


AN ABSTRACT OF THE THESIS OF

  
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(Name) (Degree) (Major)

Date thesis is presented MAY 14 1966

Title RELATIONSHIPS BETWEEN THERMAL AND  
ELECTRICAL CONDUCTIVITIES OF OCEAN SEDIMENTS  
AND CONSOLIDATED ROCKS

Abstract approved Redacted for privacy

From measurements of thermal and electrical conductivities of 64 ocean sediment samples obtained from piston cores taken off the Oregon Coast, and from 37 water-saturated sandstone samples analyzed by Zierfuss and Van der Vliet (1956), as well as 51 thermal conductivities and water contents of ocean sediments analyzed by Ratcliffe (1960), this research shows that a useful relationship can be obtained giving thermal conductivity when electrical conductivity is known. Analysis of the data was made using theoretical concepts which have been known for many years to relate thermal and electrical conductivity to porosity. The results of this research may make possible a convenient determination of in situ thermal conductivity that would give the average conductivity in materials containing large

variations in conductivity. A more exact determination of thermal conductivity from electrical conductivity can be made if the general physical properties of the materials are known.

RELATIONSHIPS BETWEEN  
THERMAL AND ELECTRICAL CONDUCTIVITIES  
OF OCEAN SEDIMENTS AND CONSOLIDATED ROCKS

by

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A THESIS

submitted to

OREGON STATE UNIVERSITY

in partial fulfillment of

the requirements for the

degree of

MASTER OF SCIENCE

June, 1966

APPROVED:

Redacted for privacy

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Date thesis is presented 5/14/66

Typed by Jamie Templeton

## ACKNOWLEDGMENTS

Sincere appreciation is extended to Dr. Joseph W. Berg, Jr., under whose direction this work was performed, for his thoughtful guidance during the development and preparation of this thesis.

The writer would like to express his thanks to Dr. LaVerne D. Kulm, who performed a microscopic analysis of the ocean sediments discussed in Section IV and offered many helpful suggestions during the course of the work, and to other staff members and graduate students of the Geological Oceanography Group, Oregon State University.

Dr. Gunnar Bodvarsson of the Geophysics Research Group, Oregon State University, read this thesis and, in addition, was responsible for many valuable suggestions throughout the course of the work.

Gratitude is also extended to Dr. Larry Boersma, Department of Soils, and to Dr. Robert E. Meredith, Department of Chemical Engineering, Oregon State University, and to Dr. George V. Keller, Colorado School of Mines, for helpful conversations during the course of the work.

This research was sponsored by the Office of Naval Research, Contract Nonr 1286 (10), and the National Science Foundation, Grant No. GP4642.

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# RELATIONSHIPS BETWEEN THERMAL AND ELECTRICAL CONDUCTIVITIES OF OCEAN SEDIMENTS AND CONSOLIDATED ROCKS

## I. INTRODUCTION

### Purpose of the Study

The object of this work is to determine whether the thermal conductivity of ocean sediments or solid rocks can be ascertained from a knowledge of their respective electrical conductivities. Thermal conductivities of earth materials are important from the standpoint of obtaining values for the heat flow through the earth's crust. The product of two experimentally observed quantities, geothermal gradient and thermal conductivity, gives the heat flow.

Current practice in ocean-bottom heat-flow measurement is to obtain a core sample of the bottom sediment simultaneously with a measurement of the geothermal gradient. The sample is later analyzed by a direct or indirect method to determine the thermal conductivity of the material in which the gradient was measured. Ratcliffe (1960) made a study of ocean sediments and derived an empirical relationship between thermal conductivity and moisture content. This water content method has been used for obtaining sediment thermal conductivities where direct conductivity-measuring equipment is not available.

In the case of heat-flow measurement on land, where thermal gradient is measured in a drill hole, thermal conductivity is determined using small samples of rock from the hole. In-situ thermal conductivity

measurements have been made in boreholes (Beck, 1965), using a large-scale adaptation of the needle-probe method (Carslaw and Jaeger, 1959), but they are time-consuming, require large amounts of electric power, and are somewhat cumbersome.

One of the properties of porous materials closely allied with thermal conductivity is electrical conductivity. Both thermal and electrical conductivity of aggregate materials can be related to the volume composition by potential theory, as can similar properties such as dielectric constant or magnetic permeability. Should a relationship between electrical and thermal conductivities of earth materials be determined within sufficiently narrow limits, several advantages could be realized over the present methods used to determine thermal conductivity for heat-flow purposes, as follows:

1. The thermal conductivity could be easily measured in situ by applying techniques presently in use in oil-well logging practice to measure electrical conductivity;
2. The thermal conductivities so determined would represent an average over a considerably larger volume of material than those obtained by present techniques using small samples; and
3. By applying such a relationship to electrical logs presently in existence for some thousands of oil wells, thermal conductivities for the materials in which these wells have been drilled could be determined.

## Previous Work

To the best of the author's knowledge, no investigation attempting to relate electrical and thermal conductivities of aggregates of materials as such has yet been conducted. However, the problem of determining the bulk conductivities (electrical or thermal) of multi-phase systems has been a subject of investigation in classical physics since the late nineteenth century.

Theoretical relationships between bulk conductivities of materials and the properties and amounts of their components exist in abundance. An excellent review of the various theoretical models proposed before 1959 (which number at least thirty), is given by Meredith (1959). Only those theories which are considered most relevant to the present study will be discussed in this thesis.

