΣX^2, and ΣXY. The normal equations are:

(2) $Na + b\Sigma X - \Sigma Y = 0$, or

(3) $a\Sigma X + b\Sigma X^2 - \Sigma XY = 0$, where "N" is number of observations.

Substituting the proper values in equations (2) and (3) and solving the equations simultaneously, constants "a" or "b" are calculated where "b" is the regression coefficient and "a" is the Y intercept. Substituting these constants into $Y = a + bX$, a predicting equation is formed.

In the present study, however, the regression coefficient "b" has been calculated and tabulated in column 6 of Form B. Hence, the only value needing to be derived is the constant "a". By substituting values into equation (2), constant "a" is determined. From these two constants, the predicting equation can be determined. For example, calculate the predicting equation for board A, Table 1. Using the regression coefficient for board A from Form B and the needed quantities ΣX, ΣY, ΣX^2, or ΣXY and substituting these values in equation (2) (or equation 3), we have:

(2) $Na + b\Sigma X - \Sigma Y = 0$

$12(a) + (12,474.257)(11.951) - 72,700 = 0$

$a = -6,364.987$

Now the predicting equation for board A is:

(4) $Y = bX - a$

$Y = 12,474.257b - 6,364.987$

Knowing the specific gravity of a specimen from the same board, its average modulus of rupture can be calculated from equation (4).

Calculation of correlation coefficient

The correlation coefficient "r" is a measure of the variability of individual test values around the average regression line or predicting equation. A high correlation coefficient indicates individual values are dispersed closely along the regression line, but a low correlation coefficient indicates individual values are widely scattered in relation to the regression line.

Calculation of correlation coefficients is fairly simple once the values of sums of products XY, ss_x and ss_y are known. These values are tabulated in Form B. The correlation coefficient for board A, Table 1 can be found by substituting values, tabulated in Form B, into the following equation:

(5) $$r = \frac{\text{sums of products } XY}{\sqrt{(ss_x)(ss_y)}}$$

thus the correlation coefficient for board A is:

(5) $$r = \frac{172.02}{\sqrt{(0.013790)(2,619,600)}}$$

$$r = 0.905$$

Another measure involving the use of the correlation coefficient is the coefficient of determination, r^2. The coefficient of determination is the proportion of the total variation or variance in modulus of rupture accounted for by the differences in specific gravity. The total variability is considered to be 1 and the coefficient of determination is 0.819, or 0.905^2; the difference, 0.181, or $1-(0.905)^2$, is a measure of the amount of variability unaccounted for by specific gravity.

At times, it is desirable to test for the significant of the correlation value; that is, to prove or disprove the hypothesis that there is no correlation present. If the hypothesis is discredited, the correlation is considered significant.

To discover whether or not an observed correlation coefficient is significantly greater than zero, the following procedure is applied to both large and small samples. This method consists in computing the value "t" from the expression

(6)
$$t = \frac{r\sqrt{N-m}}{\sqrt{1-r^2}}$$

where: m = number of constants in the estimating equation.

Thus, to determine whether or not the correlation coefficient 0.905 is significant, substitute known values into equation (6).

$$t = \frac{0.905\sqrt{12-2}}{\sqrt{1-(0.905)^2}}$$
$$= 6.73$$

Checking in a Student's "t" table at the 0.01 level of significance and 10 degrees of freedom, a t value greater than 3.169 is significantly different from zero. Therefore, the correlation coefficient 0.905 is significant.

Calculation of standard error of estimate

The standard error of estimate is similar to the standard deviation. The error of estimate is a measure of the variability of test values from the regression line. In other words, the error of estimate indicates deviation of Y from the regression of Y on X and can be determined by equation (7).

