AN ABSTRACT OF THE THESIS OF

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The statistics describing variations of turbulent motions within the so-called
inertial range of length scales depend on the scale over which the motions are varying
and the "average" rate at which the turbulent kinetic energy is being dissipated on the
molecular scale. This hypothesis stemmed from the similarity arguments published
by A. N. Kolmogorov in 1941 and implies specific scaling relations between the
average amplitude and length scale of turbulent motions. Turbulent motions agree to
a good approximation with Kolmogorov scaling provided the fluid flow admits to the
underlying assumptions.

More recently it has been recognized that the large spatial variations in the rate
of turbulent kinetic energy dissipation may be a partial explanation for deviations
from Kolmogorov scaling. This recognition is due in part to the observation that the
total volume occupied by turbulent motions of a given scale decreases as the scale
decreases. These observations imply that active small scale turbulence is intermittent.
This study aims to better understand how scaling relations describing more active
regions are different from the relations describing turbulence where the small scales
are less active. The thesis is that the relations are different.
An 18 hour segment of wind data measured in near-neutral stratification 45 meters above a relatively flat ground is analyzed. There is virtually no trend in the mean wind speed, so the describing statistics are essentially stationary. Small scale activity is measured in terms of the difference in wind speed (structure function) at a separation distance of 1/16 of a second, which translates to about a meter. The differences in wind speed are raised to the sixth power and then averaged over 4 second (50 meter) windows.

Non-overlapping windows containing a local maximum in the averaged sixth order structure function form one (MASC) ensemble of more active small scale samples and the local minima form another (LASC) ensemble of less active small scale samples. The variations in wind speed as a function of length scale within each ensemble are decomposed five different ways. Each of the five decompositions obey scaling relationships that are approximately linear in log-log coordinates. The MASC and LASC ensembles include 32% and 46% of the record, respectively.

The turbulent kinetic energy as a function of scale falls off at a slower rate in the MASC ensemble versus the LASC ensemble and in magnitude the energy is greater at all scales in the MASC ensemble. This implies the transfer rate of turbulent kinetic energy toward small scales is more rapid on average in the MASC samples. Samples in the MASC ensemble occupied 30% less of the record, implying the flattening effect on the spectral slope exhibited by the samples contained in the MASC ensemble is less influential than the steepening influence of samples of the type in the LASC ensemble. The results are robust with respect to the choice of a basis set in representing the variance as a function of scale.
The Influence of Small Scale Variability on Scaling Relationships Describing Atmospheric Turbulence

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The Influence of Small Scale Variability on Scaling Relationships Describing Atmospheric Turbulence

Chapter 1 Theoretical Background and the Data

The scale dependence of statistics describing homogeneous, isotropic, incompressible turbulence can be derived using similarity arguments. If the flow has a "similar" structure within a range of length scales then, for example, dimensional arguments can be used to show the form of the dependence of the energy on the length over which the motions are varying. These arguments rely on a set of assumptions and hypotheses initially put forth by Kolmogorov (1941).

This study analyzes the influence of the intermittency of small scale structure on the scaling laws used to characterize turbulent flows. The small scale structure is defined to be composed of more or less active regions. The scaling relationships are assessed according to multiple orthogonal decomposition of variance as well as a non-orthogonal structure function decomposition. The five different decompositions are performed in order to increase statistical confidence and to note the dependence of the results on the choice of basis set or statistical tool.

Classical theory of turbulence appeals to statistics. For example, Kolmogorov (1941) considered the differences in an individual velocity component throughout some domain, G, to represent a random variable. Combining the three velocity components leads to a vector of random variables. If the distributions corresponding to the vector of random variables are independent from the location within G then the turbulence is said to be homogeneous. Moreover, if the turbulence is homogeneous and the distributions are invariant with respect to rotations and reflections of the coordinate system then the turbulence is defined to be isotropic. Apparently, these definitions may often apply (approximately) in atmospheric boundary layer flows when the domain,
G, is sufficiently small and is not too close to a boundary or a “singularity”. This last restriction (noted in Kolmogorov, 1941) may partially indicate why modest deviations from Kolmogorov theory have been observed (Batchelor and Townsend, 1949).

The restriction of G not being too close to a singularity can be interpreted in terms of the intermittency of small scale fluctuations. As G is moved through the fluid it intermittently contains isolated locations where kinetic energy is being rapidly dissipated and Kolmogorov arguments may not hold. The concepts of fractal geometry are used to help estimate the distribution of the more singular regions, that is the intermittency of the dissipation of kinetic energy per unit mass. The concepts of fractals has been instructive in describing how the spatial coverage of relatively large amplitude motions depend on scale (Mandelbrot, 1976).

The relation to fractals is interpreted by considering a three dimensional turbulent structure with a characteristic linear length scale L. The transfer of energy to smaller scales is described in terms of this structure breaking up and giving rise to N structures of size \( L/\lambda \). The smaller “offspring” eddies will cover a volume equal to \( N \times (L/\lambda)^3 \), so if they fill as much space as the “parent” eddy, which is equal to \( L^3 \), then \( N = \lambda^3 \).

Observations suggest, however, that \( N = \lambda^D \) where \( D < 3 \) (some suggest \( D \approx 2.5 \)) where \( D \) is equivalent to the fractal dimension. These type arguments were used in developing the \( \beta \) model of intermittency (Frisch et al., 1978). The model assumption is that turbulent motions are “less space filling” as the scale of the variations decrease. Since smaller scale motions occupy less space, it follows there is less energy associated with the smaller scales than is predicted by classical Kolmogorov theory, hence a steeper fall off in the variance spectrum. Other theoretical models predict a flatter spectrum (Yakhot et al., 1989 and Yamazaki, 1990). The differences in the predicted
modifications to Kolmogorov scaling may be in part due to the quasi-singular nature of very high Reynolds number (i.e. nearly inviscid) turbulent motions.

One physical interpretation of a so called singularity is that it corresponds to an eddy microfront (Mahrt, 1988). An eddy microfront is essentially the leading edge of a wind gust meaning a sudden relatively large amplitude change in wind velocity (i.e. where the flow is nearly singular). Kinetic energy dissipation is significantly enhanced about these microfronts due to rapid deformation corresponding to large rates of strain, viscous stress and molecular diffusion of the energy.

At the microfronts, kinetic energy associated with relatively large scale motions is directly dissipated. This is in contrast to the classical concept of a continuous down-scale energy cascade (Richardson, 1922). At the downstream (leading) edge of the eddy outflow, higher momentum parcels converge and wrap up as they collide with the slower moving air ahead of the eddy microfront. The enhanced fine scale activity at the leading edge of the eddy leads to molecular friction and direct dissipation of big eddy kinetic energy, thus representing a “short circuit” in the energy cascade.

Though the statistical theory developed in Kolmorgov (1941) was intended for a “sufficiently” small domain, G, that may fit in between microfronts, the results are generally true for large domains as well, which include a number of large eddies and hence microfronts. It is possible a large domain effectively corresponds to an ensemble of small domains, most of which can be defined as containing isotropic turbulence. This suggests that the singularities or microfronts or especially large rates of energy dissipation occupy a small fraction of the turbulence.

It has been observed that for high Reynolds number turbulence (where the inertia
of the flow dominates the retarding force of molecular viscosity as in most geophysical flows) variations in the motions become more intermittent (that is less space filling) at smaller scales even within the inertial subrange. In this case the flow structure is not similar at different scales implying modest departures from the scaling laws based on self-similar structure (Kolmogorov, 1962; Frisch et al., 1978; Nelkin & Bell 1978).

An objective of this study is to isolate the regions in the flow where the dissipation “events” are concentrated and contrast this sample set with regions where the dissipation events are less concentrated. The resulting statistics of the two sample sets should be different. Toward the goal, the analysis will concentrate on atmospheric observations of turbulence collected 45 meters (≈ 150 feet) above a flat ground during the Lammeffjord Experiment (LAMEX). These atmospheric observations represent 50 hours of strong wind conditions and constitute an unusually long time series of nearly stationary conditions (Kristensen et al., 1989).

This document is divided into three main parts. The mathematics is developed in Chapter 2. Four orthogonal bases sets: Fourier, eigenvector, Haar and polynomial bases, will be defined and the structure function will be introduced. Chapter 2 also includes a discussion of the sampling strategy which appeals to the structure function statistics. Chapter 3 discusses the results of the sampling and “local” analysis applied to the LAMEX data and compares the results with previous work.

This chapter introduces the data set and discusses corresponding global statistics, which includes the Fourier and the Haar wavelet variance spectrums. The introduction to the Fourier and Haar bases sets presented in this chapter will be appealed to again in chapter 2. The data set is analyzed as whole in order to formulate a reasonable
sampling and analysis strategy. Two global characteristics of the data considered in this study include stationarity of the time series, which is inferred from the trend in the wind speed, and the scale dependence of the velocity variations.

