

AN ABSTRACT OF THE THESIS OF

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Soil moisture in a crop's root zone must be regularly monitored to accurately predict crop water requirements. Most irrigation scheduling services currently use the neutron probe to monitor soil moisture. Unfortunately, there is a degree of variability associated with use of the neutron probe creating uncertainty in measurements of soil moisture.

A review of the literature reveals there are three major types of error in estimates of average field moisture by the probe: instrument error, calibration error, and location error. Instrument error accounts for random neutron emission by the radioactive probe source. Calibration error is introduced by inaccuracies in the calibration curve. Location error is a result of estimating average soil moisture throughout a field using only point estimates.

This study attempts to provide the neutron probe user with a quick and easy means to evaluate estimates of soil moisture by the probe and to identify sources of error. A computer program is developed that quickly and easily analyzes field calibration data to evaluate the error in neutron probe measurements.

Field data from Colorado are analyzed as a numerical example of applying the mechanics of the computer model. Similar data from several California soils are analyzed using the model to show how the range of moisture content over which the probe is calibrated restricts accurate neutron probe measurements to the range of calibration data. The Colorado data are further analyzed to determine to what extent measurement errors occurring in the calibration process will contribute to calibration error in general.

In most cases, neutron probe access tubes are placed at biased locations within a field and not randomly as is assumed in the model. A method is developed to correct estimates of average field moisture by taking this discrepancy into account. Given the variability of field moisture content, the number of access tubes that must be sampled to achieve a desired level of accuracy may be calculated using the definition of the student's "t" parameter and referring to specific confidence intervals.

Uncertainty in Neutron Probe Measurements
of Soil Moisture for Irrigation Scheduling

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UNCERTAINTY IN NEUTRON PROBE MEASUREMENTS OF SOIL MOISTURE FOR IRRIGATION SCHEDULING

1. INTRODUCTION

To accurately predict crop water needs, the moisture content of soil in the crop root zone must be regularly monitored. A neutron probe is the device most frequently used by irrigation scheduling services to measure soil moisture. There is an inherent degree of variability as well as calibration error in neutron probe readings. Thus, an element of uncertainty is present in measurements of soil moisture. It is impossible to know exactly how soil water is distributed in heterogeneous field soils, causing additional uncertainty in estimates of "average" moisture content. Consequently, too much or too little water may be applied during irrigation. The result can be a loss in yield and profits. It is important to monitor soil moisture in the root zone as precisely as possible.

This study attempts to provide the neutron probe user with a quick and easy means to evaluate estimates of soil moisture by the probe and to identify sources of error. An algorithm for adjusting soil moisture estimates to compensate for neutron probe errors is also presented.

Specifically the objectives are to:

1. Verify equations recently appearing in the literature that estimate the variance in observed values of soil moisture using the probe by presenting a complete derivation of the equations;
2. Show how these equations can be used with field data to estimate the error in neutron probe measurements of soil moisture and to identify sources of error in these measurements;
3. Develop a computer program that quickly and easily analyzes field calibration data to evaluate the error in neutron probe measurements;
4. Examine the significance of deviations in certain parameters of the computer model.

Field calibration data from Colorado are analyzed as a numerical example of applying the mechanics of the computer model. Similar data from several California soils are analyzed using the model to show how the range of moisture content over which the probe is calibrated restricts accurate neutron probe estimates of moisture content to the range of calibration data. The Colorado

data are further analyzed to determine to what extent measurement errors occurring in the calibration process will contribute to calibration error in general.

In most cases, neutron probe access tubes are placed at biased locations within a field and not randomly as is assumed in the computer model. A method is developed to correct estimates of average field moisture by taking this discrepancy into account. Given the variability of field moisture content, the number of access tubes that must be sampled to achieve a desired level of accuracy may be calculated using the definition of the student's "t" parameter and referring to specific confidence intervals.

2. BACKGROUND

Farm crop productivity is inversely related to the degree of stress a crop undergoes due to limited availability of water throughout its growing season. The relationship is not linear and certain periods of growth are more susceptible to damage than others. However, it is generally desirable to minimize any depletion of soil water storage that will result in a significant loss in crop yield.

Two fundamental questions must be answered to successfully provide the crop with an adequate water supply:

(1) When must the soil water reservoir be supplemented by irrigation? and (2) How much water will need to be applied? The answers are provided by considering a water volume balance over the control volume defined by the crop, its immediate environment and the surrounding soil. This concept is shown in Figure 1.

Inputs to the control volume are irrigation water and precipitation in the form of rain or snow. Outputs are surface runoff, deep drainage to soil below the crop root zone, evaporation from the soil and plant surfaces and transpiration by the plants. The later two mechanisms are commonly combined and identified as evapotranspiration.

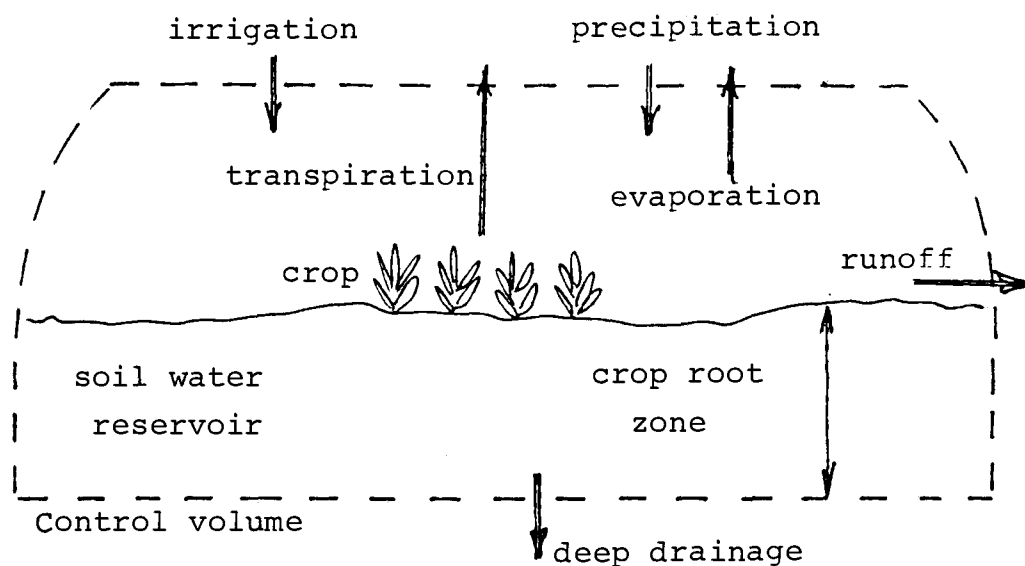


Figure 1. Control volume for a crop water balance.

Evapotranspiration (ET), or consumptive use as defined by the American Society of Civil Engineers (ASCE, 1973), is the quantity of water transpired by plants during their growth or retained in the plant tissue plus the moisture evaporated from the surface of the soil and the vegetation. The quantity is specified as a depth unit per specified time (e.g. 5.1 mm/day).

The change in water storage in the soil reservoir, ΔW_s , can be expressed as the difference between the inflows and outflows through the control volume:

$$\Delta W_s = \text{precipitation} + \text{irrigation} - \text{runoff} - \text{ET} \quad (1)$$

- drainage

Measuring precipitation by a conventional rain gauge provides an estimate of the inflow to the control volume between irrigations. Estimating the outflows during this period, normally on a daily basis, allows the calculation of the change in water storage by equation (1). When an accumulated deficit in water storage reaches a predetermined amount, an irrigation is required to replenish water supplies. The amount the irrigation must apply is equal to the accumulated negative change in water storage plus any water lost to wind losses, runoff and percolation. The percolation amount may be unavoidable losses, or a deliberate excess applied to leach undesirable minerals from the crop root zone.

An acceptable level of soil water depletion is a management decision based on characteristics of the soil, the crop and economics. Often, a 50% depletion of the total available water within the root zone is used as a limiting value. The amount of total available water, TAW, is calculated by:

$$TAW = (FC - PWP) (Z_r) , \quad (2)$$

where FC is the field capacity of the soil, PWP is the permanent wilting point of the plant and Z_r is the rooting zone depth. Field capacity, expressed as a volume of water per unit bulk volume of soil (cm^3/cm^3),

is a measure of the water held in the soil matrix by capillary action after gravitational drainage has ceased. This normally occurs two to three days following a thorough irrigation or rain. While gravitational drainage is occurring, the crop is extracting water but is doing so from the readily available water not held by soil matrix suction. There is then zero depletion of the soil water reservoir during this period. As the soil dries, water becomes increasingly hard for a plant to obtain since the plant must overcome the matrix suction in order to extract water from the soil. If too much effort is expended satisfying water needs, physiological functions of the plant become impaired. The level at which further soil moisture depletion will cause irreparable damage to the plant is called the permanent wilting point. A decrease in crop yield is experienced much sooner, somewhere around 40-50% depletion of the total available water. Some generalized acceptable levels of depletion are shown for several crops in Table 1.

The crop root zone depth is not constant, but changes as plants grow and extend their roots down into the soil to seek water. The rate of root growth is a function of many things: soil type, crop species and water availability, to name a few. Rooting depth is assumed to increase linearly from emergence of the crop

TABLE 1

Allowable root zone water depletion between irrigations for near maximum yield as applied to scheduling of "set type" sprinkler and non-automated gravity systems **

Crop	Allowable water depletion (%)	Normal root zone depth* (cm)
Alfalfa	30-50	120-180
Beans, dry	50-70	60-90
Corn	40-60	75-120
Cotton	50-65	90-120
Deciduous fruit	50-70	120-180
Potatoes	25-50	60-90
Sugar beets	30-60	90-120
Grain sorghum	50-70	90-120
Soybeans	50-60	60-90
Wheat	50-70	90-120
Vegetable crops	25-50	60-120

*for normally irrigated, deep soils

**adapted from Bausch, 1983

to full cover, and remain constant thereafter (Bausch, 1983).

In many water balance accounts, deep drainage and runoff are ignored. Evapotranspiration is estimated from climatic data. Estimates of ET are then used with measured precipitation to predict when an irrigation will

be necessary to replenish the soil water profile. Predictions by the water balance method must be frequently checked against measurements of soil moisture taken in situ.

Measurement of a soil's moisture content has traditionally been made using the feel method. A farmer uses his own judgement to evaluate moisture content by pressing a sample of soil between his thumb and finger. Alternatively, estimates may be obtained by observing crop color or turgor. Unfortunately, visible signs of crop stress do not appear until after damage has begun. Neither method indicates how much water must be applied.

Resistance blocks measure change in the electrical properties of the soil. This change can then be used to estimate soil moisture. However, resistance blocks do not function well in wet or saline soils and do not measure gravitational potential. They also measure moisture at only one point in the profile. A calibration procedure is necessary to effectively utilize resistance blocks.

Tensiometers measure soil moisture tension as an indicator of soil moisture. The effective range of tensiometers is very limited, however. At sea level they can measure tensions up to about 0.85 bars.

Relatively precise measurements can be made by gravimetric sampling. Representative soil samples are

weighed, then oven-dried at 105°C for 24 hours, which is assumed to drive off all free water. The change in weight indicates moisture content. This method is often used to calibrate instrumentation used for monitoring soil moisture by other means. To be accurate careful sampling technique is required to avoid compacting the soil sample or disturbing the sample in any way that would make it unrepresentative of the soil from which it came. Gravimetric sampling can be time consuming and cannot be repeated at the same location since previous sampling will have disturbed or removed the soil.

The temperature differential between the crop canopy and the surrounding air has been related to the air vapor pressure deficit, providing a means to define a potential evaporative rate for the vegetation. Since plant water potential can be related to soil moisture, a soil water index related to plant water potential can be devised using temperature and vapor pressure data (Idso et al., 1981; Slack et al., 1981). Remote sensing techniques such as aerial infrared photography give a quick measure of foliage surface temperature over a large area, making this an attractive guide to irrigation scheduling. To date, however, infrared thermometry is still being investigated and has not reached the stage of practical application.

Currently, soil moisture content is most often measured for irrigation scheduling using a neutron probe. A radioactive source, lowered into the soil via an access tube, emits energized neutrons which collide with hydrogen atoms in the soil. Losing energy from multiple collisions, the neutrons enter a reduced energy state, enabling their detection by a thermal neutron detector in the probe. The number of neutrons detected indicates the amount of hydrogen, and therefore presumably the amount of water, present in the volume of soil sampled. The neutron probe provides a quick and easy way to repetitively measure moisture content at the same location. The range of operating conditions over which the probe is effective encompasses most situations likely to be encountered in agricultural soils.

Despite its popularity, there are problems associated with use of the probe. Inherent in its operation is the random emission of radioactive neutrons. This random process creates variation in neutron probe readings. Certain physical and chemical properties, such as the presence of large amounts of organic matter, may also bias the neutron count. To correct for these conditions, the probe must be calibrated to the site being monitored. This is usually done using a gravimetric sampling technique. The calibration procedure may introduce measurement errors in addition to the

errors inherent in the use of the probe itself.

Almost all methods of measuring soil moisture, with the exception of infrared thermometry, provide a point estimate of moisture content. Irrigation scheduling requires evaluation of the water content throughout an entire field. Since water content varies with location in any given field, using a point estimate to approximate the true mean field moisture may produce poor results unless a sufficient number of representative locations is sampled.

Knowledge of the magnitude and source of errors associated with the neutron probe will enable an operator to interpret results more realistically and, perhaps, to compensate accordingly. More accurate measurements of soil moisture can then be obtained, increasing the level of confidence that can be placed in the scheduling of irrigations.

3. REVIEW OF LITERATURE

3.1 Theory of Moisture Measurement by the Neutron Probe

A radioactive source, lowered into the soil via an access tube, emits energized neutrons which collide with atoms in the soil. Losing energy from multiple collisions, the neutrons enter a reduced energy state, enabling their detection by a thermal neutron detector in the probe. Hydrogen atoms are primarily responsible for slowing down energized neutrons in agricultural soils. The number of neutrons detected indicates the amount of hydrogen, and presumably water, present in the volume of soil sampled.

The volume of soil sampled by the probe is not constant, but changes with moisture content. The effective volume in which 95% of the slow neutrons exist is called the sphere of influence.

Theory of Neutron Interaction

The neutron probe contains a radioisotope neutron source which emits fast (energized) neutrons. The source is generally a mixture of Beryllium powder (a potential neutron emitter) and Americium (an alpha particle emitter) in a tightly compressed pellet. The alpha particles strike the Beryllium atoms, exciting neutrons to an

average energy of five million electron volts. This increased energy is reflected in the neutron's accelerated velocity as it leaves the source.

These fast neutrons have no electrical charge, and only lose energy upon collision with other particles in the surrounding medium. A collision with a very much larger or smaller particle does not cause a significant energy loss. However, a collision with the nucleus of an atom of similar size results in the fast neutron being either absorbed into the other atom's nucleus, or bounced off with a corresponding transfer of energy to the nucleus material. After a series of collisions have reduced the neutron's energy level below 0.5 electron volts, it is referred to as a thermal or slow neutron (Int. Atomic Energy Agency, 1970).

Thermal neutrons can be identified using a Boron Trifluoride-NF₃ or a Helium-3 H₃ tube detector. The Helium-3 H₃ type is more efficient. Gas in the detector is ionized by the neutrons, creating disturbances which are capable of being amplified and transmitted as a signal and displayed electronically (Dickey, 1982).

Hydrogen atoms are the primary source of nuclei capable of slowing down accelerated neutrons in most agricultural soils. One head-on collision with a hydrogen atom can reduce a fast neutron to the thermal energy level. Head-on collisions are rare; an average

of 18 collisions at random angles with hydrogen atoms are required to reduce a neutron from a high level to a thermal energy level. By comparison, it requires an average of 115 collisions with carbon atoms to reach this state. The probability that neutrons will collide with an atom of any particular element is described in terms of area and referred to as an atom's capture cross-section. Although this cross-section does not directly relate to the physical size of a nucleus, the larger an atom's cross-section is the more chance there is of neutron-atom interaction. This concept is used to compare the relative significance of various elements in the soil (Int. Atomic Energy Agency, 1970). A common unit of cross-section is called a barn. One barn is equivalent to 10^{-24} cm^2 . Capture cross-sections for elements found in the soil are given in Table 2.

The number of thermal neutrons detected by the probe can be related to soil moisture using a calibration curve. A neutron probe calibration curve is a plot of neutron meter counts versus water-volume percentage, determined by taking probe reading and gravimetric samples at the same site. In practice, the ratio of the actual number of thermal neutrons counted in the soil to the number of thermal neutrons recorded in a standard medium is used as a measure of soil moisture. Thus the calibration curve relates that ratio to soil

TABLE 2

Capture Cross-Sections for Thermal Neutrons of the
Most Abundant Soil Elements (from Burn, 1966)

Element	Relative abundance		Capture cross section (barns)
	By weight(%)	By atom(%)	
Oxygen	46.46	60.36	0.0016
Silicon	27.61	20.45	0.16
Aluminium	8.07	6.21	0.23
Iron	5.06	1.89	2.5
Calcium	3.64	1.89	0.43
Sodium	2.75	2.49	0.5
Potassium	2.58	1.37	2.2
Magnesium	2.07	1.77	0.4
Titanium	0.62	0.27	5.2
Hydrogen	0.14	2.89	0.3
Carbon	0.09	0.17	0.0045
Chlorine	0.05	0.02	33.0

moisture. Use of a count ratio will offset the effects of radioactive decay with time, assuring that reference to the calibration curve at a later date will still be accurate.

Imaginary Sphere of Influence

The distribution of thermal neutrons around a point source in a homogeneous medium is postulated to resemble a sphere. In theory, this sphere is unbounded for a radioactive source. However, the volume within which practically all (95%) of the thermal neutrons are found is referred to as the "sphere of influence". Removal of the soil outside this sphere will have a negligible effect on the number of thermal neutrons detected (VanBavel, 1956). The radius of the sphere of influence changes with water content, since neutrons travel farther before running into hydrogen atoms in drier soils. The sphere of influence is typically described by a radius of 15 cm for a radium-beryllium source in pure water, increasing to approximately 60 cm in dry soil. The relation between the radius, R , and water-volume (percent by volume), V_w , is given by (VanBavel, 1955):

$$R = 15 \sqrt[3]{100/V_w} \text{ cm} . \quad (3)$$

Imaginary Sphere of Importance

Another description of the neutron distribution is the sphere of importance. Olgaard, 1965, has defined the sphere of importance as the "sphere around the source, in a moderating medium, which if all soil and

water outside the sphere is removed, will yield a neutron flux at the source that is 95% of the flux obtained if the medium is infinite". The sphere of importance may be a better approximation of the neutron distribution when the detector is located close to the source. A simplified expression for the sphere of importance is given by Olgaard, (1965):

$$R = 100 / (1.4 + 0.1 V_w) \text{ cm} . \quad (4)$$

Although the volume in which neutrons interact in the soil is assumed to resemble a sphere, the slow neutron flux is greatest near the source. Thus, conditions nearest to the source are of greatest influence.

3.2 Sources of Error in Neutron Probe Measurements

Early attempts to quantify sources of error in neutron probe estimates of field average soil moisture identify instrument error, timing error and location error as the major components (Hewlett, 1963). Instrument error accounts for the random emission of neutrons and electronic "noise" in the equipment. Location error is a result of using point estimates to predict field-wide moisture. Timing error refers to errors in measurement of the amount of time during which neutron counts are recorded. Before timers were installed in

the probe itself, time intervals were measured with less precision, and were a major source of concern. Now, electronic timers have reduced this error to negligible size.

More recent research has indicated that instrument error, calibration error and location error are the types of errors to consider (Sinclair and Williams, 1979; Haverkamp et al., 1984). Calibration error results from inaccuracies in the calibration equation that relates neutron probe readings to soil moisture content.

Instrument Error

Electronic "noise" is undetectable in modern equipment. Now, only the random emission of radioactive neutrons contributes to instrument error.

The number of atoms which will decay in a given amount of radioactive material during a specified time interval, Δt , tends to be distributed around a central value with a characteristic spread. This distribution can be approximated by a normal distribution where the central tendency is defined as the mean, and the variance is a measure of the dispersion of the distribution about the mean.

If a finite number of time intervals are observed, an estimate of the mean, M , and variance, s^2 , can be

calculated by (Snedecor and Cochran, 1980):

$$M = \frac{1}{N} \sum_{i=1}^N X_i = \sum_{i=1}^{\infty} (X_i \cdot P(X_i)) \quad (5)$$

and

$$s^2 = \frac{1}{N-1} \sum_{i=1}^N (X_i - M)^2 = \sum_{i=1}^N ((X_i - M)^2 \cdot P(X_i)) \quad (6)$$

where N represents the number of time intervals observed and $P(X_i)$ represents the probability that X_i number of atoms will decay during a time interval Δt_i .

Equations (5) and (6) still apply if only one time interval is observed and N is let represent the number of nuclei in a sample of radioactive material. This allows the decay process to be modeled as a binomial experiment. In a binomial experiment, each trial of the experiment has only two outcomes: success or failure. Also, the probability, p , that any one trial will result in a success must be equal to and independent of the chance of success for any other trial. For the decay process, each trial considers one particular atom. An atom undergoing decay is designated a success. The probability that any one atom will undergo decay is equal to and independent of the chance that any other atom will decay. The probability that exactly k nuclei will decay can be given by the following expression

describing a binomial distribution (Snedecor and Cochran, 1980):

$$P(k) = \frac{N!}{(N-k)! k!} p^k (1-p)^{N-k} . \quad (7)$$

The mean of all possible numbers of atoms that might have decayed from the sample during Δt is expressed by (Snedecor and Cochran, 1980):

$$M = \sum_{i=1}^N k_i \cdot P(k_i) , \quad (8)$$

where k ranges from 0 to N , the total number of atoms in the sample.

Substituting equation (7) into equation (8) and carrying out the summation, the mean can be calculated as:

$$M = p \cdot N . \quad (9)$$

The variance of a binomial distribution can be described by (Snedecor and Cochran, 1980):

$$s^2 = \sum_{i=1}^N ((k_i - M)^2 \cdot P(k_i)) . \quad (10)$$

Substituting equation (7) into equation (10) and carrying out the summation gives the variance as:

$$s^2 = N \cdot p \cdot (1-p) . \quad (11)$$

There are many nuclei in a radioactive sample of

the size found in neutron probes, but very few actually undergo decay during the length of time typically observed in counts for determining water content. The probability, p , of just one particular nucleus decaying is then very small.

Examining equation (7), the term $(1-p)^{N-k}$ can be manipulated to yield:

$$(1-p)^{N-k} = (1-p)^N / (1-p)^k = [(1-p)^{1/p}]^{Np} / (1-p)^k. \quad (12)$$

As p becomes very small, and in the limit, $(1-p)^k$ goes to one and:

$$\lim_{p \rightarrow 0} (1-p)^{1/p} = 1/e. \quad (13)$$

Equation (12) can be summarized utilizing this result:

$$(1-p)^{N-k} \rightarrow e^{-Np}. \quad (14)$$

As N becomes very large, effectively approaching ∞ , the second term of equation (7) can be evaluated:

$$\frac{N!}{k! (N-k)!} = \frac{N(N-1) \dots (N-k+1)}{k!} \quad (15)$$

$$\frac{N!}{k! (N-k)!} \rightarrow \frac{N^k}{k!} \quad (16)$$

Equation (7) can now be approximated using these results:

$$P(k) = \frac{N^k}{k!} \cdot p^k \cdot e^{-Np}. \quad (17)$$

Substituting equation (9) into equation (7) gives:

$$P(k) = \frac{M^k}{k!} \cdot e^{-M}, \quad (18)$$

which describes a Poisson distribution. Thus the Poisson distribution can approximate a binomial distribution.

Substituting equation (18) into equation (10) and carrying out the summation leads to:

$$s^2 = \sum_{i=1}^N ((k_i - M)^2 \cdot P(k_i)) = M. \quad (19)$$

The variance of all possible counts of decaying nuclei is equal to the expected value of counts. This is a result of assuming the counts to be Poisson distributed. The assumption requires that there be a large number of independent trials in which the probability of any individual trial having a successful outcome (undergoing decay) is very small.

These results can be applied to the neutron probe, assuming the number of radioactive neutrons emitted by the probe during a specified time interval to be Poisson distributed. A method for doing this is presented in Section 4.2.

Calibration Error

The neutron probe does not measure moisture content directly. Soil physical and chemical properties

influence the number of slow neutrons returning to the probe detector. Access tube installation also influences the slow neutron count. Readings taken near the soil surface are influenced by atmospheric moisture. For these reasons, neutron probes require calibration curves to indicate moisture content.

Calibration curves are developed by relating probe readings to moisture content determined by gravimetric sampling for several levels of soil moisture. Measurement errors in gravimetric sampling techniques may introduce error to the calibration curve.

Influence of Elements in the Soil

Several elements have been found to influence the count of thermal neutrons. Chlorine, boron, cadmium and iron commonly occur in many soils and affect thermal neutron activity (see Figures 2 and 3). Correction for the interaction of these elements is made by developing a calibration curve for each soil in which a neutron probe is to be used.

