AN ABSTRACT OF THE THESIS OF

RICHARD BAR THOLOMEW STOUT for the Doctor of Philosophy
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(Major) (Date)

Title: OPTIMIZATION OF IN-CORE NUCLEAR FUEL MANAGEMENT IN A PRESSURIZED WATER REACTOR

Abstract approved: Redacted for privacy

Alan H. Robinson

Fuel loading patterns which have a minimum power peak are economically desirable to allow power reactors to operate at the highest possible power density and to minimize the possibility of fuel failure. A computer code called SHUFLE was developed for pressurized water reactors which shuffles the fuel in search of the lowest possible power peaking factor. An iterative approach is used in this search routine. A radial power distribution is calculated from which the program logic selects a movement of fuel elements in an attempt to lower the radial power peak. Another power calculation is made and the process repeated until a predetermined convergence is reached. The logic by which the code decides the fuel movement is presented, along with the criteria for accepting or rejecting the move after a power calculation of the new loading pattern is made.

A 1.5 group course mesh diffusion theory method was used to obtain the power distribution for each SHUFLE iteration. Convergence
to a final loading pattern varies from about 10 to 40 shuffling iterations depending on the initial loading presented to the code. Since the typical computer running time for a one-quarter core power distribution with this 1.5 group method is only one to a few seconds, depending on the loading, convergence to a good loading pattern takes on the order of one minute on a Univac 1108.

The low computer cost plus ease of operation should make this code of considerable use in determining loading patterns with minimum power peaking for any given set of fuel elements. The program also has burnup capability which can be used to check power peaking throughout core life.

A parametric analysis study of fuel cycle costs for a PWR is also presented. Cost parameters analyzed were variation in the cost of yellow cake, enrichment, money, fabrication, and reprocessing plus changes in burnup, load factors, power densities, and the effect of forced early discharge.

Figures are presented to indicate total fuel costs as a function of burnup for these cost parameters. Linear relationships for minimum cost and optimum burnup are presented for each parameter.
Optimization of In-core Nuclear Fuel Management in a Pressurized Water Reactor

by

Richard Bartholomew Stout

A THESIS

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APPROVED:

Redacted for privacy

Associate Professor of Mechanical and Nuclear Engineering
in charge of major

Redacted for privacy

Head of Department of Mechanical and Nuclear Engineering

Redacted for privacy

Dean of Graduate School

Date thesis is presented 25 August 1972

Typed by Mary Jo Stratton for Richard Bartholomew Stout
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I would like to thank all my fellow degree candidates in C-124 for their contribution to this thesis both technical and non-technical. I would also like to acknowledge Oregon State University and the U.S. Atomic Energy Commission for the financial assistance and computer funds provided.

Finally, special thanks are due to my wife for her encouragement and assistance--she has foregone many of the luxuries of life these past few years so that I might complete this degree. Thanks, Jan.
TABLE OF CONTENTS

I. INTRODUCTION REMARKS ON NUCLEAR FUEL MANAGEMENT
   Out-of-core Nuclear Fuel Management
   In-core Nuclear Fuel Management

II. SURVEY OF FUEL CYCLE COSTS
    Introduction
    Calculational Procedures
    Conclusions of Survey

III. REVIEW OF PREVIOUS STUDIES AND PROPOSED OBJECTIVE OF INVESTIGATION
     Review of Previous Studies
     Exposed Loading Optimization Scheme

IV. NUCLEAR MODEL FOR ONE-QUARTER CORE SIMULATION
    Development of 1.5 Group Diffusion Theory Model
    Validity of 1.5 Group Model

V. LOADING AND SHUFFLING PROCEDURES FOR MINIMUM POWER PEAKING FACTORS
    Introduction to Power Peaking
    Formulation of Shuffling Optimization Procedures

VI. SPECIFIC SHUFFLING RULES OF SHUFLE
    Core Region Description
    General Shuffling Rules
    Description of SHUFLA Logic
    Description of SHUFLB Logic
VII. TESTS AND RESULTS

Initial Core  93
Exposed Core Loadings  96
Cycle Data  104
Non-equilibrium Cycles  106

VIII. SUMMARY AND CONCLUSIONS  111

BIBLIOGRAPHY  114

APPENDICES

Appendix A. ISOBURN  117
Appendix B. Listing and Input Instructions for Program SHUFLE  123
Appendix C. Reference Reactor  155
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>Economic parameters used for base case economic calculation.</td>
<td>12</td>
</tr>
<tr>
<td>2-2</td>
<td>Burnup data.</td>
<td>13</td>
</tr>
<tr>
<td>2-3</td>
<td>Plutonium price as a function of cost of yellow cake and separative work.</td>
<td>14</td>
</tr>
<tr>
<td>2-4</td>
<td>Cost in mills/kwh at 28.6 MWD/kgU for base case.</td>
<td>17</td>
</tr>
<tr>
<td>2-5</td>
<td>Linear equations of optimum fuel cycle cost.</td>
<td>23</td>
</tr>
<tr>
<td>2-6</td>
<td>Linear equations of optimum burnup.</td>
<td>23</td>
</tr>
<tr>
<td>7-1</td>
<td>Two group LEOPARD parameters for OCONEE II UO2 fuel assemblies at zero exposure.</td>
<td>89</td>
</tr>
<tr>
<td>7-2</td>
<td>Exposure values for one-quarter core.</td>
<td>90</td>
</tr>
<tr>
<td>7-3</td>
<td>Two group polynomial fits as a function of exposure for 3.0 w/o enriched OCONEE II fuel.</td>
<td>91</td>
</tr>
<tr>
<td>Table</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>-------</td>
<td>----------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>A-1</td>
<td>Parameter values used by ISOBURN.</td>
<td>121</td>
</tr>
<tr>
<td>C-1</td>
<td>OCONEE design data.</td>
<td>155</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1</td>
<td>Flow of in-core fuel management decisions.</td>
<td>6</td>
</tr>
<tr>
<td>2-1</td>
<td>Itemized costs as a function of burnup for a typical fuel cycle.</td>
<td>16</td>
</tr>
<tr>
<td>2-2</td>
<td>Variation of minimum cost and optimum burnup as a function of cost of yellow cake.</td>
<td>18</td>
</tr>
<tr>
<td>2-3</td>
<td>Variation of minimum cost and optimum burnup as a function of cost of fabrication.</td>
<td>19</td>
</tr>
<tr>
<td>2-4</td>
<td>Variation of minimum cost and optimum burnup as a function of cost of separative work.</td>
<td>20</td>
</tr>
<tr>
<td>2-5</td>
<td>Variation of minimum cost and optimum burnup as a function of interest, carrying charges, and federal tax.</td>
<td>21</td>
</tr>
<tr>
<td>2-6</td>
<td>Variation of minimum cost and optimum burnup as a function of load factor or power density.</td>
<td>25</td>
</tr>
<tr>
<td>2-7</td>
<td>Variation of minimum cost and optimum burnup as a function of burnup obtainable from a given enrichment.</td>
<td>27</td>
</tr>
<tr>
<td>2-8</td>
<td>Cost penalty for early discharge of fuel.</td>
<td>28</td>
</tr>
<tr>
<td>4-1</td>
<td>Spacial integration scheme for 1.5 group model.</td>
<td>45</td>
</tr>
<tr>
<td>4-2</td>
<td>Comparison of 2DB and 1.5 group nuclear model.</td>
<td>51</td>
</tr>
<tr>
<td>6-1</td>
<td>OCONEE one-quarter core configuration with position designation.</td>
<td>68</td>
</tr>
<tr>
<td>6-2</td>
<td>Control options available to SHUFLE.</td>
<td>71</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>6-3</td>
<td>Flow logic of SHUFLA.</td>
<td>72</td>
</tr>
<tr>
<td>6-4</td>
<td>Example of maximum radial power, local minimum and exchange of assemblies by SHUFLA.</td>
<td>76</td>
</tr>
<tr>
<td>6-5</td>
<td>Example of moving element when maximum power and position it is to go have same reactivity.</td>
<td>78</td>
</tr>
<tr>
<td>6-6</td>
<td>Flow logic of SHUFLB.</td>
<td>82</td>
</tr>
<tr>
<td>6-7</td>
<td>Illustration of local maximum and search areas $I_{\text{max}} \pm 1$, $J_{\text{max}} \pm 1$ and $I_{\text{max}} \pm 2$, $J_{\text{max}} \pm 2$ for SHUFLB.</td>
<td>85</td>
</tr>
<tr>
<td>7-1</td>
<td>Shuffle of initial core loading.</td>
<td>94</td>
</tr>
<tr>
<td>7-2</td>
<td>Shuffle of initial core loading.</td>
<td>95</td>
</tr>
<tr>
<td>7-3</td>
<td>Initial and final loadings of exposed fuel from SHUFLE.</td>
<td>97</td>
</tr>
<tr>
<td>7-4</td>
<td>Initial and final loadings of exposed fuel from SHUFLE.</td>
<td>98</td>
</tr>
<tr>
<td>7-5</td>
<td>Comparison of recommended loadings from Figures 7-3 and 7-4.</td>
<td>99</td>
</tr>
<tr>
<td>7-6</td>
<td>Initial loading presented to SHUFLE and recommended loading.</td>
<td>101</td>
</tr>
<tr>
<td>7-7</td>
<td>Initial loading and recommended loading from SHUFLE for cycle two.</td>
<td>105</td>
</tr>
<tr>
<td>7-8</td>
<td>Initial loading and recommended loading from SHUFLE for cycle three.</td>
<td>107</td>
</tr>
<tr>
<td>7-9</td>
<td>Initial and recommended loading from SHUFLE assuming one failed assembly per one-quarter core at end of cycle one.</td>
<td>109</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>7-10</td>
<td>110</td>
<td></td>
</tr>
</tbody>
</table>

**Initial and recommended loading from SHUFLE assuming two failed assemblies per one-quarter core at end of cycle one.**

**Appendix**

**Figures**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-1</td>
<td>One-quarter core x, y geometry problem solved by SHUFLE.</td>
<td>124</td>
</tr>
</tbody>
</table>
OPTIMIZATION OF IN-CORE NUCLEAR FUEL MANAGEMENT IN A PRESSURIZED WATER REACTOR

I. INTRODUCTORY REMARKS ON NUCLEAR FUEL MANAGEMENT

Nuclear Fuel Management is the term applied to the management of nuclear fuel from the earliest stages of procurement through reprocessing and recycling of the fuel.

Obviously such an all-encompassing task has many aspects which are all coupled to a greater or lesser degree. Two of the more obvious branches of nuclear fuel management are out-of-core nuclear fuel management and in-core nuclear fuel management. As the name implies, out-of-core nuclear fuel management is concerned with such things as cost of uranium, enrichment, fabrication, reprocessing, recycle, financing and related aspects which do not directly effect the fuel while in the reactor core. In-core nuclear fuel management is more directly related to the actual reactor operation. Such aspects as power density, fuel enrichment and cycle length are considered by those concerned with in-core nuclear fuel management. Although the two branches of fuel management are relatively independent, decisions in one branch do effect the other. As an example, financing and other costs effect cycle length, and cycle length and power density effect enrichment. Because of this, nuclear fuel management requires a thorough knowledge of the entire fuel cycle in order to relate how each aspect of the total cycle will effect the nuclear fuel cycle costs for a particular reactor or group of reactors. Since the fuel cycle costs
represent a significant portion of the overall nuclear power cost (40 to 50 percent) the fuel manager has a sizable financial responsibility for coordinating these functions to minimize nuclear fuel cycle costs.

Out-of-core Nuclear Fuel Management

To perform out-of-core fuel management, fuel managers need available the tools to predict which combination of variables will produce overall minimum power costs for their specific reactor over some given time period. Because each reactor will be faced with a different set of circumstances which will vary with time, it is difficult to establish hard and fast rules in this area of fuel management. However, most power reactors will operate on a typical fuel cycle and general fuel management procedures can be defined. It is possible to determine what the major cost parameters of the nuclear fuel cycle are and what variational effect each has on nuclear fuel management decisions. These parameters are scoped in Chapter II. The results of this analysis should enable the fuel manager to correlate out-of-core decisions with some of the variables of in-core fuel management.

In-core Nuclear Fuel Management

Total in-core fuel management would involve all the physical aspects which would allow optimal operation of the nuclear fuel within
the core. These would include a wide variety of parameters such as water to fuel ratio, power density, mechanical integrity of fuel clad, thermohydraulics and cycle length to mention just a few. Nuclear fuel cycle costs are insensitive to some of these parameters and many others have been fixed once a reactor has been selected. Because of this, in-core nuclear fuel management is simplified and in general for most nuclear fuel managers of a utility, the decisions can be broken into four categories;

1. Determine the desired length of subsequent power cycles.
2. Decide the fraction of the core to be replaced at the end of each cycle.
3. Set the enrichment specification of the new fuel assemblies.
4. Prescribe the physical location of the new assemblies and relocation of the remaining partially burned assemblies.

The first decision to some extent will be based on the economic analysis presented in Chapter II, but most likely will be more strongly controlled by the power requirements of the utility. If the most economic cycle length from a fuel standpoint is 15 months, the utility isn't likely to choose this length cycle if it means refueling will fall during a peak power requirement period. Most utilities prefer to refuel on an annual basis, although some flexibility can be tolerated.

Once the cycle length has been determined, decisions two and three can be made if accurate estimates of the power requirements
during the cycle are available. Decisions two and three are directly correlated in that cycle length determines enrichment and vise versa. The decision on the fraction of the core replaced is usually made first then the necessary enrichment of the new fuel assemblies is determined to allow enough excess reactivity to reach the end of the subsequent cycle. The fraction of the core replaced will be determined to a large extent by the length of the cycle. The length of the cycle together with the power density determine the approximate exposure the fuel will receive during the cycle. While it is possible to discharge a small fraction of the core and replace it with fuel with enough reactivity to reach the end of the cycle, care must also be taken so that the fuel assemblies do not receive more exposure than the mechanical integrity of the fuel can reasonably expect to endure without a high probability of failure. Because of this, the decision on the fraction of the core to discharge is considerably simplified. Usually for a normal PWR, on a yearly cycle, a third of the core consisting of the most highly burned fuel assemblies is discharged.

Having determined the fuel assemblies to be replaced, the enrichment necessary to reach the end of the next cycle can be calculated with a combination of physics codes.

The fourth decision for the nuclear fuel manager is where to place each of the fuel elements to begin the next cycle. This decision involves deciding the location of the fresh as well as exposed
assemblies. The location of the fuel elements is very important in that if a poor loading is chosen, unacceptable power spikes may result and regardless of the economics the reactor may not be able to operate at full power to avoid fuel burnout in the pins with these high power peaks.

For a reasonable core loading the end-of-life (EOL) reactivity is relatively independent of the core loading. This allows the decisions on cycle length, reload fraction and enrichment to be made almost independent of the fourth decision, the fuel loading pattern. A good loading pattern is characterized by a low peak to average power ratio throughout the cycle while maintaining shutdown requirements. More detail of this will be discussed in Chapter III, but in general the peak to average power ratio or more commonly referred to as the power peaking factor determines the maximum power at which the reactor can operate and still be within burnout safety margins. Obviously, if one loading pattern produces a lower power peaking factor it is capable of operating at a higher power density and thus reduce fuel cycle costs. Figure 1-1 is a graphical illustration of the decisions which need to be made by the fuel manager, together with some of the input data and physics codes available for each task.

USAEC report Wash 1082 Civilian Nuclear Power (1968) states that considerable studies have been done and shuffling fuel to a lower radial power peaking factor is unlikely. However, in another section
Figure 1-1. Flow of in-core fuel management decisions.
Wash 1082 states that "detailed shuffling procedure at each of the refueling outages must still be considered as one of the arts of nuclear design." Perusal of the data and computer codes available leads one to believe the second statement. In any event, the first statement pertained to predictable equilibrium fuel cycles. Considerably evidence indicates that equilibrium fuel cycles will never exist (Bupp, Brown and Busse lman, 1971); rather the reactor core will always be in a constant state of transition. Fuel element failures are inevitable and will always cause some adjustment of the fuel loading. In addition, innovations in fuel design and the possibility of plutonium recycle will cause adjustments to be made on a cycle by cycle basis. For these reasons, it is necessary that the fuel manager have some method to decide where each fuel element should be placed in the reactor for the next cycle to maintain the minimum possible power peaking factor.

The object of this study is to produce a programmed set of rules which will optimize the loading pattern for any given set of fuel elements by minimizing the radial power peaking factor.

Prior to this, it was thought useful to investigate the relative economic importance of the various fuel cycle costs, including power density. This is presented in Chapter II. Chapter III outlines some of the previous and current methods and studies in optimization of nuclear fuel loading patterns, together with an outline of the proposed philosophy of the approach of this study. A course mesh 1.5 group diffusion
theory model was the nuclear model used throughout the study. This model is described in Chapter IV. A detailed outline of the reason fuel loadings with minimum power peaking factors are desirable and necessary, plus identification of procedures which will produce such loadings, are discussed in Chapter V. General procedures employed in the fuel shuffling program, SHUFLE, are also described. For those interested in the specific details of the shuffling logic, Chapter VI presents logic flow diagrams plus examples of all the important features of the program SHUFLE. Also included are reasons why specific logic was employed and how it contributes to the entire scheme of an optimized fuel loading. The tests and results of these which indicate the validity of the shuffling logic follow in Chapter VII, and the conclusion plus recommendations for further study are in Chapter VIII. Those interested in utilizing the program SHUFLE may refer to the listing and input instructions in the Appendices. Also in the Appendices are a description of the reference reactor and a writeup of ISOBURN, the program written to determine isotopics for the economic survey.
II. SURVEY OF FUEL CYCLE COSTS

Introduction

Since fuel cycle costs represent such a significant portion of the overall nuclear power costs, prior to a discussion of optimizing loading patterns a survey of the factors affecting the fuel cycle would be useful. This chapter presents a parametric analysis study of the principal factors affecting the fuel cycle cost and optimum burnup for a typical PWR. Minimizing the fuel cycle costs must, of course, be compatible with minimizing total power costs since this is the objective. However, the current BWR and PWR reactors have been somewhat standardized in regard to core parameters such as size, fuel to moderator ratios, power densities, rod sizes and pitches and many other physical parameters. For a typical reactor, therefore, it is possible to study fuel cycle costs independent of other costs with a result that lower fuel cycle costs imply lower total power costs.

The parameters studied are cost of \( \text{U}_3\text{O}_8 \), fuel fabrication, separative work, reprocessing, interest charges on capital, public versus investor owned utilities, burnup, load factor, power density and early discharge. A typical fuel cycle flow chart was assumed for all studies, and CINCAS (McLain, Fulford, Edlund and Craig, 1968) was the economic code used. A zero dimensional one group burnup code using a fourth order Runge-Kutta integration of the burnup
equations was written to calculate isotopics and k-infinity as a function of burnup and enrichment. This code is called ISOBURN and is described in detail in Appendix A. The one group cross sections were obtained from ALTHAEA (Merrill, 1969) which calculates them using a highly modified Westcott (1960) system which corrects for heterogeneities and resonance effects. Fission product poisons were calculated for Sm$^{149}$, Sm$^{151}$, Xe$^{135}$ and three pseudo-fission products. The unit fuel cell chosen for the calculations was that of Oconee II, which is an 850 MWe PWR. This cell is typical enough of other PWR cells that the general conclusions of this study should apply to all modern pressurized water reactors in the 800 to 1100 MWe range.

The enrichments necessary to reach a given burnup were derived from the equation 2-1 (USAEC, Wash 1082, 1968):

\[
1 + \Delta k = \frac{1}{N} \sum_{n=1}^{N} k_{oo} (E_o, \frac{nB}{N})
\]

where

\[
\frac{1}{N} = \text{fraction of the core refueled}
\]

\[B = \text{maximum burnup}\]

\[E_o = \text{initial enrichment}\]

The infinite reactivities used were those calculated from ISOBURN. The \((1 + \Delta k)\) factor was normalized to a number which gave the correct enrichment for data acquired from more sophisticated codes for Oconee, Trojan, and McQuire Reactors and that given by
E. A. Mason (1971). The Oconee, Trojan and McQuire data assumed a three year scatter loading cycle.

**Calculational Procedures**

The general calculational procedure was to choose a base case which is representative of the present fuel cycle, then vary the material costs, burnups and load factor or power density, while holding all other parameters constant.

Data for the base case, which represents typical costs for an investor owned utility, are presented in Tables 2-1 and 2-2. All the indicated costs are levelized costs, present worthed to the date of start-up at a rate of 8 percent.

Two exceptions are made to holding parameters constant. The first is the price of plutonium. The price of recovered plutonium depends on the price of enriched uranium, and therefore, on the price of yellow cake, separative work and conversion. The equation used for the price of plutonium was (USAEC Wash 1082, 1968):

\[ \text{Pu value (} \$/g \text{ fissile) = } g \times U_{93} \times (1 - 0.6 P_{42}) \times (1 - WAF) \]  

(2-2)

where

\[ U_{93} = \text{value of 93 percent enriched uranium} \]

\[ g = \text{a parameter which depends on reactor type and future plutonium use} \]
Table 2-1. Economic parameters used for base case economic calculation.

