AN ABSTRACT OF THE THESIS OF

Jared S. Bahm for the degree of Master of Science in Mechanical Engineering presented on August 12, 2019.

Title: Modelling Thin Liquid Film Draining of Bubbles on Porous Media in Confined Geometry

Abstract approved: ______________________________________________________________

Deborah V. Pence

The behavior of the thin liquid film formed between a bubble and hydrophobic membrane is of high interest in applications where separating two-phase mixtures is beneficial. One such application is in-situ vapor extraction heat sinks. In these systems, high heat transfer rates are accomplished by taking advantage of the high energy associated with phase change. However, the generation of vapor may lead to dry-out and subsequent critical temperatures if the vapor bubble is not extracted proficiently. An existing model for single bubble extraction in a confined geometry theoretically predicts the bubble diameter and relevant forces acting on the bubble from inception to extinction. However, the model needed to be supplemented with empirical correlations due to the unknown conditions for bubble rupture and the behavior of the three-phase contact line on the supply and extraction surfaces. In
this work, the Stokes-Reynolds-Young-Laplace (SRYL) lubrication model is studied and adapted to the confined geometry of a growing bubble to numerically simulate the thin liquid film draining event at the extraction surface. To the author’s knowledge, this is the first implementation of the SRYL lubrication model to the special case of confined bubbles under growth. Experimental data from the existing model is used to qualitatively examine the behavior of the thin liquid film at the extraction surface. It was found that the approach velocity and bubble radius upon reaching the extraction surface was related to the formation of a hydrodynamic dimple in the thin liquid film. The hydrodynamic dimple is characterized by a barrier rim, where the thinnest part of the liquid film is no longer at the apex of the bubble. Larger radii bubbles with lower Laplace pressures are more easily deformed as they approach the extraction surface and exhibit the hydrodynamic dimple at larger liquid film thicknesses. Results show that the minimum liquid film thickness at the predicted time of rupture is relatively consistent, ranging from $\approx 3.16 \mu m$ to $\approx 2.72 \mu m$ for confined bubble gap heights ranging from $0.52 mm$ to $1.90 mm$, respectively. It is believed this is due to a balance of approach velocity, the degree of bubble deformation and resulting hydrodynamic pressure within the interaction zone of the liquid film. Further, these minimum liquid film thicknesses are outside the bounds of typical long range forces, suggesting other rupture mechanisms may occur in confined geometries. Due to the stochastic nature of liquid film rupture, the rupturing event is not modelled in this study and is suggested as future work.
Modelling Thin Liquid Film Draining of Bubbles on Porous Media in Confined Geometry

by

Jared S. Bahm

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APPROVED:

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________________________________________________________________________
Jared S. Bahm, Author
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Nomenclature

Greek Symbols

\( \alpha \) constant multiplier for \( h_0 \)
\( \chi \) non-spherical bubble correction factor
\( \Delta \) change in subsequent variable
\( \delta \) membrane thickness [m]
\( \gamma \) surface energy [J/m\(^2\)]
\( \lambda \) wavelength [\( \mu \)m]
\( \mu \) dynamic viscosity [Ns/m\(^2\)]
\( \Pi \) disjoining pressure [Pa]
\( \rho \) density [kg/m\(^3\)]
\( \sigma \) surface tension [N/m]
\( \theta \) contact angle [deg]
Roman

- \( \dot{r} \): radial velocity of bubble \([m/s]\)
- \( \dot{V}_0 \): constant volumetric flow rate \([m^3/s]\)
- \( \dot{V} \): volumetric flow rate \([m^3/s]\)
- \( A \): area \([m^2]\)
- \( Ca \): capillary number \( Ca \equiv \frac{\mu v_0}{\sigma} \)
- \( D \): diameter of bubble \([m]\)
- \( d_{ext} \): diameter of bubble contact at extraction surface \([m]\)
- \( d_{sup} \): diameter of bubble contact at supply surface \([m]\)
- \( E \): energy \([J]\)
- \( F \): force \([N]\)
- \( f \): fraction of projected wetted area
- \( g \): gravity constant \([m/s^2]\)
- \( H \): measured gap height \([m]\)
- \( h \): film height \([m]\)
\[ h_0 \quad \text{initial film height} \quad [m] \]

\[ K \quad \text{permeability} \quad [m^2] \]

\[ p \quad \text{pressure} \quad [Pa] \]

\[ R \quad \text{radius of bubble} \quad [m] \]

\[ r \quad \text{radial coordinate in film domain} \quad [m] \]

\[ R_0 \quad \text{characteristic radius of bubble} \quad R_0 \equiv H/2 \quad [m] \]

\[ r_f \quad \text{roughness ratio} \]

\[ R_L \quad \text{Laplace radius} \quad [m] \]

\[ R_p \quad \text{roughness parameter} \]

\[ r_{max} \quad \text{maximum radial coordinate in film domain} \quad [m] \]

\[ Re_f \quad \text{film Reynolds number} \quad Re_f \equiv \frac{\rho h_0 v_0}{\mu} \]

\[ S \quad \text{spreading parameter} \quad [J/m^2] \]

\[ u \quad \text{radial velocity in film} \quad [m/s] \]

\[ U_{max} \quad \text{maximum radial velocity in film} \quad [m/s] \]

\[ V \quad \text{volume} \quad [m^3] \]
\( v \) \hspace{1cm} \text{vertical velocity in film \([m/s]\)}

\( v_0 \) \hspace{1cm} \text{characteristic vertical velocity in film \([m/s]\)}

\( V_D \) \hspace{1cm} \text{volume of bubble at departure \([m^3]\)}

\( V_R \) \hspace{1cm} \text{volume of bubble at rupture \([m^3]\)}

**Subscripts**

\( o \) \hspace{1cm} \text{initial state}

\( A \) \hspace{1cm} \text{advancing}

\( B \) \hspace{1cm} \text{buoyancy}

\( CB \) \hspace{1cm} \text{Cassie-Baxter}

\( D \) \hspace{1cm} \text{departure}

\( e \) \hspace{1cm} \text{exit}

\( ext \) \hspace{1cm} \text{extraction}

\( G \) \hspace{1cm} \text{growth}

\( g \) \hspace{1cm} \text{gas}

\( i \) \hspace{1cm} \text{inlet}
\( l \) liquid

\( LG \) liquid/gas

\( LV \) liquid/vapor

\( R \) receding, rupture

\( SL \) solid/liquid

\( sup \) supply

\( SV \) solid/vapor

\( W \) Wenzel

Superscripts

\( * \) non-dimensional variable

Abbreviations

\( CFD \) computational fluid dynamics

\( DLVO \) Derjaguin-Landau-Verwey-Overbeek

\( ID \) inner diameter

\( LBP \) liquid breakthrough pressure
LEP  liquid entry pressure

OD  outer diameter

PTFE  polytetrafluoroethylene (DuPont’s Teflon™)

SRYL  Stokes-Reynolds-Young-Laplace

TPC  three phase contact
Chapter 1: Introduction

The rapid development of electronics coupled with their ever decreasing size has introduced a limiting factor of heat dissipation. Haman et al. [1] described this limiting factor in his experiments measuring the spatial and temporal variations on a microprocessor as the “power wall”. Due to the excessive heat generation within the microprocessor, its power density and performance is limited. The heat generation on the microprocessor may be hotspot limited, or power limited. In designing the architecture of a microprocessor, if it is power-limited, the power of the device must be simply reduced to maintain operable temperatures to ensure the longevity of the product. In contrast, a hotspot-limited design refers to specific junctions within the circuitry which produce high heat fluxes in localized areas. It is in the latter case that an effective spatio-temporal cooling mechanism is desired.

1.1 Background

The need to increase the capacity of the cooling mechanism while reducing its size to meet the packaging requirements of electronic devices has become an apparent and a uniquely challenging design obstacle [2, 3]. With the increase in power
density conventional cooling solutions such as air cooling with fins falls short. Other methods which have been investigated extensively are single phase liquid cooling. Most of the research in this particular cooling solution has been conducted on constant cross-section, straight micro-channels. Because of the small size of the micro-channels, the flow regime is always laminar, and thus not optimized for heat transfer [3, 4]. In addition, a relatively large volume of coolant must be contained within the thermal management system for the single-phase liquid cooling option to be viable.

In more recent years, two-phase boiling has been explored as a possible solution to thermal management of these micro devices. Due to the high enthalpy transport associated with phase change, the heat transfer rate in two-phase boiling is much higher than that of single phase liquid. The amount of bulk coolant may also be reduced which is desirable for the small packaging requirements. Despite the theoretical advantages of two-phase boiling, many challenges are involved with the complex flow behavior of two-phase boiling and are exacerbated when scaled to the micro size.

Qu et al. [5] conducted a study on two-phase boiling flow of deionized water in rectangular microchannels. The study confirmed the dominant heat transfer mechanism was forced convective boiling; however, it was also noted that the heat transfer coefficient actually declined with increasing vapor quality. It was thought this may be due to blockage of the channel by vapor bubbles, or by the thickening of the annular film by deposition of liquid droplets in the vapor core. This was
contrary to macro-scale results, and sheds light on one of the main challenges of maintaining an optimal heat transfer coefficient on the micro-scale.

As a means to mitigate against the negative effects of excess vapor within the microchannel, Zhou et al. [6] proposed local vapor venting. The vapor venting was accomplished by the use of a porous hydrophobic membrane separating a liquid and vapor channel. The operating principle being that when vapor bubbles were formed in the liquid channel, they would be vented to the vapor channel and reduce the pressure drop in the liquid channel to nearly single-phase levels.

Along the same premise, Apreotesi [7] and Salakij et al. [8] investigated local vapor extraction in fractal-like branching microchannel networks. The vapor extraction scheme differed to that of Zhou et al. [6] in that a vacuum was imposed across the hydrophobic membrane to aid in the vapor extraction from the liquid channel rather than venting to atmosphere. The results of the studies showed reduction in the pressure drop with vapor extraction; however some inconsistency in improving heat transfer was observed over the range of testing conditions [9].

This motivated a fundamental study by Fox et al. [10] to devise an in-situ vapor extraction model of single bubbles to better understand the complex mechanisms of micro-scale heat transfer. The model is based on pool boiling where the bulk fluid is stationary, and the vapor extraction point correlates to an artificial nucleation site, as would correspond to a high heat flux region. The inclusion of patterned artificial nucleation sites prevents random nucleation and subsequent clustering of adjacent
bubbles [11]. The size of the bubble diameter must also be controlled to design the optimal nucleation site array and prevent bubble interactions. Therefore the height of the extraction surface relative to the bubble diameter is of prime interest. The hydrophobic nature of the extraction surface provides the additional upward force to coerce bubble departure at smaller diameters and prevent unwanted bubble interactions at the heated surface [11].

To develop their model, Fox et al. [10], in conjunction with Juarez [12], conducted experiments for diabatic and adiabatic cases, respectively, of single bubble extraction above a single nucleation site located within a static pool of water. In these experiments, a hydrophobic, porous membrane located parallel to the heated surface was used as the extraction surface. By using high speed imaging, they were able observe the bubble dynamics as a function of time. Effective bubble diameters were assumed based on the observed bubble volumes. Bubble volume was assessed from the two-dimensional time-sequenced images, as noted in [10].

Experimental observations resulted in assigning three different regimes to describe a bubble throughout its lifetime. The first regime considers the unconfined growth of the bubble which is only in contact with the supply surface. Regime 1 begins at nucleation and ends at rupture on the extraction surface. Regime 2 is when the bubble is in contact with both the supply surface and the extraction surface and spans from rupture at the extraction surface to departure at the supply surface. Regime 3 is defined by detachment from the supply surface and the bubble is only in contact with the extraction surface. It spans from departure from the supply
surface to extinction. A diagram of the respective regimes is provided in Figure (1.1).

![Diagram of regimes](image)

Figure 1.1: Three regimes of confined vapor extraction: (a) Regime 1: growth only, (b) Regime 2: growth with extraction and (c) Regime 3: extraction only.

The theoretical model proposed by Fox et al. [10] is based upon the work of Thornicroft et al. [13], and requires knowledge of the forces acting on the bubble. Thornicroft et al. [13] conceived a detachment model for pool and flow boiling regimes by considering the forces on a vapor bubble from nucleation, growth and detachment for several orientations of the heated surface. The detachment model is in good agreement with experimental observations. Fox et al. [10] built upon Thorncroft’s pool boiling detachment model [13] and included the additional forces present for confined bubble growth due to the presence of the hydrophobic extraction surface. The proposed theoretical bubble diameter model was based on conservation of mass and conservation of momentum. The forces present on the bubble during confined extraction may then be defined for each regime in Figure (1.2).

In Regime 1, Figure (1.2)(a), the forces present are the inlet momentum, buoyancy
Figure 1.2: Forces on the three regimes of confined vapor extraction: (a) Regime 1: growth only, (b) Regime 2: growth with extraction and (c) Regime 3: extraction only.

force, growth force and surface tension force at the supply surface. In Regime 2, Figure (1.2)(b), the exit momentum and surface tension force at the extraction surface are present in addition to the inlet momentum and surface tension force at the supply surface. The growth force is assumed negligible in Regime 2 because the vertical growth rate upon contact and rupture at the extraction surface is negligible. In Regime 3, Figure (1.2)(c) only the exit momentum, buoyancy force and surface tension at the extraction surface are present.

To mathematically describe the forces acting on the bubble in the confined system, Fox et al. [10] began with the transient momentum equation offered by Thorncroft et al. [13] for unconfined pool boiling. For Regime 1, prior to the bubble reaching the extraction surface, the equation is

\[
\frac{d(mu)}{dt} = F_B + F_G + F_{\sigma,\text{sup}} + (\dot{m}u)_i \tag{1.1}
\]
where in the case of horizontal pool boiling, only the y-direction (i.e., normal to the heated surface) need be considered since the bulk fluid is assumed to be quiescent.

