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Title SAMPLING INSPECTION PLANS FOR ATTRIBUTES DERIVED FOR THE LUMBER INDUSTRY

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The purpose of this study was to derive a set of sampling plans to be used by the Western Wood Products Association for the reinspection of shipped lumber. Reinspection of a shipment of lumber occurs when a claim of grade is made by the buyer.

Each sampling plan is derived in such a way so that its operating characteristic curve would pass through two preassigned points. These two points reflect the risks involved in sampling.

The Hypergeometric Probability Distribution was used to derive the sampling plans when the lot size was 100 units. The Binomial Probability Distribution was used to derive the sampling plans when the lot size was 2,000 units. The Poisson Probability Distribution was used to derive the sampling plans when the lot size was at least 5,000 units.

The sampling plans derived in this study are acceptance plans to be used in attribute inspection.

SAMPLING INSPECTION PLANS FOR ATTRIBUTES DERIVED FOR THE LUMBER INDUSTRY

by

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SAMPLING INSPECTION PLANS FOR ATTRIBUTES DERIVED FOR THE LUMBER INDUSTRY

INTRODUCTION

Statistical quality control is relatively new when compared with quality control which is as old as industry itself. Statistical theory began to be applied effectively to quality control in the 1920's when an exact theory of sampling was developed. The first to apply the new statistical methods to quality control was Walter A. Shewhart of the Bell Telephone Laboratories who developed what is known as the control chart. Shewhart published a book in 1931, Economic Control of Quality of Manufactured Product (13), which set the pattern for subsequent applications of statistical theory and methods to quality control. Among other prioneers in applying statistical theory and methods to quality control and sampling inspection were Harold F. Dodge and Harry G. Romig; the culmination of their early work being the Dodge-Romig Sampling Inspection Tables (6).

World War II had a direct positive influence on American industry toward statistical quality control. The armed services themselves adopted statistically designed inspection procedures and established a widespread educational and training program for industrial and other personnel.

Since World War II, various sampling plans have been used in statistical quality control by both industry and the armed services.

Bad ones have been replaced by good ones, and good ones have been replaced by better ones.

However, the lumber industry today is not using any sampling plan when reinspecting shipped lumber. Currently, lumber is inspected and graded by the producer at the mill and is then shipped to the buyer. In the case of a claim of grade by the buyer, inspection is available upon request to the Western Wood Products Association. Reinspection is 100 percent inspection made by a member of the Association. The expense of a reinspection is assumed by the producer if the lot complained of is found to be more than five percent below grade, otherwise the expense of the reinspection is borne by the buyer (14).

One of the major fields of statistical quality control is what is called acceptance sampling. Items are formed into lots, a sample is drawn from the lot, and the lot is either accepted or rejected on the basis of conclusions of the quality of the sample.

Sampling plans may be further classified as to variables inspection or attribute inspection. In a variables inspection, the quality characteristics are measured on a continuous scale, i. e. expressed in numbers. In an attribute inspection, the individual items are graded as either defective or nondefective.

It is the aim of this thesis to derive a set of sampling plans, using attribute inspection, to be used in the reinspection of lumber after shipment from the producer to the buyer. Each sampling plan will reflect the value to the producer and to the buyer of the risks involved in sampling.

CHARACTERISTICS OF GOOD SAMPLING PLANS

Objectives

Any sampling plan has as its primary purpose the acceptance of good lots and the rejection of bad lots. Indeally, lots submitted at an agreed upon quality or better should be accepted, and lots submitted whose quality is less than agreed upon should be rejected.

The only way to reach these ideal conditions is to have 100 percent inspection. Due to inspection costs, this is usually not possible. However, even if complete inspections were possible, it does not insure a perfect lot because of weaknesses in either human or mechanical inspection. Monotony, fatigue, carelessness and ineptitude all contribute.

If the ideal conditions cannot be reached, they should be reached as closely as possible. If the lot quality is good, it is desirable to have the probability of acceptance be high. If the lot quality is bad it is desirable to have the probability of acceptance be small.

Likewise, if the lot quality is good, it is desirable to have the probability of rejection be low. If the lot quality is bad, it is desirable to have the probability of rejection be high. The risks involved in acceptance sampling are accepting a lot of bad quality and rejecting a lot of good quality.

This will not only protect the producer against having any lot rejected when the quality meets acceptable standards, but will protect the consumer against the acceptance of a bad lot. If a producer's product is being rejected at a high rate, the producer may take steps to improve the product quality or the supplier may be forced to find other and better sources of supply. Thus, a good sampling plan indirectly improves the quality of production with its encouragement of good quality by a high rate of acceptance and its discouragement of poor quality by a high rate of rejection, and thereby encourages the producer to keep his process under control.

One objective of any sampling plan is the minimization of sampling, inspection and administrative costs. If data are secured from only a small fraction of the aggregate, expenditures are smaller than if a 100 percent inspection is attempted (4, 2).

Formation of Lots and Drawing of Sample

A lot constitutes the sum of those individual pieces of lumber each of the same size and of the same intended grade of the lot. Each lot should be homogeneous, i. e., each lot should be representative of one process during one interval of time, so that all of the individual items have been produced under essentially the same conditions. In keeping with this principle of homogeneity, all of the individual pieces of lumber of a particular grade are or should be from the same mill. Pieces of lumber of one mill should neither be allowed to be mixed with those of different size and grade nor be allowed to mix with those of another mill.

The power of sampling plans to distinguish between good and bad lots is dependent upon variation from lot to lot. If the original lots are inspected separately, the sampling plan must accept most of the good lots and reject most of the bad ones, tending to increase the quality inspected. If the lots are mixed prior to inspection, the lots accepted will be an average value which is poorer than that which would be obtained by sampling from homogeneous lots (7, 161).

The sample from a lot should be a random sample giving every piece of lumber an equal probability of being drawn. A random sample will give an unbiased sample (1, 175). This is essential because a sample from each lot supplies the information to accept or reject the lot.

In order to insure randomness, a random number table or some other aid may be used. If a random number table is used, the individual sampling units (individual pieces of lumber) should be indexed from one to the number of items in the lot, N, and then n distinct random numbers should be selected between one and N. The sample is then defined to be that subset of the individual pieces of lumber indexed by the selected numbers.

The measurement made on each unit will be whether it is of said grade or better, or whether it is worse. The purpose of grades is to maintain a standard measure of value between mills manufacturing the same or similar woods so that a given grade will represent the same value and can be used for the same purpose regardless of the mill it came from. Also, uniform grades provide both consumers and producers of lumber with a measure by which each can determine whether he is buying or selling the lumber at full value (5, 14).

ACCEPTANCE SAMPLING

In acceptance sampling, items are formed into lots of size N, a sample is drawn from the lot, and the lot is either accepted or rejected on the basis of conclusions drawn from the quality of the sample.

Acceptance sampling does not estimate lot quality or control quality, but prescribes a procedure that, if applied to a series of lots, will give a specified risk of accepting lots of given quality. An acceptance sampling plan merely accepts or rejects lots. If the lots are of the same quality, it will accept some and reject others even though the lots are of the same quality. If some lots are better than others, it will accept the good lots more frequently than the bad ones.