<table>
<thead>
<tr>
<th>Item</th>
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<th>Value</th>
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<tr>
<td>Natural uranium net to conversion</td>
<td>$8.00/lb U$_3$O$_8$</td>
<td></td>
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<tr>
<td>Conversion from U$_3$O$_8$ to UF$_6$</td>
<td>1.30/lb U as UF$_6$</td>
<td></td>
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<tr>
<td>Separative work</td>
<td>32.00/kg sep. work</td>
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</tr>
<tr>
<td>Fabrication</td>
<td>70.00/kg contained U</td>
<td></td>
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<tr>
<td>Spent fuel shipping</td>
<td>4.50/kg discharged U</td>
<td></td>
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<tr>
<td>Reprocessing</td>
<td>25.00/kg discharged U</td>
<td></td>
</tr>
<tr>
<td>Reconversion</td>
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<tr>
<td>Pu recovery price</td>
<td>8.00/g fissile Pu</td>
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<td>Storage and shipping</td>
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<td></td>
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<td>Reprocessing</td>
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<tr>
<td>Reprocessing to sale</td>
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</tr>
<tr>
<td>Delivered to site prior to irradiation</td>
<td>3 mo.</td>
<td></td>
</tr>
<tr>
<td>Feed UF$_6$ enrichment</td>
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<td></td>
</tr>
<tr>
<td>Tails UF$_6$ enrichment</td>
<td>.3</td>
<td></td>
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<tr>
<td>Inventory charge rate prior to irradiation</td>
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<td>On and Off site carrying charge</td>
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<td>Federal income tax</td>
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<td>U$_3$O$_8$ to UF$_6$ loss</td>
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<td>Fab. losses</td>
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<tr>
<td>Uranium reprocessing losses</td>
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<td>Pu reprocessing losses</td>
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<tr>
<td>Uranium ore payment - prior to site delivery</td>
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<tr>
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<tr>
<td>Enrichment payment - prior to site delivery</td>
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Table 2-1. (Continued)

<table>
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<tr>
<td>Escalation</td>
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<td>none</td>
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<tr>
<td>Thermal efficiency</td>
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<td>1/3</td>
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<td>Power density</td>
<td>30.822 MW/MTU</td>
<td>30.822 MW/MTU</td>
</tr>
<tr>
<td>Load factor</td>
<td>0.8</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Table 2-2. Burnup data.

<table>
<thead>
<tr>
<th>Enrichment percent</th>
<th>Base burn (MWD/kg)</th>
<th>5% Increased burnup</th>
<th>5% Decreased burnup</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>14.2</td>
<td>14.9</td>
<td>13.5</td>
</tr>
<tr>
<td>2.5</td>
<td>23.4</td>
<td>24.6</td>
<td>22.2</td>
</tr>
<tr>
<td>2.8</td>
<td>27.6</td>
<td>29.0</td>
<td>26.2</td>
</tr>
<tr>
<td>3.0</td>
<td>30.2</td>
<td>31.7</td>
<td>28.7</td>
</tr>
<tr>
<td>3.2</td>
<td>32.8</td>
<td>34.5</td>
<td>31.2</td>
</tr>
<tr>
<td>3.5</td>
<td>36.6</td>
<td>38.4</td>
<td>34.8</td>
</tr>
<tr>
<td>4.0</td>
<td>42.4</td>
<td>44.5</td>
<td>40.3</td>
</tr>
</tbody>
</table>
\[ P_{42} = \text{corrective term for plutonium-242 present in recycle} \]

\( (1 - W\Delta F) = \text{discount factor for increased costs of Pu fabrication} \)

Values for these parameters have been estimated as a function of time in USAEC Report Wash 1082 (1968). The values for Pu used for this paper are listed in Table 2-3. These can be obtained by using \( g = .81, \ P_{42} = .17 \) and \( (1 - W\Delta F) = .88 \) in equation 2-2.

<table>
<thead>
<tr>
<th>Cost/lb of U(_3)O(_8)</th>
<th>Cost/kg sep. work</th>
<th>Price of Pu/g</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ 8.00</td>
<td>$32.00</td>
<td>$ 8.00</td>
</tr>
<tr>
<td>4.00</td>
<td>32.00</td>
<td>6.40</td>
</tr>
<tr>
<td>16.00</td>
<td>32.00</td>
<td>11.20</td>
</tr>
<tr>
<td>8.00</td>
<td>26.00</td>
<td>7.20</td>
</tr>
<tr>
<td>8.00</td>
<td>45.00</td>
<td>9.75</td>
</tr>
</tbody>
</table>

The second exception is the tails enrichment. At $32.00/kg through $45.00/kg separative work, 0.3 percent tails give the minimum cost. However, at $26.00/kg separative work, 0.2 percent tails is about 0.025 mills/kwhe cheaper than 0.3 percent tails. Therefore, 0.3 percent tails were used for all studies except for the $26.00/kg separative work case.

Only equilibrium cycles with one-third reloads were considered. Not all cycles were three year cycles. The length of each cycle was determined by the maximum burnup allowable for a given enrichment,
the power density, and the load factor. The time in months to achieve any burnup can easily be calculated by:

\[ \text{Months} = \frac{\text{burnup}}{\text{power density} \times \text{load factor} \times 30.42} \]  

where burnup is in units of MWD/MTU, and power density is in units of MW/MTU.

Results of the Analysis

Figure 2-1 indicates the fuel cycle cost as a function of burnup for the base case (investor owned utility). The minimum cost value occurs at 28.6 MWD/kg. At this burnup, the cost component breakdown is indicated in Table 2-4. For this case, over 97 percent of the total fuel cycle cost comes from fuel burnup, interest, fabrication, and reprocessing. The uranium cost is the greatest, and over 90 percent of this is due to cost of yellow cake and enrichment. It would seem important, therefore, to analyze the fuel cycle costs as a function of these major cost items.

The parametric analysis of these are indicated in Figures 2-2 through 2-5. As expected, decreasing the price of any cost parameter decreases the total mills/kwhe cost. With the exception of the fixed costs such as fabrication, the shapes of the cost curves are also flatter with decreased cost. The increase in fuel cycle costs due to higher interest charges, illustrated in Figure 2-5, indicates this very distinctly. For example, in the case of a public utility (6.5 percent
Figure 2-1. Itemized costs as a function of burnup for a typical fuel cycle.
Table 2-4. Cost in mills/kwhe at 28.6 MWD/kgU for base case.

<table>
<thead>
<tr>
<th>Item</th>
<th>Consumption cost</th>
<th>Financing cost</th>
<th>Total</th>
<th>% of total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uranium&lt;sup&gt;a&lt;/sup&gt;</td>
<td>1.024</td>
<td>0.356</td>
<td>1.380</td>
<td>77.4</td>
</tr>
<tr>
<td>Plutonium</td>
<td>-0.221</td>
<td>0.050</td>
<td>-0.171</td>
<td>-9.6</td>
</tr>
<tr>
<td>Fabrication</td>
<td>0.310</td>
<td>0.092</td>
<td>0.402</td>
<td>22.5</td>
</tr>
<tr>
<td>Reprocessing</td>
<td>0.105</td>
<td>0.016</td>
<td>0.121</td>
<td>6.7</td>
</tr>
<tr>
<td>Storage and shipping</td>
<td>0.019</td>
<td>0.009</td>
<td>0.028</td>
<td>1.6</td>
</tr>
<tr>
<td>Reconversion</td>
<td>0.023</td>
<td>0.002</td>
<td>0.025</td>
<td>1.4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1.26</strong></td>
<td><strong>0.525</strong></td>
<td><strong>1.785</strong></td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup>Uranium costs include cost of yellow cake, conversion and enrichment. As a percent of uranium cost, yellow cake = 52.1, conversion = 7.1, and enrichment = 40.8 for the enrichment necessary to obtain 28.6 MWD/kgU burnup.
Figure 2-2. Variation of minimum cost and optimum burnup as a function of cost of yellow cake.
Figure 2-3. Variation of minimum cost and optimum burnup as a function of cost of fabrication.
Figure 2-4. Variation of minimum cost and optimum burnup as a function of cost of separative work.
Figure 2-5. Variation of minimum cost and optimum burnup as a function of interest, carrying charges, and federal tax.
interest rate) discharge burnup from 27.8 to 38.0 MWD/kg can be tolerated with no more than 0.1 mills/kwhe penalty from the minimum; whereas, for the base case (15 percent interest), a 0.1 mill/kwhe deviation from the minimum only allows burnups from 24.6 to 32.0 MWD/kg. The burnup at which the minimum total fuel cost occurs also increases with decreased cost of yellow cake, separative work and interest, but as indicated by Figure 2-3, decreasing the cost of fabrication decreased the optimum burnup.\(^1\) This would also be true for reprocessing, reconversion and shipping and storage. This is because these items are fixed costs and reducing these reduces the burnup necessary to balance these costs against costs due to fuel depletion. The optimum burnup then occurs at less MWD/kg and, therefore, lower equilibrium enrichment.

Linear equations have been developed to calculate the minimum mills/kwhe for various cost parameters. These equations, listed in Table 2-5, were developed by least squares fit to the minimum points shown in Figures 2-2 through 2-5. Table 2-6 indicates the same relationship for optimum burnup. As an example of how to use Tables 2-5 and 2-6, consider what the cost and optimum burnup would be if U\(_3\)O\(_8\) were $12.00/lb, with the calculated plutonium price for this cost of yellow cake. All other cost parameters will be held the same as the base case. The mills/kwhe cost would be 1.120 + 0.08286 X 12 =

\(^1\)Optimum burnup is the burnup at which the minimum fuel cost occurs.
Table 2-5. Linear equations of optimum fuel cycle cost.

<table>
<thead>
<tr>
<th>Cost variable</th>
<th>Cost equation</th>
<th>Slope (mills/kwhe/unit cost variable)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{U}_3\text{O}_8 ) ($/lb)</td>
<td>( C = 1.120 + .08286 \times \text{CU}308 )</td>
<td>.08286</td>
</tr>
<tr>
<td>Fabrication ($/kg)</td>
<td>( C = 1.372 + .00587 \times \text{CFAB} )</td>
<td>.00587</td>
</tr>
<tr>
<td>Separative work ($/kg)</td>
<td>( C = 1.306 + .01489 \times \text{CSEP} )</td>
<td>.01489</td>
</tr>
<tr>
<td>Reprocessing ($/kg)</td>
<td>( C = 1.715 + .00280 \times \text{CREP} )</td>
<td>.00280</td>
</tr>
<tr>
<td>Interest (%)</td>
<td>( C = 1.311 + .0316 \times \text{PCTI} )</td>
<td>.0316</td>
</tr>
<tr>
<td>Load factor</td>
<td>( C = 1.385 + .3188 \times (1/\text{L. F.}) )</td>
<td>.3188</td>
</tr>
<tr>
<td>Burnup (% inc. or dec.)</td>
<td>( C = 1.785 + .01150 \times \text{PCT} )</td>
<td>.01150</td>
</tr>
</tbody>
</table>

Table 2-6. Linear equations of optimum burnup.

<table>
<thead>
<tr>
<th>Cost variable</th>
<th>Optimum burnup MWD/kg</th>
<th>Slope (MWD/kg/unit cost variable)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{U}_3\text{O}_8 ) ($/lb)</td>
<td>( \text{OB} = 31.4 - .35 \times \text{CU}308 )</td>
<td>- .35</td>
</tr>
<tr>
<td>Fabrication ($/kg)</td>
<td>( \text{OB} = 20.2 + .12 \times \text{CFAB} )</td>
<td>+ .12</td>
</tr>
<tr>
<td>Separative work ($/kg)</td>
<td>( \text{OB} = 35.0 - .20 \times \text{CSEP} )</td>
<td>- .20</td>
</tr>
<tr>
<td>Reprocessing ($/kg)</td>
<td>( \text{OB} = 26.8 + .073 \times \text{CREP} )</td>
<td>+ .073</td>
</tr>
<tr>
<td>Interest (%)</td>
<td>( \text{OB} = 33.4 - .32 \times \text{PCTI} )</td>
<td>- .32</td>
</tr>
<tr>
<td>Load factor</td>
<td>( \text{OB} = 36.5 - 6.29 \times (1/\text{L. F.}) )</td>
<td>-6.29</td>
</tr>
<tr>
<td>Burnup (% inc. or dec.)</td>
<td>( \text{OB} = 28.6 + .19 \times \text{PCT} )</td>
<td>+ .19</td>
</tr>
</tbody>
</table>
2.114, and the optimum burnup would be $31.4 - .35 \times 12. = 27.2$ MWD/kg. As can be seen, even if the cost of $U_3O_8$ were zero, the total fuel cost would still be 1.12 mills/kwhe and the optimum burnup would be 31.4 MWD/kg.

As long as the costs are assumed linear, any combination of costs can be calculated for a fuel cycle simply by multiplying the slope of any parameter which differs from the base case by the difference of that parameter from the base case and adding to the base case cost. As an example, assume the cost is desired for a fuel cycle where $U_3O_8$ is $12/lb$ ($4.00$ above base), and fuel fabrication is $80./kg$. Assume all other factors are the same as the base case. The base case cost is 1.785 mills/kwhe at optimum burnup of 28.6 MWD/kg.

The fuel cycle cost for this case is then:

\[
\text{Cost} = 1.785 + 0.08286 \times 4.00 + 0.00587 \times 10.00
\]

\[= 2.175 \text{ mills/kwhe} \]

\[
\text{Optimum burnup} = 28.6 - .35 \times 4.00 + .12 \times 10.00
\]

\[= 28.4 \text{ MWD/kg} \]

This can be extended to as many cost parameters as desired with the only error being caused by the extent of nonlinearity in any given parameter.

The labels on Figure 2-6 indicate only a variation in load factor. However, if one assumes that isotopics as a function of burnup are

\[2\text{Remember Pu recovery value corresponding to cost of U must be used.} \]
Figure 2-6. Variation of minimum cost and optimum burnup as a function of load factor or power density.
independent of power density, then variations in power density can be obtained from this curve by finding an equivalent load factor which will give the same total burnup per unit time by using the equation:

\[
\frac{\text{MWD}}{\text{MTU/yr}} = \frac{\text{MW/MTU}}{\text{yr}} \times \frac{365 \text{ days}}{\text{yr}} \times \text{load factor} \quad (2-4)
\]

The load factor and power density are important in that the larger these two factors are the less the capital interest charges per unit energy produced. Figure 2-6 indicates a significant savings results from operation at maximum power density.

Figure 2-7 correlates costs for the burnups as a function of enrichment given in Table 2-2. The upper curve corresponds to a five percent decrease in burnup over the base case for the same enrichment, the lower curve to a five percent increase. The 0.0115 mill/kwhe savings per percent increase in burnup is the incentive for a utility to optimize loading patterns. A 0.1 mill/kwhe change in costs for a 1000 MWe plant at a 0.8 load capacity is $701,280. Increasing the burnup by one percent could save the utility about $100,000/yr. Thus, while the cost differentials in mills/kwhe for some of these studies may seem small, when converted to dollars, the differences are quite significant.

One of the concerns of utilities and nuclear fuel suppliers is the increased cost of discharging fuel before it has achieved its expected burnup. Figure 2-8 indicates the result of early discharge for 2.8,
Figure 2-7. Variation of minimum cost and optimum burnup as a function of burnup obtainable from a given enrichment.
Figure 2-8. Cost penalty for early discharge of fuel.
3.0 and 3.2 percent enriched fuel. As an example, suppose a reactor was fueled with 2.8 percent fuel and the expected burnup was 27 MWD/kg in three years, but for some reason, such as a fuel failure, some of the fuel must be discharged at the end of two years or 18 MWD/kg. The fuel cycle cost would then be increased from the expected 1.81 to 2.33 mills/kwhe, an increase of 29 percent. If fuel had failed after only one year the cost would have been 3.65 mills/kwhe, more than double the expected cost. This curve does not take into account the added cost of a fuel failure due to increased outage time and cost of replacement power if it is necessary.

**Conclusions of Survey**

Linear relationships exist for the cost components of the fuel cycle and for the burnup at which these minimum costs occur. The slopes of these lines indicate the relative importance of that component in the overall fuel cost and can be used to accurately estimate the effect of future price changes on fuel cycle costs.

Certain things are obvious by observing Figures 2-1 through 2-8 and the linear equations in Tables 2-5 and 2-6. The most important items are the cost of yellow cake, the cost of enrichment, the financing charges and maintaining fuel integrity to obtain predicted burnup. The price of yellow cake is dependent upon the availability of uranium and the ease of mining. Most uranium reserve estimates are broken down
into given quantities at various price levels, with increasing reserves as the price increases. Figure 2-2 indicates that while increased cost of U₃O₈ is expensive, it is not prohibitive. If the cost of U₃O₈ were doubled, thus significantly increasing our uranium reserves, it would increase the fuel cycle cost by only about 0.66 mills/kwh. At the present this is certainly enough to make nuclear power noncompetitive; however, in the future this may not be true.

Since nuclear fuel inventories are large and the fuel is tied up for long periods of time, the financing charges are very important. Lower interest rates give the utility a much wider margin of choices and more tolerance for deviation from predicted patterns since the fuel cycle cost curves for the lower interest rate case has a much broader minimum.

Separative work charges will have a very great effect on the economics of nuclear power. Increasing the separative work cost from 0.2 tails and $26/kg to 0.3 tails and $32/kg, increased fuel costs by 0.114 mills/kwh, or approximately one million dollars per year for a 1000 MWe plant.

Large inventories are tied up in nuclear fuel. It is important to obtain maximum power from the fuel in as short a time period as possible. Because of this and also the even larger capital costs associated with the reactor complex, high power densities and high load factors produce more economical nuclear power costs. Thus the
subject of this thesis, to optimize loading patterns for minimum radial power peaking, allows operation at maximum power density and reduces nuclear power costs.
III. REVIEW OF PREVIOUS STUDIES AND PROPOSED OBJECTIVE OF INVESTIGATION

Review of Previous Studies

The primary objective of this study is to facilitate in-core fuel management decisions by prediction of optimum loading patterns.

Early work on this aspect of in-core fuel management concentrated on techniques which utilized burnup as an objective function to be optimized in predicting optimum core loading patterns. Wall (1965) first demonstrated that dynamic programming could be used to solve this problem. This was possible when he demonstrated that representation of burnup as the major variable involved the interrelationship between several cycles. However, Wall's approach was limited in that his dynamic programming approach was time consuming and from a practical standpoint could solve only few region problems. Wall's conclusion that fuel cycle cost is approximately inversely proportional to the average burnup is correct. Wall, however, neglected to factor in his optimization scheme that high peaking factors may limit the maximum power density, and his model unfairly favors loadings which produce high power peaking ratios. He also concluded that his method of placing the fresh and partially burnt fuels in gross relatively separated regions causes an undesirable power distribution, and that scatter loadings are better than zonal out-in or in-out loadings.  

3 These type loadings explained in Chapter V.
Other attempts were made to optimize burnup by Fagen and Sesonske (1969) and Stover and Sesonske (1969). These efforts were expanded to be able to include scatter loadings with fuel shuffling between zones. Fagen determined the shuffle patterns which maximized the core life at each reload point in the life of the reactor, thus obtaining a minimum fuel cycle cost. While Fagen's analysis would treat more zones, his nuclear model was still one dimensional and was unable to pinpoint and automate exactly where each fuel element should be placed at the beginning of the next fuel cycle.

Stover modified the dynamic programming algorithm used by Wall, using a method he described as "elimination of similar end states." Since computer storage requirements is one of the most serious difficulties of dynamic programming, by eliminating similar end states, Stover was able to handle more state variables and thus handle more zones than Wall. Stover's results were very similar to the conclusion Wall expressed. However, even if burnup were the objective function to optimize, Stover approached the problem from the standpoint of optimizing the number of fuel assemblies to replace after each cycle, when a more useful approach may have been to optimize the enrichment of a fixed number of reload assemblies.

Other attempts to utilize dynamic programming to optimize burnup have been tried (Suzuki and Kiyose, 1971). However, if power peaking is used instead as the objective function, Naft (1970) indicates
the interrelationship between operating cycles and the significance of a stage-wise process is lost. He concludes that dynamic programming is a less desirable approach in this instance.

Motoda (1970) investigated maximizing burnup through optimal distribution of power, flux, burnup and refueling rate in a continuously refueled reactor. Among his conclusions were that burnup optimization and power flattening are contradictory in the usual situation. Motado also found that the optimum power distribution is peaked in the central region and depleted in the outer region when burnup is optimized.

One of the constraints Motoda placed on his model is that fuel assemblies remained in a fixed position until discharged. This constraint was applied so that on-power refueling could be analyzed. While his conclusions are noteworthy, on-power refueling is not applicable to present generation light water reactors. Power peaking factors were also not included in the study.

More recently optimization efforts have been directed toward minimizing the radial power peaking factor. Kawai and Kiguchi (1971) continued the studies of on-power refueling by Motoda but included the effects of power peaking which they indicate must be considered, especially for large reactors.

Wade and Terney (1971) investigated the problem of optimal control theory for a reactor. They identified a "performance index"
which is non-negative and small when design objectives are met and large when they are not. Minimization of the Pontryagin function is then used to find the optimum. The performance index chosen was the least squares flux deviation. Terney (1971) indicates, however, that this is not a very sensitive parameter and does not directly limit local flux peaking, but leads to refueling schemes with maximum and minimum k-infinity regions.

Wade and Terney suggest a better choice might be to maximize the average power while limiting the peak power. Although different methods will be employed, this is exactly the objective of this study; to minimize the peak to average power ratio, in an attempt to optimize loading patterns.

Rothleder (1967) proposed a set of guidelines which are very useful to aid the designer in minimizing radial power peaking, but he did not present an automated procedure for implementing these suggestions on a refueling cycle.

Melice (1967) also presented a non-automated graphical procedure, in which he utilizes the fact that power is proportional to reactivity. Using this fact he proposed a shuffling scheme for each cycle based on experience from previous cycles. Provided the reactor is in an equilibrium cycle or near equilibrium cycle, the method may be applicable. However, the dependence on previous history of a core in a transition state can be a serious drawback.

A survey of the literature indicates that only Naft (Naft, 1970;
Naft and Sesonske, 1971) approaches the problem of minimizing the radial power peaking factors in a method which may be of use to a fuel manager.

Naft proposes a procedure for determining a loading pattern for a fuel cycle. He utilizes a function which has two components. The first, a slowly varying component, represents the overall power gradient across the core. The second, a rapidly varying component, represents the effect of an assembly's eight nearest neighbors. The composition of each assembly is represented by its material buckling. With these three parameters, Naft was able to rapidly determine the relative power of each assembly in a particular loading. He then utilized a discrete direction optimization to solve for the optimum loading pattern.

