MULTISPECIES FISHERIES. A PREDATOR-PREY MODEL FOR THE EUROPEAN HAKE AND BLUE WHITING SPANISH FISHERIES

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Abstract:

The aim of this work is to develop a predator-prey model for two species of commercial importance captured by the European fishing fleet in the European fishing grounds. In this model, the hake (*Merluccius merluccius*) represents the predator, and the blue whiting (*Micromesistius poutassou*) is the prey. Both the predator and prey population dynamics follow the Lotka-Volterra formulation. It is assumed a logistic population dynamics and a linear interaction between predator and prey populations, with two interaction coefficients: α (effect of a unit change in the prey on the percent growth rate of the predator) and β (attack rate or searching efficiency of the predator). The populations interact randomly in proportion to population density. Logistic predator-prey equations were applied to the hake and blue whiting stocks, including biomass, intrinsic rates of growth, carrying capacity and capture for both species. The goal is to maximize the present value of profit, forming the current value Hamiltonian for the maximization problem. Capture costs depending on stock and prices of hake and blue whiting and discount rate were introduced at this point. Once the theoretical model was made, landings and biomass data from both stocks over the period 1988-2014 were used for an econometric Ordinary Least Squares estimation, to determine the form taken by the predator-prey net growth functions. Optimal biomass, catches and profits were determined by solving the applied model with quadratic net growth functions. The sensitivity analysis results show that the landings profits are larger at low discount rates.

1. INTRODUCTION

The multi-species bioeconomic models try to evaluate and predict the population dynamics and economic performance of the mixed fisheries, which show ecological interdependence between two or more target species. In particular, the predator-prey models take into account the trophic relationships between two or more ecologically interdependent species in the same ecosystem. The European Union fishing fleets usually capture species in mixed fisheries. The possibilities of fishing these species must be determined jointly by using multi-species models, because catches of one species will impact the natural growth of the others. The multi-species approaches in fisheries management are increasingly recommended by scientists because single-species models cannot properly represent the complex feedbacks resulting from predatory interactions. Multi-species models can explicitly consider the interactions resulting from predation to predict the dynamics of commercial populations. This is especially important in the evaluation of management strategies.

This work develops the application of a bioeconomic multi-species predator-prey model with capture to a mixed fishery composed of two commercial species caught by the European fishing fleet operating in the EU fishing grounds. The selection criteria for the species of this model were economic (two species of commercial importance), geographic (species caught in the EU Atlantic waters by the European fishing fleet) and biological (species with a significant trophic interaction). According to these criteria, the selected predatory species was the European hake (*Merluccius merluccius*) and the selected prey was the blue whiting (*Micromesistius poutassou*). The study period is 1988-2014. Biomass and catch data from that period will be used. From Brown *et al.* (2005), costs dependent on the level of biomass are introduced into the net benefits function and the optimum levels for biomass and catches of both species are estimated. In Section 2 hake and blue whiting fisheries are described. After the construction of the theoretical model using the Maximum Principle of the Optimal Control Theory (Section 3), the applied model is resolved in Section 4, the sensitivity analysis is shown in Section 5, and finally the main conclusions are highlighted in Section 6.

2. HAKE AND BLUE WHITING FISHERIES

European hake (*Merluccius merluccius*) and blue whiting (*Micromesistius poutassou*) are marine species of commercial value in North-eastern Atlantic grounds. European hake is a demersal benthopelagic species widely distributed throughout the European waters at depths from 30 to 1,000 meters. The hake is caught in mixed fisheries by a multi-rigged fleet. The stock status was critical in the 1990s, but it has recovered after the management plans implemented by the European Union (EC 2004, 2005), and the landings have increased. Blue whiting is a demersal species of the gadus family widely distributed throughout the North Atlantic at depths from 150 to 1,000 meters. Its diet is composed mainly of crustaceans, namely copepods, krill, the larvae of decapods and a decapod known as white glass shrimp (*Pasiphaea sivado*). The primary method used to catch the blue whiting as a target species is the bottom pair trawling, and individual bottom trawling picks it up as a by-catch. This species plays a major role in fishing quota exchanges between European countries to obtain higher-value species. The blue whiting's biomass has been significantly reduced over the past several years, but now the stock status is good.