Single Sampling

A single sampling plan is characterized by a random sample of size n items drawn from a lot of size N. If the number of defective items in the sample does not exceed a number c, the lot is accepted, where an item inspected is either classified as defective or nondefective. If the number of defective items in the sample exceeds c, reject the lot.

The advantages of single sampling are many. They are easy to design, explain, and administer and the amount of sampling and inspection is constant (5, 491).

The disadvantages are: the average total inspection is usually larger than double sampling, multiple sampling and sequential sampling; psychologically, the producer may feel that it is unfair to reject a lot on the basis of just one sample.

Double Sampling

A double sampling plan may be characterized by a random sample of size n, drawn from a lot of size N. The lot is rejected if the number of defective items exceeds c_2. The lot is accepted if the number of defective items is equal to or less than c_1. If the number of defective items is between $c_1 + 1$ and c_2, then a second random sample of size n_2 is drawn from the lot. If the combined number of defective items of the two samples is greater than c_3, then the lot is rejected. If the combined number of defective items is equal to or less than c_3 the lot is accepted. Frequently, c_2 is taken equal to c_3.

The advantages of double sampling are: the average total inspection is less than single sampling except for lots of intermediate quality; psychologically, but fallaciously, the producer may feel it is fair to give the lot another chance if it is not accepted the first time.

The disadvantages are: one does not know in advance the amount of inspection that will be required; the plan is more expensive to administer than a comparable single sampling plan.

Multiple Sampling

Multiple sampling is an extension of double sampling. It is characterized by a random sample of size n, drawn from a lot of size N. The lot is accepted if, at any stage, the number of defective items equals or is less than the acceptance number, c_i. The lot is rejected if, at any stage, the number of defective items equals or is greater than the rejection number, $r_i > c_i + 1$. If the number of defective items is between $c_i + 1$ and r_i, the process continues to the next stage.

The advantage of multiple sampling is that the average sample size is generally less than single or double sampling.

The disadvantages are: higher administrative costs; it is more difficult to explain than single or double sampling plans; difficulties may arise in scheduling inspection time due to the variability of the inspection load; adequate storage facilities must be provided for the lot while multiple sampling is being done.

In comparing the advantages and disadvantages of single sampling plans with those to double, multiple, and item by item sequential sampling plans, it was decided that single sampling plans would come the closest to meeting the needs of the Western Wood Products Association (14).

OC CURVES

The discriminatory power of a sampling plan is given by its OC curve or operating characteristic curve. An OC curve shows how the probability of a accepting a lot of quality p' varies for different values of p', where p' may be viewed as the fraction defective of the lot. In other words, 100 p' percent of the items in the lot are defective items.

Certain risks are involved when sampling inspection is used. If the lot quality is good, the probability of accepting the lot should be high. If the lot quality is bad, the probability of accepting the lot should be small. If there are no defective items in the lot, i. e. $p' = 0$, the lot will always be accepted. In this case, the probability of acceptance of a lot of quality p', $L(p')$, will be equal to 1. If all of the items in the lot are defective, i. e. $p' = 1$, the lot will always be rejected. In this case, the probability of acceptance of a lot of quality p', $L(p')$, will be equal to 0. Thus, a general OC curve may be illustrated in Figure 1.

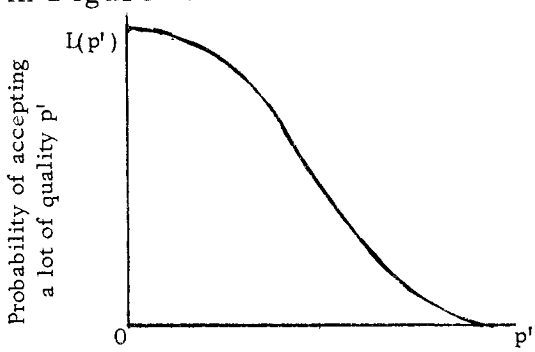

Figure 1.

It would be desirable to have a sampling inspection plan which would discriminate perfectly between good lots and bad lots. If a quality standard, p_t', can be established, then lots of a quality better than p_t' are good lots and lots of a quality worse than p_t' are bad lots. An ideal OC curve would be of the form illustrated in Figure 2. This shows that all good lots would be accepted and all bad lots would be rejected. Such a plan would exercise perfect control over the quality of inspected material.

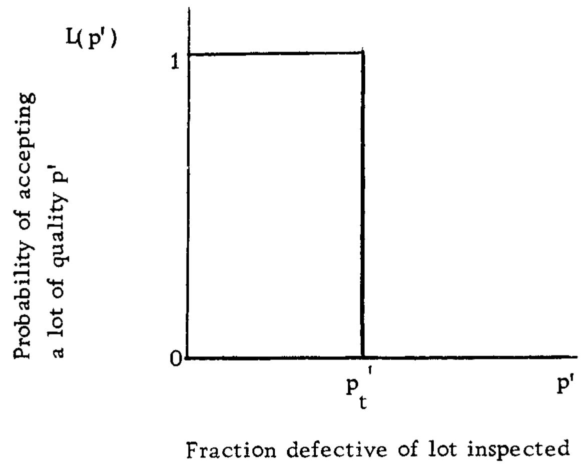

Figure 2.

This form of OC curve, however, can only be attained by perfect 100 percent inspection. The degree of approximation to this ideal OC curve depends on the sample size, n, and the acceptance number, c.

As the sample size is increased, the precision with which the sampling plan discriminates between good and bad lots is increased.

Thus, by holding c constant and by increasing n, the slope of the OC curve becomes steeper. This is illustrated in Figure 3.

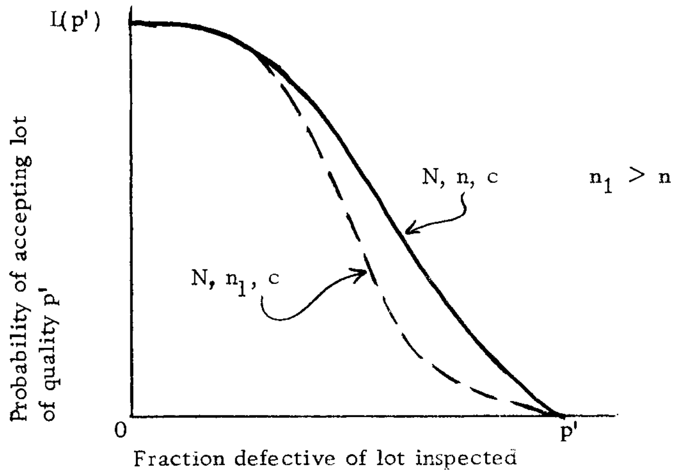

Figure 3.

The effect of varying the acceptance number, c, is illustrated in Figure 4. This shows that, by holding n constant and varying c, the OC curve is shifted to the left or right.