Section 1.1 will describe the original data set and includes the pre-processing performed in this study. A most stationary segment of the entire record is isolated according to a linear trend analysis. The energy spectra are computed for this stationary segment in section 1.2.

Section 1.1: Lammeefjord Wind Data

The entire data set considered in this study includes 50 hours of the three dimensional wind vector measured at 16 Hz by a sonic anemometer 45 meters above the ground as part of the Lammeefjord experiment (LAMEX) carried out in Lammeefjord, Denmark by the Risø National Laboratory (Kristensen et. al., 1989). The land surface around where the wind measurements are taken has only very small undulations (< 1 meter) for several kilometers upwind and is comprised primarily of root crops. This specific 50 hour window of data provided by the Risø National Laboratory is assumed to be high quality. The mean longitudinal wind speed shows little diurnal variation and has an average value of about 12 m/s.

Larger scale trend in the LAMEX 50 hour record of wind velocity is assessed by considering a sequence of linear least squares fits to ten hour windows of the longitudinal wind speed. The ten hour window is marched through the 50 hour record in one hour increments and the resulting sequence of linear least squares fits are shown in figure 1.
The sequence of lines shown in figure 1 were constructed by fitting a line of the form $Y_{k,i} = U_k + \alpha_k (2\sqrt{3}) x_i$ to the corresponding ten hour window of data where $U_k = \frac{1}{N} \sum_{i=1}^{N} U_{k+i}$ is the average longitudinal wind speed in the window and $x_i$ is a point in the interval [-0.5,0.5]. Using $x_i = (\frac{i-0.5}{N} - 0.5)$ the formula becomes

$$Y_{k,i} = U_k + \alpha_k \sqrt{3} \left( \frac{2i-1}{N} - 1 \right)$$

(1)

where $k = 0,1,...,40$ is the beginning time (in hours) of a ten hour window, and $Y_{k,i}$ is the value of the line at the $i^{th}$ point of the window starting at hour $k$. The least squares linear expansion coefficient (proportional to the slope) is given by

$$\alpha_k = \frac{\sqrt{3}}{N} \sum_{i=1}^{N} \left( \frac{2i-1}{N} - 1 \right) U_{k+i}$$

(2)
where $N = 10\text{hrs} \times 3600\text{sec/hr} \times 16\text{pts/sec} = 576,000$ pts is the number of data points in a ten hour window. The value of $\alpha^2_k$ is the variance in a ten hour window associated with linear trend.

Defining the total variance in a ten hour window as

$$\text{Var}(u_k) = \frac{1}{N} \sum_{i=1}^{N} (U_{k+i} - \bar{U}_k)^2$$

(3)

the fraction of within ten hour window variance associated with a linear trend is computed as $\alpha^2_k / \text{Var}(u_k)$. Large values of this ratio indicate strong trend. This ratio for $k=0,1,...,40$ is shown in figure 2.

Using a 5% cutoff as a tolerable fraction of variance associated with linear trend, figure 2 shows there are two regions where this condition is satisfied. The first
corresponds to hours 6 through 24 and the second is from hours 20 through 42. Hours 6 through 24 are selected for further study. Fitting a line to the 18 hours starting at the end of hour six explains 0.47% of the variance, so the influence of trend is minimal. A line fit to hours 20 through 42 explains five times that amount or 2.5% of the variance, which is a reason for selecting hours 6 through 24.

The mean longitudinal wind speed during hours 6 through 24 is 12.8 m/s. The standard deviation of the longitudinal wind speed is 2 m/s, that is 16% of the mean wind speed. Assuming the turbulence is being advected by the mean flow, Taylor’s (frozen flow) hypothesis is accepted and according to the sampling rate of 16 pts/s, data units will be converted to meters with the conversion factor $12.8 \text{ m/s} \div 16 \text{ pts/s} = 0.8 \text{ m/pt}$. A reason for converting to a length scale is that it is often easier to interpret scale dependent quantities in terms of spatial distances (which are “fixed”) versus temporal distances (which are “relative” to the wind speed). Another reason is that when the turbulent air parcel trajectories are dominated by the mean wind it is reasonable to transform temporal frequency to a spatial wavenumber. This is relevant to scaling relationships when considering data measured at a fixed point (Hunt and Vassilicos, 1991). This consideration is discussed again in chapter 3.

Section 1.2: Global Decomposition

This section introduces two distinct basis sets that can be used to decompose the variations of the time series into contributions from different scales. A (real) Fourier basis set consist of sines and cosines having different wavelengths. A Haar basis set consist of finite width step functions having different widths. (Some cosine functions are shown in figure 6 and an orthogonal Haar basis set is shown in figure
7.) The value of a transform coefficient depends on the covariance between the data and the normalized basis element used to compute the transform. The magnitude of the coefficient is determined by the similarity in shapes between the data and the basis element (a correlation) and the total variance present in the data.

A sine or cosine basis element is defined throughout the time series, so that the corresponding transform coefficient depends on a correlation involving the entire time series. The Haar in general is defined over a subsection of the time series, so the correlation corresponding to the transform coefficient may include a fraction of the entire series. In atmospheric (surface layer) turbulence, points in the flow separated by large distances or long durations are highly uncorrelated (Panofsky and Dutton, 1984).

The Fourier decomposition is useful when seeking periodicities in the data while the Haar transform provides information on the local structure in the time series. In a long time series (like more than a day), the surface layer winds exhibit variations due to local mesoscale \((10^2 \text{ km})\) forcing, as well as large scale \((10^3 - 10^4 \text{ km})\) synoptic and planetary scale forcing which might be associated with distinct spectral peaks. Fourier analysis can be used to isolate these modes of variation. When considering surface layer turbulence, length scales of interest may be much smaller \((1 \text{ cm} - 10 \text{ km})\) and the flow is not periodic in which case the wavelet decomposition is more useful.

The distribution of energy with scale determined from a Haar transform is dependent on the location in the turbulent flow. The degree of intermittency present in turbulent surface layer winds may depend on the average wind speed, the large eddies and the presence of physical boundaries. Haar (wavelet) analysis can be used to locate (in physical space) such intermittent variability (Meneveau, 1991; Farge, 1992).
This section will be divided into three subsections. The first will detail the algorithm used to compute the global Fourier spectrum and the second will present the method used to compute the global Haar spectrum. The two algorithms are applied to the 18 hour segment of the LAMEX data discussed in section 1.1. The results are presented in 1.2.3.

1.2.1 Fourier Variance Spectrum

The longitudinal, cross flow and vertical wind components of the Lammeefjord time series are denoted

\[ U_1 = u(t_1), \quad V_1 = v(t_1) \quad \& \quad W_1 = w(t_1) \]  \hfill (4)

respectively where \( t_i = (i-1/2)\delta \) seconds is the time starting from the end of hour 6 in LAMEX and \( i = 1, 2, \ldots, N \) where \( N = 18 \text{ hrs} \times 3600 \text{ s/hr} \times 16 \text{ pts/s} = 1,036,800 \text{ pts} \) is the number of points in 18 hours. Note that \( \delta = 1/16 \text{ sec} \) is the temporal sampling interval and \( \Delta = \delta \times 12.8 \text{ m/s} = 0.8 \text{ meters} \) is used to denote the spatial sampling interval.

The Fourier (sine and cosine) transformed series for the longitudinal wind is defined by

\[
\hat{U}_{2r-2} = \sum_{i=1}^{N} U_i \cos \left[ \frac{(r - 1)(i - 1)2\pi}{N} \right] \quad r = 2, \ldots, N/2 + 1 \\
\hat{U}_{2r-1} = -\sum_{i=1}^{N} U_i \sin \left[ \frac{(r - 1)(i - 1)2\pi}{N} \right] \quad r = 2, \ldots, N/2 \\
\hat{U}_1 = \sum_{i=1}^{N} U_i \quad \text{If } N \text{ odd then } r = 2, \ldots, (N + 1)/2 \]  \hfill (5)

where the conventions used in IMSL STAT/LIBRARY subroutine FFTRF are followed. The FFTRF(N, U, \( \hat{U} \)) is found in the mathematical support section of the IMSL.
STAT/LIBRARY and is a FORTRAN subroutine based on a routine in a (real) fast Fourier transform computer software package, FFTPACK, developed by Paul Swarztrauber at the National Center for Atmospheric Research and will be used to compute all Fourier transforms in this study. There are equivalent equations for $\hat{V}$ and $\hat{W}$.

