AN ABSTRACT OF THE DISSERTATION OF

Ryan W. Siegel for the degree of Doctor of Philosophy in Applied Economics, presented on June 12, 2012.

Title: Three Essays on Constrained Markets

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The three essays in this dissertation progressively answer the following questions: (a) How important are constraints? (b) Who benefits from removing constraints? (c) When does a constraint for a single market predominantly affect closely related markets? These questions are applied in the context of time, weather, and minimum wage constraints, respectively.

The first essay demonstrates that constraints matter. A data envelopment analysis capacity utilization methodology is used to measure impacts on sales from a sequential relaxation of the time and income constraints. Using a subsample bootstrap to estimate confidence intervals, results show that time matters more than income, particularly in fall and winter when other activities compete for gardening time.

The second essay shows that the poor are least likely to gain from the relaxation of non-income constraints. A theory of demand is developed in which consumers face multiple constraints. Then, a structural model is used to econometrically estimate the effect of global warming on demand, using nursery data on flowering plants. The model shows that there exists a tipping point around 64 degrees Fahrenheit, above which demand ceases to be climate-constrained.

The third essay shows that a constraint in a single market can sometimes have more profound consequences on other, more distantly related markets. First, it is proven that if a series of markets are structured like a chain— where only own and neighboring prices matter—then a shock to one market decreases with distance. The case of minimum wages in Oregon is investigated using a large panel dataset for all workers in Oregon using a first difference econometric model. It is determined that the ripple effects of the minimum have even larger effects on higher-wage earners, disconfirming the chain pattern. High substitutions between low and high wage groups may explain the pattern.

Altogether the essays further the understanding of constraints to demonstrate that (a) constraints significantly affect economic outcomes, (b) if one constraint is lifted, those individuals alternately-constrained are left behind from any benefits, and (c) constraints to a single market may have unintended and sometimes larger effects on 'farther' markets.

Three Essays on Constrained Markets

by

Ryan W. Siegel

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APPROVED:

Major Professor, representing Applied Economics

Director of the Applied Economics Graduate Program

Dean of the Graduate School

I understand that my dissertation will become part of the permanent collection of Oregon State University Libraries. My signature below authorizes release of my dissertation to any reader upon request.

Ryan W. Siegel, Author

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CONTRIBUTION OF AUTHORS

Dr. Robin Cross contributed to the research and writing of the first and second manuscripts, “Measuring and Comparing the Effects of Demand Constraints on Welfare” (Chapter 3) and “Temperature’s Tipping Point and the Effect of Climate Change on Demand for Plant Nursery Goods” (Chapter 4). Dr. Steve Buccola contributed to the research and writing of the third manuscript “Exogenous Shocks and Progressively Decreasing Effects: Sufficient Conditions and an Examination of Minimum Wage Ripple Effects in Oregon.” Dr. Rolf Färe and Dr. Shawna Grosskopf assisted in the theoretical framework and reviewed of the first essay.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF APPENDIX FIGURES

LIST OF APPENDIX TABLES

THREE ESSAYS ON CONSTRAINED MARKETS

CHAPTER 1 - INTRODUCTION AND OVERVIEW

Hermann Gossen (1854/1983) described consumers as maximizing utility subject to their earned income constraint yielding closed-form demand. Consumers face other constraint endowments, however, such as time, weather, and institutional factors. And these constraints may have a significant bearing on economic outcomes. Time limits many consumption activities such as gardening or entertainment. Cold weather, for example, may disallow certain activities, such as outdoor gardening. Minimum wages significantly affect wages for particularly low-skilled workers.

Economics has contributed a great deal to the understanding of the importance of constraints. The Samuelson-LeChatelier principle, for example, has been variously proven to show that additional constraints reduce demand's own-price elasticity. Yet, important questions remain. The three essays in this dissertation progressively answer the following questions: (a) How important are constraints? (b) Who benefits from removing constraints? (c) When does a constraint or law for a single market predominantly affect closely related markets? These questions are applied in the context of time, weather, and minimum wage constraints, respectively.

The first essay demonstrates that constraints matter. A data envelopment analysis capacity utilization methodology is used to measure impacts on sales from a sequential relaxation of the time and income constraints. Using scanner data for fifteen nursery products held at hundreds of stores in the western United States, the study shows that time constrains economic activity more than income particularly during fall and winter months when other activities compete for gardening time. Confidence intervals are generated using a nonparametric subsample bootstrap and suggest results are robust.

The second essay shows that the poor are least likely to gain from the relaxation of other constraints. First, a demand model is motivated in which consumers are income constrained until they meet a climatic constraint, a “tipping point,” where they exit the classical model and move along a new, less price-elastic climatic demand curve—reminiscent of the Samuelson-LeChatelier Principle. Using scanner data for flowering plants, the study shows that global warming stimulates demand when temperatures are below 64 degrees Fahrenheit. However, consumers making less than $36 thousand are penalized due to higher prices.

The third essay shows that a constraint in a single market can sometimes have more profound consequences on other, more distantly related markets. First, it is proven that if a series of markets are structured like a chain— where only own and neighboring prices matter—then a shock to one market decreases with distance. The case of minimum wages in Oregon is investigated using a large panel dataset for all workers in Oregon using a first difference econometric model. It is determined that the ripple effects of the minimum have even larger effects on higher-wage earners, disconfirming the chain pattern. High substitutions between low and high wage groups may explain the pattern.

Altogether the essays further the understanding of constraints to demonstrate that (a) constraints significantly affect economic outcomes, (b) if one constraint is lifted, those alternately-constrained are left behind from any benefits, and (c) constraints to a single market may have unintended and sometimes larger effects on 'farther' markets.

CHAPTER 2 - MEASURING AND COMPARING THE EFFECTS OF DEMAND CONSTRAINTS ON WELFARE

2.1. Introduction

Hermann Gossen (1854/1983) described consumers as maximizing utility subject to their earned income constraint. Economists have, since then, continued the tradition. Consumers face other constraint endowments, however, such as time, weather, and institutional factors. But how important are they compared to income? If binding, additional constraints have implications for demand estimation and inventory management. In this paper, we separate and compare the impact of time and income on demand for nursery plants, and we find time to be the more binding constraint, paralleling recent findings on the determinants of a nutritious diet (Davis and You 2011). Especially during fall and winter, when other activities compete with gardening, time is the dominant constraint on economic activity.

Economists began exploring demand under two or more constraints in response to wartime rationing (for reviews see Haneman 2004; Jackson 1991; Tobin 1952). Time constraints have also been specifically investigated. Becker (1965) elaborated a theory of household production under time and income constraints, but uses a time-price to reduce the problem to a single ‘full-income’ constraint and ignores firms. DeSerpa (1971) and Steedman (2001) model utility maximization subject to two constraints, but, like Becker, neglect discussing firms.

Samuelson sheds light on both demand and equilibrium. He proposed a model of demand under multiple, irreducible constraints (1947/1984, pp 163-171) and showed that a constraint endowment α affects equilibrium quantity q_i for good i according to the ratio of determinants

(2.1) $$\partial q_i / \partial \alpha = |\mathbf{A}_i| / |\mathbf{A}|,$$

where $\mathbf{A}$ is the excess demand Jacobian and $\mathbf{A}_i$ is the same matrix with its i^{th} column replaced with partial derivatives of inverse-demands with respect to the constraint (ibid. p 259). Closed form solutions for (2.1) are unlikely to exist (ibid.).

Färe et al. (1989) provide a nonparametric capacity utilization model to estimate output loss from a constraint. Their model is based on Johansen's (1968) concept of production capacity, i.e., the maximum producible output given an unrestricted variable input. Comparing a firm's distance to the technology frontier with, and then without, a variable input, while maintaining all other inputs fixed, indicates how restrictive the variable input is to production[1]. By sequentially relaxing variable input constraints, frontier losses attributable to each input can be measured using Data Envelopment Analysis (DEA).

In the next section, we elaborate use of the Färe et al. (1989) capacity utilization model as a non-parametric estimate of the effect of people's time and income limitations on their equilibrium demand. We then introduce datasets on time spent gardening, personal incomes, and plant sales data in hundreds of garden centers across the United States. In section 2.3, we estimate the relative impacts of time and income and test for significant differences using a subsample bootstrap by region, product type, and quarter. The paper concludes with summary findings and recommendations for further empirical and theoretical research.

2.2. Model

Denote the output vector $\mathbf{y} \in \mathrm{R}_{+}^{M}$, where M is the number of outputs, and denote the input vector $\mathbf{x} \in \mathrm{R}_{+}^{N}$, where N is the number of inputs. Technology is defined as $S = \{(x, y) : x \text{ can produce } y\}$. Technology is assumed to satisfy standard conditions like convexity and free disposability of inputs and outputs. Following Färe and Grosskopf

[1] Such measurements are related to von Liebig's Law of the Minimum which states that growth is restricted by the scarcest resource available.

(2000), the directional distance function $\vec{D}_S(\cdot)$ is the distance from an observation to the production frontier, or

$$\text{(2.2)} \qquad \vec{D}_S\left(\mathbf{x},\mathbf{y};-\mathbf{g}_{\mathbf{x}},\mathbf{g}_{\mathbf{y}}\right)=\sup\left\{\beta:\left(\mathbf{x}-\beta\mathbf{g}_{\mathbf{x}},\mathbf{y}+\beta\mathbf{g}_{\mathbf{y}}\right)\in S\right\},$$

where $\mathbf{g_x}$ and $\mathbf{g_y}$ are direction vectors.

Welfare loss is measured as the difference between the constrained and unconstrained frontier. Here we restrict (2.2) to impose non-increasing returns to scale (NIRS) and the loss is measured in the 1's direction (setting directions to zero for those that violate non-jointness). We estimate the distance function (2.2) using Färe and Grosskopf's (2000) linear programming problem adapted for NIRS[2] under the required maximum feasibility constraints (Färe et al. 2008). Time and income are treated as nondiscretionary, since we measure in the output direction only (Thanassoulis, Portela and Despic 2008 p 345).

We denote the set of input vectors *without* time (T) or income (I), i.e., free from the time or income constraint, as $\mathbf{x}_T \in \mathrm{R}_+^{\mathrm{N}-1}$ or $\mathbf{x}_I \in \mathrm{R}_+^{\mathrm{N}-1}$, respectively. Distance functions given unrestricted time or income are denoted $\hat{D}_S\left(\mathbf{x}_j,\mathbf{y};-\mathbf{g}_{\mathbf{x}_j},\mathbf{g}_{\mathbf{y}}\right)$, where constraint j is time or income. A superscript k is added to indicate that an input or output vector belongs to the k^{th} firm. Since outputs are sales, we construct a proxy for time or income related welfare loss as

$$\text{(2.3)} \qquad L_j=\frac{\sum_k\left(\hat{D}_S\left(\mathbf{x}_j^k,\mathbf{y}^k;\mathbf{0},\mathbf{1}\right)-\vec{D}_S\left(\mathbf{x}^k,\mathbf{y}^k;\mathbf{0},\mathbf{1}\right)\right)}{\sum_k \mathbf{y}^k\mathbf{1}},$$

where the denominator of (2.3) is the sum of sales in the industry ($\mathbf{1}$ is a conforming vector of ones) and the numerator is a measure of the loss of sales due to time or income. The latter is positive (since $\hat{D}_S(\cdot) \geq \vec{D}_S(\cdot)$ by Färe, Wang, & Seavert, 2010), and less

[2] We formulate NIRS by adding an observation at the origin and restricting intensity variables to sum to one to reduce computer processing time.

than total loss in sales, so impacts are understated[3]. Confidence intervals for (2.3) are constructed using the subsample bootstrap procedure provided by Simar and Zelenyuk (2007), which we adapt for the directional case[4], using 50 permutations and subsamples sized z^r where $r = 0.9$, and z is the sample size. We then test whether time (T) has a greater effect on sales than income (I). The null hypothesis is that income has a greater effect on sales loss than time, or

(2.4) $$H_0 : L_T - L_I \leq 0 .$$

2.3. Data

We define the decision making unit (DMU) as a single community which converts inputs (time, income, population density, and inventories) into a series of outputs (sales) in a given week. Inventory and sales data come from 2009 scanner data (Green Market Systems 2010) for 15 products in 193 stores in the western United States and are grouped into five categories: Annuals, Azaleas, Cypress trees, Succulents, and Vegetables (see Table 2.1). Products are aggregated by category to reduce dimensionality.

Table 2.1 – Product names and categories

Product Category	Product Size/Name
Annuals	6pk Annuals
Annuals	1.25 qt Annual assorted
Azalea, flower	2.5 qt Azalea assorted
Azalea, flower	2.25 gal Azalea assorted
Cypress trees	2.25 gal Cypress leyland
Cypress trees	3.25 gal Arborvitae emerald green
Cypress trees	2.5 qt Arborvitae emerald green
Succulent, drought	4 oz Succulent assorted
Succulent, drought	13 oz Succulent assorted
Vegetables	2.5 qt Vegetables
Vegetables	1.0 gal Vegetable/herb planter
Vegetables	1.25 qt Vegetable/herbs
Vegetables	2.5 qt Rosemary
Vegetables	1.0 pt Strawberry
Vegetables	1.0 pt Herb assorted

[3] The Cartesian distance between any two points is less than the sum of the change in each axis.

[4] In the directional case, no weights are needed, and we sum firm's revenue efficiencies.

Use of scanner data has its limitations. Customer returns introduce downward bias in recorded sales and upward bias in inventories, since returned plants are rarely returned to the shelf. In addition, not all inventory is recorded when received, biasing recorded inventories downward. Records with negative sales volumes (i.e., returns exceed purchases) are precluded from the analysis.

The Bureau of Labor Statistics (BLS) provides income and time data. Income data is collected under the Quarterly Census of Employment and Wages Program at the quarterly, county level (BLS 2011a). Time data is derived from the American Time Use Survey (ATUS), which tabulates thousands of time use diaries for individuals participating in the Current Population Survey (BLS 2011b). We aggregate time entries to the state-month level using ATUS population weights. Population density is provided by the US Census Bureau (2010). In our sample, population density ranges from 7 to 5250 people per square kilometer and averages about 926.

Figure 2.1 illustrates monthly time spent gardening and shows that spring months are busiest. To control for climactic variation, we construct temperature/precipitation regions using NOAA climate maps (NOAA 2010). We split regions by precipitation (mean/yearly total precipitation in inches) and temperature (mean daily average temperature). Figure 2.2, below, shows the stores in the final dataset and their placement in the western United States.

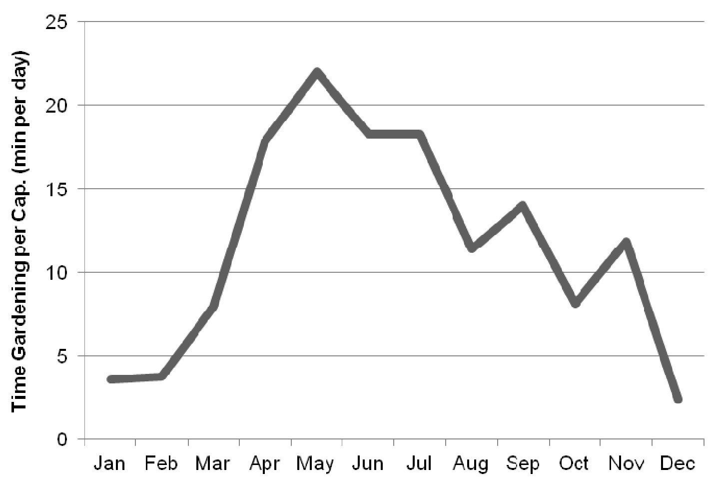

Figure 2.1 - United States mean time gardening per capita in 2009

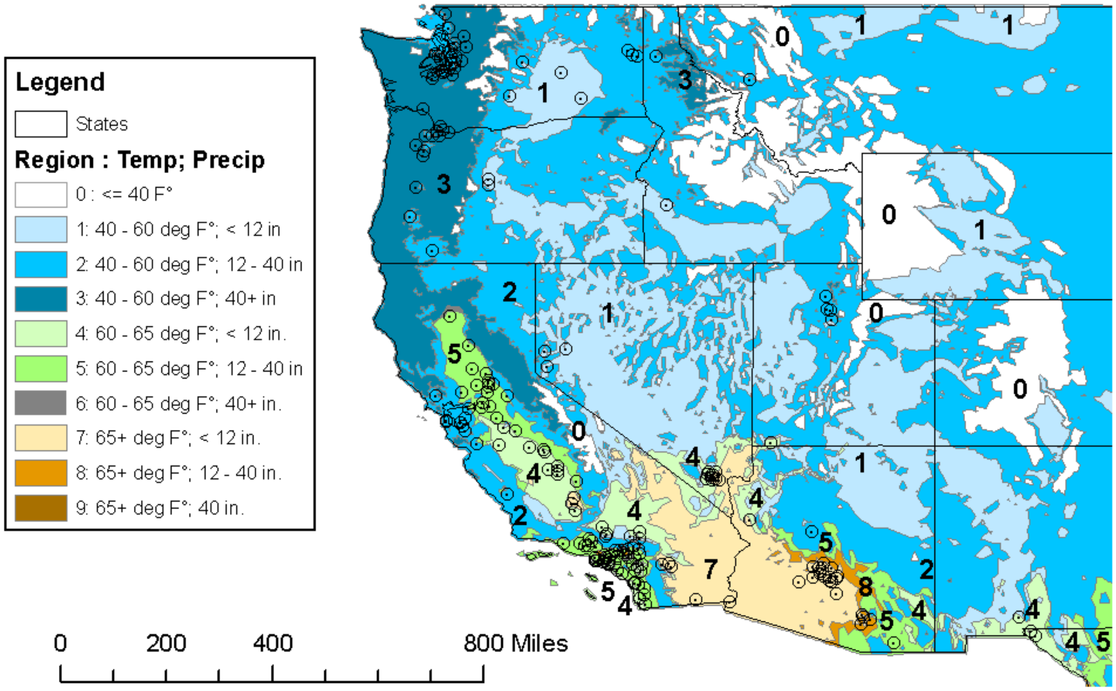

Figure 2.2 - Stores and temperature/precipitation regions

Analysis is conducted within seven temperature/precipitation regions; each with 1109 observations (community/weeks) on average. (Table 2.2 provides summary statistics on inputs and outputs used by region[5].) Within most regions, garden time and population density have coefficients of variation greater than 0.7 while income's ranges between 0.1 and 0.3.