In the following discussion, the nomenclature used by Meredith will be followed. In this system, the lower case Roman letter k is used to denote conductivities measured in the customary units for such quantities, i. e., gram-calories/cm-sec °C for thermal conductivity, and mhos/meter for electrical conductivity. The subscript e denotes an electrical quantity, the subscript t a thermal quantity. In discussion relating to two-phase systems, the subscript d refers to the discontinuous phase, or, in this case, the individual mineral grains. The subscript c refers to the continuous phase, i. e., the saturating fluid.

The subscript m refers to the bulk properties of the mixture. In the case of two-phase systems, the quantities  $\underline{k}_d$  and  $\underline{k}_m$  are normalized by dividing by the conductivity,  $\underline{k}_c$ , of the continuous phase. All normalized quantities are denoted by an upper case Roman letter K, e. g.,  $\underline{K}_{me}$  refers to the normalized bulk electrical conductivity of the two-phase mixture in question. In addition the Greek letter  $\phi$  is used to indicate porosity, expressed as the decimal fraction of the total volume occupied by void space.

A rough approximation to the conductivity of a mixture, which has been in general use for many years, can be derived by assuming a model composed of a stack of laminae of materials of different conductivities whose sides are parallel to the flow of heat or electrical current. In the case of a two-phase system, this "parallel" conductivity, which, for a given grain conductivity,  $\underline{K}_d$ , is the maximum that can be expected for the mixture, is given by

$$\underline{K}_m = \underline{K}_d + \phi(1-\underline{K}_d) . \quad (1)$$

Another approximation in common use represents a mixture by a "series" arrangement of laminae whose sides are perpendicular to the flow of heat or current. The bulk conductivity given by this model is the lowest conductivity which can be expected for the mixture for a given grain conductivity,  $\underline{K}_d$ . In the case of a two-phase system, the series conductivity is given by

$$K_m = \frac{K_d}{1 + \phi (K_d - 1)} \quad (2)$$

The earliest published work on bulk properties of mixtures was done by H. A. Lorentz (1880), L. Lorenz (1880), and J. C. Maxwell (1881), and can be found in the 1892 edition of Maxwell's classic Treatise on Electricity and Magnetism. By considering the potentials due to an assemblage of isolated spheres, Maxwell and the others arrived independently at the relation given by

$$K_m = \frac{3K_d - \phi (K_d - 1)}{3 + 2 \phi (K_d - 1)} \quad (3)$$

which has been transformed to agree with the present system of notation. The Maxwell equation is somewhat unrealistic, since it assumes that the individual spheres comprising the mixture are so far apart that their fields do not interact with one another. The relation is thus expected to apply only to aggregates of high porosity. In spite of these limitations, the equation has been successfully applied to various experimental systems.

Lord Rayleigh (1892) derived an equation for the conductivity of a mixture by assuming a cubical packing of spheres and determining the effect of the interaction of the fields of all the spheres within a finite radius of the one under consideration. I. Runge (1925) discovered an error of a factor of  $\pi$  in one of the coefficients of the equation as given by Rayleigh. The Rayleigh equation as modified by Runge is given here:

$$K_m = 1 - \frac{3(1-\phi)}{\frac{2+K_d}{1-K_d} + (1-\phi) + 0.525(1-\phi)^{10/3} \frac{K_d^{-1}}{K_d + 4/3}} \quad (4)$$

This equation is limited in application to those systems having porosities greater than 0.38, the value for cubical packing.

D. A. G. Bruggeman (1935) developed an equation by expanding the Maxwell relation in a Taylor series and neglecting all but the first two terms of the expansion. In essence his work assumes a model in which a very large range of spherical particle sizes is considered, Bruggeman's equation is

$$\phi = \frac{K_m - K_d}{K_m^{1/3} (1 - K_d)} \quad (5)$$

This equation is of more practical value than those of Maxwell and Rayleigh, since it extends to the range of low porosity.

G. E. Archie (1942) developed a semi-empirical relationship which has been useful in the interpretation of electrical well logs. In the form given here,  $\frac{K}{m}$  corresponds to the reciprocal of the quantity known to petroleum geophysicists as the formation resistivity factor. The parameter  $m$ , sometimes called the cementation factor, is an arbitrary number, usually approximately 2.0 in the electrical case, which is assumed to be constant for any particular sedimentary formation. The Archie equation is

$$K_m = \phi^m \quad (6)$$



Implicit in the Archie equation is the assumption that the electrical conductivity of the grains is zero, since the bulk conductivity given by the equation is zero for zero porosity. In practice, it is observed that more than one value of electrical conductivity is possible for a given porosity value. To explain this variation, the changes in geometry of the rock framework are taken into account. The variations in cementation or interpenetration of the grains in consolidated rocks cause changes in the value of  $\underline{m}$ .

To describe variations in the geometry of the rock framework, a factor known as the "tortuosity,"  $\underline{\tau}$ , can be introduced into the parallel conductivity equation (1). This factor provides for the increased length of the path which current must take in traversing the crooked path through the rock matrix. It can be visualized as the ratio of the average path length taken by the current to the actual length of the sample measured.

