Recently, fiber-reinforced plastics (FRP lamina) have been applied to glued-laminated (glulam) timber for the purpose of improving bending strength and stiffness. Initially, full length reinforcement using FRP lamina was developed. However, the cost of FRP lamina is a significant portion of the total cost of reinforced glulam. Therefore, it is advantageous to use reinforcement in high-moment areas of a beam. A glulam beam reinforced over less than the full length is referred to as "partially reinforced glulam."

The understanding of in-service FRP lamina-wood interactions is limited. While stress distributions in full-length reinforced beams have been studied, there is a lack of information regarding stress distributions at the end of reinforcement in partially reinforced glulam beams. Interfaces and joints in composites are known to be areas of
stress concentrations and failure initiation. The research conducted in this study investigates the stress distribution at the end of tensile FRP reinforcement experimentally and analytically.

Experimental analysis of stress distributions was performed on several partially reinforced glulam beams. Strain gage analysis was used to measure axial strain (along beam length) near the end of the FRP lamina. The analysis indicated that strain (or stress) just past the end of the FRP lamina is higher than elementary beam theory predicts.

Finite-element modeling was used to model partially reinforced glulam to investigate potential effects on stress components imposed by alternative geometries, loadings, and materials. Specifically, the effects on stress distribution due to FRP lamina thickness, FRP lamina stiffness, beam width, percent length of reinforcement, span-to-depth ratio and type of loading were investigated. A three-dimensional structural solid element was used to model wood and the FRP lamina in linear elastic analysis. Failure loads and mechanisms were beyond the scope of this thesis.

Most stress distributions were found to be singular at the end of reinforcement. In order to quantify the magnitude of each stress, average stress near the end of the FRP lamina was calculated.
The models suggest that FRP lamina thickness and stiffness have significant effects on the magnitude of stress components near the end of the FRP lamina.
Experimental and Finite-Element Analysis of Stress Distributions Near the End of Reinforcement in Partially Reinforced Glulam

by

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I. INTRODUCTION

The glue-laminated timber (glulam) industry produces beams, columns and beam-columns for use in residential and commercial structures. In 1993, the glulam industry used 260 million board feet of lumber (APA, 1995). The Tacoma Dome, Tacoma, Washington, spanning more than 500 ft, is one example of the many structures glulam is be used for (FPL, 1987).

Glulam timbers are structural members composed of two or more layers of structural lumber glued together. The laminations are typically one or two-inch nominal thickness and may be of various species and grades. Glulam beams are engineered wood products designed primarily to resist bending. In horizontally laminated glulam, high-quality laminations are placed on the tensile and compressive sides of the beam, providing stiffness and strength where it is needed most.

The advantages of glulam timbers are numerous. The main drive behind glulam is that solid timbers are more variable, limited in size, and increasingly difficult to obtain. Wood properties are naturally variable due to
differences in density, grain orientation, species, and growth features such as knots. Glulam lay-up allows for these properties to be redistributed as smaller and discontinuous defects. For example, a large knot that spans the cross-section of a timber is reduced to a knot that is no larger than the cross section of a single lamination.

Glulam can be manufactured to virtually any depth or length, limited only by manufacturing press size. Laminations can be manufactured to any length using finger-joints, which allow two pieces of lumber to be joined end-to-end. Glulam cross-sections vary from 2-1/2 x 6 in. to 10-3/4 x 81 in, while timber cross-sections range from 5 x 5 in. to 24 x 24 in. (AF&PA, 1991). The glulam manufacturing process also allows for production of curved, tapered, or cambered members.

The advantages of glulam are clear. Because of the advantages glulam offers, and the growing export market, glulam production is expected to increase dramatically over the next several years (APA, 1995). The 1995 APA report indicates total glulam production for all market segments will grow from 280 million board feet in 1995 to more than 400 million board feet by the year 2000.

Clearly, any innovation will be beneficial if it decreases costs, reduces fiber demand, and minimizes grade requirements. The objectives of glulam reinforcement
include reduction of variability, improved strength and stiffness, and reduced cost.

Glulam reinforcement facilitates size reduction as compared to conventional glulam. Production of a glulam member that is smaller in size and lighter in weight decreases shipping costs, member dead weight, preservative treatment costs, and the burden on our valuable timber resources. Furthermore, decreased cost and size allows glulam to become more competitive with steel and concrete.

**Justification of Research**

This research project involves evaluation of stress distributions near the ends of reinforcement in glulam that are partially reinforced. *Partially reinforced glulam* refers to a glulam beam that is reinforced using a fiber-reinforced plastic (FRP lamina) over less than its total length. The reinforcement can be used as tensile reinforcement when placed on the tensile side of a member or as compressive reinforcement when placed on the compressive side of a member. Reinforcement is placed either on the outer surface of the beam (top or bottom), or it is protected by one lamination of wood. A diagram of glulam reinforced partially with tensile and compressive reinforcement and having cover laminations is shown in Figure 1.
Figure 1. Partially reinforced glulam beam with a cover (bumper) lamination.
Partially reinforced glulam has been approved by ICBO Evaluation Service, Inc. and is utilized in several structures including the Lighthouse Bridge in Port Angeles, Washington (Gilham, 1995).

In composite materials, the proper evaluation of edge and end effects is crucial to reliability in service. It is known that in composite materials intralaminar ends can be the location of significant stress concentrations. Furthermore, the degree of anisotropy contributes to the extent of localized stress effects influencing the magnitude and dissipation of stress.

The FRP reinforcement and the wood in partially reinforced glulam have significantly different stiffness properties in the longitudinal direction. The intralaminar terminus of the FRP lamina represents a potential site for initiation of delamination. The structural safety and reliability of partially reinforced glulam is compromised if the stress distributions in this critical area is not fully understood.
Objectives

The objectives of the research include experimental and analytical stress analysis to identify the effects of FRP lamina on stress distributions at the end of the FRP lamina. Strain gage analysis is used to measure the strain distribution near the end of the FRP lamina in seven full-size glulam beams. The strain distribution from gages will be compared to beam theory.

Finite-element analysis will be used to predict the stress distribution at the end of the FRP lamina. A valid finite-element model (FEM) requires proper characterization of the structural components used in the model. In this case, solid wood, adhesive and FRP lamina are characterized by their respective material properties for use in the finite-element models.

These methods will be used in concert to study the stress distributions at the intralaminar FRP terminus. The stresses at this location in the bending member are thought to be influenced by several critical material and geometric features. The objective of this thesis is to examine the impact of potentially influential parameters - reinforcement thickness, reinforcement stiffness, beam width, span-to-depth ratio and loading conditions.
II. LITERATURE REVIEW

**Composite Beam Theory**

Since a glulam beam is a laminated composite, laminated composite theory may be used to predict associated bending properties. With the assumption that plane sections remain plain during bending, general beam theory may be used to analyze composite beams. While an isotropic beam has uniform properties throughout the beam, a composite beam may have nonuniform properties through the beam depth and length. The theory presented in this section illustrates the potential usefulness of reinforcement in composite members.

The beam cross section in Figure 2 is a typical cross section used with FRP lamina-reinforced glulam. This cross section will be used to show by example, composite beam theory and the benefits of using reinforcement.

![Figure 2. Typical cross section of a FRP lamina reinforced glulam beam.](image)
In order for a beam to be in static equilibrium, the net force due to tensile stress below the neutral axis must balance the net force due to compressive stress above the neutral axis. If material 2 has a higher stiffness \((E_2)\) than material 1, the neutral axis will be lower than the geometric centroid of the cross section.

One way to locate the neutral axis is by transformed section analysis. The neutral axis is calculated based on a transformed section where the area of each material is multiplied by the modular ratio of that material compared to the base material. In this case the cross sectional area of the base material, material 1, is multiplied by the modular ratio \(E_1/E_1\), and the cross sectional area of material 2 is multiplied by the modular ratio \(E_2/E_1\). The neutral axis location is calculated from equation (1).

\[
\bar{y} = \frac{\sum_{i=1}^{n} y_i A_i}{\sum_{i=1}^{n} A_i} = \frac{y_1 A_1 + y_2 A_2}{A_1 + A_2}
\]

where

- \(n\) = number of areas
- \(y_i\) = distance from the base of the cross section to the area centroid of the \(i\)th material
- \(A_1\) = cross sectional area of material 1
- \(A_2\) = transformed cross sectional area of material 2
Moments of inertia about the neutral axis are calculated from equation (2).

\[ I_i = \frac{w_i h_i^3}{12} + w_i h_i d_i^2 \]  

(2)

- \( h_i \) = actual height of the \( i \)th material
- \( w_i \) = actual width of the \( i \)th material
- \( d_i \) = distance from the area centroid of the \( i \)th material to the neutral axis of the cross section

Axial stress in each material is calculated by equation (3).

\[ \sigma_{max} = \frac{M y E_i}{E_i I_1 + E_2 I_2} \]  

(3)

where

- \( M \) = moment
- \( y \) = distance from the neutral axis to a point on the material
- \( i \) = is either a 1 for a point on material 1 or a 2 for a point on material 2

Deflection at any point in the beam can be calculated using the unit-load method (Timoshenko, 1990), which takes into account the effect of shear deflection. This method equates internal virtual work to external work (deflection).

The unit-load equation for deflection is

\[ \Delta = \int \frac{M_i M_{\lambda} dx}{EI} + \int \frac{f_{\lambda} V_{\lambda} V_i dx}{GA} \]  

(4)
where

\[ M_u = \text{moment distribution due to a unit load acting at the point where deflection is sought, in the direction where deflection is sought} \]

\[ M_L = \text{actual moment distribution} \]

\[ EI = \text{flexural rigidity} \]

\[ f_s = \text{shape factor} \]

\[ V_u = \text{shear distribution due to a unit load} \]

\[ V_L = \text{shear distribution due to the actual load} \]

\[ G = \text{shear modulus} \]

\[ A = \text{cross sectional area} \]

The form factor for shear \( f_s \) is calculated as

\[ f_s = \frac{A}{I^2} \int_A \frac{Q^2}{W^2} dA \]  

(5)

where

\[ Q = \text{first moment of cross sectional area} \]

\[ w = \text{width of cross section} \]

\[ A = \text{cross sectional area} \]

\[ I = \text{moment of inertia for the cross-section} \]

For a rectangular section, \( f_s = 6/5 \) (Timoshenko, 1990). The reinforcement cross-section is a small fraction of the total cross-sectional area for reinforced beams used in this study, and therefore, \( f_s \) is assumed to be constant over the length of the beam.
Assuming that loading and beam properties are symmetric about mid-length, the solution to half of the beam is sought and doubled (work performed on the two halves is identical). An illustrative example uses the partially reinforced beam shown in Figure 3. For a partially reinforced beam with loads at third-points, the equation for deflection must be partitioned into three segments: 1) end support to the end of reinforcement, (length \( b \)) 2) end of reinforcement to load point, (length \( a-b \)) and 3) load to center of span (length \( L/2 -a \)). Expanding the integrals over the length results in equation 6.

\[
\Delta = 2 \left[ \int_0^b \frac{M_u M_L}{E I_1} dx + \int_0^b \frac{M_u M_L}{E I_1 + E I_2} dx + \int_a^b \frac{M_u M_L}{E I_1 + E I_2} dx + \int_0^a \frac{f_x V_u V_l}{G I_A_1} dx + \int_a^L \frac{f_x V_u V_l}{G I_A_1} dx \right] 
\]

where

\( b \) = distance from the support to the end of reinforcement

\( a \) = distance from the support to the nearest load point

Substituting moment and shear into equation (6) yields:

\[
\Delta = 2 \left[ \int_0^b \frac{(P x) x}{2 E I_1} dx + \int_0^b \frac{P x (x/2)}{2 E I_1 + E I_2} dx + \int_a^b \frac{P a (x/2)}{2 E I_1 + E I_2} dx + \int_0^a \frac{f_x (P)(x/2)}{2 G I_A_1} dx + \int_a^L \frac{f_x (0)(x/2)}{2 G I_A_2} dx \right] 
\]

Evaluation and simplification of equation (7) yields:

\[
\Delta = \frac{P b^3 E_2 I_2}{3 E_1 I_1 (E_1 I_1 + E_2 I_2)} + \frac{P a}{2} \left( \frac{(3I^2 - 4a^2)}{12(E_1 I_1 + E_2 I_2)} \right) + \frac{f_x Pa}{G A_1} 
\]
Figure 3. Description of load and reinforcement locations for deflection calculation.
The effects of adding a specific reinforcement are illustrated by adding a 0.14 in. FRP lamina having an MOE of $16.6 \times 10^6$ psi to the tensile face of the central 60% of a 5-1/8 x 12 in. x 21 ft wood beam. The MOE of the wood beam is $2.0 \times 10^6$ psi. Using equations 3 and 8 leads to the results in Table 1. Clearly, deflection and bending stress in the wood are substantially reduced by adding reinforcement.

Table 1. Comparison of adding reinforcement to the tension face of a beam.

<table>
<thead>
<tr>
<th></th>
<th>Load (lb)</th>
<th>Neutral Axis Location</th>
<th>Center Deflection (in.)</th>
<th>Maximum Tensile Stress in Wood (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unreinforced</td>
<td>14643</td>
<td>6 in. Above Base</td>
<td>2.92 (L/86)</td>
<td>5000</td>
</tr>
<tr>
<td>Reinforced</td>
<td>14643</td>
<td>5.60 in. Above Base</td>
<td>2.36 (L/106)</td>
<td>3582</td>
</tr>
<tr>
<td>Reduction (%)</td>
<td>6.67</td>
<td></td>
<td>19.2</td>
<td>28.4</td>
</tr>
</tbody>
</table>

Elementary beam theory can be used to predict deflections and stress in a composite beam. This theory shows the advantages of reinforcing wood or any other material with a material which is significantly stiffer. In a material such as wood, where tensile stress is often the limiting stress, tensile reinforcement allows greater moments on the same beam size.
Reinforcement of solid wood products was tested and patented as early as the 1920’s (Krueger, 1973). In the 1960’s, and 1970’s, metal plates, cables and rods were widely investigated as reinforcement in various reinforcing schemes. These efforts were generally directed toward increasing stiffness and strength of the section.

Reinforcement Using Metal Plates

Mark (1961, 1963) and Sliker (1962) investigated the use of aluminum plates for reinforcement. Reinforcement schemes included continuous reinforcement along the compressive and tensile faces of timber (Mark, 1961), vertically and horizontally laminated glulam with 1/16 in. aluminum plates between and on the outer faces of the laminations (Sliker, 1962), and a trapezoidal wood section with a trapezoidal aluminum casing with a basal aluminum flange (Mark, 1963). In all three cases, increased stiffness and strength were observed.

Increased stiffness and strength was also acquired by adding steel plates or sections (Stern and Kumar, 1973; Coleman and Hurst, 1974; Hoyle, 1975).

Stern and Kumar (1973) used 1/16-in. steel flitch plates between vertical laminations in one study, and a U-shaped
1/16-in. section which covered three faces (two interlaminar and one edge) of the central ply of a three-ply vertical beam in the second study. The beams were nailed together.

Coleman (1974) also used three laminations of wood, with steel plates between laminations in one case and two U-shaped sections surrounding the compressive and tensile zones of the central lamination in the second case. Coleman used the U-shaped reinforcement in the central fifty percent of moment members and steel plates in the high shear regions of a shear beam. Comparisons were made between wood only, reinforced and nailed, and reinforced and glue-nailed.

Hoyle (1975) investigated the Lindal “Steelam” beam - composed of two or more vertical wood joists with toothed steel plates between the joists in the tensile and compressive zones.

As with aluminum plate reinforcement, stiffness and strength was improved in all cases.

Reinforcement Using Steel Bars and Cables

Steel bars and cables were perhaps the most extensively studied sources of reinforcement in the past. They were also found to be the most promising of metal reinforcements.

