AN ABSTRACT OF THE THESIS OF

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Heuristics based on Monte-Carlo Integer Programming (MCIP) and Tabu Search (TS) techniques were developed for generating easily implementable bucking rules that are applicable to entire stands (as opposed to individual stem classes), and for selecting the rule-set that provides the best feasible solution (given log prices and market constraints). A rule-set correspond to the definition of the attributes (e.g., minimum end-diameter and acceptable grades) that the different log-types must satisfy to be acceptable for bucking from a stem, and its role is to provide guidelines for the bucking activities.

The MCIP method consists of randomly generating the bucking rules, with the log-types' attributes subjected to certain limitations. To generate candidate solutions efficiently, several Monte-Carlo simulation controls were analyzed. Such controls include the stopping criteria, the search region, and the probability distribution used for generating

bucking rules. The TS method corresponds to a search process, with restrictions imposed on the export logs' attributes for guiding the search. The imposition of tabu-restrictions on candidate bucking rules helps the search process to avoid becoming trapped at locally optimal solutions --a condition frequently encountered in patterned searches. Refinements of Tabu Search used in this study are: [1] parallel tabu-lists of different time-span for the different tabu-restrictions, and [2] an aspiration criterion consisting of improving upon the best feasible solution.

Both rule-based models (TS and MCIP) were applied to a set of radiata pine (Pinus radiata, D.Don) stands, considering different market constraints and price sets. The TS and MCIP results were compared to solutions obtained by an iterative two-stage Linear Programming / Shortest Path model and an integer programming (IP) model that were designed for selecting the best set of stem-specific bucking patterns. The rule-based approaches lead to bucking outcomes that are reasonably close to the optimal solutions defined by the IP model. The comparatively sophisticated TS technique, however, provided solutions to the sample problems that are only marginally better than those obtained using a simple MCIP approach.

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Bucking Optimization Methods

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INTRODUCTION

The management of forest resources requires that stand valuation be accurate, and based upon intended use. Timber valuation depends on decisions regarding harvesting, bucking, and log allocation. Therefore, the management planner should be able to define the best, implementable, stand-level bucking patterns (sequence of log-types to be obtained from a stem) given log prices, grades, market restrictions, and production constraints.

Several methods have been developed for solving the optimal bucking problem. The review of the existing approaches shows that: [1] the methods that guarantee optimality through complex bucking patterns are difficult to implement, and [2] the methods that provide easy to implement bucking guidelines cannot guarantee optimality. Therefore, none of the methods provides a good estimation of the maximum potential value that can be obtained from a stand in a log marketing framework.

The objective of this study was to develop and test methods for generating feasible, near optimal bucking rules that are user-independent (and therefore should reduce the profit gap between "true" optimal solutions and "educated guesses"). In accomplishing this objective, this dissertation provides three original methodological contributions: [1] the use of an Integer Programming model for allocating bucking patterns to a stand's diameter-quality classes based on the activities generated by a Linear Programming / Shortest Path model; [2] the use of Monte-Carlo Integer Programming (a technique successfully applied to several forest management problems) for generating and selecting bucking rules; and [3] defining optimal or near optimal bucking rule-sets using Tabu Search --a promising technique for solving combinatorial problems related to forest management.

The bucking optimization methods were compared using a sample of radiata pine (Pinus radiata, D.Don) stands belonging to Forestal Chile S.A. Radiata pine plantations were selected, because in young, homogenous even-aged stands log dimensions as opposed to log grades are the major factor influencing the bucking activities. Forestal Chile S.A. was chosen because it is currently using bucking rules for its bucking decisions. The Japanese market for radiata pine logs was selected as a reference for prices and constraints applicable to log exports. This market was chosen because it represents a significant customer for Forestal Chile S.A. and also because it is a well structured market (the constraints are well defined and have been stable over time). Several variations of the market specifications and log prices were considered for comparing how the bucking optimization methods perform in different situations.

LITERATURE REVIEW

Stem-Level Bucking Optimization

Pneumaticos and Mann (1972) developed a dynamic programming (DP) algorithm to obtain the optimal bucking pattern for an individual stem given specific log dimensions, grades, and prices. This algorithm leads to the optimal stand-level solution only if no market constraints are applicable or if they are coincidentally satisfied.

Stand-Level Bucking Optimization

Eng and Daellenbach (1985) and Mendoza and Bare (1986) designed similar methods to determine the optimal bucking schedule for an entire stand subject to production constraints. Their method involves iteratively solving a linear programming (LP) master problem with new activities generated by solving DP subproblems defined based on Dantzig-Wolfe's (1961) decomposition technique. This approach is theoretically correct and computationally efficient, but the bucking schedules obtained are of little practical use since they require the bucking crew to consider an excessive amount of information. In general, this situation cannot be corrected by allowing only one bucking pattern per stem-class, because many patterns will still enter the optimal solution as similar stem classes typically have different bucking schedules. The end result of this problem is that the stand's value is usually overestimated since it is very unlikely that the bucking crew will produce the expected amount of each log-type. Another drawback of this method is that it is prescriptive and when a stem's actual taper, grade, or breakage does not permit the realization of the prescribed pattern it allows for no contingencies.

Price-Adjusted Bucking Optimization

Sessions et al. (1989) developed an algorithm to efficiently solve the problem of optimally bucking an individual tree, based on Dijkstra's (1959) shortest-path (SP) algorithm. Their bucking algorithm was used jointly with a shadow-price search mechanism for obtaining a vector of adjusted prices (one per log-type) that guarantee satisfying stand-level market constraints, while still solving the optimal bucking pattern for each stem independently. This approach requires that all of the bucking patterns be generated using a single vector of adjusted prices and it leads to profits lower than the maximum potentially obtainable (as reflected by Eng and Daellenbach's model). Sessions et al.'s algorithm has the significant advantage of being able to define bucking patterns on-site: it has been implemented on a handheld computer and can be very useful when bucking large, valuable trees. Operationally, this method requires the bucking crew to make several measurements on each tree, and when applied to smaller trees in an intensive production framework the gains in revenue may not compensate for the increase in production costs. Murphy (1993) improved upon Sessions et al.'s approach to adjusting prices by using Hooke and Jeeves (1961) pattern search procedure, and imposing restrictions on the logs' small end-diameter.

Bucking Simulation

Laroze (1985) implemented a bucking simulator for radiata pine plantations. The simulator bucks the trees following a set of rules defined by the user. A rule-set consists of the definition of the physical attributes that each log-type must satisfy (e.g., minimum end-diameter, acceptable grades, and maximum number of logs of that type that can be obtained from a stem). The simulator can easily lead to feasible solutions, but cannot guarantee that good rules will be selected: the quality of the solutions obtained is highly dependent on the user's expertise in defining an appropriate set of rules for a particular stand and market conditions. Its main advantage, however, is that the rules can be directly translated into bucking patterns that are easy to implement: the bucking crew needs only to follow a priority-list (with log-types ordered according to their profit) and check if the log's specifications are compatible with the dimensions and quality of the current section of the stem.

Monte-Carlo Integer Programming

According to Conley (1980), Monte-Carlo Integer Programming (MCIP) is a simple approach for solving combinatorial optimization problems. This technique has the advantage of not being affected by complexities or nonlinearities of the objective and/or constraint functions, and the system of equations and constraints does not need to be tested for redundancy or cycling (as in traditional optimization techniques). The MCIP method is also easier theoretically and readily implementable. The author does recognize, however, that MCIP is not a substitute for mathematical programming methods but rather a complementary technique that quickly obtains good solutions to numerous practical problems. The MCIP method relies on: [1] current computer technology that allows for rapidly exploring a problem's solution space by randomly generating a large number of solutions, and [2] the statistical properties of such randomly generated solutions.

The use of Monte-Carlo techniques in forest management has been demonstrated by Bullard et al. (1985), O'Hara et al. (1989), and Nelson and Brodie (1990). Bullard et al. (1985) formulated the problem of defining the optimal stand density through time for even-aged, mixed-species stands using a nonlinear-integer programming model with the decision variables being the number of trees cut by species and diameter class. They developed a random search algorithm for solving the problem and, for sample problems with two species, their heuristic provided near-optimal cutting strategies with very little computer time and memory. O'Hara et al. (1989) and Nelson and Brodie (1990) applied

MCIP to solve complex mixed-integer combinatorial problems involving spatially constrained, forest-level harvest scheduling plans. Both studies report, for their sample problems, solutions that are within 4% of the true optima.

Tabu Search

According to Glover (1990b), Tabu Search (TS) is a metaheuristic for solving optimization problems that can be superimposed on other procedures to prevent them from becoming trapped at locally optimal solutions. The method can be used to guide any process that employs a set of moves for transforming one solution (or solution state) into another and that provides an evaluation function for measuring the attractiveness of these moves. It uses flexible memory structures (to permit search information to be exploited more thoroughly than by rigid memory systems or memoryless systems), conditions for strategically constraining and freeing the search process (embodied in tabu restrictions and aspiration criteria), and memory functions of varying time spans for intensifying and diversifying the search (reinforcing attributes historically found good and driving the search into new regions).

Tabu Search has obtained optimal and near optimal solutions to a wide variety of classical and practical combinatorial problems. In many problem settings, TS led to consistently better solutions than other algorithms. In other cases, TS has demonstrated advantages in ease of implementation or in the ability to handle additional constraints. A detailed introduction to TS is presented by Glover and Laguna (1993). A concise description of TS's main features and a summary of computational experience for a variety of applications can be found in Glover (1989). More advanced aspects of TS and ways for applying it for solving integer programming problems are described by Glover (1990a). A presentation of TS emphasizing a perspective for guiding a user to understand basic implementation principles for solving combinatorial and nonlinear problems is provided by Glover et al. (1993).

DATA

Stand Characteristics

A set of 23 harvest-age radiata pine stands located near Concepción, Chile were selected for this study. They were selected considering different combinations of density, site class, and management conditions: the objective was to represent many diameter-height distributions. Table 1 summarizes the stand inventory information. (The set of stand tables is presented in Appendix 1).

Log Characteristics

Forestal Chile recognizes five different quality classes of tree stems. Six different log-types are used for defining the marketable products. The log-types have differential values depending on the quality class, and some classes are not suitable for some types of logs. The export logs selected for the analyses correspond to the "current" Japanese specifications for this species (December, 1992). The domestic logs are defined according to standard dimensions. Four different price sets were considered to determine solution sensitivity to relative prices. The log specifications, prices, acceptable qualities, and quality-related price adjustments are presented in Table 2.

TABLE 1. Summary of stand inventory information

Stand	Age [years]	Number Trees [/ha]	Mean DBH [cm]	Basal Area [m2/ha]	Mean Height [m]	Volume [m3/ha]
1	23	291	37.8	33.9	31.5	382.9
2	23	309	33.6	28.5	26.7	277.4
3	23	334	34.6	32.8	30.8	360.7
4	23	339	34.0	32.0	28.6	329.1
5	23	573	30.9	45.3	31.2	520.0
6	24	208	48.4	39.6	34.9	492.8
7	24	245	39.2	30.6	31.8	343.1
8	24	343	38.6	41.3	35.2	525.7
9	24	345	37.2	39.9	33.4	489.1
10	24	473	41.0	67.7	34.4	870.8
11	24	866	26.9	53.1	27.6	542.4
12	25	945	28.5	65.4	27.5	673.8
13	26	301	41.0	41.3	34.4	505.9
14	26	612	34.0	61.4	32.2	759.5
15	26	777	28.6	56.2	26.9	584.7
16	27	286	42.6	42.6	32.5	491.3
17	27	715	30.5	56.0	31.9	652.0
18	27	921	32.8	83.7	29.6	957.2
19	29	343	35.2	35.5	31.7	405.1
20	29	377	40.0	49.8	33.6	606.4
21	29	963	29.6	73.6	28.3	796.7
22	29	1015	27.1	62.4	26.1	591.9
23	30	442	39.8	58.8	30.6	655.4
Minimum	23	208	26.9	28.7	26.1	277.3
Median	25	369	34.6	45.4	31.5	525.6
Maximum	30	1015	48.4	83.8	35.2	957.3
Range	7	807	21.5	55.1	9.1	680.0

TABLE 2. Definition of log types

Log type (Product)	Length [m]	Minimum end-diameter [cm]	Prices [$/m3]				Price adjustment factors by quality class				
			Set 1	Set 2	Set 3	Set 4	1	2	3	4	5
1 (export-long)	12.1	20	44	49	54	59	1.000	0.950	---	---	---
2 (export-intermediate)	8.1	20	42	47	52	57	1.000	0.975	0.950	---	---
3 (export-short)	4.1	20	40	45	50	55	1.000	1.000	0.965	---	---
4 (domestic saw-log)	4.1	26	40	40	40	40	1.000	0.975	0.950	0.925	---
5 (domestic saw-log)	4.1	16	32	32	32	32	1.000	1.000	1.000	0.975	---
6 (domestic pulp-log)	2.5	8	25	25	25	25	1.000	1.000	1.000	1.000	1.000

Market Constraints

Typically, no constraints apply to domestic logs other than acceptable grades and dimensions. The aggregate shipment of export logs is, however, usually subjected to market constraints that guarantee certain load size attributes. The restrictions commonly encountered are: [1] a lower bound on the average end-diameter, [2] a minimum volumetric proportion of long-logs, and [3] a maximum volumetric proportion of short-logs. Eight different market sets were considered for this study: seven representing different parameters for the export restrictions plus a scenario that considered the absence of an export market. The type of logs included in each set and the parameters of the market constraints are shown in Table 3.

The combination of a market set and a price set corresponds to a market scenario. The objective of these scenarios is to compare how the bucking optimization methods perform in different situations. Market Set 6 and Price Set 1 correspond to the restrictions and prices prevailing at the time this study began (December, 1992). The average end-diameter is weighted by the number of logs, and volume is estimated using the Japanese Agriculture Standards (JAS) cubic-volume log scale described next.

Log length < 6.0 [m] : $V = L*D^2$

Log length ≥ 6.0 [m] : $V = L*(D + (L-4)/2)^2$

Where:

V : Volume [m^3].

L : Log's length rounded down to an integer [m].

D : Small end-diameter reduced to the lower even number [cm].

TABLE 3. Definition of market constraints

Set	Logs included	Minimum average end-diameter [cm] /a	Minimum volumetric proportion of long logs [%]	Maximum volumetric proportion of short logs [%]
1	All	---	---	---
2	All	24	70	10
3	All	26	70	10
4	All	28	70	20
5	All	28	60	10
6	All	28	70	10
7	All	30	70	10
8	Domestic only	---	---	---

/a: Weighted by number of logs.

DEFINITIONS

Bucking Pattern

A bucking pattern corresponds to a sequence of log-types obtained from a stem ($\mathbf{x} = [y_{(1)}, \ldots, y_{(m)}, \ldots, y_{(n)}]$; were y indicates the log-type, *(m)* the position in the stem, and n the total number of logs). If each log's attribute with respect to a market constraint *(k)* is represented by $\delta_{k(m)}$, then the technical coefficients corresponding to a bucking pattern can be represented by $\alpha_k = f_k[\delta_{k(1)}, \ldots, \delta_{k(n)}]$ (a function of the attributes of the logs obtained from the stem). Similarly, if each log has a value of $\lambda_{(m)}$, which is based on its type and attributes, then the profit corresponding to a bucking pattern is defined by $\pi = \Sigma_m \lambda_{(m)}$ (the summation of the logs' individual values).

Several mechanisms can be used for generating bucking patterns (e.g., a stem-level optimization procedure such as Sessions et al.'s, 1989). The approach used by the rule-based methods developed in this study is defined in the following section.

