The purpose of this study is to develop experimental techniques to characterize typical interconnect discontinuities, including bends, steps, T junctions, vias and pads, which are the most commonly encountered interconnections in high speed digital integrated circuits, hybrid and monolithic microwave circuits and electronic packages. The time domain reflection response of these elements is used to classify the interconnect discontinuities as distributed discontinuity elements or as lumped elements depending upon the reflected waveform. For the cases of general distributed discontinuities including bends, steps and T junctions, the distributed equivalent circuit model is characterized by the time dependent impedance profile which is extracted from the time domain reflection measurements. By using known inverse scattering techniques implemented in terms of a new algorithm based on the transfer scattering matrix method of incremental uniform sections, this nonuniform impedance profile is extracted and is used to construct distributed element circuit models to represent the interconnect
discontinuities. A circuit model consisting of lumped/distributed elements, is also developed for the interconnect discontinuities which is intended to combine the accuracy of the distributed model with the simulation efficiency of the lumped models. This hybrid mode reduces computer simulation time when used as a net list for general purpose circuit simulators, such as SPICE. For the case of discontinuities modelled as lumped elements, such as vias and wiring pads, closed form equations based on the transfer scattering matrix solution are derived and used to extract the lumped electrical parameters of these elements from the time domain reflection waveform. All of these lumped, distributed and hybrid models are validated by comparing the time domain simulation results with Time Domain Reflectometer (TDR) measurements. A procedure for extracting the excess inductances and capacitances associated with the general discontinuities from the synthesized nonuniform impedance profile or the distributed model is also presented in this report. These results for excess lumped inductances and capacitances show close agreement with the published results for these structures which are based on the electromagnetic computation of excess currents and charges and frequency domain measurements. Finally, some typical cases demonstrating the effects of interconnect discontinuities in high speed clocking systems are presented and the procedure for reducing the reflections and transmission noise voltage by chamfering the bends and junctions is described.
Time Domain Characterization Of Interconnect Discontinuities

by

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1. Introduction

High speed digital and microwave hybrid and monolithic circuits include various active and passive circuit elements interconnected by sections of strip and microstrip transmission lines having a wide range of characteristic impedances. The resulting junctions and discontinuities such as bends, or a change in strip width have associated parasitic effects, which become significant at higher frequencies, and can seriously affect the circuit performance as shown in [1] for a GaAs MESFET microwave amplifier. The trend in new digital electronic systems is toward higher clock frequencies. When the clock speed reaches into the GHz regime and the rise time drops below one hundred pico-second (ps), the useful bandwidth required from the interconnect becomes several GHz or more. When this happens, typical interconnect dimensions become appreciable fractions of a wavelength, and the finite propagation velocity of electromagnetic effects can no longer be ignored. Interconnects must be designed as transmission line, and their electrical performance needs to be predicted theoretically, and optimized by the interconnect geometry.

The use of microstrip-like transmission lines or interconnections in the electronic packaging for high speed digital circuits including VLSI is widespread, and is increasing with growing interest in high density and high speed Silicon as well as GaAs based digital integrated circuits. Although
microstrip lines can be considered as ideal delay elements in the interconnect model, they often have discontinuities, such as a bend or a step change, as shown in Figure 1. At clock frequencies high enough to be in the transmission line regime, signals will be reflected by impedance changes in the region of the discontinuities. These reflections can cause severe signal distortion and crosstalk resulting in false switching [2,3,4].

Figure 1, Interconnect discontinuities in electronic packaging such as bends and steps

From the above description, it is clear that in order to prevent signal degradation and excess crosstalk noise due to interconnect discontinuities, it is important to investigate the electrical characteristics of discontinuities. The purpose of this work is to develop an experimental technique to characterize the discontinuities from time domain measurements.
2. Review of Interconnect Discontinuities

A discontinuity is caused by an unexpected change in the cross section of the strip conductor. Whenever the uniform electric and magnetic field patterns are redistributed by the discontinuity, there are evanescent fields in the vicinity of the discontinuity. The effect of this field redistribution can usually be described by representing it as an excess lumped equivalent element, involving a capacitor, and an inductor or corresponding additional transmission lines and lumped elements inserted between sections of uniform transmission line.

In the past, numerous papers have been published for modelling various discontinuities based on quasi-static as well as full-wave solution of the electromagnetic field equations [5,6,7,8,9]. The static values of capacitances associated with discontinuities can be evaluated by finding the excess charge distribution near the discontinuity, as given by

\[ C_{\text{ex}} = \frac{1}{V} \int Q_e \, ds, \]

(1)

where \( C_{\text{ex}} \) is the excess capacitance, \( Q_e \) is the excess charge density which exists on the region of the discontinuity, and \( V \) is the constant interconnect potential.

Similarly, the inductance can be calculated by evaluating an excess current density, as given by
\[ L_{ex} = \frac{1}{I^2} \int A J_e \, ds \]  \hspace{1cm} (2)

where \( L_{ex} \) is the excess inductance, \( I \) is the total current, and \( J_e \) is the excess current distribution in the discontinuity region.

For the frequency dependent dynamic properties of discontinuities, frequency domain measurement techniques and field theoretic methods based on full wave analysis have been developed [10,11,12]. In this chapter, the existing lumped element models for the microstrip discontinuities are reviewed.
2.1 Microstrip Bend

A bend discontinuity may be formed by a change in direction of a microstrip line, a situation often encountered in the circuit layouts or circuit interconnection as shown in Figure 2-(a). The first equivalent circuit to model microstrip right angle bends was formulated by Oliner [5] and is shown in Figure 2-(b). This equivalent circuit includes a shunt capacitance to account for the excess charge accumulation at and near the corner, and series inductances to account for excess circulating currents associated with the corner region [12].

The first experimental data for the excess parameter model was reported by Stephenson and Easter's [13]. An alternative equivalent circuit replaces the inductors proposed by Oliner with additional lengths of transmission line, connected in series with the microstrip line on either side of the discontinuity, as shown in Figure 2-(c). This equivalent length \( \delta L \) is calculated as

\[
Z_{om} = \sqrt{\frac{L}{C}} ,
\]

and \( v_p = \frac{v}{\sqrt{\varepsilon_r e}} = \frac{1}{\sqrt{L/C}} , \)

where \( L \) and \( C \) are the inductance and capacitance per unit length. Then

\[
\delta L = \left( \frac{1}{L_{ex}} \right) \frac{Z_{om}}{v_p} ,
\]
Figure 2. Bends and equivalent circuits
The characteristic impedance of microstrip $Z_{om}$ is given as [28]

$$Z_{om} = \frac{376.77}{2\pi \sqrt{\epsilon_{re}}} \ln \left[ \frac{6+(2\pi-6)e^{-B}}{(w/h)+\sqrt{1+(2h/w)^2}} \right], \quad (6)$$

$$B = \frac{(30.66h)}{w^{0.7528}}, \quad (7)$$

$$\epsilon_{re} = \frac{\epsilon_r+1}{2} + (\epsilon_r-1) \left(1 + \frac{10h}{w}\right) \frac{(x \cdot y)}{2}, \quad (8)$$

$$x = 1 + \frac{\log[(w/h)^{4+(w/52h)^{2}}]}{49} - \frac{\log[(w/h)^{4+0.432}]}{18.7} + \frac{\log[1+(w/18.1h)^{3}]}{18.7}, \quad (9)$$

$$y = 0.564 \frac{\epsilon_r - 0.9}{\epsilon_r + 3} \times 0.053, \quad (10)$$

where $\epsilon_{re}$ is the effective dielectric constant,

$v$ is the velocity of light,

$w$ is the width of conductor,

$t$ is the thickness of conductor,

$\epsilon_r$ is the dielectric constant of substrate,

and $h$ is the height of substrate.

In practical circuits, microstrip bends are chamfered to compensate for the excess capacitance. Variable bend angle and the chamfers are also modelled by the same equivalent circuits as shown in Figure 2.
2.2 Microstrip Step

The width of microstrip lines is changed in many circuits to change the impedance level or when the line is connected to pads or other metallizations. A step discontinuities together within equivalent circuits are shown in Figures 3-(a), (b) and (c). The field distribution is modified due to the change in current density from one strip width to the other, and the scattered fringing fields on the front edge of the wider conductor.

Current compression results in an excess current and hence a series inductance \(L_{ex}\), and the scattered field or the excess charge can be modelled by a parallel capacitance. For the junction formed by the two different microstrip width, the total inductance \(L_{ex}\) may be separated into inductors \(L_1\) and \(L_2\) as follows

\[
L_1 = \frac{L_{w1}}{L_{w1} + L_{w2}} L_{ex},
\]

(11)

and

\[
L_2 = \frac{L_{w2}}{L_{w1} + L_{w2}} L_{ex},
\]

(12)

and the corresponding extended transmission line length may be written as

\[
\delta L_1 = L_1 \left/ \left(\frac{Z_{om1} \sqrt{\varepsilon_{re1}}}{\nu}\right)\right.,
\]

(13)

and

\[
\delta L_2 = L_2 \left/ \left(\frac{Z_{om2} \sqrt{\varepsilon_{re2}}}{\nu}\right)\right.,
\]

(14)

where \(\delta L_1 = \delta L_2 = \frac{L_{ex}}{L_{w1} + L_{w2}}\). 

(15)
Figure 3, Step discontinuity and its equivalent circuits
2.3 Vias and Pads

In order to handle denser VLSI devices and to improve higher system performance, the interconnect technologies of the multilayer board (MLB) and multichip module (MCM) have been developed to meet the increasing wiring capability requirements for electronic packaging. That is, the interconnect must have versatility to turn out of one line into another layer when the channel becomes blocked by a wire already printed there. Therefore, vias are required at turning points in the interconnection as shown in Figure 4. Vias are also required for making connections between different layers of a multilayer printed circuit board.

![Figure 4](image_url)

(a) Top view

(b) Side view

Figure 4, A physical structure of vias
In general, the inductance is usually the electrical parameter of interest to represent the lumped model of vias. Up to present time, all the issues on the discontinuity model of vias are derived from Maxwell's equation [26] and static electromagnetic computation [27]. However, the results presented in these papers treat a via as a vertical conductor or a cylindrical hole, and only via conductors are assumed to exist without any interaction with track conductors. It is also noted that the junction effects of vias with track conductors are not included in this model.

Plated through-holes provide an entry from the terminals of the components to the interconnection lines, and vias are required at turning points in the interconnection. Both of them will have a pad to which the interconnect line is joined as shown in Figure 5. Normally, if the conductor is a small, plated structure over the ground plane with low current, then it is treated as a capacitance in the equivalent circuit parameters of electronic packaging.

Figure 5, The physical structure of pads
3. Measurement Technique and Parameters Extraction

Although numerical results based on a field theoretic analysis and experimental results are available in the literature for the determination of excess parameters, these lumped element equivalent circuits do not in general lead to accurate evaluation of signal delay and distortion, and hence are not suitable for the electrical simulation of high speed digital circuits.

In recent years, full wave numerical methods have been used to characterize these elements in the frequency and time domain [8, 14, 15]. The full wave time domain analysis of microstrip discontinuities based on the transmission line method (TLM) has also been reported [8]. Some experimental work has been reported on the time domain characterization of small inductance and capacitance discontinuities [9, 16]. However, these experimental techniques are not available for distributed interconnect discontinuities such as microstrip bends and steps. In this section, an experimental technique based primarily on time domain reflection measurement is reported. This technique can be used to characterize bend, step and other discontinuities in strip and microstrip-like transmissions lines. Before presenting this experimental technique developed in this thesis, a brief description of the frequency domain and time domain reflection measurement technique is given in the following section.
3.1 Frequency Domain Measurement

Frequency domain methods for accurate measurement of microstrip discontinuities are based on the technique of incorporating the discontinuities in a microstrip resonator and measuring the change in resonance frequency [11]. The resonance method and the associated modelling techniques represent an important tool for the experimental characterization of the excess parameters when reliable theoretical data is not available [6,7,16,17].