[5] Summary statistics on inventories have been excluded due to confidentiality.

Table 2.2 - Summary statistics for inputs and outputs

Region	1	2	3	4	5	7	8
Total Observations (store/weeks)	623	1986	1139	1456	2912	1712	156
Inputs, mean (standard deviation)							
Garden Time Per Capita (minutes per day)	13.2 (18.6)	13.1 (17.4)	17.5 (21.9)	9.5 (7)	10 (5.2)	7.3 (8.2)	7.7 (6.8)
Earnings Per Capita (USD per week)	704 (86)	954 (271)	834 (115)	759 (134)	870 (128)	807 (80)	758 (36)
Population Density (persons per sq km)	256.2 (404.9)	989.9 (824)	618.7 (470)	875.4 (927.2)	1146.1 (1129.5)	1005.7 (890.7)	772.1 (669.8)
Outputs: Sales Units, % > 0							
Vegetables	43%	48%	30%	96%	99%	93%	97%
Succulents	67%	74%	67%	87%	95%	90%	96%
Azaleas	20%	53%	56%	57%	80%	22%	44%
Annuals	64%	70%	57%	99%	100%	98%	100%
Cypress	52%	60%	84%	13%	15%	3%	0%

2.4. Results

Percent welfare-loss from time averages 1.06%, i.e., the industry lost 1.06% in welfare due to people's limited time[6], as shown in Table 2.3.Welfare loss from the income constraint averages only about 0.1%. Income has stronger effects in warmer and wetter areas (0.16% to 0.2%). Time has the greatest impact in cool and hot/dry areas (regions 1, 2, 3, and 7). Altogether, time reduces welfare more than income by 0.96% overall and is significantly greater than income in all regions at the 95% confidence level.

[6] The careful reader will note that we are measuring differences of the frontiers, not actual sales, per se.

Table 2.3 - Percent of industry revenue lost due to time and income

	Region 1	Region 2	Region 3	Region 4	Region 5	Region 7	Region 8
Avg. Subsample Size	237	774	462	627	1194	743	83
Time (L_T)	1.43 (1.24, 1.63)	1.95 (1.63, 2.26)	1.46 (1.1, 1.85)	0.5 (0.35, 0.63)	0.67 (0.35, 1.07)	2.88 (2.5, 3.39)	0.5 (0.39, 0.61)
Income (L_I)	0.03 (0.02, 0.03)	0.09 (0.03, 0.12)	0.2 (0.12, 0.27)	0.12 (0.09, 0.14)	0.1 (0.06, 0.14)	0.16 (0.11, 0.19)	0.02 (0, 0.03)
Difference (L_T - L_I)	1.41 (1.21, 1.6)	1.87 (1.54, 2.2)	1.26 (0.92, 1.63)	0.38 (0.24, 0.51)	0.57 (0.24, 0.96)	2.73 (2.35, 3.24)	0.48 (0.37, 0.59)

Table presents sample estimate and 95% confidence intervals

Table 2.4 summarizes results for the difference between percent revenue loss from time over income (L_T - L_I) by region *and* product category[7]. Results suggest that time has a greater effect than income across most product types for nearly all regions.

Table 2.4 - Percent sales lost due to time over income (L_T-L_I) by region and category

	Region 1	Region 2	Region 3	Region 4	Region 5	Region 7	Region 8
All	**1.41 (1.21, 1.6)** m = 237	**1.87 (1.54, 2.2)** m = 774	**1.26 (0.92, 1.63)** m = 461	**0.38 (0.24, 0.51)** m = 627	**0.57 (0.24, 0.96)** m = 1195	**2.73 (2.35, 3.24)** m = 743	**0.48 (0.37, 0.59)** m = 84
Vegetables	**20.38 (14.07, 27.81)** m = 237	**34.21 (21.23, 47.75)** m = 774	**26.35 (20.32, 31.71)** m = 461	**19.17 (10.15, 25.53)** m = 628	**48.56 (23.77, 65.85)** m = 1195	**65.06 (53.81, 77.68)** m = 743	**16.22 (12.14, 20.6)** m = 84
Succulents	**18.32 (13.65, 23.03)** m = 237	**15.06 (7.23, 24.18)** m = 773	**13.25 (5.61, 21.4)** m = 461	**12.83 (5.45, 20.74)** m = 628	**27.56 (8.71, 44.83)** m = 1195	**60.61 (49.56, 78.12)** m = 743	**15.9 (12.7, 19.53)** m = 84
Azaleas	**8.58 (6.83, 10.33)** m = 237	**48.92 (38.44, 60.15)** m = 773	**13.31 (7.95, 18.07)** m = 460	**3.93 (0.21, 7)** m = 628	0.43 (-10.54, 8.37) m = 1195	**24.27 (16.55, 32.13)** m = 743	**6.91 (2.42, 11.51)** m = 84
Annuals	**8.81 (5.76, 12.14)** m = 237	**48.27 (35.84, 60.04)** m = 774	**18.28 (14.11, 22.4)** m = 460	**7.61 (3.48, 11.05)** m = 628	**26.44 (16.4, 35.29)** m = 1195	**73.79 (64.67, 84.73)** m = 743	**9.79 (5.92, 13.29)** m = 84
Cypress	**32.14 (28.59, 36.09)** m = 237	**28.68 (20.86, 35.66)** m = 774	**23.14 (15.19, 31.47)** m = 461	**14.37 (3.32, 21.7)** m = 627	**18.54 (10.69, 25.15)** m = 1195	**9.18 (5.37, 12.96)** m = 743	n.d. m = 84

Table presents sample estimates and 95% confidence intervals; boldface indicates significantly positive; underline indicates significantly negative; m indicates average subsample size after removing firms with no production in the relevant product category; n.d. indicates no data

[7] To control for product category, the distance function was measured in each product category direction. In order to avoid null results, store/weeks with no sales in the product category are excluded from the analysis.

Examining the difference measure by quarter reveals pronounced heterogeneity. Table 2.5 shows that time is a more significant constraint to sales than income throughout the year and across climates. In the second and third quarters, however, regions are less time sensitive. These results suggest seasonality in the time constraint.

Table 2.5 - Percent sales lost due to time over income (*L_T-L_I*) by region and quarter

	Region 1	Region 2	Region 3	Region 4	Region 5	Region 7	Region 8
All	**1.41 (1.21, 1.6)** m = 237	**1.87 (1.54, 2.2)** m = 774	**1.26 (0.92, 1.63)** m = 461	**0.38 (0.24, 0.51)** m = 627	**0.57 (0.24, 0.96)** m = 1195	**2.73 (2.35, 3.24)** m = 743	**0.48 (0.37, 0.59)** m = 84
Qtr 1	**4.91 (3.69, 6.21)** m = 70	**4.21 (2.95, 5.47)** m = 222	**4.48 (2.5, 6.68)** m = 131	**0.53 (0.24, 0.73)** m = 185	0.27 (-0.29, 0.82) m = 361	**0.39 (0.31, 0.47)** m = 223	**0.26 (0.17, 0.36)** m = 26
Qtr 2	**0.06 (0.04, 0.08)** m = 78	**0.07 (0.03, 0.12)** m = 220	**0.05 (0.01, 0.1)** m = 138	**0.08 (0.03, 0.13)** m = 182	-0.03 (-0.06, 0) m = 324	**0.53 (0.4, 0.66)** m = 212	**0.18 (0.09, 0.27)** m = 24
Qtr 3	**1.47 (1.22, 1.73)** m = 74	**0.66 (0.52, 0.78)** m = 229	**1.24 (0.94, 1.48)** m = 125	0.03 (-0.04, 0.08) m = 172	**0.21 (0.09, 0.31)** m = 327	**2.19 (1.36, 2.89)** m = 197	0 (-0.05, 0.05) m = 22
Qtr 4	**1.36 (0.06, 2.27)** m = 55	**4.56 (3.3, 5.88)** m = 215	**9.5 (6.99, 12.2)** m = 134	**0.55 (0.17, 0.82)** m = 184	**1.36 (0.93, 1.75)** m = 360	**6.68 (5.83, 7.73)** m = 221	**0.87 (0.67, 1.09)** m = 25

Table presents bootstrapped mean, 95% confidence intervals, and average subsample size; boldface indicates significantly positive; m indicates average subsample size after filtering for feasibility conditions

To test sensitivity to alternative data aggregation specifications, we re-aggregate data to the store-month level and find that it reduces magnitudes, but does not change significance levels materially. Under variable returns to scale, time has a greater effect than income in only four out of the seven regions while region 8 is, conversely, income sensitive.

2.5. Conclusions

This paper extends Färe, *et al.*'s capacity utilization measure to an equilibrium framework to measure welfare loss due to demand constraints.

We find that time is often a greater constraint than the classical budget constraint. This suggests that firms should consider people's non-monetary constraints, and should limit price specials to geographic regions and times of the year when income is the limiting factor. In wealthier communities, especially, expanded store hours or time saving features may do more than pricing to raise sales.

Empirically, it would be useful to study the same question but with greater income heterogeneity as well as in other markets. Analysis of constraint endowments other than time such as weather would be of interest. Researchers may also examine the interaction of financial and non-financial policy instruments (constraints) in shaping human behavior and find optimal arrangements in terms of timing and geography.

Theoretical research is needed to prove and parameterize welfare loss from non-economic constraints. Methods for incorporating multiple constraints in demand specifications should be investigated, especially as it pertains to issues of aggregation and econometric estimation. Results suggest that constraints fluctuate over time, which may present issues in econometric demand estimation.

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CHAPTER 3 - TEMPERATURE'S TIPPING POINT AND THE EFFECT OF CLIMATE CHANGE ON DEMAND FOR PLANT NURSERY GOODS

3.1. Introduction

We fear a warmer world. Global nutrition is expected to fall (Nelson et al. 2009), incomes decline (Dell et al. 2009), and mortality increase (Patz et al. 2005). Yet amid these fears, many harbor some guilty hope: benefits from tourism (Lise & Tol 2002), access to North Atlantic oil resources[8], or wines from a new Bordeaux[9]. Lise and Tol (2002) find that tourism peaks near 70 degrees Fahrenheit and that global warming will modify visiting patterns accordingly.

Accidental beneficiaries must satisfy an *ex ante* resource paradox: wealthy enough to capture new opportunities, but constrained by climate. The world's poor are the least likely to benefit (Mendelsohn, Dinar, & Williams 2006) with poorer populations concentrated in already warm climates (Dell et al. 2009).

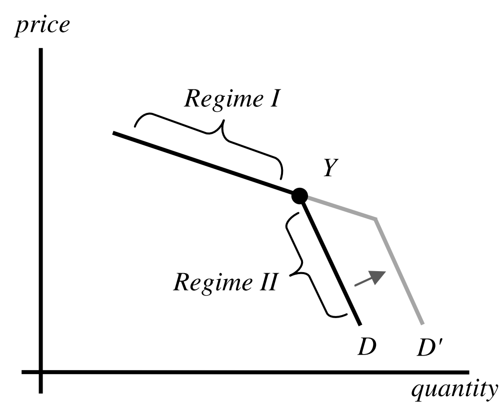

Figure 3.1 - Demand with a regime switch due to temperature.

D and D' denote demand before and after a temperature increase, respectively.

Figure 3.1 shows how consumers move down the classical demand curve (regime I) as they move along their income constraints, until they reach a climatic constraint, a

[8] See, for example, the article "Warming 'opens Northwest Passage'" on the BBC website: http://news.bbc.co.uk/2/hi/americas/6995999.stm

[9] See for example, the article "New world wines: now from the north." On the GlobalPost website: http://www.globalpost.com/dispatch/belgium/100126/winemaking-global-warming-terroir

"tipping point," (point *Y*) where they exit the classical model and move along a new, less price-elastic climatic demand curve (*Regime II*). Warmer climates shift the demand curve upwards (to *D'*) where temperature matters. Poorer people remain constrained by income, such as *Regime I,* and are prevented from the benefits of warmer weather.

This paper sheds light on how climate and income affect consumer demand and welfare in two ways. First, we find new evidence for the Lise and Tol 70-degree tipping point by examining scanner data, geocoded to integrate local climactic conditions, in a sector where the two constraints strongly interact: demand for flowering plants. However, we show that lack of income prevents most consumers from taking advantage of higher temperatures. In fact, as demand elasticity falls with temperature and price rises, consumers with household incomes of less than $36 thousand are penalized, unable to maintain existing consumption rates.

Second, we formalize the *ex ante* resource paradox to understand its implications for demand. Harkening back to the 1800s, Hermann Gossen (1854/1983) introduced the income constraint yielding closed-form statics. Samuelson (1960) showed that additional constraints reduce own-price elasticity (known as "Samuelson-LeChatelier effects") in equilibrium, and Hatta (1980) showed that they reduce welfare[10].

Stimulated by wartime rationing, Worswick (1944) graphically explored how demand and welfare depend on the interaction of constraints. Samuelson (1947/1983) setup a consumer problem with multiple inequality constraints. Becker (1965) motivated a household production framework in which time and income equality constraints are collapsed to a single constraint via a labor-leisure tradeoff. Larson and Shaikh (2001) show that if demand is affected by a separate constraint, then the latter's prices and budgets should be included in the empirical demand specification. Moffitt (1985) estimated demand under piecewise linear budget constraints, or inequality constraints, via an additive heterogeneity term.

In the next section we derive a theory of demand under multiple inequality constraints with stochastic preferences. The theory explains how poor consumers are

[10] See Appendix A for proof

precluded from the benefit of relaxed constraint endowments such as temperature. We then develop an econometric model for the important green nursery goods market. Results and conclusion follow.

3.2. Theory

Assume a representative consumer maximizes utility function $U(\cdot)$ from a vector of goods $\mathbf{x} \in \mathrm{R}^N$ subject to stochastic preferences $\boldsymbol{\alpha} \in \mathrm{R}^K$ and a vector of constraint functions $\mathbf{h}(\mathbf{x}) \in \mathrm{R}^M$. Her maximization problem is

$$\max_{\mathbf{x}} U(\mathbf{x};\boldsymbol{\alpha}) : \mathbf{h}(\mathbf{x}) \leq \mathbf{0} \ , \tag{3.1}$$

where $\mathbf{0}$ is a conformable vector of zeros.

By monotonicity, demand from problem (3.1) holds under some combination of constraints holding with equality. Equivalently, demand $\mathbf{x}^*$ is the sum of all possible conditional demands weighted by an indicator function which equals one when the conditioned set of constraints holds with equality. Expected demand is

$$E\left(\mathbf{x}^*\right) = \sum_{i=1,\ldots,Q} P_i \overline{\mathbf{x}}_i \ , \tag{3.2}$$

where $\overline{\mathbf{x}}_i$ is expected demand given that the i^{th} set of constraints hold with equality, P_i is the probability that the same set of constraints hold with equality, and Q is the total number of feasible combinations of constraints holding with equality.

The proof uses the law of iterated expectations and depends on the fact that the indicator functions and conditional demands are functions of the stochastic preference parameter (see Appendix A). The stochastic parameter can be interpreted as heterogeneity in the population. While two individuals may face identical constraints, individual preferences determine which constraints are binding.

3.3. Model

For our empirical application we assume the consumer faces two constraint endowments: temperature *T*, and a share $g \in (0,1]$ of her income *I*. In this case, (3.1) reduces to

$$\max_{x,m} U(x,m;\alpha): px + m \leq gI; rx \leq T \,, \tag{3.3}$$

where *m* is a numeraire, *x* is a scalar representing the good in question, *p* is its price, *r* is its temperature-price[11], and *α* is a stochastic preference term.