Winsauer et al. (1956) devised a method of direct measurement of the tortuosity using determinations of transit times of chloride ions through a sample subjected to an electric field. From measurements of 29 samples, Winsauer et al. obtained the following relationship involving porosity, electrical conductivity, and tortuosity:

$$\tau = \left[ \frac{\phi}{K_{me}} \right]^{0.60} \quad (7)$$

When the tortuosity factor is applied to Equation (1), the following relation results:

$$K_m = \frac{K_d + \phi(1-K_d)}{\tau} \quad (8)$$

Woodside and Messmer (1961) give as an approximation to the conductivity of a two-phase system

$$K_m = K_d^{(1-\phi)} \quad (9)$$

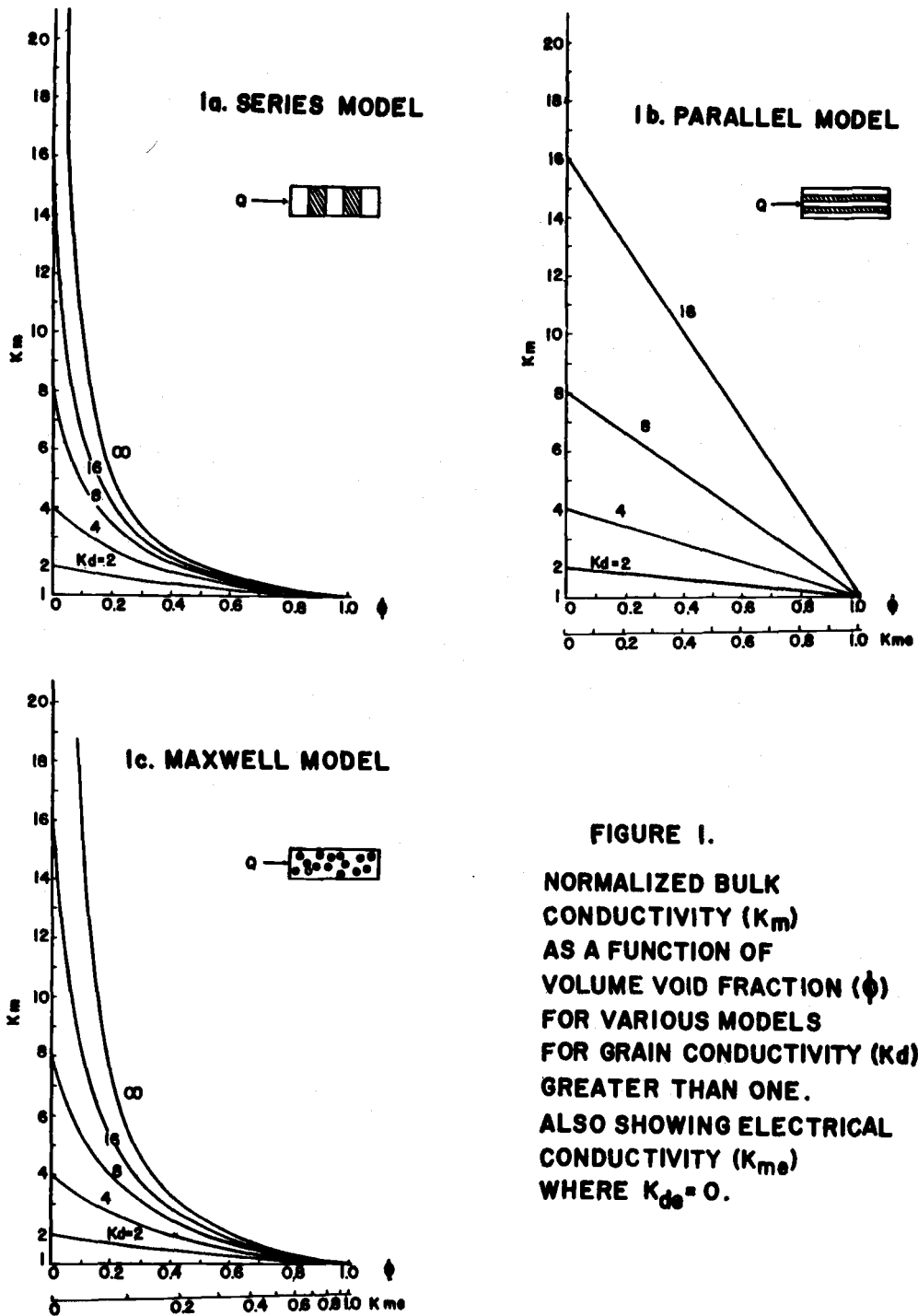
This equation is referred to as the "geometric" equation, because it postulates that the bulk conductivity of a multiphase system is given by the weighted geometric mean of the conductivities of the phases. While it is not based on a theoretical derivation, other considerations show the geometric equation to be a valid conductivity relation if the conductivities of the phases do not differ by more than an order of magnitude.