(7)
$$S_s = \sqrt{\frac{ss_y - \frac{(XY)^2}{ss_x}}{N-2}}$$

where: S_s = standard error of estimate

ss_y = sums of squares of Y variable

ss_x = sums of squares of X variable

XY = sum of the products of (X) (Y)

N = sample size

To determine the standard error of estimate for board A, substitute values from Form B into equation (7), then

$$S_s = \sqrt{\frac{2,619,600 - \frac{(172.02)^2}{0.013790}}{10}}$$

$$S_s = 217.66 \text{ psi}$$

The average deviation of Y from the regression line of Y on X is, therefore, 217.66 psi.

Calculation of confidence limits

The predicting equation for each hardboard serves primarily to estimate an average modulus of rupture (or modulus of elasticity) for a board of known specific gravity. To use the predicting equations profitably, however, it is necessary to establish the confidence that may be placed in the "best" Y computed.

The confidence limits of an average modulus of rupture of a board can be computed from equation (8).

$$(8) \quad Y_e = \bar{Y} \pm t \sqrt{S_s^2 \left[\frac{1}{N} + \frac{(x - \bar{\bar{x}})^2}{ss_x}\right]}$$

where: Y_e = estimated modulus of rupture

$\bar{Y}$ = average value of Y

t = tabulated value of Student's t-distribution

S_s^2 = standard error of estimate for sample

N = sample size

$\bar{\bar{x}}$ = average specific gravity of sample

x = specific gravity at which modulus of rupture is estimated

ss_x = sums of squares of specific gravity

It should be pointed out that equation (8) can be used whenever confidence limits for an average modulus of rupture are wanted, regardless of whether the sample is one board, 10 boards, one hardboard type, or 10 hardboard types.

For example, it is desired to locate confidence limits for board A. Assume a specific gravity of 0.980, which is within the specific gravity range of board A, but is not equal to the mean specific gravity of the board. Confidence limits determined by substituting in equation (8) are

$$Y_e = \overline{Y} \pm t \sqrt{S_s^2 \left[\frac{1}{N} + \frac{(x - \overline{\overline{x}})^2}{ss_x}\right]}$$

$$Y_e = \overline{Y} \pm 3.169 \sqrt{47,377.8 \left[\frac{1}{12} + \frac{(0.980 - 0.996)^2}{0.013790}\right]}$$

$$Y_e = \overline{Y} \pm 3.169 \quad (69.481)$$

$$Y_e = \overline{Y} \pm 220.19 \text{ psi}$$

By calculating limits for several assumed specific gravities and then connecting these points, a confidence band is formed around the regression line. Chances that the average value of another sample of board A will fall beyond the confidence limits are dependent upon Student's t-value (in this example 99% at 10 d.f.). The regression line and confidence limits (broken lines) for board A are shown in Figure 1.

At times, it is desirable to make a statement about a single specimen rather than about the average of the sample. In these instances, equation (8) is modified to include the specimen variance, as follows:

$$(9) \quad Y_e = \overline{Y} \pm \sqrt{S_s^2 \left[1 + \frac{1}{N} + \frac{(x - \overline{\overline{x}})^2}{ss_x}\right]}$$

Equation (9) can be used to determine confidence limits of a single specimen from a board, a single specimen from a group of boards, a single board from a group of boards, and so on.

For example, using data from the previous calculation, it is desired to determine the confidence limits of a single specimen from board A:

Then,

$$(9) \quad Y_e = \overline{Y} \pm t_{99\%} \sqrt{S_s^2 \left[1 + \frac{1}{N} + \frac{(x - \overline{\overline{x}})^2}{ss_x}\right]}$$

when: $\overline{Y} = 5,860$

$t_{99\%} = 3.169$

$S_s^2 = 47,377.8$

$x = 0.980$

$\bar{x} = 0.996$

$ss_x = 0.013790$

$N = 12$

$$Y_e = 5,860 \pm 3.169 \sqrt{47,377.8 \left[1 + \frac{1}{12} + \frac{(0.980-0.996)^2}{0.013790}\right]}$$

$$Y_e = 5,860 \pm 3.169\ (228.49)$$

$$Y_e = 5,860 \pm 724$$

$$Y_e = 5,136 \text{ and } 6,584 \text{ psi}$$

The answer signifies that, while the average modulus of rupture for board A is probably between $\bar{Y} \pm 220$ psi, a randomly selected individual specimen may be found to have a modulus of rupture anywhere between $\bar{Y} \pm 724$ psi. As can be seen from this example, predicting performance of individual specimens, especially those close to limits for the population, is hazardous unless the standard error of estimate is unusually small. Confidence limits for an individual specimen from board A are shown in Figure 1 (solid lines).

