AN ABSTRACT OF THE THESIS OF

Timothy J Foglesong for the degree of Master of Science in Mechanical Engineering presented on April 24, 2015.

Title: Reverse Engineering, Modeling, and Redesign of a Vibratory Conveyor

Abstract approved: ______________________________

Robert B. Stone

Abstract approved: ______________________________

John Parmigiani

Some successful products are still developed and produced without the benefit of modern design techniques. One such product is a vibratory conveyor that has been developed largely through prototype iteration. The manufacturer has collaborated with Oregon State University to introduce model-based design into the development process. This research details a proposed framework for the reverse engineering of a mature product line. The framework is applied to a vibratory conveyor that is used in processing plants for corn and other raw foods. First, the project's origin and goals are described, followed by an explanation of the reverse engineering based framework used for guiding model selection. Next the selected dynamics model, built from the dynamic properties measured during reverse engineering, is applied and refined to accurately reflect the system's measured performance. Further modeling with finite element analysis, for improving performance and durability of specific components, is also described. This work concludes with the final research outcomes - redesign of fiberglass composite elements within the dynamic system and a set of guiding principles for designing a vibratory system, specifically this manufacturer's line of vibratory equipment.

# Reverse Engineering, Modeling, and Redesign of a Vibratory Conveyor

by

Timothy J Foglesong

A THESIS

submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Master of Science

Presented April 24, 2015
Commencement June 2015

Master of Science thesis of Timothy J Foglesong presented on April 24, 2015

APPROVED:

---

Co-Major Professor, representing Mechanical Engineering

---

Co-Major Professor, representing Mechanical Engineering

---

Head of the School of Mechanical, Industrial, and Manufacturing Engineering

---

Dean of the Graduate School

I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

---

Timothy J Foglesong, Author

## ACKNOWLEDGEMENTS

*Academic*

To my advisors, Dr. Stone and Dr. Parmigiani, for their guidance and vision, providing platform to work and expand my knowledge from.

To my industry partner, A&K Development Company, for generous support and assistance.

To my research colleagues, particularly Jesus Meraz, Ryan Arlitt, and Robin Kiff, who were essential in various roles.

*Personal*

To my parents, for their unwavering support and love.

TABLE OF CONTENTS

Page

1. INTRODUCTION . . . . . . 2
   1.1. Background . . . . . . 2
   1.2. Motivation . . . . . . 3
   1.3. Context . . . . . . 4
   1.4. Thesis Roadmap . . . . . . 6
2. DRAFT: REVERSE ENGINEERING INFORMED MODEL SELECTION FOR DESIGN EXTRACTION . . . . . . 8
   2.1. Abstract . . . . . . 9
   2.2. Nomenclature . . . . . . 9
   2.3. INTRODUCTION . . . . . . 10
   2.4. BACKGROUND . . . . . . 11
      2.4.1 Reverse Engineering Techniques . . . . . . 11
      2.4.2 Reverse Engineering Calculations . . . . . . 12
      2.4.3 Model Selection . . . . . . 14
   2.5. FRAMEWORK DESCRIPTION . . . . . . 14
      2.5.1 What is a Model . . . . . . 16
      2.5.2 Required Data Categorization Table . . . . . . 19
      2.5.3 Measurement Uncertainty . . . . . . 20
      2.5.4 Advantages . . . . . . 21
   2.6. EXAMPLE CASE STUDY . . . . . . 22
      2.6.1 Background . . . . . . 22
      2.6.2 Special Considerations . . . . . . 23
      2.6.3 Application . . . . . . 24
      2.6.4 Concessions made for this specific case study . . . . . . 28
   2.7. CONCLUSION . . . . . . 29
   2.8. FUTURE WORK . . . . . . 30
   2.9. ACKNOWLEDGMENTS . . . . . . 31

TABLE OF CONTENTS (Continued)


Page

3. BRIEF BACKGROUND ON DYNAMICS MODELING PROCESS .......... 32
   3.1. Advantages .......... 33
   3.2. Disadvantages .......... 34
   3.3. Iterative Approach .......... 37

4. DYNAMICS MODELING TO INFORM DESIGN OPTIMIZATION .......... 38
   4.1. Abstract .......... 39
   4.2. Nomenclature .......... 39
   4.3. INTRODUCTION .......... 40
      4.3.1 About the device .......... 40
   4.4. BACKGROUND .......... 42
      4.4.1 Machine Function .......... 42
      4.4.2 Counterweight .......... 43
   4.5. PROBLEM DEFINITION .......... 44
   4.6. DYNAMICS MODEL DEVELOPMENT / EXPLANATION: .......... 44
   4.7. TESTING PROCEDURE .......... 48
      4.7.1 Spring Arm Testing .......... 48
      4.7.2 Spring Arm Data Analysis .......... 49
      4.7.3 Pan Suspension Testing .......... 49
      4.7.4 Pan Suspension Data Analysis .......... 49
      4.7.5 Combining Springs .......... 50
   4.8. RESULTS .......... 50
   4.9. FUTURE WORK .......... 51
   4.10. ACKNOWLEDGMENTS .......... 54

5. FURTHER DYNAMICS MODELING .......... 55

Page

5.1. Linear Potentiometer Nonlinearity ........ 55

5.2. Two Types of Load Cell Error ........ 56

5.3. Dynamic Property Testing: Damping ........ 57

5.3.1 Signal Noise: Load Cell ........ 58
5.3.2 Data Logger ........ 59

5.4. Further Improvement ........ 60

5.5. Application of the model to the problem at hand ........ 61

6. FIBER BAR ANALYSIS & REDESIGN ........ 64

6.1. Loading Condition Simplifications ........ 64

6.1.1 Mounting ........ 64
6.1.2 Load vs Displacement ........ 65

6.2. Progression of Modeling Techniques ........ 68

6.3. Justify FEA vs Traditional Methods ........ 69

7. GENERAL CONCLUSION ........ 71

7.1. Composite "Fiber Bar" Design ........ 71

7.1.1 FEA ........ 73
7.1.2 Dynamics Model ........ 74

7.2. Predictive Advice Document for Application of Design Principles ........ 75

7.2.1 Basis ........ 75
7.2.2 Predictive Advice Table ........ 76
7.2.3 Principles ........ 76

BIBLIOGRAPHY ........ 78

LIST OF FIGURES


| Figure | | Page |
|---|---|---|
| 2.1 | FLOW CHART OF FRAMEWORK STEPS. | 16 |
| 2.2 | THE SHAKER TABLE, WITH DYNAMIC COMPONENTS DIAGRAMMED. | 22 |
| 3.1 | Force-deflection plot of fiber bar under loading conditions experienced in shaker table. | 36 |
| 4.1 | DIAGRAM OF MACHINE COMPONENTS. | 41 |
| 4.2 | DETAILED VIEW OF 'DOG BONE' SUSPENSION COMPONENT. | 45 |
| 4.3 | THE DYNAMICS MODEL. | 46 |
| 4.4 | SPRING POSITIONS ON TABLE | 48 |
| 4.5 | THE DYNAMICS MODEL (TOP) DISPLAYS EVIDENT BEATS, ABSENT IN THE EMPIRICAL DATA (BOTTOM). | 51 |
| 4.6 | ARBITRARILY ADDING A DAMPING COEFFICIENT (ASTERISKS) TO THE MODEL (CIRCLES) GREATLY IMPROVES AGREEMENT WITH THE EMPIRICAL DATA (SQUARES). | 52 |
| 5.1 | Unfiltered sensor signals, load cell (top) and linear potentiometer (bottom). | 58 |
| 5.2 | Load cell output (sinusoidal); unfiltered (left) and filtered by outlier removal (right). | 60 |
| 5.3 | Dynamics model, without and with correction factor applied. | 61 |
| 6.1 | Fiber bar mounting, fixed end. | 65 |
| 6.2 | Fiber bar mounting, force application end. | 66 |

LIST OF TABLES

Table Page

2.1 QUANTITY OF MEASUREMENTS REQUIRED FOR EACH MODEL. 20

2.2 COMPONENT BREAKDOWN OF SHAKER TABLE AS PER REKOFF'S STRUCTURAL HIERARCHY.[5] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 25

2.3 DATA REQUIREMENTS OF EACH CANDIDATE MODEL. . . . . . . . . . . . 26

# REVERSE ENGINEERING, MODELING, AND REDESIGN OF A VIBRATORY CONVEYOR

# 1. INTRODUCTION

The research work reported in this thesis is the culmination of a specific industry sponsored project that reflected a larger, general class of engineering design needs affecting the redesign of existing, mature artifacts. Both documentary and methodological statement, this thesis takes the reader through the journey from initial creation of the collaboration project to final recommendations, both actual design outcomes and guidance for future design decisions. The initial step, reverse engineering, yields design information which guides the model selection process. Numerical models can be proposed only after the intended function of the product is understood, particularly how it results form the interactions of the product's components. Selecting which of these models to use is an important step in the design process as it dictates many essential aspects of the following design work, such as which data needs to be collected, how detailed the model will be and how it will be implemented (method / calculations), and what kind of output can be expected from the model and thereby what can be done with the model's output. In other words, the cost of the modeling effort. The modeling effort should also account for the considerations ('constraints') of the particular modeling situation - important design aspects, deadlines, budget, expertise, performance goals, or anything that restricts the creation and implementation of the model. The collaborative nature of the research yielded many such constraints, which largely shaped the outcomes of the redesign effort.

## 1.1. Background

The research presented in this work is the result of two years of work on an industry sponsored project. The project involved analyzing, modeling, and suggesting redesigns for an artifact system. The following chapters will describe the efforts made in reverse engineering, model selection, model application, and the resulting redesigns.

The artifact system is a vibratory conveyor table built by A&K Development Company in Eugene, Oregon. This "shaker table" is used to convey corn through a processing facility as it is automatically husked and inspected. Unlike competing company's designs, A&K's shaker table applies the agitation through a spring instead of directly to the oscillating surface ("pan"). This arrangement allows the table to handle heavier loads by pre-loading the "drive spring." Competing designs directly oscillate a sub frame to which the pan is attached with springs, so a sympathetic oscillation is created in the pan. Although these other designs use simpler oscillators, they are not robust to large loads, which reduce the amplitude of the sympathetic oscillation.

Existing redesign techniques often lack the specific guidance on the type of modeling needed and when the model is "good enough." By introducing reverse engineering (RE) into the redesign process, the tone is changed from achieving a 'best' redesign to creating an 'efficient' redesign that improves the product without incurring disproportionately high costs. Being an industry-funded research project, resource and time considerations are important; beyond a minimum acceptable level of accuracy the selected model should yield results efficiently and on schedule.

## 1.2. Motivation

For a dynamic system, A&K's shaker table design is not well optimized. Its design is more complex than those of competition and has many more design variables, which have historically been determined through iterative testing of full-scale prototypes; no formal numerical modeling as ever been performed on this system. Optimizing this system requires increasing the amplitude and frequency of the pan, which increases the flow rate of product across the table. The pan is driven through one type of springs and suspended by other types, some of which also function as dampers. Additionally, there is a counterweight with similar mass to the pan, which is driven in opposition to reduce the oscillatory forces transferred through the frame and into the floor. To optimize the shaker table, this entire dynamic system must be modeled.

The goal of the collaboration between Oregon State University and A&K Development Co. has been to analyze this 'artifact' shaker table system with numerical modeling. The resulting model can then be used to select design variables for the shaker table based on expected performance, reducing the need for prototypes to be built. The larger goal of this research, however, has been the creation and implementation of a RE framework to assist others in conducting this 'analysis - modeling - redesign' process for similarly un- or under-documented product designs.

## 1.3. Context

Before discussing the research conducted, it is important to consider the intended outcomes. From the sponsor Company's perspective, the goal of the project is to create engineered solutions for improving the performance and longevity of its unique family of vibratory conveyors. Creating effective redesigns involves more than simply manipulating specific performance requirements in a numerical model; the Company has an established design process which does not involve the techniques, terminology, and conventions generally associated with engineering design. The Company shouldn't be asked to discard the entire development process in favor of a new, foreign system. The resulting engineered solutions should therefore be restricted to individual component or subassembly redesigns, and proposed with supporting background information (principles, test and model data, etc.) just as all redesigns are in the existing development process. This involves explaining the merits of specific redesigns with the engineering principles (and accompanying terminology) that support them. Considering the alternative - proposing major redesigns without involving the Company in the decision making process - this method has higher transparency and more interaction, which allows both parties to contribute to each component redesign and creates a more trusting and cohesive collaboration. Additionally, redesigning individual components reduces or eliminates 'retooling' costs incurred in updating the manufacturing process to accommodate the new component, and allows for retrofitting of existing machines.

From the academic perspective, the goal is to offer a theoretical contribution to RE techniques in the field of engineering design. Based on the steps taken to model these shaker tables, the need was seen to formalize such a model formulation based solely on RE. Traditionally, modeling techniques in engineering design are not concerned with where the data used to populate a model comes from or what was required to obtain it. For many model applications such information does not matter, but when this data is not trivial to obtain, the economic impact of collecting the necessary data for a particular model should be considered. One of the primary concerns of RE is the cost of data collection, specifically the rate and difficulty of extracting design information from an artifact. By leveraging these RE calculations for model selection purposes, a framework can be built to formalize the process of performing model selection and implementation in tandem with RE design information extraction. Such a framework has potential to benefit engineers tasked with redesigning 'mature' products designed with little or no modeling or benchmarking.

## 1.4. Thesis Roadmap

This work is comprised of two publications, a journal article in preparation proposing the framework used to select a numerical model with reverse engineering (RE) methods and a conference paper detailing the creation of the selected dynamics model, and a report on additional modeling using finite element analysis (FEA). Supplemental information is added to further explain the following publications by explaining their interactions and appending current information to them.

The journal article in preparation (chapter two) presents a case for using RE methods to find a set of possible numerical models for a previously un-modeled product, then selecting the most efficient model to apply based on the RE data collection cost to implement each. The proposed framework considers the numerical modeling process from a RE perspective because RE is focused on real-world cost (both money and time) which, unlike accuracy or computation time, is a primary concern in the product development process. To show the use of the framework, this project is used as a case study. Three candidate models are compared by the total RE 'barrier' to obtain all of the individual measurements needed to implement each. The dynamics model, being the most efficient, is selected for further investigation.

The conference paper (DETC2014-34412) (chapter four) discusses the initial implementation of the dynamics model selected in the previous chapter. This includes details of collecting the various types of data, including system response data to directly compare to the model's output for calibration purposes. The end result is the first, most basic implementation of the dynamics model with comparison to empirical system response data and suggestions for improving the accuracy. The implementation of these suggestions in further iterations of the dynamics model is detailed in a following section.

The dynamics model, once compensated to accurately match the shaker table's output amplitude, provides a method for accurately predicting the results of various changes to the elements in the mass-spring system. Based on impact and ease of implementation, the fiberglass drive springs, referred to as 'fiber bars,' were selected for redesign. A large component of their redesign involved addressing stress concentrations along their length which were contributing to premature failure. In order to quantify the stress in a reliable and repeatable manner, FEA was used to model these fiber bars (chapter six), providing both a baseline of peak stress values in the existing bars and new values for proposed bar designs prior to their manufacture. All fiber bar redesigns that have been manufactured as of this writing were found acceptable to the Company and have been ordered in production quantities.