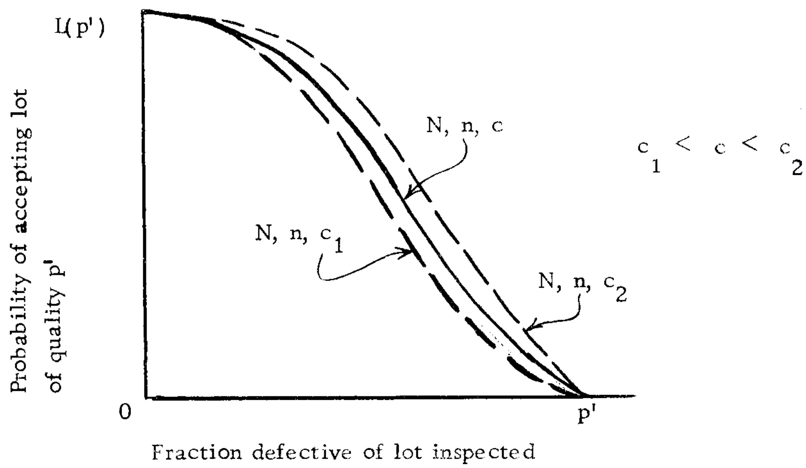

Figure 4.

An OC curve is defined by the lot size N, sample size n and acceptance number c. If the lot size N is large compared with the sample size n, the OC curve is independent of the lot size. The OC curve can then be defined by the sample size n and the acceptance number c.

Since good lots will be accepted most of the time and bad lots will be rejected most of the time, there is a chance that bad lots will be accepted some of the time and good lots will be rejected some of the time. The probability of accepting a bad lot by the consumer is commonly called the consumer's risk and is designated by β. The probability of having a good lot rejected is commonly called the producer's risk and is designated by α. A lot submitted of quality p_1' or better will be accepted equal to or greater than $1-\alpha$ percent of the time. A lot submitted of quality p_2' or worse will be accepted equal to or less than β percent of the time. These concepts are illustrated in Figure 5.

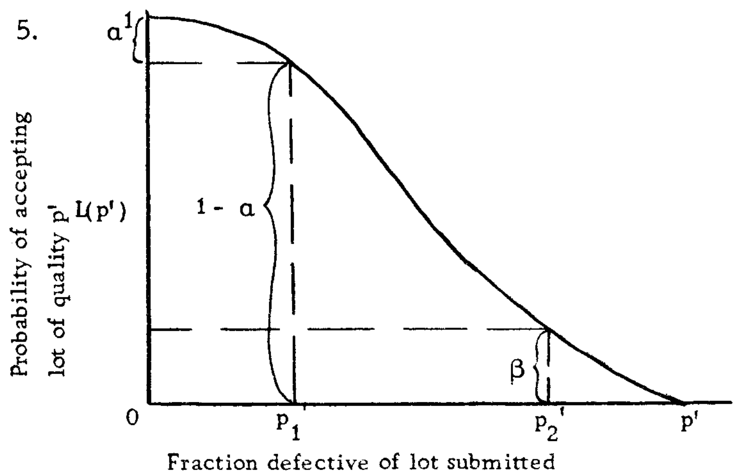

Figure 5.

Hypergeometric Probability Distribution

The hypergeometric probability distribution is concerned with X defective items in a sample of n items taken without replacement from a lot of size N items containing m defective items. The probability that there are x defective items in the sample of size n is

equation 1) $$P(x) = \frac{\binom{N-m}{n-x}\binom{m}{x}}{\binom{N}{n}} \qquad x = 0, 1, \ldots, n$$

By definition

$$\binom{N}{n} = \frac{N!}{n!\,(N-n)!}$$

and $N! = (N)(N-1)(N-2) --- 1$.

The probability of drawing c or less defectives in the sample is

equation 2) $$P(x \leq c) = L(p') = \sum_{x=0}^{c} \frac{\binom{N-m}{n-x}\binom{m}{x}}{\binom{N}{n}}$$

The situation where the sample is drawn from a lot of finite size requires the use of the hypergeometric probability distribution. This is also the case when the sample size is large compared to the lot size (5, 515).

For an accurate evaluation of the OC curve, the lot size N as well as the sample size n must be considered. This is because the OC curve for this situation depends not only on n and c but also on the lot size N.

In computing the probabilities of acceptance for the OC curve given by the hypergeometric distribution, one may either compute from the formula or use the Lieberman-Owen Tables of the Hypergeometric Probability Distribution (9). Results are given for N=2 through N=100 and for n=1 through n=50 and for a few other special values of N and n. If results are wanted for values outside the range of these values, approximations to the hypergeometric may be used.

As the lot size increases in size, the OC curve rapidly approaches the OC curve for infinite lot sizes. This leads to the discussion of the binomial distribution.

Binomial Probability Distribution

The OC curve for an infinite lot size is mathematically identical with the OC curve for drawing random samples directly from the process (7, 150). The probability of accepting a lot from a process of product quality p' is therefore the probability of accepting a lot of infinite lot size of quality p'. This probability is given by the binomial probability distribution. An OC curve, or sampling plan, is then defined by the sample size n and the acceptance number c. The probability that there are x defective items in a random sample of size n of quality p' is

equation 3) $P(x) = \binom{n}{x} (p')^{x} (1-p')^{n-x} \qquad \binom{n}{x} = 0, 1, \ldots, n.$

The probability of accepting a lot of quality p' is the probability that a random sample from an infinite universe with fraction defective p' contains c or less defectives. Thus, L(p') = P (0) + P (1) + - - - + P (c) where P (i) is the probability of i defective items in a sample of size n. This probability is given by the binomial distribution and may be written as

equation 4) $$L(p') = P(X \leq C) = \sum_{x=0}^{c} \binom{n}{x} (p')^x (1-p')^{n-x}$$

The binomial distribution may be thought of as the limit of the hypergeometric distribution as N approaches infinity. The proof is given by Duncan (7, 857-58). To compute the probabilities of acceptance for the OC curve, one may either compute from the formula directly or use the Romig _50-100 Binomial Tables_ (12). Results are given for n=50 through n=100 in steps of 5 and the range of p values in steps of .01 from .01 to .99. Also, one may use _Tables of the Cumulative Binomial Probability Distribution_ (8). If results are wanted for values outside the range of these values, approximations to the binomial may be used.

Poisson Probability Distribution

As the sample size n becomes larger and the fraction defective p' becomes proportionally smaller, np' remaining unchanged, the binomial distribution approaches the limiting form commonly called the poisson distribution (5, 398). The proof is given by Duncan (7, 857).

The probability of finding x defective items in a random sample from an infinite universe with fraction defective p', keeping within the above assumptions, is

equation 5) $$P(x) = \frac{(np')^{x} e^{-np'}}{x!} \qquad x = 0, 1, ---, n.$$

The probability of accepting a lot of quality p' is the probability that a random sample of n items from an infinite universe with fraction defective p' will contain c or less defectives. Thus,

equation 6) $$L(p') = P(x \leq c) = \sum_{x=0}^{c} \frac{(np')^{x} e^{-np'}}{x!}$$

In computing the probabilities of acceptance, one may either compute directly from the formula or use Molina's Poisson Exponential Binomial Limit (11). Results are given in individual terms or cumulative terms for np' from .001 to .01 in steps of .001, from .01 to .30 in steps of .01, from 30 to 15.0 in steps of .1, from 15 to 100 in steps of 1, and x from 1 to 153. Duncan has reprinted part of the tables (7, 911-915). Results are given for np' from .02 through 25, and x from 0 to 43.

