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ENGINEERING EXPERIMENT STATION
OREGON STATE COLLEGE
CORVALLIS, OREGON



# Review for Engineering Registration
# 3. Mechanical Engineering



By
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Associate Professor of Mechanical Engineering
Oregon State College

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# REVIEW FOR ENGINEERING REGISTRATION
# 3. MECHANICAL ENGINEERING

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## FOREWORD

This publication is the third in a series of circulars designed to assist the graduate engineer in reviewing engineering subject matter in order to prepare for registration examinations. As in the first publication of the series, most of the illustrative problems are taken from recent examinations of the Oregon State Board of Engineering Examiners, and these problems are identified in the table of contents according to the date of the examination and the number of the problem.

It is beyond the scope of these publications to give a detailed explanation of theoretical considerations and methods of analysis. For those who require additional study of specific subject material, a list of textbook references is included at the end.

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# REVIEW FOR ENGINEERING REGISTRATION
# 3. MECHANICAL ENGINEERING

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## I. INTRODUCTION

This circular has been prepared in the hope it may be of value to mechanical engineers who intend to apply for registration as professional engineers in the State of Oregon.

The form used throughout the circular presents a problem, states the fundamental principles involved, and gives the solution in detail. Alternate solutions are sometimes given, and points of interest or of particular importance are discussed.

Nearly all the problems used, of which a list is given in the table of contents, have been taken directly from written examinations given over the years 1947 to 1954 by the Oregon State Board of Engineering Examiners. Other problems have been selected in order to cover a more comprehensive range of subjects. A complete, detailed review of all fundamentals is not possible in a circular of this type, but it is expected the reader will have access to handbooks and texts in the various fields. A short bibliography of such references is included.

Many practical engineering problems require application of several fundamentals generally presented in separate individual fields; i.e., statics, air conditioning, heat transfer, engineering materials, et cetera. Solutions of such problems may be dependent upon one or more assumptions made by the engineer, and often several different solutions may result, with each being of equal value. Questions of this type often occur in professional examinations. Some are included here as examples. For purposes of review and to insure a broader coverage of fundamentals, however, most of the problems used in this circular are of specific type and are grouped in accordance with the outline given in the table of contents.

## II. TYPICAL MECHANICAL ENGINEERING PROBLEMS

Most problems encountered in mechanical engineering are either of machine-design type, involving determinations of force, stress, and proper use of engineering materials, or of thermodynamic type, dealing with transformations of energy and heat transfer.

The general approach to the solution of all problems is the same: (1) known information is listed, (2) assumptions made are stated, (3) method of solution is clearly indicated so that another engineer checking the work can follow your line of reasoning, and (4) the final answer is checked for accuracy, reasonableness, and consistency with the original assumptions.

Many mistakes result from failure to take two simple precautions—checking for consistency of units, and making a sketch or free-body diagram.

The engineering or gravitational system of units uses the foot, pound force, and second as the three fundamental units. From these three the unit of mass is derived as a slug (approximately 32.2 lb weight). This simply means that <u>one</u> slug of mass will be accelerated at the rate of <u>one</u> foot per second per second by a force of <u>one</u> pound. In any equation involving mass, the term W/g may be substituted for m, where W is pounds force (weight) and g is the acceleration due to gravity (32.2 feet per second per second). Most errors, however, come from mixing feet and inches, and seconds and minutes in the same equation. Take the calculation of a bending moment as an example.

$$\begin{aligned} M &= (PA)\,L \text{ pound-inches, moment} \\ &= (\text{pressure} \times \text{area})(\text{length}) \\ &= (\text{lb per sq ft} \times \text{sq ft})(\text{inches}) \\ &= \text{lb-in. moment} \end{aligned}$$

After writing an equation, always check to make sure each item has been used with its proper units.

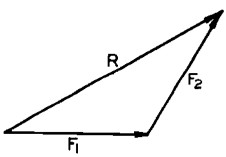
The free-body diagram used in mechanics is necessary before the fundamental equations ($\Sigma F = 0$, $\Sigma M = 0$, $\Sigma F = Ma$) can be applied. A sketch is made of the entire structure (or any part of it) and all external forces acting on the structure are shown on the free-body diagram. Only when all external forces are included can it be said that $\Sigma F = 0$, et cetera.

## III. STATICS

Statics is that portion of the subject of mechanics which concerns bodies in equilibrium under the action of external forces. The condition of equilibrium requires that the body be either at rest or moving with constant linear velocity. In other words, the acceleration of the body must be zero.

The fundamental principles of statics are expressed by the equations of equilibrium, $\Sigma F = 0$ and $\Sigma M = 0$. As indicated earlier, in applying these equations one first must be certain that all external forces acting on the body have been considered.

Forces and moments must be recognized as vector quantities. In addition to magnitude, direction and line of action of a force must be known. Being vector quantities, forces cannot be added algebraically, but must be added graphically as shown in the sketch and expressed by the vector equation

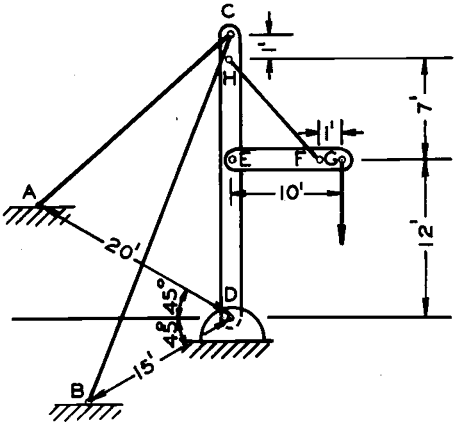



$$F_1 \; {+\!\!\!\longrightarrow} \; F_2 = R$$

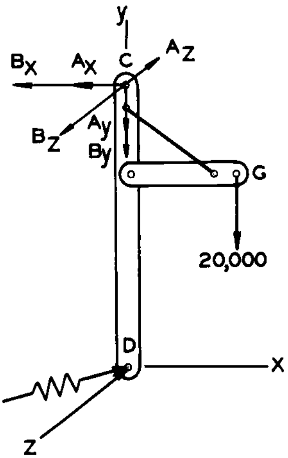
Friction is a factor in all practical problems and is often misunderstood. The direction of the friction force will always be such as to oppose the relative motion of the two bodies. The magnitude of the friction force can be any amount up to a limiting value expressed by $F = fN$, where f is the coefficient of static friction and N is the total normal force on the mating surfaces. In problems involving kinetic friction or slipping, the friction force is considered to be constant and equal to $F = f_k N$; $f_k$ being the coefficient of kinetic friction.

## 1. Boom, Three Dimensions

The structure shown has a capacity of 20,000 pounds static load at G. Points C, D, E, and H are in the same vertical plane. What is the tension in cables AC and BC when structure is loaded to capacity?




Solution: Since the cables are two-force body members, forces must act along the length of the cables. Draw a free-body diagram of the mast and boom with external forces shown. Cable tensions are shown broken down into their x, y, and z components.




$$\Sigma M_z = 0 = 20,000(10) - (A_x + B_x)(20)$$

$$\therefore \quad A_x + B_x = 10,000 \text{ lb} \qquad (1)$$

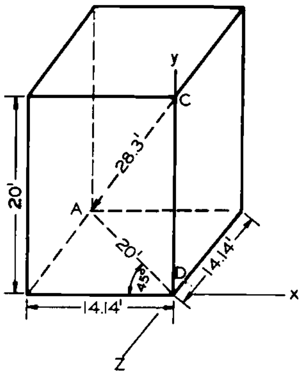
$$\Sigma M_x = 0 = 20 B_z - 20 A_z$$

$$\therefore \quad B_z = A_z \qquad (2)$$

By geometry, each component can be expressed in terms of total cable tension.

$$\text{Length of AC} = \sqrt{(20)^2 + (20)^2} = \sqrt{800} = 28.3 \text{ ft}$$

Visualize AC as the diagonal of a box. Sides of the box have lengths of 20, 14.14, and 14.14 feet, and the diagonal 28.3 feet.




$$\frac{AC}{28.3} = \frac{A_x}{14.14} = \frac{A_y}{20} = \frac{A_z}{14.14}$$

$$\therefore A_x = \frac{14.14}{28.3} AC = 0.5 \text{ AC}$$

$$A_z = \frac{14.14}{28.3} AC = 0.5 \text{ AC}$$

Similarly,

$$BC = \sqrt{(20)^2 + (15)^2} = 25 \text{ ft}$$

$$B_x = \frac{15 \cos 45}{25} BC = 0.424 \text{ BC}$$

$$B_z = 0.424 \text{ BC}$$

Substitute these values into equations (1) and (2) and solve simultaneously.

$$0.5 \text{ AC} + 0.424 \text{ BC} = 10,000$$

$$0.424 \text{ BC} = 0.5 \text{ AC}$$

$$BC = \frac{10,000}{0.848} = \underline{\underline{11,800 \text{ lb tension in BC}}}$$

$$AC = \frac{10,000}{1} = \underline{\underline{10,000 \text{ lb tension in AC}}}$$

## 2. Buoyancy, Pontoon Bridge

Steel cylinders 3 feet in outside diameter and 12 feet long overall are available for the construction of a pontoon bridge. Assume air pressure outside and inside the pontoon is standard and air and water temperatures 50°F. How much weight will one pontoon support if it is made of 3/16-inch plate and is half submerged?

Theory: Any object immersed in liquid or gas will be subjected to a vertical buoyant force equal to the weight of the liquid or gas displaced.

Solution: The pontoon is subjected to the following vertical forces:

$W$ = weight supported by pontoon
$W_t$ = weight of steel tank
$W_a$ = weight of air in tank
$B_w$ = buoyancy due to displaced water
$B_a$ = buoyancy due to displaced air

$$W_t = \frac{490}{12}\left(\frac{3}{16}\right)\left[\text{area of ends} + \text{area of cylinder}\right]$$

$$= 765\left[2(0.785)(\overline{3.0})^2 + \pi\, 3.0(12)\right] = \underline{972 \text{ lb},\ W_t}$$

$$W_a = \text{density(volume)} = \rho V$$

$$\rho = \frac{PV}{RT} = \frac{14.7(144)(1.0)}{53.3(460+50)} = 0.0777 \text{ lb/cu ft}$$

$$W_a = 0.0777\left[0.785(\overline{3.0})^2(12)\right] = \underline{6.6 \text{ lb},\ W_a}$$

$$B_w = \frac{1}{2}\text{vol}(62.4) = \frac{1}{2}\left[0.785(\overline{3.0})^2(12)\right](62.4) = \underline{2640 \text{ lb},\ B_w}$$

$$B_a = \frac{1}{2}(W_a) = \underline{3.3 \text{ lb},\ B_a}$$

$$W = B_w + B_a - W_t - W_a$$

$$= 2640 + 3.3 - 972 - 6.6 = \underline{\underline{1665 \text{ lb lifting force}}}$$

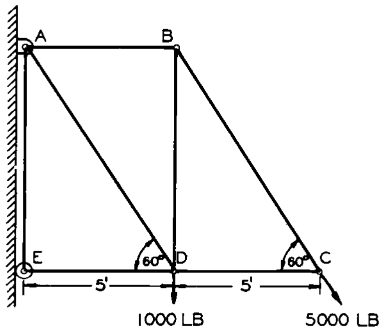
3. Truss

In the truss shown, all joints are pin joints. Wall fastener at A can take both vertical and horizontal reactions. Wall connection at E is mounted on a roller so reaction at E can be horizontal only. Determine:

a) Horizontal and vertical wall reactions at A.

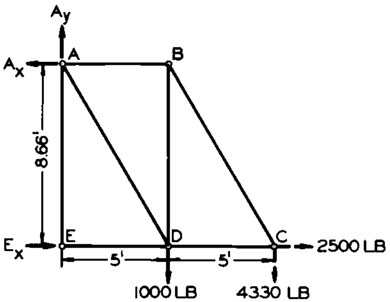
b) Horizontal reaction at E.

c) Force in each member of the truss and indicate whether tension or compression.




Theory: In truss problems it is assumed all loads are applied at a pin and each member is free to turn at the pin. This makes all members two-force body members, either in tension or compression.

Solution: First consider entire truss as a free body.




a) $\Sigma F_y = 0 = A_y - 1000 - 4330$

$A_y = \underline{\underline{5330 \text{ lb, up}}}$

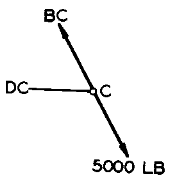
$\Sigma M_E = 0 = 8.66\, A_x - 5(1000) - 10(4330)$

$A_x = \dfrac{48,300}{8.66} = \underline{\underline{5570 \text{ lb, left}}}$

b) $\Sigma F_x = 0 = E_x + 2500 - 5570$

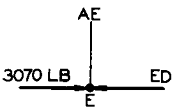
$E_x = \underline{\underline{3070 \text{ lb, right}}}$

c) Draw free-body diagrams of the separate pins.




Pin C: Link DC must have zero force because there would be no way to react the component perpendicular to BC.

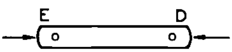
$\therefore$ BC = $\underline{\underline{\text{5000 lb tension}}}$



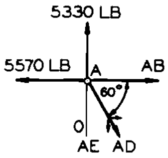

Pin E: AE must be zero since again there is nothing to react it.

$\Sigma F_x = 0$

$\therefore$ ED = $\underline{\underline{\text{3070 lb compression}}}$




Note: When the member is pushing toward the pin, as in ED, the pin is pushing back on it and the member is in compression. A free-body diagram of DE shows pins at D and E both pushing in on the member.




Pin A: First consider AD since it is the only member that can react the vertical force.

$$\Sigma F_y = 0 \qquad AD_y = 5330 \text{ lb down}$$

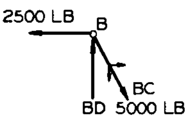
Note that $AD_x$ must be $\frac{0.5}{0.866}(5330)$

$$= 3070 \text{ lb right}$$

$$AD = \frac{3070}{\cos 60°} = \underline{\underline{6140 \text{ lb tension}}}$$

Clearly, AB must act to the right.

$$\Sigma F_x = 0, \ AB = 5570 - 3070 = \underline{\underline{2500 \text{ lb tension}}}$$

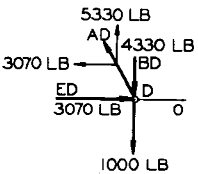



Pin B: $\Sigma F_y = 0$

$$BD = 5000 \sin 60°$$

$$= \underline{\underline{4330 \text{ lb compression}}}$$

Note: The one remaining pin may now be used as a check for errors. Magnitude of all forces acting on pin D already has been found and, if correct, should satisfy



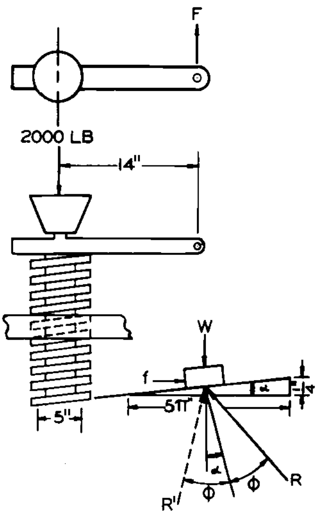

$$\Sigma F_x = 0, \qquad \Sigma F_y = 0$$

4. Screw Jack

A screw jack has 4 threads per inch. Mean diameter of threads is 5 inches. Mean diameter of bearing surface under the cap is 3.5 inches. Assuming coefficient of friction for all surfaces to be 0.06, what force at end of a 14-inch turning rod would be required to

a) Lift a 1-ton load.
b) Lower a 1-ton load.

Theory: The screw jack is equivalent to an inclined plane with a block being pushed up or down against forces of gravity and friction. Slope of the plane is $\alpha$, the angle whose tangent is lead of screw divided by circumference of thread. $\phi$ is angle of friction, R the reaction when block is being pushed up the plane, R' is reaction when load is being lowered, and f is horizontal force on the screw thread equivalent to applied force F minus friction under the cap.




Solution: a) When load is being raised

$$R \sin(\phi + \alpha) = f$$

and

$$R \cos(\phi + \alpha) = W$$

$$\therefore \tan(\phi + \alpha) = f/W$$

or

$$f = W \tan(\phi + \alpha)$$

However,

$$f = \frac{14}{2.5} F - \frac{1.75}{2.5} [0.06(2000)]$$

$$= 5.6 F - 84$$

$$\phi = \text{arc tan } 0.06 = 3.44°$$

$$\alpha = \text{arc tan } \frac{0.25}{5\pi} = \text{arc tan } 0.0159 = 0.91°$$

$$\therefore 5.6 F - 84 = 2000 \tan(3.44° + 0.91°)$$

$$F = \frac{2000(0.076) + 84}{5.6}$$

$$F = \underline{\underline{42.1 \text{ lb to lift load}}}$$

b) When load is being lowered

$$R' \sin(\phi + \alpha) = f'$$

(Note that f' is opposite in direction to f)

$$R' \cos(\phi - \alpha) = W$$

$$\tan(\phi - \alpha) = \frac{f'}{W}$$

or $$f' = W \tan(\phi - \alpha)$$

Since directions of both F and friction under the cap reverse when load is lowered, the equation for f is the same as before.

$$\therefore \quad F = \frac{2000 \tan(3.44 - 0.91) + 84}{5.6}$$

$$= \frac{2000(0.0441) + 84}{5.6}$$

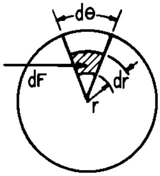
$F = \underline{\underline{30.7 \text{ lb}}}$ pushing in opposite direction to lower load

## 5. Friction Clutch

The friction disc of a single-plate clutch has an outside diameter of 10 inches and an inside diameter of 4 inches. Normal engaging force is 400 pounds. If coefficient of friction for the materials is 0.16, determine the torque which it will transmit without slipping, both when new and after the surfaces are worn in.

Theory: When new, the force is distributed evenly over the wearing surface, but wear takes place faster (because of higher velocities) at the outer parts of the disc. This decreases pressure and has the effect of reducing the holding power of the clutch.

Solution: a) With new surfaces, the normal pressure p is constant, and




$$p = \frac{P}{A} = \frac{400}{78.5 - 12.6} = 6.07 \text{ psi}$$

$dN = pdA = p(r\ dr\ d\theta)$, increment normal force

$$dF = fdN$$

$dT = rdF = f\ p\ r^2 dr\ d\theta$, representing the contribution to the total torque of the small increment area dA.

$$T = \int dT = fp \int_{\theta=0}^{\theta=2\pi} \int_{r=2}^{r=5} r^2 dr\ d\theta = f\ 2\pi\ p \int_{r=2}^{r=5} r^2 dr$$

$$= 0.16(2\pi\ p)\left[\frac{1}{3}\ r^3\right]_2^5 = 0.32\ \pi(6.07)\left[\frac{125 - 8}{3}\right]$$

T = 238 lb-in. torque when new

b) After wear has reached an equilibrium condition, the pressure p will vary inversely with the radius, and the relationship pr = k will hold true.

$$\text{Total force } P = \int pdA = \int \frac{k}{r}(r\ dr\ d\theta)$$

$$P = k \int_{r=2}^{r=5} \int_{\theta=0}^{\theta=2\pi} dr\ d\theta = 2\pi\ k(5 - 2) = 6\pi\ k$$

$$\therefore\ k = \frac{P}{6\pi} = \frac{400}{6\pi} = 21.2$$

and

$$p = \frac{k}{r} = \frac{21.2}{r}$$

Now,

$$T = \int dT = f\int p\ r^2 dr\ d\theta = f\int \frac{k}{r}\ r^2 dr\ d\theta$$

$$T = fk \int_{r=2}^{r=5} \int_{\theta=0}^{\theta=2\pi} r \, dr \, d\theta = fk \; 2\pi \left[ \frac{1}{2} r^2 \right]_2^5$$

$$T = 0.16(21.2) \left[ 2\pi \left( \frac{25 - 4}{2} \right) \right]$$

$$T = \underline{\underline{224 \text{ lb-in. torque when worn in}}}$$

## IV. DYNAMICS

Dynamics is that branch of mechanics which concerns the motion of bodies. It includes kinematics, the study of motion itself, and the relationship between displacement, velocity, and acceleration; and kinetics, the study of motion resulting from applied forces.

The equations of kinematics can be summarized for constant acceleration as follows:

$$s = s_o + v_o t + \frac{1}{2} at^2$$

$$v = v_o + at$$

$$v^2 = v_o^2 + 2\,as$$

$$v = \frac{ds}{dt}$$

$$a = \frac{d^2 s}{dt^2} = \frac{dv}{dt}$$

When acceleration is a variable, the above equations must be integrated. A graphical integration is often very simple, as in problem 2 in this chapter.

In kinetics as well as in statics a free-body diagram is again essential. The motion of a body is described by Newton's second law

$$\Sigma F = ma$$

Here, the term $\Sigma F$ represents the summation of all external forces acting on the body. m is the mass in slugs and is usually replaced by the term W/g; g being taken as 32.2 ft/sec$^2$.

A similar equation describing rotation is

$$\Sigma T = I\alpha$$

I is the moment of inertia of the mass about the center of mass, or both I and T may be taken with respect to any stationary center of rotation. $\alpha$ is the absolute angular acceleration in radians per second per second.

In general motion, both translation and rotation are present and both equations may be applied independently to the system.

By rearrangement, the above formulas may be shown in forms which are sometimes more convenient, and referred to as the equations of impulse and momentum.

$$\Sigma Ft = \Delta(mv) = m(\Delta v) \text{ provided mass is constant}$$

$$\Sigma Tt = \Delta(I\omega) = I(\Delta\omega)$$

The left-hand term is the impulse which causes the corresponding change in momentum. Note that these terms are vector quantities, and that the change in velocity will be in the direction of the applied force.

Another method of solving kinetics problems is by the law of conservation of energy. Any net work done on the body (less losses due to friction) will show up as an increase in kinetic energy. Kinetic energies of translation and rotation are

$$KE = \frac{1}{2} mv^2$$

$$KE = \frac{1}{2} I\omega^2$$

1. Projectile

An anti-aircraft gun fires vertically upward a projectile weighing one pound. Muzzle velocity is 2800 feet per second. Neglecting friction, find:

a) Maximum height to which projectile will go.

b) Velocity at one-half the maximum height in fps.

c) Horsepower developed if projectile is in gun barrel for 0.01 second.

Theory: Neglecting wind resistance, the only external force acting on the projectile is gravity. Power is the time rate of doing work. If a constant force is assumed, power increases with velocity. Maximum power will be developed as the projectile leaves the gun with maximum velocity.

Solution: a) Maximum height is reached when v = 0,

$$s = v_o t + \frac{1}{2} at^2$$

but,

$$v = v_o - gt = 0, \text{ or } t = \frac{v_o}{g}$$

$$h = v_o\left(\frac{v_o}{g}\right) - \frac{1}{2} g \left(\frac{v_o}{g}\right)^2$$

$$h = \frac{1}{2}\left(\frac{v_o^2}{g}\right)$$

$$h = \frac{(2800)^2}{2(32.2)} = \underline{\underline{122,000 \text{ ft altitude}}}$$

b) Velocity at one-half maximum height, or 61,000 feet, would have the same magnitude whether going up or coming down.

$$s = \frac{1}{2} gt^2 = \frac{v^2}{2g}, \text{ or } v = \sqrt{2\ gs}$$

$$v = \sqrt{2g(h/2)} = \sqrt{gh} = \sqrt{32.2(122,000)} = \underline{\underline{1980 \text{ ft per sec}}}$$

c) Neglecting friction and wind resistance, the free-body diagram shows the projectile acted upon by W, the force of gravity, and F, the explosive charge in the barrel.

$$\Sigma F = ma, \text{ or } F - W = \frac{W}{g} a$$

$$F = W + \frac{W}{g} a$$

$$\text{Power} = Fv = Wv + \frac{W}{g} av = Wv + \frac{d}{dt}\left(\frac{1}{2}\frac{W}{g}v^2\right)$$

Note that the first term represents power to increase the potential energy or elevation of the mass. The second term represents power required to increase the kinetic energy of the mass.

