Portfolio Analysis of Contracting Strategies for Farmer Marketing Cooperatives

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ABSTRACT

The present report illustrates the use of portfolio analysis in designing efficient market contracting strategies. The analysis shows that cooperative processing and marketing firms can partly control the risks and expected returns they face by systematically evaluating alternative contract formulae. Characteristics of several prominent contract forms are discussed.

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PORTFOLIO ANALYSIS OF CONTRACTING STRATEGIES FOR FARMER MARKETING COOPERATIVES

Steven T. Buccola and Ben C. French

INTRODUCTION

An important feature of agricultural markets of the last decade was the increasing reliance on contractual agreements which bind individual buyers and sellers to stated sales conditions over a period of time. The primary motivation for such arrangements is to obtain greater control over the physical characteristics and quantities of commodities exchanged and to reduce uncertainty over the economic terms of trade.

Farmer cooperatives--particularly processing cooperatives--are especially interested in contractual arrangements. Commitment of raw product delivery specifications is essential to minimize peak season bottlenecks, to cover member investment obligations, and to market dependably satisfactory products. Secure marketing arrangements for the processed product also are essential if the processing cooperative is to fulfill the increasingly important role of providing a secure home for members' produce. It is not surprising, therefore, that cooperative processors are among the leaders of innovation in sales contracts for finished agricultural goods. In addition to processed product and grower-member raw product contracts, cooperatives may benefit by contracting purchases from nonmembers just as do corporate processors.

All contractual decisions involve some elements of risk. Many contracts contain features or formulae such that the final cost to the buyer or revenue
to the seller is not known with certainty at the time the contract is signed. For example, the contract price may be tied to production costs or to open market prices which may assume any of a range of values. Moreover, even if the contract price is fixed, the alternative of not contracting typically is subject to a range of uncertain outcomes. Some contractors--identified as risk-neutral--will always choose, from among alternative contractual arrangements, the particular arrangement which yields maximum expected returns. However, if the contractor has some aversion to risk, as is commonly the case, there may be some trade-off between degree of risk and the expected return for various contractual arrangements.

An approach to dealing with this type of problem is the technique of portfolio analysis. A portfolio is a commitment to a particular set of business options. In the world of financial management, for example, a portfolio is a particular set of security holdings. In the case of a cooperative processor, a particular set of sales or purchase contracts might be referred to as a portfolio of contract options. Various proportions of sales might be committed under different contract terms. Portfolio analysis is a technique for choosing the particular portfolio that has the lowest degree of riskiness (as measured by the variability of returns) for each possible level of expected return from contracting. Or stated another way, it is a way of choosing the portfolio with the largest expected return for any given degree of riskiness. This process reduces the contract selection problem to a choice among a more limited set of contract portfolios than the processor may originally have considered.

The purpose of this report is to illustrate the application of portfolio analysis to a particular processing/marketing cooperative and to report some general conclusions about contracting strategies as a result of the study.
(These findings are developed in more technical detail in the referenced report by Buccola and French.) Many of the general conclusions are also applicable to noncooperative marketing firms.

**ALTERNATIVES IN CONTRACT FORMULAE**

**Types of Contract Clauses**

A marketing contract stipulates terms under which a quantity of commodity will be traded over a period of time. The terms may be as specific or general as the traders wish. Most market contracts refer under separate headings to identities of firms, quantity and quality of product, value, production or shipment timing, and place and means of transfer. Many others include finance arrangements, dockages and premiums, technical services, provisions for redress in the event of noncompliance, conditions under which the contract is dissolved, and renewability options. The first set of categories addresses questions that any two firms exchanging a commodity must answer. The object of contracting is to enable both firms to predict or control one or more of these elements better than they would be able to under "open market" conditions. However the contracting firms do not necessarily become better off as specifications under these categories become more exact. For example, the quality of raw agricultural goods often fluctuates widely; it may only be possible for contracts to specify a minimum quality that has proven frequently attainable in other years.

Quantity and value terms also may be left advantageously variable. Although hopeful of improving its ability to predict future earnings, each contractor also should hope to react as wisely as possible to future external business conditions that it cannot predict. For instance, cooperative
processors may arrange to purchase lower volumes from nonmembers in years of
high processed inventory. Similarly, they may agree with customers to sell
fewer processed goods in years of lower processed inventory or low farm yields.

Price is frequently left variable in contracts. This practice has two
potential uses. First, firms that have access to trading partners other than
their contracting partners are acutely aware of market opportunities the trading partners represent. Sellers bound to a fixed price that falls below the
"market" price in any year may consider this price difference a loss. Although
an equitable fixed price will generate roughly as many gains as losses over time,
growers or board members may not be patient enough for the long-run to occur.
In this case, it may be preferable to relate the contract price in some way to
a specifically defined market price. Second, even when there are no accessible
trading partners external to the contracting partner, a firm's stockholders or
member-growers frequently have a sense of "fair" or adequate contract price.
If adequacy is defined in terms of a rate of return over costs, the contract
price should move with changes in costs. If, on the other hand, adequacy is the
"fairness" with which a good's resale value is reflected in the good's own price,
the contract price should move with changes in resale value.