# 2. DRAFT: REVERSE ENGINEERING INFORMED MODEL SELECTION FOR DESIGN EXTRACTION

Tim Foglesong, Rob Stone, and John Parmigiani

N/A

## 2.1. Abstract

*Although modern engineering design techniques have been in use for decades, many mature manufactured products are either designed without significant engineering modeling or the detailed engineering analysis is lost. When such products need to be optimized, a model must be reverse engineered from the design drawings or product itself. If the design process was well documented, the dimensional data may be present, but other values, such as specific component properties necessary to the product's operation, may be missing or inaccurate. In this paper, the authors propose a framework for using reverse engineering to inform model selection in order to perform model-based optimization on a previously un-modeled mature or "legacy" design. The framework is then applied to a case study involving one such legacy design which has been optimized iteratively for decades and has reached the limit of "intuitive optimization." The case study demonstrates how this framework informs the selection of an optimization model based on the performance goals and types of available information.*

## 2.2. Nomenclature

$T$ - time to extract the information, estimated

$K$ - amount of information contained by the product

$K_0$ - initial amount of information contained by the product

$S$ - the product's ability to contain information

$B$ - barrier to extract information

$P$ - effort required to extract information, estimated

$F$ - information extraction rate, estimated

***system*** - the entire product

***subsystem*** - functional subdivision of the product

***assembly*** - individual functional unit within the product

***subassembly*** - collection of components

***component*** - single piece of the product, indivisible except by destructive methods

***item*** - generic higher-level division of a product in Rekoff's hierarchy

***element*** - generic constituent-level division of a product in Rekoff's hierarchy

## 2.3. INTRODUCTION

In this paper, the authors propose a framework for selecting a model to represent the function of a product for which no previous design data exists. To be clear, there is no prior knowledge of the product - the process is initiated with a reverse engineering study of a sample of the complete product. The framework begins with reverse engineering, specifically performing a planned, documented disassembly of the sample product. Numerical methods are then proposed and their functional requirements are assessed. From these functional requirements, widely accepted reverse engineering calculations [10] are performed to rank these numerical methods by summing the 'barriers' to obtaining each of the measurements for each method (providing an estimation of the total 'barrier' to the implementation of each numerical method).

The creation of this framework was motivated by an industry-sponsored project which involved creating a model for a vibratory conveyor ('shaker table'). This product utilizes a damped mass-spring system to convey items through a processing facility. Designed decades ago, this 'shaker table' has no known design documentation beyond CAD drawings. The goal of the project is to simulate the function of the table with a numerical model in order to inform future design decisions, such as optimizations for increased feed rate of corn or wear life of components which encounter high frequency cyclic loading during normal operation. The proposed framework is applied to this numerical modeling problem, with emphasis on the 'barrier' calculations used to compare the proposed models.

## 2.4. BACKGROUND

### 2.4.1 Reverse Engineering Techniques

Reverse engineering has many uses in engineering, chief among them being benchmarking and "design recovery" [1] from a competitor's product, and maintenance on legacy systems. Although a large portion of reverse engineering research is focused on the electronics and software design fields, many of the principles and processes are universal and can readily be applied to mechanical specimens. Specifically, the calculations used for planning reverse engineering operations can be applied effectively to describe the time and effort required to retrieve data from mechanical systems.

Reverse engineering is notably used in software engineering [2] and biological inspired engineering design [3][4], making use of the core principles presented in Rekoff 1985 [5]. Along with this root conceptual work, we will be using various contemporary works [6] [7] [8] [9] [10] for information on current reverse engineering techniques. These sources provide a sufficient reverse engineering background, including applications specific to mechanical engineering design, to use reverse engineering for model selection.

The first step in reverse engineering is not disassembly, instead it is planning. Regardless of the method followed or documentation used, having a plan for how the components and their initial configuration will be cataloged, and how they will be organized and stored as disassembly progresses is required to maintain organization. This is particularly important when the device may be reassembled and disassembled again in the future, or left in a state of partial disassembly for an extended period of time. Planning is important because it minimizes mistakes made in disassembly and data collection.

Disassembly can be one of the most important steps in understanding the function of the product, and therefore in deciding which types models are applicable or reasonable for analyzing its function. While Rekoff [5] suggests acquiring a number of identical copies of the device to be disassembled, Otto and Wood [6] [7] opt for a more methodical disassembly process which relies on documentation and precise cataloging of each component, including step-by-step photography, creating exploded views, and a bill of materials (BOM) list, to gain full component knowledge from a single example of the product. Both methods are valid: Otto and Woods' approach is popular in mechanical design, where components' functions and physical layout are closely related (eg when guiding material), and Rekoff's approach takes a more cautious stance, which can be beneficial when disassembling highly complex systems, such as circuits (whose components are numerous, small, and interconnected in inobvious ways). Keeping these differences in mind, we will apply aspects of both approaches in the framework and case study.

Since these methods differ so drastically, it is advisable to apply the steps which are best suited to the specific case. In the case study presented later in this paper, certain methods from each approach cannot be used, so the most applicable combined set of techniques which accomplish the necessary steps is used.

### 2.4.2 Reverse Engineering Calculations

A large part of reverse engineering is the creation and estimation of "barriers" - design elements which expressly inhibit reverse engineering. The goal of the product's design team is to create inexpensive barriers which will likely make reverse engineering more expensive to the competitor than simply designing a new product. The competitor's reverse engineering team must then find and estimate the cost (time & resources) of circumventing these barriers. This team's first job is to estimate the total time and resource cost to reverse engineer the product; if the barriers prove to be too great for the team's skills or resources under the imposed timetable, then reverse engineering is inadvisable. Therefore, the calculations behind making the decision to pursue reverse engineering efforts are very important (and well developed). The three most important quantities in the reverse engineering calculations are $T$ (time to extract the information in the product, estimated), $K$ (amount of information contained by the product), and $S$ (the product's ability to contain information). Considering the time $T$ as the ultimate measure used in the planning process, the following equation becomes central to the estimating calculations:

$$T = -BS \ln \frac{K}{K_0} \tag{2.1}$$

where $B$ (barrier to extract information) and $S$ are defined by:

$$B = \frac{P}{F^2} \tag{2.2}$$

$$S = \frac{KF}{P} \tag{2.3}$$

where $P$ (effort required - "power exerted" - to extract information, estimated) and $F$ (information extraction rate, estimated) are based on the reverse engineering team's expertise and resources. [8] [9] [10]

For our proposed framework, the most important quantity is $B$ because it is specific to individual components or pieces of data. Its inputs, $P$ and $F$, can be estimated separately for each barrier, independent of the values of $K$ and $S$, which concern the entire product. Selecting a model to simulate the product's "mechanism-of-operation" does not necessitate extracting all information it contains; our goal is "glueing" data together to "resolve a need" - ignoring superfluous information saves both time and resources. [5]

When discussing reverse engineering, it is important to remember it's an inverse process to "forward engineering," which progresses from requirements, through design, to implementation (high level of abstraction to specific solution). The process of reverse engineering starts from a product and aims to discover the high level problem abstractions the product was designed to solve by analyzing its components and their interactions.[1]

### 2.4.3 Model Selection

Model selection is largely concerned with accuracy, which leads most research to discuss errors, specifically their sources and propagation. We are specifically ignoring the abstraction and algorithmic errors inherent to the model selection and implementation steps, respectively, of all models.[11] We assume all proposed models approximate the system under consideration to a degree of accuracy that is useful for our applications. The only errors we are concerned with are those in data measurement, which we can directly avoid or choose between during reverse engineering data collection. Uncertainty propagation is not a central subject of this paper, and the authors do not provide any guidance in selecting uncertainty propagation methods; many existing works provide excellent discussions and summaries of propagation calculation methods. [12] [13]

## 2.5. FRAMEWORK DESCRIPTION

The proposed framework makes use of the aforementioned reverse engineering (RE) techniques for selecting candidate models to describe the function of a product, and the RE calculations as a metric for comparing these models. Specifically, data quantities required to implement each model are enumerated and the RE calculations from Harston & Mattson [10] are applied to one measurement for each of these quantities, providing the barrier for each measurement. The total barrier for each model can be calculated from these individual barriers. The information extraction process is limited because only a subset of the data from the product is required for numerical modeling; the goal is to find the most *efficient* model - that which is most economical, in RE terms, to implement.

The creation of this framework was motivated by an industry project requiring the development of a numerical model for an existing (previously un-modeled) product. The product, a successful design originating decades ago, has been continually updated and improved through use of the same basic design techniques which were originally used to create it. In academia and high-tech industries we often see adoption of and adherence to modern design techniques and tools, but there is a portion of the manufacturing industry which still operates in the same basic way it did 20 or 30 years ago. This project originates from one such company. Successfully transitioning from the company's original design techniques to modern methods requires current knowledge of design development and modeling, as well as design recovery and benchmarking techniques from RE. The proposed framework summarizes and applies this required knowledge, providing a method to quickly and quantitatively compare numerical models with only basic prior knowledge of their implementation and use.

The framework, summarized in figure 2.1, begins with reverse engineering, specifically planned and documented disassembly. Of these early steps, documentation take precedence; the goal of the initial disassembly is to understand how the product's components interact to perform its functions. Organizational aids, such as Rekoff's "hierarchical structure" or Otto and Woods' hierarchy tree and function structure are useful for this task, but are not the central focus of this framework, because our goal is to apply an economical model, not create an entire RE representation of the product. The primary goal of RE at this point is to discern the mechanisms of the product's functional behavior. This information is then used to propose candidate numerical models which are capable of representing the functional behavior. The inputs for each model are listed and approximate numbers required for each model counted. The RE barrier for one measurement of each type is calculated, and these individual barriers are multiplied by the number of each

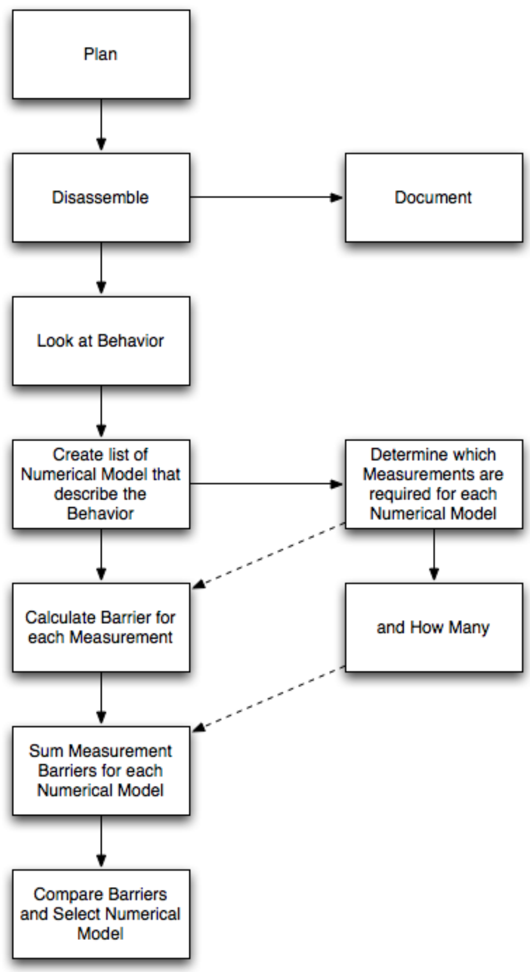


FIGURE 2.1: FLOW CHART OF FRAMEWORK STEPS.

required for a given model and added to provide a total RE barrier for implementing that model. The total RE barrier for each model can then be compared, and a model selected.

### 2.5.1 What is a Model

Once the RE analysis steps (plan, disassemble, and behavior) are complete, a list of applicable models can then be created. In this context, a model is an equation or set of equations which are capable of conveying a certain facet of the functional behavior of a system. The numerical models which are discussed in this work are: finite element analysis (FEA), motion simulation, and dynamics modeling.

FEA is the application of mechanics principles and material properties to an individual component or a set of interacting components by discretizing each component into simple geometric shapes which are approximated by a small number of nodes each. These elements propagate the material properties and mechanics principles through their individual deformations and interactions. This is made possible by approximating these elements as a set of constituent nodes which describe their shapes and storing the elements' nodes and properties in large matrices. A simple component may contain a few dozen nodes, but complex geometries, such as holes and curves require much finer discretization, and complex components can have tens of thousands of nodes. With multiple components interacting, over tens of timesteps, requiring manipulation of large matrices, calculation times can be anywhere from minutes to hours or even days for large or highly complex simulations. In general, there is an direct correlation between a FEA model's run time and the accuracy of its output.

Motion simulation, such as SolidWorks Motion or Working Model, is kinematics analysis performed by a computer - a numerical representation of the positions and orientations (and their respective velocities) of components which interact through various types of pivots, slides, and surface contact. In its simplest form, motion simulation operates on two-dimensional assemblies of rigid components, but it can be extended to describe three-dimensional assemblies with 'real' components that have compliance (stiffness) and friction properties. This form of simulation is performed in a piece-wise manner, wherein the movements and interactions are calculated in individual 'steps' or 'frames' - a computationally intensive method which allows for various types of errors, such as components 'popping through' each-other or snapping to rotation positions which are physically impossible.

Dynamics modeling is the application of dynamics equations to describe the actions of dynamic components within a system based on the forces involved. Unlike motion simulation (discussed earlier), dynamics modeling uses kinetics to describe the motion of the system. This means forces are involved in the calculations, not just interactions of spacial data. Dynamics modeling is thereby more deterministic, not relying on increasing the number of simulation steps to achieve accurate results. Consequently, the 'careful detail' stage of creating the model is *before* the calculations are run, not after. This puts more burden of careful consideration of details on the model creation step, since there is little latitude for deviation during the actual calculation steps. One major disadvantage of this characteristic is a poorly formed model will always provide absolutely correct results for the configuration it models, regardless of whether that matches the actual configuration of the system or not.

Although this list may initially contain a large variety of models, it is advisable to remove any which do not meet a minimum level of anticipated accuracy, or are simply impractical to implement. The relative 'difficulty' of implementing each model will be assessed by calculating the individual 'barriers' for each of the measurements required to populate each model's inputs, then combining these barriers (with any additional modification factors, such as anticipated measurement accuracy or special resource requirements) to create a direct comparison between the candidate models.

To better describe the application of this framework, consider the utility estimation used by Radhakrishnan and McAdams[11]. The proposed model does not identify the most truthful model, which requires applying the models, but instead focusing on data collection, which occurs before any model calculations. By incorporating reverse engineering in the modeling process, we can quantify the effort to collect the required data for each model. This method of selecting a model may seem pointless from an optimization standpoint, but for this case model accuracy and efficiency are subordinate to considerations behind the inputs to the model; RE informs us that the effort involved in collecting the necessary data is the primary deciding factor in selecting one model over another. Since the time and effort of reverse engineering measurements require more resources than the actual modeling, the most important decision is not which model to choose, but rather whether or not to create the model at all. If these reverse engineering 'costs' are greater than those of designing a new product from a simple black box flow analysis, then the exercise of model selection is unnecessary and the current design should be scrapped. If one is deciding between two satisfactory models for a given product, then the model which incurs the lowest reverse engineering 'costs' to implement should be selected.