The terms in Eq. 1.1 are defined as follows, with the net buoyant force defined by

\[ F_B = \frac{\pi D^3 g (\rho_l - \rho_g)}{6} \]  

(1.2)

the growth force, assuming the bubble grows as a sphere, defined by

\[ F_G = 2\pi \rho_l r^2 \dot{r}^2 \]  

(1.3)

and the surface tension force at the supply surface defined by

\[ F_{\sigma,\text{sup}} = \pi \sigma d_w \sin(\theta_{\text{sup}}). \]  

(1.4)

The time derivative term represents the time rate of change of momentum of the bubble whereas the term \( \dot{m}u \) represents the momentum of the vapor/gas entering the bubble.

Once the bubble ruptures at the extraction surface, Fox et al. [10] included the additional surface tension force and momentum of the extracted vapor at the ex-
traction surface. It was experimentally observed that the rate of growth was negligible and is neglected in the momentum equation used to simulate the bubble diameter for Regime 2

$$\frac{d(\mu u)}{dt} = F_B + F_{\sigma, sup} + F_{\sigma, ext} + (\dot{\mu}u)_i - (\mu u)_e. \quad (1.5)$$

At the moment of departure from the supply surface, the surface tension force term at the supply surface is assumed negligible and the inlet momentum term at the supply surface goes to zero as no more mass enters the bubble at this point. However, the exit momentum term, at the extraction surface, does not approach zero at bubble departure as does the inlet momentum term. The equation representing bubble diameter as a function of time for Regime 3 is

$$\frac{d(\mu u)}{dt} = F_B + F_{\sigma, ext} - (\mu u)_e. \quad (1.6)$$

Through experimentation, Fox et al. [10] observed the volumetric growth rate of the bubble in Regime 1 to be nearly constant and that the exit momentum term may be evaluated by the application of Darcy’s Law. However, as noted by Cappello et al. [14], the area of the bubble in contact with the extraction surface is crucial to the application of Darcy’s Law. Due to the uncertainty in the conditions necessary for rupture, the behavior of the three-phase contact (TPC) line on the supply and extraction surfaces and the difficulty in measuring the extraction
diameter by imaging, Fox et al. [10] found the need for empirical correlations in the model.

To predict rupture, a linear regression was performed on the data to attain a correlation between the rupture diameter and gap height, where gap height refers to the normal distance between the supply and extraction surfaces [10]. A similar regression was performed to assign a time-invariant extraction area. Finally, the data was fit to determine what contact angles best represented the data at both the supply and extraction surfaces.

Specifically lacking are theoretical models for (1) the thickness at which the liquid film between the bubble and the extraction surface ruptures, (2) a time-varying diameter at the extraction surface, and (3) a time-varying contact diameter at the supply surface. The latter two needs are dependent upon the contact angle at that surface.

1.2 Motivation

The purpose of the present study is to refine the model presented by Juarez and Fox et al. [12, 10] to account for the rupture process with a reduced degree of empiricism. By modeling the period of time from where the bubble comes into contact with the extraction surface to the instant of rupture and TPC line formation, it may be possible to include predictions of the bubble diameter at rupture
based on first principles. If this can be done, the model may be utilized with all working fluids and a prediction of the optimal geometry may be made based on fluid and or material properties. Specifically, the scope of this research is aimed at assessing the thin film draining event between the vapor bubble and continuous phase, and the relevant physics of the three-phase contact line on the hydrophobic extraction surface.

Currently, the most reliable method in determining the optimal confinement geometry involves computational fluid dynamics (CFD). The expense associated with such methods often deters designers. A lower cost method which yields adequate results is sought as an alternative. By creating a theoretical model which demonstrates reasonable accuracy compared to CFD methods, designers may circumvent the additional costs associated with CFD and use the model to help design more efficient two-phase heat sinks.
Chapter 2: Literature Review

This chapter serves as an examination of the current state of research applicable to the thin film draining, the mechanism(s) responsible for thin film rupture and the dynamics of the TPC line. The information is presented by broad topics relating to interfacial and colloidal science, and later concentrated to the specific nature this study. Some topics discussed are outside of the scope of this study, but are presented to give the reader some insight to the multitude of phenomena associated with two-phase vapor extraction and their contributing effects.

2.1 Physics of the Liquid/Solid/Vapor Interface

2.1.1 The Contact Angle

One may observe the spherical cap that water makes on a leaf in the morning dew. Absent of outside perturbation, this water cap may be considered in equilibrium. The nature of the water to form a spherical cap rather than spread apart and run off the leaf is the result of cohesive forces. The water shares an interface with the solid substrate of the leaf as well as the air. Thus, the system may be described
by the presence of three phases. The first attempt to characterize the equilibrium state of a solid/liquid/vapor system was offered by Young [15] in 1805. In his essay he describes the mechanical balance of surface tension forces acting on the line of three phase contact (TPC) on an ideal surface. An ideal surface being one that is absent imperfection, homogeneous, rigid and insoluble. By observing the angle that the liquid makes with respect to the solid substrate Young arrived at the following relation:

\[ \gamma_{SV} = \gamma_{SL} + \gamma_{LV} \cos(\theta) \]  

(2.1)

where \( \gamma_{SV} \) is the solid/vapor surface energy, \( \gamma_{SL} \) is the solid/liquid surface energy, \( \gamma_{LV} \) is the liquid/vapor surface energy and \( \theta \) is the contact angle measured in the liquid phase. Note that the same relation holds true when considering the surface tension \( \sigma \) rather than surface energy \( \gamma \).

In practice, surfaces are rarely ideal, and in the scope of this research, it is a safe assumption that the surface is not ideal. In 1936, Wenzel [16] formulated a variation of Young’s equation to include the effects of surface roughness. Surface roughness can have a strong effect on the surface wettability and the contact angle. To represent the effect of surface roughness, Wenzel introduced a roughness parameter defined as
where $A_{real}$ is the true surface area and $A_{geometrical}$ is the apparent projected surface area. A roughness parameter $R_p = 1$ corresponds to a perfectly flat substrate. The Wenzel model assumes the local contact angle is given by Young’s relation and that the roughness scale is much less than the size of the drop. The local contact angle calculated by the Young equation is then adjusted by the roughness parameter to yield the apparent contact angle on the rough surface as

$$\cos(\theta_W) = R_p \cos(\theta)$$  \hspace{1cm} (2.3)
\[ \cos(\theta_{CB}) = r_f f \cos(\theta) + f - 1 \]  \hspace{1cm} (2.4)

where \( f \) is the fraction of the projected area wet by the liquid and \( r_f \) is the roughness ratio of that wet area.

2.1.2 The Spreading Parameter

Knowledge of the contact angle allows classification of the solid substrate and offers insight into the behavior of the liquid phase of the system. When the contact angle is low, \( 0^\circ \leq \theta \leq 90^\circ \), the liquid is said to have high wettability. That is, the liquid tends to wet the surface which it is on. A contact angle \( \theta > 90^\circ \) indicates that is unfavorable for the liquid to spread on the surface. When the liquid is water, a low contact angle indicates the surface to be hydrophilic. Conversely, a high contact angle indicates a hydrophobic surface. The contact angle then may be thought of as the relative magnitudes of adhesive forces between the liquid and solid, and the cohesive forces within the liquid. If the adhesive forces are dominant, the surface tends to “pull” the liquid toward it and wet the surface. If the cohesive forces dominate, the liquid resists spreading on the surface [18]. This balance of forces is related to the surface tension and may be expressed as the spreading parameter

\[ S = \sigma_{SV} - (\sigma_{SL} + \sigma_{LV}). \]  \hspace{1cm} (2.5)
When $S > 0$, the liquid wets the surface completely. When $S < 0$, partial wetting occurs. Combined with Eq. 2.1, the spreading parameter may be expressed as

$$S = \sigma_{LV}(\cos(\theta) - 1).$$ \tag{2.6}

2.1.3 The Laplace Equation

The Laplace equation relates the pressure difference across a boundary of two fluids with the surface tension at the interface. A bubble or drop will naturally form a sphere in order to reduce its surface area in accordance with the laws of thermodynamics. As the surface tension pulls the bubble or drop into the spherical shape, an over-pressure is created within the bubble or drop. It is this over-pressure which gives rise to the tendency of smaller bubbles to converge with larger ones, and the phenomenon of capillary adhesion [18]. The general Laplace equation is

$$\Delta P = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$ \tag{2.7}

where $\sigma$ is the surface tension and $R_1$ and $R_2$ are the principle radii of curvature of the surface. Considering bubbles entrapped within a bulk fluid where there is
only one interface, the Laplace equation reduces to

\[ \Delta P = -\frac{2\sigma \cos(\theta)}{R}. \]  

(2.8)

The Laplace equation is also useful in estimating the liquid breakthrough pressure (LBP) or liquid entry pressure (LEP). Considering a porous membrane with cylindrical pores, the Laplace equation may be used to estimate the pressure at which the capillary forces within the pores of the membrane will no longer support the liquid-vapor interface [19]. Because the physical membrane is not made up of through cylindrical pores, but rather a scattered network of interconnected fibers, a geometric factor is included in the numerator of the Laplace equation which was empirically derived by Saffarini et al. [20].

As noted by several investigators [14, 21, 19, 8], the LEP as estimated by the Laplace equation assumes liquid is the only phase present on the low pressure side of the membrane. In the case of two-phase boiling, the presence of bubbles in liquid create additional complexity to the behavior of the thin films between the extraction bubble and the membrane. Therefore, the accuracy of the Laplace equation with respect to the LEP is questionable in two-phase boiling.
2.1.4 Hysteresis and Pinning of the Three Phase Contact Line

The contact angle is usually measured by either a sessile drop technique or captive bubble technique. The sessile drop technique is performed by applying a liquid drop to the top of a solid substrate and taking a profile image of the system at equilibrium. By contrast, the captive bubble technique is carried out by floating a gas bubble through the liquid and capturing it on the bottom surface of the solid substrate. From a theoretical standpoint, both methods should yield similar results given the material properties are kept constant. However, the sessile drop technique is usually preferred since it is easier to get a clear image.

Regardless of the technique used, the contact angle measured is not unique on real surfaces. Consider inflating the sessile drop and observing the advancing behavior of the three phase contact (TPC) line. It does not immediately advance whilst holding a constant equilibrium contact angle, rather it is the contact angle which increases until a certain threshold where the TPC line may be observed to slip forward, and the contact angle drops. The same is true if the opposite case is considered when deflating the sessile drop. The contact angle may be seen to continuously drop until a lower threshold is met and TPC line slips back. The threshold contact angles where TPC line slip occurs are the advancing and receding contact angles $\theta_A$ and $\theta_R$. The difference between these angles is the contact angle hysteresis [18]. In general, a well prepared surface which is very smooth may have a contact hysteresis of $< 5^\circ$, and rough or dirty surfaces may have a contact angle
hysteresis which exceeds $50^\circ$ [18].

The fundamental reason that hysteresis occurs is the local pinning of the TPC line. As the TPC line front encounters an appreciable blemish on the surface, the line front is pinned and distorted. If the TPC line continues to advance it will eventually break off of the blemish and move forward while dissipating some energy [18].

For the case of a bubble rupturing on a hydrophobic membrane, it is the receding contact angle which should describe the wetting front as the thin liquid film ruptures and the TPC line formation occurs.

2.2 Thin Film Draining and Rupture Mechanisms

2.2.1 Film Formation on Hydrophobic Surfaces

As a bubble approaches a solid surface, the continuous phase liquid is being displaced between the bubble and the solid. As the bubble nears contact with the surface, a thin layer of the bulk fluid exists between the bubble and the solid forming a film. The nature of the thin film differs to that of the bulk fluid as a result of the hydrodynamic draining and molecular interactions between the gaseous and solid phases. These molecular interactions include long range forces such as van der Waals forces and electrical double layer forces which arise from the electrical
potential between differently charged surfaces in aqueous solutions.

The Derjaguin Landau Verwey Overbeek (DLVO) [22] theory combines the effects of van der Waals and electric double layer forces to explain the net force between particles in close range. As two particles approach one another, the electrostatic repulsion forces increase while the attractive van der Waals forces also increase. The net force between these two interactions is explained by the dispersion theory of DLVO.

Experiments conducted on thin film draining and rupture have relied on DLVO theory to predict the rupture of the film; however, the theory uses only the capillary pressure as the driving force for thinning [19, 23]. Schulze et al. [23] discovered there were other forces termed “non-DLVO” which arise when the surface is hydrophobic. The additional hydrophobic force is thought to dramatically increase film draining and rupture since its effect is normally much stronger than the van der Waals and double layer forces. Krasowksa et al. [24, 25] suggested the mechanism of rupture of thin films on hydrophobic surfaces is due to nanobubbles trapped on the surface. The surface roughness has also been shown to increase the presence of these nanobubbles and accelerate the rupture of the thin film. With increasing surface roughness there are more scratches and grooves on the surface where, due to the high affinity to hydrophobic surfaces, air bubbles may become trapped. When the surface is submerged, the air bubbles on the surface have a tendency to redistribute along the surface and create a non-homogeneous surface condition. It is these nanobubbles which are thought to be responsible for the long
range hydrophobic forces, although there is still some debate about the stability of nanobubbles [25].

It is argued that nanobubbles should not be stable since a bubble of such small surface area would have an extremely high Laplace pressure. This implies that the bubble should dissolve almost immediately. However, several studies have supported the existence of nanobubbles and reported on their stability. Zhang et al. [26] examined the physical properties of nanobubbles on hydrophobic surfaces and reported that nanobubbles were stable on a submerged surface for hours. It was also noted that the TPC formed from the nanobubbles was irregularly shaped and the apparent contact angle was much higher than a macroscopic bubble in the same medium. Although the study was inconclusive on the reason for the high contact angle, two suggestions were made to explain the observed stability of the nanobubbles despite their unfavorable thermodynamic state. First, the nanobubbles took on a flattened shape which increased their radius of curvature compared to the height and lateral direction of the bubble. This would greatly reduce the Laplace pressure which drives the dissolution of the bubble. However, this reduction in the Laplace pressure was not enough to account for the stable existence of the nanobubbles, and it was therefore thought that they are perhaps trapped in a local minimum of the free energy landscape. Second, the time scale which characterizes the disappearance of nanobubbles is much longer than the experimental time scale used [26].

