Optimal management strategies of two species with different life history traits

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12-07-2016

Outline

- **Motivation;**
- **Literature review;**
- **Models;**
- **MSY exploitations;**

Motivation

- Biomass (Lumped growth) model: f(x) – too simplified
- Age-structured model
  - Include life history traits: mortality, growth, reproduction of individual age
  - Explore size-selective fishing,
  - Reduce risks of spawning and growth overfishing, especially for recovery program
- Ecosystem-based management
  - Quota-based control
  - Multiple species: interactions

Literature

Some examples:

- Skonhoft *et al* (2012): based on age-structured model to investage different fishing fleets targeting different age fish;

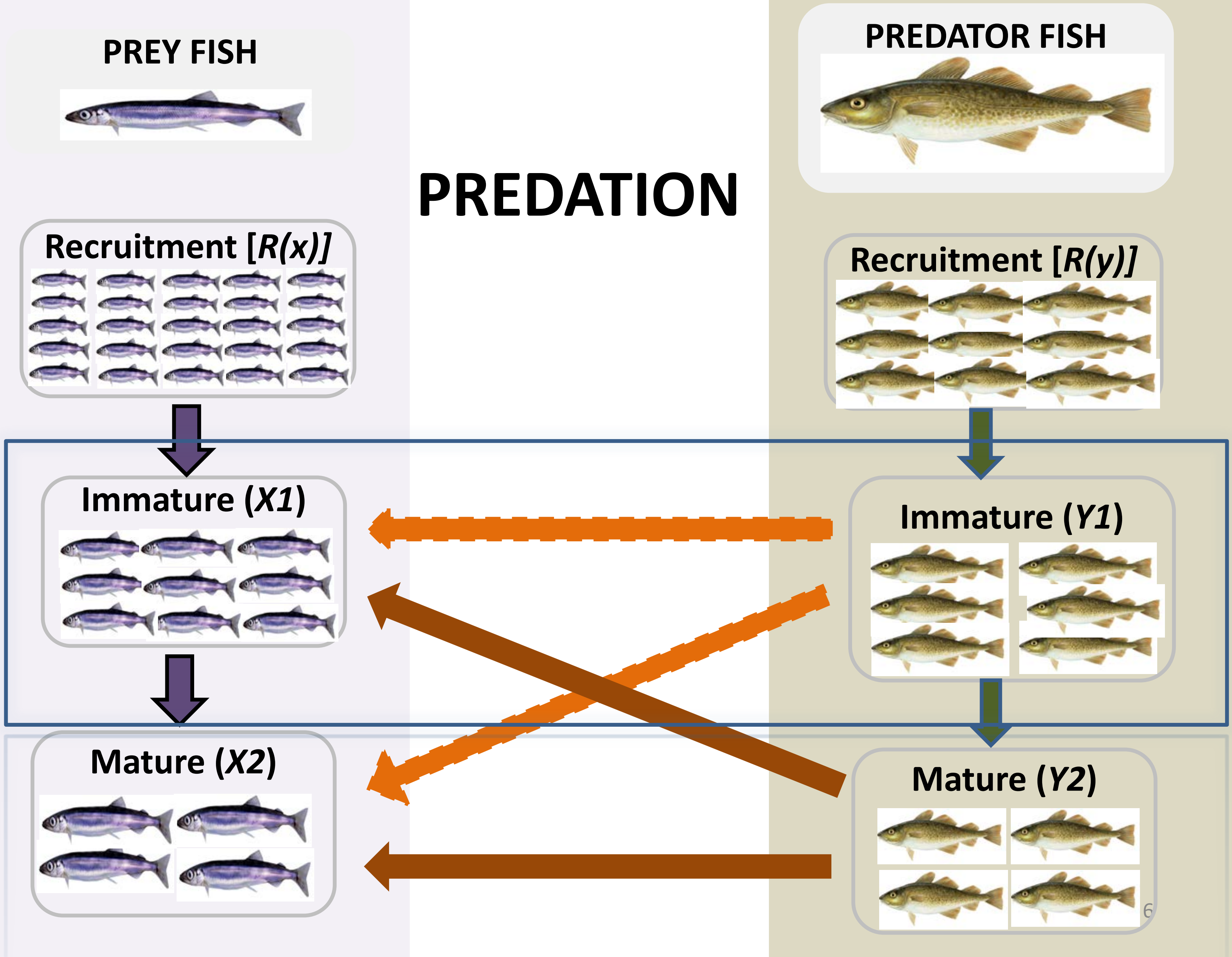
- Quaas *et al* (2013): using an age-structured bioeconmic model to explore tradable quotas of different age groups to avoid recruitment and growht overfishing;

- Tahvonen (2013): reviewing age-structued optimization in fisheries economics.