Figure 3.2 illustrates problem (3.3). In *Regime I*, prices are sufficiently high so that the choice set is unaffected by temperature. The two constraints do not intersect and demand conditional on an income constraint holds with a probability of one. Like the classical demand model, utility maximization yields a solution such as point A. As prices lower the temperature constraint cuts through the choice set to look like *Regime II*. Individuals with strong preferences for good *x* are both temperature and income constrained and demand is at point *C* with probability P_c. Alternately, demand is solely constrained by income with probability $1\text{-}P_c$ and resides on the diagonal segment to the left of C including *B*. The tipping point is where temperature ceases to influence demand, or where $gI / p = T / r$.

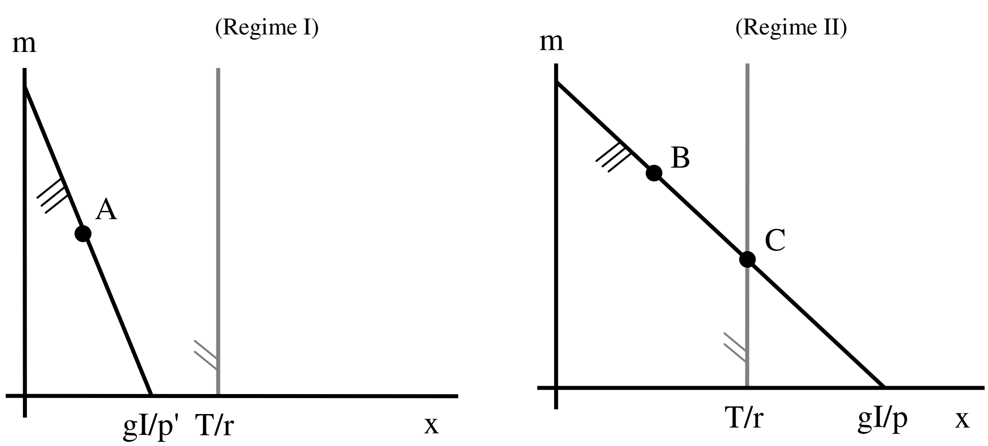

Figure 3.2 - Consumer's choice set under two constraints

[11] When the unit of analysis for temperature is weekly mean temperature, as it is in this study's empirical section, *r* may be interpreted as the average week-temperature requirement per unit of a good.

Assume utility is homothetic, Cobb-Douglas with a uniformly distributed[12] random preference parameter $\alpha \in (0,1)$ and let stores' average consumer base be of size *n*. Equation (3.2) for problem (3.3) may be econometrically specified[13] at the store *s* week *t* level of analysis as

$$nx_{st} = E\left(nx_{st}\right) + \varepsilon_{st} = \sum_{i=1}^{3} \gamma_i T_{st}^{i} \left(p_{st} / I_s\right)^{i-1} + \varepsilon_{st} \tag{3.4}$$

where ε_{st} is an error term assumed to be normally distributed, and the unknown coefficients $\gamma_1, \gamma_2, \gamma_3$ are nonlinearly constrained. Note that total sales, nx_{st}, as well as prices p_{st}, temperature T_{st} and income I_s are observed, while average consumer population *n*, income share *g*, and temperature price *r*, are not. Coefficient estimates $\hat{\gamma}_1, \hat{\gamma}_2, \hat{\gamma}_3$ are found using nonlinear least squares and are used to recover estimates of the interaction terms: *n/r*, *gr*, and *gn*. The latter are used to compute the temperature exit point, probabilities of the population being temperature-constrained, and elasticities[13]. For example, the probability that consumers are temperature-constrained is

$$P\left(\text{temperature-constrained}\right) = 1 + \frac{\gamma_2}{\gamma_1} \frac{T_{st} p_{st}}{I_{st}}. \tag{3.5}$$

3.4. Data

To estimate the effect of temperature on demand we use scanner data for 6 pack (flowering) Annuals across hundreds of stores in the United States (Green Market Systems, 2012). Data is aggregated to the store/week level so that each record indicates

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[12] The distribution may be generalized to a Kumaraswamy-distribution $F(\alpha) = 1 - (1-\alpha)^{\delta}$ for any $\delta \in \{1,2,3,...\}$. In this case equation (3) generalizes to $E[x] = \sum_{i=1,...,1+2\delta} \psi_i T^i \left(p / I\right)^{i-1}$ where $\psi_i = \psi_i(g, r, \delta)$. Robustness checks using $\delta=2$ and $\delta=3$ or triangular and curved distributions, respectively, are conducted. These progressively assume less evenly distributed preferences such that more consumers have weaker preferences for the good in question. Results using $\delta=2$ and $\delta=3$ yield confidence intervals including or at least overlapping those estimated under the inform case.

[13] See Appendix for a complete derivation. Note that the derivation assumes temperature is binding, i.e., $T/r \le gI/p$.

the total number of units sold and total revenues. Prices are imputed at the same level of analysis. The data is also paired with geographically-linked average weekly maximum and minimum temperature data from the National Oceanic and Atmospheric Administration. The simple mean is utilized as our temperature indicator. Local wage income data is available from the Bureau of Labor Statistics (BLS 2009) at the county/quarterly level.

Scanner data has limitations. Customer returns introduce downward bias in recorded sales and upward bias in inventories, since returned plants are rarely returned to the shelf. In addition, not all inventory is recorded when received, biasing recorded inventories downward. Records with negative sales volumes (i.e., returns exceed purchases) or negative inventories (unrecorded inventories) are precluded from the analysis.

Analysis is restricted to demand in February of 2009. February was selected for analysis since it is a time of year in which temperature is best modeled as a linear constraint, particularly for the large share of warm states in the sample (see Table 3.1). Results utilizing March data are qualitatively similar and suggest that temperature becomes a more important constraint endowment. In other months such as December or June, seasonal constraints predominate. An unusually warm week in the middle of winter, for example, is unlikely to stimulate outdoor gardening since the plants won't survive the following week.

Table 3.1 - Store/week observations for 6pk Annuals in February 2009 by state

	Quantity	Percent
Arkansas	5	0.4%
Arizona	101	8.5%
California	395	33.1%
Louisiana	55	4.6%
Missouri	3	0.3%
New Mexico	26	2.2%
Nevada	44	3.7%
Oklahoma	68	5.7%
Texas	491	41.2%
Utah	4	0.3%
Total	**1192**	**100.0%**

Table 3.2 provides summary statistics on the data. Stores, on average sold 439 units of 6-Pack Annuals per week in February of 2009 and consumers experienced a mean temperature of 61 degrees Fahrenheit. Mean income in our sample is $815 per week, or $3260 per month.

Table 3.2 - Summary statistics for 6-Pack Annuals in February 2009

	Observations	Mean	SD	Min	Max
Sales (units)	1168	436	511	0	3532
Temperature (°F)	1168	56	6.0	36	72
Income ($/week)	1168	815	191	480	1785
Price ($/unit)	1168	1.75	0.31	0.10	2.48

3.5. Results

Temperature matters for demand when it is below 64 degrees Fahrenheit or when incomes are above $707 per week (see Figure 3.3). The graphs in Figure 3.3 illustrate the fitted model. The top-two graphs show the tipping point with respect to temperature and income. At temperatures above 64 degrees, consumer demand is flat since only income matters for consumers, i.e., consumers enter regime I or the classical demand model. At income levels above $707 per week, returns to income decrease as more consumers are limited by temperature.

The lower-left graph in Figure 3.3 shows that demand is flatter at lower prices due to the temperature constraint. As prices increase, demand reaches a tipping point around $2 per unit where demand is no longer constrained by temperature and becomes steeper. This change in elasticity follows the Samuelson-LeChatelier principle (Samuelson, 1947/1983).

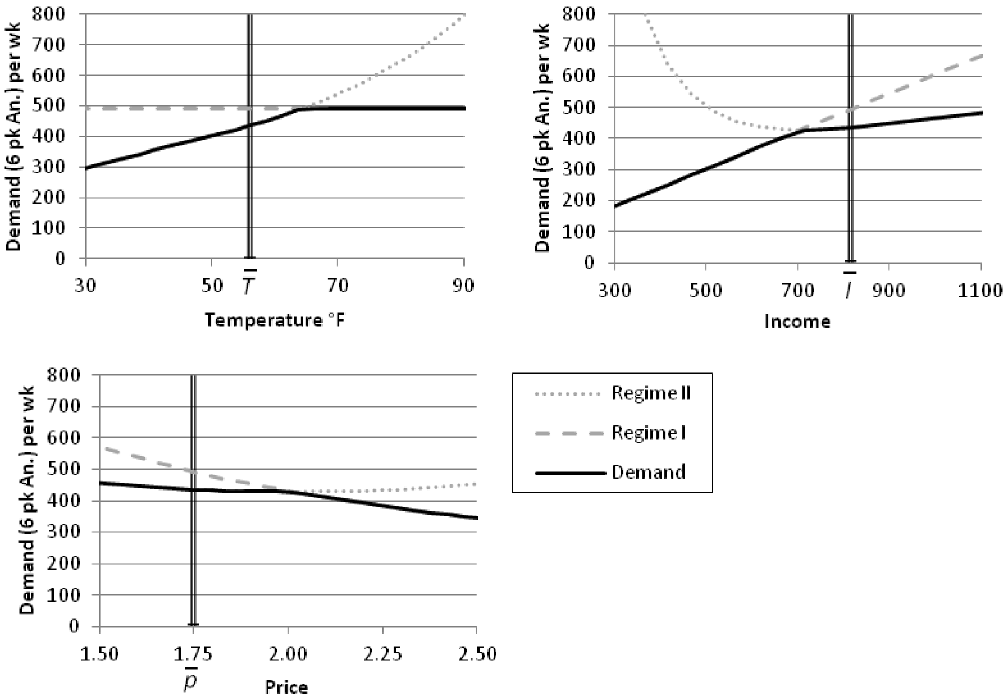

Figure 3.3 - Estimated demand across quantity, temperature, income, and price space

Vertical double-lines represent the data mean values.

Table 3.3 - Model (3.4) estimation results with confidence intervals

	Estimate	95% Confidence Interval	
Temperature tipping point (°F)	64.52	55.49	73.55
Income tipping point ($/week)	707.46	608.41	806.51
Probability income	0.87	0.75	0.99
Probability income and temperature	0.13	0.01	0.25
Price elasticity	-0.22	-0.40	-0.05
$\partial p/\partial T$ (fixed supply)	0.11	0.00	0.21
$\partial p/\partial T$ (unit elastic supply)	0.02	0.01	0.03

Estimates using parameters and variances estimated using nonlinear least squares. Data is restricted to 6-Pack (flowering) Annuals for February, 2009.

We find that the probability that individuals are constrained by their budget only is 87% whereas the probability that they are also temperature constrained is 13% (see

Table 3.3). The low probability that consumers are temperature constrained suggests a seasonality constraint. Indeed, for March, when seasonality effects likely diminish, the probability that consumers are temperature constrained is 44%.

A 1 degree Celsius (1.8 degree Fahrenheit) increase in temperature is estimated to increase prices by 2 to 11% depending on whether one assumes unit elastic or fixed supply (see Table 3.3). Figure 3.4 shows how an increase by 1 degree Celsius shifts demand and moves the tipping point to the left. Higher temperatures force demand to switch to regime I at a lower price.

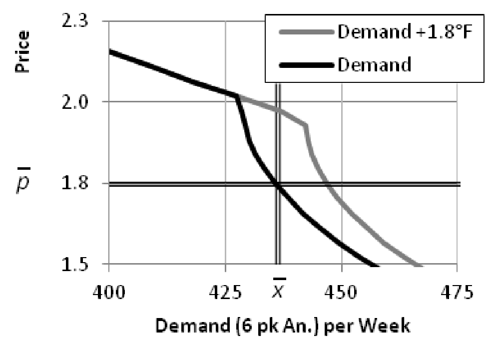

Figure 3.4 - Simulated change in demand due to temperature increase

Vertical and horizontal double-lines represent the data mean values.

3.6. Conclusion

To gain insights into how consumer demand is affected by climate change we recast classical demand theory to have two disposable constraint endowments: income and temperature. Our model shows how the effect of climate change on local demand depends on the interaction of these constraints. Temperature changes in poor communities will have no effect on demand and no benefits will accrue. In wealthy communities the population is likely to be constrained by temperature such that climate change is beneficial. Poorer individuals within such communities are disadvantaged due to higher prices.

Using plant nursery data for hundreds of stores in the United States for the month of February we find that temperatures below 64 degrees Fahrenheit affect demand. The illustration shows that there exist Samuelson-LeChatelier effects such that demand is less price-elastic when temperature constrains demand

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CHAPTER 4 - EXOGENOUS SHOCKS AND PROGRESSIVELY DECREASING EFFECTS: SUFFICIENT CONDITIONS AND AN EXAMINATION OF MINIMUM WAGE RIPPLE EFFECTS IN OREGON

4.1. Introduction

Effects of a shock are often believed to decay with distance. Labor economists have long examined the effects of minimum wages and found evidence matching such a claim. Empirical research in the early 90s found that increasing the US minimum wage spilled-over to raise wages for people just above the new minimum (Brown, 1999 pp 2147-2149). Others have shown these spillovers to be strongest on lower wage groups and weakest at higher levels (Neumark, Schweitzer, & Wascher, 2004 for the US; Maloney & Mendez, 2004 for Colombia). Minimum wages have also been found to explain wage inequality (DiNardo, Fortin & Lemieux, 1996), particularly among lower wage groups.

Other markets may exhibit the same phenomenon of progressively decreasing effects. Examples include quality/price-differentiated markets such as those for insurance, cars, or housing. A shock to the price of a low-quality insurance market may have smaller and smaller effects on higher-quality markets. Nature itself is replete with examples of this phenomenon. Dropping a stone in a pond provides the proverbial example of such "ripple" effects. Other tangible examples include heat diffusion in a large body of water or poison in an ecosystem.

However, progressively decreasing effects need not always hold in general equilibrium theory. Morishima, for example, describes conditions under which a shock to one market may have proportionately *larger* effects on other markets via complementarities (1952). And while some authors have provided conditions under which a shock to one market's price is larger than that on any other, no one has yet proven conditions for a progressively decreasing effect for the general, *n*-market case.

This paper contributes to an understanding of progressively decreasing effects in two ways. First, we review relevant results from the qualitative economics literature using excess demand Jacobians and prove that as long as markets have a chain-structure, where supply and demand are a function of only their own and two neighboring prices, the decreasing effect holds.

Second, we reexamine the literature on minimum wage spillovers and estimate these effects between 1998 and 2010 using a large panel dataset for the state of Oregon. We find that spillovers are positive across all wage groups and that they form a U shaped pattern. Spillovers decrease for wage groups close to the minimum wage and then increase for the highest wage groups. Strong substitution effects between low and high wage groups could explain the pronounced effects at the higher wage levels. Finally, we conclude.

4.2. Related Literature

Samuelson (1947/1983) posed the problem of determining the signs of comparative statics with knowledge of only the qualitative structure of an economy. To begin, Samuelson defined a general system at equilibrium as

$$\mathbf{f}(\mathbf{p};\boldsymbol{\alpha}) = 0$$

where $\mathbf{f} \in \mathrm{R}^N$ is vector of excess demand functions, $\mathbf{p} \in \mathrm{R}^N$ is a vector of endogenous price variables and $\boldsymbol{\alpha} \in \mathrm{R}^M$ vector of exogenous parameters. N is the total number of markets and M is the total number of exogenous parameters. The effect of parameters on the variables $\mathbf{p}$ can be found using the linear system

$$\sum_{j=1}^{N} \frac{\partial f_i}{\partial p_j} dp_j = -\sum_{k=1}^{M} \frac{\partial f_i}{\partial \alpha_k} d\alpha_k \quad \forall i = 1, \ldots, N.$$

Suppose that there is only *one* exogenous parameter of interest which affects the first market only so that the above equations reduce to

$$\sum_{j=1}^{N} \frac{\partial f_i}{\partial p_j} \frac{dp_j}{d\alpha_1} = \begin{cases} -\dfrac{\partial f_1}{\partial \alpha_1} & \text{if } i = 1 \\ 0 & \text{if } i = 2, \ldots, N \end{cases}, \tag{4.1}$$

or, equivalently,

$$\mathbf{Ax} = -\mathbf{b}$$

where **A** is the Jacobian $\mathbf{A} = \{a_{ij}\}_1^N = \{\partial f_i / \partial p_j\}_1^N$, $\mathbf{x} = \{dp_i / d\alpha_1\}_1^N$ and $\mathbf{b} = \{\partial f_1 / \partial \alpha_1 \quad 0 \quad \cdots \quad 0\}$. We assume throughout that the shock decreases excess demand, i.e., $\partial f_1 / \partial \alpha_1 < 0$. Note that since (4.1) assumes the exogenous parameter affects excess demand in the first market only, the vector **x** is decreasing if and only if the first column of $\mathbf{A}^{-1}$ is decreasing.