Although the exact nature of nanobubbles and the extent to which they effect the
surface dynamics is still inconclusive, it seems reasonable that some form of gas phase be present on the surface of a hydrophobic porous membrane. With two phases intermittently present at the membrane interface through bubble rupture and gas/vapor extraction the hydrophobicity of the material implies that no liquid film should be stable in direct contact with the membrane. It is also hypothesized that gas “pockets” rather than bubbles may exist within the membrane to reduce the instability of liquid films being in contact with the hydrophobic material.

2.2.2 Reynold’s Lubrication Approximation

The Navier-Stokes equations which describe the dynamical motion of fluids are generally difficult to solve in their entirety. A common problem-solving approach is to begin with the full Navier-Stokes equations and reduce them based on the physics of the problem at hand.

Considering the case of a bubble rising and contacting a solid surface, we want to examine the hydrodynamics of the thin liquid film formed between the liquid/vapor interface and solid surface under draining. The thin liquid film thickness, $h$, generally on the order of micrometers or nanometers, is much smaller in magnitude than the diameter of contact if the bubble diameter is on the order of a millimeter. As a consequence, the velocity in the liquid film flow is bound to be slow, or in other words, the Reynolds number is much less than unity. The inertial terms of the Navier-Stokes equations may then be safely neglected. It also stands to reason
that the flow of the draining film should be in the transverse direction. If cylindrical coordinates are used to define the system with the z-direction intersecting the poles of the bubble, then the r-direction would be the transverse flow direction. Applying these additional simplifications to the continuity and Navier-Stokes equations with no-slip boundary conditions at both $z = 0$ and $z = h$ yields

\[
\frac{\partial v}{\partial z} = -\frac{1}{r} \frac{\partial (ru)}{\partial r},
\]

(2.9)

\[
\frac{\partial p}{\partial r} = \mu \frac{\partial^2 u}{\partial z^2},
\]

(2.10)

and

\[
\frac{\partial p}{\partial z} = 0.
\]

(2.11)

More detailed derivations of these equations are offered by Shi et al. [27] and Manica et al. [28].

The application of no-slip boundary condition at the liquid/vapor interface produces a conundrum. How can there be no-slip if both phases cannot support shear? This topic has been explored by numerous studies in an attempt to model thin film draining and there are many factors which may support or oppose the validity of the no-slip boundary condition for the lubrication approximation [28,
Figure 2.1: A spherical bubble in contact with a solid plate defined with cylindrical coordinates. The draining of the thin film has a parabolic velocity profile in the transverse r-direction when the no-slip condition holds.

Vinogradova [29] modeled thin film draining between two rigid surfaces and concluded that the no-slip boundary condition could not be applied if at least one the surfaces were hydrophobic. The hydrophobic nature of the surface coincided with an “apparent slip” which was assumed to be from either long range forces or a decrease in viscosity in the thin layer adjacent to the hydrophobic surface.

Shi et al. [27] conducted a study on force measurements of bubbles on various superhydrophobic surfaces ($\theta > 140^\circ$) and concluded the boundary condition for the lubrication approximation was indeed no-slip based on their results and other reports [31]. The literature supports the application of the no-slip boundary condition when the fluid interface is not clean. The presence of contamination is thought to retard the no-shear effect of the interface and validate the no-slip condition [27, 30]. However, in ultra-clean water experiments, the no-shear boundary condition
was shown to be applicable, although such a level of cleanliness is often impractical [27].

Reynolds first visualized thin film draining as taking place between two parallel planes [32]. However, treating the fluid-fluid interface as a plane contradicts the axial component of the jump momentum balance [33]. In experiments, it has been shown that depending on the approach speed of the bubble a hydrodynamic dimple may be formed [34, 35]. It is believed that the dimple is formed from inward flow of the liquid upon approach which creates a radial ridge of lower film height surrounding the dimple. After some time, the dimple begins to drain in the outward radial direction. The formation of a dimple in the film greatly complicates modeling the film draining event due to the curvature of the interface within the film. In light of this, several attempts have been made to employ the Stefan-Reynolds flat film model to circumvent these challenges. Yet, as Chan et al. [36] point out, none of these attempts have been able to match the physics of experimental observation.

Lin and Slattery [33] devised a model for thin film draining which included the presence of the hydrodynamic dimple. They were able to obtain good agreement compared to experimental data, however, the boundary conditions used in their model were such that the dimpled film profile existed as an initial condition. Therefore, the approach and draining before dimpling occurred was not able to be calculated.
Later, Yiantsios and Davis [37] developed a model for thin film draining under constant buoyant force. They used a perturbation method with asymptotic analysis to derive weak forms of the solution, and then extended this to a full numerical implementation. The asymptotic scheme included solving for the inner solution within the film, and considering the shape of the bubble far outside of the interaction zone of the film. The solutions were then matched to give the corresponding boundary condition just outside of the film radius.

Similar analyses were carried out to study the bouncing behavior of a bubble drop approaching a solid surface, as well as an extension to include force modeling with atomic force microscopy [28, 35, 38, 36, 39, 40]. These models built upon the work of Yiantsios and Davis [37] and related the lubrication approximation with the augmented Laplace equation.

Yaminski et al. [34] conducted a study on two approaching bubbles and observed that for larger gaps between the approaching bubbles and rapidly expanding film formation, the dimples formed were non-axisymmetric and induced chaotic draining, which further complicates the ability to model the draining using lubrication theory. Although many studies have been conducted to model the hydrodynamic dimple, the subsequent draining models have been established from experimental values for specific materials and dimensions.

Generally, the application of the lubrication approximation is made under the pre-tense that the film is semi-infinite between two boundaries. When considering the
application of the lubrication approximation on a drop of fluid on a dry substrate, the TPC line is on the front of the flow. Imposing a no-slip boundary condition on the TPC line front with relative motion between the liquid and solid leads to a singularity in the stress and nonphysical infinite force [41]. A workaround for this is to introduce a small slip parameter in the neighborhood of the TPC where the velocity gradient is large. The difficulty in introducing such a parameter is that on smaller scales when surface tension forces are non-negligible, the local dynamic contact angle must be known to apply the correct boundary condition at the TPC line.

Another solution is to define a very thin precursor film of known height and constant value ahead of the TPC line. This method allows for the no-slip condition to be applied without risk of the singularity arising [41]. Despite the likely presence of irregular geometry in the thin film, the lubrication approximation offers good results when the contact radius is much larger than the thickness of the film.

The Reynolds lubrication approximation has been investigated as a means to model the thin film draining between a bubble and a solid surface. The literature offers a cautionary note on the validity of the boundary conditions for specific cases.
2.2.3 Rupture Mechanisms

A film may be stable, metastable, or unstable. In thin films the thickness of the film mitigates against the effects of gravity and molecular forces are dominant. Referring back to Young’s relation (Equation 2.1), and considering a macroscopic liquid film on a solid substrate, there is a liquid/vapor and liquid/solid interface associated with the film. The energy per unit area of the film may then be described as the sum of the interfacial tensions

$$E = \gamma_{LV} + \gamma_{SL}. \quad (2.12)$$

However, when the film becomes sufficiently thin, the role of intermolecular forces may have a considerable effect on the energy of the film [18, 42]. It is then appropriate to include an additional term in Equation 2.12 to account for the possible long range forces at play. The energy equation for thin films then becomes

$$E = \gamma_{LV} + \gamma_{SL} + P(h) \quad (2.13)$$

where $h$ is the film thickness, $P(\infty) = 0$ and $P(0) = \gamma_{SV} - \gamma_{SL} - \gamma_{LV}$. The additional function arises from the need to account for the energy when the liquid film thickness goes to zero. In that case, the energy of the liquid/vapor and liquid/solid interfaces must be equal to the energy of the bare solid. Within the
range of thin films, the $P(h)$ function accounts for the possible long range forces and can also be used to define the disjoining pressure [18],

$$\Pi(h) = -\frac{dP}{dh}. \quad (2.14)$$

The function $P(h)$ may then also define the stability of a the film. If $P(h)$ has positive curvature ($P''(h) > 0$), then the film is said to be stable. Likewise, if the curvature is negative ($P''(h) < 0$), then the film is unstable and prone to rupture [18, 42].

Two modes of rupture which are readily identified are rupture by nucleation and spinodal rupture. Nucleated rupture may be subdivided into homogeneous and heterogeneous nucleation. Homogeneous nucleation occurs by thermal activation whereas heterogeneous nucleation is spurred by chemical heterogeneity or local defects within the film or interface. In practical cases, heterogeneous nucleation is normally always the responsible mechanism due to the presence of contamination or molecular defects [42]. Spinodal rupture occurs by the amplification of capillary waves induced by a density fluctuations of the thin film. The competing long range forces between the liquid/vapor and liquid/solid interfaces lead to the amplification of the capillary waves at the liquid/vapor interface until the amplitude of the largest capillary wave reaches the film thickness and creates a hole in the film at the substrate [18, 43].
Although in theory these two mechanisms of rupture appear to be easy to differentiate, from an experimental view it is difficult to decouple them. Nguyen et al. [44] performed a study on dewetting using polymeric and metallic thin films. Their findings suggest multiple destabilizing forces present in film rupture due to the almost inescapable nature of heterogeneous nucleation from impurities and defects at the interfaces as well as spinodal decomposition and temperature gradients. Lessel et al. [43] also studied the dewetting of polymeric films on hydrophilic and hydrophobic substrates. They concluded that the rupture mechanism with the hydrophilic substrate was spinodal, but found the the dominant rupture mechanism to be nucleation when the substrate was hydrophobic. This was thought to be from the slip condition at the hydrophobic interface which allowed the film to resist rupture from capillary waves and give preference to a nucleation mode of rupture and subsequent dewetting. [43].

In their study of the presence of nanobubbles on hydrophobic surfaces, Krasowska et al. [25] postulated that the rupture mechanism could be due to nucleation of the submicroscopic gas bubbles or nanobubbles. Further, it was suggested that the rupture of the film could occur at any film thickness since the size of the gas nuclei vary can within a wide range. It was proposed that rupture by nucleation began at a small hole, probably at the site of the largest submicroscopic gas bubbles, and dewetting of the entire film area occurred over a very short time [25].

Further support for the theory of air-induced nucleation at hydrophobic surfaces was offered by Krasowska et al. [24] and Zawala and Kosior [45] when they studied...
the effects of surface roughness on the formation of, and dynamics of, the TPC line with hydrophobic Teflon®. The hydrophobic substrate was prepared by sanding with 100, 600 and 2500 grit sandpaper. Surface roughness for the substrates was measured to be $80 - 100 \mu m$, $40 - 60 \mu m$ and $1 - 5 \mu m$ for the 100, 600 and 2500 grit sandpapers, respectively. They found the time to rupture and TPC formation to significantly decrease with increasing surface roughness and attributed this to the increased presence of gas trapped on the roughened surface which established a Cassie state at the interface and introduced slip to the boundary condition [45].

The main conclusions from the literature suggests that for the case of a porous hydrophobic membrane, it is likely that small pockets of gas exist at the interface. If this condition is assumed, then by consequence the interface may be described as being in the Cassie state. Although there is some ambiguity to the specific circumstances of where the boundary conditions should be expected to be either no-slip, semi-slip, or full-slip, the literature implies this is directly linked to the roughness of the surface and the extent to which the Cassie state is maintained.

2.3 In Situ Vapor Extraction

The benefit of two-phase flow in microscale heat sinks is quickly offset by the risk of vapor entrainment and subsequent dryout. The nature of multi-phase flow also includes instabilities within the flow which can lead to higher pumping power
requirements. The idea of localized vapor extraction was first introduced by Zhou et al. [6] as a means to mitigate against vapor entrainment and stabilize the flow by making it single-phase [46].

In situ vapor extraction simply means to extract the vapor “in its place”. The current model proposed by Fox et al. [10] and Juarez [12] is aimed at optimizing the confined geometry to increase the heat transfer in two-phase pool boiling. The confined geometry allows for exploiting the high energy associated with phase change from liquid to vapor and removing the vapor from the heated surface when its presence retards the heat transfer by insulating and causing dryout. The challenges identified in the theoretical model were accurately characterizing the surface effects and dynamics of the bubble on the supply and extraction surfaces, which were circumvented by using empirical correlations from experimental data. The regimes established by Fox et al. [10] and Juarez [12] were tailored to accept the empirical correlations to complete the model for the lifetime of the bubble from inception to extinction. However, to remove the empirical correlations, the model may need to be separated into more distinct regimes dependent upon the working fluid and material properties.

One such example of this is the so called “induction time”, which is the minimum time the bubble must be in contact with the extraction surface to drain, rupture and form a stable two-phase attachment onto the extraction surface [47, 48]. The induction time has been studied for mineral flotation in mining applications. More specifically, the induction time is defined by three stages: (1) the thin film draining
from an initial thickness to a critical thickness; (2) the spontaneous rupture from the critical film thickness to form a nuclei hole and establish the TPC line, and (3) the spread of the TPC line from a critical radius to a stable radius on the extraction surface [48].

It has been noted in the literature that in practice the measurement of these distinct stages is quite difficult, and they are usually measured collectively and referred to as the induction time. Wang et al. [49] proposed a phenomenological model for determining the induction time between an air bubble and a hydrophobic sphere. They concluded that the time for rupture and formation of the TPC line only accounted for approximately 5% of the total time that the bubble was in contact with the hydrophobic sphere. This implies the majority of the induction time is due solely to the film draining process and further simplified the induction time to be approximately equal to the draining time [49].

In the current model by Fox et al. and Juarez [10, 12], the smaller bubble diameters grow beyond the gap height indicating that the bubble does not rupture instantaneously upon reaching the extraction surface. The conclusion is that the film between the bubble and extraction surface must be draining whilst the bubble continues to grow. As the gap heights become larger, the experimental rupture diameters reported become less than the gap height which indicates a breakdown of the spherical bubble assumption. In either case, a film must exist between the bubble and extraction surface before rupture can take place. The induction time then is of paramount interest because it accounts for the time lag between when
the bubble diameter reaches the equivalent gap height and when the bubble rup-
tures and forms the extraction surface area for which Darcy’s law may be then
applied.

