INCOME CHANGES & LABOR FORCE PARTICIPATION

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BY GARY SORENSON

FOREWORD BY MURRAY WOLFSON

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FOREWORD

Income changes and Labor Forces Participation, by Gary W. Sorenson, is not only important as a contribution to the theoretical foundations of the workings of the labor market, but it will doubtlessly trigger off further studies of a statistical nature as attempts are made to use Professor Sorenson's operational categorization of the variables.

Sorenson's analysis is, in essence, an attempt to explain the effect of a change in wages upon the supply of labor. He shows how awkward the concept is of a person being either "in the labor force" or out of it. On one hand, a wage decrease discourages workers from giving up their leisure; at the same time, by reducing their income, it gives them an incentive to greater labor in order to maintain their income levels. The notion of leisure is misleading, the author argues, because it suggests that the alternative to working for wages is doing nothing.

To sort out the differences between a person who is "in the labor force" and one who is not, it is first necessary to consider the family as a productive unit in which by one manner or another each member contributes to the welfare of the group. This notion of group welfare and its relation to individual welfare is a sticky one, and Sorenson does not minimize the difficulties. Nonetheless, the point at issue is that the family welfare is not enlarged solely by working for money wages. Rather, since the process of consumption takes time and activity, each member of the family may contribute to its utility whether or not he is "in the labor force." That is to say, the traditional conception of consumption as a passive act of enjoyment of commodities which are literally "goods," must be amended to consider the family as a sort of productive enterprise even though the product is consumer satisfaction, utility.

To be sure, the product is apparently a thing of air, but the analysis is no less real or significant than the manufacturing of washtubs and teakettles. Indeed, this is a very important advance in itself, to show that consumption is comprehensible as an analog of production, with all the efficiency concepts transposed.

The notion of efficient consumers is not new with Sorenson—he explicitly builds on the previous literature—but its application to the labor market by others is recent, and Sorenson's treatment advances the inquiry further.

Another way of saying all this is that the mother who stays home and tends to family welfare is not nonproductive of national income. Sorenson's method derives "dual prices" which explicitly impute a value to her labor based on the changing technology of the process of consumption. He is getting at fundamental changes that have been taking place in family life as well as the labor market in the United States.

Part of the problem with preceding studies has been the use of traditional calculus methods of making the optimum use of the money income and time limitations facing every family. Sorenson points out that the formal process of finding a derivative and setting it equal to zero to maximize a function, requires that the derivative *exist* and therefore precludes solutions that will examine the "corners" or "kinks" of a function. The upshot of the traditional method is that everyone who is to be considered must do some work which is sold on the market—even if it is a little bit. If a person does *not* work for money wages, then he is *out* of the labor force and somehow appears not to be optimizing family welfare. The author uses the technique of nonlinear programming associated with Kuhn and Tucker in which optimization is carried out without these artificial requirements.

Sorenson's contribution is an extremely valuable one of clearing away the underbrush so that econometric work may take place in separating the counteracting forces in labor participation resulting from wage changes. His work is elegant and correct. To be sure, it is a monograph and not a treatise. Someday the empirical work has to be done to measure participation—and I hope Dr. Sorenson will do it. Nonetheless, he provides us with important insights and he brings to bear powerful tools of analysis in a field of economics that has long needed careful investigation.

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January 5, 1970

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INCOME CHANGES AND LABOR FORCE PARTICIPATION

This monograph is focused around the controversy in the economic literature regarding the effects of income changes on labor force participation. In particular, this controversy arises from the opposing hypotheses: the "additional worker" and the "discouraged worker." Each hypothesis predicts different behavior in response to falling income or rising unemployment. Empirical analyses have failed to terminate the controversy. This study attempts to clarify the issues involved through theoretical analysis and, in so doing, to provide additional theoretical restrictions that may be incorporated in the statistical testing problem.

In 1965, Gary Becker outlined a model of consumer behavior in his article "A Theory of the Allocation of Time" (*Economic Journal,* September). This model was adopted as the basic decision model. It was modified to incorporate a family, following Paul Samuelson's suggestion in his 1956 article "Social Indifference Curves" (*Quarterly Journal of Economics,* February).

The model was first approached by way of the calculus. The character of the model, however, turns out to be such that the calculus is inapplicable. The calculus is unable to assure the satisfaction of both an income and a time constraint; theoretically, it permits the substitution of time between individuals as well as the time of a single individual when time is segmented, and it also requires each individual to work some portion of each time period.

For these reasons, the analysis was restated as a nonlinear programming problem. In contrast to Becker's analysis, this formulation of the problem has work time as an explicit endogenous variable. For analytical reasons, the Kuhn-Tucker optimal conditions were stated and assumed to hold. For post-optimality analysis, the problem was then converted to a linear programming problem with the Kuhn-Tucker conditions as the constraints; the variables are the internal shadow prices which, in this case, are the marginal utilities of time for the family members and the family marginal utility of income.

For purposes of analyzing labor force participation behavior, the wage rate parameters were varied. Two hypotheses emerge and the basis for differentiating between them hinges upon three factors: (1) the manner in which the relevant marginal utilities have changed; (2) whether the optimal activity set has been altered; and (3) whether there may be a number of time-intensive and inferior activities in the optimal activity set at positive levels. One hypothesis is a clear statement of the additional worker position, while the other is, effectively, the discouraged worker thesis.

I. THE LABOR SUPPLY AND MICRO THEORY

A systematic theory of the labor supply of recognized import in economics, comparable to the theory of market demand, has long been conspicuously missing. The most obvious explanation for this is to be found in the complexity of the labor supply concept itself and in the unavoidable dynamics of labor market analysis. This is not to say that efforts have not been made in this area. Paul Douglas [*8*, pp. 269-272]* outlines the rudiments of labor supply theory in his classic, *The Theory of Wages.* At one point Douglas quotes the mercantilist, Thomas Manly, suggesting or implying a theory of 1669 vintage. Manly said, in relation to the effect of wages on the laborer's propensity to work, "the men have just so much the more to spend in tipple and remain now poorer than when their wages were less. . . . . They work so much the fewer days by how much more they exact in wages" [*8*, p. 270]. The implication is, of course, a negatively sloping supply curve—a proposition that can be arrived at through more rigorous analysis. Douglas continues to point out the mercantilist belief that the labor supply curve is not only negatively sloping but of unit elasticity with respect to wages.

A more precise statement of a labor supply theory began to emerge in the neoclassical era. Alfred Marshall, in the tradition of Jevons, speaks of the "discommodities" of labor and the "marginal disutility" of labor [cf., *21*, pp. 140-141]. Due to the increasing marginal disutility of labor, Marshall and Jevons rationalized a rising supply price for additional labor, i.e., a positively sloped supply curve with equilibrium at the point of equality between the ratio of the marginal utility of income to the marginal disutility of labor and the wage rate. However, by 1921 the theory here was not firmly established, and Frank Knight says in regard to the supply curve of labor:

> It is usual, because superficially "natural" to assume that a man will work more, i.e., work harder or more hours per day—for a higher wage than for a lower one. But a little examination will show that this assumption is for rational behavior incorrect. In so far as men act rationally, i.e., from fixed motives subject to the law of diminishing utility—they will at a higher rate divide their time between wage-earning

*Italic numbers in brackets refer to References Cited, page 63.

> and non-industrial uses in such a way as to earn more money, indeed, but to work fewer hours. Just where the balance will be struck depends upon the shape of the curve of comparison between money (representing the group of things purchasable with money) and leisure (representing all non-pecuniary, alternative uses of time). We therefore draw our momentary supply line in terms of price with some downward slope [*16*, p. 117].

J. R. Hicks, speaking of the question of the negatively sloping supply curve of labor in 1932, says "there is no logical justification for this view" [*15*, p. 98]. He goes on to suggest[1] that the expenditure of income is a matter of habit with many expenses of a prearranged nature. The use of leisure, on the other hand, is much less a matter of habit. Regarding leisure there are fewer commitments and, says Hicks, "if the work done becomes less remunerative, it is easier to sacrifice leisure than to sacrifice income" [*15*, p. 98]. It is particularly interesting to note two things in Hicks' writing on this matter. First, he continues to accept the asserted dichotomy between working for pay (labor) and nonwork-oriented activity (leisure). While there is an apparent dichotomy in the time allocation of individuals and, as a matter of definition complete freedom in choice of language, the labor-leisure references carry the Marshallian-Jevons inspired connotation with regard to marginal (dis)utility, and this writer will challenge the necessity for such a connotation below. Second, it is interesting to note Hicks' recognition of nonphysiological, institutional restrictions on the labor supply decision—an obvious appreciation of the complexity involved in explaining, theoretically or otherwise, the determinants of the labor supply relation.

Currently, intermediate and advanced courses in micro economics frequently consider analyses of the individual labor supply schedule couched in the language and framework of neoclassical marginalism. Assuming the increasing marginal disutility of labor and the diminishing marginal utility of income, it is possible to specify the conditions under which the labor supply schedule of the individual will be either positive or negatively sloping. Two examples of this are provided by Lloyd [*20*, pp. 31-34] and Henderson and Quandt [*14*, pp. 23-24].

James Duesenberry, agreeing with Clarence Long, says:

[1] And to take issue with Lionel Robbins who, in 1930, wrote of a negatively sloping supply curve. See *Economica*, June 1930.

> The effective labor supply at any moment depends on four factors: (1) the size of the population and its composition by age, sex, ethnic origin, and geographical location; (2) labor-force participation rates for each group in the population; (3) the occupational distribution of the labor force (which influences productivity); and (4) working hours [*9*, p. 309].

He goes on to suggest that these factors are dynamic and constantly changing and "a very large part of the change must be attributed to factors which are largely independent of the balance between supply of and demand for labor" [*9*, p. 309].

Stanley Lebergott allocated a chapter of *Manpower and Economic Growth* to a discussion of the labor supply concept. In prefacing that discussion, Lebergott says, "The structure and dimensions of the labor supply are fixed by economic forces, forces that operate within political and social limits" [*19*, p. 29]. While less specific than the Duesenberry statement, the character of complexity of labor supply analysis is, again, accented. One thing is clear from this brief review: the labor force and the labor supply are not synonymous; this was emphasized earlier by Durand in 1948. Durand said:

> The supply of labor, as understood by economists, is not the same as the labor force. Labor supply is not defined as a number of workers but as a quantity of work offered in the market, taking account the number of hours which each individual is willing to work if not other variables such as efficiency and intensity of effort [*10*, p. 86].

The economist's interest and focus on the size and variation of the labor force, as distinct from the labor supply, can be attributed to the depression of the 1930's and the birth of Keynesian economics. The impact of mass unemployment on the American psyche in conjunction with the convincing denial by Keynes in 1936 that automatic market adjustment to full employment could be relied upon in the short run, stressed the requirement for improved knowledge in this area. The directions that research efforts have taken here have been quite varied. Much work has been of a strictly descriptive nature; fitting this description is John Durand's work referred to above as well as most of the work done by Gertrude Bancroft [*2*], Seymour Wolfbein [28], and even Stanley Lebergott, also referred to above. These efforts have emphasized the changing demographic

as well as economic characteristics of the labor force. While the list of research efforts bearing on the size and quality of the labor force is lengthy, the number dealing specifically with the cyclical effects of unemployment and income changes on the labor force is relatively few. Since the present project is concerned with this area, the most prominent of the latter group will be considered in some detail. Before doing this, however, a few more introductory observations are in order.

While no mention was made of this, labor supply theory in the mercantilist, classical, and neoclassical eras (what little there was) was concerned with the secular determination of labor supply. The theory attempted to relate wages—real wages in particular—to the labor supply. The population and the labor force could not be regarded as synonymous; however, the relation was regarded as close, and variation in the ratio of the latter to the former was not recognized as a problem. In speaking of this point, Philip Hauser says:

> Labor mobility in respect to labor force participation is a relatively recent phenomenon in human history. As a concept it is applicable to a mass population only in our modern urban industrial society. Consideration of changes in the population of "workers" as distinct from changes in the population in general presupposes a society in which "work" is differentiated from other activities such as play, education, worship, or courtship. In a preindustrial, folk type of society in which work is not sharply differentiated from other types of life activity, it is not possible to consider entrance to, and exit from, a working population [*1*, p. 8].

The United States economy of the 1930's (not to speak of the present character of the economy) was of the industrial type described by Hauser and distinguishes between work and other activities. In the latter years of the depression decade, serious empirical work was being done on the cyclical determinants of the labor force fluctuation.

In the late 1930's, W. Woytinsky was conducting research on the effects of unemployment on labor force participation. World War II diverted America's concern with unemployment and misery to more pressing matters. But upon returning to peacetime problems in 1945, the United States Congress saw fit to reflect on the

tragedy of the 1930's; the fruit of their efforts was the Employment Act of 1946. Displaying an awareness, if not the knowledge, of Keynesian analysis, the Employment Act of 1946 formally recognized the responsibility of the federal government to, among other things, "promote maximum employment, production, and purchasing power." Although it has swung in pendulum fashion, concern over maintaining full employment has not been reduced at the federal level of government. To the contrary, a general awareness of the effects of technological change and automation on the demand for labor has generated, since about 1961, an approach known as "manpower policy"—a policy explicitly recognizing the role of the supply side of labor market equations in affecting the equilibrium of this market. In 1964, the Senate Subcommittee on Employment and Manpower [*27*] formally recommended the merging of our 1946 type, Keynesian-oriented employment policy with the newer manpower policy in order to deal adequately with labor market developments. If only on a *de facto* basis, this wedding has probably been, or is being, achieved; at least present government policy seems to reflect recognition of both the forces of supply and demand.