The Jacobian describes how excess demand responds to prices. For the *N=3* case

$$\mathbf{A} = \begin{bmatrix} \partial f_1/\partial p_1 & \partial f_1/\partial p_2 & \partial f_1/\partial p_3 \\ \partial f_2/\partial p_1 & \partial f_2/\partial p_2 & \partial f_2/\partial p_3 \\ \partial f_3/\partial p_1 & \partial f_3/\partial p_2 & \partial f_3/\partial p_3 \end{bmatrix}. \tag{4.2}$$

As the size of the Jacobian increased beyond *N=2*, the problem of determining the signs of **x** quickly became more complex and Samuelson recommended the use of additional restrictions, such as stability, to reach any definitive conclusions (1983/1947). Research along these lines has come to be known as qualitative economics. For a more in-depth review, the interested reader may see Quirk (1981) and Lady (1995). We review relevant results for this paper.

One thoroughly investigated qualitative matrix structure is the Metzler or gross-substitute economy, so-called due to the fact that excess demand price elasticities are positive across markets. Metzler matrices are the negative of M-matrices and have negative diagonals and nonnegative off-diagonals elements, such as

$$sign(\mathbf{A}) = \begin{bmatrix} - & + & + & + \\ + & - & + & + \\ + & + & - & + \\ + & + & + & - \end{bmatrix}. \tag{4.3}$$

Hicks showed that if **A** is a Metzler matrix *and* is Hicksian stable (i.e., principal minors of **A** alternate in sign), then a rise to one market price will increase other market prices less proportionately (1939/1948) or $(p_1 / p_i)(dp_i / dp_1) < 1 \;\; \forall i > 1$. Samuelson defined stability to hold if and only if the **A** matrix has characteristic roots whose real part is negative and showed it is not equivalent to Hicksian-stability (1941). The two definitions are equivalent, however, if **A** is symmetric (ibid.) or **A** is a Metzler matrix (Lady, 1995).

Another condition imposed on matrices is diagonal dominance. Row diagonal dominance means that, for every row, the absolute value of the diagonal element is no less than the sum of the magnitude of all other entries in the same row (column). McDonald et al. (1995) showed that if **A** is a Metzler matrix[14] *and* is row diagonally dominant, then its inverse has diagonals greater in magnitude than off-diagonal elements in the same column or, by (4.1), $|dp_1 / d\alpha_1| > |dp_i / d\alpha_1| \quad \forall i > 1$, i.e., the impact on the first market's price is larger in magnitude than the impact on any other.

Another qualitative structure are Morishima-matrices which satisfy $sign(a_{ij}) = sign(a_{ji})$ and $sign(a_{ir}a_{rj}) = sign(a_{ij}) \quad \forall i, r, j$, such as

(4.4)
$$sign(\mathbf{A}) = \begin{bmatrix} - & - & + & + \\ - & - & + & + \\ + & + & - & - \\ + & + & - & - \end{bmatrix}.$$

Morishima showed that given such matrices, Samuelson's and Hicks' stability conditions are equivalent. He also proves that if demand for a commodity, which is a substitute (complement) for a numeraire/money, increases, then the prices of that commodity's substitutes rise proportionately less (more) (1952).

Another, perhaps lesser known, matrix structure is that of the form

(4.5)
$$\mathbf{A} = \{a_{ij}\}_1^n \text{ s.t. } \forall i, j \quad |a_{ij}| \le c(1 + |i - j|)^{-r} \quad \forall r > 1, c \in R .$$

These matrices are defined so that their elements are bounded by a function which exponentially decays with distance from the diagonal. The matrix

[14] McDonald et al.'s proof (1995) is for M-matrices. However, since M matrices are simply the negative of a Metzler matrix, it follows that the result holds for the latter case.

$$\text{(4.6)} \qquad \mathbf{A} = \begin{bmatrix} -1 & 0.2 & 0.01 & 0.05 \\ 0.2 & -1 & 0.2 & 0.01 \\ 0.01 & 0.2 & -1 & 0.2 \\ 0.05 & 0.01 & 0.2 & -1 \end{bmatrix}$$

satisfies (4.5) given *c=1* and *r=2*.

Jaffard (1990, cited in Gröchenig & Klotz, 2010) showed that (4.5) holds[15] if and only if its inverse $\mathbf{A}^{-1} = \{c_{ij}\}_1^n$ satisfies $|c_{ij}| \le c(1+|i-j|)^{-r}$. In words, as long as **A**'s elements are bounded by exponential decay from the diagonal, then its inverse is similarly structured. For problem (4.1), a structure of (4.5) implies that $|dp_i / d\alpha_1| < [\partial f_1 / \partial \alpha_1] c(1+|i-1|)^{-r} \quad \forall i$. Matrices with exponential decay like (4.5) make it more likely but do not guarantee effects which progressively decrease in magnitude from the diagonal. The example matrix (4.6) shows how elements may exhibit exponential decay but not progressively decrease from the diagonal because in the first column $0.01 \not> 0.05$). The next section develops sufficient conditions which guarantee positive and progressively decreasing effects.

4.3. Sufficient Conditions for Positive and Progressively Decreasing Spillovers

To fix ideas, let **f** denote excess demand functions for wage markets ordered by skill, with f_1 representing the lowest skill/wage group above minimum wage. Let **p** denote wage levels and α_1 denote the minimum wage, which is modeled as a shock to the first market only.

We provide sufficient conditions for two conceptually distinct aspects of spillovers reported in the literature. The first is the nonnegative effect of the minimum wage on higher wages, i.e.,

$$\text{(4.7)} \qquad \mathbf{x} \ge 0 \ \text{ or } \ \mathbf{A}^{-1}\mathbf{b} \le 0 .$$

[15] Jaffard's theorem is actually more general and may apply to matrices of countable dimension which are invertible and are a subset of the Hilbert ℓ^2 space.

Increasing the minimum wage is likely to have a nonnegative effect on excess demand, i.e., $\mathbf{b} \geq 0$, due to demand shifting to the right via substitution effects and/or supply shifting to the left due to fairness concerns (i.e., individuals demanding higher wages). Given that $\mathbf{b} \geq 0$, condition (4.7) holds if and only if $\mathbf{A}^{-1} \leq 0$. It is well known that a row-dominant diagonal real matrix with negative diagonal elements (i.e., demand is steeper than supply) has a negatively-signed inverse[16]. Alternatively, the inverse of any stable (in the Hicksian or Samuelson sense) Metzler matrix is negative (Lady, 1995). In summary, as long as the labor markets have small cross-market effects or are gross-substitutes and the market is stable, then increasing the minimum wage lifts all skill groups' wages[17].

The second, and central, phenomenon we wish to analyze is when spillovers have progressively decreasing effects on higher skill/wage groups. If we organize the excess demand vector by skill/wage groups then this is equivalent to saying that the elements of the vector **x** are becoming smaller in magnitude, or

(4.8)
$$\left|\frac{\partial p_1}{\partial \alpha_1}\right| \geq \left|\frac{\partial p_2}{\partial \alpha_1}\right| \geq \ldots \geq \left|\frac{\partial p_n}{\partial \alpha_1}\right|.$$

Assuming the minimum wage affects excess demand in the first market only, we find that if the matrix A is row dominant- and tri-diagonal, or

(4.9)
$$\sum_{j \neq i} \left|a_{ij}\right| < \left|a_{ij}\right| \quad \forall i \text{ and } a_{ij} = 0 \quad \forall \left|i - j\right| > 1,$$

suffices for progressively decreasing effects, i.e. (4.8).

Row-dominant diagonality means that own-effects (diagonal elements) are weakly greater than the sum of the magnitude of cross effects (off diagonal elements in

[16] Taussky (1949) documents the well known mathematical result that any row-dominant diagonal real matrix with positive diagonal elements has a positively-signed inverse. The negative case readily follows.

[17] For a more in-depth examination of conditions under which (4.7) holds, the interested reader may wish to see Simon, 1989 which treats the case of dominant diagonal matrices and the possibility that *b* have negative elements.

the same row). Tri-diagonality means that the only nonzero elements in A are on the main, first upper-, and first lower-diagonal. For the *N=4* case, a tri-diagonal matrix has the sign pattern

(4.10) $$sign(\mathbf{A}) = \begin{bmatrix} \pm & \pm & 0 & 0 \\ \pm & \pm & \pm & 0 \\ 0 & \pm & \pm & \pm \\ 0 & 0 & \pm & \pm \end{bmatrix}.$$

Tri-diagonality is reminiscent of a chain structure. Each market's excess demand is a function of own and neighboring prices only. If gross complementarities or substitutions exist between distant wage groups, then tri-diagonality does not hold. Row dominant diagonality means that the excess demand functions for each market are more significantly impacted by their own than the sum of the effects of neighboring market prices.

Condition (4.9) describes a set of markets which are relatively disconnected. In a coordinated set of markets, supply and/or demand are more likely to respond to more 'distant' as well as nearby prices, making it unlikely for (4.9) to hold. Vertical integration in a supply chain is one example of such coordination. If we assume demand for labor is fixed, then condition (4.9) implies that, for a set of skill-labor markets, relative wage concerns embedded in skill groups' supply curves only respond to their own and neighboring wages.

In his proof for upper bounds on the elements of the inverse of a tridiagonal matrix, Nabben (1999; Theorem 3.1) provides an important piece to show that (4.9) implies (4.8). We use differences and absolute values to complete our proof. In the interest of brevity, the central elements of the proof, below, refer to results already proven by Nabben (ibid.).

Theorem 1: If a matrix A is nonsingular, tri-diagonal $n \times n$ *matrix where* $\mathbf{A}^{-1} = \{c_{ij}\}_1^N$*, then, if A is row-diagonally dominant, all elements of the inverse matrix are* *pair-wise weakly decreasing from the diagonal, or* $\left|c_{i,j}\right| - \left|c_{i-1,j}\right| \geq 0 \;\; \forall i \leq j$ and $\left|c_{i,j}\right| - \left|c_{i-1,j}\right| \leq 0 \;\; \forall i > j$.

Proof of Theorem 1:

Nabben (1999) recalls the well-known result that the inverse of a tri-diagonal matrix, $\mathbf{A}^{-1} = \{c_{ij}\}_1^N$ can be described by four vectors of real numbers, $\mathbf{u} = \{u_i\}_1^N$, $\mathbf{v} = \{v_i\}_1^N$, $\mathbf{z} = \{z_i\}_1^N$, $\mathbf{y} = \{y_i\}_1^N$ where $u_i v_i = x_i y_i \;\; \forall i$ and

(4.11)
$$c_{ij} = \begin{cases} u_i v_j & i \leq j \\ z_j y_i & i \geq j \end{cases}$$

By Nabben (ibid.; in the proof for his Theorem 3.1), if **A** is tridiagonal and row-dominant diagonal then

(4.12)
$$\left|u_i\right| \geq \left|u_{i-1}\right| \quad \forall i = 2,...,N \text{ and } \left|y_{i-1}\right| \geq \left|y_i\right| \quad \forall i = 1,...,N-1 .$$

By differencing absolute values of (4.11) we may write

(4.13)
$$\left|c_{i,j}\right| - \left|c_{i-1,j}\right| = \begin{cases} \left|u_i\right|\left|v_j\right| - \left|u_{i-1}\right|\left|v_j\right| & \forall i \leq j \\ \left|z_j\right|\left|y_i\right| - \left|z_j\right|\left|y_{i-1}\right| & \forall i-1 \geq j \end{cases} .$$

Applying (4.12) to (4.13) we find

(4.14)
$$\left|c_{i,j}\right| - \left|c_{i-1,j}\right| = \begin{cases} \left(\left|u_i\right| - \left|u_{i-1}\right|\right)\left|v_j\right| \geq 0 & \forall i \leq j \\ \left(\left|y_i\right| - \left|y_{i-1}\right|\right)\left|z_j\right| \leq 0 & \forall i-1 \geq j \end{cases} .$$

or

(4.15)
$$\left|c_{i,j}\right| - \left|c_{i-1,j}\right| \geq 0 \;\; \forall i \leq j \text{ and } \left|c_{i,j}\right| - \left|c_{i-1,j}\right| \leq 0 \;\; \forall i > j ,$$

i.e., pair- and column-wise decreasing.

Q.E.D.

By assuming that the minimum wage shocks the first market only, Theorem 1 implies that (4.9) is sufficient[18] for (4.8). Theorem 1 may apply, however, to cases in which a shock occurs at any point in the market-chain. For example, if another exogenous shock α_5 occurs in the 5th labor market only, i.e., $\partial f_i / \partial \alpha_5 \neq 0$ for only $i = 5$, then the effect of the shock will be progressively decreasing for markets above and below the 5th market.

4.4. Minimum Wage Spillovers

Evidence on the spillover effects of minimum wages motivated search for conditions explaining progressively decreasing effects across a series of ordered markets. The search has produced a potentially restrictive assumption on the interaction between skill-labor markets, i.e., wages (or prices) affect excess demand in neighboring markets only. This finding motivates the authors to look more closely at the wage spillovers literature and determine if the phenomenon of progressively decreasing effects actually holds. To begin, we examine the theoretical literature on positive wage spillovers and show that it is consistent with the gross-substitutes Jacobian matrix already discussed. We then review the empirical evidence. Finally, we estimate spillovers using a large individual-level dataset for Oregon.

4.4.1. Theory for Positive Minimum Wage Spillovers

Dittrich, Knabe, and Leipold (2011) summarize three common explanations for the existence of wage spillovers. One explanation is premised on the idea that wage and skill levels are correlated. As the minimum wage increases, the price of low-skilled labor is

[18] Unfortunately, while the chain-structure (4.9) can be shown to be sufficient for decreasing effects (4.7) it may also be the proven that the chain-structure is not necessary for decreasing effects when the shock affects only a single market's excess demand function. A simple Toeplitz matrix counterexample suffices. Simply note that

$$\mathbf{A}^{-1} = \begin{bmatrix} -3 & -2 & -1 \\ -2 & -3 & -2 \\ -1 & -2 & -3 \end{bmatrix} \Leftrightarrow \mathbf{A} = \begin{bmatrix} -0.625 & 0.50 & -0.125 \\ 0.50 & -1 & 0.50 \\ -0.125 & 0.50 & -0.625 \end{bmatrix}$$

raised which may cause firms to substitute towards higher-skilled labor, thus raising the latter's wage. Another explanation is that, for firms offering better-paid jobs to workers from low-wage firms, a rise in the minimum wage causes them to offer an even higher wage to remain competitive. Alternatively, if effort is a function of relative wages, then an increase in the minimum must be accompanied by firms' raising the wage of those already making higher wages to maintain productivity.

Yet another explanation has to do with workers' fairness concerns[19] driving the demand for higher wages. An experimental study suggests, for example, that increases in the minimum wage raise what people consider to be "fair" compensation, thus shifting the labor supply curve (Falk *et al*. 2006). This explanation is similar yet distinct from the effort-explanation above. Both are relative wage-concerns yet one primarily manifests itself in productivity at work and the other in wage negotiations.

The potential mechanisms above, although derived from various theoretical frameworks (e.g., perfect competition, search, or game-theoretic models), may be expressed through this paper's excess demand framework. Substitution effects and firms' relative wage concerns may be framed as leftward shifts in supply curves. I.e., the cross-price elasticity of labor demand in one skill market with respect to another is negative. Worker's fairness concerns may be framed in terms of the effects of equilibrium wages on the labor supply functions. I.e., supply curves shift rightward due to an increase in another skill-groups' wage. In either case, the derivative of the excess demand function with respect to another skill-group's wage is negative. These ideas are consistent with the gross-substitutes economy discussed in section II of this paper, which results in positive wage spillover effects.

4.4.2. Previous Empirical Results

Two strands of literature find evidence to support positive and decreasing minimum wage spillovers. The first estimates the effect of the minimum wage on the wage distribution.

[19] A growing literature finds that fairness concerns can explain a variety of exchange behaviors (Fehr & Schmidt, 1999; Bolton & Ockenfels 2000; Sobel, 2005; Falk & Fischbacher, 2006; Fehr, Goette, Zehnder, 2008; Dohmen *et al* 2009; Abeler *& Altmann*, 2010)

For example, some researchers utilize regional-level data and regress wage-percentiles, or ratios of percentiles, on minimum wages (Dickens, Machin & Manning, 1999 for the UK; Lemos, 2004 for Brazil; Lee, 1999 for the US). Others estimate empirical and counterfactual distribution functions to make comparisons with and without minimum wages (DiNardo, Fortin & Lemiux, 1996 for the US; Dickens & Manning 2004 for the UK; Stewart, 2011 for the UK). These studies find that minimum wages tend to reduce especially lower-tail distribution wage inequality, indicating that the minimum wage does not have pronounced spillover effects on the highest wages.

Estimating the effect of the minimum wage on the wage distribution provides only preliminary evidence of decreasing spillover effects. One issue is that individual-level impacts are unrecoverable. A simple example suffices. Suppose there are two periods and the minimum wage is raised from $7 to $8 an hour. Suppose that in the first period one individual is making $8.50 while the other is making $9 an hour. In the second period, two individuals earn $9. It may be that the low-paid worker got a raise while the other did not, confirming a progressively decreasing effect of the minimum wage. Another possibility is that no decreasing effects hold. Instead, the low-paid worker got a raise, the high-paid worker retired, and a new entrant got a low-wage job.