Table 1. Hake and blue whiting biomass and landings (tons). 1988-2014

	Hake biomass (X_t)	**Blue Whiting biomass (Y_t)**	**Hake landings ($h_{t,m}$)**	**Blue Whiting landings ($h_{t,L}$)**
1988	118588	1615000	81476	558000
1989	109002	1550000	80259	627000
1990	96661	1334000	73144	562000
1991	92221	1732000	70956	370000
1992	89329	2546000	70415	475000
1993	76533	2637000	63628	481000
1994	66594	2523000	61124	459000
1995	71535	2.294000	69860	579000
1996	66257	2180000	56925	646000
1997	56283	2471000	50963	672000
1998	53453	3757000	42743	1125000
1999	60443	4611000	46984	1256000
2000	68171	4291000	49928	1412000
2001	66339	4648000	44255	1780000
2002	70492	5184000	46797	1556000
2003	76179	6934000	49906	2321000
2004	77313	6689000	53359	2378000
2005	73287	5850000	54883	2027000
2006	74754	5885000	52283	1966000
2007	83886	4672000	59960	1612000
2008	101518	3489000	64534	1246000
2009	147964	2610000	78058	636000
2010	209575	2538000	88167	540000
2011	244663	2572000	104602	105000
2012	230370	3396000	100250	384000
2013	221233	3918000	89106	626000
2014	222132	3965000	103400	1146000

Source: ICES

Blue whiting is in general the hake’s main prey in these grounds according to several studies. In particular, the blue whiting is the hake's primary prey in Portuguese waters according to Cabral and Murta (2002). In the Cantabrian Sea, the blue whiting is also noted as the hake's primary prey by Sánchez (1993) and Velasco and Olaso (1998). However, its importance as prey is moderate in the Celtic Sea according to Mahe et al. (2007).

3. THEORETICAL MODEL

The theoretical model is based on Lotka-Volterra's predator-prey model (Volterra 1926; Lotka 1932), and the logic equations used by Brown et al. (2005):

$$\frac{dX}{dt}=r_m X[1-\frac{X}{\bar{X}}]-h_m+\alpha XY \quad (1)$$

$$\frac{dY}{dt}=r_L Y[1-\frac{Y}{\bar{Y}}]-h_L-\beta YX \quad (2)$$

where X denotes the predator's stock, Y is the prey's stock; r_m and r_L denote the intrinsic population growth rates for both species, respectively; h_m and h_L are the predator and prey catches, and α and β denote the interaction coefficients. The net benefits from fishery at moment t are defined as follows:

$$\pi(X,Y,h_m,h_L)=(P_m-C_m(X))h_m(t)+(P_L-C_L(Y))h_L(t) \quad (3)$$

Where subscripts m and L denote predator and prey, P is the unit price, and C the catch unit cost. For ρ as social discount rate, the objective function is as follows:

$$J=\int_0^\infty e^{-\rho t}\pi[X(t),Y(t),h_m(t),h_L(t)]dt \quad (4)$$

Under these premises, the central manager must solve the following problem:

$$max\int_0^\infty e^{-\rho t}[(P_m-C_m(X))h_m(t)+(P_L-C_L(Y))h_L(t)]dt \quad (5)$$

s.t.

$$\frac{dX}{dt}=f(X)-h_m$$

$$\frac{dY}{dt}=g(Y)-h_L$$

$$0\leq h_m(t)\leq h_m max$$

$$0\leq h_L(t)\leq h_L max$$

$$0<X(t)$$

$$0<Y(t)$$

Given the Hamiltonian function is lineal in control variables, applying optimum control theory (Clark, 1976; Clark and Munro, 1985; Rozonoer, 1959; Spence and

Starrett, 1975), and adapting it to the multispecies case, we obtain the equilibrium equations for (5):

$$[F'(X_u)+\alpha Y_u]-\frac{[P_L-C_L(Y_u)]\beta Y_u+C_m'(X_u)[F(X_u)+\alpha X_u Y_u]}{(P_m-C_m(X_u))}=\rho \quad (6)$$

$$[G'(Y_u)-\beta X_u]+\frac{[P_m-C_m(X_u)]\alpha X_u-C_L'(Y_u)[G(Y_u)-\beta Y_u X_u]}{(P_L-C_L(Y_u))}=\rho \quad (7)$$

These two equations and constitute a system whose solution would give us the steady state optimal biomass levels for predator and prey.