From an economist's point of view the most pressing questions, given the federal government's concern for full employment and the general agreement upon the derived nature of the demand for labor, still pertain to the determinants of the supply of labor and, perhaps most importantly, to the cyclical determinants. Since Woytinsky's pioneering efforts, there have been numerous scholarly attempts at resolving these questions. Much attention has been given to the effect of unemployment on the labor force participation rate since unemployment affects income and hence, by way of conventional demand theory, affects budget constraints and levels of utility or welfare. From such effects we would expect responses by individuals at the micro level and by the population at the macro level; such responses should be reflected in labor markets if the price system has an influence. It will be contended below that existing responses to questions in this particular area have not resolved the issues. It will further be contended that our continuing relative ignorance in this area is due to two factors. First, earlier efforts to answer questions in this area have continually failed to adequately specify the set of micro relations that must exist in order to explain labor market behavior or predict responses to changing parameters. Second, there is the ever-present problem that, even given an "adequate"

set of micro relations, our existing body of data preclude careful testing of the hypotheses. With these observations in mind, the overriding, but limited, objectives of the present analysis can now be outlined.

In the analysis below, Chapter II will present a summary outline of the major existing attempts to respond to the question of the effects of income changes on labor force participation. In Chapter III, since whatever micro relations exist must arise out of a consumer-oriented model, brief mention will be made of two recent advances in the area of consumer theory. The question of the relevant micro unit will also be raised. In Chapter IV, two approaches to a model of household behavior will be developed. The first will emphasize the consumption aspects of the household, and the calculus will be relied on as the major analytical tool; the second model will be a nonlinear programming model which emphasizes the time allocation aspects of the household. Chapter V will review the hypotheses and implications of the theoretical analysis.

II. A SHORT SURVEY OF RELEVANT EMPIRICAL ANALYSES

The list of analyses which have endeavored to account for the effects of income changes on labor force participation is not extensive. Woytinsky led in 1940, and, by making certain explicit hypotheses about the character of this relationship, provided a basis for controversy which has been taken up by a handful of economists. All the work in this area has not been limited to the effects of income changes; other factors affecting work-oriented activity have been considered. In particular, the effects of demographic changes in the population on the labor force as well as the factors which might affect preferences and the effect on demand of changing technology have been considered. These factors are generally regarded as changing secularly and do not strongly influence the making of short-run employment policy; income and employment changes do influence cyclical variation. Also, most analyses discuss the effects of unemployment on participation; the reason for this emphasis is the well recognized close relationship between unemployment and the flow of income and the ready availability of aggregate labor force data. In this section the methodology and hypotheses of six such studies will be reviewed and criticized. The objective is to attempt to discern why the relevant question here (viz., what is the effect of income and/or employment changes on labor force participation) remains interesting.

In 1940, Woytinsky, in a pamphlet [*29*], expressed his so-called "additional worker" hypothesis. In 1942, after a polemical bout with Don Humphrey in the *Journal of Political Economy* Woytinsky reaffirmed his conviction that in depressions family members who otherwise would be "out of the labor force" are moved into active participation because of unemployment experienced by primary breadwinners. More completely, he says:

> The additional workers who are in the labor force because the usual earners in their families are unemployed may be described as "additional depression workers" provided that the term "depression" is not restricted to the lowest point of the business cycle. Local and partial slumps, as well as technological changes resulting in layoffs, are likely to add new jobseekers to the ranks of the unemployed even if the overall business conditions prevailing at the time are gener-

> ally good. Consequently, "additional depression workers" may be present on the labor market wherever and whenever some usual workers are out of work, but they are naturally most numerous during depressions [*30*, p. 106].

The significance of this proposition to Woytinsky lay in the fact that, if true, the number of jobs necessary to reduce unemployment to a minimum "may be appreciably less than the reported volume of unemployment would suggest" [*30*, p. 106]—an observation that was equally meaningful in the early 1960's.

The aggregate labor force relation is suggested by Woytinsky to be of the form, $S_L = S_L[f(t), g(s), h(c)]$. In this relation $f(t)$ is the trend relation and is affected by changing population and demographic factors; this portion of S_L changes slowly and predictably. The component $g(s)$ is the seasonal portion of S_L and implies recognition of the influence of demand; the impact on unemployment is not suggested as serious. Demand, of course, has a primary influence on the component $h(c)$, the cyclical portion of the relation; it is $h(c)$ that reflects the movement of additional workers due to changing business conditions. It was this hypothesized relation, $h(c)$, that Woytinsky was primarily concerned with because of its policy implications.[1]

To test his hypothesis about additional workers, Woytinsky began by defining total unemployment as consisting of the sum of "primary" and "secondary" unemployment. The former referred to "unemployment due to an inadequate number of jobs for usual gainful workers," while the latter was "unemployment accounted for by new job seekers from families in which the usual earners are unemployed," and it was this secondary or additional group that he wanted to measure. The "gainful worker" reference pertains to a concept employed by census enumerators prior to 1940. According to Seymour Wolfbein, prior to that year "census enumerators recorded the occupation of every person ten or more years of age who said that he or she followed an occupation in which money or its equivalent was earned. A gainful worker was one who reported a gainful occupation." For more information on this point see Wolfbein [*28*, pp. 17-18].

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[1] The notation here must be regarded as the author's interpretation of Woytinsky's exposition on this point. For Woytinsky's treatment cf. [*30*, pp. 110-111].

For these terms to be operationally meaningful, direct historical family data would have been required. In the absence of this data, Woytinsky attempted to use indirect cross section data in his analysis. That is, he attempted to infer from the composition of the labor force the number of "additional" workers. Given this task, he found it necessary to assume statistical independence between the chance of unemployment in a household and the number of usual earners. The purpose of this assumption was to afford the opportunity of saying something about the number of workers in a household and the incidence of unemployment. According to Woytinsky:

> The exact number of workers these families would have had if their usual earners were employed is, of course, unknown, but it may be assumed with rough approximation that, except for additional workers, there would be no substantial difference between the structure of these families and of those without unemployed members [*30,* p. 116].

Woytinsky's data appeared to support his additional worker hypothesis.

However, in 1958 Clarence Long expressed his suspicion of the Woytinsky hypothesis. In his monumental study, [*21*], Long found the additional worker hypothesis untenable with the census data. While agreeing that there are surely some workers fitting the additional worker description in conditions of rising unemployment, he said that the net effect is to discourage more prospective workers than are encouraged. Long said that this makes more sense than ever when the effect of unemployment is regarded in conjunction with the diminished real income per worker in depressions.

In methodology, Long relied extensively on his statistical analysis. Several assumed economic relations were suggested and then analyzed statistically. Among these suggested relations were the following: (1) What is the statistical relation between labor force participation and earnings? Straight correlation analysis by census year and for thirty-eight large cities was employed, and the relation was found to be inverse. (2) What is the effect of depression (unemployment) on labor force participation? This too was found to be an inverse relation. Other relations between economic or demographic factors and participation along specific

age-sex lines were also investigated. Statistical conclusions were offered as *prima facie* evidence of explanation.

Long's analysis was a contribution to the study of the labor supply relation in that it recognized the multiplicity of factors affecting this relation. Long did not, however, sufficiently detail the characteristics of a person who would likely be discouraged by rising unemployment or by falling income. He tended to regard the net observable effects as sufficient to confirm his hypothesis after deciding that a particular point in time was characterized by "depression."[2]

On the basis of casual empiricism and introspective answers to hypothetical questions, both the additional worker and the discouraged worker hypotheses seem plausible. Rather than close the door on the question of the effect of unemployment on labor force participation, Long's study merely whetted the interests of many people, and several interesting studies have been published since 1958.

For example, in 1961 W. Lee Hansen published *The Cyclical Sensitivity of the Labor Supply* [*13*, pp. 299-309]. Realizing the shortcoming of using net labor force data, as published by the Bureau of Labor Statistics on a monthly basis, to test hypotheses of the type discussed above, Hansen employed the so-called gross change data generated from the monthly *Current Population Survey*. These data have not been regularly published since 1953 because of some built-in bias they involve [cf., *23*]; however, the data do emphasize the high degree of labor force mobility displayed by the adult civilian population and also allow a finer breakdown in the characteristics of "movers." For example, these data recognize that unemployment in any month equals unemployment in the previous month plus the new disemployed, plus additions to the labor force from keeping house, in school, or the "other" categories, minus the reemployed, minus reductions from

[2]It is interesting to note in regard to the 1940 census data, that Woytinsky attributed the declining overall participation rate to a state of "balanced prosperity" in the economy. Long retaliated that the data showed eight million people either seeking work or on work relief and that this was 15 percent of the labor force. Long said 1940 was a depression year; he attributed the falling participation rate to the discouragement effect. See Long [*21*, pp. 197-198].

the labor force consisting of people going back into the three categories mentioned as additions. This is strictly an operational definition and the data allow such categorization. Thus, these data permit one to interpret, e.g., a net rise in unemployment as arising from an increase in disemployment or from additions to the labor force from the nonlabor force categories or some combination of the two.

In methodology, Hansen started by adopting the peak-trough months of the 1948-49, 1953-54, and 1957-58 business cycles (defined by the National Bureau of Economic Research) as reference points and then tabularized the percentage of the civilian labor force which each of the above-mentioned quantities consisted of at each of the months. Then Hansen asks, "What kind of impact do gross change movements exert on the level of unemployment and what is the significance of these movements for the hypotheses under consideration?"[3]

In answer, Hansen charts his data month by month (as opposed to just peak-trough months) and compares the pattern of unemployment with the patterns of additions to and reductions from the labor force. From this he concludes:

> The combined evidence on both additions to and reductions in unemployment where movement is to or from outside the labor force suggests that Woytinsky's hypothesis—that the influx of additional work-seekers raises the unemployment level in periods of high or rising unemployment—is not supported by the data for much of the postwar period. On the contrary, whatever increases in additions did occur were effectively cancelled out by increases in reductions, in precisely the manner suggested by Long's hypothesis. Consequently, the Woytinsky hypothesis must be rejected while Long's is supported by the data [*13*, p. 304].

Hansen's concentration on the two-way movement and use of gross change data must be recognized as a contribution to the literature in this area. But on completion, the Hansen analysis

[3]The hypotheses he refers to are the "additional" and "discouraged" worker hypotheses. As for his question, it seems the obverse question would be more relevant, i.e., what impact does unemployment have on the *pattern* of gross change movements? [*11*, p. 303].

could be consistent with several interpretations. The source of this uncertainty lies with the lack of a clearly stated theoretical base. Put another way, Hansen, like Long and Woytinsky, was more concerned with accounting for the data than with the behavior which generated the data; this approach leaves open the question of what was "explained."

In the November 1964 *Review of Economics and Statistics,* Kenneth Strand and Thomas Dernberg published a study of the "Cyclical Variation in Civilian Labor Force Participation." The initial objective of their study was to close the door on the Woytinsky-Long controversy. Strand and Dernberg employed multiple regression analysis in attempting to relate "demand" (the ratio of employment, E, to civilian population, P) and exhaustions (the ratio of new unemployment compensation exhaustions, X, to P) to the aggregate labor force participation rate, L/P. The signs on the coefficients were "presumed" [*25*, p. 380] and they utilized time series data over 186 months. After a "number of trials" they emerged with positive and significant coefficients for $(E/P)_t$ and $(X/P)_{t+2}$.

These results were interpreted as being consistent with *both* the Woytinsky and Long hypotheses. As demand falls, participation declines, but this leads to an increase in exhaustions in the future thus "encouraging" additional work seekers to enter the labor force; hence, both Long and Woytinsky are vindicated in part.

William G. Bowen and T. A. Finegan, in conjunction with the University of California's four-year study of unemployment and the American economy, were the next to publish comments in this area. The Bowen and Finegan paper appeared as a chapter, entitled "Labor Forcc Participation and Unemployment," in a collection edited by Arthur Ross [*4*, pp. 115-161].

As their title indicates, and like the previously described studies, their primary interest was with the effect of unemployment on labor force participation. But, Bowen and Finegan were more ambitious than this. Specifically, their objectives were:

(1) To estimate the effects of differences in local labor market conditions on the labor force participation rates of various subsets of the population—teen-age males, teen-age females, "prime age" males, married

women, and older males being the five main groups studied.

(2) To see whether this relation has changed over time.

(3) To learn more about the effects on labor force participation of factors other than unemployment—factors such as educational attainments, earnings opportunities, other income (nonlabor), and color [*4*, pp. 115-116].

Single-equation, multiple-regression techniques were used by Bowen and Finegan on cross section data for metropolitan areas derived from the 1940, 1950, and 1960 decennial censuses. The dependent variable in each of their equations was the relevant labor force participation rate. But, the independent variables were not quite so obvious to come by. They said in this regard:

> The main considerations which we would expect, a priori, to determine whether or not an individual participates in the labor force can be grouped conveniently under three familiar headings: "Tastes," "Affluence," and "Incentives" [*4*, p. 118].

