In this thesis, characteristics of coupled trigonometric transmission lines are studied, based on the general theory of nonuniform transmission lines. The four-port transmission matrix parameters of the coupled trigonometric transmission lines are derived from the even and odd mode waves. Two applications of coupled trigonometric transmission lines are presented; one, the coupled trigonometric transmission line folded all-pass networks and the other, the coupled trigonometric transmission line directional couplers. The phase shift and magnitude characteristics of these networks are studied in detail for various sets of coupling factor variation along the line and the end points of the line. Finally, design examples for a wide-band 90° phase shifter and a high-pass minimum ripple directional coupler with trigonometric transmission lines are given and their physical realization is considered.
A STUDY ON COUPLED TRIGONOMETRIC TRANSMISSION LINES

by

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A STUDY ON COUPLED TRIGONOMETRIC
TRANSMISSION LINES

I. INTRODUCTION

Coupled uniform transmission lines have been studied for a long time, and widely used as various microwaves circuit components; such as filters, directional couplers, phase shifters, delay equalizers and hybrid circuits. Most of these components make use of multisection-coupled uniform lines in order to obtain the desired circuit performance over a certain frequency range. The disadvantages of a multisection-coupled line are the degrading discontinuity effect between adjacent coupling sections, and its relatively large physical size.

Recently, theory of coupled nonuniform transmission line (abbreviated as NUTL) has been developed [13] and applied to exponentially tapered coupled transmission lines. The results show the possibility of a wide variety of characteristics to be obtained with different shapings of the coupled line.

In this paper, the characteristics of coupled NUTLs for various types of coupling factors along the line will be investigated by using trigonometric coupled lines. Unlike the exponential line and most of other exactly solvable lines, the trigonometric line provides non-monotonic as well as monotonic shapping with a
proper choice of the end points of the line. The solution for the trigonometric line is exact and easily implemented for computer calculation.

In Chapter II, the general theory of coupled NUTL is presented. First, the differential equations of a coupled NUTL are transformed to the even and odd mode equations and then its various four-port matrix parameters are derived from the assumed general solutions of the transformed equations.

In Chapter III, two applications of coupled NUTLs are presented; one the general behavior of coupled NUTL folded all-pass network and the other the coupled NUTL directional couplers. The relevant expressions for the phase shift and the coupled voltage are derived in terms of the transmission parameters.

In Chapter IV, the solution and the four-port transmission matrix parameters of the coupled trigonometric line are derived. The phase characteristics of coupled trigonometric line folded all-pass networks and the coupled voltage magnitude characteristics of coupled trigonometric line directional couplers are investigated in detail for different sets of values of the end points of the line and the coupling factor variation along the line.

In Chapter V, further studies are done on the coupling effect of the trigonometric line on its characteristics by use of three basic types of coupling variation.
In Chapter VI, practical construction of coupled NUTLs from the given specifications is considered, and two design examples of coupled trigonometric NUTLs are given: one for the high-pass directional coupler with minimum ripple and the other for the wide-band 90° phase shifter.
II. FOUR-PORT REPRESENTATION OF COUPLED NONUNIFORM TRANSMISSION LINES

In this chapter, the basic theory of general coupled non-uniform transmission lines is presented [13]. First, the differential equations of a coupled NUTL are transformed to the even and odd mode equations and then its various four-port matrix parameters are derived from the assumed general solutions of the transformed equations.

Consider a general coupled NUTL composed of two identical conductors and having reflection symmetry to one another about a longitudinal axis, as depicted in Figure 1. The wave propagation along the two conductors can be described by the following set of differential equations:

\[ \begin{align*}
\text{line 1:} & \quad E_1 + 2 \frac{\partial}{\partial x} E_1 - \frac{\partial^2}{\partial x^2} E_1 = 0 \\
\text{line 2:} & \quad E_2 + 2 \frac{\partial}{\partial x} E_2 - \frac{\partial^2}{\partial x^2} E_2 = 0
\end{align*} \]

Figure 1. Coupled nonuniform transmission line.
\[
\begin{align*}
\frac{dE_1}{dx} + L(x) \frac{dI_1}{dt} + M(x) \frac{dI_2}{dt} &= 0 \\
\frac{dI_1}{dx} + C(x) \frac{dE_1}{dt} - Cm(x) \frac{dE_2}{dt} &= 0 \\
\frac{dE_2}{dx} + L(x) \frac{dI_2}{dt} + M(x) \frac{dI_1}{dt} &= 0 \\
\frac{dI_2}{dx} + C(x) \frac{dE_2}{dt} - Cm(x) \frac{dE_1}{dt} &= 0
\end{align*}
\] (1)

where

\[E_1, E_2\] = instantaneous voltages at the distance \(x\) on line 1 and 2

\[I_1, I_2\] = instantaneous currents at the distance \(x\) on line 1 and 2

\[L(x)\] = inductance of either line at the distance \(x\) when the other line is open circuit

\[M(x)\] = mutual inductance between the two lines at the distance \(x\)

\[C(x)\] = capacitance of either line at the distance \(x\) when the other line is short-circuited

\[Cm(x)\] = mutual capacitance between the two lines at the distance \(x\)
The analysis of coupled NUTL can be done most easily by the method of Reed and Wheeler [8], in which the even mode and odd mode voltages (currents) which are linearly related to the actual line voltages (currents) are introduced; that is:

\[
\begin{align*}
\text{Even mode voltage} & \quad v_e &= E_1 + E_2 \\
\text{Odd mode voltage} & \quad v_o &= E_1 - E_2 \\
\text{Even mode current} & \quad i_e &= I_1 + I_2 \\
\text{Odd mode current} & \quad i_o &= I_1 - I_2
\end{align*}
\]

Adding and subtracting equation (1) result:

\[
\begin{align*}
\frac{dv_e}{dx} + [L(x) + M(x)] \frac{di_e}{dt} &= 0 \\
\frac{di_e}{dx} + [C(x) - C_m(x)] \frac{dv_e}{dt} &= 0
\end{align*}
\]

\[
\begin{align*}
\frac{dv_o}{dx} + [L(x) - M(x)] \frac{di_o}{dt} &= 0 \\
\frac{di_o}{dx} + [C(x) + C_m(x)] \frac{dv_o}{dt} &= 0
\end{align*}
\]

Equation (3) and (4) describe the instantaneous even mode and odd mode voltages (currents) along the coupled NUTL, which are recognized as having exactly the same form as the familiar
uncoupled transmission line equations. The actual voltages and currents on the two lines are obtained from a linear combination of the solutions of equations (3) and (4).

