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Title: A TREATISE ON STRUCTURES WITHOUT BENDING

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There are several factors which affect the cost and efficiency of a structure. The most important and basic factors are: the predominant stress, the way the structure carries the loads and the shape of the structure.

The structures in which the predominant stress is not bending have been chosen to be treated and discussed in this thesis. These structures will be called "Structures Without Bending."

These structures have been classified by grouping them into two main groups: one-way structures and two-way structures. Each group is divided into subgroups depending on the shape factor.

Some of the advantages and limitations of bending-free structures have been presented herein. The reader will also find many demonstrative examples and sketches.

A TREATISE ON STRUCTURES WITHOUT BENDING

by

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A TREATISE ON STRUCTURES WITHOUT BENDING

INTRODUCTION

1.1. Generalities

Structural design is that part of the general design which determines the structure of the building. It cannot be separated completely from the architectural design. The architectural design has to take the structural possibility into consideration and the structural design should take the architectural aspects into account as well.

If the structural possibilities were restricted due to the lack of some kinds of materials, interference will occur between the structural and architectural designs. In the past, when the structural possibilities were limited by the statical knowledge, both architectural and structural designs were overlapped in most of their effects. The design of any structure was influenced only by the choice of the ratios of the dimensions and the decoration of the structure under question. If the design range was limited, the importance of the design was not reduced.

It is easy to find several ancient monuments that are structurally the same; the only difference is that each monument has different dimensional ratios and different decorations from that of the others. Therefore each monument has different aesthetical and economical values.

Today the structural possibilities, in both structural materials and structural knowledge, give a much wider range in the design. Therefore the design becomes almost free.

The main point which will lead us in choosing a certain structure will be the economical point of view. This does not mean that other points have no effects. On the contrary, there are many other important factors; e.g., general appearance and representative and symbolic shape of the structure. These factors are usually considered as primary or basic factors just as well as are the purpose of the building and available materials and techniques. These points restrict the designer's freedom when designing the structure under question. But there are several solutions within this restricted range of design freedom from which the most economical solution should be used.

In ancient times, when only the ratios of certain dimensions were changeable, the designers felt that they needed some principles which create order and harmony. For example, they found that the most pleasing rectangle is the one which has length over width ratio equals "1.618" (Figure 1). Today, there is an enormous and huge variety of structural possibilities available in the designer's hands. Therefore the main regulating role is most likely the question of economics.

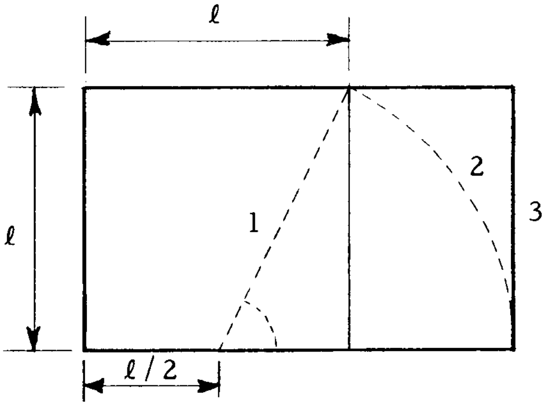


Figure 1

1.2. Factors Affecting the Cost and Efficiency of a Structure

There are several factors which affect the cost and efficiency of a structure. The following are the most important and basic factors:

1. The predominant stresses applied on the main cross sections of the structure.
2. The way the structure carries and transfers the loads to the soil.
3. The shape of the structure and its substructures or elements.

The predominant stress is defined as the kind or state of stress which is applied or exerted on most of the cross sections of the structure or its substructures. It is also of importance whether the structure carries the loads in one way or two ways. For example, a one-way slab is more expensive than a two-way slab.

This iilustrates the second factor. The shape factor has a considerable effect on the labor costs. For example, scaffolding costs are higher for a barrel shell than for a slab.

These three factors are sometimes controversial; for instance, the labor work is less costly for a flat slab than for a shell, and it is easier and less expensive to build a straight beam than a curved one. On the other hand, it is preferable to have curved structures instead of straight ones, because in this case the predominant stress would be compression or tension instead of bending. This will reduce the amount of the materials needed for construction and eventually the overall cost of the structure. It is also to be noted that the significance of a factor over another depends on the place and the country where the structure is to be built. For example, the shape factor has much more influence in the United States than it has in the Far-East. Therefore the structural designer should find a compromised solution which gives the most economical and efficient structure.

It is the aim of this thesis to treat and describe the structures in which the predominant stress is not bending. These structures will be called "Structures Without Bending".

The most important bending-free structures will be described by grouping them into two main groups according to the second factor, One-Way Structures and Two-Way Structures. Each of these two main groups will be divided into subgroups according to the shape factor. In this way the structures without bending will be classified.

1.3. Loading Conditions

On the cross sections of the structure the type of the stress may change when the load changes. Therefore it is necessary to mention the effects of the changes in loading conditions.

There is only one part of the load which is constant in magnitude while the other parts are variable. It is to be noted that some of the bending-free structures could change in shape when the loads change, since the shapes of such structures are determined by the load. This kind of shape changing should not be permitted and the structure must be designed in such a way as to secure practically a remaining constant shape.

It will be easier to deal with this question if the loads are divided into two groups, the loads of constant character and those of variable character.

The values of the loads of constant character may change in magnitude but not in character; for instance, a uniformly distributed load may change in its intensity but it remains always uniformly distributed. In other words, every change of the load may be obtained by multiplying the load by a changing parameter. So, instead of a load of constant character it may be considered as a variable load with one parameter only.

A load of variable character will change not only in its value but also in its character; for instance, a load that was uniformly distributed will change to a non-uniformly distributed load. In such a case, the load may be said to change with several parameters.

The effect of load changing becomes more significant when a structure is built using very high strength material. This is true especially if the change in the live load is very big compared with the dead load. This is the case, for instance, in suspended roofs made of high strength cables. If for example, these cables have a design stress of 172 ksi and the dead load is one-half of the full load, then the change in the cables stress is 86 ksi. The strain in the cables will be 0.003. In this case if the span was 165 ft. and the deflection of the cables under the dead load was 16.5 ft., after applying the full load the deflection will increase by 12 in. which may be objectionable. However, this is an exceptional case, and usually the change in the load magnitude gives no trouble.

Loads of changing character cannot be allowed in the case of structures which can work only in compression or only in tension. If the material used in building the structure is good for tension and for compression as well, then the changing character of the load makes no trouble but it affects the economical efficiency unfavorably.

Special care should be taken in case of having loads which change in sign as well as in character or magnitude. For example, it is impossible to build a statically determinate structure from a one-strength material if the load were changeable in sign. If the structure is statically determinate, a double-strength material has to be used. In the case of indeterminate structures, a single-strength material may be used. But in this case, the element(s), which can resist only stresses of a specific sign, should never get stresses of opposite sign.

However, for simplification purposes only, the effects of variable loading conditions will not be considered. The structures without bending will be treated on the basis of the most characteristic load so called "the main load". In the structures which have large horizontal dimensions compared with their heights, own-weight load will be characteristic, while for the structures that have large vertical dimensions as compared with their horizontal ones, the wind load will be characteristic. The sum of the dead load and the average of the live load will be considered as the design load. For this type of load, the shells are usually designed in a way not to have bending. In this case, the predominant stress on the main cross section is compression only. This state of stress will change if the snow or the wind load has changed with several parameters. In the case of loading change with several parameters, bending stresses could be obtained. But for the purpose of simplification and classification, this effect will not be considered. This does not imply that the bending stresses should not be checked.

STRUCTURES WITHOUT BENDING

2. 1. Definition

Structures without bending are the structures which resist the loads by means of one or any combination of the following stresses: Compression, tension and/or shear. In other words, the bending-free structures are the structures in which the predominant stresses, under the main load, are not bending stresses.

The previous definition does not mean that there is no bending stresses in some cross sections of the structure. The existance of such bending stresses is due to the following reasons:

1. Approximations done by the designer. These approximations depend on some general rules and, to a greater extent, on the designer's judgement.
2. Defects in workmanship and difficulties faced by the constructors when erecting the structure assigned by the designer. For example, some defects in workmanship could happen when building a parabolic arch, because after construction the shape would not have the exact shape of a parabola. The hinges and the joints of a truss are another example because the members of a truss are usually assumed to be pin-connected. It is possible to make true pin-connections using special materials and instruments,

but it is too expensive to be practical.

3. The change in load to other than the main load which the design of the bending-free structure was based upon. This effect has been discussed in 1. 3.

The first of the above mentioned reasons for the presence of bending is of minor effect if the designer has used common sense. The second reason sometimes has considerable effect. For example, the joint stiffnesses of a long-span truss result in considerable bending moments. In this case the designer should check these bending stresses by means of a proper method, moment-distribution for instance. The third reason for the presence of bending moment is sometimes of greater importance. This will not be considered here (See 1. 3.)

2. 2. Load-Shape Relationship

It will be helpful to deal with load-shape relationship in a bending-free structure by taking the free-body diagram of a small part cut from the structure under question.

1. Linear Structures: If the small part is straight, it cannot resist any load perpendicular to its axis without undergoing bending stresses (Figure 2).

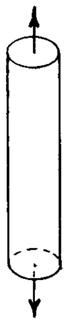
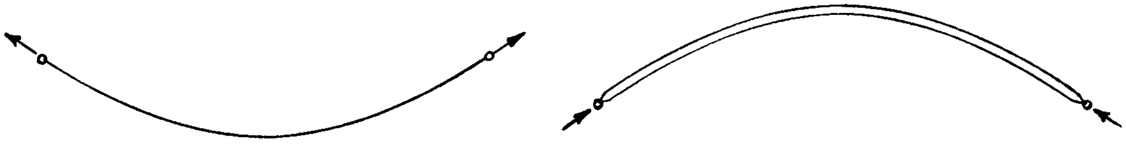


Figure 2

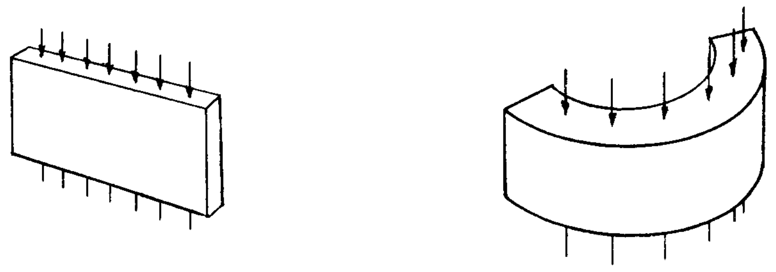
Consider a cable suspended from both its ends. This cable will take the shape of a catenary under its own weight (Figure 3-a). In this case the main stress is tension. If it is desired to use a compression resisting material, the catenary should be concave upward (Figure 3-b).



(a) (b)

Figure 3

2. Surface Structures: If the small part cut from the surface is flat, it can carry forces only in its plane without undergoing bending stresses (Figure 4-a). A curved surface structure may have a cylindrical shape if the forces act parallel to the generatrices (Figure 4-b).



(a) (b)

Figure 4

If the surface structure will carry loads perpendicular to its middle surface, then the shape of the surface should be curved in order not to have bending stresses. This surface can be one-way curved (Figure 5) or two-way curved (Figure 6) depending on loading and support conditions.

(a) tension (b) compression

Figure 5

One-way curved shells without bending have uniform normal stresses if they were loaded uniformly (Figure 5). For the same loading condition membrane shells will have normal stresses which have uniform horizontal components. Membrane shells have the state of equal normal stress in all directions if the load is perpendicular to the surface at each point.

(a) tension (b) compression

Figure 6

2.3. Bending Moment and Cost Relationship

The relation between the bending moment and the overall cost of a structure cannot be expressed by a general formula. This is true because there are too many factors involved in the problem. For example, labor cost and material cost each varies almost independently from one country to another and even from one place to another in the same country. Labor cost over material cost ratio varies widely depending on several factors that cannot be expressed in a formula. For example, this ratio depends on the place where the structure is to be built, on the availability of the materials and skilled labor, and even on the standard of living of the country.

However, an approximate relation between bending moment and material cost could be developed for each kind of material. A simply supported reinforced concrete beam will be considered as an illustrative example.

The beam is loaded by a uniformly distributed load. It is assumed that both the span and the cross section are given (Figure 7).

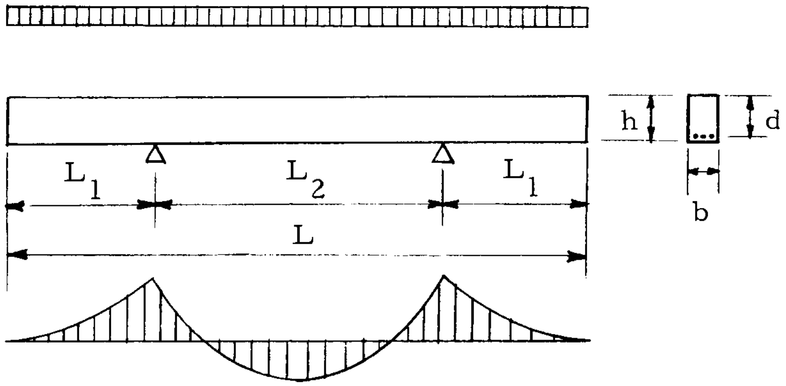


Figure 7

Since the cost of the reinforcing steel is of major significance, it is required to find the position of the supports for which the least amount of steel is used.