Model

Assumptions and model setup:

- Two species: a prey-predator Relationship
- Age- and staged-structured model with different life history traits
- Both species have commercial values


PREY FISH
PREDATOR FISH
PREDATION
Recruitment [R(x)]
Recruitment [R(y)]
Immature (X1)
Immature (Y1)
Mature (X2)
Mature (Y2)

Model

Population dynamics:

- Two ages or stages: recruitment (immature) and mature;
- Harvest mature
- Interactions:
  - Mature predator feed on prey
  - Prey loss through predation
  - Weight gain of predator by feeding on prey
- Sequential events: natural survival, harvesting and predation

Model

Prey population dynamics:

(1) Recruitment: $X_{0,t} = R_x(X_{1t}w_x) = \frac{\alpha_x SSB_{x,t}}{(1+\beta_x SSB_{x,t})}$

(2) Mature fish: $X_{1,t+1} = s_{x0}\ X_{0,t} + s_{x1}X_{1,t}(1 - f_{x,t})[1 - k_x(Y_{1,t})]$

(3) Spawning biomass: $SSB_{x,t} = X_{1,t}w_x\gamma_x$

(4) Harvesting function $H_{x,t} = f_{x,t}X_t = q_x E_{x,t}X_t$

(5) Cost function: $C_{x,t} = c_x E_{x,t}$

(6) Predation rate: $k_{x,t} = (Y_{1,t})^{\omega_1}$

Model

Predator population dynamics:

(1) Recruitment: $Y_{0,t} = R_y[Y_{1,t}w_y(X_{1,t})] = \frac{\alpha_y SSB_{y,t}}{(1+\beta_y SSB_{y,t})}$