To compute the variance associated with a particular wavelength the transform coefficients in (5) are divided by the energy contained in the (non-normalized) sine and cosine basis. $E^2(u,\lambda_f)$ is defined as the cosine & sine variance associated with the longitudinal wind speed at a wavelength

$$\lambda_f = \frac{0.8N}{1000r} \left( \frac{\text{km}}{\text{cycle}} \right)$$

where data units have been converted to kilometers in (6) with the factor $0.8/1000$ according to the discussion at the end of section 1.1. The exponent 2 in the notation $E^2(u,\lambda_f)$ is to emphasize the fact that the quantity is positive definite as well as distinguish it as a variance spectrum.

Throughout the text, variances (as a function of scale) are considered as discrete (point-wise) quantities, so total variance is equal to the sum of the point-wise quantities. This is done primarily so that decompositions are more comparable. This is in contrast to the approach of considering the variance quantity as a discrete sampling to a continuous spectrum where the total variance is recovered by continuously integrating the spectrum. A spectral density is computed by dividing $E^2(u,\lambda_f)$ by $\Delta k_f \left[ = 2\pi/(0.8N) \text{ m}^{-1} \right]$, which in this case is a constant, so it does not change the shape of the spectrum. Since equivalent relations for the other decompositions are less apparent, this operation is avoided.
Note that \( N \) is even which means there is only one (cosine) component in the transform at wavelength \( \lambda_{N/2} \) (called the Nyquist frequency) and the corresponding basis element oscillates between +1 & -1 between every point. Later results show that for LAMEX such a basis element is inefficient in representing variance implying the data is not well correlated (globally) with +1 -1 oscillations.

The normalization factors needed to determine \( E^2(u,\lambda_r) \) are

\[
N \times \sum_{i=1}^{N} \cos^2 \left[ \frac{2\pi r(i - 1)}{N} \right] \quad \& \quad N \times \sum_{i=1}^{N} \sin^2 \left[ \frac{2\pi r(i - 1)}{N} \right] \quad (7)
\]

The factor of \( N \) is needed to convert the integrated energy to an estimate of the local energy (i.e. variance) having units \( \text{m}^2/\text{s}^2 \). Using the identities \( \cos^2(*) + \sin^2(*) = 1 \) \& \( \cos(*) = \sin(* + \frac{\pi}{2}) \) it can be shown that for \( N \) even, both cosine & sine sum of squares are the same and equal to \( N/2 \) for \( r < N/2 \). For \( r = N/2 \) the cosine & sine sum of squares equal \( N \) & 0, respectively. (General results of trigonometric power series are contained in the reference Tables, Integrals, Series, and Products, pp 30-31.)

The Fourier variance decomposition is written as

\[
E^2(u, \lambda_r) = \frac{2}{N^2} \left[ (\hat{U}_{2r})^2 + (\hat{U}_{2r+1})^2 \right] \quad r = 1, \ldots, N/2 - 1
\]

\[
E^2(u, \lambda_{N/2}) = \frac{1}{N^2} (\hat{U}_N)^2
\]

Because only the energy due to the variance is considered \( \hat{U}_1 \) in the Fourier transform equations (5) is ignored.

The quantities \( E^2(u,\lambda_r) \) will in turn be summed over spectral windows centered about dyadic wavelengths: \( 0.8 \times 2^m/1000 \) [km/cycle], \( m = 1, 2, \ldots, 20 \). The Fourier spectrum summed over bands of wavelengths leads to a scale decomposition similar to the \( kE(k) \) spectrum often used in turbulence analysis (Tennekes and Lumley, 1972).
A more compact spectrum is defined as

\[
\overline{E^2}(u, \lambda_m) = \sum_{\lambda_r \in d_m} E^2(u, \lambda_r), \quad \lambda_m = \exp \left[ \frac{1}{L_m} \sum_{\lambda_r \in d_m} \log(\lambda_r) \right]
\]  

(9)

where \( d_m = 0.8 \times \left(2^{m-\frac{1}{2}}, 2^{m+\frac{1}{2}}\right)/1000 \) is the \( m \)th dyadic interval and \( L_m \) is the number of \( \lambda_r \) that fall within the \( m \)th dyadic interval. The logarithmic average of the \( \lambda_r \) that fall within the interval is selected as the "central" wavelength.

1.2.2 Haar Variance Spectrum

The Haar transform coefficients are defined as

\[
W(u, a, b) = \frac{1}{a^*} \sum_{i=1}^{a^*/2} (U_{b^*+i} - U_{b^*+1-i})
\]

(10)

where \( a = 0.8 \times \frac{a^*}{1000} \) is the dilation or width of the transform window in kilometers and \( b = 0.8 \times \left(\frac{b^* + 0.5}{1000}\right) \) is the translate or central position of the transform window in kilometers. The Haar transform coefficient (as defined above) is equivalent to computing half the difference in half window averages where the window includes \( a^* \) data points and is centered about the data position \( b^* + 1/2 \). The Haar transform coefficient is largest when the value of the signal jumps between positions \( b^* \) and \( b^* + 1 \) and is relatively constant in the two halves of the window (i.e. is correlated with an ideal step function). If the amplitude of the "jump" is small then the correlation with a step is of little consequence — the transform coefficient will be small. Potential ambiguity arises when the signal undergoes an anomalously large but smooth change. For example, significant trend (where there is no jump) across the transform window will increase the magnitude of the Haar transform.
To generate a variance spectrum the average square of the Haar transform coefficient at dilation ‘a’ is computed as

$$D^2(u, a) = \frac{1}{N - a^* + 1} \sum_{b^* = a^*/2}^{N - a^*/2} W^2(u, a, b)$$ (11)

where as with $E^2(u, \lambda_r)$ the capital letter with 2 in the exponent implies a variance (energy) spectrum. The normalization by the dilation, $a^*$, in the Haar transform (10) ensures $D^2(u, a)$ represents variance (see 2.1.2 for details). The overline in $\overline{D^2}(u, a)$ is to distinguish this spectrum from that defined in 2.1.2, which is a variance derived from an orthogonal decomposition.

The Haar spectrum will only be computed at dyadic scales (i.e. $a_m = 0.8 \times 2^m/1000$; $m=1,\ldots,19$). Because $N < 2^{20}$ the largest possible dilation is $a_{20} = 0.8 \times N/1000 = 830$ km. Note that this non-orthogonal decomposition of variance is nearly the same as a Haar orthogonal decomposition because the estimate in (11) is made from such a large number of points and because there is so little variance associated with the largest non-dyadic (that is non-orthogonal) scale.

The global Haar spectrum at a given dilation ‘a’ is very efficient to compute because of the relation

$$W(u, a, b^*) = W(u, a, b^* - 1) + U_{b^*+a^*/2} - 2U_{b^*} + U_{b^*-a^*/2}$$ (12)

for $b^* = a^*/2 + 1,\ldots,N - a^*/2$. The computational time is $O(N)$ versus $O(N\log N)$ as in most optimized transform algorithms (e.g. the fast Fourier transform).

1.2.3 Global Spectrum Results

The global Fourier and Haar spectrums of the longitudinal wind (figure 3) suggest the most energetic eddies to be on the order of one kilometer. The peak
in the Haar spectrum is at about 1600 m corresponding to approximately one minute of above (or below) average wind speed. The Fourier spectrum also begins to peak around 1 km. The peak in the Fourier spectrum extends out to about 3 km, a length scale almost two times larger than the peak in the Haar spectrum despite the fact that the Haar of scale 'a' (locally) corresponds to a Fourier spectral band centered at a wavelength only about 1/3 larger than the scale 'a'. Tests from using artificial signals suggest that in the absence of global periodicity the peak in the Haar spectrum corresponds to the dominant event (or eddy) scale while the peak in the Fourier spectrum may in part be due to the spacing of the events (Gamage, 1990).