APPLICATION OF RESULTS

Use of predicting equations and subsequent determination of confidence limits at a desired confidence coefficient of 95 or 99 per cent are of prime importance. Not only do the equations form a basis for evaluating differences in modulus of rupture or modulus of elasticity due to specific gravity, but the limits also indicate confidence that may be placed on values determined by this approach to the problem.

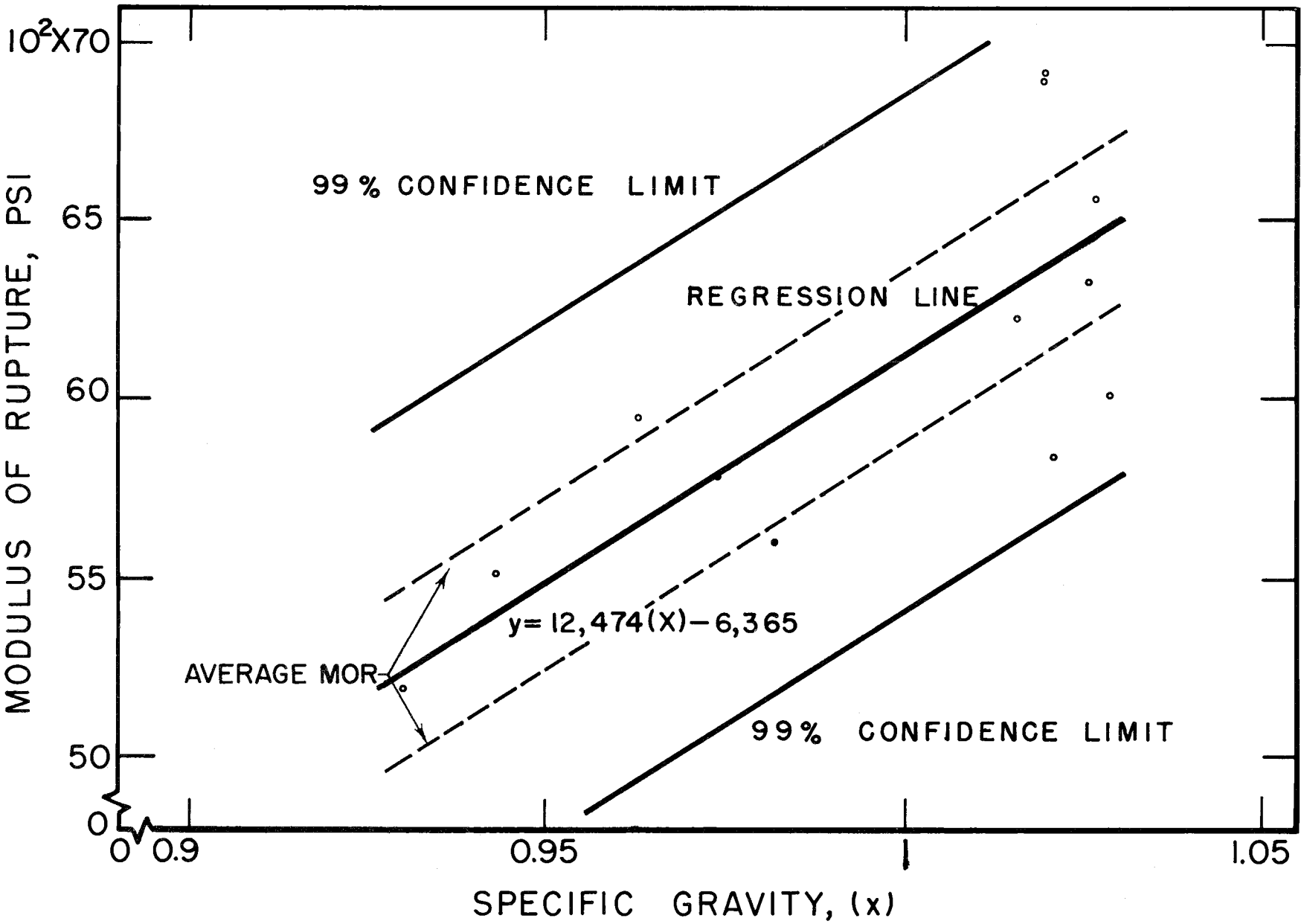

FIGURE I. SCATTER DIAGRAM OF 12 TEST VALUES FOR BOARD "A"

For benefit of the research worker, an illustrated problem, using principles of statistics described on previous pages, will be shown. The problem will be expanded to include a second type of hardboard. The new type may be a board of the same manufacture but a different thickness or treatment, or it may be of different manufacture. While these possibilities will increase the computational load somewhat, the approach to evaluate the treatment will be similar to that previously described.

Assume two hardboard types are to be evaluated. It is assumed further that test data for the two types have been tabulated as in Table 1 and 2 and Form A. Then data in Form B are as shown in Table 3.

Table 3. Regression Analysis of Illustrated Problem. Two hardboard Types Each Having 60 Test Specimens.

Board type	Degrees freedom	ss_x	Sum of prod. XY	ss_y	Regression coefficient	Regression ss
W	50	0.093436	1130.98	16,136,700	12,104.328	13,689,752.99
Z	50	0.113442	1348.79	19,927,400	11,889.659	16,036,621.10
Sum	100	0.206878	2479.77	36,064,100	- - -	29,726,374.09

