The National Council of Teachers of Mathematics published their vision of active, problem-centered instruction with a goal of conceptual understanding in 1989. Fifteen years after these reforms were proposed the changes are reflected in school policy and elementary mathematics curriculum, but only limited change has actually occurred in classroom instruction. With the belief that the classroom teacher is the key person affecting educational change, this case study examines the journey of five elementary classroom teachers as they transformed their mathematics instruction from traditional to reform-based, with the purpose of identifying the key elements that influenced the changes.

This is a multi-case study involving five elementary classroom teachers who have recently been the recipient of the Elementary Presidential Award for Excellence
in Teaching Mathematics. All of these teachers began teaching with traditional textbook programs and have changed their teaching to reform-based, problem-centered instruction. Over the course of two one-hour interviews each teacher told the story of his or her changes, explaining the influences, the key resources, the influential people, and the support they received in the process. The cases are individually presented; then all five are examined together in a cross-case analysis using a constructivist theoretical perspective.

Three key elements were found to be influential in the teachers’ change journeys. First, all five were self-motivated to make changes in their mathematics instruction. They were looking for practices that would give their students both better understanding and positive dispositions. All believed the reform-based instruction met these goals. Second, all five engaged in rich professional discussions about the changes they were making. These discussions were in groups with high levels of trust, in which the teachers freely shared concerns and successes, asked questions, and compared experiences. They were learning communities that supported the teachers’ development of pedagogy and knowledge, allowing them to become confident practitioners. Finally, all five teachers were passionate about their teaching. The learning of their students and the improvement of their teaching were the prime considerations in the changes they adopted and the knowledge and skills they developed.
A Multi-Case Study of Elementary Classroom Teachers' Transitions to Reform-Based Mathematics Instruction

by

Elizabeth Busch White

A DISSERTATION

Submitted to

Oregon State University

In partial fulfillment of the requirements for the degree of Doctor of Philosophy

Presented April 19, 2004
Commencement June 2004
ACKNOWLEDGEMENTS

When I retired from teaching elementary school five years ago, this day was nowhere on my imagined horizon. Thinking I would keep in touch with education, I pursued the idea of supervising student teachers. But thanks to the encouragement of Nancy, Karen, and Ken I found myself teaching classes to the MAT cohort. I was hooked. And I have never doubted the decision to take this road.

I first want to express my appreciation to all the denizens of Education Hall. The people that I encounter every day in the halls, at the mailboxes, on the stairs, and in the offices have, without exception, always been friendly and supportive. How fortunate I feel to have such a nurturing place to work and study. If the world knew how very special educators are, there would never be a shortage of teachers.

I would like to give thanks to the five incredible women on my committee. Karen’s mentoring of my teaching, my research, and my writing has been invaluable. She has a perfect sense of balance between giving me the independence to work things out on my own and the support of right-on suggestions that guide me to success. Nora, Eileen, Dale and Sharon have offered their clear thinking and helpful advice throughout this process. I consider all five role models – what examples they are for me in their scholarship, their service, and their caring.

Three friends who started the doctoral program with me have been important in my reaching this goal. Kathy was instrumental in getting the participants in my research. Our talks about mathematics teaching have helped me in both my own teaching and in my research. Kathy, Kirky, and Diane have been encouragers,
emotional supporters and friends through the long hours of work. It is amazing how they seem to know just when I need a call or e-mail. Their listening to me, my listening to them, and our shared experiences have made the process less lonely. They have celebrated with me as I reached each benchmark. It is great to have guides and supporters, but there is something extra special about fellow travelers.

I want to thank the five teachers who so willingly let me into their classrooms and who openly shared their stories with me. With so much bad news about schools these days, it was refreshing to spend time with these positive, upbeat, enthusiastic teachers. I hope I have represented their stories so that readers will be encouraged that education is not failing and that excellent teaching is thriving. I hope the stories will help us understand how to help teachers develop the passion that these teachers find in their teaching.

Finally, thank you to Hugh and Sarah. Without their being behind this silly idea one hundred percent, I never could have done it. Who retires from a thirty-three year career and then gets a doctorate? Their understanding and acceptance of the time and energy that flowed into this accomplishment has been what has allowed me to get through. It can never be denied that getting a doctorate is a family effort. I feel blessed by all their supportive encouragement and cheering-on.

I have kept a quote by my computer these last few months. It says, “Crossing the finish line takes only seconds. Let me enjoy the wonderful process it takes to reach it.” I have enjoyed the process, and that is primarily because all the people mentioned above had made it so wonderful.
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Chapter 1: Introduction

The Problem

The purpose of this qualitative study is to examine the elements of change as five elementary classroom teachers transformed their beliefs about learning and their practices for teaching mathematics. Change in education is a complex and slow process. Although the most effective educational innovations involve coordination of policy and commitment from all levels of the educational community – governmental, administration, classroom – the key person in affecting successful instructional change is the teacher. The classroom teacher is the agent of change because it is she who is in control at the implementation level (Ball, 1993 & 2003; Ball & Bass, 2000; Heaton, 2000; Ma, 1999; Mewborn, 2003; NCTM, 1991; National Research Council, 2000; Putnam, Heaton, Prawat & Remillard, 1992). Martin (2002) captures the importance of this when he says, “The most basic decisions about elementary teaching – what to teach and how to teach it – are done by teachers. Therefore, despite the best efforts of policymakers and administrators to create uniformity of curriculum, what children are actually taught depends on decisions we make at the classroom level” (p. 310). The cases presented in this study give a picture of five experienced elementary teachers who have adopted new
teaching practices and became proponents of reformed mathematics instruction. By taking this extended look at the stories of these five classroom teachers we can gain understanding of the influences, the motivations, and the conditions that either inhibit or contribute to teacher changes in classroom practices.

**Background**

After several years of research and examination of mathematics education, the National Council of Teachers of Mathematics (NCTM) in 1989 published their *Principles and Standards*, which proposed new goals for the teaching of mathematics. The vision presented in this, and subsequent publications from NCTM, set out a shift from linear, carefully sequenced, teacher directed mathematics to an active, problem-centered process with a goal of conceptual understanding, appreciation of relationships, and the ability to communicate mathematically.

Traditional mathematics instruction was described by Welch and quoted in NCTM's *Professional Standards for Teaching Mathematics* (1991):

> In all math classes that I visited, the sequence of activities was the same. First answers were given for the previous day’s assignment. The more difficult problems were worked on by the teacher or the students at the chalkboard. A brief explanation, sometimes none at all, was given of the new material, and the problems assigned for the next day. The remainder of the class was devoted to working on homework while the teacher moved around the room answering questions. The most noticeable thing about math classes was the repetition of this routine. (p. 1)

The different classroom vision presented by NCTM advocates, instead, more student engagement with the mathematical concepts. The reformed vision
encompasses learning representation and communication of mathematical ideas and the interpretation of the mathematical representations of others. It incorporates communicating mathematically with a range of tools such as language, diagrams, pictures, and manipulatives. In this vision understanding mathematics means not just following procedures to get “right” answers, but being able to make connections between procedural and conceptual knowledge. Heaton’s (2000) restatement of the recommended shifts presented in NCTM’s 1991 Professional Standards for Mathematics Teaching gives the essence of the recommendations:

**TOWARD...**

- classroom as mathematical communities
- logic and mathematical evidence as verification
- mathematical reasoning
- conjecturing, inventing, and problem solving
- connecting, mathematics, its ideas, and its applications

**AWAY FROM...**

- classrooms as simply a collection of individuals
- the teacher as the sole authority for right answers
- merely memorizing procedures
- an emphasis on mechanistic answer-finding
- treating mathematics as a body of isolated concepts and procedures.(pp. 6-7)

The environment in this new vision of mathematics instruction is described in NCTM’s *Principles and Standards for School Mathematics* (NCTM, 2000):

Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work and are continually growing professionals. The curriculum is mathematically rich,
offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment. Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures.

Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it. (p.3)

More than a decade after NCTM’s recommendations were initially published, states and school districts nation-wide have now integrated most of the language of NCTM’s principles and standards into their written educational policy and curriculum documents. University mathematics methods courses focus on the reformed view of mathematics instruction. Curriculum materials and textbooks reference and address NCTM’s principles and standards. However only limited changes have actually occurred in classroom instruction. The TIMSS Video Study conducted in 1995 showed that classroom instruction still followed the instruction patterns described in earlier reports. The majority of videotaped lessons from the United States started with correction of homework, followed by a teacher presentation of new problems and a demonstration of how to solve them. Students then worked independently to solve problems similar to those the teacher had demonstrated (National Research Council, 2001). Many researchers studying
classroom practice have observed that elementary teachers, even those who outwardly support reformed mathematics curriculum, continue to use predominately traditional practices in their classrooms (Grouws & Cebulla, 2000; Hargreaves, Earl, Moore, & Manning, 2001; Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, et al., 1997; Orrill & Anthony, 2003; Spillane, 2000; Wood, Cobb & Yackel, 1995).

Making the called-for changes seems difficult for teachers. First, reform-based mathematics content and pedagogy – content and pedagogy that reflect the NCTM recommendations -- represent a radical shift from the mathematics teachers did when they were in school. The curriculum has expanded from primarily the performance of operations (addition, subtraction, multiplication and division) to include more geometry, measurement, algebraic thinking, number sense, data analysis, and probability. Reform-based mathematics uses problem solving as the vehicle to develop students’ mathematical reasoning and ability to apply mathematical concepts. Right answers are still important, but the conceptual understanding, reasoning, processes, and justifications are equally critical. There is an underlying assumption that teachers will be both competent in and confident of their mathematical knowledge. Where subject matter knowledge for teaching used to be defined simply by the subject matter knowledge that students were learning – that is, by the curricular goals for students – it can now be argued that teachers need more mathematical knowledge in order to have a better perspective on where students are heading (Ball & Bass, 2000; Grouws & Cebulla, 2000). Exactly what mathematical knowledge elementary teachers need is a question currently being examined by many
educators (Ball & Bass, 2000; Heibert, Gallimore, & Stigler, 2002; National Research Council, 2001). Most agree that for complete implementation of reform-based mathematics instruction, elementary teachers need more content knowledge and more understanding of mathematics pedagogy than they did to teach mathematics traditionally (Ball & Bass, 2000; Heibert et al., 2002; Ma, 1999; National Research Council, 2001). When and how practicing classroom teachers acquire additional mathematical knowledge is still unclear.

An acceptance of reform-based mathematics instruction requires many teachers to reject their previous practices and beliefs -- both as mathematics learners and mathematics teachers. Drake, Spillane, & Hufferd-Ackles (2001) suggest that teachers have to re-form their mathematics identities. Furthermore, teachers have to re-form their teacher identities. Traditional mathematics instruction is basically behaviorist. Math skills are learned with repetition and practice. The “rightness” and “wrongness” of answers is determined by the teacher. Mathematics problems are solved by following the procedures presented by the teacher.

On the other hand, reform-based mathematics instruction is primarily constructivist. Constructivist theory permeates the pedagogy called for in teaching mathematics for conceptual understanding. Developed from writers such as Piaget, Dewey, and von Glaserfeld, constructivist teaching has as its central premise the belief that human beings create the world they know and understand from the interactions they experience with the world around them. Wood (1995) summarizes three main underlying assumptions of constructivism: 1) knowledge is actively
constructed by individuals, 2) students develop individual interpretations of science, writing, and mathematics, and 3) new meanings are created by reflection on physical and mental activity arising from problematic situations. Teaching constructively “becomes a matter of creating situations in which children actively participate in scientific, mathematical, or literacy activities that enable them to make their own individual constructions” (Wood, 1995, p. 337).

In addition to individual or personal (Matthews, 2000) construction of meaning, mathematics reformers also discuss the important role of social construction. Origins of social construction come from the work of Lev Vygotsky. According to Bedrova and Leong (1996), Vygotsky believed every part of a child’s environment influenced the development of the mind. It is through sharing and interacting with others that mental processes are acquired. Mathematical meanings evolve from students’ sharing of interpretations, as well as from contributions made by the teachers. So, with reform-based mathematics instruction, the teacher is challenged to not only structure problematic situations, but also to create opportunities for student discourse around the mathematics of the situations. Gergen (1995) reflects this thinking when he says, “Education occurs primarily through the mutual interchange – through the coordination of actions of participants within the dialogue” (p. 34).

For reform-based mathematics instruction, it is most effective not to choose one practice over the other, but to intertwine both (Cobb & Yackel, 1996). Constructivism tells us to focus attention to the mental activities of the individual
learner and social constructivism tells us to remember the influence of the culture and dynamics of the group (Bereiter, 1994). Teachers need to be able to employ both individual constructivism and social constructivism in order to develop students’ problem solving abilities and conceptual understandings to the greatest possible extent.

Therefore, teaching mathematics at the elementary level is no longer just explaining a procedure and marking answers correct or not. With reform-based mathematics instruction the teacher’s role changes from being the authority to the being a facilitator. Teachers must weave their knowledge of their learners with their knowledge of mathematics and their knowledge of teaching. It entails being open to continuous learning and growing by reflecting on practices, seeking answers, working for more understanding, and being alert to new ideas from students as well as other resources (Ball, 1994; Heaton, 1999; National Research Council, 2001; NCTM, 1991). Reform-based mathematics instruction does not propose one clear-cut format for lessons or one best way to progress through the teaching of concepts and processes. Teaching must be flexible and adaptable as the teacher and the students interact. Teachers must be comfortable with uncertainty (Heaton, 2000). Even when a school district has adopted a reform-based mathematics curriculum and reform-based materials, the way mathematics is taught in the classroom – what the teacher does and what the children do – cannot be blankly prescribed. Each teacher must be able to accept the messiness and uncertainty of this kind of teaching, knowing that she will still have to figure out much for herself (Heaton, 2000; Ohanian 1992).
The Questions

This study examined the stories of classroom teachers who made the transition from traditional to reform-based mathematics instruction. Through these teachers' stories I followed their individual paths of change. By looking at their various experiences and asking overarching questions, I looked for patterns that might suggest ways to support or enable other classroom teachers who are at different stages of change. My research questions focused on motivation for change, awareness of innovations, and support in the process of change. I sought the manner in which beliefs and pedagogy were transformed. Furthermore, I inquired about what groups or individuals, if any, who were important influences during any part of the change process.

Significance of the Study

Research over the past decade has given strong evidence that mathematics instruction modeled along the principles and standards recommended by NCTM in *Curriculum and Evaluation Standards for Teaching Mathematics* (1989), *Professional Standards for Teaching Mathematics* (1991), and *Principles and Standards for School Mathematics* (2000) is more effective for students' mathematics achievement (Cobb, Wood, Yackel, Nicholls, Grayson, Trigatti & Perlwitz, 1991; Hickey, Moore, & Pellegrino, 2001; Schoenfeld, 2002; Yackel & Cobb, 1996). Studies have shown that, compared to students in traditional mathematics programs, the students who spend more time problem solving and
discussing their mathematical thinking perform equally well in computation skills and mathematics operations and they perform significantly better on problem solving and reasoning tasks (Baxter, Woodward & Olson, 2001; Hiebert & Wearne, 1992; Schoenfeld, 2002; Yackel & Cobb, 1996). In addition, students that traditionally under-achieve in mathematics – lower socioeconomic groups, learning disabled, students of color – were also found to have improved scores after working in problem solving/discussion centered programs (Baxter, Woodward & Olson, 2001; Hickey, Moore, & Pellegrino, 2001; Hiebert & Wearne, 1992; Schoenfeld, 2002).

There is a disconnect between the research on teaching mathematics and the mathematics instruction that happens in most elementary classrooms. On the whole, teachers are not changing their practices (Grouws & Cebulla, 2000; Ma, 1999; National Research Council, 2001; Spillane, 2000). In order to improve the mathematics achievement of students, mathematics instruction in the classroom needs to change. Mathematics reformers ask: What is keeping this change from happening? What changes are happening that may be unobserved? How can change be encouraged or facilitated among elementary teachers?

Research has looked at the complex nature of educational change through the study of institutional change (Fullan, 2001; Hargreaves & Fullan, 1998) and through a lens of cognitive perspective (Spillane, 2000; Spillane, Reiser, & Reimer, 2002;). Middle school teacher changes were reported in a study of the teachers’ responses to changes in curriculum and standards (Hargreaves, Earl, Moore and Manning, 2001). Much writing has described the classrooms and instruction of teachers who have
shifted or are in the process of shifting their teaching practices (Heaton, 2000; Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver & Human, 1997; Lampert, 2001; Schifter, 1996; Schifter & Fosnot, 1993). Shifter and Fosnot (1993) include personal stories of teachers’ journeys from traditional mathematics instruction to reform-based teaching. These are the stories of teachers who were involved in a long term, supportive, and interactive summer workshop in the early 1990’s. There was follow up involvement by the workshop leaders and on-going communication among participants. This study looks instead at the journeys of five individual teachers’ that may or may not have had an organized support group or on-going mentorship. It addresses the stages even before participation in workshop and inservice training. It adds to the picture of how change comes about and progresses for classroom teachers.

*Researcher’s Connections*

For the last ten years of my career as an elementary classroom teacher I was very involved in mathematic curriculum development and inservice training of teachers. For myself, reform-based mathematics was the answer to a long personal struggle for more effective mathematics instruction. I was continuously surprised at other teachers’ lack of acceptance of these new ideas. I became interested in the research on the effectiveness of the reform-based mathematics instruction and in the implementation of educational change.
After retirement from public education, I began to teach in a teacher education program at a local university. That led me to my doctoral work. Researching this previous interest became my focus. Perhaps the seeds for this proposed study came from comments of a colleague who was the Elementary Presidential Awardee in Mathematics several years ago. She shared her dismay at being part of a collection of one hundred recognized leaders in mathematics education (one elementary and one secondary teacher from each state) in Washington D.C., wined and dined by the Department of Education, including a session with the president himself, and no one, in the whole week, asked them about the needs or issues of teaching mathematics. No one asked for recommendations for mathematics education. No one asked them to share their stories about teaching mathematics. This study gives some voice to the recognized leaders in mathematics education.

Definition of Terms

Reform-based Mathematics Instruction. Reform-based mathematics instruction is a collection of practices for teaching mathematics that incorporates the Nation Council of Teachers of Mathematics (NCTM) recommendations for building instruction around thinking and problem solving. It is built on six principles and ten standards that are key considerations in planning and teaching. An alternative term used by reformers, and occasionally in this document, is problem-centered teaching.
The NCTM Principles for mathematics instruction are listed and defined below:

- **Equity.** Excellence in mathematics education requires equity—high performance expectations and strong support for all students.

- **Curriculum.** A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.

- **Teaching.** Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.

- **Learning.** Students must learn mathematics from experience and prior knowledge.

- **Assessment.** Assessment should support the learning of important mathematics and furnish useful information to both teacher and students.

- **Technology.** Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning. (NCTM, 2000, p. 11)

*NCTM Content Standards (Content to be included at every grade level).* The NCTM Content Standards, define the contents considered to be the core of mathematics instruction:

- Number & Operations
- Algebra
- Geometry
- Measurement
- Data Analysis & Probability
**NCTM Process Standards** (Important mathematical processes that should be incorporated into mathematical work at all grade levels). The NCTM Process Standards describe processes that are important in the way mathematics is done across all the content strands:

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representations

**State Standards.** The State Standards are the set of student learning goals for each area of the curriculum that each individual state had developed. These standards are the basis for assessment done by the state’s Department of Education.

**Inservice Training.** Inservice Training is professional workshops or classes in which the participants are practicing teachers. An alternative term used is professional development.

**Summer Leadership Institute.** The Summer Leadership Institute is a generic pseudonym for the annual summer meeting to which leaders in mathematics education from around the state are invited. At the Summer Leadership Institute instructional strategies and resources are shared, issues are discussed, and activities for the support of mathematics education are planned. Most states’ mathematical education communities have a Summer Leadership Institute.
State Mathematics Center. The State Mathematics Center is a pseudonym for a state organization, whose goal is to support mathematics instruction with workshops, materials, and other resources for classroom teachers.
Chapter 2: Review of the Literature

Introduction

This study explored the important influences and compared the processes as five classroom teachers changed their mathematics instruction from traditional, text-driven, authoritative teaching to active, problem-centered student investigation. At the heart of these changes is acceptance of constructivist teaching theory. The changes require that teachers have conceptual understanding of all mathematics content areas, as well as new pedagogical skills that build that understanding into the students’ learning. Adopting reform-based mathematical instruction is a deep personal and philosophical change for most teachers. As a background for this study I have reviewed the literature on the three interconnecting areas: constructivism, change, and the mathematical knowledge required for reform-based instruction.

In the area of change, I have first summarized research on individual change, which comes mostly from the field of psychology, and then on educational reform, which looks at institutional change. Both areas are pertinent to the study because a classroom changes are related to the system, but the agents of those changes – the teachers – must each adopt the innovations individually.

Constructivism is the theoretical foundation on which the changes in mathematics instruction have been founded. It is important to be familiar with the educational application of this theory in order to understand the changes taking place in the teaching of mathematics. I have briefly summarized literature giving a
background of constructivist educational theory. Then I share what is written regarding its application to teaching.

Finally, I have reviewed the literature addressing the knowledge needed by teachers in order to teach mathematics as it is envisioned by the reform. Findings show that teachers need both mathematical content knowledge and new pedagogical skills to adopt the prescribed innovations successfully. The research on this knowledge base of teachers is very current and still being developed. Questions regarding capacity of elementary teachers to make the recommended changes are central to mathematics reform.

Change

Introduction

Much has been written over the last several decades about ways innovations can be introduced and understood by individuals and organizations. In the area of health and counseling there have been studies to help individuals become aware of new ideas that, in turn, help them change behaviors that are dangerous or unproductive. Businesses, schools, and governmental agencies have been especially interested in organizational change. In this review I have presented four models that are useful in explaining the change process as it relates to individuals: The Health Belief Model, Stages of Change (Theoretical) Model, Diffusion of Innovations Model, and the Innovation-Decision Process. I have also included Fishbein’s (1997) six lessons learned from social psychology as additional insight into change process.
Finally, I have presented a summary of research literature that focuses on institutional change as it relates to the reform efforts in schools and teaching.

**Individual Change**

*Health Benefit Model.* Researchers in the 1950’s, interested in improving people’s health decisions, examined the relationship between health beliefs and behaviors (Sheeran & Abraham, 1996, p. 23). The Health Belief Model that was developed focuses on an individual’s health behavior and the perception of threat from that behavior. This research established a correlation between beliefs and health behaviors (Sheeran & Abraham, 1996). Although the model leaves some questions about *why* an individual undertakes a specified change or not, it presents understandable constructs about an individual’s acceptance of change. As a result, it has focused researchers attentions on the psychological prerequisites of behavior and has provided a basis for practical interventions (Sheeran & Abraham, 1996). This model also sheds light on individuals’ responses to any change or innovation that presents a threat, even a perceived threat, in any area of their life. Weighing the costs against the benefits is often the criteria in the decision to make a change or not.

*Stages of Change Theory.* The Stages of Change Theory, also known as the Transtheoretical Model, was developed to help explain the phenomena of someone’s intentional change, as opposed to societal, developmental, or imposed change (Grimley, Prochaska, Velicer, Blais, & DiClemente, 1994). Looking across other theories of change, Prochaska and his colleagues described the processes and stages
individuals go through when actually making changes. This model explains change by describing nine processes and six stages. The processes and stages are integrated so that particular processes are of more or less importance during particular stages (Grimley et al., 1994). Grimley, Prochaska, Velicer, Blais, and DiClemente (1994) and Prochaska, Norcross, and DiClemente (1994) represent the progression as a spiral, not a line, because they have observed that most individuals move ahead and regress several times as they are adopting new behaviors.

The six stages of change are precontemplation, contemplation, preparation, action, maintenance, and termination. These stages are predictable and well defined. They take place over a period of time and entail completing a series of tasks before progression to the next stage. (Prochaska, Norcross & DiClemente, 1994). Being aware of the stages and matching tasks appropriate to moving through each one will help maximize progress (Prochaska et al., 1994). Nearly all change begins with precontemplation, yet only the most successful will end in attainment of the final goal. Attainment is the stage at which the old behavior or problem is completely eliminated and the new behavior is firmly in place with no going back (Prochaska et al., 1994).

The Model defines processes of change -- the actions or processes individuals do to progress from one stage of change to the next (Grimley et al., 1992). The goals of these processes are listed below as they were described by Procheska, Norcross, and DiClemente (1994):
1. Increasing information about self and problem
2. Increasing social alternatives for behaviors that are not problematic
3. Experiencing and expressing feelings about one’s problems and solutions
4. Assessing feelings and thoughts about self with respect to problem
5. Choosing and committing to act, or belief in ability to change
6. Substituting alternatives for problem behaviors
7. Avoiding stimuli that elicit problem behaviors
8. Rewarding self, or being rewarded by others, for making changes

Recognition of these processes helps explain the goal of behaviors that can be observed when someone is adopting or considering adoption of change. But change is still very complex. Each of the processes involves strategies that may employ any number of techniques (Prochaska, et al., 1994). Furthermore, change takes time -- often a long time. Time needs to be a factor in assessing change. “The vast majority of people struggle for years to find effective solutions to their problems. They try not to become demoralized by failure, although sometimes they feel they will never change. They are embarrassed or frustrated when someone comes along and tells them, ‘I... [whatever]... years ago. It was easy’” (Prochaska, et al., 1994, p. 48).

This model reiterates that some change comes from perceived problems. However, it also recognizes that many changes come from adapting to new knowledge or innovations in our personal and professional worlds. In our world today, most people are experiencing multiple innovations -- both imposed and voluntary – at any given time.

**Diffusion of Innovation Model.** The third model is called the Diffusion of Innovations Model. It looks beyond strictly individual change. It is more focused on social change than individual change. However, as Rogers and Shoemaker (1971)
write, "The structure of the social system is provided by the various individuals and group statuses which compose it." (p. 7). This model puts forth the following categories of social change: *immanent change*, with no external influence; *selective contact change*, when someone is exposed to a new idea and rejects or accepts it based on their own needs; and *directed contact change*, which is planned change that comes from outside (Rogers & Shoemaker, 1971). Directed contact change is of interest in most educational and organizational change. However, in education it is not just the imposition of change from "outsiders." It is often the introduction of innovations from one level of the organizational structure to the other.

Rogers and Shoemaker (1971) name four elements of change: the innovation itself, its communication from source to receivers, the time the ideas take to spread, and the social system through which it is diffused. An innovation is defined as any idea, practice, or object perceived as new by an individual. The perceived characteristics of the innovation determine its rate of adoption (Rogers & Shoemaker, 1971). So again, time is realized as an important factor in the change process.

One key principle in the Diffusion of Innovation Model is that change occurs within a social system, and the system’s social structure can have an important influence on the spread of new ideas. The structure of the system can act to impede or facilitate the adoption of new ideas. The group’s norms, individual’s social statuses, and hierarchy influence the behavior of individual members of that system (Rogers & Shoemaker, 1971). Furthermore, diffusion of an innovation may also
change the social structure of a system because many innovations are of a restructuring nature (Rogers & Shoemaker, 1971). Certainly these are important considerations when we reflect on educational change. We must be aware of how the changes that are recommended will change not only practices, but also the culture of the organization.

**Innovation-Decision Process.** Related to the Diffusion of Innovations Model is the Innovation-Decision Process (I-DP). It also relates individual change to the group or social structure in which the change is taking place. The model describes the process through with individuals move from first being aware of new ideas to the adoption or rejection of those ideas. The stages in this process seem parallel to the stages of change described by Procheska, et al. (1994). The I-DP consists of 1) the awareness stage, 2) the interest stage, 3) the evaluation stage, 4) the trial stage, and 5) the adoption stage. This listing implies that the process always ends in adoption decisions, yet it should be noted that, in reality, rejection is just as likely an outcome (Rogers & Shoemaker, 1971). This model further proposes that the simpler and clearer the innovation the shorter the time to adoption. More complex change takes longer.

Knowing the criteria that is most often used for the acceptance or rejection of innovations can facilitate the change process. The criteria can be used to make information clearer, answer innovators’ questions, and help match individuals with compatible innovations. They can make adoption/rejection more predictable.
Weinreich (1999) lists helpful questions from the I-DP that indicate the probability of acceptance of new ideas:

- Is the innovation better than what the individual is currently using or doing?
- Is the innovation easy to use or understandable?
- Are other people in the peer group using the innovation? If so, what is their experience with it?
- Does the innovation fit in the person’s value system and self-image?
- Is it possible to try the innovation first before committing to it?
- How much of a commitment is necessary to use the innovation?
- How much risk (monetary or emotional) is involved with adopting the innovation? (p. 96)

For voluntary or unplanned change an individual will be applying these criteria for themselves. For change instigated by others, it could be useful for the change agents to address these criteria with the audience to which change is proposed.

Finally, this model recognizes the phenomenon that in any social system, individuals will adopt new ideas at different rates. The I-DP helps explain how different categories of individuals adopt an innovation earlier or later in the process. It defines five types of innovators, with the caveat that the acceptance of innovations is more a continuum than stages. The categories are presented from the earliest to the latest adopters: innovators, early adopters, early majority, late majority and laggards.

The innovators are those who are most venturesome and are willing to take on untried and out of the norm ideas. Many of the innovations they adopt are not successful. The early adopters are the typical leaders in innovation. Peers look to them for guidance and information. Laggards are the members of a group that will never adopt the innovation or will adopt so that other innovations have been superceded by more recent ideas. These categories can be helpful to change agents in
deciding a target audience for the introduction of the initial ideas or for the choice of leadership towards the change (Weinreich, 1999).

**Observations from Social Psychology.** Fishbein's (1997) observations from social psychology are further help in understanding why individuals may or may not adopt a new behavior. He presents six lessons, which add valuable food for thought to the four previously presented models. In order to relate Fishbein's lessons to change in public schools, I have substituted the word *practices* for behavior in the following discussion.

First, even deeply established *practices* can be changed if interventions are well planned and theoretically based. No change is impossible. But implementation does not happen effortlessly. Effectiveness of interventions needs to be designed with the target population in mind (Fishbein, 1997). Second, all information is not equal in effecting change. However, information about performing the *practice*, about groups who support the *practice*, and/or about ways to overcome barriers to *using the practice*, can be effective (Fishbein, 1997). Fishbein is saying is that it is important to consider those individuals most directly effected by the change and to match information to their needs and concerns so that the change makes sense and possibilities are clear.

A third lesson from social psychology is that prior to developing any planned innovation it is necessary to determine whether the failure to perform the *practice* in question is due to a lack of intention, the absence of skills and abilities, or to the presence of environmental constraints. The interventions or the direction of
implementation should then be clearly directed to address these factors. In education, this is one of the most important factors in making changes. And it is directly related to the fourth lesson. Fishbein (1997) writes that the most effective interventions will be those directed at changing specific practices. He recommends focusing on particular practices and not aiming change at more generic goals, which does not necessarily change any specific practice.

A fifth lesson addresses attitudes and norms in relation to change. It prescribes the importance of determining if the decision to adopt or reject an innovation is more influenced by attitude or by norms. Wagner (1969), in his study of attitude change, concurs when he says, “There is no better way to predict [an individual’s] response to a stimulus than to know his attitude toward it (p.2). Individual attitude is at the core of the predisposition toward an idea or practice (Wagner, 1969). Fishbein (1997) adds that group norms must also be considered in the acceptance of change in practice (Fishbein, 1997). These are important considerations in educational change because the reform being proposed in mathematics, for example, requires changes in both attitudes and norms by teachers, policy makers, students and the community (Heibert, Carpenter, Fennema, Fuson, Wearne, Murray, et al, 1997).

Fishbein’s last lesson relates to the complexity of change. Implementation of change cannot be unthinkingly proposed. Change proponents must realize that changing practice takes time (Fishbein, 1997, p. 88). Change needs to be carefully designed and planned. It needs to be given time to take effect. This lesson reinforces
the consideration of time put forth in the other models. Putting change in perspective, Fishbein states, “Interventions cannot be looked at as a ‘quick fix.’ Change is not an all-or-nothing, immediately occurring phenomenon. We must become more realistic in our expectations about the amount of... change one can expect a given intervention to produce in a given time period” (p. 89).