In a fluctuating, imperfectly predictable economic environment, alternative
contract formulae cannot be ranked by objective criteria. Individual decision
makers differ not only in their concept of a fair expected return, but also in
their willingness to endure uncertainty or random movement in these returns over
time. The most a third party can do is to point out to potential marketing contractors the behavior they can expect of alternative contract formulae. No one
set of contract terms can satisfy all the requirements that a person would gen-
erally consider desirable in a contract.
Alternative Contract Price Clauses

A set of alternative contract price clauses, with certain characteristics one might expect of each, is presented in Table 1. Each category represents a class of pricing formulae; many pricing formulae in turn require that suitable values be assigned the indicated constants. It is likely a firm would initially indicate preference for a general category, and only subsequently be concerned with formula modifications and values for constants. Thus, a firm may be attracted to sales prices that reflect its costs, and only later realize there are many ways of designing a cost-related price.

Fixed or "negotiated" prices are frequently used in one- to three-year contracts. If the majority of transactions in a commodity are priced through such negotiations, the fixed price becomes the market price. This is the case with many processing vegetables such as tomatoes, bush beans, and sweet corn.

Despite its appearance, market price formulae may be designed in alternative fashions. Some formulae designate a particular time, place, and form at which the market price is observed. Others require that an average be calculated over time periods, places, and product forms. Some market price contracts stipulate a premium or discount over observed prices.

Frequently, contract prices are established by a scale or formula that relates the contract price to various economic indicators (the market price proxy in Table 1). Commonly used indicators are cost indexes, prices of related products, quality indicators, inventory levels, and total production. Formulae which relate contract price to inventory level reflect an attempt to approximate the excess demand facing the cooperative processor.
Table 1. Alternative Pricing Formulae for Marketing Contracts