Bucking Algorithm

The bucking algorithm starts at the base of each class-representative tree and analyzes the first log in a priority-list to see if the log's specifications are compatible with the dimensions and quality of the current section of the stem (e.g., are end-diameter and quality within acceptable ranges). If compatible, the algorithm cuts that type of log from the tree and moves to the next section. If incompatible, it continues with the next log in the list. This routine is repeated until all the log-types have been examined. If the log-type being examined can be obtained from the current stem section it will be bucked without considering upper stem bucking decisions. Once a log-type is rejected for a particular stem section it will not be considered again for any of the remaining sections of the tree. A log-type of lower priority will never be bucked before a log-type of higher priority. Figure 1 shows a graphical representation of this algorithm. This process is repeated for all diameter-quality classes.

Bucking Rules

For the purposes of this study, a bucking rule consists of [1] a priority-list of log-types based on net return (a log with higher profit is located higher in the list) and [2] three key attributes for each type of log: minimum end-diameter, acceptable quality

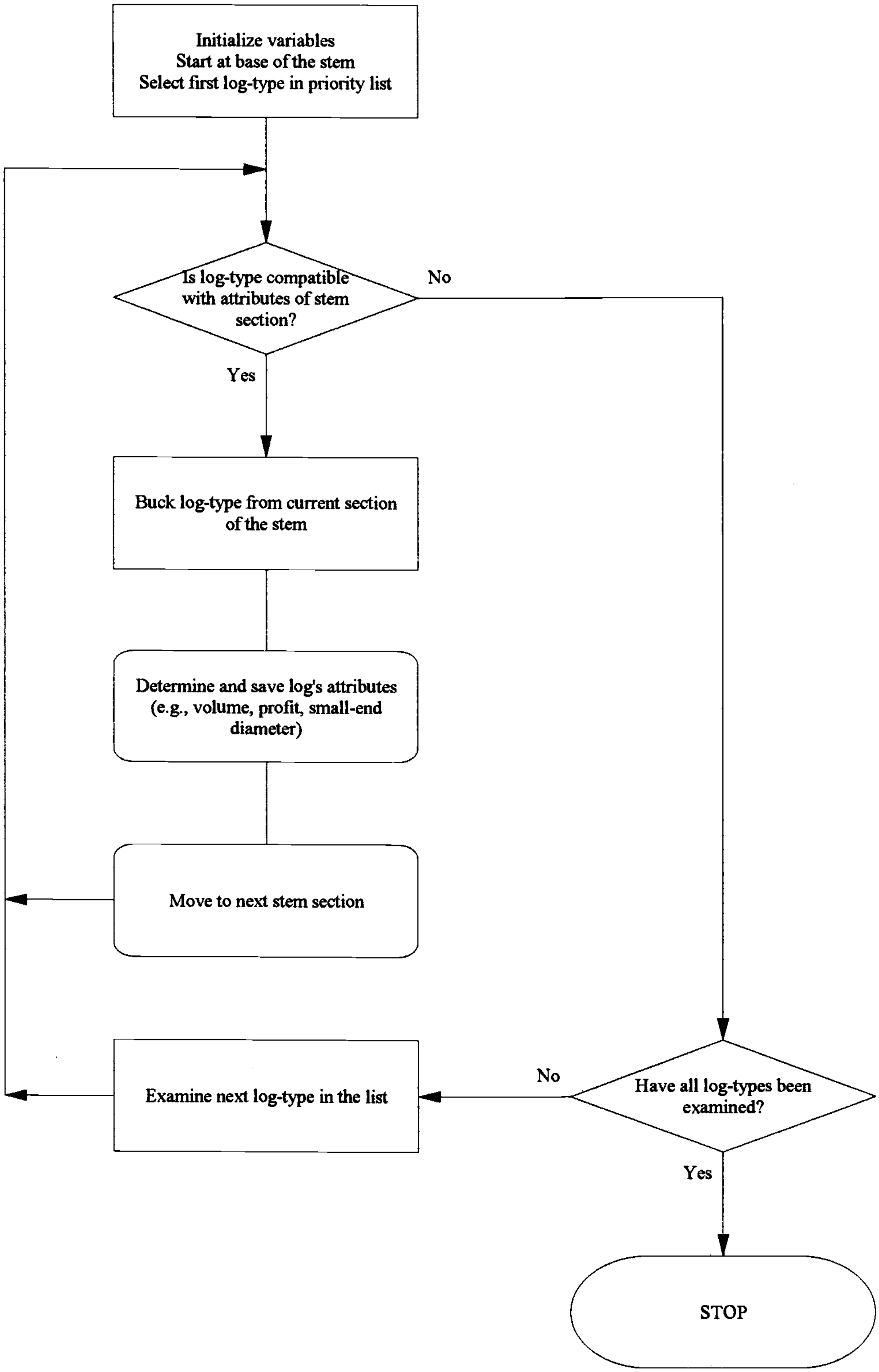

Figure 1. Bucking algortihm applied to a class-representative stem

range, and maximum number of logs of that type that can be obtained from an individual tree.

Logs are prioritized on the basis of net return due to practical considerations: [1] the most profitable logs are the long export logs which are also required in a large proportion, [2] it is an ordering that is generally consistent with the company's priorities (at individual stands) and permits readily implementable production incentives based upon log-types (to be applied across stands), and [3] it reduces the complexities in monitoring the bucking crews' performance.

The attributes used for defining bucking rules were selected because of the market constraints considered. For example, [1] increasing the minimum end-diameters increases the average end-diameter; [2] changing the minimum end-diameter for a log-type will modify its volumetric proportion (e.g., a lower minimum end-diameter tends to increase its total volume); [3] limiting the range of acceptable quality classes can modify both the volumetric proportions and the average end-diameter, but it is a more selective control since it does not apply to all the stem classes (this parameter is particularly useful when changing the minimum end-diameters may produce an over-reaction); and [4] limiting the number of logs of a certain type that can be obtained from a stem has an impact only in a subset of the diameter classes, therefore it is useful as a fine-tuning control. Other key log characteristics for defining bucking rules were analyzed and later omitted since they did not contribute to improved solutions: a parsimonious set of specifications was preferred.

The controls not used were: maximum end-diameter, maximum height in the tree where a certain kind of log could be obtained, and sections of the stem allowable for obtaining a type of log (e.g., a certain log-type could only be the first or second one bucked).

Rule-Based Bucking Methods

To generate feasible solutions when using bucking rules in combination with the bucking algorithm previously described, the parameters corresponding to the log-types usually have to be defined more restrictively than the minimum standards specified by the customers (e.g., imposing an end-diameter larger than 20 cm. for the long export logs, allowing intermediate export logs to be produced only in quality 1 trees, and not permitting more than one short export log from a stem).

A rule-based method consists of two stages: [1] a procedure for generating the parameters corresponding to the attributes defined for each log-type, and [2] using the bucking algorithm for evaluating the performance of specific parameters with respect to profitability and feasibility. Log parameters are applied across all stem classes, and only one bucking pattern is considered for each stem class for a given functional evaluation. Each pattern is generated by applying the bucking algorithm to each class-representative tree on the basis of the bucking rules. These methods generate integer solutions, since all of the trees in a stem class are bucked the same way. But rather than selecting one distinct pattern per stem class, one rule-set is selected per stand. (By contrast, an IP model selects one bucking pattern per stem class from among several patterns specially defined considering stem-specific characteristics.)

Two bucking-rules methods, based on Monte-Carlo Integer Programming and Tabu Search techniques, were considered in this study. Their features, and the characteristics of Linear Programming and Integer Programming models designed for solving the optimal bucking problem, are presented in the next section.

SOLUTION METHODS

Linear Programming Method

The bucking optimization problem for an individual stand can be represented by the following LP model, based on the formulation developed by Eng and Daellenbach (1985).

Maximize: $\Sigma_i \Sigma_j \ \pi_{ij} \, x_{ij}$

Subject to: $\Sigma_i \Sigma_j \ \alpha_{ijk} \, x_{ij} \ (\leq, =, \geq) \ \beta_k \quad [\mu_k] \qquad (k = 1, \ldots, K)$

$\Sigma_j \, x_{ij} \ \leq \ \eta_i \qquad (i = 1, \ldots, I)$

$x_{ij} \ \geq \ 0 \qquad \forall_{i,j}$

Where:

x_{ij} : Number of stems in class **i** bucked by pattern **j** (decision variable).

π_{ij} : Profit corresponding to a stem in class **i** bucked by pattern **j**.

α_{ijk} : Contribution of a stem in class **i** bucked by pattern **j** to constraint **k**.

β_k : Market constraint **k**.

μ_k : Dual variable (shadow-price) corresponding to constraint **k**.

η_i : Number of stems in class **i**.

The optimal bucking problem is solved using an iterative two-stage LP/SP procedure, similar to the method developed by Eng and Daellenbach (1985) with delayed column generation based on Dantzig-Wolfe's (1961) decomposition technique. The approach used in this study differs in the use of the SP algorithm developed by Sessions et al. (1989) for generating the activities to be included in the LP model (instead of the DP algorithm developed by Pneumaticos and Mann, 1972).

A diagram of the iterative two-stage LP/SP method is presented in Figure 2. This chart was adapted from Mendoza and Bare (1986). Figure 3 shows a network representation of the optimal stem-bucking problem: the nodes indicate the sections where the tree can be bucked and the arcs correspond to the different log-types. This diagram was adapted from Sessions et al. (1989).

In each iteration, a candidate bucking pattern (activity) is generated by solving a shortest-path problem for each stem class, based on the logs' specifications and current values of the dual variables; the class-representative tree is optimally bucked considering log prices adjusted by shadow-prices derived from the previous LP solution. The SP algorithm seeks to define the sequence of log bucking decisions that indirectly maximizes profits. The iterative process finishes when no new economically relevant activity is generated. The solution obtained by this method corresponds to the maximum value that can be obtained from each individual stand given a set of prices and market constraints--and no limitations on the bucking patterns.

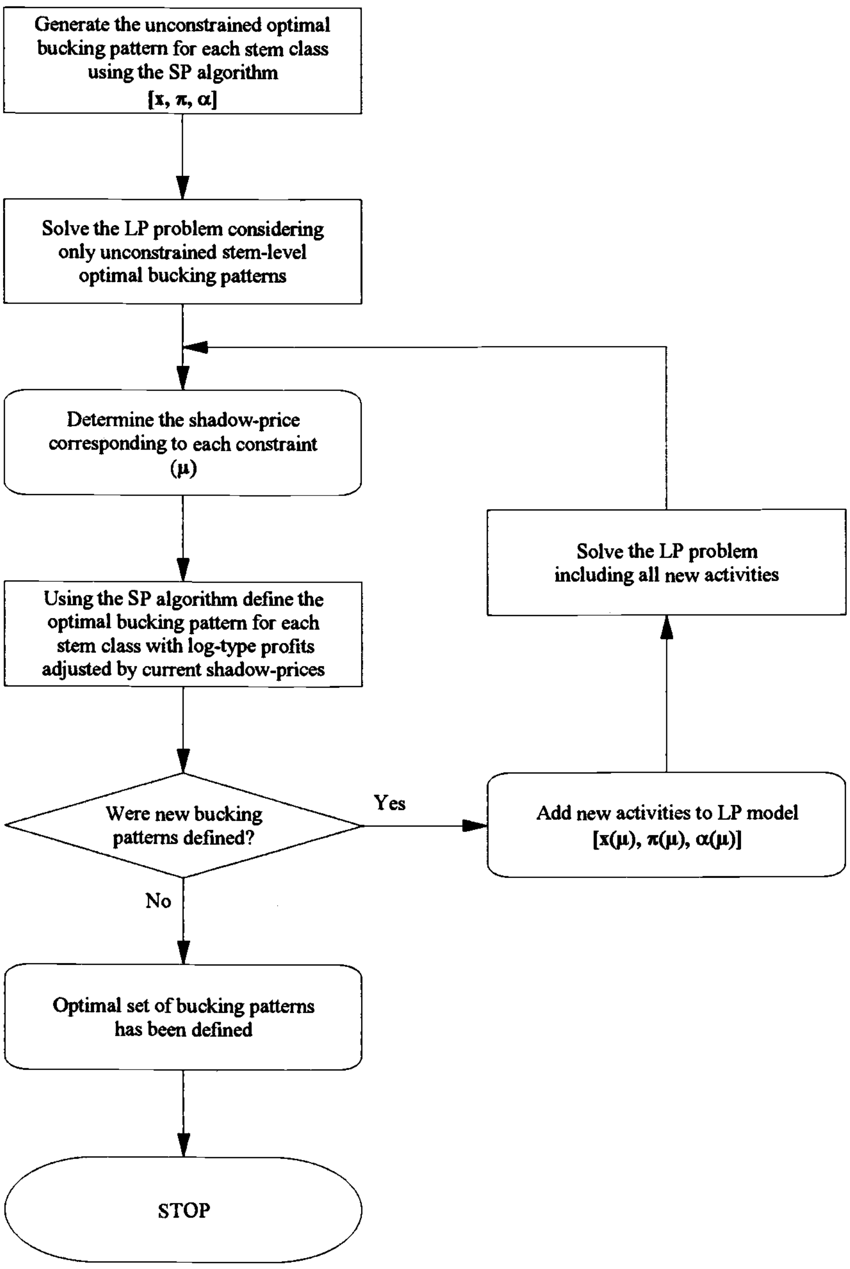

Figure 2. Iterative two-stage LP/SP method

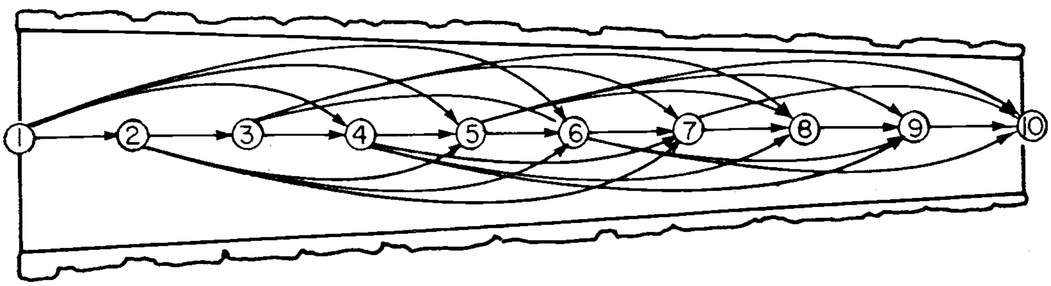

Figure 3. Network representation of the optimal stem-bucking problem.

Integer Programming Method

The integer programming version of the bucking optimization problem for an individual stand can be represented by the following multi-dimensional knapsack model.

Maximize: $\Sigma_i \Sigma_j (\eta_i * \pi_{ij}) x_{ij}$

Subject to: $\Sigma_i \Sigma_j (\eta_i * \alpha_{ijk}) x_{ij} \ (\leq, =, \geq) \ \beta_k$ $\quad (k = 1, \ldots, K)$

$\Sigma_j x_{ij} \leq 1$ $\quad (i = 1, \ldots, I)$

$x_{ij} \in \{0, 1\}$ $\quad \forall_{i,j}$

Where:

x_{ij} : Binary decision variable related to bucking all stems in class **i** by pattern **j**.

π_{ij} : Profit corresponding to a stem in class **i** bucked by pattern **j**.

α_{ijk} : Contribution of a stem in class **i** bucked by pattern **j** to constraint **k**.

β_k : Market restrictions.

η_i : Number of stems in class **i**.