Educational Reform

**Background.** Widely implemented and/or large-scale change is not an old tradition in education. Up until the mid century public schools evolved slowly, with changes taking place here and there, but without concentrated effort until Sputnik grabbed the country’s attention (Fullan, 2001; Hall & Hord, 2001; Ruddick, 1991). Since the 1950s huge amounts of time, money, and attention have been paid to improve, reform, change, and restructure public schools. In the 1960’s there was much pressure for innovation (Fullan, 2001). Behavioral objectives became the core of curricular programs and instructional processes in the 1970s (Hall & Hord, 2001). However, studies of these innovations showed little success despite all the well-intentioned efforts (Fullan, 1993). Change was not taking hold. There was a period with little headway. In the early 1980s schools were beginning to again try some innovative ideas, but the changes were too small and too late for those wanting major school improvement (Fullan, 1993). The benchmark for the beginning of a renewed and re-energized effort was the publication in 1983 of The Nation at Risk, calling for government involvement in fixing failing schools. “The solution was seen as
requiring large-scale governmental action” (Fullan, 1993, p. 2). What followed is what Hall & Hord (2001) refer to as “waves” (p. 23) of reforms. This was a new era for educational change.

Over time the efforts became more and more top-down. Curricular mandates, student competencies, and teacher responsibilities were imposed from state and national levels. Efforts have moved from school reform to district reform to system reform. However change has been elusive. In all this time “the pressure to reform has increased, but not yet the reality” (Fullan, 2001, p. 6). Perhaps this is because we have ignored the wisdom of Miles, who wrote in 1964, in his book, *Innovations in Education*, that a new social practice takes 50 years (MacDonald, 1991). “Would-be innovators… have been lucky to get five years to accomplish the job. It isn’t enough” (MacDonald, 1991, p. 8). The end of the 1990s has come and gone with many proposed changes, but few making any significant impact on schools or classrooms.

Fullan has examined educational change over a course of many years. He remains optimistic about the possibilities of change. He believes there is more appreciation of the complexity of change, which helps achieve more successful adoption. He has observed several good examples of successful change. Fullan and others who study educational change believe that now present, and missing in earlier reform efforts, is an understanding of the central role of the individual (Fullan, 2002; Hall & Hord, 2001; Hargreaves, Earl, Moore, & Manning, 2001; Spillane, Reiser, & Reimer, 2002; Windschitl, 2002). Previously, implementing agents failed to notice or
intentionally ignored individuals. They too often selectively attended to policies consistent with their own interests and agendas (Spillane, et al., 2002). Change literature now widely reports the need to identify capacity in those implementing the change(s). There is recognition of the critical importance for the innovators to be able to make sense of innovations and for innovations to make sense. Unfortunately, warn Spillane, Reiser, and Reimer (2002), “Sense-making is fraught with ambiguity and difficulties...[and] provides numerous opportunities...for the transformation of...ideas about changing practice” (p. 391). Current reform literature matches the models of change, discussed earlier, in calling for understandable and useable information. Attention to attitudes and beliefs are important, as well as incorporation of knowledge that builds a foundation for the change.

When change involves change in practice – as called for in educational change – it must be clear what that means before it can be supported and facilitated. However, the complexity of teaching means educational change is seldom a single entity. It is complex even if is just an innovation in a classroom. Fullan defines three key dimensions in the implementation of any new educational practice or program: 1) use of new or revised materials, 2) use of new teaching approaches and, 3) often a change of beliefs. In individual change models, the measure of change is in an actual, observable change in behavior. In education, the change has to occur in practice along these three dimensions if it is to have any chance of affecting the outcome (Fullan, 2001). Fishbein’s (1997) lessons recommend each change be specific. For that lesson to apply to educational change each must be specific across
the three dimensions – the materials used, the teaching approaches, and the teacher’s beliefs. For the success of any educational change, proponents of the implementation need to be aware of the stage of change the teacher, administrator, or policy maker is at for each dimension. The Innovation-Decision Process stages can be applied for each dimension. It is extremely complex, but understanding the complexity may make it less of a barrier.

Fullan (2001) has further identified a set of interactive elements that together, over time, contribute to the process of change. These factors involve the characteristics such as need, clarity, complexity and practicality. The factors include the various influences of the district, school, community, administration, classroom, and of government agencies. The more factors that support a change, the more likely the change will occur. The more factors that are against the change, the less effective the process. Just as the final stage of individual change is not always attained, school change can be elusive. Even if all the factors work together, success still requires ownership and passion on the part of the change adopter.

With passion and ownership changes can really stick. With all change, emotion and feelings are key players in adoption. They cannot be discounted. Feelings and emotion are why each individual’s role needs to be recognized and made part of the change decisions. Change is, at best, uncertain. It can be risky. When people try out new ideas they cannot always know what will happen. Teachers, feeling responsible for the learning of their students, can feel particularly vulnerable to uncertainty or uncharted waters. Fullan’s (2001) study of change
recognizes an *implementation dip* – where things can seem to get worse instead of better. This is when there must be a feeling of ownership, a sense of being supported, and a deep sense of caring about the change one is attempting (Fullan, 2001).

Andy Hargreaves and Michael Fullan, together and independently, have closely observed and comprehensively studied educational change for more than a decade. In their most recent writings they both suggest that the focus should be on *reculturing* schools rather than *restructuring* schools. They both believe that effective changes are those that keep in mind the core (as in the *heart* of) purpose of schools -- the education of children. They recommend collaborative relationships, with moral purpose in the forefront of our thinking, that operate within and between the multiple layers of the educational structure (Fullan, 2001; Hargreaves, 1997). In conclusion, we need to build capacity and support for the individuals who will be directly and personally impacted.

*Constructivism*

*Background*

The term *constructivism* seems to be the educational label of the day. Over the past decade-and-a-half the concept of constructivism has mushroomed as the focus of educational researchers, curriculum designers, teachers, teacher educators, educational philosophers, and psychologists (Applefield, Huber, & Moallem, 2000; Matthews, 2000; Phillips, 1995 & 2000; Tobin, 2000). It is a concept that is still being developed into an educational theory, with much debate as to how it translates
into classroom practices. The one central concept that seems to be agreed upon by all
"constructivists" is that all human knowledge is *constructed*. Knowledge is the
result of a learner’s activity and not of passive reception of information or

The concept of constructed knowledge is really not a new idea. Many trace
the ideas back to the writings of Immanuel Kant over two hundred years ago
According to Brooks and Brooks (1993), Kant concluded that one cannot infer new
relationships among objects, events, or actions unless one has previously formed
views through which perceptions can be organized. However, it was the
philosophers of the twentieth century that brought constructivism to the forefront of
our educational thinking. Constructivist learning theory has developed on its own
trajectory, separate from the field of philosophy (Howe and Berv, 2000).
“Constructivist learning theory has its primary roots in the work of Jean Piaget and
Lev Vygotsky. Many claim that John Dewey also held a constructivist theory of
learning” (Howe & Berv, 2000, p. 30). The work done by Thomas Kuhn on
paradigms and scientific revolutions has been an important constructivist influence
in science education (Boubourides, 1998; Howe & Barv, 2000; Matthews, 2000;
Phillips, 1995). Ernst von Glaserfeld’s radical constructivism strongly influenced
the teaching of science and mathematics (Matthews, 2000; Phillips, 1995).

Educational constructivism is divided into what Matthews (2000) calls
“personal constructivism” (p. 169), which is grounded in the ideas of Piaget and van
Glaserfeld, and social constructivism, with origins in the work of Lev Vygotsky (Matthews, 2000). Personal constructivism is the view that students actively construct their own ways of knowing as they work to understand the world in accordance with their personal experience (Cobb, 1994). Piagetian theory describes this process in terms of adaptation and organization. Adaptation is a process of assimilation and accommodation. Ideas and events new to a system are assimilated into mental structures. New and unusual mental structures are accommodated into the mental understanding. The balancing of this assimilation and accommodation results in a state of temporary cognitive stability called equilibrium (Brooks & Brooks, 1993; Piaget, 1985/1975). Piaget’s studies of children’s development ranged over several decades. As reported by Brooks and Brooks (1993), by at the end of his long career he had developed a theory that encompassed more than a simple discussion of assimilation, accommodation, and equilibrium. He moved beyond his original work on stages of development. His later theories offered a model of dynamic equilibrium, best characterized as successive coordination and progressive equilibrations. Equilibration is a process, not a finished state. Successions of constructions produce new mental structures as they correct and complete pre-existing structures (Piaget, 1985). Piaget laid the groundwork for further theories that, along with Piaget’s work, have changed cognitive psychology and the study of teaching and learning.

Ernst von Glaserfeld, whose work was greatly influenced by Piaget, developed radical constructivism. The educational implications of von Glaserfeld’s
beliefs can be summarized into two main principles. First, knowledge is not acquired passively, but is actively built by the individual. Second, the function of knowing is adaptive and serves to organize the experienced world. Learning does not discover reality. We do not find truth. Radical constructivism holds that we only construct viable explanations for our experiences (Matthews, 2000). Knowledge, therefore, cannot be transferred with words. According to von Glaserfeld, all good teachers know that the guidance they provide students remains tentative and can never approach absolute determination. In radical constructivism there is often more than one solution to a problem and different solutions can be approached from different perspectives” (Boudourides, 1998).

From the field of science, Thomas Kuhn’s ideas have provided other major influences in constructivist thought. His theory of scientific development includes anomalies and crises that cause one to look more critically at the established norm and develop a new paradigm or conceptual understanding (Boudourides, 1998). Boudourides explains, “Scientists construct, not discover, ‘what is really there’ by means of persuasion and social justification in order to arrive at a sort of consensus around the emerging new research tradition” (p. 3). Therefore, according to Kuhn, although knowledge is created within the individual, that knowledge is influenced by, and a part of, a larger body of knowledge – knowledge that is community-generated and community-maintained (Boudourides, 1998).

Although Kuhn was most known for the idea of changing paradigms, he also supplied the idea of a community-generated/community-maintained body of
knowledge. That leads us to the other constructivism – social constructivism. John Dewey has been given the title of constructivist, after the fact, because so much of what he wrote melds so smoothly into constructivist theory. Solomon (2000) reports that Dewey believed that language is an instrument of social co-operation and participation, and that it is critical in the development of the mind. Dewey also sounds constructivist when he says “[Learning] cannot take place through the direct conveyance of beliefs, emotions and knowledge.” (Quoted in Solomon, 2000, p. 291). This representation of Dewey’s beliefs fits closely with Applefield, Huber and Moallem’s (2000) description of social constructivism. Applefield, Huber and Moallem (2000) express the constructivist belief that it is through the cognitive give and take of these social interactions that one constructs personal knowledge. Social constructivists propose “the context in which learning occurs is inseparable from emergent thought” (Applefield et al., 2000, p. 4). These precepts are a shift from traditional educational beliefs because they place students’ thoughts and interactions as key elements in learning. As these are applied to educational issues there are two instructional implications. First, the implication students have an active role in learning (Bredo, 2000). Second, is the interest in students “being allowed to redefine or discover new meanings for the objects with which they interact” (Bredo, 2000, p. 132). This leads to educational practices that value classroom discourse and discussion and, additionally, value the importance of students’ ideas and suppositions.
The primary foundations for social constructivism are the theories of Vygotsky. As reported by Boudourides (1998), his main relevance to constructivism is his work in the areas of thought, language, and their mediation by society. Vygotsky believed that every part of a child's environment influenced the development of his mind. According to Bedrova and Leong (1996), a child's attempts to learn and society's attempts to teach through parents, teachers, and peers all influence the way a child's mind works. Furthermore, it is through sharing and interacting with others that mental processes are acquired (Bedrova & Leong, 1996). Bedrova and Leong further explain Vygotsky's *zone of proximal development* (ZPD), which is a social learning process. In the ZPD theory, a child starts with some knowledge – a preconception or undeveloped concept. Whatever that knowledge allows the child to do or understand is the *independent performance level*. This represents what the child can do by himself. With assistance, from a person who has more knowledge or skill or understanding in the knowledge that is being developed, the child can begin to do or know more. This is an *assisted level of performance*. Eventually, the child, with this guidance, will attain an *independent level* with new knowledge. These are not steps, but a continuum of performances. The system is dynamic and always changing (Bedrova & Leong, 1996). The importance for constructivist theory is the role of the interaction with others in the construction of new learning or the development of a new mental construct.
Cobb (1994), doing research in mathematics education, sees a growing disillusionment with a constructivism that only addresses personal construction of learning. Cobb makes a case that individual and social construction actually work best together:

Both these perspectives are of value in the current era of educational reform that stresses both students’ meaningful mathematical learning and the restructuring of the school while simultaneously taking issues of diversity seriously....The challenge of relating actively constructing students, the local microculture, and the established practices of the broader community requires that adherents to each perspective acknowledge the potential positive contributions of the other perspective. (p. 18)

In most elementary classroom this schism does not seem to be a problem. Teachers, watching children tackle the understanding of a new idea, recognize the interplay between the inward grappling and the social exchanges. Bereiter (1994) puts it succinctly when he states, “Constructivism tells us to pay close attention to the mental activities of the learner, and [social constructivism] tells us to pay close attention to cultural practices in the learner’s milieu” (p. 21). Teachers need to mix and match for the best route to students conceptual understanding (Bereiter, 1994).

Classroom Practices

How do the tenets of constructivism and social constructivism get applied to classrooms? Howe and Barv (2000) name two premises of constructivist learning theory: (1) learning takes as its starting point the knowledge, attitudes, and interests students bring to the learning situation, and (2) learning results from the interaction between these characteristics and experience in such a way that learners construct
their own understanding, from the inside, as it were,” (pp. 30-31). Brooks and Brooks (1993) name five principles of constructivist teaching:

1. Posing problems of emerging relevance. Relevance can emerge through teacher mediation.
2. Structuring learning around primary concepts or conceptual clusters.
3. Seeking and valuing students’ points of view.
4. Adapting curriculum to address students’ suppositions
5. Assessing student learning in the context of teaching.

These principles are like a scaffold onto which activities can be placed. Curriculum designers and teachers use these as a guide to selecting and creating activities and environments for learning. The strength of these principles for teaching are confirmed by the most current research on learning and reported in How People Learn (Bransford, Brown & Cocking, 2000). In the book, Bransford, Brown and Cocking explain classroom implications based on educational research:

- Teachers must draw out and work with the preexisting understanding that their students bring with them.
- Teachers must teach some subjects in depth, providing multiple examples in which the same concept is at work and providing a firm foundation of factual knowledge
- Metacognitive skills should be integrated into the curriculum in a variety of subject areas.

Bransford, Brown, and Cocking (2000) further describe the classroom environments that research indicates are the most effective:

- Classrooms and school should be learner-centered.
- Attention must be given to what is taught (subject matter knowledge), why it is taught (understanding), and what competence or mastery looks like.
• Ongoing assessments are essential. They allow teachers to grasp the students’ preconceptions, understand where the students are in the development of their thinking, and design instruction accordingly. Formative assessments help both teacher and student monitor progress.

• Learning is fundamentally influenced by the context in which it takes place. A community-centered approach requires the development of norms that support core learning values.

These findings seem to confirm that constructivist teaching practices and effective teaching are not only compatible, but also inseparable.

In mathematics particularly, constructivist teaching has been a guiding principle in the effort to reform and improve instruction. In a discussion on the improvement of mathematics instruction, Ball (1993) writes,

Teaching and learning would be improved...if classrooms were organized to engage students in authentic tasks, guided by teachers with deep disciplinary understandings. Students would conjecture, experiment, and make arguments; they would frame and solve problems; and they would read, write, and create things that mattered to them. Teachers would guide and extend students’ intellectual and practical forays, helping them to extend their ways of thinking and what they know as they develop disciplined ways of thinking and encounter others’ texts and ideas (p. 374).

This vision was not driven by constructivism, but the discipline of mathematics itself. It is the importance of understanding in mathematics and the centrality of the generation of understanding in constructivist theory that makes for a good match (von Glaserfeld, 1991). Behaviorist teaching, that aims only at training students to come up with correct answers, separates the operation from understanding and fails to teach mathematics (von Glaserfeld, 1991). The examination of students’ concepts allows for the testing of viability in our models and representations. Moreover, in
allowing students to construct their own understanding, it provides the teacher opportunity to gain a new and fuller understanding (von Glaserfeld, 1991).

Why, one could ask, is constructivist teaching not uniformly practiced? Why do some educators resist or oppose constructivist teaching? The pathways from the theory is not always clear and many of the recommendations coming to teachers have not been clear in terms of how or why this is a better way of teaching. Some teachers see it as just another fleeting idea that will blow away on the winds of the next "fad". Most certainly folk pedagogy is at play (Olson & Bruner, 1996). Folk pedagogies are the long standing, deeply believed notions one has about teaching, learning, roles, and human behavior. Any new theory has to compete with the already-in-place folk theories (Olson & Bruner, 1996). "A theorist convinced that children construct their own knowledge will have to confront the established view that knowledge is imparted; the theorist convinced that aptitude for learning is a matter of prior knowledge will have to confront the entrenched view that readiness is a matter of fixed abilities (Olson & Bruner, 1996). And, of course, they will have to confront these same entrenched views with the children and parents as well (Olson & Bruner, 1996). In mathematics, the folk pedagogy of parents created the math wars that completely stopped and turned a well-established, successful mathematics reform in the state of California (Talbert, 2003). When von Glaserfeld (1991) refers to the "sophisticated techniques developed by Professor Skinner and his followers...[showing] that in many tasks almost flawless performance can be achieved by the methodical management of stimuli and reinforcement" (p. xvi) he is
addressing a widely accepted (to this day) folk pedagogy. “The didactic view manages the child from the outside, from a third-person perspective, rather than attempting to ‘enter the child’s thoughts’ and to cultivate understanding” (Olson & Bruner, 1996, p. 18). Constructivists are still facing the folk pedagogical belief that a child’s mind is a *tabula rosa*. Many educators, indeed many adults in general, still believe that knowledge is *put into* the learner’s mind (Olson & Bruner, 1996).

Constructivism has added positive dimensions to the practices of teaching. These practices were built on several constructivist theories, which came from several different sources. Constructivist learning theory conscripted many good teaching ideas from the preceding theories of cognitive psychology and progressive education (Phillips, 1995). Mathematics reformers believe that constructivism has introduced effective and meaningful practices into the educational process and opened educators up to the promising possibilities of children’s contributions to their own learning (Phillips, 1995).

*Teacher Knowledge for Mathematics Reform*

*Content Knowledge*

A key element in the realization of the NCTM vision for reformed mathematics instruction is most certainly the role of the teacher (NCTM, 1991; Putnam et. al., 1992; Ball, 1993 & 2003; Ma, 1999; Ball & Bass, 2000; Heaton, 2000; National Research Council, 2000; Mewborn, 2003). This is evident in the following excerpts from the *Principles and Standards* (NCTM, 2000): All students
have access to high-quality, engaging mathematics instruction...knowledgeable teachers...continually growing as professionals...complex mathematical tasks chosen carefully by teachers....teachers helping students make, refine, and explore conjectures...with the skilled guidance of their teachers (p. 3). However, the skills and knowledge required to do this kind of teaching has not been easy to pin down. There is an underlying assumption that teachers will be both competent in and confident of their mathematical knowledge. However, Ball and Bass (2000) suggest that subject matter knowledge for teaching is typically defined simply by the subject matter knowledge that the students are to learn—that is, by the curricular goals for students. Ball and Bass would add to this requirement. They argue that teachers need to know more mathematics in order to have a broad perspective on where their students are heading (Ball & Bass, 2000). Ball and Bass state the concern: “The content and nature of the mathematical knowledge needed in practice is insufficiently understood” (p. 87). Only recently has the question of specifically what knowledge elementary teachers need, been given focused attention. Groups of mathematicians, educators, researchers are now looking at this issue in order to help insure teachers have the mathematical knowledge they need to teach their students more effectively (National Research Council, 2001; CBMS, 2001; & Hiebert, Gallimore, & Stigler, 2002; Ball, 2003). State licensing for elementary teachers have tended to describe mathematics requirements in terms of credit hours or generic coursework – not in specific knowledge (Kilpatrick, Swafford, & Bradford, 2001). States and universities further relied on mandated standardized tests to determine
adequate mathematics knowledge for teaching (Kilpatrick et al., 2001). In Adding It Up, Kilpatrick, Swafford and Bradford (2001) conclude, “Many students in grades pre-K to 8 continue to be taught by teachers who may not have appropriate certification at that grade and who have at best a shaky grasp of mathematics” (p. 4). The research reported by Liping Ma (1999) confirms this assessment. In her comparison of United States and Chinese elementary teachers she reports qualitative differences in the teachers’ approach to mathematics understanding. Of Chinese teachers, Ma (1999) says, “Obviously these teachers are not mathematicians. Most of them have not even been exposed to any branch of mathematics other than elementary algebra and elementary geometry. However they tend to think rigorously, tend to use mathematical terms to discuss a topic, and tend to justify their opinions with mathematical arguments” (p. 105). She further states, “only teachers who are acculturated to mathematics can foster their students’ ability to conduct mathematical inquiry….teachers must have [the ability] first” (Ma, 1999, p. 106). Ma found teachers in the United States particularly weak in their general attitude toward mathematics. She reported that most believed in nonmathematical ways of approaching the new idea and did not investigate the idea independently. Ma summarizes, “Considered as a whole, the knowledge…of the U.S. teachers was clearly fragmented” (p. 107).

Teachers’ understanding of the big mathematical ideas that run through the elementary curriculum and build a strong foundation for students’ further mathematical development has become an area of recent research. Recommendations
of the Conference Board of the Mathematical Sciences (2001) have worked to
delineate key mathematical understandings for elementary (K-4) and middle (5-8)
level teachers. The Conference Board of Mathematical Sciences (CBMS)
recommends coursework in which teachers acquire “a rich network of concepts
extending into the content of higher grades: a strong facility in making, following,
and assessing mathematical argument, and a wide array of mathematical strategies”
(CBMS, 2001, pp. 17-18). These recommendations look qualitatively different than
typical competency lists that are performance based. These recommendations are
conceptually based and aim for understanding rather than merely procedural
proficiency. CBMS recommends teachers need to develop their mathematical
knowledge in a way that is concrete and experientially based. CBMS recognizes the
presence of mathematical anxiety in many elementary teachers, and believes that
once elementary teachers experience “their own capacities for mathematical thought,
their anxiety [will be] transformed into energy for learning” (CBMS, 2001, p. 24).
Appearing in much of the literature on mathematical knowledge is the different kind
of mathematical knowledge necessary to teach mathematics to children.

**Pedagogical Knowledge**

Discussions among mathematicians, educators, and researchers have begun to
look at key understandings that are necessary for teachers in order to support their
students’ mathematical development. What is coming to light is that effective
teachers have content knowledge that is different than mere ability to do
mathematics themselves. Heibert, Gallimore and Stigler (2002) call this “craft knowledge” (p. 3). Ma (1999) also describes an in depth understanding of mathematics that is related to pedagogy. She calls it Profound Understanding of Fundamental Mathematics (PUFM). It has four properties. First, the teachers have a sense of connectedness. They are able to help students make connections among mathematical concepts and procedures, from simple to complicated. This is related also to making connections between different mathematical operations and between different domains (Ma, 1999). This connectedness allows the teachers to teach topics in relationships rather than isolated from each other. Second, Ma explains that teachers with PUFM appreciate different facets of an idea and various approaches to a solution, as well as the approach’s advantages and disadvantages. This understanding allows teachers to lead their students to a flexible understanding of the discipline. Third, PUFM gives a teacher awareness of simple but powerful concepts and principles of mathematics that Ma calls “basic ideas” (p. 122). Using these basic ideas, the teachers with PUFM revisit and reinforce the most important mathematical leaning students need to understand. Finally, Ma says, PUFM teachers have “longitudinal coherence” (p. 122). This is “a fundamental understanding of the whole elementary curriculum...[and they] are ready at any time to exploit an opportunity to review crucial concepts....They also know what students are going to learn later, and take opportunities to lay the proper foundation for it” (Ma, 1999, p. 122). Using PUFM, teachers interweave procedural and conceptual topics into “knowledge packages” (Ma, 1999, p. 115) in which they consider all of the items to have varying
status. Each package contains central pieces. These central pieces weigh more than other parts of the package (Ma, 1999). This kind of understanding of mathematics is important in effectively helping students make sense of mathematics and learn it with understanding.

Ball and Bass (2000) have also looked at teachers' content knowledge and how it gets used in teaching. They have observed what they call pedagogical content knowledge. It is an understanding that is different than a mathematician has. It is knowledge that is special to the teaching of elementary mathematics (Ball & Bass, 2000). Pedagogical content knowledge is developed by teachers over time as they teach the same topics to children of certain ages, or by researchers as they investigate the teaching and learning of specific mathematical ideas (Ball & Bass, 2000). This special understanding of mathematics, like Ma’s packages, helps teachers anticipate students’ questions, partial understandings, and misunderstandings. The pedagogical content knowledge helps teachers because, with it, there are patterns and predictability to students’ thinking (Ball & Bass, 2000). This knowledge coincides with another kind of mathematical understanding. Ball & Bass (2000) refer to decompression – or a teacher’s ability to “deconstruct one’s own mathematical knowledge into less polished and final form” (p. 98). Ball and Bass 2000) point out that mathematics is a discipline in which compression is central. Because of that compression, one’s ability to discern how learners are thinking in the beginning of their understanding is often obscured. It is important to consider what Ball and Bass have come to understand: “Because teachers must be able to work with content for
students in its growing, not finished, state, they must be able to do something perverse: work backward from mature and compressed understanding of the content to unpack its constituent elements” (Ball & Bass, 2000, p.98).

Aside from a pedagogical understanding of mathematics, reform-based mathematics instruction requires other pedagogical skills different from traditional teaching. The Professional Standards for Teaching Mathematics (1991) lists the following responsibilities of teachers: selecting or creating mathematical tasks that are interesting and challenging to students, promoting and managing classroom discourse that supports mathematical thinking and reasoning, creating a rich mathematical environment, and assessing the activities in the classroom for both instructional effectiveness and student growth (NCTM, 1991). Unpacking these teaching standards uncovers the need for knowledge and skills beyond mathematical understanding, but necessary for reform-based instruction.

The new pedagogical skills are addressed in much of the literature on mathematics reform. Many point out that in order to select appropriate tasks, teachers need to know their students. They need to know developmentally how most students at a particular age/grade level would typically solve a problem (Hiebert et al., 1997). Teachers need to use their knowledge of children in order to word a task appropriately and to set expectations for the mathematics their students can do. An expert teacher can choose or write tasks that are accessible to all students; they can find or develop problems that can ask the same kind of mathematical thinking, but can be altered for children of differing levels of skill or understanding (Van de
Walle, 2003). Finally, the teacher needs to know her students’ interests and experiences so that she can choose tasks that will be meaningful and interesting.

Heibert has also written, with others, (1997 & 2002) about skills necessary for the adoption of mathematics innovations. First, because reform-based mathematics calls for the orchestration of discourse around mathematical ideas, teachers must have knowledge of group dynamics, skills at questioning and guiding, ability to listen supportively to all students, and the art of balancing teacher control with student autonomy. Encouraging student discourse allows the insertion of unplanned ideas into lessons. Teachers have to be flexible, to pick up on teaching opportunities, to follow a new line of mathematical thinking that she had not necessarily planned for, and be able to think on her feet to decide at what point to leave a misconception or to correct a student’s mistake (Heibert et al, 1997; Ball & Bass, 2000, Heaton, 2000). “Teachers must also be perceptive and skillful in analyzing the culture of the classroom, looking out for patterns of inequality, dominance, and low expectations that are primary causes of nonparticipation by many students” (NCTM, 1991, p. 34). These are different teaching skills than used in the more traditional structure of teacher presentation and student listening. Teachers need to continually decide when and how much information to share. When will help stop a student from thinking for himself? When will information move him along in his learning? These teachers have to “swim in a sea of uncertainty every day as they make their classroom decisions” (Ohanian, 1993). They must be comfortable with uncertainty. They must be willing to take risks. They must trust their students’ ability
to think mathematically and, at the same time, trust themselves to be able to keep the
discussion mathematically focused. They must be comfortable sharing authority and
power in the classroom. Much is written on the importance in a reform-based
mathematics classroom to get the students to work toward mathematical evidence as
verification in order to determine correctness of a solution of conjecture. Teachers
need to remove themselves from a position of authority and so that they are no
longer the final word on correctness. The final word on correctness is provided by
the logic of the subject and the students’ explanations and justifications that are build
on this logic (Heibert et al, 1997; NCTM, 1991).

Another skill that is often reported as important is related to a teacher’s
mathematical knowledge, but at a special application level. Ball and Bass (2000)
write that teachers need the ability to recognize alternative solutions and to
appreciate the mathematical thinking the student may be demonstrating. In a
traditional mathematics classroom every student is expected to follow a set
procedure to find the right solution. In a problem-centered classroom, teachers see
many different solutions. They need to support the children’s thinking and guide
their justification for the process they developed. Teachers in problem-centered
classrooms need to look at solutions and ask questions like this: Does it work? Will it
work for similar problems? Will it always work? They need to “be able to see and
hear from someone else’s perspective, to make sense of a student’s apparent error
or appreciate a student’s unconventionally expressed insight” (Ball & Bass, 2000,
Beyond a skill, this is an attitude of believing in all students' potential and respecting them as mathematical thinkers and problem solvers.

Finally, I must mention that in order to teach mathematics as recommended in *Principles and Standards* (1989 and 2000) a teacher must have, besides knowledge of mathematics, besides knowledge of their students, besides the ability to manage meaningful and inclusive discourse, and besides particular attitudes and beliefs about learning mathematics, something more. Both Ball (1994) and Noddings (1990) add the affective elements of patience, acceptance, generosity in listening to and caring about other human beings, confidence, trust, and imagination. Ball (1994) additionally includes being open to see the world from another's perspective, enjoying the humor, sympathizing with the confusion, and caring about the frustration and shame of others. Teachers must be open to continuous learning and growing by reflecting on their practices, seeking answers, working for more understanding, and being open to new ideas from students as well as other resources (Ball, 1994; Heaton, 1999; Kirkpatrick, Swafford & Findell, 2001; NCTM, 1991).

The importance of this caring is summed up in this passage from *Teaching Problems and the Problems of Teaching* (Lampert, 2001):

> The risk of public failure and its impact on relationships in classrooms is rarely acknowledged in the same text with analysis of how one makes progress in understanding the subject matter or increased one’s skills in producing mathematical representations. What we can see in the teaching practice described...is that the work of maintaining productive relationships with and among students must include simultaneous attention both to academic identity and to progress. The fragility of individual identity in the school context is a problem for the teacher because it can get in the way of improving academic performance....If a student is unable to feel that it is
safe to have and express ideas, or even to answer a simple question, then performance will not be improved. The work of establishing an environment in which students feel safe to do academic work with one another is a daily business requiring constant attention. (p. 2)

Summary

In summary, the literature confirms that the reform of mathematics instruction is a Herculean task. The innovations are based on a totally new view of teaching and learning mathematics. The change requires new skills and practices that are significantly different than those used by the teachers in the past. It requires new beliefs about learning, as well as new beliefs about mathematics itself. It requires changes in a teacher’s relationships with his or her students, and changes in a teacher’s relationship with knowledge. In other words, it requires deep and meaningful change on the part of teachers. The literature on change – both individual and institutional change – clarifies the difficulty of making such comprehensive changes, even with help and support. Educational change is even more complex because it requires the change to happen at many levels. Educational change – to take hold -- requires adoption by teachers, administrators, and government officials. In addition if requires acceptance by parents, community, and the media.

To add to the complexity, the reforms recommended for mathematics instruction are based on an educational philosophy that is radically (no pun intended) different than traditional educational theory. Constructivist teaching requires the setting up of environments and the structuring of lessons in a way that students investigate problems and build their knowledge. It requires different pedagogical
skills and different classroom organization. To be constructivist, teachers must accept new premises about how students learn and how to support that learning. It is a change in traditional beliefs and traditional teaching practices.