<table>
<thead>
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<th>Pricing Formula</th>
<th>Description</th>
<th>Advantages</th>
<th>Disadvantages</th>
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<tbody>
<tr>
<td>Fixed or Negotiated Price</td>
<td>Buyer pays seller a fixed and stated price.</td>
<td>Eliminates price uncertainty.</td>
<td>Does not relate to a supply or demand variable, nor to any measure in the light of which both parties may consider P &quot;fair.&quot;</td>
</tr>
<tr>
<td>(a) ( P = a_0 )</td>
<td>(a) Price may be invariant over time.</td>
<td>May reflect competitive conditions if buyer and seller are equally powerful negotiators.</td>
<td></td>
</tr>
<tr>
<td>(b) ( P = a_1 + b_1 t )</td>
<td>(b) Price varies over time ( t ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market Price</td>
<td>Buyer pays seller a market price ( MP ) defined as to time period, place, and product specifications.</td>
<td>Eliminates possibility of lost market opportunity for either party.</td>
<td>Market price may not exist, represent a small proportion of transactions, or be highly unstable.</td>
</tr>
<tr>
<td>( P = MP )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market Price Proxy</td>
<td>Buyer pays seller a price determined by value of some agreed economic indicator. For example:</td>
<td>May be used to &quot;simulate a market&quot; if none exists.</td>
<td>Requires extensive statistical analysis or significant assumptions to verify the market model.</td>
</tr>
<tr>
<td>(a) ( P = a_2 + b_2 CPI )</td>
<td>(a) Price may vary with consumer price index CPI.</td>
<td>(a) Stabilizes buyer and seller purchasing power.</td>
<td>(a) Poorly related to any specific industry conditions.</td>
</tr>
<tr>
<td>(b) ( P = a_3 - b_3 I )</td>
<td>(b) Price may vary with inventory level ( I ) of buyer or seller.</td>
<td>(b) Stabilizes buyer and seller inventories; has theoretical merit as price determinant.</td>
<td>(b) Does not explicitly consider current production or future supply/demand expectations.</td>
</tr>
<tr>
<td>Cost-Plus</td>
<td>Buyer pays seller the latter's unit production cost ( UCS ) plus a premium. Cost may be specified to include total cost or only variable cost. Price may be determined by:</td>
<td>May discourage seller from making cost-saving technical changes. Seller may pad costs.</td>
<td></td>
</tr>
<tr>
<td>(a) ( P = UCS + K, K &gt; 0 )</td>
<td>(a) Adding a constant ( K ) to costs.</td>
<td>(a) Guarantees seller a fixed gross margin.</td>
<td></td>
</tr>
<tr>
<td>(b) ( P = (k)UCS, k &gt; 1 )</td>
<td>(b) Multiplying costs by a number ( k ) greater than one.</td>
<td>(b) Guarantees seller a rate of gross margin over cost.</td>
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<th>Advantages</th>
<th>Disadvantages</th>
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<tr>
<td><strong>Sales-Minus</strong></td>
<td>Buyer pays seller a portion of the commodity's unit resale value $URB$. Price may be determined by:</td>
<td></td>
<td>May discourage buyer from vigorous marketing efforts.</td>
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<tr>
<td>(a) $P = URB - L, L &gt; 0$</td>
<td>(a) Subtracting a constant $L$ from the buyer's unit resale value</td>
<td>(a) Guarantees buyer a fixed return over the indicated input.</td>
<td></td>
</tr>
<tr>
<td>(b) $P = (\mathcal{E})URB, 0 &lt; \mathcal{E} &lt; 1$</td>
<td>(b) Multiplying the buyer's unit resale value by a constant $\mathcal{E}$ less than one.</td>
<td>(b) Guarantees buyer a rate of return over the indicated input.</td>
<td></td>
</tr>
<tr>
<td><strong>Profit Share</strong></td>
<td>Buyer pays seller a portion of net unit profits $URB-UCB$ earned from the commodity's resale. Profit calculation may include or exclude fixed expenses. Price may be determined by:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) $P = (URB-UCB) - Z, Z &gt; 0$</td>
<td>(a) Subtracting a constant $Z$ from the buyer's unit profit.</td>
<td>(a) Buyer transfers entire profit risk to seller, an advantage if buyer risk in other activities is already near psychological limit.</td>
<td>May discourage buyer from vigorous profit-seeking behavior.</td>
</tr>
<tr>
<td>(b) $P = (URB-UCB)z, 0 &lt; z &lt; 1$</td>
<td>(b) Multiplying buyer's unit profit by a constant $z$ less than one.</td>
<td>(b) Buyer transfers part of profit risk to seller, an advantage if buyer is not willing to bear all additional profit risk.</td>
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<th>Advantages</th>
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<td><strong>Market Price, Cost-Plus Combination</strong></td>
<td>Buyer pays seller cost-plus unless this differs too much from the market price, in which event a compromise price is calculated. Maximum cost-plus, market price difference, and/or compromise price may be calculated on a constant or proportionate basis. Some combinations are:</td>
<td>Eliminates possibility of extreme market opportunity loss under cost-plus pricing. Shares advantages of market price and cost-plus.</td>
<td>Shares disadvantages of market price and cost-plus.</td>
</tr>
<tr>
<td>(a) ( P = (k)UCS ) if and only if (-M \leq [MP - (k)UCS] &lt; M; MP \pm M ) otherwise</td>
<td>(a) If cost-plus and market price differ by more than a constant ( M ), the market price plus/minus this constant is used.</td>
<td>(a) Buyer and seller can stipulate a fixed maximum market opportunity loss.</td>
<td>(a) Price acquires variance of market price when &quot;stuck&quot; at the maximum price differential. May have lower or higher coefficient of variation than market price when &quot;stuck.&quot;</td>
</tr>
<tr>
<td>(b) ( P = (k)UCS ) if and only if (-(m)MP \leq [MP - (k)UCS] \leq (m)MP; MP \pm (m)MP ) otherwise</td>
<td>(b) If cost-plus and market price differ by more than a percentage ( m ) of the latter, the market price plus/minus this percentage is used.</td>
<td>(b) Price acquires coefficient of variation of market price when &quot;stuck&quot; at the maximum price differential. May have lower or higher variance than price when &quot;stuck.&quot;</td>
<td>(b) Buyer and seller cannot set a fixed maximum market opportunity loss. Buyer's and seller's protection from opportunity loss falls as opportunity loss rises.</td>
</tr>
<tr>
<td>(c) ( P = (k)UCS ) if and only if (-(m)MP \leq [MP - (k)UCS] \leq n [MP - (k)UCS] ) otherwise</td>
<td>(c) If cost-plus and market price differ by more than a percentage ( m ) of the latter, the market price less a percentage of this difference is used.</td>
<td>(c) The price does not &quot;stick&quot; at the maximum cost-plus, market price differential; consideration of costs is retained beyond this point.</td>
<td>(c) Neither fixed nor proportionate maximum opportunity loss is guaranteed.</td>
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Cost-plus pricing and sales-minus pricing are commonly used in U.S. business. Frequently found in government contracts for heavily capitalized or innovative goods in which market prices are hard to find or meaningless, cost-plus pricing is especially popular with producers in inflationary periods. Sales-minus prices are identical to revenue shares, and are sometimes used by cooperatives for paying growers in "secondary pools."

Profit share arrangements are a simple extension of sales-minus formulae with the buyer's costs also measured. They are usually associated with cooperative associations' valuation of member product, but need not be restricted to cooperatives alone. An interesting application of this concept is in profit-share labor contracts.

Cost-plus, market price combinations are useful in cases where firms wish to take advantage of the security of cost-plus arrangements, but fear the opportunity costs of paying or receiving a price that is too different from the market price. Combination formulae are limited only by the imagination of contract participants.

PORTFOLIO SELECTION

Associated with each of the various contract price formulae in Table 1 is an expected level of return to buyer or seller and a particular degree of risk owing to variability in costs, market prices and other variables which determine the final price realized for the contracted good. In the fixed-price contract there is no price uncertainty but the sure return from such a contract must be compared with returns from alternative contracts which may have higher expected returns, but involve some risk. Often a combination of contracts may offer a lower degree of risk for a given level of expected (average) return than would be obtained from exchanging all the commodity under a single contractual arrangement.
The objective of portfolio analysis is to select from all the possible portfolios (combinations of contractual arrangements) the particular set of portfolios having the least risk for any given level of expected return from contracting. Expected return is the mean value of all possible monetary outcomes. Risk is measured by the variance of returns around the expected return. Variance is defined as the sum of squared differences between each possible return and the expected return.

**Characterizing Portfolios**

To illustrate the procedure, suppose a processing firm is considering a portfolio arrangement (call it strategy A) under which 30 percent of its processed product is sold on a market price basis and 70 percent on a three-year, cost-plus contract basis. Management wants to know how much it can expect to earn on this strategy over a three year period and how risky this strategy may be.