The circuit layout for characterization of right angle bend is shown in Figure 6-(a); a similar arrangement can be used to calculate the equivalent circuit parameters of a step discontinuity, as shown in Figure 6-(b). If the resonators are made with an overall effective length of an even number of half wavelengths \( \frac{\lambda}{2} \), and the arms are equal electrical length, then the voltage at the corner is zero, and the effect of the shunt capacitance can be ignored. The resonance calculation is then given as

\[
2L_1 + 2\delta L + 2\delta L_g = n \left( \frac{\lambda_m}{2} \right).
\] (16)
Figure 6. Circuit lay-out for measuring equivalent circuits
While for the odd number of half wavelength $\frac{\lambda}{2}$ resonance, the presence of a voltage maximum at the corner causes the corner susceptance $B$ to increase the electrical length by an amount "$\delta L'$" on either side of the corner, where

$$jB = 2Y\tan(\beta \delta L'),$$

(17)

so that the second equation is given as

$$2L2 + 2\delta L + 2Lg + \frac{\lambda m}{\pi} \tan^{-1}\left(\frac{B}{2YO}\right) = (n + 1) \frac{\lambda m}{2}.$$ (18)

As $\varepsilon_{re}$ ' and $Lg$ ' are known from the same measurement technique, the excess parameters can be determined from the two frequency resonator. By using the resonance method, a curve fitting equation for the excess inductance and capacitance has been derived by Jansen et al [23]. These expressions are widely used for the frequency domain simulation of microwave circuits, as given by

$$C_{ex}/pf = 0.001 \text{ (h/mm)} ((10.35 \varepsilon_r + 2.5) \text{ (w/h)}^2 + (2.6 \varepsilon_r + 5.64) \text{ (w/h)}) ,$$

(19)

and

$$L_{ex}/nh = 0.22 \text{ (h/mm)} (1 - 1.35 \exp (-0.8 (w/h)^{1.39} )) ,$$

(20)

for the microstrip corner without chamfering, and as

$$C_{ex}/pf = 0.001 \text{ (h/mm)} ((3.93 \varepsilon_r + 0.62) \text{ (w/h)}^2 + (7.6 \varepsilon_r + 3.8) \text{ (w/h)}) ,$$

(21)

and

$$L_{ex}/nh = 0.44 \text{ (h/mm)} (1 - 1.062 \exp(-0.177 (w/h)^{0.947})) ,$$

(22)

for the 50% chamfered microstrip right-angle bend.
3.2 Time Domain Reflection Measurement

3.2.1 Transmission Line Characteristics

Time domain reflectrometry (TDR) techniques have been applied widely to characterize transmission line systems [16, 18, 19, 20, 21, 22]. A typical set up for measuring an unknown characteristic impedance 'ZL' is shown in Figure 7.

![Figure 7, Measurement set up for unknown impedance](image.png)

The time domain reflection measurements are made by a TDR system, which involves transmitting a pulse with finite rise time into the test fixture from a 50 ohm transmission line, and displaying the reflected signal. If the pulse encounters a discontinuity, the reflection travels back to the sending point, where it is compared in time, and amplitude with the original signal.
This ratio is called the voltage reflection coefficient \( \Gamma \), and it is related to transmission line impedance by the equation,

\[
\Gamma = \frac{V_r}{V_i} = \frac{Z_L - 50}{Z_L + 50}. \tag{23}
\]

The characteristic impedance of an unknown uniform transmission line is then given by,

\[
Z_L = 50 \frac{1 + \Gamma}{1 - \Gamma}. \tag{24}
\]

The propagation delay per unit length ' \( t_{pd} \) ' can be calculated by measuring the round trip propagation time ' \( T_{pd} \) ' which is the time period between the signal travelling from the beginning point to the end point, or the propagation time difference between two different length of transmission line. The effective dielectric constant is given by

\[
t_{pd} = \frac{T_{pd1} - T_{pd2}}{2L} = \frac{\delta T_{pd}}{2L} \tag{25}
\]

and \( \varepsilon_{re} = t_{pd} \nu \) \tag{26}
The measured results for uniform interconnects on a STYCAST model substrate are compared with the closed form solutions derived by Hammerstad and Jensen [13] and are shown in Table (1).

Table (1):

<table>
<thead>
<tr>
<th>Line Width (w), $\epsilon_r = 12.9$, $h=0.5''$</th>
<th>1''</th>
<th>0.5''</th>
<th>0.25''</th>
</tr>
</thead>
<tbody>
<tr>
<td>Characteristic Impedance:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measurement (ohm)</td>
<td>31.8</td>
<td>45.6</td>
<td>59.65</td>
</tr>
<tr>
<td>Calculation</td>
<td>30.9</td>
<td>44.5</td>
<td>59.34</td>
</tr>
<tr>
<td>Propagation Delay (ns/inch)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measurement</td>
<td>0.25</td>
<td>0.24</td>
<td>0.23</td>
</tr>
<tr>
<td>Calculation</td>
<td>0.252</td>
<td>0.244</td>
<td>0.237</td>
</tr>
</tbody>
</table>
3.2.2 Multiple Reflections

One of the advantages of TDR measurement techniques is its ability to handle cases involving more than one discontinuity. These discontinuities can be a source of error unless distributed multiple reflections are considered in TDR measurement. In this section, a simple mathematical analysis is discussed which can be used to estimate the error caused by two cascaded transmission lines with different characteristic impedances if the multiple reflections are neglected. A typical TDR test fixture for this multiple discontinuity is illustrated in Figure 8. The reflection coefficients corresponding to the two junctions of the transmission line are

\[ \Gamma_1 = \frac{Z_{o1} - Z_0}{Z_{o1} + Z_0}, \]  
\[ \Gamma_2 = \frac{Z_{o2} - Z_0}{Z_{o2} + Z_0}. \]  

The mismatch reflection at junction \( a \) generates a reflected wave \( V_{ra} \) and a transmitted wave \( V_{ia} \), where

\[ V_{ra} = \Gamma_1 \cdot V_i = \frac{Z_{o1} - Z_0}{Z_{o1} + Z_0} V_i, \]  
\[ V_{ia} = (1 + \Gamma_1) V_i, \]

and the reflection from the second discontinuity is,

\[ V_{rb} = \Gamma_2 \cdot (1 + \Gamma_1) \cdot V_i. \]
(a) multiple discontinuity reflections

(b) reflection observed

Figure 8, Multiple discontinuities measurement
However, \( V_{rb} \) still is not equal to \( V_{r2} \), since a re-reflection occurs at the first discontinuity. If we just consider the first incident and the first reflected wave, then the wave that returns to the monitoring point is

\[
V_{r2} = (1 + \Gamma') \cdot V_{rb} \\
= (1 - \Gamma_1) \cdot (1 + \Gamma_1) \cdot \Gamma_2 \cdot V_i \\
= \Gamma_2 \cdot (1 - \Gamma_1^2) \cdot V_i,
\]

where \( \Gamma' = \frac{Z_0 - Z_{oi}}{Z_0 + Z_{oi}} = -\Gamma_1 \) . \hfill (32)

In fact, the reflection coefficient at the second discontinuity is

\[
\Gamma_2 = \frac{V_{r2}}{(1 - \Gamma_1^2) \cdot V_i}. 
\]

On the TDR scope, "\( \Gamma_2' \) " is normally used to determine the reflection coefficient at the second discontinuity by the following equation

\[
\Gamma_2' = \frac{V_{r2}}{V_i}. \quad \text{ (34)}
\]

The error of second reflection coefficient can then be estimated by

\[
\delta \Gamma = \Gamma_2 - \Gamma_2' = \frac{V_{r2}}{V_i} \left[ \frac{\Gamma_1^2}{1 - \Gamma_1^2} \right], \quad \text{ (35)}
\]
and equation (35) is used to demonstrate the error determined by the factors $V_{r2}$ and $\Gamma_1$. After solving $Z_{o1}$ in terms of $V_{r2}$ and $Z_{o1}$, the equation listed below can be used to adjust the data from TDR measurement

$$V_{r2} = \Gamma_2 (1 - \Gamma_1^2) V_i,$$  \hspace{1cm} (36)

where $V_i = 1$ volt, $Z_o = 50$ ohm, then

$$Z_{o2} = Z_{o1} \left[ \frac{200 Z_{o1} + V_{r2} (Z_{o1} + 50)^2}{200 Z_{o1} - V_{r2} (Z_{o1} + 50)^2} \right].$$ \hspace{1cm} (37)

Distributed reflections are encountered for the case of the nonuniform transmission lines. By using time domain reflectometry (TDR) techniques, most of the interconnect discontinuities such as bends, steps and T junction, result in a nonuniform impedance profile in the discontinuity region. This nonuniform impedance which describes the distributed nature of these discontinuities in the time domain can can be treated as a cascaded uniform transmission-lines model. These nonuniform impedance profiles used to model the discontinuities can be obtained from the TDR measurements by using basic inverse scattering techniques as shown in the next chapter.
4. Modeling Procedure

The circuit models used to represent the interconnect discontinuities are shown in Figure 9. The excess lumped element model has been used for frequency domain simulation of RF and microwave circuits [10,12]. However, the signal delay due to discontinuity effects is not accurately modelled by lumped element equivalent circuits. For high speed digital circuits, this can be significant especially for interconnects having multiple discontinuities such as bends. In addition, to describe the distributed nature of the discontinuity in time domain, distributed or the hybrid lumped/distributed models as shown in Figure 9-(b),(c) may be better for modelling and accurate simulation of interconnects in high speed digital circuits and electronic packaging. These distributed models can be obtained from either time domain electromagnetic computations or from experimental data, and can be used for time domain simulations. The procedure for deriving these models from the measurement data is presented in this chapter.
Figure 9, The equivalent circuit models of interconnect discontinuities
4.1 Distributed Model Synthesis

For a step input, the characteristic impedance profile for a low reflection case is given by

\[ Zo(t) = \frac{1 + \rho(t)}{1 - \rho(t)} \] (38)

where \( Zo(t) \) is the nonuniform impedance profile, and \( \rho(t) \) is the measured reflection coefficient. For an input waveform having a finite rise time, the characteristic impedance is readily extracted from the reflected waveform for this low reflection approximation case. For the general case of the distributed reflection or multiple discontinuities, an algorithm based on a piecewise constant impedance profile must be formulated.

Consider an equivalent circuit for a nonuniform transmission line modelled as a cascade of uniform transmission lines as shown in Figure 9-(b), where each section of the transmission line is uniform and lossless. Furthermore, the characteristic impedance and propagation delay time of ith transmission line are denoted as \( Zo(i) \) and \( Tpd(i) \), respectively. The voltage \( V(x,t) \) and current \( I(x,t) \) down the line is a function of \( x \), the distance from the initial point.

If an incident voltage \( Vin(t) \) propagates along this nonuniform transmission line, the distributed discontinuities result in multiple reflections. The reflected voltage at the initial point \( x=0 \) can be expressed as the superposition of all the reflected components as given by,
\[ V(x=0, t) = V_{in}(t) + V_{in}(t) \rho_{01} u(t-2T_d) + V_{in}(t) \sum_{i=1}^{n} (\prod_{j=0}^{i-1} T_{j,j+1}) \]

\[ \rho_{i,i+1} (\sum_{j=0}^{i-1} T_{j+1,j}) u(t-2 \sum_{j=0}^{i} T_d) \left\{ 1 + \sum_{k=0}^{i-1} \rho_{k+1,k} \sum_{l=k+1}^{n} \rho_{l,l+1} \prod_{j=k+1}^{l-1} \right\} \]

\[ T_{j,j+1} T_{j+1,j} \left[ u(t-2 \sum_{j=k}^{l-1} T_d) + \sum_{m=0}^{n} \rho_{m+1,m} \sum_{r=m+1}^{r-1} \prod_{j=m+1}^{r-1} \right] \]

\[ T_{j+1,j} \left( u(t-2 \sum_{j=k}^{r} T_d) - 2 \sum_{j=m}^{r} T_d \right) + \ldots \ldots \ldots \ldots \ldots \]

(39)

where

\[ \rho_{i,i+1} = \frac{Z_{o(i+1)} - Z_{o(i)}}{Z_{o(i+1)} + Z_{o(i)}} \]

(40)

\[ \rho_{i+1,i} = -\rho_{i,i+1} \]

(41)

\[ T_{i,i+1} = \frac{2 Z_{o(i+1)}}{Z_{o(i+1)} + Z_{o(i)}} = 1 + \rho_{i,i+1} \]

(42)

\[ T_{i+1,i} = \frac{2 Z_{o(i)}}{Z_{o(i+1)} + Z_{o(i)}} = 1 + \rho_{i+1,i} \]

(43)

\( \rho_{i+1,i}, \rho_{i,i+1} \) are the backward and forward reflection coefficients at the junction of ith and i+1th transmission line section, and \( T_{i+1,i}, T_{i,i+1} \) are the corresponding transmission coefficients.
The single summation term includes all the contributions from the reflections at other discontinuities in the discrete nonuniform transmission line, and the multiple summation terms represent the contributions of multiple reflections arriving at the input port. However, it is possible to generate arbitrary waveforms by employing a cascade of uniform transmission lines. Two general procedures for finding the nonuniform impedance profile corresponding to the TDR data are presented at the following sections. The first procedure is based on layer-peeling method [24]. The second approach is somewhat similar to the iterative approach where the algorithm is based on the transfer scattering matrix.
4.1.1 Impedance Profile Synthesis

The principle of synthesis of discrete transmission lines presented here is based on the layer-peeling method [24, 25]. The reflected voltage given by equation (39) represents an arbitrary waveform in terms of a collection of pulses with short pulse width. These pulses consist of the positive or negative amplitude which are related to the reflection coefficients of the discrete nonuniform impedance profile of interconnect discontinuities in the time domain.