A change in chlorine content of 0.0074 g/g of soil solution has been shown to reduce the slope of the calibration curve by as much as 10 percent. An equal effect can be attributed to the addition of 100 ppm. of boron to a soil solution (Holmes, 1957; Benz, 1965). Iron has

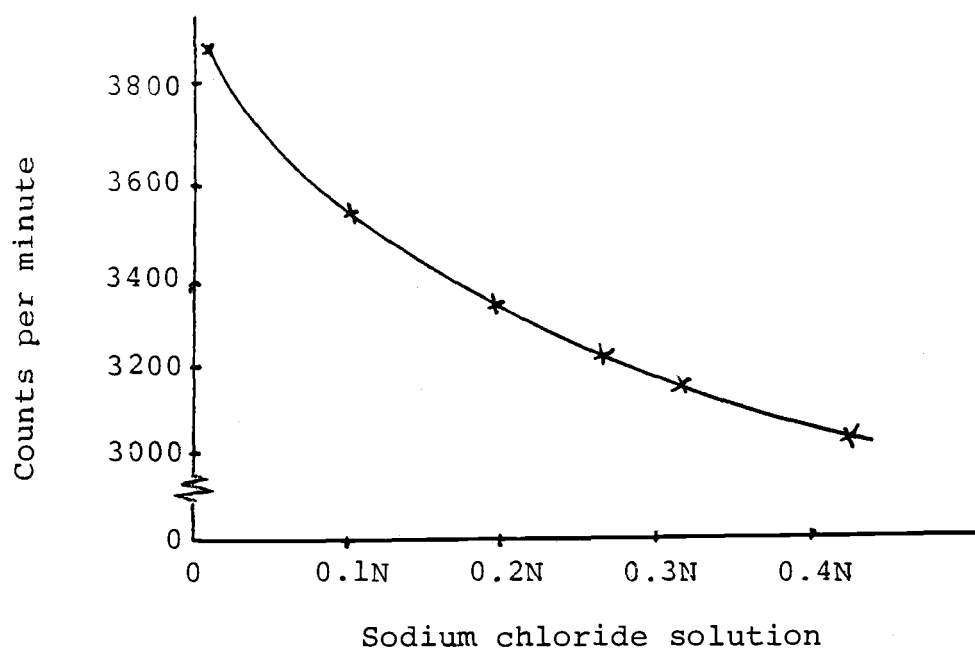


Figure 2. Effect of sodium chloride on counting rate in water (Holmes, 1956).

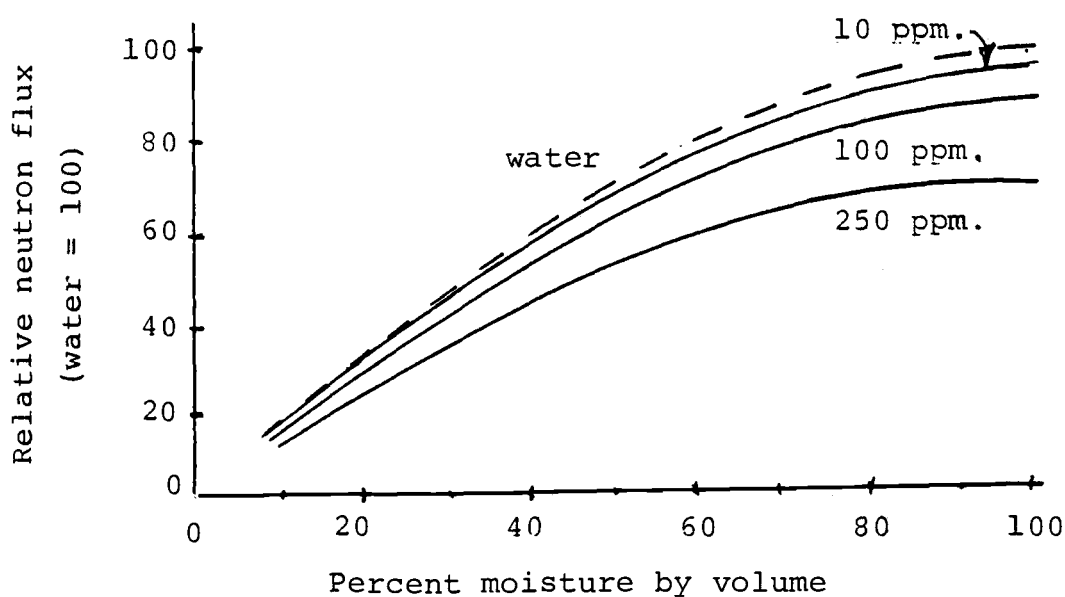


Figure 3. Effect of boron concentration on the slow neutron flux at differing moisture percentages (McHenry, 1967).

a relatively large capture cross-section. The iron content in natural soils may be as high as 7% by weight. A 9% reduction in neutron counts is attributed to an iron content of 7% (Burn, 1966).

Hydrogen present in the soil as a component of clay lattice and organic matter will bias a calibration curve (Figure 4). The neutron probe does not distinguish between hydrogen bound by colloidal matter and that occurring as free water. The result is an overestimation of the free water content. Correction for the influence of bound hydrogen causes a parallel shift in the calibration curve (Babalola, 1971).

Generalized factory curves are supplied to probe purchasers, but are obtained under laboratory conditions and do not account for unique physical and chemical properties found in field soils. Figure 5 illustrates the discrepancy of a factory curve for a Campbell Pacific Nuclear (CPN) probe when compared with curves obtained from field soils using the same probe.

Influence of Soil Bulk Density

Dry bulk density of the soil around the source will influence the thermal neutron count. The more tightly compacted the soil, the more chance there is for neutron interaction. Bulk density influences both the macroscopic cross-section for slow neutrons and the volumetric

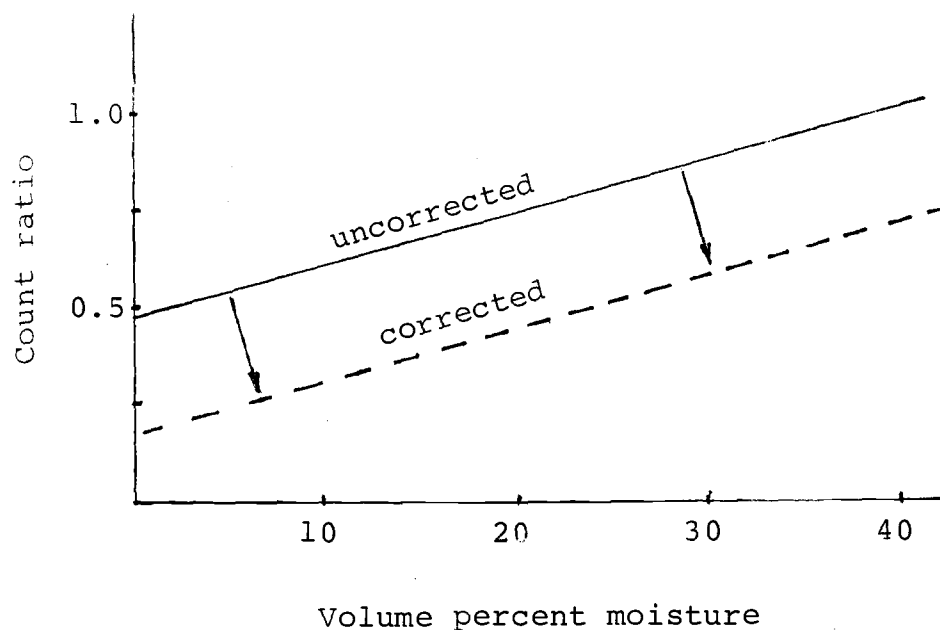


Figure 4. Effect of bound hydrogen on a field calibration curve (Babalola, 1971)

content of hydrogen bound to soil solids (Holmes, 1966). Much work has been done deriving expressions to compensate for bulk density (Lal, 1974; Greacen, 1979, 1976; Rahi, 1981; Marais, 1960, 1961, 1962; Karsten, 1973; Holmes, 1966; Olgaard, 1967, 1968). Others have accounted for bulk density by including soil texture among the parameters to consider in calibration (Gornat, 1972; Lal, 1973).

Olgaard (1965) showed the partial derivative of the count rate, CR, with respect to dry soil density, ρ , to be independent of soil type. Values ranging from 1.0 to 1.8 g/cc, for five soil types, were used to formulate the following relationship, which corrects all

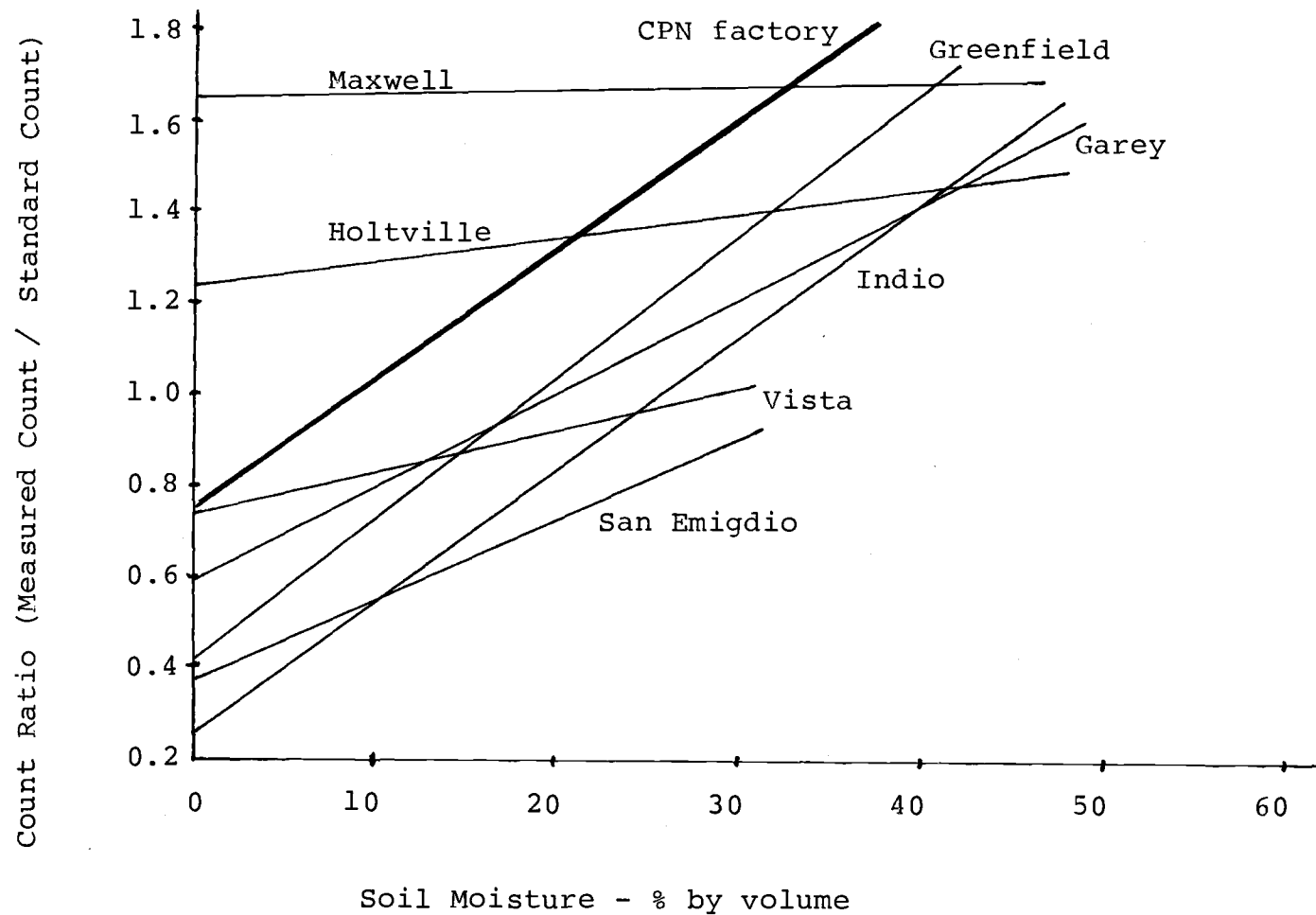


Figure 5. Neutron probe calibration curves for several common California agricultural soils and the CPN Factory Curve (Schwankl, 1975)

densities to a median value of 1.4 g/cc:

$$CR(1.4, V_w) = CR(\rho, V_w) + (1.4 - \rho) \frac{\partial CR}{\partial \rho} V_w, \quad (20)$$

where V_w is the volumetric water content as a percent.

Not everyone agrees on the effect bulk density has on a calibration curve. Most evidence suggests a parallel shift in the curve (Figure 6), but does not separate the effects of bulk density and elements with high capture cross-sections. Greacen, in a 1976 study of soils having a high absorption cross-section, showed Olgaard's model to be in error when predicting calibration curves for these soils. Greacen instead suggests a simplified procedure to correct for the effect of bulk density. This method proposes the count rate at constant total water content to be proportional to the square root of bulk density. The study also suggests that although hydrogen in forms other than free water causes a parallel shift, dry soil density affects the slope of the calibration curve. The relation that Greacen developed corrects an observed count rate, CR_f , obtained at some dry density, ρ , to a chosen standard dry density, ρ_s . The corrected count rate, CR_s , is given by (Greacen, 1976):

$$CR_s = CR_f (\rho_s / \rho)^p, \quad (21)$$

where p ranged from 0.44 to 0.52 for the soils studied.

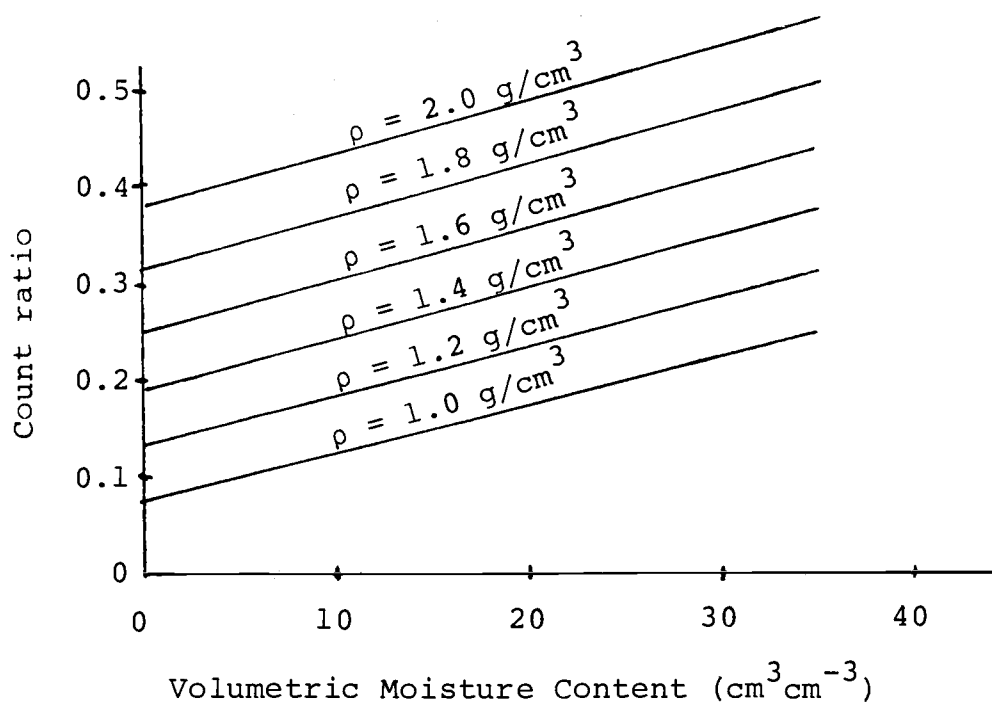


Figure 6. Effect of soil wet density on a neutron probe calibration curve (Lal, 1974)

Influence of Atmospheric Moisture on Neutron Probe Readings Taken Near the Soil Surface

When neutron probe readings are taken near the soil surface the sphere of influence extends out into the atmosphere. Correction must be made to account for the reduced quantity of hydrogen in the air. The geometry no longer resembles a sphere, as the radius above ground is larger than that in the soil. This effect is illustrated by Figure 7.

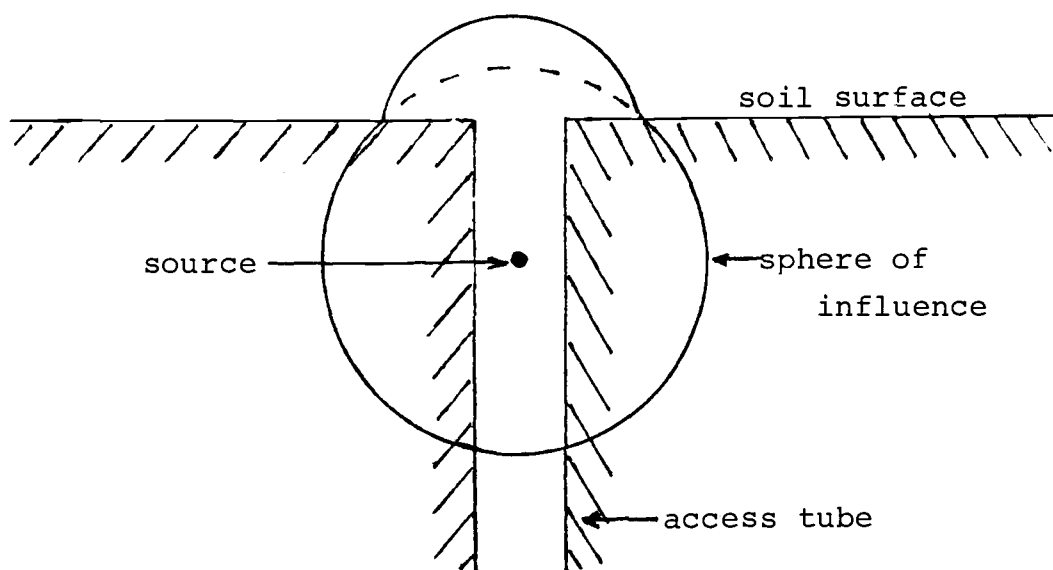


Figure 7. Geometry of theorized slow neutron flux at soil surface

An aluminum tray full of soil placed over the opening eliminates the need for correction (Hanna, 1980), but is impractical in many field operations and requires that the soil in the tray fully resemble the soil in the field below. Correction of the calibration curve for readings near the surface causes a parallel shift as illustrated by Figure 8 (Dickey, 1982).

Influence of the Access Tube

Access tubes are inserted into the soil to allow lowering of the probe source into the ground. The tube introduces an air gap around the source (a function of tube diameter) and a possible barrier to neutron

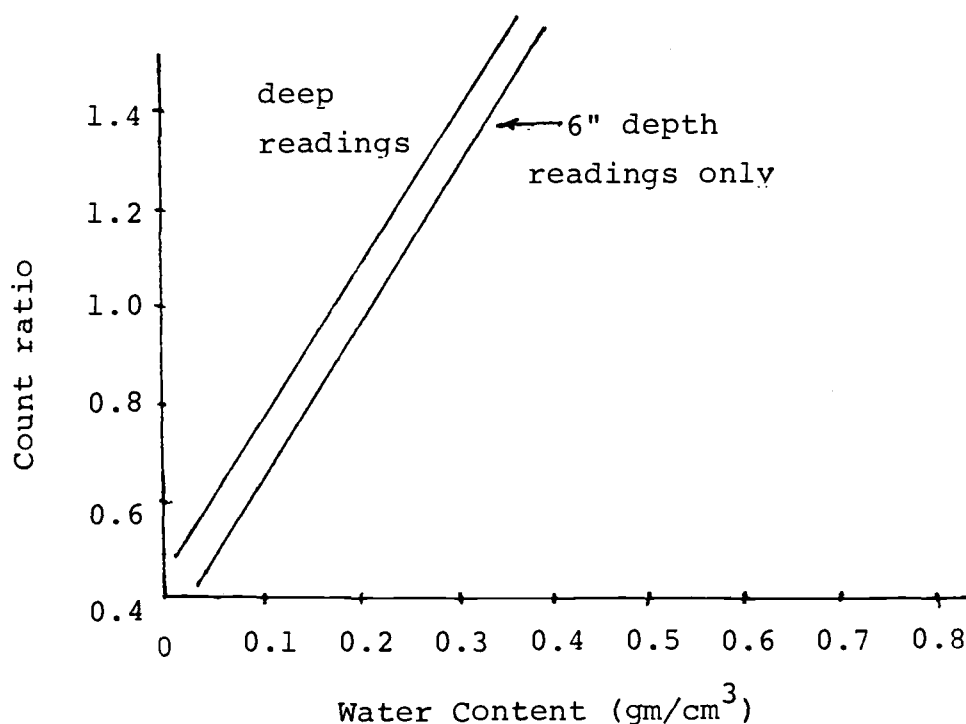


Figure 8. Effect of near surface readings on a calibration curve (Dickey, 1982).

penetration in the form of the tube wall, especially when the tube material contains hydrocarbons, as does PVC tubing.

The count rate was shown to be reduced by at least 15% for PVC access tubes (Marais, 1960), while aluminum tubing closely simulated an unlined hole. Two California soils, represented as sites 1 and 2 in Figure 10, show the count ratio to decrease 30% at 0.3 inches of water per foot moisture content when PVC access tubes are used, and 35% at 0.5 inches of water per foot.

Access tubes introduce an air gap around the source. Since conditions closest to the source are weighted most in probe readings, the air gap may significantly influence slow neutron counts. Hydrogen is present in reduced quantities in the air. Therefore, neutron counts decrease as the diameter of the access tube increases for a constant sized source. Figure 9 illustrates the effect of access tube diameter on count rates for a constant sized source.

Installation of access tubes into the soil can cause empty spaces to form between the tube and soil. Error attributed to such voids is a function of the bulk density and moisture level (Richardson, 1972). A saturated void produces a much larger error than an empty one.

The installation of access tubes may distort cracks already present in some clay soils. In such cases, special procedures are suggested to correct calibration curves (Greacen, 1979).

Careful installation of the access tube can greatly reduce errors from cracks around the tube. Techniques have been developed for installing tubes in certain soils. Displacement of the soil by driving a pointed tool steel driving rod into the ground before dropping the access tube into place is recommended for stony and

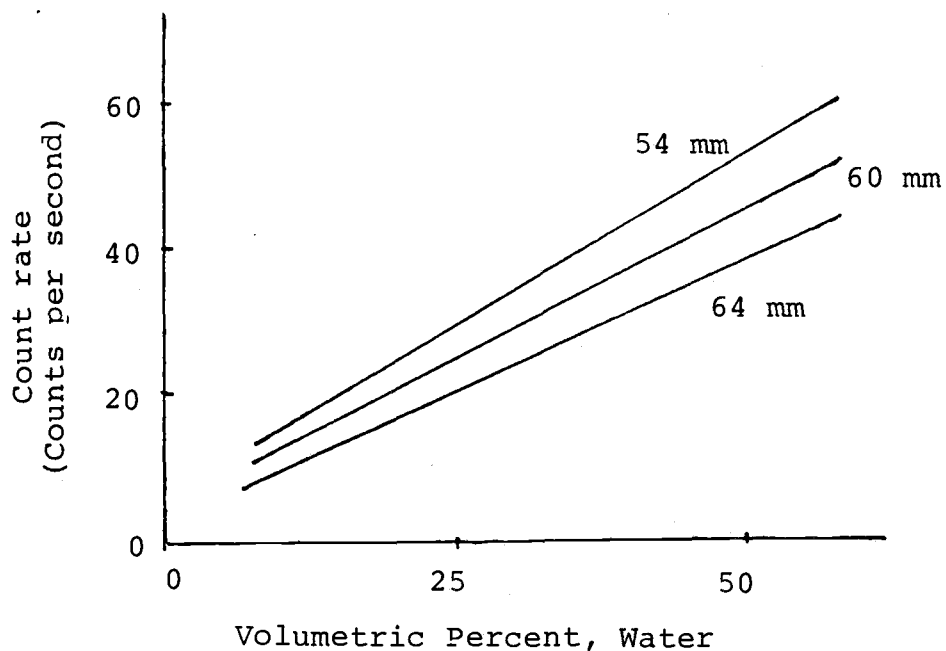


Figure 9. Variation of calibration curves with diameter of access tube (Int. Atomic Energy Agency, 1970)

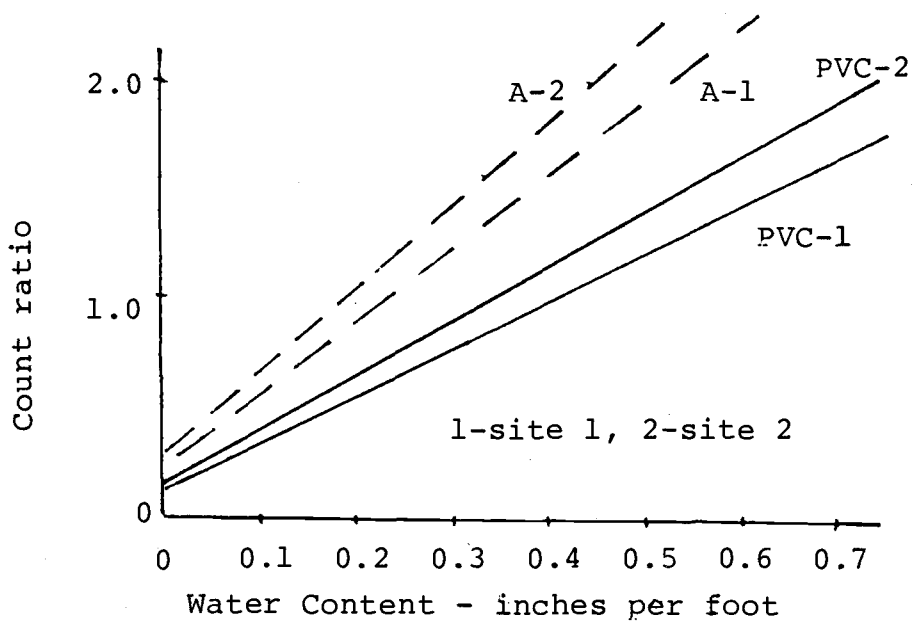


Figure 10. Changes in the calibration curve for different access tube materials (2" diameter). A-Aluminum, PVC-plastic sprinkler pipe (Dickey, 1982)

bouldery soils (Cline and Jeffers, 1975; Carpenter, 1973). This improves contact between the tubing and the surrounding soil.