The bucking patterns included in the model are: [1] the complete set of activities generated by solving the LP version of the bucking optimization using the iterative two-stage process recently described; and [2] for each stem class a bucking pattern based only on domestic-logs (if it had not been included in the previous set). The set of relevant activities defined for the LP problem using the decomposition technique does not guarantee that all the relevant activities for the IP version will be considered. In particular, since domestic log production is not subjected to constraints, bucking patterns that yield only domestic logs may not be generated for every stem class by the SP algorithm given the shadow-price adjustments. These additional activities do, however, provide more flexibility to the IP formulation precisely because they are not subject to market restrictions.

An IP model was considered because its solution represents the best outcome obtainable from a mechanized harvester enhanced with optimal bucking features, or from optimal-bucking machinery implemented with a scanner in a sort-yard. Consequently, the

IP solutions correspond to more realistic upper bounds for solution methods based on bucking rules.

Monte-Carlo Integer Programming Method

The MCIP approach randomly generates the parameters for each log-type, and then selects the rule-set that provides the best feasible solution given the log prices and market constraints. A new set of parameters is defined at each iteration (functional evaluation). Log parameters are applied across all stem classes, and only one bucking pattern is considered for each stem class for a given simulation. Each pattern is generated by applying the bucking algorithm to buck the class-representative tree on the basis of the bucking rules.

Three types of MCIP models were considered in this study. Their features are described next.

Uniform distribution model *(MC-1)*

The parameters corresponding to the export logs are generated independently, based on uniformly distributed random numbers. The ranges considered for these parameters are: [1] minimum end-diameter from 20 to 30 cm. (approximated to the lower even number); [2] maximum number of long-logs in a stem from 1 to 3, of intermediate-logs from 0 to 4, and of short-logs from 0 to 6; and [3] in 50% of the cases, the lower bound for the quality range was fixed at 1 and the upper limit varied from 1 to the maximum permissible for the log-type; in the remaining 50% of the cases, the upper bound was fixed at the highest quality allowed for the log-type and the lower limit varied from 1 to that value. Since no market restrictions apply to domestic logs, their parameters are fixed at the minimum standards accepted by the customers.

Ad-hoc distribution model *(MC-2)*

This is the same as MC-1, except that the export logs' minimum end-diameter is generated according to a discrete, non-uniform frequency distribution that is specially designed for obtaining a larger number of feasible solutions. For every market set an ad-hoc distribution was defined for each export log based on the diameter distribution corresponding to the feasible solutions obtained using MC-1. The points considered for the minimum end-diameter distribution are 20, 22, ..., and 30 cm. The frequency defined for each diameter corresponds to the actual percentage obtained for such point, approximated to a multiple of 5% (and eventually re-adjusted to obtain a sum of 100%).

Neighborhood search model *(MC-3)*

This is the same as MC-1, except that when a feasible solution better than the incumbent one is obtained, the bucking rules generated for the next 10 simulations or 5 feasible solutions (whichever occurs first) are concentrated in a narrower range: within ± 1 unit from the current parameters for every log-type (1 unit is equivalent to 2 cm. for the minimum end-diameter). In this case, the objective is to generate more iterations in the neighborhood of the currently best rule-set obtained by the Monte-Carlo process.

Initial iterations

The first two functional evaluations considered in the MCIP models are not generated randomly. The first iteration considers domestic logs only and its goal is to guarantee that at least one feasible solution will be generated (feasibility is guaranteed since there are no market constraints on the domestic log sales). This solution also represents a lower bound for the problem at hand. The second rule-set corresponds to the minimum standards defined for each log-type. If the solution obtained satisfies the market constraints then it is, in general, an optimal or near optimal solution.

Tabu Search Method

The TS heuristic will be presented from an implementation perspective, mainly through a detailed description of the algorithm shown in Figures 4 and 5. (The interested readers are referred to Glover (1989, 1990a) for the fundamental principles of Tabu Search.)

After initializing the variables, the TS method performs its first functional evaluation. A functional evaluation (*n* in the diagrams) consists of defining a set of bucking rules and applying the bucking algorithm to determine the feasibility and value (profit less penalty for infeasibility) associated with the rule-set. The first functional evaluation (N1) considers domestic-logs only and guarantees that at least one feasible solution is generated. The solution also represents a lower-bound for the problem at hand and thus the profit corresponding to N1 is used for initializing the aspiration criterion.

<u>Aspiration criterion</u>: The aspiration criterion used for overruling a tabu status is based on the profit corresponding to the currently best feasible solution found by the search process. A tabu rule-set is assigned a non-tabu status if it is feasible and achieves a profit higher than the profit corresponding to the aspiration criterion. This aspiration criterion helps to insure that the tabu rule-set is not in a path that leads to a cycle. A candidate rule-set that meets the aspiration criterion automatically redefines the profit level required for overruling a tabu status.

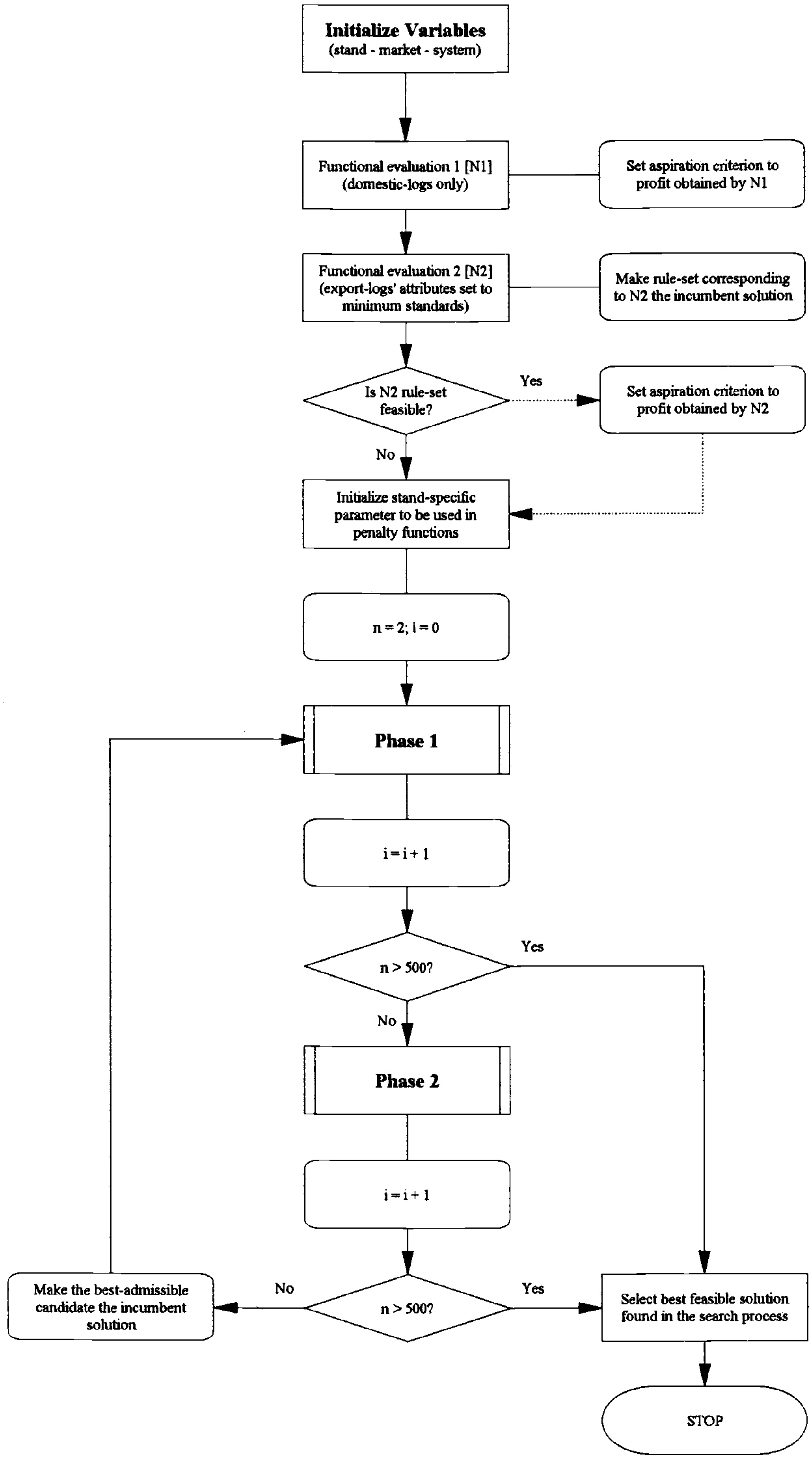

Figure 4. Description of the Tabu Search method

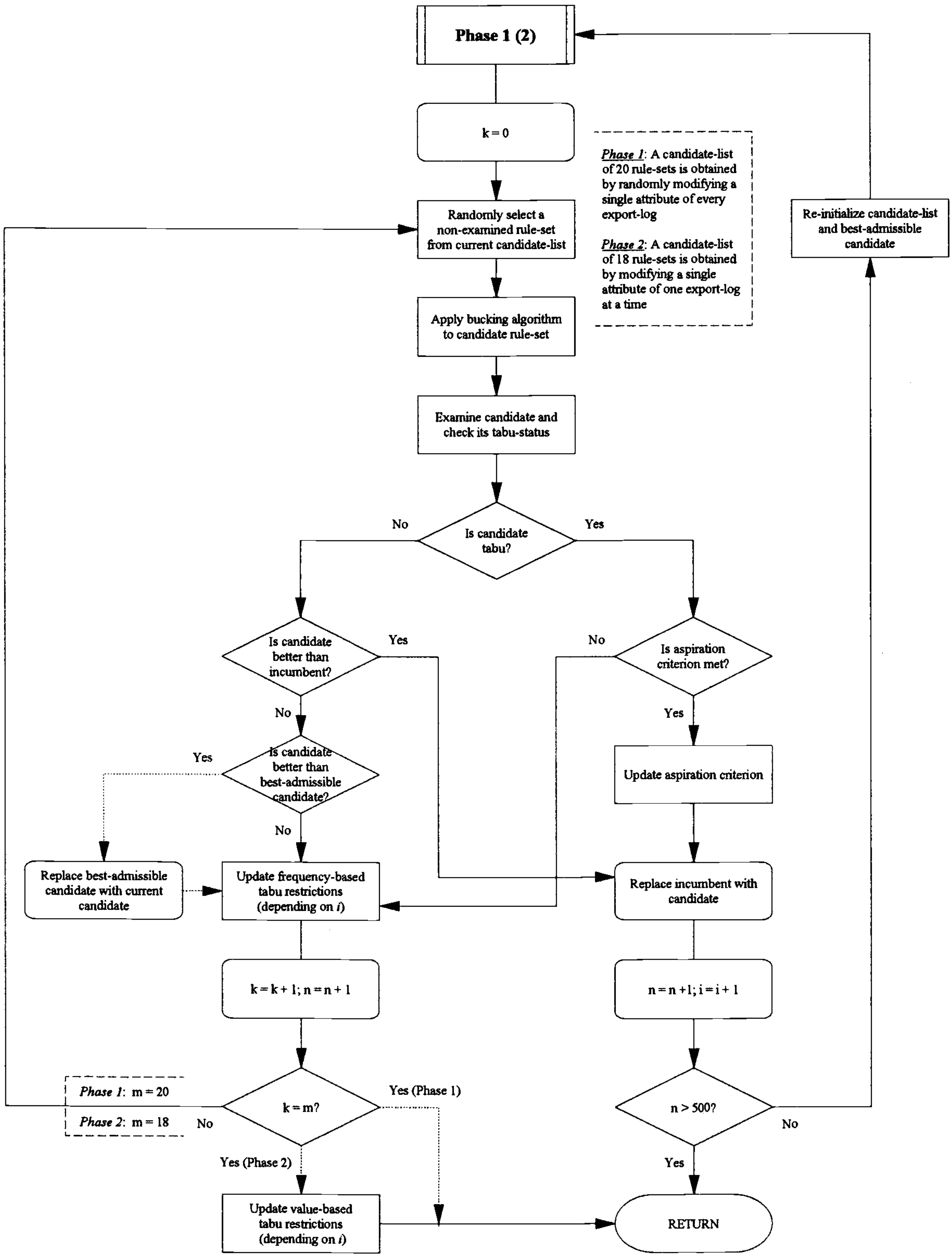

Figure 5. Phases 1 and 2 of the Tabu Search method

Tabu status: A candidate rule-set is consider tabu if it violates either a value-based tabu restriction or at least two of the frequency-based restrictions. The tabu restrictions and the mechanism for defining a tabu status are based upon user experimentation.

Frequency-based tabu restrictions: The frequency-based restrictions correspond to [1] the joint distribution of the export-logs' end-diameters, and [2] the acceptable quality range for each export-log. Parallel lists are used for these restrictions (each list records when a specific attribute related to a restriction becomes tabu and when its tabu condition should be released). A particular joint distribution of end-diameters becomes tabu after it has been evaluated 3 times, and the distribution remains tabu for 5 iterations. The quality range corresponding to a particular export-log becomes tabu after being examined twice, and remains tabu for 3 iterations. For the effects of determining a rule-set's tabu-status, the acceptable quality range is considered independently for each log-type. These restrictions correspond to short-term memory constraints and help the search process avoid concentrating itself in a particular region of the solution space.

Value-based tabu restriction: This tabu restriction is based on the values of locally optimum rule-sets. The value of the objective function corresponding to a local optimum solution remains tabu for 10 iterations. Through this restriction, all of the rule-sets that have a value equivalent to the value of one of the active local optima are considered tabu (thus this restriction does not apply to specific attributes of the rule-sets, but rather to all of the possible attribute combinations that result in certain values of the objective function). The value-based tabu restriction corresponds to a long-term memory constraint and its role is to favor diversification in the search process. This proved to be the most effective tabu restriction because it effectively eliminates irrelevant bucking rules from being considered as admissible candidates (e.g., if a rule specifies that an export-log's end diameter must be at least 22 cm., but the actual minimum end-diameter obtained for such log is 28 cm., then rule-sets that differ from the original bucking rules only by imposing a minimum end-diameter of 24 or 26 cm. will result in the same bucking patterns).

Iteration: A new iteration is started each time the incumbent solution is replaced and also when a candidate-list has been fully examined. The number of iterations elapsed between two solutions is used to indicate their "distance". A larger number of iterations indicates a larger distance and, therefore, a lower likelihood that the search will return to a previously visited solution.

In the second functional evaluation (N2), the attributes of all log-types (export and domestic) are set to the minimum standards allowed by the customers. The solution obtained usually corresponds to a super-optimum, infeasible solution (however, if it is feasible the aspiration criterion is updated to reflect the new higher profit solution). The rule-set corresponding to N2 is selected as the initial incumbent solution for starting the search procedure. The profit differential between N2 and N1 is used as a stand-specific parameter in the penalty function. Once such a parameter has been established, the profit corresponding to N2 is properly adjusted for infeasibility using the penalty function. The adjusted profit is set as the value for the initial incumbent solution.

<u>Incumbent solution</u>: The incumbent solution corresponds to the rule-set used as the basis for generating new candidate bucking rules. The value of the objective function (profit less penalty factor) corresponding to the incumbent solution is used for selecting among such candidates. The incumbent solution is updated whenever a non-tabu rule-set of higher value is found. It is also replaced by a tabu rule-set that satisfies the aspiration criterion.

<u>Penalty function</u>: The penalty function is used for quantifying, in monetary terms, a rule-set's degree of infeasibility. Its role is to help the search process converge to a good (optimal or near optimal) feasible solution by reducing the profit corresponding to infeasible rules (and therefore their attractiveness). The penalty function is comprised of 3 distinct components (one for each constraint).

