Flow structures and their contribution to turbulent dispersion in a randomly packed porous bed based on particle image velocimetry measurements

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An experimental study was undertaken to explore the evolution of flow structures and their characteristics within a randomly packed porous bed with particular attention to evaluating turbulent scalar dispersion. A low aspect ratio bed of 4.67 (bed width to spherical solid phase particle diameter) with fluid phase refractive index matched to that of the solid phase was used in order to obtain time resolved two component particle image velocimetry data. Results are based on detailed velocity vector maps obtained at selected pores near the bed center. Pore, or large scale, regions that are associated with the mean flow were identified based on Reynolds decomposed velocity fields, while smaller scale structures embedded within pore scale regions were identified and quantified by combining large eddy scale decomposition and swirling strength analysis. The velocity maps collected in distinctive pore geometries showed presence of three types of flow regions that display very different mean flow conditions, described as regions with tortuous channel like flow, high fluid momentum jet like regions, and low fluid momentum recirculating regions. The major portion of pore space is categorized as tortuous channel flow. Time series of instantaneous velocity field maps were used to identify mean and turbulent flow structures based on their spatial scales in the different regions. Even though regions exhibit varied Eulerian statistics, they show very similar eddy characteristics such as spinning rate and number density. The integral scale eddy structures show nearly a linear rate of increase in their rotation rate with increasing pore Reynolds number, indicating a linear decrease in their time scales. The convective velocities of these eddies are shown to reach an asymptotic limit at high pore Reynolds numbers, unique for each flow region. Detailed Eulerian statistics for the identified flow regions are presented and are used to predict mechanical dispersion through the use of estimated Lagrangian statistics. Contributions from each of the flow regions are presented and the recirculating regions are shown to contribute most to the overall longitudinal dispersion, whereas the tortuous channel regions contribute most to the transverse dispersion. The overall dispersion estimates agree well with global data in the limit of high Schmitt number.

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I. INTRODUCTION

Porous media flows are encountered in many engineering applications such as chemical reactors, chromatography columns, advanced heat exchangers, catalytic reactors, and filter systems. The general flow characteristics in a porous bed are discussed in many textbooks such as Bear, Scheidegger, and others. In most applications these flows are laminar. However, for some important applications these flows can also extend into the turbulent flow regime. Fixed bed catalytic reactors used to synthesize chemicals from gaseous reactants, as well as high temperature nuclear gas reactors...
and other heat exchanger types are further examples. To design for high performance mixing and dispersion properties of these flows are needed.

The identification and understanding of the complex patterns of fluid motion in porous beds can be useful to formulate models for predicting transport properties in the turbulent flow regime. The ability to identify large scale and small scale flow structures in turbulent flow has developed greatly over the past several years (Adrian et al., Adrian et al., Zhou et al., Zhou et al., and Chakraborty et al.) because of advances in direct numerical simulation (DNS) as well as improved time resolved particle image velocimetry (PIV) and other transient measurement methods. Coherent structures, thought to be important for understanding turbulent dynamics, have been successfully identified for cases such as isotropic turbulence (Jiménez et al.), turbulent free shear layers (Rogers and Moser), and wall bounded turbulent flows (Robinson, Brook and Hanratty, Head and Bandyopadhyay, Smith et al., and Adrian et al.). These scales are important since they play an important role in turbulence transport and mixing. A better understanding of integral scale dynamics could help to develop better models applicable to complex turbulent flows in porous media. This is especially important due to complex flow geometries where flow simulations require very high spatial resolution to accurately reproduce the flow dynamics.

Velocity field measurements in porous media mostly for creeping or steady inertial flow regimes have been experimentally obtained using non-intrusive techniques such as (i) PIV (Northrup et al., Saleh et al., and Patil and Liburdy), (ii) PTV (Huang et al., Lachhab et al., Moroni and Cushman, Peurrung et al., and Stephenson and Stewart), (iii) LDA (Johnston et al. and Yarlagadda and Yoganathan), (iv) NMR (Elkins and Alley, Ren et al., Ogawa et al., and Sederman et al.), and (v) PET (Khalili et al.). Of these methods PIV has advantages of full field data acquisition with relatively high temporal resolution. However, the application of optical techniques like PIV requires matching refractive index matching of the liquid phase to that of the solid phase. This leads to additional issues in obtaining reliable accurate data based on imaging particle tracking. A detailed analysis of PIV based refractive index mismatching uncertainties is given by Patil and Liburdy.

In this paper detailed flow structures found in turbulent porous media flows are identified, analyzed, and quantified in detail. Results are based on time resolved, two component PIV to obtain velocity field measurements. The mean flow is used to identify several different flow types determined by local pore geometry. Flow structures are identified using various decomposition techniques and quantified using critical point analysis. The contribution of flow structures to turbulent Taylor dispersion is estimated by indirectly computing their Lagrangian statistics. Results are presented for pore Reynolds numbers ranging up to $Re_{pore} = 3964$, where the Reynolds number is based on the porous bed hydraulic diameter, $D_H = \phi D_B/(1 - \phi)$ and average pore, or interstitial, velocity, $V_{int} = V_{Darcy}/\phi$, where $V_{Darcy} = Q/A_{bed}$, with $Q$ being the volumetric flow rate and $A_{bed}$ the bed cross section normal to the mean flow direction. Results are scaled with bed averaged variables, $D_H$ and $V_{int}$, and are shown as a function of $Re_{pore}$. The overall mechanical dispersion, based on area averaging within different mean flow types, is compared with global average results in the literature.