If acceleration is assumed constant, as

$$a = \frac{v}{t}$$

then $$P = Wv + \frac{W}{g}\left(\frac{v}{t}\right)(v) = Wv + \frac{W}{g}\left(\frac{v^2}{t}\right) \text{ ft-lb/sec}$$

$$= 1.0(2800) + \frac{1.0}{32.2}\left(\frac{2800^2}{0.01}\right) \text{ ft-lb/sec}$$

$$hp = \frac{2800 + 24,400,000}{550} = \underline{\underline{44,300 \text{ hp}}}$$

Note that the power required to increase elevation is negligible, and also that this answer is just double the average horsepower developed.

$$hp_{avg} = \frac{\text{work}}{\text{time}} = \frac{1/2\ mv^2}{0.01} = 22,150 \text{ hp}$$

2. Variable Acceleration

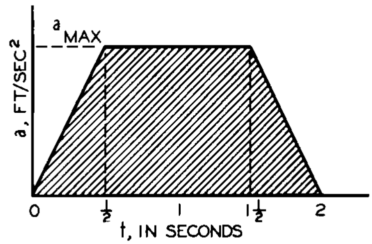
An elevator cage, rope, and passengers weigh 3000 pounds. Maximum hoisting speed is 12 feet per second. In bringing the cage up to full speed, acceleration is increased uniformly during the first 1/2 second, remains constant for 1 second, and decreases to zero during the next 1/2 second.

a) What is maximum acceleration?

b) What is maximum force in cable during hoisting?

c) If counterweight weighs 3000 pounds (and necessarily moves equal and opposite to cage), what torque must be supplied to the 18-inch diameter drum around which cable is wrapped?

Solution: Graphical integration is readily used in the solution of this problem to determine value of maximum acceleration needed.




a)

$$v = \int a\,dt = \text{area under a versus t curve}$$

$$v_{at\ 2\ sec} = 12 = \frac{1}{2}\left[\frac{1}{2}(a_{max})\right] + 1(a_{max}) + \frac{1}{2}\left[\frac{1}{2}(a_{max})\right] = 1\frac{1}{2}(a_{max})$$

$$a_{max} = \frac{12}{1.5} = \underline{\underline{9\ \text{ft/sec}^2,\ \text{maximum acceleration}}}$$

b) The force pulling up on the cable T must be greater than the weight in order to accelerate the mass W/g.

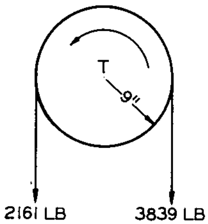
$$\therefore \Sigma F = \frac{W}{g}(a)$$

$$T - 3000 = \frac{3000}{32.2}(9)$$

$$T = 839 + 3000 = \underline{\underline{3839\ \text{lb tension}}}$$

during the 1 second the elevator is being accelerated at the maximum rate of 9 ft/sec$^2$.

c) Draw a free-body diagram of the drum and note that the counterweight must be accelerated also. This means the tension in the cable supporting it will be reduced to 839 pounds.




$$\Sigma T = 0$$

$$T + 2161\ R - 3839\ R = 0$$

$$T = \frac{9}{12}(3839 - 2161)$$

$$T = \underline{\underline{1260 \text{ lb-ft maximum torque}}}$$

during time mass is accelerating

Note that after the elevator has reached its normal speed of 12 feet per second, the only torque or power needed is that to overcome friction.

3. Mine Hoist, Energy

The winding drum of a mine hoist is 12 feet in diameter. Its radius of gyration is 5 feet, and its weight is 7-1/2 tons. A cage weighing 5 tons is raised vertically by it. If the cage is being hoisted at rate of 50 feet per second, at what distance from surface should power be shut off in order that cage may stop at surface without braking? Neglect friction and weight of cable.

Theory: Both cage and drum have kinetic energy when cage is in motion. After stopping, this kinetic energy is transferred to increased potential energy of the cage.

Solution:

$$\text{Kinetic energy of cage} = \frac{1}{2} M v^2 = \frac{W v^2}{2g}$$

$$= \frac{10,000}{2(32.2)}(50)^2 = \underline{389,000 \text{ ft-lb}}$$

$$\text{Kinetic energy of drum} = \frac{1}{2} I_o \omega^2 = \frac{1}{2}(k^2 \frac{W}{g}) \omega^2$$

$$= \frac{1}{2}\left[25(\frac{15,000}{32.2})\right](\frac{50}{6})^2 = \underline{404,000 \text{ ft-lb}}$$

Gain in potential energy of cage = 10,000 s ft-lb

$$\therefore \; 10,000\ s = 389,000 + 404,000$$

$$s = \underline{\underline{79.3 \text{ ft to stop}}}$$

4. Turbine, Energy

The rotor in a steam turbine weighs 20,000 pounds and has a radius of gyration of 14 inches. With the rotor turning at 1800 rpm, the steam is shut off. Average coefficient of friction in the bearings is 0.005, and diameter of bearings is 8 inches. Neglecting fluid friction of steam about turbine blades, determine elapsed time and total number of revolutions in coming to rest.

Theory: Since there is no gain in potential energy, all of the kinetic energy must be lost in bearing friction.

Solution:

$$\text{Energy lost to friction} = F r\, \theta$$

$$\text{Kinetic energy} = \frac{1}{2} I\omega^2 = \frac{1}{2}(k^2 \frac{W}{g})\, \omega^2$$

$$F r\, \theta = \frac{1}{2}(k^2 \frac{W}{g})\, \omega^2$$

$$0.005\ W(\frac{4}{12})\ \theta = \frac{1}{2}(\frac{14}{12})^2\ \frac{W}{g}\left[1800(\frac{2\pi}{60})\right]^2$$

$$\theta = \frac{14^2}{0.48\ g}(60\pi)^2 = 71,600\ (2\pi) \text{ radians}$$

$$\theta = \underline{\underline{71,600 \text{ revolutions to stop}}}$$

Use the impulse and momentum method to determine elapsed time.

$$(\Sigma T)t = \Delta(I\omega) = k^2\ \frac{W}{g}\,(\omega - 0)$$

$$\frac{4}{12}\left[0.005(20,000)\right]\ t = (\frac{14}{12})^2\ (\frac{20,000}{32.2})(1800\ \frac{2\pi}{60})$$

$$t = \frac{3(1.168)^2 60\pi}{0.005(32.2)} = 4770 \text{ sec}$$

$$t = \underline{\underline{1.33 \text{ hours to stop}}}$$

5. Pile Driver

A 900-pound pile driver is released from a height of 25 feet above the top of a 400-pound pile. The blow drives the pile into the bed of the bay a distance of 4 inches. Assuming the hammer and pile cling together after impact, compute the average resistance to penetration.

Theory: In this type of problem one may be tempted to use incorrectly the law of conservation of energy. Actually, this is an impact condition and considerable energy is lost when the hammer strikes the pile and before the pile is driven into the ground. During impact, the law of conservation of momentum must be applied.

Solution:

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$$

$$\frac{900}{g} v_1 + 0 = \frac{1300}{g} v$$

$$v = \frac{900}{1300} v_1 = 0.692 v_1$$

This is the velocity of the combined hammer and pile after impact. Just before impact the velocity of the hammer was $v_1 = \sqrt{2 gh}$, as obtained using conservation of energy during fall of hammer.

Loss in potential energy = gain in kinetic energy

$$Wh = \frac{1}{2}\left(\frac{W}{g}\right)(v_1^2)$$

$$v_1 = \sqrt{2 gh} = \sqrt{2(32.2)(25)}$$

$$v_1 = 40.1 \text{ ft/sec before impact}$$

and $$v = 0.692(40.1) = 27.8 \text{ ft/sec after impact}$$

Now applying the energy equation (loss in potential energy + loss in kinetic energy = work done against friction)

$$1300(\frac{4}{12}) + \frac{1}{2}(\frac{1300}{32.2})(27.8)^2 = F(\frac{4}{12})$$

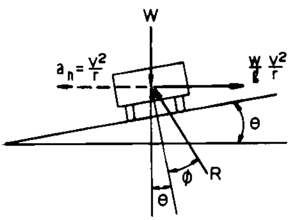
$$F = \underline{\underline{48,000 \text{ lb}}}, \text{ average force}$$

Had the problem been worked assuming all of the potential energy available in the hammer was used to drive the pile a distance of 4 inches, the answer would have been (incorrectly), F = 68,400 lb.

6. Super Elevation, Car on Curve

A 4500-pound Cadillac is rounding a banked highway curve. The highway is 30 feet wide and elevated 2 feet on the outside. The curve has a radius of 500 feet. Coefficient of friction between the rubber tires and the macadam is 0.6. What speed can be traversed safely without skidding on the curve?

Theory: The car is acted upon by three forces:




a) Force of gravity, W.

b) Inertia force, $\frac{W}{g}(\frac{v^2}{r})$, (opposite in direction to normal acceleration).

c) Reaction at the road, R.

This reaction could be broken down into a normal component and a friction component, but it will be more convenient to break it down into horizontal and vertical components.

Solution:

$$\Sigma F_v = 0; \; R \cos(\theta + \phi) = W$$

$$\Sigma F_h = 0; \; R \sin(\theta + \phi) = \frac{W}{g}(\frac{v^2}{r})$$

Dividing the second equation by the first

$$\tan(\theta + \phi) = \frac{v^2}{gr}$$

$$\theta = \arc \tan \frac{2}{30} = 3.82°$$

$$\phi = \arc \tan 0.6 = 31.0°$$

$$\tan(34.82) = \frac{v^2}{32.2(500)}$$

$$v^2 = 32.2(500)(0.697) = 11,200$$

$$v = 106 \text{ ft/sec}$$

$$= 106\left(\frac{3600}{5280}\right) = \underline{\underline{72 \text{ mph max speed}}}$$

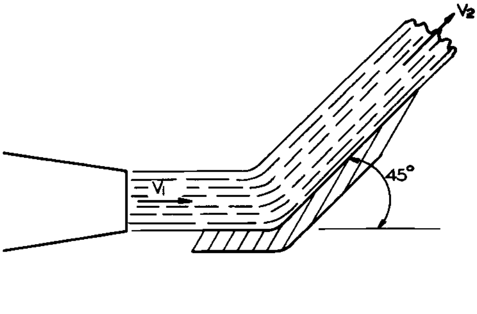
7. Nozzle and Vane

One hundred and eighty gallons per minute of water from a nozzle 2 square inches in area strikes a smooth vane, as shown. Determine the force of the water on the vane if it is:

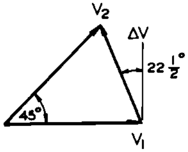
a) Fixed.

b) Moving in the direction of the water jet with a velocity of 20 feet per second.

Theory: This type of problem is easily solved by the method of impulse and momentum. Although the speed of the water stream remains unchanged, the direction is changed and, therefore, exerts a force on the vane. Solve for the force of the vane acting on the water. Force of the water on the vane will be equal and opposite.



Solution: a)




$$F(dt) = d(mv) = m(dv)$$

$$F = \frac{m}{dt}(\Delta v) = \frac{m}{dt}(v_2 \longrightarrow v_1)$$

Note that force F must act in the direction of $\Delta v$, or 22-1/2° from the vertical ($v_2 = v_1 \mathbin{+\!\!\rightarrow} \Delta v$).

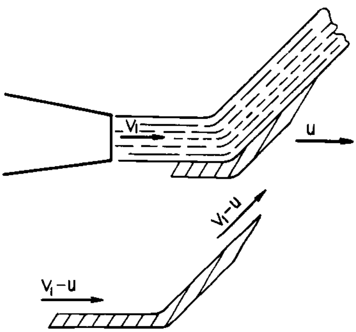
$$F = \frac{180(8\frac{1}{3})}{60(32.2)}(2\ v_1\ \sin 22\text{-}1/2°)$$

$$v_1 = \frac{180}{60(7.5)(2/144)} = 28.8\ \text{ft/sec}$$

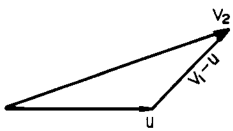
$$F = \underline{\underline{17.0\ \text{lb}}}$$

force of water acting <u>down</u> and to the <u>right</u> on the vane. (Equal and opposite to $\Delta v$.)

b) When vane is moving, a smaller mass of water moves across it per unit of time, with a velocity relative to the vane of (v - u). The final absolute velocity of the fluid must first be found.

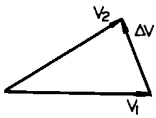



The absolute velocity of the fluid leaving the vane is equal to absolute velocity of vane plus, vectorially, the relative velocity of the fluid with respect to the vane.




$$v_2 = u \mathbin{+\!\!\rightarrow} (v_1 - u)$$

Here it will be easier to work with the x and y components.




$$(v_2)_x = u + (v_1 - u)\cos 45$$

$$= 20 + 8.8(0.707) = 26.22$$

$$(v_2)_y = (v_1 - u)\sin 45 = 6.22$$

Again,

$$\Delta v = v_2 - 1$$

$$\Delta v_x = (v_2)_x - v_1 = 26.22 - 28.8 = -2.58 \text{ (to left)}$$

$$\Delta v_y = (v_2)_y = 6.22$$

$$\therefore \Delta v = \sqrt{6.22^2 + 2.58^2} = 6.74 \text{ ft/sec}$$

The mass of water undergoing momentum change is equal only to

$$\frac{m'}{dt} = \frac{m}{dt}\left(\frac{v_1 - u}{v_1}\right) = \frac{180(8\frac{1}{3})}{60(32.2)}\left(\frac{8.8}{28.8}\right) = 2.37 \text{ slugs/sec}$$

$$F = \frac{m'}{dt}\Delta v = 0.237(6.74) = \underline{\underline{1.62 \text{ lb}}} \text{ opposite in direction to } \Delta v$$

## V. STRENGTH OF MATERIALS AND MACHINE DESIGN

Strength of materials is the determination of stresses in different members of a structure or machine; whereas machine design is the proportioning of that member so the stress does not exceed certain allowable values. In both cases simplifying assumptions are made which result in easily used formulas. In design, a factor of safety is involved and may be combined, with results of past experience, in the form of empirical formulas.

Handbook formulas are dangerous because it is difficult to tell how they have been derived, and there is often no reason or consistency in the units used. Care must be taken to check the answer to see if it is reasonable.

Two basic types of stresses are considered — shear and normal. Shear represents the type of failure where one plane slips parallel to an adjacent plane. Normal stress is tension or compression, representing the resistance to being pulled apart or pushed together.

The method of superposition is used extensively whereby the total stress at a point is determined by adding together the stresses at that point caused by each force or loading condition considered separately. Deflections may be determined in a similar manner.

Most of the formulas are restricted in their use because of simplifying assumptions made in their derivations.

The equation $S = F/A$, used for normal shear stresses, is true only when the force acts through the centroid of the area.

The formulas $S = Mc/I$, $\Delta = PL^3/c\,EI$, $S_s = VQ/Ib$, give only true stress when loading is below the proportional limit of the material, through the centroid of the cross section, and in the plane of a principal axis.

The torsion formula $S_s = Tc/J$, applies only to circular cross sections.

None of the above equations applies at a support or at a concentrated load. They do not take into account stress concentrations, temperature effects, or residual stresses.

In many practical situations, an exact stress analysis may be impossible, and the answer obtained will depend upon the assumptions made by the designer. No solution is complete without a statement of the assumptions involved, and perhaps some justification for these assumptions.

1. Stress in Beam

Two men are carrying a 1-1/8 inch square steel reinforcing bar 36 feet in length. The bar weighs 4.31 lb/ft.

a) If they pick it up at the ends, what will be the maximum stress in the bar?

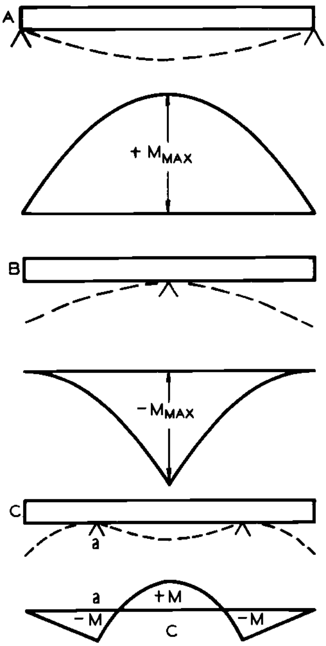
b) If it is assumed each man takes hold of the bar at the same distance from the end, between what points may this be done in order that the bending stress in the bar cannot exceed 20,000 psi?

Theory: Bending stresses produce both tension and compression in the member. When loaded as in sketch A, the bending is called positive and the upper fibers are in compression, while the lower fibers are in tension.

When loaded as in sketch B, the stresses are reversed, with tension on top of the beam. In either case the magnitude of the outer fiber stress is a maximum at the center of the beam where the bending moment is maximum.

When loaded as in sketch C, it is apparent that neither the positive moment nor the negative moment will be as high as in the other two methods of loading.

Note that the units are always inches and pound-inches in these problems.



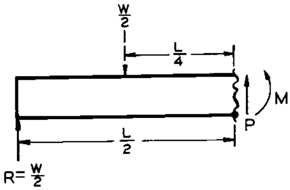
Solution: a)

$$S = \frac{Mc}{I}$$

I = moment of inertia of the cross section about its neutral axis.

$$I = \frac{bd^3}{12} = \frac{1\frac{1}{2}(1\frac{1}{8})^3}{12} = 0.1333 \text{ in.}^4$$

M = internal resisting moment at cross section. This is determined by visualizing the beam cut at the section to be considered and applying the laws of statics.




Note that at the cut section there must be a vertical force up (shear), as well as an upward moment M. These represent internal stresses which, after cutting, become external to the free body being considered.

$$\Sigma M_P = 0$$

$$\frac{W}{2}(\frac{L}{2}) - \frac{W}{2}(\frac{L}{4}) - M = 0$$

$$M = \frac{WL}{8} = \frac{[4.31(36) \text{ lb}][36(12) \text{ in.}]}{8} = 8380 \text{ lb-in.}$$

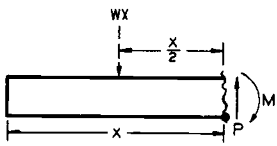
c = outer fiber distance from neutral axis

$$= \frac{d}{2} = \frac{1\frac{1}{8}}{2} = 0.563$$

$$\therefore \quad S = \frac{Mc}{I} = \frac{8380(0.563)}{0.1333} = \underline{\underline{35,400 \text{ psi}}},$$ provided

the yield point of the steel has not been exceeded.

b) Consider a section of the beam to the left of a a as a free body




$$\Sigma M_P = 0 = wx(\frac{x}{2}) - M = 0$$

$$M = wx(\frac{x}{2}) = \frac{wx^2}{2}$$

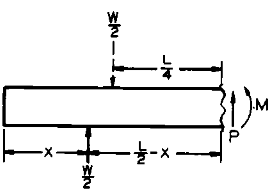
$$\therefore\ S = 20,000 = \frac{Mc}{I} = \frac{\frac{4.31}{12}(\frac{x^2}{2})(0.563)}{0.1333}$$

$$x^2 = 26,400$$

$$x = 162 \text{ in.} = \underline{13.5 \text{ ft}}$$

This is the largest x may become in order than S not exceed 20,000 psi at the supports.

Now consider one-half of the beam as a free body.




$$\Sigma M_P = 0 = \frac{W}{2}(\frac{L}{2} - x) - \frac{W}{2}(\frac{L}{4}) - M = 0$$

$$M = \frac{W}{2}(\frac{L}{2} - x - \frac{L}{4}) = \frac{W}{2}(\frac{L}{4} - x)$$

$$= \left[\frac{4.31(36)\text{ lb}}{2}\right]\left[\frac{36(12)\text{in.}}{4} - x\right]$$

$$= 77.5(108 - x)$$

$$S = 20,000 = \frac{Mc}{I} = \frac{77.5(108 - x)(0.563)}{0.1333}$$

$$x = 108 - 61 = 47 \text{ in.} = \underline{3.92 \text{ ft}}$$

This is the smallest x may become in order that S not exceed 20,000 psi at the center of the beam.

$\therefore$ x must be greater than 3.92 ft and less than 13.5 ft

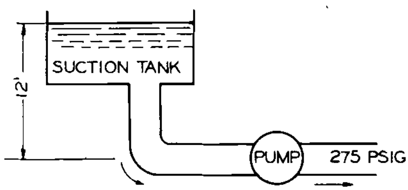
2. <u>Torsion, Power</u>

a) A two-stage centrifugal pump is to deliver 300 gpm of water against a pressure of 275 psig with a suction head of 12 feet. Neglecting velocity head and assuming an overall efficiency of 75%, what horsepower would be required?

b) With a permissible shearing stress of 10,000 psi for low carbon steel and a speed of 3600 rpm, what diameter solid shaft would be required, considering design load torsional stresses only?

c) If the maximum torsional stress during starting is three times that at design load, what size shaft would be required? (Reference: Marks, <u>Torsion</u>, p. 451.)

<u>Theory</u>: Neglecting velocity head (energy to increase kinetic energy of the water) and pipe friction, the only work done is against the pressure difference between suction and discharge. The low specified shearing stress has allowed for keyways.




<u>Solution</u>: a)

$$\text{Power} = F(v) = pA(\text{ft/sec}) = p\left[A(\text{ft})/\text{sec}\right] = pQ$$

where Q is the volume rate of flow in cubic feet per second.

$$Q = 300 \text{ gpm} = \frac{300}{7.481(60)} = 0.668 \text{ cu ft/sec}$$

$$p = \Delta p = 275 - 12\left(\frac{62.4}{144}\right) = 270 \text{ psi} = 38,900 \text{ lb/sq ft}$$

$$hp = \frac{pQ}{550(0.75)} = \frac{38,900(0.668)}{550(0.75)} = \underline{\underline{63 \text{ hp}}}$$

Note the magnitude of the error caused by neglecting velocity head as specified in the problem

$$p' = \frac{d}{dt}(\text{kinetic energy}) = \frac{d}{dt}(\frac{1}{2} Mv^2) = Mav = \frac{W}{g}(\frac{v^2}{t})$$

Assuming a 1-1/2 inch diameter pipe with 2.03 square inch area

$$v = \frac{0.668 \text{ cu ft/sec}}{2.03/144 \text{ sq ft}} = 47.3 \text{ ft/sec}$$

$$\therefore p' = \frac{W}{gt}(v^2) = \frac{300(8.33)}{32.2(60)}(47.3)^2 = 289 \text{ ft-lb/sec}$$

$$hp' = \frac{289}{550} = \underline{0.525 \text{ hp}}$$

The error is less than 1%.

b) Assuming all the 63 horsepower is transmitted through the shaft, find the torque.

$$hp = \frac{T\theta}{550} = \frac{\frac{T}{12}(2\pi \frac{N}{60})}{550} = \frac{TN}{63,000} \quad \begin{matrix}(\text{T is in lb-in.})\\(\text{N is in rpm})\end{matrix}$$

$$T = \frac{63,000 \text{ hp}}{N} = \frac{63,000(63)}{3600} = 1100 \text{ lb-in.}$$

$$S = \frac{Tc}{J} = \frac{Tr}{\frac{1}{2}\pi(r)^4}$$

$$r^3 = \frac{T}{\frac{\pi}{2}(S)} = \frac{1100}{\frac{\pi}{2}(10,000)} = 0.070$$

$$r = 0.412 \text{ or } d = 0.824$$

A <u>7/8-inch diameter</u> shaft would be needed.

c)

$$S = \frac{Tc}{J} = \frac{T}{\pi(r)^3}$$

If the maximum torque during starting is three times design torque, then $r^3$ or $d^3$ must be three times as large in order for the stress to remain the same.