Minimum end-diameter: $p_\theta = (t_\theta - a_\theta)*\Delta\pi$

Minimum proportion of long-logs: $p_{vl} = (t_{vl} - a_{vl})*e_v*[r_l + \Delta\pi/(a_{vl}*e_v)]$

Maximum proportion of short-logs: $p_{vs} = -(t_{vs} - a_{vs})*e_v*[r_s + \Delta\pi/(a_{vs}*e_v)]$

Where:

p : Component penalty [$/ha].

t : Target value.

a : Actual value.

r_l : Return corresponding to long export-logs [$\$/m^3$]

r_s : Return corresponding to short export-logs [$\$/m^3$]

$\Delta\pi$: Profit differential between N-2 and N-1 and solutions [$/ha].

e_v : Total exportable volume [m^3/ha].

θ : Sub-index representing minimum end-diameter.

ν_l : Sub-index representing minimum volumetric proportion of long-logs.

ν_s : Sub-index representing maximum volumetric proportion of short-logs.

If a constraint is satisfied, the respective component takes a zero value. The total penalty corresponds to the sum of the individual components.

The search process begins after the initialization steps have been completed. The procedure alternates between two phases. Phase 1 guides the search process through a low intensity exploration of relatively large areas of the solution space. This goal is accomplished by moves that induce diversity. By contrast, Phase 2 concentrates the search within narrow regions in order to find superior solutions among subtly different bucking rules.

In Phase 1, a single attribute of each export-log can be modified: the changes consist of a ±1 unit variation with respect to the values corresponding to the incumbent rule-set (for the minimum end-diameter 1 unit corresponds to 2 cm.). The attribute selected for change is randomly done with unequal probabilities. The probability of selecting the minimum end-diameter to be modified is set to 40%; the probability of selecting the acceptable quality range is set to 20%; the probability of selecting the maximum number of logs is set to 20%; and the probability of leaving the attributes of a specific export-log unchanged is also set to 20%. The attribute selected for change is randomly increased or decreased by 1 unit, with consistency checks performed to guarantee that the new values are within the tolerance defined by the customers. The bucking rules resulting from this modification of the attributes of the incumbent solution are called 3D neighborhood rules.

A candidate-list of 20 randomly generated 3D neighborhood rules is defined at each iteration (*i* in the diagrams). Such candidates (*k* in the diagrams) are sequentially examined, i.e., their tabu status is checked, and the bucking algorithm is applied to determine their feasibility and value. Each candidate examined represents a functional evaluation.

The non-tabu candidates are compared with the incumbent solution. A candidate with a value higher than the incumbent value automatically replaces the incumbent solution and starts a new iteration of Phase 1 (therefore leading to a new candidate-list generated on the basis of the attributes corresponding to the updated incumbent rule-set). If the candidate's value is not higher, its attributes are used for updating the frequency-based tabu restrictions. In the latter case, the candidate rule-set is also compared against the best-admissible candidate found so far in the list (and replaces it if the current candidate is better).

Best-admissible candidate: The best-admissible candidate corresponds to the non-tabu candidate rule-set that achieves the highest value that is also inferior to the incumbent value (the latter condition guarantees that it is not a rule-set equivalent to the incumbent solution). The best-admissible candidate is re-initialized at each new iteration, except when passing from Phase 1 to Phase 2. Its role is to provide a move that will allow the search process to escape from local optimality traps.

A tabu candidate is checked to see if it satisfies the aspiration criteria. If it does, the aspiration criteria is updated to reflect the new profit standard, the incumbent solution is replaced with the candidate rule-set, the candidate-list and the best-admissible candidate are re-initialized, and a new Phase 1 iteration is started. If the candidate does not satisfy the aspiration criteria its characteristics are used for updating the frequency-based tabu restrictions.

The search process leaves Phase 1 when a candidate-list has been fully examined and no rule-set improves upon the incumbent solution. It also leaves Phase 1 when 500 or more functional evaluations have been performed. In the first case, the search proceeds to Phase 2; in the latter, the search is finished.

Phase 2 is characterized by allowing only one attribute of a single export-log to be modified at a time. The bucking rules generated by this mechanism are called 1D neighborhood rules. In this phase all of the 18 (3 log-types x 3 attributes per log-type x 2 possible changes per attribute) 1D neighbors of the incumbent solution are included in the candidate-list. The elements of the candidate-list are examined in a random sequence. The procedural steps corresponding to Phase 2 differ from Phase 1 only in the following aspect: in Phase 2 a local optimum is defined (with respect to a 1D neighborhood) after examining all the rule-sets in the candidate-list without improving upon the incumbent solution. This local optimum is used to update the tabu restriction related to local optima conditions.

After leaving Phase 2, the incumbent solution is replaced by the best-admissible candidate. This step allows the search process to continue exploring the solution space, while relying on the tabu conditions for not returning to local optima already visited. The search process continues alternating between phases 1 and 2 until the maximum number of functional evaluations have been performed. A total of approximately 500 functional evaluations (not an exact number because of the size of the candidate lists) was selected as the stopping criterion. (Other termination criteria could have been chosen, in particular one that stops the search process after a maximum number of non-improving iterations are performed.) Once the process is finished, the best feasible solution found during the search is selected (with its corresponding profit).

RESULTS

Comparison of MCIP Models

Different price sets and market constraints were considered for comparing the MCIP models. The comparison between models was made in terms of the number of functional evaluations and feasible solutions required to achieve the best solution, the total number of feasible solutions generated, and the average profit obtained in the different scenarios. The same computational effort (500 functional evaluations) was used for all the models and scenarios. The market sets corresponding to no-export restrictions and domestic-logs only were not included because the lack of constraints makes these cases trivial from the perspective of this analysis.

The three models achieve essentially the same average profit in all of the scenarios considered, even though MC-2 performs slightly better (Table 4). The differences between MC-1 and MC-3 can be attributed to the stochastic nature of the solutions.

The number of feasible solutions diminishes as the market scenarios become more restrictive, especially with respect to increments in average diameter (Table 5). As expected, MC-2 generated more feasible solutions than the other two models in every case. However, such additional feasible solutions did not contribute to significantly

TABLE 4. Average profit obtained by the MCIP models considering several market constraints and price sets /a

Market Set /b	Price Set 1			Price Set 2			Price Set 3			Price Set 4		
	MC-1	MC-2	MC-3	MC-1	MC-2	MC-3	MC-1	MC-2	MC-3	MC-1	MC-2	MC-3
	[$/ha]											
2 : (24; 70; 10)	19428	19453	19464	20903	20995	20939	22462	22519	22413	23894	24015	24004
3 : (26; 70; 10)	18834	18846	18839	20039	20042	20044	21261	21286	21270	22447	22531	22465
4 : (28; 70; 20)	18366	18403	18378	19342	19369	19277	20296	20328	20231	21230	21318	21375
5 : (28; 60; 10)	18300	18320	18304	19192	19265	19181	20014	20215	20063	21017	21083	20874
6 : (28; 70; 10)	18277	18310	18297	19057	19104	19119	20047	20086	20014	20921	21055	20808
7 : (30; 70; 10)	17851	17885	17846	18379	18416	18404	18912	18956	18912	19399	19522	19578

/a: Based on 500 functional evaluations per stand.

/b: (28; 70; 10) indicates an average end-diameter > 28 cm., volumetric proportion of long logs > 70%, and volumetric proportion of short logs < 10%.

TABLE 5. Comparison of MCIP models in terms of the total number of feasible solutions, and the number of feasible solutions and functional evaluations required to obtain the best solution, considering several market restrictions and Price Set 1 /a

Market Set /b	MC-1			MC-2			MC-3		
	Feasible solutions	Best at feasible	Best at evaluation	Feasible solutions	Best at feasible	Best at evaluation	Feasible solutions	Best at feasible	Best at evaluation
2 : (24; 70; 10)	126	64	259	177	70	211	134	48	165
3 : (26; 70; 10)	87	46	229	121	66	261	93	34	171
4 : (28; 70; 20)	51	29	246	87	47	265	61	37	245
5 : (28; 60; 10)	46	21	244	75	30	233	58	29	170
6 : (28; 70; 10)	42	19	235	69	23	173	54	26	211
7 : (30; 70; 10)	15	7	221	34	17	181	26	16	192

/a: Average over 23 stands, using 500 functional evaluations per stand.

/b: (28; 70; 10) indicates an average end-diameter > 28 cm., volumetric proportion of long logs > 70%, and volumetric proportion of short logs < 10%.

higher profits nor to achieving the best solution in fewer functional evaluations. The extra feasible solutions correspond mainly to interior points.

MC-3 performed slightly better with respect to the average number of functional evaluations required to achieve the best solution, but does not clearly dominate the other models. It was observed that in the more restrictive market sets, better feasible solutions were hard to find given the limited sampling of the neighborhood of a known good solution. Due to the rule's discrete nature, the neighbor solutions were either infeasible or inferior, and therefore several simulations were lost. Even when better solutions were obtained during the neighborhood search stage, the final solution was almost always generated in the fully random phase.

No model consistently required, on average, fewer feasible solutions to obtain the best solution. As expected in a random process, the best feasible solution occurs approximately at the median of the feasible solutions; and the best solution occurs, on average, around 250 simulations (or 50% of the total number of functional evaluations).

Based on these results, MC-1 was selected due to its simpler formulation. MC-2 has an important drawback-- whenever the market restrictions change, a new ad-hoc distribution must be defined.

Analysis of Termination Criteria for MC-1

An analysis was done to determine the variability of the solutions obtained using MC-1. Five repetitions were carried-out for each stand, considering 13 different termination criteria (defined on the basis of a maximum number of functional evaluations and/or a maximum number of feasible solutions). Table 6 presents the results corresponding to Price Sets 1 and 4, and Market Set 6. The results indicate that using 100 feasible solutions as the stopping criterion leads to the best average solutions and the lowest spread (defined as the difference between the largest and smallest profit obtained among the five repetitions for each stand, expressed as a percentage of the mean profit; for example, the figure 52.2 in row 1 indicates that 52.2% of the 23 stands had a five repetition spread that was within 0.5% of the mean profit). The solution time for this criterion is, however, more than 4.5 minutes per stand (using a 486/33Mhz micro-computer).

By relaxing the termination criterion to 1000 functional evaluations or 50 feasible solutions, the average solution time for the test stands was reduced to approximately 2 minutes. Both criteria show similar performance with respect to the mean profit and variability of the solutions. As expected, the quality of the solutions decreases as the computational effort decreases, but not significantly. For Price Set 1, the mean profit loss is less than 0.35% when considering 500 functional evaluations (nearly 1 minute of solution time per stand) with respect to 100 feasible solutions as the termination

TABLE 6. Analysis of the variability of the average profit obtained by MC-1, based on five repetions for different termination criteria, two price sets, and Market Set 6

Termination criteria /a	Average profit per repetition 1	2	3	4	5	Mean	Solution time [sec/stand]	Percentage of solutions with spread less than /b 0.5 [%]	1.0 [%]	3.0 [%]
	[$/ha]									
Price Set 1										
[1000, +]	18302	18308	18313	18257	18325	18301	118	52.2	73.9	95.7
[500, +]	18283	18277	18311	18253	18287	18282	59	52.2	78.3	95.7
[350, +]	18266	18261	18304	18192	18270	18258	42	43.5	65.2	95.7
[200, +]	18229	18219	18186	18190	18187	18202	24	26.1	60.9	95.7
[100, +]	18124	18164	18167	18127	18155	18147	12	30.4	69.6	95.7
[+ , 100]	18339	18338	18340	18350	18342	18342	278	65.2	82.6	100.0
[+ , 75]	18327	18330	18354	18332	18341	18337	214	65.2	82.6	100.0
[+ , 50]	18301	18303	18321	18308	18322	18311	132	60.9	78.3	100.0
[+ , 35]	18312	18303	18306	18319	18286	18305	89	39.1	69.6	100.0
[+ , 20]	18220	18290	18247	18235	18272	18253	56	30.4	69.6	95.7
[1000, 100]	18251	18330	18272	18270	18308	18286	107	47.8	73.9	95.7
[500, 50]	18287	18200	18299	18232	18279	18260	53	52.2	73.9	95.7
[250, 25]	18229	18249	18265	18197	18239	18236	26	26.1	56.5	95.7
Price Set 4										
[1000, +]	20966	21122	21032	20844	21060	21005	118	26.1	39.1	65.2
[500, +]	20702	20946	20797	20833	20935	20843	59	8.7	17.4	65.2
[350, +]	20658	20630	20619	20869	20723	20700	42	8.7	17.4	73.9
[200, +]	20573	20737	20890	20741	20779	20744	24	13.0	26.1	56.5
[100, +]	20594	20412	20384	20150	20186	20345	12	4.3	17.4	39.1
[+ , 100]	21127	21095	21168	21109	21073	21114	278	39.1	60.9	95.7
[+ , 75]	21075	21052	21108	21036	20991	21053	214	21.7	52.2	82.6
[+ , 50]	21001	21082	21102	21039	20967	21038	132	17.4	34.8	82.6
[+ , 35]	20995	20971	20980	20883	21007	20967	89	8.7	30.4	87.0
[+ , 20]	20864	20841	20659	20777	20827	20794	56	26.1	39.1	60.9
[1000, 100]	20886	21091	21033	21042	21068	21024	107	17.4	34.8	65.2
[500, 50]	20898	20730	20880	20959	20862	20866	53	21.7	34.8	65.2
[250, 25]	20649	20903	20752	20516	20494	20663	26	13.0	30.4	60.9

/a: [500, 50] indicates a maximum of 500 functional evalutaions or 50 feasible solutions; + indicates no limit.

/b: Spread corresponds to the difference between the largest and smallest profit obtained for each stand, expressed as a percentage of the profit averaged over the five repetitions.

criterion. For Price Set 4, the corresponding mean profit loss is less than 1.3%. Using a combination of either a maximum number of functional evaluations or feasible solutions does not improve the quality of the solutions compared to using each criterion separately (given a similar solution time).

Comparison of TS and MC-1 Behaviors

These rule-based methods were compared using the same criteria and conditions used for comparing the MCIP models. Table 7 presents the results corresponding to this analysis.

As expected, the number of feasible solutions diminishes as the market scenarios become more restrictive (especially with respect to increments in average diameter); but the market constraints do not affect both techniques equally. The reduction observed in the TS method is proportionally less than the reduction experienced by the MC-1 approach. In the most restrictive scenario, TS generates approximately 1/3 of the feasible solutions that it does in the less restrictive case. The MC-1 approach, however, generates only 1/9 of the feasible solutions when comparing the same market sets. Once the TS method has selected a set of feasible bucking rules, it is very likely that several feasible solutions will be found among the neighbors (candidate list) visited at each iteration. Moreover, because of the penalty functions, it is likely that the process will choose to move from one feasible solution to another (unless the tabu restrictions do not allow it). For this reason, the TS method produces considerably more feasible solutions than the MC-1 approach.

TABLE 7. Comparison of TS and MC-1 models in terms of the total number of feasible solutions, and the number of feasible solutions and functional evaluations required to obtain the best solution, considering several market restrictions and Price Set 1 /a

Market Set /b	TS Feasible solutions	TS Best at feasible	TS Best at evaluation	MC-1 Feasible solutions	MC-1 Best at feasible	MC-1 Best at evaluation
2 : (24; 70; 10)	272	76	159	126	64	259
3 : (26; 70; 10)	220	75	213	87	46	229
4 : (28; 70; 20)	202	70	223	51	29	246
5 : (28; 60; 10)	178	71	231	46	21	244
6 : (28; 70; 10)	148	49	206	42	19	235
7 : (30; 70; 10)	110	37	163	15	7	221

/a: Average over 23 stands, using 500 functional evaluations per stand.

/b: (28; 70; 10) indicates an average end-diameter > 28 cm., volumetric proportion of long logs > 70%, and volumetric proportion of short logs < 10%.