The amount of steel needed for a cross section, resisting a bending moment M, is given by:

$$A_s = \frac{M}{jdf_s}$$

The value of j varies between 0.85 and 0.96, where d and f_s are constant in this example. Since the variation of j is small, it could be considered as constant and equal to 0.9. Therefore:

$$A_s = \frac{M}{C}$$

Assuming that only the required amount of steel is used at each cross section, the volume of the steel needed for the whole beam is given by:

$$V_s = A_s L = \frac{ML}{C} = K(LM)$$

This means that the amount of steel is proportional to the area under the bending moment diagram. Therefore the least amount of steel corresponds to the minimum area under the bending moment diagram.

The minimum area under the bending moment diagram is obtained by locating each support at a distance (L_1) from the beam end in such a way that

$$\frac{L_1}{L} = 0.235 \cong 0.25$$

This example shows that the material cost decreases with the bending moment. It follows that one of the advantages of structures without bending is the saving in material cost.

ONE-WAY STRUCTURES WITHOUT BENDING

One-way structures without bending will be divided into two groups, linear structures and surface structures. The most important one-way structures without bending will be treated and described in the following paragraphs.

3. 1. One-Way Linear Structures Without Bending

The predominant stress on the main cross sections is either tension or compression. If a double-strength material is to be used, it is preferable to have the linear structures undergo tensile rather than compressive stress because the structures under tension regain their shapes after reducing or removing the loads if the stress did not exceed the proportional limit. While, due to the buckling effect, the structures under compression will regain their shapes only if the stress did not violate the critical limit. Therefore the material of the structures under compression is not fully used. Unfortunately the use of structures under tension is not always possible.

Theoretically it is possible to build structures with tensile stresses only. For example, a tubular column can be sealed and prestressed by interior air pressure to a certain limit so that the stresses in the column will be tensile stresses even after applying the loads (Figure 8). But the application of such structures is very limited in practice.

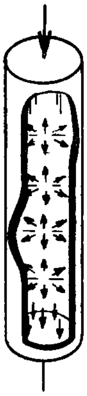


Figure 8

It is also to be mentioned that in case of loads with variable character, the problem of maintaining a constant shape arises in case of structures under tension and the solution of such problems increases the cost.

3. 1. 1. One-Way Straight Line Structures

Straight line structures are the most practical structures without bending. This is true because of their simple shape which reduces the labor cost and the material can be fully used as was mentioned previously. Since these structures can be easily prefabricated, the overall cost decreases appreciably especially if the similar members are used in large numbers.

If the straight member is under tensile stress, the cross section will be constant along the whole length of the member. This is not the case for members under compression because of the buckling effect. In order to prevent buckling, the cross section of the member should be variable. Theoretically the increase of the cross sectional area should be done gradually from the member's ends to the center (Figure 9). Although the required material is minimum, this solution is not economically the best since the labor cost will be higher. In addition this shape may not be pleasant.

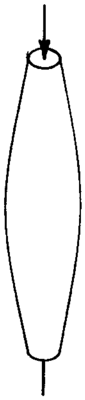


Figure 9

There are many ways of changing cross sections which sometimes depend on the material used in construction. It is to be noted that the change in cross section should be made as continuous as possible. Abrupt changes may also be used if they were reasonably made. It would be helpful to give some examples for different kinds of materials.

In the case of metals the change in cross section could be made by welding metallic parts at the middle part of the element (Figure 10).

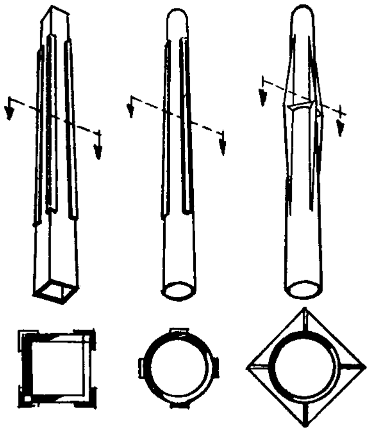


Figure 10

For wooden columns the change can be made as shown in Figure 11.

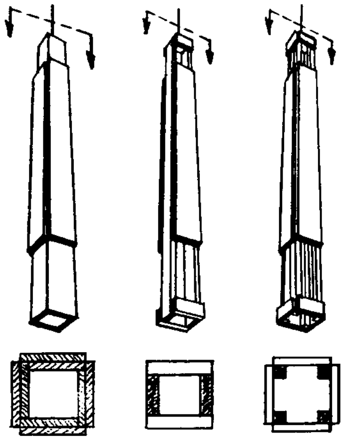


Figure 11

In case of reinforced concrete columns, the change may be easily made using molds which can be shaped as desired (Figure 12). This could be more economical if the columns were prefabricated.

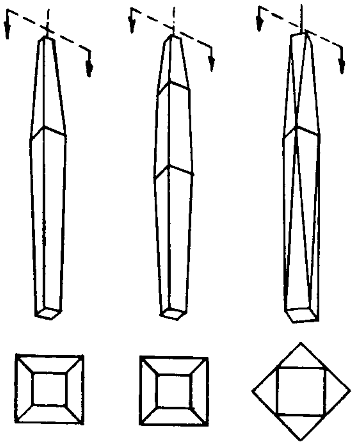


Figure 12

If the direction of the principle axis of a column cross section is different in the lower level from that of the higher level, the buckling may be prevented even without increasing the cross section. This could be done by increasing the smaller moment of inertia and reducing the bigger one constantly. This is illustrated in Figure 13.

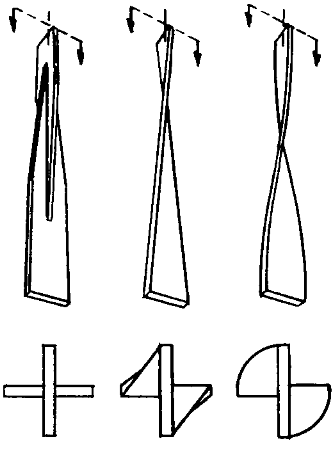


Figure 13

Changing the cross section is not always the best solution to prevent buckling. Larger cross sections are sometimes more economical. For example, if the number of similar columns is not large enough, variable cross sections might not be justified. From the aesthetic point of view, it is sometimes perferable to have columns with larger constant cross sections rather than columns with variable cross sections.

From the economical point of view, it would be advantageous to use materials which have only tensile strength for supporting compressive forces. A theoretical example of this has been described in 3.1 (Figure 8). Practically, this principle could be applied by filling a tubular column with a material without strength, like sand for instance, and by applying the load directly on the sand. The tube will undergo hoop tensile stresses and longitudinal compressive stresses due to the friction which is of minor effect. Take for example a paper cylinder filled with sand and apply a reasonable axial load. If the paper is cut by a razor blade in the transverse direction, the load-bearing capacity of the column will not be decreased. But if the cut is made longitudinally, the paper will be torn along the whole length and the column will collapse (Figure 14). In practice, reinforced concrete columns with spiral reinforcement are based on the same principle.

In the preceding paragraphs single elements of straight line structures have been treated. Structures which are combined from two or more elements will be discussed in the following paragraphs.

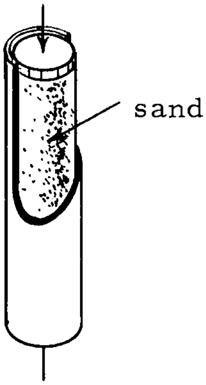


Figure 14

Composite straight line structures without bending should have the loads applied only at their joints. For structures made from two elements, the loads which produce compression or tension in both elements will not be considered.

If the force does not fall between the two elements and it has permanent direction, then the stresses will always be compressive in one element and tensile in the other. Therefore a proper single-strength material may be used for each member (Figure 15a). If the direction of the load is variable, several solutions could be made. If it is desired to use single-strength materials, the problem can be solved by adding another member to the structure (Figures 15b and 15c). In this case only two members work at a time. Using a double-strength material the problem can be easily solved. A structure made from two members could be used. In this case the structure is statically determinate (Figure 15d). If more than two elements were used, the structure will be statically indeterminate (Figure 15e). Obviously the structure illustrated in Figure 15b is more practical if single-strength materials are to be used, and that illustrated in Figure 15d is more practical for double-strength materials. It has been shown that even the simplest problem can be solved in several ways and in some cases it is practical to choose a non-conventional solution.

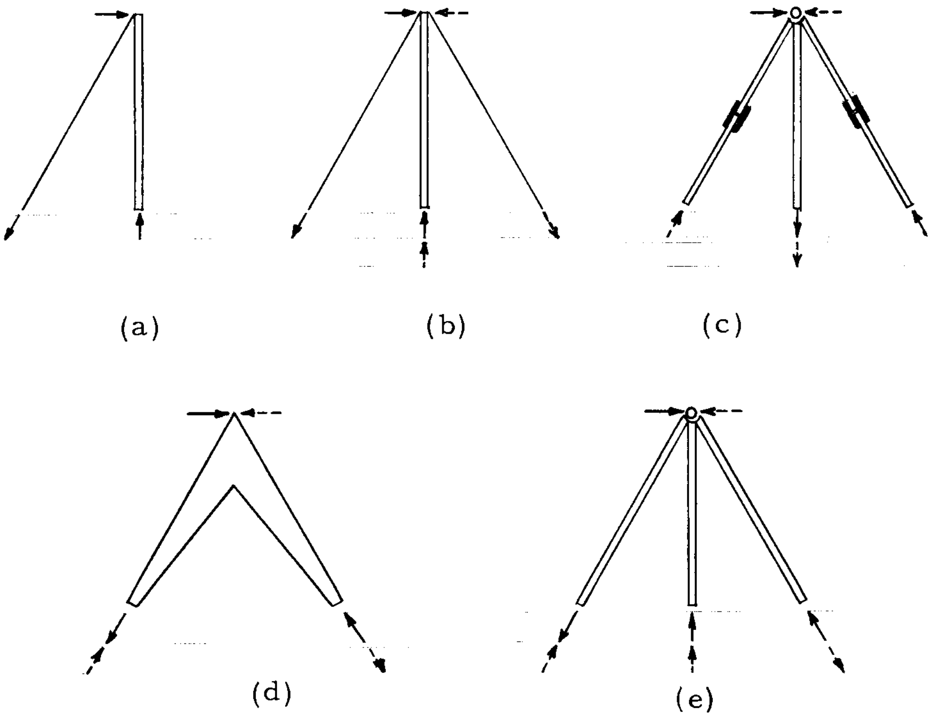


Figure 15

The sketch of the V-pillars illustrated in Figure 15d shows that if the bars are not pin-connected, it will be logical to increase the cross section toward the common point. Since such kind of pillars are practical in use, they should be investigated further.

The dimension perpendicular to the plane of the V-pillar is not always constant (Figure 16a). If there is a secondary moment perpendicular to the plane of the V-pillar, the width may be increased upward (Figure 16b), or downward (Figure 16c).

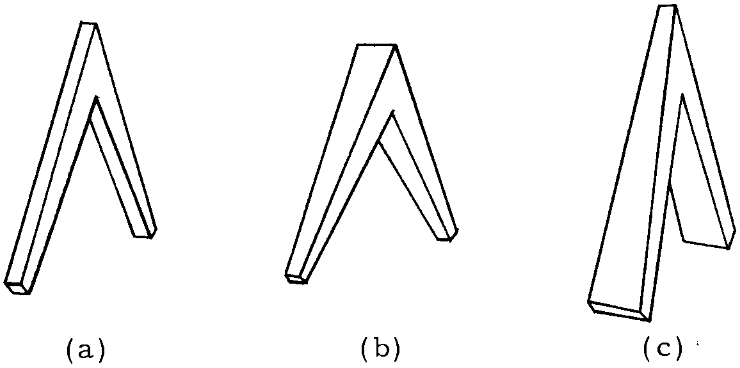


Figure 16

The so called Y-pillars which are composed of three elements may also be increased in width toward their ends. The dimension perpendicular to the plane of the Y-pillar may be constant (Figure 17a), increasing upward (Figure 17b) or downward (Figure 17c) following the direction where the secondary moment increases.