To compute these values we first specify a net margin equation which reflects the price terms of the contracts, the cost considerations, and the provisions of sales allocated to each arrangement. Such an equation might appear as follows:

\[
NM_c = Q_p \left[ 0.3MP_p + 0.7M(\text{MP}_t + \text{VC}_p) \right] - Q_p \text{VC}_p - FC_p + OR - QC_p
\]

where

- \(NM_c\) is cooperative net margin
- \(MP_p\) is market price per ton of processed product
- \(MP_t\) is market price per ton of raw product
- \(VC_p\) is variable costs per ton of processed product
- \(FC_p\) is fixed costs of processed product
- \(Q_p\) is tons of processed product produced for sale

(1)
X is tons of raw product needed to produce a ton of processed product

OR is other sources of cooperative revenue

OC is other sources of cooperative costs

m is a proportion representing the premium over coop variable cost paid to the cooperative under a cost-plus sales contract (m > 1)

The term in square brackets in equation (1) computes the average price received for the commodity sold. Thirty percent is sold at the market price, $MP_t$, and 70 percent is sold at a price determined by multiplying variable processing cost ($MP_t X + VC_p$) by a mark-up factor $m$, the latter greater than 1.0. The average price is multiplied by the total quantity sold, $Q_p$, to obtain total revenue. To this is added revenue from other sources and from it is subtracted all fixed and variable costs.

Some of the elements of equation (1) may be regarded as constant for planning purposes; others may take on values which vary with unforeseeable changes and, thus, may be regarded as random. In this case, $MP_t$, $MP_p$, $VC_p$, OR, and OC may be regarded as random variables and the other terms, FC, $Q_p$, $X$, and $m$, are specified as known quantities at any given time. The expected value of net margin, $E(NM)$, is determined by setting the random variables at their expected (mean) values. The multiplications, additions, and subtractions indicated by equation (1) are then carried out.

The variance of expected net margin associated with portfolio A is computed from the variance of each of the random variables and the covariances between pairs of random variables. The covariance is a measure of the degree to which a pair of random variables such as prices and costs move together or against one another over time. A statistical theorem states that the variance
of a sum of random variables (such as expected profit) is equal to the sum of 
the variances of each of the random variables plus twice the sum of the covari-
ances between all pairs of random variables. Letting \( \text{var}(\ ) \) indicate variance,
\( \text{cov}(i,j) \) the covariance of each pair of random variables, and \( \Sigma \) the sum of 
all these pairs, this theorem is shown as
\[
\text{var} (N_{Mc}) = \text{var}(0.3Q_{MP}) + \text{var}(0.7mQ_{MP X}) + \text{var}(0.7mP_{VC}) + \\
\text{var}(Q_{VC}) + \text{var}(OR) + \text{var}(OC) + 2 \Sigma \text{cov}(i,j) .
\] (2)

If written out fully, the term \( \Sigma \) in equation (2) would include covar-
i\( \leq j \)
iances between the following pairs of variables:
\[
\text{cov}(MP_{p}, MP_{t}) \quad \text{cov}(MP_{t}, OR) \\
\text{cov}(MP_{p}, VC_{p}) \quad \text{cov}(MP_{t}, OC) \\
\text{cov}(MP_{p}, OR) \quad \text{cov}(VC_{p}, OR) \\
\text{cov}(MP_{p}, OC) \quad \text{cov}(VC_{p}, OC) \\
\text{cov}(MP_{t}, VC_{p}) \quad \text{cov}(OR, OC).
\]

Some of the covariances may be zero or close to zero so the terms may be ignored.
Where the covariances are significant they may have an important influence on 
the variance of profit. If a covariance is negative, indicating that the vari-
ables tend to move in opposite directions, it reduces the overall variance of 
profit. For example, if rises in the market price, \( MP_{p} \), are associated with 
declines in other revenue, \( OR \), this would tend to reduce overall profit risk.
If, on the other hand, they tend to rise and fall together, profit risk is 
accentuated.

Equations provide the means for analyzing the behavior of a wide assortment 
of sales contract strategies involving the market price and cost-plus options.
Management needs to estimate the expectation (mean) and variance of the principal
random variables involved, \( MP_p, MP_t, VC_p, OR, OC \), and to supply values for the constants \( FC_p, Q_p, X, \) and \( m \) (Nelson, Casler, and Walker). The proportions associated with the two sales contract options are then free to vary. If strategy B is defined as 60 percent market price, 40 percent cost-plus, the corresponding expectation and variance are easily found by substituting 0.6, 0.4 for 0.3, 0.7, respectively in (1) and (2). Similarly, strategy C may be defined as 100 percent market price and 0 percent cost-plus.

Selecting the Best Portfolio

Suppose management has isolated several "reasonable" strategies A through F and wishes to leave to the board of directors the task of selecting the one it thinks best. This is conveniently done in the form of a diagram such as in Figure 1, which compares the expected profit and associated variance (E-V) for these alternative portfolios.