The TS approach performed slightly better with respect to the average number of functional evaluations required to achieve the best solution, but it does not clearly dominate the MC-1 method. The number of feasible solutions required to obtain the best solution proved to have no significance as a comparison criteria. In absolute terms, the TS method requires more feasible solutions. In relative terms, however, TS finds the best solution at approximately one third of the total feasible solutions generated. In the MC-1 method, as expected in a random process, the best feasible iteration occurs approximately at the median of the feasible solutions.

Variability of TS and MC-1 Solutions

An analysis was done to compare the variability of the solutions obtained using the TS and MC-1 models. Five repetitions (with different random seeds) were carried-out for each stand, considering 500 functional evaluations as the stopping criterion. Table 8 presents the results obtained for Market Set 6 given all price sets. The results indicate that both methods have similar spreads. However, as export prices increase there are more opportunities for similar bucking rules to show subtle differences. Since TS is based on exploring neighborhood solutions, it can identify such differences better than the MC-1 method that randomly explores the solution space. For this reason, the TS approach is slightly more consistent than the MC-1 method at higher export prices.

TABLE 8. Variability of the average profit obtained by TS and MC-1 models for Market Set 6 /a

Price Set	Average profit per repetition 1	2	3	4	5	Mean	Percentage of solutions with spread less than /b 0.5 [%]	1.0 [%]	3.0 [%]
	[$/ha]								
TS									
1	18319	18329	18327	18298	18331	18321	39.1	73.9	100.0
2	19246	19264	19211	19241	19214	19235	13.0	34.8	87.0
3	20145	20257	20136	20184	20234	20191	21.7	47.8	91.3
4	21119	21103	21179	21110	21134	21129	21.7	30.4	65.2
MC-1									
1	18283	18277	18311	18253	18287	18282	52.2	78.3	95.7
2	19179	19072	19084	19137	19067	19108	21.7	52.2	78.3
3	19921	20064	20013	20040	19932	19994	8.7	34.8	78.3
4	20702	20946	20797	20833	20935	20843	8.7	17.4	65.2

/a: Based on 500 functional evaluations per stand.

/b: Spread corresponds to the difference between the largest and smallest profit obtained for each stand, expressed as a percentage of the profit averaged over the five repetitions.

Profit Distribution of TS and MC-1 Solutions

Figure 6 shows the profit distribution corresponding to the feasible solutions obtained by the TS method in Stand 20 (the stand of median profit), given Price Set 4 and four market sets that differ in the required average end-diameter. This Figure indicates a strong tendency of the feasible solutions to concentrate in the upper end of the profit distribution. This situation can be explained by the following factors: [1] the search process usually starts at a super-optimum, infeasible solution (the bucking rule that corresponds to the log-type's minimum standards), and moves towards feasibility (based on the penalty function) until it will likely find a frontier solution; [2] given a frontier feasible solution, the TS method explores candidate rules within its neighborhood, many of them resulting in feasible solutions of similar profit; [3] after examining the candidate list, the movement selected by TS will typically be to another frontier pointer; and [4] the candidate neighbors visited in Phase 2 correspond to single changes in one parameter of the bucking rules, many of them leading to the same current solution (e.g., if a rule says that the end-diameter for a log-type must be at least 24 cm. but the actual minimum end-diameter obtained is 28 cm., then changing the rule from imposing a minimum end-diameter of 24 to 26 cm. has no effect in the resulting bucking patterns).

Figure 7 shows the profit distribution corresponding to the feasible solutions obtained by MC-1 for the same stand, price set, and market sets previously mentioned. As the market sets become more restrictive, the solutions become more concentrated in the

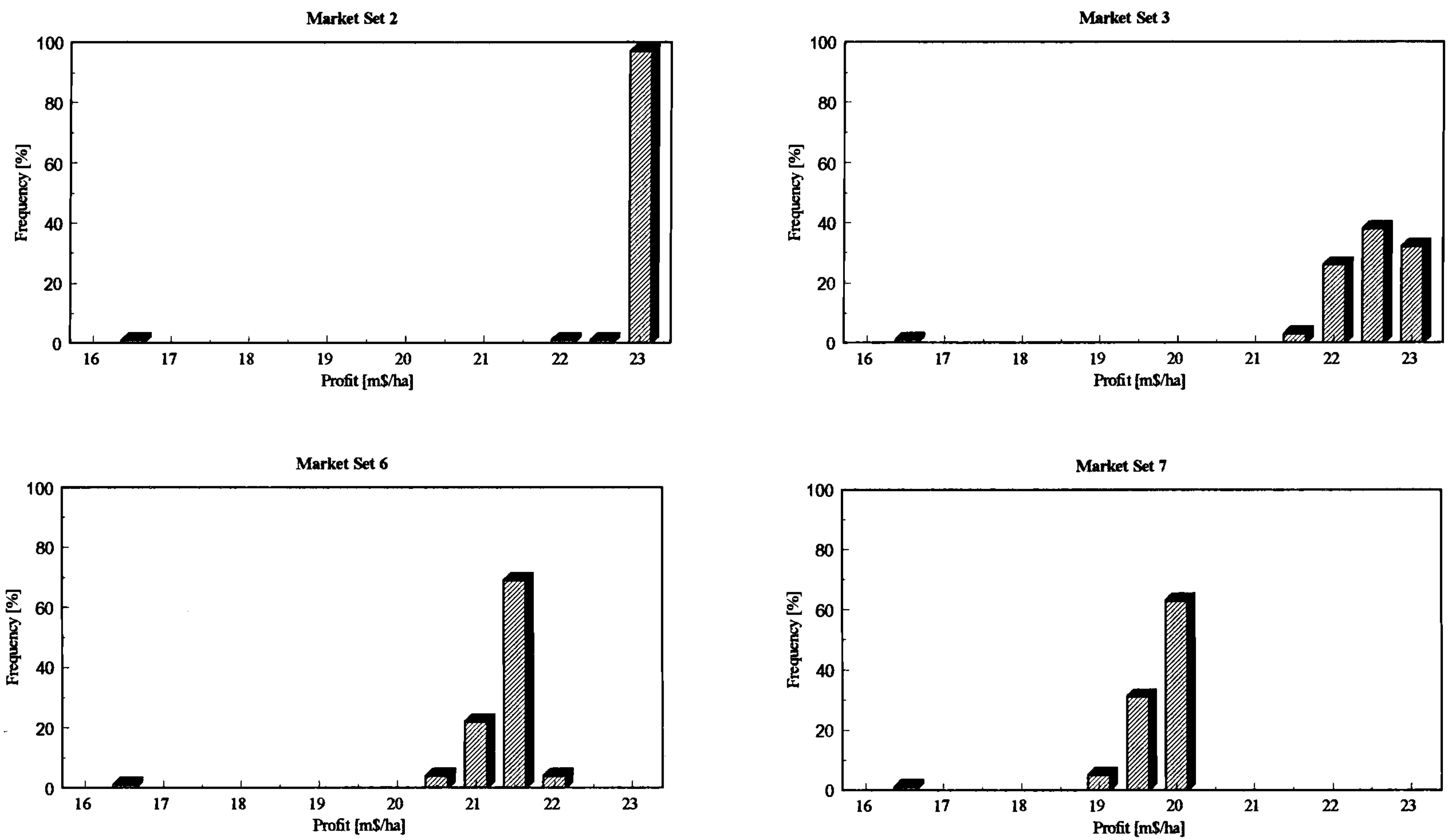

Figure 6. Profit distribution corresponding to feasible solutions obtained by TS in Stand 20, for four different market sets and Price Set 4

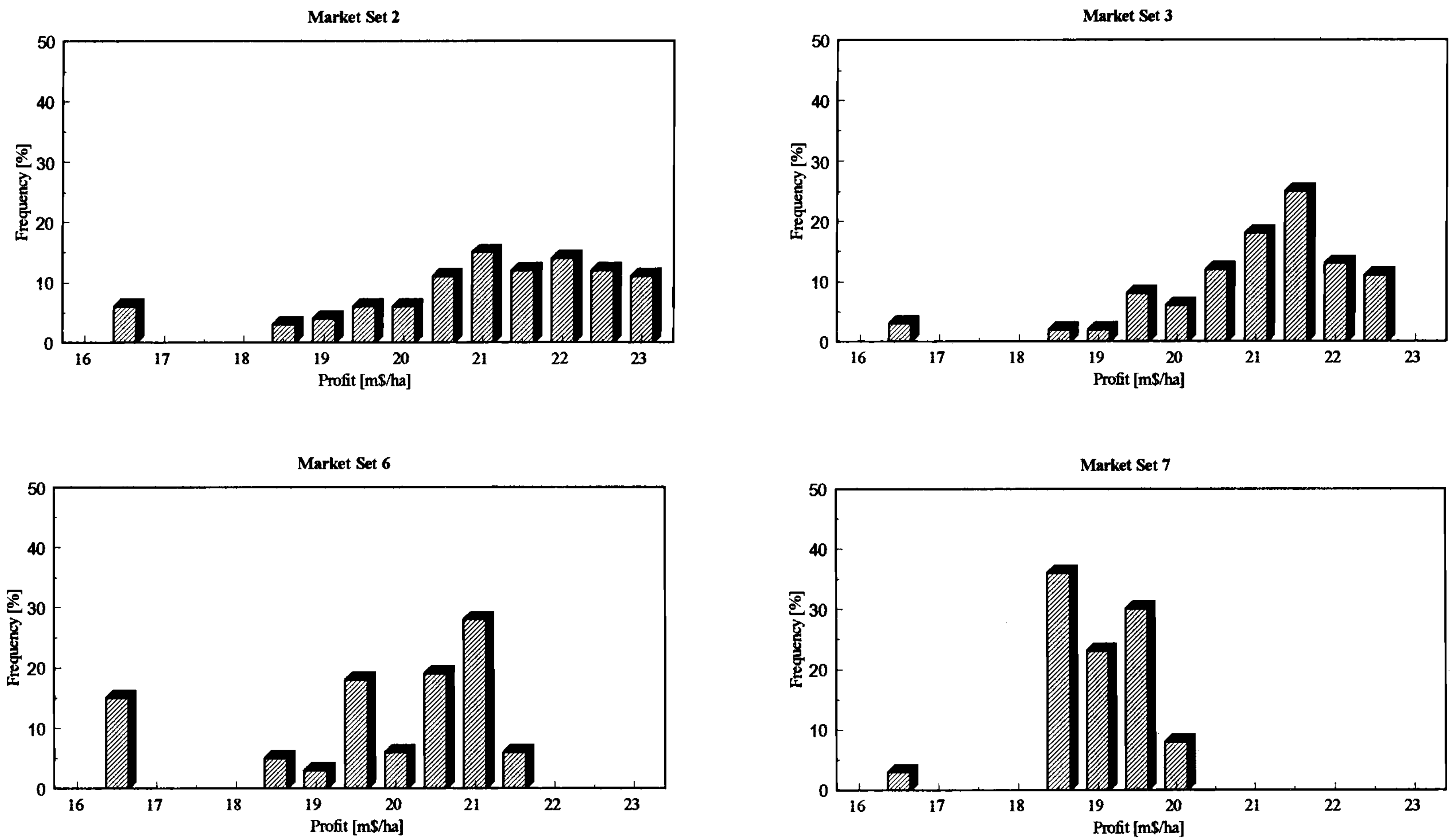

Figure 7. Profit distribution corresponding to feasible solutions obtained by MC-1 in Stand 20, for four different market sets and Price Set 4

neighborhood of the mean profit because fewer feasible options are available. The distribution also shows that a significant proportion of the feasible solutions obtained correspond to interior points. Nevertheless, there are always several solutions in the upper end of the distribution that explain the robustness shown by the MC-1 method. The points in the lower end of each profit distribution correspond to bucking rules that yield only domestic logs.

Comparison of LP and IP Solutions

The solutions corresponding to the linear and integer programming formulations of the optimal bucking problem were compared for each scenario. The comparison was made in terms of average profit, volume, and actual parameters corresponding to the market restrictions.

The differences in profit are negligible between LP and IP, ranging from 0 to 1.8% (Table 9). This result was expected since the LP solutions are almost integer due to the problem's structure: only 3 market constraints, an average of 145 activities, and approximately 50 stem classes in each stand. The profit differential tends to increase as the market constraints become more restrictive, and especially as the relative price of export logs increases. Even though it cannot be guaranteed that all of the relevant activities

TABLE 9. Percentage differences in average profit bewteen LP, IP, TS, and MC-1 solutions for different market and price sets /a

Market Set /b	Price Set 1				Price Set 2			
	LP/IP	IP/TS	IP/MC-1	TS/MC-1	LP/IP	IP/TS	IP/MC-1	TS/MC-1
1 : None	0.00	0.60	0.61	0.01	0.00	0.65	0.66	0.02
2 : (24; 70; 10)	0.12	1.94	2.46	0.53	0.45	2.08	2.85	0.78
3 : (26; 70; 10)	0.43	2.44	2.59	0.15	0.79	2.86	3.25	0.39
4 : (28; 70; 20)	0.36	2.86	2.77	-0.10	0.84	3.56	3.64	0.08
5 : (28; 60; 10)	0.46	2.45	2.59	0.15	0.52	3.46	3.74	0.29
6 : (28; 70; 10)	0.46	2.35	2.54	0.20	0.96	2.79	3.13	0.35
7 : (30; 70; 10)	0.59	1.59	1.87	0.28	0.89	2.21	3.24	1.05
8 : No export logs	0.00	0.20	0.20	0.00	0.00	0.20	0.20	0.00

Market Set /b	Price Set 3				Price Set 4			
	LP/IP	IP/TS	IP/MC-1	TS/MC-1	LP/IP	IP/TS	IP/MC-1	TS/MC-1
1 : None	0.00	0.69	0.71	0.03	0.00	0.73	0.76	0.03
2 : (24; 70; 10)	0.47	2.42	3.11	0.71	0.39	3.01	3.96	0.97
3 : (26; 70; 10)	1.06	3.27	3.83	0.57	0.75	4.51	5.09	0.61
4 : (28; 70; 20)	0.99	4.19	4.78	0.62	1.14	5.27	5.48	0.22
5 : (28; 60; 10)	0.89	4.08	4.57	0.51	1.25	4.71	5.73	1.06
6 : (28; 70; 10)	1.12	3.66	4.73	1.11	1.63	3.79	5.69	1.97
7 : (30; 70; 10)	1.10	3.49	4.37	0.91	1.81	3.66	4.36	0.73
8 : No export logs	0.00	0.20	0.20	0.00	0.00	0.20	0.20	0.00

/a: Based on 500 functional evaluations per stand.

/b: (28; 70; 10) indicates an average end-diameter > 28 cm., volumetric proportion of long logs > 70%, and volumetric proportion of short logs < 10%.

for IP model are being considered, the small profit differential between the linear and integer solutions suggests that at least the most relevant activities are included.

The average end-diameter restriction has the highest impact on the outcome (Table 10). The LP and IP solutions have no slack for the end-diameter market specification, but do show some slack for the two volumetric constraints. For an average end-diameter of 28 cm., the maximum proportion of short logs is more restrictive than the minimum volumetric proportion of long logs. It can be observed that the IP solutions have less slack for the short-logs constraint in most of the market sets. This behavior was observed across price sets.

The LP solutions consistently produce more export volume than the IP solutions. This is a direct consequence of the higher flexibility implicit in the LP formulation that allows more activities to be used for satisfying the constraints. The IP solutions are characterized by a larger volume of domestic saw-logs, especially of the kind with higher return. Therefore, the differences in profit are less than the differences in exportable volume.