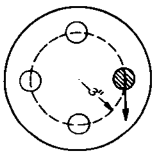
$$d^3 = 3(0.824)^3 = 1.683, \ d = 1.19 \text{ in.}$$

A 1-3/16 inch diameter shaft would do.

3. Shaft Coupling, Key

Two shafts, 2 inches in diameter, are connected by a keyed, rigid-flange coupling. The 2 halves of the couplings are bolted together with 4 bolts equally spaced in reamed holes on a 6-inch bolted circle. Bolts, keys, and shaft all are made of SAE 1035 steel. If the keys are 3 inches long, determine size of keys required to transmit 75% as much torque as the shaft can transmit. Also determine required size of bolts to transmit the total capacity of the shaft.

Theory: In bolted couplings it is assumed the shear stress is equal across the cross section of each bolt. If some bolts are farther from the center than others, then their stress is greater in direct proportion to the radial distances. In this problem all bolts have the same stress. Also, the torque transmitted by the key is calculated by assuming the shear stress is uniform at a cross section tangent to the surface of the shaft.



Solution: It must be understood that the problem refers to allowable stresses and not loads to cause failure. Since shaft, key, and bolts are of the same material, strength is not a factor in this problem, and the allowable stress in shear will be taken simply as S.

For the shaft: $S = \frac{Tc}{J} = \frac{T}{\frac{1}{2}\pi(r)^3}$, or $T = \frac{1}{2}\pi\ (S)$

For the key: $S = \frac{F}{A} = \frac{T/r}{3x} = \frac{T}{3x} = \frac{0.75(\frac{1}{2}\pi S)}{3x}$

$$x = \frac{0.75\,\pi}{6} = \underline{\underline{0.393 \text{ inch}}}, \text{ required width of key}$$

(This is not a standard key. Actually, a 1/2-inch square key would be used and the length reduced to $\frac{0.393}{0.5}(3) = 2.36$ inches.)

For the bolts: 4 bolts each 3 in. from center

$$T = 4(r\,F) = 4(3\,SA) = 12\,S(\frac{\pi}{4}\,d^2)$$

$$T = \frac{1}{2}\pi(S) = 12\,S(\frac{\pi}{4}\,d^2)$$

$$d^2 = 1/6 = 0.167$$

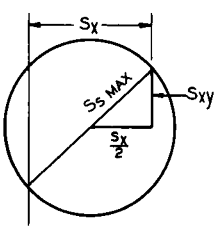
$$d = \underline{\underline{0.408 \text{ inch}}}, \text{ required diameter}$$

(Actually, standard 7/16-inch bolts would be used.)

4. Combined Bending and Torsion in Hollow Shaft

A hollow shaft is 2.5 inches OD and 2.0 inches ID. This shaft has a speed of 300 rpm and is subjected to a maximum torsional moment of 500 ft-lb and a maximum bending moment of 1000 ft-lb. Loads are steady. Shaft material has an elastic limit in tension of 33,000 psi. Is the shaft strong enough to meet usual design standards?

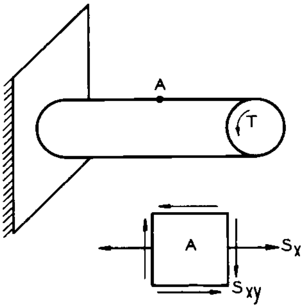
Theory: This is a combined stress problem where the shaft is subjected both to normal stresses due to bending and to shear stresses due to torsion. Since failure will be in shear, the maximum shear stress resulting from the combined bending and torsion must be determined. Also, since the shaft is rotating, some concern must be given to fatigue in bending.



Note the danger of using a handbook formula such as

$$d_o = \sqrt[3]{5.1\sqrt{M^2 + T^2}/S_s(1 - n^4)}$$

There is some uncertainty about the factor $(1 - n^4)$. Is it in the numerator or denominator? It is better to work from first principles and check the work against such a formula.




Looking down on point A, the stress condition on a small element would appear as in the sketch below the shaft. Using Mohr's circle it is apparent that

$$S_{s\,max} = \sqrt{\left(\frac{S_x}{2}\right)^2 + (S_{xy})^2}$$

$$= \sqrt{\left(\frac{1}{2}\frac{Mc}{I}\right)^2 + \left(\frac{Tc}{J}\right)^2}$$

$$= \sqrt{\left[\frac{32\ Md_o}{2\ \pi(d_o^4 - d_i^4)}\right]^2 + \left[\frac{16\ Td_o}{\pi(d_o^4 - d_i^4)}\right]^2}$$

$$= \frac{16\ d_o}{\pi(d_o^4 - d_i^4)}\sqrt{M^2 + T^2}$$

The ASME code for design of shafting recommends the formula be modified to take care of fatigue and shock by the factors $K_m$ and $K_t$.

$$S_{s\,max} = \frac{5.1\ d_o}{(d_o^4 - d_i^4)}\sqrt{(K_m M)^2 + (K_t T)^2}$$

For steady loads gradually applied, use

$$K_m = 1.5 \text{ and } K_t = 1.0 \text{ (See Marks or Spotts)}$$

$$M = 1000 \text{ lb-ft} = 12,000 \text{ lb-in. moment}$$

$$T = 500 \text{ lb-ft} = 6000 \text{ lb-in. torque}$$

$$\therefore S_{s\,max} = \frac{5.1(2.5)}{(\overline{2.5}^4 - \overline{2.0}^4)} \sqrt{\left[1.5(12,000)\right]^2 + (6000)^2}$$

$$S_{s\,max} = \underline{10,520 \text{ psi}}$$

The elastic limit in shear = 1/2 EL in tension.

$$\therefore \text{EL shear} = \frac{33,000}{2} = 16,500 \text{ psi}$$

The factor of safety is only 16,500/10,520 = 1.56, and should be from 2 to 3.

ASME code recommends $S_{s\,max}$ not exceed 8000 psi.

5. Stack with Wind Loading

A self-supporting stack made of 5/16-inch thick steel plate is 8 feet in mean diameter and 80 feet high. The steel plate weighs 0.283 pounds per cubic inch. If the stack is subjected to a wind pressure of 26 pounds per square foot of projected area, what is the maximum stress in the steel at base of stack?

Theory: This is a case of axial stress combined with bending. It is treated by the method of superposition. The stress at any one point equals the sum of the stresses caused by each type of loading considered separately.

Solution: Weight of the steel is uniformly distributed and the resultant total load acts through the centroid of the cross section. Because of this, the stress caused by the weight is equal at all points at the base.

$$S_1 = \frac{P}{A} = \frac{\rho V}{A} = \frac{\rho HA}{A} = \rho H$$

(Note that the units are taken in inches.)

$$S_1 = 0.283(80)(12) = \underline{272 \text{ psi compression}}$$

The resultant force of the wind acts halfway up the stack and causes bending stresses which are greatest at the base.

$$S_2 = \frac{Mc}{I} = \frac{(PA\frac{H}{2})(\frac{D}{2})}{\pi r_{avg}^3 t}$$

The approximate formula for I may be used here. The exact formula would be

$$I = \frac{1}{4}\pi(R_o^4 - R_i^4)$$

$$S_2 = \frac{\left[26(8)(80)\text{ lb } \frac{80(12)}{2}\text{ in.}\right]\left[\frac{8(12)}{2}\text{ in.}\right]}{\pi\left[4(12)\right]^3 \frac{5}{16}} = \underline{3540 \text{ psi}}$$

Maximum stress will be where the compression due to weight adds to the compression due to bending.

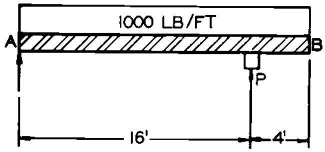
$$S = S_1 + S_2 = 272 + 3540$$

$$S = \underline{\underline{3810 \text{ psi compression}}}$$

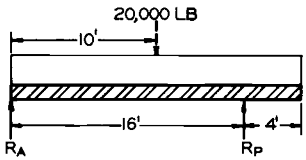
6. Beam Design — Deflection Given

One end of a 12-inch, 31.8-pound standard steel I-beam 20 feet long is supported on a wall. It is to be supported 4 feet from the other end at the middle of a transverse beam 10 feet long, simply supported at its ends. The under surface of the 20-foot beam and the top surface of the supporting beam are initially in the same horizontal plane.

It is desired that when the longer beam is subjected to a uniform load of 1000 pounds per foot, the free end shall settle 0.20 inch (as nearly as possible, but not exceeding). Select the most economical wide-flange steel beam conforming with AISC standards that will safely accomplish this with the least reduction of head room.



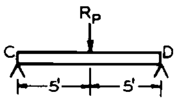

Solution: Consider the beam AB as a free body.




$$\Sigma M_A = 0 = 10(20,000) - 16\, R_p$$

$$R_p = 12,500 \text{ lb}$$

$R_p$ represents the force with which beam CD supports beam AB.




Now consider the beam CD. The same force $R_p$ is acting at the center of the span.

$$y = \frac{PL^3}{48\, EI} \quad \text{or } I = \frac{PL^3}{48\, Ey}$$

$$I = \frac{12,500(120)^3}{48\left[30(10^6)\right]0.20} = 75 \text{ in.}^4 \quad \text{required moment of inertia}$$

From the AISC manual an 8 inch by 6-1/4 inch, 24-pound WF section has $I = 82.5 \text{ in.}^4$. Although this beam will be satisfactory for the given deflection, it must be checked for strength and buckling.

$$S = \frac{Mc}{I} = \frac{\left[\frac{12,500}{2}\left(\frac{120}{2}\right)\right]\frac{7.93}{2}}{82.5} = \underline{18,000 \text{ psi}}$$

This value is below the 20,000 psi stress allowed by AISC. Check for compression in the flanges.

$$\frac{\ell d}{bt} \text{ must not exceed } 600$$

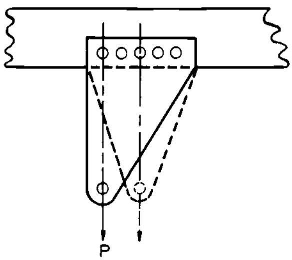
Effective length is 5 ft or 60 in.

$$\frac{\ell d}{bt} = \frac{60(7.93)}{6.5(0.398)} = \underline{184} \qquad \text{OK}$$

∴ 8 by 6-1/4 inch, 24-pound WF section would be chosen.

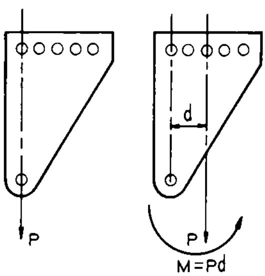
7. Eccentrically Loaded Rivets

The plate shown is to be bolted to a machine frame by 5, 1/2-inch diameter bolts equally spaced, with a pitch of 1-1/2 inches. (Consider area of each equivalent to 0.2 square inch.) What load P may be exerted without exceeding the allowed shear stress of 8000 psi? What would be the safe load if construction was that shown by the dotted lines?



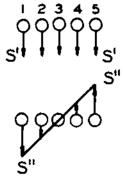

Theory: All problems of eccentric loading are treated in the same manner. The existing force system is replaced by an equivalent force system which can be more easily handled.

The force is first moved parallel to itself, through the centroid of the area which is being stressed—in this case, the 5 bolts. The stress caused by the centroidal force and by the moment then can be determined for any bolt and combined by superposition.




Equivalent Force Systems

Solution: a)




S' = stress caused by centroidal force

$$S' = P/A = \frac{P}{5(0.2)} = P \text{ (down on all bolts)}$$

S'' = stress caused by external moment, Pd = 3P

Note that outer bolts have highest stress, and the stress in any bolt is proportional to its distance from the centroid. (This is true assuming bolts fit tightly into the holes and a slight rotation of the clamp would cause each bolt to be strained in proportion to its distance from the center of rotation.)

$$F_2 = \frac{1}{2}F_1;\ F_3 = 0;\ F_4 = \frac{1}{2}F_1;\ F_5 = F_1$$

$$M = \Sigma M_c = \Sigma F_c \ell_c$$

$$M = F_1(3) + F_2(1\tfrac{1}{2}) + F_3(0) + F_4(1\tfrac{1}{2}) + F_5(3)$$

$$= F_1(3) + (\tfrac{1}{2}F_1)(1\tfrac{1}{2}) + (\tfrac{1}{2}F_1)(1\tfrac{1}{2}) + F_1(3)$$

$$M = 7\tfrac{1}{2}F_1$$

$$\therefore\ 7\tfrac{1}{2}F_1 = 3P$$

$$F_1 = 0.4P$$

$$S'' = \frac{0.4P}{A} = \frac{0.4P}{0.2} = 2P$$

For bolt 5, S'' is in opposite direction to S', but for bolt 1, S'' and S' are in the same direction and are additive.

$$\therefore S_1 = S'' + S' = P + 2P = 3P$$

if $S = 8000$ psi

P = <u>2667 lb maximum load</u>

b) The load here acts through the centroid of the stressed area and is equally distributed, therefore

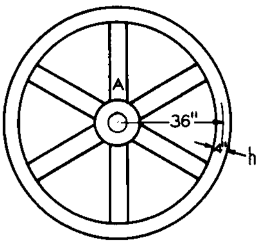
$$S = \frac{P}{A} = \frac{P}{5(0.2)} = P$$

$$\therefore \quad P = \underline{\underline{8000 \text{ lb allowable load}}}$$

8. Stresses in Flywheel

Find the direct tensile stress in the arms and "hoop," and bending stresses in the rim of a cast-iron flywheel which has 6 arms and rotates at 300 rpm. Radius r = 36 inches to center of rim; width of rim b = 12 inches; depth of rim h = 4 inches; and cross-sectional area of arms $A_1$ = 10 in.$^2$. Cast iron weighs 0.256 lb/in.$^3$.

Theory: When thickness of rim is small compared to the radius of the flywheel, stresses may be determined using the approximate equations for "hoop" stress and bending.




These equations are given in Timoshenko, Strength of Materials, Vol. 2 (or may be taken from Spotts). Simplified formulas are often used as given in Marks, and are shown for comparison at the end of this problem.

Solution: Formulas for a 6-spoke flywheel (from Timoshenko):

$$H = \frac{2/3}{0.0203\left(\frac{r^2}{h^2}\right) + 0.957 + \frac{A}{A_1}}$$

$$\frac{r^2}{h^2} = \frac{36^2}{4^2} = 81$$

$$\frac{A}{A_1} = \frac{12(4)}{10} = 4.8$$

$$H = \underline{0.0902}$$

$$F_1 = \frac{w r^2 n^2}{35,200}(H), \text{ tensile force in each arm}$$

w = 0.256(12)(4) = 12.29 lb per inch of rim

r = 36-inch radius

n = 300 rpm

$$F_1 = 3670 \text{ lb force in arm}$$

$$F = F_1\left(\frac{1}{H} - 0.866\right) = 3670\left(\frac{1}{0.0902} - 0.866\right)$$

$$F = \underline{\underline{37,600 \text{ lb hoop force in rim}}}$$

$$M = 0.0889\, F_1 r$$

$$= 0.0889(3670)(36)$$

$$M = \underline{11,720 \text{ lb-in. bending moment in rim}}$$

Tensile stress in spoke is

$$S = \frac{F_1}{A_1} = \frac{3670}{10}$$

$$S = \underline{\underline{367 \text{ psi tensile stress in arms}}}$$

Hoop stress in rim is

$$S = \frac{F}{A} = \frac{37,600}{12(4)}$$

$$S = \underline{\underline{784 \text{ psi hoop stress}}}$$

Bending stresses in rim are

$$S = \pm \frac{6M}{bh^2}$$

$$= \pm \frac{6(11,720)}{12(4)^2}$$

$$S = \pm\ \underline{\underline{367 \text{ psi bending stress}}}$$

Maximum tensile stress in rim is

$$S_t = 784 + 367 = \underline{\underline{1151 \text{ psi max tension}}}$$

An alternate solution using simplified equations (from Marks) is

$$S_t = \frac{WR\, n^2}{5800\, NA_1} \quad \text{(tensile stress in arm)}$$

W = $2\pi r(0.256)(12)(4)$ = 2780 lb, weight of rim
n = 300 rpm
N = 6, number of arms
A = 10 sq in., area of one arm
R = 3 ft radius

$$S_t = \frac{2780(3)(300)^2}{5800(6)(10)} = \underline{\underline{2160 \text{ psi arm tension}}}$$

$$S_h = \frac{V^2}{10}$$

$$= \frac{\left[2\pi(\frac{36}{12})(\frac{300}{60})\right]^2}{10} = \underline{\underline{888 \text{ psi hoop tension}}}$$

$$S_b = \frac{WR^2 n^2}{450\, N^2 Z}$$

where $Z = \frac{I}{C}$ of rim section $= \frac{bd^2}{6} = \frac{12(4)^2}{6} = 32 \text{ in.}^3$

$$S_b = \frac{2780(3)^2(300)^2}{450(6)^2(32)} = \underline{\underline{4350 \text{ psi bending stress}}}$$

Maximum rim tension is

$$S_h + S_b = 888 + 4350 = \underline{\underline{5240 \text{ psi tension in rim}}}$$

Note the large difference between the two sets of answers. Formulas from Spotts (Timoshenko) show more nearly what the stresses would be under ideal conditions with all arms loaded uniformly. Because of uncertainties of symmetry of loading and residual stresses inherent in the cast structure, very low allowable stresses are generally used.

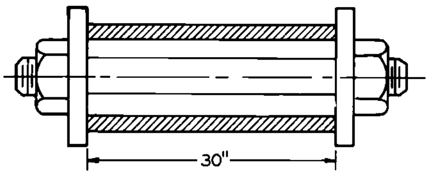
## 9. Two Materials with Thermal Stress

A common method of providing rigidity to a frame while still keeping the weight low, is to use a rod inside a tube, as shown in the sketch. The tube is of brass and the rod of steel.

a) As the nuts are tightened, what will be the ratio of the stress in the brass to the stress in the steel?

b) If the thread pitch on the rod is 1/16 inch, what stresses will result from 1/2 turn of one of the nuts?

c) If the materials are initially under stress from having been tightened, determine what change in stress would occur in each material if the temperature was to increase 30°F.




Brass tube:

- A = 0.50 sq in.
- E = 12,000,000 psi
- $\alpha = 10.4(10^{-6})$ in./in./°F
- (Thermal coefficient of expansion)

Steel rod:

- A = 0.10 sq in.
- E = 30,000,000 psi
- $\alpha = 6.5(10^{-6})$ in./in./°F

Theory: An increase in temperature will cause metal to expand, giving an apparent strain. However, there is no increase in stress accompanying this increase in length. On the other hand, if the metal is prevented from elongating, then a compressive stress will accompany the increase in temperature.

This compressive stress will equal that caused by first letting the metal expand due to the temperature change and then, while the temperature remains constant, pushing back the metal to its original length.

Solution: As the nuts are tightened, two things happen: (1) the steel increases in length due to tension, and (2) the brass shortens due to compression. The length of the steel is the same as the brass, both before and after loading. The tensile force in the steel also equals the compressive force in the brass.

a)

$$\frac{S_b}{S_s} = \frac{F/0.5}{F/0.1} = \frac{1}{5}$$

b)

$$\Delta_b + \Delta_s = \frac{1}{2}(0.0625) = 0.03125 \text{ in.}$$

$$\frac{P_b L_b}{A_b E_b} + \frac{P_s L_s}{A_s E_s} = 0.03125$$

$$P_b = P_s = P$$

$$P\left[\frac{30}{0.5(12)(10)^6} + \frac{30}{0.1(30)(10)^6}\right] = 0.03125$$

$$P = \frac{0.03125}{15(10)^{-6}} = 2080 \text{ lb force}$$

$$S_b = \frac{P}{A_b} = \frac{2080}{0.5} = \underline{\underline{4160 \text{ psi compression in brass}}}$$

$$S_s = \frac{P}{A_s} = \frac{2080}{0.1} = \underline{\underline{20,800 \text{ psi tension in steel}}}$$

c) Change in stress due to temperature will be independent of initial stress provided the unit remains rigid and the proportional limits of the material are not exceeded.

Find change in length due to temperature rise.

$$\Delta_{bt} = 30(10.4)(10)^{-6} = 312(10)^{-6} \text{ in./in.}$$

$$\Delta_{st} = 30(6.5)(10)^{-6} = \underline{195(10)^{-6}} \text{ in./in.}$$

$$\text{Difference} = 117(10)^{-6} \text{ in./in. interference}$$

This means the brass increases in compression and the steel increases in tension.

Again,

$$\Delta_{bt} + \Delta_{st} = 117(10)^{-6}$$

$$\frac{P_t}{A_b E_b} = \frac{P_t}{A_s E_s} = 117(10)^{-6}$$

$$P\left[\frac{1}{6(10)^6} + \frac{1}{3(10)^6}\right] = 117(10)^{-6}$$

$$P = \frac{117(10)^{-6}}{0.5(10)^{-6}} = 234 \text{ lb added to each}$$

$$\Delta S_b = \frac{234}{0.5} = \underline{\underline{468 \text{ psi higher compression}}}$$

$$\Delta S_s = \frac{234}{0.1} = \underline{\underline{2340 \text{ psi higher tension}}}$$

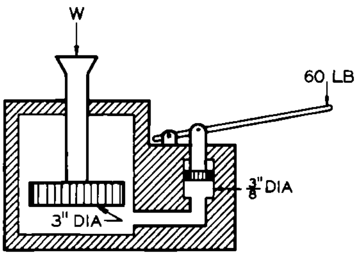
10. Hydraulic Jack

The ram of a hydraulic jack is 3.0 inches in diameter and the pump piston is 0.375 inch in diameter. Ratio of pump handle is 12 to 1, and overall efficiency is 80%.

a) With a pressure of 60 pounds on the handle, what would be the lifting capacity of the jack?

b) If the ram cylinder is made of cold-drawn steel tubing with an allowable tensile stress of 30,000 psi, what wall thickness should be used?

Theory: Hydrostatic pressure is the same on both pistons. Therefore, the force on each is proportional to the area or the square of the diameter of each piston.




Solution: a)

$$W = 0.80\left[60\left(\frac{12}{1}\right)\left(\frac{3}{3/8}\right)^2\right] = \underline{\underline{36,800 \text{ lb}}}$$

b) Using the thin-walled cylinder formula

$$S = \frac{pr}{t} \qquad p = \frac{F}{A} = \frac{60(12)}{\frac{\pi}{4}(3/8)^2} = 6520 \text{ psi}$$

$$t = \frac{pr}{S} = \frac{6520(3/2)}{30,000} = \underline{\underline{0.326 \text{ in., use tubing with 3/8-in. wall}}}$$

Note the ratio t/d = 0.326/3 = 0.109. For this ratio it is not necessary to use the thick-walled cylinder formula as the average value obtained by the approximate formula is about 5% too low. If the ratio was greater than 0.1, the Lamé formula should be used (see Marks, p. 423).

11. Column

Using the New York City Building Code formula

$$\frac{P}{A} = \frac{18,000}{1 + \frac{1}{18,000}\left(\frac{l}{r}\right)^2}$$

determine the safe axial loads on a 14 WF 78 section used as a column under the following conditions:

a) Hinged ends and a length of 30 feet.
b) Built-in ends and an unsupported length of 50 feet.
c) Built-in ends and a length of 50 feet braced at mid-point.

Theory: Column action is the tendency for a compression member to fail due to instability rather than strength. Long, slender columns always fail in this manner without regard for the strength of the material. Short compression members, on the other hand, fail by exceeding their elastic or ultimate strength. In between these two extremes, both factors contribute to failure. It is this intermediate column range for which most of the empirical formulas have been devised. The New York City Building Code limits the use of the above formula for main members to slenderness ratios ($l/r$) from 60 to 120.

This formula is the same as that used by the AISC for $l/r$ values between 120 and 200, except that they restrict its use to secondary members. When used for main members, AISC specifications multiply this formula by $(1.6 - \frac{l/r}{200})$.