Let us first review some basic characteristics of the nonuniform and lossless transmission line characterized by the local characteristic impedance $Z_0(x)$ and the local propagation constant $r$. The signal in terms of voltage or current propagating along this line can be described by the following equations,

\[
\frac{d^2V(x)}{dx^2} = r^2 V(x) \quad ,
\]

and

\[
\frac{d^2I(x)}{dx^2} = r^2 I(x) \quad .
\]

The general equations for current $I(x)$ and voltage $V(x)$ at any point $x$ down the transmission line can be expressed by
\[ V(x) = V_i e^{-rx} + V_r e^{+rx}, \]  

\[ I(x) = \frac{V_i e^{-rx}}{Z_0(x)} - \frac{V_r e^{+rx}}{Z_0(x)}, \]  

where \( V_i e^{-rx} \) represents a voltage waveform traveling in the positive \( x \) direction, and \( V_r e^{+rx} \) represents a voltage waveform propagating in the negative \( x \) direction. So voltage and current along this line can in general be considered as the sum of two oppositely traveling waveforms at any instant.

Consider the following definitions. The incident parameter \( a \) of the \( i \)th uniform transmission line is defined as the normalized incident voltage as given by

\[ a(i) = \frac{1}{2} \left( \frac{V_i e^{-rx}}{\sqrt{Z_0(i)}} \right) = \frac{1}{2} \left( \frac{V(x)}{\sqrt{Z_0(i)}} + \sqrt{Z_0(i)} I(x) \right) ; \]  

the reflected parameter \( b \), is defined as

\[ b(i) = \frac{1}{2} \left( \frac{V_r e^{+rx}}{\sqrt{Z_0(i)}} \right) = \frac{1}{2} \left( \frac{V(x)}{\sqrt{Z_0(i)}} - \sqrt{Z_0(i)} I(x) \right) ; \]  

and the parameters \( a \) and \( b \) are related by the equation

\[ b(i) = S(i) a(i) , \]  

where \( S(i) \) is called the scattering element or, more commonly, the reflection coefficient \( \rho_{i-1,i} \).
According to the definitions of $a$ and $b$, the voltage and current at any point $x$ along the transmission line can also be described in terms of the incident parameter $a$ and the reflection parameter $b$ as

\[ V(x) = \sqrt{Z_0(i)} \left( a(i) + b(i) \right) \]  
\[ I(x) = \frac{(a(i) - b(i))}{\sqrt{Z_0(i)}}. \]  

For a nonuniform transmission line having equal propagation delay time is shown as Figure 10. A further useful result is obtained by considering the continuity of voltage and current at the interface of $i$th transmission line and $i+1$th transmission line.

Figure 10, A lattice diagram used for the cascaded uniform transmission line analysis
The boundary conditions at the jth time step for the oncoming incident/reflected parameters $a_n(i+1,j)/b_p(i+1,j)$ and the outgoing incident/reflected parameters $a_p(i+1,j)/b_n(i+1,j)$ must satisfy the following equations

$$\sqrt{Zo(i+1)} \left[ a_p(i+1,j) + b_p(i+1,j) \right] = \sqrt{Zo(i)} \left[ a_n(i+1,j) + b_n(i+1,j) \right]$$ (53)

and

$$\frac{1}{\sqrt{Zo(i+1)}} \left[ a_p(i+1,j) - b_p(i+1,j) \right] = \frac{1}{\sqrt{Zo(i)}} \left[ a_n(i+1,j) - b_n(i+1,j) \right]$$ (54)

where $a_n(i+1,j) = a_p(i,j) * D$ \hspace{1cm} (55)

$b_n(i+1,j) = b_p(i,j) * D^{-1}$ \hspace{1cm} (56)

$D f(t) = f(t - Td)$ : is the time delay operator \hspace{1cm} (57)

$D^{-1} f(t) = f(t + Td)$ : is time advance operator \hspace{1cm} (58)

Equations (53) and (54) can be rewritten for the incident and reflected parameters as

$$a_p(i+1,j) = \frac{\sqrt{Zo(i+1)}}{\sqrt{Zo(i+1)}} \left[ a_n(i+1,j) + b_n(i+1,j) \right] - b_p(i+1,j)$$ (59)

$$b_p(i+1,j) = \frac{\sqrt{Zo(i+1)}}{\sqrt{Zo(i)}} \left[ - a_n(i+1,j) + b_n(i+1,j) \right] + a_p(i+1,j)$$ (60)
New equations which are useful for the iterative process are given by and formed from equations (53) and (54) as

\[ a_{i+1} = \left( \frac{Z_0(i)}{Z_0(i+1)} + \frac{Z_0(i+1)}{\sqrt{Z_0(i)}} \right) a_n(i+1,j) + \left[ \frac{\sqrt{Z_0(i)}}{\sqrt{Z_0(i+1)}} - \frac{\sqrt{Z_0(i+1)}}{\sqrt{Z_0(i)}} \right] b_n(i+1,j) \]

where

\[ = (1 - \rho_{i,i+1})^{1/2} a_n(i+1,j) - \rho_{i,i+1} (1 - \rho_{i,i+1})^{1/2} b_n(i+1,j) \]  

and

\[ b_{i+1} = \left[ \frac{Z_0(i)}{\sqrt{Z_0(i+1)}} - \frac{\sqrt{Z_0(i+1)}}{\sqrt{Z_0(i)}} \right] a_n(i+1,j) + \left[ \frac{\sqrt{Z_0(i)}}{\sqrt{Z_0(i+1)}} + \frac{\sqrt{Z_0(i+1)}}{\sqrt{Z_0(i)}} \right] b_n(i+1,j) \]

where

\[ = - \rho_{i,i+1} (1 - \rho_{i,i+1})^{1/2} a_n(i+1,j) + (1 - \rho_{i,i+1})^{1/2} b_n(i+1,j) \]  

The process of iterative computation for the recovery of the nonuniform impedance profile is based on equations (48)-(58), (61) and (62). The reflection coefficients of the first section at each time step of 2Td is determined in terms of the reflected voltage at the reference point and the amplitude of the input voltage as a function of time. By successively computing the reflection coefficients and peeling off the associated section, the characteristic impedance \( Z_0(i) \) for \( i=1,2,3,\ldots,n \) can be recovered from \( \rho_{i-1,i} \). The algorithm, which is used for step or ramp input waveform is given below.
Step (1): input the reference voltage and the reflected voltage measured by TDR at the initial point

Case (i): step input - The rise time of the pulse is smaller than the propagation delay of the distributed transmission line section (Tpd).

- Input source

\[ a_{n(1,1)} = \frac{V_{in}}{\sqrt{Z_{0}(0)}} \]  \hspace{1cm} (63)

\[ a_{n(1,i)} = 0 \text{ for } i = 2, 3, \ldots , n \text{ (let } Z_{0}(0) = Z_{0}) \]  \hspace{1cm} (64)

where \( V_{in} \) is the amplitude of input voltage

- The reflected voltage is discrete at every time step \( 2T_{d} \),

\[ b_{n(1,i)} = \frac{V_{in} - V_{r(0,i*2T_{d})}}{\sqrt{Z_{0}(0)}} \text{ for } i = 2, 3, \ldots , n \]  \hspace{1cm} (65)

Case (ii): ramp input - The rise time (\( T_{r} \)) of the pulse is not much small then the propagation delay of the distributed transmission line section (\( T_{pd} \)). A lattice diagram is shown in Figure 11.
partition the rising edge of input voltage into \( k \) intervals with equal time step \( 2T_d \),

\[
\begin{align*}
an(1,i) &= \frac{V_{in}(0,i*2Td)}{\sqrt{Zo(0)}} \quad \text{for } i = 1,2,3,\ldots,k \quad (k*2T_d=Tr) \quad (66) \\
an(1,i) &= 0 \quad \text{for } i = k+1, \ldots, n \quad (67)
\end{align*}
\]

. let the round trip delay time from measured point (initial point) to the region of the discontinuity at least over \( m*2T_d \) (\( m>k \))

\[
\begin{align*}
b_n(1,i) &= 0 \quad \text{for } i = 1,2,3,\ldots, m \quad (68) \\
b_n(1,i) &= \frac{V_{in} - V_{r}(0,i*2Td)}{\sqrt{Zo(0)}} \quad \text{for } i = m+1,\ldots, n \quad (69)
\end{align*}
\]
Step (2): determine the reflection coefficient

\[ \rho_{i-1,i} = \frac{a_n(i,1)}{b_n(i,1)} \]  \hspace{1cm} (70)

Step (3): recover the characteristic impedance

\[ Z_{o(i)} = Z_{o(i-1)} \frac{1 + \rho_{i-1,i}}{1 - \rho_{i-1,i}} \]  \hspace{1cm} (71)

Step (4): calculate the incident and reflected parameters of next section

\[
\begin{bmatrix}
  a_{p(i+1,j)} \\
  b_{p(i,j)}
\end{bmatrix} =
\begin{bmatrix}
  1 & 0 \\
  0 & D
\end{bmatrix}
\begin{bmatrix}
  (1-\rho_{i,i+1})^{1/2} & -\rho_{i,i+1} \\
  \rho_{i,i+1} & (1-\rho_{i,i+1})^{1/2}
\end{bmatrix}
\begin{bmatrix}
  D & 0 \\
  0 & 1
\end{bmatrix}
\begin{bmatrix}
  a_{p(i,j)} \\
  b_{p(i+1,j)}
\end{bmatrix}
\]  \hspace{1cm} (72)

Step (5): the index of (i,j) is noted at the position i and instant time j as the following expressions

\[ a_{p(i+1,j)} = a_{p} \left( x = (i+1) \delta x + \varepsilon, t = (i+1) + 2 (j-1) T_{pd} \right), \]  \hspace{1cm} (73)

and \[ b_{n(i,j+1)} = b_{n} \left( x = (i) \delta x - \varepsilon, t = (i) + 2 (j) T_{pd} \right). \]  \hspace{1cm} (74)

for \( i = i+1, (i=1,2,3,\ldots,n), \) then back to step (2).
4.1.2 Synthesis Procedure Based on the Transfer Scattering Matrix

In this section, we will extend the concepts of the scattering matrix discussed in the last section to develop a computer algorithm for arbitrary waveform synthesis. The algorithm then is used for the modelling of interconnect discontinuities.

Figure 12, Scattering parameter of a two port network

For the two port network shown in Figure 12, the scattering matrix \( \mathbf{S} \) is defined to relate the incident and reflected parameters at input and output ports as

\[
\begin{bmatrix}
  b_1 \\
  b_2
\end{bmatrix} = \begin{bmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{bmatrix} \begin{bmatrix}
  a_1 \\
  a_2
\end{bmatrix}.
\]

(75)
Elements along the main diagonal of the scattering matrix are reflection coefficients, and the off-diagonal elements are transmission coefficients. From the definition, the incident and reflected parameters of the input port can be expressed in terms of the incident and reflected parameters of the output port as,

\[
\begin{bmatrix}
  b_1 \\
  a_1
\end{bmatrix} = T \begin{bmatrix}
  a_2 \\
  b_2
\end{bmatrix},
\]  

(76)

where the matrix

\[
T = \begin{bmatrix}
  T_{11} & T_{12} \\
  T_{21} & T_{22}
\end{bmatrix} = \frac{1}{\text{Det} S} \begin{bmatrix}
  -D_{21} & S_{11} \\
  -S_{22} & 1
\end{bmatrix}
\]  

(77)

is defined as the transfer scattering matrix of the two port network.

