AN ABSTRACT OF THE THESIS OF

YING-CHUAN TSENG for the MASTER OF SCIENCE
(Name) (Degree)
in CIVIL ENGINEERING presented on April 19, 1974
(Major) (Date)

Title: MODELING HYDRAULIC DREDGE SPOIL FATE

Abstract approved: Redacted for privacy W. L. Schroeder

Hydraulic dredging is and most certainly will remain a primary means of maintaining and improving ship channels and for land reclamation in near-shore water. The study of which this thesis describes a part is intended to develop a method of predetermining the immediate disposition of sand spoils from hydraulic dredging discharged in water.

The results presented herein indicate that the longitudinal forward velocity field induced in still water by a model dredge discharge can be obtained from available results of jet flow studies for any discharge velocity. When combining this velocity field with particle settling velocity, particle trajectory can be computed. It is shown that a particle trajectory based on hydraulic sorting by particle size should also be workable in the case of a cross flow in the receiving water. A computer program which predicts particle trajectory for the quiescent receiving water condition is used to compute comparisons.
with experimental results. It was found that the theoretical framework proposed compares reasonably with those obtained in experiments in quiet water. However, in cross-flow conditions the comparisons between the experimental and theoretical results are not good. These observed differences are attributed to deceleration of the jet due to the cross flow and to the boundary conditions in the experimental model.
Modeling Hydraulic Dredge Spoil Fate

by

Ying-Chuan Tseng

A THESIS

submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Master of Science

June 1974
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Date thesis is presented  April 13, 1974
Typed by Mary Jo Stratton for Ying-Chuan Tseng
ACKNOWLEDGMENTS

This investigation was done with the support of the Sea Grant Program at Oregon State University, funded by the National Oceanic and Atmospheric Administration. It is the first part of a two-part study of Hydraulic Dredge Spoil Fate. The present investigation started in September 1972 and was completed in December 1973.

Sincere appreciation and gratitude are expressed to Dr. W. L. Schroeder, major professor and advisor, for his invaluable guidance, constant encouragement, and patient reviewing of the manuscript during the period of this study.

Thanks are due to Dr. J. R. Bell and Dr. R. D. Layton for serving as committee members.

It is not possible to name all of those who contributed to the completion of this thesis; however, thanks are given to Donald N. Thackery, Thomas Hanley and Thomas M. Connor for their help in the laboratory and with drawings and to Mary Jo Stratton for typing the manuscript and the final text.

This thesis is dedicated to the writer's parents and family whose support and encouragement made his graduate study possible.
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Plate 1. Hydraulic Dredge Spoiling Operation.
MODELING HYDRAULIC DREDGE SPOIL FATE

I. INTRODUCTION

Dredging in the Pacific Northwest

Dredging is one of the most extensive construction activities in the rivers and harbors of the Pacific Northwest. The basic function of a dredge is to excavate underwater solids and transport them to other locations. Space for creating fill areas on land or in marshes to dispose of the great quantities of dreding spoils is in short supply.

Because of increasing environmental pressure, there is a rapidly growing tendency to shift from traditional land disposal of dredge material to offshore or ocean disposal. For example, in Oregon alone the volume of material dredged ranges up to 30 million cubic yards per year. Some of this is deposited on land. About half of the spoil, however, is dumped back into water or immediately adjacent to it (7). The possible opposing consequences of this material on water quality and on the aquatic environment are of serious concern to the public and to the agencies charged with protecting the quality of the environment.

Subaqueous disposal of the large amount of waste material generated presents a potentially major problem. Whether simple disposal or incorporation into engineering works is proposed, the
immediate fate (trajectory from discharge to bottom deposition) of the spoils should be known in advance. Properties of spoil deposits incorporated in engineering works must be known to permit design of structures for various conditions. The present state of knowledge concerning these matters is inadequate to deal with the problem faced.

With a heavy economic commitment to maritime trade, the benefits of successful planning and control of the dredging programs become immediately evident.

**Purpose**

Hydraulic dredging is and most certainly remains a primary means of maintaining and improving ship channels and for land reclamation in near shore waters. However, indiscriminate wasting of dredge spoils is no longer a satisfactory solution to disposal of large quantities of what amounts to waste material. There are instances when the physical situation will permit subaqueous placement of such material but these instances are not physically well-defined. Consequently, opposing interest groups have little in the way of factual information or proven predictive methods with which to resolve their disagreements as to the impact of dredge spoiling on a water body. The study of which this thesis is a part is intended to develop a reliable method of computing the immediate fate of sandy spoils discharged in water. The work reported in this thesis was
intended to produce a practical mathematical framework for the method.