## THEORETICAL BACKGROUND FOR DATA ANALYSIS

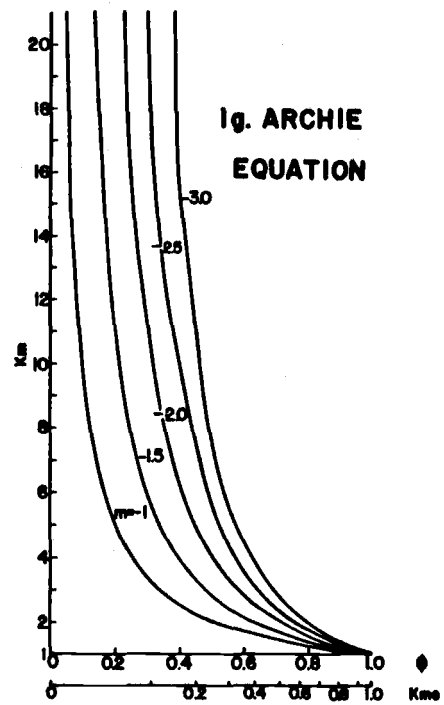
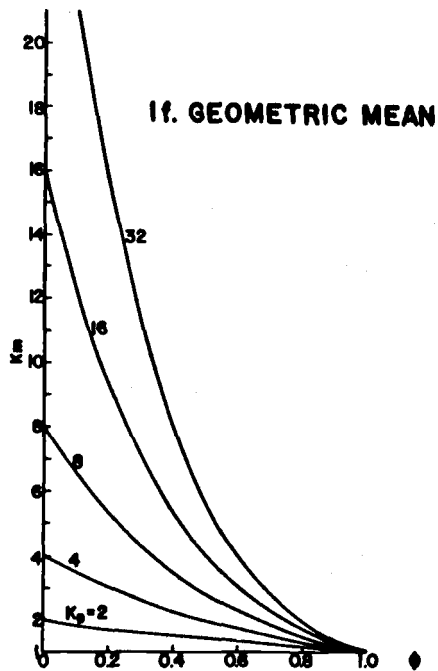
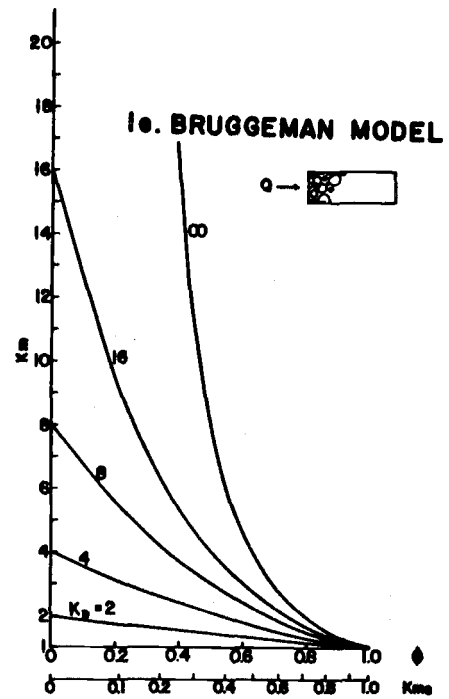
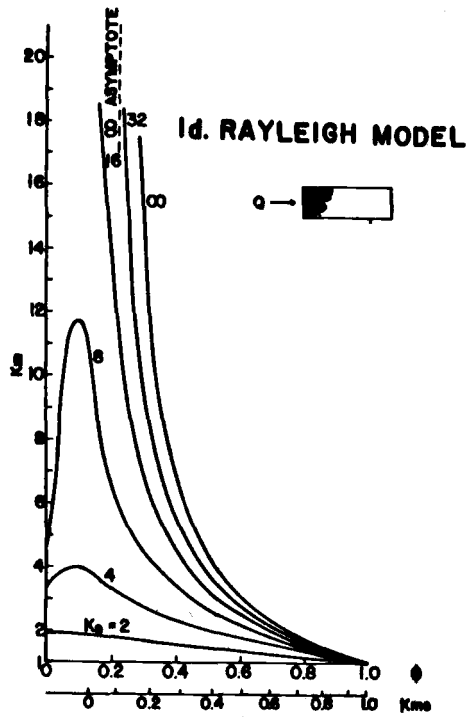
### Thermal Conductivity

In the case of the thermal conductivity of earth materials saturated with water solutions, one would expect the conductivity of the discontinuous phase to be greater than that of the continuous phase (i. e.,  $\underline{K}_{dt} > 1.0$ ). Furthermore, a wide range of values of  $\underline{K}_{dt}$  would be expected, since the thermal conductivity of water at room temperature and one atmosphere pressure is about 1.40 mcal/cm-sec° C (Nukiyama and Yoshizawa, 1934), and mineral conductivities range from about 3.6 to about 17mcal/cm-sec° C for clay and quartz, respectively (Birch, Schairer, and Spicer, 1942, and Beck, 1965).

Figure 1, a through g, shows theoretical curves of thermal conductivity as a function of porosity for various values of  $\underline{K}_{dt}$  which might be encountered in practice. Curves are given for the series, parallel, Maxwell, Rayleigh, Bruggeman, and Woodside and Messmer models. Archie's equation is also plotted to show the effect of variation of his parameter  $\underline{m}$ . A scale giving electrical conductivity, the significance of which is discussed in a later section of this thesis, also appears at the bottom of the graphs for some of the models.



**FIGURE 1.**  
 NORMALIZED BULK  
 CONDUCTIVITY ( $K_m$ )  
 AS A FUNCTION OF  
 VOLUME VOID FRACTION ( $\phi$ )  
 FOR VARIOUS MODELS  
 FOR GRAIN CONDUCTIVITY ( $K_d$ )  
 GREATER THAN ONE.  
 ALSO SHOWING ELECTRICAL  
 CONDUCTIVITY ( $K_{me}$ )  
 WHERE  $K_{d0} = 0$ .