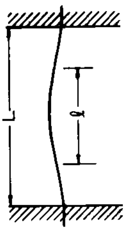
Solution: a) From AISC Manual:

$A = 22.94$ sq in.
$r = 3.00$ in., least radius of gyration
$l/r = 30(12)/3 = 120$

$$\frac{P}{A} = \frac{18,000}{1 + \frac{1}{18,000}(120)^2} = 10,000 \text{ psi}$$

P = 10,000(22.94) = 229,400 lb, safe load

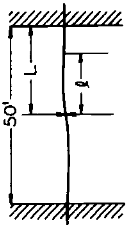
b) Considering the fixed ends to be perfectly rigid, equivalent length of column = 1/2 L.




$$\frac{\ell}{r} = \frac{\frac{1}{2}(50)(12)}{3} = 100$$

P = 22.94(11,570) = 265,000 lb, fixed ends

c) Top and bottom halves of column are symmetrical and equivalent to a column 25 feet long, fixed at one end, and restrained laterally at the other.




$$\frac{\ell}{r} = \frac{0.7(25)(12)}{3} = 70$$

P = 22.94(14,120) = 324,000 lb, braced

Note that since ends are never perfectly rigid, it would be better practice to assume equivalent lengths of about 0.75 L in part b, and 0.85 L in part c. This would somewhat reduce the allowable loads.

## 12. Eccentric Loading and Stress Concentration

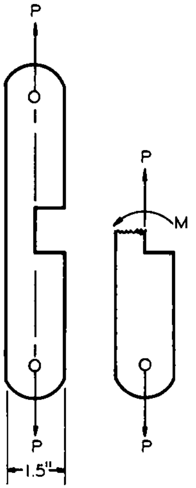
A piece of strap iron of 1-1/2 by 1/4-inch cross section is used in tension to support a load of 1500 pounds. It is later found necessary to cut away part of the piece, as shown.

a) What is the ratio of the maximum stresses for the two conditions if the load is the same?

b) Criticize manner in which change was made. How could the part be improved from a stress standpoint without changing its basic shape from that shown?

Solution: a) Before cutting, the stress was

$$S_1 = \frac{P}{A_1} = \frac{P}{bd} \qquad \begin{aligned} b &= 1/4 \text{ in.} \\ d &= 1\text{-}1/2 \text{ in.} \end{aligned}$$




After cutting, the force condition at midsection could be considered as shown (neglecting stress concentration)

$$M = \frac{1}{4} d\,(P)$$

$$S_2 = \frac{P}{A_2} + \frac{Mc}{I}$$

$$= \frac{P}{b(\frac{1}{2}d)} + \frac{\frac{1}{4} Pd\,(\frac{1}{4}d)}{\frac{b(d/2)^3}{12}}$$

$$= \frac{2P}{bd} + \frac{6P}{bd} = 8\,\frac{P}{bd}$$

$$\frac{S_2}{S_1} = \frac{8}{1}$$

b) The sharp corners will cause stress concentrations. This will be serious under conditions of impact or repeated loadings. There should be fillets at each of the sharp inside corners.

## 13. Gear Drive, Design

A 5000-pound load is being raised at the rate of 450 fpm by a drum hoist having a drum diameter of 12 inches. Between the drum and hoist engine is a speed reducing unit which consists of the following gear train: On the drum shaft is a 52-tooth gear which meshes with a 24-tooth pinion on an idler shaft; compounded with this pinion is a 60-tooth gear meshing with a 15-tooth pinion on a second idler shaft; compounded with this pinion is a 42-tooth gear which meshes with a 16-tooth pinion of the engine shaft. All gears are of 3 pitch.

a) Sketch the rig from drum to engine shaft.

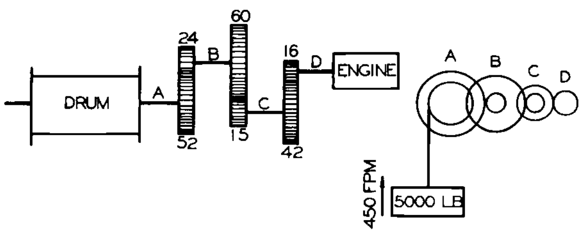
b) Assuming 88% efficiency in the rig, compute the horsepower delivered by the engine and its rpm.

c) Using 7500 psi allowable stress in shear, compute the required standard shaft size for drum and engine shaft, neglecting bending stresses.

d) Using reasonable allowable tooth stresses, determine the face widths of the cast steel spur gears.

Theory: The Lewis formula for determining width of teeth considers a gear tooth as a cantilever beam loaded at the tip of the tooth. Modifications have been proposed by the American Gear Manufacturers Association to take into account stress concentration and more nearly actual loading conditions. The Lewis formula, however, is satisfactory for most applications.

Solution: a)




b)

$$\text{hp at drum} = \frac{FV}{550} = \frac{5000(450/60)}{550} = 68.2\text{ hp}$$

$$\text{hp at engine} = \frac{68.2}{0.88} = \underline{\underline{77.5\text{ hp delivered by engine}}}$$

$$\text{Engine speed} = \frac{450}{1.0(\pi)}\left(\frac{52}{24}\right)\left(\frac{60}{15}\right)\left(\frac{42}{16}\right) = \underline{\underline{3260 \text{ rpm at engine}}}$$

c) At the drum: To take into account the keyways in the shafting, allowable stress should be reduced 25%.

$$S = \frac{Tc}{J} = \frac{Tr}{\frac{1}{2}\pi(r)^4} = \frac{2T}{\pi r^3}$$

$$r^3 = \frac{2T}{\pi S} = \frac{2(5000)(12/2)}{\pi\left[0.75(7500)\right]} = 3.40 \text{ in.}^3$$

$$r = 1.50 \text{ in.}, \quad d = 3.0 \text{ in.}$$

Use 3-inch diameter shafting for drum.

At the engine:

$$T = \frac{1}{0.88}\left[30,000\left(\frac{24}{52}\right)\left(\frac{15}{60}\right)\left(\frac{16}{42}\right)\right] = 1500 \text{ lb-in.}$$

$$r^3 = \frac{T}{\frac{\pi}{2}(S)} = \frac{1500}{\frac{\pi}{2}\left[0.75(7500)\right]} = 0.169$$

$$r = 0.552 \text{ in.}, \quad d = 1.104 \text{ in.}$$

Use 1-1/8 inch shaft at engine.

d) Using the Lewis formula (see Marks) for the drum gear pinion teeth

$$W = \frac{FS\,yc}{P_d}$$

or

$$F = \frac{WP_d}{S\,yc}$$

W = working load = hp(33,000)/V

$$= \frac{68.2(33,000)}{\pi\left[\frac{52}{3(12)}\right]\left[\frac{450}{\pi(1)}\right]} = 3470\text{-lb load}$$

$P_d$ = 3-in. diameter pitch

S = 18,000 psi working stress for cast steel

y = 0.30 form factor, 24-tooth, 14-1/2°

$$c = \frac{600}{600 + V} = \frac{600}{600 + 648} = 0.482 \text{ velocity factor}$$

$$F = \frac{3470(3)}{18,000(0.30)(0.482)} = \underline{\underline{4.0 \text{ inches for drum gear}}}$$

For the engine drive gear:

$$W = \frac{77.5(33,000)}{\pi \frac{16}{3(12)}(3260)} = 562\text{-lb load}$$

$$y = 0.242 \text{ for 16-tooth gear}$$

$$c = \frac{600}{600 + 4550} = 0.1164$$

$$F = \frac{562(3)}{18,000(0.242)(0.1164)} = \underline{\underline{3.32 \text{ inches for engine wear}}}$$

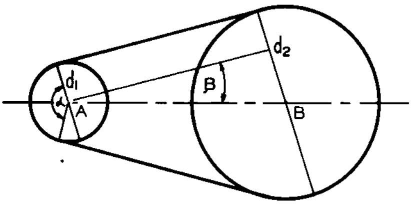
The other pair of gears may be found in a similar manner.

## 14. V-Belt Drive for Air Compressor

Design a V-belt drive for a radial 6-cylinder air compressor which has a load demand of 40 horsepower at operating speed of 300 rpm. Motor is 50 hp, 1800 rpm, synchronous motor. Compressor is operated intermittently by automatic controls and starting occurs with compressor unloading valves open.

Theory: V-belts are made in 5 sizes, and the horsepower ratings are given in tables (see Spotts). Sheaves should be as large as conditions permit, but belt speeds greater than 5000 fpm should not be used. Net horsepower ratings are found by applying two correction factors—$K_1$ for arc of contact less than 180°, and $K_2$ service factor, depending on type of load, power unit, and starting method. The distance between shafts is generally greater than the diameter of the large pulley, but less than the sum of both diameters.

Solution: Assume a belt speed of 4000 fpm. Find motor sheave diameter, $d_1$




$$d_1 = \frac{V}{\pi N} = \frac{4000}{\pi(1800)} = 7.08$$

Take $d_1 = 7$ in.

then $$d_2 = 7\left(\frac{1800}{300}\right) = 42 \text{ in.}$$

Let center-to-center distance be 48 in., then

$$\sin \beta = \frac{21 - 3.5}{48} = 0.365$$

$$\beta = 21.4°$$

Therefore,

$$\alpha = 180 - 2\beta = 137.2°$$

From Spotts:

$$K_1 = 0.82 \text{ for } 137°$$

$$K_2 = 0.67 \text{ synchronous motor normal torque}$$

Use a B-belt section (7-inch sheave too small for C size).

Hp from table = 5.3 per belt at 4000 fpm

Net hp per belt = $5.3\, K_1 K_2$

$= 5.3(0.82)(0.67) = 2.91$ hp/belt

No. of belts = 40/2.91 = 13.7 belts (use 14)

Note that 40 hp is used rather than 50 hp because the excess power used in starting is accounted for in $K_2$.

Length of belts, L:

$$L = 2(48 \cos 21.4) + \pi 42(\frac{223}{360}) + \pi 7(\frac{137}{360})$$

$$= 89.4 + 81.7 + 8.4$$

Length = 179.5 inches (use 15-ft belts)

To summarize: Use 14 size B belts 15 feet in length each. Sheaves are 7 and 42 inches in diameter on shafts 48 inches apart.

## 15. Car Puller, Design

A car puller with a drum 12 inches in diameter is driven by a 7-1/2 hp, 1760 rpm electric motor through a 30 to 1 reduction gear. Find the following:

a) Speed of cable in feet per minute.
b) Tension in cable in pounds.
c) Size of plow steel cable recommended.
d) Size of mild steel drum shaft, considering an allowable torsional shear of 12,000 psi.

Solution: a)

$$\text{Cable speed} = \pi D(\text{speed of drum})$$

$$= \pi(\frac{12}{12})(\frac{1760}{30}) = \underline{\underline{184 \text{ fpm cable speed}}}$$

b) Cable tension, assuming 7-1/2 hp is being used.

$$\text{Power} = FV$$

$$hp = \frac{FV}{33,000}$$

$$F = \frac{33,000 \text{ hp}}{V} = \frac{33,000(7\text{-}1/2)}{184} = \underline{\underline{1345 \text{ lb cable tension}}}$$

If an efficiency of 75% is assumed for the motor and reduction gear combined

$$F = 0.75(1345) = \underline{\underline{1010 \text{ lb cable tension}}} \text{ (assuming 75\% eff)}$$

c) A factor of safety of 5 or more should be used for wire rope. Therefore, breaking strength should be approximately 6700 pounds, or 3.35 tons.

From tables of Standard Hoisting Rope 6 by 19 (Marks):

1/4-inch diameter, breaking strength = 2.39 tons
5/16-inch diameter, breaking strength = 3.71 tons

5/16-inch , 6 by 19 plow steel cable should be satisfactory.

Check this size cable for the 12-inch diameter drum.

Minimum diameter = 30 D = 30(5/16) = 9.36 in.
Recommended diameter = 45 D = 45(5/16) = 14 in.

Wire rope of 5/16-inch diameter would be satisfactory, whereas a larger rope (3/8 inch) would require a larger drum.

d) Determine size of shaft. Because of the probability of suddenly applied loads, it would be well to double the torque value calculated. Since the drum may be keyed to the shaft, it is customary to reduce the allowable stress by 25% to allow for the reduced area and stress concentration.

$$S_s = \frac{Tc}{J} = \frac{T}{\frac{1}{2}\pi(r)^3}$$

$$r^3 = \frac{T}{\frac{1}{2}\pi(S)} = \frac{2\,[1345(6)]}{\frac{1}{2}\pi\,[0.75(12,000)]} = 1.14$$

$$r = 1.043 \qquad d = 2.086$$

Because of the allowance made for impact and keyway, a 2-inch diameter shaft would be satisfactory.

## 16. Flywheel Design

Determine rim thickness for a cast iron, 10-inch width flywheel of a punch press designed to use a maximum input of 3000 ft-lb of energy per punching operation. Computed energy supplied by the belt drive during the punching operation is 200 ft-lb. Other available data are:

> Maximum outside diameter of flywheel = 40 inches
> Operating speed = 175 rpm
> Permissible variation in speed during punching operation = 12.5%

Solution:

> Initial speed = 175 rpm
> Final speed = 0.875(175) = 153 rpm
> Energy loss of flywheel = 3000 - 200 = 2800 ft-lb

$$\frac{1}{2} I \omega_i^2 - \frac{1}{2} I \omega_f^2 = 2800$$

$$I = \frac{2(2800)}{\omega_i^2 - \omega_f^2} = \frac{5600}{\left(\frac{2\pi}{60}\right)^2 (175^2 - 153^2)} = 70.8 \text{ slug ft}^2$$

$$I_{mass} = \rho \frac{\frac{10}{12} \pi (d_o^4 - d_i^4)}{32} = \frac{450}{32.2} \left[\frac{2.62(3.33^4 - d_i^4)}{32}\right] = 70.8$$

$$d_i^4 = 3.33^4 - \frac{70.8(32)}{13.98(2.62)} = 123.2 - 61.8 = 61.4$$

$$d_i = 2.79 \text{ ft}$$

$$t = \frac{3.33 - 2.79}{2} = \underline{\underline{0.27 \text{ ft, or } 3.24 \text{ inches thick}}}$$

17. Gas Turbine Blade and Shaft

A gas turbine blade 6 inches long with a uniform section of 0.23 square inch, is mounted on a disk 18 inches in diameter. The blading material has a density of 0.278 pounds per cubic inch.

a) If the allowable centrifugal stress at the base of the blade is 15,000 psi, what is maximum rpm of the rotor?

b) If turbine rotor develops 5000 hp at 4800 rpm, what outside diameter hollow steel shaft would be required if inside diameter is 2/3 of outside diameter? Consider torsional forces only and allow a shearing stress of 8000 psi.

Theory: When rotating at constant velocity, the metal at any cross section is subjected to an inertia or centrifugal force due to the mass of metal outward from that section. Maximum stress will occur at base of blade.

Solution: a)

$$S = \frac{P}{A} = \frac{\frac{W}{g} r \omega^2}{A}$$

$$W = 0.278(0.23)(6) = 0.384 \text{ lb}$$

r = radial distance to center of mass (see note below)

$= 18/2 + 3 = 12$ in. $= 1$ ft (if g is in ft/sec$^2$)

$\omega = n(\frac{2\pi}{60})$ radians per second

$A = 0.23$ in.$^2$ (must be in inch units)

$$S = 15{,}000 = \frac{\frac{0.384}{32.2}(1)(0.105\, n)^2}{0.23}$$

$$n^2 = 26{,}300{,}000 \quad n = \underline{\underline{5130 \text{ rpm}}}$$

Note. This is neglecting stress concentrations at base of blade. Also, had the cross-sectional area not been constant, the force P would have had to be determined by integration instead of by using the center of mass.

b) Find torque in shaft.

$$hp = \frac{TN}{63{,}000}$$

$$T = \frac{63,000 \text{ hp}}{N}$$

$$= \frac{63,000(5000)}{4800} = 65,600 \text{ lb-in. torque}$$

$$S_s = \frac{Tc}{J} = \frac{Tr_o}{\frac{1}{2}\pi(r_o^4 - r_i^4)}$$

$$= \frac{T}{\frac{1}{2}\pi r_o^3 \left[1 - (2/3)^4\right]} = \frac{T}{1.26 r_o^3}$$

$$r_o^3 = \frac{T}{1.26 S} = \frac{65,600}{1.26(8000)} = 6.51$$

$$r_o = 1.865 \text{ in.}, \quad d = 3.73$$

Use 3-3/4 inch OD hollow shaft

Alternate solution using Marks:

Diameter solid shaft = $d_s = \sqrt[3]{5.1(T/S_s)} = 3.46$ in.

For $\frac{d_i}{d_o} = 0.667$

diameter hollow shaft = 1.08 (diameter solid shaft)

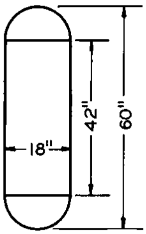
$$d = 3.46(1.08) = 3.74 \text{ inches}$$

## 18. Internal Pressure in Tank

A closed steel shell 18 inches in diameter and 60 inches in length is completely filled with water at 68°F and 30 psig. Temperature of surrounding air can get how high before there is serious danger of permanent deformation of shell? Shell and heads are made of 1/4-inch thick steel plate, SAE 1015. Heads are hemispherical.

Theory: As temperature of water and steel rises, both tend to expand due to thermal effects. Since the water expands more than the steel, internal pressure will build up. Assuming temperature does not affect strength of steel, and yield point is 35,000 psi, the steel will expand elastically (in addition to thermal expansion) by a definite amount before yielding. Pressure of the water necessary to strain the steel can be found, and then temperature of water at that pressure determined. This will be a trial and error solution or one of successive approximation.

Solution: Determine initial volume of tank.

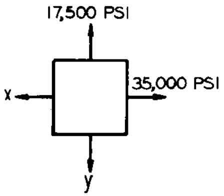



V = vol cylinder + vol sphere

$$= 0.785(18)^2(42) + 0.524(18)^3$$

$$= 10,700 + 3055 = 13,755 \text{ in.}^3$$

Find increase in diameter due to 35,000 psi hoop stress and 35,000/2 psi longitudinal stress.




The unit strain circumferentially is affected by both stresses.

$$\epsilon_x = \frac{S_x}{E} - \mu \frac{S_y}{E}$$

$\mu$ = Poisson's ratio = 0.28 for steel

$$\epsilon_x = \frac{35,000}{30(10)^6} - 0.28\left[\frac{17,500}{30(10)^6}\right] = 0.00117 - 0.000163$$

$$\epsilon_x = 0.00101 \text{ in./in. circumferentially}$$

Similarly,

$$\epsilon_y = \frac{S_y}{E} - \mu \frac{S_x}{E} = 0.000584 - 0.000283$$

$$\epsilon_y = 0.000301 \text{ in./in. longitudinally}$$

Since the diameter of a circle increases in proportion to the circumference

$$\epsilon_d = \epsilon_x = 0.00101 \text{ in./in.}$$

$\therefore$ Increase in diameter $= \epsilon_d(d) = 0.00101(18) = 0.01818$ in. and increase in length $= \epsilon_y(42) = 0.0003(42) = 0.0126$ in.

$$\Delta V_{cyl} = L\,\Delta A + A\,\Delta L = L(\pi d_{avg}\,\Delta r) + \frac{\pi}{4} d_2^2\,(\Delta L)$$

$$= \pi L\left[18 + \frac{0.0182}{2}\left(\frac{0.0182}{2}\right)\right] + 0.785(18.018)^2(0.0126)$$

$$= 3.1416(42)\left[18,009(0.00909)\right] + 3.21$$

$$\Delta V_{cyl} = 21.6 + 3.21 = 24.81 \text{ cu in.}$$

The two ends make up a sphere whose radius has increased by $\Delta r$.

Actually, there is a portion of the tank (where the cylinder attaches to the spherical ends) that conforms to none of these assumptions. The increase in radius of the cylinder is greater than the increase in radius of the hemisphere. This change from one to the other tends to cancel out, and the change in volume will be calculated as though they were a perfect cylinder and a perfect sphere.

$$\Delta r = r\,\epsilon_s$$

$$= r\left(\frac{S_s}{E} - \mu\frac{S_s}{E}\right)$$

$$= r\,\frac{S_s}{E}(1 - \mu)$$

$$= 9\left[\frac{17,500}{30(10)^6}(1 - 0.28)\right]$$

$$\Delta r = 0.00378$$

$$\Delta V_{heads} = 4\pi r^2 (\Delta r) = 4\pi (9)^2 (0.00378) = 3.85 \text{ cu in.}$$

Total increase in V = 24.81 + 3.85 = 28.66

Increase in volume of the tank, in addition to thermal expansion, is 28.7 cubic inches. Since thermal expansion is relatively small, $6.3(10)^{-6}$ in./in. °F, it will be neglected as a first assumption.

Determine internal pressure necessary to cause a hoop stress of 35,000 psi in the steel tank.

$$S = \frac{pD}{2t}$$

$$p = \frac{2tS}{D} = \frac{2(1/4)(35,000)}{18.25} = 960 \text{ psi}$$

$$p = 960 + 14.7 = 975 \text{ psia}$$

At 68°F and zero gage pressure, the specific volume of the water is $V_f = 0.01605$ cu ft/lb.

Since the final volume = 13,755 + 28.7 cu in., the final specific volume is

$$V_f = 0.01605 \left(\frac{13,755 + 28.7}{13,755}\right)$$

$$= 0.01605 \left(1 + \frac{28.7}{13,755}\right)$$

$$V_f = 0.01605 + 3.35(10)^{-5}$$

Net increase in specific volume must be $3.35(10)^{-5}$ cu ft/lb.

Since both temperature and pressure affect specific volume, refer to the compressed liquid table in the steam tables (Keenan and Keyes). The increased pressure of 975 psi causes a reduction in specific volume of $5.1(10)^{-5}$ at about 100°F. Therefore, the increase in temperature must cause a total increase of $3.35 + 5.1 = 8.45(10)^{-5}$ cu ft/lb.

Referring now to the temperature table:

| | |
|---|---|
| At 68° | $V_f = 0.01605$ |
| adding | $0.0000845$ |
| gives | $V_f = 0.0161345$ |

It is seen that this corresponds to a temperature of 100°F, or a temperature rise of 32°F.

The effect of this $\Delta T$ upon the expansion of the steel must be checked.

Coefficient of expansion of steel = $6.3(10)^{-6}$ in./in. °F

$$\Delta r_{cyl} = 9\left[6.3(10)^{-6}(32)\right] = 0.00182 \text{ in.}$$

Note that this change in radius of the cylinder is 182/910, or 20% of change due to stress and, therefore, too large to be ignored.

Assume a 35°F rise in temperature and determine the change in volume due to expansion of the steel.

$$\Delta V_{cyl} = L(\pi\, d\, \Delta r) + \frac{\pi}{4} d^2(\Delta L)$$

$$\Delta r = 9\left[6.3(10)^{-6}(35)\right] = 0.00199 \text{ in.}$$

$$\Delta L = 42\left[6.3(10)^{-6}(35)\right] = 0.0093 \text{ in.}$$

$$\Delta V_{cyl} = 42\,\pi(18)(0.00199) + \frac{\pi}{4}(18)^2(0.0093)$$

$$= 4.73 + 2.37 = 7.10 \text{ cu in.}$$

$$\Delta V_{heads} = 4\,\pi\, r^2(\Delta r) = 4\,\pi(9)^2(0.00199) = 2.03$$

$$\Delta V_t = 7.10 + 2.03 = 9.13 \text{ cu in.}$$

Therefore, the total increase in volume due to both stress and temperature is

$$\Delta V = 28.66 + 9.13 = 37.79 \text{ cu in.}$$

Again the increase in specific volume is

$$0.01605\left(\frac{37.79}{13,755}\right) = 4.41(10)^{-5}$$

Add to this the $5.1(10)^{-5}$ reduction due to pressure.

$$\Delta V_f = 4.41 + 5.1 = 9.51(10)^{-5} \text{ cu ft/lb}$$

$$V_f = 0.01605 + 0.000095 = 0.016145 \text{ cu ft/lb}$$

From the temperature tables this corresponds to a temperature of 102°F, or a temperature rise of 34°. The assumed $\Delta T$ of 35° was quite close. Therefore,

temperature should not be allowed to reach <u>102°F</u>

## VI. FLUID MECHANICS

Fluid mechanics deals with both incompressible and compressible fluids. The same general principles apply to both, and the basic equilibrium and dynamical equations have the same form for all fluids.