Gravimetric Sampling Technique

In theory, a calibration curve relates count ratios to the corresponding true moisture content at each level of moisture content. In practice, only estimates of the true moisture content are available as provided by gravimetric sampling. This produces a significant possibility for error in predictions made using the calibration equation.

Periodic checks of field calibration by Soil Conservation Service technicians in California most often find poor gravimetric sampling technique to be responsible when inaccurate calibration occurs (Schwankl, 1983). Wet soil samples should be weighed immediately in the field and care taken to obtain undisturbed, representative sample volumes. Special apparatus have been designed to aid in collecting soil samples (Lutz, 1947), but careful technique remains the best guarantee of accuracy.

Location Error

Even well-defined soil types exhibit a large degree of variability in soil moisture throughout a

field. Error is introduced into values of mean field moisture when point measurements are used to calculate the mean.

Soil moisture varies with depth as well as horizontal distance. It has been shown that the spatial variability of soil water content may be approximated by a normal distribution (Nielsen, et al., 1973). Values of selected soil properties are tabulated in Table 3. Water content averaged over depth yields a higher standard deviation (4.8 versus 4.5) than average horizontal values for the study. A more complete evaluation of the variability of water content in a 150 hectare field is presented in Table 4, as a function of soil-water pressure head and depth. Variations are much larger when soils dry and negative pressure heads create a suction effect within the soil matrix.

Mean moisture content for water held at 0.1 bars of soil matrix suction was found to be 35.82 and 34.36 percent on a gravimetric basis for two sample trials conducted on the University of Arizona's Research Farm (Gajem et al., 1981). A total of 900 samples were collected over an 85 hectare area for the trials. The standard deviations were 0.40 and 0.47 for the respective trials. Water held at 15 bars of tension was found to have a mean moisture content of 12.50 and

Table 3

Values of Estimated Means, Standard Deviations, and Coefficients
of Variation for Soil Properties (Warrick and Nielsen, 1983)

Parameter	Mean	Standard Deviation	Coefficient of Variation (%)
Bulk Density (g/cm ³)	1.3 1.4 1.5	0.09 0.095 0.11	6.9 6.8 7.3
Water Content (%) at zero tension (cm ³ /cm ³)	40/45* 47	4.5/4.8 4.8	11/11 10
Sand/Silt/Clay (%) (From four sources).	53/28/19 59/29/12 26/27/47 24/30/45	15/9.1/6.8 22/18/6.4 11/6/8 14/8/10	28/32/36 37/62/53 42/22/17 58/27/22
Unsaturated Hydraulic Conductivity (cm/dy) (90% and 60% of saturation)	0.63/0.0026	1.75/0.011	280/420

* at 30 cm depths/combined depths

Table 4

Average Soil-Water Characteristic Values for 120 Soil
Cores from each Soil Depth in a 150-Hectare Field
(Nielsen, et al., 1973)

Character- istics	Soil Water Pressure Head (cm)	Values at different depths Soil Depth (cm)					
		30.5	61.0	91.4	121.9	152.4	182.9
θ	-0	0.402	0.441	0.449	.0475	.0470	.0483
σ		0.045	0.039	0.034	0.035	0.040	0.044
CV.....		11.3	8.8	7.6	7.4	8.4	9.1
θ	-10	0.396	0.433	0.440	0.465	0.465	0.476
σ		0.042	0.035	0.031	0.033	0.038	0.042
CV.....		10.6	8.2	7.0	7.1	8.2	8.9
θ	-30	0.387	0.413	0.423	0.449	0.451	0.461
σ		0.039	0.035	0.031	0.033	0.039	0.051
CV.....		10.2	8.4	7.2	7.3	8.6	11.0
θ	-60	0.368	0.381	0.389	0.420	0.420	0.428
σ		0.041	0.042	0.040	0.043	0.054	0.077
CV.....		11.2	11.0	10.3	10.3	12.7	18.1
θ	-90	0.351	0.352	0.358	0.394	0.386	0.390
σ		0.046	0.050	0.050	0.055	0.070	0.094
CV.....		13.2	14.1	14.0	13.9	18.0	23.5
θ	-120	0.340	0.332	0.337	0.376	0.364	0.379
σ		0.051	0.055	0.056	0.062	0.077	0.102
CV.....		15.0	16.6	16.7	16.6	21.1	26.9
θ	-150	0.330	0.318	0.321	0.362	0.346	0.365
σ		0.053	0.058	0.059	0.067	0.081	0.106
CV.....		15.9	18.2	18.5	18.6	23.1	29.0
θ	-200	0.320	0.302	0.303	0.346	0.324	0.348
σ		0.054	0.059	0.060	0.072	0.083	0.109
CV.....		17.0	19.6	19.9	20.7	25.5	23.8

* θ is the mean water content (cm^3/cm); σ is the standard deviation of the sample mean; and CV is the coefficient of variation (percent).

11.80 percent, with standard deviations of 0.90 and 0.80 for the same two trials.

3.3 Computer Simulation of a Theoretical Distribution of Slow Neutrons About a Point Source

Olgaard has developed a computer program (MOPS) which uses a chemical analysis of soil samples to generate a theoretical distribution of the slow neutron flux about a point source (Olgaard, 1965). The soil samples provide information to calculate capture cross-sections for the elements present. Utilizing atomic theory, the cross-sections are used to determine the slow neutron flux. The results are then adjusted by multiplication with a correction factor. This factor is a ratio of the observed counts by a probe in water to the number of counts predicted by the program for the same medium, water. The final output is a function of the probe type and the diameter and type of access tube used in the water.

Model predictions compare well with some experimental data (Olgaard, 1965), but do not account for human error or soil heterogeneity, two of the largest sources of error in determining soil moisture in practical applications. As shown by Greacen (1976) the model also fails to predict results well for soils with high absorption cross-sections.

4. PREDICTING ERROR USING A STATISTICAL MODEL

Equations have been developed to predict the variance in estimates of water content by the probe (Sinclair and Williams, 1979; Haverkamp et al., 1984). An estimate of errors can be derived from variances by referring to specific confidence limits. The equations calculate three components of variance: the instrument component, the calibration component, and the location component. The total variance is the sum of the three components.

In the model presented in this chapter, moisture content is integrated over the soil profile using a numerical technique to obtain the total volume of water stored in the soil. An additional component of variance is created by using a numerical technique to approximate the integration.

4.1 Using Variances to Estimate Error

Statistics defines the variance of a random variable as the sum of the squares of deviations about the true mean (Snedecor and Cochran, 1980). The true mean is not usually known, but can be estimated by the sample mean. Repeating the sampling process under identical conditions provides multiple estimates of the true mean.

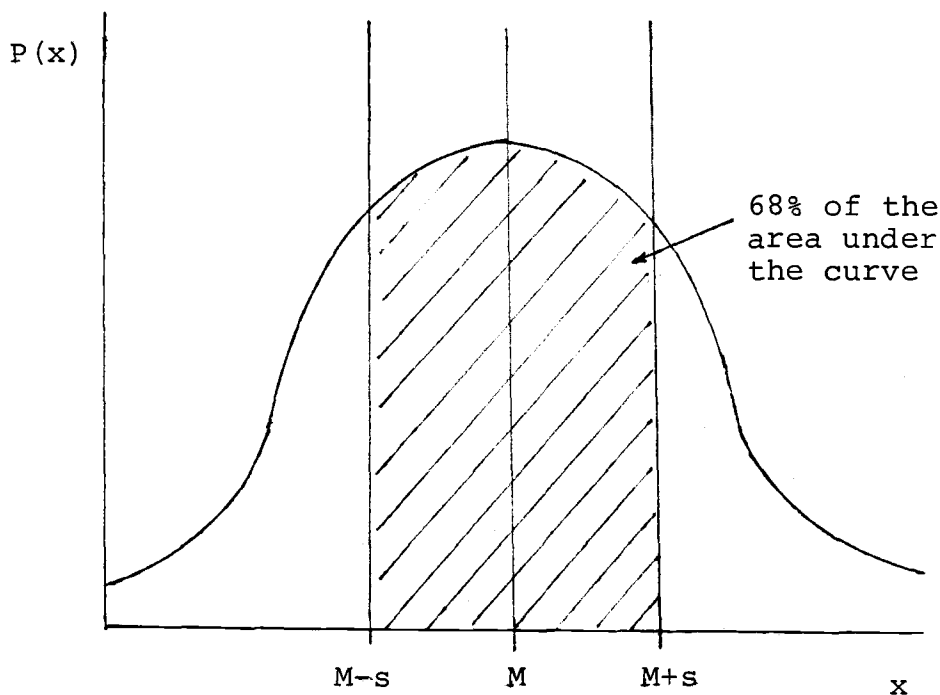
If the sample mean itself is considered a random variable, the variance of sample means about the true mean can be described. If it is assumed that sample values are normally distributed about their respective means, then the sample means are normally distributed about the true mean.

The normal distribution extends to plus and minus infinity. However, the probability that any one estimate will fall within a specified distance from the true mean can be calculated by integrating the area under the normal curve (see Figure 11). Approximately 68% of the area under the curve lies within plus or minus one standard deviation from the mean. Almost 95% of the area is contained within plus or minus two standard deviations.

Although it is impossible to know with absolute certainty where the true mean is located, it is 95.44% certain that the true mean lies within plus or minus two standard deviations of the sample mean. Thus, the probability of error in predictions of mean moisture may be calculated from the sample variance by specifying a level of confidence for the error estimate.

4.2 Development of Equations

A complete derivation of the equations to predict the variance in soil moisture estimates is given in Appendix A. The following is a brief summary.



M - mean

s - standard deviation

Figure 11. The normal probability curve.

The variables below are described since their definitions are specific to these equations, and may not correspond to what is considered common nomenclature. The descriptions will remain constant throughout this text and are as follows:

θ - true volumetric water content; its value is unknown and masked by errors

- n - the true neutron count ratio corresponding to θ ; its value is also unknown and masked by errors.
- $\hat{\theta}$ - a value of volumetric water content observed in situ, usually in a calibration process (a gravimetric measurement).
- \hat{n} - An observed value of count ratio corresponding to $\hat{\theta}$, and normally associated with the calibration process.
- $\hat{\theta}_0$ - a value of volumetric water content determined by the linear regression (calibration) equation.
- \hat{n}_0 - the count ratio for which $\hat{\theta}_0$ is defined by the calibration curve.
- u - measurement errors in the observed neutron count ratio, \hat{n} .
- v - measurement errors in the observed value of volumetric water content, $\hat{\theta}$.
- e - an error term that includes v as well as deviations from an exact linear model defined by the regression line; errors are assumed to be normally distributed with an expected value of zero.
- \bar{N} - the average of p replications of count rate obtained during a counting time of T_c seconds at a fixed location in the field.
- \bar{N}_s - the average of q replications of count rate, obtained during a counting time of T_s

seconds in a standard absorber.

The following equalities help clarify the above descriptions:

$$\hat{\theta} = \theta + v, \quad (22)$$

$$\hat{n} = n + u = \bar{N} / \bar{N}_s. \quad (23)$$

The true relationship between volumetric water content and neutron count ratios is assumed to be linear in the region encountered in agricultural applications, and is of the form (Haverkamp, 1983):

$$\theta = a_0 + a_1 n + e, \quad (24)$$

where a_0 and a_1 are coefficients. In practice, the relationship is given by a linear regression of the calibration data (i.e. paired values of $\hat{\theta}$, \hat{n}) as:

$$\hat{\theta}_0 = \hat{a}_0 + \hat{a}_1 \hat{n}_0, \quad (25)$$

where

$$E(\hat{a}_0) = a_0 \quad (26)$$

$$E(\hat{a}_1) = a_1. \quad (27)$$

Subtracting equation (25) from equation (24):

$$(\theta - \hat{\theta}_0) = a_0 - \hat{a}_0 + a_1 n - \hat{a}_1 \hat{n}_0 + e. \quad (28)$$

Mean values of moisture contents and count ratios averaged from k access tube locations at one depth, can be defined:

$$\langle \hat{\theta}_o \rangle = \frac{1}{k} \sum_{i=1}^k \hat{\theta}_{o,i} , \text{ the mean measured} \quad (29)$$

soil moisture;

$$\langle \hat{n}_o \rangle = \frac{1}{k} \sum_{i=1}^k \hat{n}_{o,i} , \text{ the mean measured count} \quad (30)$$

ratio;

$$\langle \theta \rangle = \frac{1}{k} \sum_{i=1}^k \theta_i , \text{ the true mean of soil} \quad (31)$$

moisture at the sites where measurements were made;

$$\langle n \rangle = \frac{1}{k} \sum_{i=1}^k n_i , \text{ the true mean count ratio} \quad (32)$$

that would have been recorded at the measurement sites.

When mean values are substituted into equation (28), the error term cancels since its mean value is assumed to be zero:

$$(\langle \theta \rangle - \langle \hat{\theta}_o \rangle) = a_o - \hat{a}_o + a_1 \langle n \rangle - \hat{a}_1 \langle \hat{n}_o \rangle . \quad (33)$$

Squaring both sides of equation (33), taking expected values, making use of statistical definitions and theorems and assuming neutron counts to be Poisson distributed results in (Haverkamp, 1983):

$$\begin{aligned}
 s^2(\hat{\theta}_0) = & s^2(\hat{a}_0) + (\hat{a}_1^2 - s^2(\hat{a}_1)) \cdot \frac{1}{k \bar{N}_s^2} s^2(L_0) \\
 & + (\hat{a}_1^2 - s^2(\hat{a}_1)) \cdot \frac{1}{k \bar{N}_s} \left[\frac{\langle \hat{n}_0 \rangle}{p T_c} + \frac{\langle \hat{n}_0 \rangle^2}{q T_s} \right] \\
 & + \langle \hat{n}_0 \rangle^2 s^2(\hat{a}_1) + 2\langle \hat{n}_0 \rangle s(\hat{a}_0, \hat{a}_1), \quad (34)
 \end{aligned}$$

where $s^2(\hat{a}_0)$, $s^2(\hat{a}_1)$ and $s(\hat{a}_0, \hat{a}_1)$ are the sample variances and covariance of the intercept and slope of the regression line and $s^2(L_0)$ is the variation in neutron count rates due to spatial variability of moisture content.

Equation (34) identifies three major components of variance:

a) the instrument component (35)

$$s^2_I(\hat{\theta}_0) = (\hat{a}_1^2 - s^2(\hat{a}_1)) \cdot \frac{1}{k \bar{N}_s} \left[\frac{\langle \hat{n}_0 \rangle}{p T_c} + \frac{\langle \hat{n}_0 \rangle^2}{q T_s} \right]$$

b) the location component (36)

$$s^2_L(\hat{\theta}_0) = (\hat{a}_1^2 - s^2(\hat{a}_1)) \cdot \frac{s^2(L_0)}{k \bar{N}_s^2}$$

c) the calibration component (37)

$$s_c^2(\langle \hat{\theta}_O \rangle) = s^2(\hat{a}_O) + \langle \hat{n}_O \rangle^2 s^2(\hat{a}_1) + 2\langle \hat{n}_O \rangle s(\hat{a}_O, \hat{a}_1)$$

Equation (35) predicts the variance in soil moisture estimates due to random neutron emission. The variance in a count ratio $\langle \hat{n}_O \rangle$ due to random neutron emission, $s^2(\langle \hat{n}_O \rangle)$, can be calculated by the following relation (Appendix, page 116):

$$s^2(\langle \hat{n}_O \rangle) = \frac{1}{k \bar{N}_S} \left\{ \frac{\langle \hat{n}_O \rangle}{p T_C} + \frac{\langle \hat{n}_O \rangle^2}{q T_S} \right\} \quad (38)$$

Utilizing this equality, equation (35) can be written as:

$$s_I^2(\langle \hat{\theta}_O \rangle) = (\hat{a}_1^2 - s^2(\hat{a}_1)) s^2(\langle \hat{n}_O \rangle). \quad (39)$$

The variance in soil moisture estimates is related to the variance in count ratios due to random emission by the square of the slope of the calibration curve. The calibration curve directly relates soil moisture to count ratios. Thus a steep slope magnifies deviation in count ratios (the independent variable) when projected onto the vertical axis displaying soil moisture. A flat slope tends to minimize the effect of deviations in the independent variable when projected onto the vertical

axis. This is illustrated in Figure 12. The variance of the slope is subtracted from the slope squared, since its contribution to the variance in soil moisture estimates is classified as calibration error.

Equation (36) predicts the variance in soil moisture estimates due to the spatial variability of count ratios. The spatial variability of count ratios is a result of moisture content being nonuniformly distributed throughout a field. The square of the slope of the calibration curve relates the spatial variability of count ratios (deviation in the independent variable) to the corresponding variance in soil moisture estimates (measured on the vertical axis). The variance in the slope is subtracted from the slope squared since its contribution to variance in soil moisture estimates is classified as calibration error.

Equation (37) predicts the variance in soil moisture estimates due to the variances and covariance of the slope and intercept of the calibration equation. The variances and covariances are calculated from calibration data. The calibration component is also a function of count ratio. This relationship is explored further in section 5.3.

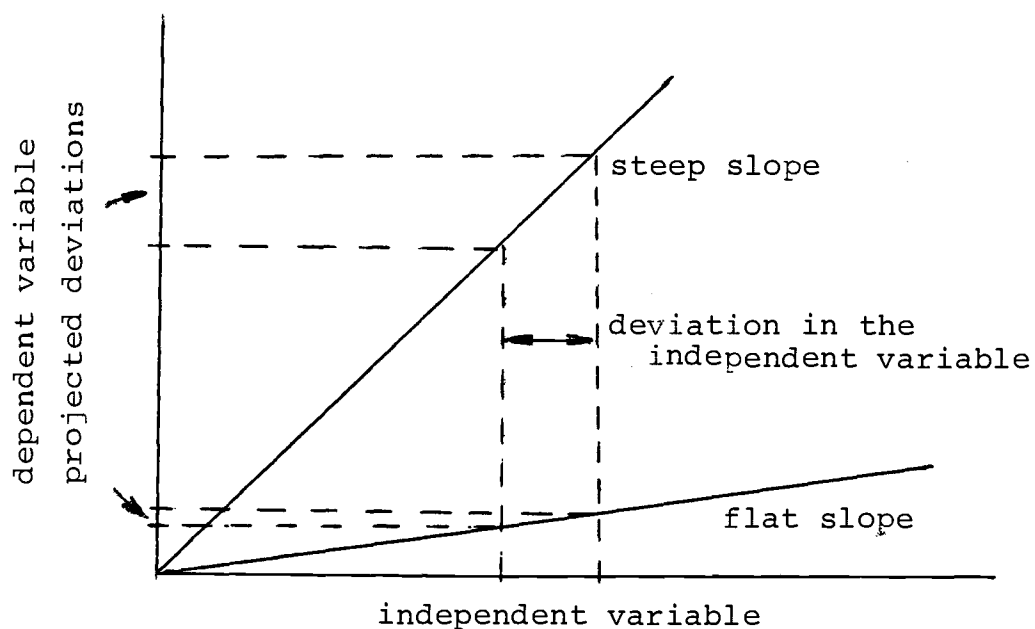


Figure 12. Effect of slope on projecting deviations from the independent to dependent variable.

Relative Magnitude of Components of Variance

The relative effect of the instrument, location and calibration components of variance have been estimated using the relation described by equation (34) for neutron probe calibration data collected from a grassy plot located on the Campus of the University of Grenoble (Haverkamp, 1984). The top 50 cm of soil consisted of a fine loam. Medium textured sands were found beneath the loam. Soil moisture measurements were taken at two depths using the probe.

The results are shown in Figure 13. The estimated variance in soil moisture measurements by the probe is

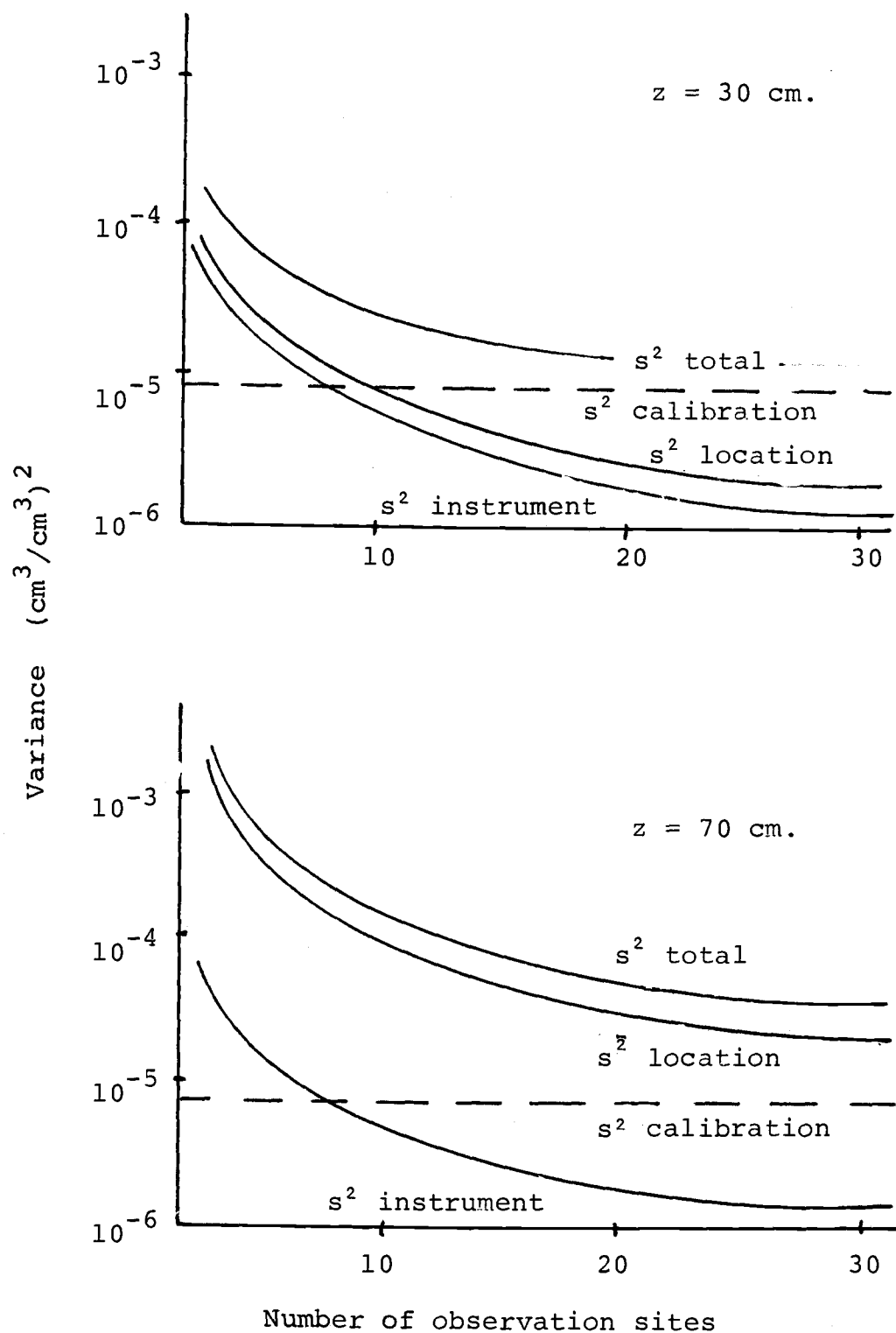


Figure 13. Relative effects of errors.
(Haverkamp, 1984).

plotted against the number of sampling locations for the two depths. In all cases the instrument component was shown to be negligible. At a depth of 30 cm. soil conditions were determined to be relatively homogeneous. The location component fell below the calibration component as the number of sampling locations was increased. At the 70 cm. depth the soil was of a more heterogeneous nature and the location component remained the dominant component of variance for all sampling locations.