At smaller scales both spectrums have a $-2/3$ slope in log-log coordinates corresponding to the inertial subrange where the theories of homogeneous isotropic
Figure 4 Global variance spectrums of the cross stream wind.

turbulence are more applicable. Isotropic turbulence is characterized by 1-d spectra in the downstream direction in which the ratio of the cross stream component (v or w) to the downstream component u is 4/3 (Batchelor, 1953). For this data the ratios are approximately 4/3 in the inertial region centered around a scale of 10 m, though the flow is subject to anisotropic forcing from the large (∼ 1 km) scales. Note that the −2/3 reference lines are at identical positions in figures 3 through 5. The fall off in the Fourier spectrum at the smallest scales could be the result of the electronic output filter on the sonic anemometer or could be a resolution problem or related to the temporal sampling. The Haar does not rapidly fall off at the smallest scale. The large difference at the smallest scale is thought to be a result of the local broad band pass nature of the Haar transform at the smallest scale.
The meandering of the variance spectrum of the cross stream wind at larger scales is indicative of larger scale "swirling" motions leading to slight changes in the mean wind direction. The spectral peak in the vertical velocity is at smaller scales, about 100 m. The variation in vertical velocity is most coherent on this scale which implies approximately 50 meters (4 seconds) of upward or downward motion. The peaks of the u, v, and w spectrums are at about 800 m, 200 m, and 100 m, implying the most energetic or variable motions have a u/v/w aspect ratio of approximately 8/2/1. This implies the main energy containing eddies are flattened considerably, presumably because of the ground 45 m below, though the flattening affect of the near-neutral stratification could be a factor.
Chapter 2  Mathematical Methods

Quantities introduced in this chapter describe the variation of the longitudinal wind speed over scales much smaller than the (∼ km) primary energy containing scales. The variance quantities are estimated by forming an ensemble of statistics corresponding to different neighborhoods in the flow. The size and positions of the neighborhoods affect the ensemble statistics. In this study the neighborhoods corresponding to 50 meter non-overlapping windows phase locked about anomalous small scale variability are used to generate the statistics. From this point on, only the longitudinal wind speed variations are decomposed.

Sample sets of non-overlapping data windows of length \( L = 2^6 = 64 \) points = 4 seconds \( \approx 50 \) meters are used to partition the flow. This choice is motivated by the results presented in 1.2.3 which suggest that 100 meters is the scale of maximum variability in the vertical velocity, thus half that length scale (50 m) should be approximately the width of an updraft or downdraft.

Fourier, eigenvector, polynomial and Haar bases sets represent four distinctive methods for completely and orthogonally decomposing the total variance. The Fourier decomposition is the most widely used and efficient at providing information on wave like periodicities. The eigenvector decomposition is also widely used and is unique in that it optimizes variance explained with each successive eigenmode. Another unique basis set is one comprised of orthogonal polynomials, which are used to decompose the data signal in order to further assess the dependency of the results on using a specific basis set. The Haar is an orthonormal wavelet with compact support (Daubechies, 1989) and is an efficient means to completely decompose the variance within a data
window of dyadic length. These four orthogonal decompositions are detailed in section 2.1.

Differences in velocity at two distinct points is referred to as a structure function. The variance and higher moments of the structure function depend on the distance separating the two points. The dependence on scale of the higher moments of point-wise differences is difficult to estimate due to the accumulation of variance (or degrees of freedom) as the separation distance is increased. Estimates of high order structure function scaling exponents are not made in this study. A method for generating structure function statistics is presented in section 2.2.

A strategy for selecting the positions of sample windows is discussed in section 2.3. The strategy is to transform the longitudinal wind so that small scale variability is emphasized. Segments of anomalous small scale activity will then be grouped into two different sample sets, so as to form two different ensembles.

Section 2.1: Orthogonal Variance Decompositions

This section describes four methods to orthogonally decompose a window containing \(2^M\) equally spaced data points. Using the notation of (4), the LAMEX longitudinal wind component within a window starting at time \(k\delta\) is

\[
U_{k+j} = u(t_{k+j}) , \ j = 1, 2, ..., 2^M
\]

\[k = 0, 1, ..., N - 2^M\]

(13)

where 'j' is the within window index, \(M = 6\) and \(N = 1,036,800\). Requiring that the windows do not overlap implies a maximum of \(N / 2^M = 16,200\) distinct windows. In this study time \(t_{k+j} = k\delta + (j - 1/2)\delta\) is converted to a distance, say \(-x_{k+j} = k\Delta + (j - 1/2)\Delta\) where \(\Delta = 0.8\) meters. Ideally, the flow patterns are considered to be
"frozen", so sitting at the sensor at $t = x = 0$ the patterns arriving at a later time, say at time $T$, are sitting upstream at a position $-X$ at time $t = 0$. Assuming the mean wind will carry that pattern toward the sensor at a (constant) rate $U$ implies $T = U \times X$. From a statistical point of view, Taylor's (frozen flow) hypothesis implies the ensemble of patterns that move pass the sensor in a time $T$ are on average the same that are distributed upstream over a distance $X$.

The Fourier decomposition was outlined in 1.2.1, so only the essentials are restated in 2.1.1. A Haar transform coefficient was introduced in 1.2.2, though using the Haar to completely and orthogonally represent the data suits a more formal development as presented in 2.1.2. In 2.1.3 some of the properties of the eigenvectors of the lagged covariance matrix are shown which furnish a natural coordinate system in which to describe the samples. Orthogonal polynomials are more subject to finite resolution problems (as are eigenvectors), though it is still possible to generate an orthogonal basis set as discussed in 2.1.4. In section 2.1.5 the relevant quantities are summarized.

2.1.1 Fourier Decomposition

The sine and cosine transforms used in this study are presented in 1.2.1 and are repeated here. The Fourier (sine and cosine) transformed series for a window starting at time $k\delta$ are defined as

$$
\hat{U}_{k,2r-2} = \sum_{j=1}^{2^M} U_{k+j} \cos \left[ \frac{(r-1)(j-1)2\pi}{2^M} \right] \quad r = 2, \ldots, 2^M - 1 + 1
$$

$$
\hat{U}_{k,2r-1} = -\sum_{j=1}^{2^M} U_{k+j} \sin \left[ \frac{(r-1)(j-1)2\pi}{2^M} \right] \quad r = 2, \ldots, 2^M - 1
$$

(14)
The Fourier variance decomposition is written in this case as

\[ E_k^2(u, \lambda_r) = \frac{2}{2^M} \left[ \left( \hat{U}_{k,2r} \right)^2 + \left( \hat{U}_{k,2r+1} \right)^2 \right] \quad r = 1, \ldots, 2^{M-1} - 1 \]

\[ E_k^2(u, \lambda_{2^{M-1}}) = \frac{1}{2^M} \left( \hat{U}_{k,2^M} \right)^2 \]  

(15)

where the wavelength \( \lambda_r = (0.8 \times 2^M \div r) \) is in meters.

2.1.2 Haar Decomposition

When considering a window containing \( 2^M \) data points the orthogonal Haar basis set (shown in figure 7 with \( M = 6 \)) can be used to completely decompose the
A Haar basis is defined as
\[ h_j(a_m, b_{mn}) = \begin{cases} 
-1, & (n-1)2^m < j \leq (n - \frac{1}{2})2^m \\
+1, & (n - \frac{1}{2})2^m < j \leq n2^m \\
0, & j \leq (n-1)2^m \text{ or } j > n2^m 
\end{cases} \]

where \( m = 1, \ldots, M \); \( n = 1, \ldots, 2^{M-m} \)

where \( m \) corresponds to dilation \( a_m = 2^m \Delta \) and the translation number \( n \) corresponds to the basis element of dilation \( a_m \) centered at \( b_{mn} = (n-0.5) \times a_m \). Note that in figure 7 the tick marks at the bottom are located at \( b_{mn} \) and their length is proportional to \( a_m \).

The Haar transform of longitudinal wind is
\[ W_k(u, a_m, b_{mn}) = \frac{1}{2^m} \sum_{j=1}^{2^m} U_{k+j} h_j(a_m, b_{mn}) \]
or after substituting in the values for \( h_j(m, n) \) it follows that

\[
W_k(u, a_m, b_m) = \frac{1}{2^m} \sum_{j=1}^{2^{m-1}} \left( U_{k+n2^m+1-j} - U_{k+(n-1)2^m+j} \right)
\]  

(18)

The within window variance is partitioned into the \( M \) different dilations after averaging \( W^2_k(u, a_m, b_m) \) over all \( 2^{M-m} \) translates. A decomposition of the total variance is precisely written as

\[
\frac{1}{2^M} \sum_{j=1}^{2^M} (U_{k+j} - \bar{U}_k)^2 = \sum_{m=1}^{M} D^2_k(u, a_m)
\]  

(19)

\[
D^2_k(u, a_m) = \frac{1}{2^{M-m}} \sum_{n=1}^{2^{M-m}} W^2_k(u, a_m, b_m)
\]

where \( \bar{U}_k = \frac{1}{2^M} \sum_{j=1}^{2^M} U_{k+j} \) is the window arithmetic mean.