Tastes were recognized as unmeasurable and, therefore, taken as given. However, affluence was interpreted in "net" terms; this was done by adjusting the gross financial position of the individual or family for the number of family members or for age. Hence, for example, reductions in net affluence due to unemployment of primary breadwinners may encourage additional workers to enter the labor force. Their motivational force, "incentives," is also put in net terms. For example, the expected monetary wage rate for working must be placed in juxtaposition with the monetary and nonmonetary costs of going to work. These latter costs are influenced by such things as the number of young children in a family or, perhaps, the loss of welfare payments or the low expected likelihood of work (i.e., the discouragement effect).

Except for "unemployment" and "schooling," the list of independent variables adopted for each of the five[4] age-sex categories

[4]These categories were (1) males, ages 14-19, (2) males, ages 25-54, (3) males, age 65 and over, (4) single females, ages 14-19, and (5) married women; see [*4*, pp. 123-125].

selected for analysis varied somewhat. Regarding unemployment, it is interesting to note that the sign of the coefficient for this variable, when regressed against the relevant labor force participation rate, was negative (and significant in most cases) in each of the five groups for the three census years. The obvious conclusion on the part of Bowen and Finegan was, of course, that rising unemployment "discourages" labor force participation. The authors went one step further here. They interpreted this obversed inverse relationship between labor force participation and unemployment as evidence that rising or chronic periods of unemployment contribute to what has come to be called "hidden" unemployment.[5]

Woytinsky was not unrepresented in the Bowen and Finegan equations, however. Each regression equation included one or more variables reflecting income. In the case of "older" males, income was reflected by median income the preceding year of all males who worked for fifty to fifty-two weeks and by "other" nonwage income. For married women similar variables were employed. In each of these cases the coefficient signs were negative, suggesting that the loss of family income tended to encourage participation and support the additional worker hypothesis. The conclusion, then, agrees in part with both Woytinsky and Long.

While not in our chronological sequence, another labor force study must be mentioned. In 1958, Richard N. Rosett published an analysis entitled "Working Wives: An Econometric Study" [*6*]. Rosett's broad objective was to "determine whether the decision of a wife to enter the labor force is influenced by the demographic and economic characteristics of the household of which she is a member." The independent variables Rosett considered included the husband's wage rate, the wife's wage rate, changes in debt holdings, presence and age of children, and education. He used data from the 1954 *Survey of Consumer Finances* to infer something of the effect of the independent variables listed above on the labor force participation of wives.

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[5] More will be said of hidden unemployment presently. However, as it is currently used, the term refers to some unknown number of persons who prematurely withdraw from the labor force due to low expectations toward employment opportunities and is interpreted as an involuntary predicament for persons in this group. For more information on this see [*26*].

Ordinary single-equation, multiple-regression techniques were employed by Rosett in his statistical analysis, but the signs of the coefficients were anticipated by developing sign conditions which emerged from an economic optimization model, and this was the novel aspect of Rosett's study. He assumed that the basic decision-making unit was the household, consisting of two or more individuals, but, in the event of more than two, the labor force decision applied only to the husband and wife. The household was assumed to maximize a conventional ordinal utility function, $U = U(Y, P)$, where Y = income and P = labor force participation of the wife (the husband's participation rate was assumed to be 1). The budget constraint for the household was $Y = H + R + WP$, where H = husband's wage rate, R = property income, and W = wife's wage rate.

While Rosett said $U(Y, P)$ applied for the spending unit [*6*, p. 62], he clearly was not implying that he had solved the community indifference curve problem. Rather, since the husband's participation was assumed to be full time and H and R are assumed constant, the problem becomes one in which the wife chooses P such that $U(Y, P)$ is maximized.

Rosett's procedure from here was straightforward, except for some confusion on the curvature of his indifference curves [*6*, p. 91]. He developed first and second-order conditions for utility maximization; then he developed the Slutsky equations for the effect of changes in H and R, and then posited certain things which might affect U. This procedure is commendable and, what is more, it offers the opportunity to generate answers about the effect of other things on labor force behavior.

This concludes summary descriptions of the major studies which have concentrated upon the cyclical effects of unemployment on the labor force. But, for several reasons, an interest in the topic has not been concluded because the question has not yet convincingly been answered. Perhaps this is explained by the multiplicity of statistical analyses and approaches that have been performed. Such multiplicity hints of a variety of possible theoretical hypotheses, and yet little has been done to supplement or replace the usual income-leisure analysis upon which most statistical expectations are based. In the analysis to follow, we will be concerned with the development of a theoretical model and the derivation of implications regarding the labor supply relation.

Before this, however, the next chapter will outline two recent theoretical contributions to consumer theory, and the possibility of departing from the assumption of the individual as the micro unit will be considered.

III. NEW DIRECTIONS

The standard approach to developing a labor supply relation is by way of a consumer-oriented micro model in which a fixed time period is allocated between "labor" and "leisure" in a manner that maximizes utility; the analysis assumes a positive price for time. Straightforward examples of this analysis are provided in [*14*, pp. 23-24 and *20*, pp. 31-34]. This analysis has been subject to sharp criticism on the grounds that it requires a person to "consume" both income and leisure to gain utility and it requires that all consumers work, i.e., interior solutions are demanded by the analysis. Two recent and imaginative contributions to the theory of consumer behavior promise to alter the general treatment of this area; one of these offers immediate application to the problem at hand. In this chapter, these recent works will be reviewed in order to establish familiarity with the approach to be adopted in the analysis to follow. Following this, the question of the relevant micro unit will be raised.

Two Recent Developments in Consumer Theory

To the noneconomist the theory of consumer behavior that has dominated the economic literature for the last 100 years must undoubtedly appear overly restrictive in its assumptions and grossly expensive in analysis, given the limited number of meaningful theorems it generates. This writer has heard physical scientists scoff at the idea of individuals obtaining "utility" directly by consuming market goods. It is certain that a similar response would follow the careful illustration that the theory requires in its standard-treatment, interior (i.e., positive) solutions in commodity space. Yet, we should not minimize the importance of the hypothesis of negative substitution effects. Nor should we impune the originators and purveyors of the theory, as major theoretical advances seem to depend intimately on the development of new analytical techniques, and the calculus has been *the* major tool in most of the last century. But, new analytical techniques have been forthcoming, and scholars are probing the intellectual frontiers of this field. Two recent and structurally similar advances in the area of consumer theory were provided in 1965 by Gary Becker [*3*] and in 1966 by Kelvin Lancaster [*18*]. By preference and for continuity, the latter's work will be discussed first.

The major shortcomings of the conventional consumer theory to which Lancaster responds pertain to its total rigidity with respect to the introduction of new commodities–a continuing phenomenon in a dynamic economy-and its inflexible definition of consistent behavior, namely, its inability to handle intrinsic properties in commodities and hence its prohibition on rational individuals from consuming one good at one time and another good at a different time. In relation to this latter shortcoming, Lancaster says:

> Thus, the only property which the theory can build on is the property shared by all goods, which is simply that they are goods [*18*, p. 132].

After recognizing that his approach is not completely original but has been suggested in part and implicitly or explicitly elsewhere, Lancaster develops a model departing from the conventional in several important respects. First, he suggests that individuals do not consume market goods, x_j, directly. Rather, market goods are combined through a "technology" matrix to yield consumption "activities," y_k. These activities are transformed through another matrix to yield "characteristic" vectors, z_i. It is the z_i that people consume directly and, via a utility function, $U(Z)$, on which they attempt to maximize their utility. In vector and matrix form, Lancaster's model is expressed as:

$$\begin{array}{ll} \text{Maximize} & U(Z) \\ \text{s.t.} & px=k \\ \text{with} & By = z \\ & Ay = x \end{array}$$

$$x,y,z \geq 0, \quad [18, \text{p. } 136].$$

Recognizing the computational difficulties involved in such a (nonlinear programming) model, Lancaster makes some simplifying assumptions regarding the number of elements in x, y, and z, and regarding the character and rank of A and B, and proceeds to build his model within a linear programming framework. After considering the effect of price changes and efficient choices, he proceeds to the labor-leisure analysis.

Labor, or work activities, is treated as it would be in an input-output model, i.e., as entering the model with a negative sign and hence using up characteristics. The analogy here in conventional theory is the treatment of labor so as to yield negative marginal utility. Again, in a simplified version of the model, it is shown that the wage rate determines the efficient combinations of work and consumption activities, given other prices and the technologies.

This description need not be carried further. Although the full implication of Lancaster's model has not been realized, it seems clear that it will influence the treatment of conventional consumption theory. The approach does offer relief from some historically binding restrictions in this area of theory and potentially offers to yield meaningful hypotheses if some refinement can be achieved in the definition of a "characteristic" or the determination of the relevant technology matrixes can be realized.

Gary Becker may have developed a more useful model to the extent that it relies on one less transformation than does Lancaster's, is not hindered by the task of defining characteristics, and, most important for the present analysis, it includes time as a resource explicitly in the model without regarding work as yielding negative marginal utility. Although much similarity exists between the Becker and Lancaster models, it is because of these last three differences that a modification of the Becker analysis will be developed below. But first, what is the Becker model?

Becker's break with conventional theory is along two lines. First, he attempts to show, contrary to the usual treatment, that production and consumption are integrated at the level of the micro unit. Individuals or households are actually "small factories" in that they combine resources (raw material, capital goods, labor, etc.) to produce useful commodities. Second, in this production-consumption process, the micro unit's primary resource is time in that time is involved in both sides of this process as well as imposing an absolute stock constraint on the process, and hence the cost of time should be recognized explicitly in our theory. That is, the opportunity cost of time must affect the terms of trade between production and consumption and there-

fore between work and nonwork.[1] Since all of one's time is allocated either to consumption or to work, it is clear that anything that affects total consumption time will affect, residually, work time, i.e., in the aggregate, the supply of labor.

Formally, as stated in "A Theory of the Allocation of Time" [*3*, pp. 495-499], the Becker model has the following form: Individuals or households do not consume market commodities directly—at least these commodities do not directly enter the utility function. Rather, market goods and time are combined through a production function into commodities, $Z_i = f_i(x_i, T_i)$. The Z_i give rise to utility through the utility function, $U = U(Z_i \ldots Z_m)$. Becker talks in terms of the household and says it is their problem to maximize U subject to a resource constraint which is made up of both an income and a time constraint. Where marginal and average prices are equal, this constraint appears as, $\sum^{m} \pi_i Z_i = V + T\overline{w}$. Here, π_i is the sum of direct and indirect expenses per unit of Z_i, while T is the total time vector, $\overline{w}$ is the relevant wage vector, and V is the sum of "other" income. Given appropriate assumptions regarding U, the first-order conditions for a constrained maximum can be derived, and they are, in appearance, identical to the conventional conditions. Some modification is required when marginal and average prices are not equal.

Becker proceeds to discuss the various applications of the model, and he considers the effects of changes in other income, V, and in the wage vector, $\overline{w}$, first. Regarding changes in V, he points out that working time would vary inversely unless the commodity set contained a number of inferior goods (i.e., goods for which $\frac{\partial Z_i}{\partial V} < 0$) and these goods happened to be relatively time intensive. As for the effect of a change in $\overline{w}$, on time worked, Becker recognized that this depends on the relative strength of the

[1] The reference to nonwork in place of leisure is intentional. The latter term carries a positive connotation and implies the existence of a negative disposition of time, i.e., labor. The connotation is not desired. Given one's tastes for consumption activities, which use time and income, and the total available time, the level or scale of consumption actually achieved will depend, in large part, on a trade-off of time for income; no assumption regarding the character of one disposition of time over another is required.

income and substitution effects, assuming that relative time intensities of commodities vary.[2]

In terms of its relation to the objectives of this analysis, further summary is not required. Without doubt Becker's model, like Lancaster's, constitutes a major contribution to micro theory and promises to yield operationally meaningful theorems. Its major positive aspects seem to be: (1) its recognition of the complex nature of consumption; (2) the explicit recognition of time opportunity costs as a relevant variable in all micro economic decisions; and (3) the ready adaptability of the model to a nonlinear programming format under a modified set of assumptions and definitions; this flexibility allows for correction of what this writer feels is an important shortcoming of the model in its present form.

As Becker developed the model in his 1965 article, the problem characterized may be described as a classical optimization problem, Such problems, while often yielding useful qualitative information and lending themselves to analysis with the calculus, have the shortcoming of requiring interior solutions in the domain set, i.e., the necessary conditions cannot prevail if a solution vector contains elements at zero. The (implicit) assumption of interior solutions has long been awkward to rationalize, but so long as the calculus was the primary tool of analysis little could be done to dispose of this anachronism. In the framework of nonlinear programming, however, no such assumption regarding the solution vector is required.

The Question of the Micro Unit

Two other problems arise in the Becker model in its original form. These pertain to the question of the relevant micro unit and the character of the dependence within the unit if it is larger than an individual. Becker, again, referred to the household, but then expressed the relevant utility function as, $U = U(Z_i \ldots Z_m)$. In this form, however, and without further qualification, the intimation is toward a family utility function of a very general form. It is so general, in fact, that many unanswer-

[2] This last point (i.e., the effect of $\bar{w}$) will receive considerable emphasis later in the analysis, and Becker's answer here will be criticized as invalid.

ed questions appear to have been sidestepped. Some of these questions relate to the assumed character of intrafamily relations for, it should be pointed out, whatever assumptions are made here bear heavily on our ability to derive analytical statements pertinent to the very core of this analysis, i.e., whether income changes brought about by, among other things, unemployment can be expected to lead to changes in the labor force participation rate. These problems will now be considered.