Comparing equations (3) and (4) with the differential equation of an uncoupled line, we define the even and odd mode characteristic impedances, $Z_{oe}(x)$ and $Z_{oo}(x)$, as

$$Z_{oe}(x) = \sqrt{\frac{L(x) + M(x)}{C(x) - C_m(x)}}$$

$$Z_{oo}(x) = \sqrt{\frac{L(x) - M(x)}{C(x) + C_m(x)}}$$

And the even and odd mode propagation constants as

$$\beta_e(x) = \sqrt{\left[\frac{L(x) + M(x)}{C(x) - C_m(x)}\right] \left[\frac{C(x) - C_m(x)}{C(x) + C_m(x)}\right]}$$

$$\beta_o(x) = \sqrt{\left[\frac{L(x) - M(x)}{C(x) + C_m(x)}\right] \left[\frac{C(x) - C_m(x)}{C(x) + C_m(x)}\right]}$$

The even and odd mode propagation constants are equal and independent of the distance $x$ if the medium is homogeneous or if the following condition holds [7]:

$$\frac{M(x)}{L(x)} = \frac{C_m(x)}{C(x)}$$

Under this condition and for the steady-state sinusoidal excitation, equations (3) and (4) can be rewritten in terms of the characteristic impedance and the common propagation constant, as
where the voltages and currents are now phasor quantities.

Differentiation and substitution of equations (8) and (9) yield the linear second-order differential equations for the even and odd mode line voltages:

\[
\begin{align*}
\frac{d^2 v_e}{dx^2} & - \frac{Z_{oe}(x)'}{Z_{oe}(x)} \frac{dv_e}{dx} + \beta^2 v_e = 0 \\
\frac{d^2 v_o}{dx^2} & - \frac{Z_{oo}(x)'}{Z_{oo}(x)} \frac{dv_o}{dx} + \beta^2 v_o = 0
\end{align*}
\]  

which may be solved with \(Z_{oe}(x)\) and \(Z_{oo}(x)\) given by using the techniques available for the single nonuniform transmission lines.
Let $f(x)$ and $g(x)$ denote a pair of linearly independent solutions of (10) for the even mode case. Then the general solution is

$$v_e(x) = c_1 f(x) + c_2 g(x)$$

and

$$i_e(x) = -\frac{1}{j\beta Z_{oe}(x)} \{c_1 f'(x) + c_2 g'(x)\}$$

where the integral constants $c_1$ and $c_2$ are to be determined from the terminal conditions and the prime indicates differentiation with respect to $x$.

Now, we define the even-mode transmission matrix by the following relation.

$$
\begin{bmatrix}
v_e(0) \\
i_e(0)
\end{bmatrix}
= 
\begin{bmatrix}
A_e & B_e \\
C_e & D_e
\end{bmatrix}
\begin{bmatrix}
v_e(\ell) \\
i_e(\ell)
\end{bmatrix}
$$

where $\ell$ is the line length. Substituting from equations (11) and eliminating $c_1$ and $c_2$, we find

$$
\begin{bmatrix}
A_e & B_e \\
C_e & D_e
\end{bmatrix}
= \frac{1}{m_4}
\begin{bmatrix}
m_1 & j\beta Z_{oe}(\ell)m_6 \\
m_2/j\beta Z_{oe}(0) & m_5 Z_{oe}(\ell) Z_{oe}(0)
\end{bmatrix}
$$

where the $m_i$'s are
\[ m_1 = f'(0) g(0) - g'(0) f(0) \]
\[ m_2 = f'(0) g'(0) - g'(0) f'(0) \]
\[ m_3 = f'(0) g(0) - g'(0) f(0) \]
\[ m_4 = f'(0) g(0) - g'(0) f(0) \]
\[ m_5 = f'(0) g(0) - g'(0) f(0) \]
\[ m_6 = f(0) g(0) - g(0) f(0) \]

In the same manner, the even mode impedance and admittance matrices are

\[ [Z]_e = \frac{j\beta}{m_2} \begin{bmatrix} m_1 Z_{oe}(0) & m_3 Z_{oe}(\ell) \\ m_4 Z_{oe}(0) & m_5 Z_{oe}(\ell) \end{bmatrix} \] (15)

\[ [Y]_e = \frac{1}{j\beta m_6} \begin{bmatrix} -m_5/Z_{oe}(0) & m_3/Z_{oe}(0) \\ m_4/Z_{oe}(\ell) & -m_1/Z_{oe}(\ell) \end{bmatrix} \] (16)

The matrix parameters for the odd mode case can be found in the same manner as for the even mode case.

The actual four-port matrix parameters of a coupled NUTL will now be derived. They can be expressed in terms of the even-mode and odd-mode parameters. For example, from the following relations between the actual and mode quantities at the four ports.
where \(v_1, v_2, v_3, v_4\) and \(i_1, i_2, i_3, i_4\) are the actual voltages and currents at the four-port (See Figure 2), and equation (12) and a similar defining equation for the odd mode parameters, we obtain, after simple elimination procedures, the actual four-port transmission parameters as

\[
\begin{bmatrix}
  v_1 \\
v_2 \\
i_1 \\
i_2
\end{bmatrix}
= \frac{1}{2}
\begin{bmatrix}
  (A_e + A_o) & (A_e - A_o) & (B_e + B_o) & (B_e - B_o) \\
  (A_e - A_o) & (A_e + A_o) & (B_e - B_o) & (B_e + B_o) \\
  (C_e + C_o) & (C_e - C_o) & (D_e + D_o) & (D_e - D_o) \\
  (C_e - C_o) & (C_e + C_o) & (D_e - D_o) & (D_e + D_o)
\end{bmatrix}
\begin{bmatrix}
  v_4 \\
v_3 \\
i_3 \\
i_4
\end{bmatrix}
\]  \\
(18)

Figure 2. Coupled NUTL considered as four-port networks.
In the same manner the four-port impedance matrix is found to be

\[
\begin{bmatrix}
    v_1 \\
    v_2 \\
    v_3 \\
    v_4
\end{bmatrix} = \begin{bmatrix}
    (z_{11}^e + z_{11}^o) & (z_{11}^e - z_{11}^o) & (z_{12}^e - z_{12}^o) & (z_{12}^e + z_{12}^o) \\
    (z_{11}^e - z_{11}^o) & (z_{11}^e + z_{11}^o) & (z_{12}^e + z_{12}^o) & (z_{12}^e - z_{12}^o) \\
    (z_{21}^e - z_{21}^o) & (z_{21}^e + z_{21}^o) & (z_{22}^e + z_{22}^o) & (z_{22}^e - z_{22}^o) \\
    (z_{21}^e + z_{21}^o) & (z_{21}^e - z_{21}^o) & (z_{22}^e - z_{22}^o) & (z_{22}^e + z_{22}^o)
\end{bmatrix} \begin{bmatrix}
i_1 \\
i_2 \\
i_3 \\
i_4
\end{bmatrix}
\]

where \(z_{11}^e, z_{12}^e, z_{21}^e, z_{22}^e, z_{11}^o, z_{12}^o, z_{21}^o, z_{22}^o\) are the set of open-circuit impedance parameters for the even (odd) mode wave.