One of the major difficulties of Naft's approach is that his optimization approach requires a very large number of power calculations. To do this in a reasonable amount of computer time it was necessary to develop his proposed method of calculating the relative power by use of slowly and rapidly functions. This requires some a priori knowledge of what the power distribution will be for a typical loading, which he obtained from PDQ-7 (Cadwell, 1967). The two-part power calculation function gave very comparable results to PDQ-7 for an initial core which had a very smooth power profile. However, comparison of relative power on a replacement core differed from
PDQ-7 by as much as 11 percent on the interior of the core where accurate prediction of the radial power peak is essential. Regardless of the methods used, it would be very difficult to optimize a loading pattern from the radial power peaking standpoint without more accurate estimates of the actual radial power peaking ratio.

**Proposed Loading Optimization Scheme**

In all of the references cited in the section on previous related studies, some statistical method was applied in an attempt to optimize the loading pattern either from a burnup standpoint, a control poison standpoint or minimizing the radial power peaking factor. While most of the conclusions reached are applicable, these methods are not highly useful to the fuel manager because the reactor core is not treated on a fuel assembly by fuel assembly basis. Present design techniques are more likely to involve the following path toward deciding how a reactor should be reloaded.

1. Based on predicted exposure, discharge enough of the least reactive or most highly burned fuel to allow the reactor to reach the end of the next cycle with the new fuel assemblies.

2. Based on experience, predict a reasonable loading pattern for several cycles and burn to the end of total cycle length on a two dimensional FLARE (Delp et al., 1964) type code.

3. Check the radial power peaks throughout each cycle and
exchange some of the fuel assemblies in an attempt to lower the peaks to acceptable limits.

4. Continue step three until all cycles meet radial power peaking requirements.

5. Check beginning of life loading with a more sophisticated code such as 2DB (Little and Hardy, 1969) or PDQ-7 (Cadwell, 1967).

The experienced fuel manager can usually predict fairly good loadings in about three or four iterations through steps three and four, especially if the reactor is in or near an equilibrium fuel cycle. If the reactor is not in an equilibrium cycle due to fuel failure or some other reason, considerably more attempts may be necessary to arrive at an acceptable loading pattern. In any event, it is often very difficult to tell how many times one should try another loading in an attempt to produce a lower radial power peak. From a fuel manager's standpoint, it would be helpful if these two steps were automated such that a computer program shuffled the fuel to obtain the best loading possible from a radial power peaking standpoint.

To date, this approach has not been published extensively, partly because of the proprietary nature of the attempts at this approach and partly because the tools to implement this type of refueling logic involved large amounts of computer time due to the number of two dimensional power calculations which were required. This study did
not reduce the amount of two dimensional power calculations, but
utilized an accurate two dimensional power calculation which requires
a very minimal amount of computer time. This allows the shuffling
program to try a relatively large number of loading patterns and con-
verge to a best loading pattern, using very small amounts of computer
time.

The basic philosophy of the approach, discussed in detail in
Chapter V, is to predict a loading pattern to load into the shuffling
program. The program calculates the two dimensional power distribu-
tion, finds the radial power peak and by a predetermined programmed
set of logic searches for two elements to exchange in an attempt to
lower this radial power peak. If the attempt is successful, a lower
radial power peak will result and this loading now becomes the new
basis for searching for two more elements to move in search of an
even lower radial power peak. If a particular shuffle does not produce
a lower radial power peak, the shuffle is rejected and a new search is
made on the loading with the lowest radial power peak. Therefore, by
definition the very worst the final loading can be by this dynamic
program approach is the initial guess, since if it cannot find a lower
radial power peak than the first guess this will be the loading it
recommends. The final recommended loading produced will be
obtained when the program cannot exchange any two fuel elements of
the best loading found and obtain a lower radial power peaking factor.
Obviously not all combinations are tried since even with one-quarter core symmetry, there are approximately $10^{26}$ possible combinations in which the fuel could be loaded. Most of the combinations can be eliminated merely by simple power and reactivity checks and a very good loading can usually be found in less than 50 shuffle iterations, which requires about one minute of computer time on the UNIVAC 1108.
IV. NUCLEAR MODEL FOR ONE-QUARTER CORE SIMULATION

Since many shuffling iterations may be necessary to find a good loading pattern, it was necessary to develop a nuclear model which could accurately predict the radial power distribution of a one-quarter core model using only a few seconds of computer time. Several methods were explored. Very accurate power distributions could be obtained by running codes such as PDQ-7 or 2DB with one space point for each fuel pin, or one space point every few centimeters. This would be far too costly from a computer usage standpoint. If the number of space points were reduced to one per bundle and only two groups considered, the 2DB or PDQ-7 approach would be feasible. However, it was found that course mesh two group diffusion theory schemes are not very accurate, especially near the core reflector interface.

A third approach would be to construct a nuclear model using transport kernels, albedos and empirical equations to express $k$-infinity as a function of burnup with the coefficients obtained from more sophisticated codes such as Leopard (Barry, 1965), Laser (Poncelet, 1966), or Thermos-HRG (Skeen and Page, 1967). This approach has been successfully demonstrated and is currently being used in the computer code FLARE. With the appropriate kernels the FLARE model can be shown to be equivalent to one group diffusion
theory. The computer use time per calculation for a one-quarter core power calculation is small for this type of code, and thus would have been an acceptable approach to use. The fourth approach, and the one adopted, was to use a course mesh 1.5 group diffusion scheme. This model has been successfully demonstrated by General Electric (Crowther, Petrick and Weitzberg, 1969) and the Institute for Atomic Energy of Norway (Borresen, 1971). Since the 1.5 group model simulates a two group diffusion theory model for areas where the diffusion coefficients and scattering cross sections are very slowly varying, it represents a higher order than the FLARE approach and should give better results. The computer time requirements are very minimal since only one space point per fuel assembly is used. To complete a radial power distribution for the reference Oconee core required only about five seconds of computer time on the CDC 3300 using teletype printout or about one second on the Univac 1108 with high speed printout. These fast running times allow 40 to 50 shuffle iterations, usually enough for convergence to a good loading pattern, for only about one minute of computer time.

Development of 1.5 Group Diffusion Theory Model

In a 1.5 group diffusion theory model, diffusion theory is used to solve the fast flux distribution. The thermal flux is then analytically calculated assuming zero buckling in the thermal group. Since the
mean free path length of the fast group is comparatively long, this allows for large space nodes in the fast group without a breakdown of the difference equations derived from diffusion theory. This breakdown of the difference equations would occur for the thermal group since the 16 to 24 cm space nodes are several thermal mean free paths apart. For this reason the thermal group flux is calculated as though the mesh size is infinite in each node, and the thermal leakage from each node is assumed zero. This is a very good approximation for large node sizes.

The multigroup diffusion equation in $x, y$ geometry is

$$D_g(x, y) \phi_g(x, y) - \sum_{g'}^N (\nu \Sigma_f(x, y))_{g', g} \phi_{g'}(x, y) + \sum_{g'=1}^{g-1} \Sigma_s(x, y) \phi_{g'}(x, y) + S_g(x, y) = 0 \tag{4-1}$$

where

$$S_g(x, y) = \frac{1}{k_{eff}} \sum_{g'=1}^N \left( \nu \Sigma_f(x, y) \right)_{g', g} \phi_{g'}(x, y) + \sum_{g'=1}^{g-1} \Sigma_s(x, y) \phi_{g'}(x, y)$$

and

$N = $ number of energy groups

$g = $ energy group index

$\phi_g(x, y) = $ flux in group $g$ at point $x, y$

$S_g(x, y) = $ source in group $g$ at point $x, y$

$D_g(x, y) = $ diffusion coefficient for group $g$ at point $x, y = 1/3\Sigma_{g'}^{Tr}(x, y)$

$\nu \Sigma_f(x, y) = $ fission source cross section for group $g$ at point $x, y$

$\Sigma_s(x, y) = $ group transfer scattering cross section from $g'$ to $g$
\[ \Sigma^R_g(x, y) = \text{removal cross section for group } g \text{ at point } x, y \]

\[ = \Sigma^a_g(x, y) + \sum_{g'=g+1}^{N} \Sigma_s (x, y) \]

\[ \chi_g = \text{fraction of fission source neutrons appearing in group } g \]

\[ k_{\text{eff}} = \text{effective multiplication factor.} \]

For a two group model with all fissions appearing in the fast group and the \( x, y \) dependence implied but not written, the diffusion equation in the fast group becomes

\[ D_1 \nabla^2 \phi_1 - \Sigma^R_1 \phi_1 = -\frac{1}{k_{\text{eff}}} \left[ \nu_1 \Sigma_f \phi_1 + \nu_2 \Sigma_f \phi_2 \right] \tag{4-2} \]

where

\[ \Sigma^R_1 = \Sigma^a_1 + \Sigma_s (1 \rightarrow 2) \]

The solution to this equation is solved similar to the methods used in 2DB. To obtain the difference equations, the space points are placed in the center of the mesh area and integrated over the area. Thus, a typical \((i, j)\) mesh point can be represented by the block 0 in Figure 4-1. Integration of the volume from \((x_j + \delta x_j/2)\) to \((x_j - \delta x_j/2)\) and from \((y_i + \delta y_i/2)\) to \((y_i - \delta y_i/2)\), will obtain the spatial difference equations associated with each point. To integrate the leakage term of equation 4-2 the volume integral over the Laplacian is changed to a surface integral using Green's Theorem,

\[ \int D \nabla^2 \phi dV = \int D \nabla \phi \cdot d\vec{A} \tag{4-3} \]
Figure 4-1. Spacial integration scheme for 1.5 group model.

The flux gradients at the boundaries are obtained by interpolating between the two mesh points on each side of the mesh boundary.

Now if an effective diffusion constant $D_k$ is defined between the $i, j$ point 0 and any other $i, j$ point $k = 1$ to 4 (see Figure 4-1) then

$$
\bar{D}_k = \frac{D_0 D_k (\delta R_0 + \delta R_k)}{D_0 \delta R_k + D_k \delta R_0} \tag{4-4}
$$

where $\delta R$ represents either $\delta x$ or $\delta y$ depending on whether $k$ is 1, 2 or 3, 4 respectively.

Now the integration can be written

$$
\sum_{k=1}^{4} \frac{D_k A_k}{\ell_k} (\phi_k - \phi_0) - \sum_{0} R_0 \phi_0 V_0 + S_0 V_0 = 0 \tag{4-5}
$$

where all indices are for the fast group and

$$V_0 = \text{Volume associated with mesh point 0}$$
\[ \ell_k = \text{distance between mesh point } k \text{ and mesh point } 0 \]

\[ A_k = \text{area of the boundary separating mesh point } k \text{ and mesh point } 0. \]

All other symbols are as previously defined.

If equation 4-5 is solved for \( \phi_0 \), then

\[
\phi_0 = \frac{S_0 V_0 + \sum_{k=1}^{4} \frac{D_k A_k}{\ell_k} \phi_k}{\sum_{R=0}^{4} V_0 + \sum_{k=1}^{4} \frac{D_k A_k}{\ell_k}}
\]

(4-6)

The center line for a quarter core of Oconee bisects the fuel elements in both the \( x \) and \( y \) direction. Thus the boundary condition on two sides of the quarter core is a zero flux gradient. On these boundary points \( \phi_0 = \phi_1 \) or \( \phi_0 \phi_1 = 0 \). This is accomplished by setting the coefficients multiplying this term at these points to zero. The other two boundaries will always be water some distance from the core boundary and can be assumed for calculational purposes to have zero flux at some extrapolated distance from the mesh boundary. For these points an imaginary mesh interval is established just beyond the last mesh interval \( I_{\text{max}} \) and the coefficient becomes:

\[
\frac{D_k A_k}{\ell_k} = \frac{D_k A_k}{.56 R_{I_{\text{max}}}} + .71 \frac{\lambda}{T_r}
\]

(4-7)

Flux iterations are performed using equation 4-6 until the fluxes at
selected points do not change by more than the selected convergence value. These iterations are called inner iterations.

When an inner iteration has converged the total fissions are calculated and the eigenvalue is corrected by

$$k_{\text{eff}}^\nu = k_{\text{eff}}^{\nu-1} \times \frac{\text{total fissions}^\nu}{\text{total fissions}^{\nu-1}}$$ \hspace{1cm} (4-8)

The new eigenvalue is divided into the source and another iteration is performed. These are called outer iterations. The problem has converged when the ratio of total fissions from one outer iteration to the next differs from unity by less than the convergence criteria.

For faster convergence the fluxes are overrelaxed after each outer iteration with the exception of the first by the equation

$$\phi^\nu = \phi^{\nu-1} + \alpha (\phi^\nu - \phi^{\nu-1})$$ \hspace{1cm} (4-9)

where

- $\nu$ = the iteration index
- $\alpha$ = the overrelaxation factor

No overrelaxation is performed on the inner iterations.

Having completed the fast flux calculations, the thermal flux can be accurately estimated for a course mesh calculation by assuming the space points are far enough apart that the thermal leakage from the node is negligible. The diffusion equation for the thermal group

$$-D_2^\nu \phi_2^\nu + \Sigma_{a_2} \phi_2^\nu = \Sigma_{s_1} \phi_1$$ \hspace{1cm} (4-10)

reduces to
This allows the thermal flux at each node to be easily calculated. The
power produced in each node can then be calculated since the fast and
thermal fluxes and fission cross sections are all known. However,
the power distribution calculated will be in error by as much as 10
to 15 percent, especially in nodes which differ in power significantly
from those around them. Fuel bundles near the water boundaries will
also show this degree of error. The reason is that since only one
space point has been put in each fuel bundle, no provision has been
made for the fact that near the edge of the bundle the flux tends to
decrease or increase depending on the flux in the adjoining bundle.
For this reason it is necessary to adjust the fluxes in each node or
bundle based on the fluxes in each of the four bundles on its faces. In
two dimensions the average flux can be expressed as

\[
\phi_i = A \phi_i + \frac{1-A}{4} \sum_{j=1}^{4} \phi_j
\]  

(4-12)

where \( \phi_j \) is the flux at the interface between node \( i \) and node \( j \) and \( A \) is
the relative weight factor on the midpoint flux of point \( i \). Since \( \phi_j \) is
the flux at the interface of nodes \( i \) and \( j \), continuity of current and
fluxes at this interface yields the solution

\[
\phi_j = \frac{\phi_i D_i + \phi_j D_j}{D_i + D_j}
\]  

(4-13)
Using equation 4-12, the average fast flux can be calculated for each space point, then using these average fast fluxes a new thermal flux is calculated from equation 4-11. The averaged thermal fluxes are then calculated using equation 4-12 except all parameters now refer to thermal values instead of fast parameters. Power distributions are calculated using these averaged values of the fast and thermal flux, along with the thermal and fast fission rates. Only the weighting factor $A$ for the fast and thermal fluxes remains to be determined.

Borrensen (1971) determined $A_1$ (the fast group averaging factor) by requiring that the slab interface current density, calculated from the infinite difference equations, agree with the analytic value. For all of the test cells a value of about 0.3 was found. The error in current density is not terribly sensitive to $A_1$ and even if $A_1 = 1$, the current density error is only about 30 percent.

The thermal group weight factor, $A_2$, was similarly calculated requiring the finite-difference-node average flux to agree with the corresponding analytic slab average flux. Borrensen found values in the range of 0.7 to 0.76. The power distributions are more sensitive to value of $A_2$.

Validity of 1.5 Group Model

To check the accuracy of the 1.5 group course mesh method, a reference calculation using three fuel types was performed using 2DB.
Six unevenly spaced mesh intervals were distributed in each fuel assembly for the 2DB calculation. Two space points were located near each bundle edge, where the fluxes are likely to be changing more rapidly, and the remaining two space points evenly divided the remainder of the assembly. On the core-reflector boundary, additional mesh intervals were placed in the water where permitted. The variable mesh spacing within the bundle was found to give better results than the same number of mesh intervals evenly spaced throughout the bundle.

The identical problem was calculated using the 1.5 group model, with averaging factors of 0.3 for the fast group and 0.7 for the thermal group. In both cases the problem was converged to $10^{-6}$ on total fissions.

Figure 4-2 indicates the results of the comparison. The relative radial power from 2DB is the number on the top and that from the 1.5 group model the number below it. The bottom number is the fuel type. A three corresponds to 3.0 w/o enriched, a two to 2.0 w/o enriched, and a one to 1.5 w/o enriched $\text{UO}_2$ fuel for Oconee. As can be seen, the 1.5 group model agrees very well in power distribution with the more sophisticated 2DB model. The effective reactivities are identical to five figures from each code. The largest discrepancy in the radial power distribution occurs in the assemblies on or near the core reflector interface. This can be expected since the largest
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**Figure 4-2.** Comparison of 2DB and 1.5 group nuclear model.
error for the 1.5 group model can be expected when two nodes differ widely in fast scattering and thermal absorption cross sections. This causes a steep flux gradient which the 1.5 group model cannot accurately predict. In addition, the assumption that the thermal flux in water is the asymptotic solution of the fast flux is not correct. Nevertheless the error created by the model is only on the order of three to five percent for these core-reflector interface assemblies. If a more accurate 1.5 group model were desired, several possibilities could be explored to improve the model. Since the core reflector interface bundles contain most of the errors, a different size mesh spacing could be employed for the reflector nodes. Possibly the thermal weighting factors could be changed for these nodes. A third possibility would be to apply the weighting factors prior to each outer iteration so a fast averaged flux could return to the next iteration. Borrensen indicates that if this is not done, little penalty is incurred in the power distribution. The most likely solution would be to eliminate the water reflector nodes, and replace them with extrapolation distances where the fast flux will go to zero and analytically calculate albedos for the thermal group. Clearly this would give a more accurate solution near the core reflector interface.

As the equations 4-2 and 4-10 indicate, the input to this 1.5 group model are simply the fast and thermal values of the diffusion coefficient, absorption cross section, fission cross sections times the
neutron yield per fission, and the scattering cross section from the fast to thermal group. These seven values must be determined for each node of the problem. These values were obtained as a function of burnup from the Jersey Nuclear version of LEOPARD (Barry, 1965). Polynomial fits as a function of exposure were then incorporated into the 1.5 group model for each of the seven input cross sections listed above. More accurate values could be obtained if the energy yields as a function of exposure and the change in the xenon poisoning as a function of power were also incorporated in the polynomials. These are second order effects, however, and were not part of the model used for this study.

The purpose of the 1.5 group model is to calculate a radial power distribution for a given loading. This power distribution will then be used to make a decision as to where the fuel elements should be moved to lower the radial power peaking factor. Figure 4-2 clearly indicates the model is adequate for this purpose. One of the ground rules for these loadings, which will be presented in a later chapter, is that the core reflector interface elements must contain new fuel assemblies and they cannot be moved in the course of the fuel shuffling. Thus the region where the 1.5 group gives the largest error is an area which is not affected by fuel shuffling.

The most crucial aspect of fuel shuffling is to be able to accurately predict high power bundles on the core interior. The 1.5
group model does this well. Even if the model were off by three or four percent on some interior bundle powers, in all cases the model predicted the correct high power bundle. In most cases the high power bundles will be from 30 to 60 percent higher than the average power and differences of a few percent do not affect the shuffling decisions.
V. LOADING AND SHUFFLING PROCEDURES FOR MINIMUM POWER PEAKING FACTORS

Introduction to Power Peaking

At the end of each fuel cycle, utilities and/or reactor fuel suppliers must decide which fuel elements are to be discharged, what the enrichment of the replacement fuel assemblies will be and where each fuel element is to be placed to obtain a desired goal at the end of the next cycle life.

In Chapter II, two of the economic factors which affected the price of nuclear fuel were the length of time the fuel is in the reactor and the average discharge burnup of the fuel. The two are directly related by the power density of the reactor. The results of the economic analysis indicated there is great incentive to minimize fuel costs by decreasing the maximum radial power peaking factor throughout life. This allows the reactor to operate at the highest possible power density and reduces overhead and inventory costs per kilowatt produced.

In general, the power density of a reactor is limited by the ability of the fuel to remove the heat generated. This is a function of the fuel design and coolant flow, which are considerations outside the scope of this thesis. However, for current light water reactors these considerations are fairly standardized. In any event, once a utility
has purchased a reactor, very little will be done to change its basic
design, so these dimensional and coolant properties will remain
relatively fixed. What can be done then to increase the average burn-
up or power density and thus decrease the fuel costs for these standard
reactors?

The limiting factor to the power density in any reactor is the
fuel assembly with the highest power. Care must be taken so that this
bundle does not exceed the minimum critical heat flux ratio (MCHFR)
in boiling water reactors or the departure from nucleate boiling ratio
(DNBR) in pressurized water reactors. For normal operating condi-
tions it is usually desirable that the fuel bundle with the most power
have a MCHFR or DNBR of at least 1.7. This allows for a margin of
safety and permits a percent over-power without exceeding the burn-
out limits.

The relative power that any fuel element must produce to achieve
the desired overall reactor power will be determined by the loading
pattern of the fuel. Obviously if too many fuel elements with very low
reactivity are placed together, the power will tend to "die" in this
area, thus forcing the power to increase in other areas of the core to
meet total power requirements. Or if too many fuel elements with
high reactivity are placed in close proximity, a power spike will occur
in this area. Thus the fuel loading will determine the maximum
relative power in any fuel assembly and maximum power density at
which the reactor can operate.
An ideal loading would obtain the same amount of power from all assemblies (i.e., normalized power for all assemblies would be 1.0). This, of course, is not possible for a reactor which is refueled a finite number of times over any given time period. Refueling a reactor more often allows lower power peaks but economic considerations limits refueling too often, and electrical power requirements usually dictate the minimum time periods between refueling outages. In most cases these refueling outages are either on an annual or 18-month cycle. The reactivity of the fuel at the beginning of any cycle must be sufficient to allow the reactor to stay critical to the end of the cycle. At the end of the cycle, all or any part of the fuel can be discharged and replaced with new fuel. In most PWR cycles about one-third of the core is replaced, whereas in BWR's usually only one-fourth to one-fifth of the fuel is replaced. In either case, an equilibrium cycle will contain new fuel as well as fuel with a spectrum of burnup. The arrangement of these fuel elements is critical to the power peaking.