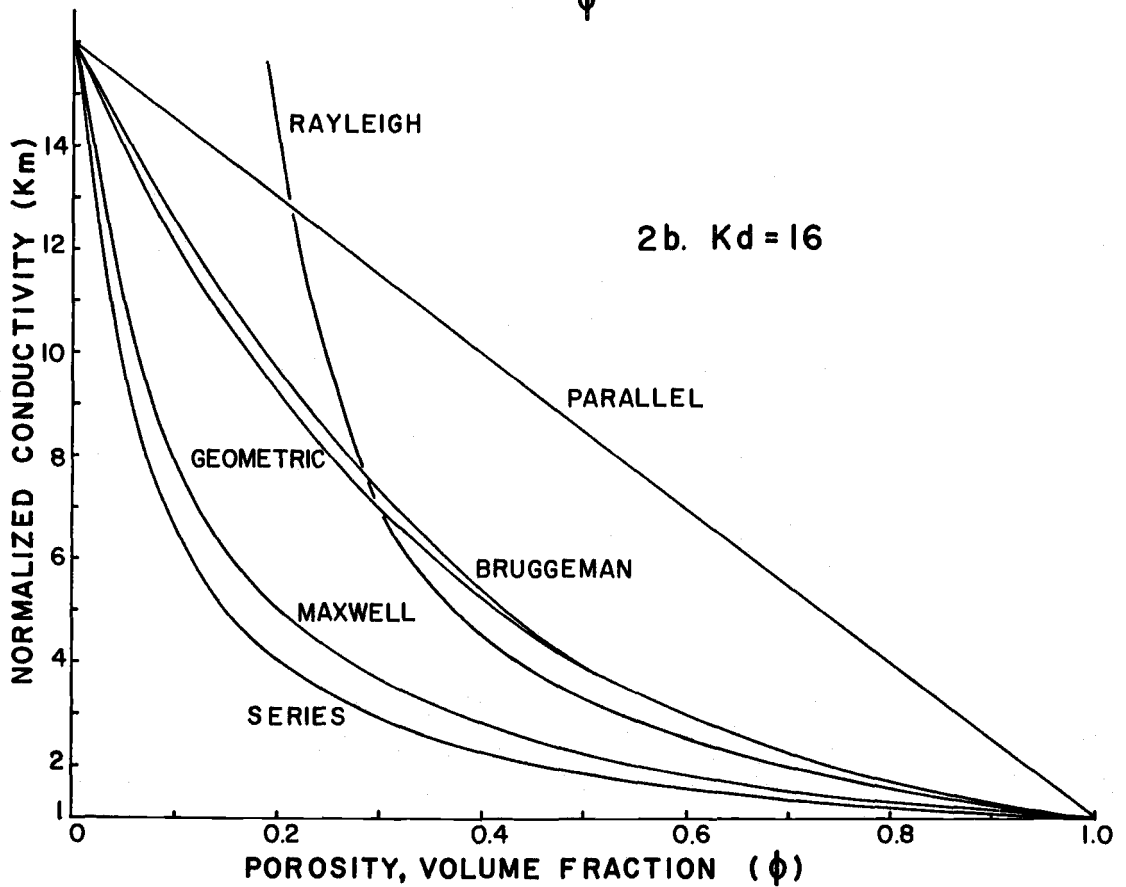
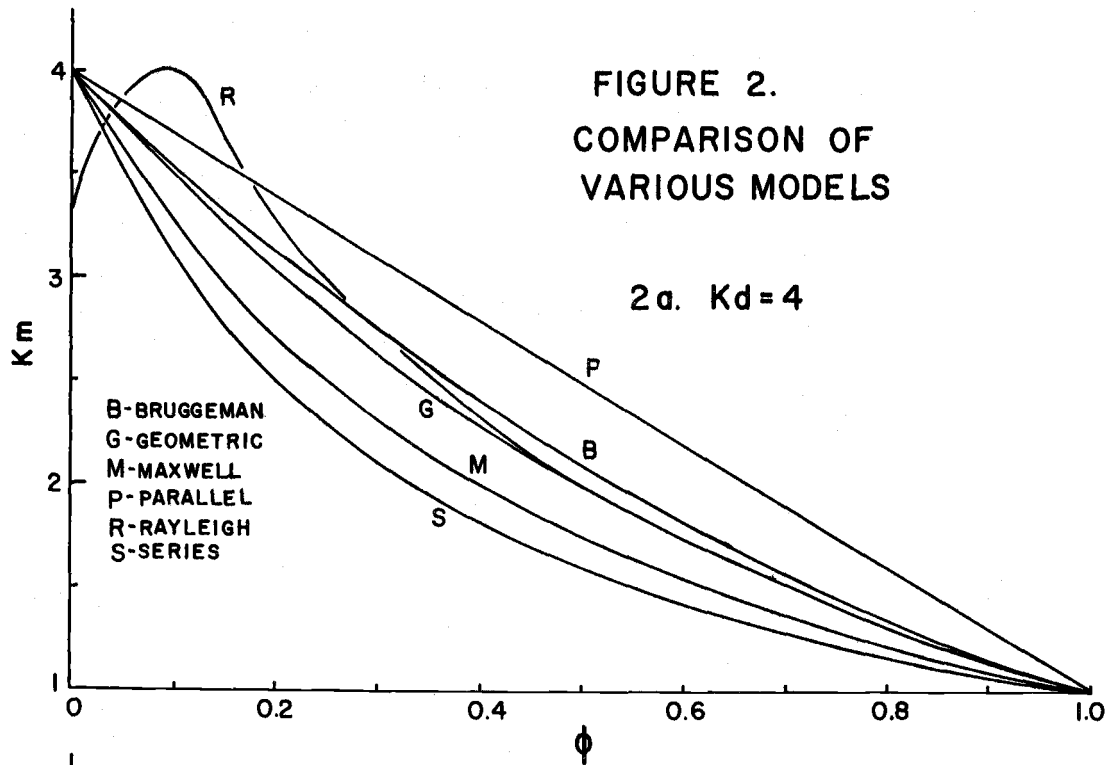
All of these equations yield curves of similar shape. In the cases of the series, Maxwell, Rayleigh, and Bruggeman equations, a limiting curve is reached as the grain conductivity,  $K_{dt}$ , approaches infinity. For the remaining equations, such a limit is not reached. The Rayleigh equation yields erratic results for porosities less than about 0.38, which is to be expected since this equation was derived with the assumption of cubical packing. The Archie equation also yields questionable results for low porosities (see Figure 1g), since it gives an infinite thermal conductivity as the porosity approaches zero.