Likewise, the admittance matrix is

\[
\begin{bmatrix}
i_1 \\
i_2 \\
i_3 \\
i_4
\end{bmatrix} = \begin{bmatrix}
    (Y_{11}^o + Y_{11}^e) & (Y_{11}^o - Y_{11}^e) & (Y_{12}^o - Y_{12}^e) & (Y_{12}^o + Y_{12}^e) \\
    (Y_{11}^o - Y_{11}^e) & (Y_{11}^o + Y_{11}^e) & (Y_{12}^o + Y_{12}^e) & (Y_{12}^o - Y_{12}^e) \\
    (Y_{21}^o - Y_{21}^e) & (Y_{21}^o + Y_{21}^e) & (Y_{22}^o + Y_{22}^e) & (Y_{22}^o - Y_{22}^e) \\
    (Y_{21}^o + Y_{21}^e) & (Y_{21}^o - Y_{21}^e) & (Y_{22}^o - Y_{22}^e) & (Y_{22}^o + Y_{22}^e)
\end{bmatrix} \begin{bmatrix}
v_1 \\
v_2 \\
v_3 \\
v_4
\end{bmatrix}
\]

where \(Y_{11}^e, Y_{12}^e, Y_{21}^e, Y_{22}^e, Y_{11}^o, Y_{12}^o, Y_{21}^o, Y_{22}^o\) are the set of short-circuited admittance parameters for the even (odd) mode wave.
In the above, from the reciprocity condition

\[ A_{e} D_{e} - B_{e} C_{e} = 1, \quad A_{o} D_{o} - B_{o} C_{o} = 1 \]

\[ z_{12}^{e} = z_{21}^{e}, \quad z_{12}^{o} = z_{21}^{o} \]

\[ Y_{12}^{e} = Y_{21}^{e}, \quad Y_{12}^{o} = Y_{21}^{o} \]
III. PROPERTIES OF COUPLED NONUNIFORM TRANSMISSION LINES

Among the many applications of the coupled NUTLs which are obtained under special conditions on the characteristic impedances and terminating impedances, of particular interest are all-pass networks and directional couplers. In this chapter, the properties of coupled NUTL folded all-pass network and coupled NUTL directional couplers will be described and several formulae to be used in latter chapters will be derived.

Jones and Bollijohn [6] show that the condition for a uniform transmission line folded all-pass network to be matched at all frequencies, is that the input and output port are terminated by the same impedance $Z_o$ given by

$$Z_o = (Z_{oe} \cdot Z_{oo})^{1/2}$$  \hspace{1cm} (21)

where $Z_{oe}$ and $Z_{oo}$ are, respectively, the even and odd mode characteristic impedances of the coupled line. Equation (21) is also the condition for a pair of coupled uniform transmission lines to be a directional coupler.

Throughout our subsequent discussions of the coupled NUTLs, it will be assumed that the condition (21) is preserved along the entire line length, or
\[ Z_{\text{oe}}(x) \cdot Z_{\text{oo}}(x) = Z_0^2 = 1 \] (22)

where \( Z_{\text{oe}}(x) \) and \( Z_{\text{oo}}(x) \) are, respectively, the even and odd mode characteristic impedances normalized to the terminating impedance.

It can be proved [13] that, under the condition (22) the even and odd mode transmission matrix parameters are mutually dual; that is

\[ A_e = D_o, \quad B_e = C_o, \quad C_e = B_o, \quad D_e = A_o \]

The degrees of coupling between the two coupled lines is usually expressed by the coupling factor \( K(x) \) which is defined by

\[ K(x) = \frac{M(x)}{L(x)} = \frac{C_m(x)}{C(x)}, \quad 0 \leq K(x) \leq 1 \]

[See equation (7)]. It can be shown [6] that this quantity is related to the even and odd mode characteristic impedances as

\[ K(x) = \frac{Z_{\text{oe}}(x) - Z_{\text{oo}}(x)}{Z_{\text{oe}}(x) + Z_{\text{oo}}(x)} \] (23)

The realizability condition \( 0 \leq K(x) \leq 1 \) can be reexpressed in terms of the ratio of the two characteristic impedances:

\[ \rho(x) = \frac{Z_{\text{oe}}(x)}{Z_{\text{oo}}(x)} = Z_{\text{oe}}^2(x) = \frac{1}{Z_{\text{oo}}(x)} = \frac{1 + K(x)}{1 - K(x)} \geq 1 \] (24)
If the coupling is symmetrical with respect to the middle point of the line, or \( K(x) = K(\ell - x) \), then it is called symmetrical coupling; otherwise it is called asymmetrical coupling.

### 3.1 Coupled Nonuniform Transmission Line Folded All-Pass Network

The coupled NUTL folded all-pass network is shown in Figure 3, in which two ports at one end of the coupled line are short-circuited to each other. We find this network a useful application as a phase equalizer or a phase shifter in the UHF and microwave frequency ranges.

![Figure 3. Coupled NUTL folded all-pass network.](image)

The various matrix parameters of this network as a two-port can be obtained by substituting the boundary conditions \( i_4 = -i_3 \), \( v_4 = v_3 \) into Equations (19) - (21). Thus, for example, the two-port transmission matrix is found to be

\[
\begin{bmatrix}
  v_1 \\
  i_1
\end{bmatrix} =
\begin{bmatrix}
  A' & B' \\
  C' & D'
\end{bmatrix}
\begin{bmatrix}
  v_2 \\
  -i_1
\end{bmatrix} =
\begin{bmatrix}
  \frac{(A D - B C)}{e \Delta} & \frac{2 A B C}{e \Delta} \\
  \frac{2 C D}{e \Delta} & \frac{(A D - B D)}{e \Delta}
\end{bmatrix}
\begin{bmatrix}
  v_1 \\
  -i_1
\end{bmatrix}
\]

(25)
where

$$\Delta = (Ae Do + B e Ce)$$

The transmission characteristics of this network are most readily understood in terms of the image parameters. From equation (25) the image impedances, $Z_{I1}$ and $Z_{I2}$ are

$$Z_{I1} = \frac{A'B'}{C'D'} = \frac{A e Do}{B e Ce}$$

$$Z_{I2} = \sqrt{A'C'} = \frac{A e Do}{C e Ce}$$

(26)

Under the condition (22), $Z_{I1} = Z_{I2} = 1$. Thus, we see that an all-pass characteristic is obtained with image impedance terminations. The image transfer function $\Phi$ in this case is calculated as

$$\Phi = \cos^{-1} A' = 2 \tan^{-1} \frac{Ce}{j Ae}$$

(27)

which gives the phase shift of the output voltage (current) with respect to the input voltage (current) under the image termination.

3.2 Coupled Nonuniform Transmission Line Directional Couplers

Coupled uniform-line directional couplers are widely used as microwave circuit components, such as antenna power splitter, local oscillator injection devices and hybrid junctions.
Coupled NUTLs behave as a directional coupler when all of its four ports are terminated by the resistance 
\[ Z_o = \sqrt{Z_{oe}(x) \cdot Z_{oo}(x)} = \text{constant} \] as shown in Figure 4 (a).