4. APPLIED MODEL

The hake and blue whiting biomass and catch data for the 1988-2014 period, published by ICES (2015a, 2015b), were used for a econometric regression by the ordinary least squares method, in order to determine the functional form of the dynamics of each fish stock. In addition to the standard quadratic form, the following exponential and potential expressions, respectively, were considered for both species:

$$X_{t+1}=\alpha X_t-\beta X_t^2+\gamma X_t Y_t-h_m \quad (8)$$

$$Y_{t+1}=\varphi Y_t-\mu Y_t^2+\omega X_t Y_t-h_L \quad (9)$$

$$X_{t+1}=\alpha e^{\beta X_t}+\gamma X_t Y_t-h_m \quad (10)$$

$$Y_{t+1}=\varphi e^{\mu Y_t}+\omega X_t Y_t-h_L \quad (11)$$

$$X_{t+1}=\alpha e^{\beta X_t+\gamma X_t Y_t}-h_m \quad (12)$$

$$Y_{t+1}=\varphi e^{\mu Y_t+w X_t Y_t}-h_L \quad (13)$$

The results of the econometric estimations for hake and blue whiting stock dynamics are shown in Tables 2 to 7.

Table 2. Results of the econometric estimations for hake quadratic stock dynamic.

Model 1: OLS, observations 1988-2014 (T = 27)
Dependent variable: endomerlu

	Coefficient	*St. deviation*	*t-statistic*	*p-value*	
xmer	1.98802	0.114474	17.3666	<0.0001	***
sq_xmer	−2.94291e-06	4.05357e-07	−5.4962	<0.0001	***

Model 1: OLS, observations 1988-2014 (T = 27)
Dependent variable: endomerlu

xy	1.83165e-09	2.39363e-08	0.0161	0.9873	

Dep.vble. mean	180815.7	S.D. of dep. vble.	84253.74
Sum of squared res.	7.23e+09	S.D. of the regression	17357.50
R-square	0.993225	Corrected R-square	0.992661
F(3, 24)	1172.854	p-value (F)	3.76e-26
Log-likelihood	−300.2893	Akaike criterion	606.5787
Schwarz criterion	610.4662	Hannan-Quinn crit.	607.7346
rho	0.488674	Durbin-Watson	0.990641

Table 3. Results of the econometric estimations for blue whiting quadratic stock dynamic.

Model 2: OLS, observations 1988-2014 (T = 27)
Dependent variable: endolirio

	Coefficient	*St. deviation*	*t-statistic*	*p-value*	
ylirio	1.49868	0.151384	9.8999	<0.0001	***
sq_ylirio	−8.21632e-08	2.2982e-08	−1.0099	0.3226	
xy	−3.12827e-06	6.59481e-07	−1.3164	0.2005	

Dep.vble. mean	4658741	S.D. of dep. vble.	2097691
Sum of squared res.	1.02e+13	S.D. of the regression	651719.3
R-square	0.985446	Corrected R-square	0.984233
F(3, 24)	541.6818	p-value (F)	3.61e-22
Log-likelihood	−398.1802	Akaike criterion	802.3605
Schwarz criterion	806.2480	Hannan-Quinn crit.	803.5164
rho	0.439072	Durbin-Watson	1.120075

Table 4. Results of the econometric estimations for hake exponential stock dynamic.

Model 3: OLS, observations 1988-2014 (T = 27)
Dependent variable: l_endomerlu

	Coefficient	*St. deviation*	*t-statistic*	*p-value*	
xmer	6.2841e-05	2.66062e-05	2.3619	0.0263	**
xy	7.13e-012	7.7206e-012	0.9235	0.3646	

Dep.vble. mean	11.97023	S.D. of dep. vble.	0.384159
Sum of squared res.	792.7804	S.D. of the regression	5.631271
R-square	0.795283	Corrected R-square	0.787094
F(3, 24)	48.55990	p-value (F)	2.45e-09
Log-likelihood	−83.93742	Akaike criterion	171.8748
Schwarz criterion	174.4665	Hannan-Quinn crit.	172.6455
rho	0.967083	Durbin-Watson	0.062242

Table 5. Results of the econometric estimations for blue whiting exponential stock dynamic.

Model 4: OLS, observations 1988-2014 (T = 27)
Dependent variable: l_endolirio

	Coefficient	*St. deviation*	*t-statistic*	*p-value*	
ylirio	2.85098e-06	5.85171e-07	4.8721	<0.0001	***
xy	8.2693e-012	5.32886e-012	1.5518	0.1333	

Dep.vble. mean	15.25821	S.D. of dep. vble.	0.447234
Sum of squared res.	815.3606	S.D. of the regression	5.710904
R-square	0.870396	Corrected R-square	0.865211
F(2, 24)	83.94738	p-value (F)	8.09e-12
Log-likelihood	−84.31655	Akaike criterion	172.6331
Schwarz criterion	175.2248	Hannan-Quinn crit.	173.4037
rho	0.876967	Durbin-Watson	0.169199

Table 6. Results of the econometric estimations for hake potential stock dynamic.