Comparison of Hypergeometric, Binomial, and Poisson

When the sample is taken from a lot which is finite the hypergeometric distribution is the correct distribution to use. When the sample is taken from a lot which is infinitely large the binomial distribution should be used. When the sample is infinitely large and the fraction defective is infinitely small, the poisson distribution is appropriate. It should be noted that it is the absolute size of the sample size n that is of importance when the lot size is large. Cowden (5, 511) suggests that the following procedures be used for approximations:

1. Replace the hypergeometric distribution by the binomial distribution when $n/N \leq .1$
2. Replace the binomial distribution by the poisson distribution when $n > 10$ and $p' \leq .1$.

DERIVATION OF SAMPLING PLANS

There are many plans available that are designed to meet certain specifications. However, it is possible to design a plan such that its OC curve passes through two designated points. Thus, a plan can be designed such that the probability of acceptance of a lot of p'_1 quality is $1-\alpha$ and the probability of acceptance of a lot of p'_2 quality is β. The two points are $(p'_1, 1-\alpha)$ and (p'_2, β), where $L(p'_1) = 1-\alpha$ and $L(p'_2) = \beta$. This is illustrated by Figure 6.

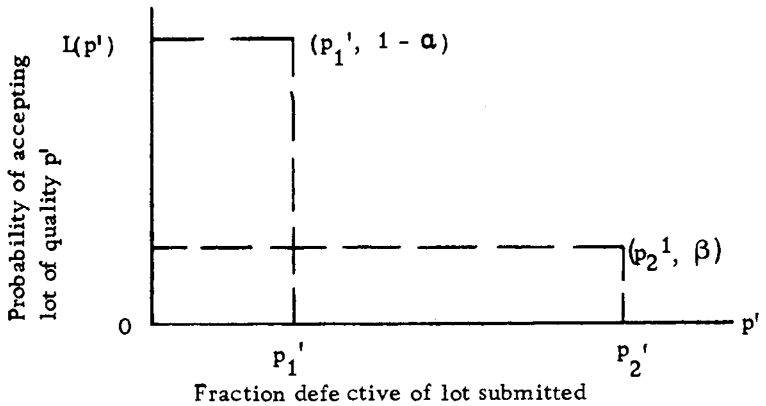

Figure 6.

By varying n and c, an OC curve can be found that will pass through these two preassigned points. Because n and c are integers, it may not be possible to meet these specifications exactly.

<u>Computations of n and c for $N \geq 5000$.</u>

A reasonable approximation for an OC curve passing through the two preassigned points can be found by using tables that give various values of np'_1, and c (2, 832) (3, 37-39). Since the n for any single sampling plan is constant, the difference between np'_1, giving $1-\alpha$ probability of acceptance and the np'_2 giving β probability of acceptance arises from the differences in p'_1 and p'_2. It is the ratio of p'_2 to p'_1 for a plan which has the acceptance number c.

To construct a sampling plan for a given p'_1, p'_2, α and β, the ratio of p'_2/p'_1 must be calculated. For the appropriate α and β, find the tabular value of np'_1, which is equal to or slightly greater than the value of the calculated ratio p'_2/p'_1. The acceptance number, c, can be read off directly. The sample size, n, is found by dividing the tabular value of np'_1 by p'_1. That is, $\frac{np'_1}{p'_1} = n$. These tables were computed from existing tables of the χ^2 distribution using the fact that for even degrees of freedom, χ^2 gives the partial sums of a Poisson series (3, 38).

For different values of p'_2, α and β, one can derive a set of sampling plans. According to the grading rules set forth by the Western Wood Products Association, a lot is considered acceptable if 95 percent or more of the lot is of said grade or better (14, 10-12). For this reason p'_1 will remain constant at .05 while varying p'_2, α and β. Selected values for p'_2 are .10, .11, .12, .13, .14, .15, .20 and .25. Selected values of α are .01 and .05. Selected values for β are .01, .05 and .10. Thus, for each value of p'_2 there will be six possible sampling plans. Each of these is designed to reflect the risks, α and β, involved in accepting or rejecting lots of fraction defective p'.

The general method of computation for the construction of a single sampling plan whose OC curve is required to pass through two points, $(p'_1, 1-\alpha)$ and $p'_2, \beta)$, using tables giving various values of np'_1 and c, as discussed previously, may be summarized as follows:

1. Compute $R = p'_2/p'_1$.
2. Find the tabular value, R_t, for selected α and β which is equal to or slightly greater than R.
3. Find c corresponding to R_t.
4. Find n by dividng the np'_1 which corresponds to R_t by p'_1.

This is the computational method that will be used for holding p'_1 constant at .05 and for varying p'_2, α and β in constructing the sampling plans.

The computations are as follows:

for $p'_1 = .05$, $p'_2 = .10$, $\alpha = .05$, and $\beta = .10$;

$$R = \frac{p'_2}{p'_1} = \frac{.10}{.05} = 2$$

$$R_t = 2.029$$

$$C = 17$$

$$n = \frac{np'_1}{p'_1} = \frac{11.633}{.05} = 233$$

for $p'_1 = .05$, $p'_2 = .10$, $\alpha = .05$, $\beta = .05$;

$$R = \frac{p'_2}{p'_1} = \frac{.10}{.05} = 2$$

$$R_t = 2.030$$

$$c = 21$$

$$n = \frac{np'_1}{p'_1} \quad \frac{14.894}{.05} = 298$$

for $p'_1 = .05$, $p'_2 = .10$. $\alpha = .05$, $\beta = .01$;

$$R = \frac{p'_2}{p'_1} = \frac{.10}{.05} = 2$$

$$R_t = 2.001$$

$$c = 31$$

$$n = \frac{np'_1}{p'_1} = \frac{23.298}{.05} = 466$$

for $p'_1 = .05$, $p'_2 = .10$, $\alpha = .01$, $\beta = .10$

$$R = \frac{p'_2}{p'_1} = \frac{.10}{.05} = 2$$

$$R_t = 2.009$$

$$c = 28$$

$$n = \frac{np'_1}{p'_1} = \frac{17.957}{.05} = 359$$

for $p'_1 = .05$, $p'_2 = .10$, $\alpha = .01$, $\beta = .05$

$$R = \frac{p'_2}{p'_1} = \frac{.10}{.05} = 2$$

$$R_t = 2.013$$

$c = 3$

$$n = \frac{np'_1}{p'_1} = \frac{21.919}{.05} = 439$$

for $p'_1 = .05$, $p'_2 = .10$, $\alpha = .01$, $\beta = .01$;

$$R = \frac{p'_2}{p'_1} = \frac{.10}{.05} = 2$$

$R_t = 2.010$

$c = 44$

$$n = \frac{30.877}{.05} = 618$$

for $p'_1 = .05$, $p'_2 = .11$, $\alpha = .05$, $\beta = .10$;

$R = 2.20$

$R_t = 2.240$

$c = 13$

$$n = \frac{8.464}{.05} = 170$$

for $p'_1 = .05$, $p'_2 = .11$, $\alpha = .05$, $\beta = .05$;

$R = 2.20$

$R_t = 2.244$

$c = 16$

$$n = \frac{10.831}{.05} = 217$$

for $p'_1 = .05$, $p'_2 = .11$, $\alpha = .05$, $\beta = .01$;