The characteristics of this directional coupler can be determined by considering the single excitation \( V_i \) applied at port 1 (Figure 4 (a)) as the superposition of the two sets of excitations applied at port 1 and 2 as shown in Figure 4 (b) and (c), the first set giving rise to the odd mode wave and the second set, the even mode wave.

![Figure 4](image)

**Figure 4.** Coupled NUTL directional coupler with even and odd excitation.
Then, the amplitude and phase of signals emerging from the four-ports are given by

\[
\begin{align*}
V_1 &= \frac{V_i}{2} (\Gamma_e + \Gamma_o) \\
V_2 &= \frac{V_i}{2} (\Gamma_e - \Gamma_o) \\
V_3 &= \frac{V_i}{2} (T_e - T_o) \\
V_4 &= \frac{V_i}{2} (T_e + T_o)
\end{align*}
\]

where \( \Gamma_e, \Gamma_o \) = reflection coefficient for the even and odd modes,

\( T_e, T_o \) = transmission coefficient for the even and odd modes.

For a single line terminated by an impedance \( Z_o \) at both ends, the reflection coefficient at the input port and the transmission coefficient are given by

\[
\Gamma = \frac{Z_{in} - Z_o}{Z_{in} + Z_o}
\]

\[
T = \frac{Z_o}{(AZ_o + B) + Z_o (Z_o C + D)}
\]

with

\[
Z_{in} = \frac{AZ_o + B}{CZ_o + D}
\]
Analogously, in the system under consideration

\[
\Gamma_e = \frac{A_e + B_e - (C_e + D_e)}{A_e + B_e + (C_e + D_e)}, \quad \Gamma_o = \frac{A_o + B_o - (C_o + D_o)}{A_o + B_o + (C_o + D_o)}
\]

\[
T_e = \frac{1}{A_e + B_e + C_e + D_e}, \quad T_o = \frac{1}{A_o + B_o + C_o + D_o}
\]

Under the condition (22), we see that

\[
\Gamma_e = -\Gamma_o
\]

\[
T_e = T_o
\]

Hence,

\[
V_1 = 0
\]

\[
V_2 = V_i \Gamma_e = V_i \frac{A_e + B_e - (C_e + D_e)}{A_e + B_e + (C_e + D_e)} \tag{29}
\]

\[
V_3 = 0
\]

\[
V_4 = V_i T_e = V_i \frac{1}{A_e + B_e + C_e + D_e} \tag{30}
\]

This shows that under the condition (22) the network shown in Figure 4 (a) behaves as a directional coupler, that is, if a signal is applied to one port, no signal emerges from the diagonally opposite port while the coupled energy emerges from the other two ports.
IV. COUPLED TRIGONOMETRIC LINE
AND COMPUTATIONAL RESULTS

In Chapter III, we have considered the general properties of two useful coupled NUTL networks, namely the coupled NUTL folded all-pass network and the coupled NUTL directional coupler, without assuming any specific variation of the characteristic impedances except for the condition (22). However, the actual detailed frequency characteristics must be investigated with some specific lines.

Coupled exponential line have been studied in detail by Yamamoto [13]. Unlike the exponential line and most of other exact solvable lines, the trigonometric line [12], which will be employed in our study of the shaping effect of the coupled NUTLs, provides non-monotonic as well as monotonic variations of the characteristic impedance and hence, it is hoped that a wider variety of frequency characteristics can be obtained with this line.

Consider a coupled NUTL in which the even and odd mode characteristic impedances (Normalized) vary according to

\[
\begin{align*}
Z_{oe}(x) & = Z_{oe} \csc^2 \mu x \\
Z_{oo}(x) & = Z_{oo} \sin^2 \mu x
\end{align*}
\]

(31)
Case II

\[ Z_{oe}(x) = Z_{oe} \sin^2 \mu x \]  
\[ Z_{oo}(x) = Z_{oo} \csc^2 \mu x \]  

where \( \mu \) is a constant and \( Z_{oe} \) and \( Z_{oo} \) are the even and odd mode characteristic impedance levels of the line at \( \mu x = \pi/2 \), satisfying the following condition [see equation (22)]

\[ Z_{oe} \cdot Z_{oo} = 1 \]

The differential equation that governs the behavior of such a line is obtained by substituting equation (31) or (32) into equation (10); in Case I:

\[ \frac{d^2 v_e}{dx^2} + 2\mu \cot \mu x \frac{dv_e}{dx} + \beta^2 v_e = 0 \]  

which can be transformed to

\[ \frac{d^2 y}{dx^2} + b^2 y = 0 \]  

with

\[ y = v_e(x) \csc \mu x \]
and
\[ b^2 = \mu^2 + \beta^2 \]

Hence, the general solution of (34) is
\[ v_e(x) = K_1 \frac{\cos bx}{\sin \mu x} + K_2 \frac{\sin bx}{\sin \mu x} \quad (35) \]

and
\[ i_e(x) = -\frac{1}{j\beta Z_{oe}(x)} \left[ \frac{d}{dx} \left( \frac{\cos bx}{\sin \mu x} \right) + K_2 \frac{d}{dx} \left( \frac{\sin bx}{\sin \mu x} \right) \right] \quad (36) \]
\[ = -\frac{1}{j\beta Z_{oe}(x)} \left[ K_1 \frac{(-b \sin bx \sin \mu x + \mu \cos bx \cos \mu x)}{2 \sin \mu x} + K_2 \frac{(b \cos bx \sin \mu x - \mu \sin bx \cos \mu x)}{2 \sin \mu x} \right] \quad (37) \]

After some manipulations with equation (36), (37), (14) and (15), the even mode transmission parameters for the case of 
\[ Z_{oe}(x) = Z_{oe} \csc^2 \mu x \] are obtained as follows:

\[ A_e = \frac{1}{b \sin \mu x_1} \left[ b \sin \mu x_2 \cos b(x_2 - x_1) - \mu \cos \mu x_2 \sin b(x_2 - x_1) \right] \]
\[ B_e = j \frac{\beta \sin b(x_2 - x_1) \sin \mu x_2 Z_{oe}(x_2)}{b \sin \mu x_1} \]
\[
C_e = j \frac{1}{\beta \cdot Z_{oe}(x_1) \sin \mu x_1} \begin{bmatrix}
\mu^2 \sin b(x_2-x_1) \cos \mu(x_2-x_1) \\
\beta^2 \sin \mu x_1 \sin \mu x_2 \sin b(x_2-x_1) \\
- \beta \cos b(x_2-x_1) \sin \mu(x_2-x_1)
\end{bmatrix}
\]

\[
D_e = \frac{1}{\beta \cdot Z_{oe}(x_2) \sin \mu x_2} \begin{bmatrix}
\beta \sin \mu x_2 \cos b(x_2-x_1) + \beta \cos \mu x_1 \sin \mu(x_2-x_1)
\end{bmatrix}
\]

where \(x_1\) and \(x_2\) correspond to the beginning and ending points of the line. We note that \(A_e\) and \(D_e\) are real whereas \(B_e\) and \(C_e\) are purely imaginary.