Other papers deal with this issue and attempt to estimate minimum wage spillovers on individuals' received wages. Prior to the mid 1990s, several papers using survey data filled out by employers in the US restaurant industry found that spillovers were restricted to individuals just above new minimum wage (Brown 1999).

More recently, Neumark, Schweitzer, and Wascher (hereafter NSW; 2004) use individual time-series data and regress changes in individual wages on changes in the minimum wage, conditional on remaining employed. By using a dummy/spline regression technique, effects of wages are identified by their position in the wage distribution. Papers utilizing this approach find the percentage change effects of the minimum wage on received wages to be especially pronounced among lower wage groups (Neumark, Schweitzer, & Wascher, 2004 for the US; Maloney & Mendez, 2004 for Colombia). Unfortunately, a decreasing percentage effect is a necessary but insufficient condition for a decreasing level effect[20].

In fact, a back-of-the-envelope recalculation of the estimates found by Neumark, Schweitzer, & Wascher (2004; henceforth NSW) in the US shows that spillover effects of the minimum wage in terms of levels are strongest at the lowest *and* highest wage groups, forming a U shape across wage groups (see Appendix for estimates). The U shape may be caused by strong substitution effects between the very low and very high wage groups. For example, as lower wages increase, firms substitute for more capital-intensive solutions which require highly skilled labor.

No one to our knowledge has estimated the level-effects of the minimum wage across wage groups using disaggregate panel data. The following section delineates the data and econometric specification to recover such effects.

4.4.3. Data

Data for this study comes from the State of Oregon's Employment Department quarterly wage record files (2011). Since all businesses covered by unemployment insurance laws are required to submit this data, over 90% of workers in Oregon are included (Personal Communication, Mrs. Peniston Workforce and Economic Research Oregon Employment Department, October 2011). The self-employed as well as some domestic service and agricultural workers are excluded (ibid.).

Records for a typical year number in the millions and are uniquely identified at the individual/job/quarter/year level from the first quarter of 1989 thru 2010. The data are subsequently aggregated to the individual/year level by summing hours and wages across

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[20] The proof necessity is as follows: assume effects are progressively decreasing, i.e. $\partial p_i / \partial \alpha_1 < \partial p_j / \partial \alpha_1 \quad \forall i > j$. Then, since wages are ordered to be decreasing, i.e. $p_j < p_i \quad \forall i > j$, it is necessary that $(\partial p_i / \partial \alpha_1)(\alpha_1 / p_i) < (\partial p_j / \partial \alpha_1)(\alpha_1 / p_j) \quad \forall i > j$, which is the definition of a decreasing percentage effect. A counterexample suffices to show it is insufficient. Suppose $(\partial p_i / \partial \alpha_1)(\alpha_1 / p_i) = 1 < 2 = (\partial p_j / \partial \alpha_1)(\alpha_1 / p_j) \quad \forall i > j$ but that $(\alpha_1 / p_i) = 1/5 < 1/2 = (\alpha_1 / p_j) \quad \forall i > j$, then $\partial p_i / \partial \alpha_1 = 5 > 4 = \partial p_j / \partial \alpha_1 \quad \forall i > j$.

jobs for each individual. While the dataset includes hours worked, wages received, as well as firm and individual identifiers, there is no demographic data. Estimating the model using first-differences removes such time-invariant effects. Individual fixed-effects are also used. Total hours worked are reported by quarter making it impossible to control for part-time versus full-time employees.

We restrict analysis to individuals who ever worked in the Accommodation and Food Services industry (NAICS code 72) during the time period for which we have data. This industry is likely to have a significant proportion of individuals directly affected by the minimum wage and also matches earlier research investigating spillovers in the restaurant-sector. A robustness check utilizing all Oregon workers produces similar results.

Outliers are removed using two filters. First we remove records according to NSW criteria: (a) wages over eight times the minimum wage, (b) wages less than half the minimum wage, (c) yearly percentage increase in wages of more than 1000%. Then, we apply a more stringent filter to ensure more sensible data. We remove records in which (a) wages increase over 300% from one year to the next, (b) wages below the minimum wage less 50 cents, and (c) total hours worked for the year are greater than 5200 or less than 260, i.e., averaging over 100 or less than 5 hours per week. These filters progressively remove 1% and 20% of the data, respectively, but do not modify the central results presented below.

Table 1 presents summary statistics by wage bin. These bin definitions match those used by NSW and are used for the econometric analysis. The data show that the individual wages and number of quarters worked are positively related to one another. The inverse relationship holds for worker's hours, employers (firms), jobs, and industries (at two-digit NAICS level). For example, people with higher wages work for fewer employers. The table also shows that real wage rates of those in lower wage groups tend to decline during our data horizon while wage rates among high-wage groups tend to rise. The opposite holds for hours. High-wage groups tend to reduce their hours and low wage groups to raise them.

Table 4.1 – Summary statistics for Oregon wage records using NSW wage bins

Wage Bin	Definition	Observations	Percent	Real Wage	Hours	Wages from primary industry	Job Count (per quarter)	Industry count	Quarter Count	Firm Count	Hours from Top Firm	Change in Hours	Change in Real Wage
1	$w - w^{min} < \$.10$	114,943	1%	$7.88	1,303	67%	1.40	1.72	3.23	1.73	68%	99	-1.99
2	$\lvert w-w^{min} \rvert < 0.1$	615,785	3%	$8.22	862	76%	1.35	1.49	2.97	1.45	78%	14	-0.63
3	$w^{min} + 0.1 < w < 1.1{*}w^{min}$	1,370,338	7%	$8.64	1,099	68%	1.37	1.65	3.22	1.73	67%	79	-0.56
4	$1.1<w/\ w^{min} <=1.2$	1,283,974	6%	$9.41	1,238	68%	1.35	1.65	3.32	1.74	67%	56	-0.44
5	$1.2<w/\ w^{min} <=1.3$	1,150,371	6%	$10.22	1,334	70%	1.33	1.61	3.40	1.70	68%	42	-0.29
6	$1.3<w/\ w^{min} <=1.5$	1,959,828	9%	$11.43	1,443	72%	1.28	1.53	3.47	1.62	71%	24	-0.12
7	$1.5<w/\ w^{min} <=2$	3,842,653	18%	$14.20	1,608	78%	1.22	1.38	3.59	1.45	78%	-4	0.16
8	$2<w/\ w^{min} <=3$	4,917,729	24%	$19.98	1,759	86%	1.15	1.22	3.71	1.28	85%	-39	0.44
9	$3<w/\ w^{min} <=4$	2,517,192	12%	$28.24	1,770	90%	1.11	1.15	3.74	1.22	89%	-47	0.82
10	$4<w/\ w^{min} <=5$	1,526,060	7%	$36.45	1,725	92%	1.10	1.12	3.76	1.20	90%	-51	1.26
11	$5<w/\ w^{min} <=6$	870,894	4%	$44.55	1,708	93%	1.09	1.11	3.76	1.16	92%	-68	2.10
12	$6<w/\ w^{min} <=8$	719,115	3%	$55.59	1,727	93%	1.08	1.11	3.74	1.13	93%	-103	4.78
All		*20,888,882*	*100%*	*$20.22*	1,569	81%	1.20	1.34	3.58	1.41	80%	-16	0.52

Table 4.2 – Summary statistics for Oregon wage records using alternate, equally spaced wage bins

Wage Bin	Definition	Observations	Percent	Real Wage	Hours	Wages from primary industry	Job Count (per quarter)	Industry count	Quarter Count	Firm Count	Hours from Top Firm	Change in Hours	Change in Real Wage
1	$0.81 > w - w^{min}$	2,088,888	10%	$8.47	1,039	70%	1.37	1.61	3.15	1.65	70%	63	-0.66
2	$2.18 > w - w^{min}$	2,088,884	10%	$9.66	1,267	69%	1.35	1.64	3.35	1.73	67%	51	-0.39
3	$3.88 > w - w^{min}$	2,088,892	10%	$11.18	1,422	72%	1.29	1.54	3.46	1.64	70%	26	-0.15
4	$5.9 > w - w^{min}$	2,088,888	10%	$13.03	1,552	76%	1.24	1.44	3.55	1.51	75%	7	0.06
5	$8.31 > w - w^{min}$	2,088,888	10%	$15.25	1,656	80%	1.20	1.34	3.63	1.40	80%	-14	0.24
6	$11.18 > w - w^{min}$	2,088,889	10%	$17.87	1,728	84%	1.16	1.26	3.68	1.32	84%	-33	0.36
7	$14.91 > w - w^{min}$	2,088,886	10%	$21.11	1,782	88%	1.14	1.20	3.72	1.26	87%	-44	0.47
8	$20.52 > w - w^{min}$	2,088,890	10%	$25.68	1,785	90%	1.12	1.17	3.74	1.22	89%	-46	0.70
9	$29.55 > w - w^{min}$	2,088,888	10%	$32.82	1,743	91%	1.10	1.13	3.75	1.21	90%	-49	1.04
10	$60.76 > w - w^{min}$	2,088,889	10%	$47.12	1,717	93%	1.09	1.11	3.76	1.16	92%	-75	2.80
All		20,888,882	100%	$20.22	1,569	81%	1.20	1.34	3.58	1.41	80%	-16	0.52

Between 1998 and 2010 the Oregon minimum wage remained above the Federal minimum. The left graph in Figure 4.1 shows that the Oregon nominal minimum wage has increased but the real wage has remained relatively stable since 1998. The right graph shows that Oregon's real minimum wage rose in 1999 and 2003 but fell in 2001 and 2008. Other years such as 2004 through 2006 witnessed relatively little change.

First implemented in 2004, Oregon's Measure 25 pegs increases in the minimum wage rate (rounded to the nearest five cents) to inflation using the city average Consumer Price Index (CPI). The calculation of the minimum wage for one year uses the August-to-August change (Minimum Wages; Employment Conditions; Minors. 653. Oregon Revised Statutes §25; 2011) in CPI for the previous two years. For these reasons, the real minimum wage continued to change after the measure took effect (see the right graph in Figure 4.1). For example, although the nominal minimum wage was boosted in 2008 to account for inflation during the years just before it, actual increases in inflation between 2007 and 2008 resulted in a decrease in the real value of that year's minimum wage.

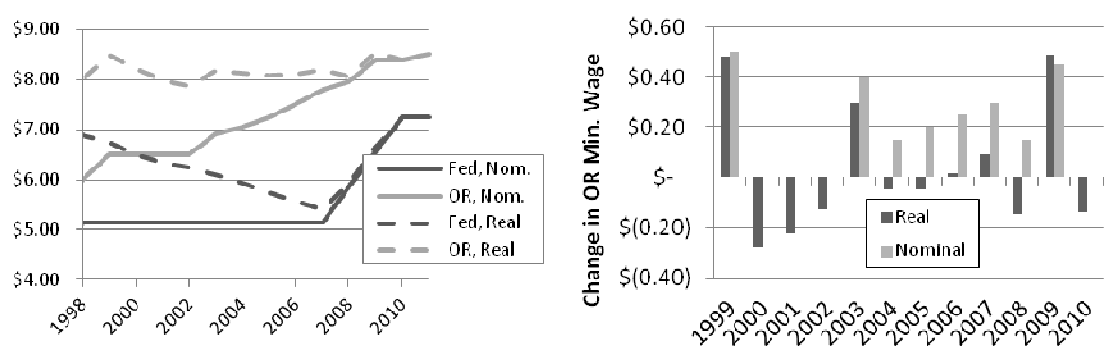

Figure 4.1 – Real (2010 USD) minimum wage levels (left) and changes (right)

4.4.4. Estimation Model

Let subscripts i and t indicate individual and year, respectively and let minimum wage and individual wages be denoted w_t^{min} and w_{it} respectively. Also, let Δ be a first-difference operator which subtracts the previous year's value from the current one. For comparability sake, we first estimate a model similar to NSW's simplest specification (ibid. equation 2), i.e.,

(4.16)
$$\frac{\Delta w_{it+1}}{w_{it}} = \sum_j R^j\left(w_{it}, w_t^{min}\right)\left(\gamma_j^{\%} + \beta_j^{\%}\frac{\Delta w_{it+1}^{min}}{w_t^{min}} + \beta_j^{\%L}\frac{\Delta w_{it}^{min}}{w_{t-1}^{min}} + \phi_j\frac{w_{it}}{w_t^{min}}\right) + \varepsilon_{it+1},$$

where ε_{it} is assumed to be a spherical disturbance, $R^j\left(w_{it}, w_t^{min}\right)$ is a function which categorizes the individual based on the distance of her wage to the minimum wage, and the remaining Greek letters are parameters to be estimated. The bins used for the $R^j(\cdot)$ function match those used by NSW and are defined in Table 1.

Model (4.16) will estimate the effect of a one percent increase of the minimum wage on the percentage change in an individual's wage or $\beta_j^{\%}$. This, however, does not correspond to the effects compared by the progressively decreasing set in (4.8).

In order to properly investigate decreasing effects we estimate a difference form of (4.16) to recover level, not percentage-effects. We drop the lagged change in the minimum wage and the relative wage (w_{it} / w_t^{min}) term for parsimony and to facilitate interpretation[21]. A vector of covariates $\mathbf{X}_{it}$ is added to include: firm count, job count, quarter count, dummy variables capturing (2-digit NAICS) industry fixed-effects, whether an individual's primary source of income comes from a new firm, and whether an individual switched industries. Although the panel is unbalanced, a sufficient number of individuals have repeated observations over time that a fixed-effect by individual may be estimated[22]. Finally, a Wooldridge test (2002) on the first-differenced model rejects the null hypothesis of no autocorrelation and motivates us to specify the error term as an AR(1) process. The standard specification for our model is

(4.17)
$$\Delta w_{it+1} = \sum_j R^j\left(w_{it}, \overline{w}_t\right)\left(\gamma_j + \beta_j \Delta w_{t+1}^{min}\right) + \boldsymbol{\delta}'\mathbf{X}_{it+1} + \eta_i + \nu_{it+1},$$

where $\nu_{it} = \rho \nu_{it-1} + \varepsilon_{it}$ and ε_{it} is a spherical disturbance term.

[21] Robustness checks find that removal of these two terms has a negligible effect on the coefficients of interest.

[22] A Hausman test suggests use of a fixed rather than a random effects model

To investigate whether the minimum wage has progressively decreasing effects on markets above the minimum wage, we examine whether

$$|\beta_3| > |\beta_4| > \ldots > |\beta_{12}|. \tag{4.18}$$

The comparison begins with the third coefficient because, in the NSW wage bins, the first and second coefficients estimate the effect on those making less than or about equal to the minimum wage, respectively.

4.4.5. Robustness Checks

Several robustness checks are conducted on model (4.17): (i) excluding all data filters, (ii) using only the NSW filters defined above, (iii) removing individual fixed effects or AR(1) error, (iv) with fixed effects but without the AR(1) error, (v) including a lagged change in the minimum wage and a relative wage term (as in model 4.16), (vi) restricting analysis to pre and post Measure 25, (vii) including the change in mean US management wage rates as a covariate, (viii) including the change in US GDP as a covariate, (ix) using alternatively constructed wage bins. The last-mentioned robustness check sorts observations by distance to the prevailing minimum wage, i.e., $w_{it} - w_t^{min}$, and redefines wage bins to each contain an equal number of observations for the entire sample. Summary statistics by these wage bins are presented in Table 2. Figure 2 shows how the number of observations significantly differs between the NSW and alternate wage bins.

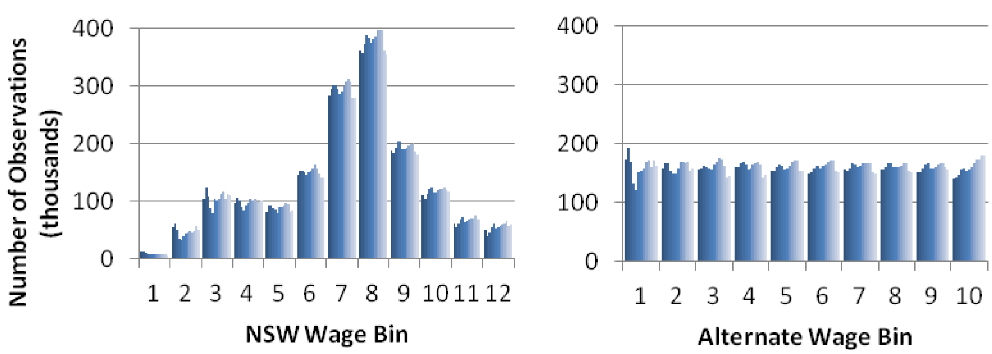

Figure 4.2 – Distribution of observations with NSW vs. alternate wage bins

Bars indicate the number of observations in the dataset (in the thousands). The darkest (leftmost) bars indicate observations (individuals) for 1998 and the lightest represent those for 2010.