The reviewed literature did give support for the implementation of change. The various change models explain the stages of change and the criteria for adoption of change. They give those proposing change information to help make innovations be more understandable and, therefore, more acceptable. After years of dealing with a variety of change efforts, schools and districts are learning more and better approaches to making effective changes. The importance of the individual, as well as the group, is recognized. Research is helping define the skills that are required for reform-based teaching, and recommending ways to support development of those skills. Mathematics instruction in the elementary classroom will not change quickly, but the literature indicates that change can come with enough time.
Chapter 3: Methodology

Introduction

This instrumental multiple-case study (Stake, 1995) explored the journey of classroom teachers as they made the transition from more traditional teaching methods to reform-based mathematics instruction. The mathematics education community developed new standards for mathematical teaching in the late 1980s (NCTM 1989). Throughout the 1990s research has repeatedly established the improved mathematical performance of students who have been taught with reformed curriculum and instruction. Curriculum frameworks and school policy across the country have incorporated the new standards, and they have become a focus of teacher training. Yet in the elementary classroom, there has been only mixed implementation of new teaching techniques and the reformed curricula have been only slowly adopted by elementary classroom teachers (Grouws & Cebulla, 2000; Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, et al., 1997; Spillane, 2000; Wood, Cobb & Yackel, 1995; and others). This study looks at five teachers who did change their instructional methods. It seeks to discover their motivations, beliefs, kinds of support, and influences. This chapter will explain who the study’s participants are and how they were selected. Recruitment and informed consent are explained. The exploration of their change journeys took place through interviews and observations. The guiding questions and the processes for these are detailed in the section on data collection. I explain my analysis process for both the single
cases and for a cross case analysis. And finally I discuss the issues of validity and reliability.

Choosing Participants

The study focused on elementary classroom teachers who have been awarded the Elementary Presidential Award for Excellence in Teaching Mathematics within the past ten years. The Mathematics Specialist in the State Department of Education provided me a list of the names and school contact information for all the Presidential Awardees from this state and two neighboring states. I focused my search on those teachers who had received their Awards after 1993. From my own knowledge, and from talking with colleagues who are active in the mathematics education community, I first eliminated awardees that had retired. Next, to address my interest in the implementation of reform-based mathematics in the regular classroom, I eliminated candidates who were not currently teaching in a self-contained classroom. A colleague and fellow doctoral student, who attended the Summer Leadership Institute, had mentioned my study to some of the awardees at the Institute. She gave me an idea of those who would be likely to agree to participate. One teacher on the list was a woman I had known several years ago through a mutual friend and mentor. I now had a shortened list. The list had a representative range of grade levels and included both male and female teachers. The school information showed that the districts of these teachers were in both rural and suburban areas. Boosted with that inside information, I called targeted Award
recipients by telephone. In the initial conversation with each, I introduced myself and outlined the research study. I explained that I was seeking “typical” elementary teachers who do not have a degree in mathematics. Also, I asked for teachers who have changed their instructional practice in mathematics from traditional to reform-based (aligned to the NCTM Principles and Standards). Five Awardees from my targeted list agreed on the phone that they met these criteria and expressed an interest in participating in the study.

After the phone interview, each volunteer was sent a questionnaire (Appendix A). The purpose of the questionnaire was to confirm that all participants met the specified criteria: classroom teaching, no mathematics degree, and a transition from traditional to reform-based mathematics instructional practices. Further, the reported information ensured diversity among participants as far as grade level, career experience, socio-economic levels of students, and location of schools. The five volunteers represented both male and female teachers, different grade levels, and different school sizes, communities, and demographics. There was no diversity in race or ethnicity, which is a reflection of the list of Awardees. Pseudonyms have been used for the participants. Descriptions of their schools and districts are generic only.

At the end of the data collection each participant was asked to provide a pseudonym for the purpose of anonymity. One chose his two sons’ names and two picked their mothers’ names. The other two were totally imaginary. It is their chosen pseudonyms by which they are referred to in all the cases and analyses.
Informed Consent

After I received the completed questionnaires from the participants, I sent each a copy of the Informed Consent Document (Appendix B). The Informed Consent document outlined the purpose and the format of the study, and gave the time commitment and the responsibilities of participation. It detailed any possible risks to participants and identified safeguards to protect them from those risks. The participants were not compensated for participation. It was reiterated that their participation was completely voluntary, it was anonymous, and that they could leave the study at any time with no negative consequences. The document gave contact information in case there were any questions or concerns before, during, or after the study. By having the document early, participants could clarify any questions or concerns and again consider the conditions of their agreement to participate. At the beginning of each participant’s first interview, which was our first face-to-face meeting, we again talked about the process, any possible questions or concerns, and their rights as participants. When all the participant’s questions were satisfied, we signed two copies of the Informed Consent so each of us had one to keep.

Data Collection

First Interview

The study for each single case entailed an initial open-ended interview. The participants were asked to respond to a general inquiry (Appendix C) about their mathematics instruction and the development of that instruction over the course of
their teaching career, with emphasis on changes and how those changes came about. The purpose of this open interview was to hear “the story” in the teacher’s own natural narrative, with his or her own emphasis and key thoughts. This interview, approximately one hour in length, was both audio and video taped. The taping freed me to listen reflectively without the distraction of notes. I wrote some interview notes immediately after the interview and then, as soon as possible, did a verbatim transcription of the tapes. For most cases the interview took place in the participant’s classroom. One interview, done during winter break, was held at the participant’s kitchen table.

Observations

Observations of the teaching took place as close as possible to the first interview. I visited the classroom on two consecutive days to observe mathematics lessons that the participant taught to his or her own class. As a guide for the observations, I created an Observation Protocol (Appendix D) that combined the listed elements from the Oregon Collaborative for Excellence in Preparation of Teachers (OCEPT) – Teacher Observation Protocol (Flick, Morrell, & Wainwright, 2001) and the National Council of Teachers of Mathematics’ Professional Teaching Standards (NCTM, 1991). These observations gave me a picture of the participant’s classroom and school environment, which became context for understanding the participants’ teaching. Since the interviews involved discussions of beliefs about instructional methods and practices in teaching mathematics, the observations served
as triangulation of the data from the interviews. The observations also became a source of additional questions for the second interview.

Second Interview

Before each participant’s second interview, I considered the data from his or her first interview and the two observations and developed a set of questions that ensured all of the study’s Guiding Questions were being addressed (Appendix E):

- What was the motivation for changing the way you taught mathematics in your classroom?

- At what point were you in your teaching career when you made these changes? Was that a factor?

- What part did your home/family life have in changing your teaching?

- Where did you get support in making your initial transition from traditional instruction to reform-based mathematics instruction?

- What was your ongoing support or encouragement? Was the support from personal or professional sources? Was it from an individual or from an organization?

- What, if any, beliefs about mathematics or about teaching and learning did you change as you changed your mathematics instruction? Describe the changes. Which came first, the change in beliefs or the change in practice? Did the changes affect your teaching of other subjects?

- When you changed the way you taught mathematics, describe the changes you saw in your students. Were there changes in the way you taught other subjects? How were the changes related to each other?

- What barriers or difficulties did you encounter when you changed your mathematics instruction?
• What resources/materials have you found most useful over time? How are these resources/materials “teacher friendly” or not? Would they be useful for less experienced or less math oriented teachers?

• Discuss some of the ongoing or reoccurring issues that you feel keep other elementary teachers from reforming their instructional practices in teaching mathematics in their classrooms.

• What changes have you seen in the way mathematics is taught at your school? In your district? In your state? To what do you attribute these changes?

• Do you believe the vision of NCTM for reform is realistic? If so, what are the key changes that would make the biggest impact on the way mathematics is taught in elementary classrooms?

• What impact did the Presidential Award have for you? What, if anything, changed for you because of the Award?

• In what ways, if any, have you strengthened your mathematics background?

Therefore, based on the response to the open prompt at the first interview, the questions at the second interview were different for each participant. This second interview, like the first, was also about an hour in length and was both audio and video taped for later transcription and analysis.

Award Application Packets

Each person nominated for the Presidential Award must submit an application packet. This packet includes artifacts and statements, from both the nominee and from others, that serve as evidence that the nominee possesses:
Subject matter competence and evidence of sustained professional growth in mathematics and in the art of teaching;

Understanding of how children learn mathematics;

Ability to engage students in direct, hands-on mathematics inquiry activities;

Ability to foster curiosity and generate excitement among students, colleagues, and parents about the uses of mathematics in everyday life;

Understanding of the relationship that science and mathematics share with each other and with learning in general as a reflection of the interconnectedness of all subject matter.

Exceptional and innovative attitude in the approach taken to teaching, and professional involvement and leadership.

I reviewed the contents of each participant’s packet to gain additional context as support for the responses given during the interviews and for the observations of the teaching.

Summary

The data collection procedures were the same for each subject. All of the data collected from the two interviews, the observations, and the application packet were used creating the case report and in the cross case analysis. The observations and the nomination information confirmed that the teachers were using reform-based, conceptually oriented, problem-centered instruction. The two interviews focused on the processes, the influences, and the supports that were a part of each teacher’s change journey.
Analysis

Introduction

I analyzed each teacher's story as a single-case study. Then, after each was analyzed singly, I did a cross-case analysis. Yin (2003) describes this as replication: “Each individual case study consists of a ‘whole’ study, in which convergent evidence is sought regarding the facts and conclusions for the case; each case’s conclusions are then considered to be the information needing replication by other individual cases” (p. 50). The cross case analysis was done by stacking (Miles & Huberman, 1994), or aligning, the findings from the five individual cases and analyzing them together. Both the single-cases and the across-case analyses complete the final report.

Single Cases

Many qualitative researchers stress the importance of case study analysis beginning with the earliest data collection and building throughout the study (e.g. Bogdan & Biklen, 1998; Lofland & Lofland, 1995; Mason, 2002; Miles & Huberman, 1994; Yin, 2003). As stated by Stake (1995): “There is no particular moment when data analysis begins. Analysis is a matter of giving meaning to first impressions as well as to final compilations” (p. 71). My formal analysis began with reading, re-reading, and reflection on data soon after each collection and transcription. The filters for this initial analysis were determined by the general categories within my guiding research questions (Appendix E): motivation, beliefs,
career, family, support, inhibitors, events, people, and practice. This was the descriptive, first-level, of coding (Miles & Huberman, 1994). As more data were collected, and with more time and reflection, these categories were refined, expanded, and collapsed. New categories were added: key people, recurring issues, pedagogy, community, and life-long learning. The original categories of family, career, and barriers were collapsed, as they were not key factors in any of the participants’ changes. The category of support was divided into initial support and ongoing support. Reading again through the data, I color coded the transcripts with a different color for each category. Then, looking again at the data and relationships, I created a matrix that intersected the above categories of motivation, early teaching, support, and teaching issues with the innovation categories put forth by Fullan (2001): materials, teaching approaches, beliefs. I added a fourth column for key influences to capture groups and people that were instrumental in the changes (Charts 2, 3, 4, 5, 6 in Appendix F). It was to this point that I analyzed each individual case. The case reports were organized around these categories, giving their interplay for each individual.

The patterns that emerged from the data led to the understanding of the cases, leading then to understanding of the phenomena being studied (Miles & Huberman, 1994; Stake, 1995). Patterns that were illuminated by the analysis of the five individual cases, when combined, led to even greater understanding about the key elements in teachers’ changes in their instructional practices. The likenesses and
differences between the cases were explored in the next step of analysis, with the goal of confirming those patterns observed in the single cases.

Cross-Case Analysis

After each individual case was analyzed independently, the five cases were analyzed together. The selection criteria built into the design of the study set up similarities between cases that allow them to be looked at as literal replications (Yin, 2003). Although these are not true replications, as each case is an entity in itself, comparing the findings of multiple, like cases is similar to comparing findings in repeated experiments (Yin, 2003). My goal for the analysis was to look across the elements that were most influential to the five participants and to examine the experiences that were common – or unique – in their transformations, in order to gain more understanding of the implications for such changes on a broader scale.

The cases, although intentionally similar to one another, were not a representative sample of elementary teachers in general. They were identified from a small, select pool of teachers who have adopted instructional practices that have been defined as a goal for the reform of mathematics instruction in elementary classrooms. The practices and the philosophy envisioned in this reform are a deep and comprehensive change from what has been traditionally accepted mathematics instruction (Ball, 1994; Heaton, 2000, Hiebert, et. al., 1997, and others). For this cross-case analysis I used the mixed strategy Miles and Huberman (1994) call “stacking comparable cases” (p. 176). After each individual case was examined and
understood separately, cases were stacked against each other for the matching of patterns, themes, and phenomena (Miles & Huberman, 1994). Word tables displayed the comparable individual case findings (Charts 2, 3, 4, 5, 6, in Appendix F). These key elements were condensed into a single Meta-Matrix (Miles & Huberman 1994) (Chart 7, page 161). This Meta-Matrix allowed me to look across the five cases for each key element. Analysis probed the similarities and differences between cases.

When I looked at these key elements of the change journeys of the five teachers together -- as they moved from their early teaching and motivation to the introduction, then early adoption, to complete and ongoing change -- I began to see each journey as a learning process. I analyzed the patterns of change through the lens of constructivist learning theory. The report of the cross-case analysis addressed the comparisons of the five teachers as a constructivist process of building new understandings and, therefore, a new schema about mathematics instructional practices.

Validity and Reliability

Introduction

Nowhere is quantitative and qualitative research more at odds then around the issues of validity and reliability. There is ongoing discussion over whether it is even appropriate to apply the terms – positivist in origin – to qualitative studies (Denzin & Lincoln, 2000; Merriam, 1998; Miles & Huberman, 1994; Stake, 1995). Yet as Merriam (1998) writes, “All research is concerned with producing valid and reliable
knowledge in an ethical manner” (p.198). The measure most cited as the standard for validity and reliability in qualitative studies is trustworthiness (Denzin & Lincoln, 2000; Merraim, 1998). It is the goal that there is enough data, collected in an uncompromised manner, and presented with enough detail, to convince the reader that the conclusions make sense (Merriam, 1998). In this section I explain the steps I have taken in this study to assure that my findings are trustworthy. I present these steps under the traditional terminology, yet I caution that they are not compatible with like-labeled quantitative categories. I have organized this section around four questions that relate to these constructs.

Validity and Reliability

Internal Validity. Internal validity assures that the research findings match reality. Do my findings capture what is really there (Merriam, 1998)? The two following questions and answers address the internal validity in the study:

1.) Were my participants truly “reform-based” teachers? What evidence do I have to make this claim? The fact that all participants were selected from a pool of recipients of the Elementary Presidential Award for Excellence in Teaching Mathematics was already strong evidence that they used reform-based teaching practices, because those practices are closely aligned to the Award’s selection criteria. For further confirmation, however, I observed each participant teaching two mathematics lessons in their classrooms. Additionally, I reviewed the teachers’ application packets for the Award. The application packets included philosophy
statements, descriptions of their teaching, and sample lesson plans or units. To evaluate the teaching I used an Observation Protocol (Appendix D) that combined key elements from the NCTM’s *Professional Teaching Standards* (1991) and from the OCEPT Teacher Observation Protocol (Flick, Morrell & Wainwright, 2001), a document developed for preservice teachers observation and identification of standards-based teaching practices.

2.) *Am I accurately portraying what my participants reported?* To make sure I accurately portrayed what the teachers reported about their change journeys I used member checking. After each participant’s case was written into the report, I sent a copy asking that the teacher identify anything he or she felt I had misrepresented. Further, I invited each participant to let me know if there was anything they would like to have removed or added. None of the participants have changed their write up.

**Reliability.** Reliability relates to the dependability and consistency of the results obtained from the collection of the data (Merriam, 1998). The following question and answer address the reliability in my study:

3.) *Do my interview questions and my observation protocol reflect the characteristics of the study they were meant to explore?* The interview questions were tried with another Presidential Awardee, not a participant in this study, during a pilot case study. That individual had also changed her teaching from traditional mathematics instruction to reform-based instruction during her career. Relating her story and her responses to the original questions allowed me to refine the Guiding Questions used for the interviews with these cases.
The Observation Protocol was developed from two sources with overlapping elements, both describing observable practices of reform-based mathematics instruction. The first source was the NCTM's Professional Teaching Standards (1991), which are directly related to the NCTM Teaching Standards on which the reform is based. The second source for my protocol was a check list developed by the Oregon Collaborative for Excellence in the Preparation of Teachers as part of a research study done in 2001. It has been piloted and put into use in teacher training programs. It has been found to be effective in identifying teachers who used standards-based (reform-based) practices. Additionally, I piloted the Observation Protocol, observing several elementary mathematics lessons.

*External Validity.* The external validity is the extent to which the findings can be applied to other situations. The following question and answer address the external validity of the study's findings:

4.) *How can I justify the trustworthiness of my findings? Can the findings be generalized?* The small, nonrandom sample of this study was selected for the purpose of understanding these particular cases, not what is generally true for all elementary teachers. The cross case analysis did uncover similarities which could be labeled as key elements in the change processes of these teachers. The cases have been presented with careful detail so that readers can determine how closely they can relate to the findings. The trustworthiness of the study will be determined by the reader. Stake (1995) refers to the generalizations from case studies as *naturalistic generalizations.* Stake explains, “Cases are not as strong a base for generalizing to a
population of cases as other research designs. But people can learn much that is
general for a case” (p. 85) The findings from this case cannot predict, but they can be
used to guide in decision making and planning.

Reflexivity of Researcher

At a conference I attended last year, practicing researchers met with small
groups of graduate students for a brief mentoring session. I was at the table with Dr.
Deborah Ball, who has done the majority of her research on teachers' practices in the
instruction of mathematics in elementary classrooms. As each of us shared our areas
of study, she briefly gave us advice on our research. She advised me to make my
classroom experience a key part of my research. That advice helped me decide to
focus my study on practicing classroom teachers. Now, in review, I believe that my
classroom experience did positively influence the carrying out of this study. From
the meeting with every participant, there was a shared vantage point from which to
explore teaching. Elementary classroom teaching is complex. It is difficult for
someone who has not taught in an elementary classroom to grasp quickly all the
levels on which a teacher operates all at the same time. I believe that, because of
my classroom background, there was that common bond that helped communication
during both the observations and the interviews. That bond added an element of
trust to the process – both on the part of the participant and on the part of the
researcher – because of the shared teaching experiences and the mutually understood
classroom realities.
I am philosophically biased toward constructivist teaching and towards mathematics instructions that uses problems and investigations to build students’ conceptual understanding. In this study, I believe that is a positive bias. It helped me understand the practices I observed and the skills and knowledge that were described in the interviews. I do not believe that filter changed the way I represented the elements that were influential in the transitions of these teachers instructional practices. And, in fact, may have added contextual insight.
Chapter 4: Results

Introduction

The purpose of this study was to examine the key elements in the transformation of elementary classroom teachers’ mathematics instruction. The investigation focused on five classroom teachers, all of whom have been recognized for their mathematics instruction by receiving the Elementary Presidential Award for Excellence in Teaching Mathematics. All five teach in a manner consistent with the vision of the National Council of Teachers of Mathematics for reformed mathematics instruction: an active, problem-centered process with a goal of conceptual understanding, appreciation of relationships, and the ability to communicate mathematically. However, over the course of their careers, all participants changed their teaching practices from a traditional linear, carefully sequenced, teacher directed approach that centered more on rote memorization than conceptual understanding. It is this very change that is the goal of current reform efforts in mathematics instruction. During the interviews and observations in this study, the five teachers reflected on their experiences: What influenced their changes and what supported them in making the transition?

These five teachers are similar in their adopted teaching practices and in their philosophies about how students learn mathematics. All of these participants are racially and ethnically the same, which is reflective of the Presidential Award recipients in the states from which the participants came. They are all very
experienced teachers, averaging about twenty-five years in the classroom. Yet their careers have not been similar (See Chart 1). The participants represent a range of grade levels and teach in schools with varying demographics. Four of the teachers are female and one is male. Because of career differences, their paths to reform-based instruction differed in several ways. Participants chose pseudonyms for the case study reports to assure anonymity for themselves and their school districts.

Chart 1. Summary of the Characteristics of the Five Teachers.

<table>
<thead>
<tr>
<th>Pseudonym</th>
<th>Gender</th>
<th>Race</th>
<th>Grade Level</th>
<th>Years Taught</th>
<th>School Size*/Community</th>
<th>School Demographics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patsy Miller</td>
<td>F</td>
<td>Caucasian</td>
<td>6th</td>
<td>31 years</td>
<td>Medium/Rural</td>
<td>Low SES</td>
</tr>
<tr>
<td>Skip Munson</td>
<td>F</td>
<td>Caucasian</td>
<td>1st/2nd/3rd</td>
<td>32 years</td>
<td>Large/Rural</td>
<td>Medium SES</td>
</tr>
<tr>
<td>Michael Patrick</td>
<td>M</td>
<td>Caucasian</td>
<td>4th</td>
<td>23 years</td>
<td>Large/Suburban</td>
<td>Low SES</td>
</tr>
<tr>
<td>Rose Sharon</td>
<td>F</td>
<td>Caucasian</td>
<td>6th</td>
<td>19 years</td>
<td>Medium/Rural</td>
<td>All SES</td>
</tr>
<tr>
<td>Jane White</td>
<td>F</td>
<td>Caucasian</td>
<td>2nd</td>
<td>22 years</td>
<td>Large/Suburban</td>
<td>Medium SES</td>
</tr>
</tbody>
</table>

*Small = Up to 100; Medium = 100 – 300; Large = 300+
#Skip teaches mathematics to the third graders this year.

Two focused interviews with each participant were the primary source of information about their motivation for change, their introduction to the innovations, and the support that they received in making the transition. During the first interview I encouraged the participant to just tell his or her “story,” with only occasional prompts from me. In the second interview, I asked more specific questions in order to gain further insight into the major influences and the kinds of support encountered
during the change process. Because all five of these teachers are now leaders in mathematics reform, I also asked them why they think changing mathematics instruction is difficult for elementary teachers, and what they believe would support more teachers in making these changes. To support the information collected from the interviews, and to get a more rounded picture of the participants' teaching practices and philosophies, I visited each of their classrooms for two observations and reviewed the personal philosophy statements and descriptions of their teaching from their Presidential Award nomination packets. Although information from the observations and the packets was occasionally referenced in the following reports, if not otherwise noted, quotations are from the interviews. When necessary for grammatical and syntactical consistency, some remarks were edited, but context and meaning of the statements were carefully preserved.

In the following sections of this chapter, the cases are presented in alphabetical order by chosen pseudonyms. The divisions in the case reports are intended to give a whole picture of each teacher's progression in change from motivation and introduction to full adoption. The divisions are as follows: A Glimpse of Their Teaching, Early Teaching, Introduction and Initial Support, Ongoing Support for the Innovations, and Continued Development and Issues in Mathematical Reform. At the end of each case report a figure summarizes that teacher's change journey in a representative diagram.
A Glimpse of Patsy’s Teaching: Connecting Fractions to the Real World

Patsy Miller submitted a description of a fractions project as part of her Presidential Award Application Packet. The following excerpts from that description capture her passion for teaching, her caring for children, and her commitment for making mathematics meaningful, interesting, and challenging. She wrote:

The study of mathematics in a school should be vibrant, rigorous and engaging to all students… A math teacher’s responsibility is to stimulate deeper understanding of concepts and principles in a non-threatening environment of exploration and application. Students often feel disconnected from math because prior experiences in this area may have been negative, therefore a teacher must provide a means of giving each student a confident feeling when faced with mathematical problems… In order to energize students to the world of mathematics a teacher must enthusiastically involve students in problem-solving activities, examine and expose students to all strands set forth in the state and national standards and give frequent opportunities to engage them in higher order thinking skills.

Typically, every year when beginning an instructional unit in the use of fractions, it has been my experience students are genuinely perplexed when they routinely ask, “Why do I have to know about fractions, what good will they do me?” The math project presented in this application was my response to that challenging question… Each small group of students completed projects that integrated other disciplines and curriculum content beyond formal math instruction. For example, all projects integrated writing, speaking art, and calculation procedures during the creation of games, skits, artwork and student made teaching tools. The springboard for the student-made projects was their visits to local businesses for the purpose of witnessing the ubiquitous nature of fractions in the workplace. Student groups then created and presented their findings to parents and staff through a variety of creative expressions. Ultimately, in the completion of classroom instruction, business field trips, group classroom projects and a parent presentation I wanted students to understand that mathematics is interwoven in the tapestry of human endeavors and experiences.
Prior to launching into field trips and project activities students were formally taught a fraction unit aligned with our district’s math continuum and State Content Standards. These lessons were designed to provide a base of student knowledge to facilitate the design of their projects and appreciation for fractions found in the workplace. Manipulatives such as MATHLAND fraction circles, student-produced fraction kits, videos, Cuisenaire rods, ratio table and blocks were used to provide concrete models of fraction concepts. Next, problem-solving connections were made through use of children’s literature selections, cooking activities, home inventories and cooperative group solutions to open-ended questions. In addition, students regularly practiced fraction concepts when engaged in the daily routine of examining the current date’s place in relation to the fractional part of the first 100 days of school expressed in decimals, percentages and fractional symbols. Lastly, computation and calculation procedures were practiced in the context of practical applications, literature assignments and problem-solving exercises.

The culminating presentation that took place in my classroom for parents and invited guests, allowed our community to see the recently completed projects and learn about the business partner field trips. An overflow crowd was treated to well-prepared presentations involving all students in seven engaging demonstrations that used a variety of methods to communicate their newly gained knowledge and appreciation of fractions. These demonstrations were scored using the State Speaking Scoring Guide and placed in their collection of Speaking Work Samples to determine benchmark achievement. (Elementary Presidential Award for Excellence of Teaching Mathematics Application Packet)

*Early Teaching*

Patsy began her career in education in a small rural school district where she was the Educational Media Specialist for fourteen years. In year fifteen she took some time to be at home with family, planning to return to the media position. However, at the end of that year, when she was ready to return, the position was not open. Since Patsy also had an elementary teaching degree, she applied for a fifth grade teaching position in the neighboring town. She was hired, but recalls feeling as
if she was “dropped right into everything.” She had very little time to get to know the fifth grade curriculum. She recalls, “I was like a first year classroom teacher.” The curriculum at that time was basically the textbook programs for her level. There was no understanding of the standards. The mathematics program, as she describes it, “mostly zeroed in on calculations and computations.” You taught with “drill and kill,” she says. Further, there was nothing about children’s developmental learning. Patsy explains, “You just assumed that if they were in the fifth grade they would be able to understand what was less than a whole. That is a huge gap when you are teaching fractions.”

It was during that first year in the classroom when Patsy was introduced to a way of teaching mathematics that she found more inspiring than the textbook instruction. One of her fifth grade colleagues had a unit in which the class made quilt books. They did the unit together. She relates, “We did a whole measurement unit teaching [the kids] how to make the books.” They made the paper, made the books, and then made the books “the next size up” to learn about ratio. With this project, Patsy realized, “I want math to be this way, because there is something they can produce [while using] the knowledge that we have taught them.” It opened her to the possibilities of what students were able to do if given the opportunity. She loved “giving kids a project, where we all get to the same place at the same time, but where they have to choose...they have to tell and reflect why something did...or did not work.” She was motivated to do more projects in her teaching.
One project she did was a science unit on wild flowers. The class collected wild flowers and dried them. They connected science to mathematics by figuring out and charting how long each kind of flower should be in the microwave to be preserved. If they were in too long they “burned to crispy critter.” If they were in too short they molded. The students learned how to chart their findings. Then, from the charts, they went to spreadsheets. Also, they looked at who in the community might use this kind of information. This project experience planted the seed for Patsy’s interest in having her students do more to connect their classroom learning to the real world. She says she wanted the students to know “who in our community uses math every day.”

In the first few years that she taught fifth grade, Patsy continued to interject projects into her math and science, but she was still tied to the textbook for the majority of her teaching. She reflects, “All our curricula were intertwined with the text. There was no other gate to go through….It was, ‘What page in the book do I need to be doing?’” She wanted to get the students more into understanding. The conversations in class seemed “one-planed” to her. She thought, “Kids didn’t have a lot of connections.” However, Patsy did not have a vision for how she could change her curriculum to accomplish that. Her training had been in traditional teaching methods. There were boxes for each math concept. There would be a box for fractions, another for measurement, and another for geometry. She remembers, “Nothing intertwined. There was never a full circle. It was all separate entities. It was all ditto sheets and packets.” When she finished a unit she put the box away for the
rest of the year and got out another box. Problem solving was working story problems at the back of the chapter. Everyone in class did the same thing. Thinking about this, she says, “I began wondering if I was asking [the kids] to do something that they were not developmentally ready to do. Was I making lessons only according to what my textbook is telling me or was I reading my kids?” (See Figure 1: Patsy Miller’s Journey, page 88)

Introduction and Initial Support

With the feelings of dissatisfaction about her mathematics program, Patsy attended a couple of workshops, and then a regional conference, sponsored by the state organization of teachers of mathematics. At these presentations she heard and saw mathematics instructional practices that incorporated the National Council of Teachers of Mathematics (NCTM) Standards: learning was centered on problem solving, focused on conceptual understanding, and connected to the real world. She immediately knew that this kind of instruction was for her. She went back to the classroom and put away her mathematics textbook, thinking, “No. I am not going to put my kids through, “You need to do thirty problems because that is how I was trained as an elementary teacher.’” The workshops gave her license to abandon the “kill and drill” teaching. What she saw at the workshops was the multi-dimensional, connected, conceptual mathematics that she wanted for her class.

The change was decisive. On the surface, Patsy’s adoption of the innovations presented in the workshops took place quickly. However, she was already looking
for a better way to teach mathematic conceptually. The workshop and conference presentations showed her methods that were not foreign ideas; they were amplifications of the project ideas with which she was already experimenting in her classroom. The values of the instructional reform matched hers also – she believed in her students having choice, conceptual understanding was important to her, and she wanted her students to make connections to other subjects and to the real world. In her own projects and activities, she had already tried many facets of this instruction. She was sure this way of teaching would improve her students’ learning as well as her own satisfaction in her teaching of mathematics. Taking this new direction in her mathematics instruction was not a difficult decision for Patsy.

*Ongoing Support for Innovation*

The ease of her decision did not mean that Patsy did not have questions, that she did not need to develop new skills, or that there was not more to know in order for her to be successful at using these innovations. Once Patsy chose this new direction for her math instruction, what support did she have for continuing down that path? How did she develop the skills and knowledge necessary to have a rich, problem centered mathematics program that addressed the important mathematics concepts that her students needed to understand? What resources would support her efforts?

As it happened, Patsy received ongoing support from many directions: the community of mathematics teachers in the state, teachers at her own school,
administrators and the school board of her district, published reform-based mathematics programs, many other professional resources, and her own initiative.

Within this support system all three dimensions of innovation – materials, teaching practices, and beliefs – discussed by Fullan (2001) were well covered. In addition, Patsy gained more mathematical knowledge and developed confidence in her mathematical understanding. As she started making changes, she was concerned about her mathematics abilities. She reflected:

> I know my math was not strong enough to go with some of my higher kids. Algebraically I didn’t feel strong. Geometry I flunked in high school... I barely made it through my classes [in college] because I had such a phobia. (First Interview)

However, she came to believe in her mathematical capabilities. She explains, “I started sitting with teachers from junior high and listening to them talk... and started to get over [my] math phobia.” She asked questions such as, “Could you have done it this way?” and “Isn’t there another way to solve this rather than the way the Greek mathematicians set it up?” She got interested in the history of mathematicians and discovered they were “real problem solvers.” She considered how algorithms were developed and applied that to her own learning and to helping her students in their learning. Today she says, “I am not a great mathematician, but I am a heck of a math teacher.” With the confidence she now has in her ability to understand mathematics, she continually pursues mathematical knowledge:

> Most of my knowledge comes from me stopping right as soon as I am uncomfortable or I am not 100% sure and going to a book or a person who has more mathematical knowledge or going to a peer who has about the same [knowledge] and asking them questions and seeing what their play is on it. (Second Interview)
Her teaching motivates her to keep learning. As she sees students struggle with concepts, she continues to struggle with them herself until she can think of other ways to present them or other connections that will allow the students to understand. Using all the resources available to her, Patsy still continues to develop her knowledge of mathematics and of how to help students understand it.