Integrating Soil Moisture Over the Profile

A unique calibration curve is assumed to be developed for each depth throughout the soil profile. Irrigation scheduling requires the integration of soil moisture over the crop root zone to obtain the total volume of water stored, V_t . The variance associated with an estimate of total water volume is a linear sum of the individual variances calculated at each depth, represented by $s_1^2(V_t)$, plus an additional error component introduced by the use of a numerical technique to perform the integration, referred to as $s_2^2(V_t)$ (Haverkamp, 1983).

Two popular numerical integration techniques are Simpson's rule and the trapezoidal rule. Simpson's rule integrates the area under a curve for an odd number of equally spaced points for which values of the function

are known. For three points, x_1 , x_2 , and x_3 , the formula for Simpson's rule is (Hamming, 1962):

$$A = \frac{1}{3} [y(x_1) + 4y(x_2) + y(x_3)] , \quad (40)$$

where A is the integrated area and y is the function of the curve. When more than three points are used in the integration, Simpson's rule becomes:

$$A = \left[y(x_1) + \frac{1}{3} \{ y(x_2) + 4y(x_3) + 2y(x_4) + \dots + 4y(x_{n-2}) + 2y(x_{n-1}) + y(x_n) \} \right] \Delta x , \quad (41)$$

where N is the total number of points and Δx the distance between points.

The error Er , attributed to use of Simpson's rule is (Hamming, 1962):

$$Er(A) = \frac{(N\Delta x) (\Delta x)^4}{180} \frac{\partial^4 y(x)}{\partial x^4} . \quad (42)$$

The trapezoidal rule of integration requires an even number of points. The trapezoidal rule may be written as (Hamming, 1962):

$$A = \frac{y(x_1) + y(x_2)}{2} \cdot \Delta x , \quad (43)$$

for two equally spaced points.

When more than two points are used in the integration, the trapezoidal rule can be written as:

$$A = \Delta x \sum_{i=1}^N y(x_i) , \quad (44)$$

where the area integrated extends $\frac{1}{2}\Delta x$ beyond each end point. The error, Er , attributed to use of the trapezoidal rule is (Hamming, 1962):

$$Er(A) = \frac{(N\Delta x)}{12} \frac{(\Delta x)^2}{\partial x^2} \frac{\partial^2 y(x)}{\partial x^2} \quad (45)$$

By assuming all values $y(x)$ to be independent of one another, the variance in estimates of area by Simpson's rule becomes:

$$s^2(A) = \left[s^2(y(x_1)) + \frac{1}{9} \left[s^2(y(x_2)) + 16s^2(y(x_3)) + 4s^2(y(x_4)) + \dots \right] \right] \Delta x^2 . \quad (46)$$

Defining the error term as the standard error of the estimate, the variance is calculated as the square of the standard error:

$$s_2^2(A) = \frac{(N\Delta x)^2 \Delta x^8}{32,400} \cdot \left[\frac{\partial^4 y(x)}{\partial^4 x} \right]^2 , \quad (47)$$

where the notation $s_2^2(A)$ refers to the variance attributed to use of the numerical technique to perform the integration.

Likewise, the variance in estimates of area by the trapezoidal rule becomes:

$$s^2(A) = (\Delta x)^2 \sum_{i=1}^N s^2(y(x_i)) . \quad (48)$$

The additional component of variance attributed to use of the numerical technique is:

$$s_2^2(A) = \frac{(N\Delta x)^2 \Delta x^4}{144} \left[\frac{\partial^2 y(x)}{\partial^2 x} \right]^2 . \quad (49)$$

Soil moisture measurements are normally taken at equally spaced depths throughout the soil profile (i.e., 10 cm., 20 cm., 30 cm., ...). The trapezoidal rule will weight the top measurement (10 cm. depth) 1.5 times those recorded at subsequent depths, if the area associated with the top measurement extends to the soil surface where no readings are taken. The concept is illustrated in Figure 14. A 1.5 weighting factor must be introduced if this is the case.

Equations (46) - (49) can be written in terms of moisture content where the function y represents the shape of the soil moisture profile and the integrated area is the total volume of water stored in a unit width of soil.

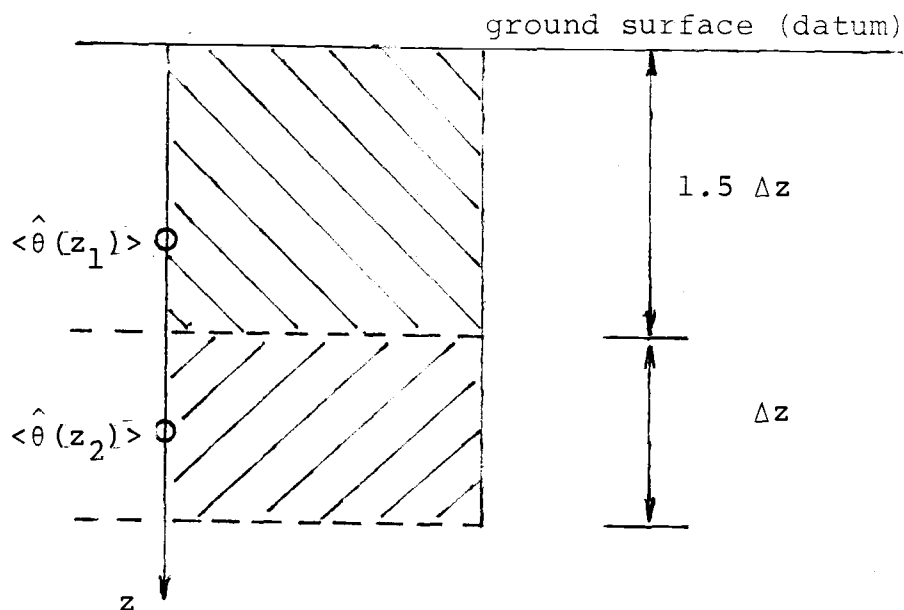


Figure 14. Weighting of area by the trapezoidal rule.

Using Simpson's rule of integration,

$$s_1^2(V_t) = \left[s^2(\hat{\theta}_o)_{\text{top}} + \frac{1}{9} \left[s^2(\hat{\theta}_o)_{\text{second depth}} \right. \right. \quad (50)$$

$$+ 16s^2(\hat{\theta}_o)_{\text{third depth}} + 4s^2(\hat{\theta}_o)_{\text{fourth depth}}$$

$$+ \dots \left. \right] \Delta z^2$$

$$s_2^2(V_t) = \frac{z^2 \Delta z^8}{32,400} \left[\frac{\partial^4 \hat{\theta}_o(z)}{\partial z^4} \right]^2. \quad (51)$$

For the trapezoidal rule:

$$s_1^2(V_t) + 1.5^2 s (\hat{\theta}_o)_{\text{top}} + \sum_{i=2}^z s^2 (\hat{\theta}_o)_i \Delta z^2_i \quad (52)$$

$$s_2^2(V_t) = \frac{z^2}{144} \Delta z^4 \left[\frac{\partial^2 \langle \hat{\theta}_o(z) \rangle}{\partial z^2} \right]^2, \quad (53)$$

where the factor 1.5 accounts for the additional area associated with the measurement closest to the soil surface. The distance between sampling depths is denoted as Δz , where the total depth integrated is z . A functional relationship is assumed to exist between water content and depth and can be described by $\langle \hat{\theta}_o(z) \rangle$. The fourth derivative of this function appears in equation (51). The true functional relationship is unknown, but a method to approximate its derivatives is given in page 61.

4.3 Evaluating Equations Using Field Data

The foregoing equations for calculating the variance in neutron probe estimates of field average soil moisture require that many of the terms be evaluated using field calibration data. The following paragraphs explain how to evaluate the terms in equations (50) - (53).

Regression Coefficients, Their Variances and Covariance

Regression coefficients for a neutron probe calibration curve may be calculated using the following equations (Haverkamp, 1984):

$$a_1 = \frac{s(\hat{n}, \hat{\theta})}{s^2(\hat{n}) - \frac{m-2}{m-1} s^2(u)} \quad (54)$$

$$\hat{a}_0 = \bar{\theta} - \hat{a}_1 \bar{n} \quad (55)$$

where m is the number of pairs of calibration observations $(\hat{n}, \hat{\theta})$, $\bar{\theta}$ is the average of the observed values of moisture content and \bar{n} is the average of the observed values of neutron counts. Measurement errors in neutron counts for calibration are attributed to random emission and denoted as u . An estimate of the variance of errors in the count ratio ($s^2(u)$) is included in equation (54) to provide a refined estimate of the slope of the regression line.

The variances and covariance of regression coefficients may be calculated as (Haverkamp, 1984):

$$s^2(\hat{a}_1) = \frac{s^2(e)}{(m-1)s^2(\hat{n}) - (m-2)s^2(u)} \quad (56)$$

$$s^2(\hat{a}_0) = \bar{n}^2 s^2(\hat{a}_1) \quad (57)$$

$$s(\hat{a}_0, \hat{a}_1) = -\bar{n} s^2(\hat{a}_1) \quad (58)$$

$$s^2(e) = \frac{m-1}{m-2} \left[s^2(\hat{\theta}) - \hat{a}_1 s(\hat{n}, \hat{\theta}) \right] - s^2(v) \quad (59)$$

The error term, e , accounts for measurement errors as well as deviations from an exact linear model. Measurement errors in gravimetrically obtained estimates of moisture content are denoted by v .

If measurement errors u and v are equal to zero, equations (54) through (59) give the classical least squares estimation of regression coefficients and their variances and covariance.

Variance of Measurement Errors ($\hat{n}_i - n_i$)

Measurement errors are errors associated with the random number of emissions of neutrons during a specified period of time. (This source of errors was discussed as instrument error in section 3.2.) The variance of measurement errors in neutron count ratios can be estimated by the average of the individual variances associated with each count ratio (Haverkamp, 1984):

$$s^2(u) = \frac{1}{m-1} \sum_{i=1}^m s^2(\hat{n}_i) \quad , \quad (60)$$

A zero mean is assumed for the distribution of count ratios around each observed value. Assuming neutron counts to be Poisson distributed (Appendix A, pp.116):

$$s^2(\hat{n}_i) = \left[\frac{\hat{n}_i}{p T_c} + \frac{\hat{n}_i^2}{q T_s} \right] / \bar{N}_s . \quad (61)$$

Variance of Measurement Errors ($\hat{\theta}_i - \theta_i$)

The variance of measurement errors in gravimetrically obtained estimates of soil moisture can be estimated by the average of the individual variances associated with each estimate (Haverkamp, 1984):

$$s^2(v) = \frac{1}{m-1} \sum_{i=1}^m s^2(\hat{\theta}_i) . \quad (62)$$

The individual distributions are assumed to have a zero mean. In practice it is impossible to know the variance associated with an individual estimate of soil moisture since gravimetric sampling will have removed the soil and repeated sampling at the exact location is impossible. The following analysis makes it possible to estimate the individual variances if the errors in obtaining sample volumes and weights are known.

Water content on a volumetric basis can be estimated from "undisturbed" soil samples using the following relation:

$$\hat{\theta}_i = \frac{M_{wi} - M_{di}}{\rho_w V_{si}} \quad (63)$$

where M_{wi} and M_{di} are the wet and dry sample weights,

ρ_w is the density of water and v_{si} is the sample volume. The associated variance for estimates of water content is determined using the statistical formula for calculating the variance of quotients and assuming measurements of weight and volume to be independent:

$$s^2(\hat{\theta}_i) = \left[\frac{s^2(M_{wi}) + s^2(M_{di})}{\rho_w} + s^2(V_{si}) \right] \frac{1}{V_{si}^2}. \quad (64)$$

Equation (64) can be used in equation (62) to evaluate $s^2(v)$.

Spatial Variability of Count Rates

The variance of neutron count rates due to the spatial variability of water content, $s^2(L_0)$, is found by subtracting the variance due to random neutron emission from the total observed variance in count rates throughout the field:

$$s^2(L_0) = s^2(\hat{N}) - \frac{\bar{N}_O}{p T_C}, \quad (65)$$

where \hat{N}_O is one particular observed count rate \hat{N} . Probe readings taken at the same time and under identical conditions but at several sample locations should be used to evaluate the total variance in count rates, $s^2(\hat{N})$.

Derivatives of the Water Distribution Profile

The partial derivative of the water distribution profile with respect to depth, $\frac{\partial^2 \hat{\theta}(z)}{\partial^2 z}$, may be evaluated using a finite difference approximation. Approximating the curve of the water profile as a straight line between two successive depths sampled, the change in water content divided by the change in depth provides an estimate of the first derivative. The change in the first derivative divided by the distance between midpoints of two successive approximations estimates the second derivative. Figure 14 illustrates the concept.

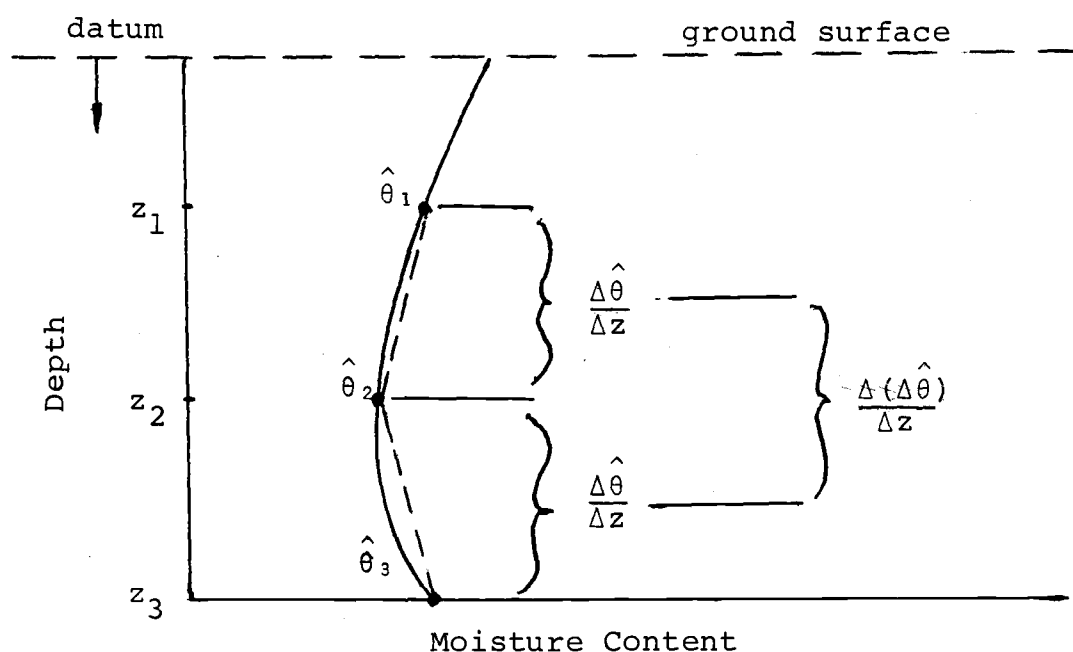


Figure 15. Approximating the second derivative of the water distribution profile with respect to depth

4.4 Summary of Equations

The equations presented are summarized in the following pages for convenience of reference.

Calculations of the regression coefficients are given by:

$$\hat{a}_i = \frac{s(\hat{n}, \hat{\theta})}{s^2(\hat{n}) - \frac{m-2}{m-1} s^2(u)} \quad (54)$$

$$\hat{a}_0 = \bar{\theta} - \hat{a}_1 \bar{n} \quad (55)$$

The variances of these coefficients are:

$$s^2(\hat{a}_1) = \frac{s^2(e)}{(m-1)s^2(\hat{n}) - (m-2)s^2(u)} \quad (56)$$

$$s^2(\hat{a}_0) = \overline{\hat{n}^2} s^2(\hat{a}_1) \quad (57)$$

$$s(\hat{a}_0, \hat{a}_1) = - \bar{n} s^2(\hat{a}_1) \quad (58)$$

in which:

$$s^2(e) = \frac{m-1}{m-2} \left[s^2(\hat{\theta}) - \hat{a}_1 s(\hat{n}, \hat{\theta}) \right] - s^2(v) \quad (59)$$

$$s^2(u) + \frac{1}{m-1} \sum_{i=1}^m s^2(\hat{n}_i) \quad , \quad (60)$$

$$s^2(\hat{n}_i) = \left[\frac{\hat{n}_i}{p T_C} + \frac{\hat{n}_i^2}{q T_S} \right] / \bar{N}_S \quad . \quad (61)$$

$$s^2(v) = \frac{1}{m-1} \sum_{i=1}^m s^2(\hat{\theta}_i) \quad (62)$$

$$s^2(\hat{\theta}_i) = \left[\frac{s^2(M_{wi}) + s^2(M_{di})}{\rho_w} + s^2(V_{si}) \right] \frac{1}{V_{si}^2} \quad . \quad (64)$$

The variance of a moisture measurement is:

$$s^2(\langle \hat{\theta}_O \rangle) = s^2_I(\langle \hat{\theta}_O \rangle) + s^2_C(\langle \hat{\theta}_O \rangle) + s^2_L(\langle \hat{\theta}_O \rangle) \quad (66)$$

where:

$$s^2_I(\langle \hat{\theta}_O \rangle) = (\hat{a}_1^2 - s^2(\hat{a}_1)) \frac{1}{k \bar{N}_S} - \left[\frac{\langle \hat{n}_O \rangle}{p T_C} + \frac{\langle \hat{n}_O \rangle^2}{q T_S} \right] \quad (35)$$

$$s^2_C(\langle \hat{\theta}_O \rangle) = s^2(\hat{a}_O) + \langle \hat{n}_O \rangle^2 s^2(\hat{a}_1) + 2 \langle \hat{n}_O \rangle s(\hat{a}_O, \hat{a}_1) \quad (37)$$

$$s^2_L(\langle \hat{\theta}_O \rangle) = (\hat{a}_1^2 - s^2(\hat{a}_1)) \frac{s^2(L_O)}{k \bar{N}_S^2} \quad (36)$$

Using Simpson's rule of integration,

$$\begin{aligned}
 s_1^2(V_t) = & \left[s^2(\hat{\theta}_O)_{\text{top}} + \frac{1}{9} \left[s^2(\hat{\theta}_O)_{\text{second growth}} \right. \right. \\
 & + 16s^2(\hat{\theta}_O)_{\text{third depth}} + 4s^2(\hat{\theta}_O)_{\text{fourth depth}} \\
 & \left. \left. + \dots \right] \right] \Delta z^2
 \end{aligned}
 \tag{50}$$

$$s_2^2(V_t) = \frac{z^2 \Delta z^8}{32,400} \frac{\partial^4 \hat{\theta}_O(z)}{\partial z^4} .
 \tag{51}$$

Alternatively, using the trapezoidal rule:

$$s_1^2(V_t) = 1.5^2 s^2(\hat{\theta}_O)_{\text{top}} + \sum_{i=1}^Z s^2(\hat{\theta}_O) \Delta z^2
 \tag{52}$$

$$s_2^2(V_t) = \frac{z^2}{144} \Delta z^4 \left[\frac{\partial^4 \hat{\theta}_O(z)}{\partial z^2} \right]^2 .
 \tag{53}$$

5. APPLICATIONS

A numerical example is used to show the mechanics of applying the statistical model using field calibration data. The algorithms in the example are incorporated into a computer program for quick and easy calculation of the variance in neutron probe measurements.

Several California soils are analyzed using the computer program to show the increased error in estimates of soil moisture outside of the range of moisture content in the calibration data.

In most cases, neutron probe access tubes are placed at biased locations within a field. A method is developed to take advantage of this fact to estimate "average" field moisture more accurately. Given the degree of variability of field moisture content it is shown how to calculate the number of access tube locations that must be sampled to achieve a desired level of accuracy.

5.1 Numerical Example

Calibration data collected by Soil Conservation Service technicians were generously provided by Ben Doerge of the Greeley, Colorado S.C.S. office. Access tubes were installed at three locations in the field: the lower end, the middle and the upper end. Four

depths were sampled. The data are summarized in Tables 5 and 6. Calibration curves are developed for each depth, and the results used to calculate the total volume of water stored in the top three feet of soil.

Data were not available to calculate calibration measurement errors u and v . Thus, these terms were set equal to zero and do not appear in the equations to calculate the regression coefficients and their variances and covariance.

The following is an estimation of the 95.44% confidence limits of a field average soil moisture estimate derived from the calibration data. The example calculations are performed for the 12" depth only. Numerical values for subsequent depths appear only in Table 7.

$$\hat{a}_1 = \frac{s(\hat{n}, \hat{\theta})}{s^2(\hat{n})} \quad (67)$$

$$\hat{a}_1 = \frac{\sum_{i=1}^m \hat{n}_i \hat{\theta}_i - \frac{\sum_{i=1}^m \hat{\theta}_i \sum_{i=1}^m \hat{n}_i}{m}}{\sum_{i=1}^m \hat{n}_i^2 - \frac{(\sum_{i=1}^m \hat{n}_i)^2}{m}} \quad (68)$$

$$= \frac{4.666448 - (3.1901)(13.08)/9}{19.0708 - (13.08)^2 / 9}$$

$$= 0.49296$$

TABLE 5

Calibration Data from Greeley, Colorado

Dated 7/6/83

Standard Counts

29,284	29,160	$T_s = 60$ seconds
29,642	29,289	$\bar{N}_s \cdot T_s = 29.244$ counts
28,956	29,195	
29,408	29,082	$\bar{N}_s = 487.3967$ counts/second
29,354	29,068	
$T_c = 60$ seconds		

Field Counts at a 12" depth

Upper end:	45,265	Middle:	44,727	Lower end:	44,264
	45,283		44,600		44,409

Field Counts at a 24" depth

Upper end:	37,685	Middle:	32,253	Lower end:	44,513
	37,373		32,327		44,272

Field Counts at a 36" depth

Upper end:	33,706	Middle:	35,279	Lower end:	44,186
	33,787		35,151		43,949

Field Counts at a 48" depth

Upper end:	35,819	Middle:	35,626	Lower end:	44,247
	35,801		35,645		44,209

TABLE 6

Seasonal Calibration Data from Greeley, Colorado

Date	Access Tube Location	12"depth		24"depth		36"depth		48"depth	
		\hat{n}	$\hat{\theta}$	\hat{n}	$\hat{\theta}$	\hat{n}	$\hat{\theta}$	\hat{n}	$\hat{\theta}$
7/6/83	Upper end	1.55	.4192	1.28	.2883	1.15	.3050	1.22	.2050
	Middle	1.53	.3100	1.10	.1933	1.18	.1717	1.22	.2892
	Lower end	1.52	.4117	1.52	.4475	1.48	.3192	1.51	.3750
7/20/83	Upper end	1.51	.4217	1.28	.3233	1.17	.2058	1.24	.1833
	Middle	1.48	.3350	1.12	.2250	1.22	.2092	1.27	.2417
	Lower end	1.46	.3908	1.50	.3483	1.50	.2925	1.50	.2808
8/15/83	Upper end	1.37	.3625	1.22	.2633	1.15	.1042	-	-
	Middle	1.34	.2550	1.01	.1650	1.20	.1992	-	-
	Lower end	1.32	.2842	1.45	.3200	1.51	.4092	-	-

$$\hat{a}_0 = \bar{\hat{\theta}} - \hat{a}_1 \bar{\hat{n}} \quad (69)$$

$$= 0.354456 - (0.49296)(1.45333)$$

$$= -0.36198$$

$$s^2(e) = \frac{m-1}{m-2} [s^2(\hat{\theta}) - \hat{a}_1 s(\hat{n}, \hat{\theta})] \quad (70)$$

$$= \frac{9-1}{9-2} [0.0038198 - (0.49296)(0.0037712)]$$

$$= 0.0022409$$

$$s^2(\hat{a}_1) = \frac{s^2(e)}{(m-1) \left[\sum_{i=1}^m \hat{n}_i^2 - \left(\sum_{i=1}^m n_i^2 \right) / m \right]} \quad (71)$$

$$= \frac{0.0022409}{(9-1) [19.0708 - (13.08)^2 / 9]}$$

$$= 0.0049203$$

$$s^2(\hat{a}_0) = \frac{1}{m} \sum_{i=1}^m \hat{n}_i^2 s^2(\hat{a}_1) \quad (72)$$

$$= \frac{1}{9} (0.093834)$$

$$= 0.010426$$

From Table 5:

$$\bar{N} = \left[\frac{45,265}{60} + \frac{45,283}{60} + \dots + \frac{44,409}{60} \right] / 6 \quad (73)$$

$$= 745.96 \text{ counts/sec}$$

$$s^2(\hat{N}) = \left[\sum_{i=1}^6 \hat{N}_i^2 - \left(\sum_{i=1}^6 \hat{N}_i \right)^2 / 6 \right] / (6-1) \quad (74)$$

$$= (3,339,054 - (4,476)^2 / 6) / 5$$

$$= 51.358 \text{ counts/sec}$$

$$\bar{n}_O = \bar{N} / \bar{N}_S \quad (75)$$

$$= 745.9600 / 487.3967 = 1.5305$$

$$s^2(L_O) = s^2(\hat{N}) - \frac{\bar{N}}{pT_C} \quad (76)$$

$$= 51.358 - \frac{745.96}{2(60)}$$

$$= 45.14 \text{ counts/sec}$$

$$s(\hat{a}_O, \hat{a}_1) = - \frac{1}{m} \sum_{i=1}^m \hat{n}_i s^2(a_1) \quad (77)$$

$$= - \frac{1}{9} (0.064357)$$

$$= - 0.0071508$$

$$s^2_L(\langle \hat{\theta}_O \rangle) = (\hat{a}_1^2 - s^2(\hat{a}_1)) \frac{s^2(L_O)}{k \bar{N}_S^2} \quad (78)$$

$$= ((0.49296)^2 - 0.0049203) \frac{45.14}{3(487.3967)^2}$$

$$= 0.000015081 (\text{in}^3/\text{in}^3)^2$$

$$s^2_C(\langle \hat{\theta}_O \rangle) = s^2(\hat{a}_O) + \langle \hat{n}_O \rangle^2 s^2(\hat{a}_1) + 2\langle \hat{n}_O \rangle s(\hat{a}_O, \hat{a}_1) \quad (79)$$

$$= 0.010426 + (1.5305)^2 (0.0049203)$$

$$+ 2(1.5305)(-0.0071508)$$

$$= 0.000062372 (\text{in}^3/\text{in}^3)^2$$

$$s^2_I(\langle \hat{\theta}_O \rangle) = (\hat{a}_1 - s^2(\hat{a}_1)) \frac{\langle \hat{n}_O \rangle}{p T_C} + \frac{\langle \hat{n}_O \rangle^2}{q T_S} \frac{1}{k \bar{N}_S^2} \quad (80)$$

$$= ((0.49296) - 0.0049203) \left[\frac{1.5305}{2 (60)} + \frac{(1.5305)^2}{10 (60)} \right]$$

$$\frac{1}{3(487.3967)^2}$$

$$= 0.000000094 (\text{in}^3/\text{in}^3)^2$$

$$\begin{aligned}
s^2(\hat{\theta}_O) &= s^2_I(\hat{\theta}_O) + s^2_C(\hat{\theta}_O) + s^2_L(\hat{\theta}_O) \quad (81) \\
&= 0.00000094 + 0.000062372 + 0.000015081 \\
&= 0.000078393 \text{ (in}^3/\text{in}^3)^2
\end{aligned}$$

From equation (30) the average count ratio is 1.5305.