$R = 2.20$

$R_t = 2.226$

$c = 23$

$$n = \frac{16.546}{.05} = 331$$

for $p'_1 = .05$, $p'_2 = .11$, $\alpha = .01$, $\beta = .10$;

$R = 2.20$

$R_t = 2.241$

$c = 21$

$$n = \frac{12.574}{.05} = 252$$

for $p'_1 = .05$, $p'_2 = .11$, $\alpha = .01$, $\beta = .05$;

$R = 2.20$

$R_t = 2.20$

$c = 26$

$$n = \frac{16.397}{.05} = 328$$

for $p'_1 = .05$, $p'_2 = .11$, $\alpha = .01$, $\beta = .01$;

$R = 2.20$

$R_t = 2.210$

$c = 34$

$$n = \frac{22.721}{.05} = 455$$

for $p'_1 = .05$, $p'_2 = .12$, $\alpha = .05$, $\beta = .10$;

$R = 2.40$

$R_t = 2.397$

$c = 11$

$$n = \frac{6.924}{.05} = 138$$

for $p'_1 = .05$, $p'_2 = .12$, $\alpha = .05$, $\beta = .05$

$R = 2.40$

$R_t = 2.442$

$c = 13$

$n = \dfrac{8.464}{.05} = 170$

for $p_1' = .05$, $p_2' = .12$, $\alpha = .05$, $\beta = .01$;

$R = 2.40$

$R_t = 2.403$

$c = 19$

$n = \dfrac{13.254}{.05} = 266$

for $p_1' = .05$, $p_2' = .12$, $\alpha = .01$, $\beta = .10$;

$R = 2.40$

$R_t = 2.455$

$c = 17$

$n = \dfrac{9.616}{.05} = 193$

for $p_1' = .05$, $p_2' = .12$, $\alpha = .01$, $\beta = .05$;

$R = 2.40$

$R_t = 2.405$

$c = 21$

$n = \dfrac{12.574}{.05} = 252$

for $p_1' = .05$, $p_2' = .12$, $\alpha = .01$, $\beta = .01$;

$R = 2.40$

$R_t = 2.431$

$c = 27$

$$n = \frac{17.175}{.05} = 344$$

for $p_1' = .05$, $p_2' = .13$, $\alpha = .05$, $\beta = .10$;

$R = 2.60$

$R_t = 2.618$

$c = 9$

$$n = \frac{5.426}{.05} = 109$$

for $p_1' = .05$, $p_2' = .13$, $\alpha = .05$, $\beta = .05$;

$R = 2.60$

$R_t = 2.630$

$c = 11$

$$n = \frac{6.924}{.05} = 139$$

for $p_1' = .05$, $p_2' = .13$, $\alpha = .05$, $\beta = .01$;

$R = 2.60$

$R_t = 2.588$

$c = 16$

$$n = \frac{10.831}{.05} = 217$$

for $p_1' = .05$, $p_2' = .13$, $\alpha = .01$, $\beta = .10$;

$R = 2.60$

$R_t = 2.603$

$c = 15$

$$n = \frac{8.181}{.05} = 164$$

for $p_1' = .05$, $p_2' = .13$, $\alpha = .01$, $\beta = .05$;

$R = 2.60$

$R_t = 2.652$

$c = 17$

$$n = \frac{9.616}{.05} = 193$$

for $p_1' = .05$, $p_2' = .13$, $\alpha = .01$, $\beta = .01$;

$R = 2.60$

$R_t = 2.615$

$c = 23$

$$n = \frac{14.088}{.05} = 282$$

for $p_1' = .05$, $p_2' = .14$, $\alpha = .05$, $\beta = .10$;

$R = 2.80$

$R_t = 2.768$

$c = 8$

$$n = \frac{4.695}{.05} = 94$$

for $p_1' = .05$, $p_2' = .14$, $\alpha = .05$, $\beta = .05$;

$R = 2.80$

$R_t = 2.895$

$c = 9$

$$n = \frac{5.426}{.05} = 109$$

for $p_1' = .05$, $p_2' = .14$, $\alpha = .05$, $\beta = .01$;

$R = 2.80$

$R_t = 2.852$

$c = 13$

$n = \frac{8.464}{.05} = 170$

for $p'_1 = .05$, $p'_2 = 14$, $\alpha = .01$, $\beta = .10$;

$R = 2.80$

$R_t = 2.795$

$c = 13$

$n = \frac{6.782}{.05} = 136$

for $p'_1 = .05$, $p'_2 = .14$, $\alpha = .01$, $\beta = .05$;

$R = 2.80$

$R_t = 2.823$

$c = 15$

$n = \frac{8.181}{.05} = 164$

for $p'_1 = .05$, $p'_2 = .14$, $\alpha = .01$, $\beta = .01$;

$R = 2.80$

$R_t = 2.874$

$c = 19$

$n = \frac{11.082}{.05} = 222$

for $p'_1 = .05$, $p'_2 = .15$, $\alpha = .05$, $\beta = .10$;

$R = \frac{.15}{.05} = 3$

$R_t = 2.957$

$c = 7$

$$n = \frac{3.981}{.05} = 79$$

for $p_1' = .05$, $p_2' = .15$, $\alpha = .05$, $\beta = .05$;

$$R = \frac{.15}{.05} = 3$$

$$R_t = 3.074$$

$$c = 8$$

$$n = \frac{4.695}{.05} = 94$$

for $p_1' = .05$, $p_2' = .15$, $\alpha = .05$, $\beta = .01$;

$$R = \frac{.15}{.05} = 3$$

$$R_t = 3.104$$

$$c = 11$$

$$n = \frac{6.924}{.05} = 139$$

for $p_1' = .05$, $p_2' = .15$, $\alpha = .01$, $\beta = .10$;

$$R = \frac{.15}{.05} = 3$$

$$R_t = 3.058$$

$$c = 11$$

$$n = \frac{5.428}{.05} = 109$$

for $p_1' = .05$, $p_2' = .15$, $\alpha = .01$, $\beta = .05$;

$$R = \frac{.15}{.05} = 3$$

$$R_t = 3.047$$

$$c = 13$$

$$n = \frac{6.782}{.05} = 136$$

for $p_1' = .05$, $p_2' = .15$, $\alpha = .01$, $\beta = .01$;

$R = 3$

$R_t = 3.048$

$c = 17$

$n = \frac{9.616}{.05} = 193$

for $p_1' = .05$, $p_2' = .20$, $\alpha = .05$, $\beta = .10$;

$R = 4$

$R_t = 4.057$

$c = 4$

$n = \frac{1.970}{.05} = 40$

for $p_1' = .05$, $p_2' = .20$, $\alpha = .05$, $\beta = .05$;

$R = 4$

$R_t = 4.023$

$c = 5$

$n = \frac{2.613}{.05} = 53$

for $p_1' = .05$, $p_2' = .20$, $\alpha = .05$, $\beta = .01$;

$R = 4$

$R_t = 4.019$

$c = 7$

$n = \frac{3.981}{.05} = 80$

for $p_1' = .05$, $p_2' = .20$, $\alpha = .01$, $\beta = .10$

$R = 4$

$R_t = 4.050$

$c = 7$

$n = \frac{2.906}{.05} = 58$

for $p'_1 = .05$, $p'_2 = .20$, $\alpha = .01$, $\beta = .05$;

