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Title: Sliding Mode Controller for Robot Manipulator

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The sliding mode controller, based on the theory of the Variable Structure System (VSS), is considered for a robot manipulator. By consideration of VSS theory, the nonlinear interactions of manipulator dynamics are suppressed by use of linear sliding equations, and VSS robustness with respect to the unknown physical manipulator parameter variations is maintained. A simple control law was designed, using a regulated derivative control algorithm. For reduction of the chattering phenomenon, which is a main disadvantage of the VSS, a new function is introduced in lieu of original sign function in the feedback control law. Position control of a two-degree of freedom manipulator is examined for the validity of the proposed controller algorithm.

Sliding Mode Controller for Robot Manipulator

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SLIDING MODE CONTROLLER FOR ROBOT MANIPULATOR

I. INTRODUCTION

1.1 Robotics

A robot is a mechanism which can be directed to do a variety of tasks without human supervision. The American Robot Industry Association (RIA) defines a robot as "a reprogrammable, multi-functional manipulator designed to move material, part, tools, or specialized devices, through variable programmed motions for the performance of variety of tasks."

The study of robotics is multidisciplinary in nature [Stokic & Vukobratovic, 1982], including the research fields of manipulators, end effectors, locomotive organs, and artificial intelligence (AI). The basic elements of robot manipulations include manipulators, controllers, sensors, and power supply. From the point of view of control engineering, controller design for the manipulator offers the challenge of correctly positioning or orienting the end effector in the workspace during the time given for the task. Various kinds of controllers, from simple chip-level controllers to those used with more complex computers, have been used. For instance, the teach pendant controller, a chip-level controller, connects the teach pendant to the controller in order to allow the operator to guide the robotic task, recording in memory each sequential command. When all the trajectory moves and job tasks have been taught, the manipulator moves without operator guidance along the same trajectories placed in memory. The advantages of this controller are that it is reprogrammable, repetitive, and is flexible with regard to various tasks without consideration of the control algorithm. On the other hand, this open-loop controller, which uses human feedback to learn the trajectories of its moves during any first operation, is not capable of coping with errors during processing. From the point of view of error control, the tasks attributed to the system's feedback controller are considerable.

1.2 Feedback Controller

The main role of feedback control is to ensure that a planned sequence of motions and forces will execute correctly in the face of unpredictable errors. The feedback controller receives status signals related to robot motion from the sensors, sending control commands to the robot manipulator. For these tasks, it must process sensory data, calculate moves in accordance with the control algorithm, and communicate with the operator. Consequently, the dynamic motions of the robotic system required for development of the control algorithm are of primary importance.

The robot manipulator is a highly nonlinear multivariable system, particularly with respect to nonlinear coupling between variable motions and physical parameter variations in accordance with the load to be carried by the arm. One common way of controlling the manipulator is to linearize the system's dynamic equation about an equilibrium point. Next, apply linear system theory to the linearized equation. The major disadvantage of this method is that linear control laws are only valid in the neighborhood of the operation point.

A second approach is feedback control by extended linearization [Baumann & Rugh, 1987]. This method can be used to overcome the disadvantages of the common linearization method, but the system must be linearized about a number of nominal trajectories and it is difficult to obtain proper gains which place the eigenvalues of the family of the closed-loop linearization at specified locations (i.e., at the left-half plane).

For purposes of control, adaptive control algorithms are useful for manipulators [Hsia, 1986], but they require prior knowledge of the range of arm parameter variations and do not specify the transient behavior of errors. In contrast, sliding mode controllers for manipulators, based on the theory of the variable structure system (VSS), seem to be very promising [Young, 1978]. The theory of the VSS has been studied in the U.S.S.R. for the last 25 years, culminating with the use of switching feedbacks based on the assumption of one of two possible values, depending on error signals or their derivatives.

The VSS algorithms differ from those used with other controllers from the standpoint of exercising discontinuous changes of control structure. In most control systems the structures are fixed during the implementation process. Even though the parameters are changed continuously in adaptive controllers, the basic system structures are maintained and feedback gains are continuous, often constant, functions of time. The basic concept of the VSS controller is that it drives a system's trajectory to the switching surface and, through proper choice of control parameters, maintains this trajectory on the switching surface throughout subsequent run time. The gains of the VSS controller are a discontinuous function of time because of the ability to switch between two proper gains. The switching surface is chosen since by restricting the original system's state trajectory to the surface, the equivalent system observes the desired system behavior.

The advantages of VSS control include its robustness with respect to unknown parameter variations or disturbances once it is in the sliding mode, and the fact that it does not require an accurate model. On the other hand, there are two principal disadvantages to VSS control. One is the specification (or selection) of the control law required to move the system to the switching surface and the other is the need to reduce chattering around the switching surface.

In this study, the sliding mode controller design for manipulators is presented. This has been accomplished by determining the proper linear sliding mode to force the system's trajectory to slide along the desired trajectory with proper switching controls, which is determined by the equivalent controls method. Simulate position control of a two degree of freedom manipulator, with consideration of manipulator parameter variations. A new function is introduced in lieu of the original sign function in order to reduce chattering and give flexibility to the design parameters.

The basic concepts of VSS are presented in Chapter II and a general design procedure is presented in Chapter III. System modeling is summarized in Chapter IV and Chapter V presents a new controller algorithm and an analysis of the simulation results of the design. Conclusions drawn from this study are presented in Chapter VI.

II. VARIABLE STRUCTURE SYSTEMS

2.1 Introduction

The main concept underlying the variable structure is the exercise of intentional structure changes through some external influence in order to obtain a desired response. Fig 2.1 illustrates this concept schematically, given the accessible state x_1, x_2, and control law u,

$$u = k_1 x_1 + k_2 x_2 \ ,$$

where

$$k_1 = \begin{cases} a_1 & \text{if condition I} > 0, \\ a_2 & \text{if condition I} < 0, \end{cases}$$

$$k_2 = \begin{cases} b_1 & \text{if condition II} > 0, \\ b_2 & \text{if condition II} < 0. \end{cases}$$

With each switch setting there are four different possible feedback structures. Each of these four different structures may correspond to a different type of system behavior and their use does not guarantee that system's stability will be preserved. A switching logic must be selected which asymptotically satisfies the stable properties of the total system, even though the individual system is not necessarily stable. In particular, VSS control allows switching between two

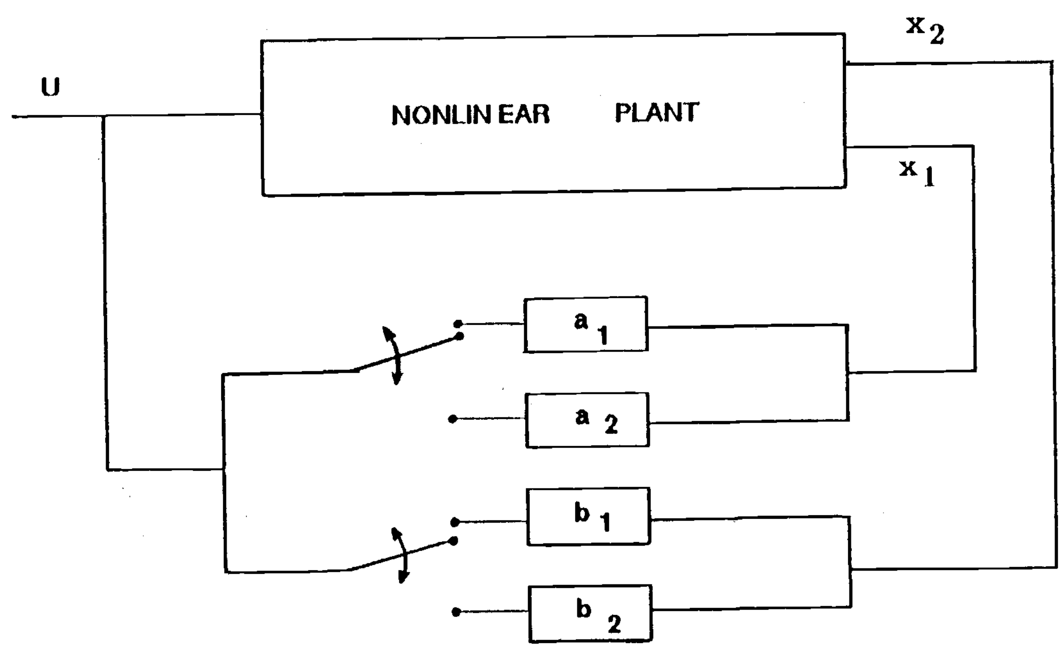


Figure 2.1 Schematic diagram of VSS

structures in such a manner that the trajectory is forced to intersect the switching surface, and then is maintained on that surface for all subsequent run time.

Moreover, we consider a second-order system described by the differential equation

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -ax_1 - u \;, \qquad a > 0 \;, \tag{2-1}$$

where

x_1, x_2 = state variables,

a = a positive constant parameter, and

u = a control function.

A variable structure control law is a piecewise linear function of x_1,

$$u = kx_1 \tag{2-2}$$

where the coefficient k is discontinuous,

$$k = \begin{cases} \beta - a \\ -\beta - a \end{cases} \tag{2-3}$$

for some positive $\beta > 1$. Then the closed-loop system takes on one of the two structures,

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\beta x_1 \tag{2-4}$$

or

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \beta x_1 \;. \tag{2-5}$$

In the first case, the phase plane trajectories of the system are ellipses, as shown in Fig. 2.2, while the second case shows a saddle portrait (Fig. 2.3). There is a unique stable region in saddle trajectories, defined by the equation,

$$x_2 + \sqrt{\beta}\, x_1 = 0 \qquad (2\text{-}6)$$

and the line $x_1 = 0$. With two lines the phase plane is divided into four regions (Fig. 2.4). For an asymptotically stable system, the control will be chosen to be

$$u = \begin{cases} (\beta - a)x_1 & \text{in regions I, III,} \\ (-\beta - a)x_1 & \text{in regions II, IV.} \end{cases}$$

The resulting closed-loop system's trajectories are composed of pieces of the trajectory of the non-switching feedback systems and are asymptotically stable (Fig. 2.5), even though the two feedback systems are, respectively, stable and unstable.

At this point, the behavior of system is determined by

$$S = c\, x_1 + x_2 = 0 \;, \qquad (2\text{-}7)$$

where $0 < c < \sqrt{\beta}$ and function $S(x,t)$ is called a switching function. The coefficient, c, must be selected so that the straight line, $S = 0$, will be between the axis, x_1, and the asymptote of the hyperbolic trajectories associated with the structure $k = -\beta - a$. The phase trajectories indicate that this total switching feedback system reaches the switching line $s = 0$ from any initial condition.

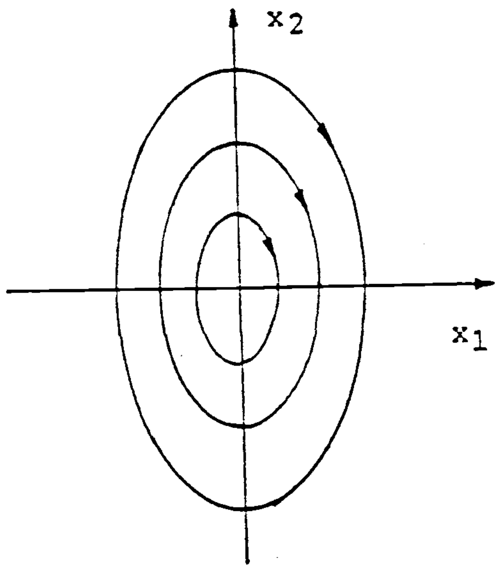


Figure 2.2 Ellipse portrait of subsystem (2.4)

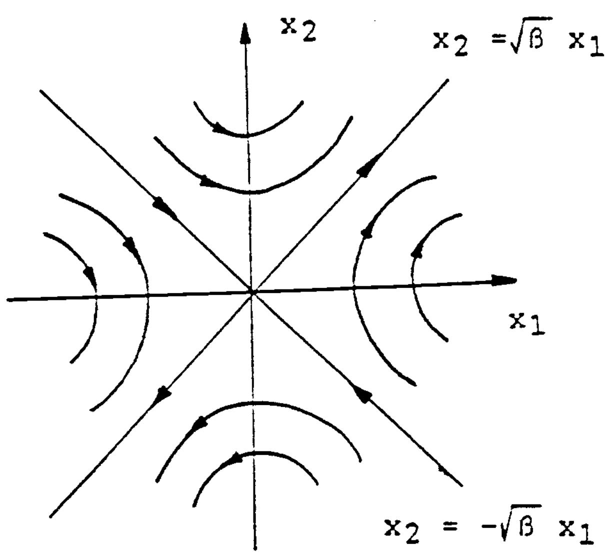


Figure 2.3 Saddle portrait of subsystem (2.5)