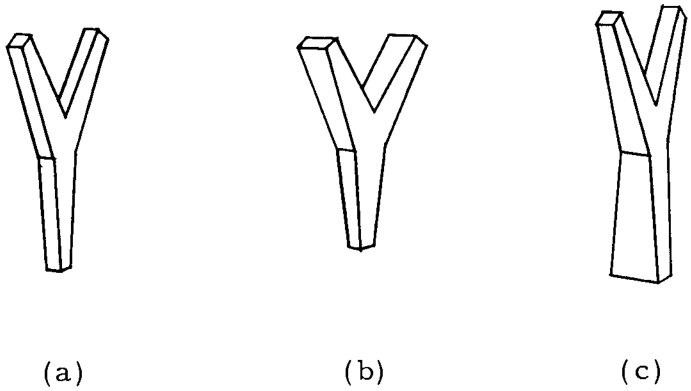


Figure 17

Comparing V-pillars with Y-pillars it is obvious that unlike Y-pillars, V-pillars can resist horizontal forces without undergoing bending stresses. The V-pillars can be constructed so that its opening could be either downward or upward. Both V- and Y-pillars are useful for reducing the span or reducing the number of supports in addition to the fact that both kinds give more useful space in the floor where they are used. But Y-pillars give more vertical space than V-pillars do.

Among the one-way structures built up from several straight elements are the one-way trusses. It would be almost impossible to present and discuss all the possible shapes of trusses. The reader is advised to refer to a proper reference in the field of theory of structures or strength of materials. But it would be advantageous to mention some viewpoints about trusses.

The economy and practicability of framework structures depend on the materials and skills that are available in the area where the structure is to be built. The materials that are usually used are steel, wood and even reinforced concrete. But steel is the most widely used material for framework construction.

One-way trusses are mainly used in bridges and roofing systems. But reinforced concrete structures with or without bending have been widely used replacing framework steel structures. This is true because reinforced concrete, as a material, is less costly, has a better formability and is easy to maintain in addition to the fact that reinforced concrete structures are aesthetically more pleasing than framework steel structures.

On the other hand, framework steel structures are sometimes more effective. For example, since the members and the detailed part of a truss could be prefabricated, the erection of such structures can be quickly achieved. This is very important especially in emergency cases. In military actions, for example, it might be required to construct a bridge within a short time. This could be very easily done especially if the truss is built up from similar members or parts (Figure 18).

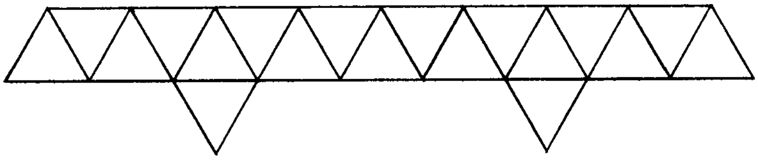


Figure 18

The work could be simplified if the truss is made from similar parts which can be assembled by means of joints and single bars (Figures 19, 20, and 21). As the size of the similar parts increases, the construction time decreases . The truss shown in Figures 19 and 20 are exactly the same; the difference is that the truss of Figure 19 is built from similar triangular parts while that of Figure 20 is built from similar rhomboidal parts which require shorter time to assemble. Furthermore, different size and spans could be made using the same parts. For example, using trapezoidal parts, different kinds of trusses could be built (Figure 21).

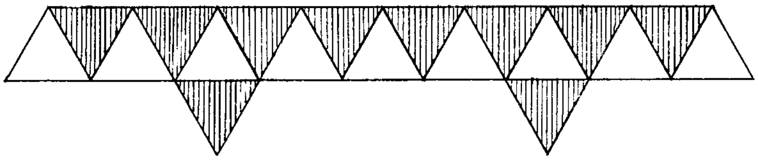


Figure 19

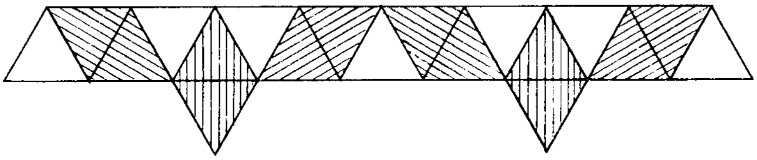
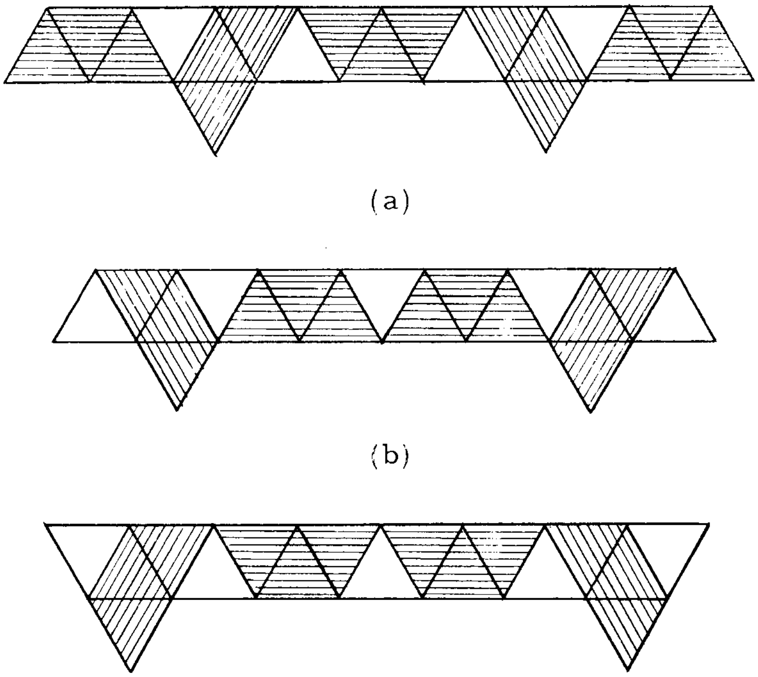


Figure 20



(a)

(b)

(c)

Figure 21

In roofing systems, one-way trusses are usually placed apart, one beside the other. In order to restrain the trusses against lateral movement, bracing members are to be used to connect the upper joints of one truss to the adjacent ones (Figure 22). The design of the covering system should be made in such a way that the loads will be transferred directly to the joints.

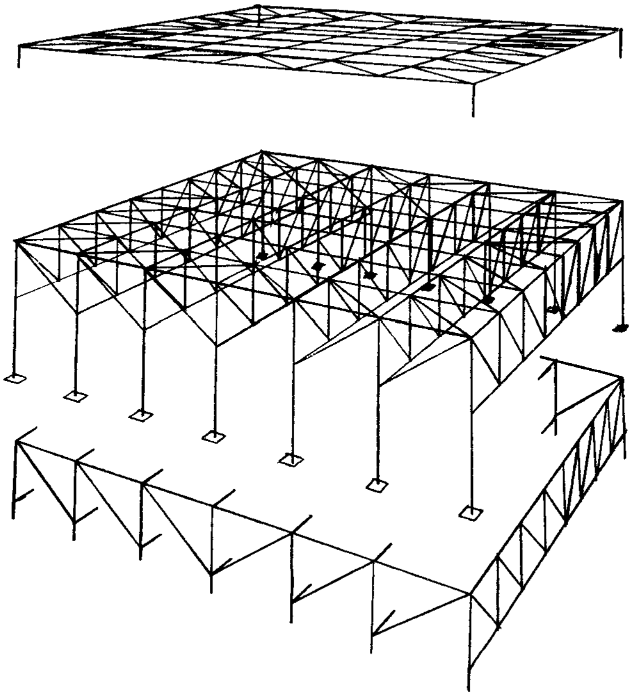


Figure 22

It is to be noted that bracing members are not considered as working members and they do not increase the capacity of the structure. These members prevent lateral displacement of each truss. Although the bracing members undergo some stresses, the loads are transferred in one direction and the structure is still behaving as one-way structure.

3.1.2. Plane Curved One-Way Line Structures

A rope which is loaded not only at its ends but also along its length will take the shape of a plane polygon if the loads were lying in the same plane (Figure 23). In case of a horizontally uniform load, the rope will have the shape of a quadratic parabola (Figure 24). If a heavy cable is used and the load is uniformly distributed along its length, it will take the shape of a catenary.

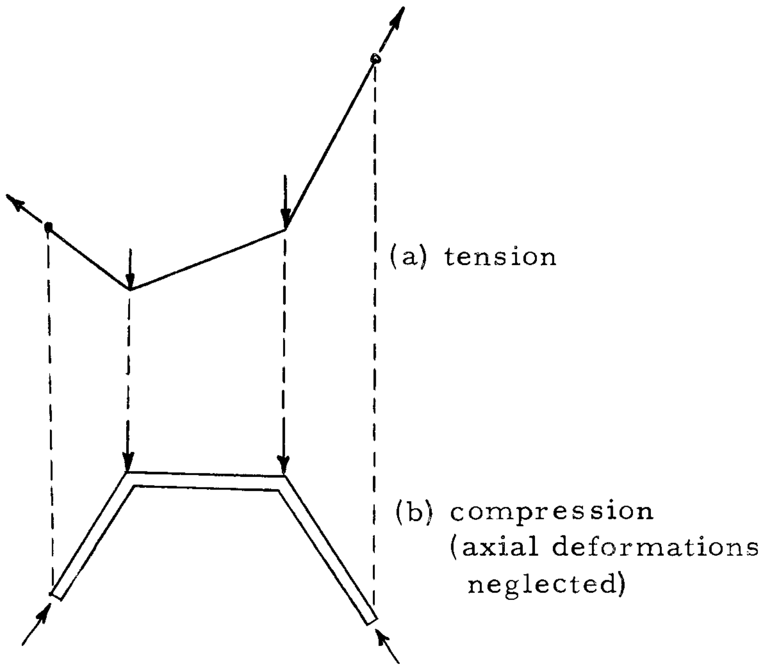


Figure 23

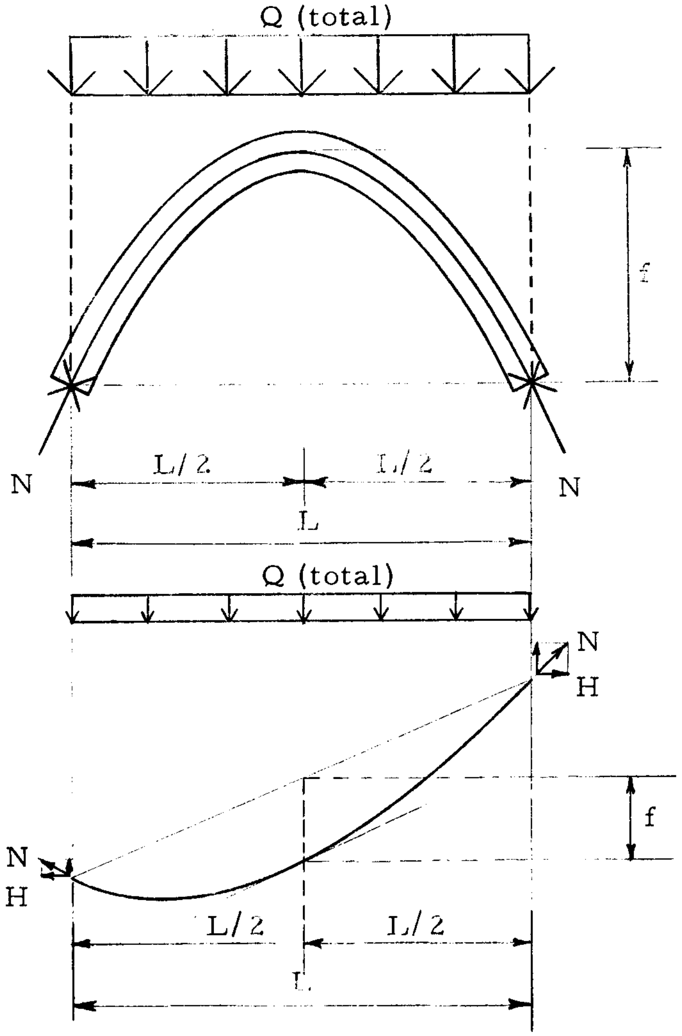


Figure 24

In the preliminary design, approximate values of the forces acting on the cross section of the structure should be found to decide whether the structure could be accomplished or not. The horizontal component of the reaction at the support could be approximated by

$$H = \frac{QL}{8f}$$

which is also equal to the axial force at a point where the tangent is horizontal. In the case where the supports are at the same level, the maximum value of the axial force could be approximately evaluated by the following formula

$$N = \frac{QL}{6f}$$

This formula shows that smaller values of f give bigger values of N. Therefore it is recommended not to use cables or arches with small cambers. Otherwise high stresses will take place which will lead to higher costs in material and construction. This is true especially in case of arches with small camber because buckling danger will exist which will increase the costs. It is usually recommended to use arches in which camber over span ratio is equal or greater than 1/8. Arches with smaller ratio could be used if it proved to be better than any other alternative.

Usually there are two major problems encountered in arch structures. The first is the question of horizontal reactions and the second is the buckling effect.

The horizontal reaction could be absorbed directly by the soil. This can be done only in case of a very good soil and this solution is usually adopted for high arches. If the supports of the arch are under the ground, the horizontal reaction can be taken using tie-beams. If the support of the arch is higher than the ground level, tie-beams can be used if they don't occupy useful space. If this cannot be done and the soil is not capable of taking the horizontal force, vertical and oblique piles are recommended. Still there are many other possible solutions to this problem and the designer should use the most pleasing and economical one.