### 2.1.3 Eigenvector Decomposition

Assume window starting positions \( k \) are contained in the set \( s_1 \). An estimate of the lagged covariance of two points within a window is defined

\[
c_{ij} = \frac{1}{S_1} \sum_{k \in s_1} U'_{k+i} U'_{k+j}
\]  

(20)

where \( S_1 \) is the number of sample windows contained in \( s_1 \) and the primes symbolize the departure from the within window mean. The lagged covariance matrix has elements \( c_{ij} = c_{ji} \) where \( i \) and \( j \) range from 1 through \( 2^M \). The lagged covariance matrix (because it is real and symmetric) will have \( 2^M \) eigenpairs, though some may be degenerate (Strang, 1980). In this study the eigenpairs (eigenvectors and the associated eigenvalues) of the lagged covariance matrix are computed using the IMSL STAT/LIBRARY FORTRAN subroutine E2CSF. Since the within window averages are
removed (so all samples have an average value = 0), at most there will be \(2^M - 1\) non-degenerate eigenpairs and in general there will be a smaller number of "significant" eigenpairs depending on the similarities across the sample set. For example, if all the samples are identical then there can only be one significant eigenpair or if the number of samples is less than \(2^M - 1\) then the number of samples puts an upper bound to the number of relevant eigenpairs.

The \(j^{th}\) element of the \(p^{th}\) normalized eigenvector is denoted \(\phi_j^{(p)}\), so \(\sum_{j=1}^{2^M} [\phi_j^{(p)}]^2 = 1\). Denoting the associated eigenvalue as \(\mu^{(p)}\) the definition of an "eigenpair" is that

\[
\sum_{j=1}^{2^M} c_{ij} \phi_j^{(p)} = \mu^{(p)} \phi_i^{(p)}
\]  

(21)
To see that the eigenvectors are orthogonal multiply both sides of (21) by \( \phi_i^{(q)} \), sum from \( i = 1 \) to \( 2^M \) and use the fact that \( c_{ij} = c_{ji} \) so the left hand side of (21) becomes

\[
\sum_{i=1}^{2^M} \sum_{j=1}^{2^M} c_{ij} \phi_j^{(p)} \phi_i^{(q)} = \sum_{j=1}^{2^M} \phi_j^{(p)} \sum_{i=1}^{2^M} c_{ji} \phi_i^{(q)} = \mu^{(q)} \sum_{j=1}^{2^M} \phi_j^{(p)} \phi_j^{(q)}
\]

(22)

and the right hand side is \( \mu^{(p)} \sum_{i=1}^{2^M} \phi_i^{(p)} \phi_i^{(q)} \) which can only hold if

\[
\sum_{i=1}^{2^M} \phi_i^{(p)} \phi_i^{(q)} = \begin{cases} 1, & p = q \\ 0, & p \neq q. \end{cases}
\]

The expansion coefficients are defined as

\[
\alpha_{kp} = \sum_{j=1}^{2^M} U'_{k+j} \phi_j^{(p)}
\]

(23)

where it follows from the fact the set of eigenvectors are orthonormal that the eigenvector expansion of a given sample is

\[
U'_{k+i} = \sum_{p=1}^{2^M} \alpha_{kp} \phi_i^{(p)}
\]

(24)

For different eigenvectors the corresponding expansion coefficients are uncorrelated across the sample set. This is shown by substituting the expression for \( c_{ij} \) from (20) into (21), use the definition of the eigenvector expansion of \( U'_{k+i} \) from (24) and the definition of the expansion coefficient from (23), so the left hand side of (21) is rewritten as

\[
\sum_{j=1}^{2^M} \left( \frac{1}{S_1} \sum_{k \in s_1} U'_{k+i} U'_{k+j} \right) \phi_j^{(p)} = \frac{1}{S_1} \sum_{k \in s_1} \left( \sum_q \alpha_{kq} \phi_i^{(q)} \right) \left( \sum_{j=1}^{2^M} U'_{k+j} \phi_j^{(p)} \right)
\]

(25)

\[
= \frac{1}{S_1} \sum_{k \in s_1} \sum_q \alpha_{kq} \phi_i^{(q)} \alpha_{kp} = \sum_q \phi_i^{(q)} \frac{1}{S_1} \sum_{k \in s_1} \alpha_{kq} \alpha_{kp}
\]

which must equal the right hand side of (21), that is \( \mu^{(p)} \phi_i^{(p)} \), implying that

\[
\frac{1}{S_1} \sum_{k \in s_1} \alpha_{kq} \alpha_{kp} = \begin{cases} \mu^{(p)} & q = p \\ 0 & q \neq p \end{cases}
\]

(26)
and the eigenvalue $\mu^{(p)}$ is the average (total) energy per sample window associated with eigenmode ‘p’.

2.1.4 Polynomial Decomposition

The finite resolution of the computer makes it difficult to precisely represent very high order orthogonal polynomials, since the scale of oscillation becomes small. In this study it is necessary to decompose the variations in a window of length 64 points. To do this with polynomials requires a 63rd degree polynomial. To force the unresolved highest order polynomials to be precisely orthogonal to all polynomials of lower order, the Gram-Schmidt orthogonalization procedure is used (Strang, 1980).
One way to construct a complete orthonormal basis set on the interval [-1,1] using 64 points is to begin by

I \[ p_i^{(1)} = \frac{2(i-1)}{63} - 1; \ i = 33, \ldots, 64 \]

II \[ p_i^{(n)} = p_i^{(1)} p_i^{(n-1)}; \ i = 33, \ldots, 64; \ n = 2, \ldots, 63 \] \hspace{1cm} (27)

III \[ p_i^{(n)} = (-1)^{n} p_{65-i}^{(n)}; \ i = 1, \ldots, 32; \ n = 1, \ldots, 63 \]

where the operations in (27) are done in the order I, II, III. Because of the odd symmetric interval, [-1,1], the odd and even degree polynomials can be treated separately (since they are already orthogonal) and the polynomials need only be made orthogonal over half the interval, say [0,1], and then the other half interval [-1,0] is set according to the symmetry [step III in (27)]. The initialization (27) essentially leads to 63 independent vectors, which can be identified as a discrete sampling of continuously defined polynomials. Gram-Schmidt orthogonalization can be used to make any linearly independent functions (or vectors as in this case) an orthogonal set.

After initializing the 63 vectors according to (27) normalize so

\[ \sum_{j=1}^{64} \left[ p_j^{(n)} \right]^2 = 1 \] \hspace{1cm} (28)

for \( n = 1, \ldots, 63 \). To Gram-Schmidt orthogonalize the set of vectors perform the following operations

\[ p_i^{(2n+1)} = p_i^{(2n+1)} - \sum_{k=1}^{n} \left[ \sum_{j=1}^{64} p_j^{(2k-1)} p_j^{(2n+1)} \right] p_i^{(2k-1)} \]

\[ p_i^{(2n+2)} = p_i^{(2n+2)} - \sum_{k=1}^{n} \left[ \sum_{j=1}^{64} p_j^{(2k)} p_j^{(2n+2)} \right] p_i^{(2k)} \] \hspace{1cm} (29)

for \( n = 1 \& i = 33, \ldots, 64 \) and use the symmetry to set \( n = 1 \& i = 1, \ldots, 32 \) [step III in (27) with \( n = 1 \)]. Next normalize \( p_i^{(3)} \) and \( p_i^{(4)} \), so (28) is satisfied. Repeat this
process for $n = 2$. Normalize $p^{(5)}_i$ and $p^{(6)}_i$, so again (28) is satisfied. Repeat these operations until all 61 vectors have passed through the Gram-Schmidt process. Note that the linear ($n = 1$) and quadratic ($n = 2$) polynomials are not altered. Because of computer round off error the process might have to be iterated once or twice to ensure all vectors are precisely orthogonal to each other.

In figure 9 the $p^{(n)}$ are plotted for $n = 1, 3, 7, 15,$ and 63. This orthogonalization procedure forces the unresolved polynomials to become more localized. The same sort of localization also occurs with the eigenvectors (figure 8).

The variance in longitudinal wind contained in a sample window starting at time $k\delta$ associated with an orthonormal polynomial of degree ‘$n$’ is

$$B_k^2(u, n) = \frac{1}{64} \left[ \sum_{j=1}^{64} U_{k+j}P_j^{(n)} \right]^2$$

2.1.5 Summary of Variance Decompositions

According to the criterion to be summarized in section 2.3, two sets of sample windows are selected. Each set is treated separately. Starting positions of sample windows of data are contained in the sets $s_1$ and $s_2$. The average variance in the longitudinal wind speed in a $s_1$ sample is defined

$$\text{Var}(u, s_1) = \frac{1}{S_1} \sum_{k \in s_1} \frac{1}{64} \sum_{j=1}^{64} (U_{k+j} - \overline{U}_k)^2$$

where $S_1$ is the number of samples in $s_1$ and $\overline{U}_k = \frac{1}{64} \sum_{j=1}^{64} U_{k+j}$. An equivalent expression holds for $\text{Var}(u, s_2)$.