$R = 4$

$R_t = 4.115$

$c = 8$

$n = \frac{3.507}{.05} = 71$

for $p'_1 = .05$, $p'_2 = .20$, $\alpha = .01$, $\beta = .01$;

$R = 4$

$R_t = 4.222$

$c = 10$

$n = \frac{4.771}{.05} = 96$

for $p'_1 = .05$, $p'_2 = .25$, $\alpha = .05$, $\beta = .10$

$R = 5$

$R_t = 4.890$

$c = 3$

$n = \frac{1.366}{.05} = 27$

for $p'_1 = .05$, $p'_2 = .25$, $\alpha = .05$, $\beta = .05$;

$R = 5$

$R_t = 4.646$

$c = 4$

$$n = \frac{1.970}{.05} = 38$$

for $p_1' = .05$, $p_2' = .25$, $\alpha = .05$, $\beta = .01$;

$$R = 5$$

$$R_t = 5.017$$

$$c = 5$$

$$n = \frac{2.613}{.05} = 53$$

for $p_1' = .05$, $p_2' = .25$, $\alpha = .01$, $\beta = .10$;

$$R = 5$$

$$R_t = 5.195$$

$$c = 5$$

$$n = \frac{1.785}{.05} = 36$$

for $p_1' = .05$, $p_2' = .25$, $\alpha = .01$, $\beta = .05$;

$$R = 5$$

$$R_t = 5.082$$

$$c = 6$$

$$n = \frac{2.330}{.05} = 47$$

for $p_1' = .05$, $p_2' = .25$, $\alpha = .01$, $\beta = .01$;

$$R = 5$$

$$R_t = 4.962$$

$$c = 8$$

$$n = \frac{3.507}{.05} = 70$$

These computations may be summarized in Table 1.

Table 1. Lot size N $\geq$ 5000, sample size, n, and acceptance number, c, for values of p_2', α and β while holding p_1' constant at .05.

p_2'	α	β	n	c
.10	.05	.10	233	17
	.05	.05	298	21
	.05	.01	466	31
	.01	.10	359	28
	.01	.05	439	33
	.01	.01	618	44
.11	.05	.10	170	13
	.05	.05	217	16
	.05	.01	331	23
	.01	.10	252	21
	.01	.05	328	26
	.01	.01	455	34
.12	.05	.10	138	11
	.05	.05	170	13
	.05	.01	266	19
	.01	.10	196	17
	.01	.05	252	21
	.01	.01	344	27
.13	.05	.10	109	9
	.05	.05	139	11
	.05	.01	217	16
	.01	.10	164	15
	.01	.05	193	17
	.01	.01	282	23
.14	.05	.10	94	8
	.05	.05	109	9
	.05	.01	170	13
	.01	.10	136	13
	.01	.05	165	15
	.01	.01	222	19
.15	.05	.10	79	7
	.05	.05	94	8
	.05	.01	139	11
	.01	.10	109	11
	.01	.05	136	13
	.01	.01	193	17
.20	.05	.10	40	4
	.05	.05	53	5
	.05	.01	80	7
	.01	.10	58	7
	.01	.05	71	8
	.01	.01	96	10

Table 1. Continued.

p'_2	α	β	n	c
.25	.05	.10	27	3
	.05	.05	38	4
	.05	.01	53	5
	.01	.10	37	5
	.01	.05	47	6
	.01	.01	70	8

It should be noted that some of the values in the table are not the same as those computed. This is so because when computing the OC curve, some of the values had to be altered slightly to give a better approximation to the two points the curve was intended to pass through.

By using Table 1, a sampling plan can be constructed such that its OC curve passes through the two points (p'_1, $1-\alpha$) and p'_2, β). By using p'_1 as the fraction defective of a lot for which the probability of rejection is α, and p'_2 as the fraction defective of a lot for which the probability of acceptance is β, the sample size and acceptance number can be read off directly to meet these specifications.

Computation of Probabilities for OC Curves for $N \geq 5000$

The OC curve is a mathematical expression stating the probability of accepting a lot as a function of the fraction defective in the lot. Since N is large compared to the lot size the OC curve need only be defined by the sample size n and the acceptance number c (5,511).

The probability of accepting a lot of quality p' is the probability that c or less defectives are found in a random sample of size n from an infinite universe with fraction defective p'. This probability can be computed from equation 6, i. e.

$$(p') = P\ (X \geq c) = \sum_{x=0}^{c} \frac{(np')^{x}\ e^{-np'}}{x!}.$$

Values of the cumulative probabilities of the Poisson distribution have been tabulated and are available for use (7) (11). Some of the probabilities in the preceeding pages have been omitted since a good graphic representation of the OC curve can be obtained from as few as five points (5, 519).

OC curves can be drawn for each sampling plan by using the fraction defective of the lot submitted, p', as the abscissa and the corresponding acceptance probability, L(p'), as the ordinate. By using OC curves one can easily visualize how the probability of acceptance varies with the quality of the lot submitted. However, when plotting the points on the graph for the OC curve, some degree of approximation must be used. Thus, the acceptance probabilities for the sampling plans previously computed can fulfill the same purpose of the OC curve and at the same time keep the accuracy of the acceptance probabilities.

for n = 233, c = 17;

p'	$P(x \leq 17) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	0.992
.05	0.951
.06	0.828
.07	0.629
.08	0.411
.09	0.229
.10	.112

for n = 298, c = 21;

p'	$P(x \leq 21) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.994
.05	.950
.06	.808
.07	.570
.08	.328
.09	.144
.10	.052

for n = 466, c = 31;

p'	$P(X \leq 31) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.997
.05	.949
.06	.751
.07	.447
.08	.184
.09	.047
.10	.011

for n = 359, c = 28;

p'	$P(x \leq 28) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.999
.05	.990
.06	.928
.07	.754
.08	.498
.09	.254
.10	.103

for n = 439, c = 33;

p'	$P(X \leq 33) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	1.000
.05	.989
.06	.825
.07	.701
.08	.403
.09	.171
.10	.052

for n = 618, c = 44;

p'	$P(x \leq 44) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	1.000
.05	.987
.06	.885
.07	.582
.09	.248
.09	.065
.10	.010

for n = 170, c = 13;

p'	$P(x \leq 13) = L(p')$
.01	1.000
.02	1.000
.03	.999
.04	.984
.05	.949
.06	.848
.07	.692
.08	.475
.09	.337
.10	.201
.11	.111

for n = 217, c = 16;

p'	$P(x \leq 16) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.992
.05	.949
.06	.799
.07	.643
.08	.435
.09	.253
.10	.131
.11	.061

for n = 331, c = 23;

p'	$P(x \leq 23) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.995
.05	.949
.06	.794
.07	.515
.08	.289
.09	.122
.10	.043
.11	.011

for n = 252, c = 21;

p'	$P(x \leq 21) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	1.000
.05	.989
.06	.943
.07	.821
.08	.630
.09	.416
.10	.238
.11	.113

for n = 328, c = 26;