Patsy’s excitement about the ideas presented at the early workshops and conferences kept her going to more. Through this participation, she first became involved with the state organization of teachers of mathematics, and then with the mathematics educators in the state’s Department of Education. Her mathematics teaching benefited from this involvement in many ways. She started attending the Summer Leadership Institute and other meetings of the organization where teaching ideas were exchanged, resources were shared, and issues were discussed. When Patsy wanted to get better at choosing problems and tasks for her class, she asked other mathematics teachers, “How do you decide what problems to use?” She was told, “You have to know what you want to assess.” That idea opened another set of doors for her. She started working with mathematics education at the state level. She became familiar with the state’s mathematics standards and with the state assessment process. She became a scorer and, eventually, a scoring director for the state problem solving assessment tasks. This involvement added to her knowledge, her resources, and her beliefs about what to teach and how to teach it.

Patsy also found a supportive professional community closer to home. Informally, she started teaching some workshops just for the people in her own
building. As she tried out new ideas in her classroom she would share discoveries with her colleagues: “I had just figured something out and thought, ‘This is so cool!’” She would discuss these things with other teachers at her school and they would say, “We could teach it to our kids like that.” They exchanged teaching ideas, planned together, and discussed teaching strategies. When they adopted a reform-based mathematics program, Mathland, they worked together on using the manipulatives in the program, revamping lessons that didn’t meet their students’ needs, and matching the program to the state’s standards. Now, after talking with the teachers in the junior high school, they have found a new mathematics program that is used across grades six, seven, and eight. They believe this will help the students bridge the transition from elementary to junior high mathematics. In Patsy’s rural school district there is just one school per level. This has allowed the teachers to work closely to build a strong local curriculum. They continue to use reform-based mathematics in their teaching. Currently, the primary grades (kindergarten through third) use Bridges, grades four and five use Investigations, and grades six through eight are using the new program, MATHThematics. They have developed a learning community in which the planning is done in teams, there is much discussion about what works and what doesn’t work, and there is a very strong level of trust in which the teachers feel comfortable sharing concerns as well as successes. The development of this professional community has been aided by strong support from the school district.
The district administrators have encouraged the teachers in their professional endeavors in several ways. The teachers have been supported with the purchase of reform-based programs, manipulatives, and professional resources that have aided them in developing a rich problem-based program. Besides their basic mathematics programs, each grade level has a professional library housed in a classroom and easily accessible to all teachers. Patsy has the mathematics library in her room:

If there is a book [a teacher] is looking for to help support how to teach anything or the standards or for math ideas, they just come to my room. If it is here they take it and if not, I know who has it. It is all centralized. (First Interview)

This year the teachers have been viewing the Annenberg Tapes on teaching mathematics. The district purchased the tapes, along with a guidebook for every teacher, for staff development. She says enthusiastically, “The tapes are tremendous…we sit around and discuss them together and that has been one of our math development programs.” These staff development sessions are another example of the full extent of administrative and district support. For several years students have been dismissed early on Fridays so the teachers have the afternoon to meet and talk. Through these regular Friday sessions the teachers have been able grow professionally as they discuss instructional practices and share how different teaching strategies are working in their classrooms. The administration and school board recognize the value of this time and continue to preserve it in the schedule.

Moreover, the district used grant funds to send the sixth, seventh, and eighth grade teachers to Montana last summer to work with the MATHThematics authors in preparation for adopting the new program. Patsy says of the district administrators,
“They are so involved. It affirms who you are. You feel like professionally they support you and trust you and support the decisions you are making.” This kind of ongoing support has certainly been a key element in the development of Patsy’s mathematics instruction. She talks in terms of the school’s and the district’s program, not just her own teaching.

Continued Development and Issues in Mathematics Reform

The teaching I observed in Patsy’s room is the result of her having the support to develop her mathematical knowledge, her teaching techniques and her beliefs about children and how they learn best. She uses manipulatives and visual mathematic activities to build understanding and then gives the students plenty of support as they make the difficult transition to the symbolic application of the concepts. On the day I observed, the students were to do some fraction exercises from their mathematics book. They had been working conceptually on fractions for several days. Patsy, aware that moving to the symbolic level is often a struggle for students, anticipated difficulties the students might have and organized the lesson to move them into work carefully. She began with a short whole class review of the concepts of fractional parts and representation. When students were ready for independent work, they were given the choice of working on their own or coming to a group that worked with the help of an assistant. There was ample time to complete the assignment and to work on related tasks, accommodating the variety of ability levels of the heterogeneous group of thirty-some students. The students grouped
themselves as they wanted. The room was buzzing, with all the students engaged in mathematical activities. Whether the students needed more or less help, all were respected. There was a sense of community without competition. There were no observable structures indicating students as superior or inferior in their abilities. The following statement demonstrates Patsy’s awareness of the fragility of students’ confidence in their mathematical abilities:

Even if someone needs me [to answer a question] ten or twelve times I will do that...even if I told him yesterday. I have to watch myself not to get irritated. If I did tell him yesterday, maybe he got a little bit of it. If I shut him down the second day he won’t come back. He thinks I will shut him down. So I have to take the time....Kids know I really care. (Second Interview)

This issue of mathematical confidence and the belief that everyone can learn mathematics is of central importance to Patsy. She gives that message to the children in her classroom and, likewise, to teacher audiences at presentations she makes. She realizes that many elementary teachers are unsure of their mathematics knowledge. She models continuous learning for them too:

Any time I do a workshop or anything these days I always open it with, “I still have huge math phobias about certain things. Very much so. And they are never going to go away until I realize that I have to tackle them. I have to learn. I have to teach myself....I am a fifty-three year old who doesn’t know her math. That is all there is to it.” (First Interview)

The teachers relate to the idea of math phobias. Yet her message to teachers is the same as to her students. You can learn mathematics. Further, she emphasizes, teachers must believe that everyone can learn mathematics. Elementary teachers may not know how to do calculus or trigonometry, she says, but they can learn about the
applications of mathematics and they can learn the concepts and applications that will help their students understand mathematics. It is important to Patsy that teachers continue to work to understand mathematics so that they can understand what is happening when their students are struggling. If a teacher stops wondering, “Why aren’t you getting this?” and “Why is this not happening?” the student is too often deemed incapable, with the following result:

Since [the student is] incapable, I can’t teach [him]. Then [he] is un-teachable. Then [he is] done! So [he must] memorize....I think that is the deal right there. If the teacher gives [a student] the death sentence that, “You are not mathematically minded,” then [the teacher] is going to approach teaching by thinking, “I don’t have anything in my toolbox that is going to help.” That is why teachers give up on kids. (Second Interview)

Patsy does not believe in giving up on kids. Teachers have to deal with all the children in their classroom. She states matter-of-factly, “You are hired to educate them. You have to find a way. You can’t just say, ‘I don’t want that one.’”

Evident in Patsy’s teaching are the skills that Ball and Bass (2000) call pedagogical content knowledge. She anticipates students’ questions, partial understandings, and misunderstandings. Some of this understanding comes from her listening to her students. Almost as a partner, and not just as an authority, she tries to figure out what they do know so they can build on that knowledge toward greater understanding. Just as Patsy herself stops and gets help when she does not understand something, she teaches her students:

Step back and say, “Okay I am done. I don’t get it.” I try to teach kids to do that. Even if they are just part way through the problem. Just put both hands on their desk. Remember what you want to say to me. When I come by just...tell me right then what
is the problem. And I can try to help them get on the right road to
go on. (Second Interview)

Pedagogical content knowledge has also developed as she intentionally constructs
her own mathematical knowledge. In order to understand concepts for herself, Patsy
has dissected them and put them back together in a way she can understand. In turn,
she is now able to take mathematical ideas, like fractions, and deconstruct them so
that she can help her students see connections and build their own understanding.

By working with teachers across grade levels in her school district as well as
working with mathematics teachers from all levels across the state, Patsy has also
developed “longitudinal coherence” (Ma, 1999, p. 22). Longitudinal coherence, or
knowledge of the developmental progression through which students build their
mathematical understanding, allows her to exploit or review crucial concepts at any
time. This allows her to work with her students in meaningful and positive ways, no
matter where on a continuum their mathematical understanding lies. She teaches a
heterogeneous class that has a wide range of ability. Using longitudinal coherence,
she can adjust the support and the expectations so that all students can move along
the continuum individually, while as a group they are working on the same concept.
Both pedagogical content knowledge and longitudinal coherence are continuously
refined as she continues to be dedicated to helping every student learn. She tells her
students, “You can teach me something every day.”

Patsy firmly states her belief that teachers must “have an innate feeling for
the willingness to be in a service-oriented profession.” This sense of service and her
passion for teaching mathematics intersect with energy and enthusiasm in the various
projects with which she is involved. She excitedly shares the awe of discovery that is taking place in a class she is teaching to classroom assistants who are (re)learning mathematics. They are discovering their own abilities in mathematics and, also, they are coming to understand how to help students be more successful. The discovery is two-way. She is the teacher, but she says, “I have learned so much by teaching these women…it makes me look at kids in a very different manner.”

In other service to the profession, Patsy is active in the state’s Council of Teachers of Mathematics. She likes the challenge of working with mathematics teachers from all levels. As an elementary teacher she queries high school teachers with such questions as, “Do you like teaching mathematics every day?” And when they share frustrations with students not being engaged in their mathematics, she invites them down to elementary classrooms to see how exciting mathematics can be. She wonders why they don’t use more projects and have more discussion in high school classes. At the same time, she is encouraging more elementary teachers to be involved with the state’s professional organization for teachers of mathematics. She is working on outreach projects that allow rural teachers to participate, via Internet, in discussions about mathematics and teaching. She is leading the efforts to reorganize mathematics workshops so that, instead of handing teachers ready-made units, they will help teachers understand the standards for each mathematical area and then to develop instructional activities themselves. Patsy not only believes that the vision of the NCTM’s Standards is realistic for elementary classrooms, she is working in many ways to make the vision a reality in classrooms across this state.
When asked what makes changing their mathematics instruction so hard for some elementary teachers, Patsy recognizes the need for more mathematical background. "Just a little. Not great big stuff," she says. They need just enough to develop their mathematical confidence. She also recognizes that some, who have been teaching longer, may need to have mathematics "reintroduced" to them. She mentions that a teacher has to be very organized to teach with problem solving. With project and problem-centered teaching there is much more to keep track of and many levels to deal with at the same time. And, further, she says, just as students need to feel supported and successful, so do teachers. She thinks that teachers may be reluctant to try new methods because, "We have so many people who have not felt success in anything." Teachers need to be supported in trying new things so they can feel successful. They also need to be dedicated to making sure the students are successful. They need to see how reform-based mathematics supports the learning of all students. They need to see how it can be done. She laughs, "They haven't seen me happy! They need to come and see someone with the energy...and how I made it work for me." She offers an invitation: "I always have my door open. Come and watch. Tell me what I could do different." To Patsy, the challenges of making mathematical concepts meaningful and understandable to students (both children and adults) is energizing and interesting collaborative work.
Figure 1: Patsy Miller’s Journey

LIFE LONG LEARNING

Professional Organizations  Creating Projects  Making Connections

Alternative Solutions and Alternative Strategies

Assessing Students’ Learning  Professional Development

Continuous Improvement – Always Looking for Better Ideas

Learning from Learners  More Mathematics

ONGOING SUPPORT

District Mathematics Programs
Discussion with District Teachers
Administration and School Board
State Teachers of Mathematics
Summer Leadership Institute
Workshops, Conferences
Journals and Professional Resources
Internet
State Department of Education

ONGOING ISSUES

What keeps teachers from changing?

Time
All that elementary teachers have to do Organization

Beliefs
Some kids can’t learn concepts
Math phobias
Need grade every day
Higher levels stop projects & discussions

Knowledge
Mathematical knowledge
Toolbox of strategies
Using manipulatives
Learning development

INITIAL SUPPORT

Workshop Materials
Interpersed Projects
Integrated Units

GATEWAY

Workshops and Regional conference of Teachers of Mathematics

MOTIVATION

Wanting the curriculum that went conceptually deeper and to be more project oriented

KEYS TO CHANGE

What is needed?

1) Classroom teachers who use reform-based mathematics working with methods classes for pre-service teachers

2) Support for new teachers from districts and math community

3) Teachers retooling by getting new skills & strategies

4) Teachers feeling success and getting students to feel success
The Case of Skip Munson

A Glimpse Into Skip’s Teaching: “Dip Sticking”

Skip Munson teaches a mixed age class of first, second, and third graders with a team of two other teachers. For math, however, the students are regrouped by grade level. Skip teaches the third grade math group. On the Monday I observed, the class was to continue work on a fractions unit that they had begun the week before. Skip had been away and a substitute had introduced the unit from the third grade level of the school’s adopted Trailblazers program. The students had been working to solve a series of pizza problems, the solutions to which required the application of their knowledge of fractions. Skip had looked over their papers and thought the students were ready to continue work on those problems. First, however, she wanted to do some dip sticking. For Skip, dip sticking means checking on students’ conceptual understanding. She explains, “Like when you check the oil in a car. Is it high? Is it low? Do I need to get more oil? Or is it okay?” She adds:

“I have learned that I have to do that on a continuous basis no matter what I am teaching. I think too many times I assume that because I have taught it, they have learned it. And too many times that is not the case... We assume because we taught that unit, we taught that lesson, or we did that activity...This is a big piece for me; the constant going back and checking on things that I have taught to see if it is still there.
(Second Interview)

Before having her class return to the problems, Skip called the group together on the rug. She said to them, “Talk to me about fractions. Tell me what you learned about fractions last week” As she listened to their responses, she realized that there
was confusion: “Maggie said you can’t have one-seventh of a pizza. Do you agree?”
“Can you explain that?” She continued to probe: “Tell me what you are thinking.” “I
want to make sure you understand...so talk to me more about that.” She was still not
satisfied: “So you think each piece is a different size?” “Tell me what else you
learned.” “In my head I heard...is that what you meant?” Finally, putting down the
papers she had originally planned they would continue with, she got out some tiles
and had the students reform into a circle. She told her class, “We are going to look at
this a little differently. We are not going to look at pizza. We are going to look at
tiles.” She counted out a group of tiles and worked with the students on how to
divide the group into fourths. She asked, “If I want to make fourths, what can I do?”
This took some trial and error. As a student made a suggestion, she would try it out.
As they worked, they discussed what “fourths” meant: “What word did I say that
made you think of four?” “I remember someone said something about equal. Are
these equal?” “Does someone have a different idea?” “Did it work to take two plus
two plus two?” “Did it work to take four?” “I have four groups. Are they fourths?”
“How can I make them equal?” And finally, when someone came up with the way
they could divide the group of tiles into fourths, she asked, “Do you understand what
he is saying? Can you say it in your own words?” And then, to give them another
task to demonstrate their understanding, she asked, “How can we group the class in
halves?” The students busily started counting and sorting themselves into two equal
groups until time ran out. As they left, Skip challenged, “Tomorrow when you come
in for math, I want you to sit on the rug in fourths.” She now knew that the class needed more work on their conceptual understanding of fractions:

I left some notes for the sub, but the sub was a former sixth grade teacher and I think he tried to do a lot of symbolic stuff with them. One of the kids told me as she left, “Mr. C. told us that the number on the top of a fraction is called a numerator and the number on the bottom is called a denominator.” And yet she was one of the kids today that had no clue about fractions...So that is what I will do tonight...go back and read Van de Walle again...See if we can do some backtracking. (First Interview)

The next day Skip brought out the fraction tiles for some concrete explorations. The students worked with partners and discussed what they could discover about the various pieces. They looked for relationships and compared the size and number of each color. In their discussion they were able to relate a whole to a dollar, fourths to quarters, and halves to fifty cents, because they were familiar with money. Someone figured out the “the pinks” [fifths] were like twenty cents because it takes five to make a dollar [whole]. When the exploration was done, Skip had them put the pieces away together, one color at a time. With each color they discussed how many pieces and what those particular pieces could be called. She felt they had made some progress toward understanding fractions, but she knew that she would have the class do more conceptual work before they returned to the pizza problems in their books.

A few weeks after her fraction unit was “finished,” Skip found an opportunity to dip stick again. For a lesson in which the students were going to work with three numbers, she passed out paper and gave the directions, “Fold this paper into thirds.” Then, by watching and listening, she checked to see who was able to remember and
apply their knowledge of thirds. When some students said, “I am not sure what thirds are,” she knew she was not through teaching the concept of fractions.

*Early Teaching*

It took only one political science class in college to change Skip’s plans of being a lawyer. To explore other career options, she started taking some education classes. When one of those classes observed a kindergarten classroom, she decided that is what she wanted to do. She majored in child development and got her elementary teaching certificate. Her first teaching job was in a kindergarten. It was a class that was 85% Spanish, non-English speaking, students. She thinks back, “I don’t remember teaching math that year. We immersed them in as much language as we could for half a day.” After only one year at that school, Skip moved. She came to the district she has now taught in for thirty-two years. She also moved “up” to first grade. When she started in that first grade, she remembers being given her mathematics “curriculum.” She describes what that curriculum was:

> What we had when I started were workbooks that had just pages and pages and pages and pages of computation, in all different ways, but that is all it was. There might have been one or two pages where they got to measure a picture of something. And one or two pages where they got to see pictures of money and write how much it was.

(Second Interview)

Even in her early years of teaching, those workbooks frustrated Skip. She thought, “There is so much out there…real life kinds of things they could be doing.” So she picked and chose only the pages she wanted to do. She brought in money the
children could count. She had them measure with paper clips. She tried to make mathematics more real.

Two concerns motivated Skip to do something different in her mathematics teaching. The first concern came from her training in child development. She thought, “You don’t just take this textbook and teach this group of children. That’s not right. You aren’t teaching them anything. You are teaching the page, but you are not teaching them.” She felt the same about all her textbooks. She changed her reading instruction right away. She had been given a reading textbook just as she had been given the mathematics workbook. She was handed the Scott Foresman Dick and Jane books and was told, “This is the first book and you take them through and then you move on to the next book.” She says, “I went down to the book room and we had several other series with primers and preprimers...I made big pockets and I let them choose the books they wanted to read.” Groups were formed from the books they picked rather than by ability. This basically created an individualized reading program. She was able to do the reading changes quickly, but she struggled with how to teach first grade mathematics more appropriately. Perhaps the reason for this was because her teacher training had included only one methods class in mathematics — and that combined mathematics and science. All her other teacher preparation classes were focused on language arts.

Skip’s other motivation for changing her mathematics instruction was her own memory of her mathematics classes. With strong emotion, she shares, “I hated all the math classes I had.” Although her young students did not seem that frustrated,
she wanted mathematics to be interesting to them. “I just kept thinking” she recalls, “I don’t want them to be like that... How can I make it more enjoyable and how can I make it more meaningful to them?” As she considered those questions, she remembered that there was one mathematics class she liked. It was her high school geometry class. “I tried to think of why I liked geometry. Was it the way it was taught? Was it the subject? I tried to look at that and think how I could reflect that in my own teaching.” In algebra she remembered always feeling intimidated. In geometry she wasn’t intimidated. She concluded it was the way it was presented. She wanted her students to like mathematics because she knew it was an important subject. “I wanted them to see that math was in life... not just a page in a workbook. I wanted them to be aware that, like reading, you can do math at home. It is not just a school thing.” Although Skip observed that the workbook lessons worked for some of her students, she says, “those were not the kids I needed to turn on.” It was “the other kids that were like me” that she really worried about. She continued to analyze her experiences and to think what made the difference in her geometry class, hoping to put that into her teaching. (See Figure 2: Skip Munson’s Journey, page 106)

*Introduction and Initial Support*

A breakthrough came for Skip, not in a mathematics workshop, but during a writing workshop. She had read Lucy Calkins’ book on the process of writing and then had signed up for a writing workshop that was presented in her district. As she remembers, “They had us writing. We went through the process. That got me really
excited that there must be a way to do this in math.” It opened up possibilities and started her thinking about the process of doing mathematics. Then, right after that writing experience, Skip took a Math Their Way class. She says, “Math Their Way really opened doors for me.” She had been introducing manipulatives in her teaching, but the textbook was still the bottom line. After the Math Their Way class she felt, “You have to stop thinking about the book. You have to bring real life experiences into the classroom.” Both the writing process and Math Their Way made the connecting of students’ own experiences central to the learning process. This matched what Skip had learned in child development classes, and now she could see it applied to her curriculum. It was the constructivist belief that you needed first to find out what children knew: “They know so much, but it is hidden. [You need] to pull it out and give them a chance to share it.” Then you “build on that and pull out more...make it so they are applying it and doing...instead of just regurgitation.”

The instructor of the Math Their Way class was a kindergarten teacher from another state. She, like Skip, had a child development background. Skip and the instructor became good friends. Their conversations about teaching mathematics to young children were helpful in framing Skip’s new thinking and in developing new teaching practices:

When “K” and I would talk, we would really talk in depth and question things. Just that idea of thinking about kids and here’s where I wanted to go and I am having trouble getting them there. That kind of thing. Not where I am in the textbook. (First Interview)
They pondered questions together. They shared stories about what happened as they used these methods in their classrooms. They read and discussed professional books and articles. It was their own action research. They learned together.

_Ongoing Support for Innovation_

Once Skip had the framework for how she wanted to teach mathematics to her children, she started learning all she could about best teaching practices and about how children learn. She continued her discussions with other professionals, including her husband, who was also an elementary teacher. She read books by authors such as Constance Kamii and Marilyn Burns. Skip says, "Marilyn Burns was another big turning point in my thinking of mathematics teaching because she did a lot of problem solving...that was probably my first introduction to that piece of getting kids to explain and talk about it." She attended workshops and conferences and continued to seek out good resources, always looking for more ways to improve her mathematics instruction.

At first, as Skip adopted these innovations, there was no one else at her school with whom she talked. Since that time, her teaching teammate took Math Their Way. That started fostering discussions. "Now," Skip says, "We can talk about everything." Only a few teachers in her district share Skip’s philosophy, so she does not get much of her support from other teachers in her school or her district. She finds that other teachers worry that they, "have to get through the unit and have to get the concepts across to the kids" for the tests. They
worry about covering the entire curriculum. Skip’s impression is that they go through the book, but do not take the time to make sure the students really understand the concepts. Skip says, “It is really hard to talk to someone who doesn’t have [the same] philosophy.” Her local support community is from her team members who do “have the same philosophies.” She shares the feeling that a lot of what she has done “was out there on a limb on my own.”

Not long after Skip started changing her instructional practices, she also started teaching multi-age classrooms. Multi-age teaching both supported and was supported by the curriculum changes she was attempting. “You can’t take the textbook and just ramrod your way through in any subject area.” This is her fourteenth year teaching a multi-age group. She shares her mixed age students with two other teachers: one is a long time teacher colleague and the other is her husband. Although they group the students by grade level for mathematics, all other subjects are taught with the first, second, and third graders mixed. Skip describes the dynamics of this grouping: “You have different ability levels, but you build the wider range of abilities into your teaching and you celebrate the differences instead of being bothered by them.” Building capacity for teaching mixed age groups has gone hand in hand with building capacity for teaching open ended, problem-centered mathematics. She says, for whatever she teaches:

Of course, all are in different places. You can’t have twenty-nine different plans. That would be wonderful, but not possible. So I really think that is my responsibility as a teacher – to try to reach each individual and try to meet their individual needs and move them forward the best I can. (First Interview)
Contrary to her experience with most other district teachers, support from building administrators has always been strong. All eight of the principals she has worked with have encouraged Skip in her teaching innovations. She describes the support as follows: “When I was out there on my limb and I felt I might drop, they were the two-by-four that would prop me up.” Two principals were particularly helpful. One woman principal, she says, “pushed and nudged me more than I did myself.” She would encourage Skip to pursue questions and follow through on ideas, where others might have just dismissed them. This helped Skip grow professionally. The other principal she found particularly supportive of her development was a former mathematics teacher and department chair at the high school. “It was really exciting to have him for a principal, because when I had content questions he would help me...[and] he was excited that I would come and ask him.” She appreciated his acceptance of her questions. That helped build her confidence and advanced her mathematical knowledge.

That level of support was not matched by the district’s central administration. That central lack of support has not kept Skip from her innovative practices, but she feels it has hindered the development of the district’s mathematics instruction in general. Although her district was considered to be on the “cutting edge” of educational practices fifteen years ago, in Skip’s opinion, that is no longer true. The last two curriculum directors have not had backgrounds or interest in mathematics. They have not been leaders in the development of district standards and have not kept the district up to date with the state’s standards. Without the leadership, other
teachers in the district have not adopted reform-based mathematics instruction. They still view mathematics primarily as arithmetic. Skip says, “The number sense part of the [curriculum] book is five pages and geometry might be one page...and the teachers think number sense is computation.” For support for her standards based teaching, Skip says, “My energies are now working with the state stuff. I feel like that is where the movement is going to be made. Districts are going to have to move. Assessment drives curriculum.”

**Continued Development and Issues in Mathematics Reform**

Skip believes that to teach mathematics constructively a teacher must know mathematics and she must know children. Skip describes this dual knowledge as follows: “You have to have the mathematical knowledge but, also, the development of that mathematical knowledge -- how it builds up...and what they have to know to get there to do it.” It is the pedagogical content knowledge Ball and Bass (2000) refer to when they describe the knowledge special to elementary teachers. They say, “ Bundles of such knowledge are built up by teachers over time as they teach the same topics to children of certain ages...” (Ball & Bass, 2000, p. 87). Heibert (2002) also discusses “craft knowledge” (p. 3), describing an in-depth understanding of mathematics that is related to pedagogy. For Skip, gaining this kind of understanding is an ongoing goal. From her early frustration with workbooks to watching her student struggle with the concept of fractions, Skip says, “I am constantly thinking, ‘How can I make this work better? What can I do?’”
Knowing that her background in mathematics was not strong, Skip has thought and thought about what made mathematics so hard for her. She has come to the following conclusions:

It was hard for me and I hated it because I didn’t understand it. It is not that I didn’t like it because I didn’t like math. I remember I didn’t like algebra because I didn’t understand it....It is [not the same as] “I don’t like squash.” I don’t like the taste of squash....I just don’t like it....I think if I had understood algebra at the time I would have liked it, because now it is still difficult, but it intrigues me. Now when I see a problem in a class I want to figure it out. (Second Interview)

Skip overcame her dislike of mathematics. She faced that dislike and, furthermore, she made it a goal to keep her students from feeling the same dislike by working to make mathematics understandable. She recognized, “I needed to learn, or relearn, or undo what I had learned...and not be intimidated by it.” In her pursuit of teaching conceptual understanding, she has built confidence in her own knowledge of mathematics. She knows that to do this, “You have to be open enough that you are willing to ask if you don’t know or when you are unsure.” An important part of the learning is the acceptance of not knowing. Skip shares the importance of this critical examination of one’s knowledge:

So if you are getting ready to teach a concept and you think you have a pretty good handle on it, but when you really start thinking about it there are some things you are not really sure of, you have to seek out help so that you truly understand it before you start presenting it or sharing it with children....Even to this day if I haven’t thought it through enough and the kids come up with something unexpected I will think, “Oh, that’s not right.” And then I will go right to the knowledge level stuff....They have the pieces, but my depth is not enough to see where those pieces fit in. So I resort to the knowledge level and expect them to regurgitate back to me. (Second Interview)
Once again, driven by the concern to teach for conceptual understanding, and not just getting surface right answers, Skip continues to observe and read and ask questions. She says, “That is what I struggle with all the time.” She continually asks herself: “What do they know and what don’t they know?” “What do I have to do?” “Where is it in the understanding of this process or this concept that I need to go back to help them?” She searches out professional publications and reads to learn more about the mathematics concepts she is teaching and about strategies that can help build students’ understanding. She still goes to conferences and workshops. She looks for resources that don’t just give her activities, but that explain appropriate ways of breaking down the important mathematical concepts for presentation to children. In her searching, she is developing the skill Liping Ma (1999) refers to as Profound Understanding of Fundamental Mathematics (PUFM): understanding connectedness among mathematical concepts and procedures, appreciating different facets and various approaches to solutions, awareness basic concepts and principles, and knowledge of longitudinal coherence in order to know where students are coming from and where they are going. Again, for Skip, content knowledge is always intertwined with knowledge of children. She continues to develop her knowledge of both areas together.

Skip believes in standards-based mathematics instruction. She is actively involved in her state assessment of mathematics. She writes and scores open-ended problems and writes conceptually oriented questions for the multiple-choice examination. She is focusing on the state level, because she believes that the state’s
assessment will be a key element in improving local districts’ curriculum and instruction. She notes that many school districts have adopted reform-based mathematics programs. She warns, however, “No curriculum is perfect. Even the ones that the National Science Foundation says are sound curricula and that follow the standards...have holes.” Therefore she also stays involved with her state organization of mathematics teachers. She presents at workshops and conferences in order to further the effort to bring the standards into classrooms and to help teachers see the need to connect the mathematics both developmentally and conceptually to their students.

When asked why reform-based mathematics has been slow to be implemented by elementary classroom teachers, Skip reflects that most elementary teachers have a stronger orientation toward language arts. She has noticed that when the teachers in her school meet to set their annual goals, it is the reading and writing scores that are first looked at. Most of the school’s goals are then focused on improvement of those scores. She notes, “Even this year. We have a math goal....It is written down, but we haven’t done anything about it.” She speculates that language arts may also be favored because so many elementary teachers are not as comfortable with their own mathematics understanding. If mathematics is not a strength, of course, people will be more reluctant to go to workshops that might just “make you feel bad.” People are often reluctant to admit they are weak in an area. There is belief that they should “know it all.” Since the part of mathematics that
elementary teachers do know well is arithmetic, that is the part of mathematics that remains the core of mathematics teaching in elementary grades.

Another issue Skip mentions is time. The deep discussions necessary to really look into one’s teaching practices take time. She recalls the most helpful discussions in her development were those that posed questions that challenged what she was doing and pulled her into new ways of thinking about her instruction. She reflects, “If someone spoon feeds it to you, you don’t do the learning, they do the learning.” But in the name of efficiency teachers often just want a textbook or activities and not the discussion. The reality of school schedules means that meetings, workshops, and classes that foster rich discussions usually happen outside of regular school hours. Many teachers will not or cannot stay late or give up evenings or weekends. Skip says, “People are not willing to give the time....It is not that they don’t have the time. They just have different priorities.” Her school has been fortunate to be part of a grant that has allowed teachers to attend staff development sessions during regular school time. It has helped get some discussions started. Unfortunately the funding will not continue past this school year.

Time is also an issue in the classroom. Teaching mathematics with conceptual understanding takes more time than teaching traditionally. After her discussion on fractions, Skip commented:

A lot of teachers are not willing to put in the time with things like I did today. Many would think that I wasted my math hour today....What did I accomplish? Because I did a lot of talking with the kids. Some teachers in this building would say I didn’t get any work done. (First Interview)
Getting through the curriculum for the sake of covering the concepts is a big worry. There is still a strong emphasis on operations. Skip says that her experience with writing questions for the state assessment has confirmed that understanding is important. The questions are not “everyday, mundane questions that you have in the curriculum…. Computation is used in getting the answer, but it is not the answer.” Teachers have to be willing to give the students time to develop conceptual understanding and not just jump to symbolic processing. This point comes back around to the issue that many elementary teachers need to take more time to develop their own conceptual understanding of mathematics.

Skip is concerned that the academic and licensing demands currently placed on student teachers are responsible for launching beginning teachers in a way that encourage getting things done to be done, and do not encourage the reflective practices that develop quality teaching over time. She asks, with all the requirements that are heaped on them, “When do they have time to think about what they are doing?” Ideally, she would like to see student teachers placed in nurturing environments in which they can observe best practices and be part of discussions that model ongoing examination of instruction and learning.