Next, let us derive the expressions for the overall transfer matrix of nonuniform transmission line which is modelled as the stepped impedance line. At the interface of i-1th and ith section as shown in Figure 13, the transfer scattering matrix is given by

\[
\begin{bmatrix}
  b_{n(i)} \\
  a_{n(i)}
\end{bmatrix} = \begin{bmatrix}
  T_{11(i)} & T_{12(i)} \\
  T_{21(i)} & T_{22(i)}
\end{bmatrix} \begin{bmatrix}
  a_{p(i)} \\
  b_{p(i)}
\end{bmatrix}.
\]  

(78)

In addition, the incident and reflected wave propagate along the uniform transmission line as,

\[
a_{n(i+1)} = b_{p(i)} e^{-rx},
\]  

(79)

and

\[
b_{n(i+1)} e^{-rx} = a_{p(i)},
\]  

(80)
For the case of lossless transmission line with the propagation delay time $T_{pd}$, equation (79) and (80) can then be rearranged in matrix form as

$$
\begin{bmatrix}
ap(i) \\
bp(i)
\end{bmatrix} =
\begin{bmatrix}
e^{-rx} & 0 \\
0 & e^{+rx}
\end{bmatrix}
\begin{bmatrix}
bn(i+1) \\
an(i+1)
\end{bmatrix}
$$

$$
= \begin{bmatrix}
e^{-sT_{pd}} & 0 \\
0 & e^{+sT_{pd}}
\end{bmatrix}
\begin{bmatrix}
bn(i+1) \\
an(i+1)
\end{bmatrix},
$$

(81)

where $rx = j\beta x = j \omega \left( \frac{x}{\upsilon p} \right) = s T_{pd}$, $s = j \omega$, and $T_{pd} = \left( \frac{x}{\upsilon p} \right)$.
With these equations, we can derive a new matrix form relating the input incident and reflected parameters of any sections to that of the adjoining section by

\[
\begin{bmatrix}
    b_{n(i)} \\
    a_{n(i)}
\end{bmatrix}
= 
\begin{bmatrix}
    T_{11}^{(i)}(s) & T_{12}^{(i)}(s) \\
    T_{21}^{(i)}(s) & T_{22}^{(i)}(s)
\end{bmatrix}
\begin{bmatrix}
    b_{n(i+1)} \\
    a_{n(i+1)}
\end{bmatrix}
\]

\[
= [T(i)] [D(s)]
\begin{bmatrix}
    b_{n(i+1)} \\
    a_{n(i+1)}
\end{bmatrix},
\]  

(82)

where \([T(i)] = 
\begin{bmatrix}
    T_{11}^{(i)}(s) & T_{12}^{(i)}(s) \\
    T_{21}^{(i)}(s) & T_{22}^{(i)}(s)
\end{bmatrix},
\]  

(83)

and \([D(s)] =
\begin{bmatrix}
    e^{-sTpd} & 0 \\
    0 & e^{+sTpd}
\end{bmatrix}.
\]  

(84)

Since the total transfer scattering matrix \([T(s)]_n\) of the discrete nonuniform transmission is given by the product of the transfer scattering matrices of \(n\) uniform transmission line, it can be expressed as

\[
\begin{bmatrix}
    b_{n(1)} \\
    a_{n(1)}
\end{bmatrix}
= [T(1)] [D(s)] [T(2)] [D(s)] \cdots [T(n-1)] [D(s)] [T(n)]
\begin{bmatrix}
    a_{n(n)} \\
    b_{n(n)}
\end{bmatrix}
\]

\[
= [T(s)]_n
\begin{bmatrix}
    a_{n(n)} \\
    b_{n(n)}
\end{bmatrix},
\]  

(85)

where the total transfer scattering matrix is

\[
[T(s)]_n =
\begin{bmatrix}
    T_{11}(s)_n & T_{12}(s)_n \\
    T_{21}(s)_n & T_{22}(s)_n
\end{bmatrix}.
\]  

(86)
For a lossless two port network, the transfer scattering elements must have the following properties (See Appendix (A.1.)),

\[
T_{11}(s)_n = T_{22}(s^*)_n, \quad (87)
\]

and

\[
T_{12}(s)_n = T_{21}(s^*)_n. \quad (88)
\]

Now let us consider the total transfer scattering matrix.

\[
[T(s)]_n = [T(1)] [D(s)] [T(2)] [D(s)] \cdots [T(n-1)] [D(s)] [T(n)]
= [T(1)] [D(s)] [T(s)]_{n-1}
= [T(1)] [D(s)] [T(2)] [D(s)] [T(s)]_{n-2}
\vdots
\vdots
= [T(1)] [D(s)] [T(2)] [D(s)] \cdots [T(n-1)] [D(s)] [T(n)]_1. \quad (89)
\]

According to the above expressions, an equation can then be found to relate the local transfer scattering matrix \([T(i)]\) and the global transfer scattering matrix \([T(s)]_i\). This is given by

\[
[T(s)]_n = [T(1)] [D(s)] [T(s)]_{n-1} \quad (90)
\]

\[
[T(s)]_{n-1} = [T(2)] [D(s)] [T(s)]_{n-2} \quad (91)
\]

\vdots

\[
[T(s)]_{n-i} = [T(i)] [D(s)] [T(s)]_{n-(i+1)} \quad (92)
\]
By rearranging equation (92), we can obtain the iterative equation which is suitable for computer programming

\[ [T(s)]_{n-(i+1)} = [T(i)]^{-1} [D(s)]^{-1} [T(s)]_{n-(i)} \]  
(93)

As we have discussed in the last section, the local transfer scattering matrix \([T(i)]\) and its inverse can be obtained from equations (53) and (54). It is noted that both of the matrices are constant coefficient matrices as given by,

\[ [T(i)] = (1 - \rho_{i,i+1})^{-1/2} \begin{bmatrix} 1 & \rho_{i,i+1} \\ \rho_{i,i+1} & 1 \end{bmatrix}, \]
(94)

and \([T(i)]^{-1} = (1 - \rho_{i,i+1})^{-1/2} \begin{bmatrix} 1 & -\rho_{i,i+1} \\ -\rho_{i,i+1} & 1 \end{bmatrix}.\]
(95)

Also, the upper and lower diagonal elements of the matrix \([D(s)]\) are defined by

\[ [D(s)] = \begin{bmatrix} e^{-sTpd} & 0 \\ 0 & e^{+sTpd} \end{bmatrix} = \begin{bmatrix} D_s & 0 \\ 0 & D_s^{-1} \end{bmatrix}, \]
(96)

where the delay and advance operators can be related to the Laplace transform of the time delay and advance operator as

\[ D_s F(s) = L \left[ D f(t) \right] = L \left[ f(t - Tpd) \right], \]
(97)

and \[ D_s^{-1} F(s) = D_s F(s) = L \left[ D^{-1} f(t) \right] = L \left[ f(t + Tpd) \right]. \]
(98)
With these definitions in mind, we can express the elements of the global transfer scattering matrix in terms of a polynomial function of the delay operators $D_s$. For example, $T_{12}(s)^n$ and $T_{22}(s)^n$ are expressed by

$$T_{12}(s)^n = \alpha_{(n-1)}e^{-s(n-1)Tpd} + \alpha_{(n-3)}e^{-s(n-3)Tpd} + \alpha_{(n-5)}e^{-s(n-5)Tpd} + \alpha_{(n-7)}e^{-s(n-7)Tpd} + \cdots + a_{(n-5)}e^{s(n-5)Tpd} + a_{(n-3)}e^{s(n-3)Tpd} + a_{(n-1)}e^{s(n-1)Tpd},$$

$$= \alpha_{(n-1)}D_s^{(n-1)} + \alpha_{(n-3)}D_s^{(n-3)} + \alpha_{(n-5)}D_s^{(n-5)} + \cdots + a_{(n-5)}D_s^{-(n-5)} + a_{(n-3)}D_s^{-(n-3)} + a_{(n-1)}D_s^{-(n-1)},$$

$$= T_{12}(D_s)^n \quad (99)$$

and

$$T_{22}(s)^n = \beta_{(n-1)}e^{-s(n-1)Tpd} + \beta_{(n-3)}e^{-s(n-3)Tpd} + \beta_{(n-5)}e^{-s(n-5)Tpd} + \beta_{(n-7)}e^{-s(n-7)Tpd} + \cdots + b_{(n-5)}e^{s(n-5)Tpd} + b_{(n-3)}e^{s(n-3)Tpd} + b_{(n-1)}e^{s(n-1)Tpd},$$

$$= \beta_{(n-1)}D_s^{(n-1)} + \beta_{(n-3)}D_s^{(n-3)} + \beta_{(n-5)}D_s^{(n-5)} + \cdots + b_{(n-5)}D_s^{-(n-5)} + b_{(n-3)}D_s^{-(n-3)} + b_{(n-1)}D_s^{-(n-1)},$$

$$= T_{22}(D_s)^n. \quad (100)$$

From equation (78), the total reflection function of the discrete transmission line can be obtained in terms of the ratio of $T_{12}(s)^n$ and $T_{22}(s)^n$ by

$$\Gamma(D_s)^n = \frac{b_{(n-1)}}{a_{(n-1)}} \bigg| a_{(n-1)} = 0 = \frac{T_{12}(s)^n}{T_{22}(s)^n}$$

$$= \frac{\alpha_{(n-1)}D_s^{(n-1)} + \alpha_{(n-3)}D_s^{(n-3)} + \cdots + a_{(n-1)}D_s^{-(n-1)}}{\beta_{(n-1)}D_s^{(n-1)} + \beta_{(n-3)}D_s^{(n-3)} + \cdots + b_{(n-1)}D_s^{-(n-1)}} \quad (101)$$
The total reflection function is a rational function of the delay operator \( D_s \). Furthermore, if the reflection function \( \Gamma(D_s)_n \) is a known rational function in which both numerator and denominator can be expressed by the polynomial function of the delay operator,

\[
\Gamma(D_s)_n = \frac{B(n) D_s^{(n-1)} + B(n-1) D_s^{(n-3)} + \ldots + B(1) D_s^{(n-1)} }{A(n) D_s^{(n-1)} + A(n-1) D_s^{(n-3)} + \ldots + A(1) D_s^{(n-1)}}. \tag{102}
\]

Once the reflection coefficient \( \rho_{1,2} \) at the first interface is given, the reflection coefficients at any stages of the cascaded uniform transmission line can then be obtained by finding the global reflection functions \( \Gamma(D_s)_i \) and following by assuming the propagation delay time \( T_{pd} \) to became large or approach to infinity. Thus, the reflection coefficients can be given by

\[
\rho_{1,2} = \frac{B(1)}{A(1)} = \Gamma(D_s)_n \bigg|_{T_{pd} \sim \text{infinite}} \tag{103}
\]

\[
\vdots
\]

\[
\rho_{i,i+1} = \frac{B^i(1)}{A^i(1)} = \Gamma(D_s)_{n-(i-1)} \bigg|_{T_{pd} \sim \text{infinite}}, \quad \text{for } i=1,2,\ldots,n-1. \tag{104}
\]

It is noted that \( B^i(1) \) and \( A^i(1) \) are the lowest order coefficients of the polynomial function for the numerator and denominator of the global reflection function \( \Gamma(D_s)_{n-(i-1)} \). Also \( \Gamma(D_s)_{n-(i-1)} \) is defined by the total reflection function \( \Gamma(D_s)_n \) via \((i-1)\) times transformation.
So far, the synthesis of a cascaded uniform transmission line structure based on the method of the transfer scattering matrix has been described. Now, a computer algorithm is presented for the realization of distributed network. Let us assume that the reflection voltage of the discrete nonuniform transmission line is given by TDR measurements. An iterative procedure can then be applied to obtain the characteristic impedances of the discrete nonuniform transmission line as follows,

**Step (1)**: Choose the reference voltage $V_{in}$ and impedance $Z_{in}$. Let the characteristic impedance of the first stage transmission line match the reference impedance.

: Obtain the total transfer scattering matrix $[T(D_s)]_n$ from the reflected voltage measured by TDR at the initial point. In a similar procedure for the layer-peeling algorithm discussed at the last section, two cases are described as follows,

Case(i): the step input response -

(a). $V_{in}(t) = V_A u(t)$, and $V_{in}(s) = \frac{V_A}{s}$  \hspace{1cm} (105)

where $V_A$ is the amplitude of input voltage.

(b). The reflected voltage is discrete for every $2Td$ time step and is given by,

$$V_{r}(t) = B(1) u(t) + B(2) u(t-2Tpd) + ... + B(i) u(t-i*2Tpd) + ... \hspace{1cm} (106)$$
Taking the Laplace transform, $V_r(t)$ can be expressed in the $s$ domain as

$$V_r(s) = \frac{B(1) + B(2) e^{-s^2T_p d} + \ldots + B(i) e^{-i s^2T_p d} + \ldots}{s}.$$  \hspace{1cm} (107)

(c). According to equation (78), the total reflection function is

$$\Gamma(D_s)_n = \frac{B(1) + B(2) e^{-s^2T_p d} + \ldots + B(i) e^{-i s^2T_p d} + \ldots}{A(1) + A(2) e^{-s^2T_p d} + \ldots + A(i) e^{-i s^2T_p d} + \ldots}$$

\hspace{1cm} = \frac{B(1) + B(2)D_s^{(2)} + \ldots + B(i)D_s^{(2i)} + \ldots}{A(1) + A(2)D_s^{(2)} + \ldots + A(i)D_s^{(2i)} + \ldots}, \hspace{1cm} (108)

where $A(1) = V_A$, $A(i) = 0$ for $i = 2, 3, \ldots, n$, \hspace{1cm} (109)
and $B(i) = V_A - V_r(i^2T_p d)$ for $i = 0, 1, 2, 3, \ldots, n$. \hspace{1cm} (110)

(d). Consider the definition of the total reflection function from equation (101). Define

$$T_{12}(D_s)_n = B(1) + B(2)D_s^{(2)} + \ldots + B(i)D_s^{(2i)} + \ldots, \hspace{1cm} (111)$$

and $T_{22}(D_s)_n = A(1) + A(2)D_s^{(2)} + \ldots + A(i)D_s^{(2i)} + \ldots$. \hspace{1cm} (112)
By applying the properties of the transfer scattering matrix given in equations (87) and (88), the total transfer scattering matrix can then be obtained in terms of the measured data as

\[
[T(D_s)]_n = \begin{bmatrix} T_{22}(D_s^*)_n & T_{12}(D_s)_n \\ T_{12}(D_s^*)_n & T_{22}(D_s)_n \end{bmatrix},
\]

where the conjugate of the delay operator \( D_s^* \) is \( D_s^{-1} \).