![Figure 1. A Hypothetical E-V Scatter and Frontier.](image-url)
It is up to the board to develop a criterion for selecting the "best" strategy from among A through F, but there is a simple preliminary elimination process to which most firms would not object. Note that strategy F provides the same net margin expectation as A but with higher variance (risk). Firms wishing to avoid unnecessary risks may reject strategy F outright. None of the originally proposed portfolios provides a net margin expectation equal to that of E and with lower variance. But a new portfolio with these characteristics, Z, can be formed by choosing the appropriate weighted average of portfolios A and B. The possibility of forming an infinite number of weighted averages similar to Z enables us to connect points D, B, Z, A, and C with a continuous line, at any point of which a unique market strategy is represented. Any points above line DBZAC are not acceptable to firms with the indicated distaste for risk-taking, since for any of these points, a new point can be found on DBZAC with equal net margin expectation and lower attendant risk.

Line DBZAC is often known as the E-V or expectation-variance frontier. An interesting property of this line is that, of the original set of six market strategies proposed by management to the board of directors, two have been automatically rejected. These two have been replaced by an infinite number of new strategies, among them the remaining four originally proposed. E-V line DBZAC contains only "efficient" portfolios; to move from one portfolio to another yielding higher expected return, it is necessary to accept higher risk as well.

Beyond constructing the E-V line, no one can choose for the cooperative board the contract portfolio it thinks best. Boards willing to suffer considerable risk may enjoy the higher profits associated with portfolios near C; portfolio C itself is the "maximum expected return" strategy. More risk
averse boards will prefer portfolios near D; portfolio D itself is a very conservative strategy which maximizes the board's certainty of what will happen. Marketing portfolios identified along the E-V line between C and D should be of special interest to farmer cooperatives since many cooperatives were formed with the express intention of reducing income fluctuations and attendant risk. It could be misleading for these coops to choose sales or purchase strategies on the basis of maximum expected return since such strategies are often so risky as to defeat the cooperative's justification for existence.

ILLUSTRATION OF PORTFOLIO ANALYSIS

The calculation of returns from alternative contracting arrangements and the application of portfolio analysis to the selection of optimal contracting strategies may be illustrated more fully by the results of a case study of an actual farmer cooperative processing firm. The firm processes a wide variety of fruits and vegetables and operates on a single-pool basis. It was considering entering into multi-year contracts for the sale of a particular line of processed tomato products and for the purchase of a portion of its corresponding raw product needs from nonmembers.

Three alternative price formulae were considered for use in both purchases and sales: market price, cost-plus, and sales-minus. The objective of the study was to develop information to aid the cooperative in deciding what proportions of the total proposed purchases and sales should be allocated to contracts with each of the alternative price formulae. The procedures described here are applicable to a wide number of agricultural goods besides tomatoes.
Specification of the Problem

To conduct the analysis it was necessary first to specify the equation which relates net margin to the parameters and variables of the portfolio problem; second to determine values to be assigned to price parameters and quantity terms, and third, to estimate expected values, variances, and covariances of the price, cost, and revenue variables.

The net margin equation. This equation is defined in a manner similar to the previous illustration in equation (1). It incorporates all sources of cooperative income and costs and takes account of the share of sales or purchases by each contracting strategy. For the case studied, it was expressed as

\[
NM_c = Q_p \left[ V_1 MP_p + V_2 m(MP_t X + VC_p) + V_3 REV_d \right] \\
- Q_t \left[ R_1 MP_p + R_2 kVC_t + R_3 MP_p \right] \\
- VC_p - FC_p - OR - OC
\]

(1)'

Recall from the discussion of equation (1) that \( MP_p \) is market price of the processed product, \( MP_t \) is the market price of the raw product, \( VC_p \) is variable cost per ton of processed product, \( FC_p \) is total fixed cost, \( Q_p \) is tons of processed product produced for sale, \( X \) is tons of raw product required to produce a ton of paste, \( OR \) is other sources of revenue, \( OC \) is other sources of cost, and \( m \) is the cost-plus markup to the distributor (a proportion greater than 1.0, representing the premium over variable costs). The additional terms introduced in equation (1)' are defined as follows:

- \( V_1, V_2, \) and \( V_3 \) are proportions of processed product sales by market price, cost-plus, and sales minus contracts, respectively \((V_1 + V_2 + V_3 = 1)\);
- \( R_1, R_2 \) and \( R_3 \) are the proportions of raw product purchases from nonmembers by market price, cost-plus, and sales-minus contracts, respectively \((R_1 + R_2 + R_3 = 1)\);
n is a proportion of the distributor's sales revenue paid to the cooperative under a sales-minus sales contract \((n > 1)\);

\(\ell\) is a proportion of the processed product's market price realized by the cooperative which is in turn paid to nonmember growers under a sales-minus purchase contract \((\ell < 1)\);

\(\text{REV}_d\) is the resale value to a distributor of processed product that is purchased from the cooperative and subsequently reprocessed;

\(Q_t\) is acres of raw product contracted for purchase from nonmember growers; and

\(\text{VC}_t\) is per acre grower variable cost.

The first term in square brackets of equation (1)' computes the average return per unit from the sale of processed product by the three alternative arrangements. The term \(m(\text{MP}_t X + \text{VC}_p)\) is the price for cost-plus sales. The actual cost of raw product would depend on the purchase portfolio ultimately adopted. However, valuing at market price \((\text{MP}_t)\) the raw product component of processing cost greatly simplifies the analysis.