When addressing all the pressure that is being put on teachers and classrooms today, Skip reiterates what she believes is central to teaching. Instead of dividing up all the teacher knowledge into compartments, we need to go back to the “development piece.” We need to first consider, she insists, “How do kids learn anything? Not just math or reading or writing or science or social studies. How do
kids learn?" With that understanding in place, we can then apply what we know in those different subject areas. Skip strongly believes that teachers need to be professionals. They need to have knowledge of subject matter and knowledge of their learners; they need to bring all that knowledge to their teaching. Skip stresses that teachers need to "get out of the factory way of thinking" and be more responsible for their classroom decisions. Good teaching does not come from just following ready-made programs. Teachers need to be professionals that continuously develop their skills and knowledge.
Figure 2: Skip Munson’s Journey

LIFE LONG LEARNING
How Mathematical Knowledge Develops in Children
Developmentally Appropriate Presentation of Mathematics
Assessing Students’ Learning       Professional Development
Continuous Improvement – Always Looking for Better Ideas
Integration of Subjects       Learning from Learners

ONGOING SUPPORT
Professional Discussions
Action Research
Workshops and Conferences
Building Administrators
State Teachers of Mathematics
State Standards & Assessment
Journals and Professional Resources

ONGOING ISSUES
What keeps teachers from changing?
Time
Professional development
outside of school
Time priorities
Instructional time
Beliefs
Math is computation
Coverage is teaching
State test is recall
Language Arts is priority
in elementary
Knowledge
Mathematical knowledge
Child development
Assessing conceptual understanding

KEYS TO CHANGE
What is needed?
1) Find time for professional development during school time
2) Teachers must examine concepts and know them thoroughly including children's probable thinking and misconceptions
3) Improve student teaching experiences
4) State Assessment will drive local teaching

INITIAL SUPPORT
Math Their Way
Professional Discussions
Professional Resources
Workshops

GATEWAY
Writing Process Training
Math Their Way

MOTIVATION
Frustration with Workbooks
Training in Child Development and Constructivist Philosophy
A Glimpse at Michael’s Teaching: Investigating Probability

After several days of activities and discussions that gave Michael Patrick’s fourth graders a variety of experiences with the concept of probability, today they were being asked to collect data and make predictions around a probability inquiry. But first, some review: “If I had a dime and dropped it on the floor, what would be the probability it would land tails up?” “Explain 50/50.” “I have a die in my hand, what is the probability, when I roll the die one time, that it will come up six?” “Defend that answer.” “What is the probability of rolling a three?” “Explain how you know.” “With the same die, what is the probability of rolling an even number?” “Think privately.” “Would it be different for odd numbers?” “Why?” “Is there another way to write 3/6?” “So like flipping a coin, you have an even chance of getting odd?” “What does that mean?” Questions, many questions, guide these students to use the mathematical concepts they have been exploring.

For today, the challenge is to collect some data and use that data to make probability predictions. Putting name cards into a large bucket, Michael asks, “How many students in the class? If I take this bucket of names around the room, how many people do you think will draw out their own name? Write down that prediction.” With much buzzing and conjecture, the students each took a turn drawing a name card. It was counted and recorded how many got their own names. Zero for the first round. New predictions were made. The whole group repeated the
drawing. Then the class was split in half, and the two smaller groups repeated the experiment. Finally table groups of four did the experiment. Then, as a class, the data were summarized on a chart and the results considered. “We went from large to pretty small groups. We had no one drawing their own name in the large group, a few in the half groups. What happened in the small groups?” Before discussing this together, the students wrote their conclusions in their journals. Finally they talked together about the data seeming to show that it is more likely to draw out your own name in the smaller group than in the larger group. As a final question, Michael asked, “In the large group, is it possible for everyone to draw his or her own name?” “No,” the students replied as a chorus. He repeated the question and then added, “It did not happen today, but is it possible?” Some yeses. Some no’s. Most were unsure. “There is a name for every person, right?” “Yes.”” So it is possible. Is it likely?” A chorus of no’s. Now they got it. Probability is a difficult concept for nine-year-olds. It will take many of these kinds of explorations and inquiries for students to understand. But today they got closer. They got closer by predicting and trying, by looking at the data and discussing what is possible and what is likely, by taking part in a concrete, doing-it-themselves experiment. This collection of questions and challenges, conjectures and reasoning, discussing together and thinking alone -- all focused on an important mathematical concept -- exemplify the vision of reform-based mathematics instruction presented in NCTM’s Professional Standards for Teaching Mathematics (1991).
Early Teaching

Michael recalls taking only one mathematics methods class in his pre-service education program. When he began teaching in his own classroom, his mathematics instruction was straight out of the textbook: “It was direct number stuff – adding, subtracting, multiplying, and dividing. It was, 'This is how I do it. Now you practice how I showed you.'” Michael was not very satisfied with his students’ mathematics performance. “I wasn’t really happy with just telling kids how to do the algorithm. And they were frustrated with it because they couldn’t remember all the rules that I told them.” However, at the time, he attributed it to, “They just weren’t listening or maybe they weren’t ready for it.” He really did not have any idea that there might be other possibilities for mathematics instruction. “I never really thought about the way I was presenting the material or anything like that.”

Over the course of the first ten years of his teaching, from 1981 to 1991, Michael taught fourth and fifth grades in three school districts. He arrived in 1990 at his third district, where he still teaches today, with a new Masters Degree in hand. All of the classes he had taken for his Masters Degree were in reading and language arts. No mathematics. It was at this point a major shift of focus occurred in Michael’s career. (See Figure 3: Michael Patrick’s Journey, page 122)

Introduction and Initial Support

In the summer of 1991, Michael registered for a mathematics workshop presented by The State Mathematics Center. He does not recall exactly how or why
he took the class. Possibly he had seen an information flier sent to the school.

Maybe, he mused, it was just an interest in finding some new ideas for teaching mathematics. There was no push in his school or district to teach mathematics any differently. The subject of the workshop was a hands-on mathematics program targeted for fifth through eighth grade. At the time, Michael was teaching fourth grade. He did not know anyone else taking the class. And the class was held in a town about thirty miles down the road. Whatever his motivation, Michael registered for the weeklong class, and his professional life took a whole new direction. He states, still with a touch of awe, “It was just amazing! It opened up a whole new world for me!”

Michael did not come to teaching with a strong background in mathematics. This is typical of most elementary teachers in American public schools (Drake, Spillane & Hufferd-Ackles, 2001; Ma, 1999; National Research Council 2001; Schifter & Fosnot, 1993). He describes his mathematics history as follows:

I was a pretty good math student in elementary school, but as I proceeded it just went downhill from there. It was all so subjective and there was nothing concrete. I remember my high school math...

I was just completely lost.

He says of himself, “I was a math phobic.” He took no mathematics in college except the one methods class required for teaching. He had no special interest in mathematics during the first years of his teaching. However, in this workshop he was introduced to activities he found exciting and interesting:

The activities made sense. I learned stuff about math that I didn’t know was true because I hadn’t been taught that way. It was very
meaningful. It was problem solving oriented. And we had fun! I wish I had had this math when I was a kid.

Michael took the ideas from that workshop back to his classroom and adapted them for his fourth graders. He took more workshops from the Math Learning Center and, eventually, the training to be a workshop leader himself. In his classroom, he put away the textbooks and began teaching mathematics primarily through investigations, projects, games, and problem solving tasks. Michael read professional journal articles to get problem and project ideas and to learn more about this kind of teaching. He brought workshop ideas into his classroom. He gathered manipulatives to support his instruction. He remembers, “Some things worked and some things I had to tweak.” He just “found more ways and more opportunities.” Michael was “very comfortable” not having a textbook, because as he used these ideas and observed the mathematics his students were doing, he felt “really successful.”

Parallel involvement in another curriculum area added momentum to the innovations Michael was incorporating into his mathematics instruction. At the time he was taking the workshops exploring new instructional strategies in mathematics, Michael was also a member of a committee that was developing a new curriculum and choosing new materials for the school district’s science program. He reflects, “Math and science really went hand-in-hand….Constructivist thinking and kid centered [teaching] all came at one time. I think one helped the other for me to take it down that road.” While constructivist teaching methods were being modeled in the mathematics workshops, a science program that relied on inquiry and discovery was being adopted by the school district. Michael made these practices a central part of
his teaching. He did not have a label for these changes until years later when he read a book on constructivism. It described constructivist teaching, and he realized, “That is what I have been doing.” In adopting the methods he had learned from his science and mathematics involvement, he had become a constructivist teacher.

Michael had found the new science and mathematic methods personally meaningful and, when he used them in his classroom, he found his students were more interested and successful. This gave him the ownership and personal commitment that are important ingredients in a teacher’s capacity to make meaningful curricular and instructional changes (Hargreaves & Fullan, 1992; Lieberman & McLaughlin, 1996; Rudduck, 1991). However, most researchers of educational change agree that for long-term and in depth change, there needs to be ongoing membership in, collaboration with, and support from a community of professional colleagues (Ball & Cohen, 1999; Cwikla, 2004; Fullan, 2001; Johnson & Brown, 1998; Lieberman & Grolnick, 1999; Nelson & Haammerman, 1996; Tinto & Masingila, 1998). Michael did not find that community in his school or in his district. Local teachers were not interested in this new way of teaching mathematics. “There wasn’t anyone in the building I could talk to that would help.” Even when new state standards promoted conceptual mathematics understanding and problem solving and made them elements in the state’s benchmark assessments, the district’s teachers remained resistant. “I tried to teach them about the scoring guide...I thought I was going to have rotten tomatoes thrown at me...one teacher came up to me afterwards and said, ‘I don’t think we should even be teaching problem solving.’”
He further recalls coming back to school one fall especially inspired from one summer conference, but when he shared the information with his staff, “they were not interested at all.” He felt really disappointed. He had to continue developing this new curriculum and acquiring these new teaching skills on his own. Today, fifteen years after the publication of the NCTM standards and more than ten years after the state’s adoption of very similar standards, only Michael and one other intermediate (grades four and five) teacher at his school teach mathematics with a problem centered approach and without a traditional textbook. That is two out of seven teachers. “I cannot,” he says, “be a prophet in my own country.” When he has offered suggestions, “they think I am trying to push a philosophy on them, and they’re not interested.” Michael overcame math phobia, changed his beliefs about mathematics learning, and developed new teaching strategies for his mathematics instruction. He believes teachers that are uninterested in changing may still be “phobics in the classroom.”

Michael did not find support from his administration either. With the exception of his first principal in this district, who was “a progressive math type,” none of his building administrators have been “curriculum kinds of principal[s].” One found out he did not use the textbook in a parent meeting. Her response was, “Don’t you think you should at least pass the books out to the students?” Michael’s sense is that his principals have not known or cared what he is doing. With the current pressures from No Child Left Behind and state report cards on schools, his administrators are focusing on test scores and, in mathematics, on arithmetic
efficiency. Their focus in that direction indicates they do not realize that this state’s tests are problem oriented, assessing conceptual understanding. Arithmetic proficiency on state examinations is a means to finding answers, not an end in itself. The district recently adopted a new mathematics textbook program that is reportedly aligned with the state’s mathematics standards. Evaluation of the program by Michael and others familiar with the standards is that the program is not aligned. The district’s curriculum director stated the expectation that all teachers would use the textbook. It is, in effect, the de facto mathematics curriculum for the district. Michael told his principal, “a real paper and pencil person,” that he would not be using the textbook. That “worried” her, but she did not insist. Michael is confident he is offering his students a better mathematics program. Parents are happy with their children’s progress. Students are coming back years later and saying, “Mr. Patrick, I am still using the problem solving we learned in your class.” Michael’s presentations and workshops are well received in other school districts around the state. He is actively involved with the state’s Department of Education and the development of state’s assessment instrument. Without local support, how did Michael develop new instructional strategies and gain the pedagogical and mathematical knowledge he needed for this transition?

Ongoing Support for Innovation

Michael found support outside of his district. Initially, he found it as a member of a regional educational consortium that was involved in the state’s
educational reform during the early 1990s. “We’d go to these meetings and [the
leader] would say, ‘Here’s what has come up lately.’” We would look at scoring
guides and proposed standards and “kick around” different ideas. “It was pretty
interesting…rubbing elbows with math people and science people from the valley.”
This experience was informative and motivating.

In the longer run, Michael found his support from the mathematics education
community. As he became involved with the Math Learning Center, first as a
consumer of their workshops and then as a presenter of their workshops. Through
that association, he developed professional relationships with many other
mathematics teachers. He started attending the annual Summer Leadership Institute,
which gathered together leaders in mathematics education from all over the state. At
this institute the participants shared teaching ideas and discussed issues around the
teaching and learning of mathematics at all levels. He became involved with the state
organization of teachers of mathematics, which also offers support in the way of
materials, resources, and workshops for teachers. He attended conferences and
state/regional forums on mathematics education. Through those experiences and
connections, Michael’s knowledge and confidence in both mathematics and teaching
grew. He feels that at the Summer Leadership Institute people made him feel
comfortable about mathematics and his teaching of mathematics. In his own words,
“It changed my whole idea about things.” This was the community of professionals
that supported his transformation. He learned from attending and presenting
workshops, from reading professional journals and resource books, from conversing
with other teachers, from participating in a mathematics Internet listserv. He learned more and more mathematics. Along the line, his initial “math phobia” was dispelled.

Continued Development and Issues in Mathematics Reform

As he gained this mathematical confidence, the learning and sharing became more and more reciprocal. He has become actively involved with the state assessments in mathematics. He now writes and evaluates problem-solving tasks. He trains scorers. And he directs the scoring centers for the annual assessment. He explains:

I have been able to work with teachers across the state that have been movers and shakers. It is always interesting to sit with that group and discuss [the] kids’ work in front of you. It is like you’re a detective looking for evidence of a good problem solver and discussing that. It is really interesting and challenging. It is a huge motivation.

Michael also writes items for the state’s multiple-choice mathematics assessments. In that effort he has become very familiar with the state standards, what each standard means, and what attainment of that standard looks like. Each assessment item needs to be very carefully related to the standard it is assessing. When Michael is leading workshops, offering information and resources to other teachers, writing assessment items, and scoring students’ mathematics problem-solving tasks for the state, he is, at the same time, learning more about his own teaching and his own students at the same time.

The self-description of his teaching that Michael wrote in his Presidential Award application packet, matches the teaching I observed in his classroom:
I am interested in the mental processes and patterns of organization that my students use while engaging in new information. If students learn a process for solving problems using strategies that they have invented, it can be applied in other situations.

And further, he says:

I hesitate to describe a series of activities as a unit because a unit sounds contrived and sometimes meaningless. I like to think of the activities as a series of experiences in which the students construct concepts and relate that learning to already familiar ideas.

In the probability activities, numerous illustrations connected the concept of probability to the student’s real world: weather predictions, finding a pair of socks in the dark, taking surveys about favorite movies, buying lottery tickets. Students worked as a whole group, in small groups, and in pairs. Answers included supporting evidence or explanation of the process for getting it. Thinking was expected. Strategies were discussed. Assessment occurred through discussion, paper and pencil responses, and reading of body language/faces. Instruction was adjusted based on the information from these assessments. The teaching was not so much a linear progression of steps in a lesson, but questions and activities flowing together. Conjectures, discourse, thinking expectations, and making connections were there as underlying currents that surfaced off and on throughout the whole lesson. I believe the transition from traditional teaching of mathematics to reform-based mathematics instruction is a total commitment. Since that first workshop caught his interest, he does not seem to have ever looked back. When asked, he said, with much conviction, that he could never go back to teaching mathematics in the traditional manner. It has taken Michael years to develop his teaching to where it is today, but I did not hear
him say that he feels he knows everything. On the contrary, he continually searches for new activities and new projects to try. He refers to his assessment activities with the state’s Department of Education and the workshops with the state’s organization of teachers of mathematics in terms of learning opportunities for himself. He puzzles about things that come up from the classroom:

You are not always sure where something is going to go. If something comes up you are not expecting, like somebody has a way to do something that you don’t know where it came from, you need to figure out where to take that.

On Rogers and Shoemaker’s (1997) continuum, Michael would fit the description of an early adopter. He has not only adopted these instructional innovations successfully, he is now a leader in the reform of mathematics education. Self-motivation carried Michael both into and through the changes that were required in his beliefs, his teaching practices, and the selection of materials he uses in his instruction. Not only were these changes not imposed on him, Michael had to find support outside his school and district. Between his own self-direction and his involvement with a community of other teachers of mathematics, he was able to address the three key dimensions Fullan (2001) describes as necessary for full implementation of new practices in the classroom: revised materials, change of beliefs, and new teaching approaches. In addition, he developed skills that Ball and Bass (2000) call pedagogical content knowledge. This is the intersection of mathematical content knowledge and knowledge of learners. He understands the important mathematical concepts and, at the same time, he can “deconstruct his own mathematical knowledge” (Ball & Bass, 2000, p. 98) in order to get the ideas across
to his fourth graders. He has developed the constructivist teaching strategy of listening carefully to his students in order to assess where they are in their understanding of a skill or concept, then thinking of questions or challenges that will allow them to build onto that understanding. This kind of teaching is based on the belief that learning is stronger and more lasting if the student *constructs* his or her own knowledge and is not just told what to do for a *right* answer. It is at the heart of the NCTM’s goal of teaching for conceptual understanding.

The innovations in Michael’s teaching now run across all curriculum areas. These were major changes. However, they were not stressful or painful or even evocative of the grief process as some suggest (Fullan, 2001). They took time. They took intentional, focused effort. They took acceptance of a certain amount of uncertainty and an openness to try new ideas. They took a change of belief from looking for mastery to looking for continuous improvement, both for himself and for his students. But Michael voluntarily dedicated personal time and energy to these changes. This was time and effort that was not compensated by his district nor, for that matter, even recognized or appreciated. Two key elements seem to have influenced Michael’s transformation. First, was a “feeling of success” when he used these new techniques in the classroom. He observed less frustration as students began to develop strategies that made sense and brought understanding. He saw enthusiasm, energy, and interest in the classroom as students participated in inquiries and mathematics projects. The feedback a teacher receives from student engagement is powerful motivation (Spillane & Louis, 2002). The second key element for
Michael was the interaction with and membership in a community of professionals who carried on rich and stimulating discussions around the learning and teaching of mathematics. Somewhere in the examination and development of his teaching practices, Michael became a life long learner and a problem solver. He views the complexity of teaching mathematics to children in a way that is both meaningful and interesting as a challenge. He continuously seeks ways to present mathematics to his students so they appreciate both its utility and its mystery. His ongoing conversations and shared inquiries around children’s work (like “detectives looking for evidence of a good problem solver”), lesson development, useful resources, and research on teaching and learning are energizing. Changing is about learning. A common mantra in school tells students that learning is exciting and rewarding. It is. Why, I wonder, is it so stressful and painful for so many teachers? Perhaps this is the question we need to pursue in finding out how to make schools into learning communities and places of continuous improvement.

I asked Michael what he believes keeps so many elementary teachers tied to traditional mathematics in their instruction and their curriculum? I have summarized his response:

I think most teachers are math phobics, like I was. The algorithm stuff is the only way they know. That is hard for people to let go of. The traditional way is so much easier because you have a recipe book. They can tell the kids how to do it, have them do it, and correct it. To teach conceptually you have to put a little more time and effort into it. Change is tough.

He believes the NCTM vision of mathematics instruction is realistic for all elementary classrooms. He notes we have changed the way we teach reading and
writing and science. He suggests, “It might just take a little bit longer for math. Maybe we view it a little differently, because there are people who say, ‘I am not good at math,’ and it is accepted. So it is a mind set kind of thing with math.” It is important, he believes, that prospective teachers see and work with teachers who use problem solving in their mathematics instruction. It is often what student teachers see in their classroom placements that have the greatest influence on the kind of teaching they will do in their own classrooms. Further, new teachers need support for this kind of teaching when they are starting out. They need to develop the understanding of their learners, classroom management strategies for working manipulatives and projects/group work, and knowledge of the mathematics standards of the state. This kind of teaching needs support early, he warns, or it may be too easy for new teachers to slip into dependency on the “recipe book” teaching a text based program offers. After that, “change is tough.”
Figure 3: Michael Patrick’s Journey

LIFE LONG LEARNING
Sharing and Exchanging Ideas with Other Teachers
Mathematical Knowledge  Connections to Real World & Other Subjects
Constructivist Teaching  Problem Posing  Manipulatives
Choosing and Writing Tasks  Classroom Management
Differentiation of Instruction  Questioning  Standards
State Assessments  Collecting Resources

ONGOING SUPPORT
Math Learning Center
Summer Institute
Mathematics Education Community
Department of Education
Workshops, Conferences, Conventions
Professional Organization
Professional Journals and Resources
Internet

ONGOING ISSUES
What keeps teachers from changing?

Time
Finding & preparing materials
Ease of using textbook “recipes”

Beliefs
Worry about state tests
View of math as algorithms & one-right-way-only
Math phobia
Problem solving is not important

Knowledge
Mathematical Content
Alternative solutions
Different strategies

Teaching Practices
Constructivist teaching
Management

KEYS TO CHANGE
What is needed?

1) Early exposure to seeing mathematics taught through problem solving
2) Training in both preservice and early teaching on methods
3) Placing student teachers with teachers using problem-centered mathematics instruction
4) Support from schools and districts

INITIAL SUPPORT
Workshops
Science Curriculum Committee
Regional Consortium

GATEWAY
Summer workshop on teaching mathematics with problem-solving

MOTIVATION
General dissatisfaction with mathematics instruction in classroom
A Glimpse of Rose’s Teaching: Balancing Checkbooks

I was looking around Rose Sharon’s classroom, when two posted signs caught my attention. They listed “INCOME” and “FINES” in unusual amounts:

<table>
<thead>
<tr>
<th>6th GRADE INCOME</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spelling Words:</td>
</tr>
<tr>
<td>Spelling Sentences:</td>
</tr>
<tr>
<td>Book Report:</td>
</tr>
<tr>
<td>Writing:</td>
</tr>
<tr>
<td>Problem Solving:</td>
</tr>
<tr>
<td>Bonus for all</td>
</tr>
<tr>
<td>4’s or Better:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6th GRADE FINES</th>
</tr>
</thead>
<tbody>
<tr>
<td>No name on paper:</td>
</tr>
<tr>
<td>Talking out:</td>
</tr>
<tr>
<td>Late paper:</td>
</tr>
<tr>
<td>Messy desk:</td>
</tr>
<tr>
<td>Late to class:</td>
</tr>
<tr>
<td>Second copy:</td>
</tr>
<tr>
<td>Borrowing equipment</td>
</tr>
<tr>
<td>Using the bathroom</td>
</tr>
</tbody>
</table>

Later, while I was observing her mathematics lesson, I noticed Rose passing out paper “dollars” as she returned assignments to students. I suspected this money was related to the posters, but did not know how. At the break after the lesson, I learned the connection. A girl approached Rose to ask for help balancing her checkbook.

Rose explained to me that the students in her class each have income and expenses
that they individually record in their checkbooks. The students are responsible for managing their funds by making sure their checkbooks balance each month. Rose has developed this system as a way for her students to both practice and apply computational skills. This makes the use of arithmetic interesting and meaningful. It connects it to a real life purpose. She described her system in her Presidential Award nominations packet with the following paragraph:

Real world math is introduced the first week of school. Each student opens their own classroom checking account. I reward them with a gift of $100.00. They learn how to complete a deposit slip and enter the transaction in a check register. Of course nothing in life is free, so they are charged rent on their desk - $18.76. They learn that checks are legal documents and must be written in ink. They record the transaction in their check register. Addition and subtraction are not isolated skills taught out of context. They are absolutely necessary. Are calculations accurate? They balance their checkbooks every three to four weeks, so 100% accuracy is required. Students have the opportunity to earn money whenever they complete a specified task. They also have several opportunities to spend money in the form of “[Sharon] Fines.” No name on your paper? - $24.95; you need a second copy of a paper? - $27.63; messy desk or notebook? - $33.78. I begin the list, but the students add fines and amounts to the list as they take control of how they want their classroom to run. One group decided that the fine for touching someone else’s food should be $500.00. They have opportunities to spend their money for enjoyment also. Free time is $50.00, but only when the entire class has earned free time. On Friday, we have an auction and students may bid on items – pencils, pens, small toys, etc. “One person’s junk is another’s treasure.” Parents have been very generous in their donations. Occasionally a Big Ticket item, like a trip to [a museum] for three or four students will go on sale. Students learn the responsibility of having a checking account, saving money for important things, and paying the price of mistakes (without too much pain). Parents are delighted with the fact that their children are learning an important life skill many adults find difficult. Checking accounts provide students with real world math and are a valuable tool for classroom. (Presidential Award Nomination Packet)
At twelve years old, sixth graders are very interested in both independence and in money that they can control. Requiring the computational skills necessary for keeping track of their accounts, while giving students a variety of ways to both earn and spend their “money,” is a developmentally appropriate and highly motivating way to address NCTM’s process standard of Connections. It represents the intention of the Principles and Standards’ (2000) recommendation:

When students can connect mathematical ideas, their understanding is deeper and more lasting. They can see mathematical connections in the rich interplay among mathematical topics, in contexts that relate mathematics to other subjects, and in their own interests and experience. Through instruction that emphasizes the interrelatedness of mathematical ideas, students not only learn mathematics, they also learn about the utility of mathematics. (p. 64)

Early Teaching

Rose had been married and started a family when she went back to school to become a teacher. After she graduated with her degree in elementary education, she looked for a job in her hometown, where she had grown up; where she had done her student teaching; and where, now, her own children were attending school. She applied for a fourth grade opening, feeling sure she would get the job. She was very surprised when they did not offer it to her. She was even more surprised, she recalls, when the superintendent told her he wanted her to teach the seventh grade math and science class. At first she thought, “He has called the wrong person.” But, no, that is position for which she was hired. “So my first job,” she says, “was teaching a block of math and science. It was exciting, but [we were] a little ahead of our time.”
Rose started teaching that junior high mathematics and science block without much support in the way of curriculum material. She reflects, “I inherited a position that did not have any equipment. There were no manipulatives or anything. There were just a couple sets of books in the classroom.” So she spent her first year of teaching going through the textbook. This was not very satisfying to her; she knew that was not the way she wanted to teach. Rose had taken mathematics methods from Dr. Sharon Ross, a leader in mathematics education and a proponent of problem solving and teaching for conceptual understanding. Rose says of Dr. Ross, “I knew that what she was telling me was powerful and would have an impact on my life.” She adds, philosophically, “But then I got into the classroom.” Rose tried to incorporate some problem solving into her teaching, but it was difficult with her “crummy set of books.” She says, “I think that student teaching never really prepares one for the reality of teaching.” (See Figure 4: Rose Sharon’s Journey, page 137)

Introduction and Initial Support

At the end of that first year, Rose was asked to participate in a summer mathematics project directed by her university methods instructor, Sharon Ross. Rose describes the project:

It was four weeks of really intensive training with a focus on problem solving and conceptual teaching for understanding, bringing in the idea of constructivism and letting kids investigate and discover things. It was four weeks with follow up. (First Interview)

The workshop, Rose remembers, “was kind of like a religious experience.” Participants were constantly thrilled as they looked at models, visualized the
concepts and, for the first time, really understood the mathematics. They would say, “Oh! That is why...!” “It was really powerful,” she says, “It had a huge impact on my curriculum.” But she adds, “It [still] didn’t help with the textbook piece.” She knew she had more to learn, but she now had the personal commitment researchers recognize as a critical element in one’s capacity to make meaningful instructional changes (Hargreaves & Fullan, 1992; Lieberman & McLaughlin, 1996; Rudduck, 1991). She looked for replacement units that she could adapt to her classroom. She found projects for her students to work on. She was enthusiastic, but when she looks back, she now reflects:

I know it was good stuff, but I didn’t do a very good job teaching it. I would throw out these manipulatives and I would throw out all these projects. Kids were working on it, but I am sure, in hindsight, that the kids were not really getting the mathematics. (First Interview)

She had the vision for problem-centered teaching, and she would continue to develop the capacity to do it well. She worked on developing her junior high school mathematics and science curricula for five years. Then Rose moved with her family to another state.

In her new hometown, Rose found a job opening for a fifth grade teacher. It was a position created by a need to relieve overcrowded classes; so school had already begun when she was hired. Her class was created by taking a few students from each of the other fifth grade classrooms. At that time, the school grouped all classes by ability. That made Rose’s class quite different. Her room became the only heterogeneous class at that level. She says, “It was a big adjustment for me and for the [other] teachers around here.” Moreover, she again found herself dealing with,
"a set of books" for the mathematics curriculum. She now had to search for projects and replacement units that she could incorporate into her fifth grade teaching. Between the mixed ability class and her teaching with projects, Rose recalls, "The things I was doing looked a bit strange."

When summer came, Rose had another opportunity to work with her university mentor. Dr. Ross was coordinating a grant-funded project "to bring together teachers to write replacement units." Rose applied, and was accepted, to the "Beyond Activities Project." She says, "The goal was to write three or four units." In the long run, they met for three summers to write just three units. She describes the project and its value to her:

In the morning we would work with a class of students. And then in the afternoon we would write lessons and talk about what happened. It was wonderful. That was probably the strongest and most powerful thing I did. As far as working with some really good teachers. And really getting into talking about curriculum and what makes a really good lesson, how the students responded and what part was engaging to them. (First Interview)

It was three summers full of learning for Rose. "The big piece for me," she says, "was getting beyond the activity to the math." She realized, "You can’t just keep having kids building that model. Eventually they have to come up with the numbers and some kind of process." One of the units they wrote "built on the idea of kids doing activities that construct their understanding of sharing and remainders." While they were working, Constance Kamii came and talked to the group about constructivism. They also sought advice from the people who were writing the *Investigations* (TERC, 1998) mathematics curriculum. Rose recalls, "We had a
chance to meet some pretty great people.” Through these interactions, they were exposed to the latest thinking about mathematics instruction.

Rose had come into teaching with some college level mathematics in her background. She had not struggled in mathematics, but she says, “I couldn’t tell you anything about the classes I took.” That is what has made her feel so strongly about teaching mathematics for understanding. After the three summers on the Beyond Activities Project, she now had pedagogical *content knowledge* (Ball & Bass, 2000), or what Heibert (2002) calls *craft knowledge*. She could take her own mathematical knowledge, break it apart, and apply it to her curriculum in a way that allowed her students to build *their* understanding of mathematics concepts. What Rose says is, “I am not a great mathematician; I just enjoy math. I think it is interesting and, over the course of the months, we [the students and I] work together.”

*Ongoing Support for Innovation*

The summer projects were inspirational for Rose. They provided her the professional community that research indicates is necessary in making deep and meaningful change in teaching practices (Ball & Cohen, 1999; Cwikla, 2004; Fullan, 2001; Johnson & Brown, 1998; Lieberman & Grolnick, 1999; Nelson & Hammerman, 1996; Tinto & Masingila, 1998). She went back to her school and used the experiences and the learning from the workshops to develop her own mathematics curriculum. She taught mathematics conceptually with many problem-solving activities and projects. She did that by herself. She did not have any group
support in her district. The teachers at her school were still teaching mathematics very traditionally. She recalls being asked, “What are you doing? Why aren’t you using the same textbook that we are using?” At that time, her only local encouragement came from her principal. He was familiar with the NCTM Standards and recognized what Rose was doing. He supported her methods of teaching. He was interested in having these innovations put into the curriculum. In fact, he asked Rose to do some after school inservice sessions for the other teachers. She smiles as she tells what happened:

The teachers would come to my classroom after school. I would show them the manipulatives that we were using and tell how I was using some of those things. They would sit patiently and play around with a few things, then they would get up and say, “This might be fine for you, but it’s not for me.” And so then they would go away.

Rose continued teaching with her replacement units and projects, and she continued teaching her mathematics to a heterogeneous group. She says, “Everybody did the same math [in her class]. We would make allowances for what kids could do, but [they all worked on] the same content area.” The rest of the teachers still grouped by ability. She remembers, “Sometimes that was brought up to me.” But she did not waver, “It has always been one of my favorite pet peeves,” she says, “Just the smart kids do the problem solving…. The rest of them haven’t learned their math facts yet so they aren’t going to be able to do these other things.” Then, as now, all her students did problem solving; all her students worked on all the mathematics content areas. Based on Rogers and Shoemaker’s (1971) categories of change agents, Rose would be classified as an Early Adopter. She led her fellow teachers in the
changes away from those traditional teaching practices. But in the beginning, only her principal supported her innovations.