Scope of the Study

In this study the major emphasis is on developing schemes for predicting immediate spoil fate where discharge in water body is planned. The approach was to (1) hypothesize an analytical model for sediment movement from discharge to deposition, and (2) investigate this model by laboratory experiment.

A concurrent field study is being done to confirm the results described herein, as they apply to field situation. The report of field study results is beyond the scope of this thesis.
II. THEORETICAL CONSIDERATIONS

Principal Factors

The rate of sediment accumulation at any point in front of a dredge pipeline may be governed by the rate of water and sediment discharge from the pipeline, sediment grain size distribution, sediment specific gravity, porosity of the resulting sediment, size of pipeline and the basin geometry (5).

A sediment fan forms where a dredge pipeline deposits its sediment load into a water body. The principal factors that affect the size, shape, position and composition of the fan are (1) density difference between inflow and basin water; (2) water discharge from pipeline; (3) nature and amount of sediment load; (4) geometry of the water basin; and (5) boundary energy condition.

In the simulation model proposed, the pipeline discharge is represented by a jet. Factors 1, 4 and 5 are assumed to be constant, while factors 2 and 3 are variable.

(1) Density difference--it is assumed that (a) inflow fluid density equals to receiving waterbody density; (b) solid specific gravity is constant; and (c) the fluid and solids are separate phases.

(2) Water discharge--the discharge is assumed to be steady and uniform.
(3) Sediment load--the sediment load may be divided into several grain size fractions.

(4) Basin geometry--the area in front of the jet mouth forms a Cartesian grid system. This grid system is used for accounting purposes.

(5) Boundary energy condition--boundary conditions are assumed to be unaffected by back currents.

In this study the mathematical model for simulating disposal of spoils in the waterbody is composed of a combination of sedimentation theory and jet theory. The theoretical model has been considered for two presumed configurations of discharge.

**Quiescent Receiving Waterbody**  
*(Ambient Velocity = 0)*

The mechanics of transport of suspended sediment can be treated considering the movement of specific sediment particles (9). For sand deposited in flowing water, the finer particles tend to be carried farther downstream than coarse particles. The dispersion of grain size of a jet discharging a sand-water mixture is clearly a function of differential settling velocity, as shown in Figure 1. Here hypothetical trajectories of particle movement are shown which are based on the assumption that they are traced by the vectorial resultant of the longitudinal velocity, \( V \), in the transition region and the settling
Figure 1a. Structure of Velocity Field in Plan View.

Figure 1b. Diagrammatic Representation of the Dispersion and Sorting of Particles.
velocity, \( V_s \). This assumption is fundamental to the model proposed herein. The simplifying assumption has also been made that particles begin to settle with a velocity, \( V_s \), only when they have transgressed the zone of no diffusion and entered the region of mixed flow. [This assumption is approximate. Uplift induced by fluid turbulence will retard the settling of particles, but the collective settling of the particles caught in relatively large eddies tends to increase the rate of settling. For a reconnaissance study the grain size composition of bottomset beds calculated by this method showed reasonable agreement with that obtained from actual bottomset samples (10).] Therefore under these conditions, for a given particle size and velocity field, the particle traveling distance can be calculated. Two approaches to determining the velocity field are presented in the following paragraphs.

Experimental results obtained in this study for the surface velocity along a jet centerline are shown in Figure 2 and compared with the results of Albertson et al. (2). The data represent a water jet discharging in water with no solid phase. It is evident that the data agree reasonably. The measured horizontal velocity profiles on jet centerline are shown in Figure 3. From these data a velocity profile for an analytical model has been represented mathematically.

The equation of the velocity profile shown is:

\[
\frac{V}{V_o} = 0.00572 \ (11 - X)^2 \ (V_s - 0.6)^2 + 0.010
\] (1)
Figure 2. Surface Velocity along Jet Centerline.
Figure 3. Measured Horizontal Velocity Profiles on Jet Centerline.
in which

\[ V = \text{Velocity (fps) at any point along the jet centerline} \]

\[ V_o = \text{Initial discharge velocity (fps)} \]

\[ V_s = \text{Particle settling velocity (fps)} \]

\[ X = \text{Distance (ft) along longitudinal direction (beyond impact point)} \]

\[ t = \text{Total traveling time (sec) for a given particle size to settle from the surface to the bottom} \]

The detailed development of this velocity model is shown in Appendix I.