For Case II, \(Z_{oe}(x) = \frac{1}{\beta} \sin^2 \mu x\), the transmission parameters are

\[
A_e' = \frac{1}{\beta \cdot Z_{oe}(x_2) \sin \mu x_2} \begin{bmatrix}
\beta \sin \mu x_1 \cos b(x_2-x_1) + \beta \cos \mu x_1 \sin b(x_2-x_1)
\end{bmatrix}
\]

\[
B_e' = j \frac{1}{\beta \cdot \beta \cdot Z_{oe}(x_1) \sin \mu x_1} \begin{bmatrix}
\mu^2 \sin b(x_2-x_1) \cos \mu(x_2-x_1) \\
\beta^2 \sin \mu x_1 \sin \mu x_2 \sin b(x_2-x_1) \\
- \beta \cos b(x_2-x_1) \sin \mu(x_2-x_1)
\end{bmatrix}
\]

\[
C_e' = j \frac{\beta \sin b(x_2-x_1) \sin \mu x_2 Z_{oe}(x_2)}{\beta \cdot Z_{oe}(x_1) \sin \mu x_1}
\]

\[
D_e' = \frac{1}{\beta \cdot Z_{oe}(x_2) \sin \mu x_2} \begin{bmatrix}
\beta \sin \mu x_2 \cos b(x_2-x_1) - \beta \cos \mu x_2 \sin b(x_2-x_1)
\end{bmatrix}
\]

(39)
Notice that

\[ A_e = D_e', \quad D_e = A_e', \quad B'_e/C'_e = Z_{oe}/Z_{oo} = B'_e/C_e \quad (40) \]

The variations of the even and odd mode characteristics impedances and the corresponding coupling factors are shown in Figure (5) for both cases.

\[ Z_{oe}(x) = Z_{oe} \csc^2 \mu x \]
\[ Z_{oo}(x) = Z_{oo} \sin^2 \mu x \]

\[ K(\theta) \]

\[ a) \quad Z_{oe}(x) = Z_{oe} \csc^2 \mu x \]
\[ Z_{oo}(x) = Z_{oo} \sin^2 \mu x \]

\[ b) \quad \text{Coupling factor variation corresponding to a)} \]

\[ c) \quad Z_{oe}(x) = Z_{oe} \sin^2 \mu x \]
\[ Z_{oo}(x) = Z_{oo} \csc^2 \mu x \]

\[ d) \quad \text{Coupling factor variation corresponding to c)} \]

Figure 5. Even and odd mode characteristic impedances of the coupled trigonometric transmission line.
4.1 Phase Shift Characteristics of the Coupled Trigonometric Lines Folded All-Pass Network.

The phase characteristic is the only quantity of interest in the all-pass network. The phase shift $\Phi$ through the coupled trigonometric line folded all-pass network is obtained by substituting $A_e$ and $C_e$ from equation (38) [or (39)] into equation (33) as

$$
\Phi = 2 \tan^{-1} \frac{C_e}{A_e}
$$

where $C_e = C_e/j = \text{real}.$

For various sets of the beginning point ($\theta_1 = \mu x_1$) and the ending point ($\theta_2 = \mu x_2$) of the line and the characteristic impedance ratio at the starting point [$\rho(x_1) = Z_{oe}(x_1)/Z_{oo}(x_1)$], the phase shift as a function of $\beta \ell (\omega \ell /v)$ is calculated with the computer and plotted in Figures 7 - 12.

Figure 7 shows the variations of the phase characteristic with $\rho(x)$ monotonically increasing and monotonically decreasing. For comparison, the case of a coupled uniform line is also shown. It is seen that monotonically increasing (decreasing) $\rho(x)$ tends to shift the phase characteristic to the right (left) with respect to the uniform line case, with the phase shift at $\beta \ell = \pi$, however, remaining the same.
Figure 8 shows how the phase characteristics for the monotonically decreasing $\rho(x)$ shift to the left continuously as $\theta_2$ increases.

Incidentally, this shifting property can be obtained by a cascade of two sections of coupled uniform lines with unequal coupling factors, as shown in Figure 6 [4].

![Figure 6](image)

**Figure 6.** Cascaded coupled uniform line folded all-pass network.

Figure 9 shows the variation of the phase characteristic for the monotonically decreasing $\rho(x)$ with different characteristic impedance ratio at the starting point of the line. It is noted that as this ratio $\rho(x_1)$ increases, the deviation from the linear phase characteristic is more pronounced, all curves, however, passing through the same point at $\beta l$ slightly less than $\pi/2$.

Figure 10 shows the case of asymmetrical non-monotonic variations of $\rho(x)$ along the line. It is seen that their phase characteristics are somewhat different from the case of monotonic variation of $\rho(x)$; in particular, the phase shift at $\beta l = \pi$ is larger (smaller) compared with the coupled uniform line. This larger phase shift means that the same phase shift can be obtained with a shorter line length.
Figure 11 and 12 show that the phase shift at $\beta l = \pi/2$ is the same as the uniform line for both concave and convex non-monotonic variations of $\rho(x)$ but the phase shifts at $\beta l = \pi$ are again different.
Figure 7. Phase shift characteristics of coupled trigonometric line folded all-pass network.
Figure 8. Phase shift characteristic of coupled trigonometric line folded all-pass network with different values of coupling factors.
Figure 9. Phase shift characteristic of coupled trigonometric line folded all-pass network with different values of $p(x_i)$. 

$$Z_{oe}(x) = Z_{oe} \csc^2 \theta$$
Figure 10. Comparisons of phase shift characteristics for two different shapes of coupling factor.
Figure 11. Phase shift characteristic of symmetrical coupling trigonometric line.
Figure 12. Phase shift characteristic of symmetrical coupling trigonometric line with different values of coupling factors.
4.2 Coupled Voltage Magnitude Characteristics of the Coupled Trigonometric Lines Directional Couplers

In the directional coupler only the magnitude of the coupled voltage is of main concern. In the system of Figure 4 (a), the coupled voltage appears only at port 2 and port 4. However, since lossless lines are terminated with identical impedances, the coupled voltage at port 4 is related to the coupled voltage at port 2 by

$$|V_4| = \sqrt{1 - |V_2|^2}$$

assuming a unity voltage source at port 1. Therefore, we will consider the frequency characteristic of the magnitude of the coupled voltage at port 2 only, which is from equation (29) (assuming a unity excitation voltage)

$$|V_2| = \sqrt{\left(\frac{A_e - D_e}{A_e + D_e}\right)^2 + \left(\frac{B_e - C_e}{B_e + C_e}\right)^2}$$

where $B_e = B_e/j = \text{real}$ and $C_e = C_e/j = \text{real}$.