Many problems of steady flow are solved by the application of the general energy equation.

| | | | | |
|---|---|---|---|---|
| [Heat added to unit weight of flowing fluid between entrance and exit.] | + | [Total work transferred to (done upon) unit weight of flowing fluid between entrance and exit.] | = | [Total gain in energy of unit weight of flowing fluid between entrance and exit.] |

In symbols, this equation becomes

$$(q_{in} - q_{out}) + \frac{W_{in} - W_{out}}{J} = (u_2 - u_1) + \frac{p_2 v_2 - p_1 v_1}{J} + \frac{V_2^2 - V_1^2}{2g\,J} + \frac{Z_2 - Z_1}{J}$$

J = 778 ft-lb per Btu and is the mechanical equivalent of heat.

For many hydraulic problems the q and W terms drop out, and it is often convenient to express the total energy of the fluid as head, or feet of fluid flowing.

$$\frac{\text{Total energy}}{\text{Unit weight}} = E = \frac{p}{\omega} + \frac{V^2}{2g} + Z$$

The first term, $p/\omega$, is called pressure head, and pressure p should be in pounds per square foot, absolute. Specific weight in pounds per cubic foot is $\omega$. The second term, $V^2/2g$, is called velocity head, with velocity in feet per second. The third term, Z, is the potential head and is the elevation in feet above some datum plane.

Special care must be taken with units in these problem as reference tables from different sources may not be consist nt. As an example, consider the calculation of Reynolds number.

$$R_N = \frac{\rho V D}{\mu}$$

The term is dimensionless and if proper units are used they will cancel out. $\rho$ must be mass density, which in engineering units would be slugs per cubic foot. V is velocity in feet per second. D would be diameter in feet. Now viscosity, $\mu$, may be found in many systems of units. Handbooks give conversion factors from one system of units to another. The real confusion, however, lies in converting to the engineering system of units. Frequently no distinction is made between pounds force and pounds mass. $\mu$ may be given in units of lb-sec/ft$^2$, or lb/in-sec. This appears to be inconsistent because, in the first case, lb refers to pounds force, and in the second case to pounds mass.

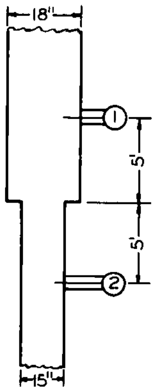
Since engineering units are generally used with a slug as the unit of mass, it is best to avoid those terms involving pounds mass. $\mu$ should have the units of slugs/ft-sec. Note that when slug is replaced with F/a = lb-sec$^2$/ft, the units for $\mu$ become

$$\frac{\frac{\text{lb}}{\text{ft}}\,\text{sec}^2}{\text{ft-sec}} \quad \text{or} \quad \frac{\text{lb-sec}}{\text{ft}^2} \quad \text{(where lb is a pound force)}$$

Once again, ρ must be in slugs per cubic foot when μ is <u>either</u> slugs/ft-sec or $lb_F$-sec/ft$^2$.

## 1. Pipeline, Pressure Drop

A vertical line of pipe includes an 18-inch pipe above a 15-inch pipe, connected by a reducer. A pressure gage in the 18-inch pipe, 5 feet above the center of the reducer, registers 7 psi. Another gage, set in the 15-inch pipe 5 feet below the center of the reducer, registers 10 psi. The flow (water) in the pipe is 5 cubic feet per second. In which direction is the flow, and what is the friction loss of head between the two gages?




Theory: This is a condition of steady flow, and the general energy equation would apply. The process may be considered adiabatic with no heat added to or lost from the fluid. No external or shaft work is done on the liquid, so the general energy equation reduces to

$$-\frac{W_{out}}{J} = \frac{p_2 v_2}{J} - \frac{p_1 v_1}{J} + \frac{V_2^2 - V_1^2}{2g\,J} + \frac{Z_2 - Z_1}{J}$$

Solution: Simplifying the equation

$$-W_f = v(p_2 - p_1) + \frac{1}{2g}(V_2^2 - V_1^2) + Z_2 - Z_1$$

Friction loss = (pressure head) + (velocity head) + (potential head)

$$V_1 = \frac{5}{A_1} = \frac{5}{\pi(9/12)^2} = 2.83 \text{ ft/sec}$$

$$V_2 = \frac{5}{\pi(15/24)^2} = 4.08 \text{ ft/sec}$$

In going from 1 to 2, the fluid increases its pressure head and velocity head (smaller area of pipe), but decreases its potential head. Since friction loss must take energy out of the fluid, the terms on the right side of the equality sign must total a negative quantity.

$$- W_f = \frac{1}{62.4}\left[144(10 - 7)\right] + \frac{1}{64.4}(4.08^2 - 2.83^2) - 10$$

$$= 6.91 + 0.134 - 10 = - 2.96$$

$$W_f = \underline{\underline{2.96 \text{ feet of head}}}, \text{ friction loss}$$

Since the change in energy terms, going from 1 to 2, is negative, our assumption that flow is down is correct.

## 2. Pump Performance at Different Speeds

A centrifugal pump having a 13.25-inch impeller has a capacity of 600 gpm when operating at a speed of 1750 rpm against a total dynamic head of 130 feet of water. Compute approximate capacity of pump when operated at a speed of 1160 rpm against a total head of 55 feet of water.

Theory: The operating conditions of a centrifugal pump at any speed can be approximated quite well from test data taken at a different speed. Mechanical similarity must exist, which means dynamic as well as geometric similarity.

Similarity relations are as follows (Marks, p. 1849):

$$\frac{Q_1}{Q_2} = \frac{N_1}{N_2}; \quad \frac{H_1}{H_2} = \frac{N_1^2}{N_2^2} = \frac{Q_1^2}{Q_2^2}; \quad \frac{P_1}{P_2} = \frac{N_1^3}{N_2^3} = \frac{Q_1^3}{Q_2^3}$$

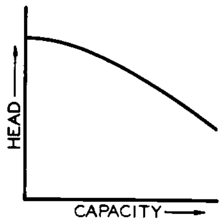
These equations in themselves tell nothing about performance of the pump. For instance, they definitely should not be used in an attempt to show how head H varies with flow Q at constant speed.

Solution: When the speed of the pump is decreased to 1160 rpm, similarity would imply that the head and flow should be reduced as follows:

$$H_2 = H_1 \left[\frac{N_2}{N_1}\right]^2 = 130\left[\frac{1160}{1750}\right]^2 = 57 \text{ ft of head}$$

$$Q_2 = Q_1 \left(\frac{N_2}{N_1}\right) = 600 \text{ gpm } \left(\frac{1160}{1750}\right) = 398 \text{ gpm}$$

Actual flow for a pump operated at a speed of 1160 rpm will vary with the head in a manner similar to the curve shown. It is apparent that if the pump is operated against a head of 55 feet of water instead of 57 feet, the capacity will be somewhat greater than 398 gpm.




The exact amount could be determined from performance curves of the pump, but would be in excess of 400 gpm.

3. Air Lift

An air lift is to be used for pumping water from a drilled well. Air from the pipe is discharged into the water at lower end of well (shown in sketch), causing water to be delivered at surface of well. If water flow is to be 200 gallons per minute:

a) What compressor capacity should be installed?
b) Compressor should have what size motor?

Theory: When air is mixed with water in the discharge pipe, the lighter mixture rises in the pipe. Initial air pressure must be greater than barometric pressure plus that due to the column of liquid shown as submergence.

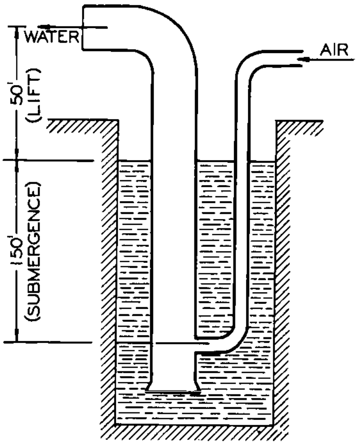
Solution: Find air pressure required. (Refer to Marks.)

$$P = B + \text{submergence}$$

$$= 14.7 + 150\left(\frac{62.4}{144}\right)$$

$P = 79.7$ psia absolute air pressure

$= 65$ psig




Determine theoretical air consumption.

$$v = \frac{l}{B \log_n(1 + s/B)}$$

$l$ = lift in feet = 50 feet

B = barometric pressure in feet of water

$= 14.7(\frac{144}{62.4}) = 34$ feet

s = submergence = 150 feet

$$v = \frac{50}{34 \log_n(5.41)} = \frac{50}{34(1.69)} = 0.87 \text{ cu ft free air/cu ft water}$$

Considering an efficiency of 50%, volume of air needed is

$$V = \frac{0.87}{0.50}(\frac{200}{7.5}) = 46.4 \text{ cu ft/min free air}$$

a) Install air compressor with twice the capacity calculated, or approximately 100 cfm with a discharge pressure in excess of 70 psig.

b) Using a single-stage air compressor with 85% efficiency and taking n = 1.3

$$hp = \frac{1}{0.85}\left[\frac{144(n)}{33,000(n-1)}\right] P_1V_1\left[\left(\frac{P_2}{P_1}\right)^{n-1/n} - 1\right]$$

$$= \frac{1}{0.85}\left[\frac{144(1.3)}{33,000(0.3)}\right](14.7)(100)\left[\left(\frac{84.7}{14.7}\right)^{0.3/1.3} - 1\right]$$

= 16.3 hp required for full capacity of air compressor. Use next larger standard size motor—<u>20 hp</u>.

4. <u>Gas Pipeline Layout</u>

A natural gas line is to run 20 miles from city A to junction B, where it splits into 2 lines; one going 10 miles to city C and the other 5 miles to city D. Initial pressure at city A is limited to 150 psig; demand at city C is 500,000 cfh (standard conditions) delivered at not less than 90 psig; and demand at city D is 250,000 cfh (standard conditions) delivered at not less than 75 psig. Select type and nearest standard size pipe required for the lines. Assume standard conditions as 14.7 psia and 60°F, and no change in temperature. Consider natural gas to be 75% $CH_4$, 21% $C_2H_6$, 3% $N_2$, and 1% $CO_2$, by volume.

<u>Theory</u>: Calculation of pressure drop in a pipe for general conditions is a complex thermodynamic problem. When isothermal conditions are assumed (and this is nearly true in a long pipeline), however, the solution may be simplified. Of course there is no <u>one</u> answer to a problem of this type as each solution will depend upon the assumptions made.

<u>Solution</u>: Pressure at junction B first must be assumed. If there is a uniform pressure drop from A to C, then pressure at B would be

$$P_B = 150 - \frac{20}{30}(60) = 110 \text{ psig}$$

The general equation for flow of gas through pipes is of the type

$$Q = 1.6156\left(\frac{T_o}{P_o}\right)\left[\frac{(P_1^2 - P_2^2)d^5}{s\ TL\ f}\right]^{1/2}$$

Friction factor f is dependent upon Reynolds number and relative roughness of the pipe. Values have been determined experimentally (see Stanton diagram in fluid mechanics texts). Considering the uncertainty of actual conditions, however, the friction factor is often given only as a function of pipe diameter. The Weymouth formula is

$$Q = 18.062\left(\frac{T_o}{P_o}\right)\left[\frac{(P_1^2 - P_2^2)d^{16/3}}{s\ TL}\right]^{1/2}$$

Q = cu ft/hr at base pressure $P_o$ and base temperature $T_o$

$T_o$ = base temperature, °F absolute

$P_o$ = base pressure, psia

$P_1$ = inlet pressure, psia

$P_2$ = outlet pressure, psia

d = internal diameter of line, inches

s = specific gravity of gas

T = flowing temperature, °F absolute

L = length of line, miles

Specific gravity of the gas is based on a weighted average.

| | Percent (by volume) | Specific gravity | |
|---|---|---|---|
| $CH_4$ | 75 | (0.554) | = 0.415 |
| $C_2H_6$ | 21 | (1.049) | = 0.220 |
| $N_2$ | 3 | (0.972) | = 0.029 |
| $CO_2$ | 1 | (1.528) | = 0.015 |
| | 100% | | s = 0.679 |

$$750,000 = 18.062\left(\frac{520}{14.7}\right)\left[\frac{(164.7)^2 - (124.7)^2}{0.679(520)(20)}\ d^{16/3}\right]^{1/2}$$

$$d^{8/3} = 917, \text{ or } d = 12.9 \text{ inches ID}$$

The next larger standard line pipe would be 14 inches OD and 13.25 inches ID.

Probable pressure should be checked at junction B. Rearranging the Weymouth formula

$$P_2 = \left[P_1^2 - \left(\frac{QP_o}{18.062\ T_o}\right)^2 \left(\frac{s\ TL}{d^{16/3}}\right)\right]^{1/2}$$

Using d = 13.25 gives

$$P_2 = 129.6 \text{ psia}$$

The line from B to C may now be determined in the same manner.

$P_1$ = 129.6 psia

$P_2$ = 104.7 psia

Q = 500,000 cu ft/hr

L = 10 miles

Using the Weymouth formula

d = 11 inches

This would require a 12-inch standard line pipe from B to C.

Similarly for the line from B to D

d = 6.92 inches

Since 7 inches is not a standard size, it would be necessary to use an 8-inch standard line pipe from B to D.

5. Use of a Weir

It is desired to measure the flow of water in a channel by use of a sharp-crested, rectangular, suppressed weir. The channel is 4 feet wide and the weir is made 3 feet high.

a) When head is measured as 6 inches, what is rate of flow?

b) Under what conditions can the use of such a weir be considered to give reasonably accurate results?

Theory: Flow over a weir is quite complicated and all flow equations contain a coefficient C, which takes into account the constriction of the nappe, friction, and velocity of approach. The general equation, neglecting velocity of approach, is

$$Q = C \frac{2}{3} b \sqrt{2g} \ (H)^{3/2}$$

Measurements are in feet and H is the head of the upstream surface above the top of the weir. Value of C will vary with proportions of H to P; head compared to height of weir. The Rehbach formula for well-ventilated, suppressed, sharp-crested, rectangular weirs is

$$C = 0.605 + 0.08(\frac{H}{P}) + \frac{1}{305\ H}$$

(See Vennard, p. 316)

Solution:

$$C = 0.605 + 0.08(\frac{0.5}{3}) + \frac{1}{305(0.5)} = 0.625$$

$$Q = 0.625(\frac{2}{3})(4)(\sqrt{64.4})\ (0.5)^{3/2} = \underline{\underline{4.72 \text{ cfs flow}}}$$

Note that the Francis formula could be used here. This reduces the terms of the above equation (using a fixed value of C = 0.62) to

$$Q = 3.33\, l\, h^{3/2} \qquad \text{(see Marks, p. 244)}$$

$l$ = width in feet h = head in feet

$$Q = 3.33(4)(0.5)^{1.5} = \underline{\underline{4.70 \text{ cfs}}}$$

b) The following conditions must be met (see Marks, p. 244):

1) Weir bulkhead must have a vertical upstream face and occupy full width of channel.

2) Crest must be level.

3) Velocity of approach should be small. Weir should be deep. (Above formula for calculating C takes this into account.)

4) Must be well ventilated. Flow must spring clear of downstream surface of the weir, with air beneath the water.

5) Head must be measured a distance upstream from weir at least equal to four times the head.

6) Side wall must extend downstream from weir to prevent spreading as water passes over crest.

7) Head must be measured accurately (usually with a "hook gage").

## 6. Turbine and Pipeline Performance and Pressure Drop

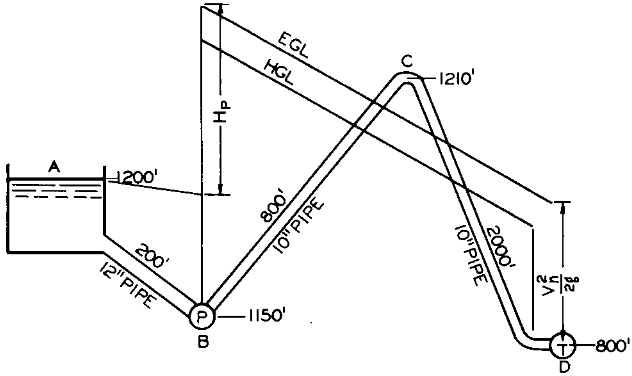
It is intended to carry water from a reservoir at elevation 1200 feet to a turbine at elevation 800 feet. Between reservoir and powerhouse site is a hill, the top of which is at elevation 1210 feet. As planned, 200 feet of 12-inch pipe conveys water to base of hill (elevation 1150), from which 800 feet of 10-inch pipe runs to top of the hill, thence 2000 feet to 4-inch nozzle of the turbine. It will be necessary to install a pump at the junction of the 10- and 12-inch pipes. Assume atmospheric pressure at the discharge side of the nozzle, friction factor of 0.020 in the Darcy-Weisbach formula, and energy loss in the nozzle of 0.108 times the velocity head of the jet. Turbine will operate under a 173-foot energy head in the jet, and turbine efficiency will be 88%, excluding nozzle loss.

a) Determine pumping head and required horsepower output of pump.

b) Determine output horsepower of turbine.

c) Compute magnitude and location of maximum and minimum pressures occurring in pipeline, assuming straight runs between the given elevations.

d) Check assumed value for friction formula, recalculate, and discuss practical implications.




Theory: The problem consists of recognizing and evaluating energy losses in pipe and nozzle, and energy relationships between static, velocity, and pressure heads.

Solution: a)

$$E_A + E_{pump} = E_D + \text{12-in. pipe loss} + \text{10-in. pipe loss} + \text{nozzle loss}$$

$$E_D = \frac{V_n^2}{2g} + 800, \text{ energy at nozzle}$$

$$\text{12-in. pipe loss} = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) \quad \text{(Darcy-Weisbach formula)}$$

Assume $f = 0.02$

$$\text{12-in. loss} = 0.02\left(\frac{200}{1}\right)\left(\frac{V_{AB}^2}{2g}\right) = \frac{4V_{AB}^2}{2g}$$

$$\text{10-in. pipe loss} = 0.02\left(\frac{2800}{10/12}\right)\left(\frac{V_{BD}^2}{2g}\right) = \frac{67.1(V_{BD}^2)}{2g}$$

$$\text{Nozzle loss} = 0.108\left(\frac{V_n^2}{2g}\right)$$

$$1200 + H_p = \left(800 + \frac{V_n^2}{2g}\right) + \frac{4V_{AB}^2}{2g} + \frac{67.1(V_{BD}^2)}{2g} + 0.108\left(\frac{V_n^2}{2g}\right)$$

$$H_p = 1.108\left(\frac{V_n^2}{2g}\right) + \frac{4V_{AB}^2}{2g} + \frac{67.1(V_{BD}^2)}{2g} - 400$$

Energy of the jet is given as 173 feet, which is all velocity head.

$$\frac{V_n^2}{2g} = \underline{173\text{ ft}}, \qquad V_n^2 = 11,150, \qquad V_n = 105.5\text{ fps}$$

Since diameter of jet is 4 inches

$$\frac{V_{AB}}{V_n} = \frac{d_n^2}{d_{AB}^2}, \qquad V_{AB} = 105.5\left(\frac{4}{12}\right)^2 = 11.74\text{ fps } V_{AB}$$

$$\frac{V_{AB}^2}{2g} = \frac{(11.74)^2}{64.4} = \underline{2.14\text{ ft}}$$

and

$$V_{BD} = 105.5\left(\frac{4}{10}\right)^2 = 16.9\text{ fps } V_{BD}$$

$$\frac{V_{BD}^2}{2g} = \frac{(16.9)^2}{64.4} = \underline{4.43\text{ ft}}$$

Therefore,

$$H_p = 1.108(173) + 4(2.14) + 67.1(4.43) - 400$$

$$= 192 + 8.56 + 298 - 400$$

$$H_p = \underline{\underline{99\text{ ft head of pump}}}$$

$$hp = \frac{\text{ft-lb per sec}}{550} = \frac{\text{lb per sec}(H_p)}{550} = \frac{\omega H_p}{550}$$

$$\omega = \text{(area of jet)(velocity of jet)(62.4)}$$

$$= 0.785\left(\frac{4}{12}\right)^2 (105.5)(62.4)$$

$$= 574 \text{ lb/sec, flow}$$

$$hp = \frac{574(99)}{550} = \underline{\underline{103 \text{ hp required of pump}}}$$

b) Determine horsepower output of turbine.

$$hp = eff\left[\frac{lb/sec(H_n)}{550}\right] = 0.88\left(\frac{\omega H_n}{550}\right)$$

$$= \frac{0.88(574)(173)}{550} = \underline{\underline{159 \text{ hp from turbine}}}$$

c) Determine maximum and minimum pressures. Maximum pressure will occur either at discharge side of pump or just before the nozzle. Both should be checked.

At discharge side of pump

$$E_A + E_P = E_B - \text{pipe losses A to B}$$

$$E_B = \frac{p}{\omega} + \frac{V_{BD}^2}{2g} + Z$$

$$1200 + 99 = \left[\frac{p(144)}{62.4} + 4.43 + 1150\right] - 8.56$$

$$p = \frac{62.4}{144}(153.1) = \underline{66.3 \text{ psi}}, \text{ gage pressure at discharge of pump}$$

At nozzle

$$\frac{p}{\omega} + \frac{V_{BD}^2}{2g} = 173 + \text{nozzle loss} = 1.108(173) = 192 \text{ ft}$$

$$p = \frac{62.4}{144}(192 - 4.43) = \underline{\underline{81.3 \text{ psi}}}, \text{ gage at base of nozzle maximum pressure}$$

Minimum pressure probably occurs at top of hill.

$$E_C = E_A + E_P - \text{loss}_{AB} - \text{loss}_{BC}$$

$$= 1200 + 99 - 8.56 - \frac{800}{2800}(298)$$

$$= 1205.4$$

but

$$E_C = \frac{p}{\omega} + \frac{V_{BD}^2}{2g} + Z$$

$$1205.4 = p(\frac{144}{62.4}) + 4.43 + 1210$$

$$p = \frac{62.4}{144}(1205.4 - 1214.4)$$

$$P = \underline{\underline{-\ 3.9 \text{ psi}}}, \text{ gage at top of hill}$$

Note that gage pressure at top of hill is a vacuum. This vacuum could not be allowed to exceed approximately -11 psi without danger of breaking the water column (theoretically, -14.7 psi).

d) The friction factor should now be checked. Note that the loss in the 12-inch pipe is small. Check first for the 10-inch pipe.

$$V_{BD} = 16.9 \text{ fps}$$

$$R_N = \frac{\rho\ Vd}{\mu}, \text{ Reynolds number}$$

$$= \frac{\frac{62.4}{32.2}(16.9)(0.834)}{3.74(10)^{-5}} = 7.36(10)^5$$

Referring to the Moody chart for friction factor and taking a relative roughness value of 0.0002

$$f = 0.015$$

Since a value of 0.020 was assumed, the 10-inch pipe loss should be (by direct proportion)

$$\frac{0.015}{0.020}(298) = 223 \text{ ft (instead of 298)}$$

Check for 12-inch pipe.

$$R_N = \frac{\frac{62.4}{32.2}(11.74)(1)}{3.74(10)^{-5}} = 6.1(10)^5$$

again

$$f = 0.015$$

Loss in the 12-inch pipe should be

$$\frac{0.015}{0.020}(8.56) = 6.42 \text{ ft}$$

Therefore,

$$H_p = 192 + 6.42 + 223 - 400$$

$$= \underline{\underline{21.4 \text{ ft}}}, \text{ head of pump}$$

(compared to 99 feet found in part a).