Change in the school has taken time, but it is happening. Rose’s school has discontinued ability grouping their classes. Now all classes are heterogeneous. In mathematics, they have adopted curriculum that supports the now-in-place State Mathematics Standards. Their curriculum, like the Standards, incorporates problem solving and process skills into all areas of mathematics: number sense, computation and estimation, statistics and probability, algebraic relationships, geometry and measurement. The school has purchased reform-based series for mathematics: *Bridges* for primary grades and *Investigations* for intermediate grades. When she taught fourth and fifth grades Rose found the adopted *Investigations* program very compatible with her instructional practices. Only because *Investigations* does not go beyond fifth grade, has Rose again had to develop her own curriculum for sixth grade. But the difference this time is the support from the state’s Mathematics Standards. They give her a framework on which to build her lessons. That framework is recognizable to the other teachers. The changes in the class organization and in the mathematics curriculum have made Rose’s teaching seem less unusual. She has a close, professional relationship with the other teachers in her school. However, it is still outside of her district where she finds the richest discussions about mathematics instruction and problem solving with students.
Continued Development and Issues in Mathematics Reform

By returning to her home state for the summer workshops and projects, Rose developed her support community away from where she was living and teaching. After she won the Presidential Award, she was invited to the State Council for Teachers of Mathematics in her new state. She has become involved with that organization; she goes to and presents at workshops and conference sessions, and she connects with others interested in mathematics instruction. She attends the Summer Leadership Institute where she can be part of the rich professional discussions with people who share her philosophy about the teaching and learning of mathematics. This group has become an important network, not replacing, but supplementing those close associations developed during the summer workshops with Dr. Ross.

In her own school district, Rose still finds uneven support of mathematics reform. Her school principals have always supported the reform. They have understood that taking on new curriculum means supplying the time, the materials, and the training for teachers to be able to implement the changes. They have provided those supportive materials. Inservice training has focused on problem solving and teaching for conceptual understanding. The teachers are using projects and problems from their adopted series. In addition, the State Standards have been helpful in educating teachers about important learning goals for all the strands in mathematics. Computation is no longer the only concept taught in the mathematics curriculum. States’ assessment focus on problem solving, had made problem solving an important element in mathematics teaching. All these things have brought about
many changes. But there are still some teachers, Rose mentions, “who are a little bit reluctant.” Perhaps some are even more than a little reluctant.

When asked why it is difficult for elementary teachers to make the changes in their mathematics instruction, Rose considers a few different issues. First, as is frequently mentioned by others (Ball, 2003; Hiebert, Gallimore, & Stigler, 2002; Ma, 1999; National Research Council, 2001), she believes, “Elementary teachers do not have enough mathematics background.” Because they are not sure of what to do, elementary teachers are dependent on the textbook. Rose says, “They don’t have enough algebra background or enough geometry background to know what they are doing in fourth, fifth, and sixth grade.” Teachers need to be able to ask good questions and listen to the responses. They need to have enough understanding of the concepts themselves so that they don’t just assume that one single right answer is understanding on the part of the student. Rose reiterates, “You really have to stop and let kids explain things and then let them explain it again before you get deep enough to realize that they don’t yet have the concept that you want them to have.”

Rose sees the State Standards and State Assessment as a mixed blessing. She is very much in favor of both. The State Standards reflect the same values as the NCTM Standards. They have been responsible for much change in the teaching of mathematics over the past several years. As Rose points out, “They have forced teachers to get away from just teaching arithmetic…. You have got to get into those process skills. You can’t just teach procedure. Kids have to learn to think and reason and communicate.” That state problem solving assessment has made problem solving
an important part of the mathematics curriculum. The scoring rubrics and guides have helped teachers understand the important elements in solving mathematical problems. On the other hand, some teachers are stressed by the state assessments. Worry causes them to push their students through the curriculum. Rose says, “If teachers haven’t embraced that step where kids construct their own understanding and maybe create their own way of doing things...they will do anything to make it happen.” As a result, Rose explains, they “let kids investigate; let them explore; let them figure it out” with open-ended lessons. But then, she says, they pull out the worksheets and “tell [the students] HOW to do it.” As a result, Rose sees students in sixth grade that will not try things in mathematics, because, she says, “They are afraid. They have no confidence that they will remember the procedure.” By rushing to the algorithms, and not allowing students to build their conceptual understanding, teachers are missing the crux of the instructional innovations in mathematics. Rose comments on the result of this:

I am convinced that the long division algorithm, the way it is traditionally taught, UNteaches [the students] everything they know about place value. Once you have taught them that and you ask them to estimate a quotient they have no idea what to do. (First Interview)

Therefore, insisting on timed facts tests and rote memorization of algorithms, not honoring the students capacity to build conceptual understanding, is counter productive in the long run. But the amount of content in the standards and the stress of state report cards tied to the assessment examinations, have perhaps overloaded teachers and made them miss the longer-term goals.
Another difficulty for elementary teachers when trying to change their instructional practices and curriculum is the lack of opportunity and time to develop their new knowledge and skills. Rose reflects, "Math is a social event." Just as we encourage students to, "Talk out your thinking," we need to give teachers a chance to talk out their thinking. Teachers in Rose’s district have not had the time to discuss the articulation of the mathematics curriculum since it was changed. They do not have time to meet and discuss what is working for the students and what needs more development. Rose says, "I think [teachers] are desperate for opportunities to grow professionally." And now, sadly, budget problems have made the ability to attend workshops and classes even less available. Other professions, Rose points out, do not expect their members to give up weekends or vacations to work on their job. And when teachers are given school time to attend a workshop or conference, they have to leave lesson plans for a substitute while they are away. Meanwhile, principals meet regularly to talk about the district. Can there be a change in these dynamics?

Rose shares a plan that has been developed by a group of educators:

They have developed a beautiful, marvelous model. They come into the school and work with the school program. They work on best practices and on articulating things through grade levels. They have a perfect, perfect professional development model.

(Second Interview)

The question is how schools can afford this kind of professional development. It is a similar problem Hargreaves and Fullan (1992) addressed in their study of teacher development more than a decade ago. They found:

Teachers’ stories show the powerful effects that collegiality and teachers working on common projects have on teacher
development. Even though these effects are corroborated to some extent by the literature, teachers’ experiences of norms of experimentations and collegiality remain rare, given the cultural conditions and constraints under which teachers work. (p. 159)

Rose believes that change in mathematics instruction is happening. She admits, “I am disappointed that it is taking so long. I feel like sometimes we are moving ahead and then something will come out…and it feels like a step back.” But her philosophy that learning is on a continuum allows her to consider that, like the kids in class, teachers are on different places. She says, “I know that over the course of the years – I hope this happens for all teachers – you are always changing.” For Rose, the changes are a matter of improving her craft. She continues to develop her skills at connecting students to mathematics understanding. She says, enthusiastically, “I like it. It is fun. It is my hobby.” It is very meaningful to her when a student says, “You know, math has never been easy for me, but I know that if anyone can help me, it’ll be you.” She feels complemented, but also, says, “Oh no. What a challenge!” Yet that is her personal challenge always: “To change kids ideas about math.”
Figure 4: Rose Sharon’s Journey

LIFE LONG LEARNING
Never Being Satisfied the Lesson is the Best it Can Be
Questioning Skills Getting All Engaged
Projects that Meaningful and Interesting to Kids
Always Learning More Mathematics
Ways to Connect Conceptual Learning to Symbolic Work

ONGOING SUPPORT
Professional Discussions
School Principal
Workshops and Conferences
State Teachers of Mathematics
State & National Standards
Professional Resources
Replacement Units
Investigations Series
Student Feedback

ONGOING ISSUES
What keeps teachers from changing?
Time
Time to find & develop and materials appropriate for students
Time for professional development
Beliefs
All can do mathematical problem solving
Quickness to teach the algorithm / not building understanding
Knowledge
Mathematical knowledge
Strategies for mixed ability

KEYS TO CHANGE
What is needed?
1) Administrative support for the changes
2) Teachers need many opportunities for good staff development: as many activities and organizations as possible
3) Time to talk about the program and align materials and expectations
4) Skill at teaching with constructivist teaching methods

INITIAL SUPPORT
Replacement Units
MATERIALS from Workshops & Classes
Principal
Student Feedback
Beyond Activities Project
Talking with Experts
Dr. Sharon Ross

GATEWAY
Summer Workshop
Dr. Sharon Ross

MOTIVATION
Dissatisfaction with Textbook
Believing Mathematics Should be Understandable
The Case of Jane White

A Glimpse at Jane’s Teaching: How Many Bugs?

Why did Jane White start her second graders’ math time by reading a story about bats? Jane gathered her class together and asked them to review what they had been learning in their bat study. She then reread a part of a story the class had heard the day before. That passage was about what bats eat and how they catch their prey. It explained that a brown bat could catch and eat six hundred bugs in one hour. Now Jane looked at her students and said, “I have been thinking. How many hours are there in a day?” The students knew the answer: “Twenty-four.” She then wondered, “How many bugs could a brown bat eat in one day?” She was challenging the class with a problem that would take some thinking on their parts. One student knew that the equation would be 600 X 24, but two-digit multiplication is beyond most second graders’ mathematical ability. Furthermore, the total amount was going to be a very large number for young children to comprehend. At this point Jane brought out a stack of three-by-five index cards and a large bag of Rice Crispies. “How many kids are there in our class?” she queried. “Twenty-four,” they all answered. This was an intentionally convenient coincidence. Jane now shared her plan with the class. She started by asking, “If I give each of you six cards and then you glue 100 Rice Crispies on each card, how many Rice Crispies will each of you have?” They figured out that would be 600. The following is a summary of what she then explained to the students:
Since there are twenty-four of you, we can then count up all the Rice Crispies on the cards. The Rice Crispies are like the bugs. And there will be 600 for each of you – or 600 X 24. We can count all the Rice Crispies and figure out how many bugs a brown bat can eat in twenty-four hours. (First Observation)

Jane shared a sample card. She showed how she made ten lines of glue across the card and put ten pieces of cereal on each line. Then she sent the students off to make their six cards of “bugs.”

For seven year olds, gluing one hundred small pieces of cereal onto a card takes time. The students worked on this project off and on over the next couple of days. It did not take long, however, before some problem solving took place. Was it faster to put a row of cereal on one line of glue at a time? Or, as some students started doing, was it faster to count out 100 pieces of cereal and then glue them on all at once? Jane left it up to the students to decide. They could choose to do the work whichever way they preferred as long as there were 100 Rice Crispies per card and each student ended up with six cards. Either way they chose, the students were beginning to get a feel for the size of the numbers. One student, getting a little tired of gluing perhaps, asked a question Jane had anticipated, “Why don’t we just figure this out with a calculator?” She honored his idea, but let him know why she preferred the counting, “You are right, we could get an answer with a calculator, but I think this way we will have better sense of how many bugs it is.”

As the students finished their gluing, Jane put up a large piece of butcher paper on which she had drawn a chart. Her chart was divided into twenty-four
sections, in four columns and six rows, with each space large enough for a student to put his or her six cards of “bugs.”

When all the cards were completed and all twenty-four sections were full, the students looked at the chart together so they could count the total number of “bugs.” They discussed how they could keep track of their counting without getting lost. They thought of some quicker, more efficient, ways they could count. When they got the final total they were able to see what 14,400 bugs looked like and have a better sense of what that amount means. In this integrated project, besides learning more about bats and their eating habits, the students gained conceptual understanding of place value by building and counting the numbers 10, 100, and 600, all the way up to 14,400. They developed and practiced counting strategies. They made a visual representation that helped them grasp the quantity of a very large amount. They worked with the concept of multiplication by solving a problem with repeated addition. Together they built a chart, which introduced them to a model for displaying data. They then used that display of data in getting the answer to their problem. Their chart was put up in the hallway and surrounded by students’ stories about bats. Much learning came from answering the question, “How many bugs can a brown bat eat in one day?” That is why Jane began a mathematics lesson with a story about bats.
Early Teaching

Jane’s teaching career started in a third grade classroom at a private school in a large metropolitan area. For her mathematics instruction, she was given a set of textbooks and teacher’s guide. Although the text was several years old, it was representative of the standard curriculum for mathematics instruction in the early 1980’s. So, following the same manner by which she had been taught mathematics, Jane plodded through the textbook page-by-page and chapter-by-chapter. “The kids would read [the instructions] and then do the practice problems that would follow the two-page lesson. Then I would assign homework.” Jane followed the teacher’s guide and tried to learn what was expected for her grade level. But the mathematics she was doing did not feel good to her.

There were so many things that [the students] weren’t getting and it bothered me. I had had a couple of math courses when I was in college...Although that is a limited amount of math training that I got, I did know that there were some opportunities for doing math games and that kind of thing. So I just wasn’t satisfied with the way that year was going. (First Interview)

It distressed Jane that her students didn’t like math. She knew they were frustrated and not “getting” it. The looks on their faces reminded her of that each day. She recalls, “Every time they opened the book there were moans and groans.” “The kids were dying,” she says, “I knew that there had to be a better way.”

Jane read some information about the annual National Council of Teachers of Mathematics (NCTM) Annual Conference being held in a nearby city that spring. She thought that might be the place to find some answers to her mathematics concerns, so she approached her principal about attending. Jane says, “She could see
that I was sincere in wanting to get better and also because I was a new teacher. She was very supportive.” The principal sponsored Jane to go to the conference (See Figure 4: Jane White’s Journey, p. 152).

Introduction and Early Support

At the conference, Jane found what she was looking for. She went to some wonderful sessions where she received ideas for many different activities she could use in her mathematics teaching. However, it was the keynote speaker that “really impressed” her. In fact, Jane declares, “That changed my life.” The speaker was Marilyn Burns, and she talked about inserting problem solving into math. Jane uses the word epiphany in describing the way some people suddenly see mathematics in a new light. The vision of teaching mathematics to children with the use of problem solving was like an epiphany for Jane. She was hooked. She came home from the conference with lots of enthusiasm and a whole new plan for her mathematics instruction. Today she laughs, “Oh my gosh, I didn’t know what I didn’t know!”

The first thing Jane did was to start using some of the activities from the conference in her classroom. They were successful. She recalls, “I could tell the kids were enjoying them a whole lot more than what we had been doing.” This was an improvement, but she realized, “there was still so much to learn.” She started looking for classes she could take. Because she lived in an urban area, she found many opportunities. She took Math Their Way classes and Box It and Bag It classes. She enjoyed these because they gave her so many good ideas and because, “there
were so many wonderful teachers around.” She was not just learning new teaching practices; she was continuously being inspired about mathematics. She took these classes and workshops for the next couple of years, incorporating what she learned into her teaching. As her teaching changed, her students changed. Now when she read their faces, she says, “I saw a real change of attitude.” She actually did some surveys, assessing their feelings about mathematics at the beginning of the year and then, again, at the end of the year. These surveys reinforced her enthusiasm for the new practices. She saw “huge differences.” The data confirmed what she was seeing in her students’ daily engagement with mathematics: “Kids developed really strong, positive attitudes toward their ability to do math.” She liked the new responses she heard when it was math time and the students said, “Yeaaaah!”

Ongoing Support for the Innovations

Jane taught a few years at the private school, and then she was hired at a public elementary school in a suburban area just outside the city. At her new school Jane had the good fortune to work with a principal who was a leader in mathematics education in the state. He had previously been with the state’s Department of Education, he had written a couple of books, and he presented workshops on teaching mathematics. He became a mentor to Jane. She says, “He encouraged me to start sharing what I had learned. I started assisting him with some workshops.” Under his leadership, Jane also served on a school district committee looking at a new mathematics adoption for the elementary school program. That was about 1987.
She recalls, “The first document he handed us to study as we went through [the adoption cycle] was a draft of the [National Council of Teachers of Mathematics] math standards.” Those standards were what NCTM published two years later in their call for reformation of the teaching of mathematics.

These NCTM Standards became the core of their new mathematics adoption. The committee decided not to adopt a textbook. Instead, Jane shares, “We wrote up our own set of expectations for our district. We compiled libraries and manipulative kits and looked for experts to come in and do some training for our staffs.” The committee developed a multi-year plan to support teachers in learning new practices and to train any new teachers who joined the district after implementation started.

Therefore, while serving on this committee, Jane learned a tremendous amount about the standards and about a problem-solving mathematics curriculum. She became a leader in her district. And then, when other districts learned of their innovative program, she had a chance to do workshops and presentations all around the state.

Jane was developing skills and knowledge that she continued to apply to the mathematics instruction in her classroom. She used more and more Math Their Way stations, small group activities, and replacement units. She describes the progression as follows:

I was still pretty traditional in terms of me at the front of the class teaching a bunch, but then I would reinforce it with giving the kids some power in choice in their selection of the reinforcement at the stations. That went really well. I think probably I got gutsier because I liked the way the kids were interacting at the stations. Then I tried some more of the problem solving in cooperative groups.

(Second Interview)
Jane credits her principal’s influence in giving her “that ability to pull out of the kids the kind of discourse that you have to have when kids were sharing on an overhead.” In order to continue transforming her instruction, Jane needed to learn more mathematics content and more teaching strategies for problem solving.

Jane’s mathematical background was similar to that of many women. She actually took a lot of mathematics in high school, but was told by a counselor, “You shouldn’t be taking these higher level math courses, you should be taking classes like personal finance, shorthand, book keeping…because who knows what your real job will be.” When she thought back on the mathematics she did have, she recalls getting good grades. However, she reflects, that does not mean she understood the mathematics: “I was taught math in the way most others were taught math. I could do pretty well because I am above average intelligence and I have a good memory. But it didn’t make sense to me.” She started relearning mathematics from the workshops she took. The instructors, while demonstrating how to present mathematics to children, were also teaching mathematics content to the participants. She recalls, “We had to know the mathematics before we could be able to teach it…. I have come to learn so much through the experience of teaching and explaining the thinking as I go along.” Now that she has taken the time to make sense of mathematics and has made discoveries on her own, Jane feels a sense of “mathematical power.” She still runs across mathematics ideas that are difficult to understand, but she is confident in her ability to figure them out. She refers to the all-too-common statements people make about mathematics: “I can’t do math” or “I was
never any good at math.” What she believes now is, “That is not true at all. We are, for the most part, just products of a mediocre to inferior math education.” She says, “It doesn’t scare me anymore the way that, you know, math used to be frightening at the very beginning. Especially that first year.”

What Jane was also learning were strategies that supported the problem solving of her students. In the workshops her principal presented, she learned the skills for coaching and for managing the classroom discussions of problems. She learned these skills for herself and she learned how to teach them to her students. She lists the skills she found most useful:

- Wait time. Learning how to be patient and not to jump in too soon. And also teaching kids how to be able to hang with it. Because disequilibrium is kind of an uncomfortable stage. So it was learning to be comfortable even in discomfort (Second Interview).

As she incorporated more and more problem solving and discussion into her teaching, Jane learned more and more about how the students dealt with the concepts at their developmental level. Over time she has learned to think through the problems ahead of time and predict the errors kids might make. That skill allows her to have some idea of what might happen and what she will do when it happens. In this way she had predicted that a student would ask about using a calculator to solve the bug problem and she had her answer ready. This is the skill Heibert (2002) calls craft knowledge. And it is the skill Ball and Bass (2000) call pedagogical content knowledge. It comes from observing and listening to children and from analyzing the children’s understanding of mathematical content at their developmental level. It is the pedagogical knowledge she uses when responding to the student who is
“looking at you with that blank stare.” She is ready to rephrase the question, semi-model the concept, or give another example that will help him or her understand.

**Continued Development and Issues in Mathematical Reform**

As Jane gained confidence in problem-centered teaching of mathematics and as she continued to learn, her support base grew. She had the support of building and district administration. She had fellow teachers she worked with on curriculum. Over the past ten years she has become more involved with mathematics teachers outside of her district. She has presented at workshops and conferences around the state. She has become active in the state’s organization of teachers of mathematics. She has served on various state committees and panels that have worked to improve mathematics instruction at all levels. She has taught elementary mathematics methods classes for local universities. She started attending the Summer Leadership Institute where she has rich professional discussions and exchanges of ideas about issues in mathematics education. She has led Internet discussions and worked on professional development websites. Jane has also become connected to teachers on a national level. Working on a state project to look for ways to integrate performance assessment into the classroom so that it is aligned to the NCTM Standards, she had what she calls, “an incredible opportunity to fly around the country and meet with teams of teachers from other states.” What made this incredible, she says, were the discussions with other teachers who were “like-minded...really supportive.” She explains, “Getting in groups of five hundred people who were all thinking of how we
could improve math was really different from what my experience was up to that point.” It was inspirational. She now had professional colleagues that “you could call on the phone, e-mail, or see a couple of times a year and know that they were like-minded.” This makes for a strong sense of community and a positive support system for changing mathematics instruction.

Jane’s enthusiasm for teaching mathematics with problem solving has made her a leader in mathematics reform. She earned a Masters Degree in Administration with the goal of becoming an instructional leader for her school district. She served a year as a Teacher on Special Assignment (TOSA). She has been instrumental in keeping her district’s elementary mathematics curriculum up to date with incorporation of the state and the national standards, first with the “radical adoption” of “no textbook,” and then with adoption of a published program that aligned with the standards. In the latter adoption she worked on a committee that “developed a resource curriculum alignment...spelling out the [state] standards.” She explains, “We said here is how it lines up and here are some holes and here are some resources you can use to supplement and here is a literature book you can use to supplement.” She posted this alignment on her website, accessible to anyone who wanted to use it. She continues presenting at workshops and conferences and she still teaches the elementary mathematics methods course at a local university. In all her experiences, Jane has come to appreciate the importance of instructional leaders continuing to teach in their own classrooms. She says, “I have intentionally stayed in the classroom because I think that I am more effective at trying to change and influence from the
inside than from being an instructional specialist or building administrator.”

Principal support is important, but to help teachers with curriculum and instructional practices, she states, “Truthfully I don’t think the principal has the best opportunities because...I think it has to be someone in the classroom.” She feels that she can be more effective working with teachers, because, as she says, “I walk the walk and I talk the talk and I am with you, brothers and sisters.” When it comes to accepting new practices teachers need to see them in action and be convinced they can work in a real live classroom. Jane leads more by modeling methods then she ever could by just talking about them.

When asked why adopting reform-based mathematics instruction is difficult for so many elementary teachers, Jane pauses to consider. She has found that when teachers have the chance to make decisions for themselves and have reasons for those decisions, change is not difficult. She shares, “When a district hires me to come in, there is always a certain percentage of teachers who are sitting there with arms folded and unhappy. It is pretty hard to get to them.” On the other hand, she finds, “When people take [workshops] on their own out of some interest or a difficult situation in their classroom, or they have some reason...it is a totally different experience.” In her own district, with the administrative support, with the choice of curricular materials, and with many opportunities for professional development, Jane has found general acceptance of new teaching practices for mathematics. And for teachers that are reluctant, Jane suggests, we need to continue to invite and
encourage them to participate. We need to wait, and then be ready to share ideas when they are ready to hear them.

She does realize, that even when someone is ready and change is voluntary, it takes time and support. There is still need for teachers to be more familiar with the standards. There are many teachers who need more confidence in their mathematical content knowledge. And, along with that content knowledge, goes the need to recognize how their students understand and apply the concepts. She says, “I think people’s expectations of what kids are capable of doing if they have the confidence and tools to use are like a glass ceiling sometimes.” Jane believes, if given time and the right tools, students are capable of working at a much higher level than what is now a typical grade level expectation. Teachers have to be willing to give students challenges and then listen to their efforts and help them build understanding. Jane accepts, “It takes time to get it all together. Or at least to start being confident or to start to feel you have the wearwithall that you can find out what you need to know.” However she is optimistic and believes the changes are happening. She, of course, is still active in promoting professional development at school, district, and the state levels. She is now part of the support system for other teachers.

An interesting note to Jane’s optimism about the transformation of mathematics teaching comes in a development she shared during our second interview. After more than fifteen years of having a standards-based mathematics curriculum, with much administrative support in the way of materials, staff development, and adoption of reform-based programs, there is a “rumor” that
problem-centered and conceptually oriented program the elementary schools have been piloting for the past two years “will not be adopted.” In this past year the high school has revised “backward” their progressive, integrated mathematics program to return to a more traditional approach that again divides courses back into algebra, geometry, and calculus. Jane speculates that pressure from that change may be pushing down through middle school to elementary grades. Only momentarily concerned, Jane reflects positively, “I think it will be fine no matter what the district ends up doing.... I think there are always ways to take whatever text you are handed and look at how you can adapt it to a constructivist kind of approach.” And, additionally, she says, “It may just be that the time line for change is a lot longer than we would want it to be.”
Figure 4: Jane White’s Journey

LIFE LONG LEARNING
How Mathematical Knowledge Develops in Children
Assessing Students’ Learning  Professional Development
Continuous Improvement – Always Looking for Better Ideas
Integration of Subjects  Internet Resources

ONGOING SUPPORT
Professional Discussions  Principal/Mentor
Workshops and Conferences  Building & District Administrators
State Teachers of Mathematics  State & National Standards
Journals and Professional Resources

ONGOING ISSUES
What keeps teachers from changing?
Time
Time to learn & make changes
Developing and collecting materials
Beliefs
What students are capable of doing
All can have mathematical power
Knowledge
Mathematical knowledge
Grade level expectations
Standards

INITIAL SUPPORT
Math Their Way
Materials from Workshops & Classes
Principal
Student Feedback

GATEWAY
NCTM Annual Meeting
Marilyn Burns Keynote
Principal Support

MOTIVATION
Dissatisfaction with Program
Student Disengagement

KEYS TO CHANGE
What is needed?
1) Instructional leadership from classroom teachers
2) Teachers need to have many opportunities, but participation must be voluntary
3) Mentorship: listen to teachers and support them when they are ready for help
4) Constructivist teaching methods
Summary

The five teachers in this case study have all made a transition from traditional to reform-based mathematics instruction. Their stories reflect journeys that start out at the beginning of their teaching careers in elementary classrooms with traditional, text-driven mathematics programs. Dissatisfaction with his or her program motivated each teacher to look for something different. What all five found, in a variety of ways, was the reform-based, problem-centered mathematics that is described in the NCTM Standards. The teachers' journeys continued with experimenting and learning, as each teacher became more and more confident with his or her newly adopted practices. The teachers found materials and resources that supported these changes. Importantly, they found or formed communities of other teachers, in which they explored, questioned, and discussed the innovations. It is in these groups they developed new pedagogical skills and gained new techniques for teaching mathematics with conceptual understanding. Over the course of several years, the teachers have developed confidence in their mathematics capabilities and in their teaching of all mathematics concepts. Yet every one of the participants talks of what he or she still have to learn.

The five participants are active leaders in the mathematics education community. All believe in and work for the wider adoption of innovations in mathematics instruction. They all believe that elementary teachers need more time and support to understand the proposed changes, learn new ways to teach mathematics, and develop their own mathematical knowledge. The participants also
recognize the need for extra support for preservice and beginning teachers. It is too easy, they say, to become textbook dependent in one’s early teaching. The five teachers all believe that changing mathematics instruction is possible, but they also understand it is difficult and complex.
Chapter 5: Cross Case Analysis

Introduction

Change seems the obvious goal of reform. The proponents for reforming mathematics instruction want teachers to change— to change their beliefs, their practices, and their instructional materials. This is not just tinkering with a different textbook or a new teaching strategy. This means a fundamental change in the way teachers perform their job. The prescribed changes require a new relationship between students and teachers, as well as a new view of learning and knowing. This is not a simple goal, nor will it be easily accomplished. It is important, I believe, to briefly address why the goal of mathematics reform came about.

Changes in the beliefs about how mathematics should be taught come from changes in our understanding of how students learn. In the past fifty years an incredible amount of research on teaching and learning has been accomplished. In the first half of the twentieth century, the theory of behaviorism held sway over educational thinking (Bransford, Brown & Cocking, 2000). Writers such as Watson, Thorndike, and Skinner influenced the development of teaching as a system of stimuli and responses, punishments and rewards, and observable behaviors (Bransford, et al., 2000). In the 1950s the new field of cognitive science began examining thinking and learning with a less simplistic view. It combined the disciplines of anthropology, linguistics, philosophy, developmental psychology, computer science, neuroscience, and other branches of psychology (Bransford et al.,
Over the past half century the number of researchers that have contributed to the study of teaching and learning has been exponential. Their theories have influenced many aspects of teaching. Reading, writing, science, and social studies have incorporated many new methods and approaches into the teaching of their disciplines. Interestingly, however, the changes have been piecemeal. The overall delivery of education -- the look of schools and the organization of classrooms -- has changed little. And the instruction of mathematics has changed least of all. The traditions of teaching as telling, learning as memorizing, along with the belief that knowledge is attained by practicing what one was told, has remained the foundational thinking in mathematics instruction (Davis, 1990).

Proponents of the reform of mathematics instruction embrace a constructivist learning theory. I, like those proponents, also embrace a constructivist learning theory. And I embrace that theory not just for the learning of mathematics, but also for all learning. I believe that the strongest, most meaningful learning is the learning that one constructs. If mathematics is “learned” by just memorizing rules, it is not made a part of the students thinking and acting repertoire. It is just performed on request. I believe that if students “learn” anything by just rote memorization or repeating scripted steps to demonstrate learning, it is not meaningful learning. Thinking constructively, I believe learning and change are parallel phenomena. We have learned from the research that for a change to occur the person changing requires a belief that the change is useful or important, information about why and what to change, new skills to make the changes, and time to practice and apply the
Changes. Meaningful change is built on learning new behaviors and gaining new understandings; constructivist learning is built on changing understandings based on gaining new meaning and new perceptions. With that thinking, I examined the change journeys of the five cases through a constructivist lens.

**Theoretical Perspective**

The theory of constructivism, although developed over hundreds of years of philosophical thought, has relatively recently come into the spotlight as an educational theory of learning and knowing. It has only been in the forefront of mathematics education for the last twenty years. Constructivism's tenets are also the tenets of the reform recommendations of mathematics instruction. Confrey (1990), a mathematics educator and proponent of reform, explains key ideas in constructivist educational theory as they relate to mathematics instruction:

- A person (or learner) constructs his or her understanding through experiences. The character of those experiences is filtered through his or her cognitive lenses (or schemas).

- A constructivist teacher must develop the skill of seeing the learners' perspective and approach all responses of the learner with the intention of fully understanding those responses' character, their origin, and their implications for making meaning for the learner.

- The differences in teachers' and learners' perceptions can be qualitatively different. They cannot be reduced to missing pieces or unfamiliar techniques. They cannot be displaced by showing the correct method.

- A constructivist teacher must reject the idea that understanding can result from knowledge that is simply passed on to the learner. The teacher must assist the learner in restructuring his or her views by providing models that connect the learner's pre-held views to the new views to be constructed.
• Learning constructively requires reflection. The learner must reflect on an activity, thinking it through and naming, in his or her mind, the symbols and images.

• The constructive process is social. Learners do not think in isolation. Communication allows a learner to consider his or her constructs alongside another’s in order to assess their comparable strengths.

Finally, and importantly, Confrey (1990) says:

The most fundamental quality of a construction is that students must believe it. Ironically, in most formal knowledge, students distinguish between believing and knowing. To them there is not contradiction in saying, “I know that such and such is considered to be true, but I do not believe it.” To a constructivist, knowledge without belief is contradictory.

My theoretical perspective in cross-analyzing the five cases is constructivism. My contention is that in order to transform their mathematics instruction, teachers must construct new concepts of teaching. In the instance of making these deep and lasting instructional changes, teachers are learners. Just as the young learners bring their schema into our classrooms, teachers bring to reform their long and deeply held schema about teaching. These must be recognized and addressed, and, for change to even begin, made personally problematic. This can be related to the important condition of disequalibrium in constructivist learning. Without that condition, knowledge will not be internalized, but will remain on the surface only. Again consider the statement, “I know that such and such is said to be true, but I don’t believe it” (Confrey, 1990). Without the condition of disequalibrium, change will not occur. The called-for reform of mathematics instruction requires that a new structure for teaching be envisioned, a new understanding of mathematics be built, and the old
methods be realized as no longer satisfactory. That is all necessary for the
construction of a new understanding, a new concept, of teaching.