The weighted average regression coefficient for the board-type regression values is 2,479.77/0.206878 = 11,986.629. The correlation coefficient "r" is $2479.77/\sqrt{0.206878\ (36,064,100)} = 0.908$, and the coefficient of determination "r^2" is 0.824. The test of homogeneity of the regression coefficient is shown in Table 4.

Table 4. Test of Homogeneity of the Regression Coefficient.

Source of variation	Degrees freedom	Sum of squares	Mean square	F
Regression due to $\bar{b}$	1	29,724,100	29,724,100	459*
Among type	1	2,300	2,300	NS**
Residual	98	6,337,700	64,700	- - -
Total	100	36,064,100	- -	- - -

* Significantly different from zero at the 1% confidence level.
** Not significant.

From this test, it is apparent that the two board types can be combined, with a common predicting equation. To determine the predicting equation for the two board types, values are substituted into equation (2).

(2) $Na + b \sum X - \sum Y = 0$

$120a + 11,986.629\ (115.418) - 669,280 = 0$

$a = -5,951.583$

The predicting equation is $Y = 11,986.629\ X - 5,951.583$ for the two hardboard types combined and is shown in Figure 2. Also in Figure 2 are plotted values (from Tables 1 and 2) for the 120 test specimens used in the illustrated problem.

The following example shows how to use the predicting equation. Assume it is known an individual specimen of hardboard has a specific gravity of 1.038, and it is known also the predicting equation for this board type is $Y = 11,986.629\ X - 5,951.583$. The following values are desired:

(a) the estimated modulus of rupture at 1.038 specific gravity.

(b) the lower confidence limit based upon the 99 per cent confidence coefficient for the estimated modulus of rupture.