Note the large difference in "pump head required" which results from the small change in "friction factor." More important, however, is the reduction in pressure which would occur at the top of the hill if such a small pump was installed.

$$E_C = E_A + E_P - \text{loss}_{AB} - \text{loss}_{BC}$$

$$= 1200 + 21.4 - 6.42 - \frac{800}{2800}(223)$$

$$= 1151.3$$

but

$$E_C = \frac{p}{\omega} + \frac{V_{BD}^2}{2g} + Z$$

$$1151.3 = \frac{p(144)}{62.4} + 4.43 + 1210$$

$$p = \frac{62.4}{144}(1151.3 - 1214.4) = \frac{62.4}{144}(-51.6)$$

$p = -27.3$ psi gage (-51.6 feet)

Since pressure cannot drop below -14.7, or a perfect vacuum, the system could not possibly operate with such a small pump. In order to provide a pressure not less than -25 feet at the top of the hill, the pump would have to supply 21.4 + (51.6 - 25), or 48 feet of head.

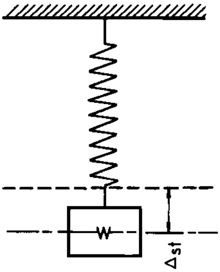
As a result of this increased head, the jet would operate under 173 + 26.6, or 200 feet of head, unless a smaller pipe was installed or a throttling valve ahead of the nozzle.

Note the vacuum existing at the top of the hill does not mean the pipe could not be filled initially with water. This low pressure is the result of the long drop down to the turbine, and would not develop until after flow had begun.

## VII. VIBRATIONS

Most vibrations are detrimental to the performance and life of a machine or structure. Many problems arise from resonance or critical speeds. A short discussion of natural frequency and resonance will help to explain the nature of vibration and how it may be reduced.

When a weight hangs from a spring, as shown, the spring will be elongated from its unstressed condition by the static deflection $\Delta_{st}$. It can be shown that if the weight is pulled down and released, the frequency of oscillation will be

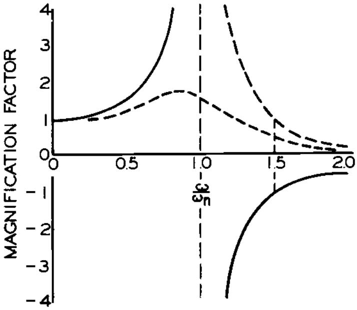



$$f = \sqrt{g/\Delta_{st}} \text{ radians per second}$$

$$= \frac{1}{2\pi}\sqrt{g/\Delta_{st}} \text{ revolutions per second}$$

and is called the natural frequency of vibration for the system.

This formula may be used for a great many types of vibration such as critical speed of rotating shafts and torsional as well as linear vibrations. One common mistake is in the units used. If g is 32.2 ft/sec$^2$, the $\Delta_{st}$ must be in feet.




Under conditions of forced vibration, such as that due to "out-of-balance," the amplitude of vibration may become excessive at one or more "critical" speeds. These critical speeds correspond to one of the natural frequencies of the supporting structure. Amplitude or vibration can be determined from the equation

$$A = \Delta_{st}\left[\frac{1}{\sqrt{(1 - \omega^2/\omega_n^2)^2 + (c\omega/k)^2}}\right]$$

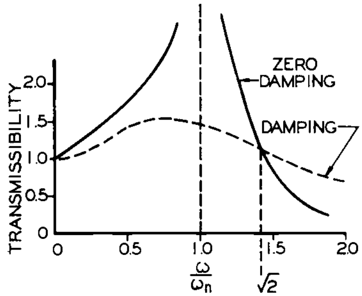
$\omega$ = frequency or distrubing force
$\omega_n$ = natural frequency of system
c = damping constant
k = spring constant

The term in the brackets is called the magnification factor, and is shown plotted against $\omega/\omega_n$ ratios. Negative values above $\omega/\omega_n$ = 1.0 merely mean the displacement is 180 degrees out of phase with the impressed force. This curve is generally shown as though all values were positive. When damping is present, amplitude is reduced for all speeds, as indicated by the dotted curve.

A similar expression can be used to indicate transmissibility, or the ratio of the force transmitted to the floor by a supporting structure to the force impressed on the structure.

$$\text{Transmissibility} = \frac{F_{tr}}{F_{imp}} = \sqrt{\frac{1 + (c\omega/k)^2}{(1 - \omega^2/\omega_n^2)^2 + (c\omega/k)^2}}$$

When transmissibility is plotted against $\omega/\omega_n$, two very important factors are apparent:




a) If the transmitted force is to be small, the natural frequency of the support must be very low compared to the frequency of the impressed force (as when mounting a machine on springs).

b) That damping is detrimental when operating at values of $\omega/\omega_n$ greater than 1.41.

## 1. Critical Speed of Rotating Shaft

A propeller shaft of a boat has an unsupported length of 76 inches between 2 self-aligning bearings. If shaft is made of steel and is 1.25 inches in diameter, what is critical speed of shaft?

Theory: At critical speed a rotating shaft becomes dynamically unstable and large vibrations are likely to develop. This occurs when the speed of rotation matches the natural frequency of the shaft considered as a beam.

Solution:

$$\omega_n = \sqrt{g/\Delta_{st}} \text{ radians/sec}$$

$\Delta_{st}$ = static deflection of shaft as a beam over simple supports

$$= \left(\frac{5\ wL^4}{384\ EI}\right)\frac{1}{12}\ \text{ft}$$

w = weight per inch = 0.283 A = $0.283(\pi r^2)$

L = length in inches = 76 inches

I = moment of inertia = $(1/4)\ \pi r^4$

$$\Delta_{st} = \frac{5(0.283\ \pi r^2)(76)^4}{384\left[30(10^6)(\frac{\pi}{4} r^4)(12)\right]} = 0.0035\ \text{ft}$$

$$\omega_n = \sqrt{32.2/0.0035} = \sqrt{9200} = 96 \text{ radians/sec}$$

$$= 96\left(\frac{60}{2\pi}\right) = \underline{\underline{917 \text{ rpm}}}$$

Note that because shaft extends through bearings as a continuous beam, the static deflection will be less than that calculated. This means the critical speed will be somewhat higher than the 917 rpm determined.

2. Resonance

A centrifugal fan installed in the stack of a powerplant and supported by a steel framework is vibrating excessively at one speed just below its operating speed. What is the probable cause, and what recommendation would you make to improve the condition?

Solution: If vibration is bad at a particular speed, but decreases both above and below this speed, the trouble is due to resonance in the supporting structure. Although "out-of balance" of the fan would aggravate the condition, it is not the principal cause of trouble since without resonance the vibration due to unbalance would become increasingly worse at higher speeds.

Balancing the fan scroll will perhaps help some, but the greatest improvement will result from stiffening the structural support upon which the fan is mounted. The result of such stiffening will be to increase the natural frequency of the support and decrease the transmissibility.

Since the $\omega/\omega_n$ ratio is less than 1.41 (see discussion at beginning of chapter), additional benefit would result from any damping which could be introduced into the support.

3. Vibration Isolation

A 2-ton machine is subjected to a disturbing force of 1000 pounds at a frequency of 2000 cycles per minute. In order to reduce force transmitted to the building to 20 pounds, the machine is to be placed on 6 equally loaded springs. Assuming motion is restrained to a vertical direction, calculate the spring constant needed for each spring.

Solution: If zero damping is assumed, the ratio of transmitted force to impressed force is

$$\frac{F_{tr}}{F_{imp}} = \frac{1}{1 - (\omega^2/\omega_n^2)}$$

$\omega$ = frequency of impressed force

$= 2000(\frac{2\pi}{60}) = 210$ radians/sec

$\omega_n$ = natural frequency of weight on spring

$$\omega_n^2 = \frac{kg}{W} = \frac{k(32.2)}{667}$$

Note when value of $\omega$ exceeds $\omega_n$, transmitted force is in opposite direction to impressed force. Therefore,

$$\frac{-F_{tr}}{F_{imp}} = \frac{1}{1 - (\omega^2/\omega_n^2)}$$

or

$$\frac{F_{tr}}{F_{imp}} = \frac{1}{(\omega^2/\omega_n^2) - 1}$$

$$\frac{20/6}{1000/6} = \frac{1}{\frac{210^2}{32.2\,k/667} - 1}$$

$k = \frac{914,000}{51}$ = 17,900 lb/ft or 1490 lb/in. for each spring

With an isolation system such as this, where frequency of the impressed force is greater than 1.41 times the natural frequency of the system, any damping would increase magnitude of the transmitted force. However, a small amount of damping would be beneficial in preventing excessive vibration while passing through the natural frequency, both when machine is being brought up to speed and when being shut down.

## VIII. ENERGY RELATIONSHIPS AND GAS PROCESSES

Thermodynamics is concerned with energy and especially the laws which govern transformation of energy. The engineer is particularly interested in the conversion of many forms of available energy into useful work. The following sections will deal with problems involving gas cycles, steam processes, air conditioning, and heat transfer. All, however, involve use of the general energy equation and the law of conservation of energy. Briefly stated, this law requires that if no energy is stored in a device, then energy entering it must equal energy leaving.

The general energy equation was discussed in the section on fluid mechanics, but a few additional comments may be helpful.

For conditions of steady flow, the general energy equation may be written in units of heat energy as Btu per pound of fluid flowing.

$$\frac{Z_1}{J} + \frac{v_{s1}^2}{2g\,J} + u_1 + \frac{p_1 v_1}{J} + Q_{in} = \frac{Z_2}{J} + \frac{v_{s2}^2}{2g\,J} + u_2 + \frac{p_2 v_2}{J} + W_{out}$$

$J$ = 778 ft-lb, mechanical equivalent of one Btu of heat energy

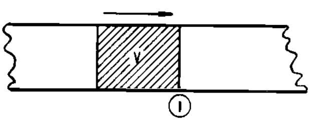
$Z/J$ = potential energy due to elevation Z in feet

$v_s^2/2g\,J$ = kinetic energy due to velocity $v_s$ in feet per second

u = internal energy in Btu or energy of molecules measured above some convenient datum plane

pv/J = "flow work" (equivalent to pressure head used in fluids)

Energy is required to move a volume of water against a given pressure past some point such as 1 on the diagram.




Energy required = F(d)

= pA(d)

= p[A(d)]

= pv

Pressure here should be in pounds per square foot absolute, and v is the specific volume or cubic feet per pound.

It is this fundamental law of conservation of energy which makes impossible a perpetual motion machine. Energy may be converted and controlled (even atomic energy), but we know of no way in which it can be created.

Once again, the problem of units may prove bothersome and one should always check for consistency of units used in any problem.

When there is no flow, energy changes are based upon end conditions or upon "change in state" of the fluid. Velocity and flow work terms drop out of the energy equation and the relationship simplifies to

$$Q_{in} = (u_2 - u_1) + W_{out}$$

For the "perfect gas" law

$$pV = w\,RT$$

A perfect gas would be one which obeyed the perfect gas laws. Most gases are nearly perfect; i.e., air, oxygen, and nitrogen. When a substance deviates appreciably from these laws it is called a "vapor" rather than a gas.

Internal energy for a perfect gas is dependent only upon temperature. Thus, the change in internal energy is

$$\Delta u = \int c_v dT = c_v(T_2 - T_1) \quad \text{(for constant } C_v\text{)}$$

Also, the change in enthalpy may be expressed as

$$\Delta h = \int c_p dT = c_p(T_2 - T_1) \quad \text{(for constant } C_p\text{)}$$

These two equations are true regardless of the path or process involved. However, care must be taken when using equations involving Q and W. Heat and work not only are dependent upon the end states, but also are affected by the process and by whether or not the process is reversible.

Properties of a fluid (internal energy, enthalpy, entropy, pressure, temperature, volume, etc.), are dependent only upon the "state" of the fluid, not upon the process by which that state is reached. Q, heat added, and $W_1$, work done by a fluid, are not properties and depend very much upon the process by which a state is reached.

Temperature and pressure must be expressed relative to an absolute zero. Thus, absolute temperature in degrees Rankine = t °F + 460 (approx). Also, absolute pressure = psig + atmospheric pressure.

Consistent units must be used with the equation

$$pV = w\,RT$$

where

p = absolute pressure in lb/sq ft
V = total volume in cu ft
w = pounds of substance
T = absolute temperature in degrees Rankine
R = individual gas constant in ft-lb force/degree Rankine/lb mass

1. Electricity to Melt Snow

a) A suburbanite is considering using electricity to melt the snow in his driveway, employing ground coils. If the driveway is 12 feet wide and 40 feet long, what would be the cost of melting a 6-inch layer of snow with electricity at 2¢ per kwhr? Assume snow to weigh 12 pounds per cubic foot at 32°F, and that 60% of the electrical energy is actually used to melt the snow.

b) If above process is completed in 3 hours, what should be the kilowatt rating of the underground heating coils?

Theory:

1 kwhr = 3413 Btu

Latent heat of fusion of water = 144 Btu/lb

Solution: a)

Volume of snow = 12(40)(1/2) = 240 cu ft
Weight of snow = 240(12) = 2880 lb
Btu to melt snow = 2880(144) = 415,000 Btu

$$\text{Electricity required} = \frac{415,000}{0.60(3413)} = 202.5 \text{ kwhr}$$

Cost = 0.02(202.5) = $\underline{\underline{\$4.05}}$

b)

$$\text{Rating} = \frac{202.5}{3} = \underline{\underline{67.5 \text{ kw}}}$$

## 2. Water-Cooled Brake

A 300-horsepower engine is given a brake test. Brakes are water cooled. At what rate must water at 80°F flow through brakes if water must not rise above 180°F?

Theory:

1 hp = 33,000 ft-lb/min

1 Btu will raise temperature of 1 lb of water 1°F

Solution:

300 hp = 300(33,000) = 9,900,000 ft-lb/min

$$Q = \frac{9,900,000}{778} = 12,720 \text{ Btu/min}$$

$$\Delta T = 180 - 80 = 100°F$$

$$Q = w\,\Delta T$$

$$w = \frac{Q}{\Delta T} = \frac{12,720}{100} = 127.2 \text{ lb/min}$$

$$w = \frac{127.2}{8.33} = \underline{\underline{15.3 \text{ gal/min}}}$$

3. Calculation of Joule's Mechanical Equivalent of Heat

It is found that 116.7 Btu are required to heat one pound of air from 32°F to 523°F at constant atmospheric pressure (14.7 psi). During this process the air expands from 12.33 to 24.66 cubic feet per pound. For the same temperature rise at constant volume, 83 Btu were required. From these data compute Joule's mechanical equivalent of heat.

Theory: For a perfect gas

$$\Delta u = c_v \, \Delta T = Q_v \quad \text{constant volume process}$$

$$\Delta_h = c_p \, \Delta T = Q_p \quad \text{constant pressure process}$$

but

$$\Delta h = \Delta u + \Delta \frac{pv}{J}$$

Solution:

$$\Delta u = Q_v = 83 \text{ Btu/lb}$$

$$\Delta h = Q_p = 116.7 \text{ Btu/lb}$$

Since $\Delta T$ is the same for both

$$Q_p = Q_v + \frac{p(v_2 - v_1)}{J}$$

$$J = \frac{p(v_2 - v_1)}{Q_p - Q_v}$$

$$= \frac{14.7(144)(24.66 - 12.33)}{116.7 - 83}$$

$$J = \underline{\underline{776 \text{ ft-lb/Btu}}}$$

4. Theoretical Draft

Find the maximum theoretical draft in inches of water obtainable from a chimney 150 feet high, atmospheric pressure 14.7 lb/sq in., temperature of outside air 60°F, temperature of chimney gases 550°F.

Theory: The buoyant force, called draft, is due to the volume of gas in the chimney weighing less than an equal volume of outside air.

Solution:
$$pV = w\,RT$$

or
$$w = \frac{pV}{RT}$$

If the same gas exists inside the chimney as outside, then

$$\frac{w_i}{w_o} = \frac{p_i V_i R_o T_o}{R_i T_i p_o V_o} = \frac{T_o}{T_i}$$

$$w_i = w_o\left(\frac{T_o}{T_1}\right)$$

$$w_o - w_i = w_o\left(1 - \frac{T_o}{T_1}\right)$$

$$\Delta w = \frac{p_o V_o}{R_o T_o}\left(1 - \frac{T_o}{T_1}\right)$$

$$= \frac{14.7(144)(150\ A)}{53.3(520)}\left(1 - \frac{520}{1010}\right)$$

$$\Delta w = 5.55 \text{ lb/sq ft}$$

Now, 12 inches water = 62.4 lb/sq ft

$$\therefore\ \Delta w = \frac{5.55}{62.4}(12) = \underline{\underline{1.07 \text{ inches of water}}}$$

The same result will be obtained using the formula from Marks, page 1120.

$$D = 0.52\ PH\left(\frac{1}{T} - \frac{1}{T_1}\right)$$

or
$$D = KH = 0.0071(150) = \underline{\underline{1.06 \text{ inches of water}}}$$

5. Balloon, Change of State

a) The pressure of a 16- by 7-inch airplane tire at sea level, standard atmospheric pressure and 70°F, is 32 psig. What will be the gage pressure in the same tire (assuming no change in volume) at an altitude of 50,000 feet where the barometer reads 3.44 inches of Hg and temperature is minus 67°F?

b) Lifting force of a balloon depends upon the difference between the weight of the gas and the air it displaces. The bag is usually loose enough that internal and external pressures are essentially equal. What weight and volume of helium will be required to lift a 500-lb load at sea level, standard atmospheric pressure, and at 85°F? Neglect weight of bag.

Theory: Both questions involve the perfect gas equation

$$pV = w\,RT$$

Solution: a) For a constant volume process

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}, \text{ or } P_2 = P_1\left(\frac{T_2}{T_1}\right)$$

$$P_2 = (14.7 + 32)\left(\frac{460 - 67}{460 + 70}\right) = 46.7\left(\frac{393}{530}\right)$$

$$P_2 = 34.6 \text{ psia}$$

However, atmospheric pressure is

3.44 in. of Hg(0.491) = 1.69 psia

Gage pressure = 34.6 - 1.7 = 32.9 psig

b) The buoyancy of the balloon = $w_{air} - w_{He}$

$$w_{air} - w_{He} = 500$$

Since $w = pV/RT$

$$\frac{pV_{air}}{R_{air}T} - \frac{pV_{He}}{R_{He}T} = 500$$

and $$V_{He} = V_{air}$$

$$\frac{14.7(144)}{53.3(545)} V - \frac{14.7(144)}{386(545)} V = 500$$

$$V(0.0728 - 0.01005) = 500$$

$$V = \frac{500}{0.0627} = \underline{\underline{7970 \text{ cu ft He required}}}$$

$$w_{He} = 0.01005(7970) = \underline{\underline{80 \text{ lb He}}}$$

6. Steady Flow, Change of State

Hydrogen gas is flowing through a pipe at a temperature of 500°F and a pressure of 210 psia. It flows through a mixing chamber where it meets a stream of carbon monoxide gas. CO entering the mixing chamber is at 110°F and 45 psia. Mixture leaving the chamber is at 35 psia and contains 70% $H_2$ and 30% CO, by volume. If chamber is considered adiabatic in operation, what is temperature of gas mixture leaving chamber?

Theory: Since the process is adiabatic, heat given up by the hydrogen must equal the increase in heat content of the CO. The change in enthalpy of a gas is equal to $C_p \Delta t$, and $C_p$ may be taken as constant over the temperature range involved.

Solution: Assume 1 mol of mixture and determine percentage by weight of each gas.

$$\frac{0.70(2) + 0.30(28)}{1.00} = 9.8 \text{ lb/mol} \quad \text{(mol wt of mixture)}$$

$$H_2 = \frac{1.4}{9.8} = 14.3\% \text{ by weight}$$

$$CO = \frac{8.4}{9.8} = 85.7\% \text{ by weight}$$

On the basis of 100 pounds of mixture

$$W_{H_2}(h_{H_2}) = -W_{CO}(h_{CO})$$

$$14.3\left[3.42(500 - t)\right] = -85.7\left[0.249(110 - t)\right]$$

$$t = \frac{26,750}{70.2} = \underline{\underline{381\ ^\circ F}}, \text{ temperature of mixture}$$

## IX. I.C. ENGINES, AIR COMPRESSORS, FANS, ORIFICES

No internal combustion engine, or any other heat engine, can convert all the energy supplied to it into available work. Maximum efficiency attainable, such as in a Carnot cycle, is dependent upon the temperatures involved, and requires completely reversible processes. Thermal efficiency for the Carnot cycle is given by

$$e = \frac{T_1 - T_2}{T_1}$$

using the highest and lowest absolute temperatures involved.

Spark ignition engines and compression ignition engines approximate Otto and Diesel cycles. Again, either of these cycles has an ideal efficiency which even the most perfect engine could not exceed. <u>Engine efficiency</u> is a comparison of the actual efficiency of an engine to its ideal cycle efficiency. For a spark ignition engine the cycle efficiency is

$$e = 1 - \frac{1}{r^{k-1}}$$

where r is the compression ratio. The value of k is often taken as the gas constant for air at room temperature (1.4). With the mixture of hot gases existing in the cylinder, however, k is often taken as 1.3 and referred to as the "hot-air standard."

The Diesel cycle efficiency is given by

$$e = 1 - \frac{1}{r^{k-1}}\left[\frac{r_c^k - 1}{k(r_c - 1)}\right]$$

where $r_c$ is the cut-off ratio.

This equation differs from the Otto cycle only in the bracketed term, and thus the Diesel cycle is always less efficient than the Otto cycle for the same compression ratio. The greater efficiency of the diesel over the spark ignition engine results from the fact that a higher compression ratio can be used.

1. Effect of Compression Ratio on Efficiency

A chemist claims he has developed an additive for gasoline which will permit the compression ratio of a certain automobile engine to be raised from 6.48 to 1, to 8.0 to 1. He further maintains this change will increase power output of the engine by 10%, and reduce fuel consumption by 12%. Are these claims reasonable enough to justify an attempt at verification by test? The additive has negligible heating value.

Solution: If the claim is based upon improvement resulting from the increased compression ratio, then engine efficiency might remain nearly the same, and the increase in thermal efficiency would be limited by the increase in theoretical cycle efficiency.

$$\text{Theoretical efficiency} = 1 - \frac{1}{r^{k-1}}$$

Using k = 1.3 (hot-air standard), the initial condition would have a theoretical efficiency of

$$e_1 = 1 - \frac{1}{6.48^{0.3}} = 42.8\%$$

and

$$e_2 = 1 - \frac{1}{8.0^{0.3}} = 46.3\%$$

The increase in cycle efficiency is

$$\frac{3.5}{42.8} = 8.17\%$$

Indicated thermal efficiency, $e_i$ = (engine efficiency, $\eta_i$) (cycle efficiency, e). Therefore, $e_i$ will increase 8.17%.

If friction horsepower remains the same, then mechanical efficiency will increase.

$$\text{bhp} = \text{ihp} - \text{fhp}$$

assuming a mechanical efficiency of 75%. An increase of 8.17% in ihp would then increase bhp by 8.17/0.75 = 10.9%.

Fuel consumption is reported on a pound per bhp-hour basis. If the same fuel will increase bhp by 10.9%, then the specific fuel consumption would be 1/1.109, or 0.902. This corresponds to a reduction in fuel consumption of 9.8%.

It is possible the additive, in addition to permitting an increased compression ratio, might control the burning rate in such a manner as to improve engine efficiency. Therefore, it would seem justifiable to run tests on his product.