However, a constructivist approach to learning implies more than just
exchanging an old schema for just any new schema. Learning constructively is to go
beyond disequalibrium and into the active building of a new construction. Noddings
(1990) explains, “This active construction implies both a base structure from which
to begin...and a process of transformation or creation which is the construction”
(p.9). As this is applied to the construction of reformed instructional practices, the
introduction of new methods – a demonstration or presentation that allows the
learner to have a mental image of these methods – can be related to assimilation. The
process of coming to an understanding of the new ideas, making them meaningful
and useful, is the process of accommodation. Equilibration is the resulting new
construction or new mental structure. However, the new state of equilibrium is not
static, but dynamic. Piaget (1980/1974) describes the process of cognitive
development as a continually broadening upward spiral in which structures become
more complex and knowledge more elaborate. In a like process, the incorporation of
reform-based methods and practices continues to be developed and refined as more
understanding and more skills are incorporated into the teacher’s repertoire of
practices. The reform-based teaching becomes, eventually, the way in which the
teachers understand themselves personally and professionally and the way in which
they view the content and context of their teaching. (Drake, Spillane, & Huffard-Ackles, 2001). Therefore, constructivist learning theory operates on the belief that
knowledge construction is a continuous process. Noddings (1990) reiterates, “It implies a process of continual revision (p. 9).” Learners building new constructions need to address their old frameworks, need to learn the new skills, and need the support of materials, tools and strategies to become successful (Noddings, 1990). They need an environment that allows them to interact with others in the social negotiation of meaning, which allows for the development of more powerful constructions (Maher & Alston, 1990). Once learning is envisioned in this way, it becomes a life-long process of always addressing one’s frameworks of knowing with continuous assimilation, accommodation, and equilibration.

Analysis

Introduction

After analyzing the five individual cases for the key elements in their journeys through change (Charts 2, 3, 4, 5, and 6, Appendix F), I have juxtaposed those findings (See Chart 7, page 161) for the cross-case analysis. For each segment of their change journeys: motivation, introduction (gateway), initial support, and ongoing support and continued learning I have compared their experiences and the interactions they considered most influential. In some of the segments there are similarities across all five cases. In some segments there are differences. Overall, constructivist elements supported the changes and aided the building of new understanding about the teaching and learning of mathematics.
## Chart 7: Meta-Matrix Comparing the Key Elements of All Five Cases

<table>
<thead>
<tr>
<th>MOTIVATION</th>
<th>Patsy Miller</th>
<th>Skip Munson</th>
<th>Michael Patrick</th>
<th>Rose Sharon</th>
<th>Jane White</th>
</tr>
</thead>
<tbody>
<tr>
<td>+Project work</td>
<td>+Ch Dev Bk/Gr</td>
<td>-Dissatisfaction</td>
<td>+Methods Class</td>
<td>-Textbooks</td>
<td>-Textbooks</td>
</tr>
<tr>
<td>+Connections</td>
<td>+Reading</td>
<td>-Kids in class</td>
<td>-Kids' attitudes</td>
<td>-Kids' attitudes</td>
<td>-Kids' attitudes</td>
</tr>
<tr>
<td>-Text program</td>
<td>-Workbooks</td>
<td>-LA Bck/Gr</td>
<td>+Dr. Ross</td>
<td>+College Class</td>
<td>+College Class</td>
</tr>
<tr>
<td>-Lack of depth</td>
<td>-Dislike of math</td>
<td>!Belief Change</td>
<td>!Belief Change</td>
<td>!Belief Change</td>
<td>!Belief Change</td>
</tr>
<tr>
<td>*Other teacher</td>
<td>*No math mths</td>
<td></td>
<td></td>
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</tbody>
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<table>
<thead>
<tr>
<th>GATEWAY OR INTRODUCTION</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>+Workshop</td>
<td>+Writing</td>
<td>+SMC</td>
<td>+Teacher Units</td>
<td></td>
</tr>
<tr>
<td>+Projects</td>
<td>+Wk/shp</td>
<td>+Workshop</td>
<td>+Math Framework</td>
<td></td>
</tr>
<tr>
<td>+Connections</td>
<td>+MTW</td>
<td>*Consortium</td>
<td>+Summer</td>
<td></td>
</tr>
<tr>
<td>+Teacher units</td>
<td>*Marilyn Burns</td>
<td>*Self</td>
<td>Wkshp</td>
<td></td>
</tr>
<tr>
<td>*Students</td>
<td></td>
<td></td>
<td>*Dr. Ross</td>
<td></td>
</tr>
<tr>
<td>*Other teacher</td>
<td></td>
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<tr>
<th>INITIAL SUPPORT</th>
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</tr>
</thead>
<tbody>
<tr>
<td>+Workshops</td>
<td>+MTW</td>
<td>+SMC</td>
<td>+Summer</td>
<td></td>
</tr>
<tr>
<td>+Teacher Units</td>
<td>+Prof. Resources</td>
<td>+Workshops</td>
<td>Wkshp</td>
<td></td>
</tr>
<tr>
<td>*Other teachers</td>
<td>+Multi-Age Class</td>
<td>+Science Curr.</td>
<td>Dr. Ross</td>
<td></td>
</tr>
<tr>
<td>*Self</td>
<td>+Manipulatives</td>
<td>+Consortium</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+MTW Instructor</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>+Projects</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>*Students</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>*Self</td>
<td></td>
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<table>
<thead>
<tr>
<th>ONGOING SUPPORT</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>+Mathland et al</td>
<td>+MTW</td>
<td>+Prof. Resources</td>
<td>+Summer</td>
<td></td>
</tr>
<tr>
<td>+Workshops</td>
<td>+Workshops</td>
<td>+SLI</td>
<td>Wkshp</td>
<td></td>
</tr>
<tr>
<td>+Manipulatives</td>
<td>+Constr.</td>
<td>+State Assessmats</td>
<td>Dr. Ross</td>
<td></td>
</tr>
<tr>
<td>+Standards</td>
<td>+Activities</td>
<td>+Manipulatives</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+Friday S/Dev</td>
<td>+Prof Resources</td>
<td>+Internet</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+SLI</td>
<td>*Students</td>
<td>*S/CTM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>*Other teachers</td>
<td>*MTW lastr</td>
<td>*SMC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>*Administration</td>
<td>*Principals</td>
<td>*S/DOE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>*S/CTM</td>
<td>*Husband</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>*S/DOE</td>
<td>*Team Members</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>*S/CTM</td>
<td></td>
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<table>
<thead>
<tr>
<th>LIFE LONG LEARNING</th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>“My door is always open. Come and watch me teach. Then tell me what I could do differently.”</td>
<td>“If you are a good teacher you are always going to be frustrated. But it is a challenge.”</td>
<td>“You are not always sure where something is going to go. If something comes up that you are not expecting, like somebody has a way to do something and you don’t know where it came from, you need to figure out where to take it.”</td>
<td>“The longer I teach the less I know.”</td>
<td>“When you go on a search for other methods that are going to be more kid friendly, it leads to giving them more power, more control, more ability to design the work that is going to take place. That is constructivism.”</td>
</tr>
</tbody>
</table>

### Motivation and Introduction to Innovations (Disequalilibrium)

Self-motivation brought all five of the participants to the edge of change. In the cases of Rose and Jane they had an idea of more effective techniques even before
attending their first workshop. The others did not have a vision for what they were going toward, they just were motivated by dissatisfaction with what they had been doing. In no case were these motivations external. The changes were not mandated nor even recommended by other people. In fact, all of five were using the officially adopted mathematics program provided by their district. Constructively speaking, they were already at a point of disequalibrium. Their earlier schema for “good teaching” – following the textbook to teach the mathematics for their grade level – was not working. Jane said, “My kid’s were dying.” Michael’s students “couldn’t remember all the rules.” And Patsy’s students weren’t making connections to the real world or to other areas of mathematics. Skip, Jane, and Rose were all dissatisfied with the presentation of mathematics in the traditional textbook programs with which they were provided.

In constructivist theory, learning is personal. When people learn constructively they personally change. Unless there is some discomfort or disequalibrium with their current experiences or understanding, they will not change. Even if someone learns (memorizes) a new vocabulary or can demonstrate certain prescribed practices, it will not change their long-term behavior nor will they accept the new learning as their own. No deep or meaningful change will occur. Therefore the fact that these five teachers were seeking new constructs for their teaching was a positive beginning for change. But that motivation was to find a “better” mathematics program; it did not mean they would accept the problem-centered, conceptually oriented instruction being suggested by the NCTM Standards.
Connected to the motivation had to be a belief that what was introduced was better. They had to have a new image of mathematics instruction and it had to match their personal criteria for better mathematics instruction.

Skip’s background in child development had given her a constructivist orientation to teaching. She only needed to be exposed to practices for teaching mathematics that matched her beliefs about children’s learning. Rose had also gained an appreciation for constructivist teaching in her college training. She had been exposed to the “powerful” methods of teaching mathematics with problem solving and teaching for conceptual understanding in her methods classes. She needed more materials and more specific pedagogical strategies so she could carry those methods into her classroom. Patsy did not have a constructivist framework, but the techniques and methods of open-ended investigations and connections to the real world matched values that were already part of her teaching. The other two teachers, Michael and Jane, were not necessarily familiar with the constructivist philosophy in teaching, but were personally captivated in their first exposures to these methods. They found the innovations interesting and exciting and took them back to try in their own classes. When they introduced the new activities to their students, they observed engagement” and “interest” and “success.” That met their goals for better instruction. Their students’ responses convinced them to continue to learn more, so they would be able to incorporate more of those ideas into their teaching. At this point all five teachers were constructing a new understanding of good mathematics instruction. From a constructivist view all were at a state of disequilibrium – feeling that their
old methods of teaching were out of balance with their goals and believing there was something better to be doing.

**Initial Support (Assimilation and Accommodation)**

The teachers moved beyond just knowing *about* the innovations. They all believed the innovations would improve their mathematics instruction. But, also, all were at a point where they had the belief without enough knowledge to implement the changes in their classrooms. They needed to build a more complete understanding of those innovations. Confrey (1990) lists some of the qualities necessary for “a powerful construction”:

1. A structure with a measure of internal consistency;
2. An integration across a variety of concepts;
3. A convergence among multiple forms and contexts of representation;
4. An ability to be reflected on and described;
5. An historic continuity;
6. Ties into various symbol systems;
7. An agreement with experts;
8. A potential to act as a tool for further constructions;
9. A guide for future actions; and
10. An ability to be justified and defended. (pp. 111-112)

Piaget (1985/1975) proposed three levels of equilibration. At the first level a very modest adjustment is made to a past structure. There is essentially an awareness of a new idea with only partial modification. Disequilibrium is not allayed (Piaget, 1985/1975). In the second level, change occurs in the old structure, but not completely. There is a “displacement of equilibrium, but with minimization of the cost and maximization of the gain” (Piaget, 1985/1975, p. 57). There is enough
adjustment to eliminate the disequalibrium, but not enough to construct a fully new structure (Piaget, 1985/1975). A third level is where complete transformation can occur (Piaget, 1985/1975). This would be the level at which Confrey’s (1990) “powerful construction” can be said to be in place. It is a construction that is both meaningful and useful. It becomes integrated into both the conscientious and unconscientious repertoire of a person’s beliefs and behavior.

Even with their beliefs in place and with their strong self-motivation, these teachers could not build a powerful construction of these instructional innovations on their own. They needed some new skills and strategies. They needed to hear from experts – or from other teachers that were using the innovations successfully. They needed to experience the innovations for themselves, with their own students, and then reflect and build understanding. They needed to interact with others to develop broader perspectives, to ask questions and then try to answer, to justify and to defend what they discovered for themselves. They needed to move up the ever-widening spiral of cognitive development toward a more powerful construction.

All five teachers in the study sought out workshops and classes to gain more knowledge; to collect more compatible activities, units and projects; to gain more content knowledge; and to learn some of the pedagogical skills that could be passed on by the workshop leaders. I make the assumption that the workshop leaders were constructivist teachers themselves. These presenters knew what would support these teachers in their new constructions. They did not just explain the innovations and strategies; they provided models and experiences. Michael described his workshop
leader: “She was teaching it the way we were supposed to be presenting it in the classroom. And it was all constructivist.” Jane said of her workshop instructor and mentor, “He was phenomenal at teaching the math, not just how to teach it.” Rose compared the discoveries and ahas in the workshops she attended to the student ahas in her classroom. The five teachers all named a similar list of skills they developed in order to learn to teach mathematics conceptually with problems: asking questions, listening (“really listening”) to students responses, probing for understanding, rephrasing, giving students time, posing problems, using manipulatives effectively, and organizing for group work and projects. They gained these skills from taking workshops, attending presentations, and by reading professional resources. All participants mentioned particular experts that were influential over the course of their changes. The experts most often named were Constance Kamii, Marilyn Burns, John Van de Walle and the National Council of Teachers of Mathematics. All these experts related mathematics teaching to elementary classrooms, which provided the real life connection for these teachers. They learned the structure of the innovations by finding model lessons and activities in particular sets of teaching materials: published replacement units, State Mathematics Center materials, Math Their Way and Mathland manipulatives and activities, and, more recently, the reform-based series Bridges, Investigations and MATHThematics. Maher and Alston (1990) sum up the benefit of all these experiences with their statement, “The idea of knowledge being built up by a subject’s well-coordinated actions…in trying to make sense of experiences is central to a constructivist position” (p. 149). These five teachers were
making sense of these new instructional practices. The activities and the experts’ advice gave them the means to experience the instructional innovations in their classrooms. But in constructivist learning there is another very important piece to sense making.

Building new and powerful constructions requires experience and reflection paired with discussion (Bauersfeld, 1995; Cobb, 1994; Confrey, 1990; Thompson & Zeuli, 1999). Thompson and Zeuli (1999) framed the complexity of building powerful new constructions when they wrote:

Students must actively try to solve problems, resolve dissonances between the way they initially understood a phenomenon and new evidence that challenges that understanding, put collections of facts or observations together into patterns, make and test conjectures, and build lines of reasoning about why claims are or are not true. Such thinking is generative. It literally creates understanding in the mind of the thinker. (p.346)

This kind of mental work requires a lot of processing. Not solitary processing, but dialogue with others who are struggling with the same problems and who are also trying to build understanding. The reform of mathematics instruction requires that the individual teacher, who will personally implement the innovations, understand those innovations for her own teaching and, at the same time, come to terms with norms developed by the mathematics education community. For reform to occur in elementary classrooms, eventually the beliefs and knowledge of the reform proponents need to be shared by the teaching community. Norms and assumptions need to be carefully examined and negotiated. This is done through discussion. In effect, as members of a group externalize their thinking in social interaction, they
internalize the process and beliefs about which they are discussing (Thompson & Zeuli, 1999).

Discussion with both professional communities and individual mentors were important factors in the change journeys of the teachers studied. Jane’s principal was a mentor to her. He introduced pedagogical skills and encouraged her in her new practices not only by being an instructor of classes she attended, but also by recruiting her to share her new perceptions by being a workshop leader herself. Jane further developed her understanding and skills by serving on the school district’s adoption committee and doing research about mathematics. That committee turned into a learning community for Jane as they ended up studying the new mathematics standards and creating together their own reform-based curriculum for the district.

Patsy also participated in a professional community at her school. She and the other teachers, at regularly scheduled Friday afternoon sessions, have discussed their curriculum and their students’ learning. Having adopted reform-based mathematics programs for the school, teaching innovations for mathematics have been a main topic of these discussions. In addition, Patsy has brought in other related mathematics resources that they have considered and processed together for even further understanding. Michael and Rose became involved in professional communities outside of their schools as active participants in intensive and long term workshop trainings. They both developed networks and relationships with professional colleagues with whom discussions around mathematics instruction were rich and ongoing. Skip’s early support was the dialogue with a mentor who asked
probing and challenging questions. She said of the conversations with her Math Their Way instructor/mentor, “When she and I would talk, we would really talk in depth and question things with that idea of thinking about kids and where we wanted to go.” Skip also had a group with whom to discuss and learn. Skip, her mentor, and some other teachers formed a study group to do some action research. She recalled how much learning took place in that group. One time she videotaped a lesson and took it to the meeting. She said, “I thought I was doing something with a Venn diagram, but we analyzed it and I wasn’t doing what I thought at all. They didn’t criticize me, but they asked me those questions that made me think.” With groups of like-minded peers, focused on the new (agreed upon) better ways of teaching mathematics to children, these teachers continued to construct their new schema.

A key element in all of these teachers’ change journeys was a parallel, but related, construction that was taking place. All five were constructing – or reconstructing – their own mathematical knowledge. None believed themselves to be strong mathematicians: three declared themselves as math phobic and two admitted that, although they had taken higher level mathematics classes, they did not feel confident about their ability to do mathematics nor in their understanding of mathematical concepts beyond elementary level expectations. All five participants talked about the need to confront their lack of mathematical knowledge and to confront their attitudes about mathematics. They all did this with the same constructivist process of 1) disequalibrium: being uncomfortable with their lack of knowledge and with their discomfort with mathematics; 2) assimilation and
accommodation: seeing a new possibility that mathematics could be understood conceptually through the ahas and self discovery during the workshop activities, and building new understanding of mathematics concepts that were different than their old understanding; and 3) equilibration: finally coming to a new understanding.

Their view of mathematics changed from seeing it as just a set of rules and facts. It became a set of concepts that could be seen in models and experienced with manipulatives and understood with planned investigations. Further, their new knowledge was built around the application of this knowledge to teaching. They all found a new level of knowing mathematics – they now understood the development of mathematical knowledge. They have all come to understand the way this conceptual knowledge is built up over time, with multiple experiences and with connecting and reconnecting it to other knowledge. They built this knowledge constructively by doing the mathematics with their students, listening to what students were saying and then seeing through their students’ eyes. They took theses new perceptions to their discussions with other mathematics teachers. And by sharing the common experiences with their like-minded colleagues, they relearned mathematics. They built a new construction that helped them understand it and helped them know how to teach it so their students understand it. A bonus of this learning has been that even though they still say they are not “mathematicians,” they are all now confident in their ability to do mathematics. And all of them believe that, since they have learned it, everyone can learn it.
**Ongoing Support and Continued Development (Continuous Building)**

For the purpose of the study, I selected these five teachers based on the practices they use in their teaching, confirmed by self-reporting, the Presidential Award criteria, and my own observations. Without any better label I call them reform-based teachers or innovators. Over the course of their teaching they have adopted these practices and made them their own. One cannot pinpoint the set time that happened. The nature of constructed knowledge does not allow it to be easily defined and tested for attainment (Steffe, 1995; von Glasersfeld, 1995). Constructed knowledge becomes integrated into the thinking and the behavior of the individual who has constructed it. Development of a new construct – or knowledge – might be said to be in place when the learner can use that new knowledge to solve problems, to relate to other concepts or principles, to make generalizations. Constructed knowledge accounts for qualitative changes in both mental or physical actions as well as changes in the learners’ images, and schemes (Steffe, 1995). It becomes a part of them, or as Drake, Spillane and Hufferd-Ackles (2001) might say, it becomes a part of their identity. In these five cases, the teaching of mathematics with problem solving and conceptual understanding has certainly become these teachers’ instructional identities. They have fully incorporated the reform-based mathematics instruction into their teaching. In addition, they have all become leaders in the mathematics education community. Beyond their classroom teaching, much of their energy is placed in teaching classes, leading workshops, and making presentations. They are active in their State Council of Teachers of Mathematics. They are involved
with their state’s Department of Education in the development of mathematics assessments. I propose their wider involvement in mathematics education and in its reform is true for two reasons. Both reasons are related to constructivism.

First, these teachers are widely considered experts in mathematics instruction. They are all proponents of mathematical reform. They are leaders in reform efforts. However, they are also philosophically constructivists. They understand readiness and disequalibrium. They know that many of their school and district colleagues are not, at this time, ready to look for a new schema for teaching mathematics. A comment by Jane reflected the feelings of all five: “It is hard when you want change to happen and you feel it is a good change. Yet I don’t think you can impose it.” In talking about other teachers in her school, Rose used the word “reluctant.” Michael and Patsy understand that some teachers are “still math phobics.” Skip acknowledged it was difficult to talk about teaching “with teachers that do not have the same philosophy.” These teachers-who-are-now-experts are available to help and support whoever is ready to change, however they are putting their energies into regional and statewide workshops and classes where people come voluntarily to get new ideas. It is those looking for new ideas that will be the ones who will change. And for the other teachers, Jane says, “We just have to keep inviting and encouraging people to participate.” For the new teacher in her building, she says, “I have taken it on myself to check in with her now and then…. It is kind of a [constructivist] trick of just listening and being willing to extend yourself when the time is right.” But while waiting and listening, Jane is teaching a mathematics
methods class at a local university and co-editing the State Council of Teachers of Mathematics journal. She is not “imposing” the innovations on those who are not ready. Skip works with her principal on some school inservice meetings, but also sees futility in “imposing” innovations. Most of her outside of classroom time is spent writing assessment tasks for the state. She believes that these assessments will build awareness of the important mathematics concepts that need to be taught. That, she hopes, will cause the disequalibrium necessary for teachers to look for the new teaching methods that she believes to be better. Patsy, Michael, and Rose also teach workshops and are active in the State Council of Teachers of Mathematics. They, like the other two, are happy to share their teaching with anyone who is interested. They say, “Come and watch me teach.” All five attend the Summer Leadership Institutes in their state. They discuss, with other enthusiastic mathematics teachers, how to be better at their craft. They share stories and resources and they plan ways to reach out to more teachers. All believe wholeheartedly in the way they are teaching mathematics. But they also understand that mandating these practices would not make the changes happen. As constructivists, they just keep inviting, encouraging, and planning experiences to support those teachers who are ready to find out more.

The second reason these five teachers are actively working outside their districts with other mathematics educators is because they want to get new ideas and learn more. By writing assessments, exchanging resources and teaching ideas during the Summer Leadership Institutes, and discussing children’s learning, they are continuing to learn more about mathematics and more about teaching. Michael and
Skip both mentioned the value of sitting with other teachers and learning as they assessed student problem solving. Patsy is teaching a mathematics class to classroom assistants and says, "I have learned so much!" These service experiences are all learning experiences.

The construction of new knowledge and new understanding is a never-ending process (Noddings, 1990). By the time these five teachers got to the point where they could identify in themselves the powerful construction of these new methods of teaching mathematics, they were in the process of building new constructions. Most educators watching them teach would consider them experts. But not one of them refers to himself or herself as an expert. They all say they are still learning. As Rose put it, "The longer I teach the less I know." Jane continues to search for other methods that are more kid friendly and will give her second graders more autonomy. She said, "I spend an incredible amount of time working to develop this classroom as a community." Skip has taught more than enough years to retire, but she loves what she is doing and is not thinking of stopping. When asked why, with the many problems schools are experiencing these days, she is still energized about education, Skip said, "If you are a good teacher you are always going to be frustrated. But [teaching] is a challenge I find exciting." Michael enjoys a similar kind of challenge. Even with more than twenty years of teaching experience and being an expert mathematics teacher, sometimes he still gets surprised. He smiled as he shared, "You are not always sure where something is going to go. If something comes up that you are not expecting, like somebody has a way to do something and you don’t know
where it came from, you need to figure out where to take it.” That is the kind of problem he still likes to discuss with other teachers. That might be his platform for new learning. Patsy invites anyone to come and watch her teach, but then she wants to learn more. “My door is always open. Come and watch me teach. And then tell me what I could do differently.” The learning – and change – continues as the process of disequalibrium, assimilation, and accommodation repeat.

Summary

The comparison of these five teachers’ change journeys through a theoretical perspective of constructivism, showed their experiences to be very similar. They were all motivated by dissatisfaction with their mathematics instructions. They all shared the disequalibrium of believing that their traditional teaching – with the materials provided by their schools -- was not working. As each was introduced to a new way of teaching, it connected to other beliefs about teaching. They perceived it as an improvement and determined to adopt it. Commitment, however, was not enough. All five had to learn new skills and come to understand mathematics in new ways. They achieved this learning with assimilation and accommodation in the form of support from lessons and teaching techniques modeled in workshops, advice and suggestions from more experienced users, real-life experiences of trying the practices with their students, reflection, and discourse with other teachers. This is parallel to the modeling, structured experiences, reflection, and discourse that can be found in a constructivist classroom.
Finally, as their new concepts (constructions) of reform-based mathematics instruction were formed, they continued to develop deeper understanding and to build additional concepts (constructions) about mathematics and about how students best learn (dynamic disequalibrium). Although all five of these teachers seem to have mastered the innovations recommended by the NCTM, none believe they have mastered all there is to know about the teaching and learning of mathematics.
Chapter 6: Conclusions

Introduction

This purpose of my study was to examine the recollections of the transition process of five elementary classroom teachers as they changed from traditional mathematics instruction to reform-based, problem-centered instruction. In this final chapter I review this multi-case study from beginning to end, first summarizing the questions and the methods, then by explaining and limitations of the study as it played out. The key findings presented are based first on the exploration of the individual cases and then a cross case analysis that looked at the five cases together. A discussion of the findings is followed by recommended applications and suggestions for further research. I close the report with a few personal thoughts.

Summary of Methods and Research Questions

The research questions were explored as a multi-case study, examining in depth the self-reported change journeys of five classroom teachers. The teachers in the study have all received the Presidential Award for Excellence in Teaching Mathematics. All five currently teach mathematics through problem-centered, conceptually focused investigations in the manner envisioned by the National Council for Teachers of Mathematics (1989, 1991 & 2000). However, they did not always teach using these methods. Over the course of their classroom careers, they changed their teaching practices from traditional, text-driven mathematics programs...
to teaching with problem solving. The story of each teacher was collected through two focused interviews, two classroom observations, and an examination of the teacher’s application packet for the Presidential Award. Over the four meetings with the teachers I was able to become familiar with the teacher’s current instructional practices and to hear a detailed recollection of the changes they have made.

The interview process used semi-open guiding questions. These questions were developed to ensure that the information gathered addressed the topics of the motivation, resources, support, and key people that were influential in each teacher’s journey. The questions further sought input about the skills and knowledge necessary to make the changes and how those were developed. Finally, as all of these teachers are leaders in the area of mathematics education, I asked what they believed would support other teachers in changing their mathematics instruction.

Cases were reported independently, with each teacher’s story analyzed on its own. The process of change was tracked from early teaching, to first awareness, to the full adoption of the innovations. Only after the five cases were examined singly were the five stories considered together. By putting the key elements of the five change journeys side by side, I looked for similarities, differences, and patterns. Using a constructivist lens, I examined the five cases as a process of creating new constructs about mathematics – how it is learned and how it is taught.
Key Findings

The teachers had both similarities and differences in the influences and the processes of their changing. However, three elements played a critical role in all five of the teachers’ acceptance of the innovations and in their adoption of the practices. Those key elements were beliefs, group discourse, and the connections to their teaching.

Beliefs

Beliefs were an important foundation for the change. Even before they were aware of the reform innovations, all five teachers were dissatisfied with their mathematics instruction and motivated to find a different – better – way to teach mathematics to their students. None of the five had strong mathematics backgrounds and none felt confident in what mathematics knowledge they did have. Three actually claimed to be phobic and intimidated by mathematics. However, all believed mathematics to be an important subject and all wanted their students to be competent in their mathematics abilities. Further, they wanted their students to have a more positive disposition about mathematics than their own when they were students.

Although they became aware of the mathematics reform in a variety of ways, when they were introduced to it all five believed it was an improvement and became committed very quickly to adopting the practices.
Community

The teachers named a variety of materials and resources that supported the new practices they were adopting. They found additional support as the NCTM’s Mathematics Standards became more widely known and incorporated into district curricula. Reform-based mathematics programs became available and were actually adopted by some of the participants’ schools. Some of these teachers had administrative support in their changes, and some did not. Although school and district support was helpful to the teachers who had it, lack of it did not deter the others. All five were committed to the new ways of teaching mathematics, and they were practicing them in their classrooms. However, as they tried out the lessons and investigations, they discovered they had much to learn about using the new methods effectively with their students. In their interviews, all of the teachers recounted the value of discussing the innovations with others as they tried them in their classrooms. They had to develop new pedagogical skills and teaching strategies. They had to develop an understanding of the way the students related to the mathematical concepts at different grade levels. For all five teachers in the study, the way they developed their new skills and knowledge was by trying the things with their students and then sharing their observations and discussing their questions with other teachers. These other teachers were like-minded peers, also working on introducing these innovations into their mathematics instruction. There was, in these relationships, the common goal of making sense of the innovations and the way they work in the classroom. There was a huge element of trust in the interactions. In these
discussions the participants reported they could admit struggles, ask questions, share both successes and failures, and evaluate together what was working and how to improve what was not working. All learned together in a way that no one could learn alone. An indication of the importance of the role of such discourse in the change process is the fact that in the cases where the participants connected to groups outside of their district, changes in their mathematics instruction happened at their school in isolation in just their own classrooms. In the cases where the teacher’s discussion group took place at their school or in their district reform-based mathematics instruction was more widely adopted in the teacher’s school. In both scenarios the discussion group was an important part of each individual teacher’s changes, but school wide change seems more likely if the discussions involve the school’s teachers in the discussions.

**Teaching Connection**

A third element of importance in the teachers’ change journeys was the centrality of teaching. These teachers are all passionate about teaching. It was the *reading of their students* that motivated these teachers to change. It was the *discussions with other teachers* that helped them learn to use the innovations effectively. It is *teaching* that is the center of their thinking and the focus of the innovations. These teachers are teaching mathematics with innovative methods, but it is evident that the most important goal for each of them is teaching in a way that best serves their students’ learning. They are expert mathematics teachers. However,
they do not love mathematics; they love teaching mathematics. In their adoption of
the instructional innovations, these teachers all had to learn or relearn mathematics.
They built that learning around their teaching. They learned to understand the
mathematics by understanding how to teach the mathematics for understanding.
They are leaders in education. They are all involved outside of their classrooms and
outside of their districts. They enjoy working with the "bigger picture" of
mathematics education. But the classroom is their love and it is where they choose to
practice their profession. They still find daily challenges in their classrooms. Or as
they all say, they continue to learn new things from their students every day.

Limitations of the Study

The selection of all Presidential Awardees was an intentional bias. I wanted
assurance that the participants were using particular teaching methods. However, that
filter could also have acted as a filter to make the teachers alike in more ways than
just their practices. There could be some elements in the process of the Award
selection that would predetermine similarity in the manner in which the teachers
changed. A wider pool of teachers using reform-based practices may have reflected
different set key elements in their change processes.

Along a similar line of thought, although efforts were made to get a diverse
group of teachers, the teachers were more alike than different. Their school
demographics varied, but the teachers were similar in race, age, and length of
careers. I believe it would add to what we have learned from this study to find
Teachers of Color who are teaching with reform-based mathematics instruction. Comparing the key elements in the change of those teachers to the five that were studied could be very instructive. Conversely, it would be important to examine any resistance to reform-based instruction by Teachers of Color. Are there cultural factors that affect the adoption of reform-based mathematics instruction?

Finally, the stories of the changes were self-reported from the participants’ memories. Although change literature reports that backsliding, or regression to previous thinking, is common in the adoption of new practices, none of these participants reported that there was any going back to traditional practices once they began the changes to reform-based instruction. After successfully adopting and using, over a long period of time, changes in which one strongly believes, the memory of some of the doubts and difficulties may be dulled. With only the participants’ personal recall of their journeys, it is impossible to tell how much the path was straightened or smoothed in the recollection.

**Discussion**

Considering the findings parallel to constructivist learning brings up the thought of who, if the teachers are learners, is the constructivist teacher that sets up the environment and guides the inquiries so that new constructions get built? This relates to a topic that came up in various discussions with participants in this study, a topic often discussed in educational reform. The topic is instructional leadership.
What part does leadership play in teachers’ construction of innovations and new instructional practices?