### 2.5.2 Required Data Categorization Table

An important part of selecting between candidate models by RE calculations is identifying the type and number of measurements used to acquire this data. A measurement's type directly affects its speed and accuracy. Stepping back a moment, we need to consider the tradeoff between speed, accuracy, and cost of a given measurement. Making the basic assumption that the simplest measurement, distance, is relatively quick to take at 'high' accuracy can lead to errors quickly; measurements beyond the range of calipers or micrometers require specialized equipment to reach a level of accuracy beyond that of a tape measure or yard stick. Likewise, measurements which are aggregated from multiple instrument outputs have multiple uncertainties which must be propagated *before* being used in a model. And digital instruments, despite boasting accurate and repeatable measurements, are often susceptible to transient error - either directly or as a result of its signal filtering - which reduces their accuracy when collecting rate-based measurements, such as velocity.

The Required Data Categorization Table (table 2.1) is a tool for organizing the measurement requirements of each candidate model. The columns, one per measurement type, allow the user to see which models require each measurement; measurements which appear frequently will probably be essential to the modeling effort. The rows show the

| | Measurement Type 1 | Measurement Type 2 | Measurement Type $j$ |
|---|---|---|---|
| Model 1 | $n_{1,1}$ | $n_{1,2}$ | $n_{1,j}$ |
| Model 2 | $n_{2,1}$ | $n_{2,2}$ | $n_{2,j}$ |
| Model $i$ | $n_{i,1}$ | $n_{i,2}$ | $\mathrm{n}_{i,j}$ |

TABLE 2.1: QUANTITY OF MEASUREMENTS REQUIRED FOR EACH MODEL.

quantity of measurements required for each model, so that once the barrier for each measurement has been calculated, the user simply multiplies by this quantity and adds across to obtain the total barrier associated with implementing each model. For model $i$, the equation is as follows:

$$B \equiv barrier, \quad n \equiv number\, of\, measurements$$

$$B_{\mathrm{Model}\, i} = B_1 * n_{i,1} \; + \; B_2 * n_{i,2} \; + \; ... \; + \; B_j * n_{i,j}$$

### 2.5.3 Measurement Uncertainty

Measurement uncertainty is a combination of systematic and random error; we are most interested in the systematic errors because they are often inherent to the specific measurement method unlike random errors, which are often caused by environmental effects on the instrument.

Just as we have showed using the estimated barrier to each quantity to be measured to rank candidate models, one can easily use measurement uncertainties for a similar comparison between models. This requires knowing the uncertainty of the specific measurement device or devices which will be used for obtaining each value. This, of course, strays into measurement uncertainty theory and the realm of metrology. Many papers in this field point to the Guide to the Expression of Uncertainty in Measurement (GUM) [14] [15] as the primary source for making these characterizations. Proper characterization also involves use of "fuzzy and interval theory models." [16] This application - fuzzy measurement theory - is well documented in a number of sources.[17][18] These methods are useful for more accurate characterization of systematic measurement errors than a normal distribution of standard deviation provides. Also of importance are the "Type B" uncertainties described by the GUM, which are *not* characterized with statistics. [14] These uncertainties are often known through familiarity with the specific measurement or piece of equipment used - in short, experience.

The degree of accuracy to which measurement uncertainty is estimated may change the outcome of model comparison, so the user should keep the level of detail consistent across all measurements.

### 2.5.4 Advantages

The greatest advantage of this model selection framework is its simplicity and quantitative output; unlike vague qualitative methods (such as a manager's preferences) or mathematically intensive uncertainty-based methods that require model benchmarking [11], this framework is easy to implement without previous modeling domain knowledge yet provides a clear quantitative ranking of candidate models. The goal is not to find the most accurate model, is it to find the most *efficient* model, in terms of RE 'cost.' It is important to remember that the key to this method is *predictive* estimation, not ultimate truth; ideally, one model is clearly more *efficient* than the others, and can be selected without further investigation.

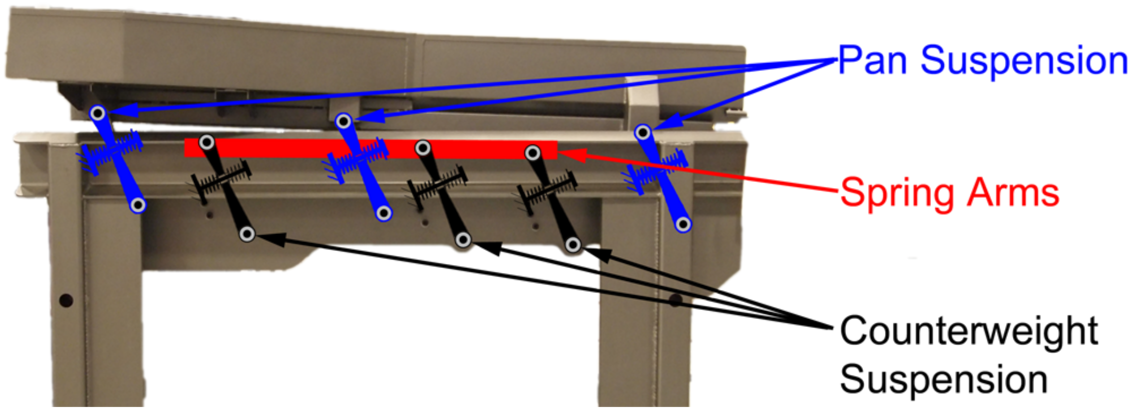


FIGURE 2.2: THE SHAKER TABLE, WITH DYNAMIC COMPONENTS DIAGRAMMED.

## 2.6. EXAMPLE CASE STUDY

The following case study is based on the industry sponsored project which prompted this research. Technical details, including the product's mode of operation and specifics of the selected modeling approach, can be found in the authors' previous paper on the subject.[19]

### 2.6.1 Background

The example product is a vibratory conveyor, referred to as a 'shaker table,' which oscillates its top surface ('pan') at an angle off of horizontal so that items placed loosely on the 'pan' will move across the surface to discrete exit lanes as preparation for processing. Oscillation is used instead of a simple conveyor belt because it prevents clogging, particularly when separating the items into lanes, and provides consistent alignment of the items for processing.

The shaker table uses a large number of coil springs and rubber bushings as the spring and damper components of its damped mass-spring system. The forcing function - displacements created by eccentric bearings on a keyed shaft, driven by a constant speed electric motor - is applied to the mass (pan) through two large drive springs. To keep the action balanced, the entire damped mass-spring system, including the forced drive springs, is mirrored by a dynamically identical copy which nests beneath the pan and is driven 180° out of phase. The mass in this system is hidden behind the frame in figure 4.4, surrounded by the 'spring arms.' All of these components, including the springs and dampers, are only visible from below the table.

### 2.6.2 Special Considerations

No real-world application of a design tool occurs under ideal conditions, and this case study was conducted under a number of constraints. First, the reverse engineering was limited to a single example unit of the product, not the preferred set of multiple identical units. Since only one unit was available, there was no way to stagger disassembly, as prescribed by Rekoff, or preserve a "good unit" for reference.[5] As a result, disassembly was kept to a minimum and the machine was often returned to original configuration to prevent this original configuration from being lost completely. This, too, introduced difficulty because the tuning techniques were not described in detail, making re-tuning after reassembly an imprecise exercise in recreation of previous assembly values. Even the original tuning did not last long; bolt torques changed over time and between operation cycles, sometimes even between cycles that were immediately consecutive. The philosophy of 'continued tuning' was adopted to mirror the machine's use in real operation conditions - the operator will re-tune the machine until it sustains an acceptable level of performance.

Being limited to a single unit suggests the use of Otto and Woods' documentation-based reverse engineering process, but this too was not possible; the sheer size of the machine (live masses over 200 pounds and greater than four feet square) and lack of access to a crane prohibited complete disassembly.[6][7] Also, the inability to view the mechanical components from outside the table or any great distance, made photography of the components' assembled configuration both difficult and confusing to outside viewers. However, these constraints did not hinder data collection because the machine's design and construction are very simple, so minimal disassembly was required to measure individual components.

### 2.6.3 Application

The first step is traditional reverse engineering. Since this example project is focused on the product's mechanical function, no other aspects of the of products design need be recovered, so the precise cataloging and BOM from Otto and Woods' approach is not necessary. The goal of this reverse engineering is simply to identify the components' interactions. To our advantage, the simplicity of the design allows much of this simplified reverse engineering to occur without complete disassembly. Due to these factors, Rekoff's "hierarchical structure" (system - subsystem - assembly - subassembly - component)[5] will be used to describe the configuration of the product's components (table 2.2).

Once the interaction of the product's components is understood, models can be suggested for simulating its function. For this example, we will be comparing three types of models: finite element analysis (FEA), motion simulation (within a solid modeling environment), and dynamics modeling. All three candidate models are capable of evaluating the oscillatory behavior of the shaker table to an acceptable degree of accuracy. Table 2.3 shows which data types are required by each model.

FEA requires a large number of dimensional measurements because it relies on material properties to determine behavior, so complete dimensions for each component is required (since no component is 'non-deforming'). For a component with a simple rectangular shape, 'complete' dimensions are three length measurements. A subgroup of the FEA model required for this particular analysis (created for other purposes) required a minimum of 24 dimensional and three material properties.

Motion simulation requires a full dimensional definition of all components. FEA and dynamics modeling require a smaller number of component dimensions than motion simulation to model the same system due to the other component properties their algo-

| Subsytem | Assembly | Subassembly | Component |
|---|---|---|---|
| frame (legs) | | | |
| | | | steel |
| | | | bolts |
| subframe (mechanism) | | | |
| | pan | | |
| | | | steel |
| | | | bolts |
| | suspension | | |
| | | | coil springs |
| | | | bolts |
| | | dog bones | |
| | | | castings |
| | | | rubber bushings |
| | drive | | |
| | | mounting frame | |
| | | | steel |
| | | | bolts |
| | | motor | |
| | | | motor |
| | | | frq drive |
| | | belt drive | |
| | | | pulleys |
| | | | belt |
| | | displacement shaft | |
| | | | keyed shaft |
| | | | mount bearings |
| | | | output bearings |
| | | drive springs | |
| | | | fiberglass bars |
| | | | fiber shims |
| | | | steel skis |
| | | | bolts |

TABLE 2.2: COMPONENT BREAKDOWN OF SHAKER TABLE AS PER REKOFF'S STRUCTURAL HIERARCHY.[5]

| | component dimensions | material properties | dynamic properties |
|---|---|---|---|
| FEA | 50 | 10 | - |
| motion simulation | 400 | - | 7 |
| dynamics model | 10 | - | 3 |

TABLE 2.3: DATA REQUIREMENTS OF EACH CANDIDATE MODEL.

rithms utilize. FEA, in particular, requires increasingly large amounts if time to process a simulation as details are added. Motion simulation, however, uses kinematics (solid body physics and component interaction), eschewing forces and moments to reduce computational complexity. This means the only way to faithfully recreate the behavior of the entire system is to model the system in its entirety. Motion simulation and dynamics modeling have the advantage of using dynamic properties instead of material properties; they use non-fundmental units to create results. Since FEA calculates the various stiffness and damping values of the components directly from their material properties, the accuracy of these material properties is very important, particularly those of non-homogeneous materials.

Now the barrier calculations for the three types of quantitative data are calculated. The results of these calculations, combined with the quantities in table 2.3, will provide the total barrier for each model; its RE ‘cost’ to implement.

What are component dimensions? They are individual linear measurements, generally taken with a set of calipers, a ruler/yard stick, or a tape measure (depending on the distance being measured). Dimensions are arguably the easiest measurements to acquire, requiring simple, readily available tools which require little training to obtain accurate data from; they will therefore be given the nominal unit value for 'effort' (a value of 1). In this case, these measurements can be performed at a rate of 60 per hour. Effort (P) and rate (F) estimations can also be made for other measurements made in this particular modeling process.

Material properties are generally known values. (In reality, we just looked them up.) Either finding data sheets via online search, requesting data from a supplier, or using an indenter-based material hardness tester requires considerably more time and effort than reading a ruler. The effort is estimated to be approximately eight times that of a dimensional measurement, and the rate is three to four per hour.

Dynamic measurements - measuring transient response to known inputs - is important for ascertaining damping values. Taking dynamic measurements is an involved process, requiring ten times the effort of taking a dimension, at two per hour.

The barrier for a single component dimension measurement is simply:

$$B = \frac{P}{F^2}$$

$$B = \frac{1}{60^2} = 2.77e^{-4}$$

And for a material property:

$$\textit{between} \;\; B = \frac{8}{3^2} = 0.889$$

$$\textit{and} \;\; B = \frac{8}{4^2} = 0.5$$

And for a dynamic property measurement:

$$B = \frac{10}{2^2} = 2.5$$

Now that we have calculated the barrier for each measurement, we need to multiply by the number of measurements required for each model (from table 2.3) and add the results to obtain the total barrier for implementing each model, yielding the following total barriers:

$$FEA : 50 * 2.77e^{-4} + 10 * .889 = 8.90$$

$$motion\ simulation : 400 * 2.77e^{-4} + 7 * 2.5 = 17.6$$

$$dynamics\ model : 10 * 2.77e^{-4} + 3 * 2.5 = 7.50$$

Motion simulation, being highly explicit in nature, requires a large number of measurements, making it the most 'expensive' model to implement. FEA and the dynamics model rely on abstractions to eliminate details which do not have significant effects on the shaker table's operating characteristics. While FEA splits these abstractions between model creation and model evaluation, the dynamics model relies on fundamental equations for evaluation, putting all abstractions *before* the calculation step, which gives its user the ability to ignore all but the most necessary inputs. A high degree of understanding is required for obtaining meaningful output from any of these models, so the dynamics model has a clear advantage in efficiency.

### 2.6.4 Concessions made for this specific case study

Since our reverse engineering team was relatively inexperienced at the onset of this project, the extraction rate $F$ values, used for calculating the barrier $B$ for any given measurement, would be much larger than for an experienced team, and generally high in variability and low in accuracy. With this in mind, the effort $P$ exerted to obtain each measurement should not be wasted, which drives us toward RE and model creation techniques which make efficient use of these measurements. Specifically, we strived to only make measurements that would be useful in the final model, and avoided large sets of measurements in an effort to reduce the impact of many small uncertainties propagating in a large model. Given these guiding constraints, we chose to first implement a simplified version of the dynamics model, using only springs and masses. We opted for this level of simplicity because it allowed easy debugging and error checking, and reduced the number of (relatively costly) dynamic property measurements to two: the spring constant of the fiber bars and that of the suspension. The masses of the pan and counterweight were given by the manufacturer and verified with a rough volume calculation, because removing such large, massive parts was not feasible (for reasons mentioned earlier). The spring constants, being static values, were relatively easy to obtain at a high accuracy. Avoiding damping in the first implementation of the model avoided the complications involved in measuring a dynamic property. In the second iteration of the dynamics model damping was added, which introduced a lower accuracy quantity to the model, due to the inherent hysteresis and signal noise of load cells.

## 2.7. CONCLUSION

The framework presented in this article provides guidance for model selection in situations where RE is required to obtain the necessary information for model implementation. Making use of existing RE calculations, it yields comparable 'barrier' values for each candidate model while requiring very little computational effort. In contrast, traditional model selection methods compare the accuracy of candidate models, requiring each model to be run and all (or at least a subset) of its inputs to be known.

When modeling an under-documented 'mature' product, RE is required to obtain many of the model inputs. This makes the 'all information up front' approach of traditional model selection methods prohibitively expensive or wasteful. The framework offers an alternative which requires little RE to implement while still providing a single-value quantitative ranking of the candidate models. This ranking directly assesses cost and time - quantities which determine the viability and scope of all engineering undertakings, particularly design projects.