Because a stream of turbulent fluid discharging into a large basin through a well-defined and stable orifice may be considered a free jet, jet flow exists whenever a major river discharges directly into a lake or ocean. In 1926, Tollmien (16) first developed a comprehensive theory for the behavior of an axially symmetrical jet issuing from a point source. This theory has been developed and tested in many laboratories by aerodynamicists, chemical engineers, heating engineers and hydraulic engineers. Corrsin and Uberoi (6), Keuthe (12), Nottage, Slaby and Gojsza (14) and Taylor (15) have been particularly helpful in developing an understanding of the axial jet in which mixing is three dimensional.

Albertson, Dai, Jensen and Rouse (2) and Bicknell (4) demonstrated satisfactory agreement between theory and observations with models in the case of the vertical plane jet where mixing takes place in only two dimensions, e.g., along a horizontal plane. There is a
difference between a plane jet and an axial jet. The difference in the deceleration of jet flow is demonstrated in Figure 4, which shows the experimental values found by Albertson et al. (2) and presented by Bates for the two types of flow. Further study of the diagram indicates that both types of jet flow pose three comparable zones of mixing. In the first of these zones, which is known as the zone of flow establishment, the core velocity is very nearly equal to the original outlet velocity. The second may be called the transition zone. In this zone the core velocity changes from a constant value to a standard rate of deceleration. The third zone may be termed the zone of established flow. In this zone, turbulent mixing takes place throughout the entire jet, and the cross-section velocity pattern resembles that of a typical probability curve, as can be seen in Figure 5.

The experimental results for surface velocity along jet center-line were compared with the experiments of Albertson et al. (2) (shown in Figure 2) which agreed reasonably. In this study we have adapted the Albertson results as an alternate approach to velocity determination. According to this work, the general form of velocity field in the zone of established flow may be stated:

$$\log_{10} \frac{V_x}{V_o} \frac{X}{D} = 0.79 - 33 \frac{Y^2}{X^2}$$  \hspace{1cm} (2)

in which:

$$V_x = \text{The longitudinal velocity in transition direction}$$
Figure 4. Center-Line Velocity in Plane Jet or Axial Jet.
Figure 5. Distribution of Forward Velocity in Plane or Axial Jet (After Bates (3)).
\[ V_o = \text{Initial discharge velocity} \]
\[ X = \text{Distance in the longitudinal direction} \]
\[ D = \text{Diameter of orifice} \]
\[ Y = \text{Depth at point} \]

In order to use this model for flow velocity in sedimentation calculations, eq. 2 may be solved by means of the computer program given in Appendix II.

**Discharge into Cross Flow**
*(Ambient Velocity \( \neq 0 \))*

Keffer and Baines (11) studied the flow of a vertical air-in-air jet directed normally to a uniform, steady ambient current for velocity ratios (jet/ambient velocity) of 2, 4, 6, 8 and 10. The integrated equations of continuity and motion along the deflected jet axis were made non-dimensional after the general method of Morton (13).

Entrainment was defined in terms of an inflow velocity, \( V_i \), which was assumed proportional to the difference between the centerline jet velocity, \( U_c \), and the ambient velocity, \( U_a \). The entrainment relation was written as

\[ V_i = E (U - U_a) \]  (3)

in which \( E \) was the entrainment coefficient.

The authors observed that the ambient flow as decelerated at the upstream surface of the jet, creating a positive pressure region, and
that separation occurred at the rear, creating a negative pressure region. They also observed that the velocity excess for the jet in an ambient current decreased much more rapidly than that reported by Albertson et al. (2) for a simple jet where the ambient velocity is zero, and that the rate of decrease increased with distance from the source.

The case of turbulent jet discharge into a cross flow is discussed by Abramovich (1). The literature on the subject is well reviewed by Fan (8). In all the cases considered the jet is deflected by two processes:

1. Entrainment of the lateral cross flow momentum. The resulting force per unit length on the jet in the direction of the cross flow is:

   \[ F_e = \rho V q_e \]  

   where \( V \) is the cross-flow velocity, \( \rho \) is the fluid density and \( q_e \) is the total entrainment per unit length of jet.

2. A net pressure force caused by eddying of the ambient fluid in the lee of the jet and by the distortion of the jet boundaries. This force per unit length is:

   \[ F_D = C_D \rho \frac{V^2}{2D} \]  

   where \( C_D \) is a drag coefficient and \( D \) is the jet diameter.

The effect of a cross flow on particle trajectory is considered in the present case of study. None of the theoretical models examined is
completely satisfactory. In the present study the particle trajectory was based on the following assumptions:

(1) Profiles of velocity normal to the jet axis are identical along the length of the jet axis and equal to the average of the actual velocities.