In Figures 2a and 2b, a comparison among the various equations being considered is presented for the extreme values of  $K_{dt}$  likely to be encountered in a study of water-saturated earth materials. The discrepancy among the values predicted by the various models appears to increase with increasing  $K_{dt}$ . The agreement between the Bruggeman and geometric curves and, for high porosities, of these two curves with the Rayleigh curve, is striking.

### Electrical Conductivity

In the case of water-solution-saturated non-metallic earth materials, one could expect the electrical conductivity of the discontinuous phase to be several orders of magnitude smaller than that

FIGURE 2.  
COMPARISON OF  
VARIOUS MODELS



of the saturating solution, and the grain conductivity,  $\underline{K}_{de}$ , would approach zero. If clay minerals were present in appreciable amounts, however, the grain conductivity could become significant (Patnode and Wyllie, 1950).

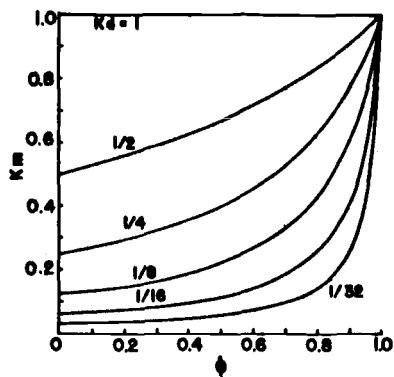
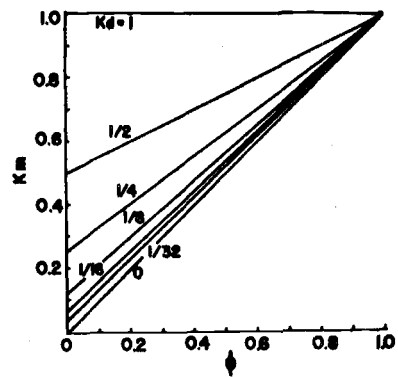
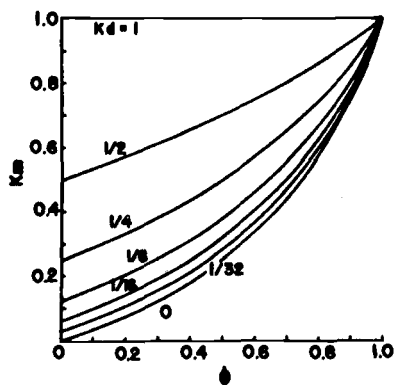
Plots of electrical conductivity as a function of porosity for various values of  $\underline{K}_{de}$  less than one are given in Figures 3a to 3g. The Rayleigh equation again gives erratic results for low porosities. The Archie equation gives zero conductivity for zero porosity for all values of  $\underline{m}$ , and therefore applies strictly only in cases where the grain conductivity is zero. Limiting curves are obtained for zero grain conductivity for all equations excepting those of the series, geometric, and Archie models.

In Figure 4, curves of the various models are plotted for comparison with a grain conductivity,  $\underline{K}_d$ , of  $1/32$ . Agreement between the Rayleigh and Bruggeman curves is still good for high values of porosity, but the geometric curve does not agree with these as it did in the case of thermal conductivity (see Figure 2).