Three general refueling patterns exist. They are the so-called out-in, in-out and scatter or shotgun loadings. The out-in pattern loads all the new fuel on or as near as possible to the periphery of the reactor. The fuel which has been in the reactor one cycle is then placed just inside the new fuel, and the fuel which has burned two cycles inside the once burned, etc. This is the procedure often used
for initial cores with the highest enrichment fuel on the outside with the lesser enriched fuel placed nearer the center of the core. This refueling method is possible for equilibrium cores, but not the most economical and also tends to cause the reactor power to "die" in the center of the core at the end of the cycle. The second method is an in to out pattern where the new fuel is loaded in the center and the older fuel loaded on the outside with the highest burned or least reactive fuel nearest the core periphery. This method causes high power peaking near the center of the core at beginning of life and is not used. The third method, referred to by such names as a salt and pepper, scatter or shotgun loading, distributes the fuel in a checkerboard pattern throughout the core. Usually new fuel is placed in all the positions on the core-reflector interface. With the larger cores now being constructed, the surface to volume ratio is such that for a one-third reload, not enough positions are available to place all the new fuel on the core-reflector interface, or as they will be referred to, the periphery positions. The several new fuel elements remaining, along with the exposed fuel, are then distributed throughout the remaining core interior. Proper arrangement of these fuel elements to obtain minimum radial power peaking throughout the core life is the subject of this thesis.

The assumption is made that minimizing the radial power peaks will minimize the overall power peak. As mentioned, this will enable
the reactor to operate at the highest possible power density and reduce power costs by obtaining maximum return per unit time on both fuel and reactor investments. Minimizing the power peak also improves the economics from the aspect of allowing increased average burnup. With present technology, the maximum exposure of fuel is limited by the fuel integrity and not by reactivity considerations. If the maximum exposure limit from a fuel integrity standpoint were increased, and the economics favorable, the enrichment could be increased to meet the reactivity requirements. Because the fuel failure rate is an exponential function of burnup, for a given average discharge exposure, it is desirable to have the range of fuel element exposures to be as small as possible. As an example, if 60 elements with an average exposure of 30,000 MWD/MTU are being discharged, it is better from a fuel failure standpoint if all the elements have exposure of 30,000 MWD/MTU than if 20 have exposures of 25,000 MWD/MTU, 20 with exposures of 30,000 MWD/MTU, and 20 with exposures of 35,000 MWD/MTU. Disregarding the fuel failure probability, the economic difference in this burnup spread is insignificant since the average burnup of each batch is the same. However, since the fuel failure rate is a steep function of burnup, the fuel elements on the upper end of the exposure spectrum may force a shutdown before the batch average has reached 30,000 MWD/MTU. If, for example, massive fuel failures were found to occur at about 33,000 MWD/MTU
and up, then in the second of the examples above, when the 20 elements at 35,000 MWD/MTU reached 33,000 MWD/MTU, fuel failure may require shutting down. The average exposure of the other 40 elements would only be about 28,000 and 23,000 MWD/MTU respectively, and the overall average would be about 28,000 MWD/MTU, thus causing an economic penalty because the highest exposure fuel forced an early discharge. The possible economic penalties of a forced early discharge are obvious both from the increased fuel cost and scheduling standpoint.

Minimizing the radial power peaking factor will distribute the power more evenly among the fuel elements, decreasing the range of exposures of a discharged batch. This will allow the fuel to reach the highest average exposure while keeping the highest exposed assemblies below some limiting exposure. Therefore, minimizing the radial power peaking factor is desirable from a fuel failure standpoint as well as enabling operation at high power densities.

Formulation of Shuffling Optimization Procedures

Theoretically for a given set of fuel elements, there is a loading pattern which will give the minimum power peak throughout the cycle life. As discussed in Chapter III, many attempts have been made to use sophisticated statistical techniques to optimize few region reactor loadings. These do not solve the problem, however, since a typical
PWR has approximately 160 to 193 fuel elements and even with one-quarter core symmetry, this leaves upward of 40 individual regions to consider. Several fuel elements will be identical, further reducing the number of unique combinations, but the number of possible loading combinations could still be astronomical and beyond any attempt to solve by analytical techniques. There is a great incentive, then, to obtain a set of rules which will make the process of deciding upon a loading pattern easier. These could be formulated into rules for a computer program which would iterate to find the loading pattern with the lowest radial power peaking factor throughout the cycle life. It is impossible to insure that the loading pattern chosen would indeed be the one with the minimum radial power peaking factor. From a practical and economic standpoint it is not necessary that it is. As long as the programmed shuffling rules produce loadings with low radial power peaks, they will be satisfactory.

An iterative programming approach was taken to produce such a program which is called SHUFLE. The program first calculates a power distribution of an initially specified loading pattern using the 1.5 group course mesh diffusion model described in Chapter IV. A search is then made for the maximum radial power peak throughout the core. Pre-programmed logic is followed to decide which two assemblies should be exchanged to lower the radial power peaking factor. The change is made and a new radial power distribution is
calculated with the 1.5 group model, and the new radial power peak identified. This new radial power peak is then compared with the lowest radial power peak of any loading which has been tried. If it is lower, the program proceeds. If it is not, the previous loading is rejected and a new search for another shuffle is made on the loading which has the lowest radial power peaking factor of any pattern tried to date in the shuffle iteration. This procedure continues until all possible programmed fuel shuffles have been tried and none give a lower radial power peaking ratio. At this point, the convergence criteria have been met.

Many of the new generation 600 to 1100 MWe PWR's have surface to volume ratios such that it is not possible to place all the new fuel elements on the core reflector interface of the reactor for a one-third reload cycle. Thus several new fuel elements will be required to be placed in the interior core positions. In the case of Oconee, five elements must be placed inside the periphery. The power peaks will almost invariably occur in these elements. The general shuffling procedure described above will always involve proper placement of these high reactivity fuel assemblies. For this reason shuffling the fuel to a minimum radial power peak is broken into two distinct sections in the program which performs the shuffling. The first section, called SHUFLA, moves the high reactivity fuel to lower the peaking factor as much as possible. The second section of the shuffling logic
performed by SHUFLB attempts to improve the radial power peaking factor by exchanging relatively high reactivity fuel in the region of the radial power peak for less reactive fuel from an area of the core which is low in power. This tends to flatten the power distribution both by lowering the reactivity in the area of the radial power peaks and by increasing those areas of low reactivity and power. In essence, the program logic tends to flatten the power distribution by proper placement of the reactivity within the core. Again after a shuffle is made, if the new power peak is greater than the minimum radial power peak, the shuffle is rejected and another search is made for a shuffle of the loading pattern with the minimum radial power peak. This procedure is continued until the convergence criteria is met for both sections of the program.

The basic philosophy which SHUFLA and SHUFLB use to determine which fuel elements to exchange in the course of searching for a loading with the minimum radial power peaking factor was derived from collating data of the radial power distributions of hundreds of loading patterns all using the same fuel. From these studies certain trends were noted from which the following general rules were derived:

1. Scatter or checkerboard loadings produce a flatter power distribution than out-in or in-out loading schemes.

2. Placing new fuel assemblies in all positions on the periphery
of the core produces the most uniform power distribution with lowest power peaking ratios.

3. Fuel assemblies of relatively high reactivity should not be placed in close proximity anywhere except on the periphery of the core or just inside those assemblies on the periphery. This means new fuel assemblies should not be placed in adjacent positions on the core interior.

4. Lower radial power peaks result if the lowest reactivity fuel is placed on all four faces of high reactivity fuel. Fuel which has been in the reactor two years of a three-year cycle should be placed on all four faces of the new fuel assemblies which are in the reactor interior.

5. Replacing a fuel element in the vicinity of a radial power peak with one of less reactivity will always lower the power peak in this area and increase the relative power in the area where the more reactive fuel element is placed. This does not mean that the overall radial power peak will be reduced since, if the power distribution is quite flat, the area receiving the more reactive element may increase enough to produce a new radial power peak which is higher than before the elements were exchanged. In most cases, however, exchanging the elements as described will lower the radial power peak.
6. New fuel assemblies placed in the reactor interior cause large perturbations on the local radial power peaking. Because of this, better results are obtained in the shuffling iteration scheme if these elements are moved only one position at a time, then a new power calculation made to determine the next move rather than moving them directly from a high power area to a low power area. This is the reason the iteration scheme is broken into two areas: that which decides on the placement of the new fuel assemblies and that which decides on the placement of all other fuel. Because of its lower reactivity, the movement of any fuel with several thousand MWD/MTU exposure does not represent such a large perturbation on the radial power distribution and can be moved considerable distances from their position in one shuffle iteration to the next.

7. Shuffling patterns which iterate by simply exchanging the highest power fuel with the lowest power fuel will give reasonable loading patterns much of the time, but are too dependent upon the initial loading pattern supplied to the shuffle program. If a bad initial pattern is chosen, this method of shuffling may result in a loading with a relatively high minimum radial power peak.

8. Requiring one-eighth core symmetry limits the available
loading patterns and will usually result in a higher radial power peak than if one-quarter core symmetry is employed. No calculations were done using one-half core symmetry which would allow even more possible loading patterns. The degree of freedom allowed by one-quarter core symmetry is sufficient to enable shuffling to very good loadings. One-eighth core symmetry does not in many cases, and for many instances, due to fuel failures and thus additional new fuel replacement, one-eighth core symmetry is not possible.

9. The lower leakage factor near the center of the core makes proper fuel placement more critical in this area. Because of this, better loading patterns can be obtained if the shuffling program uses different logic depending on which area of the core is having the radial power peaking problems.

Based on the above, the program SHUFLE was written which incorporates these observations into logical fuel shuffling statements.

SHUFLE calculates the power distribution, or power shape as the name would imply, from the 1.5 group coarse mesh diffusion model described in Chapter IV. Two elements are then exchanged, depending on a prescribed hierarchy of shuffling rules, which are discussed in detail in Chapter VI. This procedure is continued until convergence criteria indicate the program has completed its search for a loading pattern with the lowest possible radial power peak to average ratio, usually just called a radial power peak.
VI. SPECIFIC SHUFFLING RULES OF SHUFLE

Core Region Description

This chapter is a detailed discussion of the logic used by SHUFLE to converge on a fuel loading pattern which has a minimum radial power peak.

A typical PWR one-quarter core configuration such as illustrated in Figure 6-1 will help visualize the shuffling rules which are employed in SHUFLE. The dashed lines indicate the symmetry boundaries of the Oconee II core chosen for study. The core is divided into three general regions, each of which has different shuffling rules. The fuel elements on the core-reflector interface, marked with a P, will always be loaded with the most reactive fuel. These are usually new fuel assemblies, and are not shuffled throughout the shuffling iterations. This region will be referred to as the periphery region. The second region is just inside the periphery region, but outside the boundary of \( I + J \geq 9 \). These fuel elements, marked with I, are close enough to the edge of the reactor core that the shuffling rules are slightly different from those elements inside the region \( I + J < 9 \). This region will be referred to as the

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Footnote 4: Continual reference will be made to the \( I + J \) position of a fuel element. This is simply the sum of the I and J index of a particular element. As an example, the element in the upper left corner is the element at the reactor center and would have a position \( I = 1, J = 1 \), and \( I + J = 2 \).
Figure 6-1. OCONEE one-quarter core configuration with position designation.
P = Periphery region; I = Intermediate region; E = Even parity interior region; O = Odd parity interior region
intermediate region. The third region consists of all assemblies inside the boundary $I + J < 9$. This region is called the interior region. Within the interior region a fuel assembly is categorized by whether the $I + J$ position is odd or even. In Figure 6-1, the assemblies with $I + J$ even are designated E and those for $I + J$ odd are marked with an O.

**General Shuffling Rules**

In general, the shuffling rules allow any assembly in the intermediate region to be exchanged with any other element in this region or the interior region. Elements in the interior region are allowed to exchange with elements in the intermediate region but are only allowed to exchange with other interior elements with the same parity of $I + J$ (i.e., even parity exchange with even parity and odd parity with odd parity). The reason that fuel on the interior may only exchange with fuel of like $I + J$ parity is to avoid violating rule four of Chapter V, which specifies that fuel elements with low reactivity must be placed on all four faces of highly reactive fuel assemblies which are placed on the interior of the core. If the core interior is fueled with the highest burned or lowest reactivity fuel, all on odd diagonals or all on even diagonals, and the new and one cycle exposed fuel on the other diagonals, the rule of not exchanging two interior fuel elements of different $I + J$ parity will always place low
reactivity fuel on all four faces of all the high reactivity fuel on the core interior. This rule may be the single most important to preventing high power peaks in the interior region.

Since it is very difficult to describe and understand written program logic, the details of how SHUFLE uses the general rules in Chapter V to converge on a loading pattern will be described by flow charts and illustrations in Figures 6-2 through 6-4. These will be supplemented by graphical examples to illustrate the ideas and nomenclature presented in the flow charts. The overall program logic is presented in Figure 6-2. This simply describes the available options the code will follow depending on the input parameters. All decisions presented in this figure are controlled by the operator and reflect the problem he wishes to solve. In the event it is desired to shuffle the fuel, subroutines SHUFLA and SHUFLB are called from the main program. These subroutines shuffle the fuel to the minimum radial power peak possible using the observations described in Chapter V and the program logic described below.

**Description of SHUFLA Logic**

SHUFLA arranges high reactivity fuel which may be placed in the core interior. Figure 6-3 illustrates the logic flow for the shuffling iterations. The program initially checks to see if it has exhausted all its shuffling possibilities from previous iterations. If it
Start

Read initial input data

Calculate rel. power of each fuel assembly

Shuffle? Burn?

For details on decision process to determine shuffle

see flow chart on SHUFLA and SHUFLEB

Burn fuel by core exposure increment indicated

End of burn cycle?

Shuffle?

Yes

Print of core exposure and power by fuel assembly

No

Shuffle criteria exhausted?

Yes

Print of loading pattern with minimum power peak

No

Burn?

Yes

No

End
Figure 6-3. Flow logic of SHUFLA.
has, subroutine SHUFLB is called. If not, the maximum radial power element is located and compared to the lowest radial power peak from previously shuffled loading patterns. If the radial power peak is lower than previously, the program proceeds as indicated to block seven and records this loading pattern as the best it has been able to find. At this point all the iteration parameters are reinitialized because a new "best" loading has been found and the search begins for a better one. The maximum power element which is not on the periphery is selected and the search is made for a fuel element to exchange with it. If the unlikely event occurs that the maximum radial power element is located on the periphery, most likely the highest power non-periphery assembly is located adjacent to it and will be the one to be moved. Thus the restriction that the periphery fuel elements cannot be moved is no hindrance to obtaining a good loading pattern.

The search for the two fuel elements to exchange begins with the selection of the fuel element which has the lowest power in the vicinity of the maximum power non-periphery element. This element is called local minimum (block 10). In a large power reactor, the core is loosely coupled and radial power spikes are a result of the fuel which is in the immediate vicinity of the peak element and only in a remote way dependent upon what may be several fuel assemblies away. For this reason, on any single shuffling iteration best results
are obtained if the fuel is not moved far from its position of the previous iteration. \( I_{\text{max}} \) and \( J_{\text{max}} \) are the positions of the fuel with the maximum radial power peak. It was found if an area \( I_{\text{max}} \pm 2, J_{\text{max}} \pm 2 \) were searched for a local minimum, this would involve sufficient area to determine in which direction the element with maximum radial power should be moved for the next iteration. Figure 6-4 illustrates an example of the local minimum. The maximum power of 1.444 is in position 4, 4. This means the 24 elements in the area from \( I = 2 \) to 6 and \( J = 2 \) to 6 are searched for a local minimum. Within this area the lowest radial power is .758 at position 6, 5, or 5, 6 and these become the local minimum.

Since the maximum power element is not in the intermediate region of the core, the program proceeds to box 15 of Figure 6-3 and determines that the parity of the maximum radial power fuel element is even. This fuel assembly is then moved to a position of the same parity one position closer to the local minimum (i.e., move fuel in 4, 4 to 5, 5 and 5, 5 to 4, 4). If both the maximum and local minimum had been in the intermediate zone, the check at box 14 would have routed the logic to box 16 and the maximum element would simply have been moved one position closer to the local minimum without regard to parity. If through this route one of the fuel elements to be shuffled was an element on the periphery, the assembly adjacent to it would be shuffled instead, as indicated by blocks 18 and 20 of Figure 6-3.
Figure 6-4. Example of maximum radial power, local minimum and exchange of assemblies by SHUFLA.
Once the two elements to be exchanged in the shuffle have been determined, block 21 checks if the reactivity of the local minimum fuel assembly is less than that of the maximum power assembly. Obviously it will not improve the radial power peaking to insert an assembly with equal or higher reactivity in the position of the peak radial power than the one presently occupying the position. If the reactivity of the assembly to be exchanged is less than the assembly with the radial power peak, the shuffle is made. If not, this means that there is too much reactivity in the area of the maximum radial power element and some elements of high reactivity need to be moved away from the maximum power assembly. If this occurs, since the element which was to be exchanged with the maximum power assembly has already been determined to be of equal or higher reactivity than the maximum power element, it is the logical choice to be moved. The program simply assigns temporarily to this element the status of maximum power as indicated in block 22, Figure 6-3, and returns to block 10 in search of a position to move this element. In all cases, this procedure will move this high reactivity assembly away from the actual maximum power assembly and reduce the radial power peaking factor in that area. If it does not create a higher radial power peak in another area of the core, then it will reduce the overall radial power peak. An example of this situation is illustrated in Figure 6-5. Note that the maximum radial power of 1.402 occurs at position 2, 6. The
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Radial power distribution
Fuel type

Figure 6-5. Example of moving element when max power and position it is to go have same reactivity.
local minimum of this position is .783 in position 3, 8. The program then designates an exchange of 2, 6 and 3, 7. However, since these are the same fuel type with zero exposure, the fuel assembly in 3, 7 has the same reactivity as the one in 2, 6 and nothing would be gained by exchanging these two assemblies. Therefore, the program temporarily assigns the status of maximum power to 1.305 in position 3, 7 and searches for a move of this element. The local minimum of 1.305 is .724 in position 5, 7. The program attempts to exchange 3, 7 and 4, 7 but 4, 7 is on the periphery so the nearest element to the periphery, 4, 6, is chosen instead to exchange with 3, 7. In this case, this particular shuffle lowered the radial power peak to 1.381 which occurred at position 4, 2, so the shuffle was quite successful.

The shuffle logic for SHUFLA has been described if the new radial power peak is lower than any previous loading pattern. What if it is not? This decision is depicted in block four of Figure 6-3. In most cases, this will result in a rejection of the previous shuffle. However, in the event that the new radial power peak appears in the reciprocal position from the power peak of the best loading to date, the program will shuffle this reciprocal position element in an attempt to find an overall lower radial power peak.

The reason for this is that if the one-quarter core loading is

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5 Reciprocal positions are positions with indices (I, J) and (J, I). Example positions (7, 3) and (3, 7) are reciprocal positions.
symmetrical, or very nearly symmetrical, two elements in reciprocal positions will have the same or very nearly the same power. If one of these assemblies is shuffled to a position to lower its radial power, it is possible for this to be a good move and yet increase the relative power in the reciprocal position from which it was moved, because the program must have a constant total power with each iteration. It is most likely, then, that moving the element which has this increased maximum and at the same time retaining the shuffle of its reciprocal element from the previous shuffle iteration will give a lower overall radial power peak. In many cases, this could be accomplished by requiring one-eighth core symmetry. However, in most cases one-eighth core symmetry is too restrictive to shuffling and when it is advantageous this method will determine it at only the cost of one additional shuffle iteration.

If the maximum radial power of any shuffle iteration is greater than the maximum radial power of the best loading (i.e., the one with the lowest radial power peak) on any previous iteration, and it is not in the reciprocal position, the shuffle is rejected and the best loading is recalled to look for further shuffles of this loading pattern. On the second iteration after a new best loading has been found, the program proceeds to block 11, Figure 6-3, and attempts to move the second highest power element. If it is the third or higher iteration, the program goes to block 13 and attempts to move the maximum power
element to another position. This procedure continues until all possible moves of the maximum power element in the area of $I_{\text{max}} \pm 2, J_{\text{max}} \pm 2$ have been tried, remembering that only those elements in positions of like parity may be exchanged for elements in the inner region. At this point, subroutine SHUFLA has exhausted its shuffle logic and for further shuffle iterations block one of Figure 6-3 will send the program to subroutine SHUFLB to employ a new set of logic in an attempt to lower further the radial power peaking factor.

Description of SHUFLB Logic

Subroutine SHUFLA has essentially satisfied rule three of Chapter V and has spread out the high reactivity fuel as far as possible, except for those assemblies on the periphery of the core, which it is not allowed to move. To further reduce the radial power peaking it will be necessary to rearrange the exposed lower reactivity fuel around the assembly which has the radial power peak. This is what subroutine SHUFLB does. The general objective of SHUFLB is to move less reactive fuel into the region which has the maximum radial power peak, and move more reactive fuel into the area which has low radial power factors. Figure 6-6 is a flow logic illustration of SHUFLB.

Up to block 29 of Figure 6-6, SHUFLB is very similar to SHUFLA in that it finds the maximum radial power element and
Figure 6-6. Flow logic of SHUFLB.
Determine H-J parity of local max and search for element of same parity in interior region or any element in intermediate region which has lowest power.