The notable trends from induction time experiments by Hewitt et al. [50] indicate
that the induction time decreases with increasing contact angle and decreasing
bubble size. The conclusion from these trends in terms of the current study is
that the induction time is decreased with increasing hydrophobicity and higher
Laplace pressure in smaller bubbles. These trends also appear to be consistent
with the experiments carried out by Fox et al. and Juarez [10, 12] with regards to
the bubble size.
Chapter 3: Review of the Current Model

This thesis is based on the previous work of Fox et al. [10] and Juarez [12]. The diabatic experiment by Fox et al. [10] was conducted in approximately 150 ml of distilled water which was boiled vigorously with submerged cartridge heaters for 30 minutes to degas the water. The heaters in the water tank also maintain the water at a temperature of $99.0 - 99.2 \, ^\circ C$ which was measured with a K-type unsheathed thermocouple. The adiabatic experiment performed by Juarez [12] was carried out at room temperature, and there was no mention of degassing the water in this experiment. Both the diabatic and adiabatic experiments were performed in the same test chamber.

The extraction surface was formed from a hydrophobic polytetrafluoroethylene (PTFE) membrane from Sterlitech with a specified pore diameter of $0.45 \mu m$, porosity of 55% and thickness of $15.6 \mu m$. The membrane also included a laminated support layer. The membrane was secured to the bottom of a glass tube measuring $4.2 mm$ OD, $2.1 mm$ ID and a vacuum tube was attached to the opposite end to create a $35 \, kPa$ and $14.4 \, kPa$ pressure differential across the membrane for the diabatic case and adiabatic case, respectively.

The supply surface for the diabatic case was created with a $675 \mu m$ thick silicon
substrate and an artificial nucleation site measuring 30\(\mu m\) in diameter and 100\(\mu m\) depth was machined into the substrate. The nucleation site was heated with a high-repetition-rate pulsed laser at 81.4 \(W/cm^2\). The supply surface for the adiabatic case was manufactured from a 1\(mm\) thick aluminum plate with a 0.5\(mm\) through hole which served as the orifice for air to be injected into the bulk fluid. The air was injected at a constant volumetric flow rate of 90 \(mm^3/s\).

More detailed experimental setup and procedures are outlined in the work done by Fox et al. [10] and Juarez [12].

A notable difference in the development of the force balance model between the two experiments is the nature of the TPC at the supply surface. For the diabatic case, the TPC is not constant with bubble growth. As the bubble grows, the TPC on the supply surface spreads out. For the adiabatic case, the TPC was effectively pinned on the orifice which made the diameter of the bubble on the supply surface constant throughout Regime 1 and Regime 2. The high surface energy associated with aluminum made this possible, and eased calculation of the surface tension force on the supply surface as compared to the diabatic case [12].

During the experiments the types of bubbles produced were categorized. Type 1 bubbles were those which reached the extraction surface and ruptured before departure. Type 2 bubbles coalesced with the previous bubble prior to departure. Type 3 bubbles departed from the extraction surface prior to rupturing. Fox et al. [10] indicated that Type 1 bubbles were the primary interest in the model since they
were observed to be the predominant type of bubble in small gap heights where bubble depart at small diameters in high frequencies. It was also anticipated that Type 1 bubbles would be the most useful in spatio-temporal cooling [10].

3.1 Bubble Diameter Model

Both the diabatic and adiabatic studies included a bubble diameter model which serves to predict the diameter of the bubble as a function of time throughout the lifetime of the bubble from inception to extinction. This was based on control volume analysis where separate equations were utilized for the respective regimes outlined in Figure 1.1. The proposed bubble diameter equations for the three regimes are as follows:

\[ D_1(t) = \sqrt[3]{\frac{6\dot{V}_0 t}{\pi}} \]  
(3.1)

where \( \dot{V}_0 \) is the constant volumetric flow rate from the supply surface. This equation is valid for unconfined growth defined by Regime 1.

\[ D_2(t) = \sqrt[3]{\frac{6}{\pi} \left[ V_R + \dot{V}_0(t - t_R) - \frac{K\Delta p A_{ext,2}}{\mu g \delta}(t - t_R) \right]} \]  
(3.2)

where \( V_R \) and \( t_R \) denote the volume and time at rupture, respectively. This equa-
tion is valid from the time of rupture to the time of departure (Regime 2). A linear variation of the extraction area from zero to $A_{\text{ext},3}$ is assumed. The relation is of the form

$$A_{\text{ext},2} = A_{\text{ext},3} \frac{t - t_R}{t_D - t_R}. \quad (3.3)$$

From experimental data it was determined the extraction rates in Regime 3 remained nearly constant. This allowed for the assumption that the extraction diameter also remained fairly constant, and could be estimated as $D_{D} = d_{\text{ext},3}$, where $D_{D}$ is calculated by the relation

$$D_{D} = 0.81H \ (\pm 0.03\text{mm}) \quad (3.4)$$

for the diabatic case, and

$$D_{D} = 0.84H \ (\pm 0.14\text{mm}) \quad (3.5)$$

for the adiabatic case. Applying this modification into Equation 3.2 yields

$$D_{2}(t) = \sqrt[3]{\frac{6}{\pi}} \left[ V_{R} + V_{0}(t - t_{R}) \right] - \frac{K\Delta p A_{\text{ext},3}}{\mu_{\theta} \delta(t_{D} - t_{R})(t - t_{R})^{2}} \right] \quad (3.6)$$
where \( t_D \) denotes time at departure. The final equation for the bubble diameter model is given by Equation 3.7 and is valid from the time of bubble departure to extinction (Regime 3)

\[
D_3(t) = \sqrt[3]{\frac{6}{\pi}} \left[ V_D - \frac{K \Delta p A_{ext,3}}{\mu_g \delta} (t - t_D) \right].
\] (3.7)

In summary, the combination of Equations 3.1, 3.6 and 3.7 are used to predict the diameter of the confined bubble from inception to extinction as a function of time.

### 3.2 Force Balance for Confined Extraction

Although briefly introduced in the introduction section of this thesis, the relevant forces acting on the confined bubble will be revisited here. Again, Fox et al. and Juarez [10, 12] began with a modified version of the momentum equation for pool boiling offered by Thorncroft et al. [13] and altered it to include terms for confined extraction. The momentum balance equation for Regime 1 is

\[
\frac{d(mu)}{dt} = F_B + F_G + F_{\sigma, sup} + (\dot{mu})_i
\] (3.8)

where the time derivative represents the time rate of change of momentum of the
bubble and the term \( \dot{m}u \) represents the momentum of the vapor/gas entering the bubble. The additional terms in Equation 3.8 are the net buoyant force, \( F_B \), growth force, \( F_G \), and surface tension force on the supply surface, \( F_{\sigma,\text{sup}} \). They are defined as follows:

\[
F_B = \frac{\pi D^3 g (\rho_l - \rho_g)}{6}, \quad (3.9)
\]

\[
F_G = 2\pi \rho_l \dot{r}^2 \dot{r}, \quad (3.10)
\]

and

\[
F_{\sigma,\text{sup}} = \pi \sigma d_{\text{sup}} \sin(\theta_{\text{sup}}), \quad (3.11)
\]

The term \( \dot{r} \) in Equation 3.10 is determined from

\[
\dot{r} = k n l^{n-1} \quad (3.12)
\]

where \( n = 1/3 \) for both the adiabatic and diabatic cases. For the adiabatic case, \( k = 0.00277 \, m/s^{1/3} \) and \( d_{\text{sup}} \) is the orifice diameter. For the diabatic case, \( k = 0.00215 \, m/s^{1/3} \) and \( d_{\text{sup}} = D \sin(\theta_{\text{sup}}) \).
In Regime 2, the bubble has reached the extraction surface and ruptured, introducing an additional surface tension force on the extraction surface and exit momentum term. The momentum balance for Regime 2 is then

\[
\frac{d(mu)}{dt} = F_B + F_{\sigma, sup} + F_{\sigma, ext} + (\dot{mu})_i - (\dot{mu})_e
\]  

(3.13)

where \((\dot{mu})_e\) represents the momentum of the vapor/gas exiting the bubble through the extraction surface. The surface tension force on the extraction surface, \(F_{\sigma, ext}\), is defined by

\[
F_{\sigma, ext} = \pi \sigma d_{ext} \sin(\theta_{ext}).
\]  

(3.14)

Regime 3 begins at the moment the bubble detaches from the supply surface. At this stage, the surface tension term at the supply surface is assumed negligible and the inlet momentum term at the supply surface goes to zero. The momentum balance for Regime 3 is

\[
\frac{d(mu)}{dt} = F_B + F_{\sigma, ext} - (\dot{mu})_e.
\]  

(3.15)

Each force and momentum term was computed every time step in conjunction with the bubble diameter model to determine the appropriate values for the diameters in each term. Also, it was noted from Fox et al. [10] that Equation 3.12 was
derived assuming a spherical bubble growth normal to the supply surface and further reasoned that $\dot{r}$ should go to zero when the bubble first makes contact with the extraction surface. However, from experimental results it was observed that some bubbles ruptured at a diameter less than the gap height, indicating the bubble may not be spherical. As a result, the growth force in the model is extinguished at the time of rupture to alleviate ambiguity.

3.3 Data Analysis

A review of the experimental data from Fox et al. [10] and Juarez [12] offers insight into the variance between the theoretical model and experimental observations. It also serves as a way to identify potential for improvement in the current model.

3.3.1 Diabatic Case

As a result of the uncertainty of the TPC line behavior at the supply and extraction surfaces, empirical correlations were made from the experimental data to estimate the rupture and departure diameters of the bubble to fit the theoretical model. Through linear regression, the empirical equations for the rupture and departure diameters, respectively, are
\[ D_R = 0.83H + 0.16 \ (\pm 0.08mm) \] (3.16)

and

\[ D_D = 0.81H \ (\pm 0.03mm) \] (3.17)

where \( H \) is the measured gap height from the supply surface to the extraction surface. It was noted that these correlations were only valid for the range of gap heights used in the experiment and were not expected to predict bubble diameters as the gap height approached zero. The average bubble rupture diameters and departure diameters from the experiments conducted by Fox et al. [10] are presented in Table 3.1.

Table 3.1: Average bubble rupture diameters and departure diameters from experiments, correlations, bubble model and/or static force for the diabatic case.

<table>
<thead>
<tr>
<th></th>
<th>Diabatic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H ) (mm)</td>
<td>0.52 0.75 1.07 1.40 1.60 1.67 1.90 ( \infty )</td>
</tr>
<tr>
<td># Bubbles</td>
<td>109 142 79 93 50 41 9 37</td>
</tr>
<tr>
<td>( D_R ) (mm)</td>
<td>Experiments 0.56 0.72 1.09 1.32 1.52 1.53 1.71 N/A</td>
</tr>
<tr>
<td>Rupture Correlation</td>
<td>0.59 0.78 1.05 1.32 1.49 1.54 1.74 N/A</td>
</tr>
<tr>
<td>( D_D ) (mm)</td>
<td>Experiments 0.42 0.58 0.93 1.13 1.30 1.35 1.53 1.53</td>
</tr>
<tr>
<td>Departure Correlation</td>
<td>0.42 0.61 0.87 1.13 1.30 1.35 1.54 N/A</td>
</tr>
<tr>
<td>Force Balance</td>
<td>0.44 0.64 0.93 1.22 1.41 1.47 1.69 1.54</td>
</tr>
</tbody>
</table>

The data in Table 3.1 shows good agreement (\( \sim 9\% \) and \( \sim 7\% \)) between the theoretical model and the experimental rupture and departure diameters, respectively.
However, as may be seen from the work of Fox et al. [10], the time at which the theoretical model predicted the diameters was not in good agreement with experiments, especially in the smaller 0.52\(mm\) gap height case where the time was over-predicted by \(\approx 60\%\) for both rupture and departure. This may be attributed to the area of extraction used in Darcy’s Law.

3.3.2 Adiabatic Case

Similar to the diabatic case, empirical correlations were made for the adiabatic bubble diameters at rupture and departure, respectively, from experimental data and linear regression

\[
D_R = 0.62H + 0.84 \ (\pm 0.15mm) \tag{3.18}
\]

and

\[
D_D = 0.84H \ (\pm 0.14mm). \tag{3.19}
\]

A summary of the experimental findings in the adiabatic case are presented in Table 3.2.

The adiabatic case also shows good agreement with the predicted bubble rupture
Table 3.2: Average bubble rupture diameters and departure diameters from experiments, correlations, bubble model and/or static force for the adiabatic case.

<table>
<thead>
<tr>
<th></th>
<th>Adiabatic</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (mm)</td>
<td>1.22 2.14 3.25 ∞</td>
</tr>
<tr>
<td># Bubbles</td>
<td>10 10 10 10</td>
</tr>
<tr>
<td>$D_R$ (mm) Experiments</td>
<td>1.53 2.29 2.80 N/A</td>
</tr>
<tr>
<td></td>
<td>Rupture Correlation</td>
</tr>
<tr>
<td>$D_D$ (mm) Experiments</td>
<td>0.97 1.91 2.68 2.88</td>
</tr>
<tr>
<td></td>
<td>Departure Correlation</td>
</tr>
</tbody>
</table>

and departure diameters as compared with the experimental data ($\sim 6\%$ for rupture and departure). Similar to the diabatic case, the time at which the model predicted the respective diameters was shifted, over-predicting the time at which the events occurred compared to the experiment.
Chapter 4: Proposed Model for Liquid Film Drainage

This chapter serves as the outline to the proposed modifications to the current model to include the period of bubble contact with the extraction surface to the instant of rupture and subsequent TPC line formation.

The insight gained from the literature review has suggested that the interfacial dynamics at the extraction surface are complicated and depend on several factors. To the author’s knowledge, there is no generic relation which can be made to characterize all of the possible draining regimes and rupture mechanisms for all materials. Rather, a set of specific conditions or assumptions must be employed to the model to limit the number of possibilities that may occur.