The paper first describes the experimental approach; then the results of the Reynolds decomposition of the fluctuating quantities are presented; this is followed by a detailed description of identified flow regions and a presentation of their flow variance and integral vortical structures; the paper concludes with an analysis of how the turbulence within the different flow regions contribute to the overall dispersion.

II. EXPERIMENTAL SETUP

The experimental setup used for the PIV measurements is shown schematically in Fig. 1. The methods for obtaining and analyzing the velocity vector field data is discussed in detail by Patil and Liburdy. Briefly the test section was fabricated using a square cross section duct ($W \times W$: 70 mm x 70 mm inside dimensions) with transparent Pyrex® walls. The duct was 90 mm long and was filled with 15 mm diameter, $D_H$, optical grade Pyrex beads to give overall fluid porosity of 0.45. All velocity data presented were collected near the bed center, which was shown to be sufficiently far from the walls to prevent wall effects, see a previous study by Patil and Liburdy for details.
The sampling rate for each vector field ranged from 80 to 400 Hz, from the lowest $Re_{pore}$, value of 54 to the highest value of 3964. The velocity vector spatial resolution ranged from 66 to 80 vectors per bead diameter, depending on the magnification, or approximately 0.2 mm. Seed particle position displacements were determined using the multi-grid, multi-pass adaptive correlation method with a central differencing based window offset method. A high accuracy subpixel peak fitting algorithm specific to Dantec® software was also used. A moving average validation scheme was used to reject outliers, with a vector rejection rate of less than 1%.

III. UNCERTAINTY ANALYSIS

The uncertainties of the velocity data were analyzed for the various contributions and are explained in detail elsewhere by Patil and Liburdy. The uncertainty estimates for displacements in both the longitudinal, $\Delta y$, and transverse, $\Delta x$, directions are given in Table I. The results of the total error are 1.19% in the cross stream or transverse direction and 1.0% in the downstream or

<table>
<thead>
<tr>
<th>Error source</th>
<th>$\Delta x$</th>
<th>$\Delta y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max displacement</td>
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<td>9.9 pixels</td>
</tr>
<tr>
<td>Magnification, $\sigma_{\text{mag}}$</td>
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<td>0.03 pixels</td>
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<td>Refractive index mismatch error, $\sigma_R$</td>
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<tr>
<td>In-plane loss of image pairs (bias), $\sigma_{\text{bias}}$</td>
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<td>0.016 pixels</td>
</tr>
<tr>
<td>Finite number of image pairs (random), $\sigma_{\text{rms}}$</td>
<td>0.025 pixels</td>
<td>0.025 pixels</td>
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<tr>
<td>Out-of-plane motion (perspective), $\sigma_p$</td>
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<td>0.079 pixels</td>
</tr>
<tr>
<td>Total error, $\sigma_T$</td>
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<td>0.1 pixels</td>
</tr>
<tr>
<td>Total error (% of max)</td>
<td>1.19%</td>
<td>1.0%</td>
</tr>
</tbody>
</table>

longitudinal direction. The largest contributions are a result of out of plane motion and refractive index mismatch. It should be noted that the refractive index was matched to within 0.0005.

IV. RESULTS

A randomly packed bed of solid particles will necessarily result in a wide distribution of local pore sizes and flow geometries. The consequence is a very unique set of flow conditions within individual pores. Figure 2 illustrates four selected, highly variant, pore geometries in the randomly packed bed test section using spherical beads. Overlaid on the pore geometry is the instantaneous velocity vector map obtained for $Re_{pore} = 3964$. Based on the instantaneous velocity field maps

FIG. 2. Pore geometries and instantaneous velocity vector plots at $Re_{pore} = 3964$; note that only every alternate vector is shown. Reprinted with permission from V. A. Patil and J. A. Liburdy, Phys. Fluids 25, 043304 (2013). Copyright 2013 AIP Publishing LLC.
regions are identified based on their pore length scale, i.e., regions with size in the order of the hydraulic diameter, $D_H$, which are a consequence of local pore geometry. However, these regions are not isolated and must be considered in relationship to their associated flow boundaries. For example, Fig. 2(a) shows a flow through a region that is not too unlike that through a meandering or tortuous channel. This flow is relatively uniform in velocity distribution over most of its domain. In contrast, Fig. 2(b) illustrates the existence of flow separation due to higher velocities through a constricted flow from the upper right hand inlet, with flow separation and impingement near the solid boundaries. An illustration of strong recirculation is shown in Fig. 2(c) typically found on the downstream side of sphere. Fig. 2(d) shows a very strong constriction between spheres leading to region with locally high momentum compared to its surroundings, typical of a jet flow.

A. Flow description using Reynolds decomposition

By applying Reynolds decomposition to the time series of the instantaneous flow fields within the pores it is possible to better interpret the variations of the mean and turbulent flow characteristics. Examples of the mean flow conditions for three values of $Re_{pore}$ for each of the selected pores are shown in Fig. 3. The two dimensional velocity field vectors are shown (only alternate vectors are shown for clarity) overlaid with shading based on the velocity magnitude distribution, which is normalized by the overall average interstitial velocity, $V_{int}$, for each $Re_{pore}$.

Three general observations can be made concerning the mean flow distributions within these pores. (i) Low mean velocity recirculation regions that form due to flow separation persist from low to high $Re_{pore}$ values as noted in the third row of Fig. 3, (ii) High momentum, jet like regions form due to the constricted inlet to the pore which is seen to occur from low to high $Re_{pore}$ values, shown in the fourth row of Fig. 3, with somewhat greater mean velocity separation at the lower $Re_{pore}$ values. (iii) The majority of the flow regions have neither recirculation nor jet-like regions and have mean velocity values very close to that of $V_{int}$.