(a) Modulus of rupture:

$$\text{MOR or } \bar{Y} = 11,986.629\ (1.038) - 5,951.583$$

$$\bar{Y} = 6,490.5 \text{ psi}$$

(b) the lower limit for MOR with the 99 per cent confidence coefficient:

$$Y_e = \bar{Y} - t_{99\%} \sqrt{S_s^2 \left[1 + \frac{1}{N} + \frac{(x - \bar{x})^2}{ss_x^2}\right]}$$

$$Y_e = 6,490.5 - 2.616 \sqrt{53,728.94 \left[1 + \frac{1}{120} + \frac{(1.038 - 0.962)^2}{0.206878}\right]}$$

$$Y_e = 6,490.5 - 2.616 \sqrt{55,676.7753}$$

$$Y_e = 6,490.5 - 2.616 \quad (235.97)$$

$$Y_e = 6,490.5 - 617.3$$

$$Y_e = 5,873.2 \text{ psi}$$

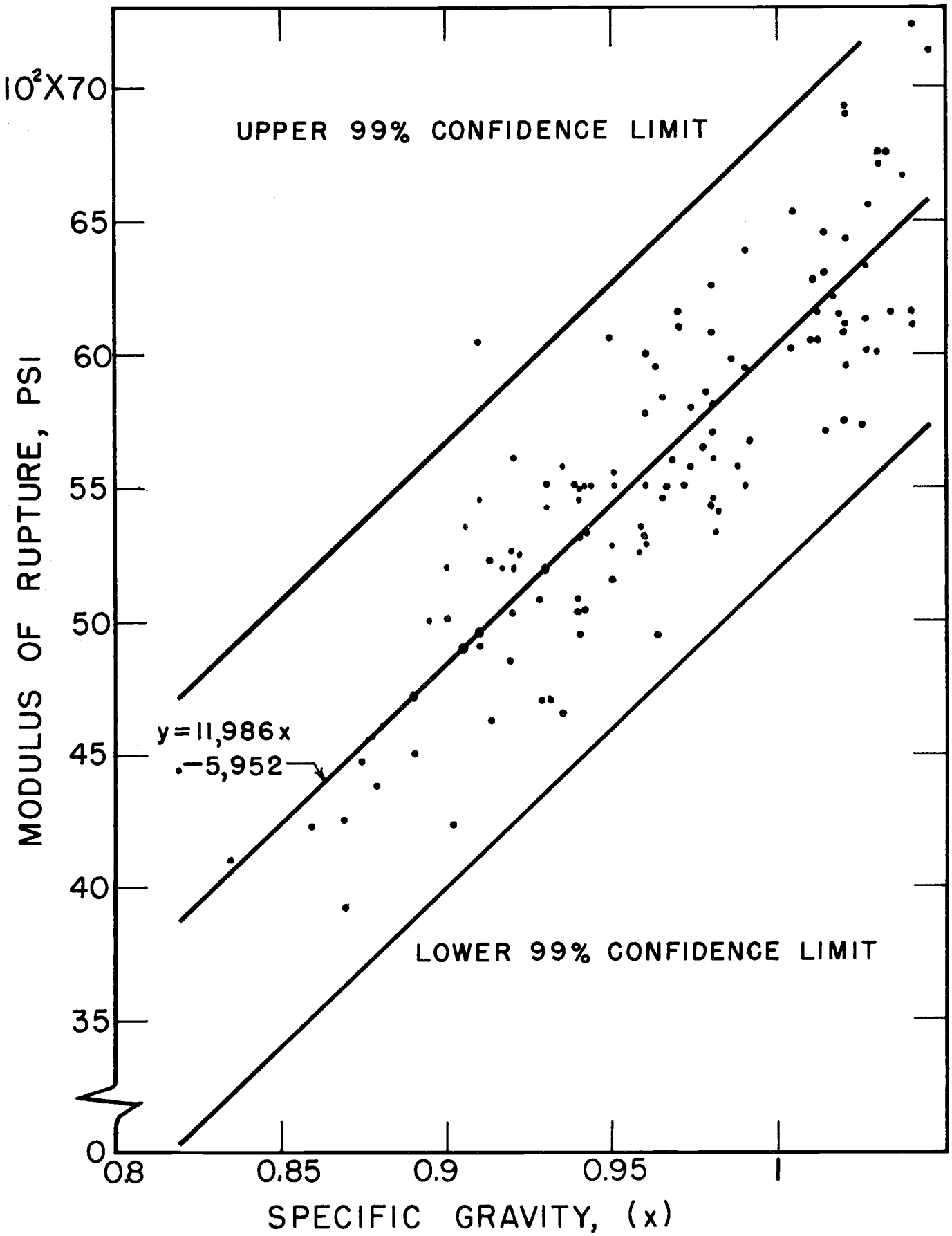

FIGURE 2. SCATTER DIAGRAM OF 120 TEST VALUES FOR ILLUSTRATED PROBLEM

Thus, if another specimen of hardboard with a specific gravity of 1.038 were taken, the chances are its modulus of rupture would be close to 6,490 psi and would fall below 5,873 psi only once out of 100 times. Predicting equations and confidence limits for specific gravity-modulus of elasticity relationships are used in a like manner.

Should the research worker desire to combine various board types, the predicting equations can be found in a manner similar to the illustrated problem. To find confidence limits to the equation, however, requires modification of the standard error of estimate.

The standard error of estimate is modified to include error terms not present when working with one board from a given board type. The terms are: error due to boards, error due to board types, and error due to directional properties of some boards. It is possible to have one form of error without the other, or have both error terms equal zero, depending on the board type. Equations (8) and (9) are now modified to read:

$$\text{(8a)} \qquad Y_e = \overline{Y} \pm t \sqrt{S_s^2 \left[\frac{1}{N} + \frac{(x - \overline{x})^2}{ss_x}\right] + S_t^2 + S_b^2 + S_d^2}$$

$$\text{(9a)} \qquad Y_e = \overline{Y} \pm t \sqrt{S_s^2 \left[1 + \frac{1}{N} + \frac{(x - \overline{x})^2}{ss_x}\right] + S_t^2 + S_b^2 + S_d^2}$$

The method of determining the standard error of estimate, S_s^2; error due to board type, S_t^2; error due to boards, S_b^2; and error due to direction, S_d^2; is illustrated in Table 5. The mathematics involved in calculating S_s^2, S_t^2, S_b^2, and S_d^2 are self-explanatory and should not present difficulty in solving for these relationships.

To determine confidence limits around the regression equation calculated for two hardboard types in the illustrated problem, substitute values determined previously as in Forms A, B, and C, and in Table 5. Then:

$$Y_e = \overline{Y} \pm t_{99\%} \sqrt{S_s^2 \left[\frac{1}{N} + \frac{(x - \overline{x})^2}{ss_x}\right] + S_t^2 + S_b^2 + S_d^2}$$

FORM C

Test of Homogeneity of Regression Coefficients

Parallel vs. Perpendicular, Within Hardboard Type

X : Specific gravity Y : Modulus of rupture

Direction	Deg. freedom	ss_x	Sum of prod. XY	ss_y	Regression coefficient b	Regression ss
Par.	25	0.040683	512.28	7,663,700	12,591.992	6,450,625.66
Perp.	25	.052754	618.70	8,472,900	11,728.021	7,256,126.59
Sum	50	.093437	1130.98	16,136,600		13,706,752.25

Weighted Average of b's, $\bar{b}$ = 12,104.199

Sources of variation	Degrees freedom	Sum of squares	Mean square	F
Regression due to $\bar{b}$	1	13,689,606.99	13,689,606.99	270.43*
Between b's	1	17,145.26	17,145.26	.34
Residual	48	2,429,847.75	50,621.83	--
Total	50	16,136,600.00	--- ---	--

* Significantly different from zero at 99% confidence level.

Table 5. Analysis of Variance Calculations.