The pertinent transmission parameters are given in equation (38) for Case I: $Z_{oe}(x) = Z_{oe} \csc^2 \mu x$ and in equation (39) for Case II: $Z_{oe}(x) = Z_{oe} \sin^2 \mu x$.

For various sets of the beginning and ending points of the line $(\theta_1, \theta_2)$ and the coupling factor at the beginning point $[K(x_1)]$,
$|V_2|$ as a function of frequency ($\beta \ell$) is computed for both cases and plotted in Figures 13-16.

Figure 13 shows the variation of the frequency characteristic of the coupled voltage for the coupling factor increasing monotonically along the line, starting with different values at the beginning of the line. It is seen that this directional coupler possesses a high pass characteristic, and at frequencies sufficiently high the coupled voltage approaches a constant value. It is also noted that the larger the coupling factor at the beginning of the line, the higher the mean value of coupled voltage, accompanied with a higher ripple voltage.

Figure 14 shows the frequency characteristic of $|V_2|$ at three different increasing rates of coupling along the line, all starting with zero coupling at the beginning of the line, which should give the smallest ripple voltage for the same $\theta_1$ and $\theta_2$ as shown in Figure 13. It is seen that the mean coupled voltage can be increased by increasing the rate of coupling factor along the line.

Incidentally, coupled uniform line directional coupler possesses band pass characteristic instead of high pass characteristic as shown in Figure 14. However, high pass characteristic can be obtained by cascading coupled uniform lines having unequal coupling factors as in the case of cascaded coupled uniform line folded all-pass networks.
The asymptotic mean coupled voltage in the pass band of the coupled trigonometric line directional coupler will now be derived. As the frequency increases, \[ b \ell = \sqrt{(\mu \ell)^2 + (\beta \ell)^2} \rightarrow \beta \ell \rightarrow \infty. \]

Hence, as \( \beta \rightarrow \infty \), the transmission parameters in equation (38)

\[
\begin{align*}
A_e & \rightarrow g \cdot \cos b \ell, \quad B_e \rightarrow g \cdot Z_{oe} (x_2) \sin b \ell \\
C_e & \rightarrow g \cdot \frac{1}{Z_{oe} (x_1)} \sin b \ell, \quad D_e \rightarrow \frac{1}{g} \cos b \ell
\end{align*}
\]

where

\[ g = \left( \frac{\sin \theta_2}{\sin \theta_1} \right) = \sqrt{\frac{Z_{oe} (x_1)}{Z_{oe} (x_2)}} \]

Therefore, from equations (29) the average value of \( |V_2|^2 \) becomes, as \( \beta \rightarrow \infty \),

\[
\text{Avg} \ |V_2|^2 \rightarrow \frac{\left( 1 - \frac{1}{2} \right)^2 + \left( Z_{oe} (x_2) - \frac{1}{Z_{oe} (x_1)} \right)^2}{\left( 1 + \frac{1}{2} \right)^2 + \left( Z_{oe} (x_2) + \frac{1}{Z_{oe} (x_1)} \right)^2} (42)
\]

For the line with \( Z_{oe} (x) = Z_{oe} \sin^2 \mu x \), \( Z_{oe} (x) \) in the above expression is replaced by \( Z_{oo} (x) = Z_{oo} \csc^2 \mu x \). These asymptotic expressions should explain the variation of the mean value level in Figures 13 and 14.

The frequency characteristic for the symmetrical coupling case are shown in Figure 15 and 16 for both concave and convex coupling variations along the line. Both cases show a band pass
characteristic as in the uniform coupled line case, with decreasing amplitudes, however, in the higher multiple bands.
Figure 13. Coupling characteristics of coupled trigonometric line directional coupler for various values of coupling factor at the beginning of the line.
Figure 14. Coupling characteristics of coupled trigonometric line directional coupler for various degree of coupling factor.

$$Z_{oe}(x) = Z_{oe} \sin^2 \theta \quad K(x) = 0$$
Figure 15. Coupled voltage characteristics of trigonometric line directional couplers with convex symmetric coupling.
$$Z_{oe}(x) = Z_{oe} \sin^2 \mu x$$

Figure 16. Coupled voltage characteristics of trigonometric line directional coupler with concave symmetric coupling.
V. COMPARISONS OF CHARACTERISTICS OF COUPLED NONUNIFORM TRANSMISSION LINES FOR THREE TYPES OF COUPLING VARIATION

In the previous chapter we have seen that both in the coupled NUTL folded all-pass network and directional coupler, asymmetrical coupling gives rise to frequency characteristics quite different from the uniform line case, whereas symmetrical coupling and uniform coupling shows similar characteristics except for minor differences. Therefore, we will investigate in this chapter the case of asymmetrical coupling in more detail.

Three basic types of coupling variation along the line are considered. They are, as shown in Figure 17:

1. Coupling factor increases (or decreases) with concave shape along the line.
2. Coupling factor increases (or decreases) linearly along the line.
3. Coupling factor increases (decreases) with convex shape along the line.

Figure 17. Three types of coupling factor variation.
All having the same coupling factor at the beginning point and at the ending point of the line.

In actual calculations, the end points of the line were chosen as follows:

Case of monotonic increasing

Type 1 variation: \( Z_{oe}(x) = Z_{oe} \sin^2 \mu x, \theta_1 = 90^\circ, \theta_2 = 135^\circ \)

Type 2 variation: \( Z_{oe}(x) = Z_{oe} \csc^2 \mu x, \theta_1 = 30^\circ, \theta_2 = 45^\circ \)

Type 3 variation: \( Z_{oe}(x) = Z_{oe} \csc^2 \mu x, \theta_1 = 45^\circ, \theta_2 = 90^\circ \)

Case of monotonic decreasing

Type 1 variation: \( Z_{oe}(x) = Z_{oe} \sin^2 \mu x, \theta_1 = 45^\circ, \theta_2 = 90^\circ \)

Type 2 variation: \( Z_{oe}(x) = Z_{oe} \csc^2 \mu x, \theta_1 = 135^\circ, \theta_2 = 150^\circ \)

Type 3 variation: \( Z_{oe}(x) = Z_{oe} \csc^2 \mu x, \theta_1 = 90^\circ, \theta_2 = 135^\circ \)

In case of coupled trigonometric line directional couplers, the coupling factor at the beginning point (or ending point) was chosen as zero for the case of monotonic increasing (or decreasing). Computational results are plotted in Figures 18 - 21.

Figures 18 and 19 are the phase characteristics of all-pass networks. Examination of these curves show that

1. All curves in one set have the same point of inflection.
2. This point of inflection occurs at $\beta l$ slightly less (greater) than $\pi/2$ for the monotonically increasing (decreasing) coupling variation, whereas it occurs exactly at $\beta l = \pi/2$ for the uniform coupling and the symmetrical coupling.