Lantos (1970) used phenol-resorcinol formaldehyde adhesive to fix square or round steel rods placed between
the two outer laminations, along the full length of the beam in both the tensile and compressive zones.

Dziuba (1985) placed various amounts of steel rods in the tensile zone of glulam to determine the effect of percentage reinforcement on cross-sectional area of reinforcement.

Krueger and Sandberg (1974) used a woven steel wire / epoxy composite to reinforce the tensile side of glulam.


Bohannon (1962) prestressed the wood in the outer tension lamination of glulam using 3/8-in. steel strands that were held in tension between steel blocks on the ends of the beam. Prestressing the tensile zone caused the tensile zone to initially be in compression and ultimately experience significantly less tensile stress than a non-prestressed wood beam. Prestressing wood members follows the prestressing of concrete that has taken place since the 1800’s.

All of the reinforcement techniques using metal bars and cables were successful in increasing stiffness and strength. Gardner (1991) has patented a reinforcing system using high-strength deformed steel reinforcing bar made for concrete reinforcement. Gardner uses epoxy to fix the
reinforcing bar into pre-milled groves centered along the outer glueline of the tensile and compressive zones.

**Reinforcement Using Fiber Reinforced Plastics**

Perhaps the most promising material for reinforcement is fiber reinforced plastics (FRP lamina).

Early work in this area was conducted by Wangaard (1964) and Biblis (1965). Both Biblis and Wangaard used fiberglass-reinforced plastic strips (Scotchply™ 1002) on the tensile and compressive sides of solid wood samples and performed bending tests to evaluate theoretical analysis of wood-fiberglass beams.

Spaun (1981) investigated the use of E-glass, a low cost fiberglass with intermediate strength properties and a Young's modulus of $10.6 \times 10^6$ psi. Spaun fabricated a composite with a nominal 2 x 6 in. wood core, with two 3 ft pieces of wood finger-jointed together, covered by a fiberglass layers on both the tensile and compressive sides (0, 3.5 and 7% cross-sectional area), with a single 1/8-in. veneer lamination of E-glass as the outside layers.

Rowlands, et al (1986) examined ten adhesives (epoxies, resorcinol formaldehydes, phenol resorcinol formaldehydes, isocyanates and a phenol-formaldehyde) and several fiber reinforcements (unidirectional and cross-woven glass,
carbon, and Kevlar®). Rowlands, et al (1986) may have been the first investigators to produce and test internal reinforcement with carbon or Kevlar®.


American Laminators of Drain, Oregon, owns the patent rights to fiber-reinforced glulam. This product is termed FiRP™ glulam and is sold commercially. The product has been approved by the ICBO Evaluation Service, Inc., a subsidiary corporation of the International Conference of Building Materials (ICBO, 1995). The product approval includes the use of partial length reinforcement.

**Summary of Reinforcement**

Numerous successful reinforcement schemes have been developed over the past 35 years, and most technologies offered enhanced static strength and stiffness. None of the technologies has yet to be used on a wide-scale basis. The economic practicality and feasibility was not realized until recently. In fact, economic feasibility was not discussed in papers until Moulin, et al (1990), and
Tingley (1990). These results are in contrast to Van de Kuilen (1991), who concluded that "the cost of glass fiber reinforced beams is extensively higher than a timber beam with equivalent properties"... with a price difference of 2 to 2.5 times.

For cost purposes, it would be beneficial to partially reinforce beams instead of applying full length reinforcement. For many applications, the central portion of glulam beams is placed under the highest moment, therefore, the critical region for tensile stress, is also the central region. Then, materials and performance are optimized by placing the reinforcement where it is needed. Unfortunately, stress distributions around the tails of reinforcement may be complex and may complicate design with partial reinforcement.

**Stress Distributions**

**Partially Reinforced Glulam**

The literature search in the area of reinforced wood products produced no insight as to what happens to stress distributions near the end of reinforcement in partially reinforced members. The most significant information is in the area of bonded joints. In particular, single and double lap-joints appear to come closest to the problem at hand.
Additional insight may be obtained from literature pertaining to laminated composites with a broken lamina (Gupta, 1995) and from Saint-Venant end effects in composites. Although these solutions may provide insight to the problem, they by no means provide a suitable model for predicting stresses near the tail of reinforcement. The problem of the lap-joint and double lap-joint, and Saint-Venant effects will be discussed to provide insight to the solution, while a finite-element model will be developed to predict the stress distribution.

**Single and Double Lap-Joints**

Stress distributions around adhesive joints have been thoroughly studied for adherends of the same or similar materials, some have studied the case of dissimilar adherends (Cheng, et al, 1991). In addition, single and double lap joints are typically loaded in tension and studied for tensile loading.

The single lap-joint is a joint where two materials are overlapped and joined at the overlap. When tensile loads are applied to the adherends, the loading is not collinear; for this reason, a bending moment is also applied to the joint. This loading leads to a more complex stress state than would occur if the loading were collinear. The moment
causes a stress normal to the adherend surfaces (peel stress).

Adams and Wake (1984) present several closed-form analyses of this problem. These analyses clearly show maximum adhesive stresses near the end of the joint.

Numerical techniques (FEM) are used to model the single lap-joint as well. Finite-element analysis by Crocombe and Adams (1981) used two-dimensional linear analysis to show the stress distributions across the thickness of the adhesive. The analysis found that peel stress and shear stress in the adhesive increase significantly near the ends of the joint.

Chen, et al (1991) used two-dimensional elasticity theory, in conjunction with the variational principal of complementary energy to analyze the stress distribution in single lap-joints under tension. Their results show that shearing and normal stresses are higher in joints with non-identical adherends than in joints with identical adherends.

In addition to increased shearing and normal stresses at the ends of the joint, interlaminar free-edge stresses are higher at the edges of the lamination. Edge and end-effects likely combine at corners of the joint to cause the most critical stress state in the joint.

Three-dimensional elasticity theory predicts an interlaminar stress singularity at free edges in laminates
(Choo, 1990). In fact, near free edges, there exists a three-dimensional stress state which can lead to delamination (Chawla, 1987).

**Saint-Venant Effects**

Saint-Venant’s principle pertains to the distribution of stress in the neighborhood of stress concentrations, and the manner in which the effect of the stress concentration diminishes with increasing distance from the concentration. Horgan and Simmonds (1994) present characteristic decay lengths of stress in terms of geometric and material properties.

Localized stress effects in highly anisotropic materials extend over much greater distances than in isotropic materials (Horgan and Simmonds, 1994). Horgan (1982) found (theoretically) that the rate of stress decay in a fiber-reinforced strip is four times greater than that in a isotropic strip, a decay length of four times the strip width.
III. EXPERIMENTAL ANALYSIS

To investigate stress distributions at the end of the FRP lamina, experimental and analytical methods will be utilized. Experimental analysis includes full-scale testing of partially reinforced glulam with foil strain gages mounted internally and externally near the end of the FRP lamina. This analysis was limited to seven beams of various sizes and degree of FRP lamina.

**Beam Manufacturing and Description**

All beams were manufactured by American Laminators, Inc., Drain, Oregon. The manufacturing process for reinforced beams is nearly identical to that of conventional glulam. The FRP lamina is passed through the same glue spreader as the wood lamina, and uses the same press settings (time and pressure) as conventional glulam. Partially reinforced glulam does add a step to the process; the FRP lamina must be indexed to center along the beam length. In members having thick reinforcement wood spacers are used to complete the ends of the FRP lamina lamination. Beam size, lamination setup, and degree of reinforcement are given in Table 2 for beams tested.
Table 2. Beam size, lamination combination, and degree of reinforcement. All beams manufactured as AITC combination No. 5 – FiRP Glulam.

<table>
<thead>
<tr>
<th>Beam</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (in.)</td>
<td>42</td>
<td>42</td>
<td>42</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Width (in.)</td>
<td>8 3/4</td>
<td>8 3/4</td>
<td>8 3/4</td>
<td>2 1/2</td>
<td>6 3/4</td>
<td>5 1/8</td>
<td>5 1/8</td>
</tr>
<tr>
<td>Length (ft)</td>
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<td>53</td>
<td>53</td>
<td>21</td>
<td>21</td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>Lumber Grade and Species</td>
<td>L-1 Df</td>
<td>L-1 Df</td>
<td>L-1 Df</td>
<td>L-1 Df</td>
<td>L-1 Df</td>
<td>L-1 Df</td>
<td>L-1 Df</td>
</tr>
<tr>
<td>Lumber MOE (10^6 psi)</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Tensile FRP Thickness (in.)</td>
<td>0.75</td>
<td>1.05</td>
<td>1.05</td>
<td>0.14</td>
<td>0.07</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>Tensile FRP Length (%)</td>
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<td>60</td>
<td>60</td>
<td>60</td>
<td>40</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>Tensile FRP E (10^6 psi)</td>
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<td>11.6</td>
<td>11.6</td>
<td>11.6</td>
<td>11.6</td>
<td>11.6</td>
<td>11.6</td>
</tr>
<tr>
<td>Compressive FRP Thickness (in.)</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>0.07</td>
<td>0.14</td>
<td>0.07</td>
<td>N/A</td>
</tr>
<tr>
<td>Compressive FRP Length (%)</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>60</td>
<td>40</td>
<td>80</td>
<td>N/A</td>
</tr>
<tr>
<td>Compressive FRP E (psi)</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>16.6</td>
<td>16.6</td>
<td>16.6</td>
<td>N/A</td>
</tr>
<tr>
<td>Filler (in)</td>
<td>0.75</td>
<td>1.05</td>
<td>1.05</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Bumper</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Gage Location Relative to Interface Height</td>
<td>0 in.</td>
<td>0 in.</td>
<td>0 in.</td>
<td>0.75 in. above</td>
<td>0.75 in. above</td>
<td>0.75 in. above</td>
<td>0.75 in. above</td>
</tr>
</tbody>
</table>

a Df: Douglas-fir
b CSA: cross-sectional area.
c FRP: fiber reinforced plastic
Strain Gage Placement

Foil strain gages were bonded to the wood surface using an epoxy adhesive. Gages were either mounted on the sides of the beam on the lamination above the FRP lamina, or they were mounted internally (at the FRP lamina-Wood interface), on the lamination above the FRP lamina. Internal gages were mounted and wired prior to beam lay-up. Finger joints, knots and other defects were avoided in gage placement as these features lead to localized stress concentrations.

Gages were mounted on the wood surface in the region near the end of the FRP lamina, and aligned in the longitudinal direction of the laminations. On all beams, one row of gages was centered at the end of the FRP lamina, with one gage at the end of the FRP lamina and one gage every 6 in. from the end of the FRP lamina in both directions extending for 6-ft end-to-end. On some beams, an additional gage was placed at 3 in. past the FRP lamina. A photo of mounted gages is shown in Figure 4. The number of gages used on each beam was limited because only twenty channels of signal conditioning were available. Seven of the twenty channels were used for deflection transducers and other strain measurements during all tests. More strain gages were used on some beams since multiple cycle testing was performed, i.e. the beam was partially loaded with one
Figure 4. Strain gages mounted near the end of the FRP lamina on a partially reinforced glulam beam.
set of gages being monitored and then reloaded to failure when the second set of gages was monitored.

**Strain Measurements**

The strain gages had a 1.00-in. gage length and 0.25-in. gage width (JP Technologies, type PA60-1000BA-120). The resistance rating was 120 ohms ($\Omega$). This large gage size was used because wood is a nonhomogeneous material. The transverse direction is especially nonhomogenous due to density variation across growth rings.

Each gage was used in an active quarter-bridge configuration. A compensating gage mounted on a Lucite® block completed the half bridge. Precision resistors making up the remaining half bridge were provided by a Vishay 2100 Strain Gage Signal Conditioner. The signal conditioner provided a two-volt excitation and a gain of 500. The output of the signal conditioner was connected to a Rockland Model 432 filter (unity gain 1-Hz fourth-order Butterworth low-pass filter). The filter minimized signal noise induced from a variety of sources, such as strain gage lead wires. Unshielded strain gage wires were brought out of the beam on the same side and kept short (less than 3 ft). These were connected to a shielded multi-conductor cable which ran from the beam to the signal conditioner. A block-diagram of the strain measurement setup is shown in Figure 5.
Figure 5. Block-diagram of the strain measurement setup.
Experimental Results and Discussion

Figures 6 through 12 show the strain distributions for gages located either at the wood-FRP lamina interface or 0.75 in. above the FRP lamina for locations 36 in. before the end of the FRP lamina to 36 in. beyond the end of the FRP lamina. Each figure shows strain distributions at three load levels. Theoretical strain levels are also plotted on each figure for a solid wood beam and for a beam with full length FRP lamina at the highest plotted load level. Theoretical strain is calculated by combining Hooke’s law \( \sigma = E \varepsilon \) and the flexure formula \( \sigma = \frac{My}{I} \), yielding \( \varepsilon = \frac{My}{EI} \), where \( M \) is moment, \( y \) is distance from the neutral axis, \( E \) is Young’s Modulus for wood and \( I \) is the moment of inertia for the cross section.

Several observations can be made about the stress distribution shown in Figure 6 for beam 1. First, the stress distribution toward the center of beam length from the end of the FRP lamina is in rough agreement with theoretical stress levels at this location for a fully reinforced beam. Stress levels beyond the end of the FRP lamina are close to theoretical stress levels for a nonreinforced beam. An important observation is the apparent lack of a stress perturbation at the end of the FRP
Figure 6. Stress distribution near the end of the FRP lamina in beam 1.
Figure 7. Stress distribution near the end of the FRP lamina in beam 2.
Figure 8. Stress distribution near the end of the FRP lamina in beam 3.
Figure 9. Stress distribution near the end of the FRP lamina in beam 4.
Figure 10. Stress distribution near the end of the FRP lamina in beam 5.
**Figure 11.** Stress distribution near the end of the FRP lamina in beam 6.
Figure 12. Stress distribution near the end of the FRP lamina in beam 7.
lamina. This result occurred because the closest working gages to the end of the FRP lamina are plus or minus 6-in., in fact the FRP-lamina end effects on stress were missed altogether.

The stress distribution for beam 2 is shown in Figure 7. The stress distribution prior to the end of the FRP lamina appears to be lower than predicted by theory, while the stress levels several inches beyond the end of the FRP lamina are in good agreement with theory. This data does capture a rise in strain between the gages located 0 to 6-in. past the end of the FRP lamina.

The stress distribution for beam 3 is shown in Figure 8. The stress distribution prior to the end of the FRP lamina is lower than predicted by theory, while the stress levels several inches past the end of the FRP lamina are very close to theoretical stress levels. A rise in stress is apparent between 0 and 6 in. from the end of the FRP lamina, with the greatest rise at the end of the FRP lamina. The rise in stress at 3 in. past the end of the FRP lamina is similar in magnitude to the rise found on beam 2 at the same location.

The stress distribution for beam 4 is shown in Figure 9. The stress levels several inches before and several inches beyond the end of the FRP lamina approach levels
predicted by theory. There is no apparent rise in stress shown on this graph.

The stress distribution for beam 5 is shown in Figure 10. The stress levels several inches before the end of the FRP lamina are very close to levels predicted by theory. Stress levels beyond the end of the FRP lamina are lower than predicted by theory; the lower stress levels beyond the end of the FRP lamina may be due to a support located 3 ft away. There appears to be a rise in stress at the gage located 6 in. beyond the end of the FRP lamina.

The stress distribution for beam 6 is shown in Figure 11. The stress levels several inches before and after the end of the FRP lamina are well in agreement with theory. A rise in stress is apparent at a location 3 in. beyond the end of the FRP lamina.

The stress distribution for beam 7 is shown in Figure 12. The stress distribution several inches before and after the end of the FRP lamina are close to stress levels predicted by theory. No stress is indicated at a location 18 in. beyond the end of the FRP lamina; this is due to a reaction support at this point. A small rise in stress may be indicated at 3 in. beyond the end of the FRP lamina.