(2) Mature predator fish: $Y_{1,t+1} = s_{y0}Y_{0,t}+s_{y1}Y_{1,t}(1-f_{y,t})$

(3) Spawning biomass: $SSB_{y,t} = Y_{1,t}, w_y\gamma_y$

(4) Weight gain: $w_{y1,t} = w_{y1}^{0} + (w_{x1}X_{1,t})^{\omega_2}$

(5) Harvesting function: $H_{y,t} = f_{y,t}X_t = q_yE_{y,t}Y_t$

(6) Cost function: $C_{x,t} = c_yE_{y,t}$

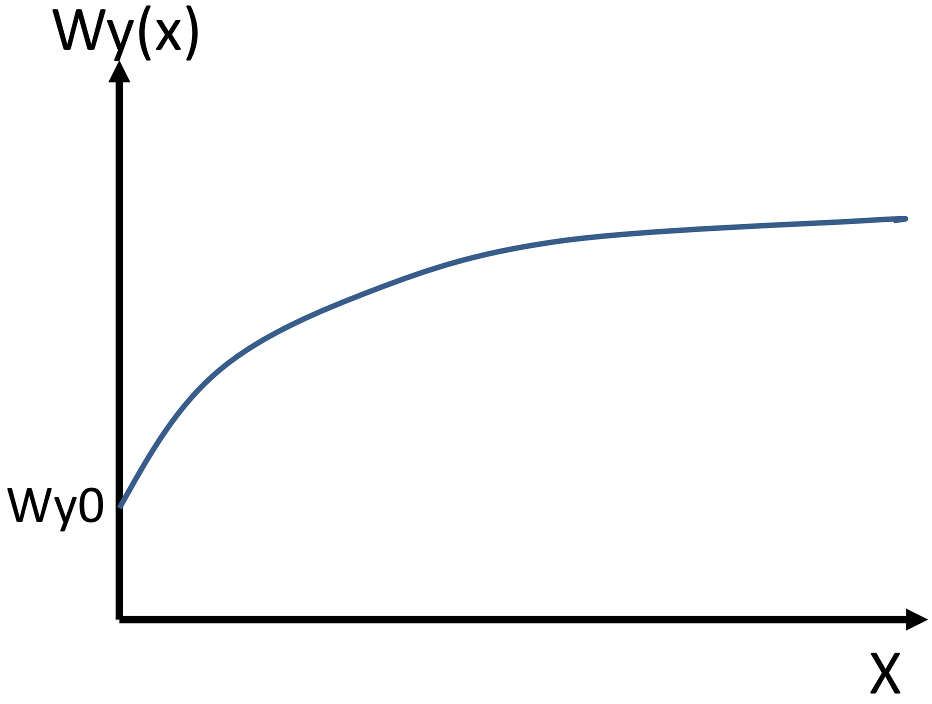
Model

Weight gain function:

$$w_{y,t} = w_y^0 + (w_x X_t)^{\omega_2}$$

$$w'_y(x) > 0$$

$$w''_y(x) < 0$$



Objective function

Maximizing sustainable yeild:

$$B = H_x w_x + H_y w_y = f_y X w_x + f_y Y w_y = q_x E_x X w_x + q_y E_y Y w_y$$

Lagrangian function :

$$L = f_x X w_x + f_y Y[w_y(X)] + \lambda\{s_{x0} R_x(X w_x) + s_x X(1 - f_x)[1 - k_x(Y)] - X\}$$

1st order necessary conditions:

$$(I)\ \frac{\partial L}{\partial f_x} = Xw_x - \lambda s_{x1}X[1 - k_x(Y)], \qquad 0 \leq f_x < 1$$

$$(\mathrm{II})\ \frac{\partial L}{\partial f_y} = Yw_y(X) - \mu s_{y1}Y\ , \qquad 0 \leq f_y < 1$$

$$(\mathrm{III})\ \frac{\partial L}{\partial X} = 0$$

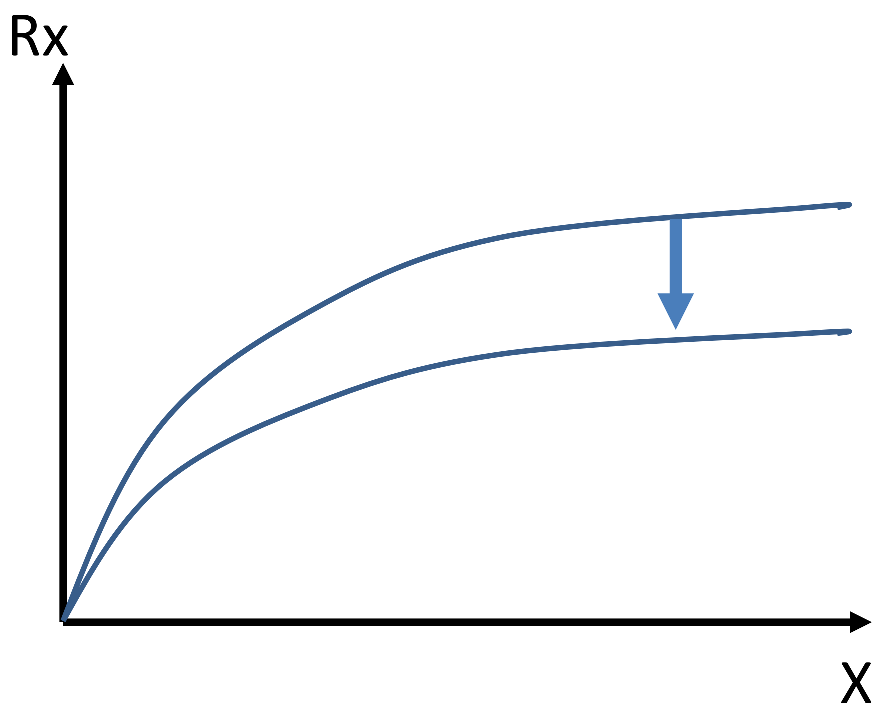
$$f_x w_x + f_y Y'[w_y(X)] + \lambda\{s_{x0}R'(X) + s_{x1}(1 - f_x)[1 - k_x(Y)] - 1\} + \mu s_{y0}R'[(w(X)] = 0$$