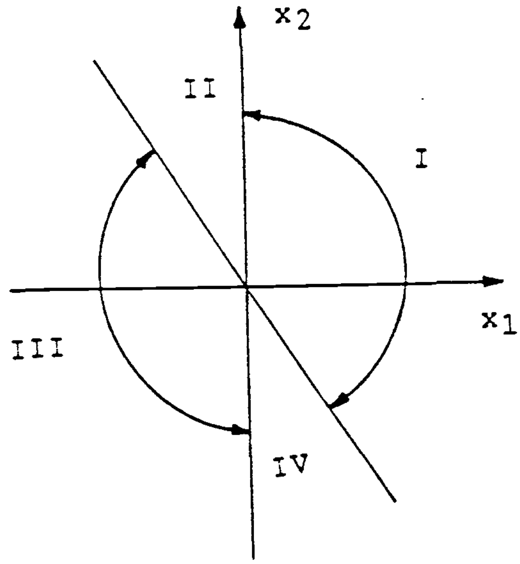


Figure 2.4 Four regions of VSS

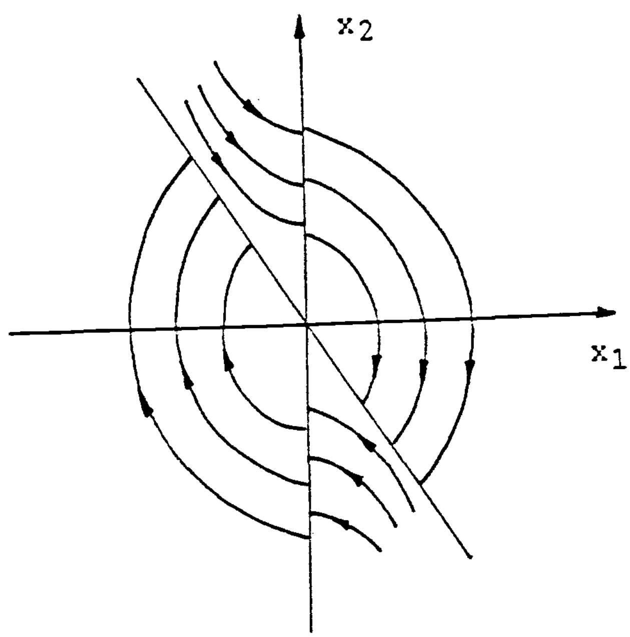


Figure 2.5 Asymptotically stable portrait of VSS

From Eqn. (2-7), it may be seen that

$$\dot{x}_1 + cx_1 = 0 \quad \text{or} \quad \dot{x}_1 = -cx_1$$

because the $\dot{x}_1$ was assigned to x_2. This equation is called the <u>sliding equation</u>. The solution of the sliding equation is

$$x_1(t) = x_1(t_0)\exp(-c(t-t_0)) ,$$

where t_0 is the time at which the trajectory intersects the switching line. This means that the solution is stable if $c > 0$, and the system's response time is dependent on parameter c.

This sliding line of Eqn. (2-7) does not belong to the two subsystems. Once the system reaches the sliding line, the fast switching control keeps the trajectory near the switching line and the system maintains stability as it moves down the switching line to the origin, i.e., the system is in the <u>sliding mode</u>. Consequently, the system response is determined only by the slope of the switching line in the sliding mode, i.e., it is dependent on parameter c, but independent of the controls, system parameters, or other disturbance. This invariance property of the sliding mode is extremely important in the view of the use of automatic controls.

From the Eqn. (2-7), the invariant conditions of sliding mode can be obtained such that

$$S(x,t) = 0 ,$$

and

$$\dot{S}(x,t) = 0 \ . \qquad (2\text{-}8)$$

The next problem concerns the conditions that guarantee the existence of the sliding mode. The existence of a sliding mode on the switching surface implies the stability of the nonlinear systems's state trajectory to the switching surface in the neighborhood of the switching surface. This means that the conditions of existence resemble stability problems. In this case, it is convenient to consider the second Lyapunov method and for a single input system, as given in Eqn. (2-1), a suitable Lyapunov candidate is chosen,

$$V(x,t) = S^2(x,t) \ , \qquad (2\text{-}9)$$

which is globally positive definite. Thus,

$$\dot{V}(x,t) = 2\ S(x,t)\ \dot{S}(x,t) \ . \qquad (2\text{-}10)$$

$\dot{V}(x,t)$ is negative definite, if

$$S(x,t)\dot{S}(x,t) < 0 \ . \qquad (2\text{-}11)$$

In consequence, the existence conditions of sliding mode satisfy the inequality (2-11). Note that this condition is a sufficient condition for existence because Lyapunov's method implies a sufficient, but not necessary condition. The above discussion considers only the ideal sliding modes. In actual systems, all switching control functions have non-ideal characteristics, such as delay or hysteresis.

2.2 General Sliding Mode

In the last section it was shown that sliding mode is an important and useful VSS property. In this section, the general existence condition of the sliding mode is considered [Itkis, 1976]. Given that a general system is specified by a nonlinear equation,

$$\dot{x}_i = f_i(x_1, x_2, \ldots, x_n, t), \quad i=1,2,\ldots,n \;, \qquad (2\text{-}12)$$

where the function, f_i is discontinuous on the switching surface

$$S: \; s(x_1, x_2, \ldots, x_n) = 0 \qquad (2\text{-}13)$$

in the phase space $X(x_1, x_2, \ldots, x_n)$. Assume that the left and right hand limits of the functions $f_i(x_1, x_2, \ldots, x_n, t)$ from either side exist,

$$f_i^- (x_1, \ldots, x_n, t) = \lim_{X \to -H} f_i(x_1, \ldots, x_n, t) \qquad (2\text{-}14)$$

and

$$f_i^+ (x_1, \ldots, x_n, t) = \lim_{X \to +H} f_i(x_1, \ldots, x_n, t) \;, \qquad (2\text{-}15)$$

where +H and -H are the half-space defined by S. Then the derivative of the function, s, along the trajectories of Eqn.(2-12) is

$$\frac{ds}{dt} = \sum_{i=1}^{n} \frac{\partial s}{\partial x_i} \cdot \frac{dx_i}{dt}$$

$$= \sum_{i=1}^{n} \frac{\partial s}{\partial x_i} \cdot f_i = (f \cdot \text{grad } s) \qquad (2\text{-}16)$$

where $f = (f_1, \ldots, f_n)$ and

grad s = the gradient of the switching surface s=0.

By Eqns. (2-12), (2-14), and (2-15), the limits

$$\lim_{x \to -H} \frac{ds}{dt} = (f^{-} \cdot \text{grad } s) \tag{2-17}$$

and

$$\lim_{x \to +H} \frac{ds}{dt} = (f^{+} \cdot \text{grad } s) \tag{2-18}$$

exist and represent the rate of change of s along the trajectory. At each point of s=0, the signs of the limits in Eqns. (2-17) and (2-18) may represent one of the following seven relations, which are illustrated by phase portraits in Figs. 2.6 and 2.7:

$$\lim_{x \to +H} \frac{ds}{dt} < 0 > \lim_{x \to -H} \frac{ds}{dt} , \tag{2-19a}$$

$$\lim_{x \to +H} \frac{ds}{dt} > 0 < \lim_{x \to -H} \frac{ds}{dt} , \tag{2-19b}$$

$$\lim_{x \to +H} \frac{ds}{dt} > 0 > \lim_{x \to -H} \frac{ds}{dt} , \tag{2-19c}$$

$$\lim_{x \to +H} \frac{ds}{dt} < 0 < \lim_{x \to -H} \frac{ds}{dt} , \tag{2-20a}$$

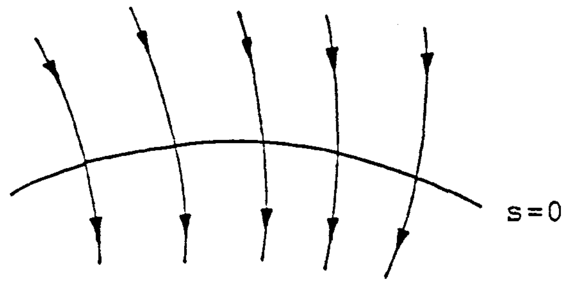
$$\lim_{x \to +H} \frac{ds}{dt} = 0 < \lim_{x \to -H} \frac{ds}{dt} , \tag{2-20b}$$

$$\lim_{x \to +H} \frac{ds}{dt} < 0 = \lim_{x \to -H} \frac{ds}{dt} , \tag{2-20c}$$

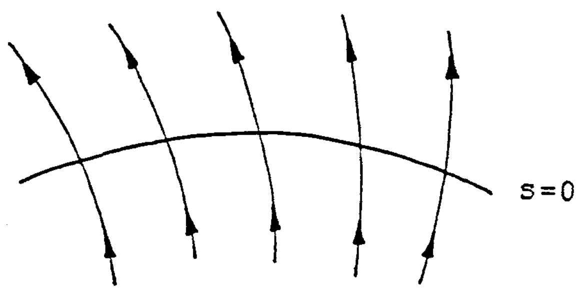
and

$$\lim_{x \to +H} \frac{ds}{dt} = 0 = \lim_{x \to -H} \frac{ds}{dt} . \tag{2-20d}$$

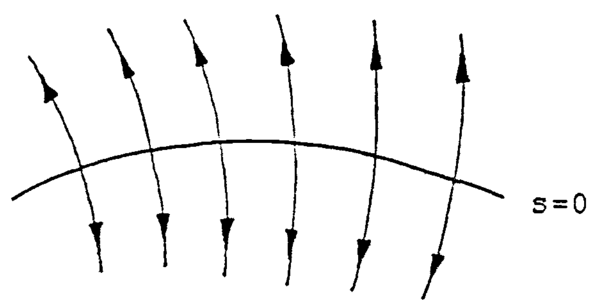
The phase portraits of Figs. 2.6 and 2.7 correspond, respectively, to Eqns. (2-19) and (2-20). As may be seen in the phase portraits, the trajectories of



(a)



(b)



(c)

Figure 2.6 Phase portraits of Eqn. (2-19)

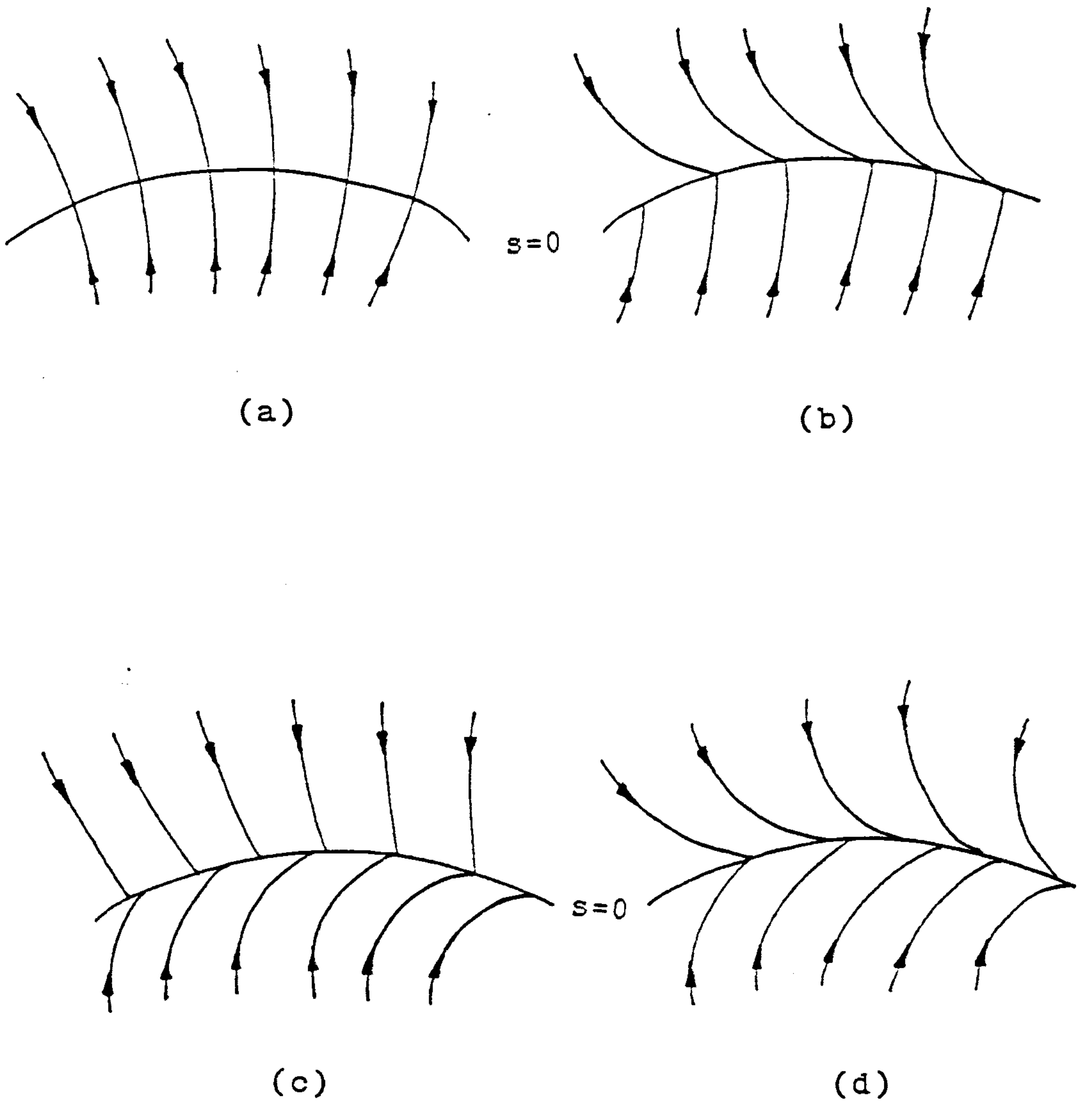


Figure 2.7 Phase portraits of Eqn. (2-20)