Is reactivity of this fuel element lower than that of local maximum?

Has this shuffle already been tried?

Is either local max or fuel to be exchanged with local maximum on a symmetry boundary of 1/2 fuel?

Exchange local max and selected fuel element + symmetry element of element on boundary.

Exit to power calculation.

End of shuffle iteration.

Select next highest local max in area of max power fuel.
compares it to previous loadings to see if the new loading pattern is better from a radial power peaking standpoint than any previous loading. If it is, this becomes the best loading. If not, it rejects the loading and reverts back to the best loading pattern. Block 29 of Figure 6-6 begins the search for an element to shuffle. First, the highest power element in the area \( I_{\text{max}} \pm 1, J_{\text{max}} \pm 1 \) is found. This element is referred to as the local maximum since it is the assembly with the second highest radial power in the vicinity of the maximum radial power element. The local maximum element is illustrated in Figure 6-7. Referring to block 32, Figure 6-6, the parity of this local maximum is determined and the element of lowest power with the same parity in the inner region or any parity in the intermediate region if found. Again the radial power peaking factor cannot be reduced if this lowest radial power element had a higher reactivity than the local maximum element. Therefore, if the reactivity of the minimum radial power element with the same parity is higher than the local maximum element, the choice is rejected (block 33) and the next lowest radial power element satisfying parity requirement is found (block 37). This continues until all elements satisfying the requirements for this local maximum have been tried in a shuffle or rejected for failure to meet reactivity requirements. This means there are no fuel assemblies throughout the core which can be exchanged with the local maximum element and will not violate the set of rules in
Figure 6-7. Illustration of local max and search areas $I_{\text{max}} \pm 1, J_{\text{max}} \pm 1$ and $I_{\text{max}} \pm 2, J_{\text{max}} \pm 2$ for SHUFLB.
Chapter V, and which will also lower the radial power peaking factor.

As an example, assume in Figure 6-7 that all shuffles of the local maximum element in position 3,2 have been tried. The next highest radial power of 1.011 occurs at position 2,3. This assembly then becomes new local maximum as far as shuffling is concerned, and a new search for assemblies of lower reactivity to exchange with this assembly begins. If none can be found which satisfy requirements or are tried and do not lower the radial power peaking factor, then the next highest radial power assembly with a radial power of .989 in position 4,4 is selected and the search continues. This procedure is continued until all the elements in the area $I_{\text{max}} + 1, J_{\text{max}} + 1$ have been tried as the local maximum element. The program then proceeds to block 39 from 40 of Figure 6-6 and expands the search area for the local maximum to $I_{\text{max}} + 2, J_{\text{max}} + 2$ and proceeds as before.

Although the search area for the local maximum element is bounded, the fuel assembly to exchange with it is found by searching the entire core, and may be any element which satisfies parity and reactivity requirements.

When all possible combinations have been tried, or eliminated by violation of rules, to shuffle the local maximum in the area $I_{\text{max}} + 2, J_{\text{max}} + 2$, the program has exhausted the programmed shuffle logic and ends. A one card change in the program would allow this search to the area $I_{\text{max}} + N, J_{\text{max}} + N$, but would require additional shuffles.
and would most likely not lower the radial power peaking significantly.

In fact, when the area $I_{\text{max}} + 1$, $J_{\text{max}} + 1$ has been entirely searched, the radial power peaking factor for the best loading is usually very near the radial power peaking factor of the best loading after $I_{\text{max}} + 2$, $J_{\text{max}} + 2$ has been entirely searched for a shuffle.

When the shuffling iteration has converged, SHUFLA will then print out the best loading it has found and burn to a specified burnup or quit, depending on the problem input.
VII. TESTS AND RESULTS

The validity and feasibility of SHUFLE was checked using OCONEE II as a reference design. OCONEE is a Babcock and Wilcox 850 MWe PWR. Although it differs slightly from the current generation of 1100 MWe PWR's, it is very similar and all conclusions will be applicable to any large PWR.

SHUFLE has the ability to optimize either an initial core loading of unexposed fuel or a core loading of any combination of exposed fuel. The results of shuffling of several combinations of both an initial and exposed core are presented in this chapter.

The initial core consisted of fuel with the physical parameters and identification listed in Table 7-1. A core loading of 1.5, 2.0 and 3.0 weight percent enriched fuel was chosen as typical enough of an initial core enrichment to demonstrate the capabilities of SHUFLE for an initial core.

The exposed core fuel elements were those listed in Table 7-2 and were selected to approximately represent an equilibrium cycle with respect to average burnup in each exposure group. The initial U-235 enrichment of each element was 3.0 w/o and the polynomials used to represent the change in each cross section as a function of burnup are presented in Table 7-3.

6Refer to Appendix C for complete design details of the reference model.
Table 7-1. Two group LEOPARD parameters for OCONEE II UO\textsubscript{2} fuel assemblies at zero exposure.

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Table 7-2. Exposure values for one-quarter core.

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<tr>
<td>22.0 (15)</td>
<td>11.93 (14.25)</td>
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Average of total loading = 11.3 MWD/kg BOL.
Table 7-3. Two group polynomial fits as a function of exposure for
3.0 w/o enriched OCONEE II fuel.

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<th>A_4</th>
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P = A_0 + A_1x + A_2x^2 + A_3x^3 + A_4x^4

P = Fitted parameter

x = Exposure in MWD/kg

All fits include equilibrium xenon

R^2 correlation coefficient > .99 for all parameters.
SHUFLE will reorganize the fuel in search of a minimum radial power peak, but to begin the search an initial loading must be specified. It is not necessary that this loading be a good one; however, to comply with the loading pattern observations in Chapter V certain basic loading rules should be observed. These are:

1. Load all periphery positions with the most reactive fuel.

2. Group the remainder of the fuel by either reactivity or exposure into two groups. For the demonstration case one group contained 1.5 w/o enriched fuel in the case of an initial core and two year exposed fuel in the case of an exposed core. The other group consisted of 2.0 w/o enriched fuel and the remainder of the 3.0 w/o enriched fuel in the case of an initial core and the one year exposed and the remainder of the unexposed fuel in the case of an exposed core.

3. Load one of the groups of item two in the odd parity positions of the core interior and the other on the even parity positions of the interior. Which group occupies the odd positions and which the even positions was usually determined by the exposure of the central element of the core (i.e., 1, 1 position). Since the parity of this position is even, which ever exposure group this central element is in occupies the even parity positions.
4. The intermediate region of the core may be loaded with either group of fuel.

5. Do not locate unexposed fuel in close proximity on the core interior.

It should be emphasized that these loading rules are not compulsory. SHUFLE will relocate the fuel in search of a minimum radial power peak regardless of the initial loading it is given. However, violation of these rules may result in a converged loading pattern with an unacceptably high radial power peak.

Initial Core

The results of some of the initial core loadings are presented in Figures 7-1 and 7-2. In each case the fuel type by position presented to SHUFLE is the top number in each block, which represents an assembly location. The bottom number is the fuel type which SHUFLE would place in the position to optimize the loading. Several different initial loadings were calculated and, although the results were not identical, they were very similar and all converged to a loading with a peak radial power of about 1.37 to 1.40, which is apparently near the minimum achievable with these three particular enrichments in this core. This is excluding the use of burnable poisons which could be used on the most reactive fuel assemblies in the core interior and intermediate regions to further reduce the radial power peak. In all
Loading presented to SHUFLE. Radial power peak 1.78 at 1, 3 and 3, 1

Final loading of SHUFLE. Radial power peak 1.40 at 1, 3 and 3, 1

25 iterations 26 sec computer time

Figure 7-1. Shuffle of initial core loading.
Loading presented to SHUFLE. Radial power peak = 1.55

Final loading of SHUFLE. Radial power peak = 1.37 at 6, 2 and 2, 6.

24 iterations 17 sec computer time

Figure 7-2. Shuffle of initial core loading.
cases the radial power peak occurred in a type three fuel (3.0 enriched).

It is possible to present SHUFLE with a loading it cannot shuffle to a reasonable radial power peak. However, in all cases where this happened, the loadings presented to SHUFLE were not reasonable loadings. As an example, if all the number three type fuel not on the periphery were loaded near the core center, SHUFLE could not find optimum positions to place all five of these elements. However, this is only of academic interest since this is an unreasonable loading to present to SHUFLE and violates rule five of the general loading rules.

Exposed Core Loadings

Using the fuel assemblies listed in Table 7-2, several loadings were run to check both total shuffling and shuffling of only the exposed fuel. Again the only restrictions on the initial core loadings presented to SHUFLE were the five rules listed earlier in the chapter.

To test if SHUFLB would converge to the same loading regardless of the initial loading presented to it, an attempt was made to load the fuel as differently as possible without violating the five loading rules. Figures 7-3 and 7-4 illustrate the results of these tests. Note that no two positions were loaded with the same exposure of fuel. The converged loading is the lower figure in each position. Figure 7-5
Exposure in MWD/kg of initial loading to SHUFLE

Loading recommended by SHUFLE.  Radial = 1.30

Figure 7-3.  Initial and final loadings of exposed fuel from SHUFLE.
### Exposure in MWD/kg of initial loading to SHUFLE.

### Loading recommended by SHUFLE. Radial = 1.29

**Figure 7-4. Initial and final loadings of exposed fuel from SHUFLE.**

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Recommended loading from Figure 7-3. Radial = 1.30

Recommended loading from Figure 7-4. Radial = 1.29

Figure 7-5. Comparison of recommended loadings from Figures 7-3 and 7-4.
reillustrates Figures 7-3 and 7-4 by comparing the final loading of each run. It is readily noted that the loadings are not identical, indicating the convergence is somewhat dependent upon the initial loading supplied to SHUFLE. However, the radial power peaks of 1.30 and 1.29 for the two loadings are essentially identical and well within any calculational uncertainty. In addition, while SHUFLE did not converge to the same pattern, Figure 7-5 illustrates that it came very close. SHUFLE tended to put about the same exposure elements in similar positions. As an example, fuels with exposure 24.5 and 20.0 MWD/kg were initially loaded in position 4, 3 on the two successive runs. SHUFLE placed fuel with exposure 19.0 and 20.0 MWD/kg in this position for the respective runs. However, in other positions such as 1, 2 the fuel on the two runs was loaded at 23.0 and 19.0 MWD/kg respectively and SHUFLE did not change them, but fuel in other positions in the area was changed to compensate for this exposure difference.

Figure 7-6 illustrates a loading using exactly the same fuel as in Figures 7-3 through 7-5 (i.e., that in Table 7-2), but the new fuel is arranged in a different manner. The radial power peak of the loading initially was 1.44 and as indicated SHUFLE rearranged the fuel to achieve a radial power peak of 1.31, although the loading is not exactly the same as the other converged loading patterns. This again illustrates that the optimized loading is dependent upon the loading
Exposure in MWD/kg of initial loading to SHUFLE.  
Radial = 1.44

Recommended loading from SHUFLE.  Radial = 1.31
55 iterations 35 sec.  UNIVAC 1108

Figure 7-6. Initial loading presented to SHUFLE and recommended loading.
initially presented to SHUFLE. Nevertheless, the radial power peaks are very good and the dependency upon the loading presented to SHUFLE will not cause any problems from a practical standpoint. In addition, just as for the initial core loading of unexposed fuel, care must be taken not to bunch the new unexposed fuel in any part of the core or SHUFLE may not find a pattern which is close to the optimum. It will, however, improve the loading in any event. As an example, one run was made in which the initial radial power peak was about 2.0 because new assemblies were placed in adjacent positions 6, 5, 6, 6, 5, 6, and 5, 7. SHUFLE rearranged the assemblies, but was only able to achieve a radial power peak of 1.40. This is certainly an acceptable radial from a practical standpoint, but is about eight percent above the optimum for the set of elements listed in Table 7-2, which is near 1.29.

Radial power peaks change throughout the cycle as the core accumulates exposure. Often the peak will even shift from one element to another. This raises the question as to where in the cycle does the maximum radial power peak occur and at what exposure should shuffling be performed to minimize this radial power peak.

All of the shuffling was done at equilibrium xenon and zero cycle exposure. To insure that this was the period in the cycle to perform the shuffling, the same loading patterns as illustrated in Figures 7-3 and 7-4 were run at zero cycle exposure non-xenon equilibrium and at
400 MWD/MT and 500 MWD/MT average cycle exposure. The result was that shuffling at non-xenon zero exposure gave equally good answers at the end of the shuffle, but as the elements began to accumulate exposure the radial power peak increased above those shufflings performed at xenon equilibrium. In each case the total cycle radial power peaks were higher when the shuffling was done at non-xenon equilibrium and, in general, non-xenon shuffling tended to load the core too heavily near the center. The shuffling done at 400 and 500 MWD/MT converged to the same total cycle radial as the xenon equilibrium zero exposure shufflings did. Beyond about 500 MWD/MT the radial power peak is never greater than at the beginning of the cycle and the radial power peaks are not a problem. Thus, these cases illustrated that shuffling at the beginning of cycle life with equilibrium xenon will produce loadings which will not give a radial power peak higher than shufflings done at any other cycle exposure. It must be noted that when the shufflings were performed at 400 or 500 MWD/MT cycle exposure, it was not intended that the fuel would be burned to this exposure then shuffled in the reactor. Rather, only that the shuffling calculations were performed with this assumed exposure. The amount of exposure added to each element was calculated using the relative power as calculated from the 1.5 group model and adding to each element the product of the average core exposure desired and the relative power factor. The shuffling calculations
would then be performed with fuel of this exposure. When the shuffling calculations were complete the same amount added to each assembly would be subtracted and the cycle would be burned. In the case of the zero exposure equilibrium xenon shufflings, these procedures were not required, of course.

**Cycle Data**

The next major question was whether SHUFLE could handle successive cycles and not incur problems in some future cycle because of the loading it produced in a previous cycle. To check this the loading in Figure 7-4 was chosen as cycle 1 and burned to 10,000 MWD/MTU. The fuel assemblies which had been in the core three cycles were discharged and replaced with new 3.0 w/o enriched assemblies. The fuel was grouped as described earlier in this chapter and loaded into SHUFLE by the five loading rules. The loading pattern given to SHUFLE and the converged loading pattern for cycle two is presented in Figure 7-7. The radial for the initial loading pattern was 1.43 and SHUFLE rearranged the fuel to a peak radial of 1.37 which is quite acceptable. SHUFLE was not able to shuffle the fuel to the 1.30 radial of cycle one because the fuel exposures at the beginning of cycle two were different than at the beginning of cycle one. This is because the fuel for cycle one was not at equilibrium exposure and as additional cycles are calculated, the exposures change to approach
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| 0     | 18.86 |      |      | 12.27 | 21.16 | 9.72 | 22.64 | 11.06 | 0
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| 19.7  |      |      |      | 9.27 | 20.46 | 0   | 10.42 | 0   |
| 0     | 21.16 |      | 23.06 | 9.27 | 23.06 | 12.80 | 23.57 | 0   |
| 9.48  | 21.16 |      | 23.06 | 9.27 | 23.06 | 12.80 | 23.57 | 0   |
| 21.72 |      | 20.46 | 12.8 | 0   | 12.56 | 0   |       |     |
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Exposure in MWD/kg of initial loading presented to SHUFLE. Radial = 1.43
Recommended loading from SHUFLE. Radial = 1.37

Figure 7-7. Initial loading and recommended loading from SHUFLE for cycle two.
equilibrium. At the end of cycle two the same procedure was followed to evaluate cycle three. Figure 7-8 illustrates cycle three data with a radial power peak of 1.35. Note that cycle three requires charging only 14.25 new fuel elements per one-quarter core rather than the 15 fresh assemblies in cycle one and two. This is because the assembly in the core center has accumulated discharge exposure and requires replacement. Because of its unique position in the core, it represents only one-fourth bundle in a one-quarter core configuration. Barring complications due to fuel failure, the number of fresh fuel assemblies charged each cycle will be in a sequence of 60, 60 and 57 for the entire core.

Figures 7-4, 7-7 and 7-8 illustrate that SHUFLE can handle successive cycles and find acceptable radial power peaking ratios for each cycle.

Non-equilibrium Cycles

One of the purposes of this study was to produce a code which would be able to find a good loading pattern in a short amount of time. As stated earlier, for an equilibrium cycle most utilities will have several good cycles computed in advance so the need for this type of code may have limited use, but can still be useful in searching for an optimum loading pattern. What happens when unexpectedly at the end of a cycle several fuel elements have failed and the core must depart
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Exposure in MWD/kg of initial loading presented to SHUFLE. Radial = 1.71
Recommended loading from SHUFLE. Radial = 1.35

Figure 7-8. Initial loading and recommended loading from SHUFLE for cycle three.
from an equilibrium cycle? This is a situation where SHUFLE would be extremely useful in determining the best cycle loading patterns.

To demonstrate SHUFLE's ability in this situation, two different situations are illustrated in Figures 7-9 and 7-10. In Figure 7-9, it was assumed that at the end of cycle one, four fuel assemblies, which were not planned to be discharged, failed and required replacement. This would require one additional new assembly to be charged in a one-quarter core. The failed fuel assemblies were presumed to be those with the highest exposure among those elements which were not planned to be discharged. Figure 7-9 illustrates the initial and converged loading patterns from SHUFLE. It is noted that the peak radial power for this cycle was 1.35, which is quite good considering the amount of fresh fuel necessarily loaded. Figure 7-10 illustrates the same situation except it was presumed eight assemblies failed and two additional fresh assemblies per one-quarter core were charged at the beginning of the cycle. This meant 17 fresh assemblies were charged instead of the usual 15 per one-quarter core. Again, SHUFLE arranged the fuel to achieve a peak radial power throughout the cycle life not to exceed 1.35. These two illustrations clearly indicate SHUFLE is capable of finding very acceptable loading patterns even for a non-equilibrium cycle situation.
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</tr>
<tr>
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<td>21.72</td>
<td>10.42</td>
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<td></td>
</tr>
<tr>
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<td>0</td>
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<td></td>
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</tr>
<tr>
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<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Exposure in MWD/kg of initial loading presented to SHUFLE. Radial 1.63
Recommended loading from SHUFLE. Radial 1.35

Figure 7-9. Initial and recommended loading from SHUFLE assuming one failed assembly per one-quarter core at end of cycle one.
<table>
<thead>
<tr>
<th></th>
<th>21.78</th>
<th>0</th>
<th>23.06</th>
<th>0</th>
<th>22</th>
<th>9.72</th>
<th>19.7</th>
<th>0</th>
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<td>19.7</td>
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<td></td>
</tr>
<tr>
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<td>0</td>
<td>10.42</td>
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<td>22.64</td>
<td>9.27</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>10.42</td>
<td>18.86</td>
<td>0</td>
<td>22.64</td>
<td>9.27</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23.06</td>
<td>10.42</td>
<td>18.86</td>
<td>0</td>
<td>22.64</td>
<td>9.27</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23.06</td>
<td>10.42</td>
<td>18.86</td>
<td>0</td>
<td>22.64</td>
<td>9.27</td>
<td>0</td>
<td></td>
<td></td>
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<tr>
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<td>10.42</td>
<td>18.86</td>
<td>0</td>
<td>22.64</td>
<td>9.27</td>
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<td></td>
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</tr>
<tr>
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<td>10.42</td>
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<td>22.64</td>
<td>9.27</td>
<td>0</td>
<td></td>
<td></td>
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<tr>
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<td>10.42</td>
<td>18.86</td>
<td>0</td>
<td>22.64</td>
<td>9.27</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23.06</td>
<td>10.42</td>
<td>18.86</td>
<td>0</td>
<td>22.64</td>
<td>9.27</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Exposure in MWD/kg of initial loading presented to SHUFLE. Radial 1.44
Recommended loading from SHUFLE. Radial = 1.35

Figure 7-10. Initial and recommended loading from SHUFLE assuming two failed assemblies per one-quarter core at end of cycle one.
VIII. SUMMARY AND CONCLUSIONS

Nuclear fuel management is a vital aspect of economic nuclear power. Because of the large amount of money involved in nuclear fuel management it is vital that the nuclear fuel manager have available the information and tools to optimize power costs for a reactor. Linear relationships are available in Chapter II which help the fuel manager estimate the effects of various out-of-core management decisions to optimize cost and burnup. The most important economic parameters are cost of $\text{U}_3\text{O}_8$, cost of separative work and cost of money.

Optimization of the loading pattern to minimize the power peaking factor is an important factor to in-core fuel management. An optimum loading is important from an economic aspect since it allows operation at the highest possible power density and still be within safe MCHFR or DNBR limits. High power densities reduce both capital and fuel cycle costs.

Program SHUFLE was developed to predict loading patterns which have minimum radial power peaking factors. The results of Chapter VII indicate that SHUFLE is capable of shuffling the fuel to produce good radial power peaking factors for initial reactor cores, equilibrium cycles and non-equilibrium cycles.

The key factors in the shuffling logic which enable SHUFLE to do this are:
1. Independent logic is used to shuffle the highly reactive fuel and the lower reactive fuel.

2. The shuffling logic is different for each of three zones within the core. In addition, the innermost zone is subdivided into even and odd parity positions for shuffling purposes.

3. Fuel shuffling is constructed so that the least reactive fuel is always placed on all four faces of all highly reactive fuel which is not on the periphery of the core.

SHUFLE utilizes course mesh 1.5 group diffusion theory to accurately predict the relative power of each fuel assembly. This method requires very little computer time per power calculation and allows SHUFLE to converge to a loading with a minimum radial power peaking factor in less than one minute of computer time on the UNIVAC 1108. Because of this, SHUFLE can be used by the nuclear fuel manager to inexpensively scope a series of cycle loading patterns or simply to predict the loading pattern which will produce the minimum radial power peaking factor for the subsequent cycle. One of the advantages of SHUFLE is its comparative ease of operation thus enabling the fuel manager to quickly decide on a new fuel loading pattern when fuel failures may dictate changing the proposed equilibrium loading pattern.