TABLE 10. Comparison of LP and IP solutions with respect to the fulfillment of market constraints, volume obtained per product, and profit, for Price Set 1 and all market sets

Market Set /a	Mean end-diameter [cm]	Volume long logs [%]	Volume short logs [%]	Volume [m3/ha] Export logs	Saw logs	Pulp logs	Total	Profit [$/ha]
LP Solution								
1 : None	23.7	63.3	10.6	367.4	79.8	77.0	524.2	20194
2 : (24; 70; 10)	24.3	70.1	8.9	336.6	107.5	78.8	522.9	19919
3 : (26; 70; 10)	26.0	70.0	9.8	290.9	149.9	80.5	521.3	19420
4 : (28; 70; 20)	28.0	70.1	19.3	246.7	191.4	81.1	519.2	18978
5 : (28; 60; 10)	28.0	60.7	10.0	234.6	203.2	81.5	519.3	18895
6 : (28; 70; 10)	28.0	70.0	10.0	228.1	208.7	81.7	518.5	18846
7 : (30; 70; 10)	30.0	70.0	10.0	162.8	268.8	83.3	514.9	18324
8 : No export logs	---	---	---	0.0	424.1	83.6	507.7	17460
IP Solution								
1 : None	23.7	63.3	10.6	367.4	79.8	77.0	524.2	20194
2 : (24; 70; 10)	24.3	70.8	8.7	333.9	109.8	79.2	522.9	19895
3 : (26; 70; 10)	26.0	70.5	9.3	282.5	157.4	80.7	520.6	19336
4 : (28; 70; 20)	28.0	71.3	18.4	240.8	195.9	82.0	518.7	18909
5 : (28; 60; 10)	28.0	63.1	9.6	224.3	212.6	81.4	518.3	18809
6 : (28; 70; 10)	28.0	71.2	9.6	217.3	218.2	82.0	517.5	18759
7 : (30; 70; 10)	30.0	71.5	9.2	148.1	282.1	83.3	513.5	18216
8 : No export logs	---	---	---	0.0	424.1	83.6	507.7	17460

/a: (28; 70; 10) indicates an average end-diameter > 28 cm., volumetric proportion of long logs > 70%, and volumetric proportion of short logs < 10%.

Comparison of IP, TS, and MC-1 Solutions

In the absence of market constraints (sets 1 and 8), the IP solutions are slightly better than the results obtained using the TS and MC-1 approaches (Table 9). This situation occurs because the bucking patterns obtained using the Sessions et al.'s SP algorithm are more efficient than the patterns defined following a priority-list of logs. In Market Set 1, TS is able to generate solutions marginally better than MC-1. It does this by finding that, in some stands, bucking rules different than the minimum standards defined for each log-type can lead to log allocations that take better advantage of the JAS cubic-volume log scale. Tabu Search is more likely to find such rules due to its neighborhood search nature.

The profit differences observed in the market-constrained scenarios are due to the loss of flexibility implicit in the use of bucking rules. The rule-based methods allow only one bucking pattern to be considered for each stem class, generated on the basis of stand-level log specifications; whereas the IP model selects among several bucking patterns for each stem class, all of them defined on the basis of stem-specific characteristics. The latter factor becomes more important as the constraints become more restrictive. It also explains why TS and MC-1 solutions have considerably more slack for the market constraints than the IP solutions (Tables 10 and 11).

TABLE 11. Comparison of TS and MC-1 solutions with respect to the fulfillment of market constraints, volume obtained per product, and profit, for Price Set 1 and all market sets /a

Market Set /b	Mean end-diameter [cm]	Volume long logs [%]	Volume short logs [%]	Volume [m3/ha] Export logs	Saw logs	Pulp logs	Total	Profit [$/ha]
TS Solution								
1 : None	23.0	70.7	7.5	363.4	82.2	73.0	436.4	20073
2 : (24; 70; 10)	24.3	78.4	5.0	312.6	130.2	73.0	515.8	19509
3 : (26; 70; 10)	26.2	81.2	5.6	250.5	189.4	73.0	512.9	18865
4 : (28; 70; 20)	28.1	79.2	11.3	191.3	245.0	73.0	509.3	18368
5 : (28; 60; 10)	28.1	76.6	6.5	189.5	247.2	73.0	509.7	18348
6 : (28; 70; 10)	28.1	79.3	6.0	184.4	252.3	73.0	509.7	18319
7 : (30; 70; 10)	30.1	79.6	5.6	120.8	312.5	73.0	506.3	17927
8 : No export logs	—	—	—	0.0	431.3	73.0	504.3	17425
MC-1 Solution								
1 : None	22.9	71.3	6.8	363.2	82.2	73.0	436.2	20071
2 : (24; 70; 10)	24.5	82.1	5.3	299.2	143.2	73.0	515.4	19406
3 : (26; 70; 10)	26.3	84.7	6.1	243.1	196.6	73.0	512.7	18836
4 : (28; 70; 20)	28.2	86.3	12.0	191.5	245.3	73.0	509.8	18386
5 : (28; 60; 10)	28.2	81.0	5.4	183.1	253.5	73.0	509.6	18321
6 : (28; 70; 10)	28.2	85.3	5.4	178.2	258.1	73.0	509.3	18283
7 : (30; 70; 10)	30.4	89.0	4.7	108.6	324.2	73.0	505.8	17876
8 : No export logs	—	—	—	0.0	431.3	73.0	504.3	17425

/a: Based on 500 functional evaluations per stand.

/b: (28; 70; 10) indicates an average end-diameter > 28 cm., volumetric proportion of long logs > 70%, and volumetric proportion of short logs < 10%.

The TS method consistently produces better solutions than the MC-1 approach in all market scenarios, with gains ranging from 0.15 to 1.97%. An exception is one case where MC-1 generates better solutions than TS. The profit differential between these two techniques tends to increase as the relative price of the export logs increases. The effect of the market restrictions is not well defined but the TS solutions show less slack with respect to the target values for the constraints (especially with respect to the minimum volumetric proportion of long logs). Because the TS method adapts better to the market restrictions it produces more exportable volume (explaining the higher profits it achieves). Little difference exists between these methods with respect to total volume.

The profit differential between TS and IP solutions initially increases as the market sets become more restrictive, but starting from Market Set 5 the relative differences decrease. Due to the bucking rules' lesser adaptability, TS's performance diminishes as the markets become more restrictive since fewer feasible solutions are generated. However, as the market specifications become highly constrained the proportion of exportable volume reduces significantly for both methods, making the solutions closer to the domestic-logs only scenarios where the differences between IP and TS solutions are minimal. The relationship between IP and MC-1 solutions presents a similar behavior.

Comparison of Bucking Patterns

Table 12 presents the set of bucking rules selected by the TS and MC-1 methods for Stand 20 (the stand of median profit) considering Market Set 6 and Price Set 1 (the base market scenario). Table 13 shows the bucking patterns that result from applying such rules to a subset of the stem classes. This table also presents the IP solution for the same stand and market scenario. The rule-based bucking patterns are characterized by their consistency across diameter and quality classes: it is possible to see the underlying rules imbedded in the patterns. The IP bucking patterns can be characterized by their irregular behavior, but also by their ability to take advantage of the JAS cubic-volume log scale and their sensitivity to profit differentials due to quality.

The effect that different rule-sets have on the resulting bucking patterns can be observed by comparing the schedules corresponding to the TS and MC-1 solutions for Stand 20. In this particular case, diameter classes 8, 9, and 10 show the impact of different end-diameters being required for the short export-logs (log-type 3). The effect of imposing different quality restrictions on the long export-logs (log-type 1) can be observed in diameter-quality classes 16-1 to 17-2. The different specifications for the export logs not only affect the export volume but also alter the types of domestic logs obtained (see diameter-quality classes 9-1 to 10-2).

TABLE 12. Bucking rules selected by the rule-based methods for Stand 20 given Market Set 6 and Price Set 1

Log type /a	Product	Length [m]	Minimum end-diameter [cm]	Acceptable qualities	Maximum number of logs
TS					
1	Export - long	12.1	22	1 - 1	2
2	Export - intermediate	8.1	26	1 - 2	2
3	Export - short	4.1	28	2 - 3	3
4	Domestic - sawlog	4.1	26	1 - 4	---
5	Domestic - sawlog	4.1	16	1 - 4	---
6	Domestic - pulplog	2.5	8	1 - 5	---
MC-1					
1	Export - long	12.1	26	1 - 2	1
2	Export - intermediate	8.1	28	1 - 2	3
3	Export - short	4.1	24	2 - 3	4
4	Domestic - sawlog	4.1	26	1 - 4	---
5	Domestic - sawlog	4.1	16	1 - 4	---
6	Domestic - pulplog	2.5	8	1 - 5	---

/a: The log's order indicates its priority.

TABLE 13. Bucking patterns determined by IP, TS, and MC-1 methods for selected stem classes of Stand 20, given Market Set 6 and Price Set 1

Diameter class	Quality class	Log type by stem section: IP solution 1	2	3	4	5	6	7	8	9	10	TS Solution 1	2	3	4	5	6	7	8	9	10	MC-1 Solution 1	2	3	4	5	6	7	8	9	10
3	1	5	6	6	6	6	6					5	6	6	6	6	6					5	6	6	6	6	6				
4	1	6	5	6	6	6	6	6				5	6	6	6	6	6	6				5	6	6	6	6	6	6			
4	2	6	5	6	6	6	6	6				5	6	6	6	6	6	6				5	6	6	6	6	6	6			
5	2	6	5	5	6	6	6	6				5	5	6	6	6	6	6				5	5	6	6	6	6	6			
6	2	5	5	5	6	6	6	6				5	5	5	6	6	6	6				5	5	5	6	6	6	6			
8	1	5	5	5	5	6	6	6				5	5	5	5	6	6	6				5	5	5	5	6	6	6			
8	2	5	5	5	5	6	6	6				5	5	5	5	6	6	6				3	5	5	5	6	6	6			
9	1	1	5	6	6	6						1	5	6	6	6						5	5	5	5	6	6	6			
9	2	1	5	6	6	6						4	5	5	5	6	6	6				3	5	5	5	6	6	6			
10	1	1	5	6	6	6	6					1	5	6	6	6	6					4	5	5	5	6	6	6	6		
10	2	4	5	5	5	6	6	6	6			3	5	5	5	6	6	6	6			3	3	5	5	6	6	6	6		
15	1	1	5	5	5	6	6					1	5	5	5	6	6					1	5	5	5	6	6				
15	2	3	3	3	5	5	5	6	6			2	2	5	5	6	6					1	3	5	5	6	6				
16	1	1	5	5	5	6	6					1	5	5	5	6	6					1	5	5	5	6	6				
16	2	2	4	5	5	5	6	6				2	2	5	5	6	6					1	3	5	5	6	6				
17	1	2	4	4	5	5	6	6	6			1	4	5	5	6	6	6				1	4	5	5	6	6	6			
17	2	2	4	4	5	5	6	6	6			2	2	5	5	6	6	6				1	3	5	5	6	6	6			
18	1	3	1	5	5	6	6	6				1	4	5	5	6	6	6				1	4	5	5	6	6	6			
20	1	3	2	4	5	5	6	6	6			1	4	5	5	6	6	6				1	4	5	5	6	6	6			
22	1	3	2	4	4	5	5	6	6			1	4	4	5	5	6	6				1	4	4	5	5	6	6			
24	1	3	3	3	3	3	5	5	6	6		1	2	5	5	6	6					1	2	5	5	6	6				
25	1	3	3	3	3	4	5	5	6	6	6	1	2	5	5	6	6	6				1	2	5	5	6	6	6			

CONCLUSIONS

Using rule-based methods for generating bucking patterns yields log allocations that are reasonably close to the more complex optimal IP solutions for the problems considered in this analysis. The average profits using TS were 2.4% less than the IP solutions for the base market conditions. The profit loss increases to 3.8% for the same market restrictions when export prices rise approximately 50% above the base prices, and to 5.3% in the most unfavorable case. The average profits using MC-1 were 2.5% less than the IP solutions for the base market conditions. The profit loss increases to 5.7% for the same market restrictions when export prices rise approximately 50% above the base prices (which is also very close to the most unfavorable case). The actual differences are expected to be less since implementing the IP solutions increases the production costs. The rule-based approaches also represent benchmarks for gains to be made by Murphy's (1993) method, and by mechanized harvesters with bucking capabilities (which only in the best case can achieve results equivalent to the theoretical IP solutions).

We recognize, however, that these results are valid only for the relatively simple, but real, market constraints and homogeneous radiata pine stands considered in the study. Similar bucking rules may not work for complex market structures or for heterogeneous stands. Further research is also needed for comparing the actual performance of the rule-based methods (e.g., TS and MC-1) with respect to the stem-specific methods (e.g., IP and Murphy's, 1993). This analysis will require the actual harvest of a series of plots for comparing the realized versus modeled outcomes, and the production costs implied.

The TS method generated solutions that are only slightly better than the results obtained using MC-1 (profit improvements in the constrained market sets ranged from 0.15% to 1.97%). The profit improvements are, however, consistent across all of the market scenarios examined. While further refinements in the TS method are possible, the characteristics of the problems analyzed suggest that there is not much to gain with such refinements. The structure of the solution space, with several bucking rules that provide near optimal solutions, almost ensures that the TS approach is able to find a rule-set close to the optimum bucking rules. Such structure also gives the MC-1 method a high probability of finding a good solution.

For problems similar to the ones studied, the selection of a rule-based method should consider the quality, simplicity, and solution-time trade-offs between the methods. Monte-Carlo Integer Programming is easy to understand and simple to implement computationally. Tabu Search requires greater mathematical programming expertise but provides better solutions. It was observed that TS converges very fast to good local optima. Therefore, TS's relative efficiency will likely increase if fewer functional evaluations are allowed. Tabu Search also has theoretical advantages for handling problems with more constraints that would be typical in more complex log merchandising situations (e.g., if different markets are to be supplied from one stand).

The rule-based methods require the same information that is currently being used by Forestal Chile for its bucking activities, and thus no additional stem measurements are necessary for implementation. Applying the bucking rules does not reduce the crew's productivity (Laroze, 1986), and differentiated payments per log-type may be used as an incentive to motivate the crew to follow the company's priority-list of logs. Monitoring only requires a check of the dimensions and quality of the logs produced. Moreover, bucking rules are robust in the sense that the activities are readily adaptable to the stem's particular taper, grade, and breakage. Both TS and MCIP represent significant improvements compared to the bucking simulator currently used at Forestal Chile (i.e., based on user-defined bucking rules). The rule-based approaches generate better solutions, their performance is independent of the user's expertise, and they allow for large batch processes.

We expect that either rule-based method will become very useful in applications such as: [1] stand appraisal, [2] stand-level management optimization, and [3] generating technical coefficients for forest-level models (e.g., the strategic-level of the log-merchandising model developed by Laroze and Greber, 1991). Before a rule-based method can be integrated into algorithms for defining optimal management regimes it is important to determine what termination criteria will provide an adequate balance between solution time, and the quality and robustness of the solutions achieved. Further research should also consider the impact of using ruled-based approaches within harvest planning models.