4.4.6. Results

Using their percentage-effect model, we find somewhat similar results as NSW did (see Figure 4.3). Using Oregon and US data, respectively, effects of the percent change in the minimum wage progressively decline until the fifth wage bin (or about 1.2 times the minimum wage) and until the ninth wage bin (i.e., 3.5 times the minimum wage) using Oregon and US data, respectively. In both cases, the effect of the current minimum wage progressively rises from the 9th bin onwards, except the effect on high wage bins is more pronounced in the Oregon data. Our sample, however, systematically differs from that used by NSW. Most importantly, NSW analyzed wage changes for individuals across the United States and controlled for state/year-effects whereas our own analysis is restricted to workers in Oregon without year-effects.

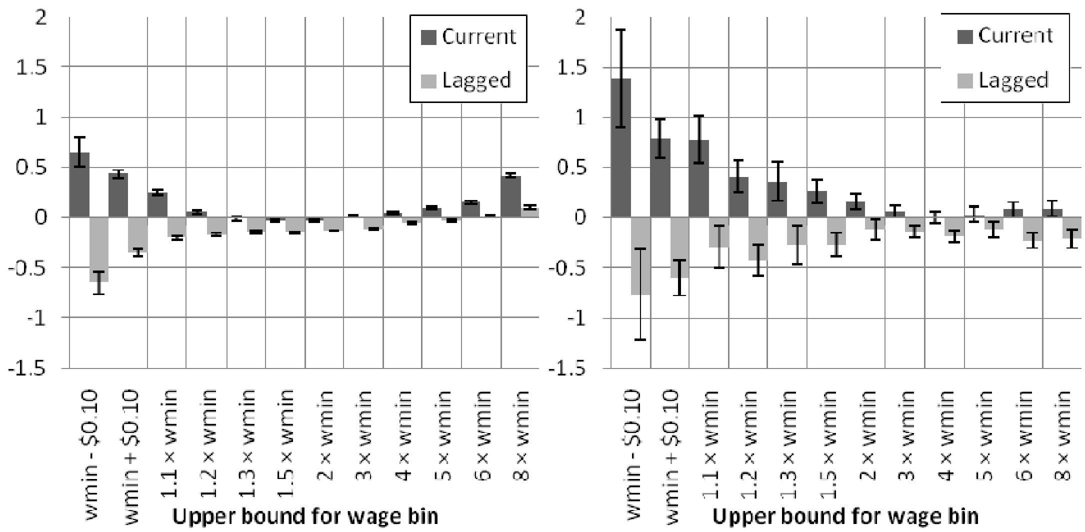

Figure 4.3 – Percentage change impacts of the minimum wage on wages by wage bin for Oregon (left) and the US (right)

95% Confidence intervals are displayed and are constructed assuming a normal distribution due to large samples. Oregon results use data described in this study and US results come from NSW (2004).

When the same model is recast in difference form (model 4.17), the graph retains the U-shape such that effects of the minimum wage are strong at the lowest levels but even higher at the highest wage categories (see Figure 4.4). This is because a given percentage impact on the higher wage categories implies a much larger level impact than that same percentage effect does on a lower wage category. Nearly all robustness checks confirm the positive and U-shape pattern (see Appendix B for all results). Removing the fixed-effects term, however, appears to play an important role in producing strong effects in the high wage categories.

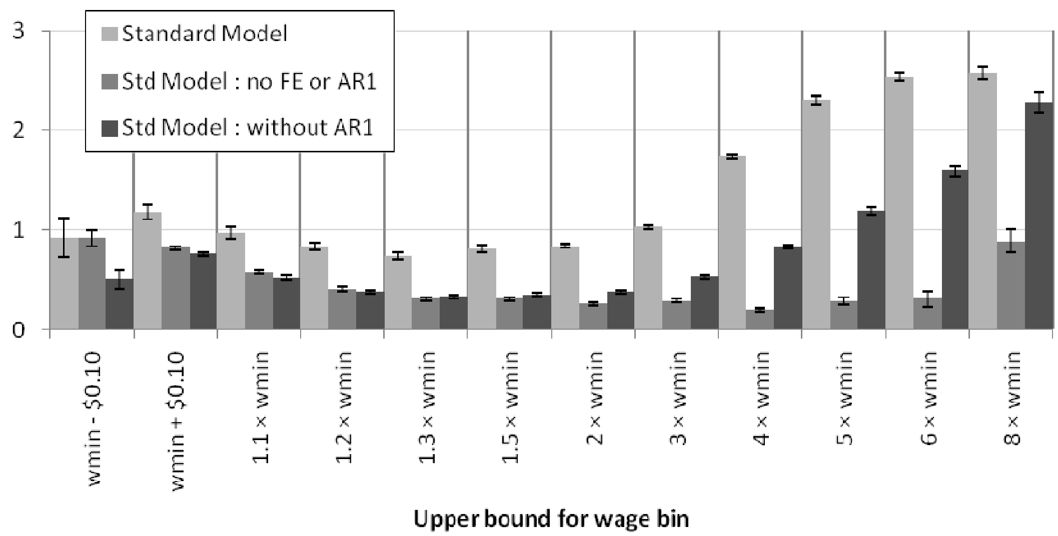

Figure 4.4 – Estimated level change impacts of the minimum wage on wages using NSW bins

95% Confidence intervals are displayed and are constructed assuming a normal distribution due to large samples.

Figure 4.5 shows the impacts by wage bin, when the latter are defined to be equally-sized. First note that under the new wage bin characterization, the number of categories is reduced to ten. The first and last wage bins roughly correspond to the first three and last three NSW wage bins. Using the alternate bins, model (4.17) continues to display a decreasing then increasing pattern. The effect in the last bin, however, falls.

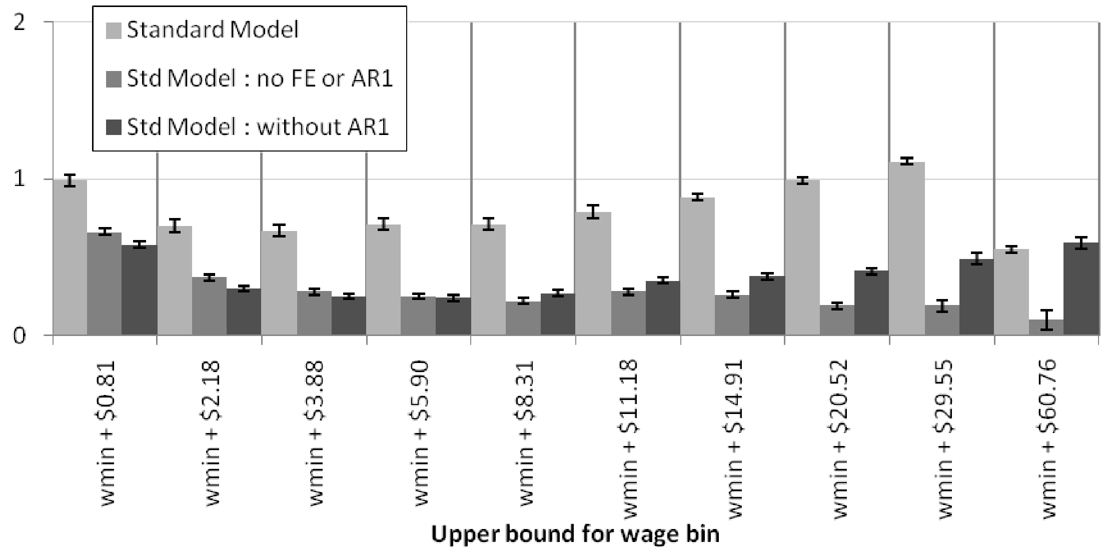

Figure 4.5 – Impacts of the change in the minimum wage on the change in wages, using alternate bins

95% Confidence intervals are displayed and are constructed assuming a normal distribution due to large samples.

4.4.7. Discussion

The results above confirm a sizeable literature that minimum wage spillovers exist and are positive. However the results with the NSW bin construction show that while the minimum wage may initially have progressively decreasing effects, they eventually rise to the point that on extremely high wages (above three times the minimum wage) they are stronger than at the lowest level.

As long as the minimum wage acts as a shock in the first market only, this finding contradicts the chain-structure (4.9) hypothesis because effects are not progressively decreasing. The U shape also precludes exponentially decreasing cross-price effects in the Jacobian, or (4.5). Furthermore, the hypothesis of a gross-substitutes, dominant-diagonal economy is precluded since the effect on the first market is not larger than the rest.

Violations of these hypotheses when employing the NSW bins suggest substantial substitution between low- and high-skilled workers' tasks—producing large positive entries in the farther off-diagonal entries in the excess demand Jacobian. As minimum wages rise, low-skilled labor costs rise as well and firms shift to more capital-intensive solutions requiring more high-skilled labor. This additional demand on high-skilled labor raises the high-skilled wages.

The discussion in section 4.4.2 demonstrated that it is possible for the minimum wage to have a U-shaped effect on wages yet still reduce inequality, depending on the inequality measure used. Minimum-wage dampening of the ratio of the 90th to the 10th percentile wage rate $\partial(w_{90} / w_{10}) / \partial\overline{w} < 0$ is equivalent to $\partial w_{90}/\partial\overline{w} < (w_{90}/w_{10})\partial w_{10}/\partial\overline{w}$. Thus, as long as the ratio of high to low wages (w_{90}/w_{10}) is large enough, a dampening effect may hold even if the minimum wage's effect on higher wages is larger than that on lower wages, i.e., $\partial w_{90}/\partial\overline{w} > \partial w_{10}/\partial\overline{w}$. In the latter case, however, the use of a difference measure would produce evidence of rising inequality.

Bins are redefined to have approximately equal numbers of individuals. Since there are 10 bins, each bin approximately represents the 5th, 15th, 25th, …, 95th percentile. Using these bins, a decreasing, then increasing effect continues to hold. The effect in the very last bin, however, drops. This may be due to the fact it spans a wider wage range (30 to 60 dollars over the minimum wage), making the model more prone to omitted variable bias. In any case, estimates using the alternate bins suggest that the minimum wage increases inequality between the 5th and 85th percentile if one uses a difference measure. A ratio measure, however, suggests otherwise[23]. Lower tail inequality (i.e., between the 5th and 15th, 25th, or 35th) percentile is reduced regardless of the measure used.

4.5. Conclusions

The intuitive notion that the effect of a shock to one market will progressively decrease with distance to other markets need not always hold in economic systems. The qualitative economics literature has shown that if an economy is of the gross-substitutes type, the effect of a shock to one market will raise the price in that market more than it will in all others. Jaffard's theorem (1990, cited in Gröchenig & Klotz, 2009) implies that as long as

[23] $\partial w_{85}/\partial\overline{w} \approx 1.11$ and $\partial w_{05}/\partial\overline{w} \approx 0.99$ yet $w_{85}/w_{05} = 32.82/8.47 = 3.87$ so that $\partial w_{85}/\partial\overline{w} \approx 1.11 < (w_{85}/w_{05})\partial w_{05}/\partial\overline{w} = 0.99(3.87) = 3.83$

an economy's cross-market price effects are bounded by a function exhibiting exponential decay with distance, a shock to one market's price will have similarly bounded effects on farther markets. This theorem does not, however, ensure effects are progressively decreasing. Instead, we find that a chain-like economic structure, in which each market is directly affected by only its own and one or two neighboring markets, is sufficient for the decreasing effect to hold.

The empirical investigation suggests that the chain-structure does not apply to labor markets in Oregon. Applying a first-difference estimator on Oregon wage data between 1998 and 2010, we find that minimum wages have strongly positive effects on the lowest and highest wage groups, forming a U-shaped pattern. Several robustness checks confirm this. Strong substitution effects across several wage categories can explain the strong effects on higher wages. Using modified, density-adjusted, wage bins we find evidence that the minimum wage reduces lower-tail wage inequality whether one uses a difference or a ratio percentile measure for workers remaining employed.

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CHAPTER 5 - CONCLUSION

This dissertation's three essays shed light on the ways in which constraints affect sales, demand, and prices. The first essay demonstrates that constraints matter and significantly affect economic outcomes. In particular, time is more important than money in the fall and winter months when other activities compete for gardening times. The second essay shows that the poor are least likely to gain from the relaxation of other constraints. This essay finds that benefits from global warming in the plant nursery sector will accrue to the wealthy and those that live in regions cooler than 64 degrees Fahrenheit.

The third essay shows that a constraint in a single market can sometimes have more profound consequences on other, more distantly related markets. It is shown that as long as markets are only affected by own and neighboring prices, then a shock to market will have progressively decreasing effects on more distant markets' prices. The case of the minimum wage constraint is investigated and it is determined that the ripple effects of minimum wages are actually greater for higher wage groups.

Broadly speaking, the research demonstrates that constraints significantly shape economic outcomes and should be accounted for. Explicitly modeling constraints raises new questions about the relevance and circumstances under which constraints matter. These questions include several: When does price matter? What constraint matters most for consumers? What constrains economic growth? To what extent are preferences versus constraints motivating outcomes? This dissertation contributes theoretically and empirically towards the dialogue around these and similar questions.

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APPENDICES

APPENDIX A – MULTIPLE CONSTRAINT RESULTS

Proof that more restrictive constraints weakly decrease market welfare.

Note: *Hatta's equations are designated using the letter H and notation follows his except that we use q instead of x, and T instead of κ*

Let $\mathbf{q} \in \mathbb{R}^N$ be a vector of length *N* indicating quantity. Let $\mathbf{D}(\mathbf{q}) \in \mathbb{R}^N$ and $\mathbf{S}(\mathbf{q}) \in \mathbb{R}^N$ be inverse demand and supply functions, respectively. Let q_j indicate the j^{th} element of **q**, and $\mathbf{q}_{(j)}$ indicate all elements in the vector **q** except the j^{th}. Define total welfare from all markets as $f(\mathbf{q}) = \sum_{j=1}^{n} \int_0^{q_j} D_i\left(q_j, \mathbf{q}_{(j)}\right) - S_i\left(q_j, \mathbf{q}_{(j)}\right) dq_j$.

Let *T* be a scalar, and *a(x)* be a single constraint function. Assume that

(a) $\mathbf{q}^*(T) = \arg\max_{\mathbf{q}}\{f(\mathbf{q}) : a(\mathbf{q}) \le T\}$ Exists and is differentiable.

(b) $\phi(T) = \max_{\mathbf{q}}\{f(\mathbf{q}) : a(\mathbf{q}) \le T\}$ And $\phi(T)$ and $f(\mathbf{q})$ are differentiable.

By (a) and (b) and Hatta's (2008) equations H33 and H34,

(A.1) $$T' \le T \Rightarrow f\left(\mathbf{q}^*(T')\right) \le f\left(\mathbf{q}^*(T)\right)$$

or $\partial f\left(\mathbf{q}^*(T')\right) / \partial T \ge 0$, like Hatta's Theorem H9 (p. 993).

By (A.1) and our definition for $f(\mathbf{q})$ as welfare, any decrease in constraint endowments, *T*, weakly decreases maximum welfare.

Q.E.D.

Derivation of Equation (3.2) $E\left(\mathbf{x}^*(\boldsymbol{\alpha})\right) = \sum_{i=1,\ldots,Q} P_i \overline{\mathbf{x}}_i$

Assume a representative consumer maximizes a utility function $U(\cdot)$ from a vector of goods $\mathbf{x} \in \mathbb{R}^{N+}$ subject to *K* stochastic preferences $\boldsymbol{\alpha} \in \mathbb{R}^K$ and a vector of *M* constraint functions $\mathbf{h}(\cdot) \in \mathbb{R}^M$. Her maximization problem is

$$\max_{\mathbf{x}} U(\mathbf{x};\boldsymbol{\alpha}) : \mathbf{h}(\mathbf{x}) \leq \mathbf{0} \ ,$$

where **0** is a vector of zeros.

Let *Q* be the possible number of combinations of *M* constraints. Let $\mathbf{x}^* = \arg\max_{\mathbf{x}}\{U(\mathbf{x};\boldsymbol{\alpha}) : \mathbf{h}(\mathbf{x}) \leq \mathbf{0}\}$. Also, let $\mathbf{x}_i^* = \arg\max_{\mathbf{x}}\{U(\mathbf{x};\boldsymbol{\alpha}) : \mathbf{h}^i(\mathbf{x}) = \mathbf{0}\}$ where $\mathbf{h}^i(\mathbf{x})$ is the $i = 1,\ldots,Q$ selection of rows from $\mathbf{h}(\mathbf{x})$. Let 1_i be an indicator function that equals 1 when the value of $\boldsymbol{\alpha}$ is such that only rows/constraints in the i^{th} case hold with equality and equals 0 otherwise.

Assume the following:

(a) The utility function is strictly quasiconcave and continuous.

(b) Utility is monotonic.

(c) All combinations of constraints form convex sets, i.e., $C^i = \{\mathbf{x} : \mathbf{h}^i(\mathbf{x}) \leq 0\}$ is convex $\forall i$.