Based on the decompositions described in 2.1.1 – 2.1.4 $\text{Var}(u, s_1)$ and $\text{Var}(u, s_2)$ can be decomposed into different components. The Fourier (sine and cosine) variance
decomposition is written

\[ \text{Var}(u, s_\eta) = \sum_{r=1}^{32} \langle E^2(u, \lambda_r) \rangle_{\eta} \]  

(32)

where the angle brackets imply ensemble averaging across the sample set \( \eta \), so

\[ \langle E^2(u, \lambda_r) \rangle_{\eta} = \frac{1}{S_\eta} \sum_{k \in S_\eta} E^2_k(u, \lambda_r) \]  

(33)

where \( \eta = 1 \) or \( \eta = 2 \) and \( E^2_k(u, \lambda_r) \) is defined in (15).

For the Haar decomposition it follows that

\[ \text{Var}(u, s_\eta) = \sum_{m=1}^{6} \langle D^2(u, m) \rangle_{\eta} \]  

(34)

where \( \eta = 1 \) or \( \eta = 2 \), \( D^2_k(u, m) \) is defined in (19) and the angle brackets imply ensemble averaging as in (33).

For the eigenvector decomposition set define

\[ \langle C^2(u, p) \rangle_{\eta} = \frac{\mu^{(p)}_{\eta}}{64} \]  

(35)

where \( \mu_{\eta}^{(p)} \) is the \( p \)th eigenvalue associated with the lagged covariance matrix as defined in (21) corresponding to the \( \eta = 1 \) or \( \eta = 2 \) sample set. The eigenvector variance decomposition is then

\[ \text{Var}(u, s_\eta) = \sum_{p=1}^{63} \langle C^2(u, p) \rangle_{\eta} \]  

(36)

The polynomial variance decomposition is

\[ \text{Var}(u, s_\eta) = \sum_{n=1}^{63} \langle B^2(u, n) \rangle_{\eta} \]  

(37)

where \( \eta = 1 \) or \( \eta = 2 \), \( B^2_k(u, n) \) is defined in (30) and the angle brackets imply ensemble averaging over sample set \( \eta \).
Section 2.2: The Structure Function

The difference in the longitudinal wind at two data points separated by '1' data intervals is defined as the structure function

\[ \psi_1(u, r) = U_{i+1} - U_k \]

(38)

where \( r = (0.8 \text{ m}) \times 1 \) is the separation distance in meters.

Homogeneous isotropic turbulence has a similar structure at different scales within a so called inertial range of length scales and this consideration has been used to theorize that higher moments of the structure function obey certain scaling laws (Kolmogorov, 1941). On the other hand it has been observed that for high Reynolds number turbulence (where the inertia of the flow dominates the retarding force of molecular viscosity as in most geophysical flows) variations in the motions become more intermittent (that is less space filling) at smaller scales even within the inertial subrange. In this case the flow structure is not similar at different scales implying modest departures from the scaling laws based on self-similar structure (Kolmogorov, 1962; Frisch et al., 1978; Nelkin & Bell 1978).

Assuming the structure function and higher moments depend only on the rate of energy dissipation \( \epsilon \), which has units \([m^2s^{-3}]\) and separation distance \( r \) [m] then dimensional analysis implies

\[ \langle \psi^n(u, r) \rangle \propto \langle \epsilon^{n/3} \rangle r^{n/3} \]

(39)

where the brackets imply averaging over all possible positions. The effects of increasing intermittency at smaller scales has lead to various modifications of this relationship based on different models. The intermittency models assume, for example,
that $\epsilon$ has a lognormal probability distribution (Obukhov, 1962) or that the distribution of $\epsilon$ can be related to the apparent fractal structure of turbulence (Mandelbrot, 1974). The modifications generally take the form

$$\langle \psi^n(u, r) \rangle \propto \langle \epsilon^{n/3} \rangle r^{n/3 - \zeta_n}$$

(40)

where $\zeta_n$ is a "universal constant" (function of $n$) and is an increasing function of $n$. The effects of intermittency on higher moment structure functions are seen in the corresponding probability density distributions of $\psi^n_i(u, r)$ at a fixed separation distance $r$. As the separation distance is decreased the distribution becomes more "peaked" while the tails of the distribution remain extended (i.e. the so called flatness factor increases). In other words, as $r$ decreases the values of $\psi^n_i(u, r)$ cluster more closely around the mean while the probability that unusually large values of $|\psi^n_i(u, r)|$ will occur remains significant. The higher moments, $\psi^n_i(u, r)$, are more sensitive to the "tails" of the distribution (i.e. occurrences of large wind speed differences) which is a reason why $\zeta_n$ becomes larger with $n$.

From (39) the 6th order structure function $(n = 6)$ divided by $r^2$ is assumed to be proportional to the square of the rate of energy dissipation. This quantity is an estimate of the variability of the energy cascade toward smaller scales in the inertial subrange, that is, energy transfer variance (Frisch et al., 1978; Nelkin & Bell 1978; Anselmet et al., 1984). In this study the variance of the transfer rate of turbulent kinetic energy toward smaller scales is estimated from the 6th order structure function (Frisch et al., 1978) for a separation distance $r = \Delta$, so it follows

$$\langle \psi^2 \rangle \propto \langle \psi^6(u, \Delta) \rangle / \Delta^2$$

(41)
where now the angle brackets imply averaging over all possible positions within a given window and \( \Delta = 0.8 \) m is the distance between data points.

Section 2.3: Sampling Strategy

An objective of this study is to demonstrate the influence of small scale "events" on scaling relationship. Based on previous studies of turbulent intermittency (see introduction) it is expected that on relatively small scales the variance is concentrated in narrow regions that are separated by broader regions. The narrow regions of concentrated small scale variance may correspond to the leading edge of a large scale eddy (microfront) or may be a small large-amplitude eddy structure. In this section it is described how segments containing anomalous small scale variability are located.

An assumption (or hypothesis) is that one source of the intermittency in the atmospheric surface layer is due to vertical advection. However, the validity of this assumption does not effect the results presented in this study, but rather it is made in order to objectively determine a sampling window width. The idea is that the vertical motion leads to air from different source levels moving into the same level. For example, air moving upwards from near the ground may be a source of enhanced small scale variability. Alternatively, air moving down from aloft may induce larger scale large amplitude velocity perturbations.

At the end of section 2.2 it was shown that the sixth order structure function may have physical relevance in describing the variance of the rate at which kinetic energy is dissipated. This is based on dimensional arguments. The rate at which energy is dissipated would be a good quantity to use for the conditional sampling in this study, but it is not available. The sixth power of a velocity difference at a small scale is
sensitive to the presence of large local velocity gradients. These large gradients may be associated with increased rates of energy dissipation.

For the above reasons the longitudinal wind is transformed using the 6th order structure function defined in section 2.2 and then averaged according to

$$\overline{\psi^6_k}(u, \Delta) = \frac{1}{64} \sum_{j=1}^{64} \psi^6_{k+j}(u, \Delta)$$

where \( k = 0, \ldots, N-65 \) and \( \Delta = 0.8 \text{ m} \) is the distance between data points. The peak in the vertical velocity variance spectrum occurs at about 100 m (figure 4). Half that scale or about 50 m ( = 64 points) should be the average width of an updraft or downdraft, which is why the averaging is performed over a 50 m window. The quantity \( \overline{\psi^6_k}(u, \Delta) \) is related to the variations in the rate of energy dissipation in a 50 m (64 point) window starting at time \( k\Delta \).

The set of starting positions of the sample windows containing anomalously large velocity differences is defined as

$$s_1 = \left\{ k^* \ni \overline{\psi^6_{k^*}}(u, \Delta) > \overline{\psi^6_k}(u, \Delta) \text{ for } |k - k^*| \leq 64 \right\}$$

where the \( k^* \) indicates the starting positions of the sample windows assumed to contain more and/or stronger small scale activity. The notation inside the curly brackets reads as "the set of all \( k^* \) such that the average sixth order structure function starting at time \( k^* \Delta \) (eqn. 42) is greater than the averages of the sixth order structure functions for all the windows starting within 64 points of \( k^* \)."

As complimentary information sample windows are selected based on the local minimum values in \( \overline{\psi^6_k}(u, \Delta) \) representing windows containing relatively depressed levels of dissipation rates. A second sample set is defined as

$$s_2 = \left\{ k' \ni \overline{\psi^6_{k'}}(u, \Delta) < \overline{\psi^6_k}(u, \Delta) \text{ for } |k - k'| \leq 64 \right\}$$
where the $k'$ define the starting positions of the windows containing a slow down in small scale activity.
Chapter 3 Results

The sampling criteria discussed in section 2.3 is applied to the 18 hour (830 km) segment of LAMEX wind data arrived at in section 1.1. The sampling strategy is designed to partition the flow according to the small scale activity. If the small scale activity varies significantly depending on the position in the flow then the flow partitioning is substantial. On the other hand, if the small scale activity is more uniform in space then the flow partitioning is poorly defined.