Case (ii): for the ramp input response -

(a). Subdividing the rising edge of input voltage into \( k \) intervals with equal time step \( 2Td \), we have

\[
A(i) = V_{in}(i*2Td), \quad \text{for } i = 1, 2, 3, \ldots, k \quad (k*2Td=Tr) \quad (115)
\]

\[
A(i) = 0, \quad \text{for } i = k+1, \ldots, n \quad (116)
\]

(b). Let the round trip delay time from measured point (initial point) to the region of the discontinuity at least subdivided by \( m*2Td \) \((m>k)\)

\[
B(i) = 0 \quad \text{for } i = 1, 2, 3, \ldots, m \quad (117)
\]

and \( B(i) = V_A - V_r(i*2Td), \quad \text{for } i = m+1, \ldots, n \quad (118) \)
Step (2): Given a known initial condition $\rho_{1,2}$, compute the global transfer scattering matrix $[T(D_s)]_{n-(i)}$ by an iterative structure as given by,

$$
\begin{bmatrix}
T_{22}(D_s)n_{-(i)} & T_{12}(D_s)n_{-(i)} \\
T_{12}(D_s)n_{-(i)} & T_{22}(D_s)n_{-(i)} 
\end{bmatrix} = (1-P_{i,i+1})^{-1/2} \begin{bmatrix}
D_s & -\rho_{i,i+1}D_s \\
-\rho_{i,i+1}D_s^{-1} & D_s^{-1}
\end{bmatrix} \begin{bmatrix}
T_{22}(D_s)n_{-(i-1)} & T_{12}(D_s)n_{-(i-1)} \\
T_{12}(D_s)n_{-(i-1)} & T_{22}(D_s)n_{-(i-1)} 
\end{bmatrix},
$$

(119)

Step (3): Successively obtain the reflection coefficients from the global reflection functions $\Gamma(D_s)_{n-(i)}$. Recover the characteristic impedance for every stage transmission lines as

the reflection coefficient given by $\rho_{i,i+1} = \frac{B^i(1)}{A^i(1)},$ \hspace{1cm} (120)

the characteristic impedance given by $Z_0(i+1) = Z_0(i)\frac{1 + \rho_{i,i+1}}{1 - \rho_{i,i+1}}.$ \hspace{1cm} (121)

Return to step(2) for $i = 2,3,\ldots,n$, \hspace{1cm}
4.3 Hybrid Lumped/Distributed Model

A lossless discontinuity can in general be represented by a cascaded ideal stepped impedance transmission line structure where the number of sections depends on the complexity of the TDR waveform. As the number of sections become large, the simulation of the interconnect on programs like SPICE becomes time consuming and the alternate hybrid model consisting of lumped as well as distributed elements may be more compatible with these simulate tools. The building blocks for this model are delay elements augmented with inductances and capacitances. Their time domain response is discussed in this section.
4.3.1 Inductive Discontinuity

In this section, the time domain response of the building block with an inductive discontinuity is presented. The network model of inductive discontinuity consisting of a series inductor and two uniform transmission lines is shown in Figure 14. The transfer scattering matrix is used to derive an characteristic equation to extract the series inductance. This characteristic equation can also be applied to characterize discontinuities like vias where the inductance accounts for the via and the transmission lines are used to account for the characteristic impedances of the track in different layers.

![Figure 14, Lumped model of vias](image)

Now, let us consider the scattering matrix of the network model. The matrix elements of any two port network can be given as (See Appendix (A.2).),
\[ S_{11} = \frac{(sL + Zo1) - Zo}{(sL + Zo1) + Zo}, \]  
\[ S_{22} = \frac{(sL + Zo) - Zo1}{(sL + Zo) + Zo1}, \]  
\[ S_{21} = \frac{2 \sqrt{Zo1 Zo}}{sL + Zo + Zo1}, \]

and \[ S_{12} = \frac{2 \sqrt{Zo1 Zo}}{sL + Zo + Zo1}. \]  

On the basis of our discussion and the definition of the transfer scattering matrix in Section 4.1.2, we see that the total transfer scattering matrix of such a network can be obtained by the product of the transfer scattering matrix for the individual component of the network as given by

\[
\begin{bmatrix}
T_{11}(s) & T_{12}(s) \\
T_{12}(s) & T_{22}(s)
\end{bmatrix}
\begin{bmatrix}
e^{-sTPd} & 0 \\
0 & e^{+sTPd}
\end{bmatrix}
\begin{bmatrix}
\frac{1}{\text{Det } S} & S_{11} \\
-S_{22} & 1
\end{bmatrix}
\]

\[ = \frac{1}{S_{21}}
\begin{bmatrix}
-\text{Det } S & e^{-sTPd} S_{11} e^{-sTPd} \\
-S_{22} e^{+sTPd} & e^{+sTPd}
\end{bmatrix}. \]  

In view of the above expression and the definition of the reflection function given by equation (101), we are then able to write the total reflection function of the input port for such a network as
With the reflection function thus obtained, we may invoke the theory of the Laplace transform for the various input excitation in the time domain. For the case of a ramp input $V_{in}(t)$ with a slope $\left( \frac{V_A}{T} \right)$, the reflection voltage is represented as

$$V_{r}(s) = \left( \frac{s + \frac{Z_{01} - Z_{0}}{L}}{s + \frac{Z_{01} + Z_{0}}{L}} \right) e^{-s^2T_{pd}} V_{in}(s),$$

where $V_{in}(s) = L \left[ V_{in}(t) \right] = L \left[ \left( \frac{V_A}{T} \right)t u(t) - \left( \frac{V_A}{T} \right)(t - T)u(t - T) \right]$

$$= \frac{V_A}{T s^2} \left( 1 - e^{-sT} \right), \quad (129)$$

and $s$ is the complex variable used to define the Laplace transform. Further, by applying the inverse Laplace transform, the reflection voltage is obtained as (See Appendix (A.3).)

$$V_{r}(t) = \left[ \frac{2\xi_L}{(r + 1)} + \frac{t - 2T_{pd}}{T} - \left( \frac{2\xi_L}{r+1} \right) e^{-T\xi_L} (t - 2T_{pd}) \right] V_{A} u(t - 2T_{pd})$$

$$- \left[ \frac{2\xi_L}{(r + 1)} + \frac{t - 2T_{pd} - T}{T} - \left( \frac{2\xi_L}{r+1} \right) e^{-T\xi_L} (t - 2T_{pd} - T) \right] V_{A} u(t - 2T_{pd} - T)$$

(130)

where the impedance ratio is

$$r = \frac{Z_{01}}{Z_{0}}; \quad (131)$$
the reflection coefficient for the impedance mismatch as

$$\Gamma_0 = \frac{r^{-1}}{r+1};$$  \hfill (132)

and the normalized inductance as given by,

$$\mathcal{L} = \frac{L}{Z_0 T (r+1)}. \hfill (133)$$

For a typical case, \(L = 0.5\ \text{nH},\ \text{VA} = 1\ \text{V}\) and \(T = 50\ \text{ps}\) with various impedance ratios \(r = 0.8, 1.0, 1.2, 1.4\), the reflection voltage is then plotted as shown in Figure 15.

![Figure 15](image-url)

Figure 15, Reflection voltage due to vias with different impedance ratio
From equation (130) and the reflection plot, we can conclude that

1) if the maximum reflection voltage caused by the inductive discontinuity is larger than the reflected voltage of impedance mismatch without the inductive discontinuity, then the value of inductance can be extracted from the measured data.

2) if $Z_{o1}$ is not less than the reference impedance $Z_o$, then the maximum reflection voltage is occurred at $2T_{pd} + T$.

Now, let us consider the reflection voltage measured at $2T_{pd} + T$. According to the equation (130), the amplitude $V_r$ can be expressed as given by

$$V_r = V_A \left\{ \frac{2\lambda L}{r+1} + \Gamma_0 - \left( \frac{2\lambda L}{r+1} \right) e^{-\lambda L} \right\}. \quad (134)$$

Moreover, if the reflection coefficient of the inductive discontinuity at that time is defined by

$$\Gamma_L = \frac{V_r}{V_A}, \quad (135)$$

then we can rewrite the equation (134) as

$$\Delta \Gamma \left( \frac{r+1}{2} \right) = \lambda L \left[ 1 - \exp \left( -\frac{1}{\lambda L} \right) \right] \quad (136)$$

where we define $\Delta \Gamma$ as the variation is the reflection coefficient as

$$\Delta \Gamma = \Gamma_L - \Gamma_0. \quad (137)$$
As we have assumed, if the inductance is extractable from the measured reflection voltage, then $\Delta \Gamma$ must be positive. Now, in order to obtain the inductance, a reasonable solution must be found from equation (136).

Given the measurable data $\Delta \Gamma$ and $r$, equation (136) can be rewritten as,

$$f(\mathcal{N}_L) = \mathcal{N}_L \left(1 - \exp\left(-\frac{1}{\mathcal{N}_L}\right)\right) - \Delta \Gamma \left(\frac{r+1}{2}\right) = 0,$$

(138)

where $\Delta \Gamma \left(\frac{r+1}{2}\right) \geq 0$ and $\mathcal{N}_L > 0$. By examining equation (138) as $\mathcal{N}_L$ respectively approaches to 0 or infinity, we obtain the following results as,

1. If $\mathcal{N}_L$ approaches 0, then $\mathcal{N}_L \left(1 - \exp\left(-\frac{1}{\mathcal{N}_L}\right)\right)$ approaches 0.

2. If $\mathcal{N}_L$ approaches infinity, then we let

$$g\left(\frac{1}{\mathcal{N}_L}\right) = \frac{1}{\mathcal{N}_L} \left(1 - \exp\left(-\frac{1}{\mathcal{N}_L}\right)\right),$$

(139)

and $\lim g\left(\frac{1}{\mathcal{N}_L}\right) = \lim g(x) \quad \text{as} \quad x \to 0$

$$= \lim \frac{1-\exp(-x)}{x} \quad \text{as} \quad x \to 0 = 1,$$

(140)
The function \( f(\mathcal{M}) \) is monotonically increasing from \(-\Delta\Gamma(\frac{r+1}{2})\) to \(1-\Delta\Gamma(\frac{r+1}{2})\). Therefore, the solution of equation (136) can be uniquely found.

By applying a quick approximate method to determine the root of the function, a computer algorithm based on Newton's Method is used to calculate the inductance as shown below,

**Step(1)**: Given the measured data \( Z_0, V_A, Z_{01}, r_L, \Gamma_0, T \) (rise time \( tr = 0.8 T \)) and an estimated value \( \mathcal{M}_0 \).

**Step(2)**: Compute \( f(\mathcal{M}(i)) \) and \( f'(\mathcal{M}(i)) \),

if \( f(\mathcal{M}(i)) \) and \( f'(\mathcal{M}(i)) \) are not equal to 0,

set \( \mathcal{M}(i+1) = \mathcal{M}(i) - \frac{f(\mathcal{M}(i))}{f'(\mathcal{M}(i))} \), \hspace{1cm} (141)

until \( |\mathcal{M}(i+1) - \mathcal{M}(i)| < \text{tolerance value} \),

save \( \mathcal{M}(i+1) \).

**Step(3)**: Calculate the value of inductance as given by

\[
L = \mathcal{M}(i+1) \left( r + 1 \right) Z_0 T. \hspace{1cm} (142)
\]
4.3.2 Capacitive Discontinuity

In this section, we deal with the time domain response of the building block with a capacitive discontinuity. The equivalent circuit model is composed of a shunt capacitance with two cascade uniform transmission lines as shown in Figure 16.

Figure 16, Lumped model of pads

The procedure for extracting the capacitance value from the measured waveform is similar to the one used for extracting the series inductance. This procedure can also be used to characterize many capacitive discontinuities such as pads in electronic packaging.

On the basis of the transfer scattering matrix, the reflection function of the input port is represented by, (See Appendix (A.4))
\[
\Gamma(s) = \frac{T_{12}(s)}{T_{22}(s)} = - \left( \frac{s + \frac{Y_0 - Y_0}{C}}{s + \frac{Y_0 + Y_0}{C}} \right) e^{-sT_{pd}}.
\] (143)

where the admittance \( Y_0 = \frac{1}{Z_{o1}} \),

the admittance \( Y_0 = \frac{1}{Z_0} \),

and the capacitance \( C \) is the lumped model of the pad.