(d) At least one constraint function monotonically increases for at least one element of **x**.

For a given $\boldsymbol{\alpha}$, $\mathbf{x}^*$ exists and is unique. This proof is standard and follows from convexity. The proof readily applies to $\mathbf{x}_i^*$ $\forall i$ by assumption (c).

Proof by contradiction. Suppose $\mathbf{x}^*$ and $\mathbf{x}'$ are both maximizers such that $U(\mathbf{x}^*) = U(\mathbf{x}')$. Then, the convex combination,

$\mathbf{x}'' = \beta\mathbf{x}^* + (1-\beta)\mathbf{x}' \;\; \forall\beta \in (0,1)$ is also feasible by assumption (c). But by strict quasiconcavity of the utility function, or assumption (a), $U(\mathbf{x}'') > U(\mathbf{x}^*)$, which is a contradiction. ■

Furthermore, for a given **α**, at least one constraint must be binding at $\mathbf{x}^*$. This proof is standard in economic theory with the added requirement that the constraint function $\mathbf{h}(\mathbf{x})$ be monotonically increasing with respect to at least one element of **x**.

Proof by contradiction. Assume no constraints hold with equality, or $\mathbf{h}(\mathbf{x}^*) < \mathbf{0}$. Then, by (d), there exists another point $\mathbf{x}'$ such that $\mathbf{h}(\mathbf{x}^*) \leq \mathbf{0}$ and $\mathbf{x}^* \leq \mathbf{x}'$. But, by monotonic utility, or (b), $\mathbf{x}^* \leq \mathbf{x}' \Rightarrow U(\mathbf{x}^*) \leq U(\mathbf{x}')$ so that $\mathbf{x}^*$ is no longer maximal, which is a contradiction. ■

Since demand is unique and must be constrained by some combination of constraints holding with equality for each **α**, demand may be written as

(A.2) $$\mathbf{x}^*(\boldsymbol{\alpha}) = \sum_{i=1,\ldots,Q} 1_i \mathbf{x}_i^* \; .$$

Taking the expectation of (A.2) one gets

(A.3) $$E\left(\mathbf{x}^*(\boldsymbol{\alpha})\right) = E\left(\sum_{i=1,\ldots,Q} 1_i \mathbf{x}_i^*\right) .$$

By linearity of the expectation function

(A.4) $$E\left(\mathbf{x}^*(\boldsymbol{\alpha})\right) = \sum_{i=1,\ldots,Q} E\left(1_i \mathbf{x}_i^*\right).$$

By the law of iterated expectations

(A.5) $$E\left(1_i \mathbf{x}_i^*\right) = E\left(E\left(1_i \mathbf{x}_i^* \middle| 1_i\right)\right),$$

where the outer expectation is with respect to the indicator function and the inner expectation conditional on the indicator function. By conditioning, and the definition of the expected value,

(A.6) $$E\left(E\left(1_i \mathbf{x}_i^* \middle| 1_i\right)\right) = E\left(1 \times \mathbf{x}_i^* \middle| 1_i = 1\right) P(1_i = 1) + E\left(0 \times \mathbf{x}_i^* \middle| 1_i = 0\right) P(1_i = 0).$$

By (A.4) thru (A.6) we find that

(A.7) $$E\left(\mathbf{x}^*(\boldsymbol{\alpha})\right) = \sum_{i=1,\dots,Q} P(1_i = 1) E\left(\mathbf{x}_i^* \middle| 1_i = 1\right)$$

or, using more compact notation,

(A.8) $$E\left(\mathbf{x}^*\right) = \sum_{i=1,\dots,Q} P_i \overline{\mathbf{x}}_i \,.$$

Q.E.D.

Expected Demand under Two goods, Two Constraints, Homothetic Cobb-Douglas Utility, and a Random Preference Parameter

Assume a consumer faces two linear constraints: $px + m \le gI; rx \le T$ where T is temperature, $g \in (0,1]$ is a share, $I \in \mathbb{R}$ is income, $m \in \mathbb{R}^+$ is a numeraire, $x \in \mathbb{R}^+$ is a scalar representing the good in question, $p \in \mathbb{R}^+$ is its price, and $r \in \mathbb{R}^+$ is its temperature-price. Assume, in addition, that utility is homothetic, Cobb-Douglas with a scalar stochastic preference parameter $\alpha \in (0,1)$. The consumer's problem is

$$\max_{x,m} U = x^\alpha m^{1-\alpha} : px + m \le gI; rx \le T\,.$$

In this application, either demand for x is constrained by temperature and equals T/r, or it is constrained by income and equals the usual Cobb-Douglas demand, $\alpha gI/p$. The latter will be the case for a given α only when $\alpha gI/p<T/r$. This implies that (A.4) may be written as

(A.9) $$x^*(a) = 1_{[\alpha gI/p<T/r]} \frac{\alpha gI}{p} + \left(1 - 1_{[\alpha gI/p<T/r]}\right)\frac{T}{r}.$$

By (A.8), expected demand is

(A.10) $$E\left(x^*\right) = P\left(\alpha < \frac{T / r}{gI / p}\right) E\left(\alpha \mid \alpha < \frac{T / r}{gI / p}\right)\frac{I}{p} + \left(1 - P\left(\alpha < \frac{T / r}{gI / p}\right)\right)\frac{T}{r}.$$

Let α have a Kumaraswamy distribution, i.e.,

(A.11) $$F(\alpha) = 1 - (1 - \alpha^{\eta})^{\delta} \quad \forall \alpha \in (0,1).$$

Let $\eta=1$ so that α has the distribution

(A.12) $$F(\alpha) = 1 - (1 - \alpha)^{\delta} \quad \forall \alpha \in (0,1)$$

with the respective density function

(A.13) $$f(\alpha) = \delta(1 - \alpha)^{\delta-1} \quad \forall \alpha \in (0,1).$$

Note that for $\delta=1$, the distribution reduces to the uniform case. In addition, for any $\delta>1$, the density function implies that there is a greater probability for α (the preference parameter on good of interest, x) to be closer to 0. In the context of the maximization problem above this implies that there is a greater probability for consumers to have weaker preferences for the good of interest.

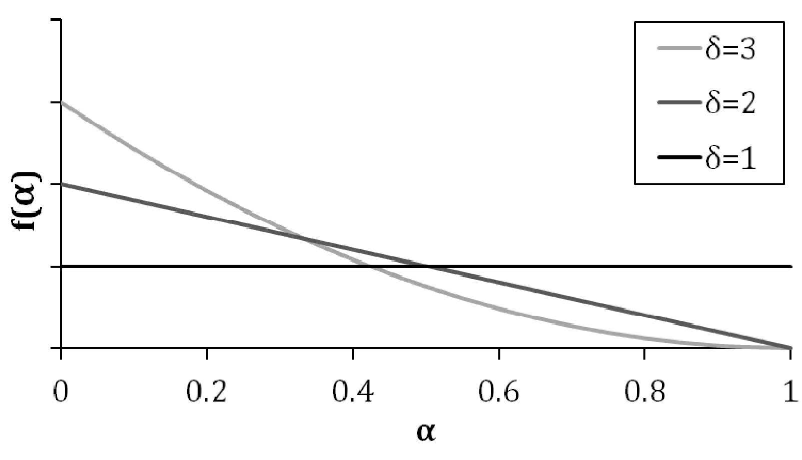

Figure A.1 – Shapes of Kumaraswamy Distributions

By (A.12) and (A.13), expected demand (A.10) may be written as

$$E\left(x^*\right)=\left[1-\left(1-\frac{T\ /\ r}{I\ /\ p}\right)^{\delta}\right]\int_0^{\frac{T/r}{I/p}}\delta(1-\alpha)^{\delta-1}d\alpha\frac{I}{p}+\left(1-\frac{T\ /\ r}{I\ /\ p}\right)^{\delta}\frac{T}{r}.$$

By simplification, inserting a population adjustment factor *n*, as well as using subscripts *s* and *t* to indicate store and time, respectively, the above equation becomes

(A.14) $$E\left(nx_{st}\right)=\frac{I_{st}r+\left(-2I_{st}r+p_{st}T_{st}\right)\left(1-\frac{p_{st}T_{st}}{I_{st}r_{st}}\right)^{\delta}+\left(1-\frac{p_{st}T_{st}}{I_{st}r}\right)^{2\delta}\left(I_{st}r+p_{st}T_{st}\delta\right)}{n^{-1}p_{st}r\left(1+\delta\right)}.$$

In the uniform case (i.e., $\delta=1$) (A.14) may be rewritten as

(A.15) $$E\left(nx_{st}\right)=\gamma_1T_{st}+\gamma_2T_{st}^{\ 2}\frac{p_{st}}{I_{st}}+\gamma_3T_{st}^{\ 3}\frac{p_{st}^{\ 2}}{I_{st}^{\ 2}}$$

such that

(A.16) $$\gamma_1=\frac{n}{r};\gamma_2=-\frac{n}{gr^2};\gamma_3=\frac{n}{2g^2r^3}.$$

We may solve the system (A.16) to show that

$$\gamma_2=-\frac{\gamma_1}{gr};\gamma_3=-\frac{\gamma_2}{2gr}\Rightarrow 2gr=-\frac{\gamma_2}{\gamma_3}\Rightarrow\gamma_2^{\ 2}=2\gamma_1\gamma_3,$$

or that the constraint

(A.17) $$\frac{\gamma_2^{\ 2}}{2\gamma_1}=\gamma_3$$

must hold on (A.15).

There is no unique solution to the system of equations in (A.16) to recover *n*, *r*, and *g* since γ_3 is not uniquely determined. However, the interaction terms *gr*, *gn*, and *n/r* may be recovered using γ_1,γ_2 by

(A.18) $$\gamma_2^{\ 2}=\gamma_1\gamma_3;gr=-\frac{\gamma_2}{\gamma_3};gn=-\frac{\gamma_1^2}{\gamma_2};n/r=\gamma_1;\Rightarrow gr=-\frac{\gamma_1}{\gamma_2};gn=-\frac{\gamma_1^2}{\gamma_2};n/r=\gamma_1.$$

For an econometric specification of (A.15) we add a spherical disturbance term ε_{st} and estimate

(A.19) $$nx_{st} = E(nx_{st}) + \varepsilon_{st} = \gamma_1 T_{st} + \gamma_2 T_{st}{}^2 \frac{p_{st}}{I_{st}} + \frac{\gamma_2{}^2}{2\gamma_1} T_{st}{}^3 \frac{p_{st}{}^2}{I_{st}{}^2} + \varepsilon_{st},$$

using nonlinear least squares to recover γ_1, γ_2.

For the case *δ=2,* (A.14) reduces to the triangular distribution case and we may write

(A.20) $$\begin{aligned} E(nx_{st}) = {} & \left(\frac{n}{r}\right) T_{st} + \left(\frac{-n}{gr^2} 2\right) T_{st}{}^2 \frac{p_{st}}{I_{st}} + \left(\frac{n}{g^2 r^3} 3\right) T^3 \frac{p_{st}{}^2}{I_{st}{}^2} \\ & + \left(\frac{-n}{g^3 r^4} \frac{7}{3}\right) T_{st}{}^4 \frac{p_{st}{}^3}{I_{st}{}^3} + \left(\frac{n}{g^4 r^5} \frac{2}{3}\right) T_{st}{}^5 \frac{p_{st}{}^4}{I_{st}{}^4} \end{aligned}, \text{ or}$$

(A.21) $$E(nx_{st}) = \varphi_1 T_{st} + \varphi_2 T_{st}{}^2 \frac{p_{st}}{I_{st}} + \varphi_3 T^3 \frac{p_{st}{}^2}{I_{st}{}^2} + \varphi_4 T_{st}{}^4 \frac{p_{st}{}^3}{I_{st}{}^3} + \varphi_5 T_{st}{}^5 \frac{p_{st}{}^4}{I_{st}{}^4},$$

where

$$\varphi_1 = \frac{n}{r}; -2\varphi_1 / \varphi_2 = gr; -2\varphi_1^2 / \varphi_2 = gn;$$

and

(A.22) $$\frac{3\varphi_2{}^2}{4\varphi_1} = \varphi_3; \frac{7}{24}\frac{\varphi_2{}^3}{\varphi_1^2} = \varphi_4; \frac{1}{24}\frac{\varphi_2{}^4}{\varphi_1^3} = \varphi_5.$$

Finally, for *δ=3*, (A.14) expected demand reduces to

(A.23) $$\begin{aligned} E(nx_{st}) = {} & \left(\frac{n}{r}\right) T_{st} + \left(\frac{-n}{gr^2} 3\right) T_{st}{}^2 \frac{p_{st}}{I_{st}} + \left(\frac{n}{g^2 r^3} \frac{60}{8}\right) T_{st}{}^3 \frac{p_{st}{}^2}{I_{st}{}^2} + \left(\frac{-n}{g^3 r^4} \frac{46}{4}\right) T_{st}{}^4 \frac{p_{st}{}^3}{I_{st}{}^3} \\ & + \left(\frac{n}{g^4 r^5} \frac{39}{4}\right) T_{st}{}^5 \frac{p_{st}{}^4}{I_{st}{}^4} + \left(\frac{-n}{g^5 r^6} \frac{17}{4}\right) T_{st}{}^6 \frac{p_{st}{}^5}{I_{st}{}^5} + \left(\frac{n}{g^6 r^7} \frac{3}{4}\right) T_{st}{}^7 \frac{p_{st}{}^6}{I_{st}{}^6} \end{aligned}, \text{ or}$$

(A.24) $$\begin{aligned} E(nx_{st}) = {} & \xi_1 T_{st} + \xi_2 T_{st}{}^2 \frac{p_{st}}{I_{st}} + \xi_3 T_{st}{}^3 \frac{p_{st}{}^2}{I_{st}{}^2} + \xi_4 T_{st}{}^4 \frac{p_{st}{}^3}{I_{st}{}^3} + \xi_5 T_{st}{}^5 \frac{p_{st}{}^4}{I_{st}{}^4} \\ & + \xi_6 T_{st}{}^6 \frac{p_{st}{}^5}{I_{st}{}^5} + \xi_7 T_{st}{}^7 \frac{p_{st}{}^6}{I_{st}{}^6} \end{aligned},$$

where

(A.25) $$\frac{5}{6}\frac{\xi_2^{\,2}}{\xi_1}=\xi_3;\frac{23}{54}\frac{\xi_2^{\,3}}{\xi_1^2}=\xi_4;\frac{13}{108}\frac{\xi_2^{\,4}}{\xi_1^3}=\xi_5;\frac{17}{972}\frac{\xi_2^{\,5}}{\xi_1^4}=\xi_6;\frac{1}{972}\frac{\xi_2^{\,6}}{\xi_1^5}=\xi_7$$, and

(A.26) $$\xi_1=n/r;\quad -3\xi_1/\xi_2=gr;\quad -3\xi_1^{\,2}/\xi_2=gn\,.$$

Derivation of comparative statics for *δ=1* (uniform) case

The following statics and probabilities are of interest and may be estimated using mean data values for T_{st}, I_{st}, p_{st} as well as interactions of estimated coefficients (from A.18)

$$gr = -\frac{\hat{\gamma}_1}{\hat{\gamma}_2}; gn = -\frac{\hat{\gamma}_1^2}{\hat{\gamma}_2}; n/r = \hat{\gamma}_1 .$$

<u>*Demand*</u>

Recall expected demand (A.15). The following is its derivative with respect to price

(A.27) $$\frac{\partial E(nx_{st})}{\partial p_{st}} = \gamma_2 \frac{T_{st}^{\ 2}}{I_{st}} + \frac{\gamma_2^2}{2\gamma_1} T_{st}^{\ 3} \frac{p_{st}}{I_{st}^{\ 2}} .$$

The price elasticity is

(A.28) $$\frac{\partial E(nx_{st})}{\partial p_{st}} \frac{p_{st}}{E(nx_{st})} = \left(\gamma_2 T_{st}^{\ 2} \frac{p_{st}}{I_{st}} + \frac{\gamma_2^2}{2\gamma_1} T_{st}^{\ 3} \frac{p_{st}^{\ 2}}{I_{st}^{\ 2}} \right) \frac{1}{E(nx_{st})} .$$

If supply (*nx*) is fixed, we can examine the effect of *T* on *p* using the implicit function theorem. Define $F(\cdot)$ as

(A.29) $$F(\cdot) \equiv n\frac{1}{r}T_{st} - \frac{n}{g}\frac{1}{r^2}T_{st}^{\ 2}\frac{p_{st}}{I_{st}} + \frac{n}{g^2}\frac{1}{2r^3}T_{st}^{\ 3}\frac{p_{st}^{\ 2}}{I_{st}^{\ 2}} - E(nx_{st}) = 0 .$$