The second term in square brackets computes the average cost per unit for the raw product purchases from nonmembers. Like the cost-plus sales option, the cost-plus purchase option \((k\text{VC}_t)\) refers to farm variable costs exclusively. The sales-minus purchase option is a "secondary pool"; participating growers receive a share \(\ell\) of the market value of processed product sales only, not of the total cooperative pool.

**Parameters and quantity terms.** The second task of the analysis was to consider possible values for the price parameters \(m, n, k,\) and \(\ell\) and the quantity terms \(Q_p\) and \(Q_t\). The cooperative is, of course, better off with higher values of \(m\) and \(n\) and lower values of \(k\) and \(\ell\). The specific values are limited by what may feasibly be achieved in contract negotiations. In the present illustration we have utilized values \(m = 1.30, n = 0.22, k = 1.30,\) and \(\ell = 0.40.\)
We have further assumed that 25 percent of the cooperative's raw product requirements are purchased from nonmember growers.

Terms $Q_p$ and $Q_c$ are the cooperative's proposed total sales and nonmember purchase volumes, respectively. They may represent the averages or sums of expected sales and purchase trends over some planning horizon. In the case study described here, it is assumed that 53,559 tons of tomatoes processed into 32 percent tomato paste are contracted for sale and that 12,680 acres of tomatoes are contracted for purchase from nonmembers. Future deviations from these estimates, if moderate, would probably not change the optimal portfolio proportions appreciably.

**Expected values, variances and covariances.** The expected value of net margin is calculated by replacing the variable items in equation (1) by their expected (average) values. The variance of the cooperative net margin is calculated as illustrated in equation (2) by using estimates of the variances and covariances of each of the variable items in the net margin equation. (See footnote 2 and the research report by Buccola and French for suggestions on calculating averages, variances, and covariances.) Note that the values of expected net margin and variance of net margin change with changes in the portfolio proportions $V_1$, $V_2$, $V_3$ and $R_1$, $R_2$ and $R_3$. For example, if the market price sales option has higher expected return than the cost-plus sales option, total expected net margin increases as $V_1$ increases and as $V_2$ decreases. Of course, the degree of risk (as measured by the variance) might increase in this case as well.

Expectations, variances, and covariances of revenue and cost variables for (1)' were estimated on the basis of educated guesses of industry and academic researchers and on historical data. For the sake of brevity, most of these are
not reported here. Expected values (means) and variances of the principal variables are shown in Table 2.

Table 2. Expected Values and Variances of Important Profit Factors

<table>
<thead>
<tr>
<th>Item</th>
<th>Unit</th>
<th>Expected Value</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processed product market price</td>
<td>$/ton raw</td>
<td>81.33</td>
<td>370.05</td>
</tr>
<tr>
<td>Raw product market price</td>
<td>$/ton</td>
<td>32.21</td>
<td>54.81</td>
</tr>
<tr>
<td>Raw product yield</td>
<td>tons/acre</td>
<td>26.35</td>
<td>0.49</td>
</tr>
<tr>
<td>Processed product revenue/acre</td>
<td>$/acre</td>
<td>2,143.04</td>
<td>256,875.54</td>
</tr>
<tr>
<td>Raw product production cost/acre</td>
<td>$/acre</td>
<td>848.73</td>
<td>45,098.61</td>
</tr>
</tbody>
</table>

a/ Processed and raw product in this case refer to 32% tomato paste and tomatoes, respectively. Data represent annual forecasts for the 1975-84 period and are expressed in 1974 dollars.

The E-V Frontier

The specifications noted above provide the basic information required to calculate E-V curves which define efficient contract portfolios. One calculation method is to compute expected returns and variances for a variety of portfolio possibilities and then construct an E-V frontier curve as described in Figure 1. A more precise and efficient procedure, employed here, is to utilize a computer routine that traces out the E-V curves and ignore points associated with inefficient portfolio strategies above it. Figure 2 shows the E-V curve that is generated using the means, variances, and other specifications listed above. The specific portfolio solutions from which the curve was plotted are given in Table 3. To generate this curve, it was assumed that the cooperative's board of directors has set a blanket prohibition against committing more than 40 percent of its processed product sales to a nonmarket price basis. This requirement is especially meaningful if market price sales represent noncontracted transactions, if the sales which are contracted are done so with a single buyer, and if the board feels that committing a high proportion of
Figure 2. E-V and Coefficient-of-Variation Curves for Cooperative's Contract Portfolio Problem.
Table 3. Efficient Portfolios of Market Price, Cost-plus, and Sales-minus Contracts for Cooperative Processed Product Sales and Raw Product Purchases

<table>
<thead>
<tr>
<th>Moments and Coefficients of Net Margin</th>
<th>Processed Product Sales\textsuperscript{a/}</th>
<th>Raw Product Purchases\textsuperscript{a/}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean\textsuperscript{b/}</td>
<td>Variance\textsuperscript{b/}</td>
</tr>
<tr>
<td>Million Dollars</td>
<td>Trillion Dollars</td>
<td></td>
</tr>
<tr>
<td>288.5</td>
<td>340.2</td>
<td>.0639</td>
</tr>
<tr>
<td>289.6</td>
<td>345.2</td>
<td>.0642</td>
</tr>
<tr>
<td>293.8</td>
<td>397.4</td>
<td>.0678</td>
</tr>
<tr>
<td>295.0</td>
<td>411.8</td>
<td>.0687</td>
</tr>
<tr>
<td>298.0</td>
<td>450.6</td>
<td>.0712</td>
</tr>
<tr>
<td>302.8</td>
<td>549.7</td>
<td>.0774</td>
</tr>
</tbody>
</table>