Data for some of the beams failed to indicate a perturbation or "stress rise." This was due to a number of factors. At the time beams were being tested and gaged, the
location and distribution of the stress rise was unknown. Gage survival was not 100% due to handling, thus some gages did not function as was the case for the gage at the end of the FRP lamina in beam 1. The strain gages used for this analysis have a one-inch gage length and are 0.25 in. wide. Because strain gages essentially measure average strain over the area they are mounted, strain measurement at a point was not possible.

At the time beams were being tested, the magnitude and occurrence of a stress rise were unknown. Data from testing does indicate that a rise in stress occurs in the region near the end of the FRP lamina. This fact provided important information for partially reinforced glulam designers and manufacturers. For the designer, the fact that stresses are higher in wood at the end of the FRP lamina than the flexure formula predicts, make it prudent to be conservative on the design of this portion of the beam. For the manufacturer, knowledge of stress rise location, provided purpose to ensure that the end of the FRP lamina does not coincide with growth defects, fingerjoints or other potentially weak points in the wood.
IV. FINITE-ELEMENT ANALYSIS

Analysis was performed with ANSYS® (ANSYS, 1995) finite-element software. Material properties based on experimental values and theoretical relations were used to characterize the wood and FRP lamina components of the model. The adhesive layers presented a special problem, and were left out of the model.

Several models are developed for beams with a wooden bumper lamination and without the bumper lamination (bumper). A sensitivity analysis was conducted to investigate the effect of changing several model parameters.

Material Orientation

Material orientations for wood are usually referred to grain direction-longitudinal, radial and tangential. The longitudinal direction is defined as the direction parallel to the grain of the wood, i.e. along the length of a stem or piece of lumber. The radial direction is defined as the direction along the radius of a tree stem, or perpendicular to a growth ring. The tangential direction is defined as the direction tangent to the circumference of a tree stem, or tangent to a growth ring. In the models and material properties described later, the longitudinal direction will be referred to as the x-direction, while the radial and
tangential directions will be combined and referred to as the transverse direction or y and z-directions in the model. Figure 13 shows these directions relative to a stem cross section. In effect, the wood was reduced to a planar isotropic material. This material is described using Hooke's law by two $E$ values, two $G$ (shear modulus) values and two Poisson ratios.

![Diagram of material directions](image)

**Figure 13.** Visual comparison of material directions used in finite-element model and material directions commonly used for wood.

The FRP lamina was assumed to be unidirectional pultrusion of high modulus fibers embedded in a polymer matrix. As such, the material directions for the FRP lamina were defined in a manner similar to wood. The fiber direction of the laminate is defined as the $x$-direction. The depth of the laminate is defined as the $y$-direction, and the width of a laminate is defined as the $z$-direction.
Material Characterization for Finite-Element Analysis

Wood

Although wood has numerous and widely varying defects and variation in properties throughout its structure, no attempt was made to model this variability, the model is deterministic with regard to materials. In addition, no attempt was made to include finger joints in the model. This source of strength variability was left out because the focus was not on failure of the beam, but the influence of the partial length FRP lamina on stress distributions near the end of the FRP lamina.

The wood material of the finite-element model was assumed to be linearly elastic. This assumption was made since the region of the beam being studied was primarily in tension, and wood in tension typically behaves as a linear elastic material to the point of failure.

Material properties for Douglas-fir were from Bodig and Jayne (1993) and the Wood Handbook (FPL, 1987). In order to simplify the model, transverse isotropy was assumed. Then, the average of tangential and radial properties were used for $E_y$ and $E_z$. Transverse elasticity ($E_y$ and $E_z$) was calculated from the longitudinal elasticity ($E_x$):

$$E_y = E_z = 0.059 \cdot E_x \text{ (FPL, 1987).}$$

Poisson's ratios $\nu_{xy}$ and $\nu_{xz}$
were made equivalent given the assumption that the transverse elastic properties were equivalent. These Poisson ratios are averages of values given in Bodig and Jayne (1983) for the ratios of \( \nu_{LR}/E_L \) and \( \nu_{LT}/E_L \): \( \nu_{xy} = \nu_{xz} = 0.169 \times 10^{-6} (E_k)/0.338 \) for \( E_L = 2 \times 10^6 \) psi. The average transverse Poisson ratio of 0.41 for softwoods in Bodig and Jayne (1983) was used. Shear moduli from Bodig and Jayne (1983) were used, where the average of \( G_{xy} \) and \( G_{xz} \) (0.115 \times 10^6 psi) were used for both transverse moduli and the value of 0.012 \times 10^6 psi was used for \( G_{yz} \). A summary of material properties used is given in Table 3. Table 3 also shows material properties for an \( E_x \) of 2.0 \times 10^6 psi.

**Table 3.** Wood property calculations and properties used in the finite-element models.

<table>
<thead>
<tr>
<th>Property</th>
<th>Property Function</th>
<th>Numerical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_x )</td>
<td>Arbitrary</td>
<td>2 \times 10^6 psi</td>
</tr>
<tr>
<td>( E_y )</td>
<td>0.059(( E_x ))</td>
<td>0.118 \times 10^6 psi</td>
</tr>
<tr>
<td>( E_z )</td>
<td>0.059(( E_x ))</td>
<td>0.118 \times 10^6 psi</td>
</tr>
<tr>
<td>( \nu_{xy} )</td>
<td>0.169 \times 10^{-6} (E_k)</td>
<td>0.338</td>
</tr>
<tr>
<td>( \nu_{xz} )</td>
<td>0.169 \times 10^{-6} (E_k)</td>
<td>0.338</td>
</tr>
<tr>
<td>( \nu_{yz} )</td>
<td>(0.47+0.35)/2</td>
<td>0.41</td>
</tr>
<tr>
<td>( G_{xy} )</td>
<td>0.115 \times 10^6 psi</td>
<td>0.115 \times 10^6 psi</td>
</tr>
<tr>
<td>( G_{xz} )</td>
<td>0.115 \times 10^6 psi</td>
<td>0.115 \times 10^6 psi</td>
</tr>
<tr>
<td>( G_{yz} )</td>
<td>0.012 \times 10^6 psi</td>
<td>0.012 \times 10^6 psi</td>
</tr>
</tbody>
</table>
Phenol resorcinol is the adhesive commonly used in conventional and FRP lamina reinforced glulam (Gilham, 1995). The adhesive layer is a thin film having a thickness of 0.002 in. The adhesive is assumed to be isotropic with an MOE similar to that of wood. Since the adhesive layers were very thin and had elastic properties similar to wood, the adhesive layer was not included in the model. Omitting the adhesive layer simplified the finite-element model and facilitated modifications of beam size and the FRP lamina length. Furthermore, this leads to further homogenization of the wood material, which is an acceptable practice given the objectives.

Further justification for omission of the adhesive layer was performed by comparing a lap joint with an adhesive layer modeled using spring elements and an analogous model where the ANSYS® “GLUE” command was used. Spring elements act in a specified direction - x, y or z. Spring elements connect adjacent nodes of the wood and FRP lamina. Spring element properties are defined by a load-deflection table, which characterizes the elastic properties of the spring. This analysis indicated that the “GLUE” command provided the same results as the spring elements; thus, omission of the adhesive layer was further justified.
Fiber-Reinforced Plastic

Fiber and Matrix Materials

Fiber reinforced plastics used in reinforced glulam include fiberglass-aramid reinforced plastics (FARP), carbon-aramid reinforced plastics (CARP) and aramid reinforced plastics (ARP). Fiber properties and descriptions following come from the Composite Handbook of Reinforcements (Lee, 1993).

Fiberglass fibers are based on silica (SiO₂), which is mixed in molten form with other elements and extruded into glass fibers with diameters ranging from 5 to 15 μm in diameter. Glass fibers are generally grouped into E-glass and S-glass, with MOE's of 10 x 10⁶ psi and 12.5 x 10⁶ psi respectively.

Aramid fiber is composed of poly(p-phenylene terephthalamide), which is extruded from a molten form into fibers, having a diameter of approximately 1 x 10⁻⁶ in. Further working leads to a family of aramids with different strength and stiffness characteristics. A common stiffness value is E = 20.3 x 10⁶ psi.

Carbon fibers are produced from polyacrylonitril precursor (Lee, 1993). Carbon fibers are formed by spinning the polyacrylonitril into fiber form, oxidizing the fibers
at 392-572°F and carbonizing at 1800-4500°F. Typical carbon fiber diameters are around 8μm, and E-values range from 43 × 10⁶ psi for high strength (HS) and 75 × 10⁶ psi for high modulus (HM) fibers.

In FRP reinforcements, the high modulus fibers are held together with matrix material that protects and supports fibers and allows stress to be distributed between fibers (Jones, 1975). A variety of polymers can be used for the matrix.

**FRP Lamina Properties**

Fibers and matrix can be combined to produce a wide range of laminate properties. The FRP lamina for reinforced glulam has fibers that are 100% aligned with the length of the FRP lamina. Young’s modulus in the axial direction of the composite can be calculated theoretically using the rule of mixtures (Jones, 1975)

\[ E_a = E_{f_i} V_{f_i} + E_m V_m \]  

(9)

where

- \( i \) = index for fiber number when more than one fiber is used
- \( E \) = Young’s modulus
- \( V \) = volume fraction and the subscripts
According to Jones (1975), Young's modulus in the transverse direction can be calculated from the equation

\[ E_t = \frac{E_f E_m}{V_m E_f + V_f E_m} \]  

where the variables are as defined above. Poisson's ratio is also calculated from the rule of mixtures equation. The in-plane shear modulus is calculated the same way as transverse Young's modulus. Tingley and Leichti (1994) describe reinforced glulam having an FRP lamina with a 65%/35% fiber/polymer ratio.

Mechanical properties of FRP lamina can be determined experimentally using ASTM D-3039 (ASTM, 1995). Carbon and aramid fiber reinforced plastic (CARP) properties used in the FEM are based on experimentally measured properties (WS&TI, 1995). Properties for aramid reinforced plastic (ARP), fiberglass and aramid reinforced plastic (FARP) and the fourth FRP lamina used in the model have the same elastic and shear moduli, and Poisson ratios as CARP with the exception of Young's modulus in the axial direction. Properties used in the FEM are given in Table 4.
Table 4. FRP lamina elastic properties used in the finite-element models.

<table>
<thead>
<tr>
<th></th>
<th>E_x</th>
<th>E_y, E_z</th>
<th>G_{xy}, G_{xz}</th>
<th>G_{yz}</th>
<th>v_{xy}</th>
<th>v_{xz}</th>
<th>v_{yz}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(10^6 psi)</td>
<td>(10^6 psi)</td>
<td>(10^6 psi)</td>
<td>(10^6 psi)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FARP</td>
<td>8.0</td>
<td>0.4</td>
<td>0.5</td>
<td>0.01</td>
<td>0.36</td>
<td>0.36</td>
<td>0.30</td>
</tr>
<tr>
<td>ARP</td>
<td>11.6</td>
<td>0.4</td>
<td>0.5</td>
<td>0.01</td>
<td>0.36</td>
<td>0.36</td>
<td>0.30</td>
</tr>
<tr>
<td>CARP</td>
<td>16.6</td>
<td>0.4</td>
<td>0.5</td>
<td>0.01</td>
<td>0.36</td>
<td>0.36</td>
<td>0.30</td>
</tr>
<tr>
<td>RP#4</td>
<td>20</td>
<td>0.4</td>
<td>0.5</td>
<td>0.01</td>
<td>0.36</td>
<td>0.36</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Design of Analytical Investigation

The effects of partial reinforcement on the concentration of stress at the end of the FRP lamina were determined for several factors and combinations of factors. Variables studied include thickness of FRP lamina, percent of beam length reinforced, beam width, third-point versus uniform loading, stiffness ratio of FRP lamina to wood and span-to-depth ratio. For each of these variables, beams with and without a bumper were modeled. The geometries and material parameters studied were selected for various reasons.

Parameters of Study

Thickness of FRP lamina was selected since thicker FRP lamina will carry a greater portion of the load, therefore more load will have to be transferred to wood at the end of the FRP lamina. The closed-form solution described by Chen, et al (1991) shows that stress distributions in adhesive
joints with non-identical adherends are a function of thickness of laminates.

Percent of beam length reinforced was considered to be an important variable, since it is not fully understood how and where stresses are transferred to the wood portion of the beam from the FRP lamina. In addition, thickness of adherends and interface length are known to be important factors contributing to stress distribution in lap joints (Cheng, et al, 1991). Although a partially reinforced beam is not a lap joint, it displays some similar characteristics.

The affect of beam width was studied since stresses at the edge are influenced by width of adherends in laminated composites (Jones, 1975).

Third-point loading was primarily used in the models because third-point loading is the prescribed method of loading for ASTM D 198 (ASTM, 1995) bending tests for lumber. Uniform loading was modeled in order to compare results to those of third-point loading since uniform loading is the predominant design assumption for most applications.

Stiffness ratio of FRP lamina to wood was studied since load carried by the FRP lamina will vary with stiffness ratio; the FRP lamina will carry a greater portion of the tensile load with a greater stiffness ratio. Stiffness
ratio is also known to be an important factor in lap joints (Cheng, et al, 1991).

Span-to-depth ratio was studied for the same reasons that percent span length of FRP lamina was studied. It is not fully understood how and where stresses are transferred to the wood portion of the beam from the FRP lamina. Interface length, which may vary with span-to-depth ratio is known to be an important factor contributing to stress distributions in lap joints (Cheng, et al, 1991).

Model Loading

Since beams of various dimensions and degree of FRP lamina are studied, a basis of comparison must be set. A reasonable approach to the problem is to use a load such that the calculated tensile stress is the same for all models at the location of the upper interface at the end of the FRP lamina, assuming a solid wood cross section at this point. That is the normal bending stress, \( \sigma = \frac{My}{I} \), is the same for all beams at the end of the FRP lamina. The distance \( y \) is included in the comparison since, while on the beams without a bumper, the interface is always at the outer fiber of the wood, while the interface for beams with a bumper and filler is located at distance less than that to the outer fiber.
In addition to using a similar loading for all models, only one parameter was varied at a time. For example, when the FRP lamina length was studied, beam size, the FRP lamina thickness, material properties and load location was held constant. The standard beam size used in this study was 5-1/8 x 12 in. x 21 ft. This beam size was used for all models except those where a beam dimension was the variable being studied. A summary of the models produced and related parameters of study are given in Table 5 for beams without a bumper and Table 6 for beams with a bumper.

**Finite-Element Model Description**

All models were meshed with an 8-node solid element (SOLID45) using ANSYS® (ANSYS, 1995). The element was defined by eight nodes, with three mutually perpendicular translational degrees of freedom at each node. Input variables used for this element include Young’s moduli $E_x$, $E_y$, and $E_z$, shear moduli $G_{xy}$, $G_{yz}$, and $G_{xz}$, and Poisson’s ratios $v_{xy}$, $v_{yz}$, and $v_{xz}$. The analysis performed was a linear elastic analysis.

All models were loaded at third-points of the beam, with the exception of one model loaded with a uniform load. The load placed on each beam was the load needed to produce an outer-fiber bending stress of 3000 psi at a location on a
Table 5. Description of finite-element models for beams without a bumper.