The shuffling logic used by SHUFLE to determine the shuffle
before each new power iteration was developed through observations
of what would improve power peaking for a given situation. Obviously
the rules are not exhaustive and further study may enable expansion
of the rules to give improved loadings.

The study was limited to pressurized water reactors with
uniform control throughout the core. Boiling water reactors have many
more fuel elements than pressurized water reactors and insert control
in discrete rods. Extension of the shuffling logic to boiling water
reactors is obviously an area for further study. Another area for
study is to expand the objective function so that it includes more
parameters than just minimizing the radial power peaking factor. As
an example, any loading pattern must be capable of meeting cold
shutdown requirements throughout the cycle or regardless of the power
peaking factor, it will be unacceptable. Further study may include
program shuffling logic which minimizes the radial power peaking,
while insuring satisfying cold shutdown requirements. Another area
of investigation is loadings which minimize the probability of fuel
failure throughout the cycle. This would require considerable data
correlating exposure, power density and fuel failure rates. If this
were available, it would be possible to develop shuffling logic which
loaded the fuel to satisfy operating constraints while minimizing the
probability of fuel failure.
BIBLIOGRAPHY


Merrill, E. T. 1969. ALTHAEA - A one dimensional two-group diffusion code with an effective four-group burnup. BNWL-462, UC-80 Reactor Technology, Battelle Memorial Institute.


APPENDIX A

ISOBURN

To conduct a survey of the economics of fuel management, isotopic concentrations of U-235, U-236, U-238, Pu-239, Pu-240, Pu-241, and Pu-242 are necessary for a wide range of initial enrichments of uranium oxide fuel. The isotopics for these heavy metals and the infinite reactivity values were calculated for the study in Chapter II by program ISOBURN. It was found necessary to develop this program on the OS3 system here at Oregon State University for two reasons. First, most programs that calculate isotopics were either too large for the OS3 computer system or would require a great deal of conversion to put on the OS3 system or both. Second, since the program may need to be run quite often, a fast, easy to operate program would be helpful.

ISOBURN is a zero dimensional burnup code which uses fourth order Runge-Kuta to integrate the basic equation

\[
\frac{dN_i}{dt} = -\sigma_a^i \ N_i \ \phi + \sigma_c^i \ N_{i-1} \ \phi - \lambda_i \ N_i
\]

where

\( N \) = number density

\( \sigma_a^i \) = absorption cross section

\( \sigma_c^i \) = capture cross section

\( \lambda_i \) = decay constant
\[ \phi = \text{flux} \]

\[ i = \text{the heavy metal isotope in the chain} \]

\[ = \text{U-235, U-236, U-238, Pu-239, Pu-240, Pu-241, Pu-242} \]

Cross sections were obtained from ALTHAEA which yields one group self shielded Wescott cross sections. These were incorporated into the program as a series of tables for several enrichments and for burnup from 0 to 40,000 MWD/MT. The cross sections for enrichments not listed in the tables were obtained by linear interpolation. Number densities are calculated within the program and are a function of the density and enrichment of the fuel, which are input parameters.

The one group flux is obtained from the equation

\[ \phi = \frac{\text{Power density}}{\sum \gamma_i \sigma_{f_i} N_i} \]  

(A-1)

where

\[ \gamma_i = \text{energy yield per fission} \]

\[ = 200 \text{ mev/fission for U-235, U-236, Pu-240, Pu-242} \]

\[ = 208 \text{ mev/fission for Pu-239} \]

\[ = 210 \text{ mev/fission for Pu-241} \]

The infinite reactivity, \( k_{\infty} \), is calculated in the usual manner for a one group equation.
\[ k_{oo} = \frac{\sum_{i} v_i f_i N_i}{\sum_{i} \sigma_{a_i} + N_i + \Sigma_a \text{(all other)}} \]  \hspace{1cm} (A-2)

\( \Sigma_a \text{(other)} \) is the absorption term for all other elements in the cell. ISOBURN breaks this down into three categories: fission products, non fuel lattice and two higher order isotopes (i.e., Np\(^{237}\) and Pu\(^{238}\)). The fission product absorption cross section was interpolated as a function of burnup from a polynomial fit of a three pseudo fission product model plus Xe\(^{135}\), Sm\(^{151}\), and Sm\(^{149}\). This method is also used in ALTHAEA and described in ANL-5800 (ANL, 1963). The non fuel absorption cross section is fitted to a first order Taylor series as a function of enrichment and held constant throughout the burnup since it is not highly burnup dependent. For the OCONEE fuel

\[ \Sigma_{a_{\text{NF}}} = 0.040 + (\text{w/o U-235 - 3.0}) \times 0.0014 \]  \hspace{1cm} (A-3)

The absorption cross sections for Np-237 and Pu-238 were obtained by analytically approximating the number densities from the calculated number density of U\(^{236}\). The cross sections were obtained from ALTHAEA. It was found that these higher order isotopes were necessary to include if \( k_{oo} \) were to accurately fit more sophisticated reactivity calculations. Errors of about 8 mk resulted at burnups of about 30,000 and 10 mk at 40,000 MWD/MT if these were not included.
Am$^{151}$ and Am$^{153}$ are also included in many burnup codes, but were not in ISOBURN since the contribution to $k_{\infty}$ is negligible up to about 40,000 MWD/MT.

After the first time step in the burnup, equilibrium values of Xe$^{135}$, Sm$^{149}$ and Sm$^{151}$ were included by the equations

\[
\Sigma^a_{\text{Xe135}} = \left[ \sum_i \left( \gamma_{1i} + \gamma_{\text{Xe}_i} \right) \sigma_{f_i} N_i \phi \right] \frac{\lambda_{135} + \sigma_{a135} \phi}{\sigma_{a135} \text{Xe135}}
\]

(A-4)

\[
\Sigma^a_{\text{Sm149}} = \sum_i \gamma_{\text{Pm}_i} \sigma_{f_i} N_i
\]

(A-5)

\[
\Sigma^a_{\text{Sm151}} = \sum_i \gamma_{\text{Sm}_i} \sigma_{f_i} N_i
\]

(A-6)

and the values used are listed in Table A-1.

All cross sections were obtained from ALTHAEA and the other parameters are listed as a function of isotope in Table A-1.

The only remaining parameter in equation A-2, $\nu_i$, was calculated by weighting four group $\nu$'s from ALTHAEA by the number of fissions in each group and assuming the number of fissions per group was relatively constant throughout the burn cycle. Since the energy spectrum does not change markedly with burnup and the $\nu$ values are not rapidly varying with burnup, this is a relatively good assumption. The values of $\nu_i$ are listed in Table A-1.
Table A-1. Parameter values used by ISOBURN.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>U-235, U-236</td>
<td>$\gamma_i + \gamma_{Xe}$</td>
<td>0.062</td>
<td>ANL 5800</td>
</tr>
<tr>
<td>U-238</td>
<td>&quot;</td>
<td>0.060</td>
<td>&quot;</td>
</tr>
<tr>
<td>Pu-239, Pu-241</td>
<td>&quot;</td>
<td>0.061</td>
<td>&quot;</td>
</tr>
<tr>
<td>Pu-240, Pu-242</td>
<td>&quot;</td>
<td>0.061</td>
<td>&quot;</td>
</tr>
<tr>
<td>Xe-135</td>
<td>$\lambda$</td>
<td>2.09 x $10^{-5}$ sec</td>
<td>LAMARSH (1966)</td>
</tr>
<tr>
<td>U-235</td>
<td>$\gamma_{Pm}$</td>
<td>0.013</td>
<td>ANL 5800</td>
</tr>
<tr>
<td>U-236</td>
<td>&quot;</td>
<td>0.023</td>
<td>&quot;</td>
</tr>
<tr>
<td>U-238</td>
<td>&quot;</td>
<td>0.023</td>
<td>&quot;</td>
</tr>
<tr>
<td>Pu-240</td>
<td>&quot;</td>
<td>0.023</td>
<td>&quot;</td>
</tr>
<tr>
<td>Pu-242</td>
<td>&quot;</td>
<td>0.023</td>
<td>&quot;</td>
</tr>
<tr>
<td>Pu-239, Pu-241</td>
<td>&quot;</td>
<td>0.0189</td>
<td>LAMARSH</td>
</tr>
<tr>
<td>U-235</td>
<td>$\gamma_{Sm}$</td>
<td>0.005</td>
<td>ANL 5800</td>
</tr>
<tr>
<td>Pu-240, Pu-242</td>
<td>&quot;</td>
<td>0.014</td>
<td>&quot;</td>
</tr>
<tr>
<td>Pu-241, Pu-239</td>
<td>&quot;</td>
<td>0.0117</td>
<td>&quot;</td>
</tr>
<tr>
<td>U-235</td>
<td>$v_{eff}$</td>
<td>2.432</td>
<td>ANL 5800 - ALTHAEA</td>
</tr>
<tr>
<td>U-236</td>
<td>&quot;</td>
<td>2.771</td>
<td>&quot;</td>
</tr>
<tr>
<td>U-238</td>
<td>&quot;</td>
<td>2.815</td>
<td>&quot;</td>
</tr>
<tr>
<td>Pu-239</td>
<td>&quot;</td>
<td>2.873</td>
<td>&quot;</td>
</tr>
<tr>
<td>Pu-240</td>
<td>&quot;</td>
<td>3.079</td>
<td>&quot;</td>
</tr>
<tr>
<td>Pu-241</td>
<td>&quot;</td>
<td>3.031</td>
<td>&quot;</td>
</tr>
<tr>
<td>Pu-242</td>
<td>&quot;</td>
<td>3.093</td>
<td>&quot;</td>
</tr>
</tbody>
</table>
Equation A-2 may now be expressed in more detail:

\[
\kappa_{oo} = \frac{\sum_{i} \nu_{i} \sigma_{f_{i}} N_{i}}{\sum_{i} \sigma_{a_{i}} N_{i} + \sum_{Xe135}^{a} + \sum_{Sm149}^{a} + \sum_{Sm151}^{a} + \sum_{(other\ FP)}^{a}} + \sum_{Np137}^{a} + \sum_{Pu238}^{a} + \sum_{NF}^{a}
\]

This equation coupled with the isotopic calculations yielded reactivities which did not differ from ALTHAEA by more than 2 mk out to 40,000 MWD/MT. Isotopics matched those of ALTHAEA within less than one percent at all enrichments and burnups. Based on this, ISOBURN was considered capable of being used as a very fast calculational tool to predict isotopics and \( \kappa_{oo} \) for all the studies conducted in this thesis.
APPENDIX B

LISTING AND INPUT INSTRUCTIONS FOR PROGRAM SHUFLE

Program SHUFLE has the capability of burning, shuffling to a minimum radial power peaking ratio or simply calculating the two dimensional radial power distribution of a one-quarter core PWR or any combination of these three. As the program is presented only problems which have half nodes on the reflected boundaries can be solved. With only slight modifications to subroutine RELPOW full nodes can be treated on these reflecting boundaries. Slight modifications to the shuffling logic would also be required, but, since PWR's always have the center line symmetry bisecting a fuel assembly, this option was not incorporated in the logic.

SHUFLE is very easy to set up and can be run with five types of cards: two control cards, fuel type cards, fuel exposure cards and burnup control cards. When preparing the input cards, Figure B-1 will be useful to illustrate the nomenclature used by SHUFLE.
Figure B-1. One-quarter core \( x, y \) geometry problem solved by SHUFLE.

**Input Instructions for SHUFLE**

<table>
<thead>
<tr>
<th>Card type</th>
<th>Variable</th>
<th>Columns</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Format (13A6)</td>
<td></td>
<td>1-78</td>
<td>Any description to identify the run may be printed on this card. Card must be repeated for each case.</td>
</tr>
</tbody>
</table>

Control Card 1

<table>
<thead>
<tr>
<th>Format (16 I 5)</th>
<th>Variable</th>
<th>Columns</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NX</td>
<td>1-5</td>
<td>Number of nodes in the ( J ) direction of Figure B-1</td>
</tr>
<tr>
<td></td>
<td>NY</td>
<td>6-10</td>
<td>Number of nodes in the ( I ) direction of Figure B-1</td>
</tr>
<tr>
<td></td>
<td>NCH</td>
<td>11-15</td>
<td>Controls which option program should run initially. The options</td>
</tr>
</tbody>
</table>
for NCH are as follows:

<table>
<thead>
<tr>
<th>NCH</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A new case is initiated.</td>
</tr>
<tr>
<td>2</td>
<td>Old fluxes used and iteration continues.</td>
</tr>
<tr>
<td></td>
<td>This is used when print out is desired then convergence continued on same problem.</td>
</tr>
<tr>
<td>3</td>
<td>Exposed fuel shuffled to minimize radial power peaking ratio.</td>
</tr>
<tr>
<td>4</td>
<td>New and old fuel shuffled to minimize radial power peaking ratio.</td>
</tr>
<tr>
<td>5</td>
<td>Burnup of fuel by exposure increment indicated on burnup card.</td>
</tr>
<tr>
<td>6</td>
<td>End of run.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Range</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAXFT</td>
<td>16-20</td>
<td>Number of fuel types for this case.</td>
</tr>
<tr>
<td>MAXI</td>
<td>21 25</td>
<td>Maximum number of inner iterations in power calculation.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Recommended value = 10.</td>
</tr>
<tr>
<td>MAXO</td>
<td>26 30</td>
<td>Maximum number of outer iterations in power calculation.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Recommended value 75 to 100.</td>
</tr>
<tr>
<td>MAXIT</td>
<td>31-35</td>
<td>Maximum number of shuffling iterations. MAXIT puts upper limit on the number of shuffles performed in a case if a very difficult problem occurs.</td>
</tr>
</tbody>
</table>
Recommended value 75 to 100.
Most normal loadings will converge in less than 50 iterations.

If polynomial cross sections are to be used $NPOLY = 1$. If cross sections input on cards $NPOLY = 0$.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Range</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$NPOLY$</td>
<td>36-40</td>
<td>Polynomial cross sections.</td>
</tr>
<tr>
<td>SIZE</td>
<td>1-10</td>
<td>Size of normal node in the $x$ and $y$ direction.</td>
</tr>
<tr>
<td>TESTIN</td>
<td>11-20</td>
<td>Converging criteria on inner iterations. Recommended value $10^{-4}$.</td>
</tr>
<tr>
<td>TESTOUT</td>
<td>21-30</td>
<td>Convergence criteria on outer iterations. Recommended value $10^{-6}$.</td>
</tr>
<tr>
<td>AA(1)</td>
<td>31-40</td>
<td>Averaging factor in fast group. Range 0 to 1.0. Recommended value 0.3.</td>
</tr>
<tr>
<td>AA(2)</td>
<td>41-50</td>
<td>Averaging factor in thermal group. Range 0 to 1.0. Recommended value 0.7.</td>
</tr>
<tr>
<td>ALPHA</td>
<td>51-60</td>
<td>Overrelaxation factor. Range 1.0 to 1.9. Recommended value 1.7. Set = 1.0 for no effect.</td>
</tr>
<tr>
<td>TFUEL</td>
<td>61-70</td>
<td>Total fuel nodes in one-quarter core. As example $TFUEL$ for Figure B-1 = 14.25.</td>
</tr>
</tbody>
</table>

NY number of these cards are required with $NX$ number of...
values on each card. Simply place the designated fuel types on the cards as they are arranged in the one-quarter core. The water nodes in the reflector region must have fuel type MAXFT + 1.

If NPOLY = 0, MAXFT + 1 number of these cards are required, if NPOLY = 1 none are required.

Cards must be in order of increasing fuel type with the values for the reflector on the last of these type cards. Each card must contain in order the following values for a particular fuel type:

\[ \Sigma_{a_1}, \Sigma_{a_2}, v\Sigma_{f_{1}}, v\Sigma_{f_{2}}, \Sigma_{s}(1, 2), D_{1}, D_{2}. \]

If NPOLY = 0 none of these type cards are required. If NPOLY = 1, NY number of these cards are required with NX number of exposure values in GWD/MT on each card. Arrange as fuel exposure of each assembly appear in the one-quarter core.

If NCH = 5 on any control card, burnup cards are required.

Put a 5 in this position on each burnup card

Input incremental burnup in units of GWD/MT
Input as many burnup cards as desired.

Control card

| NCH | 1-5 |

This card can be inserted at any point after the exposure cards and will direct the path the program takes according to the value of NCH discussed on first control card.
** ALT MOVE DECIDES MOVES OF FUEL WHEN REACTIVITY ARE SAME **

```fortran
C SUBROUTINE ALTMOVE

COMMON/VX,NX,NYM1,NYM2,NCH,SAVE(10,10),EX(10,10),ITYPE(10,10)

1, RE(10,10), MAXFT
COMMON/LOGIC/ EXS(10,10), SAV(10,10), IMX, JMX, IMN, JMN, LIMX, LIMN

1, IMOVE, JMOVE, IMXL, JMXL, PMAX, PMIN, NMAX, NMIN, NPOLY, PMAXL, RES(10,10)

IF (IMX = JMOVE) 110, 100, 150

100 IF (JMX = JMOVE) 130, 150, 150

110 JMOVE = JMOVE + 1
IF (ITYPE(IMOVE + 1, JMOVE), EQ, MAXFT) GO TO 120

120 JMOVE = JMOVE + 1
IF (ITYPE(IMOVE + 1, JMOVE), EQ, MAXFT) GO TO 150

130 IMOVE = IMOVE + 1
GO TO 150

IMOVE = IMOVE + 1
IF (ITYPE(IMOVE + 1, JMOVE), EQ, MAXFT) GO TO 140

140 IMOVE = IMOVE + 1
IF (ITYPE(IMOVE + 1, JMOVE), EQ, MAXFT) GO TO 150

CONTINUE

PRINT 160
END

C 160 FORMAT ('0EXCHANGE THE FOLLOWING TWO FUEL ELEMENTS/1', I, J, EXPALT)

160 FORMAT (10CHARS=INVAT)


END CUR
```
**ELT CARDS,1,720612,37508,**

```plaintext
000001 C *** CSCALC CALCULATES CROSS SECTIONS AND C(i,j) THRU ***** CSC 1
000002 C *** C4(i,j) TO CALCULATE THE RELATIVE POWER FOR EACH NODE ***** CSC 2
000003 C.
000004 SUBROUTINE CSCALC
000005 COMMON/ALL/VX,NV,XM1,NM1,MC,SAVE(10,10),EX(10,10),ITYPE(10,10) CSC 3
000006 1 , RE(10,10), MAXFT
000007 COMMON/NUCLEAR/SIGA(10,10),SIGFNU(10,10,2),SIGS(10,10) CSC 4
000008 1 , C(10,10),C5(10,10),SIGTR(2),A(2),C(2) CSC 5
000009 3 , AREA,MAX,MAXI,TESTOT,TESTIN,ALPHA,T FUEL CSC 6
000010 3 , SIGF(2,10),SIGF(2,10),POD(2,10) CSC 7
000011 COMMON/LOGIC/EXS(10,10),SAV(10,10),IMX,IMX,INN,IMN,LMX,LMX CSC 8
000012 1 , IMOVE, JMOVE, IMX, JMX, JMX, LMX, LMX CSC 9
000013 COMMON/B hastily, (max,10),MAXIT,MAXI,MAX POLY,RES(10,10) CSC 10
000014 DIMENSION SIGA(10,10) CSC 11
000015 EQUIVALENCE (SIGA(1),SIGA(1)) CSC 12
000016 DO 130 I=1,NV CSC 13
000017 DO 130 J=1,NX CSC 14
000018 IF (NPOLY.EQ.0) GO TO 110 CSC 15
000019 C
000020 C *** POLYNOMIAL_ FITS FOR 3,0 W/O ENVIRONMENT C** FUEL ************ CSC 16
000021 C *** THESE DATA MAY BE CHANGED FOR ANY ENR OR REACTOR ************ CSC 17
000022 C
000023 IF (ITYPE(1,J).EQ.MAXFT) GO TO 100 CSC 18
000024 EX2=EX(1,J)*EX(1,J) CSC 19
000025 EX3=EX(1,J)*EX2 CSC 20
000026 EX4=EX(1,J)*EX3 CSC 21
000027 SIGA(1,J)=9.046707E-3*1.04479*E-4*EX(1,J)-1.0468E6*EX2 CSC 22
000028 SIGA(1,J)=8.32636*E4*EX(1,J)*1.497E4*EX2 CSC 23
000029 1 + 2.4044E-6*EX3-2.0867E8*EX2 CSC 24
000030 SIGFNU(1,J,1)=6.37457E-3*6.39586E5*EX(1,J)*3.5867E7*EX2 CSC 25
000031 SIGFNU(1,J,2)=1.34856E-1*1.1543*E-3*EX(1,J)*9.2979E9*EX2 CSC 26
000032 1 + 1.1656E-5*EX3 CSC 27
000033 SIGS(1,J)=1.2*1.251E-2*2.8721E-5*EX(1,J)*4.9878E6*EX2 CSC 28
000034 D(1,J,2)=3.9779E-1*1.0606E6*3*EX(1,J)*3.7810E8*EX2 CSC 29
000035 1 + 4.2554E-07*EX3 CSC 30
000036 D(1,J,1)=1.444444*2.3386E-3*EX(1,J) CSC 31
000037 GO TO 120 CSC 32
000038 D(1,J,1)*1.561 CSC 33
000039 D(1,J,2)=3.1218 CSC 34
```
**CROSS SECTION VALUES WHEN READ IN ON CROSS SECT CARDS**

```plaintext
N=EX(I,J)+.01
SIGA1(I,J)=SIGA1(I,N)
SIGA2(I,J)=SIGA2(I,N)
SIGFNU(I,J,1)=SIGFNU(1,N)
SIGFNU(I,J,2)=SIGFNU(2,N)
SIGS12(I,J)=SIGS(N)
D(I,J,1)=D(I,1,1)+D(I,2,1)
D(I,J,2)=D(I,1,2)+D(I,2,2)
```