Barrier calculations include collection rate as well as the required effort, yielding a more complete estimation of the 'cost' of implementing each model than either value would alone. Collection rate does not account for required expertise and resources, and effort does not assess the time required. Barrier is an existing RE value which incorporates both and is already is use for estimating the 'cost' of RE information extraction from a product. Using the proposed framework, an engineer can easily compare both time and effort required by various models with a single number that can quickly and easily be presented to colleagues and managers during project planning.

## 2.8. FUTURE WORK

We see many directions future work could take. Most useful would be the creation of taxomony to describe the difficulty and speed of various measurement methods. Such a taxonomy would both increase the accuracy of barrier calculations and provide quantitative guidance for selecting measurement equipment. It could even help estimate the propagated uncertainty for a specific model based only on its inputs. The consideration of manufacturing tolerances in reverse engineering measurements could also prove useful in estimating the distribution of a specific measurement value based on a small number of sample measurements.

## 2.9. ACKNOWLEDGMENTS

Authors would like to thank A&K Development Company for their support of this research. Any opinions or findings of this work are the responsibility of the authors and do not necessarily reflect the views of the sponsors or collaborators.

# 3. BRIEF BACKGROUND ON DYNAMICS MODELING PROCESS

The result of the framework described in chapter two is the selection of a dynamics model - specifically one which uses vibrations analysis principles to estimate the position of each of the system's two masses at any given point in time based on certain initial conditions. Dynamics models use forces to describe the movements of components within a system, allowing for accurate simulation of complex behavior with relatively low computational overhead. The model used here starts from a know state and calculates the position of each mass within the system based on the input from the 'forcing function,' which drives the system's movement. This 'initial value problem' uses the following initial conditions to numerically solve for the positions of each mass:

initial position of each mass: $x_1 = 0, x_2 = 0$
initial velocity of each mass: $x_1' = 0, x_2' = 0$
initial acceleration of each mass: $x_1'' = 0, x_2'' = 0$
initial position of the forcing function: $x_0 = 0$

These initial conditions and the system's dynamic equation are evaluated with a fourth-order Runge-Kutta predictor-corrector (in this case, MatLab's ODE45() function) to yield the positions of each mass (x1 and x2) at any given time, starting from t=0.

This form of model has a number of advantages and disadvantages.

## 3.1. Advantages

The dynamics model has a number of significant advantages over the alternatives. First, it is the simplest model. The visual structure of the dynamics model is a symbolic diagram of dynamic elements, where as the structures of the FEA and motion simulation models are 3-D solid models, requiring representations of a very large percentage of the components to produce. This simplicity makes the dynamics model robust to errors in the representations of specific components, either from changes made to the system or inferior data used in the representation. Second, errors are relatively easy to spot - good dynamics modeling technique generally yields good results. The other models considered can produce completely incorrect output that is difficult to detect. The background knowledge required to correctly construct on of these models is much larger than that required to input information and build a valid model, so anyone with basic solid modeling experience can create a model which will produce output, but that model may not accurately represent the real-world system on which it is based.

Last and most important, the dynamics model is robust. FEA and motion simulation also require fine detail, necessitating component-by-component accuracy as the system is modified, which requires continuous time and effort to be expended throughout the modeling process. The amount of data collection and degree of modeling accuracy needed to create and maintain these models also makes them much more susceptible to error, either in data collection or application, and less adaptable to changes in the system they represent. The dynamics model, however, is relatively easy to change, requiring measurements and allowing for greater attention to detail when collecting and analyzing data. The dynamics model's other important functional strength is that its accuracy is decoupled from its computational cost. The other two methods perform calculations on discretized units of the components in the system, requiring a discretization and interaction methods, as well as the level of detail, to be chosen. Higher levels of detail create a higher computational cost (not always resulting in higher accuracy results) which makes each simulation run take longer. The dynamics model derives its accuracy from the accurate and proper representation of the model and correct application of principles. As a result, it has a very low computational cost, which means it runs quickly - allowing model outputs that require many consecutive model iterations, such as frequency response curves, to be generated frequently.

A frequency response curve - the output amplitude of the system along a range of operating frequencies - requires the model to be run many times. Consider the shaker table system: its operating range is between 45 and 65 Hz on the motor driver, with the drivers display precision being 0.1 Hz. Taking one reading every 1.0 Hz, the frequency response curve would require 21 readings. Assuming the FEA and motion simulation model each require between 30 and 60 minutes to run a single iteration, those methods would require at least one day to provide an entire frequency response curve for one system configuration. The dynamics model requires less than ten seconds to run an iteration, yielding a frequency response curve in less than four minutes.

## 3.2. Disadvantages

The dynamics model also has some disadvantages which should be noted. First, a high degree of understanding of the system being modeled is required to create a useful dynamics model, The simplicity and inherent accuracy of dynamics methods are thanks to their application of fundamental principles on a large scale. Where as the FEA and motion simulation methods apply principles to small discretizations of the individual components of the system, dynamics modeling generalizes the behavior of entire components and assemblies as dynamic components of the system, making applications of principles simple and low-intensity for a computer. The high degree of understanding is required to correctly generalize the systems components, translating them into a dynamics model which mimics the systems behavior and component interactions while ignoring its overall form and the form of its components. Currently the definition of system components must be performed by an engineer, and may require a number of iterations to achieve a functioning model which correctly represents the systems behavior. The other methods require the human to focus on rigorously recreating the details of the assemblies and components, requiring a relatively low degree of understanding of the systems dynamic behavior. These methods can even be implemented with very little understanding of their mechanisms of operation, although that would likely result in erroneous model output. Stated another way, a large portion of the human's effort in these other methods is spent on low understanding / speciality work and, although understanding is required, relatively little time or effort is spent on it. In contrast, dynamics modeling requires the majority of human effort to be spent on the creation and tuning / updating of the model - work which requires constant (and consistent) consideration of how the dynamics model works.

A large part of this 'translation' from the system's configuration to a dynamics model representation / simplification requires utilizing a number of simplifying assumptions. These include, for example, assuming that the masses in the dynamic system (which are large constructs of welded steel) are perfectly stiff and do not have their own spring stiffness properties, and that the frame is perfectly stiff and does not deflect much. Also, the springs - both fiber bars and coil springs - are assumed to be linear. Testing shows that the fiber bars have a slight curvature in their force-deflection plot (figure 3.1), but not

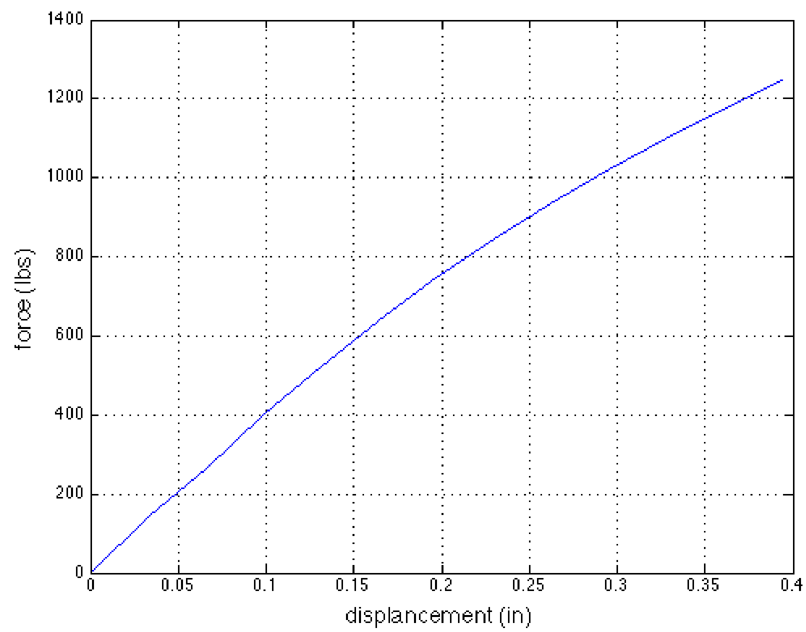


FIGURE 3.1: Force-deflection plot of fiber bar under loading conditions experienced in shaker table.

enough to make a linear approximation unreasonable. Furthermore, newer bar designs exhibit even less force-deflection curvature. Lastly, the drive mechanism is assumed to always provide a perfect sinusoidal forcing function which dominates the dynamic system. Since the system operates near its natural frequency, this assumption is both reasonable and helpful because it allows the model input to be a simple sinusoid without requiring the inclusion of feedback force from the driven dynamic components.

It is also important to note that the dynamics model scales easily, requiring little modification to add details, including nonlinear dynamic properties and additional dynamic components. This flexibility is a byproduct of the model representing component interactions without using the components' geometries, so adding detail to the model only requires modifying a set of dynamic expressions. Being able to modify the model quickly compliments the iterative approach taken to increase the efficiency of the RE measurements.

## 3.3. Iterative Approach

An iterative approach was selected for building the dynamics model, starting from a very basic model and adding details to reach the desired level of accuracy. The simple model included essential measurements and property values that are were likely to change, such as spring constants and masses. Details were then added, with each addition being the next most critical or effective value for increasing model fidelity. This strategy helped ensure that the time and resources spent in measuring the model values were not be wasted

The following conference paper (chapter four) describes the initial model in great detail, including the efforts taken to obtain the data used to create this model.

# 4. DYNAMICS MODELING TO INFORM DESIGN OPTIMIZATION

Tim Foglesong, Rob Stone, and John Parmigiani

Proceedings of the ASME 2014 International Design Engineering Technical Conferences
& Computers and Information in Engineering Conference

## 4.1. Abstract

*This paper presents the methods employed in modeling a vibratory conveyor for use in model-based design optimization. The conveyor, essentially a large table whose top oscillates at an angle off of horizontal, uses springs between the drive mechanism and the tabletop to directly apply a sinusoidal excitation. These springs prevent the system from losing response amplitude as load is increased. The manufacturer is having difficulty optimizing performance and reliability in newer designs, and has requested a model-based approach to the design optimization. This study discusses the initial steps taken in modeling the original mechanism design, specifically the dynamic model and experimental determination of the necessary spring constants. The first full iteration of the model starts with low detail and simplified geometry, with a plan to add complexity as needed to improve accuracy. In the initial model, the parallel springs in the tabletop suspension are combined, bypassing the spring mounting geometry, and tested as one large spring. The drive mechanism springs, bars of fiber reinforced plastic (FRP), are more meticulously tested in a tensile testing machine. The resulting spring constants are used in the initial model to calculate the sinusoidal response of the tabletop at any given input frequency. The deflection response per time of the tabletop is then measured and compared to the model. Conclusions detail the initial model's accuracy and Future Work examines how to bring it in closer agreement with the real machine's sinusoidal response.*

## 4.2. Nomenclature

$a$ Amplitude

$frq$ Frequency

$m$ Mass

$k$ Spring constant

## 4.3. INTRODUCTION

Reverse engineering is largely practiced in industry, but not as prevalent in academia. In industry, it is generally associated with comparing competing designs or obtaining features, but has a wide range of uses, which also include duplicating irreplaceable parts and replace or update data for current designs [20]. In the case of this paper, reverse engineering is used to develop a dynamic model in order to enhance optimization. Some of the essential measurements needed to create a complete dynamics model, in this case spring constants, are simply not known or vary widely from the prescribed nominal value. This paper details the process of starting from the machine and a few nominal values and creating a complete dynamics model. Specifically, the progression includes developing the model from the device's function, determining which values are needed in the model, finding the needed values, including experimental techniques and results, and comparing the model output to data from the machine running. What makes this project unique is the control over every aspect of both the dynamic model and the data collection and interpretation. Balancing model complexity and data accuracy is the primary concern in this stage of research; useful optimization requires an effectively detailed model, but increasing the number of variables increases the number of required tests (and compounding uncertainties).

### 4.3.1 About the device

The mechanical device is a vibratory conveyor — a large steel 'shaker table' that oscillates its top surface at an angle off from horizontal (Figure 4.1). As the table top (referred to as the 'pan') rises and falls, the loose material sitting on the table 'hops'

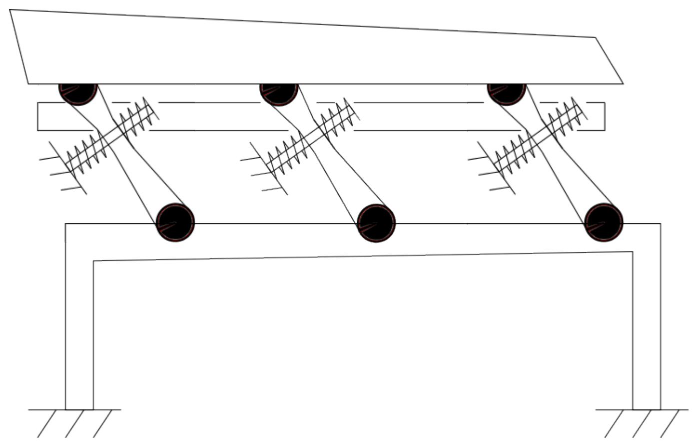

FIGURE 4.1: DIAGRAM OF MACHINE COMPONENTS.

across the surface. When the surface rises, the material gains momentum so that, when the surface starts to descend, the material loses contact and is briefly in free fall. Since the surface is oscillating at an angle and the material falls vertically, the material will always land slightly 'downstream' of where it started (closer to the output end of the table). Although the displacement is small, the material traverses the entire conveyor surface in a matter of seconds because the pan oscillates at ten to fifteen hertz.

Most vibratory conveyors use expensive offset-mass oscillators to provide a base excitation to the pan, a simple mass-spring system, which vibrates consistently when the load is constant and the vibrator units are synced properly. Additionally, this scheme requires a massive subframe to mount the oscillators with large rubber isolators to properly support it. The device discussed in this paper does not use base excitation, instead applying force directly to the pan through another spring, which provides more force under heavier load due to preloading, and does not require timing adjustments.

The device being analyzed is previously well established and has a track record for reliability, but its manufacturer would like to increase performance and reduce material costs while maintaining or improving reliability. The manufacturer's next generation design, currently under development, is proving difficult to optimize. Model-based optimization has been chosen as an alternative to the current development process. The first step, presented in this paper, discusses the development of a dynamics model of the root design as practice for modeling the newest design. This model will be used to inform changes to stabilize performance and increase reliability. The efforts presented in this paper are a first step toward modeling the newer design for use with computer-based design optimization.

## 4.4. BACKGROUND

### 4.4.1 Machine Function

Traditionally, vibratory conveyors use offset-mass oscillators attached either directly to the vibrating surface or to a 'live' subframe to induce oscillation. These oscillators rotate a pair of offset masses, geared 180° out of phase, to produce a linear oscillation. Although reliable, these systems are difficult to balance and require a substantial energy input to rotate the masses fast enough to maintain usable oscillation under load. To keep flow consistent, active control systems are used to monitor performance and adjust for changing load. Unlike these standard offset-mass vibrators, the shaker table discussed in this paper uses eccentric bearings keyed to a drive shaft to oscillate the pan and an equal mass 'counterweight' in opposition. The eccentric drive transfers its force through a set of composite beams, specifically fiber reinforced plastic (FRP). This configuration can maintain or even improve performance under high load; the FRP bars are loaded in parallel with the suspension springs that provide the normal force to support the load, so the load on the pan preloads the bars. When combined with the deflections from the eccentric bearings, this preload increases the deflection in the FRP bars, thereby increasing their total force output. The counterweight, meanwhile, vibrates in opposition to the pan; whenever the pan encounters a large acceleration, the counterweight accelerates in the opposite direction, minimizing the force transferred to the subframe. As a result, the subframe has less mass and requires much smaller isolators than those of oscillator-based conveyors.