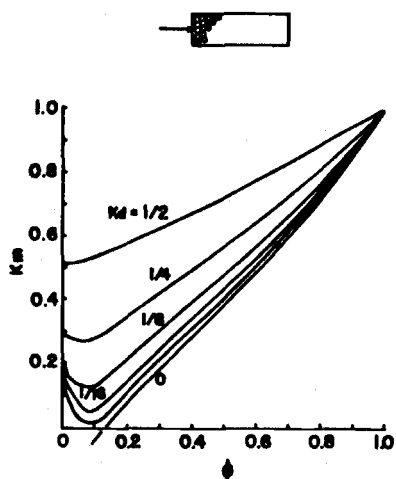
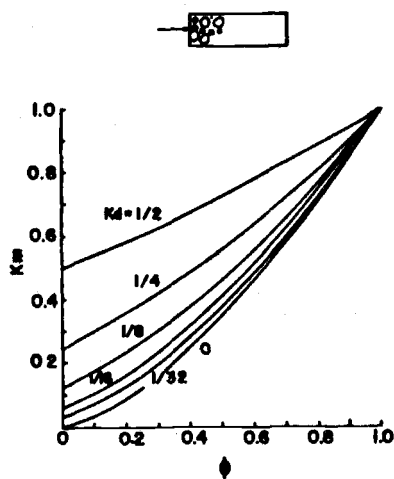
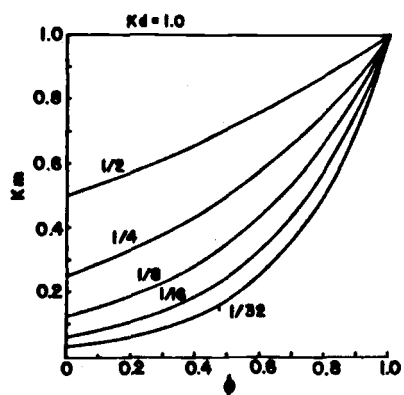
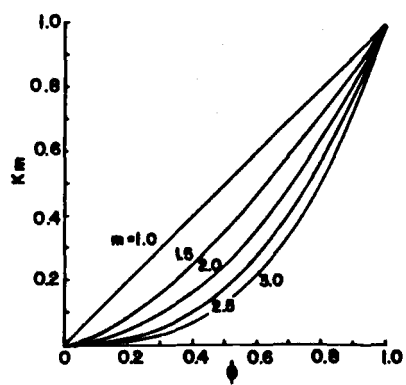
### Electrical and Thermal Conductivity

By assuming a zero electrical conductivity for the grains of the materials under consideration, it is possible to derive direct relationships between electrical and thermal conductivity for those models in



**3a. SERIES MODEL****3b. PARALLEL MODEL****3c. MAXWELL MODEL**

**NORMALIZED BULK  
CONDUCTIVITY ( $K_m$ )  
AS A FUNCTION OF  
VOLUME VOID FRACTION ( $\phi$ )  
FOR VARIOUS MODELS  
FOR GRAIN CONDUCTIVITY ( $K_d$ )  
LESS THAN ONE.**