3. The slope at the point of inflection (which is proportional to the group delay time at the corresponding frequency) is maximum and increases in the order of type 3, 2, 1 variation or as the area $\int_{x_1}^{x_2} K(x) \, dx$ increases.

4. The phase characteristics for the case of increasing coupling are more linear compared with the opposite case.

Figures 20 and 21 show the coupled voltage $|V_2|$ of the directional coupler with three different coupling variations. Examination of these curves shows that

1. All curves have the same average value as the frequency increases.

2. The ripple increases in the order of type 3, 2, 1 variation or as the area $\int_{x_1}^{x_2} K(x) \, dx$ increases.

3. The ripple decreases as the frequency increases.

4. The same sets of curves are obtained in Figures 20 and 21. This indicates that inversion of the coupling variation $K(x)$ along the line of a NUTL directional coupler does not change the amplitude characteristics. That this is the actual case can
be proved by noting that inversion of $K(x)$ means inversion of the

NUTL and hence $A_e$ and $D_e$ are interchanged while $C_e$ and $D_e$

remain the same. From equation (29), we see that interchange

of $A_e$ and $D_e$ is immaterial to the value of $|V_2|$. 
Figure 18. Comparison of phase characteristics of coupled trigonometric line folded all-pass network for different types of monotonically decreasing coupling variations.
Figure 19. Comparison of phase characteristics of coupled trigonometric line folded all-pass network for different types of monotonically decreasing coupling variations.

\[ Z_{oe}(x) = Z_{oe} \csc^2 \mu x \]

- \( \theta_1 = 135^\circ \), \( \theta_2 = 135^\circ \)
- \( \theta_1 = 90^\circ \), \( \theta_2 = 135^\circ \)
- \( \theta_1 = 45^\circ \), \( \theta_2 = 90^\circ \)
Figure 20. Comparisons of high pass directional coupler characteristic for different types of monotonically increasing coupling variation.

\[ Z_{oe}(x) = Z_{oe} \csc \mu x \]

- \( \theta_1 = 135^\circ, \theta_2 = 150^\circ \)
- \( \theta_1 = 90^\circ, \theta_2 = 135^\circ \)
- \( \theta_1 = 45^\circ, \theta_2 = 90^\circ \)

\[ |V_2| \]

\[ \beta l \text{ in radian} \]
Figure 21. Comparisons of high pass directional coupler characteristic for different types of monotonically decreasing coupling variation.
VI. PRACTICAL CONSIDERATION
AND DESIGN EXAMPLE

So far we have not considered the physical geometry of the coupled NUTLs; in this chapter we will consider the problem of actual construction of the coupled NUTLs from the given specifications.

Since the NUTLs considered in this paper satisfies the condition
\[ Z_{oe}(x) \cdot Z_{oo}(x) = \text{constant} = Z_o^2 \]
and the coupling factor is related to the two mode characteristic impedances by
\[
K(x) = \frac{Z_{oe}(x) - Z_{oo}(x)}{Z_{oe}(x) + Z_{oo}(x)} = \frac{\rho(x) - 1}{\rho(x) + 1}
\]

Only one of the four quantities, \( Z_{oe}(x), Z_{oo}(x), K(x) \) and \( \rho(x) \) is sufficient to specify the coupled lines. For convenience, throughout this chapter, it will be assumed that the even mode characteristic impedance \( Z_{oe}(x) \) is specified and we will consider the problem of realizing
\[ Z_{oe}(x) = Z_{oo} \csc \mu x \] or \[ Z_{oe}(x) = Z_{oe} \sin^2 \mu x, \]
in the given range \( x_1 \leq x \leq x_2 \).

In Chapter 3, we have mentioned that the physical realizability requires \( 0 \leq K(x) \leq 1 \). Therefore,
\[
Z_{oe}(x) \geq Z_o > 0 \quad (x_1 \leq x \leq x_2)
\]

This is the only restriction for \( Z_{oe}(x) \).
Coupled NUTLs can be realized in the form of open wire line, coaxial line or stripline. However, in this thesis only the stripline coupler will be considered. Stripline couplers have the advantages of light weight, small size and facility of construction, compared with other configurations. Also, design formulae are available in the literature in the form of coupled uniform striplines, which can be applied to the design of coupled nonuniform lines.

The characteristic impedances $Z_{oe}$ and $Z_{oo}$ of infinitesimally thin coupled stripline have been calculated by Cohn [3] who presents the information in the form of nomograms. The approximate values of $Z_{oo}$ and $Z_{oe}$ for thick strips and strips printed on dielectric sheets are also given.

For a moderate degree of coupling, the coplanar structure, Figure 22 (a), is used [11]. The broadside coupled stripline pair, Figure 22 (b), permits a higher degree of coupling. The slit coupled stripline, Figure 22 (c) and (d) is another approach to the construction of the line. When zero coupling factor at a certain point on the line is desired, as in the high pass coupled NUTL directional coupler with minimum ripple, then the conventional structure cannot be used, since the two coupled lines must be infinitely apart. However, the use of slit-coupled configurations allow this condition to be realized by making the slit width zero.
In the following are given design examples of the nonuniform coupled strip line folded all-pass network and directional couplers.

6.1 Coupled Stripline All-Pass Network

Among the many applications of coupled NUTL folded all-pass networks, wide band 90° differential phase shifters will be described. Schiffman [10] used coupled uniform folded network to construct 90° wide-band differential phase shifters. The network configuration and phase characteristics are shown in Figure 23. The phase shift through the uniform line, $\phi_2$, is represented by the straight line of a slope $k$ through the origin, where $k$ is the ratio of the length of the
uniform line to that of the coupled lines. $\Phi_1$ is the phase shift through the coupled line. The output phase difference $\Delta \Phi = \Phi_2 - \Phi_1$, can be made approximately equal to $90^\circ$ for a desired frequency range.

![Diagram of phase shifter](image)

**Figure 23.** A $90^\circ$ differential phase shifter.

Lately, Cristal [4] has shown that a wider band $90^\circ$ differential phase shifter with minimum phase error can be obtained by cascading two or more coupled uniform lines with unequal coupling factors (see Figure 6).

Examining the phase characteristics of coupled trigonometric line all-pass networks shown in Figures 7 - 11 and 19 we note that curve of Figure 7 for $\rho(\ell) = 5$ is most linear in the widest frequency range. If the corresponding coupler replaces the uniform coupler
in the Schiffman configuration, we obtain the differential phase response as shown in Figure 24 which shows that the phase shift is $90^\circ \pm 5^\circ$ over a $2.8 : 1$ band width, as compared with Schiffman $90^\circ \pm 4.8$ over a $2.34 : 1$ band width. Hence, a wider band width is realized by using the coupled trigonometric line.

![Figure 24. Differential phase response of trigonometric coupled line 90° phase shifter.](image)

Since $K(x) \neq 0$ over the entire length for this network, the simpler structure, coplanar or broadside coupling configuration of Figure 22 (b) may be used. For the purpose of illustration, let us employ the latter configuration (Figure 25).