Based on these observations, the primary picture for turbulent porous media flow is visualized as flow paths consisting of primarily parallel and interconnecting tortuous channel flow interspersed with high momentum jet like flow as well as recirculating flow regions. This description of randomly packed porous media flow has led to the idea of dividing the velocity map into three primary mean flow regions. The pore scale regions identified here are (i) tortuous channel flow, (ii) recirculation flow, and (iii) jet flow. The mean velocity fields are described as: nearly uniformly distributed spatially within the regions with values near the average interstitial velocity for the tortuous channel flow, lower and higher respectively compared to $V_{int}$; low mean enclosed regions of swirling velocity for the recirculating flow; and high mean velocities within the jet flow regions. These flow regions are further divided latter and are used here to determine their role in mechanical dispersion within the bed. Both the mean and turbulent aspects of the regions are evaluated.

The turbulent characteristics of the various regions can be studied by looking at statistical descriptions of the fluctuating velocity field maps. Reynolds decomposed fluctuating longitudinal velocity, $v'$, and lateral velocity, $u'$, presented in terms of the variances, $v'^2$ and $u'^2$, normalized with $V_{int}^2$, are shown in Figs. 4 and 5, respectively. The longitudinal component is aligned with the mean flow direction through the bed, while the lateral component is normal to this in the plane of the measurements. Several general observations can be made concerning the fluctuating velocity variance distributions within these pores: (i) as $Re_{pore}$ increases the distributions of the variances tends to be more homogeneous within the tortuous channel regions, see the first row of Figs. 4 and 5; (ii) in tortuous channel regions the lateral velocity variance, $u'^2$, increases with increasing $Re_{pore}$ until it is approximately equal to the longitudinal velocity variance, $v'^2$, in the larger pore regions, seen by comparing the first row of Figs. 4 and 5; (iii) recirculation regions with low fluid momentum exhibit lower values of the fluctuating velocity variances for all $Re_{pore}$ evaluated, see the third row of Figs. 4 and 5; (iv) the high fluid momentum jet regions of the constricted flows are associated with higher longitudinal (but not lateral) velocity variances, $v'^2$, shown in the second and fourth rows of Figs. 4 and 5.

The temporal behavior of the turbulence within different pore regions was evaluated using local correlation functions. In particular, the temporal autocorrelation function based on the longitudinal
fluctuating velocity component, $v$, was used to evaluate the Eulerian integral time scale, $T_{E,v}$. Similarly, the lateral fluctuating velocity component, $u$, was used to evaluate the Eulerian integral time scale, $T_{E,u}$. Figure 6 shows contour plots of the longitudinal time scale normalized by the longitudinal convective time scale defined as $D_H/V_{int}$. This convective time scale represents a measure of the average time of transport along a bed length equivalent to the hydraulic diameter, where $D_H(= \phi D_B/(1 - \phi))$. Nevertheless general observations can be made concerning the Eulerian timescale distributions within the indicated pores: (i) as $Re_{pore}$ increases the contour maps of the normalized integral time scale in tortuous channel regions tends to become more homogeneous within a pore, as is shown in the first row of Fig. 6; (ii) regions of recirculation exhibits higher values of the integral timescales for all $Re_{pore}$ values, as indicated in the third row of Fig. 6; (iii) as $Re_{pore}$ increases the values of the normalized integral time scale tend to decrease throughout the pores, shown in the first row of Fig. 6. Similar conclusions can be drawn based on the lateral Eulerian time scale, $T_{E,u}$.

Patil and Liburdy\textsuperscript{30} have studied the turbulent characteristics of these pore regions by looking at their pore area averaged statistical turbulence quantities such as Eulerian integral scales, turbulent...
kinetic energy, and turbulent production rate (that is to say results averaged over the entire pore, not the different regions identified above). They found that these averaged quantities, when scaled with hydraulic diameter, $D_h$, and average interstitial velocity, $V_{int}$, exhibited asymptotic behavior at large $Re_{pore}$ values beyond approximately 2000. Remarkably these asymptotic values are very similar from pore to pore. In the current work, the turbulence quantities are examined based on the identified flow regions by using weighted area averaged quantities, within each flow region, such as the tortuous channel, jet-like or recirculating regions.

B. Characterization of flow regions

The mean flow variance and turbulence quantities that contribute to dispersion were determined for the different flow regions within pores based on the regions previously shown. The ability to identify proper criterion for identifying these regions is difficult at best. Quantitative discriminators based on some fundamental flow pattern would need to be determined. However, for this initial study it was decided to use a reasonable observable criterion for each region based
on mean flow features as a method of delineating the features and seeing how the dispersion related quantitative measures within these features varied. The recirculating regions were identified where the mean velocity is significantly less than the spatially averaged interstitial velocity within a pore, $V < 0.3 V_{\text{int}}$, while being constrained by closed mean streamlines. An example is shown in Figs. 7(a) and 7(b) which indicates enclosing streamlines overlaid with contours of mean velocity magnitudes normalized by the mean interstitial velocity. It can be noted that since the mean velocity outside of these regions tends to approach the average interstitial velocity, there is also a fairly high shear component along these dividing streamlines. This latter condition is illustrated in Figs. 7(c) and 7(d) which shows contours of mean vorticity being very large at the boundaries of a recirculation features (here vorticity is being used as an indicator of local shear rate).