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APPENDICES

APPENDIX A

Stand Tables

Stand table: Stand 1

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
22	0	0	3	0	3	6	0.2	25.2	2.0
24	3	0	0	0	0	3	0.1	26.3	1.2
26	3	0	3	0	0	6	0.3	27.3	3.0
28	5	5	5	0	0	15	0.9	28.2	9.1
30	8	8	3	0	0	19	1.3	29.0	13.6
32	15	0	0	3	0	18	1.4	29.8	15.1
34	23	8	5	0	0	36	3.3	30.5	34.9
36	20	10	5	5	0	40	4.1	31.2	44.5
38	13	5	3	0	0	21	2.4	31.8	26.5
40	38	3	0	0	0	41	5.2	32.4	58.4
42	20	5	0	0	0	25	3.5	33.0	40.0
44	15	5	0	3	0	23	3.5	33.5	41.0
46	10	3	0	0	0	13	2.2	34.0	25.7
48	8	0	3	0	0	11	2.0	34.5	24.0
50	3	0	0	0	0	3	0.6	35.0	7.2
52	5	3	0	0	0	8	1.7	35.4	21.1
70	0	0	0	3	0	3	1.2	38.7	15.6
Totals	189	55	30	14	3	291	33.9	31.5	382.9

Stand table: Stand 2

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
16	1	0	0	0	1	2	0.0	18.8	0.3
18	1	0	0	0	5	6	0.2	20.1	1.1
20	1	0	0	0	3	4	0.1	21.3	0.9
22	6	0	0	0	4	10	0.4	22.3	3.0
24	11	0	0	0	3	14	0.6	23.3	5.2
26	10	1	4	0	2	17	0.9	24.2	7.6
28	17	0	2	4	0	23	1.4	25.0	12.4
30	25	4	2	4	1	36	2.5	25.7	22.9
32	21	3	6	5	0	35	2.8	26.4	26.0
34	27	1	1	2	0	31	2.8	27.1	26.7
36	27	6	4	2	0	39	4.0	27.7	38.5
38	18	2	3	2	0	25	2.8	28.3	28.1
40	18	2	0	3	0	23	2.9	28.8	29.1
42	13	1	0	1	0	15	2.1	29.3	21.3
44	5	2	0	1	0	8	1.2	29.8	12.7
46	6	1	0	1	0	8	1.3	30.3	14.1
48	7	0	0	0	0	7	1.3	30.7	13.6
50	1	1	0	0	0	2	0.4	31.1	4.3
52	3	0	0	0	0	3	0.6	31.5	7.0
54	1	0	0	0	0	1	0.2	31.9	2.6
Totals	219	24	22	25	19	309	28.5	26.7	277.4

Stand table: Stand 3

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
16	0	0	0	0	2	2	0.0	22.4	0.3
18	1	2	1	0	0	4	0.1	23.8	0.8
20	0	1	0	0	2	3	0.1	25.0	0.8
22	3	1	0	1	2	7	0.3	26.1	2.4
24	3	3	1	0	3	10	0.5	27.0	4.3
26	6	2	0	1	8	17	0.9	27.9	8.8
28	23	5	2	1	1	32	2.0	28.8	19.9
30	16	4	0	3	0	23	1.6	29.5	16.8
32	25	4	5	1	0	35	2.8	30.2	29.8
34	31	3	3	1	0	38	3.5	30.9	37.3
36	34	2	0	1	0	37	3.8	31.5	41.5
38	30	8	4	4	0	46	5.2	32.1	58.6
40	19	2	0	1	0	22	2.8	32.6	31.5
42	19	3	2	1	0	25	3.5	33.1	40.1
44	10	0	1	0	0	11	1.7	33.6	19.7
46	8	2	2	0	0	12	2.0	34.1	23.8
48	6	0	0	0	0	6	1.1	34.5	13.1
50	1	0	0	0	0	1	0.2	34.9	2.4
52	1	0	0	0	0	1	0.2	35.3	2.6
56	2	0	0	0	0	2	0.5	36.0	6.2
Totals	238	42	21	15	18	334	32.8	30.8	360.7

Stand table: Stand 4

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
16	0	0	0	0	1	1	0.0	20.1	0.1
18	0	0	0	0	1	1	0.0	21.5	0.2
20	0	0	0	0	2	2	0.1	22.7	0.5
22	0	0	1	1	11	13	0.5	23.8	4.1
24	0	2	0	3	3	8	0.4	24.8	3.1
26	0	6	5	4	4	19	1.0	25.7	9.1
28	1	11	10	8	2	32	2.0	26.6	18.3
30	7	7	15	3	2	34	2.4	27.4	23.0
32	8	11	16	1	0	36	2.9	28.1	28.5
34	16	20	13	4	0	53	4.8	28.8	48.5
36	15	17	19	1	0	52	5.3	29.5	54.6
38	8	6	10	0	0	24	2.7	30.1	28.7
40	10	3	5	0	0	18	2.3	30.6	24.2
42	2	5	2	0	0	9	1.2	31.2	13.6
44	8	1	4	0	0	13	2.0	31.7	21.9
46	6	3	3	0	0	12	2.0	32.2	22.5
48	3	0	0	0	0	3	0.5	32.6	6.2
50	2	2	2	0	0	6	1.2	33.1	13.6
54	1	0	1	0	0	2	0.5	33.9	5.4
56	0	0	1	0	0	1	0.2	34.3	3.0
Totals	87	94	107	25	26	339	32.0	28.6	329.1

Stand table: Stand 5

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
16	0	0	0	0	8	8	0.2	21.7	1.2
18	0	0	0	0	17	17	0.4	23.5	3.6
20	8	0	4	0	17	29	0.9	25.0	8.0
22	17	8	8	4	13	50	1.9	26.4	17.6
24	13	0	8	0	25	46	2.1	27.8	20.2
26	13	0	8	0	0	21	1.1	29.0	11.3
28	17	13	8	8	4	50	3.1	30.1	32.4
30	21	13	8	0	0	42	3.0	31.1	32.3
32	44	17	17	0	0	78	6.3	32.1	70.5
34	21	13	8	4	0	46	4.2	33.0	48.2
36	58	8	8	8	0	82	8.3	33.9	99.0
38	17	4	4	0	0	25	2.8	34.7	34.4
40	17	8	0	8	0	33	4.1	35.5	51.5
42	13	4	4	0	0	21	2.9	36.2	36.9
44	4	4	0	0	0	8	1.2	36.9	15.7
46	13	0	0	4	0	17	2.8	37.6	37.2
Totals	276	92	85	36	84	573	45.3	31.2	520.0

Stand table: Stand 6

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
28	2	0	2	0	0	4	0.2	29.9	2.6
34	4	7	0	0	0	11	1.0	31.8	11.1
36	4	2	0	0	0	6	0.6	32.3	6.9
38	2	0	0	2	0	4	0.5	32.9	5.2
40	16	0	2	0	0	18	2.3	33.3	26.4
42	13	9	2	0	0	24	3.3	33.8	39.3
44	11	7	2	0	0	20	3.0	34.2	36.4
46	7	0	2	0	0	9	1.5	34.6	18.1
48	11	2	0	2	0	15	2.7	35.0	33.3
50	13	0	0	2	0	15	2.9	35.4	36.5
52	13	2	0	0	0	15	3.2	35.7	39.8
54	11	2	2	0	0	15	3.4	36.1	43.4
56	6	2	0	2	0	10	2.5	36.4	31.4
58	6	4	0	4	0	14	3.7	36.7	47.5
60	2	2	0	7	0	11	3.1	37.0	40.3
62	0	0	0	2	0	2	0.6	37.3	7.9
64	2	0	0	2	0	4	1.3	37.5	16.9
66	4	0	0	7	0	11	3.8	37.8	49.8
Totals	127	39	12	30	0	208	39.6	34.9	492.8

Stand table: Stand 7

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
22	0	0	1	0	0	1	0.0	27.3	0.4
26	2	0	1	0	1	4	0.2	28.7	2.1
28	3	2	0	0	1	6	0.4	29.3	3.8
30	4	2	2	1	1	10	0.7	29.9	7.4
32	21	3	1	1	0	26	2.1	30.4	22.2
34	24	0	0	0	0	24	2.2	30.8	23.5
36	30	4	3	0	0	37	3.8	31.3	41.3
38	15	1	2	0	0	18	2.0	31.7	22.6
40	20	1	0	1	0	22	2.8	32.1	31.1
42	25	1	1	1	0	28	3.9	32.4	44.0
44	10	4	5	0	0	19	2.9	32.8	33.2
46	16	2	2	1	0	21	3.5	33.1	40.4
48	9	0	0	3	0	12	2.2	33.4	25.4
50	3	2	0	1	0	6	1.2	33.7	13.9
52	5	0	1	0	0	6	1.3	33.9	15.1
54	2	0	0	1	0	3	0.7	34.2	8.2
64	1	0	0	0	0	1	0.3	35.3	4.0
68	0	0	0	1	0	1	0.4	35.7	4.5
Totals	190	22	19	11	3	245	30.6	31.8	343.1

Stand table: Stand 8

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
22	0	0	0	2	9	11	0.4	27.2	4.0
24	0	0	0	2	9	11	0.5	28.5	5.0
26	0	4	2	4	4	14	0.7	29.7	7.7
28	0	0	0	7	0	7	0.4	30.8	4.6
30	0	2	0	0	0	2	0.1	31.8	1.6
32	4	4	0	2	0	10	0.8	32.7	9.2
34	18	22	2	2	0	44	4.0	33.6	47.0
36	18	13	7	0	0	38	3.9	34.5	46.7
38	20	18	0	0	0	38	4.3	35.3	53.2
40	22	9	2	0	0	33	4.1	36.0	52.3
42	33	7	0	0	0	40	5.5	36.7	71.2
44	27	2	2	0	0	31	4.7	37.4	61.7
46	18	7	0	0	0	25	4.2	38.0	55.3
48	16	0	4	0	0	20	3.6	38.6	48.9
50	4	0	0	0	0	4	0.8	39.2	10.8
52	7	2	0	0	0	9	1.9	39.7	26.6
54	4	0	0	0	0	4	0.9	40.3	12.9
56	0	2	0	0	0	2	0.5	40.8	7.0
Totals	191	92	19	19	22	343	41.3	35.2	525.7

Stand table: Stand 9

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
18	0	0	0	0	7	7	0.2	23.5	1.5
20	0	0	0	0	10	10	0.3	25.0	2.7
22	0	0	0	0	7	7	0.3	26.4	2.5
24	0	0	3	0	17	20	0.9	27.6	8.7
26	3	0	0	0	0	3	0.2	28.8	1.6
28	10	3	3	0	10	26	1.6	29.8	16.7
30	10	3	7	0	0	20	1.4	30.8	15.2
32	23	13	3	0	0	39	3.1	31.8	34.9
34	10	0	3	3	0	16	1.5	32.6	16.6
36	7	0	0	3	0	10	1.0	33.5	11.9
38	23	0	0	0	0	23	2.6	34.2	31.2
40	17	3	0	0	0	20	2.5	35.0	30.8
42	23	3	3	3	0	32	4.4	35.6	55.2
44	20	7	3	0	0	30	4.6	36.3	58.0
46	33	0	0	0	0	33	5.5	36.9	70.8
48	17	3	0	0	0	20	3.6	37.5	47.5
50	13	0	0	0	0	13	2.6	38.1	34.0
52	3	3	0	0	0	6	1.3	38.6	17.2
54	7	0	0	0	0	7	1.6	39.1	21.9
56	3	0	0	0	0	3	0.7	39.6	10.2
Totals	222	38	25	9	51	345	39.9	33.4	489.1

Stand table: Stand 10

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
20	0	0	0	0	6	6	0.2	24.1	1.6
22	0	0	6	0	6	12	0.5	25.5	4.1
24	6	0	0	0	6	12	0.5	26.8	5.1
26	0	0	6	0	0	6	0.3	28.1	3.1
28	11	0	28	0	6	45	2.8	29.2	28.3
30	0	0	11	11	6	28	2.0	30.3	21.0
32	0	11	6	0	0	17	1.4	31.2	14.9
34	6	6	6	0	0	18	1.6	32.2	18.4
36	22	0	0	6	0	28	2.9	33.1	33.0
38	28	0	6	6	0	40	4.5	33.9	53.8
40	22	0	11	6	0	39	4.9	34.7	59.5
42	22	6	22	0	0	50	6.9	35.4	85.8
44	22	11	0	6	0	39	5.9	36.1	74.9
46	11	0	0	0	0	11	1.8	36.8	23.5
48	11	11	6	6	0	34	6.2	37.4	80.5
50	6	6	0	0	0	12	2.4	38.0	31.3
52	6	0	6	0	0	12	2.5	38.6	34.4
54	11	0	0	6	0	17	3.9	39.2	53.4
56	6	0	0	0	0	6	1.5	39.7	20.5
58	11	0	0	0	0	11	2.9	40.2	40.9
64	0	6	0	0	0	6	1.9	41.7	28.2
66	6	0	0	0	0	6	2.1	42.1	30.2
70	6	0	0	0	0	6	2.3	43.0	34.8
74	6	0	0	0	0	6	2.6	43.7	39.5
82	6	0	0	0	0	6	3.2	45.2	50.1
Totals	225	57	114	47	30	473	67.7	34.4	870.8

Stand table: Stand 11

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
16	16	26	0	34	0	76	1.5	22.1	11.8
18	16	34	0	21	0	71	1.8	23.5	14.9
20	15	25	0	24	0	64	2.0	24.7	17.4
22	36	53	0	21	0	110	4.2	25.8	37.8
24	30	29	0	9	1	69	3.1	26.8	29.3
26	26	23	0	19	0	68	3.6	27.7	35.0
28	39	23	0	13	0	75	4.6	28.5	46.1
30	39	21	0	9	0	69	4.9	29.3	50.0
32	48	21	0	4	0	73	5.9	30.0	61.6
34	38	15	0	5	0	58	5.3	30.7	56.6
36	34	16	0	0	0	50	5.1	31.3	55.8
38	11	10	0	0	0	21	2.4	31.9	26.6
40	13	14	0	0	0	27	3.4	32.4	38.5
42	9	8	0	1	0	18	2.5	33.0	28.8
44	4	3	0	0	0	7	1.1	33.5	12.5
46	9	1	0	0	0	10	1.7	33.9	19.7
Totals	383	322	0	160	1	866	53.1	27.6	542.4

Stand table: Stand 12

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
16	7	35	1	25	1	69	1.4	20.1	9.8
18	9	23	8	19	1	60	1.5	21.7	11.6
20	15	29	0	19	1	64	2.0	23.1	16.3
22	14	23	11	19	1	68	2.6	24.4	22.1
24	29	26	8	11	1	75	3.4	25.6	30.4
26	45	19	8	13	0	85	4.5	26.8	42.3
28	45	23	13	8	0	89	5.5	27.8	53.3
30	55	26	0	9	0	90	6.4	28.7	63.9
32	55	14	0	4	0	73	5.9	29.6	60.8
34	56	9	0	2	1	68	6.2	30.5	65.9
36	47	13	0	2	0	62	6.3	31.3	69.1
38	35	7	0	1	0	43	4.9	32.0	54.6
40	26	6	0	2	0	34	4.3	32.7	48.9
42	27	3	0	0	0	30	4.2	33.4	48.6
44	9	4	0	0	0	13	2.0	34.0	23.5
46	9	1	0	0	0	10	1.7	34.6	20.1
48	3	0	0	1	0	4	0.7	35.2	8.9
50	2	0	0	0	0	2	0.4	35.8	4.9
52	1	0	0	0	0	1	0.2	36.3	2.7
54	1	0	0	1	0	2	0.5	36.8	5.9
56	0	2	0	0	0	2	0.5	37.3	6.4
60	1	0	0	0	0	1	0.3	38.2	3.8
Totals	491	263	49	136	6	945	65.4	27.5	673.8