It is more economical to change the cross section in direct ratio with the axial force if the arch is secured against buckling in some way or another. This means the cross section should increase toward the supports. But if buckling danger does exist, the cross section should increase toward the crown. In this case a proper constant cross section would be logical.

For cable structures the problem of absorbing the reaction is rather difficult. Since the tensile strength of the soil is small, it is too expensive to take the reaction by the soil. If the anchorage points of the cable are slightly higher than the ground level, the best solution is to use piles. But in this case the piles will undergo bending stresses (Figure 25).

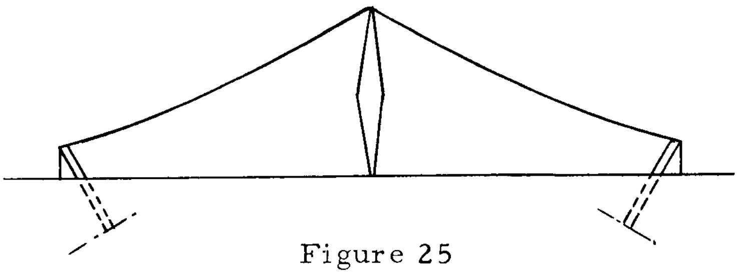


Figure 25

If the anchorage points are high enough, the reactions could be taken by the outside walls which will resist these reactions by bending

(Figure 26). In the case where the anchorage points are under the ground level, the reactions could be taken either by piles or by beams under compression. These beams could be secured against buckling either by the surrounding soil or by cross beams. The same principle is used in case of a one-way curved plate under tension as shown in Figure 33.

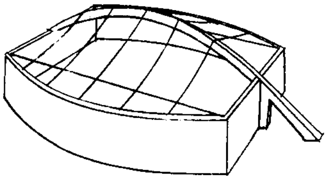


Figure 26

In addition to their simple stress action, plane curved one-way structures prove to be very useful as roof systems for covering large areas such as airplane hangars (Figure 27), colliseums, etc.

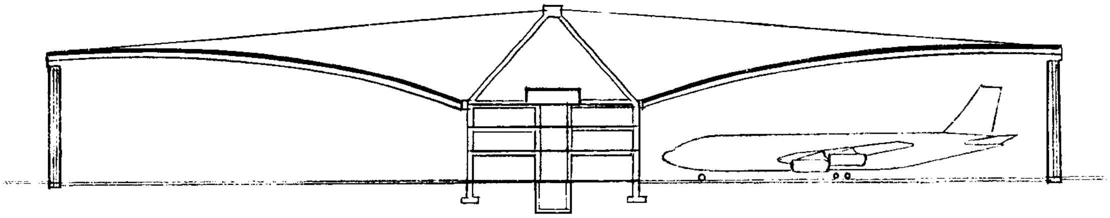


Figure 27

3.1.3. Space Curved One-Way Line Structures

If the forces acting on a cable are not parallel to each other, the cable will be curved in space. If these forces were concentrated forces, the cable will take the shape of a space polygon. Such a system of forces cannot be produced by weights. Therefore these structures are usually considered to be secondary structures or stiffening structures.

This kind of structure is very limited in practice. Therefore further detail will not be presented. As a demonstrative example Figure 28 shows two space curved cables carrying a waterpipe crossing a river.

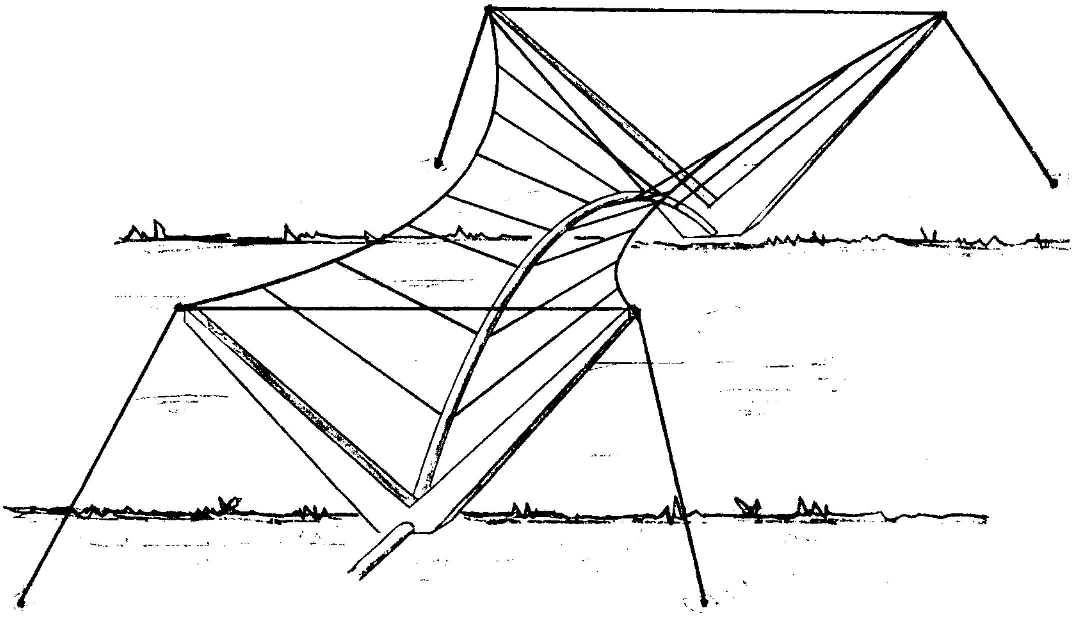


Figure 28

3.2. One-Way Surface Structures Without Bending

Surface structures will be divided into two groups: namely, plane surface structures and curved surface structures. Since plane surface structures without bending cannot carry loads perpendicular to their plane, they will be called wall structures.

3.2.1. One-Way Wall Structures

Wall structures behave actually the same as one-way linear structures without bending. The only difference is that wall structures have two dimensions. The buckling danger is usually treated differently. This could be done by increasing the moment of inertia instead of increasing the thickness. Figure 29 illustrates the different ways with which buckling danger could be prevented.

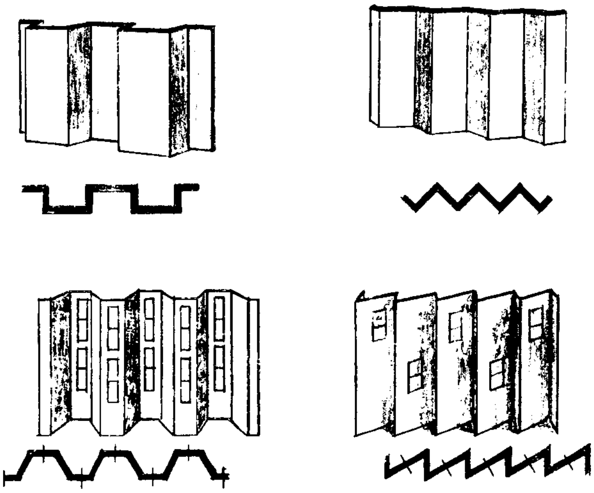


Figure 29

Curved walls could be treated as plane wall structure if the curvature is perpendicular to the direction of the stress action (Figure 30).

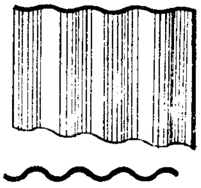


Figure 30

3.2.2. Curved One-Way Surface Structures

Curved one-way surface structures are usually curved in one direction. The direction of the main curvature follows the direction of the stress trajectories. In other words, one-way curved surface structure should be curved in one way only and the two edge generatrices constitute the springing lines (Figure 31). The directrice could be either a circular arc, parabola or a catenary, depending on the main load function. Circular and parabolic directrices are used most often.

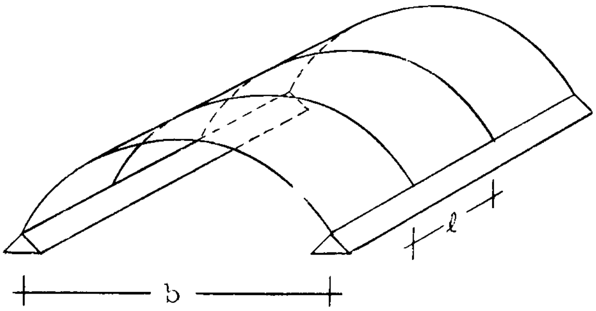


Figure 31

Sometimes the use of the structure requires the establishment of some side openings or side entrances. This could be done by increasing the thickness of the shell where the openings exist. That is, the surface structure should have the required cross sectional area everywhere. Figure 32 shows two different ways of maintaining the required cross section where the side openings exist.

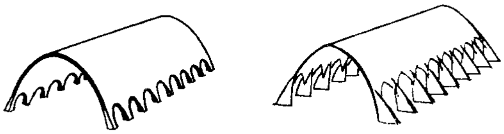


Figure 32

Everything which has been said about curved line one-way structures without bending applies to curved one-way surface structures without bending, since both kinds have the same type of stress behavior.

Curved one-way surface structures are usually built with hogging curvature. In this case a compressive strength material must be used. The buckling effect appears to be a major problem. Buckling could be prevented either by increasing the cross section or by corrugating the surface to increase the moment of inertia.

These surface structures could also be built with sagging curvature if a tensile strength material is to be used (Figure 33). The sketch shown in Figure 33 represents an underground storage structure roofed by a steel plate under tension. The reactions at the anchorage points are taken by compressive members. The cross girders and the surrounding soil prevent these members from buckling.

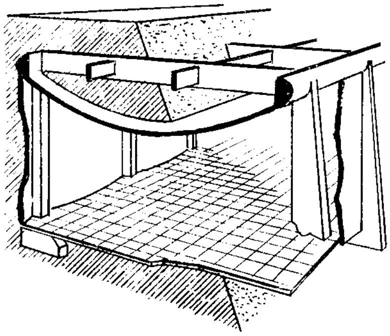


Figure 33

TWO-WAY STRUCTURES WITHOUT BENDING

Two-way structures without bending include linear structures and surface structures. Linear structures will be divided into two groups; namely, straight line structures and curved line structures. Surface structures will be also divided into two groups according to their behavior rather than their curvatures. These two groups are two-way membrane structures and two-way shell structures.

When speaking about membrane and shell structures without bending, the term "two-way" will be dropped simply because all kinds of membrane and shell structures behave as two-way structures.

4.1. Two-Way Linear Structures Without Bending

The predominant stress in the main cross sections is either tension or compression. Everything that has been said about one-way linear structures regarding material use and buckling applies to two-way linear structures.

Obviously one single linear member cannot be a two-way structure. A two-way linear structure without bending should be built with at least two linear elements. These elements should be assembled and connected properly to form a real two-way linear structure. For example the structure shown in Figure 34 cannot be considered as a two-way structure, while the structure shown in

Figure 35 is a two-way structure. The difference between these two structures is that each member of the first structure carries the load (or part of it) individually, while all members of the second structure carry the load cooperatively.

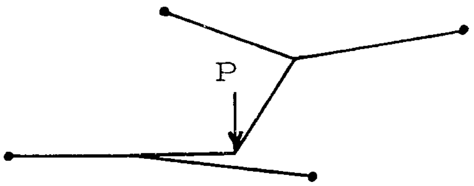


Figure 34

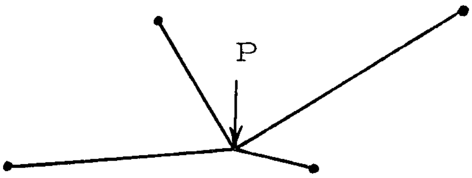


Figure 35

4.1.1. Two-Way Straight Line Structures

Since these structures have regained their importance, more details will be presented.

The simplest two-way straight line structure without bending is the one constituted from three bars not in a common plane carrying one concentrated load (Figure 36).

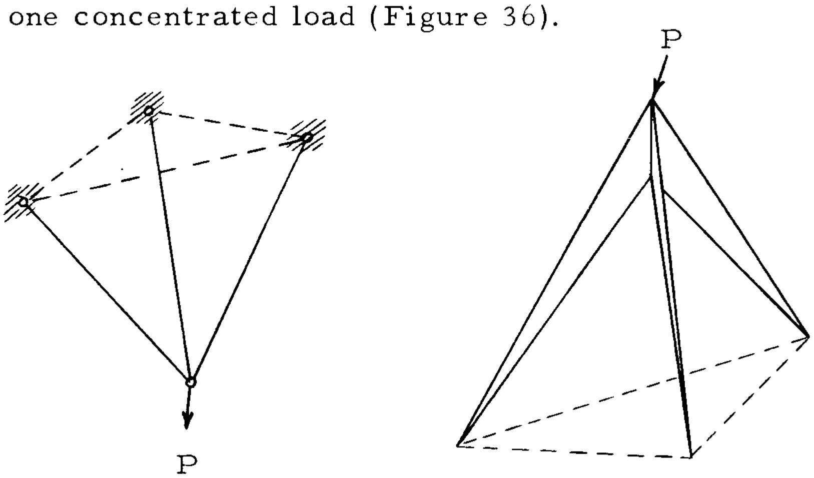


Figure 36

A two-way bar-structure cannot be supported by less than six bars. The arrangement of these six bars is also of importance. In order to illustrate the supports arrangement and the way of maintaining a constant shape of a two-way cord-structure, the structure shown in Figure 37 will be discussed.