The high momentum jet-like regions are found to appear downstream of flow constrictions as shown in the instantaneous velocity vector plot presented in Fig. 8. The fluid velocity vectors for this flow are highly directional with distinctively larger magnitudes than the surroundings. At sufficiently large values of $Re_{\text{pore}}$ the jet-like flow structures are observed to exhibit a cross stream transient flapping motion. This implies a lower frequency component to the fluctuating variance.
The demarcation of a high velocity region from the surroundings is identified in the mean velocity vector map as illustrated in Fig. 8(a). The separating streamlines are identified in Figs. 8(b) and 8(c). The high values of mean vorticity coincide with the separating streamlines. The local mean velocity in this region is seen to be larger than the average interstitial velocity within a pore. Subsequently, the larger velocity magnitudes, \( V > 0.7 \) \( \bar{V}_{int} \) coupled with a high shear region bordering the flow is used to identify with these regions.

The third flow region, identified as tortuous channel flow, is less distinctive than the previous two in that the mean flow is more uniform within the pore and mean streamlines indicate a flow in the mean flow direction through the bed. Consequently, this flow type is assigned to regions that are neither recirculating nor jet-like. As a result, the tortuous channel flow regions have neither identified dividing streamlines nor high shear regions that delineate the other two pore scale regions. This region is more indicative of a channel flow with a relatively uniform mean velocity and is typically found in nearly all pores as either a partial region, or filling the entire pore. The mean velocity and variances within these regions can be seen for comparison in Figs. 3–5.

FIG. 6. Longitudinal Eulerian time scale, \( TE, V_{int}/D_h \), contour maps for four pores over a range of \( Re_p \) values.
FIG. 7. Recirculation regions enclosed by separating streamlines, shown in (a) and (b) overlaid by the mean velocity contours in the recirculation region for pores shown in Figs. 2(a) and 2(c); in (c) and (d) the same streamlines are shown overlaid with mean vorticity contours, \( \Omega \) (note that vectors are used to indicate flow direction only, not magnitude).

1. Flow variance

To quantitatively describe the mean flow structure the variance of the mean velocity field in both the longitudinal and transverse directions was determined for each of the three regions. Results are presented as region area averaged values. The region area average is noted as \( \langle \rangle_R \). (Note that \( V_{\text{int}} \), the average interstitial velocity, is the mean longitudinal velocity and the mean lateral velocity is zero.) The results are shown versus \( R_{\text{pore}} \) in Fig. 9. For the transverse mean variance the results show nearly constant values over the range of \( R_{\text{pore}} \) beyond approximately 1000. The transverse mean variance is essentially zero for the recirculating regions due to the circular mean flow feature having fluid velocity closer to zero. The tortuous channel and jet-like flows have a relatively large value of normalized variance, approximately 0.25–0.30. The longitudinal mean variance is distinctly different. The recirculating flow has a very large longitudinal variance relative to the other regions, approximately five times larger. This is due to the relatively low mean velocity in this region, which when subtracted from the average interstitial velocity, \( V_{\text{int}} \), results in significantly large values. The longitudinal variance for the tortuous channel and constricted jet regions are similar and are significantly lower. In general, for high \( R_{\text{pore}} \) values the mean velocity variance is essentially uniform for all flow regions, with the recirculating regions exhibiting much higher longitudinal values due to the low mean velocity.

The flow region area weighted average fluctuating velocity variance for both velocity components, for each flow region versus \( R_{\text{pore}} \) is shown in Fig. 10. As with the mean velocity variance, results indicate an asymptotic limit at sufficiently large values of \( R_{\text{pore}} \), although the limiting value depends on the flow region. The transverse component of the variance is consistently smaller than the longitudinal component for the recirculating and jet-like flow regions. However, for the tortuous channel flow the longitudinal and lateral components are essentially identical, indicative of nearly isotropic behavior of the fluctuating variance. This same isotropic condition for this flow region is also shown in the maps of fluctuating variances in the first rows of Figs. 4 and 5.
Each of the flow regions was evaluated for the associated integral time scale versus $Re_{pore}$, where the time scale is normalized with the convective time scale, $V_{inf}/D_H$. The area weighted average results for the different flow regions are given in Fig. 11 for both transverse and longitudinal directions. For each region the normalized integral time scale approaches an essentially constant value for sufficiently high values of $Re_{pore}$. These data show that beyond $Re_{pore}$ of approximately 1000 the time scale is constant with values of approximately 0.15 for the tortuous channel and jet-like flows, and a significantly higher value of 0.24 for the recirculating regions. Both velocity components show very similar values. The larger value associated with the recirculating regions is due to the relatively slow moving fluid and lower turbulence levels. This results in regions of the flow where fluid is retained for significant periods of time before eventually passing through the bed. The nearly equal integral time scales based on both $u$ and $v$ for the tortuous channel flow indicates a measure of isotropic behavior. However, the jet like regions have very different $u$ and $v$ variances, as seen in Fig. 10, with higher longitudinal variance, $v$, yet a lower transverse variance, $u$, when compared with the tortuous channel flow. Despite this difference the resulting integral time scales are comparable for both regions in both directions. This may be due in part to the observed flapping motion of the jet, mentioned previously, extending the transverse time scales.