The first set ($s_1$ sample set) includes 5,213 samples while the second set ($s_2$ sample set) includes 7,431 samples. The individual samples are 50 m (or 4 sec.) windows of longitudinal wind speed data measured every 1/16 second or about every 0.8 meters. The non-overlapping windows in the first set are centered around local maximums in the 6th order structure function — intensified small scale activity corresponding to enhanced rates of energy dissipation. The second set of windows are centered around local minimums in the 6th order structure function — slow downs in small scale activity.

The average longitudinal wind speed in both sample sets is 12.8 m/s (which is identical to the global average) while the average vertical and cross stream wind components are both very close to zero. Hence, there is no systematic difference between the mean wind vectors of the two sets. The eigenvectors of both sample sets (see figure 8 for $s_1$ eigenvectors) are approximately sinusoidal at the larger scales. The eigenvectors for the two sample sets are virtually identical. Eigenfunctions of random phase samples approach Fourier modes (Lumley, 1970) suggesting that the samples selected in this study contained random phase events. At the smallest scales
the eigenvectors are characterized by the finite resolution problem as are the discrete orthogonal polynomial basis set.

The average variance of the longitudinal wind speed in the \( s_1 \) samples is 0.78 m\(^2\)/s\(^2\) and the average variance is 0.43 m\(^2\)/s\(^2\) in the \( s_2 \) samples. The four orthogonal variance decompositions presented in section 2.1 reproduced the same variance values to at least six digits. The standard error in the spectral estimates (defined as the standard deviation divided by the square root of the number of estimates) is very small (< 1% of the spectral estimate) due to the large number of estimates. This is the main reason for not adding additional conditions to the sampling criteria, since that would have reduced the number of samples.

![Figure 10 Fourier decomposition of variance.](image-url)
When plotted in log-log coordinates each of the four decompositions are nearly linear, particularly in the center regions, implying that variation decreases with scale according to some scaling exponent. At smaller scales the scaling relationships break down. The poor resolution of the discrete basis elements at the smaller scales may contribute to this break down, though there may be another contributing factor.

Hunt and Vassilicos (1991) describe Taylor's hypothesis in terms of 'inside' and 'outside' trajectories. The word trajectories refers to a Lagrangian description of the flow. An inside trajectory loosely describes the small scale eddy motion and the outside trajectory describes the large scale eddy motion. If the inside trajectory is dominant then the transformation from a temporal frequency to a spatial wavenumber is not valid. This situation may occur when the average wind speed is very small.
If this is the case then dimensional analysis implies a $-2$ slope (i.e. $E(\omega) \propto \omega^{-2}$), but if the trajectory is dominated by the outside trajectory then $E(\omega)$ will depend on some outer velocity scale, say an average wind speed $u_0$, and it follows that $E(\omega) \propto \epsilon^{2/3} u_0^{2/3} \omega^{-5/3}$.

The Fourier, Haar, eigenvector, polynomial and structure function variance decompositions are discussed in that order in the following. See sections 2.1 and 2.2 for definitions of the notation used in figures 10 through 17.

The Fourier decomposition is sensitive to variations in the variance spectra across different samples, so it is difficult to precisely determine scaling exponents for the two sample sets. Both sample sets obey an approximately $5/3$ scaling as predicted for isotropic turbulence. Though it is generally not appropriate to precisely quantify deviations from this relationship, the influence of the small scale variability on the sign of the deviations is demonstrated. The slope of the curves in figure 10 are an estimate of the scaling exponents. Because of the noise in the Fourier decomposition a scaling exponent is best estimated from only a narrow range of wavelengths. The slope of the Fourier variance decomposition for wavelengths ranging from about 7 — 40 meters is shown in figure 11. The global spectrum results in 1.2.3 suggest inertial range scaling between 10 m and 100 m. The decomposition of a 50 meter window reduces the inertial range scaling region further because (1) the window is less than 100 m and (2) the largest and smallest scales in the 50 m window are not accurately resolved.

The two points straddling the wavelength of 20 meters are selected for the estimation of a scaling exponent. The scale of 20 meters is small enough to be less influenced by the variations on the scales greater than and equal to the window
width (50 m), yet large enough to be less affected by the smallest scale (unresolved) variations. These two points represent variance quantities corresponding to three wavelengths. The average of these two points corresponding to the $s_1$ sample set is 1.66 while for the $s_2$ sample set the average of the two points is 1.82.

A statistically more formal approach is to estimate the log-log slopes of the variance decompositions from additional spectral estimates in order to increase the degrees of freedom. For example, using wavenumbers 2–10 (wavelengths 25.6 m – 5.1 m) yields linear regression slopes of $-1.70 \pm 0.0104$ and $-1.84 \pm 0.0057$ for sample sets $s_1$ and $s_2$, respectively. The test for difference (Steele and Torrie, 1980 section 10.8) based on a Student's 't' test yields $t = 11.3$ with 16 degrees of freedom, which is well beyond the 99.5% significance level.
An estimate of a scaling exponent is derived from a linear regression fit to the Haar decomposition of variance for all but the smallest dilation. The slopes of the two lines are $-0.54 \pm 0.0212$ and $-0.73 \pm 0.0104$ for sample sets $s_1$ and $s_2$, respectively. The test for difference based on a Student’s ‘$t$’ test yields $t = 8.2$ with 6 degrees of freedom, which is beyond the 99.5% significance level.

Because there are no explicit length scales associated with the eigenvector and polynomial decompositions, the scaling relationships are defined in terms eigenvector number ‘$p$’ and polynomial degree ‘$n$’. An estimate of a scaling exponent is derived from a linear regression fit to the eigenvector decomposition of variance for eigenvectors 1–10. The slopes of the two lines are $-1.47 \pm 0.0227$ and $-1.67 \pm 0.0208$ for sample sets $s_1$ and $s_2$, respectively. The test for difference based on a Student’s ‘$t$’
test yields $t = 6.6$ with 16 degrees of freedom, which is beyond the 99.5% significance level. An estimate of a scaling exponent is derived from a linear regression fit to the polynomial decomposition of variance for polynomial degrees 1–10. The slopes of the two lines are $-1.41 \pm 0.0171$ and $-1.61 \pm 0.0102$ for sample sets $s_1$ and $s_2$, respectively. The test for difference based on a Student’s ‘$t$’ test yields $t = 10.2$ with 16 degrees of freedom, which is beyond the 99.5% significance level.

Finally, the variance of the structure function is considered. Figure 18 shows the average variance of the structure function as a function of separation distance $r$. The corresponding (log-log) slope of the structure function variance is shown in figure 19. An estimate of a scaling exponent is derived from a linear regression fit to the structure function variance for separation distances greater than 3 m. The slopes of
the two lines are $-0.58 \pm 0.0055$ and $-0.76 \pm 0.0026$ for sample sets $s_1$ and $s_2$, respectively. The test for difference based on a Student’s ‘t’ test yields $t = 28.8$ with 116 degrees of freedom, which is far beyond the 99.5% significance level.

Section 3.1: Discussion

Sampling regions where small scale activity is more active leads to a flatter variance spectrum (smaller scaling exponent) in all of the five decomposition methods relative to sampling regions where the small scales are less active. The significance of the differences based on the Student’s ‘t’ test are in excess of 99.5% for each of the five decompositions. For the purpose of discussion, the samples corresponding to enhanced small scale activity are denoted $MASC$ and the samples that are less
active on small scales are denoted LASC. Over 5,000 MASC samples and over 7,000 LASC samples were captured corresponding to about 80% of the record. The average variance decompositions of the two sample sets were characterized by slopes in log-log coordinates that were different by an amount ranging from 0.1–0.2 within a specified range of length scales.

Though the structure function corresponds to a spectral window with a given shape (so the shape of the sampled variance spectrums might have been predicted) the 6th order structure function provided a description of the intermittency that would have been difficult to obtain by sampling the data directly in terms of spectra or variance quantities. The minimums and maximums in the 6th order structure function lead to sample sets whose variance spectrums were only modestly different. In this sense, the
6th order structure function described the intermittency quite different than the variance spectrums. From another point of view, even modest deviations from "Kolmogorov scaling" (where scaling relations are well defined) are rarely observed.

The actual value of a scaling exponent estimate depended on the method of decomposition. For example, the Fourier decomposition of the MASC (LASC) samples has a scaling exponent estimated to be 1.70 (1.84) where as the scaling exponent estimated from the polynomial decomposition of the MASC (LASC) samples is 1.41 (1.61). The differences in scaling exponent estimates between the two sample sets are more consistent.