While noted economists like Debreu [*5*, p. 50] and Becker (noted above) have alluded to the "family" or "household" as the relevant micro unit, analysis of consumer demand has proceeded to consider the behavior of individuals. The reason for this, of course, lies in the knotty problem of making interpersonal comparisons of the utility which is involved in the assumption of a joint welfare function. Recall, now, in Richard Rosett's study of "Working Wives . . .," mentioned above, the model made use of a "family" utility function, $U = U(Y,P)$. However, the structure of the model made the function essentially the wife's. Yet, the family does, on the basis of casual empiricism, seem to be the appropriate micro unit. Sociologists are convinced of this as evidenced by a recent collection edited by Nye and Berardo [*22*]. One chapter, essentially an historical survey, discussed the "Economic Framework for Viewing the Family" [*22*, pp. 223-268]. Yet, the analytical problems remain. Fortunately, a partial solution has been suggested by Paul Samuelson.

Samuelson, in his 1956 article on "Social Indifference Curves" [*24*], while denying the possibility of such curves, admits that the "fundamental unit on the demand side is clearly the family." He goes on to recognize "discernible decentralization" in the family decision-making process, although the well-being of any single family member is related to the group. Thus, the family utility or social welfare function might be expressed as:

$$U = F[U_1(x_1 \ldots x_m), \ldots U_i \quad (x_1 \ldots x_m), \ldots U_n (x_1 \ldots x_m)]$$

$i = 1 \ldots n$ and assuming $\frac{\partial F}{\partial U_i} > 0$ for all i. It is recognized that if children are represented in F, then they must be rather "mature," i.e., they act rationally. This particular framework allows for what Samuelson calls quasi-independence in family decisions.

And the generality of the expression does seem to allow, through F, for joint or consensus decisions.

The problem is to maximize $U = F((U_1 \ldots . U_n)$ assuming F is continuous and $\frac{\partial F}{\partial U_i} > 0$ for all i and that there is a joint family income, I which constrains the problem. In this framework the primary joint task of the family is the intrafamily allocation of income. And Samuelson points out in this regard that arithmetic or arbitrary rules of income allocation are generally incompatible with the maximization of a function like F. Due to the very nature of the problem (and particularly to the assumption that $\frac{\partial F}{\partial U_i} > 0$), there is the implication that "income must always be reallocated among the members of our family society so as to keep the 'marginal social significance of every dollar' equal" [*24*, p. 11]. That is, the ratio of the marginal utility of income (as derived through family consensus) for family members must equal unity. Any departure from this implies that income can be reallocated such that a dollar spent by some family member will increase family utility more at the margin than is lost by taking a dollar from some other member. What this points up, according to Samuelson [cf., *24*, pp. 13-14], is that (1) any distribution of income cannot be once-and-for-all; and (2) the original distribution must be determined with the final equilibrium configuration in mind. Samuelson says this calls for a system providing for lump-sum intrafamily transfer of income, i.e., if F is to be maximized, some functional relation must exist to provide for an income distribution which satisfies the equality of marginal utility of the income condition mentioned above. Before proceeding, please note that, by itself, this last proposition *is not* a value judgment, but it is required by the character of the problem. The implications of this restriction will be considered in some detail below.

Given the analytical difficulties of handling social welfare functions, the framework just considered seems particularly appropriate for a starting point in analyses of the labor supply question. In this area the relevant decision-making unit does appear to be the family. Some of the recent labor economics texts, like Lester or Chamberlain, are paying lip service to this proposition. However, empirical studies

(like those alluded to above) of factors affecting the aggregate labor supply have either focused on the aggregate participation rate or, if they dealt with the separate components of the labor supply, proceeded as if each component were independent of any other. This seems a gross oversimplification and it is one of the primary objectives of the analysis below to take explicit, if partial, recognition of the intrafamily dependence of labor force participation decisions.

IV. THEORETICAL ANALYSIS

In this chapter we will draw upon Gary Becker's model, outlined above, in the hope of generating answers to the kind of questions in which we have indicated an interest. In particular, we hope to clarify the old controversy between the "encouraged worker" and "discouraged worker" hypotheses. As a first approach to this task, we will attempt to apply the calculus to the Becker model as modified to incorporate the family as the micro unit. The Becker analysis was expressed in terms of continuous functions and, for the purposes at hand, the implications of this assumption should be reviewed. Following this, the model will be approached from a programming perspective. In particular, a nonlinear programming model will be developed in an effort to derive possible empirical implications regarding the relation between time allocation and income changes.

Drawing upon the Samuelson analysis expressed in "Social Indifference Curves" [*24*], the Becker model can be applied to a family. A family will be defined as consisting of m individuals (and m may equal 1) bound together by any sort of legal or extralegal ties and attempting to maximize a family utility function. For simplicity of exposition in the analysis, we shall assume $m = 2$. The family ordinal utility function is,

$$U = F[U_1(Z_j), U_2(Z_j)], \; k = 1, 2. \tag{1}$$

As in Samuelson's suggestion outlined above, we recognize some degree of intrafamily decentralization in decision-making; therefore we assume $\frac{\partial F}{\partial U_k} > 0$ for all k (again, in this case k runs from 1 to 2 but in general runs to m).

The usual assumptions regarding U_k are: (1) U_k is a concave single valued increasing function over the domain of Z_j and $\frac{\partial^2 U_k}{\partial Z_j \partial Z_i} \neq 0$; and (2) $U_k \epsilon\, C^2$; that is, the U_k have continuous second partial derivatives with respect to Z_j. Without making any

restrictive assumptions on the character of F except that it is an increasing monotone transformation on the U_k, by making the usual assumptions about U_k we can say that $F \epsilon C^2$ on Z_j.[1]

The U_k are a function of the Z_i, where

$$(2) \qquad Z_j = f_j(x_{1j} \ldots x_{gj}; \quad T_{1j} \ldots T_{rj}) \; j = 1 \ldots n.$$

In (2) the elements x are market commodities required in the production of Z_j. We will denote the whole set of market commodities required for Z_j as a vector, x_j; hence x_j is a subset of the universal set of market commodities, X. Similarly, the elements, T_{hj}, are components of the time set required for Z_j; we will denote this in vector form as T_j and will recognize that the components of T_j may not apply to consecutively running hours since the production and consumption of some Z_j may require preparatory efforts involving time. Equation (2) can be written,

$$(2') \qquad Z_j = f_j(x_j T_j), \; j = 1 \ldots n.$$

If it was not clear in the summary of the Becker model provided above, the Z_j are not commodities per se. They are consumption activities (or processes) which, in most cases, involve market commodities in some combination and time. Examples of such activities range from an income intensive evening out for dinner at a fine restaurant where the market inputs include the food and atmosphere purchased as well as some portion of the services of the family automobile and the use of one's finer attire; such an activity is not regarded as equivalent to a quick stop at the local drive-in. Another example worth mentioning might be an unemployed worker playing pool for an afternoon. This involves minimal market expenditures but is time intensive.

There is a "technology" governing the production of the Z_j; this is recognized in the f_j. Assuming the production technology of (2') is linear, we can say:

[1] The assumptions we wish to avoid are that F is simply the sum of the U_k or any other particular form. Also the assumption that $F \epsilon C^2$ will be later weakened to $F \epsilon C^1$.

$$(3) \qquad x_j = b_j Z_j$$

$$T_j = t_j Z_j,$$

where, again, x_j and T_j are column vectors on the left and b_j and t_j are vectors on the right of technical coefficients, while Z_j is a scalar. To this point, except for the focus on the family as opposed to the individual, the model is as Gary Becker expressed it. In searching for the empirical implications of this model, Becker mentioned the apparent adaptability to a programming framework. However, he proceeded to rely on geometry and the calculus, making some required simplifying assumptions in the process. For purposes of the present analysis, we will proceed on the same track in order to discern the implications of this approach. The development of a programming variant will follow.

The Calculus Approach

For both analytical convenience and acceptability of assumptions, assume that our individual utility functions are expressed as:

$$(4) \qquad U_1 = U_1(Z_j), j = 1 \ldots j_1$$

$$U_2 = U_2(Z_j), j = j_1+1 \ldots j_n.$$

This breakdown in notation is sufficiently general and yet permits our pointing out, e.g., a particular pair of Z_j, one entering U_1 and the second entering U_2, may, for all intents and purposes, be the "same" activity. This, of course, implies that the production technologies, f_j, for a common activity may be different between family members. But, this is as it should be; we have no reason to

believe that it requires the same expenditure of time for father to, say, read the evening paper as it would mother, although market expenditures *may* be equal.

The family's problem is to maximize (1), but since we assumed

$\frac{\partial U}{\partial U_k} > 0$ this is tantamount to maximizing (5) *if the budget constraints are independent.* For the first family member we would have,

(5) $$U_1 = U_1(Z_1, Z_2 \ldots Z_{j_1}) \text{ to be maximized}$$

s.t.

(6) $$\sum_{j=1}^{j_1} p'_j x_j = Y_1 + g_1 + w'_1 T_{w_1}$$

and

(7) $$\sum_{j=1}^{j_1} T_j = T_{c_1} = T_1 - T_{w_1}.$$

Here, T, T_c, T_w, p_j, and w_1 are all interpreted as vectors. T_1 = total time, T_{c_1} = consumption time, T_{w_1} = working time, p_j = prices appropriate for x_j, and w_1 = the relevant wage vector for the first family member. Y_1 and g_1 are "intrafamily income transfer" and "other income," respectively.

Regarding these definitions, more explanation is required. Both T_1 and T_{w_1} are treated as vectors in the same manner as T_j because there are both qualitative and economic differences in consuming or working during one interval during the day or week rather than another. That is, while the sum of the components in T_1 equals 24 hours if we are dealing in days or 168 if our time period is a week, it may be more costly to indulge one's self in consumption between, say, the hours of eight and five on a weekday rather than in some other interval. The vehicle through which we determine the indirect costliness of consumption is w_1; this is a vector of wage rates which is regarded as applying to the components of T_1 and therefore has the same number of components as T_1. This vector, w_1, reflects the fact that, for any set of personal and skill characteristics of an individual, the wage rate applicable for prime weekday hours is likely greater than for Sunday or "after-hour" times when one could occupy himself at a "secondary" job. Becker points out the necessity of recognizing "productive consumption" in this model, i.e., sleeping, eating, and so forth. He suggests that for commodities such as these the "opportunity cost of time is less because these commodities indirectly contribute to earnings" [*3*, p. 503]. That is, failure to

recognize the need for productive consumption in some, perhaps physiologically determined, magnitude will tend to lower the components of w_k ultimately—prudence and rationality require some consumption.

Equation (6) is the goods constraint for the first family member. In (6), $\sum_{i=1}^{j_1} p_j x_j$ is the sum of expenditures on market goods by the first family member which is equal to the sum Y_1 = transfer income received by the first family member from other members, g_1 = any "other" (nonwage) income received by this member, and $w_1 T_{w_1}$ = the wage income of this member. An important assumption is implied in this constraint. The parameter Y_1 is, again, the net transfer income received by the first family member. Therefore, we are assuming that the family has a method for optimally allocating joint family income among family members.[2] The parameter Y_k need not be positive for each k then, i.e., $Y_k \lesseqgtr 0$ would indicate an amount transferred from k to other family members. We do know, defining

$$I_k = Y_k + g_k + w'_k T_{w_k}$$

$$\sum_{k=1}^{2} I_k = \sum_{k=1}^{2} (Y_k + g_k + w'_k T_{w_k}) = \text{total family income.} \tag{8}$$

Constraints (6) and (7) are not independent and, following Becker [*3*, pp. 496-497], let us combine these into one. Solving for T_{w_k} in (7) and substituting in (6) we get:

$$\sum^{n} p'_j x_j + w'_k \sum^{n} T_j = Y_k + g_k + w'_k T. \tag{9}$$

From (3) we can write (9) as:

$$\Sigma\ (p'_j b_j + w'_k t_j) Z_j = Y_k + g_k + w'_k T_k = \hat{I}_k. \tag{10}$$

Where the components of w_k are constant, Becker terms this constraint "full income"[cf., *3*, pp. 497-498], which is supposedly

[2] This assumption is rather casually inserted here. However, recalling Samuelson's discussion on the requirements for income allocation necessary to discuss the maximization of a function like $U = F(U_1 \ldots U_m)$, further elaboration on this point is demanded and will follow.

the maximum money income achievable. For the kth family member the ultimate constraint is:

$$(11) \qquad S_k(Z_j) = Y_k + g_k + w_k' T_k - \Sigma\,(p_j b_j + w_k' t_j) Z_j = 0.^3$$

In order to continue developing this variant, an important observation and an assumption are required at this point. Constraint (11) was derived from (6) and (7) by way of (3). But let us consider (7) as an inequality (i.e., $\Sigma T_j \leqq T_k$) then, using (3), express (6) and (7) as $\Sigma p_j' b_j Z_j = Y_k + g_k + w_k' T_{w_k}$ and $\Sigma t_j Z_j \leqq T_k$. Provided both of these constraints are binding, an optimal solution must occur on the subset of (6) in the intersection of (6) and (7). However, in (11) there is no assurance that this condition will be satisfied.