Research on educational reform includes leadership as an important component of effective change (Fullan, 2001; Hargreaves, Earl, & Manning, 2001). However in an elementary school the location for instructional leadership is not well defined. Where is the responsibility for instructional leadership? And how is that leadership developed? Much was written in the 1990s about instructional leadership. Most of the writings from that time assigned the role of instructional leader to the principal. But typically in elementary schools that has not been the reality. Spillane, in his studies of mathematics and science reforms, has noted that administrators have a different frame of mind from teachers when looking at the suggested innovations (Spillane & Louis, 2002; Spillane, Reiser & Reimer, 2002). Often principals’ multiple responsibilities remove them from instruction and make leadership in instructional practices less authentic. Instructional leadership and teacher evaluation can be incompatible roles. Spillane (2000) also found that administrators often looked at reform from a budgetary standpoint and not from an instructional framework. They can overlook issues of practice and understand reform only in terms of purchased materials and textbooks. On the other hand, the teacher’s role in instructional leadership is gaining attention (Fullan, 2001; Spillane & Louis, 2002).

All of the participants in the study are leaders in mathematics education. But they began their careers rather ordinarily. In the words of Confrey (1990), they have built powerful constructs. They have all gone beyond just adopting prescribed
innovative practices. They have made the practices their own. They are now spokespersons for that change. Mathematics reform is an important part of their identity. But so is their classroom teaching. At their schools they often feel they walk a careful balance. While they have knowledge and experience that make them a valuable resource for change in mathematics instruction, they feel they must be careful not to be change agents or experts, which would set them apart from their colleagues. And even in schools where they are acknowledged leaders and catalysts for change, they are cautious in not wanting overt recognition for that role.

Schools will need to figure out this dilemma if they want to improve the quality of teaching and learning. Master teachers must be encouraged to stay in the classroom and not move to administrative positions. Teachers with passion for teaching need to be nurtured by supporting their pedagogical development. But when teachers develop extraordinary skills and highly effective practices there needs to be a way these can be shared. Perhaps this is what Fullan (2001) has in mind when he calls for reculturation of education – a way for teachers to share and discuss professional issues and practices in a way that focuses on improved learning for their students.

**Recommended Applications**

The findings in this study indicate considerations for the planning of staff development. The important role of discussions in the acquisition of new pedagogical skills and the making sense of new practices would recommend
planning and allocation of time for teachers to have focused discussions around new practices. However, key to the effectiveness of the discussions recounted by the participants were the like-mindedness of the group – all had the same goals in adopting the changes – and the level of trust. Discussions were open and tough questions were addressed collaboratively. Finally, effective discussions need leadership to steer the group to the next question or to recognize what could move the members to the next level of understanding. The leader must be carefully chosen without losing the earlier-named elements of trust and openness. The success of such groups would seem highly valuable in teachers’ learning, professional growth, and the adoption of change.

A second recommendation, taken from comments by all five participants, would be to make sure all student teachers are placed in classrooms where they can observe and experience children learning mathematics with problem solving. For the prescribed reform practices to be learned, they need to be observed. Student teachers need to see reform-based mathematics instruction and have a chance to discuss it with the teacher. They need to be able to try it for themselves with the support of a teacher who can guide them in its use.

Finally, an application that I believe could be valuable in increasing the adoption of teaching mathematics conceptually would be to develop summer workshops, when teachers have time to come together to create conceptual mathematics units. This idea is based on the experiences of one of the teachers in the study. She participated in a similar workshop in which the teachers taught in the
morning and discussed the teaching and wrote lessons in the afternoon. If this workshop were taught in conjunction with a school whose students are low performers in mathematics, it could serve as a dual benefit.

Recommendations for Further Research

Using the groundwork of this research, I would also like to pursue research applying a constructivist framework to the implementation of change. As one looks at teaching constructively, one looks for where the learner is in their understanding. Is it possible to build acceptance for the idea that teachers are on a continuum of development – and, as in a constructivist classroom, address the misconceptions and the partial understandings in a way that challenges the learner’s thinking and builds readiness for new construction.

I also would like to follow the students that leave Oregon State University and see what happens with their mathematics instruction as they begin their teaching careers. Knowing the perspective with which the class is taught, it would be valuable in the future development of our courses to see if we have been successful in helping our students build a reform-based practice. Again, if so what were the key influences and, if not, what elements were missing?

Concluding Thoughts

Doing this research had a benefit I did not expect. We are going through a time when almost all school news is bad news: budget crises, testing pressures,
crowded classrooms, and students with ever expanding physical and emotional issues. These issues were not absent in any of the classrooms or the schools in this study. They have large classes, troubled students, a range of student abilities with little help, and shrinking school budgets. And yet, observing and interviewing these teachers was an incredibly positive and energizing experience. The passion these teachers feel for what they do is contagious. The caring for their students, the problem-solving mentality, and the joy of learning make all of their classrooms places any student or any adult would love to be. These five teachers are examples for all of us. They are demonstrating an enthusiasm for teaching and learning that makes school an exciting place to be; a sense of humor that puts the extremes in education in balance; and a belief that all students can learn that makes them dedicated to single student’s success. I feel extremely fortunate to have seen these classrooms and observed this kind of teaching. It gave me hope for our schools.
REFERENCES


APPENDICES
QUESTIONNAIRE

Please fill out this short questionnaire and return it within one week. Return to Liz White, Education Hall #111, OSU, Corvallis Oregon 97331. A stamped, addressed envelope has been provided.

The initial criteria for selecting participants will be 1) elementary classroom teachers who 2) do not have a degree in mathematics and 3) have changed their mathematics instruction from a traditional to a reform-based approach. However, if you are interested in participating in this study, but are not currently in a classroom or you do have a degree in mathematics, I encourage you to fill out this form and send it to me. Depending on the responses, my criteria may broaden and you may be selected.

1. Briefly describe your current teaching assignment? Then give a brief overview of your teaching career. List other levels you have taught, including how long at each level. Include other districts or states in which you have taught. (Use the back of this page, if necessary.)

2. Math Background: Do you have a degree in mathematics? _____YES _____NO

3. Please list 3-5 ways you have changed your mathematics instruction from how you began teaching to how you teach now. (Use the back of this page, if necessary.)

4. Please indicate: _____FEMALE _____MALE

5. What is your race/ethnicity?

6. Put an X in each column to indicate the demographics of the school in which you now teach. Put a √ next to any demographic in which you have substantial historical experience.

   SCHOOL SIZE: LOCATION: SOCIOECONOMIC:
   ___Small (to 100) ___Urban ___High
   ___Medium (100-300) ___Suburban ___Medium
   ___Large (300+) ___Rural ___Low

Name* ____________________________ Date __________________________

Phone Number: ____________________________ E-mail: __________________________

School ____________________________

School District ____________________________

*Only the researchers will know who does or does not return this form. If you become a participant in the study, your name will not be used. A pseudonym will be used to maintain confidentiality.

THANK YOU FOR YOUR INTEREST AND FOR TAKING THE TIME TO FILL THIS OUT.

______Please check if you have a copy of your award application/nomination packet that you would be willing to share with the researcher.
INFORMED CONSENT DOCUMENT

Project Title: A Multiple-Case Study of Elementary Teachers' Transitions to Reform-Based Mathematics Instruction

Principal Investigator: Dr. Karen M. Higgins, Associate Professor, School of Education

Student Researcher: Liz White, Doctoral Student, School of Education

PURPOSE

This is a research study. The purpose of this research study is to gain understanding about the elements of educational change at the classroom level by examining the paths of five elementary teachers as they transformed their beliefs about learning and teaching mathematics. The purpose of this consent form is to give you the information you will need to help you decide whether or not to be in the study. Please read the form carefully. You may ask any questions about the research, what you will be asked to do, the possible risks and benefits, your rights as a volunteer, and anything else about the research or this form that is not clear. When all of your questions have been answered, you can decide if you want to be in this study or not. This process is called “informed consent.” You will be given a copy of this form for your records.

You are being invited to participate in this research study because, by being a recipient of an award for excellence in mathematics instruction, you have been recognized by the mathematics community for your enthusiasm and dedication in the teaching of mathematics at the elementary level. The transition you have made in your teaching is the goal of mathematics reform. You will be one of five teachers in this in-depth study.

PROCEDURES

If you agree to participate, your involvement will entail a total of approximately four hours over the next few months. Two interviews would be approximately one hour in length each. The interview will be audio and/or video taped. Between the interviews you would be observed teaching two consecutive or related mathematics lessons on two different days. These would be regular classroom lessons that are a part of your normal teaching. The observations would not entail any time outside your regular teaching. Another hour would be required to review and give feedback on the initial analysis of the interviews and observations. This feedback could be done in the form of an interview or in writing. In addition, if available, the researcher would like to review the application packet you submitted for your award.
The time line for this study will need to be negotiated based on our schedules. The goal would be to have no more than a month between the first interview and the second interview. The initial write up and review by you would be within a month after the second interview. Below is a table summarizing all the pieces of the study:

<table>
<thead>
<tr>
<th>ELEMENT OF THE STUDY</th>
<th>LENGTH OF TIME</th>
<th>LOCATION</th>
<th>DATA COLLECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interview #1</td>
<td>One hour</td>
<td>Classroom or quiet place of your choice</td>
<td>Notes and audio and/or tape</td>
</tr>
<tr>
<td>Review of Award Nomination Packet</td>
<td>As available</td>
<td></td>
<td>Notes</td>
</tr>
<tr>
<td>Observation #1</td>
<td>Class Period</td>
<td>Classroom</td>
<td>Notes</td>
</tr>
<tr>
<td>Observation #2</td>
<td>Class Period</td>
<td>Classroom</td>
<td>Notes</td>
</tr>
<tr>
<td>Interview #2</td>
<td>One hour</td>
<td>Classroom or quiet place of your choice</td>
<td>Notes and audio and/or tape</td>
</tr>
<tr>
<td>Review and feedback on initial analysis</td>
<td>About one hour</td>
<td>Your choice</td>
<td>Interview or written feedback (e-mail or letter)</td>
</tr>
</tbody>
</table>

**RISKS**

Your only foreseeable risk is embarrassment if a reader of the paper is not only offended by your comments, but also identifies you as the source. To minimize this possibility, the researcher will not name your school or even your state in the article, and you will be given a pseudonym. Before publication, you will be given the opportunity to review the article to discover and modify controversial or possibly compromising statements.

**BENEFITS**

There will be no direct personal benefit for participating in this study. However, the researcher anticipates that increased understanding of the factors that enhance or hinder a classroom teacher’s transformation of beliefs and practices will advance educational reform. This study will be a way of making known the lessons and insights you gained from your experiences with change.

**COSTS AND COMPENSATION**

You will not have any costs for participating in this research project. You will not be compensated for participating in this research project.
CONFIDENTIALITY

Records of participation in this research project will be kept confidential to the extent permitted by law. However, federal government regulatory agencies and the Oregon State University Institutional Review Board (a committee that reviews and approves research studies involving human subjects) may inspect and copy records pertaining to this research. It is possible that these records could contain information that personally identifies you. Any references, either in the notes or in the final write up, will be with pseudonyms for you, your school, and your district. Pseudonyms will be used for any people or places you name in the interviews. Audiotapes and videotapes will be kept in a locked file and only the two named researchers will have access to them. The audiotapes and videotapes will be destroyed after this study is completed. No one other than the researchers will know who did or did not participate in this study. In the event of any reports or publications from this study, your identity will not be disclosed. Results will be reported in a summarized manner in such a way that you cannot be identified.

VOLUNTARY PARTICIPATION

Taking part in this research study is voluntary. You may choose not to take part at all. If you agree to participate in this study, you may stop participating at any time. During the interviews you are free to decline answering any question asked by the researcher. If you decide not to take part, or if you stop participating at any time, your decision will not result in any penalty or loss of benefits to which you may otherwise be entitled. If you decide to withdraw from the study after any interview or observations have taken place, all data collected from you will be destroyed and not used in any part of the study.

QUESTIONS

Questions are encouraged. If you have any questions about this research project, please contact Karen Higgins at (541) 737-4201 or by e-mail at higginsk@orst.edu. You may also contact Liz White at (541) 737-8573 or by e-mail at whiteliz@onid.orsu.edu.

If you have questions about your rights as a participant, please contact the Oregon State University Institutional Review Board (IRB) Human Protections Administrator at (541) 737-3437 or by e-mail at IRB@oregonstate.edu.

Your signature indicates that this research study has been explained to you, that your questions have been answered, and that you agree to take part in this study. You will receive a copy of this form.
RESEARCHER STATEMENT

I have discussed the above points with the participant or, where appropriate, with the participant’s legally authorized representative, using a translator when necessary. It is my opinion that the participant understands the risks, benefits, and procedures involved with participation in this research study.

(Signature of Researcher)  (Date)
The prompt for the first interview was the following open inquiry:

Please tell me the “story” of your mathematics teaching. How did you start out? How did you change the way you teach mathematics now? Please give as much detail as you remember about who and what influenced you and what you consider the milestones in this transition.

I let the story unfold in the participant’s own words. If there was a lag or it the story ended before the allotted hour, I used the Guiding Questions as prompts or probes for more information.
### Observation Protocol

<table>
<thead>
<tr>
<th>NOTES:</th>
<th>ANTICIPATED PRACTICES:</th>
</tr>
</thead>
</table>
| 1) Organization  
   a) Collaborative  
   b) Groupings  |
| 2) Problem/Task  
   a) Multiple Solutions/Answers  
   b) Inquiry/Discovery  
   c) Time  
   d) Richness  |
| 3) Questions  
   a) Open  
   b) From teacher and students  
   c) Probing/Clarifying  |
| 4) Discourse  
   a) Alternative ideas encouraged  
   b) Sharing of ideas  
   c) Questions  
   d) Probes/Prompts  
   e) Clarifications  
   f) Non judgmental/Safe  
   g) Active Listening/Interchange of ideas  |
| 5) Thinking Expectations  
   a) Evidence/Justifications/Proofs  
   b) Reasons  
   c) Conjectures  
   d) Metacognition Encouraged  
   e) Respect for students’ input  |
| 6) Mathematics Content  
   a) Clear mathematics goals  
   b) Focus on important mathematics  
   c) Accuracy of presentation  
   d) Focus on math making sense  |
| 7) Pedagogy  
   a) Knowledge of students  
   b) Useful strategies for student success  
   c) Developmental understanding  
   d) Clear performance expectations  
   e) Procedures that support problem solving  |
| 8) Representations  
   a) Use of models, manipulatives, drawing, etc  
   b) Variety of material available/used  |
| 9) Assessment  
   a) On-going: learning, thinking, disposition  
   b) Self Assessment encouraged |
GUIDING QUESTIONS FOR INTERVIEW

The second interview will be semi-structured. The following are the guiding questions that the researcher will use to organize statements and to probe for more information:

- What was the motivation for changing the way you taught mathematics in your classroom? Were the changes mandated or self-initiated?

- At what point were you in your teaching career when you made these changes? Was that a factor?

- What part did your home/family life have in changing your teaching?

- Where did you get support in making your initial transition from traditional instruction to reform-based mathematics?

- What was your ongoing support or encouragement? Was the support from personal or professional sources? Was it from an individual or from an organization?

- What, if any, beliefs, about mathematics or about teaching and learning did you change as you changed your mathematics instruction? Describe the changes. What came first, the change in beliefs or the change in practice? Did these changes affect your teaching of other subjects?

- When you changed the way you taught mathematics, describe any changes you saw in your classroom. Did your students change? Did your teaching in other subjects change? What did you like or dislike about these changes?

- What barriers or difficulties did you encounter when changing your mathematics instruction?

- What resources/materials have you found most useful over time? How are these resources or materials "teacher friendly" or not? Would they be useful for less experienced or less math oriented teachers?

- Discuss some of the ongoing or recurring issues you come across as you teach mathematics. Include issues in your classroom and also the school/district/community. How do you deal with these?
• What **issues or dilemmas** have you seen for other teachers? How do these issues or dilemmas impact mathematics instruction in your school/district/state?

• How much change have you seen in *the way mathematics is taught in the past ten years*? What changes have you seen statewide? What changes at your school? To what do you attribute these changes? What do you find about the changes encouraging or discouraging?

• What do you think might be the “**key changes**” that would make the biggest impact on the way math is taught in elementary classrooms?

• Do you think the changes recommended by the NCTM are possible on a widespread basis? What is needed for this to happen at the elementary level? What elements of reform-based mathematics instruction do you think can **realistically** be implemented on a wide scale?

• What **impact** did your award have for you? What, if anything, changed for you because of the award?

• In what ways, if any, have you strengthened your background in mathematics?
APPENDIX F
## Chart 2: Summary of Key Elements in Patsy Miller’s Change Journey

<table>
<thead>
<tr>
<th>CATEGORY</th>
<th>MATERIALS</th>
<th>TEACHING APPROACHES</th>
<th>BELIEFS</th>
<th>INFLUENCES</th>
</tr>
</thead>
<tbody>
<tr>
<td>EARLY TEACHING AND</td>
<td>Math Units in boxes</td>
<td>Text driven</td>
<td>Student choice</td>
<td>Math methods classes</td>
</tr>
<tr>
<td>TRAINING</td>
<td>Ditto &amp; worksheets</td>
<td>Project work</td>
<td>Integration</td>
<td>Media background</td>
</tr>
<tr>
<td></td>
<td>Traditional text</td>
<td>Connections to</td>
<td></td>
<td>Other 5th gr. teacher</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Community</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MOTIVATION</td>
<td>Lack of depth in text</td>
<td>Project work – quilts</td>
<td>Real world connection</td>
<td>Students</td>
</tr>
<tr>
<td></td>
<td>Each unit separate – no connections</td>
<td>Science unit</td>
<td>to math</td>
<td>Other teachers</td>
</tr>
<tr>
<td></td>
<td>Student projects</td>
<td></td>
<td>Student thinking only</td>
<td>Self motivated</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>one-planed</td>
<td></td>
</tr>
<tr>
<td>INITIAL SUPPORT</td>
<td>Workshop materials</td>
<td>Projects</td>
<td>This was better</td>
<td>Workshops</td>
</tr>
<tr>
<td></td>
<td>Teacher-made units</td>
<td>Problem Solving</td>
<td>Students attitudes</td>
<td>Regional Conference</td>
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<td></td>
<td></td>
<td>Manipulatives</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td>Visual Math Activities</td>
<td></td>
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<tr>
<td>ONGOING SUPPORT</td>
<td>Mathland</td>
<td>Problem Solving</td>
<td>Multiple experiences</td>
<td>State math community</td>
</tr>
<tr>
<td></td>
<td>MATHThematics</td>
<td>Projects</td>
<td>Nurturing &amp; trust</td>
<td>District teachers</td>
</tr>
<tr>
<td></td>
<td>SMC Workshops</td>
<td>Models</td>
<td>Math confidence</td>
<td>District administrators</td>
</tr>
<tr>
<td></td>
<td>Manipulatives</td>
<td>Discovery lessons</td>
<td>Accepting all ideas</td>
<td>Van de Walle</td>
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<tr>
<td></td>
<td>NCTM Materials</td>
<td>NCTM Standards</td>
<td>Student choice</td>
<td>Annenberg Tapes</td>
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<tr>
<td></td>
<td></td>
<td>State Standards</td>
<td>Never give up</td>
<td></td>
</tr>
<tr>
<td>RECURRING ISSUES FOR</td>
<td>No one program fits all</td>
<td>Multiple approaches:</td>
<td>Some give up on kids</td>
<td>Class for assistants</td>
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<tr>
<td>OTHER TEACHERS</td>
<td>Knowing standards</td>
<td>must find one that</td>
<td>Discussions needed</td>
<td>State math community</td>
</tr>
<tr>
<td></td>
<td></td>
<td>works</td>
<td>Math phobias</td>
<td>Internet</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Math knowledge</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Standards</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Honor kids thinking</td>
<td></td>
<td></td>
</tr>
<tr>
<td>KEY CHANGES NEEDED</td>
<td>Workshop on developing lessons</td>
<td>See it in action:</td>
<td>Belief ALL can learn</td>
<td>State math org.</td>
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<tr>
<td></td>
<td>Standards</td>
<td>open door</td>
<td>Support &amp; mentoring</td>
<td>Building work</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Methods classes with</td>
<td>Feeling success</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Classroom teachers</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mentoring &amp; support</td>
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Chart 3: Summary of Key Elements in Skip Munson’s Change Journey

<table>
<thead>
<tr>
<th>CATEGORY</th>
<th>MATERIALS</th>
<th>TEACHING APPROACHES</th>
<th>BELIEFS</th>
<th>INFLUENCES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EARLY TEACHING AND TRAINING</strong></td>
<td>Child development Textbooks/workbooks Reading books in Pockets</td>
<td>Developmentally Appropriate (ECE) ELL class – no math Textbook/workbooks Added manipulatives Individualized reading</td>
<td>Text driven program inappropriate Hated math – wanted student to like it Math important</td>
<td>Preservice training – only 1 mth/sci class LA emphasis Constance Kamii</td>
</tr>
<tr>
<td><strong>MOTIVATION</strong></td>
<td>Workbooks</td>
<td>Added real objects Real world Constructivist</td>
<td>Needed to like math Needed to be understandable Way taught important</td>
<td>Constance Kamii ECE training Child Development</td>
</tr>
<tr>
<td><strong>INITIAL SUPPORT</strong></td>
<td>Real life items like Money &amp; buttons Math Their Way</td>
<td>Writing process Reading Program MTW – no longer textbook driven</td>
<td>Constructivist Hands-on Meeting student needs Teaching for understanding</td>
<td>MTW Instructor Lucy Calkins MTW Self motivated Marilyn Burns</td>
</tr>
<tr>
<td><strong>ONGOING SUPPORT</strong></td>
<td>Math Their Way Workshop materials Manipulatives Professional resources</td>
<td>MTW Stations &amp; Activities Problem Solving Constructivist Multi-age grouping</td>
<td>Constructivism Meeting each student’s needs Listening to kids</td>
<td>Principals Team members Husband &amp; friends Professional resources</td>
</tr>
<tr>
<td><strong>RECURRING ISSUES FOR OTHER TEACHERS</strong></td>
<td>No program complete Materials don’t teach conceptually Time to find materials Filling holes</td>
<td>Time Focus on operations Avoidance of school math goals Jump to symbolic Rush to cover</td>
<td>Number sense means computation Saying right answer = to understanding State tests are Computation</td>
<td>Time &amp; priorities Math backgrounds District administration State Assessment</td>
</tr>
<tr>
<td><strong>KEY CHANGES NEEDED</strong></td>
<td>Better programs State standards &amp; assessments</td>
<td>Professional developmt during school hours Student teaching Examining concepts Integration</td>
<td>Conceptual understanding How children learn Math knowledge</td>
<td>State assessments Leadership Teachers as professionals</td>
</tr>
<tr>
<td>CATEGORY</td>
<td>MATERIALS</td>
<td>TEACHING APPROACHES</td>
<td>BELIEFS</td>
<td>INFLUENCES</td>
</tr>
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<td>-----------------------------------------------</td>
<td>----------------------------------------------</td>
<td>-----------------------------------</td>
<td>---------------------------------</td>
</tr>
<tr>
<td><strong>EARLY TEACHING AND TRAINING</strong></td>
<td>Text program 10 years</td>
<td>Focus on operations</td>
<td>Kids not paying attn.</td>
<td>LA emphasis</td>
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<td></td>
<td>One math class</td>
<td>Show and tell</td>
<td>Needed to learn rules</td>
<td>Masters in LA</td>
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<td></td>
<td></td>
<td>Rules</td>
<td>Never questioned</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>program/teaching</td>
<td></td>
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<tr>
<td><strong>MOTIVATION</strong></td>
<td>Not happy with text program</td>
<td>Dissatisfied with kids performance</td>
<td>Looking for new ideas</td>
<td>Self motivated</td>
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<td></td>
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<td>New school district</td>
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<td><strong>INITIAL SUPPORT</strong></td>
<td>Workshop materials</td>
<td>Problem solving investigations</td>
<td>Fun &amp; interesting</td>
<td>Self motivated</td>
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<td>Manipulatives</td>
<td>Inquiry in science</td>
<td>Made sense</td>
<td>Consortium</td>
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<td></td>
<td>Science kits</td>
<td></td>
<td>Learning more while</td>
<td>Science committee</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>doing activities</td>
<td>SMC workshops</td>
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<tr>
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<td></td>
<td>NCTM journals</td>
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<tr>
<td><strong>ONGOING SUPPORT</strong></td>
<td>SMC Notebooks</td>
<td>Workshop sessions</td>
<td>Best practices</td>
<td>State Scoring</td>
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<td>Professional resources</td>
<td>Workshop leader training</td>
<td>No more phobia</td>
<td>Writing Assessments</td>
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<td>Internet</td>
<td>training</td>
<td>Learn from</td>
<td>Dept of Edn</td>
</tr>
<tr>
<td></td>
<td>Workshop materials</td>
<td>Professional groups</td>
<td>discussions</td>
<td>State Math Center</td>
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<td></td>
<td>Conference materials</td>
<td>Sum Leadership Inst</td>
<td>Only way to teach –</td>
<td>S/CTM</td>
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<tr>
<td></td>
<td>Manipulatives</td>
<td>Constructivist</td>
<td>would not go back</td>
<td>Students</td>
</tr>
<tr>
<td></td>
<td>NCTM Standards</td>
<td>State assessments</td>
<td>Math tied to real world</td>
<td></td>
</tr>
<tr>
<td></td>
<td>State Standards</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>RECURRING ISSUES FOR OTHER TEACHERS</strong></td>
<td>Time to make/find materials</td>
<td>Classroom mgmt</td>
<td>State assessment</td>
<td>Administrative</td>
</tr>
<tr>
<td></td>
<td>Management of materials</td>
<td>Focus on arithmetic proficiency</td>
<td>(misplaced) focuses</td>
<td>misunderstanding</td>
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<tr>
<td></td>
<td>Text easy – recipe bk</td>
<td>Lack of math knowledge so teach algorithms</td>
<td>on arithmetic</td>
<td>Time</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Only 1 way to get</td>
<td>Knowledge</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>THE right answer</td>
<td>Classroom</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Lack of knowledge</td>
<td>management</td>
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<td></td>
<td></td>
<td>Math phobias</td>
<td></td>
</tr>
<tr>
<td><strong>KEY CHANGES NEEDED</strong></td>
<td>Adoption of problem solving rather than text</td>
<td>Training</td>
<td>Need to see it early</td>
<td>University training</td>
</tr>
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<td></td>
<td>Standards</td>
<td>Mentoring new tchrs</td>
<td>Hard to change after</td>
<td>Cooperating teachers</td>
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<tr>
<td></td>
<td></td>
<td>Student teacher placement where</td>
<td>get into text</td>
<td>District mentoring</td>
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<tr>
<td></td>
<td></td>
<td>PS being used</td>
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</table>
## Chart 5: Summary of Key Elements in Rose Sharon’s Change Journey

<table>
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<tr>
<th>CATEGORY</th>
<th>MATERIALS</th>
<th>TEACHING APPROACHES</th>
<th>BELIEFS</th>
<th>INFLUENCES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EARLY TEACHING AND TRAINING</strong></td>
<td>Crummy old texts No manipulatives</td>
<td>Text based Some problem solving</td>
<td>This is not the way to teach math and science</td>
<td>Dr. Sharon Rose Methods Class</td>
</tr>
<tr>
<td><strong>MOTIVATION</strong></td>
<td>Uninteresting &amp; not problem oriented</td>
<td>Few problems Not conceptual Ideas from methods Class Text driven</td>
<td>Could not tell what she learned in her math classes Should teach for understanding</td>
<td>First year teaching</td>
</tr>
<tr>
<td><strong>INITIAL SUPPORT</strong></td>
<td>Replacement units Projects/problems she found herself State framework for Mathematics</td>
<td>Constructivist Manipulatives Projects &amp; problems</td>
<td>Children must build own understanding All students can PS Mixed ability groups Standards</td>
<td>Dr. Sharon Ross School principal Workshop teachers</td>
</tr>
<tr>
<td><strong>ONGOING SUPPORT</strong></td>
<td>Replacement units Units and projects from resources Workshop materials Investigations program</td>
<td>Constructivist Manipulatives Projects &amp; problems Long term projects (checkbooks)</td>
<td>Connect conceptual to symbolic Mixed ability groups Standards ALL can do math Goal: change kids ideas about math</td>
<td>Beyond Activities Dr. Sharon Ross S/CTM Principals Students</td>
</tr>
<tr>
<td><strong>RECURRING ISSUES FOR OTHER TEACHERS</strong></td>
<td>Time: text is easier Use investigations then go to worksheets &amp; algorithms Lacking alignment</td>
<td>Tell students HOW Teach algorithms Too quick to accept answer as knowing Timed tests Lack alignment</td>
<td>Think PS for bright kids only State test worries - hurry learning Must know facts before doing other math areas</td>
<td>Time Need for discussions</td>
</tr>
<tr>
<td><strong>KEY CHANGES NEEDED</strong></td>
<td>Good materials &amp; Resources</td>
<td>Alignments of instruction Time to discuss curric &amp; practices Questioning skills Listening skills</td>
<td>More math knowledge More conceptual Understanding</td>
<td>Administrative support</td>
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</tbody>
</table>
Chart 6: Summary of Key Elements in Jane White’s Change Journey

<table>
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<tr>
<th>CATEGORY</th>
<th>MATERIALS</th>
<th>TEACHING APPROACHES</th>
<th>BELIEFS</th>
<th>INFLUENCES</th>
</tr>
</thead>
<tbody>
<tr>
<td>EARLY TEACHING AND TRAINING</td>
<td>Old traditional text &amp; teachers guide</td>
<td>Page by page</td>
<td>Text was the curriculum</td>
<td>1st year teacher</td>
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<td></td>
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<td>following text</td>
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<tr>
<td></td>
<td></td>
<td>Getting to know grade</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>level expectations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MOTIVATION</td>
<td>Kids moans &amp; groans</td>
<td>Kids not understanding</td>
<td>Kids not getting math</td>
<td>College methods</td>
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<tr>
<td></td>
<td></td>
<td>No engagement in the activities</td>
<td>There must be a better way to do this</td>
<td>Class (games), Students, Self</td>
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<tr>
<td>INITIAL SUPPORT</td>
<td>NCTM Conference Activities MTW stations</td>
<td>Stations</td>
<td>Better: kids were engaged</td>
<td>Marilyn Burns Principals, MTW</td>
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<td>Teacher directed still</td>
<td>Kids liked math</td>
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<td>MTW &amp; BIOI activities</td>
<td>Attitude surveys</td>
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<td>Didn’t know what I didn’t know – lot to learn</td>
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<td>ONGOING SUPPORT</td>
<td>MTW and other workshop materials</td>
<td>Problem solving</td>
<td>Math is multidimensional</td>
<td>Principal/Mentor, Math community</td>
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<td>NCTM Standards</td>
<td>Stations</td>
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<td>Department of Edn, NCTM Standards</td>
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<td>District guidelines</td>
<td>Cooperative groups</td>
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<td>Integrated, National connections</td>
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<td>Adopted materials</td>
<td>Discussions</td>
<td></td>
<td>Technology, Masters in Admin</td>
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<td></td>
<td>Teacher-made and</td>
<td>Constructivist</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>replacement units</td>
<td>Student helpers</td>
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<td>State Standards</td>
<td>Units</td>
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<tr>
<td></td>
<td>Internet</td>
<td>Integrated</td>
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<tr>
<td>RECURRING ISSUES FOR OTHER TEACHERS</td>
<td>Return to traditional text Knowledge of Standards</td>
<td>Organizing group work</td>
<td>Voluntary participation &amp; choice Underestimating kids PS ability</td>
<td>Classroom based instruction leadership</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Managing discussions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>KEY CHANGES NEEDED</td>
<td>Multiple resources</td>
<td>Constructivist teaching Problem solving skills</td>
<td>Comfort with disequilibrium Belief in students’ PS abilities Support when ready Keep inviting</td>
<td></td>
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</table>