Model 5: OLS, observations 1988-2014 (T = 27)
Dependent variable: l_endomerlu

	Coefficient	*St. deviation*	*t-statistic*	*p-value*	
xyL_xmer	2.01014e-012	2.43724e-013	8.2476	<0.0001	***

Dep.vble. mean	11.97023	S.D. of dep. vble.	0.384159
Sum of squared res.	1070.873	S.D. of the regression	6.417743
R-square	0.723472	Corrected R-square	0.723472
F(2, 24)	68.02305	p-value (F)	9.94e-09
Log-likelihood	−87.99664	Akaike criterion	177.9933
Schwarz criterion	179.2891	Hannan-Quinn crit.	178.3786
rho	0.985883	Durbin-Watson	0.068781

Table 7. Results of the econometric estimations for blue whiting potential stock dynamic.

Model 6: OLS, observations 1988-2014 (T = 27)
Dependent variable: l_endolirio

	Coefficient	*St. deviation*	*t-statistic*	*p-value*	
xyL_ylirio	2.03629e-012	2.34986e-013	8.6656	<0.0001	***

Dep.vble. mean	15.25821	S.D. of dep. vble.	0.447234
Sum of squared res.	1618.020	S.D. of the regression	7.888698
R-square	0.742810	Corrected R-square	0.742810
F(2, 24)	75.09263	p-value (F)	3.83e-09
Log-likelihood	−93.56849	Akaike criterion	189.1370
Schwarz criterion	190.4328	Hannan-Quinn crit.	189.5223
rho	0.978454	Durbin-Watson	0.075548

Based on the results shown in tables 2-7 and taking into consideration also the value of R^2 adjusted, the quadratic expression is most appropriate for the natural dynamic of predator and prey in these grounds (value for R^2 is higher than 0.98 for both species). On other hand, only that expression shows a positive predator-prey interaction coefficient for predator while at the same time negative for prey, which would be typical of a trophic relationship. As a consequence, the quadratic forms as follows for predator and prey, respectively:

$$\dot{X} = \alpha X - \beta X^2 + \gamma XY - h_m = 1.98802\, X - 0.0000029429 X^2 + 0.00000000183165\, XY - h_m \quad (14)$$

$$\dot{Y} = \varphi Y - \mu Y^2 - \omega XY - h_L = 1.49868\, Y - 0.0000000821632\, Y^2 - 0.00000312827\, XY - h_L \quad (15)$$

Starting from the theoretical model, and once we know that the stock dynamics are quadratic, we can set the applied model including the growth functions and the cost functions (16) and (17) in the expressions (6) and (7). We obtain the expressions (18) and (19):

$$C_m(X_t) = a - bX_t; \qquad C_m{}'(X_t) = -b \quad (16)$$

$$C_L(Y_t) = c - dY_t; \qquad C_L{}'(Y_t) = -d \quad (17)$$

$$[(\alpha-2\beta X_u)+\gamma Y_u]-\frac{[P_L-(c-dY_u)]\omega Y_u+(-b)[(\alpha X_u-\beta X_u^2)+\gamma X_u Y_u]}{P_m-(a-bX_u)}=\rho \quad (18)$$

$$[(\varphi-2\mu Y_u)-\omega X_u]+\frac{[P_m-(a-bX_u)]\gamma X_u-(-d)[(\varphi Y_u-\mu Y_u^2)-\omega Y_u X_u]}{P_L-(c-dY_u)}=\rho \quad (19)$$

And the hake and blue whiting biomass levels corresponding to the maximum sustainable yield (MSY) are given as follows:

$$X_{mrs}=\frac{\frac{\alpha}{\gamma}-\frac{\varphi}{2\mu}}{\frac{-\omega}{2\mu}+\frac{2\beta}{\gamma}} \quad (20)$$

$$Y_{mrs}=\frac{\alpha-2\beta X_{mrs}}{\gamma} \quad (21)$$

The equations (18) and (19) system was solved by iterations in order to determine the optimum values of the hake and blue whiting biomasses, substituting in the equations the 2001-2014 average prices in constant 2014 monetary units (P_m=4619.23 €/tonne; P_L=399.13 €/tonne), the cost functions and the discount rate rho = 0.05. The MSY biomasses were calculated from the expressions (20) and (21). The shadow prices were calculated from the commutation functions, introducing the discount rate rho = 0.05, the 2001-2014 average prices in constant 2014 monetary units and the capture cost functions. The benefits produced by the capture of both species were calculated from the expression (3), introducing the 2001-2014 average prices, the cost functions and the optimal capture values. The results are shown in Table 8.