2. S.I. Engine Calculations

a) If a mixture of gasoline and vapor in an engine cylinder has an ignition temperature of 550°F, what is the maximum compression ratio which could be used without preignition taking place if k equals 1.3? Assume initial temperature and pressure at start of compression stroke as 100°F and 14.7 psia. What percentage clearance volume would give this compression ratio?

b) If above engine is single acting, 4 cylinder, 4-stroke cycle, and is to produce 100 bhp at 1200 rpm, what should be its bore and stroke? Assume engine efficiency of 50%, mechanical efficiency 75%, indicated mean effective pressure in cylinder 75 psig, and ratio of stroke to diameter 1.5.

Solution: a) The compression process is described by the relation, $PV^{1.3} = C$. Since $PV/T = C$, the following relationship holds:

$$\frac{V_1}{V_2} = \left(\frac{T_1}{T_2}\right)^{1/1.3-1} = \left(\frac{550 + 460}{100 + 460}\right)^{1/0.3}$$

$$\frac{V_1}{V_2} = \underline{\underline{7.11 \text{ to } 1 \text{ compression ratio}}}$$

If clearance volume is C and displacement volume D, compression ratio would then be

$$\frac{V_1}{V_2} = \frac{D + C}{C} = 7.11$$

$$\frac{C}{D} = \frac{1}{6.11} = \underline{\underline{16.4\% \text{ clearance volume}}}$$

b) There will be 2 power strokes each revolution. Information on engine efficiency is unnecessary since ihp can be obtained from the mean effective pressure.

$$\text{imep} = \frac{\text{ihp}(33,000)}{\text{LAN}}$$

L = stroke in feet
A = piston area in square inches
N = power strokes per minute

Now,

$$\text{ihp} = \frac{\text{bhp}}{e_m} = \frac{100}{0.75} = 133.3 \text{ ihp}$$

$$75 = \frac{133.3(33,000)}{\text{LA}(1200)(2)}$$

$$\text{LA} = 24.5 \text{ ft, in.}^2$$

If

$$L = 1.5 \text{ d in.} = \frac{1.5 \text{ d}}{12} \text{ ft}$$

$$\text{LA} = \frac{1.5 \text{ d}}{12}(0.785 \text{ d}^2) = 0.0981 \text{ d}^3$$

$$d^3 = \frac{24.5}{0.0981} = 250$$

$$d = \underline{\underline{6.3 \text{ inches}}}$$

$$L = \underline{\underline{9.45 \text{ inches}}}$$

3. Two-Stage Unit From Two Single-Stage Air Compressors

A certain plant has installed near each other 2 single-stage, double-acting air compressors which have a discharge of 200 psia. The bores and strokes of the 2 units are 6 by 8 inches and 10 by 12 inches, respectively. Rpm are 200 and 150, respectively, and both have a clearance volume of 4%. It is proposed to connect these machines to operate as one 2-stage unit. Assuming perfect intercooling, inlet pressure 15 psia, inlet temperature 70°F, and n = 1.3, calculate the following:

a) Intercooler pressure for equal work per stage.

b) Speed of each unit if capacity of 2-stage machine equals the sum of the original capacities.

c) Horsepower of original and proposed installations.

Theory: Work done in a single-stage air compressor is given by the formula

$$W = \frac{nw' RT_1}{1 - n}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

With perfect intercooling, air entering the second stage would be at the same temperature as air entering the first stage. Now, if it is assumed the outlet pressure of the first stage equals the inlet pressure of the second stage, then for equal work in both stages, $p_i = (p_1 p_4)^{1/2}$.

Solution: a)

$$p_i = \sqrt{200(15)} = \underline{\underline{54.7 \text{ psia}}} = \underline{\underline{40 \text{ psig}}}$$

b) Original capacity of small compressor

$$V_1 = \left[1 + C - C\left(\frac{p_2}{p_1}\right)^{1/n}\right] V_D N$$

C = clearance volume

$V_D$ = displacement volume

Because compressor is double acting, the displacement volume = 2 AL.

$$V_{D_1} = 2(0.785\ \frac{36}{144})(\frac{8}{12}) = 0.262 \text{ cu ft/cycle}$$

$$V_1 = \left[1 + 0.04 - 0.04(\frac{200}{15})^{1/1.3}\right] 0.262(200) = 39.1 \text{ cfm}$$

Similarly,

$$V_{D_2} = 1.09 \text{ cu ft/cycle}$$

and

$$V_2 = 122 \text{ cfm}$$

Total capacity of the two compressors is

$$122 + 39 = 161 \text{ cfm}$$

Capacity of the first stage, as proposed, must be the same as the total capacity.

$$161 = \left[1 + 0.04 - 0.04(\frac{54.7}{15})^{1/1.3}\right] 1.09(N_1)$$

$$N_1 = \underline{\underline{159 \text{ rpm}}}, \text{ speed of large compressor}$$

Since the temperature of air entering the second stage is the same as that entering the first stage, its volume will be reduced inversely as the pressure, or

$$V_2 = V_1(\frac{P_1}{P_2}) = 161(\frac{15}{54.7}) = 44.1 \text{ cu ft/cycle}$$

and

$$44.1 = \left[1 + 0.04 - 0.04(\frac{200}{54.7})^{1/1.3}\right] 0.262\ N_1$$

$$N_1 = \underline{\underline{181 \text{ rpm}}}, \text{ speed of small compressor}$$

c) When operated separately, the sum of the horsepower needed is

$$hp = \frac{W_1 + W_2}{33,000}$$

$$= \frac{np_1 V_1}{33,000(1-n)} \left[ \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.3(15)(144)(161)}{33,000(-0.3)} \left[ \left(\frac{200}{15}\right)^{0.3/1.3} - 1 \right]$$

$$= \underline{\underline{-\ 37.4\ hp}} \text{ operated separately}$$

The minus sign merely means that work is done <u>on the air</u>.

When operated as a 2-stage compressor, each unit requires the same horsepower (see part a).

$$hp = 2 \left[ \frac{1.3(15)(144)(161)}{33,000(-0.3)} \right] \left[ \left(\frac{54.7}{15}\right)^{0.3/1.3} - 1 \right]$$

$$= \underline{\underline{-\ 31.8\ hp}} \text{ as 2-stage unit}$$

## 4. Fan Performance, Different Speed and Temperature

A multivane centrifugal fan is rated to deliver 17,500 cfm of air at 68°F, and static pressure of 1 inch when running at 256 rpm. Under rated conditions, the shaft load is 4.54 horsepower. It is proposed to change speed of fan to 300 rpm and have air temperature increased to 150°F. Determine cfm, static pressure, and shaft horsepower.

<u>Theory</u>: In comparing the performance of centrifugal fans, the relations below apply for fans of geometrically similar design, constant density, and constant <u>orifice ratio</u>. The conventional orifice ratio is taken as

$$O = \frac{Q}{D^2 \sqrt{p}}$$

$$Q \propto D^3 N \qquad p \propto D^2 N^2 \qquad hp \propto D^5 N^3$$

For a given fan at constant orifice ratio and constant density

$$Q \propto N \qquad p \propto N^2 \qquad hp \propto N^3$$

Solution: First assume the speed changes, but not the temperature (density).

$$Q = 17,500\left(\frac{300}{256}\right) = 20,500 \text{ cfm}$$

$$p = 1.0\left(\frac{300}{256}\right)^2 = 1.375 \text{ in. of } H_2O$$

$$hp = 4.54\left(\frac{300}{256}\right)^3 = 7.32 \text{ hp}$$

When temperature increases, density decreases, and although the volume remains constant, pressure and horsepower vary inversely as the density, or as 1/T.

$$Q = \underline{\underline{20,500 \text{ cfm}}}$$

$$p = 1.375\left(\frac{460 + 68}{460 + 150}\right) = \underline{\underline{1.19 \text{ in. water}}}$$

$$hp = 7.32\left(\frac{528}{610}\right) = \underline{\underline{6.35 \text{ hp}}}$$

## 5. Orifice, Gas Flow Measurement

A constant speed ventilating fan delivers a maximum of 8000 cfm of air at 68°F and 30 inches of Hg absolute in a circular duct 53 inches in diameter. It is desired to measure the flow delivered by the fan at 3/4, 1/2, and 1/4 maximum flow by using a set of concentric sharp-edged orifices, each of which is to have a pressure drop of 0.25 inches of water. Flow will be controlled by a separate damper. What should be the diameters of the orifices in inches?

Theory: The general equation for velocity is given by the following formula:

$$v_{s2} = C\sqrt{2\,gJ(h_1 - h_2)} \left[ \frac{1}{\sqrt{1 - (A_2/A_1)^2 (v_1/v_2)^2}} \right]$$

Section 1 is upstream from orifice
Section 2 is at orifice
h = enthalpy
v = specific volume, cu ft/lb

The second term may be thought of as a velocity-of-approach factor and becomes equal to 1 when approach velocity is zero.

For pressure drops not over 1%, or 4 inches of water, specific volume is nearly constant and the formula simplifies to

$$v_{s2} = C\sqrt{2g(RT_1/p_1)(\Delta p)} \left[ \frac{1}{\sqrt{1 - (A_2/A_1)^2}} \right]$$

The discharge coefficient C may be taken as 0.60 for any Reynolds number above 100,000 and for any diameter ratio up to $D_2/D_1 = 75\%$. (The error will be less than 1-1/2%.)

Flow ($Q = v_{s2}A_2$), and air at normal conditions (70 °F and 14.7 psia) reduces to the convenient formula

$$Q = 4005\ CA_2\ \sqrt{H} \left[ \frac{1}{\sqrt{1 - (A_2/A_1)^2}} \right] \text{ cu ft/min}$$

A is in square feet
H is pressure drop in inches of water

For area ratios less than 20%, the velocity-of-approach factor may be ignored. Area ratios greater than 50% are not recommended.

Solution: Using the simplified equation at 3/4 flow

$$Q = 4005\ CA_2\sqrt{H} \left[ \frac{1}{1 - A_2^2/A_1^2} \right]^{1/2}$$

$$= 4005(0.60)\, A_2 \sqrt{0.25} \left[ \frac{1}{1 - (A_2/15.9)^2} \right]^{1/2}$$

$$= 1202 \left[ \frac{A_2}{(1 - A_2^2/252)^{1/2}} \right]$$

This equation may be rearranged to give

$$A_2 = \frac{Q}{1202 \left[ 1 + \frac{Q^2}{(1202)^2(252)} \right]^{1/2}}$$

For 3/4 flow, Q = 6000 cfm.

$$A_2 = \frac{6000}{1202 \left[ 1 + \frac{(6000)^2}{(1202)^2(252)} \right]^{1/2}} = 4.76 \text{ sq ft}$$

$d = \sqrt{A/0.785} = 2.46$ ft = 29.5-in. diameter for 3/4 flow

Area ratio = 4.76/15.9 = 30%. For the smaller orifices, the velocity factor can probably be neglected.

For 1/2 flow, Q = 4000 cfm.

$$Q = 4005\, C(A_2 \sqrt{H})$$

$$4000 = 4005(0.6)\, A_2(0.5)$$

$$A_2 = 3.33 \text{ sq ft} \qquad (21\% \text{ of } A_1)$$

$d_2 = 2.06$ ft = 24.7-in. diameter for 1/2 flow

For 1/4 flow, Q = 2000 cfm, and

$d_2 = 1.46$ ft = 17.5-in. diameter for 1/4 flow

# X. COMBUSTION, POWERPLANTS, AND AUXILIARY EQUIPMENT

Combustion is the rapid oxidation of solid, liquid, or gaseous fuels. In using combustion equations it would be well to keep in mind the following facts:

1. Total weight of each substance must be the same before and after chemical combination.

2. Avogadro's law (approximate for real gases) states that at a given pressure and temperature all gases have the same number of molecules in a given volume.

3. Composition of air is approximately 20.9% oxygen and 79.1% nitrogen by volume, or 23.1% oxygen and 76.9% nitrogen by weight.

4. Partial pressure of a constituent is its percentage by volume multiplied by the total pressure of the mixture.

5. In treating combustion products, it must be specified whether or not proportions are on a dry basis or a wet basis. Moisture formed may exist as vapor or may condense out of the products of combustion.

## 1. Natural Gas Combustion

Natural gas flowing in a pipeline at 3000 fpm, 75 psig, and 42°F, has been found to have the following composition by volume: $CH_4$ (methane), 78.8%; $C_2H_4$ (ethylene), 9.7%; and $N_2$, 11.5%. Find the following:

a) Density of gas flowing in the pipeline in pounds per cubic foot.

b) Cubic feet of theoretical air required to burn 1 cubic foot of gas, both at 60°F and 14.7 psia.

c) Size of pipeline necessary to supply 87,800,000 Btu per hour if the higher heating value is 1000 Btu per standard cubic foot.

Solution: a) Determining the density of a mixture of gases amounts to calculating a weighted average.

| Gas | Proportion by volume | Molecular weight | Product |
|---|---|---|---|
| $CH_4$ | 0.788 | 16 | 12.60 |
| $C_2H_4$ | 0.097 | 28 | 2.71 |
| $N_2$ | 0.115 | 28 | 3.22 |
| Mixture | 1.0 mol | = | 18.53 lb/mol |

At the temperature and pressure given, a mol of gas occupies the following volume:

$$V = \frac{RT}{p} = \frac{1544(502)}{89.7(144)} = 60 \text{ cu ft}$$

R = universal gas constant

$$\text{Density} = \frac{w}{V} = \frac{18.53}{60} = \underline{\underline{0.309 \text{ lb/cu ft}}}$$

b)

$$CH_4 + 2O_2 = CO_2 + 2H_2O$$

1 part $CH_4$ requires $\frac{1}{0.209}(2) = 9.56$ parts air, by volume

$$C_2H_4 + 3O_2 = 2CO_2 + 2H_2O$$

1 part $C_2H_4$ requires $\frac{1}{0.209}(3) = 14.32$ parts air, by volume

1 cu ft mixture requires $0.788(9.56) + 0.097(14.32)$

$= \underline{\underline{8.91 \text{ cu ft air}}}$

c)

$$\text{Vol (standard conditions)} = \frac{87,800,000}{1000} = 87,800 \text{ cu ft/hr}$$

$= 1463$ cfm

Vol (line conditions) = 1463(502/520)(14.7/89.7) = 232 cfm

$$Q = vA$$

$$232 = 3000(0.785\, d^2)$$

$d^2 = 0.0982 \text{ ft}^2$, $d = 0.314$ ft = $\underline{3.77 \text{ inches}}$

Use a $\underline{\underline{\text{4-inch standard line pipe}}}$ (ID = 4.026 in.)

## 2. Boiler Test

Data from a boiler test included the following:

| Coal - ultimate analysis | | Exhaust gas analysis | |
|---|---|---|---|
| Carbon | 73.44% | $CO_2$ | 12.9% |
| Hydrogen | 4.76 | $O_2$ | 6.1 |
| Oxygen | 6.28 | CO | 0.6 |
| Nitrogen | 1.45 | Boiler room temp, 82°F | |
| Sulphur | 0.71 | Outside air temp, 40°F | |
| Ash | 11.85 | Exhaust gas temp, 375°F | |
| Moisture | 1.51 | Steam flow, 96,840 lb/hr | |
| Coal-fired, 10,183 lb/hr | | Bar pressure, 28.6 in. Hg | |
| Higher heating value, 12,375 Btu/lb | | Steam pressure, 265 psig | |
| Ash and refuse, 1500 lb/hr | | Steam temp, 525°F | |
| Carbon in ash, 20% | | Feedwater temp, 210°F | |

Find the following:

a) Percentage excess air.

b) Weight of stack gases (including moisture) per pound of fuel.

c) Percentage of heating value of 1 pound of fuel lost to water vapor formed during combustion.

Solution: a) Determine theoretical air required.

$C + O_2 = CO_2$ 1 lb C requires $\frac{32}{12}\left(\frac{1}{0.231}\right) = 11.52$ lb air

$2H + \frac{1}{2}O_2 = H_2O$ 1 lb H requires $\frac{16}{2}\left(\frac{1}{0.231}\right) = 34.5$ lb air

$S + O_2 = SO_2$ 1 lb S requires $\frac{32}{32}\left(\frac{1}{0.231}\right) = 4.33$ lb air

$$w_{th} = 0.7344(11.52) + 0.0476(34.5) + 0.0071(4.33) - \frac{0.0628}{0.231}$$

$= \underline{9.86 \text{ lb air/lb fuel (theoretical)}}$

Find actual air used. This is done by noting percentage of nitrogen in the flue gas. If we start with 1 mol of flue gas, the nitrogen present will be 1.00 - 0.196 = 0.804 mol (from exhaust analysis).

Weight of $N_2$ = 28(0.804) = 22.5 lb $N_2$/mol of flue gas

Weight of carbon in 1 mol of flue gas is

$12(CO + CO_2) = 12(0.129 + 0.006) = 1.62$ lb carbon/mol flue gas

Since there is unburned carbon in the ash, the amount of carbon actually burned per pound of coal must be determined.

Carbon in ash = 0.20(1500) = 300 lb/hr
Carbon in coal = 0.7344(10,183) = 7480 lb/hr
Carbon burned/lb coal = 7180/10,183 = 0.706 lb/lb coal

Amount of flue gas formed from 1 pound of coal is

$$\frac{0.706}{1.62} = 0.436 \text{ mol flue gas/lb coal}$$

Weight of $N_2$ in flue gas is

$$22.5(0.436) = 9.8 \text{ lb } N_2\text{/lb coal}$$

but the nitrogen in the fuel carries over into the flue gas and amounts to

$$0.0145 \text{ lb/lb of coal}$$

Weight of $N_2$ coming from air is

$$9.8 - 0.0145 = 9.786 \text{ lb } N_2 \text{ from air/lb coal}$$

$$\text{Weight of air} = 9.786/0.769 = \underline{12.72 \text{ lb air/lb coal}}$$

$$\text{Excess air} = \frac{12.72 - 9.86}{9.86}(100) = \underline{\underline{29\% \text{ excess air}}}$$

b) Since there is 0.436 mol flue gas/lb coal

$$\text{Weight} = [44(0.129) + 32(0.061) + 28(0.006) + 28(0.804)]\ 0.436$$
$$= 13.2 \text{ lb dry gas/lb coal}$$

$$2H + \frac{1}{2}O_2 = H_2O$$
$$2 \text{ lb} + 16 \text{ lb} = 18 \text{ lb}$$

1 lb H results in 9 lb $H_2O$

$\therefore$ Moisture = 0.0151 + 9(0.0476) = 0.44 lb moisture/lb coal

Total weight of stack gases = 13.64 lb wet gas/lb coal

c) If $H_2O$ present in the stack gases condenses, approximately 1100 Btu per pound would be given up. This heat is lost when the water remains as vapor in the stack.

$$\text{Loss} = \frac{0.44(1100)}{12,375}(100) = \underline{3.9\% \text{ loss to moisture}}$$

Note that this does not include the sensible heat contained in the water vapor because of the stack temperature of 375 °F.

3. Steam Powerplant, Preliminary Design

It is desired to make a preliminary design for a new 30,000-kw steam powerplant. Steam pressure has been set at 600 psig, maximum steam temperature leaving the superheater 750°F, and the lowest economical exhaust pressure 1.0 in. Hg absolute. Feedwater enters the boiler at 650 psig and 275°F, the engine (or internal) efficiency of the turbine is 85%, and the mechanical and generator losses are 8%. Find the following:

a) Steam rate in pounds per kwhr.

b) Internal diameter in inches of pipe required at turbine throttle; assuming 10% overload on turbine, steam velocity of 200 fps in pipe, and neglecting pressure drops.

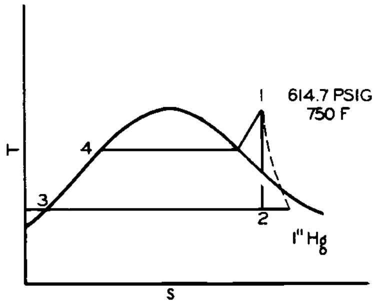
c) Capacity of condenser cooling water pump in gpm if all the steam at overload condition is condensed to liquid at 70°F, and a 15-degree rise in cooling water is allowed.

Theory: Changes in heat content or enthalpy can be taken from steam tables or charts (such as the Mollier diagram). Engine efficiency is the ratio of the actual thermal efficiency to the ideal thermal efficiency or, in this case, the ratio of the actual decrease in enthalpy to the theoretical decrease in enthalpy.

Solution: Theoretical heat drop through the turbine is

$$h_1 - h_2 = 1377 - 863 = 514 \text{ Btu/lb}$$

These values are most easily found on the Mollier diagram.




Condition 2 has the same entropy, and is vertically below 1 on the diagram.

Actual heat drop = 0.85(514)
= 436 Btu/lb

Btu necessary to develop 1 kwhr

$$\frac{3413}{0.92} = 3710 \text{ Btu/hr}$$

$$\text{Steam rate} = \frac{3710}{436} = \underline{\underline{8.5 \text{ lb/kwhr}}}$$

b)

v = 1.0 cu ft/lb, specific volume from steam table

$$\text{Volume of flow} = \frac{8.5(1.10)(30,000)(1.10)}{3600} = 85.7 \text{ cu ft/sec}$$

$$\text{Area} = \frac{V}{v_s} = \frac{85.7}{200} = 0.428 \text{ sq ft}$$

$$d^2 = \left(\frac{A}{0.784}\right) = \frac{0.428}{0.784} = 0.546 \text{ ft}^2$$

$$d = 0.743 \text{ ft} = \underline{\underline{8.9 \text{ inches inside diameter}}}$$

c) Loss of heat in condensing steam equals increase in heat of cooling water.

$$\text{Loss from steam} = W(h_2 - h_3)$$

$$h_2 = 1377 - 436 = 941 \text{ Btu/lb}$$

$$h_3 = 38 \text{ Btu/lb (from steam tables)}$$

With 8-1/3 lb per gallon, cooling water needed is

$$\frac{8.5(1.10)(30,000)(941 - 38)}{15(60)(8.33)} = \underline{\underline{33,800 \text{ gpm}}}$$

4. Vertical condenser

A vertical condenser for a process industry is composed of 20, 1-1/2 inch aluminum tubes 10 feet long and with 0.049-inch walls mounted in tube sheets 15 inches in diameter. Steam at atmospheric pressure enters the tubes at the top by means of a header, and leaves as 100°F condensate from the bottom header at the rate of 10.2 seconds per 1000 cc. Cooling water at 55°F enters at the bottom between the tubes and the shell and leaves out the top at 209°F at the rate of 7.0 seconds per gallon.

a) What rating in square feet and heat transfer capacity in Btu per hour would you give this unit?

b) What would be the logarithmic mean temperature difference between the two fluids?

c) Find the heat transfer coefficient in Btu per square foot per hour per degree F.

Theory: The usual equation for calculating flow of heat through a material is

$$Q = CA\,\Delta t = kA\,\frac{\Delta t}{L}\ \text{Btu/hr}$$

where C is the conductance, transmittance, or overall coefficient of heat transmission, and k is the thermal conductivity of the material per inch of thickness.

In many cases, such as in condensers, temperature difference $\Delta t$ is not uniform over the entire surface area.

In evaluating a mean temperature difference to use in equation above, it can be shown that for parallel or counterflow conditions when w, c, and C are constant

$$\Delta t_m = \frac{\Delta t_1 - \Delta t_2}{\log_e(\Delta t_1/\Delta t_2)}$$

(Called logarithmic mean temperature difference.)

Solution: a) The surface area of both tubes and tube sheets are considered. Since steam is condensing on the inside of the tubes, the inside surface area should be used.

$$\text{Area of tubes} = 20\left(\frac{\pi\ 1.402}{12}\right)(10) = \underline{73.5 \text{ sq ft}}$$

$$\text{Area of tube sheets} = 2\left[\frac{0.785(15)^2}{144} - 20\left(\frac{0.785(1.5)^2}{144}\right)\right] = \underline{1.98 \text{ sq ft}}$$

$$\text{Surface area} = \underline{\underline{75.5 \text{ sq ft}}}$$

Heat transferred could be determined from either the steam or the cooling water. However, no information as to the quality of the entering steam is given. It is probably not saturated, so it will be necessary to make the calculation based upon the cooling water.