### 4.4.2 Counterweight

The counterweight, as mentioned above, receives input force in a sinusoid that is inverse to the one the pan sees, so whenever the pan's FRP bar pair is under maximum compression, the counterweight's bar pair is experiencing maximum tension. Since the pan and counterweight are vertically stacked completely parallel, this means their momentums at any point should be equal and opposite. As a result, the frame and floor provide very little horizontal force (ideally zero) so that, unlike the traditional offset mass system, this design does not need to be anchored to a concrete slab to operate.

A half-speed, three-phase electric motor running at 208V drives a keyed drive shaft via a belt drive (approximately 2:1 ratio). The shaft is keyed to a set of four eccentric bearings, two pairs set at 180° offset from each other. The bearings are set in pillow blocks, which are attached to the FRP bars. The other end of each bar is fixed to a mass, either the pan or counterweight, so the loading condition of the bars resembles a free end column load. Since each bar's displacement is applied directly by the eccentric bearing and pillow block, there are equal axial and transverse displacements applied to the bar ends each rotation. Also, the bearings do not prevent the bars from bending, so standard operation produces relatively large bending displacements perpendicular to the material plane.

There are two 'masses' in the system: the top surface pan and the counterweight. Their masses and loading conditions are similar, as are the spring arms that attach them to the device's frame. Six of these spring arms, called 'dog bones,' suspend the pan and restrict its motion to a small tangential arc (Figure 4.1). The counter weight is also suspended by six dog bones, identical to those on the pan.

The dog bones have four springs each; one rubber collar press fit into either end and a pair of coil springs mounted along the span (Figure 4.2). The larger of these coil springs is compressed against a tab protruding from the frame, and the smaller spring sits opposite it, compressed only by the bolt that runs through the larger spring. The smaller coil spring prevents the bolt and larger coil from moving freely as forces change during the cycling process. There is speculation that the smaller coil spring, or even both coil springs, are not needed, but this is beyond the scope of the current project phase.

## 4.5. PROBLEM DEFINITION

The presented problem is the quantitative representation of the device's mechanical operation. The FRP drive springs/bars are of particular interest. Obtaining the stiffness of these members and quantifying effects of variations in their geometry and mechanical properties is key to informing decisions about improvements to the drive mechanism. The client is interested in better utilizing the current FRP 'springs,' but would ultimately like to reduce the number used, reduce their size, or even replace them with a smaller or lower cost alternative.

## 4.6. DYNAMICS MODEL DEVELOPMENT / EXPLANATION:

The first step of the project is to create a model of the shaker table mechanism that is accurate enough to predict performance while minimizing detail. Avoiding unnecessary detail is essential to creating a useful model. In this case, many easy-to-acquire details are irrelevant or have low impact on the overall dynamic response, and many avenues of testing were available, requiring a compromise between cost and data quality. Even high quality data can be made irrelevant by a single low accuracy/precision variable. To achieve

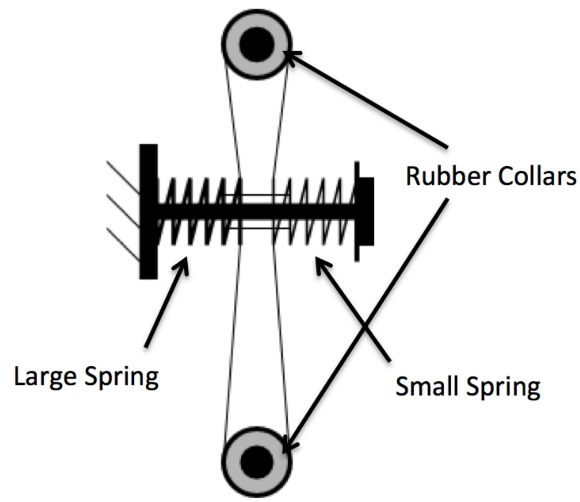


FIGURE 4.2: DETAILED VIEW OF 'DOG BONE' SUSPENSION COMPONENT.

high precision, high detail models require consistently high accuracy (and precision) for every constant value; acceptably small errors in a large number of interacting values quickly compounds to large uncertainties and low precision. For this reason, a highly simplified model was developed to begin this project, with the understanding that higher model complexity and more experiments would be necessary to improve the accuracy to necessary levels. For instance, each of the six dog bones suspending the pan has four discrete springs, but all 24 are considered to be one in this model, and are tested as such.

The model also simplifies the system from full three dimensional with displacements and rotations to a linear system, with only one dimensional displacement. These simplifications allow for easy accommodation of the system's two degrees of freedom, since the geometric details that complicate adding the second degree of freedom have been eliminated by simplification (Figure 4.3).

The dynamics model presented herein is simplified by combining springs and reducing the geometry to a two degree of freedom linear system. The structure of the mechanism lends itself rather easily to these simplifications. In this vibratory conveyor, the drive springs are fiber reinforced plastic (FRP) bars under modified free end offset

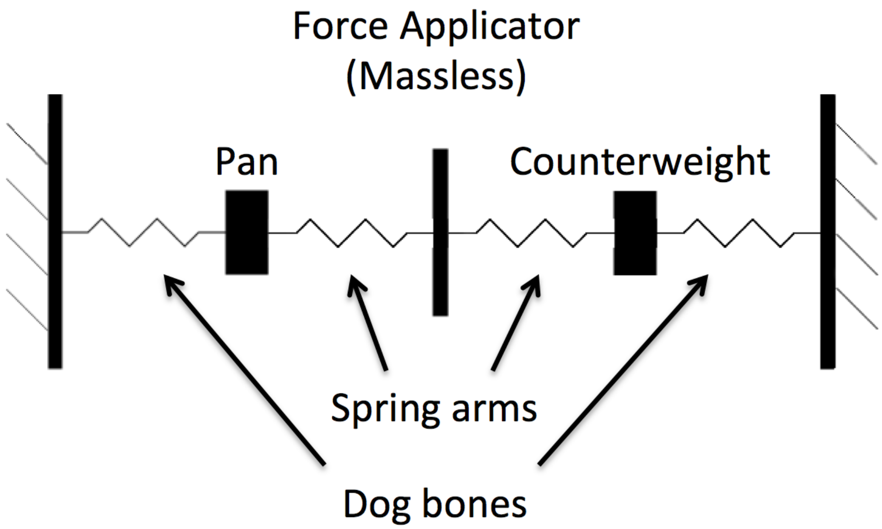


FIGURE 4.3: THE DYNAMICS MODEL.

column loading to transfer the driving sinusoidal agitation to the table surface. These bars are run in parallel pairs, which are combined in the dynamics model. The array of springs in the pan suspension is also completely parallel, and is likewise combined in the model. Further, the drive mechanism applies force to the pan and counterweight in exact opposition, they are set 180° out of phase, so at any point in time, their drive springs are receiving equal and opposite deflection from the eccentric bearing drive. This maps directly to a linear system in which the pan and counterweight move in opposition because the sinusoidal application of the excitation can be handled by the forcing function definition, which defines the position of the mass-less 'force applicator' that sits between the FRP drive springs for the pan and counterweight (Figure 4.3).

The dynamics model takes the standard form for damped mass-spring systems (Equation 4.1):

$$m * x'' + l * x' + k * x = 0 \tag{4.1}$$

where $m$ refers to the mass of either the pan or the counterweight (same value within 1.6%), $l$ refers to the damping constant, which is omitted in this model, and $k$ refers to the spring constant of the spring arms (FRP bars) and dog bones. Figure 4.4 identifies where the various springs reside in the machine; the sets of dog bones that make up the pan suspension and counterweight suspension are attached to their respective masses and the subframe. The spring arms are connected in pairs to the drive mechanism and either of the two masses.

The system is assumed to be forcing function dominated, so the right-hand-side can be represented by a standard sinusoidal expression (Equation 4.2):

$$a * cos(2\pi * frq) \tag{4.2}$$

where the amplitude, $a$, of the forcing function is simply the offset of the eccentric bearings (0.016in) and the frequency, $frq$, is based on the frequency supplied by the electronic motor controller (set by the user), the motor's design, and the pulley ratio of the belt drive. The three-phase induction AC motor has four poles per phase, which means each full AC cycle going into the motor causes 0.5 turn (the poles are alternating opposites, so the precession of a given pole is caused by either a peak or trough in the time vs voltage waveform); the frequency from the motor driver is halved when the motor converts the electrical energy into mechanical energy [21]. The belt drive is approximately a 2:1 ratio from the motor to the keyed shaft, so actual drive frequency is 0.25 of the frequency supplied by the motor driver.

To obtain the sinusoidal response, the dynamic model is evaluated as a differential equation, using a 4th order Runge Kutta predictor-corrector. Since the starting conditions acceleration, velocity, and position are known to be zero, and the process can continue indefinitely, this is an initial value problem. The resulting solution is the pan's and counterweight's positions with respect to time, starting when the drive motor is activated [22][23].

Nominal values for mass were obtained from the manufacturer. The the spring constants of spring arms and dog bones were measured directly with electronic instruments.

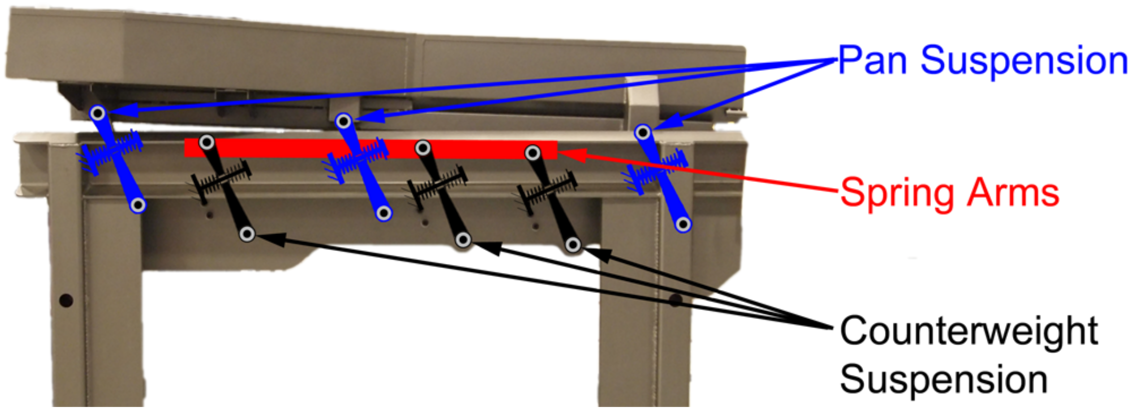


FIGURE 4.4: SPRING POSITIONS ON TABLE

This procedure is described in the next section.

## 4.7. TESTING PROCEDURE

Since the model constants associated with the motor drive are nominal values and the masses of the pan and counterweight were treated as nominal values for the first model, the only model constants determined from direct experimentation were the spring constants of the FRP spring arms and dog bone suspension. Next, the testing procedures and necessary data analysis to attain the spring constants are discussed for each of these spring systems.

### 4.7.1 Spring Arm Testing

The spring arms were tested in an Instron 4505 with a 100kN load cell. In order to match the mounting and loading conditions present in the shaker table, the pillow block and associated hardware used at the 'driven' end of the spring arm in the machine was attached to a clevis that mounted to the load cell on the underside of the cross bar. For the fixed end, a plate with a vertical block was bolted to the base of the Instron, preventing translation and rotation in all three axes (again with consistent mounting hardware).

Each sample was deflected from zero to ten millimeters at five millimeters per minute. Although the load rate was much slower than the machine produces, it allowed for precise data collection. Aside from load rate, the axial load test setup matched the spring arms' loading conditions in the shaker table.

Four matched spring arms (a full set for the device) were tested individually. The four bars were assumed to be physically identical, so that the measured variation would match that encountered by the operating mechanism.

### 4.7.2 Spring Arm Data Analysis

The compression data was analyzed in MATLAB. A first degree polynomial was fitted to each run (individually), then the first degree coefficients (slopes) of these polynomials were treated as a simple data set (mean and standard deviation taken). The smallest $R^2$ of the linear fits was 0.989. The data, however, was not expressly linear. Second degree (quadratic) polynomial fits yielded high conformance, with $R^2$ values above 0.995.

### 4.7.3 Pan Suspension Testing

The pan suspension was tested "en masse" – a 500 lb load cell was affixed to the center of the pan and a linear potentiometer with 0.5in of travel was mounted to the frame, with its wiper actuator mounted to the pan. The potentiometer and load cell were mounted perpendicular to the back of the pan, in order to maintain the dynamics model's linear paradigm. Data was collected with a National Instruments USB6211 acquisition unit at 1kHz. The load cell was powered externally with a digital power supply, and the potentiometer was operated from the DAQ's 5V rail.

### 4.7.4 Pan Suspension Data Analysis

The data acquired from the pan suspension initially had an unacceptably high level of noise for spring constant calculations. After taking measures to reduce radio interference, the noise was still unacceptable. Since samples were taken at 1kHz, well above the operating frequency of the mechanism, the data could be filtered. A moving average was used with some success, but was scrapped in favor of a Butterworth low pass filter. Using a fourth order Butterworth filter, high frequency noise was more effectively removed. Additionally, a forward pass / backward pass scheme removed the filter lag. Filtering improved both the force test data and the empirical system response data (collected with the linear potentiometer only). From the low pass filtered system response data, relatively accurate response amplitudes were attained, and could be directly compared to the modeled system response [24]. Comparisons are in Figures 4.5 and 4.6 in the Results section.

### 4.7.5 Combining Springs

All combined springs are in parallel in the mechanism. This simplifies data collection because the pan suspension springs can be tested as one – taking the effective spring constant of the entire pan suspension assembly at once. This eliminates both the high uncertainty from compounding a large number of spring measurements and the large effects of errors in spring mounting geometry measurements. Unlike the model, the device's springs are mounted at a variety of angles. Some change angle and load characteristics as they are loaded. Model errors in the recreation of the geometry can have large impact on force vectors and loads throughout the mechanism, and introduce unacceptably large uncertainties.

## 4.8. RESULTS

Based on the geometry used to collect the effective spring constant data from the pan suspension and spring arms, the values could be used directly in the linear dynamics model. This linear model, lacking damping, produced severe beats with large maximum amplitudes near the natural frequency, which is approximately 15hz (Figure 4.5). As a result, the model's amplitude increase is much more pronounced near the natural frequency

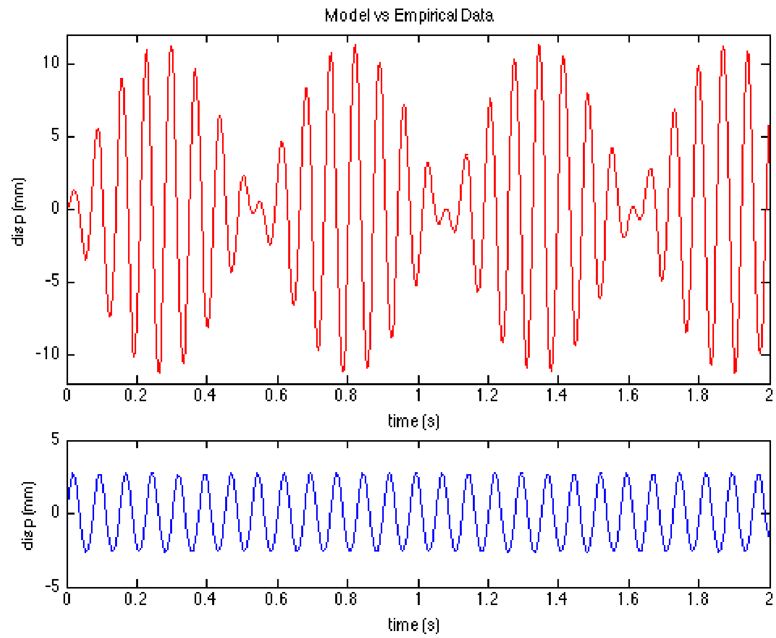


FIGURE 4.5: THE DYNAMICS MODEL (TOP) DISPLAYS EVIDENT BEATS, ABSENT IN THE EMPIRICAL DATA (BOTTOM).

than that of the machine (Figure 4.6). Although the model only fits below 8Hz, arbitrarily adding a constant damping coefficient (Figure 4.6) eliminates the runaway frequency response.