Table 8. Optimum levels of biomass, catches, MSY biomass, shadow prices and benefits

	Hake	**Blue Whiting**
Optimal biomass	281,000 t.	4,871,500 t.
Optimal catch	47,800 t.	1,068,700 t.
MSY biomass	336,922 t.	2.706,178 t.
Shadow prices	3044.54 €/t.	171.40 €/t.
Benefits	1052 mill. €	193 mill. €
Total benefits	1245 mill. €	

5. SENSITIVITY ANALYSIS

The sensitivity of the model was analyzed performing different simulations of variation of the discount rate and the prices of both species, recalculating the benefits generated by the capture of both species. The results of the sensitivity analysis are shown in Table 9.

Table 9. Optimum benefits under different discount rates and prices.

Simulations		π_m	π_L	π_{total}	var. π_m	var. π_L	var. π_{total}
rho	-0.05	1,098.15	146.92	1,245.07	104.4%	76.3%	100.0%
	0	1,079.19	166.47	1,245.66	102.6%	86.4%	100.1%
	0.05	**1,052.26**	**192.57**	**1,244.83**	**100.0%**	**100.0%**	**100.0%**
	0.1	1,034.46	207.96	1,242.42	98.3%	108.0%	99.8%
	0.15	1,016.14	222.36	1,238.51	96.6%	115.5%	99.5%
P_m	**P_L**						
4400	399.13	1,000.78	171.26	1,172.04	95.1%	88.9%	94.2%
4500	399.13	1,031.18	174.13	1.205.31	98.0%	90.4%	96.8%
4619.23	**399.13**	**1,052.26**	**192.57**	**1,244.83**	**100.0%**	**100.0%**	**100.0%**
4700	399.13	1,070.04	201.55	1,271.60	101.7%	104.7%	102.2%
4800	399.13	1,099.85	204.79	1,304.65	104.5%	106.4%	104.8%
P_L	**P_m**						
300	4619.23	1,014-79	126.48	1,141.27	96.4%	65.7%	91.7%
350	4619.23	1,020.35	173.57	1,193.92	97.0%	90.1%	95.9%
399.13	**4619.23**	**1,052.26**	**192.57**	**1,244.83**	**100.0%**	**100.0%**	**100.0%**
450	4619.23	1,064.03	223.09	1,287.12	101.1%	115.9%	103.4%
500	4619.23	1,078.32	241.87	1,320.19	102.5%	125.%	106.1%
P_m	**P_L**						
4700	450	1,026.52	302.37	*1,328.89*	97.6%	157.0%	106.8%
4500	350	1,090.15	78.35	1,168.51	103.6%	40.7%	93.9%
4619.23	**399.13**	**1,052.26**	**192.57**	**1,244.83**	**100.0%**	**100.0%**	**100.0%**
4700	350	1,070.90	152.30	1,223.21	101.8%	79.1%	98.3%
4500	450	1,036.97	208.99	1,245.96	98.5%	108.5%	100.1%

As we can see in the table, as the rate of discount diminishes, the total net benefits increase, being greater the sensitivity for the hake's profits. Regarding prices, the increase in the price each species always generates an increase in the net benefits overall, with a higher sensitivity shown by blue whiting's benefits. The higher net benefits are obtained by a simultaneous increment in the price of both species.

6. CONCLUSIONS

This research work has developed the application of a multispecies bio-economic model of the predator-prey type to the mixed fishery of hake and blue whiting of the EU Atlantic waters. These species display a significant ecological predatory interdependence, along with a commercial importance for the European fishing fleet which makes them suitable for applying this type of model. Optimal biomasses, optimal catches, MSY biomasses, shadow prices and benefits were determined for both species of this mixed fishery.

The sensitivity analysis results reveals that the selection of discount rates close to zero leads to more efficient managing results for this fishery than the use of high discount rates. It is also remarkable the high sensitivity of the optimal solutions to changes in the price of hake. The sensitivity analysis of the benefits results shows that the maximum benefit of the hake fishery and the maximum benefit of the mixed fishery are reached under the minimum discount rate.

According to the results of this study, the management of the mixed fishery of hake and blue whiting on the EU Atlantic waters should base the objectives and technical measures in the use of discount rates close to zero, since both the larger effective capture and the maximum economic benefit are reached whit a low pressure over the resources.

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