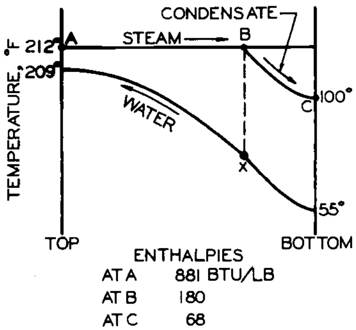
The water measured is at 209°F, corresponding to 59.9 lb/cu ft, or 8.0 lb/gal (from steam tables).

$$Q = w\ \Delta t$$

$$= \frac{8.0}{7.0}(3600)(209 - 55)$$

$$= \underline{\underline{633{,}000 \text{ Btu/hr}}}, \text{ heat capacity}$$

b) Temperature change in the condenser will vary in the manner shown in the sketch. Generally, the amount of undercooling is less than 4°F, and is neglected in determining the log mean temperature difference. In this case it will be necessary to calculate $\Delta t_m$ in two parts.




From A to B the steam will be considered at 212°F, while from B to C the temperature of the condensate will drop from 212°F to 100°F.

In order to determine the approximate temperature of the cooling water at x, assume rise in temperature is proportional to the heat removed. Then

$$t_x = 55 + (209 - 55)\left(\frac{180 - 68}{881 - 68}\right) = 76°\text{F}$$

$$\underset{C \to B}{\Delta t_m} = \frac{\Delta t_1 - \Delta t_2}{\log_e(\Delta t_1/\Delta t_2)} = \frac{(212 - 76) - (100 - 55)}{\log_e(136/45)} = \frac{91}{1.105} = 82.3\,°F$$

$$\underset{B \to A}{\Delta t_m} = \frac{(212 - 76) - (212 - 209)}{\log_e(136/3)} = \frac{133}{3.82} = 34.9\,°F$$

Now, using a weighted average of the two values

$$\Delta t_m = 82.3(0.138) + 34.9(0.862) = \underline{\underline{41.5\,°F}}$$

Had the undercooling been neglected, however, the value obtained would have been

$$\Delta t_m = \frac{(212 - 55) - (212 - 209)}{\log_e(157/3)} = 38.5\,°F$$

A larger error would have resulted if the calculations had been based only upon the end conditions.

$$\Delta t_m = \frac{(100 - 55) - (212 - 209)}{\log_e(45/3)} = 15.5\,°F$$

c) The heat transfer coefficient must be calculated using the log mean temperature difference.

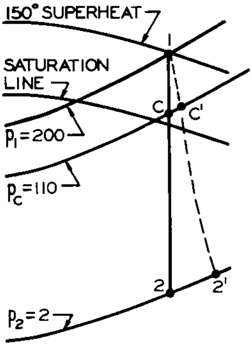
$$U = \frac{Q}{A\,\Delta t_m} = \frac{633,000}{75.5(41.5)} = \underline{\underline{202 \text{ Btu/sq ft, hr, °F}}}$$

5. Steam Nozzle Design

Calculate throat and exit areas of a nozzle to discharge 20 pounds per second of steam when the initial condition is 200 pounds per square inch absolute pressure and 150°F superheat. Discharge pressure is 2 pounds per square inch absolute. Friction loss is 3%.

Theory: Pressure at the throat cannot drop below the critical pressure, which will be 0.55 $p_1$ for superheated steam. As pressure drops further, the cross-sectional area of the nozzle must increase or the nozzle must become divergent.

Solution:




$p_c = 0.55\ p_1 = 110$ psia

From the Mollier diagram:

$h_1 = 1285$ Btu/lb; $s_1 = 1.641$

$h_c = 1227$ Btu/lb

$h_{c'} = 1227 + 0.03(58) = 1229$ Btu/lb

$h_2 = 953$ (16% moisture)

$h_{2'} = 953 + 0.03(332) = 963$ Btu/lb
(15% moisture)

Find velocities at each section.

$$\text{Velocity at throat} = 224\sqrt{h_1 - h_{c'}}$$

$$= 224\sqrt{56} = \underline{1680 \text{ fps}}$$

$$\text{Velocity at exit} = 224\sqrt{h_1 - h_{2'}}$$

$$= 224\sqrt{322} = \underline{4020 \text{ fps}}$$

Determine specific volume at each section.

$$v_{c'} = 4.47 + \frac{2}{10.7}(0.123) = 4.49 \text{ cu ft/lb}$$

(by interpolation from superheat table)

$$v_{2'} = 0.85(174) = 148.2 \text{ cu ft/lb}$$

Area at throat

$$A_{c'} = \frac{W_s v_{c'}}{V_{c'}} = \frac{20(4.49)}{1680} = 0.0535 \text{ sq ft}$$

$$A_{c'} = \underline{\underline{7.70 \text{ sq in. throat area}}}$$

$$A_{2'} = \frac{20(148.2)}{4020} = 0.738 \text{ sq ft} = \underline{\underline{106.3 \text{ sq in. exit area}}}$$

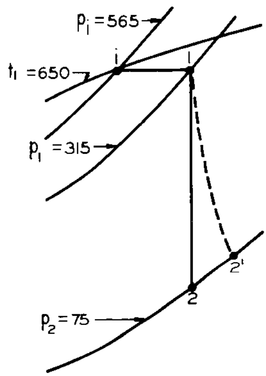
6. Steam Turbine and Turboblower—Steam and Power Requirements

A steam turbine-driven centrifugal blower operates at 3600 rpm. The turbine is a 2-row impulse type with a pitch diameter of 20 inches, nozzle angle of 20°, and nozzle throat area of 0.220 square inch. It operates initially on steam at 550 psig and 650°F, with a back pressure on the exhaust of 60 psig.

a) If steam chest pressure under a certain load is 300 psig, estimate steam requirements of the turbine in pounds per hour.

b) If a 7-inch diameter flow nozzle placed in the blower intake indicates a pressure drop of 1.8 inches of Hg at 75°F, and the blower is compressing the air to a static pressure of 24 inches of water, what would you estimate as the horsepower requirements of the blower?

Theory: The steam processes involved are best visualized on a Mollier diagram. Although initial pressure is 550 psig, it is throttled (constant enthalpy) to 300 psig in the steam chest.




Since back pressure on the nozzle (75 psia) is less than critical pressure ($p_c = 0.55\ p_2$ = 173 psia), flow through the nozzle is independent of back pressure and may be calculated using Grashof's formula (Marks, p. 335).

Flow of steam through the nozzle and turbine may be assumed to be isentropic, 1 to 2. Due to losses, however, the end point will actually be at 2', with a somewhat higher enthalpy than 2.

Solution: a) Using Grashof's formula to determine steam flow (A = throat area, sq in., $p_1$ = reservoir pressure, psia)

$$w = 0.0165\ A\ p_1^{0.97}$$

$$= 0.0165(0.220)(315^{0.97})$$

$$= 0.951\ \text{lb/sec} = \underline{\underline{3420\ \text{lb/hr}}}$$

b) Airflow through the nozzle can be determined from

$$Q = CA\sqrt{2gH}$$

$$H = \text{ft of fluid flowing} = \frac{h}{12}\left(\frac{847}{P/RT}\right)$$

or

$$Q = CA\sqrt{2g\left(\frac{h}{12}\right)\left(\frac{847\ RT}{P}\right)}$$

$$= 0.98\left[\frac{0.785(49)}{144}\right]\sqrt{2(32.2)\left(\frac{1.8}{12}\right)\left[\frac{847(53.3)(535)}{(14.7 - 1.8)(0.491)(144)}\right]}$$

$$Q = 89.6\ \text{cu ft/sec}$$

Theoretical adiabatic horsepower required is

$$P = \frac{k}{550(k-1)}\ p_1\ Q\left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right]$$

Using $k = 1.395$ for normal air, and $p_2 = 24$ in. water

$$P = \frac{1.395}{550(0.395)}\ (14.7)(144)(89.6)\left[\left(\frac{15.566}{14.7}\right)^{0.283} - 1\right]$$

$$P = 19.6\ \text{hp}$$

Efficiency of the blower can be estimated from the table in Marks (p. 1900) as e = 66% (for 5400 cfm).

At the operating load, horsepower required by the blower would be

$$P = \frac{19.6}{0.66} = \underline{\underline{29.7\ \text{hp}}}$$

However, if the blower is to be capable of absorbing the full capacity of the turbine, it will need to be larger.

## 7. Steam Pipeline Layout

The boiler house of an industrial plant contains two 150-horsepower, HRT, oil-fired boilers. Process steam is to be supplied to a new building through an insulated steel pipeline. Using Unwin's formula, compute size of pipe required by the following set of data, neglecting condensation and roughness of pipe:

| | |
|---|---|
| Boiler pressure, psig | 155.3 |
| Superheat, degrees F | 71.58 |
| Steam flow, lb/min | 80 |
| Pressure drop, % of gage pressure | 3 |
| Length of pipe, equiv feet | 994 |

Solution: The Unwin-Babcock formula is commonly used for flow of steam in clean steel pipes, as follows:

$$\Delta p = 0.000131\left(1 + \frac{3.6}{d}\right)\left(\frac{L\bar{V}\omega_m^2}{d^5}\right)$$

d = inside pipe diameter in inches
L = 944 ft = length of pipe
$\omega_m$ = 80 lb/min = rate of steam flow
$\bar{V}$ = 3.0335 cu ft/lb = average specific volume
$\Delta p$ = pressure drop = 0.03(155.3) = 4.65 psi

Solve by trial and error.

$$L\bar{V}\omega_m^2 = 994(3.0335)(80)^2 = 20,500,000$$

| d | $1 + \frac{3.6}{d}$ | $\frac{L\bar{V}\omega_m^2}{d^5}$ | $\Delta p$ |
|---|---|---|---|
| 3.6 | 2.0 | 34,200 | 8.95 |
| 4.026 | 1.895 | 19,340 | 4.80 |

A standard 4-inch steel pipe would cause a pressure drop of 4.80 psi, or 3.09% of boiler gage pressure. This should be satisfactory.

## 8. Fuel Cost Analysis, Turboelectric Steam Plant

A 2500-kilowatt, 3600-rpm turboelectric steam plant operates on steam at 600 psig and 750 °F total temperature, with a condenser vacuum of 28.5 inches of Hg. At full load its steam rate is 12.3 pounds per kwhr. Average daily load on the plant is 43% of full load, under which condition total steam consumption is 65% of that at full load. Using reasonable boiler efficiencies for equipment of this capacity, calculate average fuel cost in mills per kwhr for the following fuels:

| Fuel | Estimated cost | Higher heating value |
|---|---|---|
| a) Bituminous coal | $5.75/ton | 11,750 Btu/lb as fired |
| b) Fuel oil, PS 400, No. 6 | $1.90/barrel | 18,500 Btu/lb |
| c) Natural gas | $0.35/1000 cu ft | 1,050 Btu/cu ft |
| d) Hogged wood (wood waste) | $2.25/200 cu ft unit of 4000 lb with 45% water | 8,900 Btu/lb dry |

Solution:

Enthalpy of entering steam = 1378 Btu/lb

Enthalpy of condensate = 60 Btu/lb

Heat from steam to produce 1 kwhr at average load

$$h = \frac{12.3(2500)(0.65)(1378 - 60)}{2500(0.43)} = 24,500 \text{ Btu/kwhr}$$

Cost of fuel:

a) For coal (use efficiency = 80%):

$$\frac{24,500(\$5.75)}{0.80(11,750)(2000)} = \$0.0075 \text{ or } \underline{\underline{7.5 \text{ mills/kwhr}}}$$

b) For fuel oil (use efficiency = 85%):

$$\frac{24,500(\$1.90)}{0.85(18,500)(8.33)(42)} = \$0.00845 \text{ or } \underline{\underline{8.45 \text{ mills/kwhr}}}$$

c) For natural gas (use efficiency = 80%):

$$\frac{24,500(\$0.35)}{0.80(1050)(1000)} = \$0.0102 \text{ or } \underline{\underline{10.2 \text{ mills/kwhr}}}$$

d) For hogged wood (use efficiency = 70%):

$$\frac{24,000(\$2.25)}{0.70(8900)(4000)(0.55)} = \$0.00402 \text{ or } \underline{\underline{4.02 \text{ mills/kwhr}}}$$

## XI. AIR CONDITIONING AND REFRIGERATION

### 1. Walk-In Cooler

A walk-in cooler has inside dimensions of 6 by 8 by 7 feet high. Walls, ceiling, and floors are constructed on outside and inside with two thicknesses of 1/2-inch plywood sheathing. The 2- by 4-inch stud space is filled with redwood bark fiber packed to a density of 4 pounds per cubic foot. Inside temperature is 35°F and outside temperature 81°F. To allow for load, air changes, lights, etc., assume total load 50% greater than that due to heat leakage. Evaporating temperature is 25°F, using Freon 12. Find refrigerating capacity required and specify horsepower of the motor to drive the compressor.

Solution:

$$U = \frac{1}{1/f_i + x_1/k_1 + x_2/k_2 + x_3/k_3 + 1/f_o}, \text{ coefficient of heat transmission}$$

Since both walls are indoors, assume an average value of 1.65 for both $f_i$ and $f_o$, the surface conductances (Heating, Ventilating, and Air Conditioning Guide).

$k_1 = k_3 = 0.8$ for plywood

$k_2 = 0.28$ for redwood bark

$$U = \frac{1}{1/1.65 + 1/0.8 + 3.62/0.28 + 1/0.8 + 1/1.65}$$

$U = 1/16.6 = 0.0602$ Btu/hr, sq ft, °F

Calculate the area as that of a room intermediate in size between the inside and outside dimensions.

$$\text{Length} = 8 + \frac{5.62}{12} = 8.47 \text{ ft}$$

Width = 6.47 ft
Height = 7.47 ft

$$\text{Area} = 2\left[6.47(7.47)\right] + 2\left[8.47(7.47)\right] + 2\left[6.47(8.47)\right]$$

$$= 332.8 \text{ sq ft}$$

Heat leakage = 332.8(0.0602)(81 - 35) = 922 Btu/hr

Total load = 922(1.5) = 1380 Btu/hr

If refrigerating unit is assumed to operate two-thirds of the time, capacity of the unit should be

$$\frac{1380}{2/3} = \underline{\underline{2070 \text{ Btu/hr}}}$$

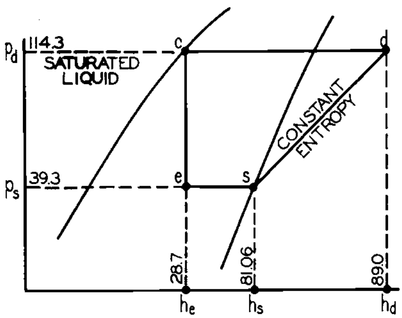
A ton of refrigeration is 200 Btu/minute. Therefore, capacity required is

$$\frac{2070}{200(60)} = \underline{\underline{0.1725 \text{ tons of refrigeration}}}$$

The refrigeration cycle is best shown on the p-h, pressure-enthalpy chart.

Saturated vapor from the evaporator is compressed from s to d. The condenser cools and condenses the vapor from d to saturated liquid at c.

In passing through the expansion valve the pressure drops




from c to e, and some of the liquid vaporizes. The remainder vaporizes in the evaporator e to s, where heat is removed from the refrigerated space.

Suction and discharge pressures are selected to give the desired temperatures in the evaporator and condenser.

$$p_s = 39.3 \text{ psia, corresponding to } 25\,°\text{F}$$

$$h_s = 81.06 \text{ Btu/lb (saturated vapor)}$$

If temperature available for cooling the condenser is assumed to be 90 °F, then

$$p_d = 114.3 \text{ psia}$$

$$h_d = 89.0 \text{ Btu/lb (superheated)}$$

$$h_c = 28.7 \text{ Btu/lb (saturated liquid)}$$

Heat absorbed in evaporator

$$h_s - h_e = 81.06 - 28.7 = 52.36 \text{ Btu/lb}$$

Freon circulated

$$\frac{2070}{52.36} = 39.6 \text{ lb/hr}$$

Work done by compressor

$$h_d - h_s = 89.0 - 81.06 = 7.94 \text{ Btu/lb}$$

$$W = \frac{7.94(39.6)}{2545} = 0.123 \text{ hp}$$

If an efficiency of 60% is taken for the compressor, then motor horsepower required would be

$$\text{hp} = \frac{0.123}{0.60} = 0.206 \text{ hp}$$

A 1/4-hp motor should be satisfactory.

Note: This analysis is based upon a simple saturation system. Vapor leaving the evaporator would be taken practically as having 10°F of superheat.

2. Ammonia Refrigeration

In connection with an industrial process, it is desired to cool 10,000 cfm of air at 15 psia from 90°F dry bulb and 80°F wet bulb temperature, down to 55°F dry bulb temperature by passing the air over the evaporator coils of an ammonia refrigeration plant.

a) If saturation temperature of liquid $NH_3$ must be 20°F below the lowest air temperature, what should be the suction pressure in the evaporator in psig?

b) If $NH_3$ enters the evaporator as saturated liquid and leaves as saturated vapor, what quantity of ammonia should be circulated in pounds per minute?

c) If it takes 300 square feet of surface area in the coil to cool the above air (plus 15% additional pickup), what should be the capacity of the plant in tons of refrigeration?

d) What would be the heat transfer coefficient of the evaporator coil under these conditions?

Solution:

a) Saturation temperature = 55 - 20 = 35°F. Therefore, the suction pressure required in the evaporator is 51 psig. (See tables or p-h chart.)

b) Increase in enthalpy of $NH_3$ evaporating at 35°F is

$$h_{fg} = 540.5 \text{ Btu/lb}$$

A pound of dry air at the given conditions has 139 grains of water vapor with it (see psychrometric chart). The vapor pressure of the water is 0.455 psi, leaving the air pressure at 14.505 psi.

$$\text{Specific volume} = \frac{wRT}{P} = \frac{1.0(53.3)(460 + 90)}{14.505(144)}$$

$$= 14.0 \text{ cu ft/lb dry air}$$

139 grains = 139/7000 = 0.0199 lb moisture, which also is present and occupies the same volume as the dry air. The General Electric psychrometric chart gives total heat for both air and moisture in terms of Btu/lb of dry air.

Weight of dry air is

$$w = \frac{10,000}{14.0} = 715 \text{ lb dry air/min}$$

Total heat given up by the air is

$$\Delta h = 43.5 - 23.2 = 20.3 \text{ Btu/lb dry air}$$

$\therefore$ Heat given up by air = 715(20.3) = 14,500 Btu/min

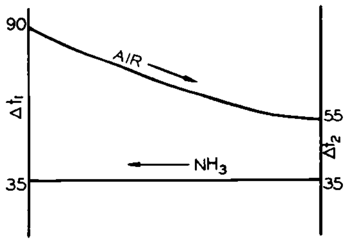
$$NH_3 \text{ required} = \frac{14,500}{540.5} = \underline{\underline{26.8 \text{ lb/min}}}$$

c) Refrigerating capacity is

$$\frac{14,500(1.15)}{200} \doteq \underline{\underline{83.5 \text{ tons}}}$$

d) Heat transfer coefficient is defined as

$$U = \frac{H}{A\,\Delta t_m}$$




where $\Delta t_m$ is the mean temperature difference. Here, the mean $\Delta t$ must be determined as the log mean temperature difference.

$$\Delta t_m = \frac{\Delta t_1 - \Delta t_2}{\log_n \frac{\Delta t_1}{\Delta t_2}}$$

$$= \frac{55 - 20}{\log_n \frac{55}{20}} = \frac{35}{1.01} = 34.6\,°F$$

$$U = \frac{14,500(1.15)(60)}{300(34.6)} = \underline{\underline{98.9 \text{ Btu/hr}}}, \text{ sq ft, } °F$$

3. Barley Dryer

For manufacture of beer, barley is dried by direct contact with air at 175°F. Air is heated to 175°F by heat exchange with hot furnace gases. It is proposed to eliminate the heat exchanger by directly mixing the hot gases with sufficient outside air to produce a temperature of 175°F, and drying the barley with this mixture.

a) Will the latter system work favorably under the following conditions:

| | |
|---|---|
| Outside air | 75°F dry bulb, 70°F wet bulb |
| Barley | 110°F |
| Fuel | methane gas burned with 20% excess air |
| Furnace gases | 2500°F (assume $c_p$ for furnace gases and air constant at 0.24 Btu/lb F) |

b) By which system will the barley dry faster for a fixed rate of warm air flow?

Give supporting data for answers a and b above.

Solution: From an overall view it is clear that the second method introduces moisture from the burned gas into the drying air. For the same temperature of drying air, this higher humidity would slow the rate of drying the barley.

Consider the first system.

Outside air: 75°F db, 70°F wb, 68°F dp, 78% RH, 102 grains SpH.

When heated to 175°F: 175 db, 92.5 wb, 68 dp, 5% RH, 102 grains SpH.

At wet bulb temperature (92.5°F), air could hold 238 grains of moisture, or a pound of dry air to become saturated could pick up 238 - 102 = 136 grains of water.

Consider the second system. Moisture formed by burning the H in $CH_4$ will be

$$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$$

or $\quad$ 2 mols water per mol $CH_4$

When burned with 20% excess air

$$1 \text{ mol } CH_4 \longrightarrow 1 \text{ mol } CO_2 + 0.4 \text{ mol } O_2 + 2 \text{ mol } H_2O + 9.03 \text{ mol } N_2$$

$$16 \text{ lb} \longrightarrow 44 + 0.4(32) + 2(18) + 9.03(28)$$

$$16 \text{ lb fuel} \longrightarrow 345.3 \text{ lb furnace gas}$$

or $$\frac{345.3}{16} = 21.6 \text{ lb gas/lb fuel used}$$

In cooling the hot gases from 2500°F to 175°F by mixing with outside air at 75°F, the following heat balance would apply (assuming $c_p = 0.24$ for both):

$$W_a c_p (175 - 75) = 21.6\, c_p (2500 - 175)$$

$$W_a = 21.6\left(\frac{2325}{100}\right) = 506 \text{ lb mixing air/lb fuel}$$

Weight of air supplied to burn fuel is

$$\frac{2.4(29/0.21)}{16} = 20.75 \text{ lb air/lb fuel burned}$$

Thus total air is 527 lb/lb fuel.

Total moisture in the final mixture is $W_f$ water from fuel, plus $W_a$ moisture from air.

$$W_f = \frac{2(18)}{16} = 2.25 \text{ lb/lb fuel}$$

$$W_a = 527\left(\frac{102}{7000}\right) = 7.68 \text{ lb/lb fuel}$$

$$W_f + W_a = 9.93 \text{ lb water/lb fuel, total moisture}$$

In order to determine vapor pressure of water, and thus the dew point, it will be necessary to express the quantities on a mol basis.

$$9.93 \text{ lb water} = \frac{9.93}{18} = 0.552 \text{ mol water}$$

$$1 \text{ mol } CH_4 \longrightarrow 10.43 \text{ mols dry gases}$$

$$1 \text{ lb } CH_4 \longrightarrow \frac{10.43}{16} = 0.653 \text{ mol dry gases}$$

$$\text{Hot mixture} = \frac{506}{29} \text{ air} + 0.653 \text{ dry gas} + 0.552 \text{ water} = 18.65 \text{ mols}$$

$$\text{Vapor pressure} = \frac{0.552}{18.65}(30) = 0.888 \text{ in. Hg}$$

This vapor pressure corresponds to a dew point of 75°F, specific humidity of 132 grains/lb dry air, and relative humidity of about 6-1/2%.

The available drying capacity by the second method would be 106 grains/lb compared to 136 by the first method. Since air leaving the dryer will not be saturated, this difference in drying capacity will be even more pronounced.

Although the second system would be satisfactory and require less fuel, a longer drying period would be required for the same rate of warm air flow.

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