p'	$P(x \leq 26) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	1.000
.05	.990
.06	.931
.07	.774
.08	.529
.09	.298
.10	.134
.11	.050

for n = 455, c = 34;

p'	$P(x \leq 34) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	1.000
.05	.989
.06	.911
.07	.687
.08	.387
.09	.155
.10	.047
.11	.011

for n = 138, c = 11;

p'	$P(x \leq 11) = L(p')$
.01	1.000
.02	1.000
.03	.998
.04	.987
.05	.949
.06	.861
.07	.731
.08	.577
.09	.416
.10	.269
.11	.166
.12	.101

for n = 170, c = 13;

p'	$P(x \leq 13) = L(p')$
.01	1.000
.02	1.000
.03	.999
.04	.990
.05	.949
.06	.848
.07	.693
.08	.507
.09	.337
.10	.201
.11	.104
.12	.055

for n = 266, c = 19;

p'	$P(x \leq 19) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.994
.05	.948
.06	.815
.07	.595
.08	.364
.09	.183
.10	.079
.11	.029
.12	.010

for n = 196, c = 17;

p'	$P(x \leq 17) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.999
.05	.989
.06	.945
.07	.837
.08	.588
.09	.503
.10	.329
.11	.194
.12	.103

for n = 109, c = 9;

p'	$P(x \leq 9) = L(p')$
.01	1.000
.02	1.000
.03	.993
.04	.986
.05	.948
.06	.874
.07	.762
.09	.482
.11	.243
.13	.101

for n = 139, c = 11;

p'	$P(x \leq 11) = L(p')$
.01	1.000
.02	1.000
.03	.999
.04	.988
.05	.949
.06	
.07	.731
.09	.405
.11	.139
.13	.052

for n = 217, c = 16;

p'	$P(x \leq 16) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.992
.05	.949
.07	.643
.09	.254
.11	.036
.12	.016
.13	.010

for n = 164, c = 15;

p'	$P(x \leq 15) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.999
.05	.990
.07	.879
.09	.593
.11	.284
.13	.100

for n = 793, c = 17;

p'	$P(x \leq 17) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.999
.05	.990
.07	.860
.09	.529
.11	.214
.13	.053

for n = 282, c = 23;

p'	$P(x \leq 23) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	1.000
.05	.990
.07	.803
.09	.365
.11	.085
.13	.011

for n = 94, c = 8;

p'	$P(x \leq 8) = L(p')$
.01	1.000
.02	1.000
.03	.998
.04	.985
.05	.950
.06	.882
.07	.785
.08	.659
.09	.529
.10	.405
.12	.209
.14	.096

for n = 109, c = 9;

p'	$P(x \leq 9) = L(p')$
.01	1.000
.02	1.000
.03	.993
.04	.986
.05	.948
.06	.874
.07	.762
.09	.482
.11	.243
.12	.101
.14	.062

for n = 170, c = 13;

p'	$P(x \leq 15) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.999
.05	.989
.06	.954
.08	.748
.10	.419
.12	.111
.14	.050

for n = 136, c = 13;

p'	$P(x \leq 19) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.999
.05	.990
.06	.947
.08	.671
.10	.292
.12	.080
.14	.012

for n = 165, c = 15;

p'	$P(x \leq 15) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.999
.05	.989
.06	.954
.08	.748
.10	.419
.12	.111
.14	.050

for n = 222, c = 19;

p'	$P(x \leq 19) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.999
.05	.990
.06	.947
.08	.671
.10	.292
.11	.161
.12	.080
.14	.012

for n = 79, c = 7;

p'	$P(x \leq 7) = L(p')$
.01	1.000
.02	.999
.03	.997
.04	.986
.05	.952
.07	.801
.09	.552
.11	.374
.13	.198
.15	.097

for n = 94, c = 8;

p'	$P(x \leq 8) = L(p')$
.01	1.000
.02	1.000
.03	.998
.04	.985
.05	.950
.07	.785
.09	.529
.10	.405
.12	.209
.14	.096
.15	.058

for n = 139, c = 11;

p'	$P(x \leq 11) = L(p')$
.01	1.000
.02	1.000
.03	.999
.04	.988
.05	.949
.07	.731
.09	.405
.11	.139
.13	.052
.15	.014

for n = 136, c = 13;

p'	$P(x \leq 13) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.999
.05	.990
.06	.961
.08	.814
.10	.507
.12	.251
.14	.097
.15	.056

for n = 40, c = 4;

p'	$P(x \leq 4) = L(p')$
.01	1.000
.02	.999
.03	.992
.04	.976
.05	.947
.07	.848
.10	.629
.14	.324
.16	.235
.18	.112
.20	.100

for n = 109, c = 11;

p'	$P(x \leq 11) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.998
.05	.990
.07	.913
.09	.718
.11	.463
.13	.252
.15	.108

for n = 193, c = 17;

p'	$P(x \leq 17) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.999
.05	.990
.07	.860
.09	.529
.11	.214
.13	.053
.15	.012

for n = 53, c = 5;

p'	$P(x \leq 5) = L(p')$
.01	1.000
.02	.999
.03	.994
.04	.978
.05	.947
.07	.829
.10	.563
.14	.251
.16	.150
.18	.089
.20	.048

for n = 80, c = 7;

p'	$P(x \leq 7) = L(p')$
.01	1.000
.02	1.000
.03	.997
.04	.983
.05	.949
.07	.797
.10	.453
.14	.129
.16	.060
.18	.027
.20	.010

for n = 58, c = 7;

p'	$P(x \leq 7) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.998
.05	.990
.07	.945
.10	.771
.14	.437
.16	.293
.18	.184
.20	.107

for n = 71, c = 8;

p'	$P(x \leq 8) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.998
.05	.990
.07	.944
.10	.716
.14	.340
.16	.202
.18	.111
.20	.055

for n = 96, c = 10;

p'	$P(x \leq 10) = L(p')$
.01	1.000
.02	1.000
.03	1.000
.04	.998
.05	.990
.07	.920
.10	.633
.14	.216
.16	.103
.18	.044
.20	.014

for n = 27, c = 3;

p'	$P(x \leq 3) = L(p')$
.02	1.000
.04	.976
.05	.951
.10	.714
.15	.423
.20	.213
.25	.097

for n = 38, c = 4;

p'	$P(x \leq 4) = L(p')$
.02	.999
.04	.980
.05	.954
.10	.668
.15	.327
.20	.125
.25	.040

for n = 53, c = 5;

p'	$P(x \leq 5) = L(p')$
.02	.999
.04	.981
.05	.947
.10	.562
.15	.205
.20	.045
.25	.010

for n = 37, c = 5;

p'	$P(x \leq 5) = L(p')$
.02	1.000
.04	.996
.05	.989
.10	.830
.15	.519
.20	.253
.25	.102

for n = 47, c = 6;

p'	$P(x \leq 6) = L(p')$
.02	1.000
.04	.997
.05	.989
.10	.804
.15	.442
.20	.173
.25	.050

for n = 70, c = 8

p'	$P(x \leq 7) = L(p')$
.02	1.000
.04	.998
.05	.990
.10	.729
.15	.279
.20	.062
.25	.101

Computations of n and c for N = 2000

When the lot size N equals 2000 items, the cumulative binomial distribution, equation 6, will be used to approximate the OC curve passing through the two preassigned points (p_1', $1-\alpha$) and (p_2', β). A reasonable approximation can be found by using tables of the cumulative binomial probability distribution (8).