\textsuperscript{a/} Values listed under V_1, V_2, V_3 are percentages of 53,559.31 tons (200,048 55-gallon barrels) of tomato paste contracted for sale; values listed under R_1, R_2, R_3 are percentages of 12,680 acres of raw tomatoes contracted for purchase. (12,680 acres at 26.35 tons/acre, or 334,080 tons of tomatoes at 5.46% solids, produce 53,559.31 tons paste 32%.) Only 25% of tomatoes are assumed contracted with nonmember growers.

\textsuperscript{b/} The mean, variance, and coefficient of variation shown here refer to the sum of profits over the 10-year planning horizon. Average annual figures might just as well be used.
sales to one buyer would put it in a disadvantageous negotiating position. In the frontier shown, the market price provides the highest revenue expectation and variance of all sales options. Thus, the prohibition in these frontiers against exclusively cost-plus or sales-minus sales has eliminated many low-gain, low-risk strategies from the cooperative's choice set.

A close look at this E-V frontier reveals a variety of efficient marketing portfolios. Although open market sales predominate at all points because of the constraint \( V_2 + V_3 \leq 0.40 \), all three sales options are employed simultaneously in the low-risk, low-gain region. As portfolios with higher net margin expectations are considered, the cost-plus sales option is gradually eliminated. The sales-minus sales option declines more slowly and does not disappear until portfolios with the highest net margin expectation are considered. On the raw product purchase side, the sales-minus option is the only efficient option in the low-risk, low-expected return region; this option is gradually joined by the cost-plus, then market price formulae as net margin expectation increases. The cost-plus purchase contract is a "middle range" performer. It neither maximizes expected gain nor minimizes risk, but performs sufficiently well in each category to be included in some efficient portfolios.

**Alternative Risk Measures**

The measurement of risk by the variance associated with expected net margin, while mathematically proper, may present interpretation problems for many decision makers. Such large abstract numbers are difficult to compare in a meaningful sense.

An alternative measure is the coefficient of variation. The coefficient of variation expresses the standard deviation (the square root of the variance)
as a proportion of the mean or expected value. If the values about the mean are distributed approximately normally, the coefficient of variation may be interpreted specifically in terms of the probability of various relative deviations from the mean value. For example, if the coefficient of variation is .2, it would mean that there is about one chance in six that the actual return would be more than 20 percent below the expected value. (See any elementary statistics text, such as Mendenhall, pp. 132-135.)

The coefficient of variation associated with various portfolios shows whether an increase in risk from one portfolio to another is high or low in relation to the increase in promised long-run earnings. Coefficient-of-variation curves do not always, or even usually, run parallel to variance (E-V) curves. In the frontier shown in Figure 2, the former curve (represented by the dotted line) rises more slowly than the latter, suggesting that risk relative to long-run earnings increases less quickly than does the risk of earnings itself.

Coefficient-of-variation readings may be useful when it comes time for the cooperative to select a specific portfolio from the efficient set. In the presently illustrated situation, for example, the board may decide it wishes to maximize net margin subject to the restriction that relative risk (coefficient of variation) not exceed .07. This would suggest there would be only about one chance in six of a return of more than seven percent below the expected return. The selected portfolio is then roughly 80 percent market price sales, 20 percent sales-minus contract sales.

Evaluation of Major Effects

A review of the effects of changes in coefficients of variation and covariances in this study suggests some general conclusions with respect to how
variations in these values may influence portfolio choices. These conclusions apply specifically only to the tomato processing case studied. However, the authors believe they generally apply to a wide range of other food processing industries as well.

1. Coefficient-of-variation effects

   a) Cash costs of food processing have significantly lower coefficients of variation than do market prices of the raw products handled or final products produced. Because of this, risk averting processors derive benefit from favoring contract sales on a cost-plus basis if the expected net margins under cost-plus and market price options are about equal.\(^5\)

   b) The coefficients of variation of market prices of agricultural products tend to decrease as these products move through the market chain and/or become further processed. Observing this effect alone, risk averse processors would tend to favor sales-minus over market price contracts, all expected values equal.\(^6\)

2. Covariance Effects

   c) The strong positive covariances found between the market prices of raw products and those of finished products tend to discourage simultaneous use of sales-minus contracts and market price contracts for final product sales.

   d) But these same positive covariances tend to encourage sales-minus contracts as sales instruments when they are already employed as purchase instruments and vice versa. This is because positive covariances between revenue and cost variables are negative covariances insofar as they affect total cooperative net margin; that is, they
reduce net margin variance and increase the expected satisfaction of the risk averse processor.

e) For the same reason as (d), market price contracts as sales instruments perform well alongside market price contracts as purchase instruments. Furthermore, market price contracts perform well alongside sales-minus contracts provided one is a sales instrument and one is a purchase instrument.