(2) The flow regime is completely turbulent.

(3) Jet longitudinal velocity field is expressed by Albertson's equation (eq. 2).

(4) Ambient flow is at $90^\circ$ to the initial jet axis.

(5) Particles are within the jet influence throughout the trajectory.

(6) The particle trajectories are the vectorial of longitudinal jet velocity and cross-flow velocity.

(7) The direction of the horizontal longitudinal velocity at the beginning of a time increment is the same as that of the horizontal component of the velocity vector computed at the end of the previous time increment.

The solution of particle trajectories is illustrated by the scheme shown in Figure 6. In this solution it should be noted that particle settling velocity is of major importance, since the particle traveling time is determined by settling velocity. For this study settling velocities computed from Stoke's Law were used.
\[ \Delta X_i = (V_i \cos B_i)(\Delta t) \]
\[ \Delta Z_i = (V_i \sin B_i)(\Delta t) \]

\[ X = \sum_{i=1}^{n} \Delta X_i \]
\[ Z = \sum_{i=1}^{n} \Delta Z_i \]

WHERE:

- \( V_i \) (i=1,n) = HORIZONTAL VELOCITY ALONG DEFLECTED JET
- \( B_i \) (i=1,n) = ANGLE OF DEFLECTION FROM INITIAL JET AXIS
- \( V_{xi}^* \) (i=1,n) = HORIZONTAL VELOCITY AT START OF TIME INCREMENT
- \( V_c \) = AVERAGE CROSS FLOW VELOCITY
- \( X \) = DISTANCE IN LONGITUDINAL DIRECTION
- \( Z \) = DISTANCE IN HORIZONTAL DIRECTION

Figure 6. Calculation of Deflected Jet Trajectory.
III. EXPERIMENTAL SET-UP AND PROCEDURES

Experimental Set-Up

The experiments described herein were performed in a 28.5 ft by 18.5 ft by 2 ft deep basin in Graf Hall at Oregon State University, Corvallis, Oregon. The water depth in the basin was maintained at 1.4 ft throughout the experiments. Figure 7 is a photograph of the experimental set-up.

A well-graded sand was used as the spoil material. It was stored in an 8 ft by 2 ft by 2 ft deep water tank (see Figure 7). The sand was discharged to the basin by means of a small jet pump submerged in the sand tank. An electric water pump with a capacity of 50 gallons per minute was connected to the jet pump to provide the necessary flow. The rapid movement of water rushing through the jet pump caused the sand to become suspended and discharge along with the water. The mixture of sand and water was pumped through the pipeline and then discharged into the basin. The discharge jet was parallel to the surface of the water approximately one jet diameter above the water surface. The jet diameter was 1.062 inch. The basin bottom was laid out in a Cartesian coordinate grid system with the X-axis along the direction of discharge and the Z-axis horizontal and the Y-axis vertical. Three-inch diameter aluminum cans one inch
Figure 7. Experimental Set-Up--Hydraulic Dredge Spoil Fate Physical Model.
deep were used to collect the sediment at the grid coordinate points. The grid system was shown in Figure 8. The bottoms of the cans were filled with lead to a depth of about 1/4 inch in order to keep them in place on the floor of the basin.

To maintain a constant depth in the basin, all water pumped by the electric pump came out of the basin and was consequently discharged back into the basin so that it was a closed system. It was assumed that the recirculation currents resulting had no effect on the results.

The laboratory experiments were divided into two parts; the first part consisted of discharging into a quiescent basin; the second part consisted of discharging into a cross flow. The apparatus for producing the cross flow was a 4-ft long manifold of 1.5 inch diameter galvanized pipe, with discharge nozzles. A 1.5 inch to 0.5 inch adapter was installed at each discharge point. The water used in the manifold was pumped out of the basin and consequently discharged back into the basin through the manifold so that the basin was a closed system. The current was regulated by a gate valve on the discharge side of the pump. Two plywood sheets were set vertically along each side of the cross flow to simulate a channel. Figure 9 shows the apparatus for simulating the cross flow. The sand and water mixture was discharged at right angles to the cross flow and the particles were collected in the sample cans as in the previous experiments.
Figure 8. Cartesian Coordinate Grid System of Basin Floor.
Figure 9. Apparatus for Simulating the Cross-Flow System.
Experimental Procedures

Velocity measurements in the basin were taken prior to both the first and the second set of experiments. The measurements were obtained with a Neyrpic precise velocity meter. The unit consists of a propellant-type meter which ties to a timer-counter unit that records the number of revolutions of the propellor in a given period of time. The velocity in fps was determined from the calibration formula,

\[ V_{fps} = 0.00918 + 0.74572n \]

where \( n \) is the number of revolutions per second.