Given the ramp input excitation as described in the last section, the reflection voltage can be obtained via the inverse Laplace transform by

\[
V_r(t) = - \left[ \left( \frac{2\kappa c}{q+1} \right) + \rho_0 \left( \frac{t - 2T_{pd}}{T} \right) - \left( \frac{2\kappa c}{q+1} \right) e^{\frac{-1}{T\kappa c}(t - 2T_{pd})} \right] VA u(t - 2T_{pd})
\]

\[
+ \left[ \left( \frac{2\kappa c}{q+1} \right) + \rho_0 \left( \frac{t - 2T_{pd} - T}{T} \right) - \left( \frac{2\kappa c}{q+1} \right) e^{\frac{-1}{T\kappa c}(t - 2T_{pd} - T)} \right] VA u(t - 2T_{pd} - T)
\] (146)

where the admittance ratio

\[
q = \frac{Y_01}{Y_0},
\] (147)

the reflection coefficient for the admittance mismatch as

\[
\rho_0 = \frac{q - 1}{q + 1},
\] (148)

and the normalized capacitance is defined by,

\[
\kappa c = \frac{C}{Y_0 T (q + 1)}.
\] (149)
For a typical case, $C = 0.5$ pf, $V_A = 1$ v and $T = 50$ps with various admittance ratios $q = 0.8, 1.0, 1.2, 1.4$, the reflected voltage is then plotted as shown in Figure 17.

Figure 17, Reflection voltage due to a pad with different impedance ratio
Next, let us denote the reflection coefficient measured at time $2T_{pd} + T$ by $\rho_c = \frac{V_r}{V_A}$. By examining equation (146) at time $2T_{pd} + T$, a simple formula which used to calculate the normalized capacitance is given by

$$\Sigma \rho \left( \frac{n+1}{2} \right) = \kappa C \left[ \exp\left(\frac{-1}{\kappa C}\right) - 1 \right]$$ (150)

where

$$\Sigma \rho = \rho_c + \rho_0.$$ (151)

Further, the value of capacitance can be recovered from the normalized capacitance by solving equation (150).

In the work we have described so far, we have shown two simple cases for synthesizing hybrid lumped/distributed network by measuring the reflection voltage in the time domain. At this point, we note that by combining capacitive and inductive discontinuity models with uniform transmission line models, it is possible to synthesize arbitrary TDR waveforms in the time domain.
4.4 Excess Parameter Lumped Element Model

The method presented in this section is used to extract the excess inductances and capacitances associated with the discontinuities. It is derived from the time domain characteristics of microstrip discontinuities obtained from the reflection measurements in the quasi-TEM approximation. It is further assumed that the transmission line behavior is lossless or low-loss at all frequencies. If the characteristic impedance is known from measurements and the propagation delay of the signal is measured or computed, these parameters can be related to an uniform interconnect inductance and capacitance by

\[ L = Z_0 \cdot T_{pd}, \quad (152) \]

and \[ C = \frac{Z_0}{T_{pd}}. \quad (153) \]

For the case of nonuniform test line or if discontinuities exist in the test line, the waveforms measured by TDR are not uniform and show discontinuities. The total inductance and capacitance can then be estimated by examining the subinterval within the measured waveform corresponding to the discontinuity, and using the following expressions

\[ L = \frac{1}{2} \int_{t_a}^{t_b} Z_0(t) \, dt = \frac{1}{2} \sum_{i=0}^{n} Z_{0i} \delta t, \quad (154) \]

and \[ C = \frac{1}{2} \int_{t_a}^{t_b} \left( \frac{1}{Z_0(t)} \right) \, dt = \frac{1}{2} \sum_{i=0}^{n} \left( \frac{\delta t_i}{Z_{0i}} \right), \quad (155) \]
where $Z_{0i}$ is the characteristic impedance for the time subinterval, and $t_i$ is the $i$th time subinterval, and $[t_a, t_b]$ is the round trip propagation delay time.

As indicated above the integrals are evaluated by subdividing the interval from $t_a$ to $t_b$ with an appropriate number of subintervals, and applying a simple procedure, such as Simpson's rule or the trapezoidal rule,

\[
\text{the trapezoidal rule : } \int_{t_i}^{t_{i+1}} f(t) \, dt = \frac{f(t_i) + f(t_{i+1})}{2} \delta_{t_i},
\]

\[
\text{Simpson's 1/3 rule : } \int_{t_i}^{t_{i+2}} f(t) \, dt = \frac{\delta_{t_i}}{3} [ f(t_i) + 4f(t_{i+1}) + f(t_{i+2}) ].
\]

4.4.1 Case (1): Excess Parameters For A Microstrip Bend

Consider the microstrip having the bend shown in Figure 18. The test fixture for this case was made on scaled STYCAST substrate having a dielectric constant $\varepsilon_r = 12.9$. In order to exclude the SMA connector-to-microstrip transient in the integration and extract the discontinuity excess parameters, a reference line having the same length as the test line was used. These excess inductance and excess capacitance are then given by

\[
L_{ex} = L_b - L_{beq},
\]

\[
C_{ex} = C_b - C_{beq},
\]
Figure 18, Test fixture used for characterizing a bend by TDR
where $L_{ex}$ is the excess inductance,

$C_{ex}$ is the excess capacitance,

$L_b$ is the total bend inductance,

$C_b$ is the total bend capacitance,

$L_{beq}$ is the inductance of the corresponding values of a straight strip having the length $Leq = L - w$,

and $C_{beq}$ is the capacitance of the corresponding values of a straight strip having the length $Leq = L - w$.

In Figure 19, two TDR waveforms, representing a reference line, and a microstrip bend in the time domain are shown. The Figure illustrates the distributed nature of the discontinuity for the microstrip bend. The redistributed current and charge effectively results in a nonuniform line, and hence the equivalent characteristic impedance is a function of position or time. At a sufficiently large distance from the nonuniform discontinuity region, the current distribution corresponds to an infinite uniform line with constant characteristic impedance. The propagation delay in the bend is not the same as the corresponding straight line. In most interconnect bends, it is found that the propagation delay time of a microstrip with a bend is larger than the corresponding straight line.
Figure 19, Measured waveforms for a bend and a reference line
The excess inductance and capacitance for the bend then are given by,

\[ L_{ex} = \frac{1}{2} \int_{ta}^{tb} Z_0(t) \, dt - Z_0 \left[ \frac{(tb-ta) - \delta t}{2} \right], \]

(160)

and

\[ C_{ex} = \frac{1}{2} \int_{ta}^{tb} \left( \frac{1}{Z_0(t)} \right) \, dt - \left( \frac{1}{Z_0} \right) \left[ \frac{(tb-ta) - \delta t}{2} \right], \]

(161)

where \( \delta t \) is the time difference in propagation delay between the bend and the reference line having the same length.

The above equations can be used to calculate the excess parameters of any bend discontinuity including the arbitrary angled bends and chamfered bends.

4.4.2 Case (2): Excess Parameters For A Microstrip Step

A similar method was applied to extract the excess inductance and excess capacitance for the microstrip step discontinuities. The test fixture of the asymmetrical discontinuity structure is shown in Figure 20. Two reference lines are now required for disembedding and evaluating the excess parameters, and the equations used to determine these excess parameters are then given by,
Figure 20. Test fixture used for characterizing step discontinuities
\[ L_{ex} = (L_{w1-w2} + L_{w2-w1}) - (Z_{o2} t_{pd2} + Z_{o1} t_{pd1}) L, \]  
\[ C_{ex} = (C_{w1-w2} + C_{w2-w1}) - \left( \frac{t_{pd2}}{Z_{o2}} + \frac{t_{pd1}}{Z_{o1}} \right) L, \]

where

\( L_{w1-w2} \) : total inductance of the step line when \( w1 \) is followed by \( w2 \),
\( L_{w2-w1} \) : total inductance of the step line when \( w2 \) is followed by \( w1 \),
\( C_{w1-w2} \) : total capacitance of the step line when \( w1 \) is followed by \( w2 \),
\( C_{w2-w1} \) : total capacitance of the step line when \( w2 \) is followed by \( w1 \),
\( Z_{o1} \) : characteristic impedance of reference line 1,
\( Z_{o2} \) : characteristic impedance of reference line 2,
\( t_{pd1} \) : propagation delay per unit for line 1,
\( t_{pd2} \) : propagation delay per unit for line 2,
\( L \) : total length of line.

Waveforms obtained with a TDR for two reference lines and two step-type microstrip lines are shown in Figure 21.
Figure 21, Measured waveforms for step and reference lines
The excess inductance and capacitance are given by

\[ L_{\text{ex}} = \frac{1}{2} \left\{ \frac{1}{2} \left[ \int_{t_a}^{t_b} Z_{O1}(t) \, dt + \int_{t_a}^{t_b} Z_{O2}(t) \, dt \right] - \frac{Z_{O1} + Z_{O2}}{2} \left[ \frac{tb-ta}{2} \right] + \frac{Z_{O2} \delta t_2 - Z_{O1} \delta t_1}{2} \right\}, \]

(164)

and

\[ C_{\text{ex}} = \frac{1}{2} \left\{ \frac{1}{2} \left[ \int_{t_a}^{t_b} \left( \frac{1}{Z_{O1}(t)} \right) \, dt + \int_{t_a}^{t_b} \left( \frac{1}{Z_{O2}(t)} \right) \, dt \right] - \frac{1}{2} \left[ \frac{1}{Z_{O1}} + \frac{1}{Z_{O2}} \right] \left[ \frac{tb-ta}{2} \right] + \frac{\delta t_2}{2 Z_{O2}} - \frac{\delta t_1}{2 Z_{O1}} \right\}, \]

(165)

where

- \( Z_{O1}(t) \): measured impedance in the region of discontinuity when \( w_2 \) is followed by \( w_1 \),
- \( Z_{O2}(t) \): measured impedance in the region of discontinuity when \( w_1 \) is followed by \( w_2 \),
- \( \delta t_1 \): the difference of propagation delay between the step line \( L_{w_1-w_2} \) and the reference line \( L_2 \),
- \( \delta t_2 \): the difference of propagation delay between the step line \( L_{w_1-w_1} \) and the reference line \( L_2 \).
5. Results

5.1 Excess Model Parameters

The excess parameters of a right angle bend obtained by the method presented in section 3.3 are compared with published results based on electromagnetic computational results and the frequency domain experimental data are shown in Figure 22. These excess capacitance and inductance are normalized with respect to the substrate height for w/h ratios of 0.5, 1.0, and 2.0. Figure 23 shows the excess capacitance and excess inductance data for chamfered right angle bends include the 45 degree chamfer. Results for the excess capacitance and inductance of a typical step discontinuity are obtained by maintaining the normalized width of the test line at w/h_2=0.5, and varying the width of the other strip across the discontinuity according to w_1/h = 1, 1.5, and 2. Again, the time domain measurements lead to results that are in a good agreement with theoretical values shown in Figure 24.
(a) the normalized excess inductance of a right angle bend

(b) the normalized excess capacitance of right angle bends

Figure 22, The normalized excess parameters of right angle bends
Figure 23-(a), The excess inductance of the right angle bends with chamfer

Figure 23 -(b), The excess capacitance of the right angle bends with chamfer
(a), The excess inductance of a step discontinuity

(b), The excess capacitance of a step discontinuity

Figure 24, The normalized excess parameters of a step discontinuity
5.2 Lumped/Distributed Element Model of Bends

The results for the piecewise uniform transmission line model and corresponding hybrid lumped distributed element model are presented here. They refer to a typical microstrip-like interconnect having a right angle bend, which is commonly seen in high speed digital circuits. The w/h ratio is equal to 1 and the characteristic impedance of the interconnect is approximately equal to 46.8 ohm. TDR measurements and distributed network extraction technique presented in chapter 4 were used to extract the electrical parameters of the line including the bend structure. In order to validate the accuracy of the modeling procedure, measurement were made with input pulse having two different rise times of input pulses, Tr=28ps and Tr=800ps. The nonuniform impedance profiles of the distributed transmission lines model, which represents the bend discontinuity, were obtained and used as net-list for SPICE-like simulator, as shown in Figure 25. Finally, MWSPICE is employed to analyze the transient response of the synthesized circuit model.

Figure 25, Impedance profiles with different rise times (-) tr=28ps (o) tr=800ps
Comparison of signal reflections for the case of Tr=28ps, obtained from TDR measurements and computer simulation of both distributed and hybrid models are shown in Figure 26. The agreement between measured and simulated response is excellent.

Figure 26, The reflected voltage due to the bend discontinuity (w/h=1)
5.3 Distributed Element Model of the T junction

The T junction is of practical importance, and often occurs in the microwave hybrid and monolithic integrated circuits. The equivalent circuit model in the frequency domain for the T junction for symmetrical three port based on measurements was proposed by Stinehelfer [28]. A microstrip T junction, and its equivalent circuit model are shown in Figure 27. An equivalent circuit consisting of nonuniform transmission lines as shown in Figure 28-(a) may be more suitable for time domain simulation as compared to the model based on the experimental techniques in the frequency domain [11, 28, 29] or electromagnetic computational techniques [10, 30, 31].
The time domain reflection measurement was performed to observe the reflections due to the T junction. The test fixture of the microstrip T junction is made on the STYCAST substrate. The w/h ratio for the main line is 2, and the w/h ratio for the stub line is 0.5. The characteristic impedances are approximately equal to 29.5 ohm and 58.8 ohm for the main line and the
stub arm, respectively. The measured waveform for voltage vs time is shown in Figure 29. Again, the reflected waveform exhibits the distributed nature of the discontinuity. By applying the distributed network synthesis method, an equivalent circuit model of the discontinuity for the T junction as shown in Figure 28 is obtained. These two distributed sections having the same nonuniform impedance profile are used to connect the main line and the stub arm, and account for discontinuity region of the T junction. The results of simulation and measurement in the time domain are also shown in Figure 29.