The (50 m) samples were much smaller than the (~km) big eddies, so it is likely
that the samples contained in the two different sets were systematically positioned with respect to the big eddies. From the results of a related study (Mahrt and Howell, 1992), many of the MASC samples correspond to structures located immediately ahead of the big eddies. The region ahead of a large eddy structure appears to be associated with more active small scales. On the other hand, many of the LASC samples are located “in” the eddy, behind the leading edge represent eddy substructure (see fig. 21).

The spatial distribution of the samples is also related to the distance separating two adjacent samples. The samples are not uniformly spaced, though the fact there are about 30% more LASC samples than MASC samples implies the latter are more widely spaced. The average distance between MASC samples is 160 meters and the distances range from 50 to 500 meters. The average distance separating the LASC
samples is 110 meters where the distances range from 50 to 370 meters. *MASC* samples separated by more than 300 meters occurred 150 times while *LASC* samples were separated by more than 300 meters only 6 times. These numbers suggest that the *MASC* samples occurred much more intermittently than the *LASC* samples.

The probability density distributions of the distance separating samples along with the associated sample amplitudes, that is the mean value of the sixth order structure function associated with a sample, suggest that the greater distance separating two *MASC* samples was correlated with a larger amplitude upstream sample. In other words, if a *MASC* sample does not hit the sensor for an unusually long time then the chances of a large amplitude sample hitting the sensor increases. However, this result...
is statistically uncertain at this point, so it is left as a conjecture. An alternative study is required to properly quantify this hypothesis.

A sample from each sample set is shown in order to clarify the sampling method, describe the intermittency of the small scale variability, and show what the real data looks like. Two sequential MASC samples are separated by about 500 meters, which is the largest distance separating any two sequential MASC samples. The sample upstream (observed at the later time) was the largest amplitude MASC sample and is shown in figure 20.

The wind gust at the center of the MASC sample in figure 20 corresponds to a change in the wind speed of more than 7 m/s over four data intervals, so the
average rate of increase in the wind speed at the sensor is close to 30 m/s². The final burst corresponds to an increase in the wind speed from 12.4 m/s to 16.7 m/s in 1/16 of second implying an acceleration of about 70 m/s² on the time scale of the sampling interval. The sudden increase in the wind speed is followed by a more gradual decrease resulting in a typical “ramp” structure. There are two smaller size structures with about half the amplitude immediately downstream of the big gust.

About 100 km upstream or 2.5 hours after the explosive sample (figure 20) a LASC sample with very quiescent small scale variability was observed and is shown in figure 21. The mean wind speed of the quiescent LASC sample is significantly above average suggesting it may be associated with the interior of a “sweep” of high momentum air from aloft. The largest accelerations between data points in this
particular LASC sample is about 2 m/s² and the average acceleration is less than 1 m/s². This LASC sample is part of an eddy that is varying over a length scale considerably larger than the length of the sample. MASC samples like the one shown in figure 20 occur very intermittently. LASC samples like the one shown in figure 21 occurred more frequently. The rest of the samples have amplitudes somewhere between these two samples.

The distribution of other samples around the locations of the two specific samples shown in figures 20 & 21 describe how the regions of assumed rapid dissipation are concentrated. Small amplitude LASC samples are clustered close to the position corresponding the MASC sample shown in figure 20. On the other hand, the next large amplitude MASC sample (like the one shown in figure 20) after the position of the sample shown in figure 21 occurs about 1600 m upstream, that is, it was not observed until a couple of minutes later.

Section 3.2: Conclusion

There is no claim that any of the scaling exponent estimates made in this study are in some way "universal". The results of this study include observational evidence that the effect of large variability in the fine scale structure, as represented by the rapidly dissipating MASC samples, is to flatten the spectral slope or decrease the scaling exponent. This result is in agreement with theoretical results presented in Yakhot et al. (1989) and Yamazaki (1990). Other models have predicted a more rapid fall off of velocity variance with decreasing scale (Obukhov, 1962; Mandelbrot, 1974; Frisch et al., 1978). This tendency is consistent with the slowly dissipating LASC samples where large amplitude smaller scale motions occupy a smaller fraction
of the area. These predicted tendencies in the spectral slope were generally made for a (globally) averaged flow where as in this study the flow was partitioned.

Considering the continuous Fourier transform, the occurrence of singularities in the function being transformed acts to “whiten” the spectrum, that is, to flatten the spectral slope. A very smooth function on the other hand will have a Fourier spectrum that decays rapidly with decreasing scale. The recognition that high Reynolds number turbulence is associated with intermittently spaced dissipative events implies a more singular behavior or flatter slope. However, atmospheric turbulence is associated with a “mixed regime” (Obukhov, 1962) or is a “non-ideal flow” (Hunt and Vassilicos, 1991), so the precise departures from Kolmogorov scaling, though typically small, may vary considerably even when large ensembles are taken.

The fact that there is some inconsistency in the literature over the sign of the deviations from Kolmogorov scaling (in the case of velocity variance) is probably due to (1) different researchers analyze different types of “non-ideal” high Reynolds number isotropic turbulence, (2) the size of the departures with respect to sampling problems and (3) the different ways of representing scaling exponents. For example, Anselmet et al. (1984) estimates a scaling exponent for the second order structure function to be 0.71 for three different laboratory flows, which is closer to the current estimate for the LASC samples. Other researchers focus more on direct numerical simulations of turbulence to estimate the so called “intermittency exponent” (Vincent and Meneguzzi, 1990; Hosokawa and Yamamoto, 1990). The different ways of representing scaling relationships include the use of multifractal spectrum (Meneveau and Sreenivasan, 1991), structure function statistics (Nelkin and Bell, 1978) and Fourier spectrums (Yakhot et al., 1989).
Kolmogorov scaling for idealized turbulence in an inertial range of length scales is based on Kolmogorov's second hypothesis (Kolmogorov, 1941), which basically states that very high Reynolds number isotropic turbulence results in a range of length scales over which velocity fluctuations only depend on the scale of the variations and the average rate of kinetic energy dissipation $\epsilon$. In this study, atmospheric wind data measured 45 meters above the surface yield global spectrum results in 1.2.3 which suggest that the motions within an assumed inertial range are close to isotropic. However, the turbulence is subject to large (outer) scale forcing, so the assumptions that Kolmogorov's second hypothesis is not completely satisfied. Although these assumptions are probably never completely satisfied in geophysical turbulence, the deviations from Kolmogorov scaling are found to be small even in this mixed regime (Obukhov, 1962).

Deviations from Kolmogorov scaling are in part due to the large variations of the rate at which kinetic energy is dissipated. To study the influence of these variations an 18 hour segment of longitudinal wind data is sampled according to the occurrence of more or less active small scales assumed to be related to the change in wind speed over 1/16 sec., which is the sampling rate. The rate of energy transfer toward small scales is measured in terms of the sixth order structure function at a minimum separation distance (1/16 sec.) averaged over 50 meter windows.

Windows containing a local maximum of the mean sixth order structure function are selected as more active small scale $MASC$ samples and local minima are associated with less active small scale $LASC$ samples. The $MASC$ samples have a flatter variance spectrum on average than the $LASC$ samples as computed in 5 different decompositions, 4 of them orthogonal. The difference in spectral slopes was on
the order of 10% and the actual differences were at the 99.5% significance level. Because the fine scale variability is greater in the MASC samples it is concluded that the corresponding effect on scaling relationships for spectra is to flatten the slope implying enhanced rates of energy transfer toward smaller scales. On the other hand, small scale variations in the LASC samples are smaller amplitude and less space filling and the corresponding spectrum has a steeper slope.

The sampling strategy was designed to yield this result, though if the spectrum did not systematically vary with the position, the sampling strategy would have failed. The use of the sixth order structure function in obtaining MASC samples provides a range of scales where a scaling relationship still hold (e.g. figure 19). Moreover, the scaling relations are found to hold for the different basis sets.

Understanding the relationship between small scale variability and the dissipation of kinetic energy should provide insight for the modeling of turbulent flows. A description of how the small scale variability is related to the distribution of velocity variance within the inertial range may help along these lines. This study demonstrates that regions in the flow associated with more active small scale variability have a (≈ 10%) flatter spectral slope and occupy (≈ 30%) less space than regions where the small scales are less active. The (99.5%) significance level in the difference between the two slopes implies that transfer rate of energy toward smaller scales varies considerably throughout the turbulent flow, which in turn must be related to variations in the rate at which turbulent kinetic energy is eventually dissipated.
Bibliography