Becker recognized the problem of requiring $\Sigma t_j Z_j \leqq T_k$; he says at one point "I assume for simplicity that the amount of each dimension of time used in producing commodities is less than the total available . . ." so that this problem can be ignored [cf., *3*, n. 2, p. 498]. This can be partially rationalized on the basis of the recognized necessity for productive consumption; yet we must observe that an assumption like this is something less than desirable. Becker goes on to suggest that the incorporation of $\Sigma t_j Z_j \leqq T_k$ is not difficult; this is true, but, as will be shown by way of the programming variant, the implications are somewhat different than if we assume that time constraints are satisfied. In addition, as will be shown below, proceeding by way of the calculus effectively reduces this assumption to $\Sigma t_j Z_j < T_k$, i.e., each family member works. Yet, as it is not the realism of the assumptions that makes a theory important but rather the degree to which the theory permits meaningful interpretation of observed behavior, we will proceed to assume for the time being that, in (11), $\Sigma w_k' t_j Z_j \leqq \Sigma w_k' T_k$. Unfortunately, (11) embodies even more troublesome difficulties; these will be brought out below.

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[3]Two important assumptions are embodied in constraint (11). First, (11) could be written as a weak inequality indicating possible saving. For the moment we shall assume a constant, Keynesian-type marginal propensity to consume and consider the "income" component of (11) as net spending income. Second, as stated, the constraints are linear. Hence, average and marginal prices are equal. This need not be retained and, if required, nonlinear constraints can be introduced.

Now, due to our definition of Y_k and the character of the present problem, it should be clear that the constraints $S_k(Z_j)$ are not independent. Only if the transfers, Y_k, could be made on a once-and-for-all basis could the individual utility functions be maximized separately subject to the respective constraint. But, in the discussion of Samuelson's social indifference curve article, it was indicated that if the problem is to maximize $U = F(U_1, U_2)$ and the assumption that $\frac{\partial U}{\partial U_k} > 0$ is made—and there is no apparent reason not to assume this—then a system providing for income transfers, Y_k, so as to keep $\frac{\partial U}{\partial I_1} = \frac{\partial U}{\partial I_2} =$ constant, must be provided. It is this requirement—and this one alone—that provides any reason to believe that unemployment or income changes for one family member will have any effect on other family members.

It is now appropriate to consider the precise character of the problem facing the family. However, this requires a return to, and critical examination of, Samuelson's condition for a maximum of (1). For, while the equality of marginal utility of income of family members is a valid and necessary condition for a maximum of F when continuous functions are assumed, Samuelson's approach to this problem is excessively casual. His analysis will now be reviewed and the implications examined and criticized. A solution to the problem will then be suggested and incorporated in the family's problem.

Again, "the fundamental unit on the demand side is clearly the 'family' " [*24*, p. 9]. But, in western culture we do observe a certain intrafamily decentralization. Yet, "blood is thicker than water" and,

> . . . the preferences of different members are interrelated by what might be called a "consensus" or "social welfare function" which takes into account the deservingness or ethical worths of the consumption levels of each of the members. The family acts *as if* it were maximizing a joint welfare function [*24*, p. 10].

Assuming that each family member has his own ordinal utility indicator function, u^i, the joint function can be expressed

$U = f(u^i, u^2 \ldots)$. Assuming $\frac{\partial u}{\partial u^i} > 0$, the model displays what Samuelson calls quasi-independence. By virtue of this quasi-independence, Samuelson says:

> The only joint consensus decisions that have to be made by the family have to do with the allocation among the different individuals of the total family income I. If this is properly broken down into $I = I^1 + I^2 + \ldots$, then each member confronted with market prices $(P_x, P_y, \ldots)$ can be counted on to maximize his own ordinal $u^1(X^i, Y^i, \ldots)$—and each will be led, as if by an invisible hand, toward maximization of $U = f(u^1, u^2 \ldots)$ [*24*, p. 10].

This joint allocation problem requires a system providing for the lump-sum transfers referred to above. For family income, I, to be optimally allocated:

> Income must always be reallocated among the members of our family society so as to keep the "marginal social significance of every dollar" equal, i.e.,
>
> $$\frac{\frac{\partial U}{\partial u^2} \cdot \frac{\partial u^2}{\partial I^2}}{\frac{\partial U}{\partial u^1} \cdot \frac{\partial u^1}{\partial I^1}} = 1, \text{etc. [24, p. 11]}.$$

It is suggested here that all that is required for a maximum of U is that income be reallocated through lump-sum transfers in a manner such that the above ratios are unity; the quasi-independent nature of the problem would do the rest. The manner for achieving this result is not outlined; however, it is pointed out that these transfers cannot be once-and-for-all and must be performed with the final equilibrium configuration in mind. The rub in this whole problem lies in the implied two-stage approach. That is, allocate income, I, among I^i such that the above ratios are satisfied, then let each member maximize his own utility function subject to his own constraint. Yet, consider the partial derivatives

$$\frac{\partial u^i}{\partial I^i} \text{ and hence } \frac{\partial U}{\partial u^i} \cdot \frac{\partial u^i}{\partial I^i}.$$

By way of the standard treatments of consumer theory (and this

would include the Becker and Lancaster approaches), income does not enter directly into the utility function but contributes to total utility by way of the commodities it permits the consumer to consume. The marginal utility of income is the rate of change of total utility for marginal changes in income, and the relevant value of this derivative is specified by the utility function, an optimal vector of commodities, and the respective market prices, i.e., $\frac{\partial u^i}{\partial I^i} = \frac{\partial u^i}{\partial x_j} / p_j$ for some commodity index j over all j and with $\frac{\partial u^i}{\partial x_j}$ *evaluated at the optimal value of the commodity.* Hence, the values of those partial derivatives, $\frac{\partial U}{\partial u^i} \cdot \frac{\partial u^i}{\partial I^i}$, in Samuelson's equilibrium condition are not specified until the optimal commodity vector is determined, which implies, for the present analysis, that the determination of the optimal income transfers and commodity vector *must take place simultaneously.* Fortunately, this is possible if proceeding with the calculus variant is warranted on other grounds.

Because of the implications of the above consideration, it is necessary to consider (8) as the relevant income constraint in the problem of maximizing $U = F(U_1, U_2)$. By (8) and (10), total family income is:

$$\hat{I} = \sum_{k=1}^{2} I_k = \sum_{k=1}^{2} (Y_k + g_k + w_k' T) = \sum_{j=1}^{n} (p_j b_j + w_k' t_j) Z_j. \tag{12}$$

From the definition of the intrafamily transfers, Y_k, it is clear $\sum_{k=1}^{m} Y_k = 0$. The presentation of (12) as the relevant constraint is equivalent to regarding the family as having a joint checking account which is exhausted in each period on total expenditures, $\sum_{J=1}^{n} (p_j b_j + w_k' t_j) Z_j$. Family members are required to draw upon the account in a responsible manner which leads to the maximum of F. And, on reflection, this does not seem to be a distortion of the way families behave in fact. In effect this says that the family is a collection of rational individuals who are aware, either intuitively or through discussion, of the restrictions

posed by the household utility function and of the necessity for drawing on I in a manner such that some final equilibrium configuration is satisfied.

The family's problem may be expressed as:

(13) Maximize

$$U = F(U_1, U_2) = F[U_k(Z_j)],\ k = 1,2$$

s.t.

$$\sum_{k=1}^{2} S_k(Z_j) = \sum^{2} Y_k + \sum^{2} g_k + \sum^{2} w_k' T_k - \sum^{n} (p_j b_j + w_k' t_j) Z_j = 0$$

$$k = 1,2 \quad j = 1 \ldots n.$$

In (13), p_j, t_j, b_j, and w_k are vectors, as is T_k, where $T_k = T_{w_k} + T_{c_k}$ and $T_{c_k} = \sum^{n} t_j Z_j$ over the appropriate interval of j. The vector T is now being subscripted, not to imply that the total time vectors are different but to suggest that each individual is subject to the constraint of a total time vector. As was expressed in (4), the index j is broken into two intervals, one interval for each family member.

Since the assumptions we have made regarding the functions in (13) seem to permit, we will treat this as a classical optimization problem and proceed to form a Lagrangian function,

$$L(Z_j, \lambda) = F[U_1(Z_j), U_2(Z_j)] + [\sum^{2} S_k(Z_j)]. \tag{14}$$

Again, each U_k, S_k is defined over a particular interval of j.

To wit: $j = 1 \ldots, j_1$ for U_1, S_1

$j = j_1 + 1 \ldots . j_n$ for U_2, S_2

With this proviso then, the familiar necessary conditions for the maximization of (14) are,

$$
(15) \qquad
\begin{array}{ccccc}
\dfrac{\partial F}{\partial U_1} & \cdot & \dfrac{\partial U_1}{\partial Z_1^o} & = & \lambda\pi_1 \\
\cdot & & & & \cdot \\
\cdot & & & & \cdot \\
\cdot & & & & \cdot \\
\dfrac{\partial F}{\partial U_1} & \cdot & \dfrac{\partial U_1}{\partial Z_{j_1}^o} & = & \lambda\pi_{j_1} \\
\dfrac{\partial F}{\partial U_2} & \cdot & \dfrac{\partial U_2}{\partial Z_{j_1}^o + 1} & = & \lambda\pi_{j_1+1} \\
\cdot & & & & \cdot \\
\cdot & & & & \cdot \\
\cdot & & & & \cdot \\
\dfrac{\partial F}{\partial U_2} & \cdot & \dfrac{\partial U_2}{\partial Z_n^o} & = & \lambda\pi_n
\end{array}
$$

$$
\overset{2}{\Sigma} Y_k + \overset{2}{\Sigma} g_k + \overset{2}{\Sigma} w_k' \mathrm{T}_k - \overset{n}{\Sigma}(c_j + w_k' t_j) Z_j^o = \mathrm{O}.
$$

Again, $\pi_j = (c_j + w_k' t_j)$ is the sum of two scalar products. The superscript on the Z_j now denotes the values of Z_j for which these conditions hold. From (15) comes the two familiar propositions that the marginal rates of substitution equal the ratios of the "prices," π_j, and the marginal utility of income, λ, equals the ratio of the family marginal utilities, evaluated at Z_j^o, to the prices, π_j.

The equations provided in (15) are the usual necessary conditions from which we would derive theoretical demand curves and the other qualitative statements regarding the effects of parameter changes. These conditions are intuitively attractive because the "prices," whose ratios we might consider in (15), consist of both the direct market expenditures and the indirect cost of time per unit of Z_j. Yet, there are reasons to believe this outward attractiveness is oversimplified and deceiving. Consider the following: First, if a set, Z^o, exists which satisfies (15) we know $Z_j > 0$ for all j since the calculus can only lead us to an interior solution in

activity space. Secondly, we know $\sum_{j=1}^{n} t_j Z_j$ = "total consumption time" of all household members. However, we do not know anything of the relation between $\sum^{n} t_j Z_j$ and total family time, and, more importantly, we know nothing of the relation between *separate* consumption times and the individual time constraints; clearly the hours of one family member are not substitutable for the hours of another member yet there is nothing in constraint (11) or (12) which prohibits substitution. A third, and similar, difficulty arises out of the specification of T_k as a vector with components of fixed size. In the discussion above we followed Becker in assuming, e.g., $\sum^{J_1} t_j Z_j < T_1$. But if we extend this assumption and apply it to each component of the T_k—and it would have to be an assumption since there is nothing in the constraint nor in the calculus that assures satisfaction of the separate time constraints—then, since the calculus provides only interior solutions, each family member is required to *both* consume and work during *each* interval of time. This is absurd. The essence of these remarks is to say effectively that the combination of the expenditure and time constraints by way of the production technologies is misleading and the result, the full income constraint, (11) or (12), leads us to question the existence of even a feasible solution, not to mention an optimal solution.

In spite of these difficulties, suppose we assume, for the sake of argument, that Z^o is optimal and satisfies all the time constraints—all stock time constraints are satisfied in the aggregate and in terms of the separate components. Can the kind of empirical implications we are looking for (or any other kind) be drawn from the model?

Since it is the effect of wage and/or income changes that has been our concern, let us confine the analysis to just such considerations and apply the Slutsky analysis to (15). The set of equations, (15), provides the necessary conditions for a constrained maximum of family utility, and we have assumed that the separate time constraints are not violated. On this assumption, then, total consumption time of the two family members is, $\sum_{j=1}^{j_1} t_j Z_j = T_{c_1}$ and $\sum_{j=j_1+1}^{n} t_j Z_j = T_{c_2}$, respectively. Work time is found resid-

ually in this case and is $T_{w_1} = T_1 - T_{c_1} > 0$ for the first family member and $T_{w_2} = T_2 - T_{c_2} > 0$ for the second--but these concepts are vectors. Consider the set of equations derived by taking the total differential of (15). These are,

$$
\begin{aligned}
(16) \qquad & F_{11}dZ_1 \ldots + F_{1j_1}dZ_{j_1} - \pi_1 d\lambda = \lambda d\pi_1 \\
& \vdots \qquad\qquad\qquad\qquad \vdots \\
& F_{j_1+1}dZ_{j_1+1} \ \ldots + F_{nn}dZ_n - \pi_n d\lambda = \lambda d\pi_n \\
& -\pi_1 dZ_1 \ \ldots - \pi_n dZ_n = H.
\end{aligned}
$$

In the last equation of (16), $H = dI + Zd\pi$.