## 4.9. FUTURE WORK

This paper presents the first iteration of a reverse engineered model, which can benefit from a number of improvements. One such improvement is measuring the pan and counterweight masses. Pan and counterweight mass data for every machine built would be much more useful for using the model as an optimization tool. Ideally, every measured piece of data in the model would have an associated standard deviation, but that is not always possible. Its not likely every pan and counterweight could be weighed,

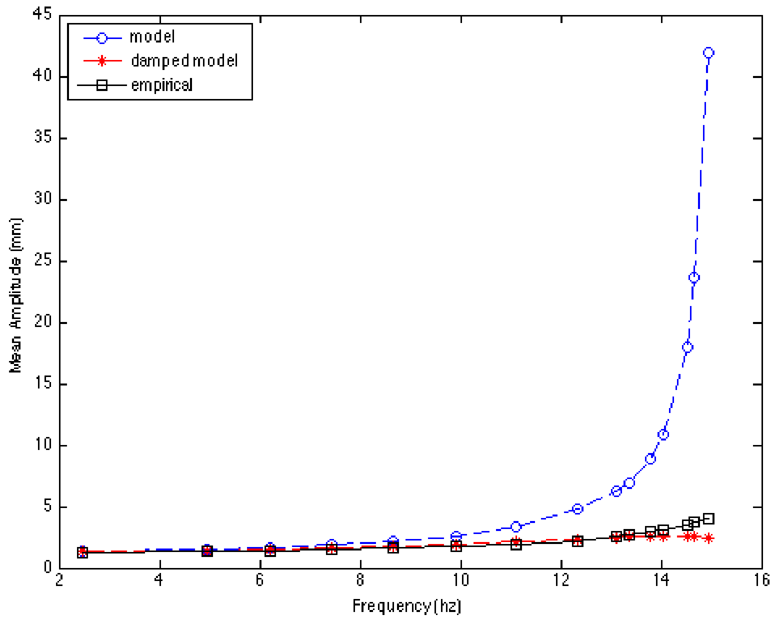


FIGURE 4.6: ARBITRARILY ADDING A DAMPING COEFFICIENT (ASTERISKS) TO THE MODEL (CIRCLES) GREATLY IMPROVES AGREEMENT WITH THE EMPIRICAL DATA (SQUARES).

so the nominal design values may be the only masses available. In that case, a reasonable standard deviation for each mass value would have to be assumed.

Another improvement is reducing sensor noise. The noisiness of the current data signals used to calculate the spring constant of the pan suspension means the resulting spring constant data is not perfect. A complete re-test of the suspension's spring response has the potential to improve the model's accuracy, if additional measures are taken to further reduce noise. One option is trying different sensors. Others have tried a variety of sensors, and selected the highest fidelity data after running the experiment [25]. Although this technique provides the best data available from the chosen range of sensors, it also requires simultaneous collection from the range of sensors. Adding sensors increases setup and processing times, as well as overall complexity. Some sensors, such as high speed cameras, cannot be used with a standard data acquisition unit for simultaneous data capture.

High speed cameras, like potentiometers, measure position directly and are therefore a reasonable option for capturing position over small time intervals. High speed video has been used successfully on past studies of cyclic motion, and allows for simultaneous position tracking of dozens of data points [26][27]. The only issue is high speed photography requires even, high intensity lighting and careful preparation for obtaining correctly scaled measurements. Additionally, specialized software is needed to reliably track datum points.

Sensor data can also be improved by preprocessing. Load cells are inherently noisy, an issue which has reduced the reliability of the pan suspension spring constant testing data. To this point, all load cell data has been filtered in post processing, but filters can also be accomplished with an analog circuit. Circuits are commonly used to enhance analog signals *before* conversion to digital data, and can increase voltage range (enhance digital resolution) as well as decreasing noise [28]. Although improvements in digital signal processing have made such circuits less necessary in many situations, the small resistance variations provided by a load cell's strain gauges make the data more difficult to precisely digitize and exacerbate the effect of any outside noise. A filter/amplifier circuit has the potential to greatly improve the analog signal coming from the load cell.

But most important, creating a method to load the pan in a consistent way over a range of load rates will enable testing of the suspension's damping constant. Damping is dependent on the velocity, the first time derivative of position, so simply applying the amount of force haphazardly will not give accurate data. To obtain good damping data, loads need to be applied at a variety of consistent rates, so the damping response can be found across a range of velocities. This will likely require a purpose-built load application device. Another option is to remove an individual 'dog bone' from the device and test it in the aforementioned Instron tensile tester at a variety of load rates. This many be insufficient, however, because the tensile tester uses a pair of lead screws to move the cross head, and is therefore unable to apply loads at rates consistent with the table's drive mechanism.

## 4.10. ACKNOWLEDGMENTS

Authors would like to thank A&K Development Company for their support of this research. Any opinions or findings of this work are the responsibility of the authors and do not necessarily reflect the views of the sponsors or collaborators.

# 5. FURTHER DYNAMICS MODELING

The damping discussed at the end of chapter four has since been implemented. This required a relatively large amount of effort, which was predicted by the framework calculations that indicated most of the dynamics model's total barrier was due to dynamic properties. In practice, this meant quite a bit of specialized testing, both in the equipment used and the data analysis performed. The assemblies which provide the damping in the dynamic system (the dogbones) had previously been tested in parallel; they were isolated from all other dynamic elements and their displacement was measured when subjected to a static force of known magnitude. The equipment used for collecting this data was very simple: a linear potentiometer was used for capturing frequency response, and a load cell for the force, which applied by a screw-based apparatus. The analysis of this data was also very simple: the force was applied very slowly and the final measurement was taken in a static state over a few seconds (four to ten seconds) at 1000 samples per second. This measurement was definitively static and the load cell was allowed to dwell at the measurement state for at least two seconds, so measurement hysteresis was well compensated for. Also, since the final measurement position was approached with continuous, un-reversed compression applied to the load cell, there was no possibility of load cell error due to force-reversal.

## 5.1. Linear Potentiometer Nonlinearity

The linear potentiometer that measured position (and from that speed) of the pan in frequency response, static force, and damping testing has only one source of error, nonlinearity. Although designed to have linear response characteristics, this potentiometer is just a variable resistor; it is an analog component that measures a continuum of positions, not distinct distinct units or steps of position (as a linear encoder does). Since it is analog, the resistance along the wiper's track is not perfectly uniform, so the resistance response is not perfectly linear. To account for this, six resistance measurements were taken at known displacements along the wiper's travel, and used as a direct (linearly interpolated) "conversion table" for the voltage outputs of the linear potentiometer.

## 5.2. Two Types of Load Cell Error

The load cell was a more problematic sensor to use. It suffers from two different types of hysteresis, as well as providing a very small change in voltage compared to the required excitation voltage to operate it. Additionally, it has much higher signal noise ratio than the linear potentiometer, is very susceptible to outside signal noise (RF interference), and will be damaged if loaded beyond 150% of measurement capacity or left loaded for extended periods of time. The first type of hysteresis the load cell suffers from is 'short-term transience' - essentially the change in output signal lagging behind the force value it is indicating as the force is applied. This does not affect steady-state 'static' measurements, but is an issue for time-based measurements (such as transient load response for determining a damping constant). The second type of hysteresis is 'load creep' or 'load-reversal' lag, which is the load cell failing to correctly report the magnitude of a reduction or reversal in the applied force. This is most noticeable when a load cell is fully unloaded after a load, either compressive or tensile, has been applied. The load cell, with no load applied (even when removed from the test apparatus entirely) still reports a small load of the same type as was just applied (i.e. compressive if compressive, tensile if tensile). This 'shift' in the reading will disappear and the unloaded reading will go back to zero, but it takes a significant amount of time (greater than ten minutes). Waiting for this longer-term hysteresis to dissipate is generally unsuitable (or unreasonable) for static measurements, and definitively impossible for measurements of time-based phenomena. When taking steady-state measurements, such as the quasi-static force-displacement curves of the fiber bars taken on the Instron tensile test machine, the time between tests is short enough (less than twenty seconds) that the second type of hysteresis (load-reversal lag) can be assumed to have not dissipated. As a result of that and the consistent load direction and similar load magnitudes of each successive test, this second type of hysteresis can be circumvented by simply discarding the first test of each batch, which did not have a test immediately before it, so the load cell did have time to recover from the second type of hysteresis, resulting in a changing reported baseline (unloaded) value.

For dynamic tests, however, load cell hysteresis is difficult to account or adjust for. The force response as a function of time is critical to these tests, and loads are often dwell or are reversed; these characteristics trigger both types of load cell hysteresis. In the case of testing the dog bones' damping, the system is loaded cyclically at operating frequencies.

## 5.3. Dynamic Property Testing: Damping

Stiffness is the force reaction to a certain displacement. Damping is the first time derivative; the force reaction to a certain velocity of displacement. As such, all damping measurements must be made at constant, known velocity. In order to take many of these measurements, the system must be forced cyclically (sinusoidally), achieving constant velocity across the center of the movement range and rapidly accelerating and changing direction at the extremes. This technique mitigates the long-term hysteresis, but occurs

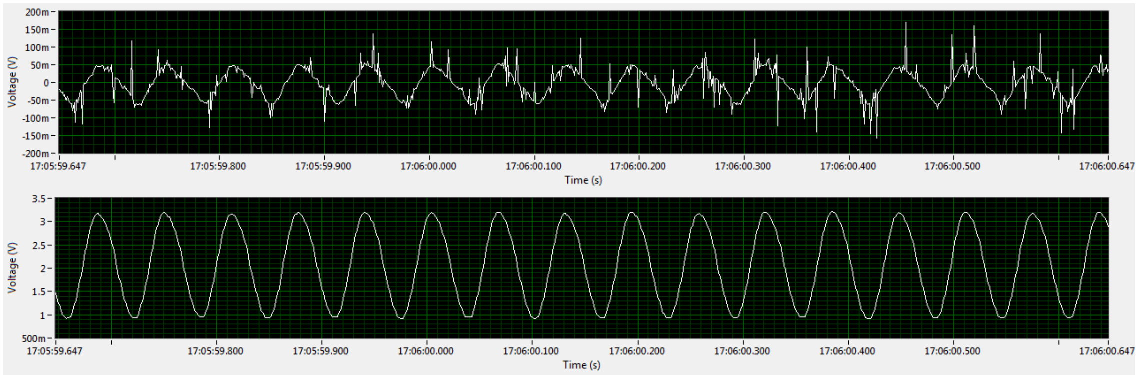


FIGURE 5.1: Unfiltered sensor signals, load cell (top) and linear potentiometer (bottom).

very quickly - to obtain the necessary velocities - and includes frequent load reversals. This means the peak force values, both minimum and maximum, are higher than what is reported by the load cell. At the same time, the load is being reversed twice per cycle, creating more amplitude error from hysteresis. Since damping is measured at constant velocity, only data from the center of the sinusoid (constant velocity as the system traverses the middle of its total travel range) is used for calculating the damping constant. This point in the travel range is farthest from the ends, so experiences less severe effects from the load-reversal hysteresis. However, it is not immune to short-term transience (from load application). This resulting reduction in accuracy is considered to be smaller than that of the signal noise and the temporal lag smaller than that of a frame-averaging filter which could be used to mitigate the signal noise.

### 5.3.1 Signal Noise: Load Cell

The load cell has a very small signal voltage: 3mV/V. This means that for each volt of excitation, three millivolts of output signal will be generated at the load cell's maximum rated measurable force. For this testing setup, there is a 500lb load cell with a 14V excitation put across it, yielding 0.042V output at 500lbs of tension, and -0.042V output at 500lbs of compression. The linear potentiometer, in comparison, sweeps the full range of its excitation voltage across its one half inch displacement (in this case, zero volts to five volts). Figure 5.1 shows 'raw' data from both the load cell (top) and the linear potentiometer (bottom). The potentiometer produces a very smooth output, with no lag or signal noise, but the relatively weak signal from the load cell is highly affected by noise.

### 5.3.2 Data Logger

The National Instruments 6211 USB data acquisition module (DAQ) used to collect the data has a 16-bit Analog-to-Digital Converter (ADC) with a selectable +/-200mV to +/-10V range. This DAQ has enough resolution to surpass that of the load cell and see all of the signal noise. The National Instruments "Signal Express" software used for data collection has a number of tunable filters. At first, a fourth-order IIR Butterworth lowpass filter was applied to the live data, but this added time-domain lag to the signal. A two-pass Butterworth filter was later used to view data without creating this lag. By adding 'backward' or 'reverse' passes to the filter, the transition lag is introduced in both directions, which effectively removes it. This technique cannot be performed on live data because it requires the data after the current point to be known. Both of these filters degraded the data, however, by averaging the lower amplitude 'good' values with higher amplitude noise, smoothing the noise into the data instead of removing it.

Figure 5.2 demonstrates the state of the unfiltered load cell signal (left). Although very noisy near the peaks, the signal is relatively acceptable through the center of the sinusoidal oscillation. As previously described, this is the useful portion of the data because it is the region of the oscillation over which the system's velocity is constant, so damping ($force = f(\frac{dx}{dt})$) is isolated from higher-order effects. Simply removing the outliers (most erroneous data points), the important data points - those at the center of

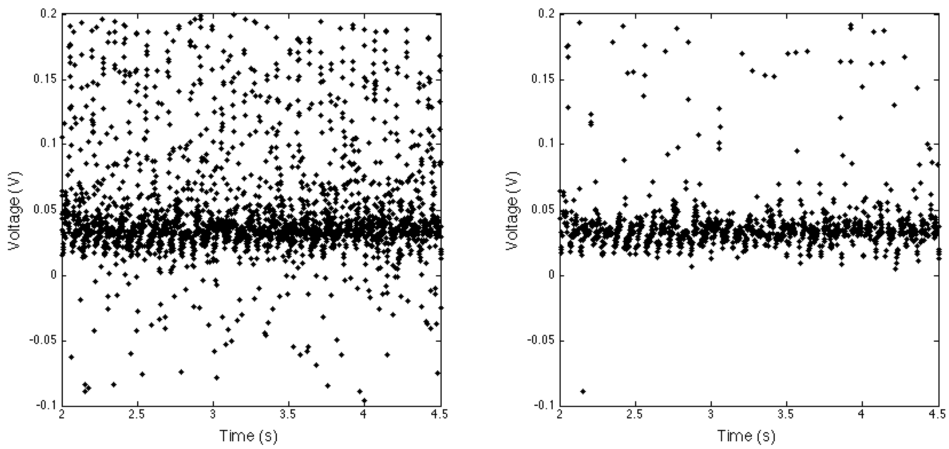


FIGURE 5.2: Load cell output (sinusoidal); unfiltered (left) and filtered by outlier removal (right).

the sinusoid - became acceptably consistent without the need for averaging-based filtering.