<table>
<thead>
<tr>
<th>Variable Studied</th>
<th>FRP Thickness (in.)</th>
<th>FRP Length (%)</th>
<th>FRP MOE ($10^6$ psi)</th>
<th>Beam Size (in. x in. x ft)</th>
<th>Stiffness Ratio</th>
<th>Span:Depth</th>
<th>Load (lb)</th>
<th>My/I at End FRP (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRP Thickness</td>
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<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>14643</td>
<td>3000</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>14643</td>
<td>3000</td>
</tr>
<tr>
<td></td>
<td>0.21</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
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<td>21</td>
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<td>3000</td>
</tr>
<tr>
<td></td>
<td>0.28</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>14643</td>
<td>3000</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>14643</td>
<td>3000</td>
</tr>
<tr>
<td>FRP Length</td>
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<td>5.125x12x21</td>
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<td>21</td>
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<td></td>
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<td>21</td>
<td>11714</td>
<td>3000</td>
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<td></td>
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<td>60</td>
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<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>14643</td>
<td>3000</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>80</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>29286</td>
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</tr>
<tr>
<td>Beam Width</td>
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<td>16.6</td>
<td>3.125x12x21</td>
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<td>21</td>
<td>8929</td>
<td>3000</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>14643</td>
<td>3000</td>
</tr>
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<td></td>
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<td>Loading Scheme</td>
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<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>14643</td>
<td>3000</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>18303</td>
<td>3000 (uniform)</td>
</tr>
<tr>
<td>Stiffness Ratio</td>
<td>0.14</td>
<td>60</td>
<td>8.0</td>
<td>5.125x12x21</td>
<td>4.0</td>
<td>21</td>
<td>14643</td>
<td>3000</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>60</td>
<td>11.6</td>
<td>5.125x12x21</td>
<td>5.8</td>
<td>21</td>
<td>14643</td>
<td>3000</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>14643</td>
<td>3000</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>60</td>
<td>20.0</td>
<td>5.125x12x21</td>
<td>10.0</td>
<td>21</td>
<td>14643</td>
<td>3000</td>
</tr>
<tr>
<td>Span to Depth Ratio</td>
<td>0.14</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x10</td>
<td>8.3</td>
<td>10</td>
<td>30750</td>
<td>3000</td>
</tr>
<tr>
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<td>60</td>
<td>16.6</td>
<td>5.125x12x15.5</td>
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<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>14643</td>
<td>3000</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x27</td>
<td>8.3</td>
<td>27</td>
<td>11389</td>
<td>3000</td>
</tr>
</tbody>
</table>
Table 6. Description of finite-element models for beams with a bumper.

<table>
<thead>
<tr>
<th>Variable Studied</th>
<th>FRP Thickness</th>
<th>FRP MOE (10^6 psi)</th>
<th>Beam Size (in. x in. x ft)</th>
<th>Stiffness Ratio⁴</th>
<th>Span:Depth</th>
<th>Load (lb)</th>
<th>My/I at End FRP (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRP Thickness</td>
<td>0.07</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>20023</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>20535</td>
</tr>
<tr>
<td></td>
<td>0.21</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>21058</td>
</tr>
<tr>
<td></td>
<td>0.28</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>21594</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>22144</td>
</tr>
<tr>
<td>FRP Length</td>
<td>0.14</td>
<td>40</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>13690</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>20535</td>
</tr>
<tr>
<td>Beam Width</td>
<td>0.14</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>20538</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>60</td>
<td>16.6</td>
<td>10.75x12x21</td>
<td>8.3</td>
<td>21</td>
<td>43073</td>
</tr>
<tr>
<td>Stiffness Ratio</td>
<td>0.14</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>4.0</td>
<td>21</td>
<td>20535</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>5.8</td>
<td>21</td>
<td>20535</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>20535</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>10.0</td>
<td>21</td>
<td>20535</td>
</tr>
<tr>
<td>Span to Depth Ratio</td>
<td>0.14</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x21</td>
<td>8.3</td>
<td>21</td>
<td>20535</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>60</td>
<td>16.6</td>
<td>5.125x12x27</td>
<td>8.3</td>
<td>27</td>
<td>15971</td>
</tr>
</tbody>
</table>
solid wood beam analogous to the location of the end of the FRP lamina on the reinforced beam.

Element Mesh

The "standard" model type used for beams without a bumper is shown in Figure 14. The "standard" model type used for beams with a bumper is shown in Figure 15. One-quarter of the beam was modeled in both cases since load, dimensions and boundary conditions were symmetric about mid-length and mid-width of the beam. In both types of models, the region of the beam near the end of the FRP lamina was finely meshed to provide a clear picture of the stress distributions in this region and to reduce element aspect ratio. In order to reduce model size and analysis time, the mesh was scaled so that mesh density increases from top to bottom, mid-width to edge, and from 2 in. either side of the end of the FRP lamina to the end of the FRP lamina. A side view of mesh distribution at the end of the FRP lamina is shown in Figures 16 and 17 for beams without and with bumpers respectively. The Mesh distribution across the width is shown in Figure 18 and is identical for both model types.

Note that a two-inch gap is located at the end of the FRP lamina in the models having a bumper. In a manufactured beam, the gap length may be greater and the gap shape may be more wedge shaped beyond the end of the FRP lamina.
Figure 14. The standard finite-element model for a partially reinforced glulam without a bumper having simply supported end conditions.
Figure 15. The standard finite-element model for a partially reinforced glulam with a bumper having simply supported end conditions.
Figure 16. Side view of finite-element mesh at end of the FRP lamina for a beam without a bumper.
Figure 17. Side view of finite-element mesh at the end of the FRP lamina for a beam with a bumper.
Figure 18. Finite-element mesh for beams with and without a bumper; mesh density increases near the FRP lamina end and toward the edges.
Although the model does not produce the same shape found in a manufactured beam, the effect of the gap shape opposite the end of the FRP lamina likely has little effect on stresses closer to the end of the FRP lamina.

**Model Verification**

The final model for a beam with a bumper and a beam without a bumper were verified in several ways. Verification was performed using solid wood beam models where the same material properties are used as in the standard model type. The verification was used to verify element performance, boundary conditions and meshing. All models represented a 5-1/8 x 12 in. x 21 ft beam loaded at third-points. These results should indicate that boundary conditions, material properties and element selection are adequate.

Theoretical beam deflection was compared to model deflection for solid wood beams and partially reinforced beams. Derivation of theoretical beam deflection for a composite beam composed of isotropic materials was given in Chapter 1.

Table 7 compares centerline deflection and maximum bending stress predicted by theory to values from the finite-element model for 1) the model without a bumper and without FRP lamina (the FRP lamina was made nonfunctional by
reducing E values to 10 psi), 2) the model without a bumper with 0.14 in. of $16.6 \times 10^6$ psi MOE along 60% of the length, 3) the model with a bumper without FRP lamina (FRP lamina assigned properties for wood), and 4) the model with a bumper with 0.14 in. of $16.6 \times 10^6$ psi MOE along 60% of the length.

Table 7. Closed-form and finite-element model values for centerline deflection.

<table>
<thead>
<tr>
<th>Beam Description</th>
<th>FRP MOE (10^6 psi)</th>
<th>FRP Length (%)</th>
<th>Load (lb)</th>
<th>Closed-form $\delta_{cu}$ (in.)</th>
<th>Model $\delta_{cu}$ (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-1/8 x 12 in. x 21 ft No Bumper No FRP</td>
<td>N/A</td>
<td>N/A</td>
<td>14643</td>
<td>2.923</td>
<td>2.935</td>
</tr>
<tr>
<td>5-1/8 x 12 in. x 21 ft No Bumper 0.14 in. FRP</td>
<td>16.6</td>
<td>60</td>
<td>14643</td>
<td>2.36</td>
<td>2.38</td>
</tr>
<tr>
<td>5-1/8 x 12.14 in. x 21 ft 1.5 in. Bumper No FRP</td>
<td>N/A</td>
<td>N/A</td>
<td>20535</td>
<td>3.90</td>
<td>3.97</td>
</tr>
<tr>
<td>5-1/8 x 12.14 in. x 21 ft 1.5 in. Bumper 0.14 in. FRP</td>
<td>16.6</td>
<td>60</td>
<td>20535</td>
<td>3.514</td>
<td>3.57</td>
</tr>
</tbody>
</table>

*Centerline deflection.

Aspect ratios at the end of the FRP lamina were held around 2:1; yet for the SOLID45 element, ANSYS® solutions exhibited little effect due to much higher aspect ratios (30:1). To ensure that effects of high aspect ratio in portions of the mesh did not influence stress concentrations near the end of the FRP lamina, a closed-form solution for stress was compared to model stress for a partially
reinforced beam without a bumper with an FRP lamina of negligible stiffness (100 psi). The maximum difference between FEM and closed-form bending stress at the FRP lamina/wood interface level, over a 2-in. length, centered at the end of the FRP lamina, and half-width of the beam was 0.35%.

The agreement between closed-form solutions and FEM results verified that boundary conditions were correct and element performance and aspect ratios were adequate.

**Finite-Element Model Results**

Before comparing effects of various parameters, it is important to have an understanding of the distribution and magnitude of the stress components near the end of the FRP lamina. It is also important to compare stresses predicted by the FEM to mechanical properties for wood used in the beam. Since material properties used in the FEM were for Douglas-fir, mechanical properties of Douglas-fir were used as a guideline as to what value of stress is considered to be significant.

Table 8 shows values of mechanical properties of interior west Douglas-fir from the Wood Handbook (FPL, 1987). Note that properties given in the Table 8 are for small, clear, straight-grained specimens at 12% moisture content.
Since the effect of FRP lamina on stress distributions at the end of the FRP lamina is localized, and the FRP lamina should be manufactured such that no defects occur at the end of the FRP lamina, properties from small clear samples are appropriate for comparison purposes.

For models with a bumper and for models without a bumper, the distribution of each stress was plotted for the scenario that caused the greatest stress at the end of the FRP lamina. These stress distributions were compared to mechanical properties in Table 8. The stresses found to be most significant were studied further for all scenarios investigated.

Since models built for glulam with a bumper and without a bumper, results from these models will be dealt with separately.

### Table 8. Strength properties for small, clear, straight-grained samples interior west Douglas-fir Wood Handbook, 1987).

<table>
<thead>
<tr>
<th>Moisture Condition</th>
<th>Maximum Tensile Stress Parallel to Grain (psi)</th>
<th>Compression Perpendicular to Grain (psi)</th>
<th>Maximum Shear Parallel to Grain (psi)</th>
<th>Maximum Tensile Strength Perpendicular to Grain (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 30%</td>
<td>7700</td>
<td>420</td>
<td>940</td>
<td>290</td>
</tr>
<tr>
<td>12%</td>
<td>12600</td>
<td>760</td>
<td>1290</td>
<td>350</td>
</tr>
</tbody>
</table>
Methodology for Stress Distribution Characterization

Innumerable options exist for characterizing the stress distributions at the end of the FRP lamina. Some options include comparison of maximum stresses, comparison of average stress over an arbitrary distance in a given direction at the end of the FRP lamina, comparison of stresses at a consistent location relative to the end of the FRP lamina or a combination of these or other options.

The basis for comparison chosen for this study was to compare the average stress over a given distance near the end of the FRP lamina at the outer fiber, or wood surface at the plane of the FRP lamina-wood interface.

In order to calculate average stress values, a polynomial curve fit was used to describe the stress distribution at or near the end of the FRP lamina. The best-fit polynomial for each stress distribution was integrated over an arbitrary distance to calculate the average stress. This type of approach is common in analysis of stress distributions near small notches and holes in laminated composites (Whitney and Nuismer, 1974) and peeling stresses near free edges of laminated composites (Kim and Soni, 1984). The reasoning behind calculating an average stress near discontinuities lies within a material’s inherent ability to redistribute the large stress
concentrations predicted from theory (Whitney and Nuismer, 1974).

**Glulam Without a Bumper**

All stress levels presented include the stress distribution for the load applied to the model, which would produce a 3000 psi bending stress in a solid wood beam at the same location as the location of the end of the FRP lamina in the reinforced beam.

For summary plots and tables, the ratio of stress predicted in the model was compared to the 3000 psi bending stress since all stresses, with the exception of $\sigma_x$, are zero at the outer surface of the beam. Thus, the "stress ratio" was calculated by equation (11).

$$Stress \ Ratio_y = \frac{\sigma_y}{3000}$$  \hspace{1cm} (11)

**Maximum Stress Levels and Stress Distribution**

The six stress components $\sigma_{ij}$ were plotted for the model with an FRP lamina thickness of 0.35 in. (see Table 5 for model details). The stress components for this model were investigated first to find out which stresses were significant, because the combination of variables used in this model produced the greatest levels of stress at the FRP
lamina terminus. The maximum stress levels near the end of the FRP lamina occurred at the wood-FRP lamina interface plane; therefore, all stress distributions for models discussed in this section are distributions at the wood-FRP lamina interface plane and occurring on the wood lamina.

Figure 19 shows the stress distribution in the wood for $\sigma_x$ in the neighborhood of the end of the FRP lamina along the outside edge of the beam. The distribution clearly shows a dramatic increase in $\sigma_x$ at the end of the FRP lamina. In fact, $\sigma_x$ appears to be singular at this location. The distribution of $\sigma_x$ before the end of the FRP lamina is fairly uniform in magnitude and lower than the 3000 psi stress that would occur if this were an unreinforced beam. The rise in stress quickly decays past the end of the FRP lamina and nearly approximates the predicted 3000 psi level at 2 in. past the end of the FRP lamina. The magnitude of $\sigma_x$ at the end of the FRP lamina is clearly greater than the maximum tensile strength for Douglas-fir (Table 8). Therefore, $\sigma_x$ was investigated for all models over 0.05 and 0.15 in. distance just beyond the end of the FRP lamina at the outside edge of the beam.

Figure 20 shows the distribution for $\sigma_y$ at the end of the FRP lamina, at the outside edge of the beam on the wood surface above the FRP lamina. The distribution for $\sigma_y$
Figure 19. Distribution of $\sigma_x$ near the end of the FRP lamina on the wood at the outside edge of the beam for glulam without a bumper. The stress distribution is for 0.35-in. thick FRP lamina, the FRP combination resulting in the highest level of all stresses.
Figure 20. Distribution of $\sigma_y$ near the end of the FRP lamina on the wood at the outside edge of the beam for glulam without a bumper. The stress distribution is for 0.35-in. thick FRP lamina, the FRP combination resulting in the highest level of all stresses.
indicates that the solution is singular at the end of the FRP lamina. The magnitude of \( \sigma_y \) rapidly decreases to zero over a 0.75 in. distance beyond the end of the FRP lamina. The magnitude of \( \sigma_y \) at the end of the FRP lamina appears to be greater than the maximum tensile strength for Douglas-fir perpendicular to grain (Table 8). Therefore, \( \sigma_y \) was investigated in all models at the outside edge of the beam, over a 0.05 and 0.15 in. distance up to the end of the FRP lamina. Stress \( \sigma_y \) was integrated prior to the end of the FRP lamina for two reasons. First of all, there appears to be a greater average stress in this region than beyond the end of the FRP lamina. Secondly, \( \sigma_y \) may be an important factor leading to the FRP lamina peeling off of the beam.

Figure 21 shows the distribution for \( \sigma_z \) at the end of the FRP lamina at mid-width of the beam. The distribution and magnitude of \( \sigma_z \) are very similar to that of \( \sigma_y \), however the stress appears to be distributed along a greater length of the beam. Stress \( \sigma_z \) approaches zero at approximately 1.5 in. beyond the end of the FRP lamina. Since the maximum value of \( \sigma_z \) may be greater than the maximum tensile strength of Douglas-fir perpendicular to grain, \( \sigma_z \) was also investigated for all models. The \( \sigma_z \) stress was be averaged over a 0.05 and 0.15 in. distance beyond the end of the FRP lamina at mid-width of the beam; the stress was distributed
Figure 21. Distribution of $\sigma_z$ near the end of the FRP lamina on the wood at the mid-width of the beam for glulam without a bumper. The stress distribution is for 0.35-in. thick FRP lamina, the FRP combination resulting in the highest level of all stresses.
over a greater length of the beam beyond the end of the FRP lamina, and it was highest at mid-width of the beam.