In (16) there are $n + 1$ linear equations in $n + 1$ unknowns, dZ_j and $d\lambda$, but the coefficients of this system should be made clear before proceeding. The family utility function is, again, $U = F[U_1(Z_j), U_2(Z_j)]$. The total differential of this is,

$$dU = F_{u_1}dU_1 + F_{u_2}dU_2.$$

But since

$$dU_1 = U_{1_1}dZ_1 \ \ldots + U_{1_{j_1}}dZ_{j_1} = \sum_{j=1}^{j_1} U_{1_j}dZ_j$$

and

$$dU_2 = U_{2_{j_1+1}}dZ_{j_1+1} \ \ldots + U_{2_n}dZ_n = \sum_{j=j_1+1}^{n} U_{2_j}dZ_j$$

and where U_{1_j} and U_{2_j} are first partial derivatives, dU becomes,

$$dU = F_{u_1}\left(\sum_{j=1}^{j_1} U_{1_j}dZ_j\right) + F_{u_2}\left(\sum_{j=j_1+1}^{n} U_{2_j}dZ_j\right).$$

Now, d^2U is,

$$d(dU) = F^2_{u_1}dU_1dU_1 + F_{u_1u_2}dU_1dU_2 + F_{u_2u_1}dU_2dU_1 + F^2_{u_2}dU_2dU_2.$$

The family utility function, F, embodies the interrelatedness of family satisfactions. However, we, like Samuelson, assumed a degree of intrafamily independence in decision making; this amounts to assuming that the middle two terms of d^2U are zero, i.e., $F_{u_1u_2}dU_1dU_2$, $F_{u_1u_2}dU_2dU_1 = 0$. This being the case, d^2U breaks down to,

$$d^2U = F^2_{u_1}dU_1dU_1 + F^2_{u_2}dU_2dU_2$$

$$= F^2_{u_1} \sum_{j=1}^{j_1} \sum_{r=1}^{j_1} \frac{\partial^2 U_1}{\partial Z_j \partial Z_r} dZ_j dZ_r$$

$$+ F^2_{u_2} \sum_{j=j_1+1}^{n} \sum_{r=j_1+1}^{n} \frac{\partial^2 U_2}{\partial Z_j \partial Z_r} dZ_j dZ_r, \; j,r = 1 \; \ldots n.$$

As an example and assuming j and $r = 1$, the notation in (16) is $F_{11} = F^2_{u_1} \dfrac{\partial^2 U_1}{\partial Z_1^2}$. In matrix and vector form (16) appears as,

$$(16') \quad \begin{bmatrix} F_{11} & \cdots & F_{1j_1} & O & & O & -\pi_1 \\ \vdots & & & \vdots & & \vdots & \vdots \\ F_{j_1 1} & \cdots & F_{j_1 j_1} & O & & O & -\pi_{j_1} \\ O & & O & F_{j_1+1 j_1+1} & \cdots & F_{j_1+1 n} & -\pi_{j_1+1} \\ \vdots & & \vdots & \vdots & & \vdots & \vdots \\ O & & O & F_{n j_1+1} & \cdots & F_{nn} & -\pi_n \\ -\pi_1 & \cdots & -\pi_{j_1} & -\pi_{j_1+1} & \cdots & -\pi_n & O \end{bmatrix} \begin{bmatrix} dZ_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ dZ_n \\ d\lambda \end{bmatrix} = \begin{bmatrix} \lambda d\pi_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \lambda d\pi_n \\ H \end{bmatrix}$$

If we define Δ as the matrix of coefficients in (16′), dZ as the vector of differentials consisting of the differentials dZ_j and $d\lambda$; and letting Θ represent the right side of (16′), we have,

$$(17) \qquad \Delta dZ = \Theta.$$

The matrix, Δ, is a bordered Hessian matrix of order $n + 1$. Assuming $|\Delta| \neq 0$, we can solve for dZ thusly:

$$(18) \qquad \Delta^{-1}\Delta dZ = \Delta^{-1}\Theta$$

$$dZ = \Delta^{-1}\Theta.$$

For any particular differential in (18), e.g., dZ_1, we would obtain an expression of the form,

$$(19) \qquad dZ_1 = \frac{D_{11}d\pi_1 + \cdots + \lambda D_{mn}d\pi_n + D_{n+1.1}(-\overset{2}{\Sigma}dY_k - \overset{2}{\Sigma}dg_k}{|\Delta|}$$

$$\frac{-\overset{2}{\Sigma}w_k dT - Td\overset{2}{\Sigma}w_k + Z^o d\pi}{|\Delta|}$$

where D_{sv} is the appropriate cofactor from Δ.

We are interested in the sign of $\partial Z_j/\partial I_k$ where, again, $\hat{I}_k = Y_k + g_k + w_k T$. Total income, $\hat{I}_k$ can change due to changes in other income, g_k, or in the wage "rate," w_k. To obtain these effects it is necessary to "jolt" the equations like (19) by changing g_k or w_k. It is at this juncture where the final breakdown comes in the application of the calculus to the model at hand because, due to the specification of the model, some of the necessary differentials emerge as vectors. It is possible to obtain, e.g., from (19), the partial derivative $\frac{\partial Z_1}{\partial g_1}$ by following the usual procedure of holding other differentials to zero and varying g_1. The result would be,

$$(20) \qquad \frac{\partial Z_1}{\partial g_1} = \frac{-D_{n+1.1}}{|\Delta|}.$$

This is the pure income effect whose sign is indeterminant because the cofactor in the numerator is unsigned. However, using (19) as an example again, consider what happens when we attempt to differentiate dZ_1 with respect to w_1.

From the definitions above, w_1 was specified a vector because there apparently is a difference in the way firms value the time of individuals with various job-skill profiles in various times of the day or week. If we attempt to differentiate dZ_1 with respect to w_1 we are, then, differentiating a scalar with respect to a vector;

the result is a vector and would have no meaning for the purposes for which it is being derived. That is, because of the structure and concepts of the model, the primary result required in order to observe the effect on consumption or work time of changes in the wage vector is not meaningful. The analytical sacrifices we were willing to accept by assuming that Z^o satisfied the time constraints turn out not to be helpful after all.

Gary Becker's integration of production and consumption at the micro level and in a manner that incorporates time as the primary scarce resource must be regarded as a contribution to the body of theoretical literature in economics. However, this innovation, along with the specification of the family as the micro unit, has sufficiently complicated the analysis so the calculus is not a useful tool for extracting the implications in which we are interested. Should we simplify the model to the point where this tool would be applicable, we would be back to an individual-oriented, single-wage, single-time-period model that still *requires* the individual to work and, in addition, still requires the assumption that the time constraint is not violated. In order to take meaningful advantage of Becker's contribution and to emphasize the role of time allocation, a programming framework is required.

The Allocation and Programming Variant

The observations above have pointed out the necessity of approaching a problem of the type we have formulated with an analytical tool other than the calculus. More specifically, the character of the problem invites a programming perspective and nonlinear programming in particular. In this section a programming version of this modified Becker model will be developed; the objective is to discern the theoretical differences in the model's implications as compared to the calculus model. Return now to equations (1), (2'), (3), and assumption (4). We have:

(1) $$U = F[U_1(Z_j), U_2(Z_j)]$$

(2') $$Z_j = f_j(x_j, T_j),\ j = 1 \ldots n$$

(3) $$x_j = b_j Z_j$$
$$T_j = t_j Z_j$$

(4) $$U_1 = U_1(Z_j),\ j = 1 \ldots j_1$$
$$U_2 = U_2(Z_j),\ j = j_1 + 1 \ldots n.$$

Regarding (1) we only need assume now $F \epsilon C^1$, i.e., F has continuous first partial derivatives. However, this does not exclude the possibility that $F \epsilon C^2$. Also, the U_k and F are assumed to be concave. Each activity, Z_j, involves unit market expenditures $c_j = p_j' b_j$, where p_j and b_j are vectors of market prices and technological coefficients, respectively. The market budget constraint facing an individual would be, then,

$$\sum^{n} c_j Z_j - w_k' T_{w_k} = g_k. \tag{21}$$

The time constraint for each family member is

$$\sum^{n} t_j Z_j + T_{w_k} \leqq T_k. \tag{22}$$

This inequality, (22), needs some explanation. It says, in effect, that consumption time plus work time must not exceed but *may* fall short of total time. This latter contingency may seem strange for it recognizes that one may consume "pure leisure," i.e., consume time only and *no* market commodities. In terms of a programming framework this is recognized as slack in the time constraint in which case time is a free good. However, from the definition of the consumption activities, there is nothing to preclude some Z_j from involving time only and no market commodities, in which case Z_j is "pure leisure" and enters the utility function. On this basis constraint (22) would always be an equality and time would not possibly be a free good. In addition, all references to time, such as t_j, T_{w_k}, etc., are now recognized as scalars.

Let us now recognize that each family member probably has several occupational choices potentially open to him or her and a wage rate is associated with each occupation. The time spent in each occupation is variable; we will denote these times and wage rates as T_{ks} and w_{ks}, respectively. In terms of the present model the family's problem can now be expressed as,

(23) Maximize

$$U = F[U_1(Z_j), U_2(Z_j)]$$

s.t.

$$\sum_{j=1}^{j_1} t_j Z_j + \sum^{s} T_{1s} = T_1$$

and hence,

$$\nabla_z V(Z, T_w, \lambda)/T^o = 0$$

$$\nabla_{t_w} V(Z, T_w, \lambda)/T^o_{ks} = 0$$

where, e.g., $_z V(Z, T_w, \lambda)$ is the gradient vector of the Lagrangian function with respect to the Z_j. In addition, we know from the Kuhn-Tucker conditions that

$$\frac{\partial V(V, T_w, \lambda)}{\partial \lambda} = 0 \text{ or } \lambda = 0 \text{ for } \lambda_k \text{ and } \lambda_3$$

and therefore $\nabla_\lambda V(Z, T_w, \lambda)\lambda^o = 0$.

The immediate questions that emanate from consideration of (24) are, first, what is the economic interpretation of the λ and, second, what are their dimensions in the present problem? Hadley, again, provides our frame of reference.

In [*12*, pp. 72-73] it is pointed out that the Z^o and λ^o are, in general, functions of $G = g_1 + g_2$ and T_k. On the assumption that Z^o_j and λ^o are continuous functions of G and T_k in some ϵ neighborhood of G^* and T^*_k, values generating Z^o and λ^o, Hadley shows that:[5]

$$\frac{\partial F[U_k(Z^o)]}{\partial G} = \lambda^o_3 \text{ and } \frac{\partial F[U_k(Z^o)]}{\partial T_k} = \lambda^o_k.$$ [6]

That is, λ^o_3 is the rate of change of the family concensus function with respect to income, *ceteris paribus*—it is the marginal utility of income for the family. Also, λ^o_k is the rate of change of family

---

[5] Technically, the primal variables and the Lagrange multipliers are functions of whatever appears on the right side of the constraints; in the present case, as now stated, this is G and the T_k.

[6] At another point Hadley shows [*12*, p. 199] that the existence of, e.g., $\frac{\partial F[U_k(Z^o)]}{\partial G}$ is intimately related to the satisfaction of the Kuhn-Tucker constraint qualification and the rank of the matrix of first partial derivatives of the constraints with respect to Z_j and T_{ks}.

utility with respect to the kth time constraint–it is the (weighted) marginal utility of nonwork time of the kth family member. More on this point will follow presently.

Consideration of the dimensions of λ_k and λ_3^o is the next topic of analysis. We could develop analytical expressions for λ_k^o and λ_3^o by relying on the proof of Hadley [*12*, pp. 64-65] involving the implicit function theorem. This proof demonstrates that one can take, in this case, and three rows from (25) for positive variables, and, if the Jacobian implicit to those equations is nonsingular, unique expressions for the multipliers, λ, can be obtained. However, we can proceed by another tack that implicitly relies on the Hadley proof. That is, if an optimal solution vector, Z^o, exists to our problem (23), the Kuhn-Tucker conditions expressed in (25) can be regarded as a linear programming problem in the differentials of the Lagrangian function. Since the marginal utilities, $\frac{\partial F}{\partial U_k} \cdot \frac{\partial U_k}{\partial Z_j^o}$, are constants at Z^o, this problem amounts to finding values of λ_1, λ_2, and λ_3 which minimize $dR = dT_1 + dT_2 \lambda_2 + dG\lambda_3$ subject to (25).[7]

With these observations, let us make some simplifying assumptions and then proceed to consider the linear programming problem embodied in (25). Regarding the optimal conditions for the T_{ks}, e.g., $\lambda_1 - w_{ks}\lambda_3 \geqslant 0$, since the first (each) family member has a set of possible occupational choices, there will be a corresponding set of conditions like these and, since λ_1 and λ_3 enter each condition, it is clear that if the wage rates, w_{1s}, differ, only one (if any) occupation will be chosen. This applies to the second member as well. Assume, for simplicity, that we know that for the second family member $\lambda_2 - w_{2s}\lambda_3 > 0$ for all occupations–the second member does not work in the present solution. And

[7]The reason that the differentials emerge here is to be found by way of the Kuhn-Tucker constraint qualification [cf., *12*, pp. 199-202]. This qualification imposes some restrictions on the character of the feasible region surrounding Z^o (if it exists). This feasible region is confined to a neighborhood around Z^o in the intersection of the hyperplanes derived from the constraints and the objective function and that pass through Z^o these hyperplanes are expressed in differentials. Also, only if the constraint qualification is satisfied (and the assumptions made in the present model assure this) is a unique solution vector, guaranteed.

assume that we know, for the first family member, that $\lambda_1 - w_{11}\lambda_3 = 0$ and $\lambda_1 - w_{1s}\lambda_3 > 0$ for all other occupations–the first family member works in occupation number one. In addition, assume for convenience that the index j still runs from 1 to n and j_1 still is the upper end of the index for the first family member.