## 5.4. Further Improvement

By reducing the model calibration range to the operation range of the shaker table, unpredictable behavior at lower frequencies was avoided. The shaker is never run at these lower frequencies, so the frequency response in that region is unnecessary for creating an effective dynamics model. In the operating range, between 600 and 800 RPM, the shaker table and the model exhibit similar frequency response curves, suggesting a simple correction factor to make the model match the empirical data (see figure 5.3). First, a phase shift was applied to account for losses in the drive mechanism. A ten percent reduction in drive frequency, attributable to the belt drive slipping and the motor spinning slower than its driven frequency, accounts for the mismatch in curvature along the horizontal axis. Second, the difference in amplitude between the empirical data and the model ('difference' on figure 5.3), averaged over the operating range, was directly subtracted as

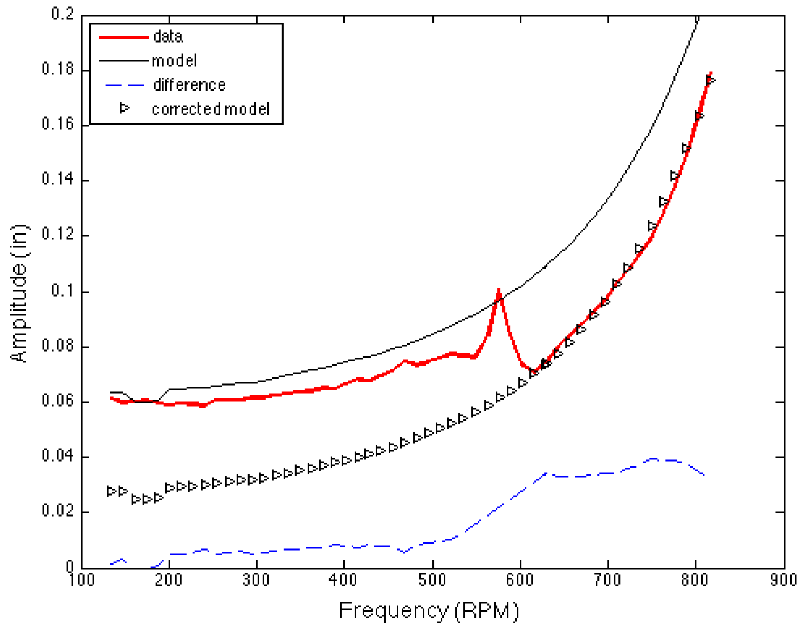


FIGURE 5.3: Dynamics model, without and with correction factor applied.

a correction factor for frictional losses in the shaker table. These corrections shifted the model's output, bringing it into agreement along the shaker table's entire operation range.

## 5.5. Application of the model to the problem at hand

The dynamics model required two important improvements to be useful for working with the sponsor Company. First was the relatively easy task of converting the output data to the units used by the Company - lengths and masses in imperial units, and frequency in RPM instead of Hz. The second, however, was the very involved process of adapting the model to the Company's larger machines - considered crucial for project success.

The test machine located in Corvallis, although useful for design recovery and early testing, was not the machine for which redesigns were originally intended. The Company's 'color sorter' shaker table is produced in three standard sizes: eight lane, twelve lane, and sixteen lane. The test machine in Corvallis was a twelve lane varient, with pan and counterweight masses of 250 and 254 pounds, respectively. The sixteen lane variant, which is the most important in the product line, has pan and counterweight masses of 622 and 634 pounds, respectively. The difference in mass changes the natural frequency of the system, requiring stiffer springs to achieve the same operating points (desired amplitude and frequency). Changing the pan and counterweight masses in the model to those of the sixteen lane variant is an essential step in designing fiber bars for this larger variant of the system.

Why did the project start with analysis of the 'incorrect' variant of the machine? The twelve lane test machine was select for testing in Corvallis based on access into the host facility and delivery equipment requirements. This decision was made with the knowledge that the model would need to be adjusted to simulate this larger machine. The sixteen lane shaker table, being the most important, is the primary concern for performance optimization. Any sweeping redesign to optimize this variant would break the manufacturing parity with the smaller shaker table variants, likely increasing manufacturing costs. This situation dictates a shift of focus to optimizing an individual assembly or component for each variant. The main 'spring' - a bar of unidirectional fiber glass, has a large effect on the shaker table's natural frequency, so designing this 'fiber bar' for the sixteen lane shaker is an efficient method to improve performance without drastically changing the table's overall form. The smaller tables will need their own fiber bar designs if optimized performance is to be realized from them, but the Company's primary goal is to optimize the performance of largest shaker table, and this will be accomplished through composite design.

The unidirectional e-glass bars, produced by Gordon Composites, are manufactured in sheets, then cut to size and laminated in stacks to achieve the desired layup thickness. Stress concentrations created by the loading conditions in the shaker table were causing early failure of the fiber bars. The Company was exploring a tapered fiber bar, which would have a wedge-shaped profile through the thickness, making the end where failure occurred thicker and therefore stiffer. FEA analysis of this design quickly showed that a smaller stress concentration at the opposite end, previously inconsequential, would create a new failure point for the fiber bars. With this in mind, a double-tapered bar design was created to prevent failure at both stress concentrations. By making the ends of the bar thicker than the center, with the thicknesses based on the severity of the stress concentrations, a more robust bar design was created. After validation in FEA, prototypes were built and tested, with long-term test specimens lasting the entirity of a three-month (over 2000 hr) continuous operation test without failing. Double-tapered bars designed specifically for the sixteen lane shaker are now being installed in newly manufactured shaker tables.

# 6. FIBER BAR ANALYSIS & REDESIGN

Finite Element Analysis (FEA) was selected to perform analysis of the fiber bars. Starting with creating a model that recreated the force-displacement curve of the existing fiber bars, this method was later used to design fiber bars

## 6.1. Loading Condition Simplifications

### 6.1.1 Mounting

Both ends of the fiber bar are mounted with two $\frac{7}{16}$ inch bolts spaced $2\frac{11}{16}$ inch apart. These bolts affix the fiber bar to the rest of the machine through a set of smaller ($\frac{1}{8}$ inch) fiberglass 'shims' placed immediately on either side of the bar, with $\frac{3}{16}$ inch thick steel plates outside of those (see figure 6.1). Although these eliminate the stress concentrations around the bolts, they make a new stress concentration near the end of the steel plates. Since the bar has freedom of rotation along the axis parallel to its width (length being the longest dimension, and thickness the shortest) and the shims and plates are oriented flat to the bar's largest face (perpendicular to the thickness, through which the bolts run), the straight line stress concentration made by the end of the plates manifests whenever the bar enters a bending mode. This happens during the compression step of each cycle of the machine. The details of this assembly are well represented in the model, receiving the same level of detail as the shape and anisotropic properties of the bar. The fixed end of the bar test assembly is simply a pair of steel blocks outside the plates, with their ends affixed to a large steel plate with its opposite face fully fixed in space. This configuration geometrically matches the test setup used in the Instron machine to test individual bars, which was designed to match the degree-of-freedom isolation the bar experiences when installed and operating in the machine. The forced end has a single pillow block mounted

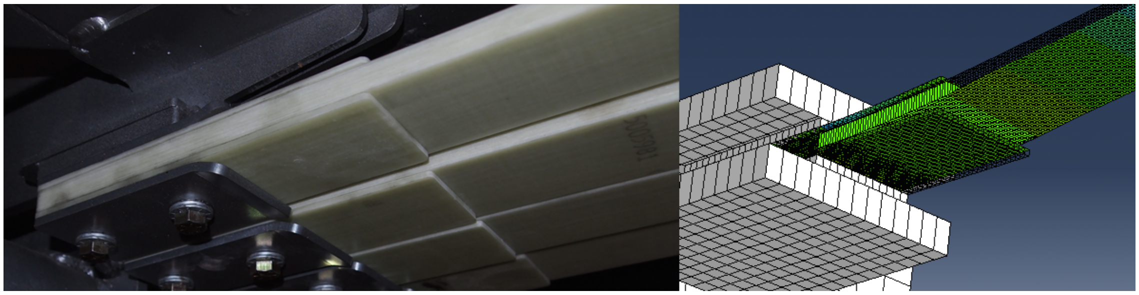

FIGURE 6.1: Fiber bar mounting, fixed end.

to one of the steel plates, while the other has nothing attached to it. The pillow block matches the geometry of those used in the machine, and the Instron test setup uses one such pillow block for consistency. Further details about the pillow block are in the next section (load vs displacement).

### 6.1.2 Load vs Displacement

Originally, a constant load was to be applied, but this was later substituted for a constant displacement. At first, the load was applied as a distributed pressure on half of the pillow block's inside face. This strategy did not allow for the rotational freedoms allowed in the actual system, providing no reference line for the limited rotational freedom. Load application to a single point or line was then used, but this caused high stresses and unrealistic amounts of deflection. Changing material properties of the component that the force was applied to solved the deflection issue, at the expense of model accuracy.

Constant displacement was eventually used because it better reflected the 'real' load application in the system. In order to achieve a certain pan displacement, the operator will run the system at whichever speed is required. Since the system is forcing-function dominated, we will use the earlier assumption that the drive mechanism can provide any force necessary to maintain its perfect sinusoidal deflection pattern to the forced end of

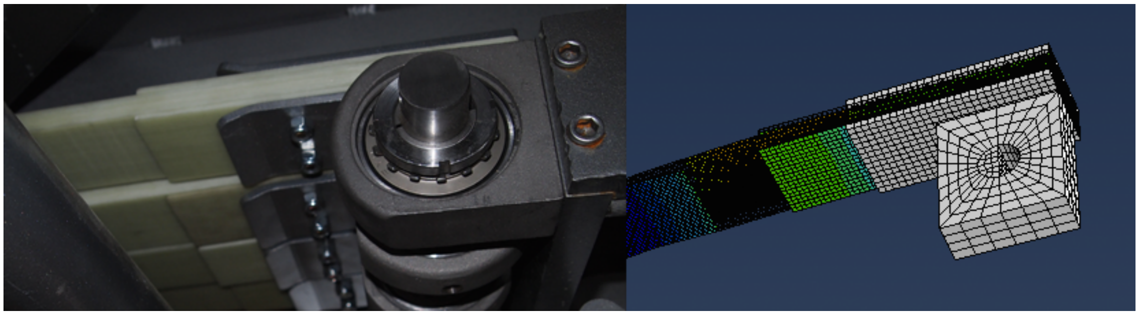

FIGURE 6.2: Fiber bar mounting, force application end.

the fiber bars. From this information we know that regardless of mass of spring constant, the system will always be run near a certain total pan displacement (peak to trough). Correcting for the pan's angle and estimating the maximum degree to which the drive and pan can be 'out of sync' (pan and drive both providing compression to the fiber bars), it was determined that a ten millimeter deflection was a reasonable maximum to use for testing, both in real-world physical tests and FEA simulations.

To mimic the loading and rotational constraints experienced in the system, a pin was added in the pillow block. For simplicity, only half of the pin was modeled (see figure 6.2). This allowed the center axis line (upon which the deflection was applied) to be easily selected and verified. Furthermore, the 'tie' command which is used to transfer force between components, matches the displacement of nodes regardless of tension or compression, so using a full pin would have created a unrealistic stress conditions on the inside of the pillow block, since the entire surface making contact would be receiving force from the pin, both in tension and compression. Another option would be to use surface contact, but this would include unnecessary details in the simulation, such as surface friction and contact forces, which increase computation time without providing meaningful information about the fiber bar's stress or deflection.

All component surface contacts in the FEA model use the tie command, except for the unconstrained contact between the shims and the fiber bar. Beyond the end of the steel plates, the shims are only held against the surface of the bar by their own stiffness (beneath the plates they are clamped). Since the bar and shims are free to deflect separately, a using the tie command to transfer force would be inappropriate because this contact is regularly broken when the bar bends away from a given shim. At the same time, the bar is bending toward that shim's twin, mirrored across the bar, and both the bar and shim deflect. Since the bar and shim are not affixed to each other at that point, their surfaces are free to slide past one-another. Both the loss of contact and presence of sliding contact require the use of surface contact, since these phenomena are important to the bar's deflection and the severity of the stress concentration caused by the plates.

Major assumptions in FEA modeling:

- Bolt hole stress concentrations ignored
- Rounded corners on fiberglass components ignored
- No slip between contact surfaces under steel plates (plate to shim, shim to bar)
- Deflection inconsistencies in pillow block due to half pin not rotating to match force direction ignored
- Listed lamina thickness includes epoxy
- Listed properties are well represented by FEA model

## 6.2. Progression of Modeling Techniques

The FEA model was developed through iterative improvement starting with a simple, geometry-only model which did not include the composite layup. The model was improved continuously to include all appropriate subsystem details, and eventually came to accurately reflect the fiber bar's behavior in the shaker table.

At first, the fiberglass composite bar was represented with a solid component, which allowed for correct geometric attachments (thickness included), but did not allow a composite layup to be used, only anisotropic (directional) material properties. Next, shell elements were used, which gave the bar no physical thickness - forcing the loading conditions to be oversimplified. Attempting to make the shell three-dimensional created only the perimeter of the bar, which does not work for composite stacking. The final construction method for (correctly) modeling the fiber bar was creating the flat shell, then expanding it into a three-dimensional solid. This provided both proper thickness allowance for mounting conditions and the ability to use a composite layup. To allow these dissimilar model properties, a "continuum shell" eight node reduced integration element type was used. The other components, all steel, were modeled with tetrahedron elements.

The element size sensitivity of the fiber bar model was then calculated. Using strictly cube-shaped elements (for best deflection properties) resulted in a non-converging model; stiffness continued to increase as element size decreased. Inquiry with experienced FEA users revealed that separating a composite layup into elements which interface either between the lamina or inside lamina causes "shear lock." [31] This phenomenon makes the simulation of the composite layup stiffer than it should be, which contributed to the lack of convergence because discretizing the fiber bar model into smaller elements created more element interfaces inside the layup, which increased the "shear lock" effect. The solution to "shear lock" was using only one element through the thickness of the fiber bar model, regardless of the elements' aspect ratio. Sensitivity analysis was performed starting with cubic elements with edge lengths equal to the full height of the composite layup, then equally reducing the lengths of the sides which didn't traverse the layup's height. From cubic elements the side lengths were reduced to one half, one third, one quarter, one fifth, and one sixth of the layup height. Overall stiffness converged at elements with the variable side lengths at one quarter of the height of the layup. Although the aspect ratio of these elements is not ideal, the results converge well because "shear lock" has been eliminated, greatly increasing the accuracy of the model.