Figure 22 shows the distribution for $\sigma_{xy}$ at the end of the FRP lamina in the wood at the outside edge of the beam. The distribution of $\sigma_{xy}$ shows much greater levels of stress over the 2 in. prior to the end of the FRP lamina than over the 2 in. beyond the end of the FRP lamina. The distribution also appears to be singular at the end of the FRP lamina. The maximum $\sigma_{xy}$ is greater than the maximum shear parallel to grain for Douglas-fir (Table 8). Hence, $\sigma_{xy}$ will be averaged over a 0.05 and 0.15 in. distance prior to the end of the FRP lamina at the outside edge of the beam.

Figure 23 shows the distribution for $\sigma_{yz}$ at the end of the FRP lamina at the outside edge of the beam. The magnitude of $\sigma_{yz}$ is much smaller than the maximum shear strength parallel to grain (Table 8), which is much lower than the maximum shear strength perpendicular to grain. For these reasons, $\sigma_{yz}$ was not investigated further.

Figure 24 shows the distribution for $\sigma_{xz}$ at the end of the FRP lamina at the outside edge of the beam. Although the magnitude of $\sigma_{xz}$ is close to the maximum shear parallel to grain for Douglas-fir (Table 8), it is very localized, and lower in magnitude than $\sigma_{xy}$. Since $\sigma_{xy}$ and $\sigma_{xz}$ are both
Figure 22. Distribution of $\sigma_{xy}$ near the end of the FRP lamina on the wood at the outside edge of the beam for glulam without a bumper. The stress distribution is for 0.35-in. thick FRP lamina, the FRP combination resulting in the highest level of all stresses.
Figure 23. Distribution of $\sigma_{yz}$ near the end of the FRP lamina on the wood at the outside edge of the beam for glulam without a bumper. The stress distribution is for 0.35-in. thick FRP lamina, the FRP combination resulting in the highest level of all stresses.
Figure 24. Distribution of $\sigma_{xz}$ near the end of the FRP lamina on the wood at the outside edge of the beam for glulam without a bumper. The stress distribution is for 0.35-in. thick FRP lamina, the FRP combination resulting in the highest level of all stresses.
shear parallel to grain and $\sigma_{xy}$ is much greater than, and less isolated than $\sigma_{xz}$, only $\sigma_{xy}$ will be investigated.

The distributions of $\sigma_x$, $\sigma_y$, $\sigma_z$ and $\sigma_{xy}$ across the beam width and along the length are shown in Figures 25, 26, 27 and 28 respectively. In these figures, the x and y-axes are not to scale. Figure 25 shows that $\sigma_x$ is distributed uniformly across the width. Figure 26 shows that $\sigma_y$ is uniformly distributed across the width, however, it decreases slightly toward the outside edge. Figure 27 shows that $\sigma_z$ decreases at the outside edge by about one-third the level at mid-width. Figure 28 shows that $\sigma_{xy}$ is distributed uniformly across the width, while decreasing slightly at the outside edge.

Effect of Reinforcement Thickness

All beams in these models were 5-1/8 x 12 in. x 21 ft and had FRP lamina over 60% of the length on the tensile side. The models were subjected to third-point loads. The FRP lamina thicknesses are given in Table 5.

The stress distributions for $\sigma_x$, $\sigma_y$, $\sigma_z$ and $\sigma_{xy}$ are plotted in Figures 29 to 32.

Polynomial curve fits were used to average stress levels over a 0.05 and 0.15 in. distance prior to the end of
**Figure 25.** Typical distribution of $\sigma_x$ along the beam length and across beam width near the end of the FRP lamina for beams without a bumper. Location relative to the end of the FRP lamina is not to scale.
Figure 26. Typical distribution of $\sigma_y$ along the beam length and across beam width near the end of the FRP lamina for beams without a bumper. Location relative to the end of the FRP lamina is not to scale.
Figure 27. Typical distribution of $\sigma_z$ along the beam length and across beam width near the end of the FRP lamina for beams without a bumper. Location relative to the end of the FRP lamina is not to scale.
Figure 28. Typical distribution of $\sigma_{xy}$ along the beam length and across beam width near the end of the FRP lamina for beams without a bumper. Location relative to the end of the FRP lamina is not to scale.
Figure 29. The distribution of $\sigma_x$ is plotted for various FRP thickness on a beam without a bumper. Best fit polynomials are plotted through the finite-element data.
Figure 30. The distribution of $\sigma_y$ is plotted for various FRP thickness on a beam without a bumper. Best fit polynomials are plotted through the finite-element data.
Figure 31. The distribution of $\sigma_z$ is plotted for various FRP thickness on a beam without a bumper. Best fit polynomials are plotted through the finite-element data.
Figure 32. The distribution of \( \sigma_{xy} \) is plotted for various FRP thickness on a beam without a bumper. Best fit polynomials are plotted through the finite-element data.
the FRP lamina (\(\sigma_y\) and \(\sigma_{xy}\)) or beyond the end of the FRP lamina (\(\sigma_x\) and \(\sigma_z\)). Coefficients for the polynomial fits are given in the Appendix, Table A1. The best fit polynomials are plotted through the FEM data in Figures 29 to 32.

The effect of FRP lamina thickness on \(\sigma_x\), \(\sigma_y\), \(\sigma_z\) and \(\sigma_{xy}\) are given in Table 9 and shown graphically in Figures 33 to 36. It is apparent that FRP lamina thickness does have an effect on stress distributions at the end of the FRP lamina. The stresses all increase with increasing FRP lamina thickness, but approach a common maximum value with greater FRP lamina thickness. The distance between the 0.05 and 0.15 in. averaged stress levels is an indication of stress localization. For example, the magnitude of \(\sigma_y\), Figure 34, decreases by half when averaged over 0.15 in. compared to 0.05 in.; this indicates that \(\sigma_y\) is very localized. The magnitude of \(\sigma_{xy}\) (Figure 36), on the other hand, is very close for the 0.05 and 0.15 in. averages; this indicates that the \(\sigma_{xy}\) is not very localized.

Effect of Stiffness Ratio

All beams represented in these models were 5-1/8 x 12 in. x 21 ft and had a 0.14-in. thick FRP lamina over 60% of
Table 9. Stresses and stress ratios at the end of the FRP lamina for various FRP thicknesses for glulam without a bumper.

<table>
<thead>
<tr>
<th>FRP Thickness (in.)</th>
<th>0.05 in. Average Stress at Applied Load (psi)</th>
<th>0.15 in. Average Stress at Applied Load (psi)</th>
<th>Stress Ratio (Equation (11))</th>
<th>Stress Ratio (Equation (11))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>14720</td>
<td>11590</td>
<td>4.91</td>
<td>3.86</td>
</tr>
<tr>
<td>0.14</td>
<td>19670</td>
<td>15220</td>
<td>6.56</td>
<td>5.07</td>
</tr>
<tr>
<td>0.21</td>
<td>22510</td>
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Figure 33. The effect of the FRP lamina thickness on $\sigma_x$ near the end of the FRP lamina is plotted for average stress from the end of the FRP lamina to 0.05 and 0.15 in. past the end of the FRP lamina.
Figure 34. The effect of the FRP lamina thickness on $\sigma_y$ near the end of the FRP lamina is plotted for average stress from the end of the FRP lamina to 0.05 and 0.15 in. prior to the end of the FRP lamina.
Figure 35. The effect of the FRP lamina thickness on $\sigma_z$ near the end of the FRP lamina is plotted for average stress from the end of the FRP lamina to 0.05 and 0.15 in. past the end of the FRP lamina.
**Figure 36.** The effect of the FRP lamina thickness on $\sigma_{xy}$ near the end of the FRP lamina is plotted for average stress from the end of the FRP lamina to 0.05 and 0.15 in. prior to the end of the FRP lamina.
the length. The models were subjected to third-point loads. Stiffness ratios were given in Table 5.

The stress distributions at the end of the FRP lamina for various stiffness ratios are shown in Figures 37 to 40 for $\sigma_x$, $\sigma_y$, $\sigma_z$ and $\sigma_{xy}$ respectively. It is apparent that stiffness ratio has a significant effect on these stress distributions.

Polynomial curve fits were used to average stress levels over a 0.05 and 0.15 in. distance prior to the end of the FRP lamina ($\sigma_y$ and $\sigma_{xy}$) or beyond the end of the FRP lamina ($\sigma_x$ and $\sigma_z$). Coefficients for the polynomial fits are given in the Appendix, Table A2. The best fit polynomials are plotted through the FEM data in Figures 37 to 40.

The effects of stiffness ratio on $\sigma_x$, $\sigma_y$, $\sigma_z$ and $\sigma_{xy}$ are given in Table 10 and shown graphically in Figures 41 to 44. The stresses all increase with increasing stiffness ratio.

While $\sigma_x$, $\sigma_z$ and $\sigma_{xy}$ increase significantly with increased stiffness ratio, $\sigma_y$ increases much less; this may be a result of having $E_y$ as a constant for all FRP applications.
Figure 37. The distribution of $\sigma_x$ is plotted for various FRP-to-wood stiffness ratios on a beam without a bumper. Best fit polynomials are plotted through the finite-element data.
Figure 38. The distribution of $\sigma_y$ is plotted for various FRP-to-wood stiffness ratios on a beam without a bumper. Best fit polynomials are plotted through the finite-element data.
Figure 39. The distribution of $\sigma_z$ is plotted for various FRP-to-wood stiffness ratios on a beam without a bumper. Best fit polynomials are plotted through the finite-element data.
Figure 40. The distribution of $\sigma_{xy}$ is plotted for various FRP-to-wood stiffness ratios on a beam without a bumper. Best fit polynomials are plotted through the finite-element data.
Table 10. Stresses and stress ratios at the end of the FRP lamina for various stiffness ratios for glulam without a bumper.

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<td>Stress at Applied Load (psi)</td>
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<td>Stress Ratio (Equation (11))</td>
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Figure 41. The effect of the FRP-to-wood stiffness ratio on $\sigma_x$ near the end of the FRP lamina is plotted for average stress from the end of the FRP lamina to 0.05 and 0.15 in. past the end of the FRP lamina.
Figure 42. The effect of the FRP-to-wood stiffness ratio on $\sigma_y$ near the end of the FRP lamina is plotted for average stress from the end of the FRP lamina to 0.05 and 0.15 in. prior to the end of the FRP lamina.
Figure 43. The effect of the FRP-to-wood stiffness ratio on $\sigma_z$ near the end of the FRP lamina is plotted for average stress from the end of the FRP lamina to 0.05 and 0.15 in. past the end of the FRP lamina.
Figure 44. The effect of the FRP-to-wood stiffness ratio on \( \sigma_{xy} \) near the end of the FRP lamina is plotted for average stress from the end of the FRP lamina to 0.05 and 0.15 in. prior to the end of the FRP lamina.
Effect of Reinforcement Length

All beams represented in these models were 21 ft long, 5-1/8 in. wide, 12 in. high and had 0.14 in. thick FRP lamina. The models were subjected to third-point loads. The FRP lamina lengths are given in Table 5.

Figures 45 to 48 show the stress distributions $\sigma_x$, $\sigma_y$, $\sigma_z$ and $\sigma_{xy}$ for various lengths of FRP lamina (percent of total span length). It is apparent from these graphs that the length of the FRP lamina does not have a major impact on stress distributions. However, it appears that for the 80% reinforced beam length, stress levels are increased slightly. Since the other FRP lamina lengths are approximately the same and different from the 80% reinforced length, it is reasonable to justify that the effect of the support 2 ft away was influential.

Effect of Beam Width

All beams represented in these models were Width x 12 in. x 21 ft and had a 0.14-in. thick FRP lamina over 60% of the length. The models were subjected to third-point loads. The beam widths are given in Table 5.

Figures 49 to 52 show the stress distributions $\sigma_x$, $\sigma_y$, $\sigma_z$ and $\sigma_{xy}$ for beam widths ranging from 3.125 in. to 10.75
Figure 45. The distribution of $\sigma_x$ is plotted for various FRP lamina lengths on a beam without a bumper.
Figure 46. The distribution of $\sigma_y$ is plotted for various FRP lamina lengths on a beam without a bumper.
Figure 4.7. The distribution of $\sigma_z$ is plotted for various FRP lamina lengths on a beam without a bumper.
Figure 48. The distribution of $\sigma_{xy}$ is plotted for various FRP lamina lengths on a beam without a bumper.
Figure 49. The distribution of $\sigma_x$ is plotted for various beam widths on a beam without a bumper.
Figure 50. The distribution of $\sigma_y$ is plotted for various beam widths on a beam without a bumper.
Figure 51. The distribution of $\sigma_z$ is plotted for various beam widths on a beam without a bumper.
Figure 52. The distribution of $\sigma_{xy}$ is plotted for various beam widths on a beam without a bumper.
in. It is apparent from these graphs that beam width has very little influence on stress distributions at the end of the FRP lamina.

**Effect of Span-to-depth Ratio**

All beams represented in these models had a 5-1/8 x 12 in. cross section and had a 0.14-in. thick FRP lamina over 60% of the length. The models were subjected to third-point loads. The span-to-depth ratios are given in Table 5.

Figures 53 to 56 show the stress distributions $\sigma_x$, $\sigma_y$, $\sigma_z$ and $\sigma_{xy}$ for span-to-depth ratios ranging from 10 to 27. It is apparent from these figures that span-to-depth ratio has very little influence on stress distribution at the end of the FRP lamina. However, the stress levels tend to be higher for the span-to-depth ratio of 10. It is likely that the higher value for this ratio is due to the proximity of the end support, which was 2 ft from the end of the FRP lamina.

**Effect of Loading Conditions**

All beams represented in these models were 5-1/8 x 12 in. x 21 ft and had a 0.14-in. thick FRP lamina over 60% of the length.
Figure 53. The distribution of $\sigma_x$ is plotted for various span-to-depth ratios on a beam without a bumper.
Figure 54. The distribution of $\sigma_y$ is plotted for various span-to-depth ratios on a beam without a bumper.
Figure 55. The distribution of $\sigma_z$ is plotted for various span-to-depth ratios on a beam without a bumper.
Figure 56. The distribution of $\sigma_{xy}$ is plotted for various span-to-depth ratios on a beam without a bumper.
Figures 57 to 60 show the stress distributions $\sigma_x$, $\sigma_y$, $\sigma_z$ and $\sigma_{xy}$ for third-point and uniform loading conditions. These figures show that uniform loading results in virtually identical stress levels as third-point loading.

**Glulam With a Bumper**

All stress levels presented include the stress distribution for the load applied to the model. This load would produce a 3000 psi bending stress in a solid wood beam at the same location as the location of the end of the FRP lamina, at the upper surface of the FRP lamina, in the reinforced beam.

For summary plots and tables, the ratio of stress predicted in the model were compared to the 3000 psi bending stress since all stresses, with the exception of $\sigma_x$, are zero at the outer surface of the beam. The stress ratio is calculated by equation (10).

Since FRP lamina thickness and stiffness ratio were the only variables found to be influential on stress levels for glulam without a bumper, it is reasonable to assume the same holds true for glulam with a bumper. However, to ensure that this is the case, two models will be compared for the effects of width and span-to-depth ratio.
Figure 57. The distribution of $\sigma_x$ is plotted for uniform and third-point loading on a beam without a bumper.
Figure 58. The distribution of \( \sigma_y \) is plotted for uniform and third-point loading on a beam without a bumper.
Figure 59. The distribution of $\sigma_z$ is plotted for uniform and third-point loading on a beam without a bumper.
Figure 60. The distribution of $\sigma_{xy}$ is plotted for uniform and third-point loading on a beam without a bumper.
**Maximum Stress Levels and Stress Distribution**

Figures 61 to 66 show stress distributions for $\sigma_{ij}$ at the end of the FRP lamina at the level of the upper wood/FRP lamina interface. All stress distributions were taken at the outside edge of the beam, with the exception of $\sigma_z$, which was taken at mid-width of the beam.