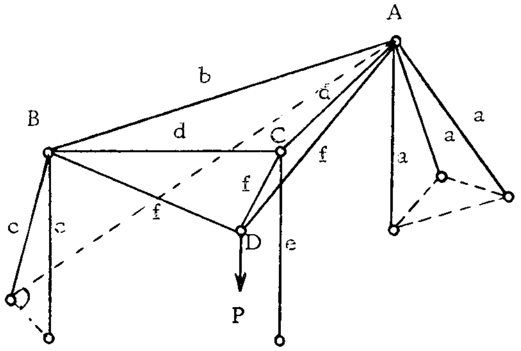


Figure 37

The structure of Figure 37 is presented here to show how the basic rules could be applied on such kinds of structures.

It is obvious that a point in space can be fixed if it is connected with other fixed points by means of at least three rigid bars provided that these three bars should not be in one plane. Therefore point A in Figure 37 is fixed by the three bars indicated by the letter a. These bars do not lie in one plane. Point B is also fixed because bar b connects it with the already fixed point A and the two bars c connect it with the ground provided that the plane formed by the bars c does not contain bar b. Point C is connected with the fixed points A and B by bars d and with the ground by bar e provided that bar e does not lie in the plane ABC. Point D is now fixed by connecting it to the three fixed points A, B and C by means of bars f.

Let us consider now the effect of removing one of the supports. It is obvious that the tetrahedron ABCD is of constant shape. Considering this tetrahedron as a rigid body, its means of support will be investigated (Figure 38). Point A is fixed by the three bars indicated by the letter a. If bars c and bar e do not exist, the body is free to rotate around point A in all directions. If bar e alone is missing and if the plane formed by the two bars c does not pass through A, the body can rotate around the axis AB only. If bar e is added, the body will be fixed provided that bar e does not lie in the plane ABC.

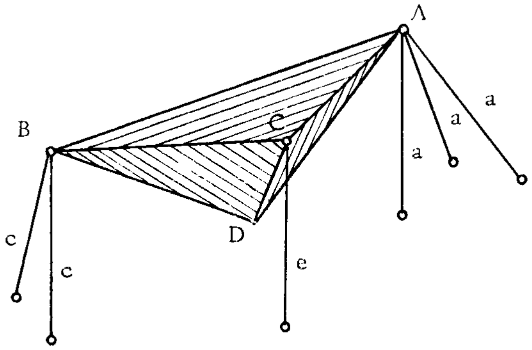


Figure 38

The above described method of constructing a stable two-way straight line structure is not the only possible method. There are several other methods which will not be presented here; the reader is advised to refer to a proper reference.

The uses of two-way straight-line structures are too numerous to be explained satisfactorily in this thesis. Therefore only a few typical structures will be presented.

Two-Way Trusses. A two-way truss is essentially a series of trusses intersecting in a consistent grid pattern and rigidly connected at the points of intersection. Such systems are statically indeterminate and the structural analysis may be quite complex (Figure 39).

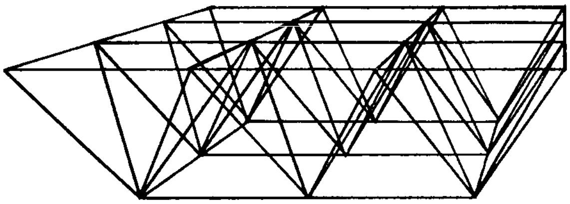


Figure 39

Fabrication and erection considerations require a careful study of the joints and their details. However, two-way truss systems may provide several significant advantages and benefits. These include savings in weight of the structure, reduced depth of construction, repetition of detail due to symmetry, and stronger definition in architectural form.

Steel-Framed Spherical Domes. A dome roof is a very economical structural solution when a large round or square-shaped area must be covered with a high roof without interior columns.

Its applications are found particularly in such structures as auditoriums, arenas, stadiums, exposition halls, etc.

Of the several types of space structures considered, the dome has the longest history as a roof type. Its construction can be traced to the days of the Roman empire when stone was the principal construction material and the shell theory of domes was intuitively recognized.

Interest in dome construction was sporadic until after World War II. Then improvements in structural analysis and welding techniques, as well as the availability of alloy steels, created a resurgence of interest, and the steel-framed dome has emerged as an important and economical structural form.

Very recently, important weight savings and new structural patterns have appeared with the introduction of the lamella and geodesic domes. The most recent lamella dome, which was beautifully built, is the dome of the United States pavilion in Expo 67.

Steel-Framed Shells. This kind of structure is usually made from steel triangular mesh. If the horizontal projection of this mesh is a mesh composed of regular triangles, then the actual mesh has a correct shape although the traingles themselves are not regular. In the case of such a steel-framed shell, it could be expected that the bar-forces are not of equal magnitude. However, the framing could be designed in such a way that, under the main load, the horizontal components of the bar-forces will be equal. In this case the characteristic shape of the shell is a result of the calculations. Only the height of the shell can be regulated by changing the value of the horizontal component of the bar-force which could be previously assumed. Thus, the design freedom will be limited, but the resulting structure is economical and has a nice appearance. The structure shown in Figure 40 is a typical example of stee-framed shells. It could be noticed that the triangles are of different size, but their horizontal projections are of the same size.

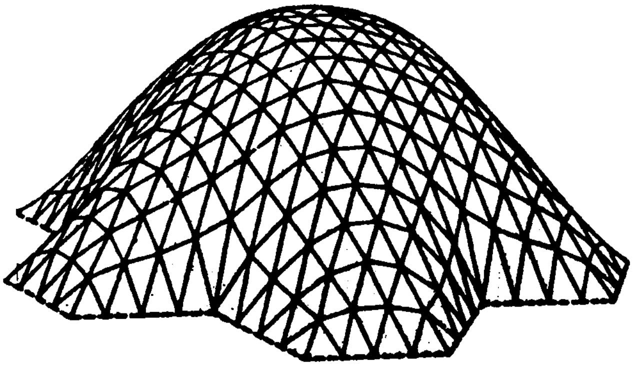


Figure 40

Finally, two-way straight line structures are of great importance in structural, mechanical and aeronautical engineering. They are used as roofing systems, towers, bridges as well as airplane constructions.

4.1.2. Two-Way Plane Curved Line Structures

A two-way curved line structure is usually built from steel cables which constitute a space mesh. If the horizontal projection of the cable mesh is regular, then the cable mesh is said to be a regular mesh. If this mesh is loaded along its cables by a system of parallel forces, the cables will take the shape of plane curves. If the forces are not parallel the cables will be curved in space.

Two-way plane curved line structures are not necessarily mesh structures. Figure 41 shows another way of building a two-way cable structure which is used as a roof.

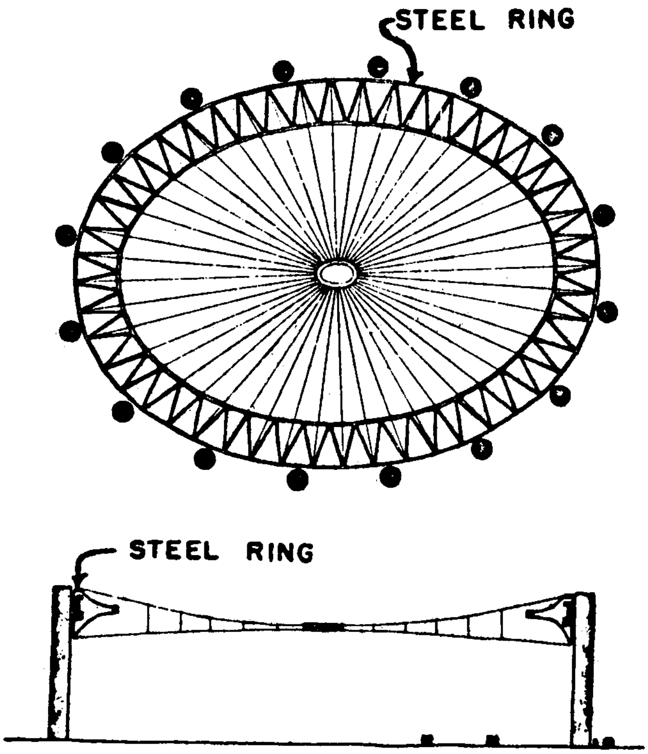


Figure 41

Usually two-way curved line structures are built from steel cables which possess tensile strength. But these structures could also be built from compressive strength materials. For example a roof could be made using an arch-grid, but buckling danger coupled with construction difficulties make it so expensive that the structure will not be economical.

4.2. Two-Way Surface Structures Without Bending

The first surface structure had been built in 1923 in Germany. This signaled the opening of a new era of freedom in architectural design. In the few years since the construction of that barrel shell roof, the size, types and shapes of concrete shells have grown and multiplied until today there is a concrete shell roof for nearly every type of building. Shell roofs are now used for such divergent structures as churches, service stations, airplane hangars, auditoriums, industrial buildings, water reservoirs and stores.

Despite their spectacular beauty, shell roofs often prove to be the most economical means of roofing buildings. Simplified design procedures and improved framing techniques have made them highly competitive with other roof systems long thought to be the lowest in cost. One reason for this surprising economy is the structural action of shell roofs.

As was mentioned before, surface structures will be divided into two groups: namely, membrane structures and shell structures. Before doing this, the following definitions will be introduced by considering a differential element cut from a surface structure.

If the differential element was taken from a plane surface, then it cannot carry loads perpendicular to its surface without undergoing bending moment. Points of a surface of this type will be called "Plane Points" (Figure 42). Plane surfaces have of course plane points everywhere, but a curved surface may have plane points in some particular part(s). For example, if in a curved surface there are several points which have one common tangent plane, these points are considered to be plane points of this particular surface.

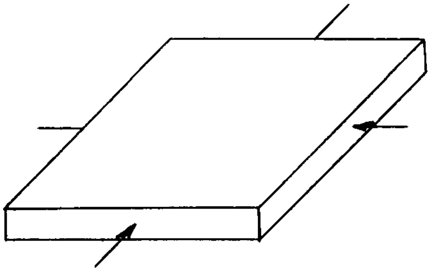


Figure 42

If the differential element is concave in one direction and convex in the other, then, in case of dead load, it will usually undergo tensile stresses in the first direction and compressive in the other. Points of such a surface will be called "Hyperbolic Points" (Figure 43).

If the small element is convex in both directions, under dead loads it will usually be under compression. Or, if the element is concave, it will undergo tensile stresses. Points of such a surface will be called "Elliptic Points" (Figure 44).

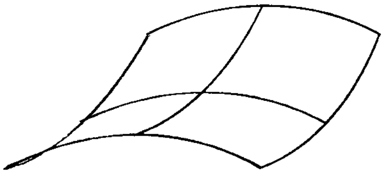


Figure 43

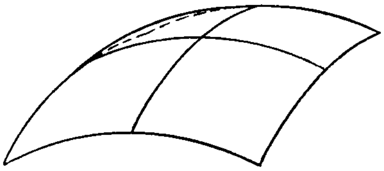


Figure 44

For the previous mentioned cases the load alone will not determine the stresses because this is a function of the way of supporting the structure as well.

If the differential element is limited by a straight line in one direction, then the load will determine the stresses perpendicular to this straight line. These stresses will vanish at the edge if the load has no component perpendicular to that edge. Points of such a surface will be called "Parabolic Points" (Figure 45).

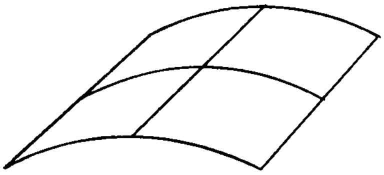


Figure 45

Also included in the last case is the surface which has a curved edge instead of the straight one. But in this case each point of this curved edge should be a plane point, i. e., each point of the curved edge should be a point of tangency of one common tangent plane. If the load has no component perpendicular to the tangent plane at the edge, there will be no normal stresses at that edge.

4.2.1. Membrane Structures

A real membrane loaded on its surface will have a curved surface just as well as a cable loaded along its length which will take a curved shape under that load. If the boundary curve of a membrane is a plane curve, the membrane will have a plane surface if it is loaded only at the boundary.

The structural behavior of a membrane can be better understood by considering the following example.

Let us make a soap-film test with a square shaped wire. Due to the surface tension effect, the film will have the tendency to contract itself to the smallest possible surface. If the edges constitute a plane polygon or a plane curve, the minimal surface will be a plane surface (Figure 46). In this case the membrane is undergoing tensile stresses of equal magnitude at each point and in all directions.