Eqn. (2-19) intersect the switching surface, s=0, but do not remain on the switching surface after contacting the switching surface. While Eqn. (2-20) shows that the trajectories do not leave the neighborhood of the switching surface, s=0, following intersection, equality in each equation occurs when the trajectories have an asymptotic approach to the switching surface. If the four inequalities are combined in a single condition, then

$$\lim_{x \to +H} \frac{ds}{dt} \leq 0 \leq \lim_{x \to -H} \frac{ds}{dt} . \qquad (2\text{-}21)$$

The function s generates a one-parameter family of switching surfaces parallel to S, given by

$$S_s: s(x_1, \ldots x_n) = s$$

and +H and -H are associated, respectively, with the sets,

$$\bigcup_{s>0} S_s \text{ and } \bigcup_{s<0} S_s .$$

Then, Eqn. (2-21) changes to

$$\lim_{s \to 0} s \frac{ds}{dt} \leq 0 \qquad (2\text{-}22)$$

This inequality may also be obtained from the second method of Lyapunov. Suppose that the Lyapunov function for the system of Eqn. (2-12) is

$$V(x_1, \ldots, x_n) = [s(x_1, \ldots, x_n)]^2 \cdot \qquad (2\text{-}23)$$

This function is positive semidefinite globally and equality occurs if s=0. The derivative of Eqn. (2-23) is negative semidefinite when it meets Eqn. (2-22). In other words, Eqn. (2-22) is a sufficient condition for the system of Eqn. (2-12) to have a Lyapunov function and Eqns. (2-11) and (2-22) have the same meaning.

III. GENERAL DESIGN PROCEDURE OF VSS

The design of a variable structure system controller is a two-step process. The first step is to design the switching surface such that the behavior of the system on the switching surface results in the desired response, i.e., motion is asymptotically stable. The second step is to design the control law to steer the system state to the switching surface, maintaining itself for all subsequent run time in the sliding mode. This control law must be configured to guarantee the existence and invariance condition of the VSS.

3.1 Switching Surface Design

To obtain a sliding equation, Utkin [1977] used an equivalent control method, which technique is introduced in this section. Consider the following general nonlinear single input system equation:

$$\dot{x}(t) = f(x,t) + B(x,t)u(t) \quad . \qquad (3\text{-}1)$$

If this equation can be changed into the general nonlinear controllable canonical form, then

$$\dot{x}(t) = A(x,t)x(t) + B(x,t)u(t) \qquad (3\text{-}2)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \cdots & 1 \\ a_1(x,t) & a_2(x,t) & \cdots & \cdots & a_n(x,t) \end{bmatrix}$$

$$B = [0 \; 0 \; \ldots\ldots \; 1]^T .$$

Assume that a sliding mode exists on the switching surface, $S(x,t) = Cx(t)=0$, where $C = \text{diag}[c_1 \; c_2 \ldots c_n]$. Even though a nonlinear switching surface is possible, a linear surface is effective and convenient. If the state trajectory intersects $S(x,t) = 0$ at $t = t_0$, then $S(x,t) = 0$ for all $t \geq t_0$. This implies that

$$\frac{d\,S(x,t)}{dt} = 0 \qquad (3\text{-}3)$$

for all $t \geq t_0$. The derivative of $S(x,t)$ is given by

$$\dot{S}(x,t) = \frac{\partial s}{\partial t} + \frac{\partial s}{\partial x}\dot{x} = 0 \; . \qquad (3\text{-}4)$$

Then, substitute Eqns. (3-2) and (3-3) to Eqn. (3-4),

$$\dot{S}(x,t) = \frac{\partial s}{\partial x}[A(x,t)x(t) + B(x,t)u(t)] = 0 \; , \qquad (3\text{-}5)$$

and define $G = \frac{\partial s}{\partial x}$, then

$$\dot{S}(x,t) = GAx + GBU = 0 \; . \qquad (3\text{-}6)$$

Assuming that GB is nonsingular for all x and t, by the equivalent control technique, solving for $U_{eq}(t)$

$$U(t) = U_{eq}(t) = -(GB)^{-1}GAx \; . \qquad (3\text{-}7)$$

Inserting this expression into the original system dynamics, Eqn. (3-2) is given by

$$\dot{x}(t) = A(x,t)x(t) - B(x,t)[GB(x,t)]^{-1}GA(x,t)x(t) \ . \qquad (3\text{-}8)$$

This is the system ideal sliding equation on the intersection of the switching surface. Eqn. (3-8) is equivalent to Eqn. (3-2) when the system is in the sliding mode.

For the sequel, this development may be summarized as follows. Given that the nonlinear system Eqn. (3-1) is restricted to the switching surface, $S(x,t) = 0$, and $GB(x,t)$ is nonsingular for all x and t, then the original system response restricted to the switching surface is given by Eqn. (3-8).

To illustrate this procedure, consider a simple ideal pendulum. The dynamic equation is

$$ml\ddot{\Theta} = -mg\sin\Theta + u \qquad (3\text{-}9)$$

where Θ is the angle of pendulum bob, u is the control force, and g is the gravity force. For the sake of simplicity, $m = 0.1$ Kg, $l = 10$ m, and $g = 10$ m/s^2. If state is assigned by the following $x_1 = \Theta$ and $x_2 = \dot{\Theta}$, then the state space model is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\sin x_1/x_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \ . \qquad (3\text{-}10)$$

For the control object, $x_1 \to 0$, $x_2 \to 0$, as $t \to \infty$, assume a switching surface of the form

$$s = c_1\, x_1 + c_2\, x_2 = 0 \ . \qquad (3\text{-}11)$$

By the equivalent control method, the system while restricted to s=0 is

$$\dot{x} = [I - B(GB)^{-1}G]\ Ax\ . \tag{3-12}$$

By simple calculation, the equivalent system is

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -c_1/c_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \tag{3-13}$$

or

$$\dot{x}_1 = x_2\ , \tag{3-14}$$

$$\dot{x}_2 = -(c_1/c_2)x_2 \tag{3-15}$$

with the surface imposed state constraint that

$$x_2 = -(c_1/c_2)x_1\ . \tag{3-16}$$

Eqn. (3-16) was derived from Eqn. (3-11). In the sequel,

$$\dot{x}_1(t) = -(c_1/c_2)x_1(t)$$

with x_2 as determined by Eqn. (3-16). The response on the switching surface is

$$x_1(t) = x_1(t_0)\exp[-(c_1/c_2)(t-t_0)]$$

and

$$x_2(t) = -(c_1/c_2)x_1(t), \qquad t \geq t_0$$

where t_0 is the time at which the trajectory intersects the switching surface.

At this point, the following conclusions may be drawn:

1) If c_2 is always a unit value, the system response depends on c_1, i.e., the slope of the sliding equation in the phase plane.

2) The system emulates a reduced-order, while the trajectories remain along the switching surface.

3) As previously mentioned, the system responses are independent of the control, u, in the sliding mode.

3.2 Control Law Design

Generally, the feedback control law, under conditions of the switching surface of the VSS,

$$s = c_1 x_1 + \ldots\ldots\ldots + c_{n-1} x_{n-1} + x_n = 0 \ , \qquad (3\text{-}17)$$

is given as

$$u(x,t) = k(x,t)x(t) \qquad (3\text{-}18)$$

with

$$k_i(x,t) = \begin{cases} \alpha_i(x,t) & \text{if } sx_i > 0 \\ \beta_i(x,t) & \text{if } sx_i < 0 \ . \end{cases}$$

In many cases, $\alpha_i(x,t)$ and $\beta_i(x,t)$ may be constant.

To verify this form, let the feedback control be

$$u = k_1 x_1 + \ldots\ldots + k_n x_n \qquad (3\text{-}19)$$

with ss is given by

$$\begin{aligned} s\dot{s} &= s[\ c_1 \dot{x}_1 + \ldots + c_{n-1} \dot{x}_{n-1} + \dot{x}_n\ ] \\ &= s[c_1 x_2 + \ldots + c_{n-1} x_n + a_1(x,t)x_1 \\ &\quad + a_2(x,t)x_2 + \ldots. + a_n(x,t)x_n \\ &\quad + u] < 0 \ . \end{aligned} \qquad (3\text{-}20)$$

Insert Eqn. (3-19) into Eqn. (3-20),

$$s\dot{s} = s[(a_1+k_1)x_1 + (a_2+k_2+c_1)x_2 + \ldots$$
$$\ldots\ldots + (a_n+k_n+c_{n-1})x_n] < 0 \, . \qquad (3\text{-}21)$$

Then, to select the proper gain, k_i, force each term of the above equation to be negative (i.e., satisfying the existence condition of the sliding mode):

$$\begin{aligned} &s\ (a_1 + k_1)\ x_1 < 0 \\ &s\ (a_2 + k_2 + c_1)\ x_2 < 0 \\ &\qquad \vdots \\ &s\ (a_n + k_n + c_{n-1})\ x_n < 0 \, . \end{aligned} \qquad (3\text{-}22)$$

As a consequence, gains are expressed as

$$\begin{aligned} &k_1 < -a_1 && \text{if } sx_1 > 0 \\ &k_1 > -a_1 && \text{if } sx_1 < 0 \\ &k_i < -(a_i+c_{i-1}) && \text{if } sx_i > 0 \\ &k_i > -(a_i+c_{i-1}) && \text{if } sx_i < 0 \, . \end{aligned} \qquad (3\text{-}23)$$

Assume that $a_i(x,t)$ is continuously diffentiable and δ_i denotes the bound of each function, $a_i(x,t)$, such that

$$\|\ a_i(x,t)\ \| < \delta_i \, . \qquad (3\text{-}24)$$

Then inequality, Eqn. (3-23), can be expressed as

$$\begin{aligned} &k_1 < -\delta_1 && \text{if } sx_1 > 0 \\ &k_1 > -\delta_1 && \text{if } sx_1 < 0 \\ &k_i < -(\delta_i+c_{i-1}) && \text{if } sx_i > 0 \\ &k_i > -(\delta_i+c_{i-1}) && \text{if } sx_i < 0 \, . \end{aligned}$$

This illustrates that VSS in the sliding mode exhibits the following robustness property: If the bound δ_i is sufficiently conservative, then control will exist despite variations in the parameter, $a_i(x,t)$.

In summary, one possible feedback control is

$$u = \sum_{i=1}^{n} k_i x_i$$

where

$$k_i = \begin{cases} -\alpha_i & \text{if } sx_i > 0 \\ -\beta_i & \text{if } sx_i < 0 \end{cases}$$

and

$$\alpha_1 > \delta_1 \quad , \beta_1 < \delta_1$$

$$\alpha_i > \delta_i + c_{i-1} \quad , \beta_i < \delta_i + c_{i-1} \ .$$

For ease of understanding, the previous pendulum example may be used. The control and sliding equations are

$$u = k_1 x_1 + k_2 x_2$$

and

$$s = c_1 x_1 + x_2 = 0 \ .$$

For the existence condition of the sliding mode,

$$s\dot{s} = s(c_1\dot{x}_1 + \dot{x}_2)$$

$$= s(c_1 x_2 - \sin x_1 + u)$$

$$= s(c_1 x_2 - \sin x_1 + k_1 x_1 + k_2 x_2) < 0 \ . \quad (3\text{-}25)$$

For the validity of Eqn. (3-25), each term of the right-hand side is forced to be negative,

$$s(-\frac{\sin x_i}{x_i} + k_1)x_1 < 0$$

and

$$s(c_1 + k_2)x_2 < 0 \ .$$

Obviously, $\| -\sin x_1/x_1 \| < 1$. Thus, the compact form of k_i is given by

$$k_1 = -\alpha \ \mathrm{sgn}(sx_1)$$

and

$$k_2 = -\beta \ \mathrm{sgn}(sx_2)$$

where sgn(.) is the sign function, $\alpha > 1$, and $\beta > c_1$.