Figure 61 shows the distribution of $\sigma_x$ near the end of the FRP lamina. This distribution is very similar to the distribution for the analogous beam without a bumper (Figure 19); it is singular at the end of the FRP lamina and levels off to the predicted bending stress in a solid wood section at about 2 in. beyond the end of the FRP lamina. The rise in stress at the node located at about 1.7 in. is due to the gap at 2 in. past the end of the FRP lamina (Figure 17). The magnitude is greater than the maximum tensile strength for Douglas-fir (Table 8); therefore $\sigma_x$ will be investigated over a range beyond the end of the FRP lamina at the outside edge of the beam.

Figure 62 shows the distribution of $\sigma_y$ near the end of the FRP lamina. $\sigma_y$ is singular and compressive just before the end of the FRP lamina and is tensile beyond the end of the FRP lamina. Since compressive stress is unlikely to cause failure and the tensile stress is less than the
Figure 61. Distribution of $\sigma_x$ near the end of the FRP lamina on the wood at the outside edge of the beam for glulam without a bumper. The stress distribution is for 0.35-in. thick FRP lamina, the FRP combination resulting in the highest level of all stresses.
Figure 62. Distribution of $\sigma_y$ near the end of the FRP lamina on the wood at the outside edge of the beam for glulam with a bumper. The stress distribution is for 0.35-in. thick FRP lamina, the FRP combination resulting in the highest level of all stresses.
Figure 63. Distribution of $\sigma_x$ near the end of the FRP lamina on the wood at mid-width of the beam for glulam with a bumper. The stress distribution is for 0.35-in. thick FRP lamina, the FRP combination resulting in the highest level of all stresses.
Figure 64. Distribution of $\sigma_{xy}$ near the end of the FRP lamina on the wood at the outside edge of the beam for glulam with a bumper. The stress distribution is for 0.35-in. thick FRP lamina, the FRP combination resulting in the highest level of all stresses.
Figure 65. Distribution of $\sigma_{yz}$ near the end of the FRP lamina on the wood at the outside edge of the beam for glulam with a bumper. The stress distribution is for 0.35-in. thick FRP lamina, the FRP combination resulting in the highest level of all stresses.
Figure 66. Distribution of $\sigma_x$ near the end of the FRP lamina on the wood at the outside edge of the beam for glulam with a bumper. The stress distribution is for 0.35-in. thick FRP lamina, the FRP combination resulting in the highest level of all stresses.
maximum tensile strength perpendicular to grain for Douglas-fir (Table 8), $\sigma_y$ will not be investigated further.

Figure 63 shows the distribution of $\sigma_z$ near the end of the FRP lamina at mid-width of the beam. The stress $\sigma_z$ is compressive prior to the end of the FRP lamina and tensile beyond the end of the FRP lamina. The tensile stress beyond FRP lamina is greater than the maximum tensile strength of Douglas-fir perpendicular to grain. Thus, $\sigma_z$ will be investigated for all models and averaged over a 0.05 and 0.15 in. distance beyond the end of the FRP lamina.

Figure 64 shows the distribution of $\sigma_{xy}$ near the end of the FRP lamina at mid-width of the beam. The distribution of $\sigma_{xy}$ is very similar to that in the analogous beam without a bumper (Figure 20). Maximum values for $\sigma_{xy}$ are greater than values for shear strength parallel to grain for Douglas-fir (Table 8). Thus $\sigma_{xy}$ will be investigated for all models and averaged over a 0.05 and 0.15 in. distance prior to the end of the FRP lamina.

Figure 65 shows the distribution of $\sigma_{yz}$ near the end of the FRP lamina at the outside edge of the beam. The magnitude of $\sigma_{yz}$ is much less than the shear strength of Douglas-fir parallel to grain (Table 8), which is much less
than shear strength perpendicular to grain, therefore $\sigma_{yz}$
will not be considered further.

Figure 66 shows the distribution of $\sigma_{xz}$ near the end of
the FRP lamina at the outside edge of the beam. The
magnitude and area covered by $\sigma_{xz}$ is much less than $\sigma_{xy}$,
therefore, $\sigma_{xz}$ will not be considered further.

Figures 67 to 69 show three-dimensional plots of $\sigma_x$, $\sigma_z$
and $\sigma_{xy}$ along the length of the beam near the end of the FRP
lamina and across the half-width of the beam. Distributions
are shown for the surface above the FRP lamina and the
surface below FRP lamina.

It is apparent in Figure 67 that $\sigma_x$ is greater along
the surface above the FRP lamina than below the FRP lamina.
This is due to the bumper bending about the gap, reducing
the stress on the upper portion of the bumper near the end
of the FRP lamina. Figure 67 also shows that the maximum $\sigma_x$
occurs at the outside edge of the beam.

Figure 68 shows that $\sigma_z$ is tensile and greater in
magnitude on the upper surface than on the lower surface.

Figure 69 shows that $\sigma_{xy}$ is nearly two times greater on
the surface above the FRP lamina than on the surface below
the FRP lamina.
**Figure 67.** Typical distribution of $\sigma_x$ along the beam length and across beam width near the end of the FRP lamina for beams with a bumper.
Figure 68. Typical distribution of $\sigma_z$ along the beam length and across beam width near the end of the FRP lamina for beams with a bumper.
Figure 69. Typical distribution of $\sigma_{xy}$ along the beam length and across beam width near the end of the FRP lamina for beams with a bumper.
Effect of Reinforcement Thickness

All beams represented in these models were 5-1/8 x 12 (+ FRP lamina thickness) in. x 21 ft and had FRP lamina over 60% of the length. The models were subjected to third-point loads. The FRP lamina thicknesses are given in Table 6.

Polynomial curve fits were used to average stress levels over a 0.05 and 0.15 in. distance prior to the end of the FRP lamina for $\sigma_{xy}$ and beyond the end of the FRP lamina for $\sigma_x$ and $\sigma_z$. Coefficients for the polynomial fits are given in the Appendix, Table A3.

Figures 70 through 72 show the effect of FRP lamina thickness on $\sigma_x$, $\sigma_z$ and $\sigma_{xy}$. It is apparent from these figures that thickness does influence the stress distribution at the end of the FRP lamina.

Polynomial curve fits were used to average stress levels over a 0.05 and 0.15 in. distance prior to the end of the FRP lamina for $\sigma_{xy}$ and beyond the end of the FRP lamina for $\sigma_x$ and $\sigma_z$. Coefficients for the polynomial fits are given in the Appendix, Table A3. The best fit polynomials are plotted over the data points on Figures 70 through 72.

The effect of FRP lamina thickness on stress distribution are given in Table 11 and shown on Figures 73 to 75 for $\sigma_x$, $\sigma_z$ and $\sigma_{xy}$. The magnitudes of all stress
Figure 70. The distribution of $\sigma_x$ is plotted for various FRP thickness on a beam with a bumper. Best fit polynomials are plotted through the finite-element data.
Figure 71. The distribution of $\sigma_z$ is plotted for various FRP thickness on a beam with a bumper. Best fit polynomials are plotted through the finite-element data.
Figure 72. The distribution of $\sigma_{xy}$ is plotted for various FRP thickness on a beam with a bumper. Best fit polynomials are plotted through the finite-element data.
Table 11. Stresses and stress ratios at the end of the FRP lamina for various FRP thicknesses for glulam with a bumper.

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<th>FRP Thickness</th>
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<td>399</td>
<td>0.162</td>
<td>0.133</td>
</tr>
<tr>
<td>0.35</td>
<td>540</td>
<td>444</td>
<td>0.180</td>
<td>0.148</td>
</tr>
<tr>
<td>0.07</td>
<td>1090</td>
<td>1049</td>
<td>0.363</td>
<td>0.350</td>
</tr>
<tr>
<td>0.14</td>
<td>1569</td>
<td>1513</td>
<td>0.523</td>
<td>0.504</td>
</tr>
<tr>
<td>0.21</td>
<td>1932</td>
<td>1865</td>
<td>0.644</td>
<td>0.622</td>
</tr>
<tr>
<td>0.28</td>
<td>2215</td>
<td>2144</td>
<td>0.738</td>
<td>0.715</td>
</tr>
<tr>
<td>0.35</td>
<td>2475</td>
<td>2395</td>
<td>0.825</td>
<td>0.798</td>
</tr>
</tbody>
</table>
Figure 73. The effect of the FRP lamina thickness on $\sigma_x$ near the end of the FRP lamina is plotted for average stress from the end of the FRP lamina to 0.05 and 0.15 in. past the end of the FRP lamina.
Figure 74. The effect of the FRP lamina thickness on $\sigma_z$ near the end of the FRP lamina is plotted for average stress from the end of the FRP lamina to 0.05 and 0.15 in. past the end of the FRP lamina.
Figure 75. The effect of the FRP lamina thickness on $\sigma_{xy}$ near the end of the FRP lamina is plotted for average stress from the end of the FRP lamina to 0.05 and 0.15 in. prior to the end of the FRP lamina.
distributions increase significantly with FRP lamina thickness.

**Effect of Stiffness Ratio**

All beams represented in these models were 5-1/8 x 12.14 in. x 21 ft and had FRP lamina over 60% of the length. The models were subjected to third-point loads. The stiffness ratios are given in Table 6.

Figures 76 to 78 show the stress distributions for $\sigma_x$, $\sigma_z$ and $\sigma_{xy}$ near the end of the FRP lamina for various stiffness ratios. It is apparent from these plots that stiffness ratio does have a significant effect on stress distributions at the end of the FRP lamina.

Polynomial curve fits were used to average stress levels over a 0.05 and 0.15 in. distance prior to the end of the FRP lamina ($\sigma_y$ and $\sigma_{xy}$) or 0.05 and 0.15 in. beyond the end of the FRP lamina ($\sigma_x$ and $\sigma_z$). Coefficients for the polynomial fits are given in the Appendix, Table A4. The best fit polynomials are plotted over the FEM data on Figures 76 through 78.

The effects of stiffness ratio on stress distribution are given in Table 12 and shown on Figures 79 to 81 for $\sigma_x$. 
**Figure 76.** The distribution of $\sigma_x$ is plotted for various FRP-to-wood stiffness ratios on a beam with a bumper. Best fit polynomials are plotted through the finite-element data.
Figure 77. The distribution of $\sigma_z$ is plotted for various FRP-to-wood stiffness ratios on a beam with a bumper. Best fit polynomials are plotted through the finite-element data.
Figure 78. The distribution of $\sigma_{xy}$ is plotted for various FRP-to-wood stiffness ratios on a beam with a bumper. Best fit polynomials are plotted through the finite-element data.
Table 12. Stresses and stress ratios at the end of the FRP lamina for various stiffness ratios for glulam with a bumper.

<table>
<thead>
<tr>
<th>Stiffness Ratio</th>
<th>0.05 in. Average Stress at Applied Load (psi)</th>
<th>0.15 in. Average Stress at Applied Load (psi)</th>
<th>Stress Ratio (Equation (11))</th>
<th>Stress Ratio (Equation (11))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.05 in. Average</td>
<td>0.15 in. Average</td>
<td>Stress Ratio (Equation (11))</td>
<td>Stress Ratio (Equation (11))</td>
</tr>
<tr>
<td>4.0</td>
<td>14310</td>
<td>11340</td>
<td>4.77</td>
<td>3.78</td>
</tr>
<tr>
<td>5.8</td>
<td>16480</td>
<td>12970</td>
<td>5.49</td>
<td>4.32</td>
</tr>
<tr>
<td>8.3</td>
<td>18790</td>
<td>14700</td>
<td>6.26</td>
<td>4.90</td>
</tr>
<tr>
<td>10</td>
<td>20070</td>
<td>15680</td>
<td>6.69</td>
<td>5.23</td>
</tr>
<tr>
<td>4.0</td>
<td>268</td>
<td>243</td>
<td>0.089</td>
<td>0.081</td>
</tr>
<tr>
<td>5.8</td>
<td>303</td>
<td>256</td>
<td>0.101</td>
<td>0.085</td>
</tr>
<tr>
<td>8.3</td>
<td>343</td>
<td>280</td>
<td>0.114</td>
<td>0.094</td>
</tr>
<tr>
<td>10</td>
<td>369</td>
<td>303</td>
<td>0.123</td>
<td>0.101</td>
</tr>
<tr>
<td>4.0</td>
<td>1115</td>
<td>1054</td>
<td>0.372</td>
<td>0.351</td>
</tr>
<tr>
<td>5.8</td>
<td>1335</td>
<td>1275</td>
<td>0.445</td>
<td>0.425</td>
</tr>
<tr>
<td>8.3</td>
<td>1569</td>
<td>1513</td>
<td>0.523</td>
<td>0.504</td>
</tr>
<tr>
<td>10</td>
<td>1697</td>
<td>1641</td>
<td>0.566</td>
<td>0.547</td>
</tr>
</tbody>
</table>
Figure 79. The effect of the FRP-to-wood stiffness ratio on $\sigma_x$ near the end of the FRP lamina is plotted for average stress from the end of the FRP lamina to 0.05 and 0.15 in. past the end of the FRP lamina.
Figure 80. The effect of the FRP-to-wood stiffness ratio on $\sigma_z$ near the end of the FRP lamina is plotted for average stress from the end of the FRP lamina to 0.05 and 0.15 in. past the end of the FRP lamina.
Figure 81. The effect of the FRP-to-wood stiffness ratio on $\sigma_{xy}$ near the end of the FRP lamina is plotted for average stress from the end of the FRP lamina to 0.05 and 0.15 in. prior to the end of the FRP lamina.
σ_z and σ_{xy}. The magnitudes of all stress distributions increase significantly with stiffness ratio.

**Effect of Reinforcement Length**

All beams represented in these models were 5-1/8 x 12.14 in. x Length ft and had a 0.14-in. thick 16.6 \times 10^6 psi FRP lamina. The models were subjected to third-point loads. The FRP lamina lengths are given in Table 6.

Figures 82 to 84 show the stress distributions, σ_x, σ_z and σ_{xy} respectively, for 60% length FRP lamina and for 40% length FRP lamina. It is apparent that length of FRP lamina does not contribute significantly to stress distributions.

**Effect of Beam Width**

All beams represented in these models were Width x 12.14 in. x 21 ft and had FRP lamina over 60% of the length. The models were subjected to third-point loads. The beam widths are given in Table 6.

Figures 85 to 87 show the stress distributions, σ_x, σ_z and σ_{xy} respectively for 5-1/8 in. and 10.75 in. beam widths. It is apparent that width does not significantly effect stress distributions at the end of the FRP lamina.
Figure 82. The distribution of $\sigma_x$ is plotted for two FRP lamina lengths on a beam with a bumper.
Figure 83. The distribution of $\sigma_z$ is plotted for two FRP lamina lengths on a beam with a bumper.
Figure 84. The distribution of $\sigma_{xy}$ is plotted for two FRP lamina lengths on a beam with a bumper.
Figure 85. The distribution of $\sigma_x$ is plotted for two beam widths on a beam with a bumper.
Figure 86. The distribution of $\sigma_z$ is plotted for two beam widths on a beam with a bumper.
Figure 87. The distribution of $\sigma_{xy}$ is plotted for two beam widths on a beam with a bumper.
**Effect of Span-to-depth Ratio**

All beams represented in these models were 5-1/8 x 12.14 in. x Length ft and had FRP lamina over 60% of the length. The models were subjected to third-point loads. The span-to-depth ratios are given in Table 6.

Figures 88 to 90 show the stress distributions, $\sigma_x$, $\sigma_z$ and $\sigma_{xy}$ respectively, for 21:1 and 27:1 span-to-depth ratios. It is apparent from these figures that span-to-depth ratio does not contribute significantly to stress distributions at the end of the FRP lamina.