Stand table: Stand 13

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
20	0	0	0	1	1	2	0.1	28.7	0.6
26	0	0	1	1	0	2	0.1	31.0	1.2
28	0	0	0	10	0	10	0.6	31.6	6.8
30	3	1	0	11	0	15	1.1	32.2	11.9
32	6	3	10	4	0	23	1.8	32.7	21.2
34	11	3	7	6	0	27	2.5	33.2	28.5
36	11	4	3	4	0	22	2.2	33.6	26.3
38	10	7	7	3	0	27	3.1	34.0	36.4
40	14	3	4	1	0	22	2.8	34.4	33.3
42	21	10	4	0	0	35	4.8	34.8	59.1
44	23	3	1	1	0	28	4.3	35.1	52.3
46	14	1	3	0	0	18	3.0	35.4	37.1
48	7	0	3	0	0	10	1.8	35.7	22.6
50	13	1	1	3	0	18	3.5	36.0	44.5
52	17	3	4	0	0	24	5.1	36.3	64.8
54	4	1	0	0	0	5	1.1	36.6	14.7
56	6	0	0	1	0	7	1.7	36.8	22.2
58	1	1	0	0	0	2	0.5	37.0	6.8
60	1	0	0	1	0	2	0.6	37.3	7.4
62	1	0	0	0	0	1	0.3	37.5	4.0
64	1	0	0	0	0	1	0.3	37.7	4.2
Totals	164	41	48	47	1	301	41.3	34.4	505.9

Stand table: Stand 14

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
16	0	0	0	0	44	44	0.9	21.4	6.6
18	0	0	0	0	28	28	0.7	23.2	5.8
20	0	0	0	0	33	33	1.0	24.8	9.0
22	0	0	0	0	39	39	1.5	26.3	13.6
24	0	6	0	0	22	28	1.3	27.6	12.2
28	6	6	17	0	11	40	2.5	30.1	25.9
30	11	0	6	6	0	23	1.6	31.1	17.7
32	11	6	17	22	0	56	4.5	32.2	50.8
34	11	0	11	0	0	22	2.0	33.1	23.1
36	11	17	6	6	0	40	4.1	34.0	48.5
38	17	0	6	0	0	23	2.6	34.9	31.9
40	44	11	0	0	0	55	6.9	35.7	86.4
42	33	6	6	0	0	45	6.2	36.4	79.4
44	11	6	0	0	0	17	2.6	37.2	33.7
46	39	0	0	0	0	39	6.5	37.8	85.7
48	33	6	0	6	0	45	8.1	38.5	109.7
50	0	6	0	0	0	6	1.2	39.1	16.1
52	6	0	0	0	0	6	1.3	39.7	17.7
54	11	0	0	0	0	11	2.5	40.3	35.5
56	6	0	0	0	0	6	1.5	40.9	21.2
64	6	0	0	0	0	6	1.9	42.9	29.0
Totals	256	70	69	40	177	612	61.4	32.2	759.5

Stand table: Stand 15

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
16	0	0	0	0	86	86	1.7	20.7	12.5
18	5	0	0	0	76	81	2.1	22.1	15.9
20	10	5	0	0	19	34	1.1	23.4	8.7
22	24	0	5	0	33	62	2.4	24.5	20.2
24	19	5	5	0	19	48	2.2	25.6	19.5
26	29	5	10	0	19	63	3.3	26.5	31.0
28	43	10	10	0	10	73	4.5	27.4	43.1
30	38	0	5	5	0	48	3.4	28.2	33.5
32	38	24	10	0	0	72	5.8	28.9	58.6
34	29	5	0	5	10	49	4.4	29.6	46.1
36	24	5	0	0	0	29	3.0	30.3	31.3
38	24	0	0	0	0	24	2.7	30.9	29.4
40	19	5	0	0	0	24	3.0	31.5	33.3
42	10	0	0	0	0	10	1.4	32.1	15.6
44	5	5	0	0	0	10	1.5	32.6	17.3
46	0	0	0	5	0	5	0.8	33.1	9.6
48	10	0	0	10	0	20	3.6	33.6	42.6
50	14	0	0	0	0	14	2.7	34.0	32.7
52	0	5	0	5	0	10	2.1	34.4	25.6
54	0	0	0	5	0	5	1.1	34.8	13.9
66	10	0	0	0	0	10	3.4	37.0	44.3
Totals	351	74	45	35	272	777	56.2	26.9	584.7

Stand table: Stand 16

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
20	0	0	0	0	2	2	0.1	28.2	0.6
26	0	0	2	0	3	5	0.3	29.8	2.8
28	3	0	3	0	2	8	0.5	30.3	5.2
30	3	2	2	0	0	7	0.5	30.7	5.3
32	5	3	5	3	3	19	1.5	31.1	16.6
34	3	0	12	2	0	17	1.5	31.4	17.0
36	0	12	5	3	2	22	2.2	31.7	24.8
38	8	7	15	0	0	30	3.4	32.0	38.1
40	3	15	10	0	0	28	3.5	32.3	39.8
42	8	17	2	0	0	27	3.7	32.6	42.7
44	7	10	0	0	0	17	2.6	32.8	29.7
46	8	12	5	0	0	25	4.2	33.1	48.1
48	5	5	3	0	0	13	2.4	33.3	27.4
50	3	2	5	2	0	12	2.4	33.5	27.6
52	3	10	3	0	0	16	3.4	33.7	40.1
54	5	0	0	0	0	5	1.1	33.9	13.6
56	3	2	2	2	0	9	2.2	34.0	26.4
58	5	5	0	0	0	10	2.6	34.2	31.6
60	2	0	0	0	0	2	0.6	34.4	6.8
62	3	0	2	0	0	5	1.5	34.5	18.2
64	3	0	0	0	0	3	1.0	34.7	11.7
66	0	0	2	0	0	2	0.7	34.8	8.3
68	0	2	0	0	0	2	0.7	34.9	8.9
Totals	80	104	78	12	12	286	42.6	32.5	491.3

Stand table: Stand 17

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
16	0	0	3	0	34	37	0.7	25.8	6.7
18	6	0	0	0	40	46	1.2	27.0	11.1
20	11	0	0	0	17	28	0.9	28.1	8.7
22	17	0	0	0	29	46	1.7	29.1	17.8
24	11	9	3	0	20	43	1.9	30.0	20.4
26	29	3	0	3	14	49	2.6	30.8	28.0
28	26	3	3	6	17	55	3.4	31.5	37.3
30	23	6	3	0	3	35	2.5	32.2	27.9
32	43	6	9	6	6	70	5.6	32.8	64.6
34	59	9	9	6	3	86	7.8	33.4	91.3
36	31	6	6	14	0	57	5.8	33.9	68.8
38	29	3	6	6	0	44	5.0	34.4	60.1
40	23	6	0	6	0	35	4.4	34.9	53.7
42	31	6	3	6	0	46	6.4	35.4	79.0
44	23	0	0	3	0	26	4.0	35.8	49.5
46	3	0	0	3	0	6	1.0	36.2	12.6
48	3	0	0	0	0	3	0.5	36.5	6.9
50	0	0	0	3	0	3	0.6	36.9	7.6
Totals	368	57	45	62	183	715	56.0	31.9	652.0

Stand table: Stand 18

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
16	2	0	0	0	31	33	0.7	16.5	3.8
18	0	0	0	0	47	47	1.2	18.5	7.7
20	0	0	4	15	67	86	2.7	20.4	19.3
22	0	0	0	20	9	29	1.1	22.2	8.6
24	2	4	4	11	4	25	1.1	23.9	9.5
26	4	2	22	9	2	39	2.1	25.5	18.5
28	0	7	7	11	0	25	1.5	27.0	14.5
30	33	20	20	4	2	79	5.6	28.4	55.5
32	27	20	9	0	0	56	4.5	29.8	47.0
34	38	22	4	0	0	64	5.8	31.1	63.2
36	31	26	7	0	0	64	6.5	32.3	73.6
38	42	24	12	6	0	84	9.5	33.5	111.7
40	67	27	7	2	0	103	12.9	34.6	156.7
42	73	30	12	2	0	117	16.2	35.7	202.5
44	15	2	0	0	0	17	2.6	36.8	33.3
46	20	6	2	2	0	30	5.0	37.8	66.0
48	4	0	0	0	0	4	0.7	38.7	9.8
50	7	4	0	0	0	11	2.2	39.6	29.9
52	4	2	0	0	0	6	1.3	40.5	18.1
58	2	0	0	0	0	2	0.5	43.0	8.0
Totals	371	196	110	82	162	921	83.7	29.6	957.2

Stand table: Stand 19

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
16	2	0	0	0	3	5	0.1	23.5	0.8
18	0	0	0	0	2	2	0.1	24.9	0.4
20	0	0	0	0	7	7	0.2	26.0	2.0
22	7	3	0	0	5	15	0.6	27.1	5.4
24	3	3	0	0	10	16	0.7	28.0	7.1
26	5	0	3	2	5	15	0.8	28.9	8.1
28	13	3	2	2	2	22	1.4	29.7	14.1
30	12	3	3	0	0	18	1.3	30.4	13.5
32	25	7	3	3	0	38	3.1	31.1	33.3
34	25	2	0	0	0	27	2.5	31.8	27.3
36	25	2	2	3	0	32	3.3	32.4	36.9
38	20	5	7	5	0	37	4.2	32.9	48.3
40	20	2	2	0	0	24	3.0	33.4	35.3
42	15	5	3	2	0	25	3.5	33.9	41.1
44	13	0	2	2	0	17	2.6	34.4	31.1
46	12	0	2	0	0	14	2.3	34.8	28.3
48	13	2	0	0	0	15	2.7	35.2	33.4
50	3	2	0	0	0	5	1.0	35.6	12.2
52	3	0	0	2	0	5	1.1	36.0	13.4
56	2	0	0	0	0	2	0.5	36.7	6.3
58	2	0	0	0	0	2	0.5	37.0	6.8
Totals	220	39	29	21	34	343	35.5	31.7	405.1

Stand table: Stand 20

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
22	0	2	0	0	2	4	0.2	26.3	1.4
24	6	0	4	0	0	10	0.5	27.4	4.3
26	6	0	2	0	0	8	0.4	28.4	4.2
28	10	4	0	0	0	14	0.9	29.4	8.9
30	6	2	0	0	0	8	0.6	30.2	6.0
32	14	4	0	0	0	18	1.4	31.0	15.7
34	30	0	0	2	0	32	2.9	31.8	32.3
36	22	8	4	0	0	34	3.5	32.5	39.4
38	36	10	2	4	0	52	5.9	33.1	68.4
40	32	8	0	2	0	42	5.3	33.8	62.7
42	24	0	0	0	0	24	3.3	34.3	39.9
44	20	0	2	0	0	22	3.3	34.9	40.9
46	24	6	0	0	2	32	5.3	35.4	65.9
48	14	0	0	0	0	14	2.5	35.9	31.8
50	6	4	0	0	0	10	2.0	36.4	25.0
52	8	2	0	0	0	10	2.1	36.9	27.4
54	2	2	0	0	0	4	0.5	37.3	6.0
56	2	0	0	2	0	6	1.5	37.7	19.5
58	0	2	0	0	2	4	1.1	38.1	14.1
60	4	2	2	0	0	8	2.3	38.5	30.5
62	3	2	0	0	0	5	1.5	38.9	20.6
64	2	0	0	0	0	2	0.6	39.5	8.9
66	2	0	0	0	0	2	0.7	39.8	10.3
68	2	0	0	0	0	2	0.7	40.1	10.4
70	2	0	0	0	0	2	0.8	40.3	11.9
Totals	277	58	16	10	6	369	49.8	33.6	606.4

Stand table: Stand 21

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
16	0	0	0	0	73	73	1.5	21.0	10.8
18	0	0	0	0	64	64	1.6	22.5	12.8
20	6	0	6	0	67	79	2.5	23.9	20.8
22	15	3	18	0	52	88	3.3	25.1	29.4
24	52	15	12	0	9	88	4.0	26.3	36.6
26	27	15	9	0	3	54	2.9	27.3	27.4
28	42	21	0	0	3	66	4.1	28.3	40.3
30	39	15	6	0	0	60	4.2	29.2	43.3
32	39	12	3	3	0	57	4.6	30.0	48.1
34	36	3	3	0	0	42	3.8	30.8	41.1
36	43	6	3	6	0	58	5.9	31.5	65.1
38	39	6	6	0	0	51	5.8	32.2	65.2
40	30	3	0	6	0	39	4.9	32.9	56.4
42	24	0	3	6	0	33	4.6	33.5	53.6
44	36	6	0	3	0	45	6.8	34.1	81.7
46	15	0	0	0	0	15	2.5	34.6	30.2
48	18	3	3	0	0	24	4.3	35.1	53.4
50	6	0	0	0	0	6	1.2	35.7	14.7
52	6	3	0	0	0	9	1.9	36.1	24.1
54	0	0	3	0	0	3	0.7	36.6	8.8
58	3	0	0	0	0	3	0.8	37.5	10.4
60	3	0	0	3	0	6	1.7	37.9	22.5
Totals	479	111	75	27	271	963	73.6	28.3	796.7

Stand table: Stand 22

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
16	51	10	1	0	0	62	1.2	21.8	9.5
18	55	12	0	1	0	68	1.7	22.8	13.8
20	67	10	2	0	0	79	2.5	23.8	20.7
22	89	5	5	1	0	100	3.8	24.6	32.7
24	98	6	0	7	0	111	5.0	25.4	44.6
26	98	11	0	0	0	109	5.8	26.0	52.7
28	93	7	4	1	0	105	6.5	26.7	60.4
30	94	5	0	4	0	103	7.3	27.2	69.3
32	91	6	3	2	0	102	8.2	27.8	79.8
34	40	3	0	0	0	43	3.9	28.2	38.5
36	33	3	0	1	0	37	3.8	28.7	37.8
38	34	2	1	0	0	37	4.2	29.1	42.7
40	21	1	0	2	0	24	3.0	29.5	31.1
42	11	1	1	0	0	13	1.8	29.9	18.8
44	12	1	0	0	0	13	2.0	30.3	21.0
46	3	1	0	0	0	4	0.7	30.6	7.1
48	1	0	0	0	0	1	0.2	30.9	2.0
50	1	1	0	0	0	2	0.4	31.2	4.3
52	1	0	0	0	0	1	0.2	31.5	2.3
56	1	0	0	0	0	1	0.2	32.0	2.8
Totals	894	85	17	19	0	1015	62.4	26.1	591.9

Stand table: Stand 23

DBH [cm]	Number of Trees per Quality Class [/ha] 1	2	3	4	5	Total	Basal Area [m2/ha]	Height [m]	Volume [m3/ha]
16	0	0	0	0	3	3	0.1	21.5	0.5
18	0	0	0	0	8	8	0.2	22.8	1.6
20	0	2	0	0	6	8	0.3	23.9	2.1
22	0	0	0	2	5	7	0.3	24.9	2.3
24	9	5	0	2	2	18	0.8	25.9	7.4
26	6	2	2	3	2	15	0.8	26.7	7.4
28	9	8	2	2	0	21	1.3	27.5	12.4
30	11	3	0	2	3	19	1.3	28.2	13.3
32	11	2	0	0	0	13	1.0	28.9	10.6
34	12	0	0	0	0	12	1.1	29.5	11.2
36	29	5	0	0	0	34	3.5	30.1	36.5
38	35	2	0	0	0	37	4.2	30.6	44.9
40	43	0	0	0	0	43	5.4	31.1	58.8
42	26	0	0	2	0	28	3.9	31.6	42.9
44	42	0	0	0	0	42	6.4	32.1	71.7
46	17	0	0	0	0	17	2.8	32.5	32.1
48	39	0	0	2	0	41	7.4	32.9	85.4
50	19	0	0	0	0	19	3.7	33.3	43.5
52	19	0	0	0	0	19	4.0	33.7	47.6
54	9	0	0	0	0	9	2.1	34.0	24.5
56	8	0	0	0	0	8	2.0	34.4	23.7
58	9	0	0	0	0	9	2.4	34.7	28.9
60	8	0	0	0	0	8	2.3	35.0	27.7
66	2	0	0	0	0	2	0.7	35.9	8.6
70	2	0	0	0	0	2	0.8	36.4	9.8
Totals	365	29	4	15	29	442	58.8	30.6	655.4

APPENDIX B

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