Finally, the feedback control law, u, is

$$u = -\alpha \ \mathrm{sgn}(sx_1)x_1 - \beta \ \mathrm{sgn}(sx_2)x_2 \ . \qquad (3\text{-}18)$$

Young shows a similar control law [Young, 1978],

$$u_i = [k_i x_1 + k_i^2 x_2 + \ldots . + k_i^n x_n] \ \mathrm{sgn}(s_i) \qquad (3\text{-}19)$$

where u_i and s_i are, respectively, the i^{th} input and sliding surface. The constant, k_i, is found using the previous knowledge of system parameter's bound and the existence condition of the sliding mode, as shown in the previous example. These inequality equations are difficult to analyze, requiring off-line numerical calculations.

Other inquiries [Slotine, 1984; Yeung & Chen, 1988] proposed control laws using the uncertain system theory. However, most of the controls based on this theory are too complex to calculate for purposes of practical applications, requiring extensive work with pencil and paper.

IV. MATHEMATICAL MODELING OF MANIPULATOR

4.1 Introduction

To control the movements of the robot arm, calculations related to torque generation, motion, and trajectory planning are required. There are two categories of manipulator modeling equations which best apply to this requirement, kinematic equations and dynamic equations.

Kinematic equations describe relationships, including position, orientation, and velocity, as well acceleration of the links of the manipulator. These equations are used for the trajectory planning of robot motion and for deriving the dynamic equations of motion. Dynamic equations are the expressions of the necessary forces or the torques to be applied to the different joints of a manipulator as a function of position, velocity, and joint acceleration. Dynamic equations have been used for this study, in this case to determine control input to each of the arm's joints.

Two major approaches are used to describe dynamic equations for a manipulator with n degree of freedom (d.o.f). The first is based on the Newton-Euler (N-E) formulation as expressed below (Luh, Walker, & Paul, 1980),

$$F_i = m_i r_i ,$$

$$N_i = J_i w_i + w_i \times (J_i w_i) ,$$

$$f_i = F_i + f_{i+1} ,$$

and

$$n_i = n_{i+1} + N_i + (p_i^* + s_i) X F_i + p_i^* X f_{i+1} , \qquad (4-1)$$

where m_i is the mass of link i,

F_i is the total force exerted on link i,

r_i is the linear acceleration of link i,

N_i is the total moment exerted on link i,

J_i is the inertia matrix of link i,

w_i is the angular velocity of link i,

w_i is the angular acceleration of link i,

f_i is the force exerted on link i by link i-1,

n_i is the moment exerted on link i by link i-1,

s_i is the location of the center of mass of link i with respect to the origin of link i coordinates

$p_i^* = p_i - p_{i-1}$ and

X stands for the vector cross product.

The second approach is based on the Lagrange-Euler (L-E) formulation as follows. The first step for the L-E formulation of robot manipulator dynamics is to find the kinetic energy, K_i, and the potential energy, P_i, of each link i, then

$$K = \sum_{i=1}^{n} K_i$$

and

$$P = \sum_{i=1}^{n} P_i \ . \qquad (4-2)$$

Defining the Lagrangian,

$$L = K - P \ , \qquad (4-3)$$

the Lagrange equations are

$$f_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\Theta}_i}\right) - \frac{\partial L}{\partial \Theta_i} \qquad i=1,\ldots,n \qquad (4-4)$$

where Θ_i is the position of joint i, $\dot{\Theta}_i$ is the velocity of joint i, and f_i is the torque applied to joint i. The dynamic equation of the manipulator may then be expressed in a compact form

$$f_i = \sum_{j=1}^{n} D_{ij}(\Theta)\ddot{\Theta}_j + \sum_{j=1}^{n} \sum_{k=1}^{n} N_{ijk}(\Theta)\dot{\Theta}_j\dot{\Theta}_k + G_i(\Theta) \qquad (4-5)$$

where D_{ij} is the coupling inertia between i and j,

N_{ijk} is centrifugal and Coriolis force at joint i due to the velocity at joint j, and

G_i is the gravity at joint i.

The inertia and gravity terms are particularly important in manipulator control since they affect positioning accuracy. Centrifugal and Coriolis forces are important when the manipulator is moving at a high rate of speed, during which time the errors they introduce are small. If Eqn. (4-5) is changed to vector form,

$$F = D(\Theta)\ddot{\Theta} + N(\Theta,\dot{\Theta}) + G(\Theta) . \qquad (4-6)$$

Assume then, that $D(\Theta)^{-1}$ exists,

$$\ddot{\Theta} = -D(\Theta)^{-1}[N(\Theta,\dot{\Theta})+G(\Theta)] + D(\Theta)^{-1}F . \qquad (4-7)$$

This equation represents a system of n nonlinear and coupled second order differential equations.

These two methods are equivalents in mathematical meaning, but the structure of the equations and the efficiency of computation differ. The Lagrange-Euler dynamic equation has the ability to clearly express the relationship between the input force and the coordinates, but this is a difficult method requiring complex calculations. While the Newton-Euler method doesn't clearly express the Coriolis or centrifugal terms and the relation between input and output, it does provide for simple and convenient calculation. From the control point of view, the L-E formulation is to be preferred for the calculation of input torques.

From the point of view of on-line control, the ordinary L-E or N-E formulation is not suitable for industrial robot manipulators. To overcome this limitation, other formulations have been suggested, including the recursive L-E, the recursive N-E, the configuration space method [Hollerbach, 1980], and parallel processing algorithms.

4.2 Lagrangian Formulation of a Two-D.O.F. Manipulator

In this section the dynamic equations for a two-d.o.f. manipulator's are derived, using the Lagrange-Euler formulation. Most commercial robot manipulators have six-d.o.f in order to move to an arbitrary point in the work space. Two-d.o.f, however, are sufficient for examining the validity of the control algorithm because the system dynamics of the two-d.o.f arm show the nonlinear interactions, such as Coriolis and centrifugal force. The schematic diagram for a two-d.o.f manipulator is illustrated in Fig. 4.1

It is assumed that the mass of a link is concentrated at the end of the corresponding link. First, the total kinetic and potential energy of the manipulator is computed to obtain the Lagrangian values. The expression of the position of the mass, M_1, in Cartesian coordinates is,

$$X_1 = L_1 \cos(\Theta_1)$$

and

$$Y_1 = L_1 \sin(\theta_1) \ .$$

The velocity of M_1 can be obtained by differentiating each position equation about t,

$$\dot{X}_1 = - L_1 \sin(\Theta_1)\dot{\Theta}_1$$

and

$$\dot{Y}_1 = L_1 \cos(\Theta_1)\dot{\Theta}_1 \ .$$

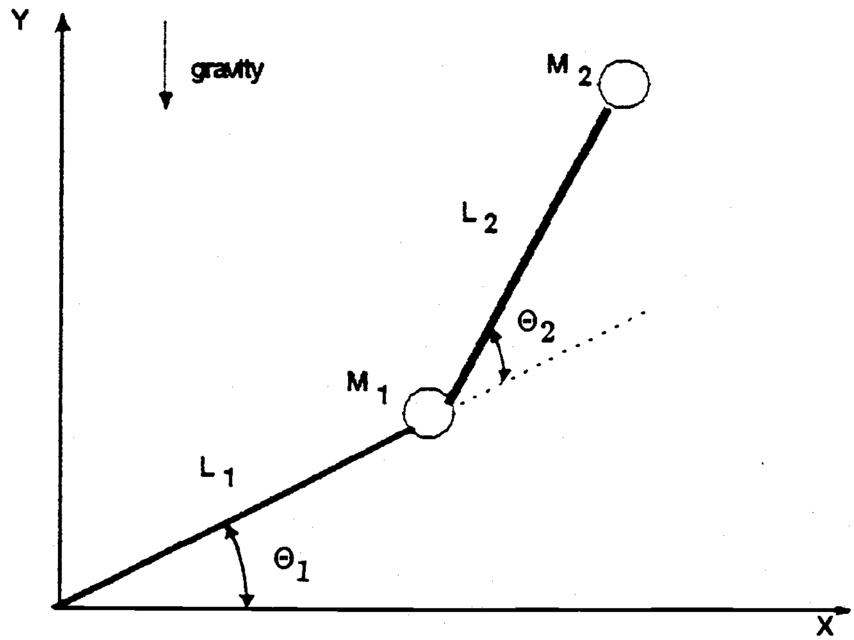


Figure 4.1 Schematic diagram of a two-link manipulator

The magnitude of the velocity squared is then

$$V_1{}^2 = \dot{X}_1{}^2 + \dot{Y}_1{}^2 = L_1{}^2\ \dot{\Theta}_1{}^2$$

The kinetic energy of M_1 is

$$K_1 = 0.5\ M_1\ V_1{}^2 = 0.5\ M_1\ L_1{}^2\ \dot{\Theta}_1{}^2$$

and the potential energy of M_1 is

$$P_1 = M_1\ g\ Y_1 = M_1\ g\ L_1\ SIN(\Theta_1)\ .$$

About the second mass, M_2, apply same method as applied with respect to mass M_1,

$$X_2 = L_1\ COS(\Theta_1) + L_2\ COS(\Theta_1+\Theta_2)$$

and

$$Y_2 = L_1\ SIN(\Theta_1) + L_2\ SIN(\Theta_1+\Theta_2)\ .$$

Each derivative is,

$$\dot{X}_2 = -L_1\ SIN(\Theta_1)\dot{\Theta}_1 - L_2\ SIN(\Theta_1+\Theta_2)(\dot{\Theta}_1+\dot{\Theta}_2)$$

and

$$\dot{Y}_2 = L_1 \cos(\Theta_1)\dot{\Theta}_1 + L_2 \cos(\Theta_1+\Theta_2)(\dot{\Theta}_1+\dot{\Theta}_2)$$

and the velocity squared is

$$V_2^2 = \dot{X}_2^2 + \dot{Y}_2^2 = L_1^2 \dot{\Theta}_1^2 + L_2^2(\dot{\Theta}_1+\dot{\Theta}_2)^2$$
$$+ 2 L_1 L_2 \dot{\Theta}_1(\dot{\Theta}_1+\dot{\Theta}_2) \cos(\Theta_2) .$$

The kinetic and potential energy of M_2 is, respectively,

$$K_2 = 0.5 M_2 V_2^2 = 0.5M_2L_1^2\dot{\Theta}_1^2$$
$$+ 0.5M_2L_2^2(\dot{\Theta}_1+\dot{\Theta}_2)^2 + M_2L_1L_2\dot{\Theta}_1(\dot{\Theta}_1+\dot{\Theta}_2)\cos(\Theta_2)$$

and

$$P_2 = M_2 g Y_2 = M_2 g L_1 \sin(\Theta_1)$$
$$+ M_2 g L_2 \sin(\Theta_1+\Theta_2) .$$

The Lagrangian L is

$$L = K - P = K_1 + K_2 - P_1 - P_2 = .5(M_1+M_2)L_1^2 \dot{\Theta}_1^2$$
$$+ .5M_2L_2^2(\dot{\Theta}_1+\dot{\Theta}_2)^2 + M_2L_1L_2\dot{\Theta}_1(\dot{\Theta}_1+\dot{\Theta}_2)\cos(\Theta_2)$$
$$- (M_1+M_2) g L_1 \sin(\Theta_1) - M_2 g L_2 \sin(\Theta_1+\Theta_2) .$$

Finally, we differentiate the Lagrangian L,

$$f_1 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\Theta}_1}\right) - \frac{\partial L}{\partial \Theta_1} = [(M_1+M_2) L_1^2 + M_2 L_2^2$$
$$+ 2 M_2 L_1 L_2 \cos(\Theta_2)] \ddot{\Theta}_1 + [M_2L_1L_2\cos(\Theta_2)$$
$$+ M_2L_2^2] \ddot{\Theta}_2 - M_2L_1L_2\sin(\Theta_2)(\dot{\Theta}_2^2 + 2\dot{\Theta}_1\dot{\Theta}_2)$$
$$+ g [(M_1+M_2) L_1 \cos(\Theta_1) + M_2 L_2 \cos(\Theta_1+\Theta_2)]$$

$$f_2 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\Theta}_2}\right) - \frac{\partial L}{\partial \Theta_2} = [M_2L_1L_2\cos(\Theta_2) + M_2L_2^2] \ddot{\Theta}_1$$
$$+ M_2 L_2^2 \ddot{\Theta}_2 + M_2 L_1 L_2 \sin(\Theta_2) \dot{\Theta}_2$$
$$+ M_2 g L_2 \cos(\Theta_1+\Theta_2)$$

V. CONTROLLER DESIGN FOR MANIPULATOR

5.1 Overview

The dynamics of an n degree of freedom manipulator, as shown in Eqn. (4-6), is as follows:

$$\Theta = D^{-1}(\Theta)[N(\Theta,\dot{\Theta}) + G(\Theta)] + D^{-1}(\Theta)U \ .$$

The state x is defined by:

$$x^T = [\Theta_1,\dot{\Theta}_1,\Theta_2,\dot{\Theta}_2 \ \ldots, \ \Theta_n,\dot{\Theta}_n]$$

$$= [x_1,x_2,x_3,x_4 \ \ldots, \ x_{2n-1},x_{2n}] \ .$$

The following space state equations are obtained

$$\dot{x}_i = x_{i+1}$$

and

$$\dot{x}_{i+1} = D^{-1}(x)[N_i(x) + G_i(x)] + D^{-1}(x)U \ . \qquad (5\text{-}1)$$

In the position control of the manipulator, the angular position error is

$$e_i = x_i - x_{id} \qquad , \qquad i=1,3,5 \ \ldots \ , \ 2n-1 \qquad (5\text{-}2)$$

where x_i is the state of the joint angle of each joint and x_{id} is the position the link must reach. The control objective is to determine the feedback control law, u_i, such that

$$e_i = x_i - x_{id} \longrightarrow 0 \quad \text{as } t \longrightarrow \infty \ . \qquad (5\text{-}3)$$

For this position control, let the feedback control, u_i, be a variable structure system control, i.e.,

$$u_i(x,t) = \begin{cases} u_i^+(x,t) & \text{if } s_i(x,t) > 0 \\ u_i^-(x,t) & \text{if } s_i(x,t) < 0 \end{cases} \tag{5-4}$$

where the switching surface in the error state is

$$s_i = c_i e_i + d_i v_i = 0 \ , \tag{5-5}$$

where v_i is the velocity of each joint, i, and c_i and d_i are positive constants. For the sake of simplicity, let $d_i = 1$, as discussed in chapter II. Then, the derivative of s_i is

$$\dot{s}_i = c_i \dot{e}_i + \dot{v}_i = 0 \tag{5-6}$$

where v_i is the acceleration of joint i.