If we denote, e.g., $\Phi_1 = \frac{\partial U}{\partial U_1} \cdot \frac{\partial U_1}{\partial Z_1^o}$, the relevant linear programming problem is:

$$
(26) \qquad
\begin{array}{ll}
t_1\lambda_1 + O + c_1\lambda_3 & \geqq \Phi_1 \\
\vdots & \vdots \\
t_{j_1}\lambda_1 + O + c_{j_1}\lambda_3 & \geqq \Phi_{j_1} \\
O + t_{j_1+1}\lambda_2 + c_{j_1+1}\lambda_3 & \geqq \Phi_{j_1+1} \\
\vdots & \vdots \\
O + t_n\lambda_2 + c_n\lambda_3 & \geqq \Phi_n \\
\lambda_1 - w_{11}\lambda_3 & = O \\
\lambda_2 - w_{21}\lambda_3 & > O \\
dT_1\lambda_1 + dT_2\lambda_2 + dG\lambda_3 & = dR \text{ (minimum)} \\
\lambda_1, \lambda_2, \lambda_3 \geqslant O. &
\end{array}
$$

The optimal vector for (26), λ^o, is a vector such that for the Lagrangian function in (24),

$$V(Z, T_{ks}, \lambda^o) \leqq V(Z^o, T_{ks}^o, \lambda^o) \leqq V(Z^o, T_{ks}^o, \lambda).$$

Solution procedures for (26) could proceed by way of adding $n + 2$ additional (surplus) variables and an identity matrix of order $n + 2$ to the constraints. However, it is clear that the inverse to the optimal basis would involve *only three* of the original rows of

coefficients. Therefore, let us deal with the linear dual to (26). This would be:

(27)

$$t_1 dZ_1 \cdots + t_{j_1} dZ_{j_1} \qquad + dT_{11} \leqq dT_1$$

$$t_{j_1+1} dZ_{j_1+1} \ldots + t_n dZ_n \qquad + O \leqq dT_2$$

$$c_1 dZ_1 \cdots + c_{j_1} dZ_{j_1} + c_{j_1+1} dZ_{j+1} \cdots + c_n dZ_n - w_{11} dT_{11} \leqq dG$$

$$\Phi_1 dZ_1^o \cdots + \Phi_{j_1} dZ_1^o + \Phi_{j_1+1} dZ_{j_1+1}^o \cdots + \Phi_n dZ_n^o + O \qquad = dU \text{ (max.)}$$

$$Z_j \geqslant O; \lambda_k, \lambda_3 \geqslant O.$$

The "prices" in the objective function, Φ_j, are the relevant marginal utilities and are constant since they are evaluated at Z^o.

For purposes of viewing the dimensions of the λ^o, we can pick any nonsingular basis matrix, B, from (27) from indices $j \epsilon J$ or $T_{ks} > 0$ and obtain values for these dual variables as follows:

(28) $\lambda' = \Phi_{j_B} B^{-1}$

where Φ_{j_B} is the vector of prices associated with the basis. Let us arbitrarily choose the columns of (27) associated with Z_1, Z_n, and T_{11}. he basis matrix is,

(29)

$$B = \begin{bmatrix} t_1 & O & 1 \\ O & t_n & O \\ c_1 & c_n & -w_{11} \end{bmatrix}$$

and

(30)

$$B^{-1} = \frac{1}{-t_n(c_1 + t_1 w_{11})} \begin{bmatrix} -w_{11}t_n & c_n & -t_n \\ 0 & -(c_1 + w_{11}t_1) & 0 \\ -t_n c_1 & -t_1 c_n & t_1 t_n \end{bmatrix}$$

By way of (28) we obtain the following:

(31)
$$\lambda_1 = \frac{w_{11}\Phi_1}{c_1 + w_{11}t_1} = \frac{w_{11}\Phi_1}{\pi_1} = w_{11}\lambda_3$$

$$\lambda_2 = \frac{c_n\Phi_1 - \pi_1\Phi_n}{-t_n(c_1 + w_{11}t_1)} = \frac{\Phi_n}{t_n} - \frac{c_n}{t_n}\lambda_3$$

$$\lambda_3 = \frac{\Phi_1}{c_1 + w_{11}t_1} = \frac{\Phi_1}{\pi_1}$$

In (31), the family marginal utility of income, λ_3, is the familiar ratio of activity marginal utilities to "total prices," π_j, where this price includes both direct market expenditures and the indirect cost of time. For the second family member who is assumed not to work in the present solution, the marginal utility of time can be seen to be related to the family marginal utility of income and, hence, is not independent of the first member's utility function. For the first member who does work, the marginal utility of nonwork time is $w_{11}\lambda_3$. Let us now recognize the multipliers expressed in (31) as optimal values since the same kind of expression would be obtained for any other nonsingular basic matrix involving the column for T_{11}. Certain post-optimality questions can now be raised—in particular, what is the effect of a change in w_{11}? Before proceeding, we should define certain analytical concepts that will prove helpful below. In the subsequent analysis it will be convenient to consider activities in terms of their "income intensiveness" or "time intensiveness" and some measure of these concepts is required. For this purpose, consider $\frac{t_j}{c_j + w_{11}t_j}$; this has the dimensions units of time/\$ and if this is "small" we would say Z_j is of low time intensity;

conversely, if $\frac{t_j}{c_j+w_{11}t_j}$ is "large" (and the largest value it could take is $1/w_{11}$) Z_j would be time intensive. Similarly, $\frac{c_j}{c_j+w_{11}t_j}$ has the dimension \$/\$ and is, therefore, a pure number and $0 < \frac{c_j}{c_j+w_{11}t_j} < 1$. If this ratio approximates 1, Z_j can be regarded as income intensive, while if it approximates zero, Z_j is of low income intensity. Of course, these are the extremes and intermediate possibilities exist. In the analysis below it will be convenient to talk of activities as being time or income intensive. Also, it will be useful to refer to the ratio of these two concepts; or, more simply, $\frac{c_j}{t_j}$. This ratio represents dollar expenditures per unit of time in Z_j and can be alluded to as being relatively large or relatively small. In addition and following the conventional calculus precedent, it will be convenient to refer to activities as being "normal" or "inferior." In the usual way, an activity is normal if an increase in income leads to an increase in consumption of the activity, while a decrease in income leads to a reduction in consumption. An inferior activity would display the opposite set of sign relations. It will not, however, be necessary to stipulate that Z^o has any dominant character and, in fact, Z^o can have both time-and income-intensive activities represented in it at positive levels.

Consider a new wage rate applying to the first occupation; this is $w_{11}^* = w_{11} + \Delta w_{11}$. We can then consider the effects on (31) and the condition $\lambda_1 - w_{11}\lambda_3 = 0$ of replacing w_{11} by w_{11}^* in (29). That is,

(29′)

$$B = \begin{bmatrix} t_1 & 0 & 1 \\ 0 & t_n & 0 \\ c_1 & c_n & -w_{11}^* \end{bmatrix}$$

and

(30')

$$B_*^{-1} = \frac{1}{-t_n(c_1+w^*_{11}t_1)} \begin{bmatrix} -w^*_{11}t_n & c_n & -t_n \\ 0 & -(c_1+w^*_{11}t_n) & 0 \\ -t_nc_1 & -t_1c_n & t_1t_n \end{bmatrix}$$

Suppose, initially, $\Delta w_{11} < 0$ and $w^*_{11} < w_{11}$. Then the optimal condition for T_{11} becomes $\lambda_1 - w^*_{11}\lambda_3 > 0$ and if T_{11} is to remain positive λ_3 must rise and/or λ_1 must fall to restore equality in this condition. In the event $\Delta w_{11} > 0$, the opposite set of sign changes must take place. The new vector, λ^*, obtained when w_{11} is replaced by w^*_{11} is,

(32) $\lambda^* = \Phi_B B_*^{-1}$

and

$$\lambda_1^* = \frac{w^*_{11}\Phi_1}{c_1+w^*_{11}t_1}$$

$$\lambda_2^* = \frac{\pi_1\Phi_n - c_n\Phi_1}{t_n(c_1+w^*_{11}t_1)} = \frac{\Phi_n}{t_n} - \frac{c_n\Phi_1}{t_n(c_1+w^*_{11}t_1)}$$

$$\lambda_3^* = \frac{\Phi_1}{c_1+w^*_{11}t_1}$$

Before examining the individual elements of λ^* we must dispose of some potentially troublesome implications of changing an element in the assumed optimal basis matrix. First, such a change could make $|B_*| = 0$ and hence B_*^{-1} would not exist; in the present case this could only happen if $w^*_{11} < 0$ and $[w_{11}t_1] = [c_1]$, a possibility we need not consider. Secondly, it is theoretically possible for such a change to require a nonfeasible solution. In the present case, again, this would mean a solution that violates the time constraints or the requirement Z_j, $T_{ks} \geqslant 0$; it would require $w^*_{11} < 0$ to lead to the last implication and it would seem to require "large" increases in w_{11} to have the former

constitute a problem. Thirdly, the potentially most troublesome implication of changing w_{11} is that such a change might alter the optimality conditions in the primal. We shall consider this presently.

In (32), if $w^*_{11} < w_{11}$ (i.e., $\Delta w_{11} < 0$) we observe the following:

1. $\lambda_1^* < \lambda_1^o$, $\Delta\lambda_1 < 0$, and $\Delta\lambda_1$ is larger the more income intensive (i.e., the larger $\frac{c_1}{\pi_1}$) is Z_1. Since λ_1 is related to nonwork time of the first family member and we have assumed concave utility functions, $\Delta\lambda_1 < 0$ implies $\Delta T_{11} < 0$—nonwork time increases.

2. $\lambda_2^* < \lambda_2^o, \Delta\lambda_2 < 0$; changes in λ_2 are related to the first member's consumption set and are smaller the less time intensive is Z_1, i.e., the smaller $\frac{t_1}{\pi_1}$.

3. $\lambda_3^* > \lambda_3^o$; $\Delta\lambda_3 > 0$; the loss of wage income *ceteris paribus* raises the family marginal utility of income which is consistent with what would be expected from the character of the utility functions.

For sufficiently small changes in w_{11} we observe from (32), $\lambda^* = w_{11}^*\lambda_3^*$ and T_{11} remains positive but smaller. However, a change in the equilibrium condition for the time worked variable for the second family member may have occurred. Suppose w_{21} was the highest wage available for this family member; then the relevant condition in the optimal solution would have been $\lambda_2^o - w_{21}\lambda_3^o > 0$. The effect of lowering w_{11} has been to increase λ_3 and decrease λ_2 which may make this condition zero and push the second member into the labor force. With no further considerations, this amounts to a clear cut theoretical statement of the Woytinsky additional worker hypothesis. That is, negative wage changes to the employed primary labor force should, through the effect on the marginal utilities of time and income, push some additional ("secondary") workers into the labor force; with no changes in tastes, increases in wages should lead to withdrawal of some or all of those secondary workers. The hypothesis is broader than this, too; it says that either secondary members enter the labor force in order to maintain a stable family income or the demand for income-intensive commodities falls.

This can be illustrated further. Consider Z_j where $1 \leqslant j \leqslant j_1$ and $j \epsilon J$, i.e., $Z_j^o > 0$. The optimal condition for Z_j^o was, $t_j\lambda_1^o - c_j\lambda_3^o = \Phi_j$. After the change in w_{11} and in terms of (32), the left side of this becomes $t_j(\lambda_1^o + \Delta\lambda_1) + c_j(\lambda_3^o + \Delta\lambda_3)$. It is clear that the optimal condition for Z_j will remain an equality only if $t_j\ \Delta\lambda_1 + c_j\Delta\lambda_3 = 0$; put otherwise, this is $\frac{c_j}{t_j} = -\frac{\Delta\lambda_1}{\Delta\lambda_3}$. If $\frac{c_j}{t_j} > -\frac{\Delta\lambda_1}{\Delta\lambda_3}$, Z_j must go to zero or at least decline; in this case Z_j would be called normal and relatively income intense. On the other hand, if $\frac{c_j}{t_j} < -\frac{\Delta\lambda_1}{\Delta\lambda_3}$, there would be forces operating to increase Z_j and, in this case, Z_j would be referred to as being inferior and of relatively low income intensity or relatively time intense. For $j \epsilon \hat{J}$ a similar set of arguments holds. That is, for Z_j in which $j \epsilon \hat{J}$ and the optimal condition was $t_j\lambda_1 + c_j\lambda_3 > \Phi_j$; if $\frac{c_j}{t_j} > -\frac{\Delta\lambda_1}{\Delta\lambda_3}$, Z_j remains at zero and would behave as a normal activity. On the other hand, if $\frac{c_j}{t_j} < -\frac{\Delta\lambda_1}{\Delta\lambda_3}$, Z_j may go positive and display the inferior quality. For Z_j in which $j_1+1 \leqslant j \leqslant n$, the same arguments pertain since $\Delta\lambda_2 < 0$ (as was $\Delta\lambda_1$) and $\Delta\lambda_3 > 0$ for $\Delta w_{11} < 0$. The source of these effects on the activity set is, again, changes in w_{11} (for the present case) which affect the relevant marginal utilities and hence affect the optimal conditions for the Z_j. Any particular effect depends on the ratio $\frac{c_j}{t_j}$ in relation to, e.g., $\frac{\Delta\lambda_1}{\Delta\lambda_3}$. We have seen that where wages are reduced and where $\frac{c_j}{t_j}$, is larger than $-\frac{\Delta\lambda_1}{\Delta\lambda_3}$, Z_j either remains at zero or is reduced and possibly to zero; such activities can be regarded as relatively income intensive. A perfectly symmetrical counter argument holds for increases in wages.