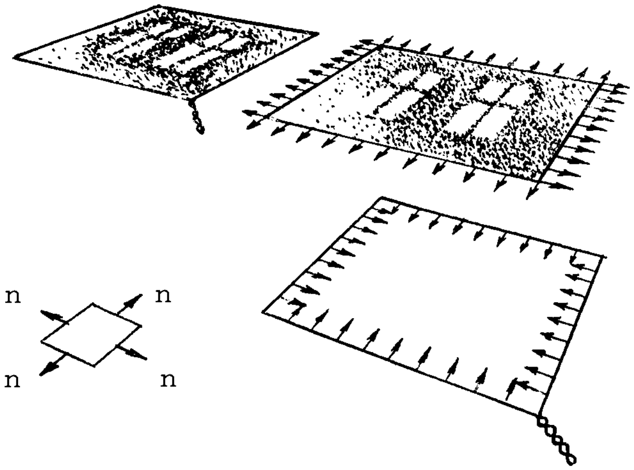


Figure 46

By means of a ventilator, let us apply air pressure on the lower face of the soap film. This soap film will become curved upward (Figure 47). And in this case also, the membrane will undergo tensile stresses of equal magnitude at each point and in all directions.

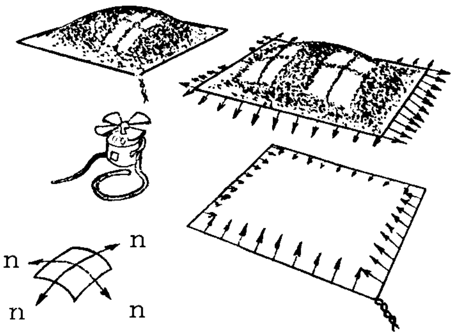


Figure 47

Let us consider the above mentioned membrane as a surface made from a stiff material. By applying air pressure on the top face, the stresses will be compressive instead of tensile.

This structural model can be built using concrete or reinforced concrete. This kind of structure is very convenient and economical for covering very large areas with or without interior column-like supports.

In the above mentioned example, the loads were perpendicular to the surface at each point. For this kind of load, the stresses at each point are equal in all directions. If the membrane carries a uniformly distributed load applied vertically over the whole surface, then the horizontal components of the stresses will be equal (Figure 48).

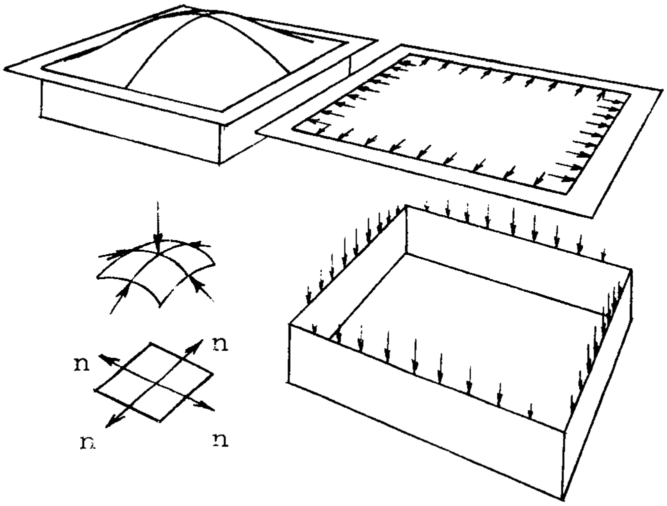


Figure 48

As is seen in Figure 48, the structure should contain edge members. These edge members will undergo bending stresses. But if the edge of the membrane is circular, the edge member will be under tensile stresses. In this case the middle surface of the membrane is a rotation paraboloid (Figure 49).

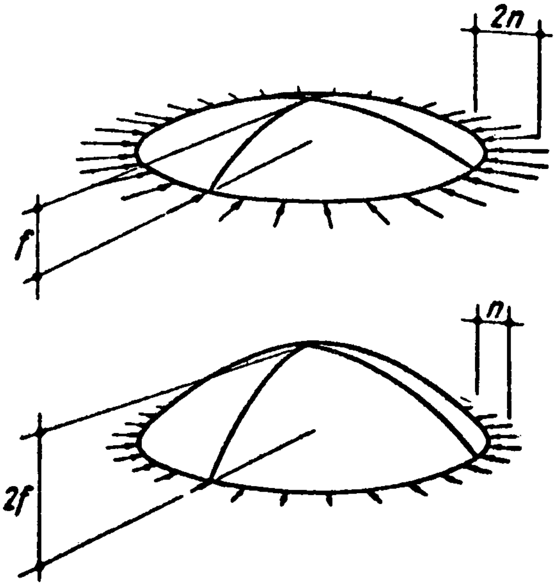


Figure 49

If the edge is a concave curve or polygon, it would be advisable to use tie beams between the ends of the concave parts in order to reduce the bending moments (Figure 50).

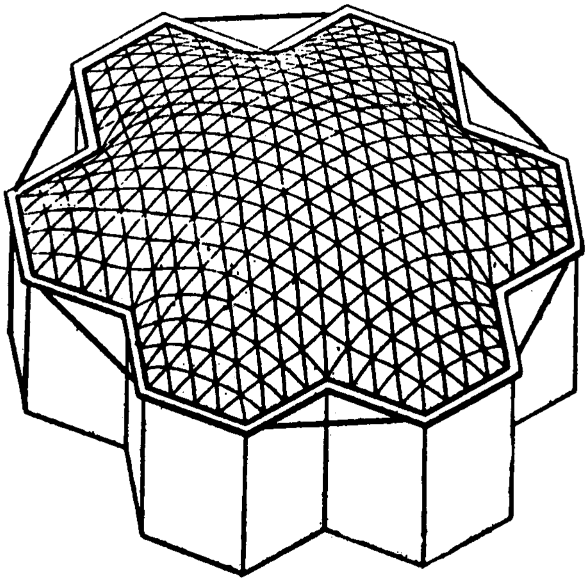


Figure 50

From the aesthetical point of view, it is preferred to have shallow membranes instead of deep ones, but the buckling effect becomes more significant for shallow membranes. By making the membrane deeper, the horizontal component of the stress decreases which in turn decreases the danger of buckling. If this could not prevent buckling, internal ribs should be used (Figure 51).

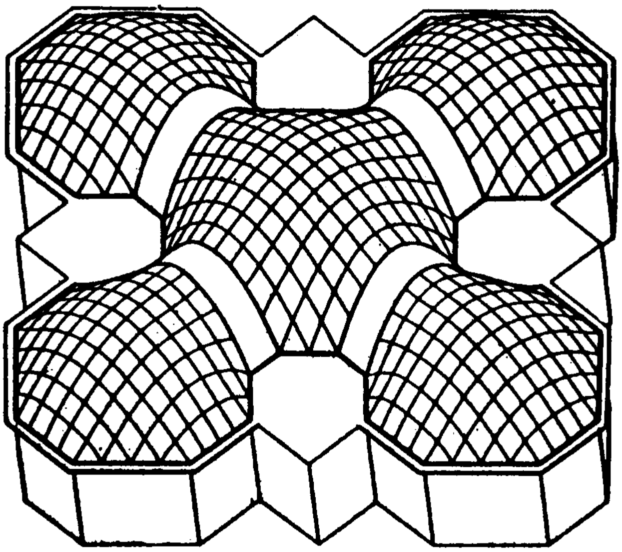


Figure 51

Membrane surfaces could be applied for gas and water tanks constructions. The statical behavior of a gas reservoir is just the same as it is in a soap film or a balloon. It might be interesting to know that the material needed for building gas reservoirs is a function of the amount of gas and it is not a function of the number of reservoirs. In other words, the amount of steel needed to build one large reservoir of a given capacity is the same for building several small reservoirs of the same capacity. The sizes of the reservoirs are usually determined depending on either some aesthetical view-points or the available plate thicknesses.

Water tanks could be built in the shape of a drop of water. Due to the surface tension, the surface of a drop of water behaves just the same as a membrane does (Figure 52). Some problems could be encountered in this kind of structure. One of these problems is due to the fact that a drop of water is always full of water, while a water reservoir cannot always be full of water. Therefore, the loading conditions will always change and the upper part of the reservoir will be subject to a different state of stress which might lead to buckling danger. This problem could be treated by installing an air valve at the top to keep the internal pressure at a certain level. If it is expected that the reservoir will be emptied very often, internal reinforcing ribs should be added. Note that the floor of the reservoir is one of the few applications to plane membranes.

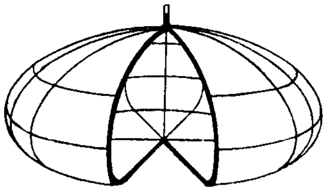


Figure 52

Sometimes it might be required to have skylights within the surface of the membrane. To explain the behavior of a membrane with a skylight, the soap film test will be introduced again. After making the soap film, let us throw a thread ring on the soap film (Figure 53a). The shape of the thread is now fully irregular. If the soap film is pierced within the thread ring, the thread will immediately take the shape of a circle (Figure 53b), because this shape corresponds to the minimum area of the membrane. Since the horizontal components of the internal forces of a membrane structure, under a uniformly distributed load, are similar to those of a plane membrane, it can be concluded that the horizontal projection of an edge without bending is always circular. The edge itself is usually not a circle and not even a plane curve. But the edge of a skylight in the center of a membrane with a circular boundary is itself a circle (Figure 53c).

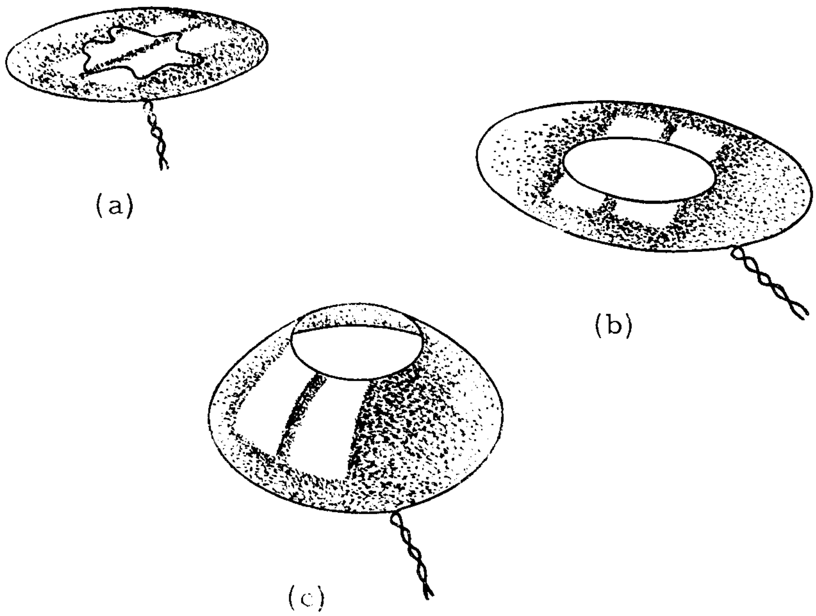

(a)
(b)
(c)

Figure 53

Membrane structures could also be built with free outside edge(s). This could be illustrated by the following soap film test. Take a tripod-shaped wire and connect the ends of the legs by loose threads (Figure 54a). By dipping these wires into a soap solution, the threads connecting the ends of the wires will take circular shapes which correspond to a minimum area (Figure 54b). If air pressure is applied, the soap film will be curved in space (Figure 54c). This corresponds to a membrane structure supported at three points.

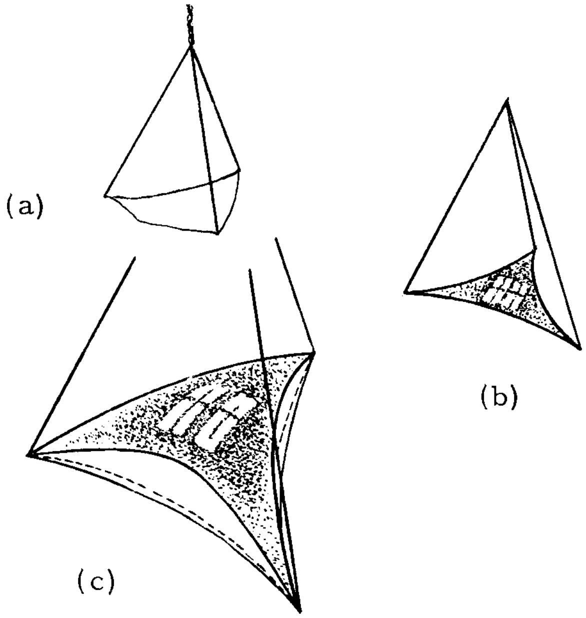


Figure 54

Membrane structures could also be supported partly at some of its edges (Figure 55). It is to be noted that where free edges exist, edge members should be used. The state of stress in these edge members depends on their shape. For example the main stress in the edge member of the structure shown in Figure 55 is compression.

