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FOREST PRODUCTS LABORATORY

In cooperation with the University of Wisconsin

MADISON, WISCONSIN

INSTRUCTIONS FOR USING FOREST PRODUCTS LABORATORY FORMULA FOR WOODEN COLUMNS

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INSTRUCTIONS FOR USING

FOREST PRODUCTS LABORATORY FORMULA FOR WOODEN COLUMNS

In order that the type of column to which the formula applies may be understood, it should be stated that for structural purposes three types of columns are recognized, namely: Long columns, which depend for their strength on stiffness; short columns which depend for their strength on the crushing strength in direction of length; and intermediate columns which depend on a combination of stiffness and crushing strength.

The load-carrying capacity of a long column is dependent upon its stiffness and its cross-sectional dimensions may therefore be determined by the well known Euler formula, $\frac{P}{A} = \frac{0.274E}{\left(\frac{L}{d}\right)^2}$.

If the slenderness ratio $\left(\frac{\text{length}}{\text{least dimension}} \text{ or } \frac{L}{d}\right)$ of a column is 11 or less, the column is considered as short. The size of a short square column required to support a given load is found by dividing the load by the allowable crushing stress of the material, and extracting the square root to find d, the side of the column.

The Forest Products Laboratory formula, it should be understood, applies to intermediate columns, which are the ones most frequently used in structural work. The formula is as follows:

$\frac{P}{A} = S\left[1 - 1/3\left(\frac{L}{Kd}\right)^4\right]$, in which P = load in pounds; A = cross-sectional area in square inches; S = maximum crushing stress (pounds per square inch) or, in case of working loads, the safe stress, for short columns; L = unsupported length in inches; K is a constant, depending on modulus

of elasticity and crushing strength, for the given species, grade, and condition of service; and d = least dimension in inches.

Intermediate columns range from an $\frac{L}{d}$ of 11 to an $\frac{L}{d}$ equal to K, above which a column is classed as a long column.

In order that the use of the formula may be clearly understood let us suppose that it is desired to determine the size of columns necessary to support the floor girder in a dwelling or store building.

The wood to be used is western hemlock, common grade. The safe crushing stress (that is, the allowable stress for short columns) when used in dry locations is 720 pounds per square inch, and the value of K is 28.3, as shown in the tables of safe working stresses prepared by the Forest Products Laboratory.

Assume a load of 20,000 pounds and the length of the column 9 feet (108 inches). First compute for a square column.

The formula

$$\frac{P}{A} = S\left[1 - 1/3\left(\frac{L}{Kd}\right)^4\right] \qquad (1)$$

is solved as follows:

Substituting d^2 for A,

$$\frac{P}{d^2} = S - \frac{SL^4}{3K^4d^4} \qquad (2)$$

multiplying through by d^4,

$$Pd^2 = Sd^4 - \frac{S}{3}\left(\frac{L}{K}\right)^4 \qquad (3)$$

transposing and dividing by S,

$$d^4 - \frac{P}{S}d^2 = 1/3\left(\frac{L}{K}\right)^4 \qquad (4)$$

completing the square,

$$d^4 - \frac{P}{S}d^2 + \left(\frac{P}{2S}\right)^2 = \left(\frac{P}{2S}\right)^2 + 1/3\left(\frac{L}{K}\right)^4 \qquad (5)$$

extracting the square root,

$$d^2 - \frac{P}{2S} = \pm \sqrt{\left(\frac{P}{2S}\right)^2 + 1/3\left(\frac{L}{K}\right)^4} \qquad (6)$$

transposing and extracting the square root,

$$d = \sqrt{\frac{P}{2S} + \sqrt{\left(\frac{P}{2S}\right)^2 + 1/3\left(\frac{L}{K}\right)^4}}. \qquad (7)$$

Substituting in equation (7) the values for P, S, and K as previously given,

$$d = \sqrt{\frac{20,000}{1440} + \sqrt{\left(\frac{20,000}{1440}\right)^2 + 1/3\left(\frac{108}{28.3}\right)^4}}.$$

Solving, $d = 5.49$, the side of the square column.

Since the nominal 6 by 6-inch column (American Lumber Standards) when surfaced is actually 5-5/8 by 5-5/8 inches, the 6 by 6-inch column would be the smallest one that could be used under the circumstances. Obviously a 5 by 5-inch column, the next lower size, would be too weak to use.

If some other form of rectangular column is desired, it is necessary to know either the ratio of least dimension to width or the least dimension. In varying widths of columns of the same thickness it is evident that the load is proportional to the width. Therefore, if the ratio is known, multiply the load by this ratio, substitute the result for P in equation (7), and solve as shown for the square column. If the least dimension is known, equation (1) may be written

$$\frac{P}{bd} = S\left[1 - 1/3\left(\frac{L}{Kd}\right)^4\right].$$

Substitute the known value of the least dimension for d in this equation and solve for b, the greater dimension.

In computing the size of a round column required, solve first for a square column. The diameter of the round column will be one-eighth greater than the side of the square column. This diameter must be taken at a point one-third of the length of the column, measured from the small end. The unit compressive stress, $\frac{P}{A}$, on the small end should also be computed, as the small end area is often the controlling factor in determining the safe load which the column may sustain. If the unit stress thus obtained exceeds the allowable crushing stress for short columns of the given species, then a column of greater diameter must be used.

Tables and curves giving the working stresses for columns of various lengths have been prepared by the Laboratory. Column strength may be determined from these without the necessity of solving the formula.