2. Integral scale vortical structure

The turbulent flow structure associated with the different flow regions was evaluated using several techniques, including large eddy scale (LES) filtering to distinguish spatial scale structures and swirl strength to identify eddy structures along with their number density and convective velocity. These data are used to estimate overall dispersion within each flow region. To observe integral scale vortical structures, the LES decomposition technique (Adrian et al. 3) was used. In this method the
FIG. 9. Flow type area averaged mean (a) longitudinal and (b) transverse mean velocity variance for different regions versus $Re_{pore}$.

translation or convection velocity of an identified vortex due to larger scale motion is filtered out from the instantaneous velocity field in order to identify vortex, or eddy, structures. The convection velocity was found by local spatial averaging of the velocity field using a moving top hat filter of specified spatial extent. This results in two vector fields, a large scale component which is used to define the local time dependent convection velocity, and a smaller scale component (the result of subtracting the large component from the instantaneous field) used to identify the local eddy structures. The integral scale eddy size, $l$, was estimated previously (Patil and Liburdy\textsuperscript{30}) to be approximately equal to 0.2$D_p$ based on the autocorrelation function. As an example of the present results, the instantaneous small scale vector map is shown in Fig. 12 overlaid with the instantaneous shear rate field, $\tilde{S}_{xy}$ and also overlaid with the instantaneous vorticity field for $Re_{pore} = 3964$. The observed discrete vorticies, indicated by regions of swirling velocity vectors, are clearly visible and strongly correspond to locations of both high vorticity and shear rate, with the latter being in regions between vortex elements. Results for $Re_{pore} = 418$ were not well defined and are not included.

An alternative method to identify local eddy events is to evaluate the swirl strength, $\lambda_{ci}$, as defined by Zhou \textit{et al.}\textsuperscript{5} and Adrian \textit{et al.}\textsuperscript{3} This was used to quantify the strength and number density of the smaller scale eddy structures. In this technique, swirling motion due to vortex flow is identified using critical point analysis on the local instantaneous velocity gradient tensor and finding the associated eigenvalues. The existence of complex conjugate eigenvalues indicates swirling motion.
at that location, and the imaginary value of the eigenvalue, $\lambda_{ci}$, is a direct measure of swirling, or rotational, strength of the vortex. Figure 13 shows, for the pore shown in Figs. 2(a) (tortuous channel region) and 2(c) (recirculating region), the instantaneous LES decomposed smaller scale velocity field with the swirl strength superimposed for $Re_{pore} = 3964$. It should be noted that the jet-like flow regions did not show any significant swirl over the entire range of $Re_{pore}$. This is possibly due to turbulent suppression by the constriction as well as flow acceleration, as is indicated in the instantaneous velocity vector plot shown in Fig. 8(a). Maps of the swirl strength associated with individual vortical motion within each flow field were determined using the following procedure. First the swirl strength, $\lambda_{ci}$, was calculated and a background threshold of swirl strength was set (nominally one tenth of the maximum value), and a high pass filter applied to the instantaneous swirl field. The flow field was then checked by comparing the location of the swirling motion from the LES decomposition (e.g., Fig. 12) with the location of high swirl strength within the region of interest. It should be noted that this comparison resulted in 100% conformation. Then the swirl strength data were processed to determine the local maximum swirl strength within each region of high swirl concentration that also coincides with identified swirling motion from the velocity data. This local maximum was used to define the swirl strength and the location of each individual swirl, or eddy. Finally, the convective velocity associated with each swirl was determined by using the
FIG. 11. Flow area averaged Eulerian integral time scale versus $Re_{pore}$ for regions with different flow regions, (a) lateral time scale and (b) longitudinal time scale.

...instantaneous LES large scale velocity associated with the location of the maximum swirl. That is, the instantaneous large scale velocity at the point of maximum swirl strength was assigned as the local convective velocity of that swirl element.

Swirl strength distributions are shown in Fig. 13 for two pores shown in Figs. 2(a) and 2(c). Corresponding histograms of swirl strength are given in Fig. 14 for the same two regions. The general trends show that as $Re_{pore}$ increases the distributions tend to converge; however, less scatter is apparent for the tortuous channel region. Both regions show peaks in the range $\lambda_{ci} = 7–10(V_{int}/D_H)$ with skewed tails at larger swirl strength values. In the previous study by Patil and Liburdy\(^{30}\) the integral length scale, $L$, found for the entire pore region from the spatial autocorrelation function, was approximately $0.1 D_H$ for all pore geometries at sufficiently large $Re_{pore}$. Consequently, based on results of Fig. 14 with $\lambda_{ci} D_H/V_{int} \sim 10$ it can be estimated that $L/V_{int} \sim 1/\lambda_{ci}$. This can be interpreted as follows: the fluid transit time to travel one integral length scale is approximately equivalent to the time of rotation of the typical swirl, or eddy.

As shown later, the convective velocity associated with eddies is an important characteristic describing the overall dispersion. The magnitude of the convective velocity for individual swirls, $V_{conv}$, was determined versus $Re_{pore}$ for the tortuous channel and recirculating regions from the large scale local eddy velocity based on LES. The histograms versus $Re_{pore}$ are shown in Fig. 15, normalized as $V_{conv}/V_{int}$ for the regions shown in Fig. 13. With increasing $Re_{pore}$ from 839 to 1815...
there is a shift to higher relative convective velocities with somewhat broader distributions for both flow regions. For the highest values of $Re_{\text{pore}}$, the distributions converge. The mean values from the histograms are shown in Fig. 16 for these two flow regions indicating an asymptotic limit for the scaled convective velocities at sufficiently high $Re_{\text{pore}}$ values. The tortuous channel flow region has a significantly higher convective velocity (approximately four times larger) when compared with the recirculating region. This large difference is due to the relatively low velocity within the recirculating region. At moderate values of $Re_{\text{pore}}$ there is evidence of a local maximum with a much stronger peak for the tortuous channel regions. This is consistent with the higher fluctuating velocity intensities associated with this region for these intermediate values of $Re_{\text{pore}}$, as shown in Fig. 10. The higher energetic fluctuations seem to be associated with a somewhat higher large scale local convective velocity. Recall that these convective velocities are the instantaneous large scale filtered values, so it can be expected that they should be correlated with the larger turbulence energy.