**Summary of Analytical Results**

Magnitudes of stresses found at the end of the FRP lamina show stress levels in wood much greater than published strength values for Douglas-fir. While these stress levels may appear to be impossible or too high, it is important to consider these factors: 1) this is a linear model based on linear material properties, 2) the length of wood affected by the stress rise is less than half an inch, and 3) there is little understanding of how individual wood cells behave due to isolated shear and tensile stresses at a microscopic level.
Figure 88. The distribution of $\sigma_x$ is plotted for two span-to-depth ratios on a beam with a bumper.
Figur e 89. The distribution of $\sigma_z$ is plotted for two span-to-depth ratios on a beam with a bumper.
Figure 90. The distribution of $\sigma_{xy}$ is plotted for two span-to-depth ratios on a beam with a bumper.
As a model based on linear material properties, it does not incorporate nonlinear material behavior.

The second factor to consider is the localized nature of the stress riser. Tensile and other strength properties for wood are based on large specimens when compared to the region of the stress rise. The Wood Handbook (FPL, 1987) gives values between 12,400 and 12,600 psi for coast and interior west Douglas-fir at 12% moisture content. Individual fiber (0.1 to 0.25 in. in length) strengths for Douglas-fir range from 49,850 psi for earlywood to 138,000 psi for latewood (Bodig and Jayne, 1982).

It is understandable that FRP lamina thickness and stiffness are related to the stress distributions at the end of the FRP lamina. FRP lamina thickness and stiffness directly relate to the amount of force carried by FRP lamina that must be transferred to wood at the end of the FRP lamina.

Although beam width was thought to influence stress distributions at the edge, the model found no significant influence on stress distributions.

Length of FRP lamina was thought to be important since the distance over which stress is transferred to the beam was unknown; i.e. how stress is transferred through the glueline by shear.
Span-to-depth ratio was found to have no influence on stress distributions when beam depth was held constant. Uniform loading and third-point loading provided essentially the same stress levels.

The lack of influence due to FRP lamina length, span-to-depth ratio and third-point versus uniform loading conditions implied that the moment distribution near the end of the FRP lamina was more important than how the beam was loaded some distance from the end of the FRP lamina. However, a small increase in stress was observed for the 80% length reinforced beam and the beam with a span-to-depth ratio of 10 where the end of the FRP lamina was only 2 ft from the support. This suggests that the effect of the support caused the change in stress levels and not the length of FRP lamina or stress levels.

While there are many similarities in stress component distribution and parameters of influence between beams with and without a bumper, there are differences. A significant difference is due to the gap at the end of the FRP lamina in beams with a bumper. When the beam with a bumper is placed in bending, the gap causes the end of the FRP lamina to be somewhat compressed through its depth. This compression causes the stress $\sigma_y$ (peel stress) to be compressive instead of tensile. This effect is beneficial since a compressive peel stress is unlikely to cause the lamina to "peel" off.
There are also notable differences in the magnitudes of stress components. The stress $\sigma_x$ is similar for small FRP lamina thicknesses in both beam types, while the stress is greater for larger FRP lamina thickness in beams with a bumper. The larger $\sigma_x$ in beams with a bumper is also likely caused by the bumper bending more at the gap. The shear stress $\sigma_{xy}$ is also larger for beams with a bumper.
V. CONCLUSION

Experimental and analytical studies of stress distributions at the end of the FRP lamina for partially reinforced glulam were performed. Based on these analyses, stress levels are higher near the end of the FRP lamina than levels expected at the same location in a solid wood member. The most important stresses to consider include $\sigma_x$, $\sigma_y$, $\sigma_z$ and $\sigma_{xy}$ for glulam without a bumper and $\sigma_x$, $\sigma_z$ and $\sigma_{xy}$ for glulam with a bumper. FRP lamina thickness and the ratio of FRP lamina stiffness to wood stiffness have the greatest effect on stress levels of the variables studied.

While this study contributes to the understanding of stress distributions at the end of the FRP lamina in partially reinforced glulam, more research needs to be performed. In order to effectively apply the results of this study, stress levels predicted by the model need to be compared to load levels that cause failure in actual tests. Further research should be aimed toward reducing stress levels at the end of the FRP lamina by comparing various shapes of the FRP lamina tail.
BIBLIOGRAPHY


APPENDIX
**APPENDIX A**

Table A1. Coefficients for polynomial curve fits to finite-element stress distributions for glulam beams without a bumper, where thickness of reinforcement is varied.

<table>
<thead>
<tr>
<th>FRP Thickness (in.)</th>
<th>Coefficients of $\sigma_{i,j} = A_6 x^6 + A_5 x^5 + A_4 x^4 + A_3 x^3 + A_2 x^2 + A_1 x + A_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_e$</td>
<td>$A_6$  $(10^6)$  $A_5$  $(10^7)$  $A_4$  $(10^8)$  $A_3$  $(10^6)$  $A_2$  $(10^5)$  $A_1$  $(10^4)$  $A_0$  $(10^3)$</td>
</tr>
<tr>
<td>0.07</td>
<td>4.156  -0.8165  6.563  -2.844  0.7468  -1.271  1.736</td>
</tr>
<tr>
<td>0.14</td>
<td>6.074  -1.194  9.591  -4.147  1.083  -1.824  2.345</td>
</tr>
<tr>
<td>0.21</td>
<td>7.242  -1.422  11.40  -4.916  1.277  -2.138  2.694</td>
</tr>
<tr>
<td>0.28</td>
<td>8.236  -1.618  12.97  -5.589  1.449  -2.416  2.989</td>
</tr>
<tr>
<td>0.35</td>
<td>8.689  -1.704  13.64  -5.860  1.515  -2.519  3.117</td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>$A_6$  $(10^6)$  $A_5$  $(10^7)$  $A_4$  $(10^8)$  $A_3$  $(10^6)$  $A_2$  $(10^5)$  $A_1$  $(10^4)$  $A_0$  $(10^3)$</td>
</tr>
<tr>
<td>0.35</td>
<td>1.328  1.192  4.172  7.333  7.043  3.915  1.376</td>
</tr>
<tr>
<td>0.28</td>
<td>1.685  1.495  5.138  8.795  8.114  4.238  1.342</td>
</tr>
<tr>
<td>0.21</td>
<td>1.193  1.077  3.801  6.747  6.531  3.608  1.176</td>
</tr>
<tr>
<td>0.14</td>
<td>0.325  0.335  1.389  2.987  3.591  2.489  9.376</td>
</tr>
<tr>
<td>0.07</td>
<td>1.474  1.302  4.439  7.461  6.631  3.246  8.063</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>$A_6$  $(10^6)$  $A_5$  $(10^7)$  $A_4$  $(10^8)$  $A_3$  $(10^6)$  $A_2$  $(10^5)$  $A_1$  $(10^4)$  $A_0$  $(10^3)$</td>
</tr>
<tr>
<td>0.07</td>
<td>0  0  0  -3.408  1.929  -4.131  4.477</td>
</tr>
<tr>
<td>0.14</td>
<td>0  0  0  -2.705  1.779  -4.614  5.914</td>
</tr>
<tr>
<td>0.21</td>
<td>0  0  0  -4.216  2.558  -6.033  7.155</td>
</tr>
<tr>
<td>0.28</td>
<td>0  0  0  -3.544  2.336  -6.068  7.757</td>
</tr>
<tr>
<td>0.35</td>
<td>0  0  0  3.587  6.288  -9.191  8.817</td>
</tr>
<tr>
<td>$\sigma_{xy}$</td>
<td>$A_6$  $(10^6)$  $A_5$  $(10^7)$  $A_4$  $(10^8)$  $A_3$  $(10^6)$  $A_2$  $(10^5)$  $A_1$  $(10^4)$  $A_0$  $(10^3)$</td>
</tr>
<tr>
<td>0.35</td>
<td>8.885  1.390  8.282  2.366  3.352  2.373  2.387</td>
</tr>
<tr>
<td>0.28</td>
<td>9.384  1.464  8.685  2.461  3.424  2.305  2.184</td>
</tr>
<tr>
<td>0.21</td>
<td>8.299  1.293  7.663  2.166  3.009  2.041  1.981</td>
</tr>
<tr>
<td>0.14</td>
<td>6.614  1.032  6.122  1.736  2.435  1.705  1.701</td>
</tr>
<tr>
<td>0.07</td>
<td>6.534  1.012  5.943  1.659  2.246  1.386  1.158</td>
</tr>
</tbody>
</table>
Table A2. Coefficients for polynomial curve fits to finite-element stress distributions for glulam beams without a bumper, where stiffness of reinforcement is varied.

<table>
<thead>
<tr>
<th>FRP:Wood Stiffness Ratio</th>
<th>Coefficients of $\sigma_{1,2} = A_6x^6 + A_5x^5 + A_4x^4 + A_3x^3 + A_2x^2 + A_1x + A_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_x$</td>
<td>$A_6$ $(10^6)$ $A_5$ $(10^6)$ $A_4$ $(10^6)$ $A_3$ $(10^6)$ $A_2$ $(10^5)$ $A_1$ $(10^5)$ $A_0$ $(10^4)$</td>
</tr>
<tr>
<td>10.0</td>
<td>0 -2.087 3.671 -2.553 9.124 -1.840 2.514</td>
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<tr>
<td>8.3</td>
<td>0 -1.948 3.420 -2.371 8.440 -1.693 2.324</td>
</tr>
<tr>
<td>5.8</td>
<td>0 -1.682 2.942 -2.029 7.166 -1.423 1.980</td>
</tr>
<tr>
<td>4.0</td>
<td>0 -1.423 2.479 -1.700 5.952 -1.168 1.659</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>$A_6$ $(10^7)$ $A_5$ $(10^7)$ $A_4$ $(10^6)$ $A_3$ $(10^5)$ $A_2$ $(10^4)$ $A_1$ $(10^3)$ $A_0$ $(10^2)$</td>
</tr>
<tr>
<td>10.0</td>
<td>0 1.811 1.173 2.946 3.685 2.521 9.380</td>
</tr>
<tr>
<td>8.3</td>
<td>0 1.575 1.045 2.701 3.496 2.480 9.377</td>
</tr>
<tr>
<td>5.8</td>
<td>0 1.176 0.825 2.271 3.143 2.377 9.177</td>
</tr>
<tr>
<td>4.0</td>
<td>0 0.8398 0.636 1.889 2.902 2.244 8.746</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>$A_6$ $(10^5)$ $A_5$ $(10^5)$ $A_4$ $(10^5)$ $A_3$ $(10^5)$ $A_2$ $(10^4)$ $A_1$ $(10^3)$ $A_0$ $(10^2)$</td>
</tr>
<tr>
<td>10.0</td>
<td>-2.001 3.102 -1.596 0.1827 1.107 -4.410 6.309</td>
</tr>
<tr>
<td>8.3</td>
<td>0.8562 -1.847 1.637 -0.8418 2.620 -5.200 6.072</td>
</tr>
<tr>
<td>5.8</td>
<td>5.476 -9.838 6.942 -2.487 5.021 -6.376 5.570</td>
</tr>
<tr>
<td>$\sigma_{xy}$</td>
<td>$A_6$ $(10^7)$ $A_5$ $(10^7)$ $A_4$ $(10^7)$ $A_3$ $(10^6)$ $A_2$ $(10^5)$ $A_1$ $(10^4)$ $A_0$ $(10^3)$</td>
</tr>
<tr>
<td>10.0</td>
<td>9.118 8.228 2.879 4.966 4.433 2.129 1.863</td>
</tr>
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<td>8.3</td>
<td>9.892 8.918 3.117 5.371 4.784 2.238 1.714</td>
</tr>
<tr>
<td>5.8</td>
<td>11.03 9.924 3.462 5.951 5.276 2.370 1.440</td>
</tr>
<tr>
<td>4.0</td>
<td>11.70 10.51 3.662 6.278 5.538 2.405 1.180</td>
</tr>
</tbody>
</table>
Table A3. Coefficients for polynomial curve fits to finite-element stress distributions for glulam beams with a bumper, where thickness of reinforcement is varied.

<table>
<thead>
<tr>
<th>FRP Thickness (in.)</th>
<th>Coefficients of ( \sigma_{i,j} = A_6 x^6 + A_5 x^5 + A_4 x^4 + A_3 x^3 + A_2 x^2 + A_1 x + A_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_x )</td>
<td>( A_6 ) ( A_5 ) ( A_4 ) ( A_3 ) ( A_2 ) ( A_1 ) ( A_0 )</td>
</tr>
<tr>
<td>0.07</td>
<td>0 ( 3.914 ) ( -5.955 ) ( 3.4 ) ( -9.264 ) ( 1.593 )</td>
</tr>
<tr>
<td>0.14</td>
<td>0 ( 5.842 ) ( -8.849 ) ( 5.019 ) ( -13.54 ) ( 2.178 )</td>
</tr>
<tr>
<td>0.21</td>
<td>0 ( 7.37 ) ( -11.1 ) ( 6.29 ) ( -16.9 ) ( 2.63 )</td>
</tr>
<tr>
<td>0.28</td>
<td>0 ( 8.704 ) ( -13.12 ) ( 7.381 ) ( -19.67 ) ( 3.007 )</td>
</tr>
<tr>
<td>0.35</td>
<td>0 ( 9.8 ) ( -14.75 ) ( 8.293 ) ( -22.05 ) ( 3.33 )</td>
</tr>
<tr>
<td>( \sigma_y )</td>
<td>( A_6 ) ( A_5 ) ( A_4 ) ( A_3 ) ( A_2 ) ( A_1 ) ( A_0 )</td>
</tr>
<tr>
<td>0.07</td>
<td>0 ( 2.563 ) ( -2.752 ) ( 1.061 ) ( -1.597 ) ( -3.0 ) ( 2.54 )</td>
</tr>
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<td>0.14</td>
<td>0 ( 3.614 ) ( -3.898 ) ( 1.514 ) ( -2.320 ) ( 7.484 ) ( 3.564 )</td>
</tr>
<tr>
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<td>0 ( 4.739 ) ( -4.728 ) ( 1.839 ) ( -2.828 ) ( 11.45 ) ( 4.353 )</td>
</tr>
<tr>
<td>0.28</td>
<td>0 ( 4.940 ) ( -5.321 ) ( 2.062 ) ( -3.135 ) ( 1.606 ) ( 5.067 )</td>
</tr>
<tr>
<td>0.35</td>
<td>0 ( 5.512 ) ( -5.941 ) ( 2.304 ) ( -3.517 ) ( 6.227 ) ( 5.621 )</td>
</tr>
<tr>
<td>( \sigma_{xy} )</td>
<td>( A_6 ) ( A_5 ) ( A_4 ) ( A_3 ) ( A_2 ) ( A_1 ) ( A_0 )</td>
</tr>
<tr>
<td>0.07</td>
<td>0 ( -5.1 ) ( -4.238 ) ( -1.317 ) ( -1.875 ) ( -0.967 ) ( 0.962 )</td>
</tr>
<tr>
<td>0.14</td>
<td>0 ( -8.025 ) ( -6.620 ) ( -2.039 ) ( -2.879 ) ( -1.534 ) ( 1.37 )</td>
</tr>
<tr>
<td>0.21</td>
<td>0 ( -10.22 ) ( -8.388 ) ( -2.569 ) ( -3.601 ) ( -1.914 ) ( 1.683 )</td>
</tr>
<tr>
<td>0.28</td>
<td>0 ( -10.67 ) ( -8.787 ) ( -2.704 ) ( -3.822 ) ( -2.047 ) ( 1.948 )</td>
</tr>
<tr>
<td>0.35</td>
<td>0 ( -12.0 ) ( -9.89 ) ( -3.06 ) ( -4.34 ) ( -2.33 ) ( 2.17 )</td>
</tr>
</tbody>
</table>
Table A4. Coefficients for polynomial curve fits to finite-element stress distributions for glulam beams with a bumper, where stiffness of reinforcement is varied.