Figure 28-(a), The equivalent circuit of the T junction with discontinuity model used for time domain simulation
One-side nonuniform impedance profile of discontinuity model

Figure 28-(b) Nonuniform impedance profile of discontinuity model

Reflection voltage due to the T junction

Figure 29, The reflected voltage due to T junction
5.4 Lumped Model of Vias and the Pad

The test fixture of a via connected with two lines is shown in Figure 30. The bottom line is a microstrip made on the STYCAST substrate, and the upper line is surrounded in the air medium.

Figure 30, A test fixture for characterizing vias

The results of time domain reflection measurements are shown as the following lists,

(1) input source: the amplitude \( V_A = 1 \text{volt} \) and the rise time \( t_r = 480 \text{ps} \),
(2) line impedance: lower line \( Z_0 = 43.5 \text{ohm} \) and upper line \( Z_{01} = 101.5 \text{ohm} \),
(3) reflection coefficients: inductive discontinuity \( \Gamma_L = 0.66 \) and impedance mismatch \( \Gamma_0 = 0.4 \).
The value of the excess inductance is found 34.6 nH from the computer algorithm as presented in the section 4.3.1. This result agrees with the Spice simulated result as shown in Figure 31.

Figure 31, Measured and simulated reflection voltage due to vias
The following example deals with modelling a capacitive discontinuity. Two uniform transmission lines connected by a pad are shown in Figure 5. The test pattern is made on the substrate called FR4 (epoxy glass) which the dielectric constant $\varepsilon_r = 4.7$ and substrate height $h = 0.05$ inch. The $w/h$ ratios of the uniform transmission line and the pad are 1.8 and 3, respectively. The length of the pad is about 0.12 inch. The waveform measured by TDR corresponds to an input source voltage of 1 V with a rise time of 120ps. The line impedance is 50.3 ohms, which together with the measured reflection coefficient of $\rho_C = 0.034$ for the discontinuity leads to the pad capacitance of 0.16pf. Again, the simulated results based on the above value of pad capacitance are in good agreement with the measured data as shown in Figure 32.

![Reflection voltage measurement and simulation](image)

*Figure 32, Measured and simulated reflection voltage due to pads*
6. Conclusion and Suggestion for the Future Work

This study explored some important modeling and simulation aspects of interconnect discontinuities for high speed digital and microwave applications. A time domain technique for the experimental characterization of the interconnect discontinuities has been presented. Techniques for extracting distributed, hybrid and lumped element models based on the inverse scattering algorithm for constructing nonuniform impedance profile from the TDR data have been formulated. A new algorithm based on the transfer scattering matrix method has been developed to extract the nonuniform impedance profile. The synthesized nonuniform impedance profile is then used for the time domain characterization of the interconnects in terms of equivalent circuit models. A procedure for extracting the excess inductance and capacitance associated with arbitrary two port discontinuities has been presented in this thesis. The accuracy of the algorithms and the validity of the models has been evaluated by comparing the SPICE simulation results with measured TDR data.

Some examples concerning the interconnect discontinuities with improved geometry for high speed clock lines have been discussed in the appendix. It is seen that the reflection voltage can be reduced by chamfering a proper area in the discontinuity region. The measurement techniques outlined here can be used to study of a wide variety of interconnect structures and discontinuities.

This study represents a first step towards the development of a complete automated procedure for modeling arbitrary discontinuities from
time domain measurements. The next phase of the work deals with the formulation of the procedure for identifying the model topology and extraction of parameters for models that are most compatible with time domain simulation. Losses due to the medium as well as radiation lead to pulse degradation and should be included in the model together with proximity effects which in general result in crosstalk between interconnects. Techniques to model those losses and proximity effects based on multichannel TDR/T measurements will constitute a significant contribution in this field, and are scheduled to be formulated in the next phase of this research program.
REFERENCES


[37] J. M. Jong, V. K. Tripathi, "Time Domain Characterization of Interconnect Discontinuities in High Speed Digital Circuits" (Submitted to IEEE *Trans. on CHMT*)
APPENDICES
A.1 The Transfer Scattering Matrix

The scattering parameter of a two port network can be expressed by,

\[ b_1 = S_{11} a_1 + S_{12} a_2 \]  \hspace{1cm} (a.1)

and \[ b_2 = S_{21} a_1 + S_{22} a_2. \]  \hspace{1cm} (a.2)

After rearranging above equations, we will have

\[ b_1 = \frac{1}{S_{21}} (-\text{Det} |S|) a_2 + \frac{1}{S_{21}} (S_{11}) b_2 \]  \hspace{1cm} (a.3)

and \[ a_1 = \frac{1}{S_{21}} (-S_{22}) a_2 + \frac{1}{S_{21}} b_2, \]  \hspace{1cm} (a.4)

where the matrix \[ [T] = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \frac{1}{S_{21}} \begin{bmatrix} -\text{Det} |S| & S_{11} \\ -S_{22} & 1 \end{bmatrix} \]  \hspace{1cm} (a.5)

is called the transfer scattering matrix of the two port network.

We note that if the network is lossless network, it must satisfy the following condition,

\[ S_{11}(s) S_{22}(s^*) + S_{12}(s) S_{21}(s^*) = 1 \]  \hspace{1cm} (a.6)

and \[ S_{11}(s) S_{12}(s^*) + S_{21}(s) S_{22}(s^*) = 0. \]  \hspace{1cm} (a.7)

For a reciprocal network, it also have the following properties,

\[ S_{11}(s) = S_{22}(s) \]  \hspace{1cm} (a.8)

and \[ S_{12}(s) = S_{21}(s). \]  \hspace{1cm} (a.9)
Since

\[ S_{22}(s) = \frac{-S_{11}(s^*) S_{12}(s)}{S_{12}(s^*)}, \]  

such that

\[
T_{11}(s) = \frac{-\text{Det}_1 S_{11}(s) S_{22}(s) - S_{12}(s) S_{21}(s)}{S_{21}(s)} \]

\[
= \frac{-\text{Det}_1 S_{11}(s) S_{12}(s) - S_{12}(s) S_{21}(s)}{S_{21}(s)} \]

\[
= \left[ \frac{-\text{Det}_1 - (|S_{11}(s)|^2) S_{21}(s) - (|S_{12}(s)|^2) S_{21}(s)}{S_{21}(s)} \right] / S_{21}(s) \]

\[
= \frac{1}{S_{21}(s^*)} = T_{22}(s^*). \]  

(a.10)

(a.11)

Also from the properties of the scattering matrix for a reciprocal, lossless network, we have

\[ S_{11}(s) = -\frac{S_{22}(s^*)}{S_{21}(s^*)}, \]  

then the transfer scattering matrix must satisfy the property as given by,

\[ T_{12}(s) = T_{21}(s^*). \]  

(a.12)

(a.13)

So we have derived the following properties used for synthesizing a cascaded uniform transmission line as,

\[ T_{11}(s) = T_{22}(s^*) \]  

(a.14)

and \[ T_{12}(s) = T_{21}(s^*). \]  

(a.15)
A.2 Scattering Parameter: Inductive Discontinuity

From the definition, \[ S_{11} = \frac{b_1}{a_1} \bigg| a_2=0 \] (a.16)

and \[ S_{22} = \frac{b_2}{a_2} \bigg| a_1=0 \] (a.17)

we can write

\[ S_{11} = \frac{Z_{in1} - Zo}{Z_{in1} + Zo} \] (a.18)

and in a similar way

\[ S_{22} = \frac{Z_{in2} - Zo_1}{Z_{in2} + Zo_1} \] (a.19)

In the case of inductive discontinuity, we have

\[ Z_{in1} = SL + Zo_1 \] (a.20)

and \[ Z_{in2} = SL + Zo \] (a.21)

Next, we consider the definition of \( S_{21} \) and \( S_{12} \) as

\[ S_{21} = \frac{b_2}{a_1} \bigg| a_2=0 \] (a.22)

and \[ S_{12} = \frac{b_1}{a_2} \bigg| a_1=0 \] (a.23)

If we connect a voltage source \( Vg_1 \) with source impedance \( Zo \) and port 2 terminated in \( Zo_1 \), then we can express \( S_{21} \) as

\[ S_{21} = \frac{2 V_2}{Vg_1} \sqrt{\frac{Zo}{Zo_1}} \] (a.24)

where \( V_2 \) is the voltage across at the port 2.

Since

\[ V_2 = Vg_1 \frac{Zo_1}{SL + Zo + Zo_1} \] (a.25)
then we have

\[ S_{21} = \frac{2 \sqrt{Z_0 Z_{01}}}{sL + Z_0 + Z_{01}} \]  
\[ (a.26) \]

In a similar fashion, we find that when port 1 is terminated in \( Z_0 \), and when a voltage source \( V_{g2} \) with source impedance \( Z_{01} \) is connected to port 2, then

\[ S_{12} = \frac{2 V_1}{V_{g2}} \sqrt{\frac{Z_{01}}{Z_0}}, \]
\[ (a.27) \]

where \( V_1 \) is the voltage across port 1.

From above derivation, we have the scattering parameter used for analyzing the inductive discontinuity as given by,

\[ S_{11} = \frac{(sL + Z_{01}) - Z_0}{(sL + Z_{01}) + Z_0}, \]
\[ (a.28) \]

\[ S_{22} = \frac{(sL + Z_0) - Z_{01}}{(sL + Z_0) + Z_{01}}, \]
\[ (a.29) \]

\[ S_{21} = \frac{2 \sqrt{Z_{01} Z_0}}{sL + Z_0 + Z_{01}}, \]
\[ (a.30) \]

and \[ S_{12} = \frac{2 \sqrt{Z_{01} Z_0}}{sL + Z_0 + Z_{01}}. \]
\[ (a.31) \]
A.3 Evaluate The Reflection Voltage Due To A Inductive Discontinuity

From the equation (143), the reflection voltage can be formulated in s domain as given by,

\[ V_r(s) = \frac{V_A}{T s^2} \left( 1 - e^{-s2T} \right) \left( \frac{s + \frac{Z_01 - Z_0}{L}}{s + \frac{Z_01 + Z_0}{L}} \right) e^{-s2Tpd} \]

\[ = \frac{V_A}{T s^2} \left( \frac{s + \frac{Z_01 - Z_0}{L}}{s + \frac{Z_01 + Z_0}{L}} \right) e^{-s2Tpd} - \frac{V_A}{T s^2} \left( \frac{s + \frac{Z_01 - Z_0}{L}}{s + \frac{Z_01 + Z_0}{L}} \right) e^{-s2(Tpd + T)} \]

\[ = \frac{V_A}{T} \left[ \frac{2Z_0 L}{(Z_0 1+Z_0)^2 s + (Z_01+Z_0) s^2} - \frac{2Z_0 L}{(Z_01+Z_0)^2 s + (Z_01+Z_0) s^2} \right] e^{-s2Tpd} \]

By applying the inverse Laplace transform, the reflection voltage is obtained as

\[ V_r(t) = \left[ \frac{2M}{r+1} + \Gamma_0 \left( \frac{t - 2Tpd}{T} \right) - \left( \frac{2M}{r+1} \right) e^{-\frac{1}{T}M(t - 2Tpd)} \right] V_A u(t - 2Tpd) \]

\[ - \left[ \frac{2M}{r+1} + \Gamma_0 \left( \frac{t - 2Tpd - T}{T} \right) - \left( \frac{2M}{r+1} \right) e^{-\frac{1}{T}M(t - 2Tpd - T)} \right] V_A u(t - 2Tpd - T) \]

(a.33)
where we define the impedance ratio is

\[ r = \frac{Z_{\text{load}}}{Z_0}, \]  

(a.34)

the reflection coefficient for the impedance mismatch is

\[ \Gamma_0 = \frac{r - 1}{r + 1}, \]  

(a.35)

and the normalized inductance is defined as given by,

\[ \mathcal{L} = \frac{L}{Z_0 T (r + 1)}. \]  

(a.36)
A.4 Evaluate The Reflection Voltage Due To A Capacitive Discontinuity

From the network as shown in Figure 16, we can write

\[ S_{11} = \frac{(SC // Zo1) - Zo}{(SC // Zo1) + Zo} \]

\[ = \frac{SC \cdot Zo1}{SC + Zo1} \cdot \frac{Zo1 - Zo}{Zo1 + Zo} \]

\[ = -\left( \frac{s - \frac{Zo1 - Zo}{C \cdot Zo1}}{s + \frac{Zo1 + Zo}{C \cdot Zo1}} \right) \]

\[ = -\left( \frac{s + \frac{Yo1 - Yo}{C}}{s + \frac{Yo1 + Yo}{C}} \right) \]  

(a.37)

According to equation (141), the total reflection function of such a network can be expressed as

\[ \Gamma(s) = \frac{T_{12}(s)}{T_{22}(s)} = -\left( \frac{s + \frac{Yo1 - Yo}{C}}{s + \frac{Yo1 + Yo}{C}} \right) e^{-s2Tpd} \]  

(a.38)

Comparing this equation with equation (142), we can obtain the reflection function analogized to equation (146) as given by

\[ V_r(t) = -\left[ \frac{2\lambda c}{q+1} + \rho_o \left( \frac{t - 2Tpd}{T} \right) - \left( \frac{2\lambda c}{q+1} \right) e^{\frac{1}{\lambda c}(t - 2Tpd)} \right] VA \ u(t - 2Tpd) \]

\[ + \left[ \frac{2\lambda c}{q+1} + \rho_o \left( \frac{t - 2Tpd - T}{T} \right) - \left( \frac{2\lambda c}{q+1} \right) e^{\frac{1}{\lambda c}(t - 2Tpd - T)} \right] VA \ u(t - 2Tpd - T) \]  

(a.39)
A.5 The Bend Analysis For Single High Speed Clock Net

When a pulse propagates along a microstrip line having a bend, a signal of amplitude no greater than the original input signal will be reflected back and is degraded, due to the interactions with a bend, will arrive at the loading end. In the case of high speed clock net, or a critical net, the signal reflections and attenuations must be minimized, and the effects of the interconnects need to be reduced by optimizing the layout geometry.