The basin velocities induced by the plunging jet in the first set of experiments were taken using a jet discharge velocity of 14.0 fps. All jet velocities were determined by using a 5 gallon bucket to gather the discharge in a known time period. The dimensions of the bucket were known and the water depth was measured in order to compute the volume of water discharged. Once the volume was known the discharge in cubic feet per second was calculated. The cross-sectional area of the jet was measured and then the jet velocity was calculated.

For the quiescent condition the induced velocities from the jet at several depths in front of the plunge point were measured at grid points on the coordinate system. This procedure resulted in a velocity profile for every coordinate point for a jet discharge velocity, \( V_o \), of 14 fps. Theoretically, for any jet discharge velocity the induced velocities
could then be calculated knowing the $V/V_o$ ($V$: induced velocities; $V_o$: discharge velocity) ratio for that point and depth in the basin.

After the spoiling experiments the sediment samples were removed from the basin and taken to an oven for drying at 100°C. The samples were analyzed using a 3 inch sieve shaker unit and U.S. Standard Sieves ranging from No. 4 to No. 200. Numbers 40, 50, 60, 70, 80, 100 and 200 were used for most samples away from the jet plunge point. The samples were shaken approximately 15 to 20 minutes and weighed using a capacity 311 grams beam balance manufactured by Ohaus Scale Corporation. The percentages of the total samples which passed each sieve were determined and the sieve analysis curve was drawn. The uniformity coefficient and the $D_{50}$, $D_{90}$, and $D_{10}$ sizes were all found from the resulting grain size distribution curves.
IV. EXPERIMENTAL RESULTS AND DISCUSSION

Quiescent Receiving Waterbody

For this investigation there were three experimental runs. In each run a different jet velocity was used. The discharge velocities were 8.4, 5.8 and 4.4 fps respectively. Figure 10 presents the sequence of jet dispersion in experimental model and Figure 11 shows a confirmation of dredge spoil accumulation in front of the jet mouth. In Figure 12 the experimental results for total weight distribution for the three cases are presented. This figure indicates the shape of the spoil mound which forms in front of the jet. The effect of greater jet velocity is to produce a more elongate mound. It should be noted that each experiment was terminated when the first collecting pan was full. Figures 13 and 14 illustrate variation of grain size distribution induced by discharge velocities. From these results the following features are noted:

1. For decreasing grain size, the particle spreading range increases.

2. Spreading does not occur to any marked degree close to the impact point. A fan-like shape is achieved if the hydraulic parameters and grain size are such that particles are carried well into the zone of established flow.
Figure 10. Jet Dispersion in Model Tests.

Figure 11. Sediment Accumulation in Model Tests.
Figure 12. Total Weight Distribution for Ambient Velocity = 0 Tests.
Figure 13. Particle Size Distribution - $D_{90}$ for Ambient Velocity = 0 Tests.
Figure 14. Particle Size Distribution - $D_{50}$ for Ambient Velocity = 0 Tests.
(3) The trajectories at the sides of the jet are shorter than those at the center, because velocities are more quickly reduced by moving normal to the jet centerline than by moving along it. The centerline distribution of particle size therefore represents a maximum displacement from the discharge point.

The grain size distribution of the sand used in the experiment is compared with the grain size distribution of the total weight of all samples collected along the jet centerline in Figure 15. These results indicate that the centerline is representative of original material in the holding basin. A summary of uniformity coefficient \( C_u \) data for tests at ambient velocity = 0 is presented in Table 1. Data for all points on the grid system in a series are presented in Appendix III. Careful study of Appendix III and Table 1 indicates that:

1. There is essentially uniform size at all data points except near impact point of jet.
2. There is a well defined mean \( C_u \).

Table 1. Summary of Grain Size Data Statistics for Ambient Velocity = 0.

<table>
<thead>
<tr>
<th>Velocity (fps)</th>
<th>Mean Uniformity Coefficient ( C_u ) ( (D_{60}/D_{10}) )</th>
<th>Standard Deviation (S)</th>
<th>Standard Error ( (S_m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.4</td>
<td>1.45</td>
<td>0.19</td>
<td>0.04</td>
</tr>
<tr>
<td>5.8</td>
<td>1.59</td>
<td>0.26</td>
<td>0.04</td>
</tr>
<tr>
<td>4.4</td>
<td>1.48</td>
<td>0.17</td>
<td>0.06</td>
</tr>
</tbody>
</table>
Figure 15. Grain Size Distribution Curves for Samples on Jet Centerline.
This illustrates that hydraulic sorting is efficient except at (near) jet impact point. Since grain size at a point is near unique even though the original material was well-graded, a theory based on hydraulic sorting should be viable.