C. Pore scale flow structure contribution to dispersion

The mechanical dispersion within a randomly distributed porous bed is a complex combination of both the mean flow and its variance and the characteristics of the turbulence. Since the flow geometry is very complex it is expected that individual pores will have their own characteristic spatial and temporal scaling that contributes to overall dispersion. However, in an attempt to assemble all of this complexity into representative longitudinal and transverse dispersion coefficients, different pore scale regions have been identified along with their relevant turbulent characteristics. A somewhat similar approach was followed by Koch and Brady\textsuperscript{31} in their development of dispersion limits in a laminar porous bed where the randomly distributed particles result in local spatial fluctuations of
velocity. At high values of Peclet number they show some regions having an asymptotic limit driven by mechanical dispersion, yet in other regions, such as “hold-up” and “boundary layer” regions with relatively low convective velocities, there is a molecular diffusion dependence. The work in this paper is related to the purely mechanical aspects of dispersion in turbulent regime, i.e., no molecular diffusion effects are considered.

In the turbulent flow regime, dispersion is related to the local mean flow variations as well as the turbulent flow structure, the former being shown to be highly dependent on the local pore geometry. Three major pore scale regions were found in porous media flows and are proposed to have significantly different contributions to dispersion: (i) tortuous channel-like with nearly uniform mean velocities, (ii) separating or recirculation flow regions with low mean velocities and confining streamlines, and (iii) jet-like flow regions with bordering high mean shear rates. These three regions are further examined below to determine their contributions to dispersion within a randomly packed porous bed. It is interesting to note that at moderate values of $Re_{\text{pure}}$, in the range of 800–2000, the values of the mean and turbulent variances, in Figs. 9 and 10, and the convective velocity, in Figs. 15 and 16, tend to increase beyond their asymptotic large $Re_{\text{pure}}$ values. Although the explanation for this is unclear, it is most likely attributed to developing turbulence within the flow.

1. Dispersion: Tortuous channel regions

In the asymptotic limit of high Peclet number, $Pe_M$, the overall dispersion of flow through a randomly packed bed is expected to be independent of molecular diffusion effects. The resultant
FIG. 14. Histogram of the normalized swirl strength, $\lambda_{ci}$, for a range of $Re_{pore}$ in (a) tortuous channel region and (b) recirculating region.

Mechanical dispersion is principally due to the spatial variation of velocity within the flow field. A reasonable estimate of the longitudinal dispersion in the mechanical dispersion regime is possible by using the Eulerian velocity statistics. If the Lagrangian velocity of a tracer that follows the fluid flow, which at time $t = 0$ is at point $(x_i, y_i, z_i)$, is $V_L(x_i, y_i, z_i, t)$, then the tracer position at an arbitrary time, $t$, is $(X, Y, Z)$ and is a function of the velocity experienced by the tracer during that time interval. If $V_L$ is a statistically stationary variable, then $(Y - \bar{Y})$ has a Gaussian probability distribution in the asymptotic limit and its variance is given by (Taylor$^{32}$ and Tennekes and Lumley$^{33}$)

$$
(Y - \bar{Y})^2 \approx 2(V_L - \bar{V}_L)^2 T_L \approx 2D_L t,
$$

(1)

where $D_L$ is the dispersion coefficient due to fluid advection alone, $T_L$ represents the Lagrangian integral scale and $(V_L - \bar{V}_L)^2$ is the Lagrangian velocity variance. Tennekes and Lumley$^{33}$ show that for homogeneous flow in the longitudinal direction and bounded flow in the spanwise direction (e.g., pipeflow) the Lagrangian velocity variance can be written as the sum of the fluctuating Eulerian velocity, $(v'^2)$, and the square of the Eulerian velocity field difference from the bulk fluid velocity. Consequently, the dispersion coefficient in Eq. (1) can be written as $T_L$ times the sum of $(v'^2)$ and $(V - V_{int})^2$, where the bulk fluid velocity has been taken as the interstitial velocity, $V_{int}$.

The tortuous channel regions exhibit relatively uniform mean velocity distributions in the limit of high $Re_{pore}$. Also, the previous data shows that the tortuous channel regions have tendencies of being relatively isotropic and homogeneous in their turbulence quantities. Consequently, using this
FIG. 15. Histogram of the distribution of normalized eddy convective velocity, $V_{conv}/V_{int}$, versus $Re_{pore}$ in (a) the tortuous channel-like and (b) the recirculating regions.

FIG. 16. Flow region averaged swirl convective velocity magnitude, $V_{conv}$ versus $Re_{pore}$ in channel-like and recirculating regions.
region’s contribution to longitudinal dispersion can be estimated as

\[ D_L = \langle v^2 \rangle^{\text{tc}} T_L + \langle (V - V_{\text{int}})^2 \rangle^{\text{tc}} T_L, \]

where \( \langle \cdot \rangle^{\text{tc}} \) represents an area averaged operator for the tortuous channel regions, with spatial averaging over the area of the fluid phase identified as tortuous channel within a pore. Similarly, for the lateral dispersion coefficient, \( D_T \), which is expected to have a spatially mean lateral velocity equal to zero, the following relationship holds:

\[ D_T = \langle u'^2 \rangle^{\text{tc}} T_L + \langle (U^2) \rangle^{\text{tc}} T_L, \]

where \( U \) is the temporal mean lateral velocity at a given location.