In an effort to maintain the utility of the model, simplification is a highly weighted factor. However, there is risk of over-simplifying, such as assuming the film is perfectly flat, which could lead to erroneous conclusions. The challenge with micro and nano-scales is the ability to accurately monitor the phenomena which are at play. Contrary to macro-scale experiments, in these scales it is often on the limit or beyond which we are capable of measuring or observing with satisfactory accuracy, necessitating further dependence on theory.

The exact crossover point from macro to micro or nano is ill-defined. Ultimately,
the use of two-phase vapor extraction is poised to be an efficient heat sink for relatively small devices. With this in mind, the scale then would tend towards the smaller scale, and assumptions relevant to this scale should be imposed.

Without actual measurements of the film thickness during the experiments of Fox et al. [10] and Juarez [12] to which to compare, a qualitative study of the thin film draining phenomenon will be conducted using the existing experimental values for rupture diameter. Because of the availability of more data in the diabatic case by Fox et al. [10], the results from this case will be used to develop the theory of confined film draining and rupture in the present thesis.

4.1 Redefining the Model Regimes

The current model of Fox et al. [10] defines three regimes in which the lifetime of the bubble takes place as may be seen in Figure 1.1. The rupture and departure diameters of the bubble were assessed through empirical means which fit nicely into those three regimes. However, in order to reduce the empiricism of the current model it is proposed to include an additional regime in the model to coincide with the draining of the liquid film between the bubble and extraction surface and eventual rupture.

The updated regimes of confined extraction are depicted in Figure 4.1 and are defined as follows: Regime 1, begins at inception and terminates once the bubble
Figure 4.1: Four regimes of confined vapor extraction: (a) Regime 1: growth only, (b) Regime 2a: growth and formation of thin film on extraction surface, (c) Regime 2b: growth with extraction, and (d) Regime 3: extraction only.

diameter is equal to the gap height. Regime 2a, begins when the bubble diameter is equal to the gap height and ends at the moment of rupture of the thin film. Regime 2b, begins at the moment of rupture and ends at the moment of departure, and Regime 3 begins at the moment of departure and ends at extinction of the bubble.
4.2 Governing Equations for Thin Liquid Film Draining

The addition of Regime 2a allows for a defined time interval to model the thin film draining and rupture. As noted in the literature review, this time interval corresponds to the induction time.

As the bubble approaches the extraction surface, the continuous liquid phase between the bubble interface and extraction surface must be displaced. However, as the bubble interface nears contact with the extraction surface, it deforms and flattens out at the apex of the bubble forming a thin film. As the bubble continues to grow, more deformation of the interface occurs and the thin film radius increases. At the same time, the forcing of the bubble into the extraction surface causes the thin film to drain, where at some time, the film would become sufficiently thin and the effects of the disjoining pressure would be significant. These additional forces act to enhance or retard the draining effect of the film depending on their attractive or repulsive nature [33]. Eventually, the thin film would drain to a stable equilibrium, or become unstable and rupture. In the present study, the extraction surface is hydrophobic which implies unstable film behavior.

Assuming axisymmetric draining, it is convenient to adopt a radial coordinate system in the region of the thin film. This may be seen in Figure 4.2.

As the bubble approaches the extraction surface and $h(r,t)$ becomes much smaller than the radius of the bubble, then creeping flow may be assumed in the re-
Figure 4.2: Axisymmetric diagram for region of thin film draining.

Assuming a no-slip boundary condition on the extraction surface and at the liquid-
vapor interface, $u = v = 0$ at $z = 0$ and $z = h(r, t)$, and observing that the pressure is independent of $z$ from Equation 4.3, Equation 4.2 can be integrated twice with respect to $z$ to give the velocity profile

$$u = \frac{1}{2\mu} (z^2 - h z) \frac{\partial p}{\partial r}. \quad (4.4)$$

After substituting Equation 4.4 into the continuity equation (Equation 4.1) and integrating from $z = 0$ to $z = h$, the equation for the evolution of the thickness of the thin film is obtained

$$\frac{\partial h}{\partial t} = \frac{1}{12\mu r} \frac{\partial}{\partial r} \left( rh^3 \frac{\partial p}{\partial r} \right). \quad (4.5)$$

To account for the local deformation of the bubble interface, the augmented Young-Laplace equation may be used to relate the curvature of the interface to the pressure across the interface

$$\sigma (\kappa_1 + \kappa_2) \equiv \frac{\sigma}{r} \frac{\partial}{\partial r} \left( \frac{rh_r}{(1 + h_r^2)^{1/2}} \right) = \frac{2\sigma}{R_L} - \Pi(h) - p \quad (4.6)$$

where $h_r = \partial h/\partial r$, $R_L$ is the Laplace radius and $\Pi(h)$ is the disjoining pressure. Due to the assumed small deformation compared to the bubble radius, $R$, the Laplace radius, $R_L \approx R$. When the film is nearly flat, $\partial h/\partial r \ll 1$, Equation 4.6 may be linearized to obtain
\[
\frac{\sigma}{r} \frac{\partial}{\partial r} \left( r \frac{\partial h}{\partial r} \right) = \frac{2\sigma}{R_L} - \Pi(h) - p. \tag{4.7}
\]

It is noted that the Young-Laplace equation is derived from energy minimization, and is valid for equilibrium models. A consideration of the capillary wave speed of the bubble interface to the characteristic approach velocities in this study can justify the use of Equation 4.7. Capillary waves of velocity \( c \) and wavelength \( \lambda \) on a spherical bubble with surface tension \( \sigma \) obey the dispersion relation \( c^2 = \frac{2\pi\sigma}{\lambda(\rho_b + \rho_c)} \), where \( \rho_b \) and \( \rho_c \) are the densities of the bubble and continuous phase, respectively. Using an upper limit of the size of deformation, \( \lambda \approx 100\mu m \), gives \( c \approx 2m/s \), which is much faster than the approach velocities of the bubble in this study. The interface of the bubble may then be considered to deform instantaneously under the presence of hydrodynamic interactions. This result allows for the safe assumption that the effects of the hydrodynamic pressure and disjoining pressure may be treated with a quasi-steady equilibrium model [28, 36].

In general, the relationship between the approach velocity and bubble radius dictates the initial thickness of the liquid film between the bubble and solid surface [28]. In the work of Carnie et al. [40], they examined the liquid film between deformable drops of radii up to \( 100\mu m \) at a maximum approach velocity of \( 13.2\mu m/s \) where the effects of the disjoining pressure were considered. However, at bubble radii above this range and higher approach velocities, the hydrodynamic pressure
is expected to be dominant over the effects of the disjoining pressure due to the formation of thicker liquid films. In the current study, the bubble radius is directly influenced by the gap height between the supply and extraction surfaces, with the smallest gap height of $0.52\,mm$ in the diabatic case. The radius of the smallest bubble considered here is then $\approx 250\,\mu m$. Since the bubble growth rate at smaller gap heights is higher, the approach of the bubble interface to the extraction surface is also be higher, leading to more appreciable hydrodynamic effects. With this assumption, the effects of the disjoining pressure are presumed negligible and Equation 4.7 then becomes

$$\frac{\sigma}{r} \frac{\partial}{\partial r} \left( r \frac{\partial h}{\partial r} \right) = \frac{2\sigma}{R_L} - p.$$  \hspace{1cm} (4.8)

The combination of Equations 4.5 and 4.7 or 4.8 form the basis of the Stokes-Reynolds-Young-Laplace, or SRYL model. Since there is no closed-form solution to Equations 4.5 and 4.7 or 4.8, the equations are generally solved numerically outside of weak perturbation solutions.

4.3 Numerical Method

The length scales of the film draining phenomenon occurs over several orders of magnitude. The radius of the bubble is assumed to be on the order of a millimeter, and the thin film thickness may drain to the thicknesses in the nanometer range. As
such, the numerical method is carried out by nondimensionalization of Equations 4.5 and 4.8.

The small interaction zone which extends to the film radius and is of thickness $h$ describes the thin liquid film that the bubble and extraction surface interact through. Then as proposed by Davis et al. [37] and Carnie et al. [40], the natural length scale in this region is $r \sim \sqrt{hR_0}$. The pressure is scaled by $\sigma/R_0$, and the time scale is set by the approach velocity $t \sim h/v_0$. The important parameters are the approach velocity $v_0$, the surface tension $\sigma$, and the liquid viscosity $\mu$. These parameters can be combined into the capillary number, $Ca \equiv \mu v_0/\sigma$, which compares the viscous force to the surface tension force. The scaled variables then take the form

$$
\begin{align*}
  h & \sim Ca^{1/2}R_0 \\
  r & \sim Ca^{1/4}R_0 \\
  p & \sim \sigma/R_0 \\
  t & \sim Ca^{1/2}R_0/v_0 
\end{align*}
$$

(4.9)

where $v_0$ is the characteristic velocity defined as the velocity of the non-deforming bubble interface at the moment it reaches the extraction surface and $R_0$ is taken to be $H/2$, or half of the gap height. More explanation of how the velocity is determined is offered later in this section.

To account for the bubble growth and change in Laplace pressure over the course
of the liquid film draining, the Laplace radius $R_L$ is taken to be the equivalent bubble radius, $R$, at any given time.

After applying the scaled parameters in Equation 4.9, Equations 4.5 and 4.7 take the form

$$\frac{\partial h^*}{\partial t^*} = \frac{1}{12r^* \partial r^*} \left( r^* h^{*3} \frac{\partial p^*}{\partial r^*} \right)$$  \hspace{1cm} (4.10)

and

$$\frac{1}{r^* \partial r^*} \left( r^* \frac{\partial h^*}{\partial r^*} \right) = \frac{2R_0}{R} - p^*$$  \hspace{1cm} (4.11)

where * denotes dimensionless units. Assuming axisymmetric draining, the computational domain of the solution begins at the axis of symmetry ($r^* = 0$) and extends to a value $r_{\text{max}}^*$ which is defined outside of the contact film radius. As noted by Klaseboer et al. [39], the choice of $r_{\text{max}}^*$ does not change the drainage dynamics of the solution and is an over-estimation of the the contact film radius to ensure that all of the drainage dynamics occur within the radial domain. The arbitrary choice of $r_{\text{max}}^*$ in other works has ranged from 70-120% of the bubble radius $R$ [28, 39]. In the present study, $r_{\text{max}}$ is chosen to be equal to $R_0$ resulting in a ratio of $r_{\text{max}}/R_0 = 1$. However, the effects of this ratio on the solution in the special case of confined geometry with bubble growth will be explored in Chapter
The coupled set of partial differential equations require one initial condition and four boundary conditions. Setting $p^* = 0$ in Equation 4.11 and solving for $h^*$ yields a parabolic profile which is a good approximation of a spherical surface in two dimensions when $r \ll R$. It takes the form

$$h^*(r^*,0) = h_0^* + \frac{R_0}{R} \frac{r^*}{2}$$

where $h_0^*$ is the initial central ($r^* = 0$) height of the film interface from the extraction surface at $t^* = 0$. It is assumed that $h_0^*$ be sufficiently large such that the bubble interface is still undeformed.

Taking advantage of symmetry, the natural boundary conditions at $r^* = 0$ are

$$\frac{\partial h^*}{\partial r^*} = 0$$

and

$$\frac{\partial p^*}{\partial r^*} = 0.$$  

At $r^* = r_{max}$ the pressure in the continuous phase tends to zero as $r^* \to \infty$ when the bulk pressure is taken to be zero. However, it may be noted that the far-field
pressure decays as $1/r^4$ and may be implemented as

$$r^4 \frac{\partial p^*}{\partial r^*} + 4p^* = 0 \quad (4.15)$$

to save computational space [28, 35, 37].

The last boundary condition serves as the driving function for the solution and is applicable at $r^* = r^*_\text{max}$

$$\frac{\partial h^*}{\partial t^*} = -v^*(r^*_\text{max}, t^*). \quad (4.16)$$

The velocity function in Equation 4.16 is obtained from Equation 3.12. Observing that the bubble is "pinned" (see Figure 4.3) on the supply supply surface, the velocity of the approaching bubble interface is taken to be equal to $2\dot{r}$.

The spatial derivatives in Equations 4.10 and 4.11 are discretized with second order central difference approximations in the interior nodes, and first order forward and backward differences at first and last nodes, respectively. The time derivative is discretized by a backward difference approximation, leading to an implicit numerical scheme.

Given the initial parabolic profile of the film interface, the numerical scheme is carried out by solving for the pressure profile, $p^*(r^*, t^*)$, in Equation 4.10. This
Figure 4.3: Depiction of the pinned bubble at the supply surface and the velocity of the bubble center of mass over the period of unconfined growth.

The pressure profile is then substituted into Equation 4.11 to solve for a new deformed interface profile. The pressure and interface profiles are solved for iteratively and the process is repeated until the desired convergence error is reached. The converged solutions of \( h^*(r^*, t^*) \) and \( p^*(r^*, t^*) \) are then stored as the initial guess values for the next time step. A successive under-relaxation iteration scheme is implemented to help convergence. For the present work, the convergence tolerance is \( 10^{-5} \) and the relaxation factor is 0.9. The radial domain is calculated between \( r^* = 0 \) and \( r^* = r^*_{\text{max}} \), where \( r^*_{\text{max}} \) is determined from scaling the ratio of \( r_{\text{max}}/R_0 \), and \( \Delta r^* = 0.025 \). The time step is \( \Delta t^* = 0.05 \).
As noted earlier, as the gap height increased, the rupture diameters reported by Fox et al. [10] were less than the gap height, indicating the bubble was not spherical. To account for the presumed “necking” of the bubble at the supply surface, a relative correction is applied. Assuming the smallest gap height in the diabatic case (0.52 mm) remained spherical and using the reported experimental rupture diameter of 0.56 mm, a non-spherical bubble correction factor is proposed as

\[ \chi = \left| \frac{H_{0.52} - D_{R,0.52}}{H_{0.52}} \right| = \left| \frac{0.56 - 0.52}{0.52} \right| \approx 0.077. \] (4.17)

The correction factor \( \chi \) is then applied to the remaining gap height rupture diameters to make the rupture diameter larger than the gap height. As an example, for the next gap height of \( H = 0.75 \text{mm} \), the corrected rupture diameter is calculated as

\[ D_{R,0.75} = H_{0.75}(1 + \chi). \] (4.18)

It should be noted, that in all practicality even the bubbles formed in the gap height of 0.52 mm have some degree of “necking”, but the smallest diameter bubble has the highest Laplace pressure, and therefore the highest tendency to remain spherical. As a result, the correction factor given in Equation 4.17 is effectively normalizing the rupture diameters in the data set of Fox et al. [10] and is a relative correction factor for the gap heights in this study. The reported experimental (Fox et al.
Table 4.1: Non-spherical bubble correction factor applied to rupture diameters for given gap heights in the diabatic case.