The general existence and reaching condition of the sliding mode is defined as

$$s_i \dot{s}_i < 0 \ . \tag{5-7}$$

The principal problem is to choose the feedback controls, $u_i^+(x,t)$ and $u_i^-(x,t)$, so that sliding mode occurs on the intersection of the switching surface of Eqn. (5-5). If the design procedure as introduced in Chapter III is followed, the time required to calculate the inequalies of each parameter is too lengthy. Moreover, during the reaching movement of the sliding mode, robustness cannot be guaranteed because the robustness of the VSS occurs only when the system trajectories are entered on the sliding mode. To overcome this problem, the method of minimizing the time required to reach the sliding mode should be considered. To reach the sliding mode quickly, a regulated derivative control algorithm is considered [Morgan, 1985]. For regulating the derivation of the switching variable, s, to a constant, p_i, define s_i under the conditions of Eqn. (5-7) of the switching surface,

$$\dot{s}_i = -p_i \operatorname{sgn}(s_i) \tag{5-8}$$

where p_i is positive constant and sgn(.) is the sign function as follows:

$$\operatorname{sgn}(s_i) = \begin{cases} 1 & \text{if } s_i > 0 \\ 0 & \text{if } s_i = 0 \\ -1 & \text{if } s_i < 0 \end{cases} .$$

For a multi-input, multi-output (MIMO) VSS, Young [1978] presented the hierarchy method to find feedback control, u_i. This approach takes a long time for calculation of the control law. Suppose the MIMO system is considered as a composition of a single-input, single-output (SISO) system when the interaction between subsystems are suppressed as disturbances in the sliding mode, the SISO VSS is applicable to the manipulator (MIMO). This is a very straightforward approach.

The steps outlined in this overview always satisfy the existence condition, $s\dot{s} < 0$, and simplify feedback conrol for fast calculations. The sliding mode equation, as discussed in chapter II, is then obtained as

$$\dot{e}_i = -c_i e_i \, . \tag{5-9}$$

These error equations represent the dynamics of a linear system when the system described in Eqn. (5.1) is in the sliding mode. Clearly, the nonlinearities of manipulator dynamics are eliminated. From this point, new relationships from Eqns. (5-6) and (5-8) can be derived

$$c_i(\dot{x}_i - \dot{x}_{id}) + \dot{v}_i = - p_i \ \mathrm{sgn}(s_i) \ ,$$

or represented with respect to the acceleration of joint i

$$\dot{v}_i = - c_i(\dot{x}_i - \dot{x}_{id}) - p_i \ \mathrm{sgn}(s_i) \ . \tag{5-10}$$

Inserting Eqn. (5-10) into the original system Eqn. (5.1), in order to find an equivalent control law, the consequent feedback control law is

$$u_i = u_{ieq} = D(x)[-c_i\dot{e}_i - p_i \mathrm{sgn}(s_i)] + N_i(x) + G_i(x) \ . \tag{5-11}$$

This feedback control law shows a simpler form than the existing control law presented in Chapter III.

5.2 Simulation of the Proposed Algorithm

This section presents the simulation of the two-d.o.f. manipulator model outlined in Chapter IV, in accordance with the design procedures established in previous sections. The dynamic equation for the matrix form is given as

$$\begin{bmatrix}\ddot{\Theta}_1 \\ \ddot{\Theta}_2\end{bmatrix} = - \begin{bmatrix}D_{11}(\Theta) & D_{12}(\Theta) \\ D_{21}(\Theta) & D_{22}(\Theta)\end{bmatrix}^{-1} \begin{bmatrix}N_1(\Theta,\dot{\Theta}) + G_1(\Theta) \\ N_2(\Theta,\dot{\Theta}) + G_2(\Theta)\end{bmatrix} + \begin{bmatrix}D_{11}(\Theta) & D_{12}(\Theta) \\ D_{21}(\Theta) & D_{22}(\Theta)\end{bmatrix}^{-1} \begin{bmatrix}u_1 \\ u_2\end{bmatrix} \ . \tag{5-12}$$

Let the states $x^T = [x_1, x_2, x_3, x_4] = [\Theta_1, \dot{\Theta}_1, \Theta_2, \dot{\Theta}_2]$ be assigned, then the individual terms of Eqn. (5-12) may be defined as

$D_{11} = (M_1+M_2)L_1^2 + M_2L_2^2 + 2M_2L_1L_2\text{COS}x_3,$

$D_{12} = M_2L_1L_2\text{COS}x_3 + M_2L_2^2,$

$D_{21} = D_{12},$

$D_{22} = M_2L_2^2,$

$N_1 = -M_2L_1L_2(x_4^2+2x_2x_4)\text{SIN}x_3,$

$N_2 = M_2L_1L_2x_3\text{SIN}x_4,$

$G_1 = [(M_1+M_2)L_1\text{COS}x_1 + M_2L_2\text{COS}(x_1+x_3)]g,$ and

$G_2 = [M_2L_2\text{COS}(x_1+x_3)]g,$

where the values of the model parameters are

$M_1 = 0.5$ Kg,

$M_2 = 1.0$ Kg,

$L_1 = 1.0$ m,

$L_2 = 0.8$ m, and

$g = 9.8$ m/s^2,

and the controls, u_1 and u_2, are external torques in newton meter (N • m).

The control objective is to find the feedback control law, u_i, such that the joint positions, $x_1 \longrightarrow x_{1d}$, $x_3 \longrightarrow x_{3d}$, and the joint velocities, x_2, $x_4 \longrightarrow 0$, respectively, reflect the following:

1) The position errors are

$e_1 = x_1 - x_{1d}$

and

$$e_2 = x_3 - x_{3d} \; ; \qquad (5\text{-}13)$$

2) The switching surfaces are

$s_1 = c_1(x_1 - x_{1d}) + x_2 = 0$

and

$$s_2 = c_2(x_3 - x_{3d}) + x_4 = 0 \ , \qquad (5\text{-}14)$$

and their derivatives are

$$\dot{s}_1 = c_1(\dot{x}_1 - \dot{x}_{1d}) + \dot{x}_2 = 0$$

and

$$\dot{s}_2 = c_2(\dot{x}_3 - \dot{x}_{3d}) + \dot{x}_4 = 0 \ ; \qquad (5\text{-}15)$$

3) From the regulated derivative control,

$$\dot{s}_1 = -p_1 \ \mathrm{sgn}(s_1)$$

and

$$\dot{s}_2 = -p_2 \ \mathrm{sgn}(s_2) \ ; \qquad (5\text{-}16)$$

4) And from Eqn. (5-15) and Eqn. (5-16),

$$c_1(\dot{x}_1 - \dot{x}_{1d}) + \dot{x}_2 = -p_1 \ \mathrm{sgn}(s_1)$$

and

$$c_2(\dot{x}_3 - \dot{x}_{3d}) + \dot{x}_4 = -p_2 \ \mathrm{sgn}(s_2) \ ,$$

or each acceleration of joint1 and joint2 is

$$\dot{x}_2 = -c_1(\dot{x}_1 - \dot{x}_{1d}) - p_1 \ \mathrm{sgn}(s_1)$$

and

$$\dot{x}_4 = -c_2(\dot{x}_3 - \dot{x}_{3d}) - p_2 \ \mathrm{sgn}(s_2). \qquad (5\text{-}17)$$

Finally insert Eqn. (5.17) into Eqn. (5.12) to find the feedback control law,

$$\begin{aligned} u_1 = \ & D_{11}[-c_1\dot{e}_1 - p_1 \ \mathrm{sgn}(s_1)] \\ & + D_{12}[-c_2\dot{e}_2 - p_2 \ \mathrm{sgn}(s_2)] \\ & + N_1(x) + G_1(x) \\ u_2 = \ & D_{21}[-c_1\dot{e}_1 - p_1 \ \mathrm{sgn}(s_1)] \\ & + D_{22}[-c_2\dot{e}_2 - p_2 \ \mathrm{sgn}(s_2)] \\ & + N_2(x) + G_2(x) \ , \end{aligned} \qquad (5\text{-}18)$$

Initial conditions of simulation are set at nonzero initial positions and nonzero velocities. Both the desired positions and velocities are zero (i.e., $x_{1d} = x_{2d} = x_{3d} = x_{4d} = 0$). The system, governed by Eqn. (5.12) with the controls given in Eqn. (5.18), is simulated utilizing a fourth-order Runge-Kutta integration method. The step size selected for this scheme is 0.05 seconds.

To assess global performance of the control laws, several simulations are performed, with appropriate changes of parameters and initial conditions. The algorithm is simulated using different configurations, including 4-block, a basic sliding mode controller, the reduction of chattering phenomena, consideration of parameter variations, and consideration of design parameters for a fast time response in the sliding mode. All of initial conditions in these simulation results are

$$x^T = [1.2,\ 0.5,\ -1.0,\ -0.5] \qquad (5\text{-}19)$$

and the constraints for each of the manipulator states are defined by $|x_1| \leq \pi$, $|x_3| \leq \pi$, $|x_2| \leq \pi/s$, and $|x_4| \leq \pi/s$.

5.2.1 Basic Sliding Mode Controller Simulation

The responses of the system governed by Eqn.(5-12), with feedback control given by Eqn. (5-18), are shown in (a) of Figures 5.1 through 5.6. The initial condition is given by Eqn. (5.19) and set at $c_1 = 0.4$,