For curve of Figure 7, $p(x_2) = 5$, $Z_{oe} (x) = Z_{oe} \csc \mu x$ with $\theta_1 = \mu x_1 = 90^\circ$ and $\theta_2 = \mu x_2 = 135^\circ$. Assume the coupled line is to
be excited by a voltage generator with an internal impedance 100Ω.

Then \( Z_{oe}(x) Z_{oo}(x) = Z_o^2 = 100^2 \) and \( \rho(x_2) = Z_{oe}(x_2) / Z_{oo}(x_2) = \)

\( (Z_{oe}/Z_{oo}) \csc^4 \theta_2 = 5 \), from which \( Z_{oe} \) (normalized) = 1.118 Ω.

Thus, the required variations of the even mode characteristic impedance along the line are as follows

<table>
<thead>
<tr>
<th>μx</th>
<th>90°</th>
<th>105°</th>
<th>115°</th>
<th>120°</th>
<th>125°</th>
<th>130</th>
<th>135°</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_{oe}(x) )</td>
<td>111.8</td>
<td>120</td>
<td>137</td>
<td>150</td>
<td>168</td>
<td>190</td>
<td>223.6</td>
</tr>
</tbody>
</table>

Figure 25. Coupler cross-section of coupled coplanar stripline.

From Gunderson and Guida [5] the ratio of \( w/b \), with constant ratio of \( s/b \), is found as follows (see figure 25)

\( (s/b = 0.3) \)

<table>
<thead>
<tr>
<th>μx</th>
<th>90°</th>
<th>105°</th>
<th>115°</th>
<th>120°</th>
<th>125°</th>
<th>130°</th>
<th>135°</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w/b )</td>
<td>0.95</td>
<td>1.10</td>
<td>7.3</td>
<td>1.5</td>
<td>1.75</td>
<td>2.0</td>
<td>2.5</td>
</tr>
</tbody>
</table>

t is found from \( t = (b - s) / 2 \), which holds for zero thickness of the strip lines.
The final shape of the coupled stripline is shown below (Figure 26), where \( l \) is the length of the line which can be determined from the operating frequency range.

![Figure 26. Width of coupled stripline.](image)

6.2 Coupled Stripline Directional Coupler

It is desired to design a high-pass coupled trigonometric line directional coupler with the following specifications:

a. high frequency voltage gain = 0.1 (20 db), with minimum ripple,

b. 3 db cutoff frequency = 500 MC,

c. the coupler is to be used with a terminating impedance 100 \( \Omega \).
The design procedures are as follows:

1. Either monotonically increasing or decreasing coupling variation can be used. Let us choose the monotonically increasing coupling. Since minimum ripple is required, the coupling factor at the beginning of the line should be zero or \( K(x_1) = 0 \) or \( Z_{oe}(x_1) = Z_{oe} \csc^2 \mu x_1 = 1 \) (Normalized value). Now choose \( \theta_1 = \mu x_1 = 90^\circ \), then \( Z_{oe} = Z_{oo} = 1 \) (normalized value). From equation (42), with \( g = \sin \theta_2 = Z_{oe}(x_2) \),

\[
|V_2|^2 = (0.1)^2 = \frac{(g-1)^2 + g^4(g-1)^2}{(g+1)^2 + g^4(g+1)^2}
\]

from which \( \theta_2 = 115.2^\circ \).

2. \( |V_2| \) as a function of frequency (\( \beta \ell \)) is plotted in Figure 28, from which the length of the line can be determined by locating the low end 3 db cut-off frequency, which is found to be \( \beta \ell = 0.312 \pi \).

From \( 2\pi \times 500 \times 10^6 \times 1/3 \times 10^8 = 0.312 \pi \), the line length = 9.36 cm.

3. The shape of the coupled line is now determined. Since normalized \( Z_{oe} = 1 \) and \( Z_o = 100 \Omega \), the actual \( Z_{oe}(x) = 100 \csc^2 \mu x \).

Hence, the following table is obtained.

<table>
<thead>
<tr>
<th>( \mu x )</th>
<th>90°</th>
<th>100°</th>
<th>105°</th>
<th>110°</th>
<th>115°.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_{oe}(x) )</td>
<td>100</td>
<td>104</td>
<td>108</td>
<td>114</td>
<td>123</td>
</tr>
</tbody>
</table>
Since \( K(x_1) = 0 \), slit-coupled stripline is preferable. The configuration shown in Figure 27 will be employed.

![Cross-section of slit-coupled stripline.](image)

**Figure 27.** Cross-section of slit-coupled stripline.

From Yamamoto [14], with constant strip spacing \( t/b = 0.1 \).

The value of \( s/b \) and \( t/b \) are found as follows:

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>90°</th>
<th>100°</th>
<th>105°</th>
<th>110°</th>
<th>115°</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w/b )</td>
<td>.08</td>
<td>.085</td>
<td>.09</td>
<td>0.10</td>
<td>0.12</td>
</tr>
<tr>
<td>( s/b )</td>
<td>0</td>
<td>.03</td>
<td>.05</td>
<td>0.08</td>
<td>0.12</td>
</tr>
</tbody>
</table>

The final shape of slit-coupled stripline directional coupler is shown in Figure 29 with an assumed value of \( b \).

It is seen that the width of stripline in both cases is only slightly nonuniform. Therefore, the assumption of one dimensional current flow in the theoretical treatment is not violated severely and hence the experimental results are expected to be in good agreement with the given specifications.
Figure 28. Characteristic of a 20 db coupled trigonometric line directional coupler.

Figure 29. Top view of slit coupled stripline, for figure 28.
VII. CONCLUSIONS

The characteristic of the coupled trigonometric lines were investigated; in particular, the phase shift characteristics of the folded line all-pass network and the coupled voltage magnitude characteristic of the directional coupler were investigated in detail.

It has been found that both in the all-pass network and in the directional coupler, asymmetrical coupling gives rise to characteristics quite different from the uniform coupling case, whereas symmetrical coupling and uniform coupling show more or less similar characteristics.

The phase curves of all-pass networks tend to shift to the left (right) for monotonically decreasing (increasing) coupling. The point of inflection is dependent on the positions of both ends of the line but not on the coupling variation between them. The maximum slope of the phase curve which occurs at the point of inflection, tends to increase as the total coupling \[ \int_{x_1}^{x_2} K(x) \, dx \]
increases.

Asymmetrical coupling gives a high pass characteristic to the coupled NUTL directional coupler. The asymptotic value of the coupled voltage is dependent on the coupling factor at both ends. Minimum ripple voltage in the pass band is obtained in the case of zero coupling at either end of the line. The smaller the quantity
\[ \int_{x_1}^{x_2} K(x) \, dx, \] the smaller the ripple voltage in the pass band.

Design examples for a wide-band 90° phase shifter and a high-pass minimum ripple directional coupler with trigonometric lines were given.
BIBLIOGRAPHY