<table>
<thead>
<tr>
<th>Diabatic</th>
<th>H (mm)</th>
<th>0.52</th>
<th>0.75</th>
<th>1.07</th>
<th>1.40</th>
<th>1.60</th>
<th>1.67</th>
<th>1.90</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_R$ (mm)</td>
<td>Experiments</td>
<td>0.56</td>
<td>0.72</td>
<td>1.09</td>
<td>1.32</td>
<td>1.52</td>
<td>1.53</td>
<td>1.71</td>
</tr>
<tr>
<td>Corrected</td>
<td>0.56</td>
<td>0.81</td>
<td>1.15</td>
<td>1.51</td>
<td>1.72</td>
<td>1.80</td>
<td>2.04</td>
<td></td>
</tr>
</tbody>
</table>

The choice of $h_0$ for each gap height was determined by using the scaling in the interaction zone $r \sim \sqrt{h_0 R_0}$ and the radial length scale $r \sim Ca^{1/4}R_0$. Solving for $h_0$ between these scales gives

$$h_0 = \alpha Ca^{1/2} R_0$$  \hspace{1cm} (4.19)

where $\alpha$ is a positive constant multiplier. The function of $\alpha$ is to increase $h_0$ and thus begin the numerical simulation earlier in time to allow for the first few time steps to normalize the interface and pressure profiles before any appreciable hydrodynamic effects are seen. In the present work $\alpha = 2$, which coincides with $h_0^* = 2$.

With $h_0$ defined, the diameter of the bubble at the beginning of the film draining regime (Regime 2a) may be determined. Equation 3.1 is then solved for time to define the time span of the film draining regime from the initial time, $t_0$, to the predicted rupture time, $t_R$. An example of this calculation is provided here:
The initial diameter of the spherical bubble at the beginning of Regime 2a is $D_0$, where $D_0$ is the gap height minus the initial offset, or $H - h_0$. The initial time of Regime 2a may then be determined as

$$t_0 = \frac{\pi (D_0)^3}{6V} = \frac{\pi (H - h_0)^3}{6V}. \quad (4.20)$$

The predicted time of rupture, $t_R$, is determined in a similar manner using the corrected bubble diameter, $D_R$. The calculation is

$$t_R = \frac{\pi (D_R)^3}{6V}. \quad (4.21)$$

The initial time and predicted time of rupture are presented in Table 4.2 for all gap heights.

Table 4.2: Initial time of Regime 2a and predicted time of rupture for given gap heights in the diabatic case.

<table>
<thead>
<tr>
<th>H (mm)</th>
<th>0.52</th>
<th>0.75</th>
<th>1.07</th>
<th>1.40</th>
<th>1.60</th>
<th>1.67</th>
<th>1.90</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_0$ (ms)</td>
<td>2.24</td>
<td>5.28</td>
<td>8.01</td>
<td>17.90</td>
<td>32.20</td>
<td>45.90</td>
<td>64.00</td>
</tr>
<tr>
<td>$t_R$ (ms)</td>
<td>2.97</td>
<td>6.91</td>
<td>10.30</td>
<td>23.00</td>
<td>41.20</td>
<td>58.70</td>
<td>80.80</td>
</tr>
</tbody>
</table>
4.4 Verification of Numerical Scheme

Chan et al. [38] proposed an analytical perturbation solution for two approaching bubbles and compared it to the full numerical solution of Equation 4.10 and a modified version of Equation 4.11. The modification of Equation 4.11 is an additional factor of $1/2$ which arises from the approach of two spherical objects, rather than a sphere approaching a flat plate. Notwithstanding, the relevant physics of the problem remain the same. A dimensionless velocity ramp $v^*(t) = v_0(1 - e^{-t})$ where $v_0 = -1$ for approaching bubbles is used to drive the solution from an initial dimensionless height of $h_0^* = 10$.

The full numerical solution of the central thickness of the thin film for this study and the results of Chan et al. [38] are compared in Figure 4.4. As reference, the non-deforming analytical solution determined by integrating the dimensionless velocity function is also provided. Note that the non-deforming interfaces would make contact with one another at a dimensionless time of $t^* = 11$.

The boundary condition at $r^*_{max}$ in the solution of Chan et al. [38] included additional terms relating the outer shape of the bubble to the film force arising from the hydrodynamic pressure in the film. They used this relation to exploit force measurements with atomic force microscopy experiments. However, in the present study, only the inner part of the solution within the film radius is of interest and the simple far-field boundary condition has been successfully employed in other works with the velocity function alone [28, 39].
Figure 4.4: Comparison of the numerical solution for the central thickness of a deformable surface versus a rigid surface. The comparison data is taken from Chan et al. [38]. All units are dimensionless.

The slight deviation in the numerical solutions depicted in Figure 4.4 may be attributed to the convergence tolerance, the choice in $\Delta r^*$, or $\Delta t^*$, none of which were explicitly stated in the work of Chan et al. [38]. The simple boundary condition used in this work may also give rise to some difference in the solution, however this is not suspected to have much influence at the central film thickness region [39]. Nonetheless, the good agreement in the numerical solutions gives confidence in the numerical method employed. For this verification, $\Delta r^* = 0.025$ and $\Delta t^* = 0.05$. The convergence tolerance is set to $10^{-5}$. From the plots in the work by Chan et al. [38], $r^*_{\text{max}}$ is chosen to be 30.
Chapter 5: Results and Discussion

As the bubble grows and approaches the extraction surface, an appreciable hydrodynamic pressure builds up in the interaction zone between the bubble interface and extraction surface. Under certain conditions, the hydrodynamic pressure in the liquid film may exceed the Laplace pressure inside the bubble and lead to reverse curvature of the bubble interface. This phenomenon is known as the “hydraulic dimple” and is characterized by the presence of a barrier rim where the thinnest part of the liquid film is no longer at the apex.

In theory, the development of a dimple should occur for any approach velocity in the absence of disjoining pressure, however if the approach velocity was very small, the resulting minimum film thickness would be unreasonably small as well [51]. The existence of a dimple also implies more complex draining phenomenon than which can be modeled by the simplified flat-film model and emphasizes the importance of the more physically realistic Stokes-Reynolds-Young-Laplace (SRYL) model.

In previous works the SRYL model was used to predict thin film draining behavior for steady bubbles approaching with constant velocity or interacting under constant force. In this study, the SRYL model has been adapted to model film draining of a confined bubble that is transiently growing within the gap and throughout
the draining process. This adaptation attempts to capture the effects that the changing Laplace pressure inside of the bubble has on the liquid film draining as it approaches the extraction surface.

Another difference in the present work comes from the velocity function used to drive the solution. In other works, the velocity at \( r_{\text{max}} \) was either determined from experimental data, or a force balance, including the liquid film pressure force, at each time step to determine the velocity at \( r_{\text{max}} \). In the confined geometry, however, the bubble is attached to the supply surface as it approaches the extraction surface and has a negative net force acting on it until departure from the supply surface.

For Type 1 bubbles, Fox et al. [10] determined the dominant force for departure from the supply surface is the surface tension force at the extraction surface, which does not exist until after the bubble ruptures and forms the TPC line on the extraction surface. In light of this, the most reasonable path forward was to use the radial growth function provided by Fox et al. [10] to describe the bubble’s approach to the extraction surface. Because this driving function acts as the coupling mechanism between the macroscopic bubble and microscopic interaction zone, the boundary condition should be consistent with the choice of \( r_{\text{max}} \).

As may be seen in Figure 5.1, a large deviation in the solution occurs if the choice of \( r_{\text{max}}/R_0 \) differs. This suggests that the radial velocity function must be scaled accordingly to match to the speed of the interface for the selected choice of \( r_{\text{max}}/R_0 \).
Figure 5.1: Comparison of the effect of scaling the radial velocity function at $r_{max}^*$ for select values of $r_{max}/R_0$: (a) dimensionless pressure profiles at predicted rupture time $t_R^*$, and (b), dimensionless film force from initial dimensionless time, $t_0^*$, to the predicted rupture time, $t_R^*$. Solid lines are with scaling applied (Equation 5.1) and dashed lines are without scaling (Equation 4.16). All solutions shown are for a gap height of 0.52 mm.

This is necessary because the radial velocity function is not directly coupled to the film force within the interaction zone as with previous works.

In Figure 5.1(a), the dimensionless pressure profiles at the predicted time of rupture show that the central pressure at the apex is within $\approx 5\%$ over the range of $r_{max}/R_0$, with or without scaling the radial velocity function at $r_{max}^*$. Although there is obvious deviation in the pressure profile farther away from the apex, the need for the scaling the radial velocity function is more apparent in Figure 5.1(b). Figure 5.1(b) shows a very large deviation of the dimensionless film force over the range of $r_{max}/R_0$ when the radial velocity function is not scaled. However,
by scaling the radial velocity function by the ratio $r_{max}/R_0$, the choice of $r_{\text{max}}^*$ becomes reasonably independent of the solution. In actual implementation, the boundary condition at $r^* = r_{\text{max}}^*$ takes the form

$$\frac{\partial h^*}{\partial t^*} = -\left(\frac{r_{\text{max}}}{R_0}\right) v^*(r_{\text{max}}^*, t^*).$$  \hspace{1cm} (5.1)

Note that Equations 4.16 and 5.1 are equivalent when $r_{\text{max}}/R_0 = 1$. For clarity, it should be distinguished that the remaining analysis is presented using a ratio $r_{\text{max}}/R_0 = 1$.

For the diabatic case, the relevant variables and parameters for all gap heights are summarized in Table 5.1. The liquid viscosity, $\mu$, and surface tension, $\sigma$, are evaluated at the saturation temperature and are $0.282 \times 10^{-3} \text{ kg/ms}$ and $58.9 \text{ mN/m}$, respectively.

Table 5.1: Variables and parameters used in the numerical simulation for all gap heights in the diabatic case.

<table>
<thead>
<tr>
<th>Diabatic</th>
<th>H (mm)</th>
<th>t_0 (ms)</th>
<th>t_R (ms)</th>
<th>t_R - t_0 (ms)</th>
<th>\dot{V} (mm^3/s)</th>
<th>R_0 (mm)</th>
<th>V_0 (cm/s)</th>
<th>Ca (10^{-4})</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (mm)</td>
<td>0.52</td>
<td>0.75</td>
<td>1.07</td>
<td>1.40</td>
<td>1.60</td>
<td>1.67</td>
<td>1.90</td>
<td></td>
</tr>
<tr>
<td>t_0 (ms)</td>
<td>2.24</td>
<td>5.28</td>
<td>8.01</td>
<td>17.90</td>
<td>32.20</td>
<td>45.90</td>
<td>64.00</td>
<td></td>
</tr>
<tr>
<td>t_R (ms)</td>
<td>2.97</td>
<td>6.91</td>
<td>10.30</td>
<td>23.00</td>
<td>41.20</td>
<td>58.70</td>
<td>80.80</td>
<td></td>
</tr>
<tr>
<td>t_R - t_0 (ms)</td>
<td>0.73</td>
<td>1.63</td>
<td>2.29</td>
<td>5.10</td>
<td>9.00</td>
<td>12.80</td>
<td>16.80</td>
<td></td>
</tr>
<tr>
<td>\dot{V} (mm^3/s)</td>
<td>31</td>
<td>40</td>
<td>77</td>
<td>78</td>
<td>65</td>
<td>52</td>
<td>55</td>
<td></td>
</tr>
<tr>
<td>R_0 (mm)</td>
<td>0.260</td>
<td>0.375</td>
<td>0.535</td>
<td>0.700</td>
<td>0.800</td>
<td>0.835</td>
<td>0.950</td>
<td></td>
</tr>
<tr>
<td>V_0 (cm/s)</td>
<td>-8.060</td>
<td>-4.590</td>
<td>-3.490</td>
<td>-2.060</td>
<td>-1.390</td>
<td>-1.100</td>
<td>-0.884</td>
<td></td>
</tr>
<tr>
<td>Ca (10^{-4})</td>
<td>3.86</td>
<td>2.20</td>
<td>1.67</td>
<td>0.986</td>
<td>0.666</td>
<td>0.527</td>
<td>0.423</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.2 depicts the evolution of the liquid film for select gap heights at the initial
time, \( t_0^* \) and \( t_0 \), and the predicted rupture time, \( t_R^* \) and \( t_R \), in dimensionless units (a) and dimensional units (b), respectively. Note in Figure 5.2(a) that the initial interface location at \( t_0^* \) is the same for all gap heights. Most notable in Figure 5.2 is that the smallest gap height of 0.52\text{mm} does not show the smallest minimum film thickness at the time of rupture, as one may expect. This result is attributed to the difference in approach velocity and Laplace pressure in confined geometry compared to freely rising fixed-size bubbles.

Figure 5.2: Axisymmetric liquid film thickness for select gap heights in (a) dimensionless units at initial time, \( t_0^* \) (dashed lines), and predicted rupture time \( t_R^* \) (solid lines), and (b) in dimensional units at initial time, \( t_0 \) (dashed lines), and predicted rupture time \( t_R \) (solid lines).