The estimate of moisture content corresponding to a count ratio of 1.5305 is found using the regression coefficients:

$$\begin{aligned}
\hat{\theta}_O &= \hat{a}_0 + \hat{a}_1 \langle \hat{n}_O \rangle \quad (82) \\
&= -0.36198 + (0.49296)(1.5305) \\
&= 0.39250 \text{ in}^3/\text{in}^3
\end{aligned}$$

For 95.44% confidence limits, the estimate of moisture content is accurate within plus or minus two standard deviations. The following result applies only to 95.44% confidence limits and must not be confused with the standard error, which corresponds to 68 % confidence limits:

$$\begin{aligned}
\% \text{error} &= 2s^2(\hat{\theta}_O)/\hat{\theta}_O * 100 \quad (83) \\
&= (0.017708)/(0.39250) * 100 \\
&= 4.51\%
\end{aligned}$$

Components of variance are summarized for 12", 24" and 36" depths in Table 7. The total volume of water stored in the top three feet of soil can be calculated

using these results and the trapezoidal rule:

$$\begin{aligned}
 V_t &= [1.5(\hat{\theta}_o)_{12''} + \hat{\theta}_o_{24''} + \hat{\theta}_o_{36''}] \Delta z \quad (35) \\
 &= [1.5(0.39225) + (0.29677) + (0.24866)] 12'' \\
 &= 13.6056 \text{ in}^3
 \end{aligned}$$

The change in moisture content divided by the change in depth approximates the first derivative of the water distribution profile with respect to depth:

$$\begin{aligned}
 \Delta(\hat{\theta}_o)/\Delta z_{12-24''} &= (-0.09547)/(12) \quad (36) \\
 &= -0.0079558 \text{ in}^3/\text{in}^3/\text{in}
 \end{aligned}$$

$$\begin{aligned}
 \Delta(\hat{\theta}_o)/\Delta z_{24-36''} &= (-0.04811)/(12) \quad (87) \\
 &= -0.004009 \text{ in}^3/\text{in}^3/\text{in}
 \end{aligned}$$

Approximating the second derivative as the change in the first derivative divided by the change in depth:

$$\begin{aligned}
 \Delta(\Delta(\hat{\theta}_o)/\Delta z) &= [(-0.0079558) - (-0.004009)] / (12) \quad (88) \\
 &= -0.00032889 \text{ in}^3/\text{in}^3/\text{in}^2
 \end{aligned}$$

The variance in total volume of water can be calculated as the sum of variances at each depth, $s^2_1(V_t)$

TABLE 7

Components of Variance For Calibration

Data From Greeley, Colorado

	12"depth	24"depth	36"depth
\hat{a}_1	0.49296	0.44097	0.43940
\hat{a}_0	-0.36198	-0.27649	-0.31816
$s^2(\hat{a}_1) \times 10^{+3}$	4.9203	4.0513	19.635
$s^2(\hat{a}_0) \times 10^{+2}$	1.0426	0.6713	3.2845
$s(\hat{a}_0, \hat{a}_1) \times 10^{+3}$	-7.1508	-5.1676	-25.220
\bar{N}	745.96	634.50	627.94
$s^2(\hat{N})$	51.36	8190.6	6929.8
$\langle \hat{n}_0 \rangle$	1.53	1.30	1.29
$s^2(L_0)$	45.14	8185.3	6924.6
$s^2_I(\langle \hat{\theta}_0 \rangle) \times 10^{+6}$	0.094	1.778	1.604
$s^2_C(\langle \hat{\theta}_0 \rangle) \times 10^{+4}$	0.6237	1.235	4.523
$s^2_L(\langle \hat{\theta}_0 \rangle) \times 10^{+3}$.01508	2.187	1.685
$s^2(\langle \hat{\theta}_0 \rangle) \times 10^{+3}$.06237	2.312	2.139
Percent error*	4.51	32.40	37.20

*For 95.44% confidence limits

plus an additional component from the use of a numerical technique to approximate the integration, $s^2_2(V_t)$:

$$s^2_1(V_t) = \left[(1.5)^2 (s^2(\hat{\theta}_O)_{12''} + s^2(\hat{\theta}_O)_{24''} + s^2(\hat{\theta}_O)_{36''} \right] \Delta z^2 \quad (89)$$

$$= \left[(1.5)^2 (6.237 \times 10^{-5}) + (2.312 \times 10^{-3}) + (2.139 \times 10^{-3}) \right] (12)^2$$

$$= 0.66115 (\text{in}^3)^2$$

$$s^2_2(V_t) = \frac{z^2}{144} \Delta z^4 \left[\frac{\partial^2 \hat{\theta}_O(z)}{\partial^2 z} \right]^2 \quad (90)$$

$$= \frac{(42)^2}{144} (12)^4 (-0.00032889)^2$$

$$= 0.02747 (\text{in}^3)^2$$

$$s^2(V_t) = s^2_1(V_t) + s^2_2(V_t) \quad (91)$$

$$= 0.66115 + 0.02747$$

$$= 0.68863 (\text{in}^3)^2$$

For 95.44% confidence limits, the estimate of total water stored in the top three feet of soil is accurate within plus or minus two standard deviations. Expressed as a percent error (for 95.44% confidence limits):

$$\begin{aligned}
 \%error &= 2s^2 (V_t)^{\frac{1}{2}} / (V_t) * 100 \\
 &= 2(0.68863)^{\frac{1}{2}} / 13.6056 * 100 \\
 &= 12.20\%
 \end{aligned}
 \tag{92}$$

Analysis of the Components of Variance

The foregoing computations and results (summarized in Table 7) indicate that at a 12" depth soil conditions are more homogeneous than at 24" or 36" depths. This is borne out by the small location error at the 12" depth.

The instrument error is negligible in all cases. Calibration error is the dominant source of error at the 12" depth. However, as the depth increases location error is the more dominant error.

Estimation of Measurement Errors

Several degrees of sample compaction are assumed for a 6" deep cylinder, 1" in diameter. In this manner measurement errors may be estimated. Weighing errors are assumed negligible compared to errors resulting from sample compaction.

At the 12" depth with $\frac{1}{4}$ " soil compaction equation (62) can be written as:

$$s^2(v) = s^2(V_s)/V_s^2 \quad (93)$$

$$= (0.14642 \text{ in}^2)/(18.84956 \text{ in}^2)$$

$$= 0.00041211 \text{ in}^2/\text{in}^2 \quad .$$

The effect of including measurement errors in calibrations will be to decrease the calibration component of variance. In this case, the calibration component decreased from $0.00006237 \text{ in}^3/\text{in}^3$ to $0.000047637 \text{ in}^3/\text{in}^3$. The calibration component is shown for several degrees of sample compaction in Table 8.

TABLE 8

Influence of Measurement Errors on Calibration Error

Soil sample compaction	$s^2(v)$	$s^2_c(\langle \hat{\theta}_o \rangle)$
0 "	0.00000000	0.00006237
1/8"	0.00010303	0.00005302
1/4"	0.00041211	0.00004764
3/8"	0.00092724	0.00003577
1/2"	0.00164842	0.00001541

Error in the Change of Water Stored

Irrigation scheduling requires calculation of the change in water storage to determine the amount of water that must be applied to replenish the soil profile. The change in water storage, ΔW_s , may be calculated by:

$$\Delta W_s = V_{t2} - V_{t1} \quad , \quad (94)$$

where V_{t1} and V_{t2} are the volumes of water calculated in storage at time 1 and time 2, respectively.

The variance associated with the sum of two random variables is the sum of the variances attributed to each variable (Chapman, 1983). The variance associated with a change in water storage can be calculated by:

$$s^2(\Delta W_s) = s^2(V_{t2}) + s^2(V_{t1}) \quad . \quad (95)$$

The change in water storage is calculated for the Greeley soil examined in section 5.1. The water stored in the top three feet of soil was calculated to be 13.6056 in. The variance associated with this estimate was 0.68863 in². At a later date, water was depleted from the soil profile. The storage was recalculated to be 11.4444 in. with an associated variance of 0.6780 in². Table 9 contains data used to recalculate the storage.

TABLE 9

Data Used to Calculate the Change in Water Storage

Date: 8/15/83

At a 12" Depth:

$$\langle \hat{\theta}_O \rangle = 0.2986 \text{ (in}^3/\text{in}^3\text{)}$$

$$\langle \hat{n}_O \rangle = 1.34$$

$$s^2_I(\langle \hat{\theta}_O \rangle) = 0.000000005 \text{ (in}^3/\text{in}^3\text{)}$$

$$s^2_C(\langle \hat{\theta}_O \rangle) = 0.000096747 \text{ (in}^3/\text{in}^3\text{)}$$

$$s^2_L(\langle \hat{\theta}_O \rangle) = 0.000015081 \text{ (in}^3/\text{in}^3\text{)}$$

At a 24" Depth:

$$\langle \hat{\theta}_O \rangle = 0.2659 \text{ (in}^3/\text{in}^3\text{)}$$

$$\langle \hat{n}_O \rangle = 1.23$$

$$s^2_I(\langle \hat{\theta}_O \rangle) = 0.000000004 \text{ (in}^3/\text{in}^3\text{)}$$

$$s^2_C(\langle \hat{\theta}_O \rangle) = 0.000278954 \text{ (in}^3/\text{in}^3\text{)}$$

$$s^2_L(\langle \hat{\theta}_O \rangle) = 0.002187000 \text{ (in}^3/\text{in}^3\text{)}$$

At a 36" Depth:

$$\langle \hat{\theta}_O \rangle = 0.2399 \text{ (in}^3/\text{in}^3\text{)}$$

$$\langle \hat{n}_O \rangle = 1.27$$

$$s^2_I(\langle \hat{\theta}_O \rangle) = 0.000000004 \text{ (in}^3/\text{in}^3\text{)}$$

$$s^2_C(\langle \hat{\theta}_O \rangle) = 0.000198920 \text{ (in}^3/\text{in}^3\text{)}$$

$$s^2_L(\langle \hat{\theta}_O \rangle) = 0.001685000 \text{ (in}^3/\text{in}^3\text{)}$$

The change in water storage is calculated to be:

$$\Delta W_s = V_{t2} - V_{t1} \quad (96)$$

$$= 11.4444 \text{ in} - 13.6056 \text{ in}$$

$$= -2.1612 \text{ in} .$$

The variance associated with the estimated change in storage is:

$$s^2(\Delta W_s) = s^2(V_{t2}) + s^2(V_{t1}) \quad (97)$$

$$= 0.68863 \text{ in}^2 + 0.6780 \text{ in}^2$$

$$= 1.3666 \text{ in}^2 .$$

Expressed as a percent error for 95.44% confidence limits:

$$\% \text{error} = 2(1.3666)^{\frac{1}{2}} / 2.1612 * 100 \quad (98)$$

$$= 108.18\%$$

To replenish the soil profile it is calculated that 2.1612 in. of water must be applied. However, the calculation may be in error by as much as 108.18% for a 95.44% level of confidence.

In practice, soil moisture is usually replenished to field capacity when irrigating. A value of field capacity is determined independently of neutron probe

measurements and is assumed to remain constant. Therefore, a zero variance is associated with the value of field capacity and the variance associated with an estimate of the change in water storage is reduced.

For example, if the total volume of water at field capacity for the Greeley soil is determined to be 14.0000 in., the change in storage is:

$$\Delta W_s = V_{t2} - V_{t1} \quad (99)$$

$$= 11.4444 \text{ in} - 14 \text{ in}$$

$$= - 2.5556 \text{ in} .$$

The variance associated with the estimate of the change in storage is:

$$s^2(\Delta W_s) = s^2(V_{t2}) + s^2(V_{t1}) \quad (100)$$

$$= 0.6780 \text{ in}^2 + 0 \text{ in}^2$$

$$= 0.6780 \text{ in}^2 .$$

Expressed as a percent error for 95.44% confidence limits:

$$\% \text{error} = \pm 2(0.6780)^{\frac{1}{2}} / s.5556 * 100 \quad (101)$$

$$= \pm 64.44\% .$$

5.2 Computer Program To Calculate Components of Variance

An error analysis should be quickly and easily performed to be of use in irrigation scheduling applications. To facilitate the foregoing computations a computer program, VARMOIS, was written in Fortran IV by the author.

The variance in soil moisture measurements by the neutron probe is calculated using field data. Appendix B presents the computer program. An example of the input format is found in Appendix D. Several examples of output files are found in Appendix E.

5.3 Effect of Count Ratio on The Calibration Component of Variance

The calibration component of variance is a function of count ratio as well as calibration data (equation 37, page 47). Calibration data from several soils were used by the computer program, VARMOIS, to generate estimates of the calibration component of variance for several values of count ratios. Appendix E contains copies of the output files. The calibration component was observed to increase orders of magnitude when the calibration equation was used to predict soil moisture content outside the range of calibration data.

Calibration data from five benchmark agricultural soils common to California were generously supplied by Larry Schwankl of the Soil Conservation Service in Davis, California. Descriptions of the soils given by the National Cooperative Soil Survey, USDA, follow.

Sandy loams characterize the Camarillo soil series. Cultivation is common. The soils are well suited to use as fruit orchards.

The Holtville soils are silty clays, typically overlying layers of silt loam to loamy very fine sand. The series is cultivated and is found in low basins in a semi-desert environment.

The Maxwell series are typically deep, somewhat poorly drained soils formed in alluvium on basin rims or fans. Maxwell soils are commonly used as rangeland. Cracking clays are common in this series.

Indio soils are exemplified by a fine, sandy loam, used for cultivation. They lie on floodplains and low basins and usually have little slope. Arid climates prevail.

The Vista soil series is described as a moderately deep, well drained soil that formed in material weathered from decomposed granitic rocks. A typical Vista soil is a coarse sandy loam, on hilly slopes at elevations of 400 to 3,900 feet in Southern California and at less than 3,500 feet in Central California.

Using equation (37) the calibration component of variance was calculated for several values of count ratios for each soil. The results are shown in Figures 16 - 20. The calibration component was observed to increase orders of magnitude when calculated for count ratios outside of the range of calibration.

Lower Limit of The Calibration Component

Equation (37) shows the calibration component of variance to be a quadratic function in terms of count ratio $\langle \hat{n}_O \rangle$:

$$s^2_c(\langle \hat{\theta}_O \rangle) = s^2(\hat{a}_O) + \langle \hat{n}_O \rangle s^2(\hat{a}_1) + 2\langle \hat{n}_O \rangle s(\hat{a}_O, \hat{a}_1) \quad (37)$$

The value of $\langle \hat{n}_O \rangle$ that minimizes this function can be found by setting the derivative of the calibration component with respect to count ratio equal to zero, where:

$$\frac{ds^2_c(\langle \hat{\theta}_O \rangle)}{d \langle \hat{n}_O \rangle} = 2\langle \hat{n}_O \rangle s^2(\hat{a}_1) + 2s(\hat{a}_O, \hat{a}_1) \quad (102)$$

The value of $\langle \hat{n}_O \rangle$ for the derivative to be zero:

$$\langle \hat{n}_O \rangle = -s(\hat{a}_O, \hat{a}_1) / s^2(\hat{a}_1) \quad (103)$$

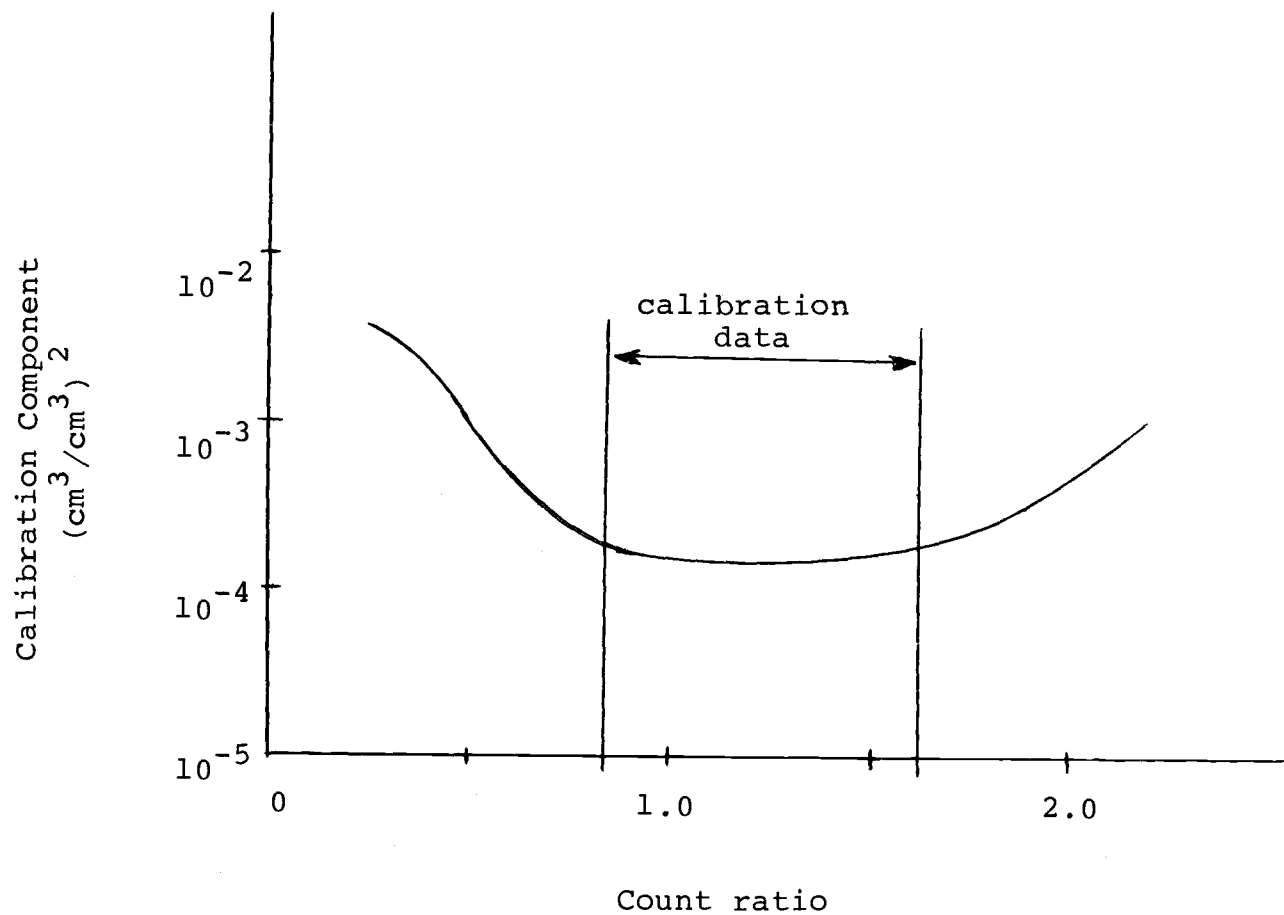


Figure 16. Effect of count ratio on the calibration component for Camarillo series soils.

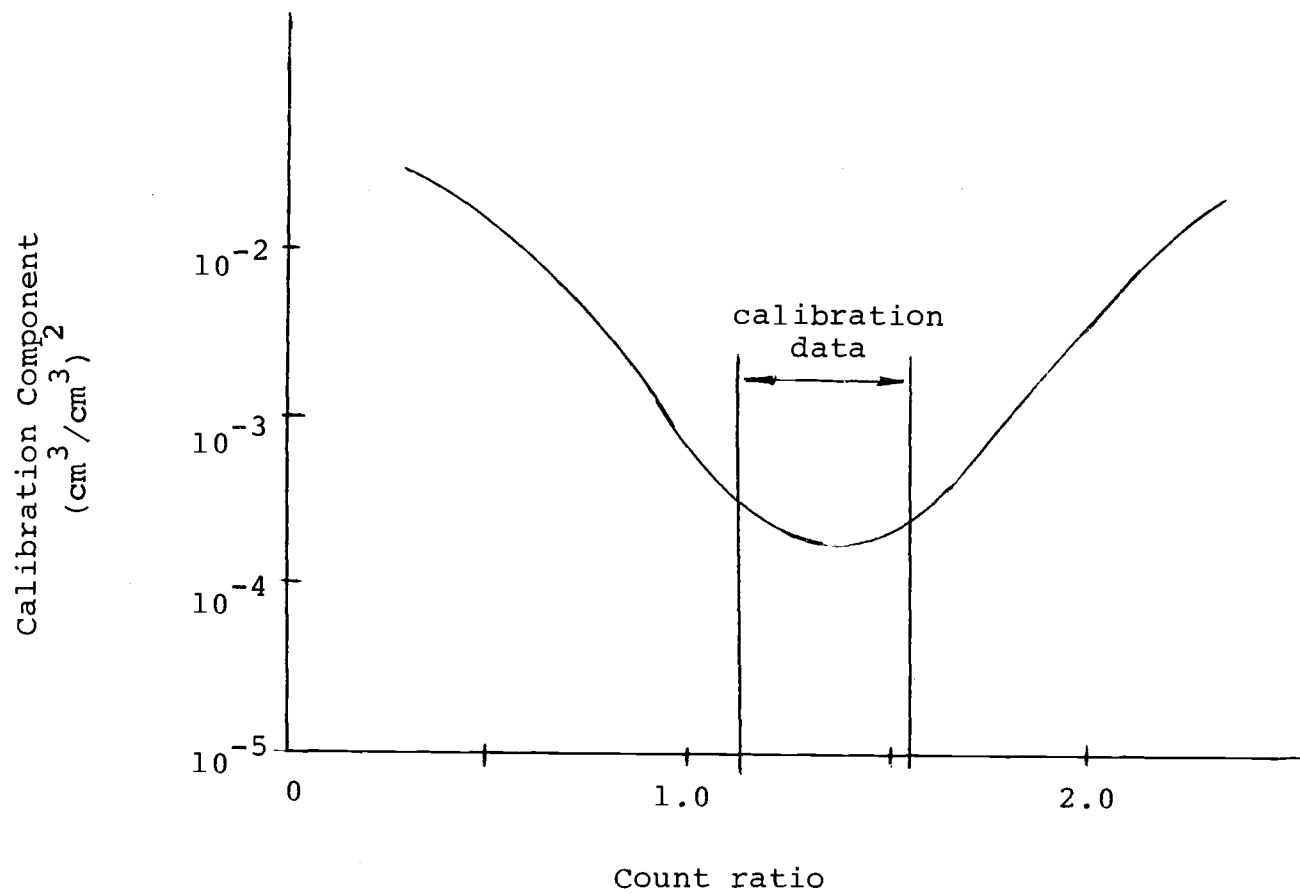


Figure 17. Effect of count ratio on the calibration component for Holtville series soils.