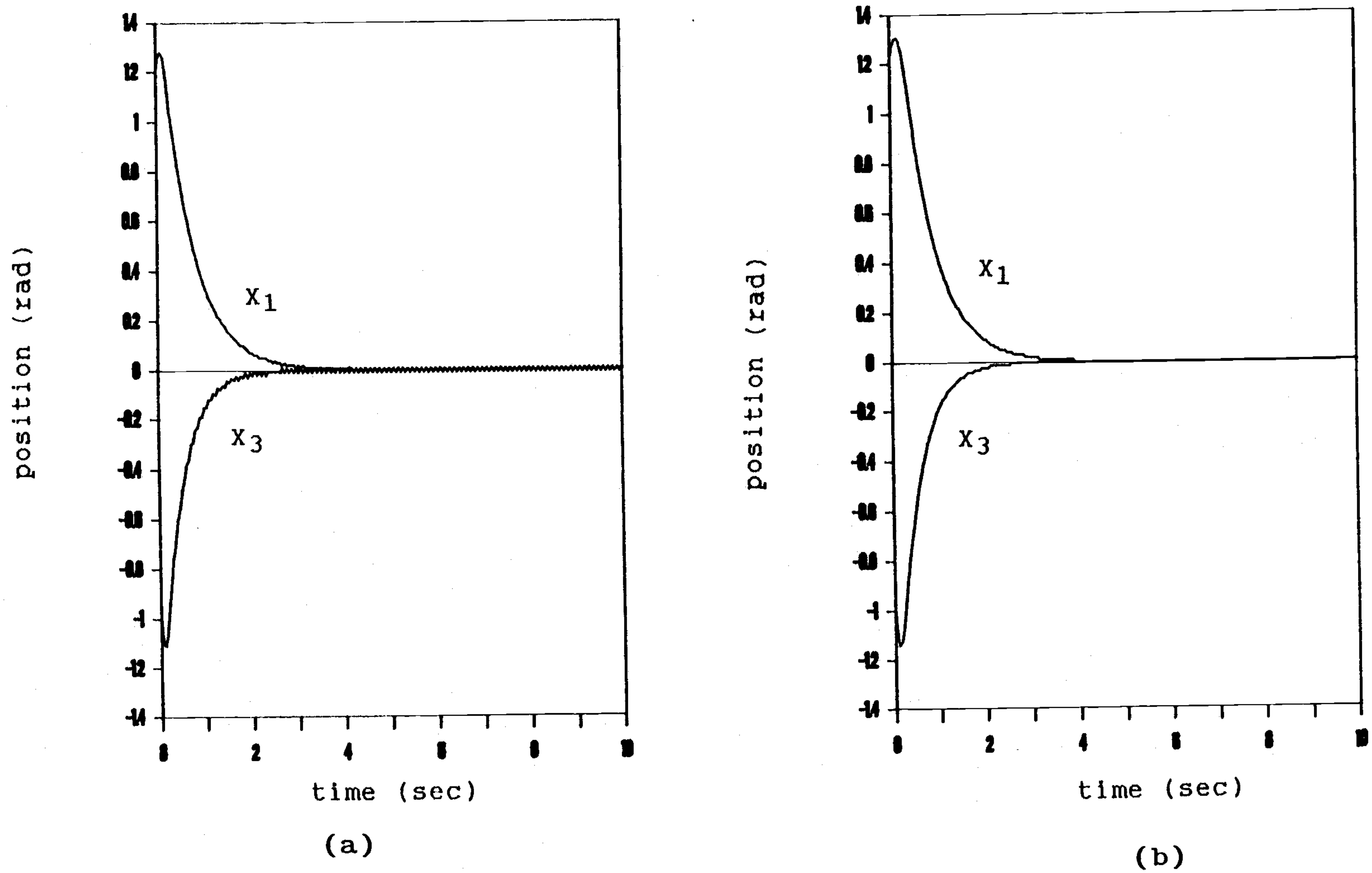
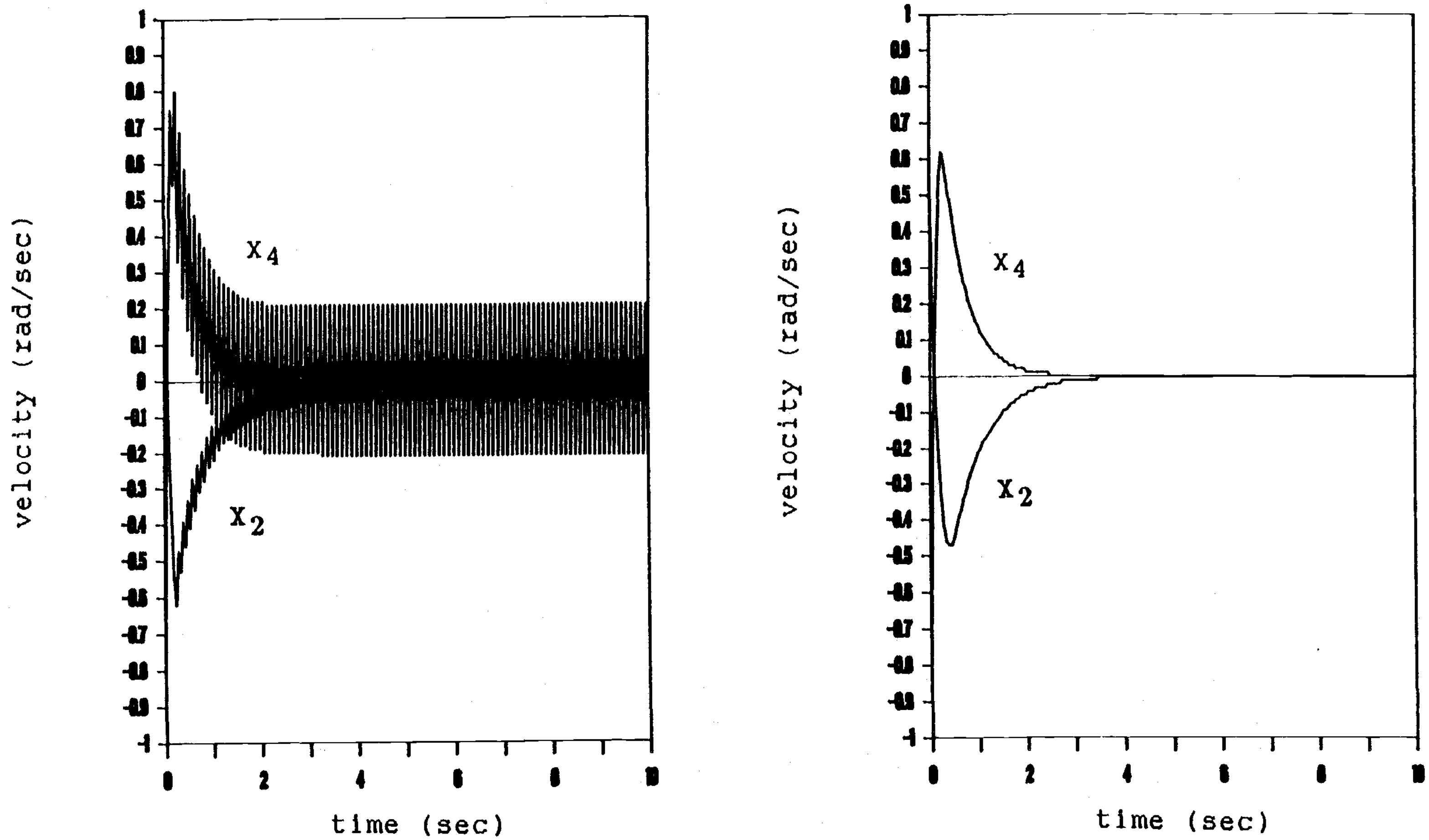
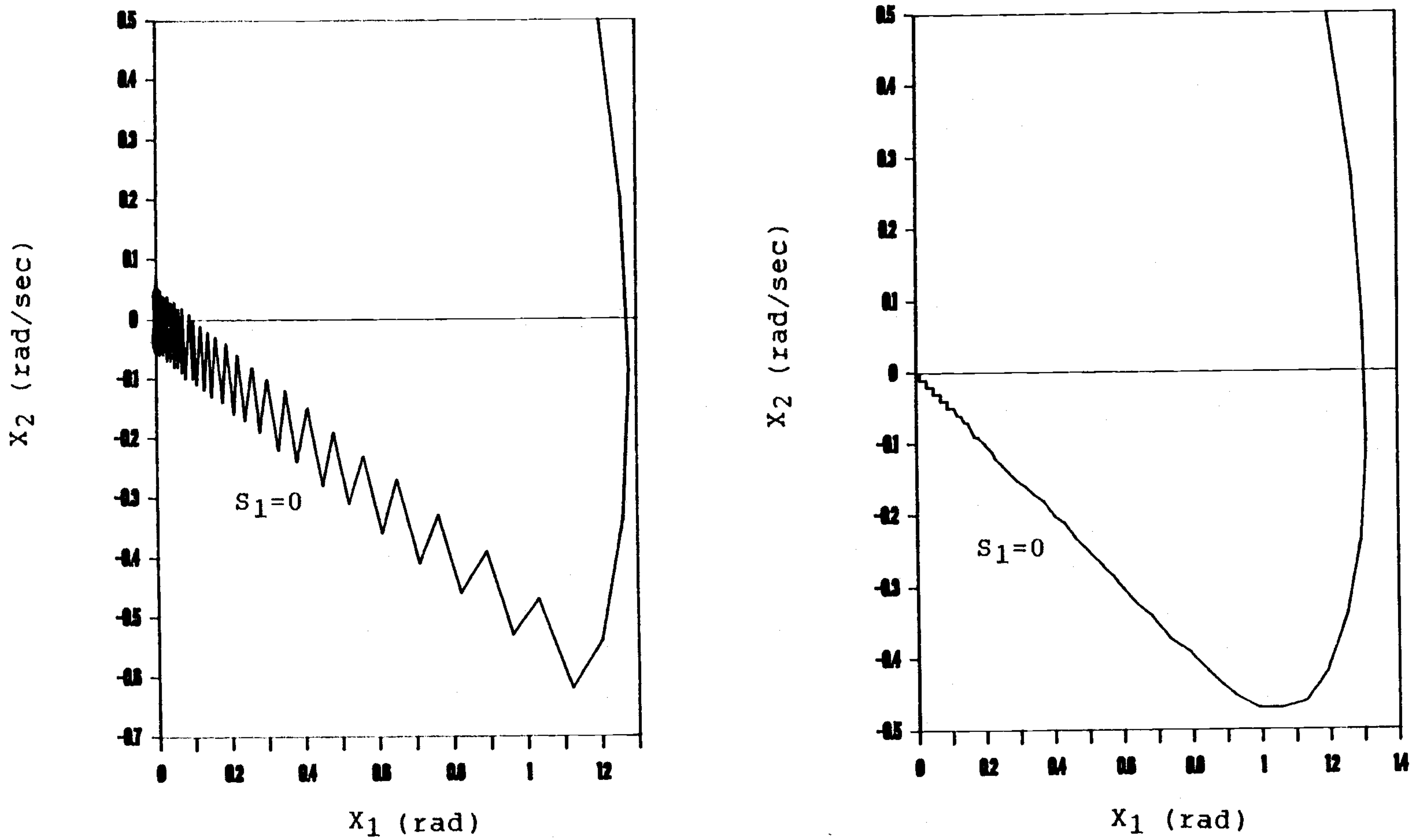


Figure 5.1 Time response of position x_1, x_3



(a) (b)

Figure 5.2 Time response of velocity x_2, x_4



(a) (b)