In the work of Manica et al. [28], they modelled the rise and interaction of bubbles with a glass surface. Two bubble radii were used (385\text{µm} and 630\text{µm}) of similar size as the bubbles being investigated in this study. During the bubble rise, the
bubbles were released sufficiently far enough away from the glass surface to reach terminal velocity, which was modelled as the balance between buoyant force and hydrodynamic drag force on the bubble. Their results showed an increase in the terminal velocity, and thus an increase in the approach velocity with an increase of bubble radius. The subsequent film draining model also showed that the smaller bubble radius drained to a smaller liquid film thickness compared to the larger bubble size. However, in this study the opposite case is true. In the confined geometry, the approach velocity, a function of the radial growth rate from the supply surface, is infinitely large in magnitude at inception of the bubble and decays with time like $t^{-2/3}$. This implies that the approach velocity is higher at smaller gap heights, i.e., smaller radii bubbles. For the range of gap heights in this study, the respective approach velocities are given in Figure 5.3.

The other aspect of the confined geometry arrangement which differs from freely rising bubble models is that the bubble is growing as it approaches the extraction surface. This means that the Laplace pressure is not constant throughout the draining process.

If the Laplace pressure were treated as a constant value based on the radius of the spherical bubble at $R_0$, then the non-dimensional Laplace pressure would be 2. However, taking into consideration the change in the bubble radius from the initial time, $t_0^*$, to the predicted time of rupture, $t_R^*$, the Laplace pressure is scaled by the ratio of $R_0/R$. How this scaling affects the non-dimensional Laplace pressure may be seen in Figure 5.4.
Figure 5.3: Velocity of bubbles at different gap heights from time at initial position \( t_0 \), to time at predicted rupture \( t_R \). Note that the negative velocity corresponds to approaching bubbles when the z-axis is taken positive in the downward direction.

From Figure 5.4, it may be observed that the variable non-dimensional Laplace pressure varies from the constant value by \( \approx 7.5\% \) at the predicted rupture time. However, this variable Laplace pressure over the course of the liquid film draining process yields appreciable differences in the solution at the time of rupture.

The time span from when the spherical bubble reaches the extraction surface to the predicted time of rupture is very short, ranging from \( \approx 0.7ms \) to \( \approx 17ms \) for gap heights of 0.52mm and 1.90mm, respectively. Naturally, the question arises whether or not the need to account for the change in Laplace pressure over such a small amount of time is necessary. In Figure 5.5, the pressure profiles and corresponding film thicknesses for select gap heights are shown comparing the
Figure 5.4: (a) Change in the non-dimensional Laplace pressure as a function of time for a gap height of 0.52\textit{mm}, and (b) the change in the non-dimensional Laplace pressure as a function of time for a gap height of 1.90\textit{mm}.

constant Laplace pressure and variable Laplace pressure. The need to account for the change in the bubble size and Laplace pressure over the time interval is apparent from Figure 5.5.

Treating the Laplace pressure as a constant yields an over-prediction of the hydrodynamic pressure within the liquid film. Consequently, the degree of deformation of the bubble interface is also over-predicted. From Figure 5.5, it may also be seen that for the smaller gap heights (0.52\textit{mm} and 1.07\textit{mm}), the constant Laplace pressure predicts the presence of a hydrodynamic dimple. However, with a variable Laplace pressure at the same gap heights, no dimple has formed and much less deformation of the bubble interface is observed.
Figure 5.5: (a) Comparison of pressure profiles at predicted rupture time $t_R$, for constant Laplace pressure and variable Laplace pressure. (b) Comparison of film thickness at predicted rupture time $t_R$ for constant Laplace pressure and variable Laplace pressure. Solid lines indicate variable Laplace pressure and dashed lines are constant Laplace pressure. For clarity, only select gap heights are shown.

In Figure 5.6, the non-dimensional and dimensional pressure profiles are presented for all gap heights with a variable Laplace pressure. Unsurprisingly, the smallest gap height of 0.52 mm produced the highest magnitude of hydrodynamic film pressure due to the higher approach velocity. Note that the pressure profile does not flatten out at the apex for the smaller gap heights compared to the larger gap heights. When the pressure profile flattens, it is an indication that the hydrodynamic film pressure is equivalent to the Laplace pressure in the bubble. This also implies that after the formation of a dimple, the film pressure within the region of the dimple is always equivalent to the Laplace pressure of the bubble.
Figure 5.6: Comparison of variable Laplace pressure profiles at predicted rupture time $t_R$. (a) Dimensionless pressure profiles for all gap heights and (b) dimensional pressure profiles for all gap heights. For clarity, only the positive radial axis profiles are shown.

The higher approach velocity of the bubbles at smaller gap heights also induces higher radial velocity within the film. With immobile boundary conditions at the extraction surface and bubble interface, the maximum radial velocity inside the liquid film is

$$U_{\text{max}} = -\frac{h^2 \partial p}{8\mu \partial r}. \quad (5.2)$$

In Figure 5.7, the maximum radial velocity and corresponding film thicknesses are presented for the smallest and largest gap heights, 0.52$\text{mm}$ and 1.90$\text{mm}$, respectively. The time interval begins approximately half way through the simulation.
and ends at the predicted time of rupture. Initially, the radial velocity in the film is very high, but decreases as the film thickness decreases.

Figure 5.7: (a) Maximum radial velocity in film and (b) corresponding film thicknesses at select times for a gap height of 0.52mm. (c) Maximum radial velocity in film and (d) corresponding film thicknesses at select times for a gap height of 1.90mm. Solid lines represent the variable Laplace pressure and dashed lines are with a constant Laplace pressure. Note that only the positive radial axis is shown.

For comparison, the case of constant Laplace pressure is also provided in Figure 5.7. Again, the effect of treating the Laplace pressure as a constant may be seen. Beyond the time that the non-deforming bubble would have reached the extraction
surface, the Laplace pressure in the bubble is decreasing with the film pressure. This lower film pressure is evident in the lower magnitude of radial velocity as compared to the constant Laplace case.

The similar magnitude of liquid film thickness at the predicted time of rupture over the range of gap heights (see Figure 5.2) is believed to stem from the inverse relationship between the approach velocity and bubble size. With the bubble size growing during the draining process, the decreasing Laplace pressure also reduces the hydrodynamic liquid film pressure resulting in bubbles at larger gap heights having a slightly smaller minimum liquid film thickness at the predicted rupture time, contrary to what is expected in the free rising bubble scenario.

To analyze the effect of a longer duration of liquid film draining on the film thickness, simulations were carried out for the smallest and largest gap heights, 0.52\text{mm} and 1.90\text{mm}, respectively, for 20% beyond the predicted time of rupture. Figure 5.8 shows the evolution of the liquid film at the initial time, predicted rupture time and 20% beyond the predicted rupture time.

As expected, the film thickness is decreased for both gap heights when the simulation is run beyond the predicted rupture time; however, the difference in film thickness of both gap heights at the predicted rupture time and 20% beyond the predicted time is still \( \approx 0.5\mu m \). This suggests that even if the calculation of the predicted rupture time using the corrected rupture diameter with the non-spherical correction factor was flawed, similar magnitudes of film thickness are expected over
Figure 5.8: Evolution of the liquid film thickness at the initial time, \( t_0 \), predicted rupture time, \( t_R \), and 20% beyond the predicted rupture time, \( t_{R120\%} \) for gap heights of (a) 0.52\( mm \), and (b) 1.90\( mm \).

In the Fox et al. [10] model, the rupture times were predicted by dividing the volume of the bubble at the time of rupture by the volumetric growth rate, \( \dot{V} \), where the volume of the bubble at rupture was determined by the empirical rupture diameter correlation of Equation 3.16. To investigate the effect of the non-spherical correction factor on the predicted rupture times for this study and that of Fox et al. [10], the respective predicted rupture times are presented in Figure 5.9 for all gap heights.

The effect of the non-spherical correction factor on the predicted rupture times is apparent from Figure 5.9. At larger gap heights, the predicted rupture times
Figure 5.9: Predicted rupture times from this study and from the model of Fox et al. [10] for all gap heights.

in this study are grossly over-estimated. This stems from $t \propto D^3$ in Regime 1, and is expected since the non-spherical correction forced the diameter of the bubble to be larger than what was observed experimentally. The larger deviation in predicted rupture times at larger gap heights is also indicative of the degree of non-sphericity of bubbles at larger gap heights. However, there is reasonably good agreement ($\sim 20\%$) in the predicted rupture times for the smaller gap heights up to 1.4mm.
5.1 Further Discussion

The smallest value for the minimum liquid film thickness observed at the predicted time of rupture was \( \approx 2.7\mu m \) for a gap height of 1.90\text{mm}. This minimum film thickness is far outside of the bounds that typical long range forces would have any appreciable effect on the film rupture [40]. However, several other factors could lead to film rupture at these larger film thicknesses.

Local defects on the extraction surface could serve as nucleation sites which protrude through the liquid film and create local thin spots within the film where long range forces may become dominant. This was not accounted for in the present draining model, as the extraction surface was treated as a perfectly flat and rigid surface. In reality, the membrane making up the extraction surface is a woven mesh of hydrophobic fibers and is expected to have some degree of deformation, which would reduce the kinetic energy of the bubble upon approach. A reduction in the kinetic energy upon approach would result in lower film pressure, and thus lower film thicknesses.

The likely presence of vapor pockets trapped on the hydrophobic extraction surface from repeated bubble bursts may also serve as effective rupturing aids to the thin liquid film. The aerophilic attraction of these nearby vapor pockets could serve to locally thin the liquid film to a critical thickness [25].

In a practical application, it is also likely there are trace contaminants in the
bulk fluid. These contaminants coupled with the radial flow in the film upon
the bubble approaching the extraction surface may also give rise to Marangoni
effects, where contaminant adsorption on the film interface may cause local surface
tension gradients, leading to instabilities in the liquid film. Marangoni stresses in
the liquid/air interface have been shown to be highly effective in rupturing the
thin film of spherical cap bubbles at similar film thicknesses as explored here [52].
In addition, the intrinsic perturbation of the moving fluid in the region of the
interaction zone may exacerbate this phenomenon.

Given the array of possible rupture mechanisms, the absolute reason for film rup-
ture is still elusive and may be stochastic in nature. Still, knowledge of the mecha-
nisms which increase the likelihood of film rupture may be exploited in a practical
implementation of an in-situ vapor extraction heat sink.

In the case of vapor extraction, any delay due to liquid film drainage on the ex-
traction surface would negate the overall performance of such a device since the
frequency of bubbles from the supply surface would be slowed by this induction
time. From a design perspective, the goal would be then to initiate film rupture as
soon as possible. This may include the addition of a protrusion on the extraction
surface such that a local thin site exists in the liquid film and initiates ruptures
sooner. An increase in the surface roughness of the hydrophobic membrane may
also encourage a consistent Cassie state at the extraction surface by increasing the
entrapment of vapor pockets. Also, the addition of surfactant into the working
fluid may aid in reducing the film draining time by lowering the surface tension at
the liquid/vapor interface.

The ultimate goal in this research was to theoretically determine the rupture time of confined bubbles and subsequent TPC line expansion on the extraction surface in the model proposed by Fox et al. [10]. The present study would have offered a replacement to the empirical rupture and departure diameters given by Equations 3.16, 3.4, 3.18 and 3.5. However, without experimental data on the thin liquid film height, only qualitative findings on the behavior of the draining of the bubbles can be assessed at this time. Further, the rupture mechanism responsible could not be definitively identified, and was not modelled in this study. Despite the shortcoming of the ultimate research goal, many new insights have been gained from the present work.
Chapter 6: Conclusions and Recommendations for Future Work

An adaption of the SRYL lubrication model has been developed for the application of a growing bubble in a confined geometry. The approach velocity which drives the solution is determined from the radial growth of the bubble which is pinned at the supply surface. The model also accounts for the variable Laplace pressure within the growing bubble during the liquid film draining process.

In using the experimental data of Fox et al. [10], it was found that for larger gap heights, the reported rupture diameter of the bubble was less than the gap height. This suggests that the bubble does not remain spherical, and necessitated that a correction be applied to the reported rupture diameters to ensure the rupture diameter was larger than the gap height to coincide with a liquid film draining event at the extraction surface. This non-spherical correction led to an over-prediction of the rupture time for bubbles at larger gap heights.

The need for the correction may be circumvented if the bubble were modelled as non-spherical, perhaps with a spline fit. However, this would introduce additional complexity in the bubble growth model and may not be necessary if the optimum heat transfer coefficient is experimentally found with smaller gap heights where it is more likely that the growing bubbles remain more spherical due to higher
Laplace pressures.

Results indicate that the predicted minimum liquid film thickness at the predicted time of rupture ranges from \( \approx 3.16\mu m \) to \( \approx 2.72\mu m \) for gap heights ranging from 0.52\text{mm} \) to 1.90\text{mm}, respectively. This finding was contrary to free rising bubble studies, where smaller bubble sizes resulted in smaller predicted minimum liquid film thickness. It is believed this is due to the inverse relationship of the bubble size and approach velocity in confined geometry as well as decreasing Laplace pressure with bubble growth.

The minimum liquid film thicknesses predicted for all gap heights studied here are outside the bounds which typical long range forces such as van der Waals forces and electrical double layer forces are dominant. Given this finding, alternate rupture mechanisms are suspected in rupturing the liquid film.