For a given value of p_1', p_2', α and β, a value of n and c must be found to meet the specifications desired. This is done strictly by trial and error. To reduce the length of time involved in finding the n and c to meet the desired specifications, the maximum magnitude of n and c can be approximated by looking at the n and c for lots of size 5000 or greater. Then, all that is necessary is to work for the required n and c by trying decreasing values of n and c.

The results are summarized in Table 2. By holding p_1' constant at .05, results are given in terms of n and c for selected values of p_2', α and β. Selected values for p_2' are .10, .15, .20 and .25. Selected values for α are .05 and .01. Selected values for β are .10, .05 and .01.

Since n and c are integers, the OC curve may not pass through the two preassigned points exactly. Thus, the α' and β' in Table 2 indicate the actual values of the producer's risk α and the consumer's risk β respectively.

Table 2. Lot size N = 2000, sample size, n, and acceptance number, c, for values of p'_2, α and β while holding p'_1 constant at .05. Also given are the actual values of α and β, α' and β' respectively.

p'_2	α	β	n	c	α'	β'
.10	.05	.10	232	17	.048	.110
	.05	.05	285	20	.055	.056
	.05	.01	452	30	.048	.011
	.01	.10	348	27	.011	.094
	.01	.05	400	30	.011	.053
	.01	.01	600	43	.008	.010
.15	.05	.10	68	6	.053	.099
	.05	.05	94	8	.045	.046
	.05	.01	140	11	.049	.009
	.01	.10	100	10	.011	.100
	.01	.05	110	11	.009	.049
	.01	.01	160	14	.014	.014
.20	.05	.10	38	4	.039	.099
	.05	.05	50	5	.038	.049
	.05	.01	68	6	.053	.011
	.01	.10	50	6	.011	.104
	.01	.05	69	8	.008	.049
	.01	.01	88	9	.012	.012
.25	.05	.10	25	3	.056	.097
	.05	.05	28	3	.049	.055
	.05	.01	41	4	.048	.014
	.01	.10	30	4	.015	.098
	.01	.05	39	5	.012	.052
	.01	.01	60	7	.009	.009

Computations of n and c for N = 100

When the sample is drawn from a lot of finite size, the cumulative hypergeometric probability distribution, equation 4, is used to approximate the OC curve which is intended to pass through the two preassigned points (p'_1, 1-α) and (p'_2, β). The Lieberman-Owen Tables of the Hypergeometric Probability Distribution (9) were used to give a reasonable approximation to the OC curve.

Again, the process of computation for values of n and c meeting the desired specifications for α, β, p_1', and p_2' is a trial and error method. The n and c must be found for $p_1' = .05$ for which the probability of acceptance is $1-\alpha$, and for the same n and c the probability of acceptance for a selected value of p_2' is β.

The results are summarized in Table 3. By holding p_1' constant at .05, results are given in terms of n and c for selected values of p_2', α and β. Selected values for p_2' are .10, .15, .20 and .25. Selected values for α are .05 and .01. Selected values for β are .10, .05 and .01.

Due to the fact that n and c are integers, the OC curve may not exactly meet the intended specifications. The α' and β' in Table 3 indicate the actual values of the producer's risk α and the consumer's risk β respectively.

Table 3. Lot size N = 100, sample size, n, and acceptance number, c, for values of p'_2, α and β while holding p'_1 constant at .05. Also given are the actual values of α and β, α' and β' respectively.

p'_2	α	β	n	c	α'	β'
.10	.05	.10	100	5	.000	.000
	.05	.05	100	5	.000	.000
	.05	.01	100	5	.000	.000
	.01	.10	100	5	.000	.000
	.01	.05	100	5	.000	.000
	.01	.01	100	5	.000	.000
.15	.05	.10	37	3	.061	.115
	.05	.05	50	4	.029	.045
	.05	.01	57	4	.055	.011
	.01	.10	45	4	.017	.101
	.01	.05	100	5	.000	.000
	.01	.01	100	5	.000	.000
.20	.05	.10	22	2	.070	.123
	.05	.05	32	3	.040	.045
	.05	.01	37	3	.061	.018
	.01	.10	34	4	.006	.010
	.01	.05	38	4	.008	.052
	.01	.01	44	4	.014	.013
.25	.05	.10	19	2	.047	.087
	.05	.05	21	2	.061	.050
	.05	.01	33	3	.040	.007
	.01	.10	23	3	.010	.105
	.01	.05	26	3	.016	.052
	.01	.01	37	4	.007	.009

PROCEDURE FOR USING SAMPLING PLANS

When lots are ready to be inspected, the producer of the lot and the consumer should come to an agreement about the specifications involved. The producer and consumer should come to an agreement on the producer's risk α and the consumer's risk β. The probability of rejecting a good lot, a lot of quality $p_1' = .05$ or better, is the producer's risk. The probability of accepting a bad lot of quality p_2' or worse is the consumer's risk. Thus an agreement must be reached for the value of p_2'.

After the specifications have been agreed upon, the lot size should be noted. By using the appropriate table according to the lot size, a value of n and c can be found meeting the agreed upon specifications of α, β and p_2'. The n indicates the size of the random sample to be taken. The c indicates the acceptance number.

Thus for a lot of size N, a random sample of size n should be taken and the lot should be accepted if the number of defective items is less than or equal to the acceptance number c. If the number of defective items is greater than the acceptance number, the lot should be rejected. An item is considered acceptable if it is of said grade or better.

Consider the following example. A lot of size 2000 units is shipped to a consumer. The consumer and the producer of the lot agree to reinspection of the lot by using statistical sampling methods. They agree on a sampling plan that will not accept more than one percent of the time lots of quality ten percent defective or worse, but would accept at least 95 percent of the time lots of quality five percent defective or better. Thus, $p'_1 = .05$, $p'_2 = .10$, $\alpha = 1-.95 = .05$, $\beta = .01$. Since the lot is of size 2000, Table 2 should be used. To find the sample size, n, to be taken and the acceptance number, c, locate the value for $p'_2 = .10$ of $\alpha = .05$ and $\beta = .01$. The n and c are read off of the table directly as $n = 452$ and $c = 30$.

After indexing each individual piece of lumber from one to 2000, a random number table may be used to select 452 random numbers from the 2000 numbers corresponding to the 2000 individual pieces of lumber. The units to be in the sample are those individual pieces of lumber whose indices correspond to the 452 random numbers.

It may be difficult to pull out some of the pieces of lumber which are to be included in the sample, but it is necessary that a genuine effort be made to separate the units to be included in the sample from the rest of the lot. This will insure an unbiased sample (1, 175).

Each individual piece of lumber in the sample must be inspected to see whether it is acceptable or not. An individual piece of lumber is considered acceptable if it is of said grade or better, otherwise it is not acceptable or defective. If the number of defective pieces of lumber in the sample is less than or equal to $c = 30$, the lot is accepted. If the number of defective pieces of lumber in the sample is greater than 30, the lot is rejected.

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