## 6.3. Justify FEA vs Traditional Methods

Although the initial bar configuration could be analyzed with mechanics equations (e.g. using only traditional mechanics methods), this would have severely increased the difficulty of performing the redesigns. FEA, despite its disadvantages compared to dynamics modeling discussed earlier, is more robust to changes in loading conditions and component geometries than mechanics equations. Features such as blended surface thicknesses (some with curved outer layers applied over) and variously mounted fiber bars in series spring arrangements are being explored. Such features would greatly complicate the mechanics equations by introducing variations in composite stiffness that are complex in both their representation and interactions. These changes would only linearly increase the complexity of the FEA model, simply requiring the solid models of the components to accurately formed and connected, but require much higher effort to incorporate in pure mechanics equations. The detailed application of forces, moments, and material properties that FEA abstracts away from the engineer, only requiring only geometry, material properties, and understanding of the calculation methods, must be performed by the engineer when using traditional mechanics equations. Directly applying mechanics equations requires much more knowledge and advanced techniques than does FEA to accommodate these increases in model complexity. Mechanics equations are a good choice for performing exacting calculations and are much more likely to yield correct results sooner and more consistently for simple cases, but cannot change as quickly or easily to adapt changes in the model as FEA can.

The ever-changing nature of the system creates changes in the physical arrangement of the components which apply load to the fiber bars. Classical mechanics methods, although feasible, would likely need to be recreated each time the system is reconfigured. FEA can simply be given the new configuration, with any new components (dimensions and properties), and perform the calculations. This makes large changes in the mounting and loading conditions on the fiber bars relatively easy to implement compared to completely restructuring mechanics equations.

# 7. GENERAL CONCLUSION

The shaker table redesign is based largely on the RE framework and methods used to select a model and gather the data to populate it. The framework, as described in chapter two, provides guidance to the engineer tasked with model selection, but is not a definitive measure of model suitability. The FEA and dynamics models had roughly similar 'total barrier' values, but dynamics was selected based on its adaptability and short computation times. This selection guided the RE efforts, which eventually expanded to include dynamic damping testing. The result was a dynamics model that predicted the frequency response of the test specimen shaker table at OSU. With this model, product variations, such as different shaker table sizes and fiber bar variations could be simulated by changing the mass and spring stiffness values, respectively. With this flexibility, performance improvements could be suggested for any of the Company's shaker tables, regardless of size or dynamic components.

## 7.1. Composite "Fiber Bar" Design

The primary goal of this industry-collabraotion project is improving the performance of the Company's "shaker table" vibratory conveyor. As discussed earlier, a full-scale redesign of the product is not feasible, so proposed redesigns are focused on improving the function and longevity of the product by refining the design of key functional components. The most prevalent of these is the main drive springs - the "fiber bars." Redesigning these allowed for precise adjustment of the machine's performance point and reduction in internal stress peaks within the "fiber bars" (increasing wear life). The shaker table's performance was the Company's primary goal for improvement, which directed redesign efforts toward the table's dynamic components. Being a damped mass-spring system, the suspension springs and dampers (the “dog bones”) and the main drive springs (the fiber bars) govern the shaker table’s oscillatory amplitude across the range of drive frequencies (frequency response). In addition they are both readily replaceable, and also the most frequently failed components.

The dog bones are cast with rubber bushings pressed into parallel holes at either end. Near the middle, coil springs provide a portion of the ‘suspension’ spring force (they are attached to the frame, which is anchored to the ground, performing the function of ‘suspending’ the mass - positioning it while allowing it to move dynamically). The rubber bushings, loaded in torsion (instead of laterally, as in traditional applications, such as automotive suspension), provide the majority of this ‘suspension’ force, as well as the damping in the system. These bushings are not normally produced in fine increments (inner / outer diameter, wall thickness), and are therefore a poor choice for ‘tuning’ the shaker table’s performance.

Of the two, the fiber bars have both more influence on the product’s performance and a higher failure rate. They are custom-built to any thickness (lamina are .060 inch thick and can finished bars can be shaved thinner if needed) and can be configured with various lamina direction ‘layup’ patterns and thickness profiles. The fiber bars are the most frequently failed component because they are subjected to multiple stress concentrations as a result of how they are mounted in the shaker table. These factors make the fiber bars the best candidate for redesign, since they contribute to the Company’s two primary design shortcomings.

### 7.1.1 FEA

Finite Element Analysis (FEA) is an important part of redesigning the fiber bars, providing easy visualization of the internal stresses and prediction of the stiffness response of complex composite fiberglass designs within the machine's specific loading conditions. A finite element model of the existing fiber bar design has been created and verified by comparison to test data, allowing for direct comparison of proposed fiber bar designs to the existing design. Each new design is created from the previous design, with carefully selected changes selected to improve the fiber bar's function within the shaker table. At first, the profile was constant along the length of the fiber bar, which resulted in reasonable performance and acceptable wear life across a narrow specification range. The problem with a uniform profile was thicker bars could handle the stresses better, but also reduced the performance. The small acceptable range in which the trade-off between performance and longevity was balanced yielded an unacceptable performance envelope for making future designs competitive. Introducing variations in the fiber bar's thickness profile solved this problem by increasing the bar's stiffness in higher stress regions (where most of the deflection was occurring) and reducing it in low stress regions. By balancing the thickness variations, new bar designs more effectively handled concentrations of high stress without ruining the shaker table's performance with excessive stiffness. In fact, thickness variation allows the stiffness of the fiber bars to be tuned specifically to individual shaker table models within the product line, allowing for improved performance in all sizes and types of shaker tables instead of compromising with a single fiber bar drive spring for all products.

The design was modeled in FEA with loading conditions identical to those of the verified 'existing design' model. The FEA simulation was run and the results were compared to the existing design to determine if the intended changes in properties were achieved.

These properties were, primarily, stress and overall stiffness (under the given loading conditions). The stress concerns are both peak value and location; the position and concentration (acuity) of the stress are very important to improving bar life. Overall stiffness determines the natural frequency of the mass-spring system, and thereby the operating point (frequency & amplitude) at which the shaker table will run. This target stiffness is determined by the dynamics model.

### 7.1.2 Dynamics Model

The dynamics model plays a less direct role in the redesigns than FEA. Although iteratively improved, with correction factors applied to bring it into agreement with the detailed test data collected from the test machine, the dynamics model does not perfectly reflect the Company's application of the fiber bars. The bars are intended for use on larger machines, which have much larger 'live' masses (from about 450 to 650 lbs, across various models) than the approximately 250 pound masses on the test machine on location at OSU. As a result, the model can be applied with a correction factor, but without comprehensive test data from the larger shaker table, there is no way to assess the model's accuracy.

The Company intends to use the redesigned fiber bars across its entire line of shaker tables. Of these, the test machine at OSU is of a good representative 'middle' size; it should provide good approximation of an average shaker table's behavior, despite not guaranteeing an accurate prediction of its amplitude behavior. Ideally, every size of every type of shaker table could be individually tested and accurately modeled, but such extensive testing is both impractical and unnecessary for this level of redesign. The behavior predictions - changes in amplitude response to drive frequency based on changes in dynamic components - provided by the dynamics model have proven reliable in providing design information for modifying the fiber bar designs to meet specified performance points on individual shaker table models. At the time of this writing, two such fiber bar designs have been tested by the Company and subsequently ordered in production quantities.

## 7.2. Predictive Advice Document for Application of Design Principles

The Company produces many prototypes, making changes to multiple aspects of the dynamic system, often based only on inspiration and intuition. Although this method has proved successful, it necessitates building and testing every design, even those which are fundamentally flawed. A document containing design guidance, in this case principles to consider when proposing redesigns, can improve the efficiency of this iterative design process by discouraging the pursuit of designs that blatantly violate one or more of these principles, and will likely be flawed. Although far from an ideal design process, this use of guiding principles will bring some of the benefits of the modern, structured design processes to the Company's iterative prototyping process with minimal change to the associated workflow and overall design cycle.

### 7.2.1 Basis

Pahl and Beitzs' principles of embodiment design [29] are well regarded resource for improving mechanical designs. After considering the principles in the categories of Force Transmission, Division of Tasks, Self-Help, and Stability & Bi-Stability, nine were selected for their specific applicability to the Comapany's shaker table designs. This subset was then mapped to recurring issues the Company encounters when modifying shaker table designs. The principles and their corresponding issues were then arranged in a table, inspired by the table used in *40 Principles: TRIZ keys to innovation* [30] to lead the reader from design issues to relevant TRIZ principles. The following table suggests specific "principles of embodiment design" for each of six categories of design shortcoming.

### 7.2.2 Predictive Advice Table

| Problem | Principles | Need | Principles |
|---|---|---|---|
| Unstable Operation | 9, 4, 1, 2, 3, 7, 5 | Higher Frequency | 7, 2, 1, 9, 4, 3 |
| Difficult to Control | 9, 4, 7, 2, 1 | Higher Amplitude | 7, 9, 3, 4 |
| Premature Breakage | 3, 6, 8, 5, 1, 2 | Longer Component Life | 3, 6, 8, 5, 1, 2 |

### 7.2.3 Principles

From *Engineering Design: A Systematic Approach* [29]

1. Flowlines of Force & Uniform Strength

   "... try to avoid all sudden changes of direction in the flowlines of force, that is the force transmission path, caused by sharp deflections and abrupt changes of cross section."

2. Direct and Short Force Transmission Path

   "If a force or moment is to be transmitted from one place to another with the *minimum possible deformation*, then the *shortest and most direct* force transmission path is the best."

3. Matched Deformations

   "... related components must be designed in such a way that, under load, they will deform *in the same sense* and, if possible, *by the same amount*."

4. Balanced Forces

   "... the associated forces must, whenever possible, be balanced out at their place of origin, thus obviating the need for a heavier construction or for reinforcing bearing and transfer elements."

5. Division of Tasks for Identical Functions

   "If increases in load or size reach a limit, a single function can be assigned to several, identical function carriers."

6. Self-Reinforcing Solutions

   ". . . the supplementary effect is obtained directly from a main or associated force and adds to the initial effect to produce a greater overall effect."

7. Self-Balancing Solutions

   ". . . the supplementary effect is obtained from an associated force, and offsets the initial effect to produce an improved overall effect."

8. Self-Protecting Solutions

   ". . . [the] supplementary effect [is derived] from an additional force transmission path that,in case of excess loading, is generally created after a given elastic deformation as taken place. As a result, the distribution of the flowlines of force is altered and the load-carrying capacity is increased."

9. Stability

   ". . . try either to ensure that disturbances cancel out or else to reduce their particular effects."

## BIBLIOGRAPHY

1. Chikofsky, Elliot J., and James H. Cross. "Reverse engineering and design recovery: A taxonomy." Software, IEEE 7.1 (1990): 13-17.

2. Chikofsky, Elliot J., and James H. Cross. "Reverse engineering and design recovery: A taxonomy." Software, IEEE 7.1 (1990): 13-17.

3. Wilson, Jamal O., and David Rosen. "Systematic reverse engineering of biological systems." ASME 2007 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference. American Society of Mechanical Engineers, 2007.

4. Nagel, Jacquelyn KS, Robert B. Stone, and Daniel A. McAdams. "An engineering-to-biology thesaurus for engineering design." ASME 2010 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference. American Society of Mechanical Engineers, 2010.

5. Rekoff, M. G. "On reverse engineering." Systems, Man and Cybernetics, IEEE Transactions on 2 (1985): 244-252.

6. Otto, K., and K. Wood. "A reverse engineering and redesign methodology for product evolution." Proceedings of the 1996 ASME Design Theory and Methodology Conference. Vol. 96. 1996.

7. Otto, Kevin, and Kristin Wood. "Product Design: Techniques In Reverse Engineering And New Product Development Author: Kevin Otto, Kristin Wood, Publisher." (2000): 1104.

8. Harston, Stephen P., and Christopher A. Mattson. "Metrics for evaluating and optimizing the barrier and time to reverse engineer a product." ASME 2009 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference. American Society of Mechanical Engineers, 2009.

9. Curtis, Shane K., Stephen P. Harston, and Christopher A. Mattson. "A generic formulaic characterization of the time to reverse engineer the tolerances of a product." ASME 2009 International Mechanical Engineering Congress and Exposition. American Society of Mechanical Engineers, 2009.

10. Harston, Stephen P., and Christopher A. Mattson. "Metrics for evaluating the barrier and time to reverse engineer a product." Journal of Mechanical Design 132.4 (2010): 041009.

11. Radhakrishnan, Rajesh, and Daniel A. McAdams. “A methodology for model selection in engineering design.” Journal of mechanical design 127.3 (2005): 378-387.

12. Hoyle, Christopher, et al. “Multi-Stage Uncertainty Quantification for Verifying the Correctness of Complex System Designs.” ASME 2011 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference. American Society of Mechanical Engineers, 2011.

13. Du, Xiaoping, Agus Sudjianto, and Wei Chen. “An integrated framework for optimization under uncertainty using inverse reliability strategy.” Journal of Mechanical Design 126.4 (2004): 562-570.

14. International Organisation of Standardisation, Guide to the Expression of Uncertainty in Measurement, ISO, Geneva, 1995.

15. Dsenfant, Michle, and Marc Priel. “Road map for measurement uncertainty evaluation.” Measurement 39.9 (2006): 841-848.

16. Reznik, Leon, and Kishor P. Dabke. “Measurement models: application of intelligent methods.” Measurement 35.1 (2004): 47-58.

17. Tern, Pedro. “Probabilistic foundations for measurement modelling with fuzzy random variables.” Fuzzy Sets and Systems 158.9 (2007): 973-986.

18. Urbaski, Micha K. “Fuzzy measurement theory.” Measurement 41.4 (2008): 391-402.

19. Foglesong, Tim, Rob Stone, and John Parmigiani. “Dynamics Modeling to Inform Design Optimization.” ASME 2014 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference. American Society of Mechanical Engineers, 2014.

20. Curtis, S. Mattson, C. Harston, S. “On Barriers to Reverse Engineering Mechanical Components”. In ASME IDETC/CIE 2010, Montreal, Quebec, Canada, DETC2010-28610.

21. http://www.ibiblio.org/kuphaldt/electricCircuits/AC/AC_13.html #xtocid1743821

22. Palm, W. “Mechanical Vibration”. New York: Wiley, 2007.

23. Inman, D. “Engineering Vibration”. New Jersey: Prentice-Hall 2001 (2008).

24. Figliola, R. Beasley, D. “Theory and Design for Mechanical Measurements”. New York: Wiley, 2011.

25. Malagò, M. Mucchi, E. Dalpiaz, G. “Condition Monitoring And Diagnostics In Heavy-Duty Wheels: A First Experimental Approach”. In ASME IDETC/CIE 2009, San Diego, California, USA, DETC2009-87333.

26. Ballard, Z. Mann, B. "Experimental And Numerical Investigations Of An Untethered, Nonlinear Spherical Buoy In A Wave Tank". In ASME IDETC/CIE 2011, Washington, DC, USA, DETC2011-48401.

27. McGehee, C. Ballard, Z. Mann, B. "Horizontal Pendulum With Sudden Changes In Platform Tilt". In ASME IDETC/CIE 2011, Washington, DC, USA, DETC2011-48423.

28. Triplett, A. Quinn, D. "Experimental Investigation Of Energy Harvesting With Essential Nonlinearities". In ASME IDETC/CIE 2011, Washington, DC, USA, DETC2011-48164.

29. Pahl, Gerhard, et al. Engineering design: a systematic approach. Vol. 157. Springer Science & Business Media, 2007.

30. Altshuller, Genrich. 40 Principles: TRIZ keys to innovation. Eds. Lev Shulyak, and Steven Rodman. Vol. 1. Technical Innovation Center, Inc., 1997.

31. Cook, Robert D. Concepts and applications of finite element analysis. John Wiley & Sons, 2007.