**TWO GROUP REACTIVITY CALCULATION WITH ZERO BUCKLING**

```plaintext
10 RE(1,J)=(SIGFNU(1,J,2)/SIGA2(1,J)*SIGS12(1,J)*SIGFNU(1,J,1))
  1/(SIGS12(I,J)+SIGA1(I,J))
```

**NODAL VOLUME WEIGHTED CALC OF PARAMETERS FOR RELPOW**

```plaintext
VOL=AREA
IF (I.EQ.1,OR,J.EQ.1) VOL=AREA/2,
IF (I.EQ.1,J.EQ.1) VOL=AREA/4,
SIGA1(I,J)=SIGA1(I,N)*SIGS12(I,J)*VOL
SIGFNU(I,J,1)=SIGFNU(1,N)*VOL
SIGFNU(I,J,2)=SIGFNU(1,N)*VOL
SIGFNT(I,J)=SIGFNU(1,J,1)+SIGFNU(1,J,2)*VOL
SIGTRW(I)=T(I,1,1)*DNY,NX,1)
SIGTRW(2)=T(I,1,2)*DNY,NX,2)
C2(I,1)=D(I,1,1)+D(I,2,1)/D(I,1,1)*5+D(I,2,1)
C4(I,1)=D(I,1,1)+D(I,2,1)/D(I,1,1)*5+D(I,2,1)
DO 140 I=2,NY
  C3(I,1)=C4(I,1,1)
  C4(I,1,1)=D(I,1,1)*D(I,1,1)+D(I,2,1)*D(I,2,1)
DO 150 J=2,NX
```

**DO 150 J=2,NX**
\[ C_1(1,J) = C_2(1,J-1) \]

\[ C_4(1,J) = 2 \cdot D(1,J,1) \cdot D(1,J,1) \]

\[ C_2(1,J) = D(1,J,1) \cdot D(1,J,1) + D(1,J,1) \]

\[ C_2(1,J) = D(1,J,1) \cdot D(1,J,1) \]

\[ C_3(1,J) = C_4(1,J) \]

\[ C_4(1,J) = 2 \cdot D(1,J,1) \cdot D(1,J,1) \]

\[ C_1(1,J) = C_2(1,J-1) \]

\[ C_1(I,J) = 1 \]

\[ C_2(I,NX) = C(I,NX,1) \cdot \text{SIZE} / (5 \cdot \text{SIZE} + 1) \]

\[ C_2(I,NX) = C(I,NX,1) \cdot \text{SIZE} / (5 \cdot \text{SIZE} + 1) \]

\[ C_4(NY,J) = C(NY,J,1) \cdot \text{SIZE} / (5 \cdot \text{SIZE} + 1) \]

\[ C_5(I,J) = C_1(I,J) + C_2(I,J) + C_3(I,J) + C_4(I,J) \]

\[ \text{RETURN} \]

\[ \text{END} \]

3. PCH i,j

END CUR
**ELT CARD 11720612, 37665**

*** DIAGMN CALLS MIN RELATIVE POWER OF SAME DIAG TYPE ******

**SUBROUTINE DIAGMN**

**COMMON/ALL/NX, NY, NXM1, NYM1, NCH, SAVE(10, 10), EX(10, 10), ITYPE(10, 10)**

1, RE(10, 10), MAXFT

**COMMON/LOGIC/ EXS(10, 10), SAV(10, 10), IMX, JMX, IMN, JMN, LIMX, LMX**

1, IMOVE, JMOVE, IMXL, JMXL, PMAX, PMIN, MAXIT, NSER, NPOLY, PMAXL, RES(10, 10)

**DEL**

**COMMON/LOG/ EXS(10, 10), SAV(10, 10), IMX, JMX, IMN, JMN, LIMX, LMX**

1, IMOVE, JMOVE, IMXL, JMXL, PMAX, PMIN, MAXIT, NSER, NPOLY, PMAXL, RES(10, 10)

100 MODD=1

IPJ=LIWMX+LMX

I=(IPJ/2)*2

IF (IPJ,EQ,1) MODD=0

NXM2=NX-2

NYM2=NY-2

PMNO=PMO

DO 170 I=1, NYM2

DO 170 J=1, NXM2

IF (RE(I, J), GE, RE(LIMX, LMX)) GO TO 170

IPJ=I+J

IF (IPJ, LE, LIMX) GO TO 170

IPJ=I+J

IF (IPJ-10) 120, 130, 130

120 MODD=1

I=(IPJ/2)*2

IF (IPJ, E3, IT) MODD=0

IF (MODD, NE, MODD) GO TO 170

IF (ITYPE(I+1, J), GE, ITYPE(LIMX, LMX)) GO TO 170

130 MODD=1

IF (ITYPE(I+1, J), EQ, ITYPE, MAXFT) GO TO 170

IF (I, EQ, 1, EQ, 1) GO TO 170

140 REPP=RE(I, J)-PMND

150 IF (REPP, LE, RE(LIMX, LMX)) GO TO 170

PMND=PMND+1

IMN=1

JMN=J

CONTINUE

170 IF (PMND, EQ, 1) GO TO 220

PRINT 250

PRINT 260, IMX, JMX, EX(LIMX, LMX), RE(LIMX, LMX), IMN, JMN

1 EX(147, 1), RE(147, 1)
IF (IMN.EQ.JMNL.AND.JMN.EQ.IMNL) GO TO 180
GO TO 200

180 IF (IMX.EQ.IMXL,A ND.JMX.EQ.JMXL) GO TO 190
GO TO 200

190 PMNDL=ABS(PMNDL-PMND)
IF (PMNDL.GT.0.004) GO TO 200
PRINT 270
SAVE(IMN,JMN)=0,
CALL SAVEEXP (SAVE,SAV)
GO TO 110

200 IMNL=IMN
JMNL=JMN
PMNDL=PMND
RETURN

210 IF (SAVE(LIMX,LJMX),E00.) GO TO 240
PRINT 280
SAVE(LIMX,LJMN)=0,
CALL LOGMAX
GO TO 100

230 NSER=NSER+1
IF (NSER.GT.21) GO TO 210
PRINT 290
GO TO 230

240 FORMAT ('EXCHANGE THE FOLLOWING TWO FUEL ELEMENTS/! I J EXP')
SAVE(K-INF)


260 FORMAT ('THE FOLLOWING TWO FUEL ELEMENTS/! I J EXP')

270 FORMAT ('يصAME MOVE AS LAST TIME, GO TO NEXT MOVE!')

280 FORMAT ('NONE OF THE TABLES OF THIS TYPE WITH MORE EXPOSURE, THAT HAVE NOT BEEN TRIED!')

290 FORMAT ('EXPAND LOCAL MAX AREA BY 1 AND CONTINUE!')
END CUR
C *** LOCMAX CALCS LOCAL MAX REL POWER IN AREA OF MAX POWER ***

SUBROUTINE LOCMAX

COMMON/ALL/XX,XY,NXYM1,NXY1,NCH,SAVE(10,10),EX(10,10),IYTYPE(10,10)

COMMON/LOGIC/EXS(10,10),SAV(10,10),IMX,JMX,INNMJMN,LIMX,LJMX

IMOVE,JMOVE,IMXLJMX,LMAXPMINPMAX,NOIT,NSER,NPOLYPMAXLRES(10,10)

LI=IMX+NSER
LJ=JMX+NSER
.

IF (ITYPE(I,J).Eq.MAXFT) GO TO 130
IF (I.EO,CMX,N,J) GO TO 130
IF (NPOLY.EG,I) GO TO 130
IF (EX(I,J),LT,1.0) GO TO 130
IF (SAVE(I,J),PMXL0C) 1301304120
PMXLOC=SAVE(I,J)

CONTINUE

FORMAT ('LOCAL MAX PWR OF ',F6.3,' AT ',I2,I2,' WITH EXPOSURE ',F8.3,F8.3)

END CUR
C **** LOCAL MIN POWER IN AREA OF MAX POWER *****

C SUBROUTINE LOCAL

COMMON/ALL/NX,NY,NXM1,NYM1,NCH,SAVE(10,10),EX(10,10),ITYPE(10,10)

i, RE(10,10), MAXIT

COMMON/LOGIC/ EXS(10,10), SAVE(10,10), IMX, JMX, IMN, JMN, LIMX, LJMX

i, IMOVE, JMNOE, IMXL, JMXL, PMAX, PMIN, MAXIT, NDER, NPOLY, PMAXL, RES(10,10)

IMOVE, JMOVE

LLI=IMX-2

LLJ=JMX-2

LI=IMX+2

LJ=JMX+2

IF (IMX.LE.3) LLI=1

IF (IMX.GT.6) LLI=8

IF (JMX.LE.3) LLJ=1

IF (JMX.GT.6) LLJ=8

PMIN=1000,

DO 110 I=LI, LLI

DO 110 J=AJJ, LJ

IF (SAVE(I,J),GE,0) GO TO 110

IF (SAVE(I,J),.GE.PMIN) 100,110,110

100 PMIN=SAVE(I,J)

IMN=1

JMN=J

CONTINUE

110 PRINT 120

PRINT 130, PMAX, RE(IMX,JMX), EX(IMX,JMX), IMX, JMX, PMIN, RE(IMN,JMN)

2, EX(IMN,JMN), IMN, JMN

END

RETURN

C FORMAT ('0 REL POWER K=INF EXPOSURE I J')


END
C *** SUBROUTINE MAX TO CALC MAX ELEMENT IN SAVE ARRAY ************
C
C SUBROUTINE MAX
C COMMON/ALL/NX,NY,NXM1,NYM1,NCH,SAVE(10,10),EX(10,10),ITYPE(10,10) MAX 4
C 1 , RE(10,10),PMAXFT MAX 5
C COMMON/LOGIC/EXS(10,10),SAV(10,10),IMX,JMX,IMN,JMN,LIMX,LJM MAX 6
C i,IMOVE,JMOVE,IMXL,JMXL,PMAX,PMIN,MAX1T,USER,NPOLY,PMAXL,RES(10,10) MAX 7
C PMAX=0, MAX 8
C DO 110 I=1,NX+1 MAX 9
C DO 110 J=1,NX+1 MAX 10
C IF (SAVE(I,J)-PMAX) 110,110,110 MAX 11
C 110 PMAX=SAVE(I,J) MAX 12
C IMX=I MAX 13
C JMX=J MAX 14
C CONTINUE MAX 15
C PRINT 120, PMAX,IMX,JMX,EX(IMX,JMX),RE(?!MX,JMX) MAX 16
C RETURN MAX 17
C FORMAT ('PMAX POWER OF','F6.3,' AT','F12.1,' WITH EXPOSURE','F6.2,' ANDMAX MAX 18
C i, K=INF OF','F6.3)
C END MAX 20
C END CUR MAX 21

3. PCH I,3
END CUR
C MOVDEC DECIDES WHAT ELEMENTS TO MOVE FOR SUBR SHAPE A

SUBROUTINE MOVDEC

COMMON ALL/NX,NY,NXM1,NYM1,NCH,SAVE(10,10),EX(10,10),IYPE(10,10)
COMMON/LOGIC/ EXS(10,10),SAV(10,10), IMX,JMX,IMN,JMN,LIMX,LMX
COMMON DATA IMOVE,JMOVE,IMXL,JMXL,PMAX,PMIN,MAXIT,USER,NPOLY,PMAXL,RES(10,10)

LOUT=0
IPJ=IMX+JMX
IF (IPJ,GE,9) LOUT=OUT+1
IPJ=IMN+JMN
IF (IPJ,GE,9) LOUT=OUT+1
MOVE=0
MOVEJ=0
IF (IMX,GT,IMO MOVEI=1
IF (JMX,GT,JMN) MOVEJ=1
IF (IMX,LT,IMO MOVEI=1
IF (JMX,LT,JMN) MOVE=1
IF (LOUT,LE,2) GO TO 130
IF (IMX=INN) 100,110,100
IF (JMX=JMN) 130,120,130
100 IF (IMX,GT,IMN) MOVEI=2
IF (JMX,LT,JMN) MOVEJ=2
GO TO 130
120 IF (IMX,GT,IMN) MOVEI=2
IF (JMX,LT,JMN) MOVEJ=2
130 IMOVE=IMX+MOVEI
JMOVE=JMX+MOVEJ
PRINT 140
PRINT 150, IM, JMX,EX(IMX,JMX),ER(IMX,JMX),IMOVE, JMOVE
RETURN
C
FORMAT ('EXCHANGE THE FOLLOWING TWO FUEL ELEMENTS!'/I J EXP
10SURE KH!')
150 FORMAT (213,F11.3,F9.3,/,213,F11.3,F9.3,/
END
C SUBR MOVE MOVES FUEL ELEMENTS AND EXPOSURES **************

SUBROUTINE MOVE (II, JJ, I, J)

COMMON/VX, NY, NYM, NYM1, NYM1, NGH, SAVE (10, 10), EX (10, 10), IYPE (10, 10)

i, RE (10, 10), MAXFT

TEMP = EX (II, JJ)
ITEM = IYPE (II, JJ)

EX (II, JJ) = EX (1, J)

IYPE (II, JJ) = IYPE (I, J)

IF (II, EQ, 1, OR, J, EQ, I) GO TO 100
IF (II, EQ, 1, OR, JJ, EQ, 1) GO TO 100
EX (I, J) = TEMP

RETURN

IF (I, EQ, 1, AND, JJ, EQ, 1) GO TO 120
IF (J, EQ, 1, AND, JJ, EQ, 1) GO TO 120
IF (II, EQ, 1, AND, JJ, EQ, 1) GO TO 120
IF (II, EQ, 1, AND, JJ, EQ, 1) GO TO 110
EX (I, J) = TEMP
EX (J, I) = TEMP

RETURN

EX (II, JJ) = EX (I, J)
EX (I, J) = TEMP

RETURN

END
** RELPOW CALCULATES THE RELATIVE POWER FOR EACH NODE ******
** USING A 1,3 GROUP COURSE MESH DIFFUSION THEORY MODEL ******

** MINIMUM DIMENSIONS ON PHI SHOULD BE PHI(NY-1,NX-1) ******

** BEGINING OF INNER AND OUTER ITERATIONS ****************************
000040   ITIN=0
000041      TEST1=PHI(2,2,1)
000042      TEST2=PHI(2,7,1)
000043      TEST3=PHI(7,2,1)
000044      TEST4=PHI(5,5,1)
000045      DO 170 I=1,NY
000046         DO 170 J=1,NX
000047      170   PHI(I,J,1)=ITF(I,J)*C3(I,J)*PHI(I+1,J,1)+C4(I,J)*PHI(I-1,J,1)
000048         1 + C1(I,J)*PHI(I,J,1,1) + C2(I,J)*PHI(I+1,J,1,1) / C5(I,J)
000049  ITIN=ITIN+1
000050      IF (ITIN.GT.MAXI) GO TO 180
000051      TEST1=ABS((PHI(2,2,1)-TEST11)/PHI(2,2,1))
000052      TEST2=ABS((PHI(2,7,1)-TEST12)/PHI(2,7,1))
000053      TEST3=ABS((PHI(7,2,1)-TEST13)/PHI(7,2,1))
000054      TEST4=ABS((PHI(5,5,1)-TEST14)/PHI(5,5,1))
000055      TESTI=MAX1(TESTI1,TESTI2,TESTI3,TESTI4)
000056      IF (TESTI.GT.TESTIN) GO TO 160
000057
000058      *** CALCULATION OF EFFECTIVE K   *******************************
000059
000060      180   DO 190 I=1,NY+1
000061            DO 190 J=1,NX+1
000062      190   TLFIS=TLFIS+P(I,J)*SIGFNT(I,J)
000063            RLA=TFLIS/TLFISL
000064            EFFK=EFFK*RLA
000065            TESTO=ABS((TLFIS-TLFISL)/TLFIS)
000066            TLFISL=TLFIS
000067            ITOUT=ITOUT+1
000068            IF (ITOUT.EQ.1) GO TO 210
000069
000070      *** ACCELERATION BY OVER RELAXATION  ******************************
000071
000072      TFLSOr=0
000073      DO 200 I=1,NY
000074            DO 200 J=1,NX
000075      200   TFLSOr=TFLSOr+SAVE(I,J)*ALPHA*(PHI(I,J,1)-SAVE(I,J))
000076         SAVE(I,J)=SAVE(I,J)+ALPHA*(PHI(I,J,1)-SAVE(I,J))
000077         GO TO 230
000078      210   TFLSOr=TLFIS
000079      DO 220 I=1,NY
000080            DO 220 J=1,NX
DO 240 I=1, NY
DO 240 J=1, NX
TF(I,J)=SAVE(I,J)*SIGNED(I,J)*TEMP
SAVE(I,J)=PHI(I,J)

C *** CHECK CONVERGENCE AND PRINT OUT OF DATA ***************

I=(ITOUT/3)*3+1
IF (I.EQ.1) GO TO 250
PRINT 410, ITOUT, ITIN, EFF, PHI(1,1), PHI(4,1), TLF, TES, TOT, RLM

DO 240 J=1, NY
DO 240 I=1, NX
SAVE(I,J)=PHI(I,J)*D(I,J)
B2(I,J)=(SAVE(I,J)+SAVE(I+1,J))/D(I,J)
PHIA(I,J)=AA(I,J)*PHI(I,J)+2*CC(I,J)*B2(I,J)

DO 270 K=1, 2
DO 270 J=1, NY
SAVE(I,J)=PHI(I,J)*D(I,J)
B2(I,J)=(SAVE(I,J)+SAVE(I,J+1))/D(I,J)
PHIA(I,J)=AA(I,J)*PHI(I,J)+2*CC(I,J)*B2(I,J)

DO 260 K=1, 2
DO 260 I=1, NY
SAVE(I,J)=PHI(I,J)*D(I,J)
B2(I,J)=(SAVE(I,J)+SAVE(I,J+1))/D(I,J)
PHIA(I,J)=AA(I,J)*PHI(I,J)+2*CC(I,J)*B2(I,J)

C *** AVERAGING FLUX OVER NODE USING INPUT AVERAGING FACTORS *****

DO 340 K=1, 2
DO 340 J=1, NY
SAVE(I,J)=PHI(I,J)*D(I,J)
B2(I,J)=(SAVE(I,J)+SAVE(I,J+1))/D(I,J)
PHIA(I,J)=AA(I,J)*PHI(I,J)+2*CC(I,J)*B2(I,J)

DO 300 K=1, 2
DO 300 J=1, NY
SAVE(I,J)=PHI(I,J)*D(I,J)
B2(I,J)=(SAVE(I,J)+SAVE(I,J+1))/D(I,J)
PHIA(I,J)=AA(I,J)*PHI(I,J)+2*CC(I,J)*B2(I,J)
C

C *** CALCULATION OF RELATIVE POWER FOR EACH NODE **************

C

TLFIS=0.
DO 330 I=1,NY
DO 340 J=1,NX
SAVE(I,J)=PHIA(I,J)+SIGFNU(I,J,1)+PHIA(I,J,2)+SIGFNU(I,J,2)

330 TLFIS=TLFIS+SAVE(I,J)
PRINT 420
AVGFIS=AVGFIS/TFJEL
PRINT 430
DO 370 I=1,NY
DO 380 J=1,NX
SAVE(I,J)=SAVE(I,J)/AVGFIS
IF (I.EQ.1 .OR. J.EQ.1) SAVE(I,J)=SAVE(I,J)*2,
IF (I.EQ.1 .AND. J.EQ.1) SAVE(I,J)=SAVE(I,J)*2
370 PRINT 440, I, SAVE(I,J), J=1,NX+1
RETURN

C

390 FORMAT (141,12X,'****** THE FUEL EXPOSURE OR FUEL TYPE OF LOAD
C IS ******')

400 FORMAT ('// ITOJT ITIN K=EFF PHI1(1,1) PHI4(4,4) FISSIONS')
1 CONVERGENCE (LAMBDA')

410 FORMAT (215,F10.5,2F11.4,E12.5,E12.4,F14.8)

420 FORMAT (7X,T1,T2,T3,T4,T5,T6,T7,T8)

430 FORMAT (I21RX,971.3)

END
SUBROUTINE SAVEARRAY

DIMENSION SAVE(1000,1000), SAV(1000,1000)

DO 100 I=1,100
  DO 100 J=1,100
    SAV(I,J) = SAVE(I,J)
  100 CONTINUE

RETURN
END
C *** SHUFFLE A SHUFFLES HIGH REACTIVITY FUEL TO MINIMIZE ********** SHA 1
C ** RADIAL POWER PEAK ********** SHA 2
C
SUBROUTINE SHFLA

COMMON/ALL/NX,NY,NXM1,NYM1,NCH,SAVE(10,10),EX(10,10),ITYPE(10,10)

1, RE(10,10), MAXFT

COMMON/LOGIC/EEXS(10,10),SAV(10,10),IMX,JMX,IMN,JMN,LIMX,LJMX

1, IMOVE, JMOVE, IMXL,JMXL, PMA XL,PMIN, MAXIT,NSER,NPOLY,PMA XL,RES(10,10)

IF (MTIME,GT,12) GO TO 320

ITNUC=ITNUC*1

IF (ITNUC,GT,MAXIT) GO TO 320
CALL RELPOW

CALL MAX

IF (PMA XL,GE,PMA XL) GO TO 110

PRINT 330

PMA XL=PMA XL

IMXR=JMX

JMXR=IMX

NRECIP=0

MTIME=1

MTPR=1

GO TO 130

IF (NRECIP,GT,0) GO TO 170

IF (IMX,LT,IMXR,AND,JMX,LE,JMXR) GO TO 120

GO TO 170

PRINT 340, PMA X

NRECIP=1

TEMP=IMOVE

IMOVE=JMOVE

JMOVE=TEMP

130 IF (ITYPE(I MX+1,J MX).EQ.,MAXFT) GO TO 140

IF (ITYPE(I MX,J MX+1),NE.,MAXFT) GO TO 220

140 PRINT 350, PMA X,I MX,J MX

SAVE(I MX,J MX)=0.