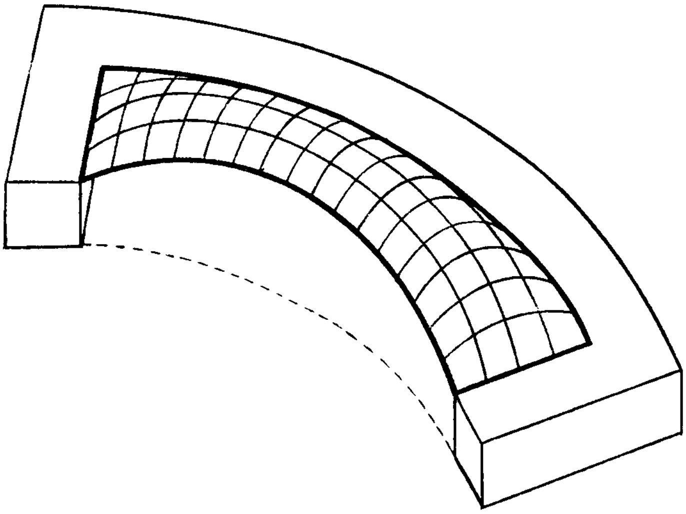


Figure 55

It is possible to build a membrane with all its edges free and support it on isolated points (column-like supports). These supports are parts of the membrane itself and they are not considered to be as columns (Figure 56).

Membranes could also be loaded at the edges. But the shape of the membrane and the edge height should change to make the membrane able to carry the edge line load without undergoing bending stresses (Figure 57).

Membranes, with edge line loads only, could be used as prefabricated column footings. The membrane footing shown in Figure 58 is composed of eight identical parts. The external (lower) edge is surrounded by a prestressed steel ring. This ring is prestressed

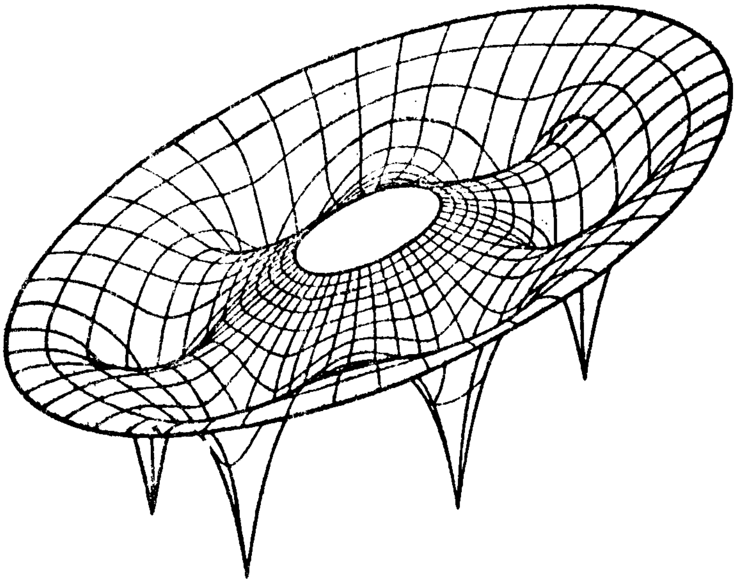


Figure 56

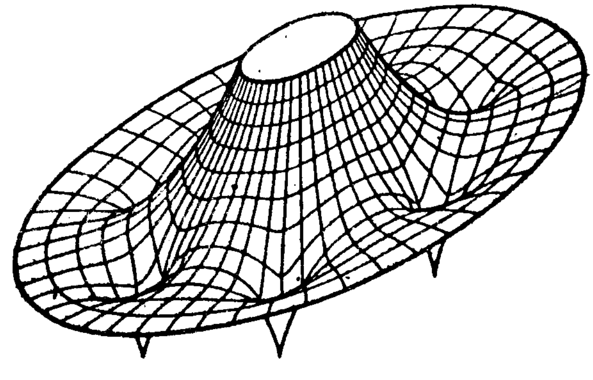


Figure 57

by the wedges driven into the joints between the membrane parts (Figure 58).

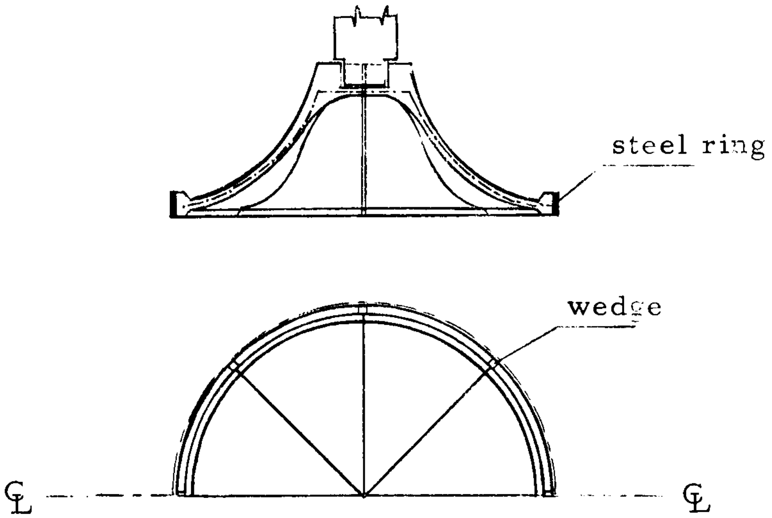


Figure 58

4.2.2. Two-Way Shell Structures Without Bending

From the standpoint of structural behavior, all shell structures are two-way structures. But if a shell is supported only by two opposite sides, then it can be considered as a one-way structure, because the way of supporting this shell gives the feeling that it is a one-way shell while it is not.

A shell structure can be in equilibrium without bending only if on its free edges, if there are such, no normal stresses are needed for the equilibrium. In the introduction of two-way surface structures, it has been shown that a differential element, cut from a shell surface, will not have normal stress at a point if

1. The point in question is a parabolic point of the surface,

2. The direction of the section is a direction without curvature, and
3. The load has no component perpendicular to the surface at that point.

Since shell structures are usually designed on the basis of the main load, the above three conditions can be restated as:

1. The free edges should have only parabolic points,
2. The edges should everywhere follow the direction without curvature, and
3. The shell must have a vertical tangent plane at each of its free edges.

Cylindrical shells in which the main curve is a semi-circle, semi-ellipse or a cycloid, are examples of such kinds of shells. Such a cylindrical shell, supported at two or more of its directrices, satisfies the above three conditions (Figure 59). But at the free

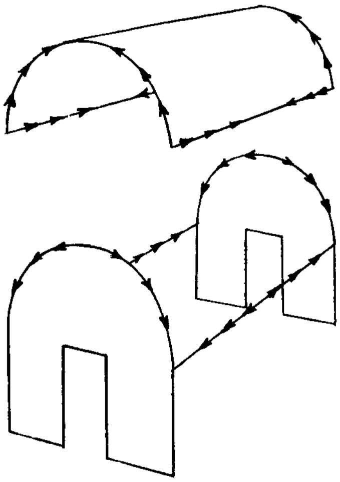


Figure 59

edges, edge members are needed to absorb the shearing stress. The only problem is the difference between the strain of the shell edge and that of the edge member. This effect could be eliminated by properly prestressing the edge member.

The free edge of a shell should not actually be straight. The shell could also have curved edges, but the three conditions mentioned previously are still to be fulfilled. As an example of this let us take the surface obtained from a torus by cutting it into two parts along a parallel circle which has only parabolic points. This parallel circle is the circle at which the torus has a tangent plane perpendicular to the axis of rotation. Two different parts will be obtained. If each part is placed in such a way that the tangential plane at the free edge is vertical, two different shell structures without bending will be obtained (Figure 60).

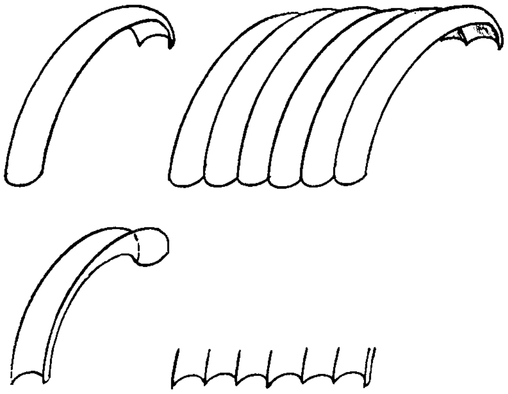


Figure 60

Two-way shells without bending are too numerous; therefore, it would be useful to divide them into subgroups. Subgrouping will be based on the structural behavior under the main load.

Rotation Shells. The middle surface of a rotation shell and its main load are axisymmetrical. If a rotation shell is supported along a parallel circle, the stress distribution is very simple. The plane passing through the axis of symmetry is a plane of symmetry for the surface, the main load and the reactions. Therefore, there can be no shearing stresses along the meridian. As a result, the meridians and the parallel circles constitute the trajectories of the principal stresses (Figure 61).

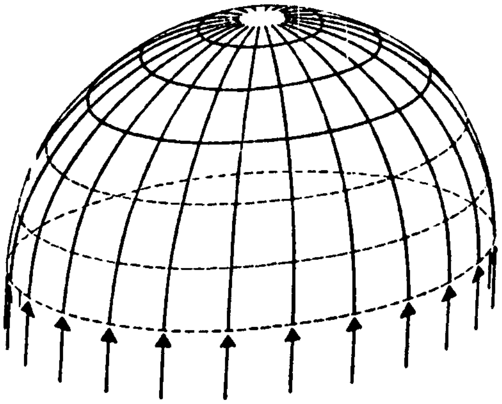


Figure 61

Under the dead load, rotation shells of hogging curvature have compressive stresses everywhere in the direction of the meridians. In the ring directions the flat parts are in compression; the steep parts may be even in tension.

The meridian should be a smooth line if the load is distributed. If, in addition to the distributed load, there is another line load acting along a parallel circle, an edge or a discontinuity in the curvature is needed at that parallel circle (Figure 62).

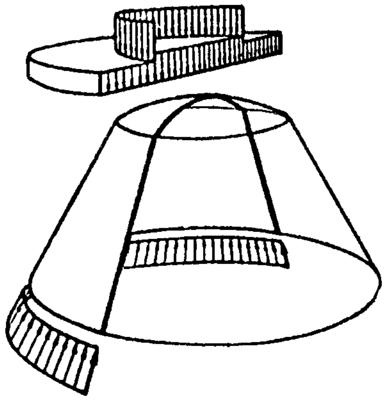


Figure 62

In fact, the smoothness of the meridian is not enough; the change of its curvature should also be continuous. In addition to this, the surface should not have inflexion points at which the tangent is horizontal. In other words, the curvature should have no horizontal intersecting tangent, because the internal stress at that point will be horizontal and cannot carry vertical loads without bending moment.

A rotation shell may have a peak point even if there is no concentrated load at that point (Figure 63). The structure of Figure 63 shows a rotation shell with a peak point but without any concentrated

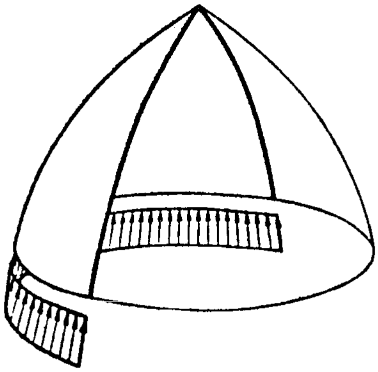


Figure 63

load while the structure of Figure 64 has an inside point-support.

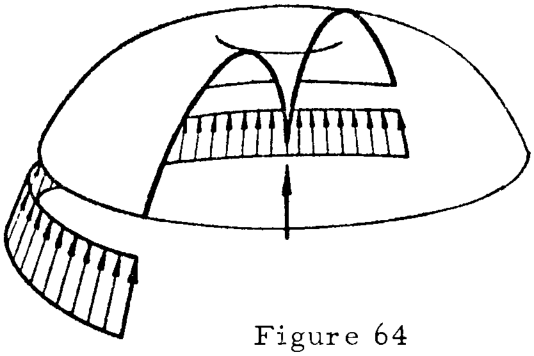


Figure 64

There are no difficulties in supporting a shell without bending if the tangent of the meridian curve is vertical at the support (Figure 61). If the tangent is not vertical, the reactions cannot be taken by a simple wall. In this case a tie-ring should be used in order to absorb the horizontal components of the reactions. But if the meridian stresses near the edge are compressive, edge disturbances will occur due to the difference between the deformation of the edge ring and that of the shell edge. This effect could be neglected if the edge ring is properly prestressed. This problem could also be avoided if the shell is supported tangentially (Figure 65).