EXPERIMENT : Specific gravity - modulus of rupture

A. PRELIMINARY CALCULATIONS

Source of variation	Total of squares (from Tables 1 and 2)	No. of items squared	Observations per squared item	Total of squares per observation (2) ÷ (4)	Sum of squares 5-correction
1	2	3	4	5	6
Correction	$(72,700+....+63,660)^2$ = 447,935,718,400	1	120	3,732,797,600	0
Type	$(72,700+...+71,760)^2$ $+(62,800+....+63,660)^2$ = 224,122,387,400	2	60	3,735,373,100	2,575,500
Board	$(72,700)^2+....+(63,660)^2$ = 44,931,097,600	10	12	3,744,258,100	11,460,500
Direction	$(35,090)^2+....+(30,670)^2$ = 22,495,919,400	20	6	3,749,319,900	16,522,300
Specimen	$(5,190)^2+....+(5,020)^2$ = 3,785,384,000	120	1	3,785,384,000	52,586,400

Table 5. (continued)

B. ANALYSIS OF VARIANCE

Variation due to:	Sums of squares	Degrees freedom	Mean square	Error*
Type	2,575,500	1	2,575,500	(1) $S_t^2 = \frac{(1) - (2)}{N} = \frac{2,575,500-1,110,600}{60} = 24,400$
Board	8,885,000	8	1,110,600	(2) $S_b^2 = \frac{(2) - (3)}{N} = \frac{1,110,600-506,200}{12} = 50,400$
Direction	5,061,800	10	506,200	(3) $S_d^2 = \frac{(3) - (4)}{N} = \frac{506,200 - 360,200}{6} = 24,300$
Specimen	36,064,100	100	360,200	(4)
Regression	29,724,100	1		
Residual	6,340,000	99	64,000	(5) $S_s^2 = (5) = 64,000$
Total	52,586,400	119		

* S_t^2, error due to board type; S_b^2, error due to boards; S_d^2, error due to machine direction; S_s^2, standard error of estimate. N = 60 specimens to a type, 12 specimens to a board, or 6 specimens to a direction.

When: $x = 0.962 = \bar{\bar{x}}$

$t_{99} = 2.618$

$N = 120$

$\bar{Y} = 5,580$ psi

$$Y_e = 5,580 \pm 2.618\sqrt{64,000\left[\frac{1}{120} + \frac{(0.962 - 0.962)^2}{0.206878}\right] + 24,400 + 50,400 + 24,300}$$

$$Y_e = 5,580 \pm 2.618\sqrt{99,633}$$

$$Y_e = 5,580 \pm 826 \text{ psi}$$

Now, when X = 0.962, the confidence limits of the predicting equation at this specific gravity are 5,580 ± 826 psi. Confidence limits of an average value of Y are determined in a like manner for other values of X. By joining the plotted points together with a line, a curve parabolic in nature is formed that describes the boundary of the confidence limits. It can be seen that as the difference between $(x - \bar{\bar{x}})$ becomes more pronounced, the error of the estimated Y_e increases and the precision of the estimate is lowered.

Illustrated in Figure 2 is the average regression line for 120 test specimens used in this problem. Also shown is the confidence boundary at a 99 per cent confidence coefficient around the average regression line.

When considering the predicting equations and their respective confidence limits, it can be seen that the wide range required to satisfy the inclusion of 99 per cent of the values does not indicate a high degree of accuracy in the prognostic value of the equation. For instance, the lower limit at the 99 per cent confidence coefficient is a negative value of 1,068 psi, a physical impossibility when the specific gravity for this value is 0.50.

CONCLUSIONS

The regression and correlation method is merely an averaging process by which an average relationship is measured. It has an advantage that it is adapted to few data. It has the disadvantage that it always assumes linear relationships

regardless of whether or not that assumption is correct.

The regression equation describes the nature of relationships between variables and shows rates of change in one in terms of others. The greatest value of a regression equation is to estimate the dependent variable in terms of one or more independent variables.

Reliability of regression and correlation estimates may be tested by the standard error of estimate based on a simple linear regression equation, $Y = a + bX$. The standard error of estimate is interpreted in about the same way as the standard error of the arithmetical mean.

The primary use of a correlation coefficient is to show with an index or number the degree of relationship between two variables. These coefficients range from + 1.0 through 0 to - 1.0. When the coefficient is + 1 or - 1, there is perfect positive or negative relationship between variables. Usefulness of this coefficient depends in part on knowledge of its limitations.

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