As presented by Koeltzsch,\textsuperscript{34} the ratio of the Lagrangian time scale, \( T_L \), to Eulerian time scale, \( T_E \), is taken to be proportional to the eddy convection velocity, \( u_{\text{conv}} \) (rather than the local mean velocity) expressed as

\[ \frac{T_L}{T_E} = 0.8 \frac{u_{\text{conv}}}{\sqrt{\langle u'^2 \rangle}}, \]

where \( u'^2 \) is the mean square value of the fluctuating streamwise velocity and \( u_{\text{conv}} \) is the streamwise convection velocity of eddies. The value of the prefactor of 0.8 was obtained by Koeltzsch\textsuperscript{34} based on examining data for boundary layer flows with a range of surface roughness values. In the present study this approach is modified for the case of porous media flows to account for the strong transverse component of fluctuating velocity, as well as the variation of the convective velocity within each flow region. This results in

\[ \frac{T_L}{T_E} = 0.8 \frac{V_{\text{conv}}}{\sqrt{(u'^2 + v'^2)}} \]

where \( u' \) and \( v' \) are the rms values of the fluctuating transverse and longitudinal velocity and \( V_{\text{conv}} \) is the magnitude of the convection velocity of identified eddies based on local swirl strength described previously. To evaluate \( V_{\text{conv}} \), the average of all of the identified swirls for a given flow region was used.

Results of using Eq. (5) for the tortuous channel regions are shown in Fig. 17 versus \( Re_{\text{pore}} \). \( T_L/T_E \) increases with increasing \( Re_{\text{pore}} \), and reaches an asymptotic value very close to 1.5 beyond \( Re_{\text{pore}} \) of 2000. Using this asymptotic result, combined with the measured value of \( T_E \) for the tortuous channel flow given in Fig. 12, the asymptotic Lagrangian time scale for large \( Re_{\text{pore}} \) values is estimated to be

\[ T_L = 0.24 \frac{D_H}{V_{\text{int}}}. \]

To help establish the validity of this relationship for the tortuous channel regions the following argument is presented. As stated previously, these regions have reasonable isotropic and homogeneous characteristics for its turbulence. Consequently, the Lagrangian time scale should be on the order of magnitude of \( T_L \sim l/3v' \), where \( l \) represents the integral length scale and \( v' \) is the rms fluctuating velocity of turbulence (Tennekes and Lumley\textsuperscript{33}). It was shown in Patil and Liburdy\textsuperscript{30} that the integral length scale for flow in this bed reaches a high \( Re_{\text{pore}} \) asymptotic limit of 0.2 \( D_H \) and \( v' \) reaches the limit 0.34 \( V_{\text{int}} \). Using these values the Lagrangian time scale is estimated to be

\[ 0.2 \frac{D_H}{V_{\text{int}}}, \]

which is reasonably close to the measured value of 0.24 \( D_H \)/ \( V_{\text{int}} \).
2. Dispersion: Recirculating regions

Flow within recirculating regions result in increased local retention times for fluid elements as they become entrapped within these regions. The overall retention time can then be a consequence of the ability of fluid elements to release from these regions due to both molecular and turbulent diffusion. The former is expected to be minimal in the limit of large Peclet numbers. The retention time, \( T_R \), in recirculation regions was estimated using the following expression based on the local retention volume (or area within a two dimensional slice), \( A_{\text{rec}} \), and the effective turbulent diffusivity, \( D_{\text{turb}} \).

\[
T_{R,\text{rec}} = \frac{R_{\text{rec}}^2}{D_{\text{turb}}}.
\]

(6)

Here \( R_{\text{rec}} \) is a size estimate taken to be the square root of the area of the recirculating region, such as shown in Fig. 2(c), to be the area enclosed within dividing streamlines. \( D_{\text{turb}} \) is estimated based on the Eulerian velocity variance and integral time scales to be \( D_{\text{turb}} = \langle v'^2 \rangle_{\text{rec}} \langle T_E \rangle_{\text{rec}} \). Based on the above retention time the longitudinal and transverse dispersion contributions due to a recirculation region can be estimated as

\[
D_{L,\text{rec}} \approx \left\langle (V - V_{\text{int}})^2 \right\rangle_{\text{rec}} T_{R,\text{rec}},
\]

(7)

\[
D_{T,\text{rec}} \approx \left\langle (U)^2 \right\rangle_{\text{rec}} T_{R,\text{rec}}.
\]

(8)

These expressions show the contributions of the mean and turbulence variance coupled with the Eulerian time scale in determining the dispersion within each flow region.

3. Dispersion: Constricted flow regions

The jet like flow regions formed due to contractions within the flow geometry forming high velocity regions with high shear boundaries are regions containing high fluid acceleration. The associated retention time, \( T_R \), of a fluid element in these regions is estimated using a similar area and diffusivity approximation given by Eq. (6). To determine the region of influence, \( R_{\text{jet}} \), is taken to be estimated from the regions’ width based on the streamline passing through the high shear region, shown in Fig. 2(d). Also, \( D_{\text{turb}} \) is the turbulent diffusivity estimated to be equal to the product of \( \langle v'^2 \rangle_{\text{jet}} \) and \( \langle T_E \rangle_{\text{jet}} \) within this region. The dispersion coefficient associated with the recirculation region is thereby estimated as

\[
D_{L,\text{jet}} \approx \left\langle (V - V_{\text{int}})^2 \right\rangle_{\text{jet}} T_{R,\text{jet}},
\]

(9)
\[ D_{T,\text{rec}} \approx \langle (U)^2 \rangle_{\text{jet}} T_{R,\text{jet}}. \]  

Because this regions have lower width, \( R_{\text{jet}} \), and hence lower \( T_{R,\text{jet}} \), the contribution to the overall dispersion is expected to be low.