Figure 5.3 Phase plane trajectory of joint 1

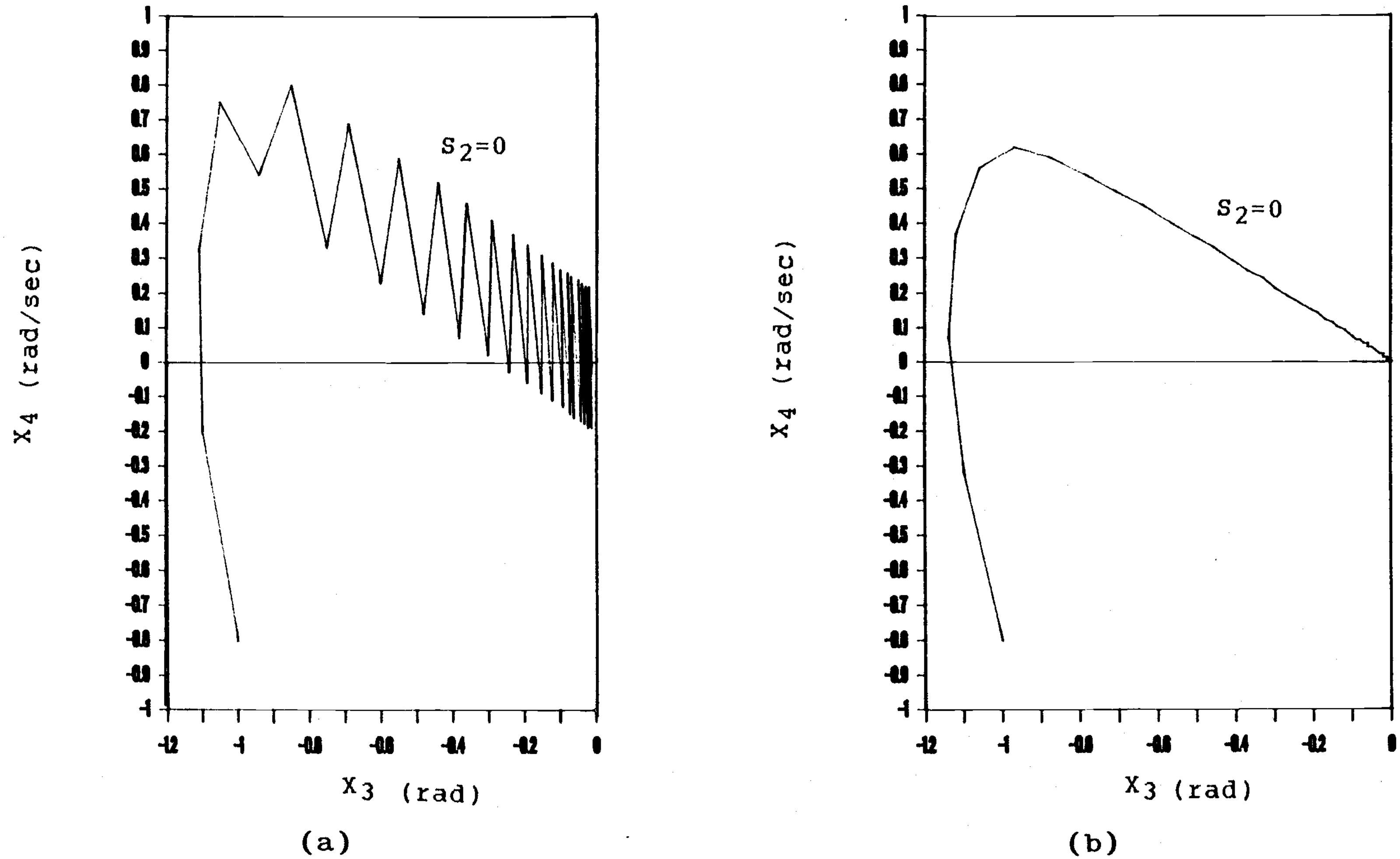


Figure 5.4 Phase plane trajectory of joint 2

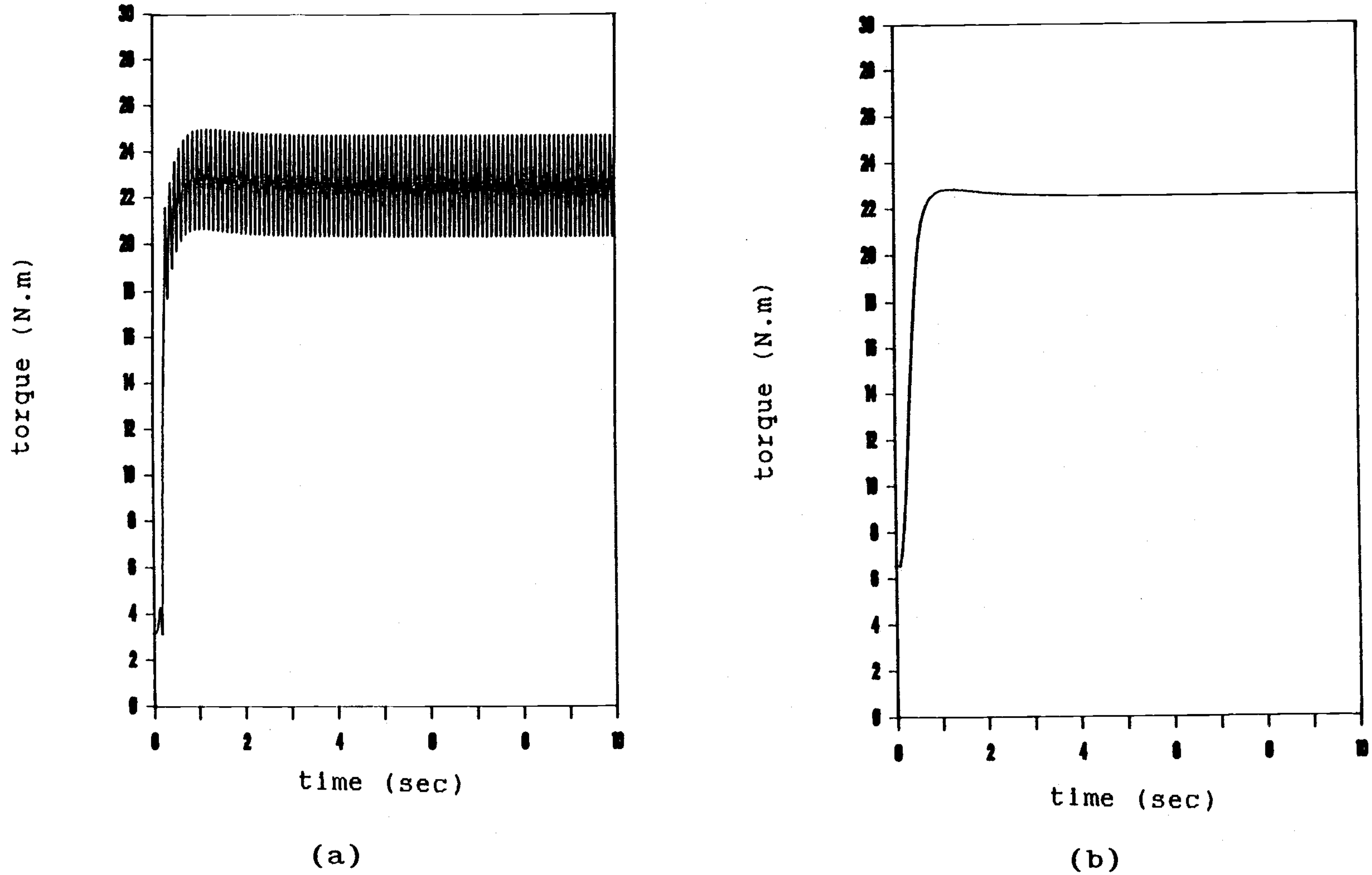


Figure 5.5 Desired torque u_1

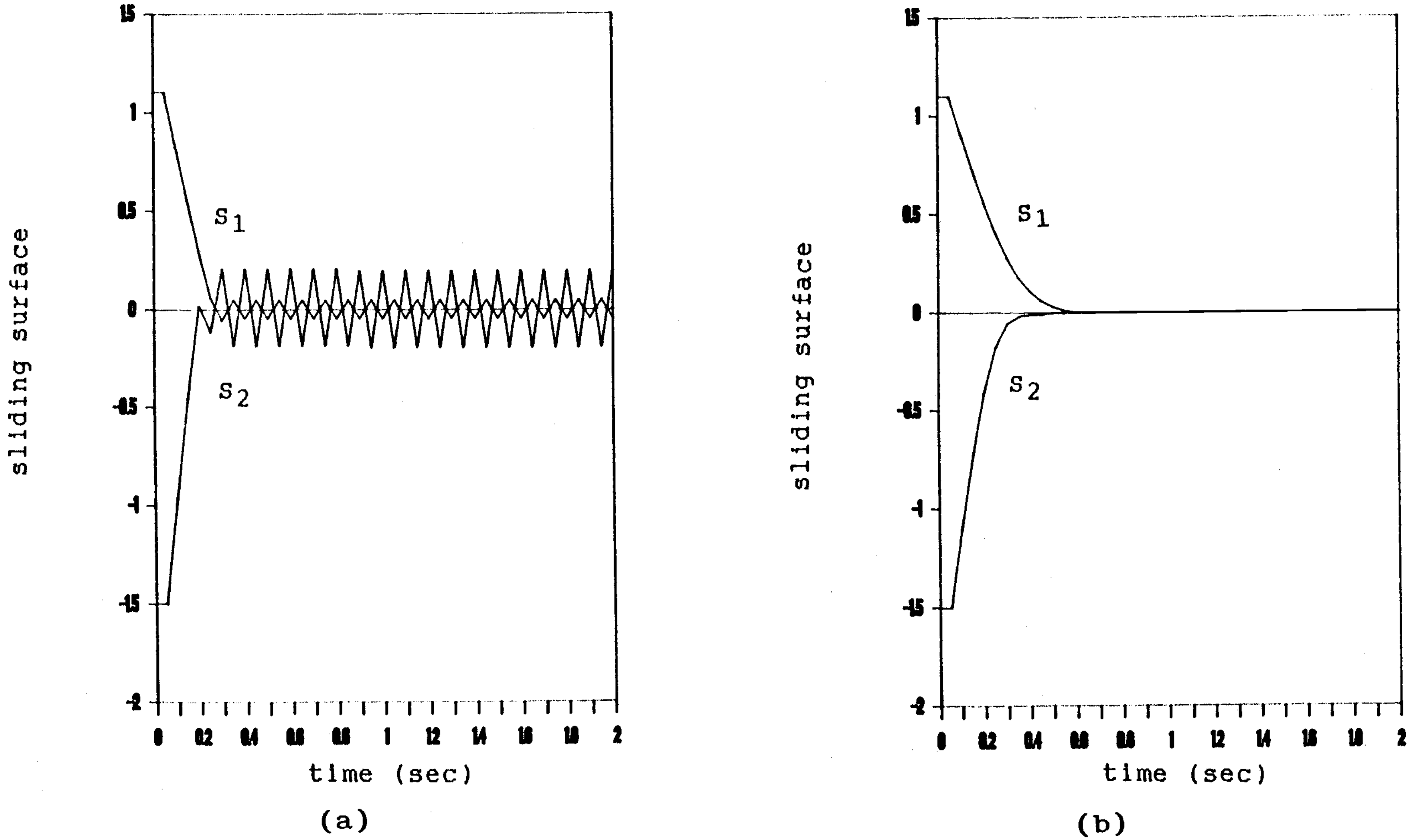


Figure 5.6 Time response of sliding mode equation s_1, s_2

$c_2 = 0.7$, $p_1 = 2.0$, and $p_2 = 4.0$. The time responses of positions x_1 and x_3 reached the desired positions within proper time limits, even though a very slight chattering was experienced. While in the time responses for velocities x_2 and x_4 (see Figure 5.2), there was a large degree of undesirable chattering. Figures 5.3 through 5.6 clearly indicate this phenomenon.

5.2.2. Reduction of Chattering Phenomena

The principal reasons for the chattering are the discontinuity of the feedback control law, which may be attributed to the sign function and the large gain parameter, p_i, within. Furthermore, this sign function is in practice difficult to realize. To remove this difficulty and to create additional design parameter flexibility, a new function was introduced in lieu of the discontinuous sign function of the feedback control law:

$$ftn(s_i) = \frac{s_i}{|s_i| + \mu_i} \qquad (5\text{-}20)$$

where μ_i is a small positive constant. This function is continuous, making the feedback control law continuous. Another continuous function could be considered, such as

$$ftn(s_i) = s_i^2/(s_i^2+\mu_i^2) \text{ or } sat(s_i)$$

where

$$\text{sat}(s_i) = \begin{bmatrix} 1 & s_i > 1 \\ s_i & -1 < s_i < 1 \\ -1 & s_i < 1 \end{bmatrix} .$$

In this study, Eqn. (5-20) is considered for reason of the flexibility of the design parameter.

To test the validity of this algorithm, the same initial conditions, the same sliding mode slopes, and the same gain parameter as the previous simulation, but with μ = 0.6, was simulated. The results of this algorithm are presented in (b) of Figures 5.1 through 5.6. Reduction of the chattering problem, which the major disadvantage of VSS control, was clearly effective with the new sign function. The original sign function and the new continuous function may be in a trade-off relation. Due to use of the continuous function, the time optimality of the total system is reduced even though chattering in the total system is reduced. The system response time, however, is more dependable using sliding slope c. This is discussed further in section 5.2.4.

Using the same design parameters, except for a change in the gain parameter, p_i, the chattering problem was considered. It is obvious that the larger the value of p_i, the more chattering is produced in the system response since large gain parameters determine there will be a large spectrum of switching value between the two switching feedback control values.

5.2.3 Consideration of Parameter Variations

The dynamic equations of the robot manipulator for the joint torques or forces given in Eqn. (5-12) consist of configuration dependent coefficients, multiplied by instantaneous velocities and acceleration. It is important that the equation coefficient be evaluated when the configuration is changed. Configuration changes (i.e., parameter variations) mainly occur when payloads are changed. In most manipulators the payload is located at the end effector. In this two-d.o.f manipulator, payload was located at the end of link L_2. Actually, when a manipulator is loaded, the parameter M_2 varies. In this simulation M_2 was varied with weights up to 6.0 Kg. When choosing the same design parameters as the simulation described in section 5-2, the results were as shown in Figure 5.7 (the time response of positions x_1 and x_3) and Figure 5.8 (the torques u_1 and u_2, in accordance with each variation). The time responses were almost identical, reflecting no concern with the weight of the payload. The torques for each joint corresponded to the payload weight. These results confirm the robustness of the sliding mode controller with respect to parameter variations.