Figures 16 through 18 show a general, though not exact, agreement between the experimental and computed centerline average particle size distribution. The computed jet centerline particle size distribution results show that the D$_{50}$ size curve measured approaches that computed using Albertson's velocity distribution, except near impact point. The largest departure from theory is thought to be due to inefficient hydraulic sorting. The D$_{50}$ size can therefore by considered representative of the particle size of a uniform sand. It should be noted that the computed results are based on the assumption that the impact point of the jet in the water coincides with the jet origin in the velocity calculations. When comparing the Albertson and measured velocities results, reasonable agreement is obtained. From a practical point of view the Albertson distribution is more applicable, since it can be expressed mathematically without the need for actual measurements.

**Discharge into Cross Flow**

In this part of the study three experiments were run using the same jet along with three ambient velocity conditions. The jet
### Table 1: Assumed Settling Velocities

<table>
<thead>
<tr>
<th>PARTICLE DIAMETER (MM)</th>
<th>ASSUMED SETTLING VELOCITY, ( V_s ) (FPS)</th>
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<td>4</td>
<td>1.460</td>
</tr>
<tr>
<td>3</td>
<td>1.278</td>
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### Figure 16

Experimental and Computed Results for Discharge Velocity - 8.4 fps and Ambient Velocity = 0.

---

*Figure 16. Experimental and Computed Results for Discharge Velocity - 8.4 fps and Ambient Velocity = 0.*
Figure 17. Experimental and Computed Results for Discharge Velocity = 5.8 fps and Ambient Velocity = 0.
Figure 18. Experimental and Computed Results for Discharge Velocity = 4.4 fps and Ambient Velocity = 0.
discharge velocity was 7.8 fps. Ambient velocities were designated maximum, medium and minimum respectively. For analysis they were specified as 0.271, 0.140 and 0.10 fps. These values were determined by the average of the velocities at 0.2 and 0.8 of the depth in the basin. Actual measurements showed that the experimental set-up did not produce uniform cross-flow conditions. The summary of grain size data for the three different cross-flow velocities is shown in Table 2. Uniformity coefficient $C_u$ data for all points on the grid system are shown in Appendix IV.

Table 2. Summary of Grain Size Data Statistics for Ambient Velocity ≠ 0.

<table>
<thead>
<tr>
<th>Average Cross Flow Velocity (fps)</th>
<th>Discharge Velocity (fps)</th>
<th>Mean Uniformity Coefficient $C_u (D_{60}/D_{10})$</th>
<th>Standard Deviation (S)</th>
<th>Standard Error (S_m)</th>
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</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>7.8</td>
<td>1.75</td>
<td>0.16</td>
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<td>Medium</td>
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<td>Minimum</td>
<td>7.8</td>
<td>1.67</td>
<td>0.06</td>
<td>0.02</td>
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From Table 2 and Appendix IV it can be seen that:

(1) There is a well defined mean $C_u$.

(2) Particle size at all data collection points is nearly uniform.

(3) Hydraulic sorting efficiency is good except near the jet impact point.

It is therefore apparent that a theory based on sorting by particle size should be workable in a cross flow as well as for the case of
ambient velocity = 0. This is to be expected since the same physical factors control both situations.

The comparisons of experimental and computed results for this case are shown in Figures 19, 20 and 21. The experimental results were obtained by plotting percent of total weight contours on grid coordinates for a given particle size range using the data collected from the containers. The actual coordinates were obtained by calculating the center of gravity of the largest percent contour line for the desired particle size range. From these figures it can be seen that there are considerable differences between experimental results and computed results. Boundary conditions in the experimental model were affected by back currents. Before experimental testing two plywood sheets were set vertically along each side of the cross-flow to simulate a channel (see Figure 9). The range of cross flow was observed to be about 4 ft in preliminary testing. When the mixtures of sand and water were discharged into the basin, however, the jet impacted on the plywood sheets and was deflected. The resulting currents created vortices, and as a result most of the sediment was deposited in a small area.