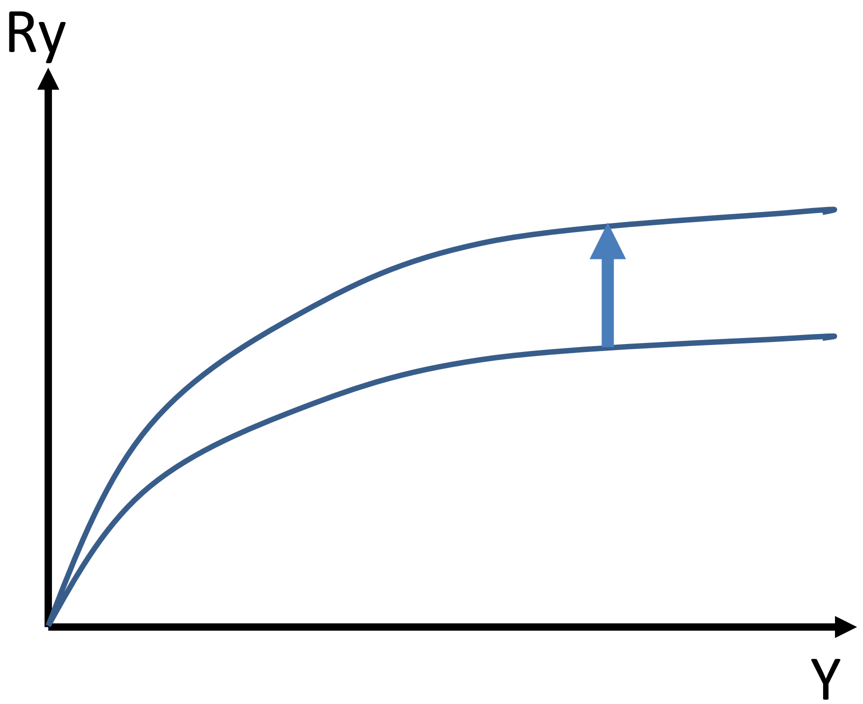
$$(I)\ \ \frac{\partial L}{\partial Y} = 0 \qquad f_y w_y(X) - \lambda s_{x1}X(1 - f_x)k_x'(Y) + \mu\left[s_{y0}R'(Y) + s_{y1}\left(1 - f_y\right) - 1\right] = 0$$

Without fishing but predation:

$$X^* = \frac{s_{x0}}{1 - s_{\mathrm{x}}[1 - k_x(Y)]} R_x(Xw_x)$$

$$Y^* = \frac{s_{\mathrm{y}0}}{(1 - s_y)} R_y[Yw_y(X)]$$





Maximizing sustainable yeild: *With fishing and predation:*

$$\frac{w_x}{s_{x1}[1-k_x(Y)]} \leq \lambda$$

$$\frac{w_y(X)}{\mu s_{y1}} \leq \mu$$

$$f_x w_x + f_y Y'[w_y(X)] + \lambda\{s_{x0}R'(X) + s_{x1}(1 - f_x)[1 - k_x(Y)] - 1\} + \mu s_{y0}R'[(w(X)] = 0$$

$$f_y w_y(X) - \lambda s_{x1}X(1 - f_x)k_x'(Y) + \mu\left[s_{y0}R'(Y) + s_{y1}\left(1 - f_y\right) - 1\right] = 0$$

With fishing and predation:

$$(I)\ \frac{\partial L}{\partial f_x} = \ Xw_x - \lambda s_{x1}X[1-k_x(Y)] \leq 0, \qquad 0 \leq f_x < 1$$

$$(\mathrm{II})\ \frac{\partial L}{\partial f_y} = Yw_y(X) - \mu s_{y1}Y \leq 0, \qquad 0 \leq f_y < 1$$

$$(\mathrm{III})\ \frac{\partial L}{\partial X} = 0$$

$$f_x w_x + f_y Y'[w_y(X)] + \lambda\{s_{x0}R'(X) + s_{x1}(1-f_x)[1-k_x(Y)] - 1\} + \mu s_{y0}R'[(w(X)] = 0$$

$$(I)\ \ \frac{\partial L}{\partial Y} = 0 \qquad f_y w_y(X) - \lambda s_{x1}X(1-f_x)k_x'(Y) + \mu\left[s_{y0}R'(Y) + s_{y1}\left(1-f_y\right) - 1\right] = 0$$