4. Total dispersion analysis

The estimates of the dispersion characteristics of the various regions were used to determine the overall dispersion for the porous bed. It is assumed that the pore scale flow regions identified in this study are representative of the dominant flow regions that can result in variations of the dispersive properties within the bed. Consequently, a typical fluid element experiences a cumulative effect of regions with these flow characteristics as it travels through the bed. The volume fraction of these pore scale regions is representative of what is experienced by a wandering ideal tracer. As representative of this the present data were evaluated based on area averaging using the two dimensional data sets. This suggests that the area average dispersion coefficients, \( D_L \) and \( D_T \), represent overall dispersion coefficients for a large randomly packed bed in the limit of large molecular Peclet number, \( Pe_M \). The area weighted average results become

\[
D_L = \frac{\sum_r D_{L,r} A_r}{\sum_r A_r} = \frac{\sum_r D_{L,r} A_r}{A_{\text{tot}}},
\]

\[
D_T = \frac{\sum_r D_{T,r} A_r}{\sum_r A_r} = \frac{\sum_r D_{T,r} A_r}{A_{\text{tot}}},
\]

where \( D_{L,r} \) and \( D_{T,r} \) represent the longitudinal and transverse dispersion coefficients and \( A_r \) represents the area of each region ‘r’, identified as tortuous channel, jet like or recirculating regions. This weighting accounts for the total bed dispersion by representing the dispersion within each region based on its contribution to the length and time scales distributed throughout the flow. Therefore individual particles will reside in individual regions based on the local time scales, providing information on local dispersion magnitudes.

The dispersion coefficients at the largest value of \( Re_{\text{pore}} = 3964 \) for the different regions are shown in Fig. 18 for both the longitudinal and transverse directions using a log scale. The longitudinal dispersion is seen to be dominated by the contribution from the recirculating region, which is an order of magnitude larger than the tortuous channel contribution. This suggests that the mechanism of retention in these regions plays a very important role in the observed scalar spreading rates within the
bed. The transverse dispersion is primarily a result of the tortuous channel like regions transporting scalars in the lateral direction in the bed. The jet like flow regions, due to its smaller length scale (on the order of the contraction region forming the jet width), is seen to play an insignificant role in the overall dispersion coefficient.

The results of the overall dispersion coefficient versus $Re_{\text{pore}}$ was obtained by combining the individual contributions using Eqs. (11) and (12) and are shown in Fig. 19 for both the transverse and longitudinal dispersion coefficients. These data are normalized by $V_{\text{int}}D_H$, which are representative of the bed velocity and length scales, and results are shown on a log-linear plot. The dashed lines in the figure represents the dispersion values suggested by Wen and Fan$^{35}$ and Delgado$^{36}$ in the limit of the molecular Peclet number becoming very large, and are shown by these authors to be consistent with global dispersion measurements in the literature. These results show that the estimates of the Lagrangian statistics based on Eulerian turbulence measurements for the present study are very close to the asymptotic limits found from the global data. The results appear to reach the limiting value for large $Pe_M$ for values of $Re_{\text{pore}}$ approximately 1500 and above. Although the close agreement could be fortuitous, the estimates of the various scaling parameters, and their corresponding weighting based on the three flow regions is shown to provide a close estimate of the overall dispersion. Results need to be further verified in larger porous bed flows, possibly using a larger representative sample of pores and flow regions to know if these results are consistent over many bed bead packing geometries.

V. CONCLUSIONS

Detailed measurements using time resolved PIV were obtained within a randomly packed porous bed using refractive index matching over a pore Reynolds number range from 418 to 3964. These data were analyzed to determine certain turbulence flow characteristics relevant to dispersion within the bed. Flow regions were identified within the bed based on their mean flow structures which were then used to show the $Re_{\text{pore}}$ dependence on the scaled turbulence quantities. These results indicate that the mean scale characteristics found in pore spaces in the turbulent flow regime ($Re_{\text{pore}} > 839$) were regions containing (i) tortuous channel flow, (ii) recirculating flow, and (iii) jet like flow.

The mean Eulerian velocity variance, fluctuating velocity variance, and Eulerian time scale for these structures were measured and found to vary significantly between the three flow regions. The longitudinal mean velocity variance in the recirculating regions was found to be approximately six times larger when compared with the other regions, while its transverse mean velocity variance was significantly lower by approximately a factor of 20. Turbulent transport is weakest in the recirculating
regions, as indicated by its lower fluctuating velocity variances and longer Eulerian time scales. The jet-like regions with its constricted flow geometry and observed flapping motion are shown to have differences in the directional variances of turbulent motion. However, the associated Eulerian time scales are relatively large, very close to those of the tortuous channel regions. Vortical, or eddy-like structures found within these flow regions were analyzed for their number density, spin rate, and convective velocity. The jet-like regions have little or no detectable swirl structure for the full Re_pore range. However, the other two flow regions both show converging histograms of swirl strength with increasing Re_pore, but with significantly different average convective velocities for swirl events. The overall longitudinal dispersion, as determined from Lagrangian scales based on Eulerian data, is found to be dominated by the retention time associated with the recirculation regions. In contrast, the lateral dispersion is shown to be dominated primarily by the characteristics of the tortuous channel flow region. The overall results are consistent with the asymptotic limit of high Schmitt number results based on global data found in the literature. The current results indicate that dispersion may be directly evaluated from local bed statistical measures at sufficiently large values of Re_pore.

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