When comparing the computed and experimental results, the computed and measured positions appear to be parallel. The spreading distance is shorter than computed. According to Keffer and Baines' (10) study, the particle distribution measured should tend
Figure 19. Experimental and Computed Results for Discharge Velocity = 7.8 fps and Maximum Ambient Cross-Flow Velocity.
Figure 20. Experimental and Computed Results for Discharge Velocity - 7.8 fps and Medium Ambient Cross-Flow Velocity.
Figure 21. Experimental and Computed Results for Discharge Velocity = 7.8 fps and Minimum Ambient Cross-Flow Velocity.
toward that shown when compared to our figures due to deceleration of the jet by the cross flow. No method is available to compute how much. In addition, it is possible that the assumption of uniform cross-flow velocity and the recirculation currents mentioned above influenced the results.
V. CONCLUSIONS

Quiescent Receiving Waterbody

(1) It is an expected and observable fact that sand particles collected at a point where hydraulic sorting has acted will have uniform size, even though the sand from which they were derived may be well graded.

(2) There is poor hydraulic sorting near the jet impact point.

(3) If the velocity field in the receiving water can be determined accurately, particle trajectories can be reasonably computed.

(4) A simple theory based on flow profiles from the experiments of Albertson et al. (2) is workable.

Discharge into Cross Flow

(1) Hydraulic sorting efficiency is as effective in a cross flow as in quiescent water.

(2) Ambient cross flows with small velocities relative to the initial discharge velocity will deflect the jet centerline.

(3) A practical predictive model cannot be developed for spoil fate if the ambient velocity is complex. In only the simplest ambient flow patterns does the possibility for a workable solution become real. This solution must include the effects of deceleration of the jet by the cross flow.
BIBLIOGRAPHY


APPENDICES
APPENDIX I

THE DEVELOPMENT OF AN EQUATION FOR A MEASURED VELOCITY PROFILE ON JET CENTERLINE

Assume the surface velocity profile shown in Figure 2 is parabolic. The curve equation can be expressed as:

\[ \frac{V}{V_o} = C (11 - X)^2 \]

when \( X = 5; \) \( \frac{V}{V_o} = 0.074 = A \)

Therefore \( A = 36C \) and \( C = 0.00206 \).

The general form for surface velocity profile is, therefore,

\[ \frac{V}{V_o} = 0.00206 (11 - X)^2 \]

The general shape of the measured profile is approximated in Figure 3.

In Figure 3, assume that for \( 0.8 < Y < 1.4 \) the curve is parabolic. Further assume that \( Y \leq 0.8, \frac{V}{V_o} = 0.010 \). For the upper part of the profile then \( \frac{V}{V_o} = C_1 (0.8 - Y)^2 \). When \( Y = 1.4 \), then

\[ \frac{V}{V_o} = 0.00206 (11 - X)^2 \]

Therefore \( \frac{V}{V_o} = 0.00206 (0.36) \) and \( C_1 = 0.00572 (11 - X)^2 \).

\[ \frac{V}{V_o} = 0.00572 (11 - X)^2 (0.8 - Y)^2 \]

But if \( V_s \) is the particle settling velocity, \( Y = 1.4 - V_s t \).
So the equation of the velocity profile on jet centerline is:

\[ \frac{V}{V_0} = 0.00572 (11 - X^2) (V_s t - 0.6)^2 + 0.010 \]
APPENDIX II

COMPUTER PROGRAM FOR THEORETICAL CALCULATION OF PARTICLE TRAJECTORIES

The listing below is based on the Albertson et al. (2) results as a theoretical framework to predict particle trajectory in quiescent water. The governing equation is written in Fortran IV for use on a CDC 3300 computer.

The governing equations are:

\[
\log_{10} \frac{V_x}{V_o} \frac{X}{D} = 0.79 - 33 \frac{Y^2}{X^2}
\]

and

\[Y = 1.4 - V_s t\]

In the program the particle settling velocity is input. The particle traveling distance in a longitudinal direction equals the traveling time times the longitudinal forward velocity. The total traveling time is computed by dividing water depth by settling velocity. Longitudinal forward velocity, \(V_x\), is a function of \(Y\) where \(Y\) is the depth of a particle below the water surface.

The program reads the input data for theoretical calculations and sets up the initial conditions for each calculation.

<table>
<thead>
<tr>
<th>Input Data</th>
<th>Format</th>
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<tr>
<td>(V_0), (V_s), (H)</td>
<td>5F6.3</td>
</tr>
<tr>
<td>(D), (DELT)</td>
<td>13</td>
</tr>
</tbody>
</table>

Input Data

Format
where:

\[ V_0 \] = initial jet discharge velocity (fps)

\[ V_s \] = particle settling velocity (fps)

\[ H \] = water depth in the basin (ft)

\[ D \] = orifice diameter (ft)

\[ \text{DELT} \] = time increment (sec)

It should be pointed out that in the program VELIMP is the horizontal component of the impact point velocity obtained for each experiment and that \( V_x \) represents longitudinal forward velocity. For the quiescent receiving waterbody condition the calculations of particle trajectories were made using the computer.