CALL MAX

IF (IMX,LT,1,AND,JMX,GT,1) GO TO 160

GO TO 130

SAVE(I MX,J MX)=0.

CALL SAVEXP (SAVE,SAV)
170  GO TO 150
180  PRINT 360, PMAX, PMAXL
190  CALL SAVEXP (svav, SAVE)
190  CALL SAVEXP (RES, PE)
190  CALL SAVEXP (EXS, EX)
190  CALL MAX
190  MTP = MTP+1
200  NRECIP = 0
210  IF (MTP, GT, 2) GO TO 210
210  PMAXS = PMAX
210  SAVE (IMX, JMX) = 0
210  CALL MAX
210  IF (PMAX, LT, 1) GO TO 320
210  IF (ITYPE (IMX+1, JMX), EQ, MAXFT) GO TO 190
210  IF (ITYPE (IMX, JMX+1), NE, MAXFT) GO TO 200
210  IF (PMAX, GT, 002) GO TO 230
210  GO TO 180
210  PMAXS = ABS (PMAXS - PMAX)
210  IF (PMAXS, GT, 002) GO TO 230
210  GO TO 180
210  MTIME = MTIME+1
210  IF (MTIME, GT, 12) GO TO 320
210  SAVE (IMNL, JMN) = 0
210  SAVE (IMNL, JMN) = 0
220  IF (NRECIP, EQ, 1) GO TO 310
220  CALL SAVEXP (RES, RE)
220  CALL SAVEXP (SAVE, SAV)
220  CALL SAVEXP (EX, EXS)
220  CALL LOCMTN
230  CALL LOGMII
230  IF (ITPR, EQ, 2) GO TO 240
240  IMNL = IMN
240  JMN = JMN
240  IF (MTIME, GT, 1) GO TO 250
240  IF (PMIN, LT, 1.1) GO TO 250
240  PRINT 370
240  GO TO 140
250  CALL MOVUEC
250  IF (RE (IMOVE, JMOVE) * RE (IMX, JMX)) 290, 260, 260
260  IF (ITYPE (MOVE, IMOVE), EQ, MAXFT) GO TO 260
260  IF (ITYPE (MOVE, JMOVE+1), EQ, MAXFT) GO TO 280
260  IMX = IMOVE
260  JMX = JMOVE
000061 PMAX=SAVE(IMOVE, JMOVE)          SHA R1
000062 PRINT 380                     SHA R2
000063 CALL LOCMIN                    SHA R3
000064 CALL MOVEI                    SHA R4
000065 IF (ITYPE(IMOVE, JMOVE)=FT) GO TO 270  SHA R5
000066 IF (ITYPE(IMOVE, JMOVE)=IF) GO TO 290  SHA R6
000067 270 PRINT 390                  SHA R7
000068 CALL ALTMOV                   SHA R8
000069 IF (IMX.EQ. IMOVE, JNOVE, JMX.EQ. JMOVE) SAVE(IMN, JMN)=0, SHA R9
000070 IF (RE(IMOVE, JMOVE).EQ.RE(IMX, JMX)) 290, 260, 260 SHA R10
000071 290 PRINT 390                  SHA R11
000072 SAVE(IMN, JMN)=0.              SHA R12
000073 GO TO 230                     SHA R13
000074 230 IF (IMOVE.EQ. IMXL, AND, JMOVE.EQ. JMXL) GO TO 300 SHA R14
000075 IMXL=IMX                     SHA R15
000076 JNXL=JNX                     SHA R16
000077 GO TO 310                     SHA R17
000078 310 IF (IMOVE.EQ. IMXL, AND, JMOVE.EQ. JMXL) GO TO 400 SHA R18
000079 300 PRINT 400                  SHA R19
000080 SAVE(IMN, JMN)=0.              SHA R20
000081 GO TO 220                     SHA R21
000082 220 CALL MOVE (IMX, JMX, IMOVE, JMOVE) GO TO 100 SHA R22
000083 GO TO 100                     SHA R23
000084 320 PRINT 410; PMAX_           SHA R24
000085 RETURN                       SHA R25
000086 FORMAT ('/ **********************************************************/ SHA R26
000087 1NEW BEST LOADING HAS BEEN FOUND **********************************************************/ SHA R27
000088 "**********************************************************/ RETURN SHA R28
000089 C SHA R29
000090 330 FORMAT ('/ MAX POWER OF',F7,3,' IN RECIPICAL POSITION. MAKE RECIPRICAL OF
000091 LAST SHUFFLE')               SHA R30
000092 340 FORMAT ('/ MAX POWER OF',F7,3,' AT POSITION',I2,12,' IS ON THE BNDRY') SHA R31
000093 1 'LOOK FOR THE NEXT HIGHEST POWER TO MOVE')                                 SHA R32
000094 350 FORMAT ('/POWER PEAK OF',F6,3,' (HOTTER THAN PREV PEAK OF',F6,3/) SHA R33
000095 1 HEAT TO LAST LOADING AND LOOK FOR NEW MOVE')                                  SHA R34
000096 360 FORMAT ('/POWER PEAK OF',F6,3,' IS THE SAME. MOVE FUEL WHERE MAX WAS')       SHA R35
000097 1 TO BE PLACED')                                                            SHA R36
000098 370 FORMAT ('/POWER PEAK IS ON BOUNDARY. LOOK FOR NEW MOVE')                       SHA R37
000099 380 FORMAT ('/THIS IS RECIPICAL OF LAST MOVE. TRY NEW MOVE')                     SHA R38
000100 390 FORMAT ('/SORRY I CAN NOT FIND A BETTER LOADING THAN THE ONE WHERE')  SHA R39
000101 400 FORMAT ('/ MAX POWER=',F7,3,' PROCEED TO SUBSEQUENT SHUFFLE FOR DIFFERENT LOADING') SHA R40
000102 410 FORMAT ('OHOWEVER K=INF S ARE THE SAME. MOVE FUEL WHERE MAX WAS')       SHA R41
000103 420 FORMAT ('OHOWEVER MOVE IS ON BOUNDARY. LOOK FOR NEW MOVE')                       SHA R42
000104 430 FORMAT ('OHOWEVER K=INF S ARE THE SAME. MOVE FUEL WHERE MAX WAS')       SHA R43
000105 440 FORMAT ('OHOWEVER K=INF S ARE THE SAME. MOVE FUEL WHERE MAX WAS')       SHA R44
000106 END CUR

3. PCH 1,3

END CUR
** SHUFFLE & SHUFFLES EXPOSED FUEL TO MINIMIZE RADIAL POWER PEAK **

SUBROUTINE SHUF, R

COMMON ALL, VX, NV, XM1, YM1, NCH, SAVE(10,10), EX(10,10), ITYPE(10,10)

1, REL(10,10), MAXFT

COMMON LOGIC/ EXS(10,10), SAV(10,10), IMX, JMX, IMN, JMN, LMX, LMX

MOVE, JMOVE, IMX, JMX, PMAX, PMIN, MAXIT, NSER, NPOLY, PMAVL, RES(10,10)

ITNUC=0, PMAVL=PMAVL+1

IF (ITNUC, GT, MAXIT) GO TO 200

CALL RELPOW

CALL MAX

ITTIME=ITTIME+1

IF (PMAVL, GE, PMAVL) GO TO 150

PRINT 230

PMAVL=PMAVL

IMX=IMX

JMX=JMX

ITIME=0

CALL SAVEXF (SAVE, SAV)

CALL SAVEXP (EX, EYS)

CALL SAVEXP (RE, RES)

CALL LOGMAX

CALL DIAGMN

IF (NSER, GT, 12) GO TO 210

IF (IMN, EQ, IMXL, AND, JMN, EQ, JMNLX) GO TO 12C

GO TO 14G

IF (LIMX, EQ, IMXL, AND, LMNX, EQ, JMNLX) GO TO 130

GO TO 14G

PRINT 246

JMN=JMNLX

JMN=JMNLX

ITIME=ITIME+1

NRECIP=0

GO TO 190

IMXLM=LMNX
JMNLN=LJMNL
IMNLN=IMNN
CALMIN MOVE (LIMX,LJMX,IMNN,JNN)
PRINT 250, ITN UC
GO TO 100
IF (ITIN,GE,1) GO TO 170
IF (JMX,LE,IMX,AND,LMX,LE,JMXL) GO TO 160
GO TO 170
PRINT 260, PMAX, PMAXL
IMNR=IMNN
JNRR=JMNL
NRECIP=1
GO TO 110
NRECIP=0
IMNN=IMNR
JMNL=JNRR
PRINT 270, PMAX, PMAXL
CALL SAVEXF (SAVE,SAVE)
CALL SAVEXP (EXS,EX)
CALL SAVEXP (RES,RE)
PMA XL=PMA XL
IMXL=IMXL
JMXL=JMXL
SAVE(IMNN,JNN)=0
CALL SAVEXF (SAVE,SAVE)
GO TO 110
PRINT 280
PRINT 290, PMAX
CALL SAVEXF (EXS,EX)
PRINT 300
DO 220 I=1, NYM1
PRINT 310, !,(EX(I,J),J=1,NMX1)
RETURN
C
FORMAT ("'************ ' NEW BEST LOADINGS HAS BEEN FOUND '****' ************")
FORMAT (''THIS IS JUST RECIPRICAL MOVE OF LAST TIME. CONTINUE'')
FORMAT ("' THE NUMBER OF THIS SHUFFLE ITERATION IS '15)

SHB 40
SHB 41
SHB 42
SHB 43
SHB 44
SHB 45
SHB 46
SHB 47
SHB 48
SHB 49
SHB 50
SHB 51
SHB 52
SHB 53
SHB 54
SHB 55
SHB 56
SHB 57
SHB 58
SHB 59
SHB 60
SHB 61
SHB 62
SHB 63
SHB 64
SHB 65
SHB 66
SHB 67
SHB 68
SHB 69
SHB 70
SHB 71
SHB 72
SHB 73
SHB 74
SHB 75
SHB 76
SHB 77
SHB 78
SHB 79
SHB 80
151

3. PCH 1,3

END CUR
C *** PROGRAM SHUFFLE *** CALCULATES RELATIVE POWER OF 1/4 CORE ***
C *** FOR A PWR AND WILL SHUFFLE FUEL TO MIN RADIAL POWER PEAK ***
C *** AS DIMENSIONED MAX PROBLEM SIZE LIMITED TO 10 BY 10 ******
C *** TO INCREASE PROBLEM SIZE SIMPLY INCREASE DIMENSIONS ******
C ** NOTE SAVE(I,J) IS AN ALL PURPOSE ARRAY AND MAY BE USED ******
C ** TO REPRESENT DIFFERENT ARRAYS DEPENDING WHERE USED ******
C
COMMON/ALL/NX,NY,NXMI,NYMI,NCH,SAVE(10,10),EX(10,10),ITYPE(10,10)
100 PRINT 260
PRINT 280
PRINT 270
ITNUC=0
TAURN=0,
C *** INPUT OF DATA SECTION TO CONTROL PROGRAM OPTIONS *******
C
READ (5,E30) 1TITL
READ (5,300) VX,NY,NCH,SAXT,MAXI,MAXO,MAXIT,NSER,NPOLY
110 READ (5,210) SIZE,TESTIN,TESTOT,AA(1),AA(2),ALPHA,TFUEL
C *** NEWSIZE**2
C MAXI=MAXF+1
C NXM1=NX-1
C NYM1=NY-1
C PMAXL=10.
C IF (NCH,EQ.2) GO TO 160
C *** INPUT OF FUEL TYPES AND CROSS SECTIONS ******
C
DO 120 I=1,NY
READ (5,320) (ITYPE(I,J),J=1,NX)
DO 120 O=LOIX
READ (5,330) (EX(I,J),J=1,NX)
EX(I,J)=ITYPE(I,J)
IF (NF(LY,C,) TO 140
DO 130 I=.101Y11
READ (7.A330) (EX(I,J),J=1,NX)
GO TO 160
DO 150 I;101AXPT
READ (5050) SIGA(I),SIOA(I),SIGEN(I),SIGEN(2),SIGS(I)
1 ,DD( 1,1)100(21)
*** PRINT OUT OF INPUT DATA AND INITIAL FUEL LOADING **********
PRINT 270
WRITE (6,290) NTT%
PRINT 270
PRINT 340, QC+MAXIT,NPOLY,NX,ND,SIZE
PRINT 350, QC+MAXO,TESTIN,TESTOT,AI,AA(2),ALPHA
PRINT 360
DO 170 I;.10NY
WRITE (6,322) (ITYPE(I,J),J=1,NX)
CC(1)='1.-AA(1)/8,
CC(2)='1.-AA(2)/8,
*** OPTIONS TO DETERMINE BURN OR SHUFFLE ***********************
GO TO (100,113,200,190,220,240), QC
CALL SHUFLA
CALL SAVEXP (3AV,SAVE)
CALL SAVEXP (EXS,EX)
CALL SHUFLEi
READ (5,380) NC
gO TO (100,113190119011010/240), NCH
*** BURNUP OF FUEL AND DETERMINE TOTAL BURNUP ***************
TBURN=0,
CALL RELPOW
CALL MAX
READ (5,380) VCH,BURN
PRINT 370, TBURN,BURN
TBURN=TBURN+BURN
DO 230 I=1,NY11
DO 230 J=1,NX11
230 EX(I,J)=EX(I,J)+SAVE(I,J)*BURN
GO TO 180
240 CONTINUE
C
250 FORMAT (7E12.3)
260 FORMAT (141,13X) "TWO DIM 1,5 GROUP DIFFUSION THEORY PROGRAM,",1X
270 FORMAT (/*
280 FORMAT (/*" THE OPTION CHOICE VALUES ARE DEFINED AS FOLLOWS
290 FORMAT (/* 1 /' START NEW CASE'" 2 USE OLD FLUXES AND CONTINUE
300 FORMAT (/* 2 /' 3 SHUFFLE EXPOSED FUEL'" 4 SHUFFLE NEW AND EXPOSED FUEL'
310 FORMAT (/* 5 BURN FUEL'" 6 END OF RUN'
320 FORMAT (/*
330 FORMAT (/*
340 FORMAT (" OPTION CHOICE: 1, MAX NUMBER SHUFFLE ITERATIONS=","13SHU
350 FORMAT (" MAX NUMBER OF INNER AND OUTER ITERATIONS RESP =","1,215,/,18SHU
360 FORMAT (" FAST AND THERMAL GROUP AVERAGING FACTORS RESP =","2F10,2,/,19ACSHU
370 FORMAT (" THE INITIAL FUEL LOADING BY TYPE IS'/")
380 FORMAT (" TOTAL BURNUP FOR THIS CASE=","1F11,2," MWD/KG'" INCREMENTSHSHU
390 FORMAT (" BURNUP TO BE ADDED=","1F7,2,' MWD/KG'/")
400 FORMAT (15,2F10,0)
END CUR
APPENDIX C

REFERENCE REACTOR

The reference reactor chosen for this study was OCONEE II, a 2568 MWth Babcock and Wilcox pressurized water reactor being built by Duke Power Company. Selection of this reactor resulted from the fact that it is typical of PWR's which will be coming on line in a few years, and large amounts of data from industry accepted computer physics and reactor simulator codes were available to the author for this particular reactor.

The core design and thermal and hydraulic data from the OCONEE Preliminary Safeguards Analysis Report (Duke Power) were the data used to make the calculations. Table C-1 is a listing of these data.
Table C-1. OCONEE design data.

## Core design, thermal, and hydraulic data

<table>
<thead>
<tr>
<th>Reactor</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated heat output, MWt</td>
<td>2,568</td>
<td></td>
</tr>
<tr>
<td>Vessel coolant inlet temperature, F</td>
<td>554</td>
<td></td>
</tr>
<tr>
<td>Vessel coolant outlet temperature, F</td>
<td>604.7</td>
<td></td>
</tr>
<tr>
<td>Core outlet temperature, F</td>
<td>605.5</td>
<td></td>
</tr>
<tr>
<td>Core operating pressure, p/sig</td>
<td>2,185</td>
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</table>

<table>
<thead>
<tr>
<th>Core and fuel assemblies</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of fuel assemblies in core</td>
<td>177</td>
<td></td>
</tr>
<tr>
<td>Number of fuel rods per fuel assembly</td>
<td>208</td>
<td></td>
</tr>
<tr>
<td>Number of control rod guide tubes per assembly</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>Number of in-core instrumentation positions per fuel assembly</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Fuel rod outside diameter, in.</td>
<td>0.430</td>
<td></td>
</tr>
<tr>
<td>Clad thickness, in.</td>
<td>0.0265</td>
<td></td>
</tr>
<tr>
<td>Fuel rod pitch, in.</td>
<td>0.568</td>
<td></td>
</tr>
<tr>
<td>Fuel assembly pitch spacing, in.</td>
<td>8.587</td>
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</tr>
<tr>
<td>Unit cell metal/water ratio (volume basis)</td>
<td>0.82</td>
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</tr>
<tr>
<td>Clad material</td>
<td>Zircaloy-4 (cold worked)</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Fuel</th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Material</td>
<td></td>
<td>UO₂</td>
</tr>
<tr>
<td>Form</td>
<td>Dished-end, cylindrical pellets</td>
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<tr>
<td>Pellet diameter, in.</td>
<td>0.370</td>
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<tr>
<td>Active length, in.</td>
<td>144</td>
<td></td>
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<tr>
<td>Density, % of theoretical</td>
<td>93.5</td>
<td></td>
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<table>
<thead>
<tr>
<th>Heat transfer and fluid flow at rated power</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total heat transfer surface in core, ft²</td>
<td>49,734</td>
<td></td>
</tr>
<tr>
<td>Average heat flux, Btu/h-ft²</td>
<td>171,470</td>
<td></td>
</tr>
<tr>
<td>Maximum heat flux, Btu/h-ft²</td>
<td>534,440</td>
<td></td>
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<tr>
<td>Average power density in core, kW/ft</td>
<td>79.60</td>
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<tr>
<td>Average thermal output, kW/ft of fuel rod</td>
<td>5.656</td>
<td></td>
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<tr>
<td>Maximum thermal output, kW/ft of fuel rod</td>
<td>17.63</td>
<td></td>
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<tr>
<td>Maximum clad surface temperature, F</td>
<td>654</td>
<td></td>
</tr>
<tr>
<td>Average core fuel temperature, F</td>
<td>1,540</td>
<td></td>
</tr>
<tr>
<td>Maximum fuel central temperature at hot spot, F</td>
<td>4,250</td>
<td></td>
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<tr>
<td>Total reactor coolant flow, lb/h</td>
<td>$131.32 \times 10^6$</td>
<td></td>
</tr>
<tr>
<td>Core flow area (effective for heat transfer), ft²</td>
<td>49.19</td>
<td></td>
</tr>
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(Continued on next page)
Table C-1. (Continued)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Core coolant average velocity, fps</td>
<td>15.73</td>
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<tr>
<td>Coolant outlet temperature at hot channel, F</td>
<td>642.8</td>
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<tr>
<td><strong>Power distribution</strong></td>
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<tr>
<td>Maximum/average power ratio, radial x local (F_{ah}^{nuclear})</td>
<td>1.78</td>
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<tr>
<td>Maximum/average power ratio, axial (F_{z}^{nuclear})</td>
<td>1.70</td>
</tr>
<tr>
<td>Overall power ratio (F_{q}^{nuclear})</td>
<td>3.03</td>
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<tr>
<td>Power generated in fuel and cladding, %</td>
<td>97.3</td>
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<tr>
<td><strong>Hot channel factors</strong></td>
<td></td>
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<tr>
<td>Power peaking factor (F_{Q})</td>
<td>1.011</td>
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<tr>
<td>Flow area reduction factor (F_{A})</td>
<td></td>
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<tr>
<td>Interior bundle cells</td>
<td>0.98</td>
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<tr>
<td>Peripheral bundle cells</td>
<td>0.97</td>
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<tr>
<td>Local heat flux factor (F_{Q''})</td>
<td>1.014</td>
</tr>
<tr>
<td>Hot spot maximum/average heat flux ratio (F_{Q nuc and mech})</td>
<td>3.12</td>
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<tr>
<td><strong>DNB data</strong></td>
<td></td>
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<tr>
<td>Design overpower % rated power)</td>
<td>114</td>
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<tr>
<td>DNB ratio at design overpower (W-3)</td>
<td>1.55</td>
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<tr>
<td>DNB ratio at rated power (W-3)</td>
<td>2.0</td>
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<td><strong>Nuclear design data</strong></td>
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<td>Fuel assembly volume fractions</td>
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<td>Fuel</td>
<td>.303</td>
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<td>Moderator</td>
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<td>Zircaloy</td>
<td>.102</td>
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<td>Stainless steel</td>
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<tr>
<td>Void</td>
<td>.012</td>
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<tr>
<td>Core dimensions (in.)</td>
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<tr>
<td>Equivalent diameter</td>
<td>128.9</td>
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<td>Active height</td>
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<tr>
<td>Unit cell H_{2}O to U atomic ratios</td>
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<tr>
<td>Cold</td>
<td>2.85</td>
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<tr>
<td>Hot</td>
<td>2.04</td>
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<tr>
<td><strong>Control data</strong></td>
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<tr>
<td>Control rod material</td>
<td>Ag-In-Cd</td>
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<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
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<tbody>
<tr>
<td>Number of full length CRA's</td>
<td>61</td>
</tr>
<tr>
<td>Number of APSR's</td>
<td>8</td>
</tr>
<tr>
<td>Worth of 61 full length CRA's ($\Delta k/k$)%</td>
<td>10.6</td>
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<tr>
<td>Control rod cladding material</td>
<td>SS304</td>
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