<table>
<thead>
<tr>
<th>FRP:Wood Stiffness Ratio</th>
<th>Coefficients of ( \sigma_{1,1} = A_6 x^6 + A_5 x^5 + A_4 x^4 + A_3 x^3 + A_2 x^2 + A_1 x + A_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_x )</td>
<td>( A_6 )</td>
</tr>
<tr>
<td>4.0</td>
<td>0</td>
</tr>
<tr>
<td>5.8</td>
<td>0</td>
</tr>
<tr>
<td>8.3</td>
<td>0</td>
</tr>
<tr>
<td>10.0</td>
<td>0</td>
</tr>
<tr>
<td>( \sigma_z )</td>
<td>( A_6 ) ((10^3))</td>
</tr>
<tr>
<td>4.0</td>
<td>0</td>
</tr>
<tr>
<td>5.8</td>
<td>0</td>
</tr>
<tr>
<td>8.3</td>
<td>0</td>
</tr>
<tr>
<td>10.0</td>
<td>0</td>
</tr>
<tr>
<td>( \sigma_{xy} )</td>
<td>( A_6 ) ((10^7))</td>
</tr>
<tr>
<td>4.0</td>
<td>0</td>
</tr>
<tr>
<td>5.8</td>
<td>0</td>
</tr>
<tr>
<td>8.3</td>
<td>0</td>
</tr>
<tr>
<td>10.0</td>
<td>0</td>
</tr>
</tbody>
</table>
APPENDIX B

ANSYS Input File for Beams Without a Bumper

/show
/prep7

BMLENGTH=21!!  !BEAM LENGTH (ft)
L=(BMLENGTH)*12/2  !!  !MODELED LENGTH (IN) INCLUDING 3 INCHES PAST SUPPORT
BMWIDTH=5.125  !!  !BEAM WIDTH (IN)
W=(BMWIDTH/2)  !MODELED BEAM WIDTH (IN)
H=12  !BEAM HEIGHT (IN)
HF=3  !HEIGHT OF FINE MESH (IN)
HT=5  !HEIGHT OF TRANSITION MESH (IN)
LENGTHRP=60  !!  !RP LENGTH (%)
LRP=(LENGTHRP/100)*L  !MODELED RP LENGTH (IN)
LLT=LRP-2
LRT=LRP+2
HRP=0.14  !!  !RP THICKNESS (IN)
LOADDIST=L/3  !!  !DISTANCE FROM CENTER TO LOAD (IN)
WDIV=8
LOAD=(((-1464314)/(WDIV+1)))  !!  !LOAD APPLIED AT TOP SURFACE NODES 32-36 IN APART (LB)

LIPE=12
LTIPE=8
LFRPIPE=0.15
LFWIPE=0.1
DDIV=12

MOEX=2E6  !!
ET,1,SOLID45

MP,EX,1,MOEX  !WOOD PROPERTIES
MP,EY,1,(0.059*MOEX)  !(AVG ET/EL AND ER/EL FROM WOOD HANDBOOK)
MP,EZ,1,(0.059*MOEX)  !"  "
MP,PRXY,1,((0.169e6)*MOEX)  !(AVG FROM vRL/EL AND vTL/EL FROM BODIG & JAYNE)
MP,PRXZ,1,((0.169e6)*MOEX)  !"  "
MP,PRYZ,1,0.41  !(WOOD HANDBOOK)
MP,GXY,1,0.115E6  !BODIG AND JAYNE
MP,GXZ,1,0.115E6  !"  "
MP,GYZ,1,0.012E6  !"  "

MP,EX,2,16.6E6  !!  !REINFORCEMENT PROPERTIES
MP,EY,2,0.4E6
MP,EZ,2,0.4E6
MP,GXY,2,0.50e6
MP,GXZ,2,0.50e6
MP, nuXY, 2, 0.36*0.4/16.6
MP, nuXZ, 2, 0.36*0.4/16.6
MP, nuYZ, 2, 0.3

!VOLUME 1
K, 1, 0, H, 0
K, 2, LOADDIST, H, 0
K, 3, LOADDIST, H, W
K, 4, 0, H, W
K, 5, 0, 0, 0
K, 6, LOADDIST, 0, 0
K, 7, LOADDIST, 0, W
K, 8, 0, 0, W
V, 1, 2, 3, 4, 5, 6, 7, 8

K, 9, LLT, H, 0
K, 10, LRP, H, 0
K, 11, LRP, H, W
K, 12, LLT, H, W
K, 13, LLT, 0, 0
K, 14, LRP, 0, 0
K, 15, LRP, 0, W
K, 16, LLT, 0, W
V, 9, 10, 11, 12, 13, 14, 15, 16
V, 2, 9, 12, 3, 6, 13, 16, 7

K, 17, LRT, H, 0
K, 18, L, H, 0
K, 19, L, H, W
K, 20, LRT, H, W
K, 21, LRT, 0, 0
K, 22, L, 0, 0
K, 23, L, 0, W
K, 24, LRT, 0, W
V, 17, 18, 19, 20, 21, 22, 23, 24
V, 10, 17, 20, 11, 14, 21, 24, 15

K, 25, 0, HRP, 0
K, 26, LOADDIST, -HRP, 0
K, 27, LOADDIST, -HRP, W
K, 28, 0, HRP, W
V, 5, 6, 7, 8, 25, 26, 27, 28

K, 29, LLT, -HRP, 0
K, 30, LRP, -HRP, 0
K, 31, LRP, -HRP, W
K, 32, LLT, -HRP, W
V, 13, 14, 15, 16, 29, 30, 31, 32
V, 6, 13, 16, 7, 26, 29, 32, 27

VGLUE, 1, 2, 3, 4, 5, 6, 7, 8

ILINES WHERE ELEMENT LENGTH IS UNIFORM
LSEL, S, LINE,, 1, 3, 2, 0
!TRANSITION ELEMENT LENGTH
LSEL,S,LINE,,29,31,2,0
LSEL,A,LINE,,34,38,4,0
Lesize,ALL,LTlPE,,1,1

LSEL,S,LINE,,26,28,1,0
LSEL,A,LINE,,61,62,1,0
Lesize,ALL,LTlPE,,2,1

LSEL,S,LINE,,25,25,1,0
Lesize,ALL,Ltipe,,0.5,1

LSEL,S,LINE,,15,15,1,0
LSEL,A,LINE,,18,22,4,0
LSEL,A,LINE,,54,58,2,0
Lesize,ALL,LtRPIPE,,25,1

LSEL,S,LINE,,13,13,1,0
Lesize,ALL,LtRPIPE,,0.04,1

LSEL,S,LINE,,42,44,1,0
Lesize,ALL,LFWlPE,,0.04,1

LSEL,S,LINE,,41,41,1,0
Lesize,ALL,LFWlPE,,25,1

LSEL,S,LINE,,7,11,2,0
LSEL,A,LINE,,17,23,2,0
LSEL,A,LINE,,35,39,2,0
Lesize,all,,DDIV,75,1

LSEL,S,LINE,,5,17,12,0
LSEL,A,LINE,,33,33,1,0
Lesize,ALL,,DDIV,1/75,1

LSEL,S,LINE,,2,4,2,0
LSEL,A,LINE,,8,8,1,0
LSEL,A,LINE,,14,16,2,0
LSEL,A,LINE,,20,20,1,0
LSEL,A,LINE,,30,32,2,0
LSEL,A,LINE,,36,40,4,0
LSEL,A,LINE,,48,52,4,0
LSEL,A,LINE,,56,60,4,0
Lesize,ALL,,WDIV,0.05,1

LSEL,S,LINE,,4,16,12,0
LSEL,A,LINE,,32,40,8,0
LSEL,A,LINE,,52,60,8,0
LSEL,A,LINE,,12,24,12,0
Lesize,ALL,,WDIV,20,1
LSEL,S,LINE,,45,51,2,0
LSEL,A,LINE,,53,59,2,0
LESIZE,ALL,,,2,1,1

ALLSEL
MAT,1
VMESH,1,5,1
MAT,2
VMESH,6,8,1

ALLSEL

NSEL,S,LOC,X,0,0
DSYM,SYMM,X
NSEL,S,LOC,Z,0,0
DSYM,SYMM,Z

FINISH

/SOLU
ANTYPE,STATIC
ALLSEL
NSEL,S,LOC,X,L,L
NSEL,R,LOC,Y,0,0
D,ALL,UY,0
D,ALL,UZ,0

ALLSEL

NSEL,S,LOC,X,LOADDIST,LOADDIST
NSEL,R,LOC,Y,H,H
F,ALL,FY,LOAD

allsel

SOLVE

/post1
ANSYS Input File for Beams With a Bumper

/prep7

BMLENGTH=21 !BEAM LENGTH (ft)
L=(BMLENGTH)*12/2 !MODELED LENGTH (IN) INCLUDING 3 INCHES PAST SUPPORT
BMWIDTH=5.125 !BEAM WIDTH (IN)
W=(BMWIDTH/2) !MODELED BEAM WIDTH (IN)
HRP=0.07 !RP THICKNESS
HBUMP=1.5 !BUMPER THICKNESS
HB=HRP+HBUMP !DISTANCE FROM TOP OF RP TO BOTTOM OF BUMPER
H=12-HBUMP !HEIGHT ABOVE RP
LENGTHRP=60 !RP LENGTH (IN)
LRP=(LENGTHRP/100)*L !MODELED RP LENGTH (IN)
LLT=LRP-2 !DISTANCE TO FINE MESH LEFT OF END RP
LRT=LRP+2 !DISTANCE TO FINE MESH RIGHT OF END RP
LOADDIST=L/3 !DISTANCE FROM CENTER TO LOAD (IN)
WDIV=8
LOAD=((-20023/4)/(WDIV+1)) !LOAD APPLIED AT TOP SURFACE NODES 32-36 IN APART (LB)
LIFE=12
LTIME=8
LFRPIPE=0.15
LFWIPE=0.1
DDIV=11

MOEX=2E6

ET,1,SOLID45

MP,EX,1,MOEX !WOOD PROPERTIES
MP,EY,1,(0.059*MOEX) !AVG ET/EL AND ER/EL FROM WOOD HANDBOOK
MP,EZ,1,(0.059*MOEX) ! "
MP,nuXY,1,((0.169e-6)*MOEX)*0.059 !AVG FROM \nuRL/EL AND \nuTL/EL FROM BODIG & JAYNE
MP,nuXZ,1,((0.169e-6)*MOEX)*.059 ! "
MP,nuYZ,1,0.41 !WOOD HANDBOOK
MP,GXY,1,0.115E6 !BODIG AND JAYNE
MP,GXZ,1,0.115E6 ! "
MP,GYZ,1,0.012E6 ! "

MP,EX,2,16.6E6 !REINFORCEMENT PROPERTIES
MP,EY,2,0.4E6
MP,EZ,2,0.4E6
MP,GXY,2,0.50e6
MP,GXZ,2,0.50e6
MP,GYZ,2,0.01e6
MP,nuXY,2,0.36*0.4/16.6
MP,nuXZ,2,0.36*0.4/16.6
MP,nuYZ,2,0.3
K,41,LRT,-HRP,0
K,42,L,-HRP,0
K,43,L,-HRP,W
K,44,LRT,-HRP,W
K,45,LRT,-HB,0
K,46,L,-HB,0
K,47,L,-HB,W
K,48,LRT,-HB,W
V,25,26,27,28,33,34,35,36
V,26,29,32,27,34,37,40,35
V,29,30,31,32,37,38,39,40
V,30,41,44,31,38,45,48,39
V,21,22,23,24,41,42,43,44
V,41,42,43,44,45,46,47,48

VGLUE,1,2,3,4,5,6,7,8,9,10,11,12,13,14

!LINES WHERE ELEMENT LENGTH IS UNIFORM
LSEL,S,LINE,,1,3,2,0
LSEL,A,LINE,,6,10,4,0
LSEL,A,LINE,,46,50,4,0
LSEL,A,LINE,,64,68,4,0
LESIZE,ALL,LIPE,,,1,1

!TRANSITION ELEMENT LENGTH
LSEL,S,LINE,,29,31,2,0
LSEL,A,LINE,,34,38,4,0
LSEL,A,LINE,,90,94,4,0
LSEL,A,LINE,,97,100,3,0
LESIZE,ALL,LTIP,E,,,1,1

LSEL,S,LINE,,26,28,1,0
LSEL,A,LINE,,61,62,1,0
LSEL,A,LINE,,72,75,3,0
LESIZE,ALL,LTIP,E,,,2,1

LSEL,S,LINE,,25,25,1,0
LESIZE,ALL,LTIP,E,,,0.5,1

LSEL,S,LINE,,15,15,1,0
LSEL,A,LINE,,18,22,4,0
LSEL,A,LINE,,54,58,2,0
LSEL,A,LINE,,77,80,3,0
LESIZE,ALL,LFRPIPE,,25,1

LSEL,S,LINE,,13,13,1,0
LESIZE,ALL,LFRPIPE,,0.04,1

LSEL,S,LINE,,42,44,1,0
LSEL,A,LINE,,83,83,1,0
LSEL,A,LINE,,85,88,3,0
LESIZE,ALL,LFWIPE,,0.04,1

LSEL,S,LINE,,41,41,1,0
LSEL,A,LINE,,81,81,1,0
LESIZE,ALL,LFWIPE,,25,1

LSEL,S,LINE,,7,11,2,0
LSEL,A,LINE,,17,23,2,0
LSEL,A,LINE,,35,39,2,0

lesize,all,,,72,1

LSEL,S,LINE,,5,17,12,0
LSEL,A,LINE,,33,33,1,0
LESIZE,ALL,1,,1/72,1

LSEL,S,LINE,,2,4,2,0
LSEL,A,LINE,,8,8,1,0
LSEL,A,LINE,,14,16,2,0
LSEL,A,LINE,,20,20,1,0
LSEL,A,LINE,,30,32,2,0
LSEL,A,LINE,,36,40,4,0
LSEL,A,LINE,,48,52,4,0
LSEL,A,LINE,,56,56,1,0
LSEL,A,LINE,,66,66,1,0
LSEL,A,LINE,,73,78,5,0
LSEL,A,LINE,,82,86,4,0
LSEL,A,LINE,,92,98,6,0
LESIZE,ALL,,,WDIV,0.05,1

LSEL,S,LINE,,4,16,12,0
LSEL,A,LINE,,32,40,8,0
LSEL,A,LINE,,52,60,8,0
LSEL,A,LINE,,12,24,12,0
LSEL,A,LINE,,70,70,1,0
LESIZE,ALL,,,WDIV,20,1

!RP DEPTH AND FILLER DEPTH
LSEL,S,LINE,,45,51,2,0
LSEL,A,LINE,,53,59,2,0
LSEL,A,LINE,,91,95,4,0
LSEL,A,LINE,,89,93,4,0
LESIZE,ALL,,1,1,1

!BUMPER DEPTH
LSEL,S,LINE,,63,71,2,0
LSEL,A,LINE,,74,76,2,0
LSEL,A,LINE,,79,79,1,0
LSEL,A,LINE,,96,99,3,0
LSEL,A,LINE,,84,87,3,0
ALLSEL

MAT,1
VMESH,1,5,1
VMESH,9,14,1
MAT,2
VMESH,6,8,1

AUTOMATICALLY SCALED AND SPACED
ALLSEL

NSEL,S,LOC,X,0,0
DSYM,SYMM,X
NSEL,S,LOC,Z,0,0
DSYM,SYMM,Z

FINISH

/SOLU
ANTYPE,STATIC
ALLSEL
NSEL,S,LOC,X,L,L
NSEL,R,LOC,Y,0,0
D,ALL,UY,0
D,ALL,UZ,0

ALLSEL

NSEL,S,LOC,X,LOADDIST,LOADDIST
NSEL,R,LOC,Y,H,H
F,ALL,FY,LOAD

allsel
SOLVE

/POST1