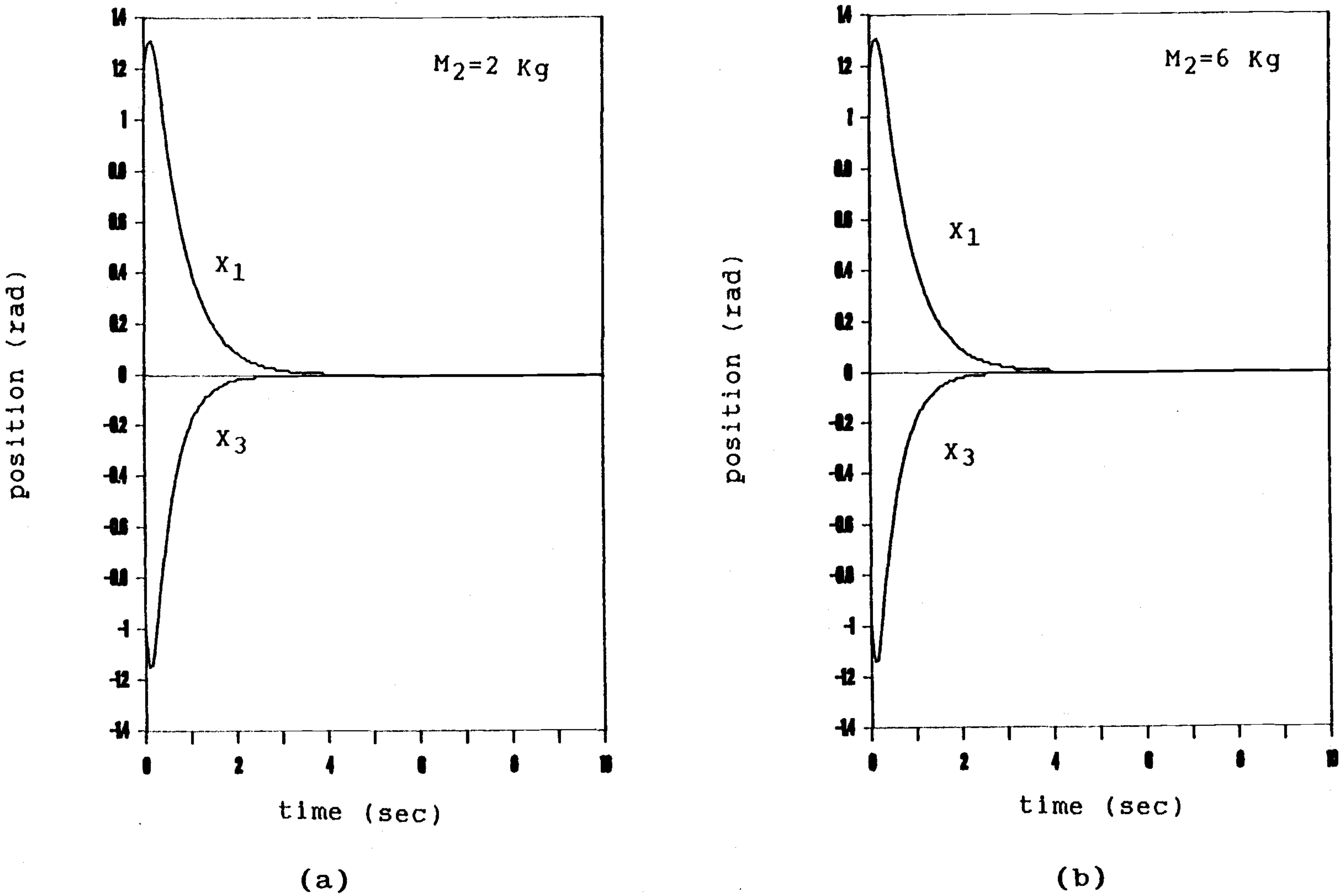


Figure 5.7 Time response with respect to payload

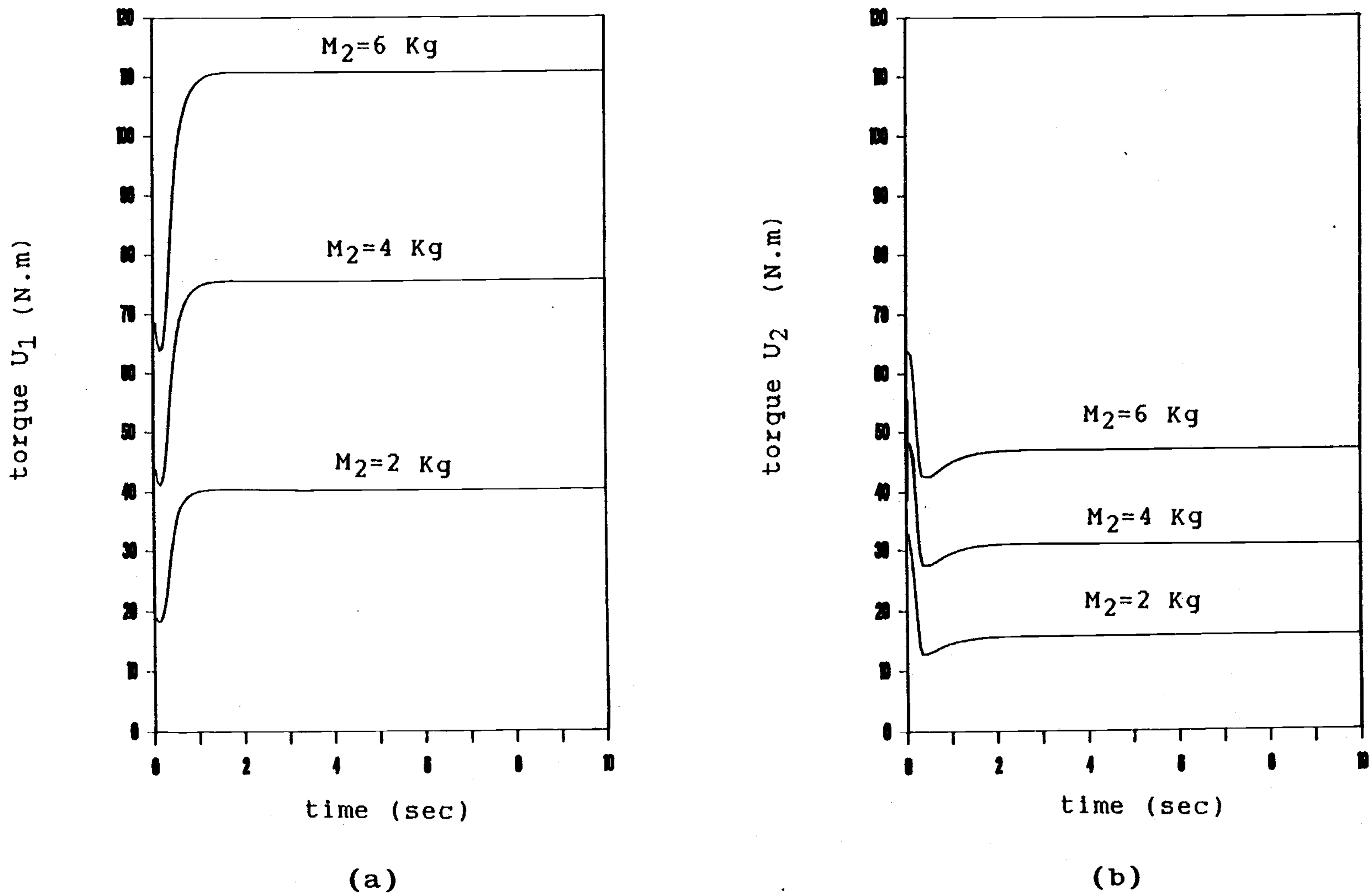
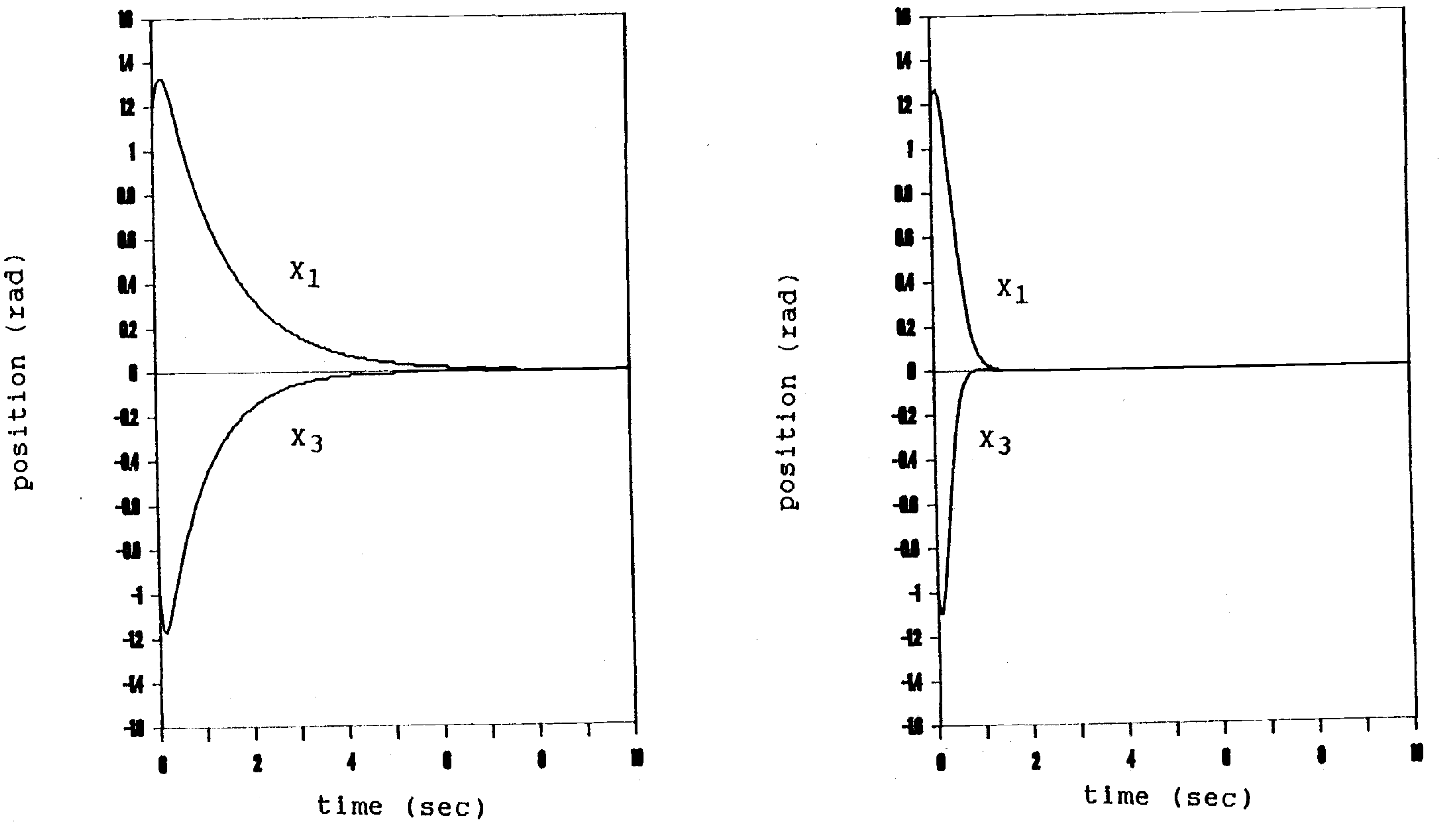


Figure 5.8 Desired torque with respect to payload

5.2.4 System Response In Sliding Mode

The design parameter, sliding slope c_i in the sliding mode controller, is related to the system's response time, as discussed in Chapters II and III. If c_i is changed, the feedback control law of Young's algorithm must be recalculated at the cost of considerable calculation time. However, the controller tested for this study proved to be easily changeable, without the need for complex calculations.

The simulation results are shown in Figure 5.9. When c_i was set at half value (c_1 = 0.25, c_2 = 0.35) of the original c_i specified in section 5.2.1, the response time was nearly doubled (Fig. 5.9(a)); when c_i was set at four times the value (c_1 = 2.0, c_2 = 2.8), the response decreased to double the duration of the original response time (Fig. 5.9(b)). Consequently, the choice of a larger value for c_i can speed up the system motion in the sliding mode.



(a) (b)

Figure 5.9 Comparison of system response

VI. CONCLUSION

The goals of this work were threefold. First, design a sliding mode controller based on the theory of the Variable Structure System; second, reduce the chattering phenomenon connected with the sliding mode controller; and last, determine a simple feedback control law for fast calculation of the control algorithm. It has been demonstrated that the design methodology presented in this study is valid for the control of a robot manipulator.

The Variable Structure System control law eliminates the nonlinear dynamic interaction of the manipulator between each of its joints when the system trajectory reaches the sliding mode. Due to substitution of the continuous function, $ftn(s_i)$, in place of the discontinuous original sign function for the feedback control law of the sliding mode controller, the main disadvantage of the VSS was eliminated and the flexibility of design parameters was enhanced. One of the most important properties of the VSS is its robustness with respect to unknown manipulator parameter variations, and this was confirmed in this simulation.

In comparison to existing control laws, the sliding mode control would seem to be suitable for complex nonlinear systems. The simulation results provided in this study show that the proposed algorithm can possibly be applied to a commercial n degree of freedom robot manipulator. However, implementation of the control algorithm should be an open question. In the case of the adaptive control scheme, a great number of them have not to date been tried on real robots to test their effectiveness [Hsia, 1986]. Neither has the sliding mode controller been applied to a n d.o.f. robot system. The principal obstacles to this application are the construction of an effective and simple manipulator model for real-time control and to determine the sampling time of the control algorithms. Due to sensor inaccuracy, a VSS observer must be considered and for digital computer implementation, discrete time VSS control should be considered.

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