The implied possible effects of wage changes on the optimal conditions in the activity set rings a familiar tone. That is, falling wages and income lead to reduced consumption of income-intensive activities–the "income effect" here implies that such activities are normal in the usual sense of the term. Conversely, falling wages and income lead to increased consumption of time-

intensive activities–the "income effect" is negative and implies that such activities are inferior in the usual sense and there is nothing in the assumptions underlying U_k and F which preclude the presence of inferior activities. Another, and perhaps better, way of describing the effects just observed is to say that activities with "large" $\frac{c_j}{t_j}$ become relatively expensive, while those with "small" $\frac{c_j}{t_j}$ become relatively inexpensive, and the implied direction of adjustment is just what we would expect.

Before terminating this discussion on the effects of wage changes, let us pursue one more thread. Rather than confine the assumed wage change to w_{11}, it might be worth while to consider the effects of a "general" wage change. That is, what if both w_{11} and w_{21} (w_{ks} in general) change uniformly?

Begin by assuming that an optimal solution exists for a problem like (23); in the Kuhn-Tucker conditions of (25) there is a set of activities, $j \epsilon J$, that enter Z^O positively and a second set, $j \epsilon J$, which enters Z^O at the zero level. In addition, assume $\lambda_1^o - w_{11}\lambda_3^O = 0$ and $\lambda_2^o - w_{21}\lambda_3^O > 0$–the first family member works in the first occupation and the second member does not work for pay. In the initial equilibrium the vector, λ^o, is defined as in (31). Now, suppose there is a negative uniform wage change for all occupations. In this case, even if the wage change did not alter the optimal conditions (i.e., the Kuhn-Tucker conditions), we would observe the same changes in λ as are expressed in (32). That is, $\Delta\lambda_1 < 0$, $\Delta\lambda_2 < 0$, and $\Delta\lambda_3 > 0$. The first member works less but is still in the labor force. But in this case, the reduction in λ_2 and increase in λ_3 are less likely to make $\lambda_2^* - w_{21}^* \lambda_3^* = 0$ since w_{21} has fallen also. Further, if any of the optimal conditions are affected, time-intensive activities will be affected positively (i.e., increase) and income-intensive activities will be affected negatively (i.e., decrease) as before. On the other hand, for a uniform increase in wages we would observe $\Delta\lambda_2 > 0$, $\Delta\lambda_3 < 0$, and $\Delta w_{21} > 0$. The sign of $\lambda_2^* - w_{21}^* \lambda_3^*$ is not clear but, from the analysis above, those time-intensive activities that had behaved like inferior activities for wage reductions will, if affected, tend to be reduced in their consumption. A uniform wage increase has, then, generated an increase in λ_2 (i.e., $\Delta\lambda_2 > 0$) and this implies that consumption time has been reduced. But whether the change in λ_2 is sufficiently large, in conjunction with a change in λ_3 of the

opposite direction, to make $\lambda_2^* - w_{21}^* \lambda_3^* = 0$ is not clear. However, if there are sufficient time-intensive and inferior activities represented in $j \epsilon J$, $j_1+1 \leqslant j \leqslant n$, there is reason to believe that the second family member may enter the labor force. For a uniform change in wages, then, the hypothesis that emerges is effectively Long's discouraged worker hypothesis. And the choice between these hypotheses depends on whether one assumes that wage changes affect only the employed family member or are uniform, and whether the optimal conditions are affected. If Z^o includes inferior and time-intensive activities at the positive level, the implications that emerge for any specific change are accentuated.

V. EMPIRICAL IMPLICATIONS AND CONCLUSIONS

Wage rates (or average earnings) and unemployment are but two of many recognized factors that influence the aggregate labor supply relation. In this monograph the focus on these two variables, rather than on the multitude of possible determinants of the labor supply, is explained by apparent short-term effects of changes in these variables. While any variation in the labor supply asks for explanation, it is the short-run variation of the labor force that can affect indexes of the nation's economic health in such a manner so as to generate appeals for corrective counter measures. In particular, changes in the measured rate of unemployment, which can result from a change in short-run supply as well as from a change in short-run demand, is regarded as indicative of changes in the health of the economy and, by way of the Employment Act of 1946, can initiate implementation of a countering policy. Also, changes in the short-run labor supply can have an indirect effect on price levels in the system and, because of a national commitment to "stable" prices, can summon counteraction in labor markets. For these reasons, then, changes in wage rates and/or unemployment rates can play a prominent role in determining short-term economic policy.

As was indicated in Chapter II, considerable effort has been spent on explaining the character of the relation between the aggregate labor force participation rate or other, more specific participation rates and the variation in wage rates (or average earnings) and unemployment. However, these efforts have been along statistical lines, and little or no advance has been realized in the theoretical model upon which the interpretation of any statistical result is based. As mentioned above, the standard income-leisure model from which labor supply implications emerge has been subject to some very pointed criticism due to several properties of the model. These properties are, again, (1) the inherent necessity to generate interior solutions in commodity space and, therefore, the requirement that everyone work; (2) the specification of the model in a manner such that the micro unit is restricted to the individual when the general consensus among economists is that a family is the relevant unit of analysis in explaining variation in the labor supply; and (3) although less disagreeable than properties (1) and (2) above, the model has utility obtained by "consuming" income and leisure when utility is customarily re-

garded as deriving from the consumption of "commodities," and thereby the model establishes a very indirect reasoning process en route to the maximization of utility.

Gary Becker's model, in his 1965 article [3], appeared to offer relief from the restrictions posed by these properties if it was modified to incorporate the family. The explicit inclusion of time as a resource constraint and the specification of consumption activities as being "produced" by the individual through production functions in which both market goods and time are inputs was particularly innovative and promised empirical implications not available in the standard model.

In Chapter IV and as a first approach to the modified Becker model, we attempted to apply the calculus to the model, but, for a variety of reasons, this approach began breaking down before optimal conditions could be derived. In particular, the calculus could not assure us that both time and income constraints were not violated for an individual. When the problem is expressed in terms of a family, Becker's "full income" constraint treats time as additive within the family, implying that the time of one family member may be substituted for another. By assuming that the constraints are not violated, analysis is permitted to proceed to the establishment of the necessary optimal conditions. But, the derivation of post-optimal implications from the necessary conditions was complicated and even precluded by the vector-defined character of the model.

In recognition of these problems, the economic problem around which the model was built was viewed as a nonlinear (concave) programming problem. With this approach, the time each family member works is an endogenous variable in the system, and, hence, the problem is to allocate time between work and consumption activities in a manner that maximizes family utility. Each time constraint is recognized explicitly and the opportunity to "trade" time between family members does not exist. By way of the Kuhn-Tucker analysis, the optimal conditions for a maximum contain the Lagrangian multipliers which, in the present analysis, are imputed prices or opportunity costs of time and income, and, under appropriate assumptions, these multipliers are unique.

In order to consider the theoretical effects of post-optimal

parameter changes, it was necessary to convert the problem to a linear programming analysis in which the Kuhn-Tucker conditions from the nonlinear problem are the constraints and the Lagrangian multipliers are the endogenous variables. The objective function in this case is expressed in differential form because the Kuhn-Tucker conditions are only expected to hold in some neighborhood around the optimal solution, and this neighborhood must lie in the intersection of the hyperplanes formed from the constraints and the family utility function evaluated at the optimal point. In this form the equilibrium of the system could be displaced by imposing various "shocks" on it. For purposes of the present analysis, it is the variation of wage rates that is important.

This post-optimality analysis led to a set of empirical implications regarding the effects of wage changes on labor force participation. These implications were as follows: First, wage reductions experienced by, what we will call, employed primary wage earners in a family will affect the relevant marginal utilities of income and time in a manner so as to *increase* the participation of secondary wage earners, assuming that the wage changes do not alter the optimal conditions in the activity set. This is, of course, the additional worker hypothesis, and it postulates that labor force participation varies inversely with wages. The hypothesis does not predict any marked shift in the demand for market commodities. Second, if wage changes are assumed to be uniform in a family and all optimal conditions are recognized as possibly being affected by such changes and if the family's activity set is dominated by "normal" activities (i.e., activities for which consumption varies positively with income), it is less likely that secondary wage earners will alter their work status and more likely that these wage changes and subsequent changes in the vector, λ will push a family's consumption away or toward income-intensive activities and toward or away from time-intensive activities.

Put otherwise, this hypothesis is close to the Long position and says that labor force participation may vary positively with the demand for income-intensive market purchases. The strongest form of this second hypothesis emerges in the case where the optimal activity set includes many time-intensive inferior activities. In this case wage reductions are more likely to increase the consumption of income-intensive activities. Wage increases, on

the other hand, would produce the opposite set of sign relationships. This form of the second hypothesis supports the prediction that *both* labor force participation and the consumption of income-intensive market commodities vary positively with wage changes.

From a theoretical perspective, these hypotheses effectively polarize the Woytinsky and Long controversy. Perhaps more importantly, an additional discriminating dimension has been added by the "either/or" character of the hypotheses. That is, either wage changes affect labor force participation inversely or the demand for certain classes of activities must vary in a predictable manner. For policy purposes and in relation to the problems of empirically testing and interpreting the hypotheses, this additional dimension is a welcome adjunct because it simplifys the specification of a statistical relation and provides a broader framework within which to interpret what empirical data reveal. As a further testimony to the merit of the hypotheses emerging from the above model, we might point out that only one hypothesis need be tested because the failure to refute one implies refutation of the other. This, of course, greatly simplifies the problems of testing.

The present analysis will not undertake the task of empirical testing. However, from the point of view of methodology, it is important that the predictions generated above lend themselves to meaningful testing, and a brief discussion of this problem is appropriate. One approach to this problem would be say, to concentrate on the second of the above hypotheses and, after making appropriate aggregation assumptions, specify a regression relation between the aggregate labor force participation rate, on the one hand, and a measure of wage rates as well as a measure of the demand for some income-intensive class of market commodities. The Bureau of Labor Statistics publishes, on a monthly basis in *Employment and Earnings*, participation rate data as well as a set of figures on average hourly earnings in various industries. Of this set, average hourly earnings in manufacturing is the most common surrogate for the general reference "wage rate." For the last variable in this analysis, estimated retail sales in durable goods stores seems to be a usable proxy. These data appear in the *Survey of Current Business* monthly.

Although regression results from such a procedure might prove

interesting and informative, they could not provide a convincing explanation of labor force participation rate variation. The reason for this lies along two lines. First, while theory specified wages and expenditures on income-intensive commodities as independent variables in the labor force participation relation, we must be aware that theory could also specify a host of other partial relations that would be reflected in a more general labor (force) supply relation. Some variables in this unspecified set can be conceived as influencing the effects of wage or income-intensive market expenditure changes on labor force participation. To implicitly hold this other set constant may produce grossly misleading results. In short, the failure to more completely specify the supply relation can be expected to produce erroneous or biased regression coefficients.

Second, there is the simultaneity problem and all that goes with it. Economically, there must be a demand relation that, with its supply counterpart, explains the relevant observable data. In particular, these two relations together must certainly account for average hourly earnings as well as unemployment, which is usually included in estimates of the supply relation. The mere existence of a demand relation, which confronts us with the difficulties of simultaneous estimation, also raises the identification problem. That is, if we had proceeded to estimate a single supply equation—and this is the technique employed by those researchers reviewed in Chapter II who used regression techniques—there would be the overriding question of whether the results pertained to the supply relation or to the unrecognized demand relation.

With these observations and for purposes of this presentation, it is sufficient to say that the concepts involved in the model's predictions have empirical counterparts, although the available aggregates may be less desirable than, say, precise micro data. Also, the model's predictions, although expressed in a partial equilibrium framework, are simple and unambiguous. To the extent that one or the other of the above hypotheses fails to be refuted by empirical data, we can say that the model has enlarged our understanding of the behavior that underlies a portion of the labor supply decision, and in terms of making rational policy decisions it is an understanding of the behavior that is important—not merely an awareness of statistical correlations.

In conclusion, two features of the above model are worth point-

ing out. First, the character of the model does not preclude its use for deriving other partial effects in relation to time allocation. Examples of other questions might include the effects of changes in debt-income ratios or family size. Second, it may be recalled that the latter of the model's two hypotheses was suggested to be stronger if a number of time-intensive inferior activities were included in the optimal solution set at positive levels. Given the manner in which consumption activities were defined as being "produced" by combining time and market commodities through some production function and given the role that inferior activities could play in the analysis, it seems clear that the phenomenon of inferior activities should be subjected to additional economic inquiry. In the area of manpower analysis and particularly for understanding observed labor force behavior, the inferior activity concept seems to offer a useful frame of reference, especially when analysis proceeds along socio-economic lines.

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