Due to the presumed stochastic nature of film rupturing, the rupturing event was not modelled in the present study. Further limitations include the treatment of the extraction surface. The extraction surface was modelled as an ideal surface; however, the membrane making up the extraction surface likely has some degree of deformation as the hydrodynamic film pressure builds upon bubble approach. This deformation of the extraction surface may have stronger implications on the liquid film draining behavior for smaller gap heights, where the approach velocity and subsequent liquid film pressures are higher. These items are suggested for future studies.
Bibliography


APPENDICES
%% SRYL Adapted Model for Confined Bubble

clear; clc; clf; close all;

%% Define Parameters

prompt = 'Enter "a" for adiabatic or "d" for diabatic : ';
str = input(prompt,'s');
if isempty(str)
    str = 'a'; % No input defaults to adiabatic
end

if str == 'a'

%%%% Adiabatic properties

sigma = 0.072 ; % Surface tension water [N/m] (Adiabatic case)
mu_L = 1*10^-3; % Dynamic viscosity water [N*s/m^2] [kg/m*s]
rho_L = 998; % Density water
rho_G = 1.2; % Density air
drho = rho_L-rho_G;
orf = 0; % Neglect orifice offset
H = 1.22/1000; %Gap Height [m]
RD = 1.53/1000;
Vdot = 90./(1000^3); %[m^3/s] Juarez rate (adiabatic)
K = 0.00277; %rdot constant 'k'

elseif str == 'd'

%%% Diabatic properties

sigma = 0.072 ; % Surface tension water [N/m] (Adiabatic case)
mu_L = 1*10^-3; % Dynamic viscosity water [N*s/m^2] [kg/m*s]
rho_L = 998; % Density water
rho_G = 1.2; % Density air
drho = rho_L-rho_G;
orf = 0; % Neglect orifice offset
H = 1.22/1000; %Gap Height [m]
RD = 1.53/1000;
Vdot = 90./(1000^3); %[m^3/s] Juarez rate (adiabatic)
K = 0.00277; %rdot constant 'k'

end
%%% Diabatic Properties

\[ \sigma = 0.0589; \quad \text{% Surface tension water @ 100\textdegree C (Diabatic case)} \]
\[ \mu_L = 0.282 \times 10^{-3}; \quad \text{% Dynamic viscosity water \([\text{N}\cdot\text{s}/\text{m}^2]\) \([\text{kg}/\text{m}\cdot\text{s}]\)} \]
\[ \rho_L = 958; \quad \text{% Density water} \]
\[ \rho_G = 0.95; \quad \text{% Density air} \]
\[ \Delta \rho = \rho_L - \rho_G; \]
\[ \text{orf} = 0; \quad \text{% Neglect orifice offset} \]
\[ H = [0.52, 0.75, 1.07, 1.40, 1.60, 1.67, 1.90]/1000; \quad \text{% [m]} \]
\[ \text{RD} = [0.56, 0.808, 1.15, 1.508, 1.723, 1.800, 2.040]/1000; \]
\[ V_{\text{dot}} = [31, 40, 77, 78, 65, 52, 55]/(1000^3); \quad \text{% [m}^3/\text{s]} \]
\[ K = 0.00215; \quad \text{%r} \]

\text{end}

%% Get values at nondeforming 'contact' for scaling

\[ \alpha = 2.0; \quad \text{%h0 multiplier} \]
\text{for } z = 1:\text{length}(H)

\[ t_{\text{contact}}(z) = \text{round}((H(z)^3\pi)/(6\cdot V_{\text{dot}}(z)),3,'Significant'); \]
\[ R_0(z) = \text{round}(H(z)/2,3,'Significant'); \]
\[ V_0(z) = \text{round}((2/3)\cdot K\cdot t_{\text{contact}}(z)^{-2/3},3,'Significant'); \]
\[ C_a(z) = \text{round}(\mu_L\cdot V_0(z)/\sigma,3,'Significant'); \]
\[ h_00(z) = \text{round}(\alpha\cdot C_a(z)^{(1/2)}\cdot R_0(z),3,'Significant'); \]
% Time at h00 offset
\[ t_{h00}(z) = \ldots \]
\[ \text{round}((H(z) - h00(z))^3 \pi)/(6 \times V_{dot}(z)), 3, 'Significant'); \]
% Time at corrected rupture
\[ t_{rupture}(z) = \text{round}((RD(z)^3 \pi)/(6 \times V_{dot}(z)), 3, 'Significant'); \]
end

%% Regime I code for plotting
R_{max} ratio = 1; \quad \% \text{Ratio} \ r_{max}/R_{0}
for \ z = 1: length(H)
    \ dr = 0.025; \quad \% \text{space step}
    \ dt = 0.05; \quad \% \text{Time Step}

    % Adjusted for gap heights
    \ r_{max} \ raw = R_{max} \ ratio/(Ca(z)^{(1/4)});
    \ % \text{Rounds} \ r_{max} \ \text{for} \ dr
    \ r_{max}(z) = \text{floor}(r_{max} \ raw) + \text{ceil}( \ldots
    \quad \quad \quad \quad \quad \quad (r_{max} \ raw - \text{floor}(r_{max} \ raw))/dr) \times dr;

    \ r\{z\} = 0:dr:r_{max}(z);
    \ N = \text{length}(r\{z\});

    % Max dimensionless time (choose from scaling real time)
    % goes until corrected rupture
    \ t_{max} \ raw = (t_{rupture}(z) \times (V_{0}(z))/(Ca(z)^{(1/2)} \times R_{0}(z)));
    \ % \text{Rounds} \ t_{max} \ \text{for} \ dt
    \ t_{max}(z) = \text{floor}(t_{max} \ raw) + \text{ceil}( \ldots
    \quad \quad \quad \quad \quad \quad (t_{max} \ raw - \text{floor}(t_{max} \ raw))/dt) \times dt;
% Offset time for run in
\[ t_{\text{startraw}} = \frac{(t_{\text{h00}}(z) \cdot V_0(z))}{(C_0(z)^{1/2} \cdot R_0(z))}; \]

% Rounds tmax for dt
\[ t_{\text{start}}(z) = \text{floor}(t_{\text{startraw}}) + \ldots \]
\[ \text{ceil}( (t_{\text{startraw}} - \text{floor}(t_{\text{startraw}}))/dt) \times dt; \]
\[ t\{z\} = t_{\text{start}}(z) \cdot dt : \text{tmax}(z); \]
\[ S = \text{length}(t\{z\}); \]

% ND driving velocity function — accepts dimensionless time
\[ V_t = @(t, C_0, R_0, V_0) \left(-2K/(3 \cdot V_0^{1/3} \cdot C_0^{1/3} \cdot R_0^{2/3})\right) \ldots \]
\[ \times t^{(\frac{-2}{3})}; \]

% Instantaneous Bubble radius — Function accepts ...
\[ R_t = @(t, C_0, R_0, V_0, V_{\text{dot}}) \ldots \]
\[ (1/2) \times \left((6 \cdot V_{\text{dot}} \cdot C_0^{1/2} \cdot R_0 \cdot t)/(\pi \cdot V_0)\right) \ldots \]
\[ \times (1/3); \]

% Initial nondimensional height of interface (unperturbed)
\[ h_0(z) = h_{00}(z)/(C_0(z)^{1/2} \cdot R_0(z)); \]
\[ h\{z\} = \text{zeros}(N, S); \]
\[ h\{z\}(:, 1) = h_0(z) + \ldots \]
\[ (R_0(z)/R_t(t\{z\}(1), C_0(z), R_0(z), V_0(z), V_{\text{dot}}(z))) \cdot r\{z\}^{2/2}; \]
\[ P\{z\} = \text{zeros}(N, S); \]
dhdt{z} = zeros(N,S);
% Initial approach speed of interface
dhdt{z}(:,1) = ones(N,1)*Vt{z}(1),Ca(z),R0(z),V0(z));

F.film{z} = zeros(1,S);
dPdr = zeros(N,S);
Umax{z} = zeros(N,S);
NDR{z} = zeros(1,S); % Nondeforming reference at apex
NDR{z}(1) = h0(z);

figure(5); hold on; title('Interface Profile');
xlabel('r'); ylabel('h');
plot(r{z},h{z}(:,1),'--');
figure(6); hold on; title('Pressure Profile');
xlabel('r'); ylabel('P');
figure(8); hold on; title('dh/dt as f(r,t)');
xlabel('r'); ylabel('dhdt');
ylabel('$$\mathbf{\textbf{\frac{\partial h}{\partial t}}}$$',...
       'Interpreter', 'latex');

w = 0.9; %SOR parameter
tol = 10e-5; % tolerance
flag1 = 1; % For contact time of NDR

index_NDR(z) = 1; % Temporary ND time index for NDR contact

% Begin loop for drainage
for n = 2:S

% Non-deforming reference at apex
NDR{z}(n) = NDR{z}(n-1) + dt*(Vt(t{z}(n),Ca(z),RO(z),V0(z)));
Pkp1 = [P{z}(::,n-1), P{z}(::,n-1)]; %Set initial guess
hkpl = [h{z}(::,n-1), h{z}(::,n-1)]; %Set initial guess

err = 1; % Reset error for next loop
k = 0;
while err > tol
k = k+1;
%Get pressure profile from Stoke's eqn
for i = 2:N %N:-1:2
if i == N
Pkp1(i,2) = ((r{z}(end)/dr)*Pkp1(i-1,2))/...
( (r{z}(end)/dr) + 4); %BC
else
ap = -(2*r{z}(i)*hkpl(i,2)^3)/(dr^2);
av = (hkpl(i,2)^3)/(2*dr) + ... 
((3*r{z}(i)*hkpl(i,2)^2)... 
/(4*dr^2))*(hkpl(i+1,1)-hkpl(i-1,2))... 
- (r{z}(i)*hkpl(i,2)^3)/(dr^2);
ae = -(hkpl(i,2)^3)/(2*dr) - ... 
((3*r{z}(i)*hkpl(i,2)^2)/(4*dr^2))... 
*(hkpl(i+1,1)-hkpl(i-1,2))... 
- (r{z}(i)*hkpl(i,2)^3)/(dr^2);
b = 12*r{z}(i)*(hkpl(i,2) - h{z}(i,n-1))/dt;
Pkp1(i,2) = (aw*Pkp1(i-1,2) + ae*Pkp1(i+1,1) ...
+ b)/ap;

end

end

Pkp1(1,2) = Pkp1(2,2);

Pstar = Pkp1(:,2);
Pkp1(:,2) = Pkp1(:,1) + w*(Pstar - Pkp1(:,1));
local_err = sum(abs(Pkp1(:,2) - Pkp1(:,1)));
Pkp1(:,1) = Pkp1(:,2); %Update Pressure

% Calculate deformation from augmented Laplace eqn
for i = 2:N
    if i == N
        hkp1(i,2) = h{z}(i,n-1) + dt*Rmax_ratio ...
* (Vt{z}(n),Ca(z),R0(z),V0(z));
    else
        ap = -2/(dr^2);
        aw = 1/(2*r{z}(i)*dr) - 1/(dr^2);
        ae = -1/(2*r{z}(i)*dr) - 1/(dr^2);
        % Without disjoining Pressure
        b = ...
        2*(R0(z)/(Rt{z}(n),Ca(z),R0(z),V0(z),Vdot(z)))...
        - Pkp1(i,2);
        hkp1(i,2) = (aw*hkp1(i-1,2) + ae*hkp1(i+1,1) ...
        + b)/ap;
    end

end
hkpl(1,2) = hkpl(2,2);
hstar = hkpl(:,2);
hkpl(:,2) = hkpl(:,1) + w*(hstar - hkpl(:,1));
err = sum(abs(hkpl(:,2) - hkpl(:,1)));
hkpl(:,1) = hkpl(:,2); % Update h profile
end

disp(k);

% Store h and P profiles
h{z}(:,n) = hkpl(:,2);
P{z}(:,n) = Pkpl(:,2);
dhdt{z}(:,n) = (h{z}(:,n) - h{z}(:,n-1))/dt;

% Film force
Ff = 0;
for i = 1:N-1
    Ff = Ff + ( (P{z}(i+1,n) + P{z}(i,n))/2 ) ... 
          *(pi*(r{z}(i+1)^2 - r{z}(i)^2));
end
F_film{z}(n) = Ff + pi*r{z}(end)^2*P{z}(end,n);

% get dP/dr for radial velocity and shear stress
for i = 1:N
    if i == 1
        dPdr(i,n) = (P{z}(i+1,n) - P{z}(i,n))/dr;
    elseif i == N
        dPdr(i,n) = (P{z}(i,n) - P{z}(i-1,n))/dr;
    else
        dPdr(i,n) = (P{z}(i,n) - P{z}(i-1,n))/dr;
    end
end
else
  dPdr(i,n) = (P{z}(i+1,n) - P{z}(i-1,n))/(2*dr);
end
end

% Max Radial velocity in film
for i = 1:N
  Umax{z}(i,n) = -(h{z}(i,n)^2*dPdr(i,n))/8;
end

% Check NDR for contact
if NDR{z}(n) <= 0 && flag1 == 1
  disp(strcat('Non-Deformed bubble made contact at time ...
  =''...
  ,num2str(t{z}(n))));
  % Store time index at non-deforming contact
  index_NDR(z) = n;
  flag1 = 0;
end

% (stops simulation if solution goes negative)
if any(h{z}(::,n) < 0.0001)
  sprintf(strcat('Simulation stopped at time = ''...
  ,num2str(t{z}(n))));
  % Make current time step equal zero before storing
  h{z}(::,n) = zeros(N,1);
  P{z}(::,n) = zeros(N,1);
\[ dhdt\{z\}(\cdot,n) = \text{zeros}(N,1); \]
\[ F_{\text{film}}\{z\}(n) = 0; \]
\[ dPdr(\cdot,n) = \text{zeros}(N,1); \]
\[ U_{\text{max}}\{z\}(\cdot,n) = \text{zeros}(N,1); \]
\[ \text{break} \]

\text{else}

\% If you want to monitor the solution plot here:
\% 
\% figure(5); plot(r\{z\},h\{z\}(\cdot,n));
\% figure(6); plot(r\{z\},P\{z\}(\cdot,n));
\% figure(8); plot(r\{z\},dhdt\{z\}(\cdot,n));

\text{end}
\text{end}

\text{figure(7); hold on;}
\text{title('Central Film Thickness vs time'); xlabel('t^*');
ylabel('$$\mathbf{h^*(0,t^*)}$$', 'Interpreter', 'Latex');
ylab = get(gca, 'ylabel');
ypos = get(ylab, 'Position');
set(ylab, 'Rotation', 0,'Position', ...
    ypos,'HorizontalAlignment', ...
    'right', 'VerticalAlignment','middle' );
plot(t\{z\},h\{z\}(1,:));

\text{figure(5); plot(r\{z\},h\{z\}(\cdot,n));
figure(6); plot(r\{z\},P\{z\}(\cdot,n));
figure(8); plot(r\{z\},dhdt\{z\}(\cdot,n));
end % End gap height for loop}