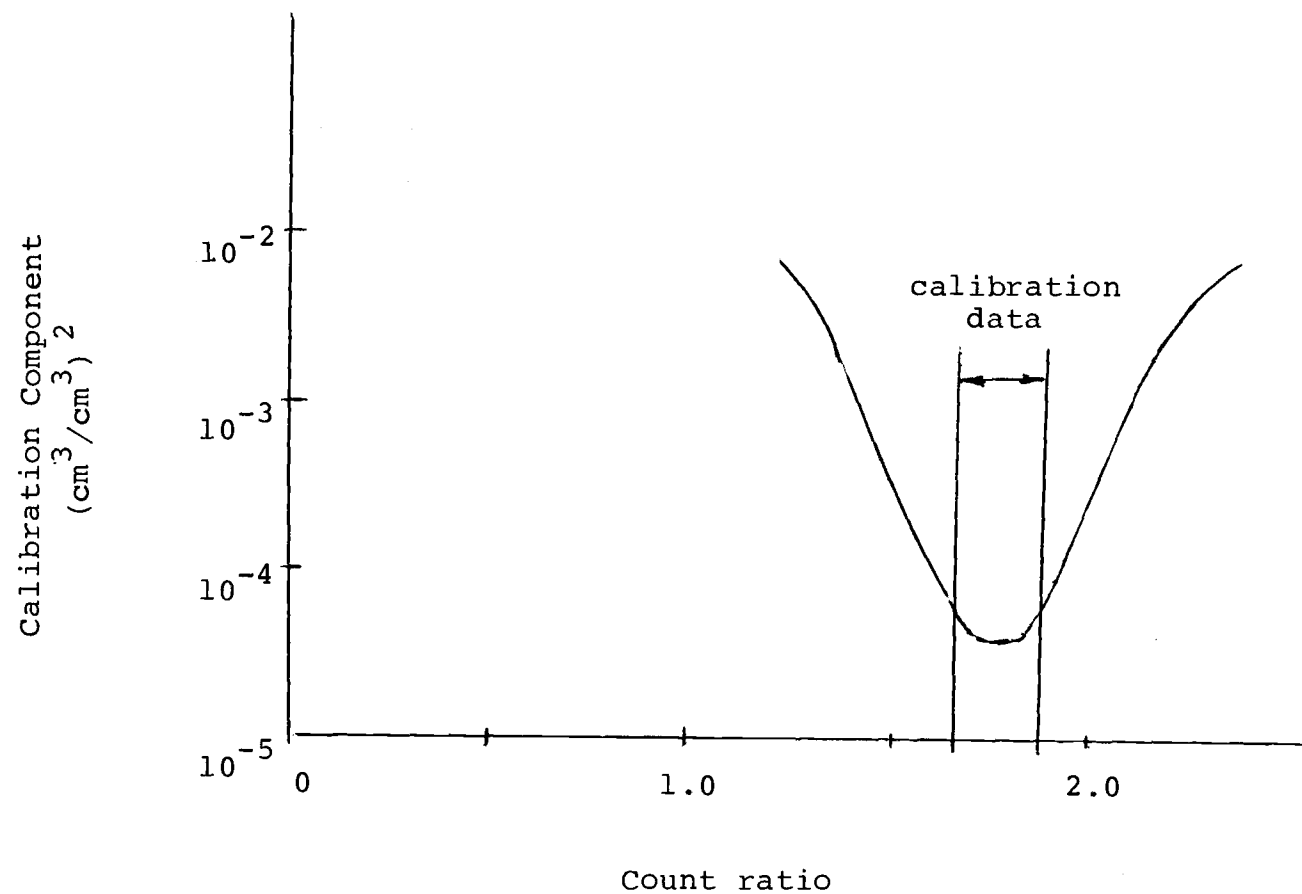


Figure 18. Effect of count ratio on the calibration component for Maxwell series soils.

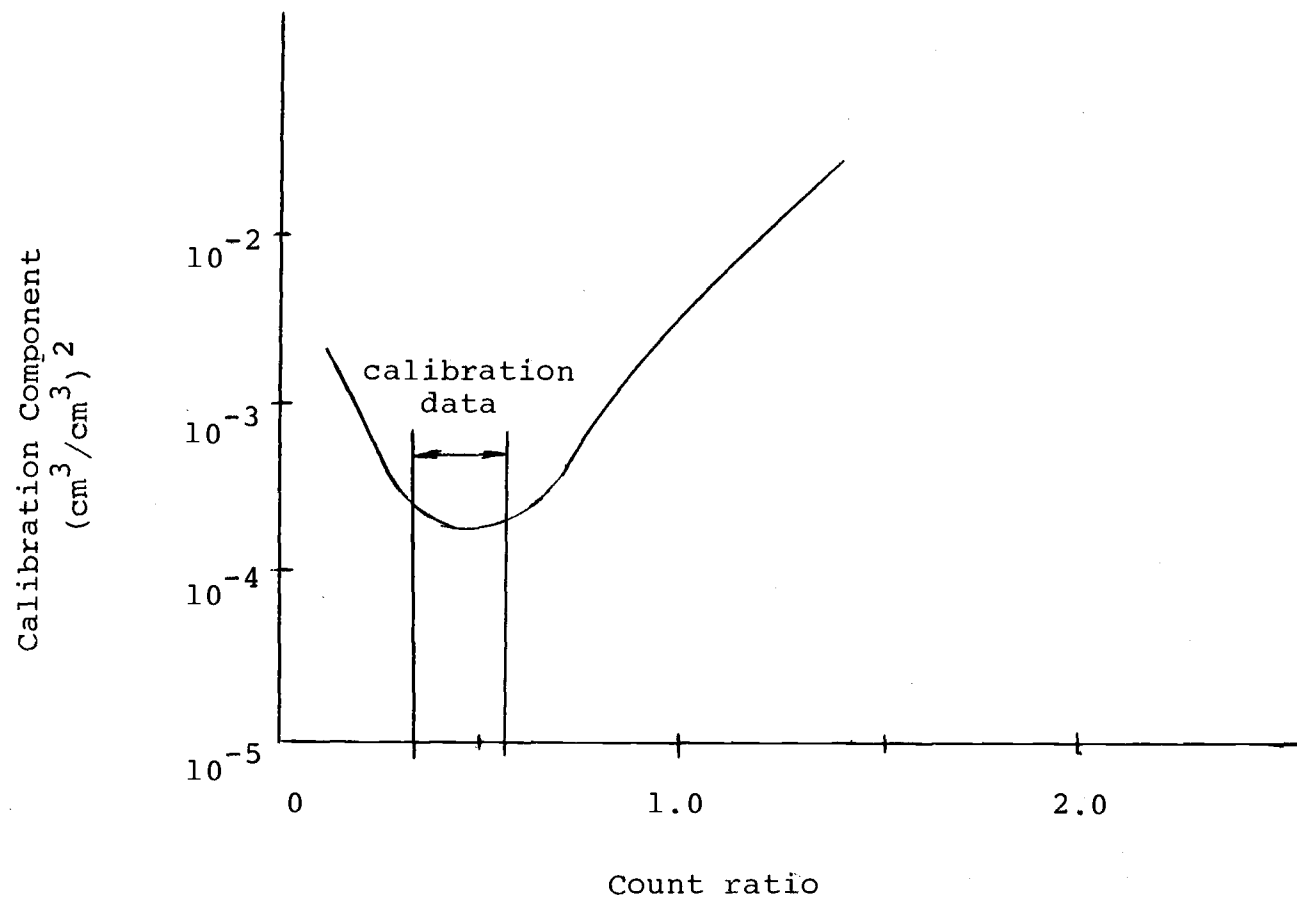


Figure 19. Effect of count ratio on the calibration component for Indio series soils.

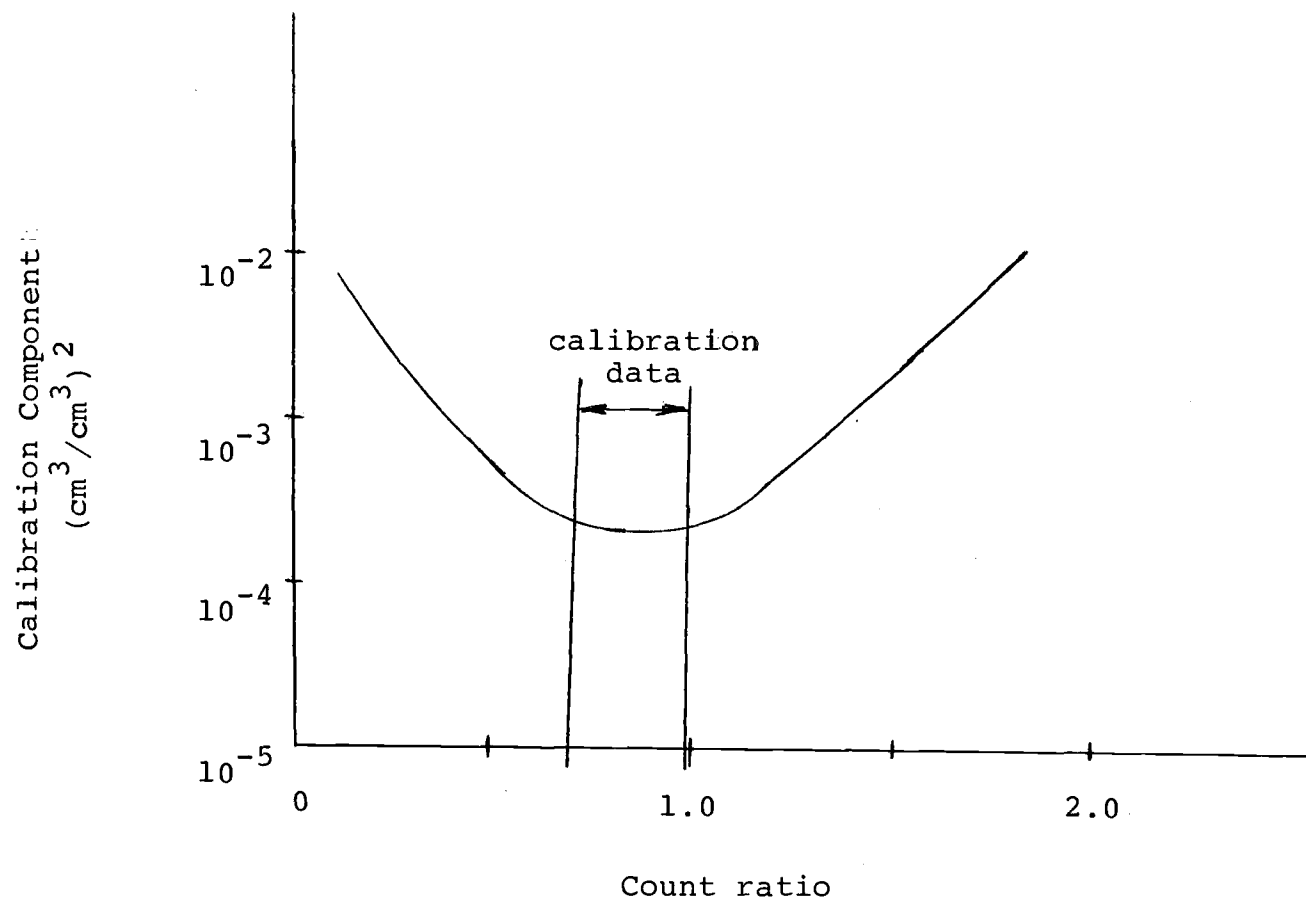


Figure 20. Effect of count ratio on the calibration component for Vista series soils.

Substituting this value back into equation (37):

$$s^2_C(\langle \hat{\theta}_0 \rangle) = s^2(\hat{a}_0) - s(\hat{a}_0, \hat{a}_1)^2 / s^2(\hat{a}_1) . \quad (104)$$

Figures 16 - 20 show the minimum calibration component is achieved for values of $\langle \hat{n}_0 \rangle$ in the middle of the range of calibration data. That minimum value may be calculated using equation (104).

5.4 Location Bias

Access tubes are rarely placed at random in a field. Usually, the intent is to deliberately bias the location. One access tube placed in a location that visually represents "average" field conditions may indicate true mean field moisture more accurately than several randomly placed access tubes. The statistical model overestimates the location component of variance when a biased access location is used since the model assumes all sampling to be random.

The location component may be adjusted to reflect the use of biased sample locations by replacing the actual number of biased locations, k in equation (36), by the number of random locations that would be required to duplicate the biased estimate. For

example, if it can be determined that one access tube placed in a location that visually represents average field conditions estimates mean field moisture as accurately as 20 randomly placed access tubes. The location component can be reduced by a factor of 20 since equation (36) shows the location component to be inversely proportional to k . Unfortunately, it is usually not possible to determine the number of equivalent random locations using calibration data.

An adjusted location error may be approximated by relating soil moisture at a biased location to the mean of the soil moisture of the original calibration locations. The case where a constant relation exists between the biased estimate and the sample mean is described below.

A sample of neutron count rates consists of count rates $N_1 \dots N_j$ with a mean \bar{N} , and a standard deviation, s_N . Each count rate is observed at a unique location $1 \dots j$. The standard deviation and distribution of count rates are assumed to remain constant with time.

The count rate observed at the biased access location is denoted N_* . The relationship between the mean count rate and N_* can be written:

$$\bar{N} = N_* \pm d , \quad (105)$$

where d is the difference between N_* and the sample mean. The mean count rate can also be estimated using only the biased count rate to reflect a change in soil moisture by:

$$\bar{N} + \Delta = N_* + \Delta \pm d , \quad (106)$$

where Δ is the change in count ratio due to a change in moisture content. The estimation of mean count rate is as accurate as if all j locations were sampled. The location component of variance may be adjusted accordingly by substituting j number of locations instead of 1 for term k in equation (36).

5.5 Calculating the Number of Access Tubes To Achieve A Required Accuracy

Many times it is of interest to know the number of access tubes that must be sampled to achieve a predetermined degree of accuracy. The number of required sampling locations can be calculated from the following analysis.

Assuming sample estimates of mean soil moisture, \bar{x} , to be normally distributed about the true mean moisture content, μ , with a variance, s^2 , the quantity $n^{\frac{1}{2}}(\bar{x}-\mu)/s$ follows the t distribution with $n-1$ degrees of freedom,

where n is the number of sample estimates. The probability that the quantity will be contained within the interval t_α and $-t_\alpha$ is given by (Chapman, 1983):

$$P \left\{ -t_\alpha \leq \frac{n^{1/2}(\bar{x} - \mu)}{s} \leq t_\alpha \right\} = 1 - \alpha. \quad (107)$$

Multiplying by s and dividing by $n^{1/2}$:

$$P \left\{ -t_\alpha s/n^{1/2} \leq (\bar{x} - \mu) \leq t_\alpha s/n^{1/2} \right\} = 1 - \alpha. \quad (108)$$

Adding \bar{x} to the quantities within the parentheses:

$$P \left\{ \bar{x} - t_\alpha s/n^{1/2} \leq \mu \leq \bar{x} + t_\alpha s/n^{1/2} \right\} = 1 - \alpha. \quad (109)$$

Defining d :

$$d = t_\alpha s/n^{1/2}, \quad (110)$$

squaring and rearranging variables:

$$n = t_\alpha^2 s^2 / d^2 \quad (111)$$

allows the calculation of the number of sampling locations necessary for an estimate of mean soil moisture to be within d units of the true mean. In the context of irrigation scheduling, n would be the number of access tube locations.

The spatial variability of soil moisture can be calculated from the spatial variability of count rates, $s^2(L_0)$, using equation (36) for $k=1$ sampling locations.

At a 24" depth for Greeley, Colorado, the spatial variance in soil moisture is calculated to be:

$$\begin{aligned}
 s^2_L(\langle \hat{\theta}_O \rangle) &= (\hat{a}_1 - s^2(\hat{a}_1)) \frac{s^2(L_O)}{k \bar{N}_s^2} & (112) \\
 &= \{ (0.44097)^2 - (0.004051) \} \frac{(8185.3)}{(1)(487)^2} \\
 &= 0.0065606
 \end{aligned}$$

Mean soil moisture content was calculated to be 0.29677 cm³/cm³ on 7/6/83. If a minimum of 10% deviation due to location error is acceptable, the parameter d becomes 0.029677 cm³/cm³.

The number of sampling locations necessary to predict mean moisture content for this spatial distribution is:

$$\begin{aligned}
 k &= t_{\alpha}^2 s^2 / d^2 & (113) \\
 &= (1.96)^2 \frac{(0.0065606)}{(0.029677)^2} \\
 &= 28.616 \rightarrow 29 .
 \end{aligned}$$

The number of access tubes must be increased from 3 to 29 to estimate mean soil moisture within a 10% location error. This analysis assumes all locations are chosen randomly and does not consider the calibration or instrument component of error.

6. CONCLUSIONS

A review of the literature shows there are three components of variance to consider in an analysis of the error for neutron probe measurements of soil moisture: the instrument component, the calibration component, and the location component. Instrument error accounts for random neutron emission by the probe source. Calibration error is introduced by inaccuracies in the calibration curve. Location error is a result of estimating average soil moisture throughout a field using only point estimates.

The variance in neutron probe estimates can be calculated using field calibration data. Relating variances to specific confidence limits, an estimate of errors can be obtained. The calculations were incorporated into a computer program to quickly and easily evaluate probe estimates.

An analysis of field calibration data from Greeley, Colorado, shows the instrument error to be negligible in all cases. Where soil conditions are relatively homogeneous, the calibration error is the dominant source of error. For heterogeneous soil conditions, the location error is the most significant component of error.

Usually, calibration data do not provide information to calculate the magnitude of calibration measurement errors. However, if measurement errors can be estimated, a refined estimate of the regression coefficients and their variances can be obtained. The result is to decrease the calibration component of error.

When the neutron probe is used to estimate soil moisture beyond the range of calibration data, the calibration component is substantially increased.

The statistical model overestimates the location component of error when a biased access tube location is used to predict average soil moisture since the model assumes all locations are randomly sampled. The location component may be adjusted by substituting the number of random locations that would need to be sampled to duplicate a biased observation for the number of biased locations actually sampled. Calibration data can sometimes be used to approximate an adjusted location error component.

The number of access tubes that must be sampled to achieve a predetermined degree of accuracy can be calculated from the definition of confidence limits for a student's "t" distribution.

Knowledge of the magnitude and source of errors associated with the neutron probe will enable an operator to compensate accordingly. More accurate measurements of soil moisture can then be obtained.

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APPENDIX

APPENDIX A

Derivation of Equations to Predict Variances in Soil Moisture Measurements Using The Neutron Probe

The variables below are described since their definitions are specific to these equations, and may not correspond to what is considered common nomenclature. The descriptions given will, however, remain constant throughout this text and are as follows:

θ = true volumetric water content; its value is unknown and hidden by errors

n = a true neutron count ratio corresponding to θ ; its value is also unknown and hidden by errors

$\hat{\theta}$ = a value of volumetric water content observed in situ, usually in a calibration process

\hat{n} = an observed value of count ratio corresponding to $\hat{\theta}$ and normally associated with the calibration process

$\hat{\theta}_0$ = a value of volumetric water content determined by the linear regression (calibration) equation

\hat{n}_0 = the count ratio for which $\hat{\theta}_0$ is defined

u = measurement errors in the observed neutron count ratio, \hat{n}

v = measurement errors in the observed value of volumetric water content, $\hat{\theta}$

e = an error term including v as well as deviations from an exact linear model defined by the calibration equation, assumed to be normally distributed with an expected value of zero

\bar{N} = the average of p replications of count rate, N , obtained during a counting time of T_C seconds at the same location

\bar{N}_s = the average of q replications of count rate, N_s , obtained during a counting time of T_s seconds in a standard absorber

The following equalities help clarify the above descriptions:

$$\hat{\theta} = \theta + v \quad (98)$$

$$\hat{n} = n + u = \bar{N} / \bar{N}_s \quad (99)$$

The true relationship between volumetric water content and neutron count ratios is assumed to be linear in the region encountered in agricultural applications, and of the form (Haverkamp, et al., 1983):

$$\theta = a_0 + a_1 n + e \quad , \quad (100)$$

where a_0 and a_1 are linear coefficients. In practice, the relationship between volumetric water content and neutron count ratios is given by a linear regression of

the calibration data (i.e. paired values of $\hat{\theta}, \hat{n}$) as:

$$\hat{\theta} = \hat{a}_0 + \hat{a}_1 \hat{n}_0, \quad (101)$$

where

$$E(\hat{a}_0) = a_0, \quad (102)$$

and

$$E(\hat{a}_1) = a_1. \quad (103)$$

These concepts are visualized in Figure 16.

Subtracting equation (101) from equation (100) yields:

$$(\theta - \hat{\theta}) = a_0 - \hat{a}_0 + a_1 n - \hat{a}_1 \hat{n}_0 + e_0. \quad (104)$$

This can be rearranged to read:

$$(\theta - \hat{\theta}) = (a_0 - \hat{a}_0) + \hat{a}_1 (n - \hat{n}_0) + n(a_1 - \hat{a}_1) + e_0, \quad (105)$$

since

$$\begin{aligned} \hat{a}_1 (n - \hat{n}_0) + n(a_1 - \hat{a}_1) &= \hat{a}_1 n - \hat{a}_1 \hat{n}_0 + na_1 - \hat{a}_1 n \quad (106) \\ &= a_1 n - \hat{a}_1 \hat{n}_0. \end{aligned}$$

It can be shown that $\hat{\theta}_0$ is an unbiased estimator of θ since taking expected values of equation (104) gives $E(\theta - \hat{\theta}) = 0$:

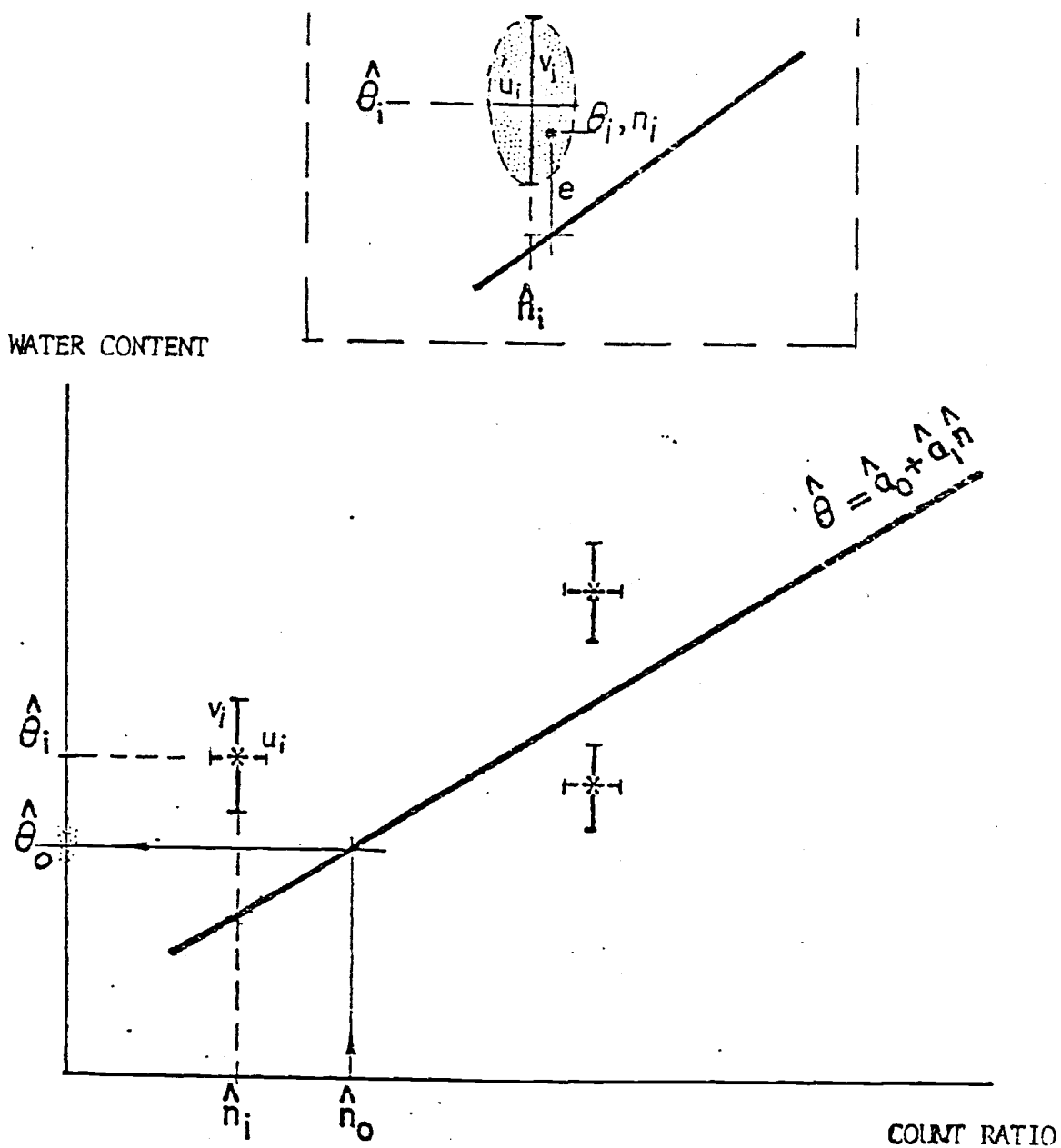


Figure 21. Parameters and errors in calibration curves (Haverkamp, 1982).

$$\begin{aligned}
E\{(\theta - \hat{\theta}_0)\} &= E\{(a_0 - \hat{a}_0)\} + E\{a_1(n - \hat{n}_0)\} + E\{n(a_1 - \hat{a}_1)\} \\
&\quad + E\{e\} \quad (107) \\
&= E\{a_0\} - E\{\hat{a}_0\} + E\{a_1\}[E\{n\} - E\{\hat{n}_0\}] \\
&\quad + E\{n\}[E\{a_1\} - E\{\hat{a}_1\}] + E\{e\} \\
&= a_0 - a_0 + a_1(n - n) + n(a_1 - a_1) \\
&= 0.
\end{aligned}$$

It is assumed that e is independent of \hat{a}_0 , \hat{a}_1 and \hat{n}_0 and that \hat{a}_0 and \hat{a}_1 are independent of \hat{n}_0 .