For discharge into cross flow conditions calculations were made as follows:

1. For the first time increment obtain the longitudinal forward velocity at any axial distance along the jet, \( V_x^* \) from the computer solution for the governing equations for a given particle size. This is done as in the quiescent receiving waterbody situation. For that condition this velocity is designated \( V_x \).

2. Determine the average cross-flow velocity, \( V_c \).

3. Determine the vectorial sum of \( V_x^* \) and \( V_c \).

4. The vectorial sum of \( V_x^* \) and \( V_c \) is taken as the velocity vector during the time increment.
(5) Compute the horizontal traveling distance as the horizontal velocity times the traveling time during the time increment.

(6) Repeat steps 1 through 6 assuming the direction of travel at the beginning of any time increment is the same as the direction of travel at the end of the previous time increment.

The trajectory calculations for discharge into cross-flow condition were made manually, step by step, for each time increment.
PROGRAM

List
01  PROGRAM DREDGE
02  DIMENSION X(100), Y(100), VX(100), S(100)
03  REAL X, Y, VO, VS, H, D
04  VELIMP = C
05  X(1) = 0.
06  Y(1) = 0.
07  TM = 0.
08  REAL(60, 1) VO, VS, H, D, DELT, N
09  1 FORMAT (5F6.3, 13)
10  WRITE(61, 3)
11  3 FORMAT (2X, 'TIME', 20X, 'VX', 20X, 'S')
12  VSDELT = VS*DELT
13  DO 20 I = 1, N
14  IF(I.EQ.1) GO TO 31
15  VX(I) = (VO*D)/X(I)*(10.**((0.79-33.*(Y(I)**2.))/(X(I)**2.)))
16  GO TO 32
17  31 VX(I) = VELIMP
18  32 DX = X(I) + VX(I)*DELT
19  DY = Y(I) + VSDELT
20  J = I + 1
21  X(J) = DX
22  Y(J) = DY
23  T = H/VS
24  TM = TM + DELT
25  WRITE(61, 4) TM, VX(I), DX
26  4 FORMAT (/2X, 3(F10.5, 10X))
27  IF(TM.GE.T) GO TO 40
28  20 CONTINUE
29  40 WRITE(61, 41)
30  41 FORMAT (/2X, 'TRAVEL TIME', 20X, 'TRAVEL DIST')
31  WRITE(61, 6) T, DX
32  6 FORMAT (/4X, F6.3, 20X, F10.5)
33  END
APPENDIX III

UNIFORMITY COEFFICIENT DATA FOR THE QUIESCENT RECEIVING WATERBODY CONDITION

$V_o = 8.4$ fps and Ambient Velocity $= 0$

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Excluding * points:

$\Sigma C_u = 36.37$; $N = 25$; $\overline{C_u} = 1.45$

$S = 0.19$; $S_m = 0.04$
Appendix III. (Continued)

\[ V_o = 5.8 \text{ fps and Ambient Velocity} = 0 \]

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Excluding * points:

\[ \Sigma C_u = 57.34; N = 36; \overline{C_u} = 1.59 \]

\[ S = 0.26; S_m = 0.04 \]
Appendix III. (Continued)

\[ V_0 = 4.4 \text{ fps and Ambient Velocity} = 0 \]

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Excluding * points:

\[ \Sigma C_u = 13.29; \ N = 9, \bar{C_u} = 1.48 \]

\[ S = 0.17; \ S_m = 0.06 \]
APPENDIX IV

UNIFORMITY COEFFICIENT DATA FOR THE CROSS FLOW CONDITION

$V_o = 7.8$ fps and Maximum Ambient Velocity

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Excluding * points:

$\Sigma C_u = 31.52; \ N = 18; \overline{C_u} = 1.75$

$S = 0.16; \ S_m = 0.04$

$V_o = 7.8$ fps and Medium Ambient Velocity

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Excluding * points:

$\Sigma C_u = 23.80; \ N = 16; \overline{C_u} = 1.66$

$S = 0.169; \ S_m = 0.044$
Appendix IV. (Continued)

$V_o = 7.8$ fps and Minimum Ambient Velocity

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Excluding * points:

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$S = 0.06; \ S_m = 0.02$