Using the implicit function theorem we may write

(A.30) $$\left.\frac{\partial p_{st}}{\partial T_{st}}\right|_{nx} = -\frac{\partial F / \partial T_{st}}{\partial F / \partial p_{st}} = -\frac{n\frac{1}{r} - \frac{n}{g}\frac{1}{r^2}2\frac{T_{st}p_{st}}{I_{st}} + \frac{n}{g^2}\frac{1}{2r^3}3\frac{T_{st}^{\ 2}p_{st}^{\ 2}}{I_{st}^{\ 2}}}{-\frac{n}{g}\frac{1}{r^2}\frac{T_{st}^{\ 2}}{I_{st}} + \frac{n}{g^2}\frac{1}{2r^3}2\frac{T_{st}^{\ 3}p_{st}}{I_{st}^{\ 2}}} .$$

If supply *y* is a function of price, i.e., $y = y(p)$ and at *p* the system is at equilibrium, then we may define $G(\cdot)$ as

(A.31) $$G(\cdot)\equiv n\frac{1}{r}T_{st}-\frac{n}{g}\frac{1}{r^2}T_{st}{}^2\frac{p_{st}}{I_{st}}+\frac{n}{g^2}\frac{1}{2r^3}T_{st}{}^3\frac{p_{st}{}^2}{I_{st}{}^2}-y(p_{st})=0\,.$$

By the implicit function theorem

(A.32) $$\frac{\partial p_{st}}{\partial T_{st}}=-\frac{\partial G/\partial T_{st}}{\partial G/\partial p_{st}}=-\frac{n\frac{1}{r}-\frac{n}{g}\frac{1}{r^2}2\frac{T_{st}p_{st}}{I_{st}}+\frac{n}{g^2}\frac{1}{2r^3}3\frac{T_{st}{}^2p_{st}{}^2}{I_{st}{}^2}}{-\frac{n}{g}\frac{1}{r^2}\frac{T_{st}{}^2}{I_{st}}+\frac{n}{g^2}\frac{1}{2r^3}2\frac{T_{st}{}^3p_{st}}{I_{st}{}^2}-\frac{\partial y(p_{st})}{\partial p_{st}}}\,.$$

If supply is unitary elastic, i.e., $\frac{\partial y(p_{st})}{\partial p_{st}}\frac{p_{st}}{y(p_{st})}=1$ and supply equals demand, then

(A.33) $$\frac{\partial p_{st}}{\partial T_{st}}=-\frac{\partial G/\partial T_{st}}{\partial G/\partial p_{st}}=-\frac{n\frac{1}{r}-\frac{n}{g}\frac{1}{r^2}2\frac{T_{st}p_{st}}{I_{st}}+\frac{n}{g^2}\frac{1}{2r^3}3\frac{T_{st}{}^2p_{st}{}^2}{I_{st}{}^2}}{\frac{n}{g}\frac{1}{r^2}\frac{T_{st}{}^2}{I_{st}}+\frac{n}{g^2}\frac{1}{2r^3}2\frac{T_{st}{}^3p_{st}}{I_{st}{}^2}-\frac{nx_{st}(p_{st})}{p_{st}}}\,.$$

<u>*Probabilities*</u>

By our Kumaraswamy distribution (A.12)

(A.34) $$P\left(\theta<\frac{T_{st}/r}{I_{st}g/p_{st}}\right)=1-\left(1-\frac{1}{rg}\frac{T_{st}p_{st}}{I_{st}}\right)^{\delta}$$

and

(A.35) $$\frac{\partial P\left(\theta<\frac{T_{st}/r}{gI_{st}/p_{st}}\right)}{\partial p_{st}}=\frac{\partial\left[1-\left(1-\frac{T_{st}/r}{gI_{st}/p_{st}}\right)^{\delta}\right]}{\partial p_{st}}=\delta\left(1-\frac{T_{st}p_{st}}{rgI_{st}}\right)^{\delta-1}\frac{T_{st}}{rgI_{st}}\,.$$

Assuming $\delta=1$ we may write

(A.36) $$P\left(\text{only income-constrained}\right)=P\left(\theta<\frac{T_{st}/r}{I_{st}g/p_{st}}\right)=\frac{1}{gr}\frac{T_{st}p_{st}}{I_{st}}\,,$$

(A.37) $$\frac{\partial P\left(\text{only income-constrained}\right)}{\partial p_{st}} \frac{\partial P\left(\theta < \dfrac{T_{st} / r}{gI_{st} / p_{st}}\right)}{\partial p_{st}} = \frac{T_{st}}{grI_{st}},$$

and its complement

(A.38) $$\frac{\partial P\left(\text{weather-constrained}\right)}{\partial p_{st}} = \frac{\partial P\left(\theta \geq \dfrac{T_{st} / r}{gI_{st} / p_{st}}\right)}{\partial p_{st}} = -\frac{T_{st}}{grI_{st}}.$$

APPENDIX B – NEUMARK, SCHWEITZER & WASCHER (2004) REVISITED

Derivation of Level Effects from Neumark et al. (2004)

Neumark, Schweitzer, &Wascher (hereafter NSW; 2004) estimate the model

(B.1)
$$\frac{\Delta w_{y+1,qi}}{w_{yqi}} = \sum_j \beta_j \left[R^j \left(w_{yqi}, \overline{w}_{sy} \right) \Delta w_{y+1}^{min} / w_y^{min} \right] + \sum_j \beta_j{}^L \left[R^j \left(w_{yqi}, w_{sy}^{min} \right) \Delta w_y^{min} / w_{y-1}^{min} \right] \\ + \sum_j \gamma_j R^j \left(w_{yqi}, w_{sy}^{min} \right) + \sum_j \phi_j R^j \left(w_{yqi}, w_{sy}^{min} \right) \frac{w_{yqi}}{w_y^{min}} + \eta_q + \varepsilon_{yqi}$$

The same authors estimate effect of a percent change of the minimum wage on the percent change of an individual's wage, or β_j. Twelve of these are estimated, one for each wage bin. These bins are defined according to the distance of an individual's wage to the minimum wage (see Table B.1).

Table B.1 – Neumark et al. (2004) estimates of the effect of a percentage increase in the minimum wage on the percentage change of an individual's wage.

Wage Bin	β_j	Robust SE
w<wmin-$.10	1.39	0.25
wmin- $.1 < w < wmin + .1	0.79	0.10
wmin+ $.1 < w < 1.1*wmin	0.78	0.12
1.1<w/wmin<=1.2	0.41	0.08
1.2<w/wmin<=1.3	0.36	0.10
1.3<w/wmin<=1.5	0.26	0.06
1.5<w/wmin<=2	0.16	0.04
2<w/wmin<=3	0.06	0.03
3<w/wmin<=4	0.00	0.03
4<w/wmin<=5	0.03	0.04
5<w/wmin<=6	0.08	0.04
6<w/wmin<=8	0.09	0.04

Minimum wage is shortened using "wmin" and wage as "w"

To get a rough estimate of the level-effect, we simply post-multiply the β_j terms by w^{min}/w. Since the time period analyzed is between 1979 to 1997 we use the average real federal minimum wage of $6.74. This is clearly incorrect since minimum wages not to mention population sizes vary by state so that our estimates are most likely a lower bound. Average wages within each bin are calculated using the mean value of the upper and lower bound. For example, in the third wage bin we use the value $7.13 = \left[(6.74+0.1) + (1.1 \times 6.74) \right] / 2$. Thus we assume a uniform distribution of observations within each wage bin. For the first wage bin we assume a lower bound of half of the minimum wage, per the filter the authors apply (ibid.).

The result of the calculations can be read in Table B.2 and are sketched out in Figure B.1. It can be seen that once the effects are converted into level-form, that the minimum wage appears to have a U-shaped pattern which is pronounced at the lower and highest wage groups.

Table B.2 – NSW (2004) percentage and imputed level effects of the minimum wage on wages.

Wage Bin	$\partial\%\Delta w$ / $\partial\%\Delta mw$	Mean Wage	$\partial\Delta w$ / $\partial\Delta mw$
w< wmin -$.10	1.39	$ 5.01	1.03
wmin - $.1 < w < wmin + .1	0.79	$ 6.74	0.79
wmin + $.1 < w < 1.1* wmin	0.78	$ 7.13	0.82
1.1<w/ wmin <=1.2	0.41	$ 7.76	0.47
1.2<w/ wmin <=1.3	0.36	$ 8.43	0.45
1.3<w/ wmin <=1.5	0.26	$ 9.44	0.36
1.5<w/ wmin <=2	0.16	$ 11.80	0.28
2<w/ wmin <=3	0.06	$ 16.86	0.15
3<w/ wmin <=4	0.00	$ 23.60	0.00
4<w/ wmin <=5	0.03	$ 30.35	0.14
5<w/ wmin <=6	0.08	$ 37.09	0.44
6<w/ wmin <=8	0.09	$ 47.21	0.63

Minimum wage is shortened using "wmin" and wage as "w"

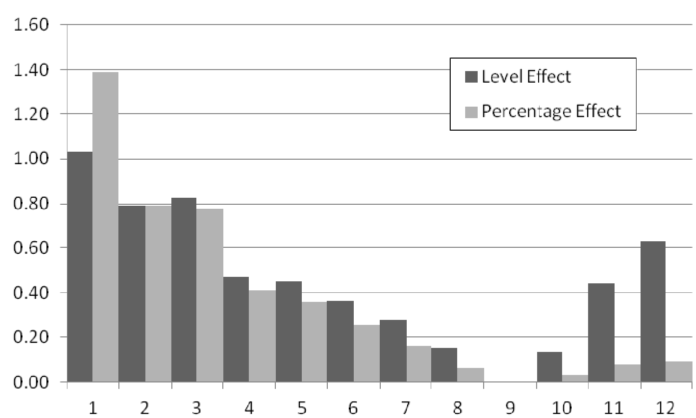

Figure B.1 – NSW (2004) percentage and imputed level effects of the minimum wage on wages.

Table B.3 – Robustness Checks for Level Estimates

Wage Bin	Standard Model	no filters	only NSW filters	no FE or AR1	no AR1	lagged Δw^{min} & relative wage	pre measure 25	post measure 25	with USA management wages	with USA GDP
1	0.92 (0.1)	1.89 (11.28)	1.46 (0.04)	0.92 (0.04)	0.51 (0.05)	1.21 (0.13)	2.73 (0.21)	1.58 (0.14)	0.78 (0.11)	1.65 (0.1)
2	1.18 (0.04)	1.78 (11.31)	1.3 (0.03)	0.82 (0.01)	0.76 (0.01)	0.96 (0.06)	2.51 (0.1)	0.89 (0.06)	0.92 (0.05)	1.21 (0.05)
3	0.97 (0.03)	0.5 (7.53)	1.09 (0.02)	0.58 (0.01)	0.52 (0.01)	0.82 (0.03)	2.1 (0.06)	0.79 (0.03)	0.81 (0.03)	0.96 (0.03)
4	0.84 (0.02)	0.86 (7.55)	0.97 (0.02)	0.41 (0.01)	0.38 (0.01)	0.74 (0.03)	1.94 (0.05)	0.74 (0.03)	0.7 (0.03)	0.81 (0.02)
5	0.74 (0.02)	0.49 (7.68)	0.88 (0.02)	0.31 (0.01)	0.33 (0.01)	0.69 (0.03)	1.84 (0.05)	0.73 (0.03)	0.63 (0.03)	0.73 (0.02)
6	0.81 (0.02)	0.63 (5.89)	0.96 (0.02)	0.31 (0.01)	0.35 (0.01)	0.77 (0.02)	1.93 (0.03)	0.82 (0.02)	0.69 (0.02)	0.79 (0.02)
7	0.84 (0.01)	0.09 (3.98)	1 (0.01)	0.26 (0.01)	0.38 (0.01)	0.93 (0.01)	1.74 (0.02)	0.92 (0.01)	0.71 (0.01)	0.82 (0.01)
8	1.03 (0.01)	0.19 (3.39)	1.18 (0.01)	0.29 (0.01)	0.53 (0.01)	1.27 (0.01)	1.51 (0.02)	1.13 (0.01)	0.82 (0.01)	0.98 (0.01)
9	1.74 (0.01)	0.45 (4.75)	1.91 (0.01)	0.2 (0.01)	0.83 (0.01)	1.99 (0.01)	2.86 (0.02)	1.74 (0.02)	1.41 (0.01)	1.63 (0.01)
10	2.3 (0.02)	4.91 (6.12)	2.45 (0.02)	0.29 (0.02)	1.19 (0.02)	2.64 (0.02)	3.99 (0.03)	2.24 (0.02)	1.89 (0.02)	2.12 (0.02)
11	2.54 (0.02)	12 (8.1)	2.68 (0.02)	0.31 (0.04)	1.59 (0.03)	2.87 (0.02)	4.87 (0.04)	2.43 (0.03)	2.07 (0.02)	2.37 (0.02)
12	2.57 (0.03)	22.16 (8.8)	2.81 (0.03)	0.89 (0.06)	2.28 (0.05)	3.83 (0.03)	2.93 (0.05)	1.81 (0.03)	2.08 (0.03)	2.53 (0.03)

Estimates for the effect of the change in the minimum wage on the change in wage. (Standard errors are displayed in parenthesis).

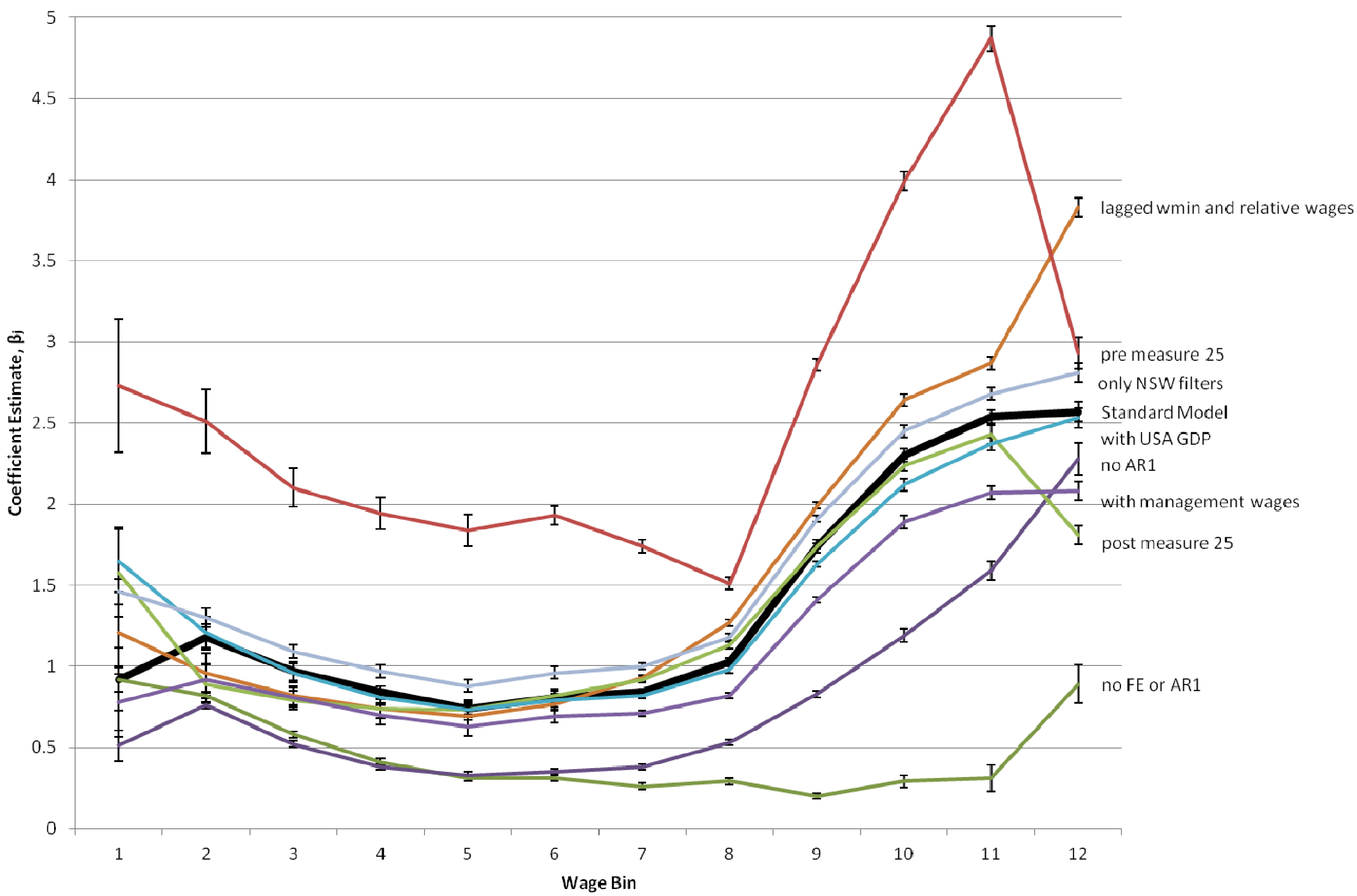

Figure A.4 – Robustness Checks for Level Estimates

95% Confidence intervals are constructed assuming a normal distribution due to large sample sizes. Results for the robustness check without filters are omitted since they are too large to be displayed.