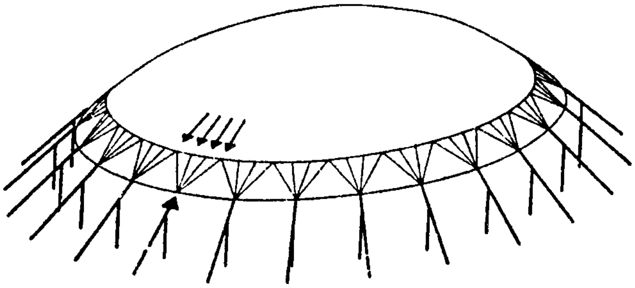


Figure 65

Hyperbolic Parabaloids. The rapid growth of interest in hyperbolic paraboloid shells is due largely to the economical use of construction materials, the simplicity of the structural action and to the inherent beauty of the hyperbolic paraboloid. Economy in the construction and design of hyperbolic paraboloids allows the architect to depart from the conventional practice of forcing all structures to conform to networks of linear members and to make imaginative use of the many graceful shapes that may be developed.

The doubly curved surface of the hyperbolic paraboloid may be developed in several ways, but for the purpose of simplification, the following procedure will be considered.

Let us take a horizontal square and lift one of its corners vertically. A space quadrate is obtained. Let us now divide each of the opposite sides into equal number of parts. If each of the division points of one side is connected by a straight line to the corresponding division points of the opposite side, a hyperbolic paraboloid surface is obtained (Figure 66).

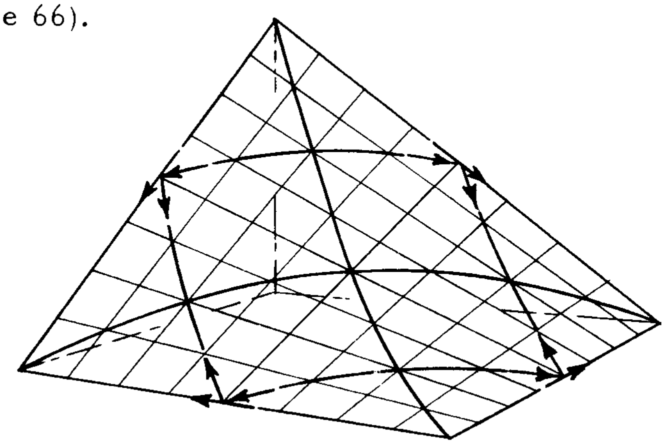


Figure 66

The structural behavior of a hyperbolic paraboloid shell, under a uniformly distributed load, can be explained by cutting the surface with vertical planes parallel to the diagonals of the base square. The intersection curves constitute the trajectories of the principal stresses. The curves that are concave upward are the trajectories of the compression stresses and vice versa. The principal stresses are of equal magnitude. Therefore the stress resultant at each point is pure shear and the shell transfers its loads to the edges by means of shearing stress.

A hyperbolic paraboloid surface could also be obtained by lifting two opposite corners of the base square (or rectangle) and proceeding similarly (Figure 67).

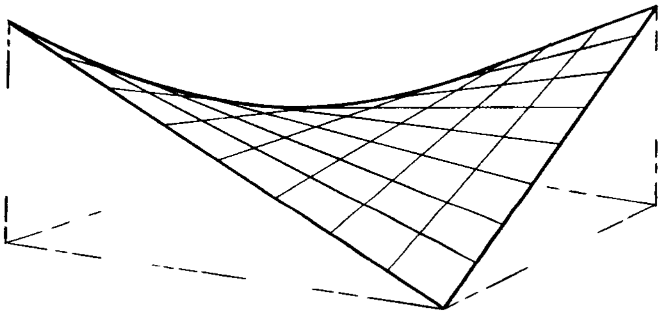


Figure 67

A variety of roof forms may be developed either by the use of the entire warped surface (Figure 66 or Figure 67) or by combining parts of it in various ways. A few of these are illustrated in Figure 68.

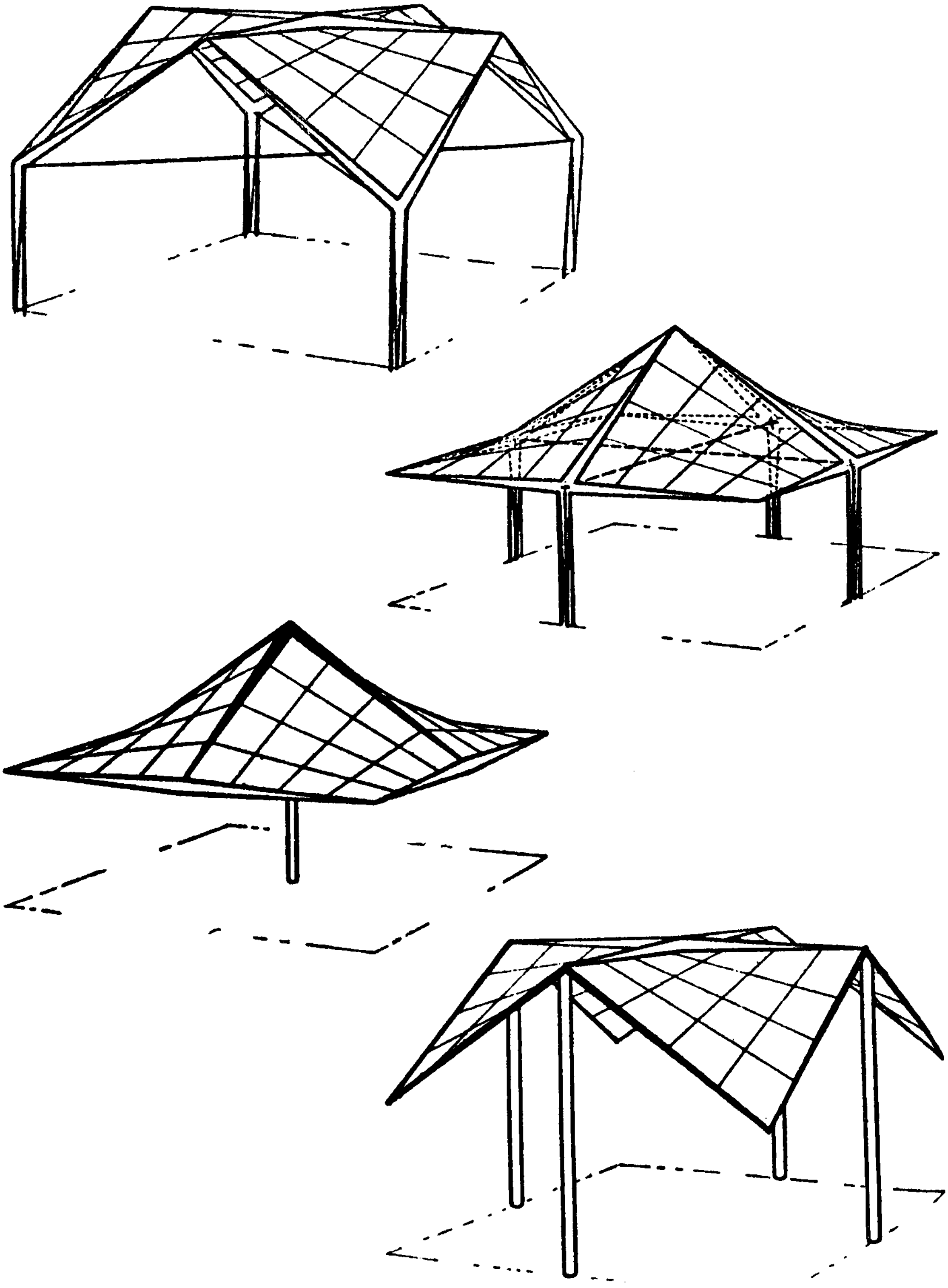


Figure 68

Since the stresses at the edges of hyperbolic paraboloid shells are shearing stresses only, edge members are needed to absorb those shearing stresses. The edge members will undergo compressive stresses if the direction of the edge shearing forces is toward the support. If the edge shearing forces are acting away from the support, the edge member will undergo tensile stresses. The stress of an edge member varies linearly from zero value at its free end to a maximum value at the other end which is connected with the support. Figure 68 shows how the cross section of each edge member varies linearly with the stress. Note that the edge members drawn with solid lines are under tensile stresses and those drawn with double lines are under compressive stresses.

If the hyperbolic paraboloid shell is supported by vertical columns, the supports should be made at the intersection points of the edge members. Also provided is that, at the support, at least one of the intersecting members should not be horizontal in order to prevent the members from undergoing bending stresses.

Due to the weight of the edge members, a concentrated line load will be exerted along the boundary of the shell. This will create some stress disturbances at the edges; therefore, edge members of light weight are to be used.

Translation Surfaces. A translation surface is obtained by moving a generating curve over an orthogonal curve (directrix). The planes of the generating curve, whatever their position, must always remain parallel to one another. Generating and orthogonal curves may have their curvatures in one direction (Figure 69a) or in opposite directions (Figure 69b)

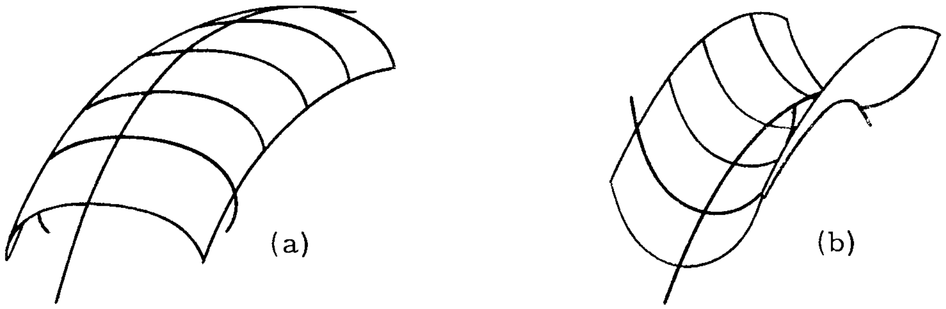


Figure 69

Hyperbolic paraboloid surfaces, discussed previously, can also be produced by translation if both main curves are quadratic parabolas of opposite curvatures.

A translation shell of elliptic surface could be supported at its four corners. In this case edge arches are needed. The horizontal reactions of these edge arches should be taken by tie members (Figure 70). If a translation shell is supported at two opposite sides, edge arches should be used at the free sides. These edge arches

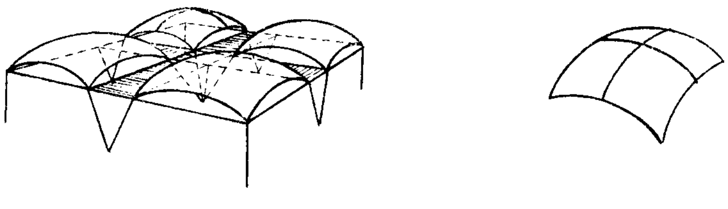


Figure 70

could be eliminated if each point of the edge curves is a plane point (Figure 60). (See 4.2.2.)

Conoid Surfaces. A conoid surface is obtained by moving a straight line along two curves so that it always remains parallel to a given plane. One of the main curves is usually an arc concave upward which lies in a vertical plane. The other main curve is usually a horizontal straight line (Figure 71).

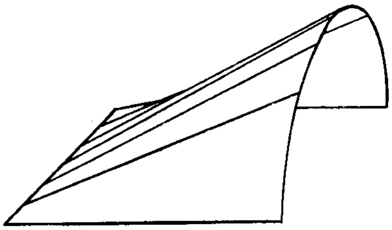


Figure 71

Motion Surfaces. A motion surface is obtained by moving a generating line along another line; the motion is now a translation plus a simultaneous rotation. This motion can be defined by a law of motion which has very broad limits. Motion surfaces include every previously discussed surface as a special case.

The use of such surfaces is very limited in practice. Therefore further details will not be presented.

CONCLUSION

The classification of structures without bending was based upon the last two factors mentioned in 1.2. The first factor is described by the way a structure carries the loads (one-way or two-way). This factor has a major effect on the material cost. The second factor is the shape factor and it has a considerable effect on the labor cost.

The structures without bending have been divided into two groups, one-way structures and two-way structures. Each of these two groups has been divided into subgroups depending on the shape factor.

Comparing one-way structures with two-way structures, it could be said that the material cost of a one-way structure is higher than that of a two-way structure, provided that both structures are of the same shape group and both serve the same purpose. For example, a one-way straight line structure requires more materials than does a two-way straight line structure.

Comparing the structures according to their shapes, it is obvious that labor costs are higher for surface structures than that of linear structures.

For linear structures, the labor cost of space curved line structures is the highest, then comes that of plane curved line structures, while the labor cost of straight line structures is the lowest.

As for surface structures, the labor cost is higher for doubly curved surface structures, lower for one-way curved surface structures and lowest for plane surface structures.

The above mentioned factors are not the only factors which restrict the designer's freedom in adopting one solution or another. Sometimes the aesthetical value of the structure has a considerable effect. For example, shell structures are usually more pleasing than any other roofing systems. Two-way linear structures are commonly preferred over one-way structures.

It could be noticed that there are some contradictions between the factors mentioned above. For example, in addition to the fact that surface structures are aesthetically more preferable, their material costs are relatively low. On the other hand, the labor costs of surface structures are much higher than that of linear structures.

Therefore, for a given case the designer should find a compromise solution which leads to a pleasing economical structure.