![Reflection (Vref) and transmission noise (Vn)](image)

Figure 33, Reflection and transmission noise due to the bend discontinuity

In Figure 33, the maximum reflection and signal attenuation due to the discontinuity of the bend with the circuit schematic as shown in Figure 34, are simulated in the time domain by using MWSPICE.
It can be used to describe and characterize the electrical performance of a bend in high speed digital circuits. The following example deals with the minimum reflection from a discontinuity obtained by chamfering the corner of the bend. In Figure 35, the TDR measurement results are used to determine the minimum reflection voltage of a bend with chamfered corner found for the width to substrate height ratios of \( w/h = 2, 1 \). These results show that minimum reflection voltages will be reached when the excess characteristic impedance \( Z_{ex} \) associated with the discontinuity is
approximately equal to the characteristic impedance of the uniform transmission line. By using simple circuit analysis which interconnects a series excess inductance and a parallel excess capacitance with two semi-infinite uniform transmission lines having the characteristic impedance 'Zo', it is easy to verify this result by the following calculations and assumptions

the excess characteristic impedance: \[ Z_{ex} = \sqrt{\frac{L_{ex}}{C_{ex}}} \] (a.40)

the input impedance at point DD' is \[ Z_{in} = \frac{1}{SC_{ex}} \left( SL_{ex} + Zo \right) \] (a.41)

the reflection coefficient at point DD' is

\[ \Gamma_{dd'} = \frac{Z_{ex} - Zo}{Z_{ex} + Zo} = \frac{S \left( L_{ex} + C_{ex} Zo^2 \right) - S^2 L_{ex} C_{ex} Zo^2}{S \left( L_{ex} - C_{ex} Zo^2 \right) + S^2 L_{ex} C_{ex} Zo^2 + 2 Zo} \] (a.42)

if \[ \omega^2 << \frac{1}{Zo^2 L_{ex} C_{ex}} \], then for minimum reflection

\[ Zo^2 \sim \frac{L_{ex}}{C_{ex}} = Z_{ex}^2 \] (a.43)

It is noted that the excess characteristic impedance approaches the desired values of 'Zo' when the length of chamfer 'wc' is approximately equal to 1.8w ~ 1.82w.
Figure 35, Maximum reflected voltage vs the excess impedance of the bend discontinuity

( The uniform line impedance for w/h=2 is 32 ohms, for w/h=1 is 47 ohms )
A.6 Application of T junction in Multiple High Speed Clock Distribution

The conventional means of wiring a regular clocking net where all the loads are tapped on the signal line would adversely affect the electrical performance of high speed digital systems. The clock skew increased due to the variation of the arrival time of the clock signal to each receiver, will increase the machine cycle time and slow the rate of data processing [32]. Also there will be discontinuities of the transmission line associated with parasitic capacitance loaded at the junctions of the line section which will impair the pulse waveform. However, one of major concerns in high speed clocking system design is to minimize the clock skew and maintain the fidelity of signal transmission.

A traditional way to eliminate the clock skew is to interconnect the loads radially, with all lines having the same length, as shown in Figure 36. It is interesting to observe the transient voltage in the time domain due to multiple reflections of the interconnect mismatch. To simplify the analysis, the equivalent circuits shown in Figure 37-(a) and (b) can be used to represent the transient and dc circuits. The reflection lattice diagrams are a universal tool for analyzing reflections on transmission line systems with an impedance mismatch, as shown in Figure 38.
Figure 36, A schematic of circuit layout having equal length interconnects

Figure 37, Equivalent circuits of radial circuit layout

(a) transient circuit  
(b) steady state circuit
Figure 38. Lattice diagram analysis for radial circuit layout

From the lattice diagram, the step response can be derived by the following procedures. Denoting the driving end reflection coefficient by

$$\Gamma_s = \frac{(Rs / \frac{Z_0}{N-1}) - Z_0}{(Rs / \frac{Z_0}{N-1}) + Z_0} = \frac{(2-N)Rs - Z_0}{N Rs + Z_0}, \quad (a.44)$$

the loading end reflection coefficient is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}, \quad (a.45)$$
the initial incident voltage is
\[ V = \frac{Z_0}{N R_s + Z_0} E \], \quad (a.46)

the step amplitude is \( E \), the loading end voltage at \( i \)th reflection is \( V(i) \), then

1\( \text{th} \) \( V(1) = (1 + \Gamma L) V \), \quad (a.47)

2\( \text{nd} \) \( V(2) = [ V \Gamma L \Gamma s + V \Gamma L + \Gamma s (N - 1) ] = V \Gamma L \Gamma s' (\Gamma L + 1) \), \quad (a.48)

3\( \text{rd} \) \( V(3) = [ V \Gamma L \Gamma s' \Gamma s + V \Gamma L^{2} \Gamma s' + \Gamma L^{2} \Gamma s' \Gamma s (N -1)] (\Gamma L + 1) \)
\[ = V \Gamma L^{2} (\Gamma s')^{2} (\Gamma L + 1) \], \quad (a.49)

\( \vdots \)

nth \( V(n) = V \Gamma L^{n-1} (\Gamma s')^{n-1} (\Gamma L + 1) \), \quad (a.50)

where
\[ \Gamma s' = \Gamma s + (N -1) (1 + \Gamma s) \], \quad (a.51)

Further, the loading end voltage after complete nth reflection can be given by

\[ V_n = V (1 + \Gamma L) u(t - T_{pd}) + V (1 + \Gamma L) \Gamma L \Gamma s' u(t - 3T_{pd}) + \cdots + \]
\[ V \Gamma L^{n-1} (\Gamma s')^{n-1} (\Gamma L + 1) u(t - (2n - 1) T_{pd}) \]
\[ = V (1 + \Gamma L) \sum_{m=0}^{n-1} (\Gamma L \Gamma s')^{m} u(t - (2m-1)T_{pd}) \] \quad (a.52)

and the sum of transient voltage is
\[ V_n = V (1 + \Gamma L) \left[ \frac{1 - (\Gamma L \Gamma s')^{n}}{1 - \Gamma L \Gamma s'} \right] . \quad (a.53) \]
For a typical digital circuit, input impedance $Z_L \gg Z_0$, and $\frac{1}{\Gamma L \Gamma_s'} > 0$.

In general, reflections between driver and receiver will reach a steady state after approximate $10T_{pd}$. Therefore, a linear equation

$$t = 2.33nT_{pd}$$  \hspace{1cm} (a.54)

is used to construct the timing relation between the number of reflections at the loading end and the propagation delay time of the interconnects. Then a general equation can be derived to express the transient voltage due to the interconnect discontinuities at the loading end.

$$V_n = \frac{V(1+\Gamma L)}{1-\Gamma L \Gamma_s'} \cdot \frac{V(1+\Gamma L)}{1-\Gamma L \Gamma_s'} \cdot e^{-n \ln\left(\frac{1}{\Gamma L \Gamma_s'}\right)}$$

$$= \frac{V(1+\Gamma L)}{1-\Gamma L \Gamma_s'} \cdot \frac{V(1+\Gamma L)}{1-\Gamma L \Gamma_s'} \cdot e^{-\frac{t \ln\left(\frac{1}{\Gamma L \Gamma_s'}\right)}{2.33T_{pd} \Gamma L \Gamma_s'}}$$

$$= V_{ss} - V_{ss} \cdot e^{-\frac{t}{\tau}}$$  \hspace{1cm} (a.55)

where steady state voltage

$$V_{ss} = \frac{V(1+\Gamma L)}{1-\Gamma L \Gamma_s'} = E \left[ \frac{Z_L}{N} \right]$$  \hspace{1cm} (a.56)

and time constant

$$\tau = \frac{2.33T_{pd}}{\ln\left(\frac{1}{\Gamma L \Gamma_s'}\right)}.$$  \hspace{1cm} (a.57)
By the square root theory for the rise time of cascaded structures, if a clock signal having a nonzero rising time ($t_r$) travels along this radial transmission line, then the result of finite rise time at the loading end will be given approximately by the following equation

$$Tr = \sqrt{tr^2 + (2.2t)^2}.$$  \hspace{1cm} (a.58)

For comparison, the data from the SPICE simulation is shown in Figure 39.

Figure 39, The rise time of pulse waveform due to radial circuit layout vs the characteristic impedance of interconnects.
Equation (a.58) demonstrates that the rise time of pulse waveforms will not only depend on the electrical characteristics of the digital device, but also will be influenced by the loading effects and the characteristics of the interconnects such as the characteristic impedance and the propagation delay. Then, the increased rise time is treated as extra line delay in a timing analysis. For high speed clocking circuits, it is essential for signal pulses to have a fast rising/falling time in order to achieve a high repetition rate, and the circuit delay should be short in order to reduce the machine cycle time. However, even though the radial interconnects can reach the requirement of minimum clock skew, interconnect discontinuities can cause a significant timing problem. Therefore, an approach which accounts for the additional delay due to the interconnect discontinuities can improve on the rise time of the high speed digital system.

As described above, interconnects need to be carefully designed in order to minimize the clock skew and reduce signal reflections. An interconnect structure based on the concept of impedance matching, shown in Figure 40, can achieve this goal. To avoid reflections, the lines separate into two trees with the characteristic impedance twice the impedance of each incoming branch. Let each end point terminate with an impedance equal to the characteristic impedance of the line, so that the reflection coefficient at every branching point will approach zero ideally for the initial incident waveform as

\[ \text{Zo}(w2) = 2\text{Zo}(w1), \quad (a.59) \]

then \[ \Gamma_b = \frac{\text{Zo}(w2)/\text{Zo}(w2) - \text{Zo}(w1)}{\text{Zo}(w2)/\text{Zo}(w2) + \text{Zo}(w1)} = 0. \quad (a.60) \]
Equation (4) can be used to calculate the characteristic impedance associated with packaging parameters. A quick equation can be derived from a simple formulas for impedance calculations, and is used to select the width of transmission line at each branching level for typical packaging structures, as shown in Figure 41 as given by,

Kaupp's Equation [33]:

\[ Zo = \frac{87}{\sqrt{\varepsilon r + 1.41}} \ln\left(\frac{5.98h}{0.8W_i + t}\right) \] (a.61)

Then by taking \( \frac{\delta Zo}{\delta W} \), a parametric equation is given by

\[ W_{i+1} = W_i - (W_i + 1.25 t) \ln\left(\frac{5.98h}{0.8W_i + t}\right) \] (a.62)

Figure 40, A circuit schematic for high speed clock distribution
A test fixture for obtaining the optimizing geometry with chamfer was made on the STYCAST substrate for the case of $W_1 = 1\"$ and $W_2 = 0.5\"$, with resultant characteristic impedance $Z_{w1} = 30$ ohm, and $Z_{w2} = 60$ ohm. Figure 42 show the waveforms measured by TDR with different chamfered geometry. For this case, it is seen that the reflection caused by the discontinuity of interconnects, the branching point with the V type chamfer can minimize reflections, and thus improve upon the traditional design approach. It is noted that the optimizing length ' $W_b$ ' with a V type chamfered branch point is approximately $3.5 \sim 4 \times W_2$. For the above case, similar empirical expressions for the chamfers need to be calculated for other media and impedance.
Figure 42, Reflection voltage measured by TDR for the T junction with different chamfered structure.