INTERPRETING CONTRACT PORTFOLIO STUDIES

The principal advantage of portfolio analysis of contracts is that it summarizes in an orderly way the long-term profitability and risk of alternative contract portfolios. However, portfolio analysis has important shortcomings as well. The most important difficulty is selecting the set of price parameters and other assumptions that are appropriate to a given situation. For example, cost-plus markups, \( m, k \), and sales-minus markdowns, \( n, \ell \), utilized in E-V curves should be those jointly acceptable by the cooperative and its trading partners. Selection of these price parameters is often a matter of negotiation between buyer and seller. Since the amount a buyer and seller wish to trade on a given set of price terms may diverge, it may be useful to construct several tentative E-V frontiers as negotiations proceed. The "best" portfolio indicated in each sequential frontier provides the amount a cooperative wants to trade under the currently considered price terms. Roughly speaking, a contract agreement is reached when the amounts the buyer wishes to purchase and the amounts the seller wishes to dispose of become equal for a given set of price parameters.
Another difficulty is that it may be impractical to include in an E-V curve all the information a cooperative feels important in decision making. For example, it is difficult to elicit from portfolio analysis an optimal duration of contract or to compare transaction arrangements of different durations. Specifically, portfolio analysis requires that all contract options considered be defined over the identical planning horizon and that total traded quantity be fixed, possibly at the average quantity expected to be traded over the planning horizon.

An inevitable problem that a cooperative encounters is selection of the decision rule necessary for identifying the "best" contract strategy. This final decision criterion seems best placed in the hands of the cooperative board. Even so, it is important that board members represent not their personal attitudes toward risk taking, but their most accurate assessment of the proclivities toward risks of their membership constituency. Such proclivities are often expressed in general terms in the cooperative's articles of incorporation or by-laws, such as in the intention to "maximize returns" or to "provide a secure home." However it is usually not possible to find a choice criterion that will please all concerned.

Above all, it is not likely that portfolio analysis alone will convince a firm to market some of its goods by contract. The decision to employ contracts arises essentially from a belief that they promote income security for farmer-members of cooperatives, and thus promote one of the most fundamental purposes of cooperative organization. Once this belief is adopted, contract portfolio analysis can, despite its inherent simplifications, provide a valuable analytic tool for cooperative policy makers.
FOOTNOTES

1/ Unlike corporate firms, cooperatives may not wish to deduct from revenues the market value of raw products transferred from members. The result is what can be called a "net margin" rather than "profit" function.

2/ Expectation and variance are easily calculated. Suppose the possible returns and their probabilities are: 30 percent chance of $10 million, 45 percent chance of $12 million, and 25 percent chance of $14 million. The expectation of this distribution is found by multiplying each probability by its payoff, then summing: $(.3)(10) + (.45)(12) + (.25)(14) = \$11.9$ million.

Its variance is found by multiplying each probability by the square of the distance between its payoff and the expected payoff: 

$(.3)(10-11.9)^2 + (.45)(12-11.9)^2 + (.25)(14-11.9)^2 = \$2.001$ million.

The covariance, Cov ($X,Y$), between two variables $X$ and $Y$ may be calculated as $\text{Cov}(X,Y) = \sigma_X \sigma_Y \rho_{XY}$, where $\sigma_X$ and $\sigma_Y$ are, respectively, the square roots of the variances of $X$ and $Y$ and $\rho_{XY}$ is the correlation between $X$ and $Y$. See the discussion of Nelson, Casler, and Walker, pp. 4-5 to 4-15, on development of subjective probabilities of occurrences.

3/ The set of weighted average portfolio proportions that represent $Z$ is found by multiplying strategy B's proportions by distance $YA$, strategy A's proportions by distance $YB$, summing these products across contract options, and dividing each by distance $AB$. Thus, if $A = .2X_1 + .8X_2$, $B = .8X_1 + .2X_2$, $AB = 10$, $YB = 3.3$, and $YA = 6.7$, then $Z = .6X_1 + .4X_2$. $Z$ falls below the straight line connecting $B$ and $A$ since a weighted average variance is formed with the squares of these fractional weights. (See the referenced book by Sharpe.)
A more general method of developing a set of efficient portfolios is known as stochastic dominance analysis. This method involves forming a large number of potential portfolios and calculating the cumulative probability function of each. The E-V method is much simpler to calculate and in any event is equally valid if variables are approximately normally distributed.

In this and subsequent statements, "favoring" an option is used to mean that a higher portfolio proportion is assigned to this option than to other options. It is not meant that this option is chosen to the exclusion of other options, unless the decision maker is extremely risk averse or is risk-neutral.

For example, the share \( n \) of revenue from reprocessed tomato paste which a paste processor would have to receive from a distributor to equate the expected sales-minus price of paste to the expected market price of paste was 25.3 percent. At this revenue share, the variance of the sales-minus price was $746,269, and of the market price, $2,568,755.
REFERENCES


Errata for Bulletin 655

The second and third lines of equation (1)', p. 16 should read

\[-Q_t \left[ R_1^{\text{MP}} Y + R_2 k^{\text{VC}} + R_3^{\text{MP}} Y \right]\]

\[- VC_p - FC_p + OR - OC\]

and the list of definitions on p. 17 should include

Y is farm tomato yields in tons per acre;

k is cost-plus markup to the farmer (k > 1).