Squaring equation (104) gives:

$$\begin{aligned}
(\theta - \hat{\theta}_0)^2 &= (a_0 - \hat{a}_0)^2 + \hat{a}_1^2(n - \hat{n}_0)^2 + n^2(a_1 - \hat{a}_1)^2 \quad (108) \\
&\quad + e^2 + 2(a_0 - \hat{a}_0)\hat{a}_1(n - \hat{n}_0) + 2(a_0 - \hat{a}_0) \\
&\quad n(a_1 - \hat{a}_1) + 2\hat{a}_1(n - \hat{n}_0)n(a_1 - \hat{a}_1) \\
&\quad + e[(a_0 - \hat{a}_0) + \hat{a}_1(n - \hat{n}_0) + n(a_1 - \hat{a}_1)].
\end{aligned}$$

Taking expected values of equation (108) produces:

$$\begin{aligned}
E\{(\theta - \hat{\theta}_0)^2\} &= E\{(a_0 - \hat{a}_0)^2\} + E\{\hat{a}_1^2(n - \hat{n}_0)^2\} \quad (109) \\
&\quad + E\{n^2(a_1 - \hat{a}_1)^2\} + E\{e\} \\
&\quad + E\{2(a_0 - \hat{a}_0)\hat{a}_1(n - \hat{n}_0)\} + E\{2(a_0 - \hat{a}_0) \\
&\quad n(a_1 - \hat{a}_1)\} + E\{2\hat{a}_1(n - \hat{n}_0)n(a_1 - \hat{a}_1)\}
\end{aligned}$$

$$+ E\{e \left[(a_0 - \hat{a}_0) + \hat{a}_1 (n_0 - \hat{n}_0) + n (a_1 - \hat{a}_1) \right]\}$$

Examining equation (109) term by term:

$$\begin{aligned} E\{(\theta - \hat{\theta}_0)^2\} &= E\{(\hat{\theta}_0 - \theta)^2\} = E\{(\hat{\theta}_0 - E(\hat{\theta}_0))^2\} \quad (110) \\ &= \text{var}(\hat{\theta}_0) \end{aligned}$$

$$\begin{aligned} E\{(a_0 - \hat{a}_0)^2\} &= E\{(\hat{a}_0 - a_0)^2\} = E\{(\hat{a}_0 - E(\hat{a}_0))^2\} \quad (111) \\ &= \text{var}(a_0) \end{aligned}$$

$$\begin{aligned} E\{\hat{a}_1^2 (n - \hat{n}_0)^2\} &= E\{\hat{a}_1^2\} E\{(\hat{n}_0 - n)^2\} \quad (112) \\ &= E\{\hat{a}_1^2\} E\{(\hat{n}_0 - E(\hat{n}_0))^2\} \\ &= \overline{\hat{a}_1^2} \text{var}(\hat{n}_0) \end{aligned}$$

$$\begin{aligned} E\{n^2 (a_1 - \hat{a}_1)^2\} &= E\{n^2\} E\{(\hat{a}_1 - a_1)^2\} \quad (113) \\ &= E\{n^2\} E\{(\hat{a}_1 - E(a_1))^2\} \\ &= \overline{n^2} \text{var}(\hat{a}_1) \end{aligned}$$

$$E\{e^2\} = E\{(e - E(e))^2\} \quad (114)$$

$$= \text{var}(e)$$

$$\begin{aligned} E\{2(a_0 - \hat{a}_0)\hat{a}_1(n - \hat{n}_0)\} &= 2E\{(a_0 - \hat{a}_0)\hat{a}_1\}E\{(n - \hat{n}_0)\} \quad (115) \\ &= 2E\{(a_0 - \hat{a}_0)\hat{a}_1\}E\{n_0\} - E\{\hat{n}_0\} \\ &= 2E\{(a_0 - \hat{a}_0)\hat{a}_1\}[n_0 - n_0] \\ &= 0 \end{aligned}$$

$$\begin{aligned}
E\{2(a_0 - \hat{a}_0)n(a_1 - \hat{a}_1)\} &= 2E\{n\} E\{(a_0 - \hat{a}_0)(a_1 - \hat{a}_1)\} \quad (116) \\
&= 2E\{n\} E\{(\hat{a}_0 - a_0)(\hat{a}_1 - a_1)\} \\
&= 2E\{n\} E\{(\hat{a}_0 - E(\hat{a}_0))(\hat{a}_1 - E(\hat{a}_1))\} \\
&= 2 \overline{n} \text{cov}(\hat{a}_0, \hat{a}_1)
\end{aligned}$$

$$\begin{aligned}
E\{2\hat{a}_1(n - \hat{n}_0)n(a_1 - \hat{a}_1)\} &= 2E\{\hat{a}_1(a_1 - \hat{a}_1)\}E\{n(n - \hat{n}_0)\}^* \\
&= 2E\{\hat{a}_1(a_1 - \hat{a}_1)\}E\{n\} E\{(n - \hat{n}_0)\} \\
&= 2E\{\hat{a}_1(a_1 - \hat{a}_1)\}E\{n\} [E(n) \\
&\quad - E(\hat{n}_0)] \\
&= 2E\{\hat{a}_1(a_1 - \hat{a}_1)\} E\{n\} [n - n] \\
&= 0 \quad (117)
\end{aligned}$$

$$\begin{aligned}
E\{e(a_0 - \hat{a}_0) + \hat{a}_1(n - \hat{n}_0) + n(a_1 - \hat{a}_1)\} &= E\{e\}E\{(a_0 - \hat{a}_0) \\
&\quad + \hat{a}_1(n - \hat{n}_0) + n(a_1 - \hat{a}_1)\} \quad (118)
\end{aligned}$$

$$= 0 .$$

Var denotes the variance and cov the covariance of the random variable within the corresponding parenthesis.

*Note that n is fixed while \hat{n}_0 is considered random.

Substitution of the relations derived in equations (110) through (118) back into equation (109) results in:

$$\begin{aligned} \text{var } (\hat{\theta}_0) = & \text{var } (\hat{a}_0) + \overline{a_1^2} \text{var } (\hat{n}_0) + \overline{n^2} \text{var } (\hat{a}_1) \\ & + \text{var } (e) + 2 \overline{n} \text{cov } (\hat{a}_0, \hat{a}_1) \end{aligned} \quad (119)$$

The variance of \hat{a}_1 can be written as:

$$\begin{aligned} \text{var } (\hat{a}_1) = & E(\hat{a}_1^2) - (E(\hat{a}_1))^2 \\ = & \overline{\hat{a}_1^2} - (\overline{\hat{a}_1})^2. \end{aligned} \quad (120)$$

Rearranging terms,

$$\overline{\hat{a}_1^2} = \text{var } (\hat{a}_1) + (\overline{\hat{a}_1})^2. \quad (121)$$

Substituting this expression into equation (119):

$$\begin{aligned} \text{var } (\hat{\theta}_0) = & \text{var } (\hat{a}_0) + \text{var } (\hat{n}_0) [\text{var } (\hat{a}_1) + (\overline{\hat{a}_1})^2] \\ & + \overline{n^2} \text{var } (\hat{a}_1) + \text{var } (e) \\ & + 2 \overline{n} \text{cov } (\hat{a}_0, \hat{a}_1). \end{aligned} \quad (122)$$

Since n is fixed (given),

$$\text{var } (n) = 0. \quad (123)$$

By definition,

$$\begin{aligned} \text{var } (n) = & E(n^2) - (E(n))^2 \\ = & \overline{n^2} - (\overline{n})^2 \end{aligned} \quad (124)$$

Substituting equation (124) into equation (123) :

$$\overline{n^2} - (\overline{n})^2 = 0 , \quad (125)$$

or

$$\overline{n^2} = (\overline{n})^2 . \quad (126)$$

Introducing this result into equation (122) gives:

$$\begin{aligned} \text{var } (\hat{\theta}_0) &= \text{var } (\hat{a}_0) + \text{var } (\hat{n}_0) \text{var} [(\hat{a}_1) + (\overline{a_1})^2] \\ &\quad + (\overline{n})^2 \text{var } (\hat{a}_1) + \text{var } (e) \\ &\quad + 2 \overline{n} \text{cov } (\hat{a}_0, \hat{a}_1) . \end{aligned} \quad (127)$$

Expanding the third term in equation (127) :

$$\begin{aligned} \text{var } (\hat{n}_0) [\text{var } (\hat{a}_1) + (\overline{a_1})^2] &= \text{var } (\hat{n}_0) [\text{var } (\hat{a}_1) \\ &\quad + (E(a_1))^2] \\ &= \text{var } (\hat{n}_0) \text{var } (\hat{a}_1) \\ &\quad + \text{var } (\hat{n}_0) (E(\hat{a}_1))^2 \end{aligned} \quad (128)$$

Substituting this result back into equation (127) :

$$\begin{aligned} \text{var } (\hat{\theta}_0) &= \text{var } (\hat{a}_0) + \text{var } (\hat{n}_0) \text{var } (\hat{a}_1) + \text{var } (\hat{n}_0) \\ &\quad (E(\hat{a}_1))^2 + (\overline{n})^2 \text{var } (\hat{a}_1) + \text{var } (e) \\ &\quad + 2 \overline{n} \text{cov } (\hat{a}_0, \hat{a}_1) \end{aligned} \quad (129)$$

Grouping like terms together:

$$\begin{aligned} \text{var } (\hat{\theta}_0) &= \text{var } (\hat{a}_0) + \text{var } (\hat{n}_0) [E(a_1)]^2 + \text{var } (\hat{a}_1) \\ &\quad [\text{var } (\hat{n}_0) + \overline{(n_0)^2}] + \text{var } (e) \\ &\quad + 2 \overline{n} \text{cov } (\hat{a}_0, \hat{a}_1) . \end{aligned} \quad (130)$$

Adding and subtracting $\text{var } (\hat{a}_1)$ to the third term of equation (130) will not change the equality:

$$\begin{aligned} \text{var } (\hat{\theta}_0) &= \text{var } (\hat{a}_0) + \text{var } (\hat{n}_0) [(E(\hat{a}_1))^2 + \text{var } (\hat{a}_1) \\ &\quad - \text{var } (\hat{a}_1)] + \text{var } (\hat{a}_1) [\text{var } (\hat{n}_0) + \overline{(n_0)^2}] \\ &\quad + \text{var } (e) + 2 \overline{n} \text{cov } (\hat{a}_0, \hat{a}_1) . \end{aligned} \quad (131)$$

From the definition of the variance of a_1 ,

$$E(\hat{a}_1^2) = (E(\hat{a}_1))^2 + \text{var } (\hat{a}_1) . \quad (132)$$

This relation can be replaced into equation (131) to give:

$$\begin{aligned} \text{var } (\hat{\theta}_0) &= \text{var } (\hat{a}_0) + \text{var } (\hat{n}_0) [E(a_1^2) - \text{var } (\hat{a}_1)] \\ &\quad + \text{var } (\hat{a}_1) [\text{var } (\hat{n}_0) + \overline{(n_0)^2}] + \text{var } (e) \\ &\quad + 2 \overline{n} \text{cov } (\hat{a}_0, \hat{a}_1) . \end{aligned} \quad (133)$$

Using sample estimates to evaluate equation (133), \hat{n}_0 will approximate n , \hat{a}_1^2 will replace $\overline{\hat{a}_1^2}$ and s^2 will replace var . The covariance for the sample estimate is

estimated by $s(\hat{a}_0, \hat{a}_1)$. The expected value of \hat{a}_1^2 becomes $\overline{\hat{a}_1^2}$ for the sample estimate. The use of the following equality is also incorporated into the results:

$$\text{var } (\hat{n}_0) + (\overline{\hat{n}_0})^2 = \overline{\hat{n}_0^2} \quad (134)$$

Using sample estimates:

$$\begin{aligned} s^2(\hat{\theta}_0) = & s^2(\hat{a}_0) + s^2(\hat{n}_0) [\hat{a}_1^2 - s^2(\hat{a}_1)] \\ & + s^2(\hat{a}_1) [\overline{\hat{n}_0^2}] + s^2(e) + 2 \overline{\hat{n}_0} s(\hat{a}_0, \hat{a}_1) \end{aligned} \quad (135)$$

Usually, only one value of \hat{n}_0 is available to estimate $\overline{\hat{n}_0}$. Equation (135) is then written as:

$$\begin{aligned} s^2(\hat{\theta}_0) = & s^2(\hat{a}_0) + s^2(\hat{n}_0) [\hat{a}_1^2 - s^2(\hat{a}_1)] + s^2(\hat{a}_1) (\hat{n}_0) \\ & + s^2(e) + 2 \hat{n}_0 s(\hat{a}_0, \hat{a}_1) \end{aligned} \quad (136)$$

The statistical formula for calculating the variance of quotients is given by (Sinclair, 1979):

$$\begin{aligned} \text{var } (x/y) = & (\mu/n)^2 (\text{var } (x)/\mu^2 + \text{var } (y)/n^2 \\ & - [2 \text{ cov } (x, y)] / (\mu n) , \end{aligned} \quad (137)$$

where x and y are any random variables with means μ and n respectively.

Using this formula to calculate the sample variance of the count ratio \hat{n}_0 results in:

$$s^2(\hat{n}_O) = s^2\left(\frac{\overline{N_O}}{\overline{N_S}}\right) = \left(\frac{\overline{N_O}}{\overline{N_S}}\right)^2 \left(\frac{s^2(\overline{N_O})}{(\overline{N_O})^2} + \frac{s^2(\overline{N_S})}{(\overline{N_S})^2} \right) \quad (138)$$

The means, $\overline{N_O}$ and $\overline{N_S}$, are considered to be independent so that their covariance is zero.

Expressing the definition of $\overline{N_O}$ given at the beginning of this Appendix in equation form yields:

$$\overline{N_O} = \frac{1}{p T_C} \sum_{i=1}^p \sum_{j=1}^{T_C} \hat{N}_{Oij} \quad (139)$$

Assuming neutron counts to be Poisson distributed:

$$s^2(\overline{N_O}) = \frac{1}{p T_C} \overline{N_O} \quad (140)$$

A similar process gives the variance in the mean standard count rate as:

$$s^2(\overline{N_S}) = \frac{1}{p T_C} \overline{N_S} \quad (141)$$

Replacing equations (141) and (140) into equation (138):

$$s^2(\hat{n}_O) = \left(\frac{\overline{N_O}}{\overline{N_S}}\right)^2 \left[\frac{\overline{N_O}}{p T_C \overline{N_O}^2} + \frac{\overline{N_S}}{q T_S \overline{N_S}^2} \right] \quad (142)$$

Rearranging terms and carrying out the multiplication:

$$\begin{aligned} s^2(\hat{n}_O) &= \frac{1}{\overline{N_S}} \left[\frac{\overline{N_O}^3}{\overline{N_O}^2 \overline{N_S} p T_C} + \frac{\overline{N_O}^2 \overline{N_S}}{q T_S \overline{N_S}^3} \right] \\ &= \frac{1}{\overline{N_S}} \left[\frac{\overline{N_O}}{\overline{N_S}} \frac{1}{p T_C} + \frac{\overline{N_O}^2}{\overline{N_S}} \frac{1}{q T_S} \right] \end{aligned} \quad (143)$$

Realizing \hat{n}_O is the ratio of $\overline{\hat{N}_O}$ to $\overline{N_S}$:

$$s^2(\hat{n}_O) = \frac{1}{\overline{N_S}} \left[\frac{\hat{n}_O}{pT_C} + \frac{\hat{n}_O^2}{qT_S} \right] \quad (144)$$

Substituting equation (144) into equation (136) :

$$s^2(\hat{\theta}_O) = s^2(\hat{a}_O) + \frac{1}{\overline{N_S}} \left[\frac{\hat{n}_O}{pT_C} + \frac{\hat{n}_O^2}{qT_S} \right] (\hat{a}_1 - s^2(\hat{a}_1)) \quad (145)$$

$$+ s^2(\hat{a}_1) \hat{n}_O^2 + 2 \hat{n}_O s(\hat{a}_O, \hat{a}_1) + s^2(e) \quad .$$

Mean values of moisture content and count ratios , averaged from k access tube locations at one depth, can be defined by:

$$\langle \hat{\theta}_O \rangle = \frac{1}{k} \sum_{j=1}^k \hat{\theta}_{O,j} \quad (146)$$

$$\langle \hat{n}_O \rangle = \frac{1}{k} \sum_{j=1}^k \hat{n}_{O,j} \quad (147)$$

$$\langle \theta \rangle = \frac{1}{k} \sum_{j=1}^k \theta_j \quad (148)$$

$$\langle n \rangle = \frac{1}{k} \sum_{j=1}^k n_j \quad (149)$$

When mean values are substituted into equation (104), the error term cancels since its mean value is assumed to be zero:

$$(\langle \theta \rangle - \langle \hat{\theta}_O \rangle) = a_O - \hat{a}_O + a_1 \langle n \rangle - \hat{a}_1 \langle \hat{n}_O \rangle \quad (150)$$

Following the same analysis to derive equation (145), equation (150) becomes:

$$s^2(\langle \hat{\theta}_0 \rangle) = (\hat{a}_1^2 - s^2(\hat{a}_1)) s^2(\langle \hat{n}_0 \rangle) + s^2(\hat{a}_0) + \langle \hat{n}_0 \rangle^2 s^2(\hat{a}_1) + 2\langle \hat{n}_0 \rangle s(\hat{a}_0, \hat{a}_1) . \quad (151)$$

Realizing the count ratio \hat{n}_0 to be the ratio of \hat{N}_0 to N_s and assuming each observation of \hat{n}_0 to be independent of another:

$$s^2(\langle \hat{n}_0 \rangle) = \frac{1}{k} s^2\left(\frac{\hat{N}_0}{N_s}\right) . \quad (152)$$

Applying the statistical formula given to calculate the variance of quotients:

$$\begin{aligned} s^2(\langle \hat{n}_0 \rangle) &= \frac{1}{k} \left(\frac{\hat{N}_0}{N_s}\right)^2 \left[\frac{s^2(\hat{N}_0)}{\hat{N}_0^2} + \frac{s^2(N_s)}{N_s^2} \right] \\ &= \frac{1}{k N_s^2} \left[s^2(\hat{N}_0) + \frac{\hat{N}_0^2}{N_s^2} s^2(N_s) \right] \\ &= \frac{1}{k N_s^2} \left[s^2(\hat{N}_0) + \langle \hat{n}_0 \rangle^2 s^2(N_s) \right] . \end{aligned} \quad (153)$$

The random emission of neutrons and the variation of count rates due to spatial variability of moisture content both contribute to the variance of \hat{N}_0 :

$$s^2(\hat{N}_0) = s^2(L_0) + \frac{\hat{N}_0}{pT_c} , \quad (154)$$

where $s^2(L_O)$ is the spatial variability of neutron count rates.

The variance in standard counts is unaffected by spatial variability and remains constant. Equation (153) is now written as:

$$s^2(\langle \hat{n}_O \rangle) = \frac{1}{k\bar{N}_S^2} \{ s^2(L_O) + \frac{\bar{N}_O}{pT_C} + \langle \hat{n}_O \rangle^2 \frac{\bar{N}_S}{qT_S} \} \quad (155)$$

Rearranging and multiplying through by \bar{N}_S^2 and \bar{N}_S gives:

$$\begin{aligned} s^2(\langle \hat{n}_O \rangle) &= \frac{1}{k\bar{N}_S^2} s^2(L_O) + \frac{1}{k\bar{N}_S} \left\{ \frac{\bar{N}_O}{\bar{N}_S} \frac{1}{pT_C} \right. \\ &\quad \left. + \langle \hat{n}_O \rangle^2 \frac{\bar{N}_S}{\bar{N}_S} \frac{1}{qT_S} \right\} \\ &= \frac{1}{k\bar{N}_S^2} s^2(L_O) + \frac{1}{k\bar{N}_S} \left\{ \frac{\langle \hat{n}_O \rangle}{pT_C} + \frac{\langle \hat{n}_O \rangle^2}{qT_S} \right\} \end{aligned} \quad (156)$$

Substituting equation (156) into equation (151):

$$\begin{aligned} s^2(\langle \hat{\theta}_O \rangle) &= s^2(\hat{a}_O) + (\hat{a}_1^2 - s^2(\hat{a}_1)) \left\{ \frac{1}{k\bar{N}_S^2} s^2(L_O) \right. \\ &\quad \left. + \frac{1}{k\bar{N}_S} \left(\frac{\langle \hat{n}_O \rangle}{pT_C} + \frac{\langle \hat{n}_O \rangle^2}{qT_S} \right) \right\} + \langle \hat{n}_O \rangle s^2(\hat{a}_1) \\ &\quad + 2 \langle \hat{n}_O \rangle s(a_O, a_1) . \end{aligned} \quad (157)$$

Carrying out the multiplication:

$$\begin{aligned}
 s^2(\hat{\theta}_O) &= s^2(\hat{a}_O) + (\hat{a}_1^2 - s^2(\hat{a}_1)) \frac{1}{k\bar{N}_s^2} s^2(L_O) \\
 &\quad + (\hat{a}_1 - s^2(\hat{a}_1)) \frac{1}{k\bar{N}_s} \left\{ \frac{\langle \hat{n}_O \rangle}{pT_c} + \frac{\langle \hat{n}_C \rangle}{qT_s} \right\} \\
 &\quad + \langle n_O \rangle^2 s^2(\hat{a}_1) + 2 \langle n_O \rangle s(\hat{a}_O, \hat{a}_1) \quad (158)
 \end{aligned}$$

Equation (158) identifies three components of variance:

a) the instrument component (159)

$$s^2_I(\hat{\theta}_O) = (\hat{a}_1^2 - s^2(\hat{a}_1)) \frac{1}{k\bar{N}_s} \left\{ \frac{\langle \hat{n}_O \rangle}{pT_c} + \frac{\langle \hat{n}_O \rangle^2}{qT_s} \right\}$$

b) the calibration component (160)

$$s^2_C(\hat{\theta}_O) = s^2(\hat{a}_O) + \langle \hat{n}_O \rangle^2 s^2(\hat{a}_1) + 2\langle \hat{n}_O \rangle s(\hat{a}_O, \hat{a}_1)$$

c) the location component (161)

$$s^2_L(\hat{\theta}_O) = (\hat{a}_1^2 - s^2(\hat{a}_1)) \frac{s^2(L_O)}{k\bar{N}_s^2}$$

A unique calibration curve is assumed to be developed for each depth sampled within the soil profile. Irrigation scheduling requires the integration of soil moisture estimates over the crop root zone to obtain the total volume of water stored, V_t . Numerical

methods are used to approximate the integration. The total variance in an estimate of total water within the crop root zone is a linear combination of the individual variances calculated at each depth plus an additional component attributed to the numerical approximation. Expressing the variance estimates of water content as $s^2(V_t)$ allows the total variance to be given as:

$$s^2(V_t) = s^2_1(V_t) + s^2_2(V_t) . \quad (162)$$

Expressing the total volume of water stored in the crop root zone as:

$$V_t = \sum_{i=1}^d \theta_i(z) \Delta z_i , \quad (163)$$

where d is the number of depths sampled and Δz_i the distance between each depth, defines the variance, $s^2_1(V_t)$:

$$s^2_1(V_t) = s^2(\Delta z_1 \theta_1 + \Delta z_2 \theta_2 + . . . + \Delta z_d \theta_d) . \quad (164)$$

The statistical formula for calculating the variance of random variables x and y with corresponding coefficients k and l is given by (Sinclair, 1979):

$$\text{var} (kx + ly) = k \text{ var} (x) + l \text{ var} (y) . \quad (165)$$

Assuming estimates of moisture content at each depth to be independent of each other, the sample variance $s^2_1(V_t)$ becomes:

$$s^2_1(V_t) = \Delta z^2 \sum_{i=1}^d s^2(\theta_i(z)) \quad (166)$$

Use of the trapezoidal rule to integrate equally spaced depths over the soil profile weights the area associated with the top measurement 1.5 times greater than the area corresponding to lower depths. This is illustrated in Figure 14.

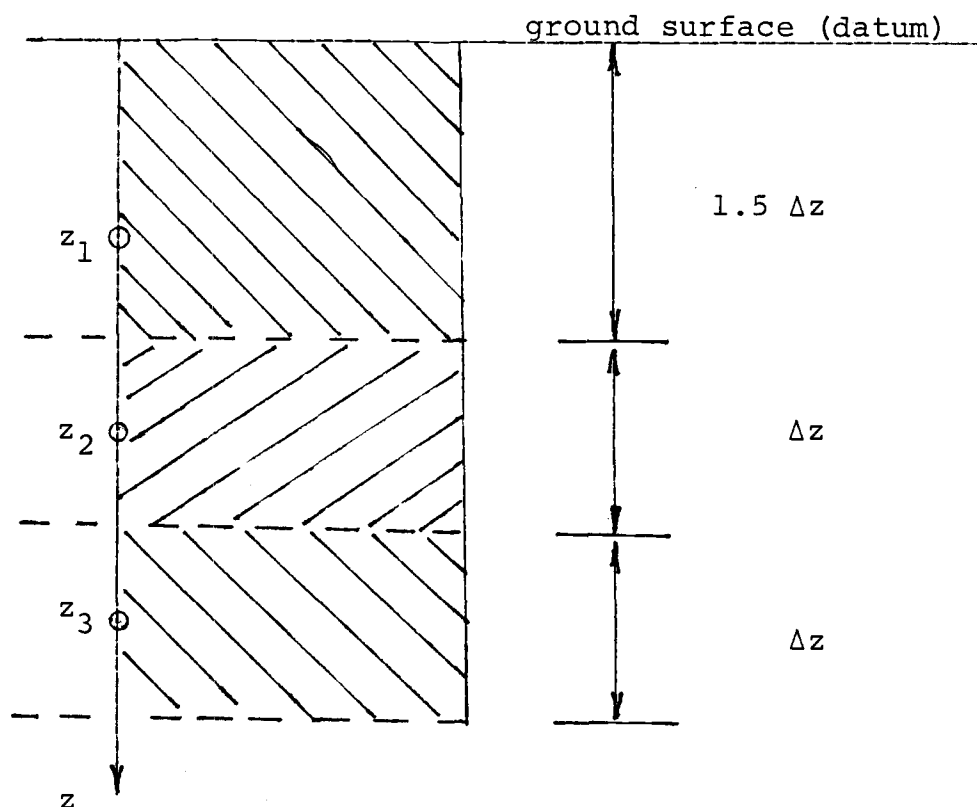


Figure 14. Weighting of area by the trapezoidal rule.

When using the trapezoidal rule, equation (166) should be corrected accordingly:

$$s^2_1(V_t) = \{(1.5)^2 s^2(\theta_1(z)) + \sum_{i=2}^d s^2(\theta_i(z))\} \Delta z^2 \quad (167)$$

The error introduced by use of the trapezoidal rule to approximate the integration is (Hamming, 1973):

$$s_2(V_t) = \frac{z}{12} \Delta z^2 \left(\frac{d^2(z)}{d^2 z} \right) . \quad (168)$$

Since the variance is the square of the error term:

$$s^2_2(V_t) = \frac{z^2}{144} \Delta z^4 \left(\frac{d^4(z)}{d^4 z} \right) . \quad (169)$$

If Simpson's rule is used to perform the numerical approximation:

$$V_t = \{\theta_0 + 1/3(\theta_1 + 4\theta_2 + 2\theta_3 + \dots + 4\theta_{d-1} + \theta_d)\} \Delta z \quad (170)$$

where θ_0 is the unrecorded water content at the soil surface, assimilated to the value θ_1 measured at z_1 (Haverkamp, 1983). The associated variance then becomes:

$$s^2_1(V_t) = \{s^2(\theta_0) + 1/9\{s^2(\theta_1) + 16s^2(\theta_2) + 4s^2(\theta_3) + \dots + s^2(\theta_d)\}\} \Delta z^2 . \quad (171)$$

The variance associated with use of Simpson's rule is given by (Hamming, 1973):

$$s^2_2(V_t) = \frac{z^2 \Delta z}{32,400} \left\{ \frac{d^4(z)}{d^4z} \right\}^2 \quad (172)$$

APPENDIX B

Computer Program VARMOIS

```

PROGRAM VARMOIS (INPUT,OUTPUT,TAPES,TAPE6,TAPE2=INPUT,
1TAPE3=OUTPUT)
DIMENSION SV(20),WW(20),DW(20),CM(20),PR(20)
CMT=XHST=XT2=PRT=AT1=AT2=AT3=0.0
TVARPR=TAD=TCOVUB=0.0

C
C INPUT OF PARAMETERS, FIELD DATA (SAMPLE VOLUME, WET AND DRY SAMPLE
C CALCULATE MOISTURE CONTENT (VOL. DECIMAL FRACTION), AND COUNT RATIO
C
      READ(5,*) K,RW,M,P,Q,TC,TS,AVCR
      DO 150 J=1,M
        READ(5,*) SV(J),WW(J),DW(J),PR(J)
        PR(J)=PR(J)/AVCR
150      CM(J)=(WW(J)-DW(J))/(RW*SV(J))
        WRITE(6,100)K,RW,M,P,Q,TC,TS,AVCR
100      FORMAT(////,25X,"ESTIMATION OF VARIANCE FOR MOISTURE MEASURE
1,///,14X,"K=",I2,"", RW=",F4.1,"", M=",I2,"", P=",F5.1,"", Q=",F5.1,
2", TC=",F5.1,"", TS=",F5.1,"", AVCR=",F8.1,/,14X,"SAMPLE VOLUME",
33X,"WET WEIGHT",5X,"DRY WEIGHT",5X,"MOISTURE",6X,"COUNT RATIO",/)
        DO 125 J=1,M
125          WRITE(6,126)SV(J),WW(J),DW(J),CM(J),PR(J)
126          FORMAT(14X,F10.2,F14.2,F15.2,F15.7,F14.4)

C
C CALCULATE AVERAGE COUNT RATIO, SAMPLE VOLUME, MOISTURE CONTENT
C
      DO 200 J=1,M
        CMT=CMT+CM(J)
        PRT=PRT+PR(J)
200      CONTINUE
      XM=FLOAT(M)
      AVCRM=AVCR/TS
      CMAV=CMT/XM
      PRAV=PRT/XM

C
C CALCULATE STATISTICAL PARAMETERS
C
      DO 350 J=1,M
        AT1=AT1+(CM(J)-CMAV)*(PR(J)-PRAV)
        AT2=AT2+(PR(J)-PRAV)**2
350      AT3=AT3+(CM(J)-CMAV)**2

C
C DETERMINE MOISTURE VARIANCE OF SAMPLES, LEAST SQUARE COEFFICIENT, A1
C
      VARCH=AT3/(XM-1.)
      A1LS=AT1/AT2

```

DETERMINE PARAMETERS, COEFFICIENTS A1 AND A0

```

      DO 300 J=1,M
300      TVARPR=TVARPR+(PR(J)/(P*TC)+PR(J)**2/(Q*TS))/AVCRM
      VARU=(1.0/(XM-1.0))*TVARPR
      A1UB=(A11/(XM-1.0))/((AT2/(XM-1.0))-(XM-2.0)/(XM-1.0)*VARU)
      A0LS=CHAV-A1LS*PRAV
      A0UB=CHAV-A1UB*PRAV

```

DETERMINE VARIANCES OF MEASUREMENT ERRORS, COUNT RATIOS

```

      DO 400 J=1,M
      XMST=XMST+((CM(J)-(A0LS+A1LS*PR(J)))**2)
      XT2=XT2+PR(J)**2
400      CONTINUE
      VARV=0.0
      VARPR=AT2/(XM-1.0)
      COVPRCM=A11/(XM-1.0)
      VARE=(XM-1.0)/(XM-2.0)*(VARCM-A1UB*COVPRCM)-VARV
      VARA1UB=VARE/((XM-1.0)*VARPR-(XM-2.0)*VARU)
      DO 450 J=1,M
      TAO=TAO+VARA1UB*PR(J)**2
450      TCOVUB=TCOVUB+VARA1UB*PR(J)

```

CALCULATE MEAN SQUARE ERROR

CALCULATE VARIANCE OF COEFFICIENTS

```

      XMSE=XMST/(XM-2.0)
      VARA0LS=(XMSE*XT2)/(XM*AT2)
      VARA1LS=XMSE/AT2
      COVLS=(-PRAV*XMSE)/AT2
      COVUB=TCOVUB/(-XM)
      VARA0UB=TAO/XM

```

DETERMINE COMPONENT VARIANCE

DETERMINE TOTAL VARIANCE

DETERMINE LOCATION EFFECT

```

666  READ(2,*)PO,VARPO
      IF(PO.EQ.0.0) GO TO 999
      UBINST=(A1UB**2-VAR1UB)*(PO/(P*TC)+PO**2/(Q*TS))/(K*AVCRM)
      XINSTLS=(A1LS**2-VAR1LS)*(PO/(P*TC)+PO**2/(Q*TS))/(K*AVCRM)
      UBCALIB=VARA0UB+PO**2*VARA1UB+2.*PO*COVUB
      CALIBLS=VARA0LS+PO**2*VARA1LS+2.*PO*COVLS
      TVARUB=(A1UB**2-VAR1UB)*VARPO+VARA0UB+PO**2*VARA1UB
      +2.*PO*COVUB
      TVARLS=(A1LS**2-VAR1LS)*VARPO+VARA0LS+PO**2*VARA1LS
      +2.*PO*COVLS
      UBLOC=TVARUB-UBINST-UBCALIB
      XLOCLS=TVARLS-XINSTLS-CALIBLS

```

OUTPUT RESULTS

```

      WRITE(6,899)PO,VARPO
899  FORMAT(/,18X,"FOR COUNT RATIO =",F7.4,1X,"WITH VARIANCE ="
      1,F10.7,/)
      WRITE(6,901)AO1S,A0UB
901  FORMAT(/,31X,"LEAST SQUARES",2X,"UNBIASED TREATMENT",/,21X,"AO=",
      17X,F12.8,3X,F12.8)
      WRITE(6,902)A11S,A1UB
902  FORMAT(21X,"A1=",7X,F12.8,3X,F12.8)
      WRITE(6,903)VARAO1S,VARA0UB
903  FORMAT(21X,"VAR(A0)=",2X,F12.8,3X,F12.8)
      WRITE(6,904)VARA11S,VARA1UB
904  FORMAT(21X,"VAR(A1)=",2X,F12.8,3X,F12.8)
      WRITE(6,898)COV1S,COVUB
898  FORMAT(21X,"S(A1,A0)= ",F12.8,3X,F12.8)
      WRITE(6,905)XINST1S,UBINST
905  FORMAT(21X,"INST COMP=",F12.8,3X,F12.8)
      WRITE(6,906)CALIB1S,UBCALIB
906  FORMAT(21X,"CALB COMP=",F12.8,3X,F12.8)
      WRITE(6,907)XLOC1S,UBLOC
907  FORMAT(21X,"LOC COMP =",F12.8,3X,F12.8)
      WRITE(6,908)TVAR1S,TVARUB
908  FORMAT(21X,"TOT. VAR =",F12.8,3X,F12.8)
      WRITE(6,909)XMSE
909  FORMAT(21X,"MEAN SQUARE ERROR =",F12.8)
      WRITE(6,888)VARCH,VARPR,COVPRCH,VARU,VARE
888  FORMAT(5F12.8)
      GO TO 666
999  STOP
      END

```

APPENDIX C

Variable List

Computer Program VARMOIS

<u>Variable</u>	<u>Format</u>	<u>Description</u>
K	INTEGER	Number of location sites
RW	REAL	Density of water
M	INTEGER	Number of sample pairs
P	REAL	Count replications
Q	REAL	Count replications (Standard Absorber)
TC	REAL	Counting time
TS	REAL	Counting time (Standard Absorber)
AVCR	REAL	Standard Count
SV	REAL	Sample volume
WW	REAL	Wet sample weight
DW	REAL	Dry sample weight
PR	REAL	Neutron probe count
CM	REAL	Moisture content
AVCRM	REAL	Standard count rate
CMAV	REAL	Average of moisture samples
PRAV	REAL	Average of count samples
VARCM	REAL	Variance of moisture samples
VARPR	REAL	Variance of count samples

VARU	REAL	Variance of measurement errors u
COVPRCM	REAL	Covariance of counts and moisture samples
AlLS	REAL	Least square estimate of a_1
AOLS	REAL	Least square estimate of a_0
AlUB	REAL	Unbiased treatment for a_1
AOUB	REAL	Unbiased treatment of a_0
VARAlLS	REAL	Variance of a_1 , least squares
VARAOLS	REAL	Variance of a_0 , least squares
VARAlUB	REAL	Variance of a_1 , unbiased
VARAOUB	REAL	Variance of a_0 , unbiased
COVLS	REAL	Covariance of a_1 , a_0 least squares
COVUB	REAL	Covariance of a_1 , a_0 unbiased
XMSE	REAL	Mean square error

APPENDIX D

Sample Input File

Computer Program VARMOIS

8
1.
8
2.
10.
60.
60.
21627.

195.47	358.4	337.8	11470.5
195.47	353.4	326.4	14392.5
195.47	335.7	309.6	15336.5
195.47	330.4	302.1	15728.5
195.47	302.5	369.5	6895.5
195.47	317.	309.9	7011.
195.47	290.6	280.6	6894.
195.47	310.1	303.3	6530.3

K (Number of sample locations)
 RW (Density of water)
 M (Number of sample pairs)
 P (Count replications)
 Q (Count replications in a standard absorber)
 TC (Counting time)
 TS (Counting time in a standard absorber)
 AVCR (Standard count)

SV	WW	DW	PR
(Sample volume)	(Wet weight)	(Dry weight)	(Count Ratio)
1	1	1	1
2	2	2	2
3	3	3	3
.	.	.	.
.	.	.	.
.	.	.	.
M	M	M	M

Figure 18. Description of VARMOIS Input File

APPENDIX E

Estimates of Variance

From California Data

(Camarillo Series)

ESTIMATION OF VARIANCE FOR MOISTURE MEASUREMENTS

K= 8, RW= 1.0, M=16, P= 2.0, Q= 10.0, TC= 30.0, TS= 30.0, AVCR= 19878.0

MOISTURE	COUNT RATIO	MOISTURE	COUNT RATIO
.2805000	1.1300	.1940000	1.0500
.2750000	1.3000	.2440000	1.1400
.3400000	1.3700	.3045000	1.2400
.2270000	1.4300	.3600000	1.3200
.4225000	1.5000	.3920000	1.4100
.4300000	1.6100	.4160000	1.2200
.4175000	1.6300	.3000000	1.1900
.1622000	.8100	.2560000	1.0800

FOR COUNT RATIO = .4000 WITH VARIANCE = .0020860

	LEAST SQUARES	UNBIASED TREATMENT
A0=	-.08796820	-.08831681
A1=	.31466917	.31494219
VAR(A0)=	.00723915	.00723515
VAR(A1)=	.00432432	.00432193
S(A1,A0)=	-.00552162	-.00551856
INST COMP=	.00000013	.00000013
CALB COMP=	.00351375	.00351181
LOC COMP =	.00019740	.00019776
TOT. VAR =	.00371128	.00370970
MEAN SQUARE ERROR =	.00301986	

FOR COUNT RATIO = .4000 WITH VARIANCE = .0000327

	LEAST SQUARES	UNBIASED TREATMENT
A0=	-.08796820	-.08831681
A1=	.31466917	.31494219
VAR(A0)=	.00723915	.00723515
VAR(A1)=	.00432432	.00432193
S(A1,A0)=	-.00552162	-.00551856
INST COMP=	.00000013	.00000013
CALB COMP=	.00351375	.00351181
LOC COMP =	.00000297	.00000297
TOT. VAR =	.00351685	.00351491
MEAN SQUARE ERROR =	.00301986	

(Camarrillo Series)

ESTIMATION OF VARIANCE FOR MOISTURE MEASUREMENTS

K= 8, RW= 1.0, M=16, P= 2.0, Q= 10.0, TC= 30.0, TS= 30.0, AVCR= 19878.0

MOISTURE	COUNT RATIO	MOISTURE	COUNT RATIO
.2805000	1.1300	.1940000	1.0500
.2750000	1.3000	.2440000	1.1400
.3400000	1.3700	.3045000	1.2400
.2270000	1.4300	.3600000	1.3200
.4225000	1.5000	.3920000	1.4100
.4300000	1.6100	.4160000	1.2200
.4175000	1.6300	.3000000	1.1900
.1622000	.8100	.2560000	1.0800

FOR COUNT RATIO = 1.3000 WITH VARIANCE = .0020860

	LEAST SQUARES	UNBIASED TREATMENT
A0=	-.08796820	-.08831681
A1=	.31466917	.31494219
VAR(A0)=	.00723915	.00723515
VAR(A1)=	.00432432	.00432193
S(A1,A0)=	-.00552162	-.00551856
INST COMP=	.00000049	.00000049
CALB COMP=	.00019105	.00019095
LOC COMP =	.00019704	.00019740
TOT. VAR =	.00038858	.00038884
MEAN SQUARE ERROR =	.00301986	

FOR COUNT RATIO = 1.3000 WITH VARIANCE = .0000327

	LEAST SQUARES	UNBIASED TREATMENT
A0=	-.08796820	-.08831681
A1=	.31466917	.31494219
VAR(A0)=	.00723915	.00723515
VAR(A1)=	.00432432	.00432193
S(A1,A0)=	-.00552162	-.00551856
INST COMP=	.00000049	.00000049
CALB COMP=	.00019105	.00019095
LOC COMP =	.00000261	.00000261
TOT. VAR =	.00019415	.00019405
MEAN SQUARE ERROR =	.00301986	

(Holtville Series)

ESTIMATION OF VARIANCE FOR MOISTURE MEASUREMENTS

K= 8, RW= 1.0, M=16, P= 2.0, Q= 10.0, TC= 30.0, TS= 30.0, AVCR= 19878.0

MOISTURE	COUNT RATIO	MOISTURE	COUNT RATIO
.2700000	1.3900	.3310000	1.3300
.3550000	1.4100	.3325000	1.4400
.3975000	1.3100	.3346000	1.5300
.1750000	1.3000	.3444000	1.5100
.2075000	1.1800	.2900000	1.2900
.4001000	1.4100	.2231000	1.2600
.3931000	1.5100	.4034000	1.3700
.2863000	1.4900	.4125000	1.5500

FOR COUNT RATIO = .4000 WITH VARIANCE = .0020860

	LEAST SQUARES	UNBIASED TREATMENT
A0=	-.24322163	-.24537249
A1=	.40608376	.40762836
VAR(A0)=	.04254893	.04262307
VAR(A1)=	.02181885	.02185686
S(A1,A0)=	-.03038274	-.03043568
INST COMP=	.00000019	.00000020
CALB COMP=	.02173375	.02177162
LOC COMP =	.00029828	.00030082
TOT. VAR =	.02203223	.02207264
MEAN SQUARE ERROR =	.00385539	

FOR COUNT RATIO = .4000 WITH VARIANCE = .0000327

	LEAST SQUARES	UNBIASED TREATMENT
A0=	-.24322163	-.24537249
A1=	.40608376	.40762836
VAR(A0)=	.04254893	.04262307
VAR(A1)=	.02181885	.02185686
S(A1,A0)=	-.03038274	-.03043568
INST COMP=	.00000019	.00000020
CALB COMP=	.02173375	.02177162
LOC COMP =	.00000448	.00000452
TOT. VAR =	.02173843	.02177634
MEAN SQUARE ERROR =	.00385539	

(Holtville Series)

ESTIMATION OF VARIANCE FOR MOISTURE MEASUREMENTS

K= 8, RW= 1.0, M=16, P= 2.0, Q= 10.0, TC= 30.0, TS= 30.0, AVCR= 19878.0

MOISTURE	COUNT RATIO	MOISTURE	COUNT RATIO
.2700000	1.3900	.3310000	1.3300
.3550000	1.4100	.3325000	1.4400
.3975000	1.3100	.3346000	1.5300
.1750000	1.3000	.3444000	1.5100
.2075000	1.1800	.2900000	1.2900
.4001000	1.4100	.2231000	1.2600
.3931000	1.5100	.4034000	1.3700
.2863000	1.4900	.4125000	1.5500

FOR COUNT RATIO = 1.4000 WITH VARIANCE = .0020860

	LEAST SQUARES	UNBIASED TREATMENT
A0=	-.24322163	-.24537249
A1=	.40608376	.40762836
VAR(A0)=	.04254893	.04262307
VAR(A1)=	.02181885	.02185686
S(A1,A0)=	-.03038274	-.03043568
INST COMP=	.00000081	.00000081
CALB COMP=	.00024219	.00024261
LOC COMP =	.00029767	.00030021
TOT. VAR =	.00054066	.00054363
MEAN SQUARE ERROR =	.00385539	

FOR COUNT RATIO = 1.4000 WITH VARIANCE = .0000327

	LEAST SQUARES	UNBIASED TREATMENT
A0=	-.24322163	-.24537249
A1=	.40608376	.40762836
VAR(A0)=	.04254893	.04262307
VAR(A1)=	.02181885	.02185686
S(A1,A0)=	-.03038274	-.03043568
INST COMP=	.00000081	.00000081
CALB COMP=	.00024219	.00024261
LOC COMP =	.00000387	.00000391
TOT. VAR =	.00024687	.00024733
MEAN SQUARE ERROR =	.00385539	

(Indio Series)

ESTIMATION OF VARIANCE FOR MOISTURE MEASUREMENTS

K= 9, RW= 1.0, M=18, P= 2.0, Q= 10.0, TC= 30.0, TS= 30.0, AVCR= 19878.0

MOISTURE	COUNT RATIO	MOISTURE	COUNT RATIO
.0275000	.2900	.0740000	.3900
.0400000	.3900	.1250000	.4000
.0225000	.3300	.0685000	.3400
.0342000	.3100	.0672000	.3100
.0120000	.3200	.0500000	.3200
.0816000	.5500	.0925000	.4500
.0675000	.4800	.3550000	.5500
.0217000	.3600	.0888000	.4700
.0576000	.2900	.0516000	.3700

FOR COUNT RATIO = .4000 WITH VARIANCE = .0020860

	LEAST SQUARES	UNBIASED TREATMENT
A0=	-.15164591	-.15198287
A1=	.58769166	.58856815
VAR(A0)=	.00460657	.00460846
VAR(A1)=	.02983531	.02984756
S(A1,A0)=	-.01147002	-.01147473
INST COMP=	.00000038	.00000038
CALB COMP=	.00020421	.00020429
LOC COMP =	.00065785	.00065997
TOT. VAR =	.00086244	.00086464
MEAN SQUARE ERROR =	.00354576	

FOR COUNT RATIO = .4000 WITH VARIANCE = .0000327

	LEAST SQUARES	UNBIASED TREATMENT
A0=	-.15164591	-.15198287
A1=	.58769166	.58856815
VAR(A0)=	.00460657	.00460846
VAR(A1)=	.02983531	.02984756
S(A1,A0)=	-.01147002	-.01147473
INST COMP=	.00000038	.00000038
CALB COMP=	.00020421	.00020429
LOC COMP =	.00000994	.00000997
TOT. VAR =	.00021452	.00021464
MEAN SQUARE ERROR =	.00354576	

(Indio Series)

ESTIMATION OF VARIANCE FOR MOISTURE MEASUREMENTS

K= 9, RW= 1.0, M=18, P= 2.0, Q= 10.0, TC= 30.0, TS= 30.0, AVCR= 19878.0

MOISTURE	COUNT RATIO	MOISTURE	COUNT RATIO
.0275000	.2900	.0740000	.3900
.0400000	.3900	.1250000	.4000
.0225000	.3300	.0685000	.3400
.0342000	.3100	.0672000	.3100
.0120000	.3200	.0500000	.3200
.0816000	.5500	.0925000	.4500
.0675000	.4800	.3550000	.5500
.0217000	.3600	.0888000	.4700
.0576000	.2900	.0516000	.3700

FOR COUNT RATIO = 1.4000 WITH VARIANCE = .0020860

	LEAST SQUARES	UNBIASED TREATMENT
A0=	-.15164591	-.15198287
A1=	.58769166	.58856815
VAR(A0)=	.00460657	.00460846
VAR(A1)=	.02983531	.02984756
S(A1,A0)=	-.01147002	-.01147473
INST COMP=	.00000158	.00000159
CALB COMP=	.03096772	.03098044
LOC COMP =	.00065665	.00065877
TOT. VAR =	.03162595	.03164080
MEAN SQUARE ERROR =	.00354576	

FOR COUNT RATIO = 1.4000 WITH VARIANCE = .0000327

	LEAST SQUARES	UNBIASED TREATMENT
A0=	-.15164591	-.15198287
A1=	.58769166	.58856815
VAR(A0)=	.00460657	.00460846
VAR(A1)=	.02983531	.02984756
S(A1,A0)=	-.01147002	-.01147473
INST COMP=	.00000158	.00000159
CALB COMP=	.03096772	.03098044
LOC COMP =	.00000874	.00000877
TOT. VAR =	.03097804	.03099079
MEAN SQUARE ERROR =	.00354576	