**3d. RAYLEIGH MODEL****3e. BRUGGEMAN MODEL****3f. GEOMETRIC MEAN****3g. ARCHIE EQUATION**

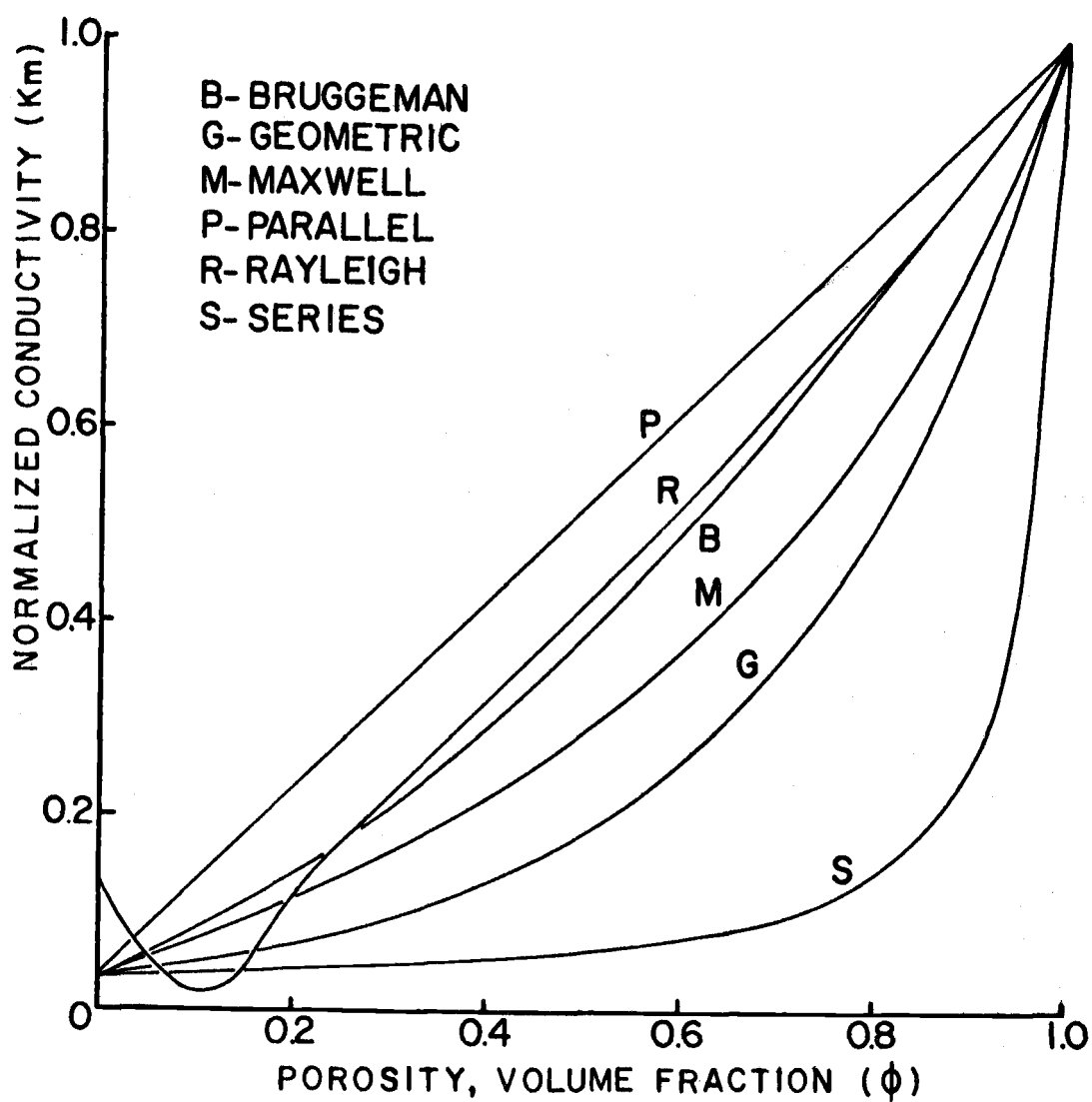


Figure 4. Comparison of various models for  $\underline{K}_d = 1/32$

which a limiting curve exists for  $\underline{K}_{de}$  equal to zero. This is done by eliminating porosity between the thermal and electrical conductivity equations with  $\underline{K}_{de} = 0$ . This has been done in Equations 10 to 13 below.

$$\text{(Parallel)} \quad K_{mt} = K_{me} (1 - K_{dt}) \quad (10)$$

$$\text{(Maxwell)} \quad K_{mt} = \frac{K_{me} (K_{dt} + 1) + K_{dt}}{2 K_{me} K_{dt} + 1} \quad (11)$$

$$\text{(Bruggeman)} \quad K_{me} = \left[ \frac{K_{mt} - K_{dt}}{K_{mt}^{1/3} (1 - K_{dt})} \right]^{3/2} \quad (12)$$

$$\text{(Archie)} \quad K_{mt} = K_{me} \frac{m_t}{\underline{m}_e} \quad (m_c \neq 0) \quad (13)$$

The curves which result are similar to those of Figure 1. An additional scale for electrical conductivity, assuming zero grain conductivity, replaces the porosity scale in applicable parts of Figure 1 to give thermal conductivity directly from electrical conductivity. Since the Rayleigh equation is not amenable to analytical elimination of porosity, the scale for Figure 1d was determined graphically from Figure 3d. The scale for the Archie equation (Figure 1g) is based on the assumption of an  $\underline{m}_e$  value of 2.0.

In addition to Equations 10 to 13, "hybrid" relations can be derived, where the electrical conductivity is described by one model and the thermal conductivity by another. For example, Equations 14 and 15 can be derived by using the Maxwell equation to represent electrical conductivity and the Bruggeman and geometric models,

respectively, for thermal conductivity. Equation 16 is derived by using the Archie equation for electrical conductivity and the geometric model for thermal conductivity.

$$\text{Maxwell-Bruggeman} \quad K_{me} = \frac{3 K_{de} K_{mt}^{1/3} (1-K_{dt}) + (K_{mt}-K_{dt})(1-K_{de})}{3 K_{mt}^{1/3} (1-K_{dt}) - 2 (K_{mt}-K_{dt}) (1-K_{de})} \quad (14)$$

$$\text{Maxwell-Geometric} \quad K_{me} = \frac{\ln K_{dt} (1+2 K_{de}) + \ln K_{mt} (K_{de}-1)}{\ln K_{dt} (1+2 K_{de}) - 2 \ln K_{mt} (K_{de}-1)} \quad (15)$$

$$\text{Archie-Geometric} \quad K_{me} = \left[ 1 - \frac{\ln K_{mt}}{\ln K_{dt}} \right]^{me} \quad (16)$$

( $K_{de}$  has not been assumed to equal zero in Equations 14 and 15.)

From an examination of the theoretical curves (see Figure 1), one would expect an electrical conductivity measurement to be a rather sensitive indicator of the thermal conductivity for low porosities, such as those which might be encountered in consolidated rocks. A difficulty would exist, however, in determining thermal conductivity from an electrical conductivity measurement alone, since for any value of electrical conductivity, an infinite range of thermal conductivities is possible because of variations in the grain thermal conductivities. The success of any electrical method for measuring thermal conductivity would depend on the range of grain conductivities encountered in a given situation. Experimental work is necessary to

determine whether, in the case of actual ocean sediments or consolidated rocks, the variation of  $\underline{K}_d$  is small enough to permit determination of the thermal conductivity within sufficiently narrow limits for practical use.

## EXPERIMENTAL METHODS - OCEAN SEDIMENTS

In order to determine the practical usefulness of electrical conductivity measurements in finding thermal conductivities of marine sediments, and to determine the applicability of some of the theories discussed in the previous section of this thesis, laboratory measurements of thermal conductivities, electrical conductivities, water contents and porosities were made. Sediment samples were obtained from four piston cores taken in about 2800 meters of water, 150 miles west of Newport, Oregon (see Figure 5). Sediments in the cores represented typical marine clays and silts, with some occurrences of sand layers, or "turbidites." The cores were about twelve feet long and 2 1/2 inches in diameter. They were cut into sections about four inches long for measurement purposes. The sections were numbered consecutively from the top of the core.

In the case of PC-100, measurements were conducted aboard the research vessel Yaguina, within 24 hours after the core was taken. The other cores were measured in a land laboratory three to four weeks after coring. Cores were hermetically sealed and maintained in a horizontal position at room temperature during storage.

The following is a description of the experimental techniques used in making the measurements required.

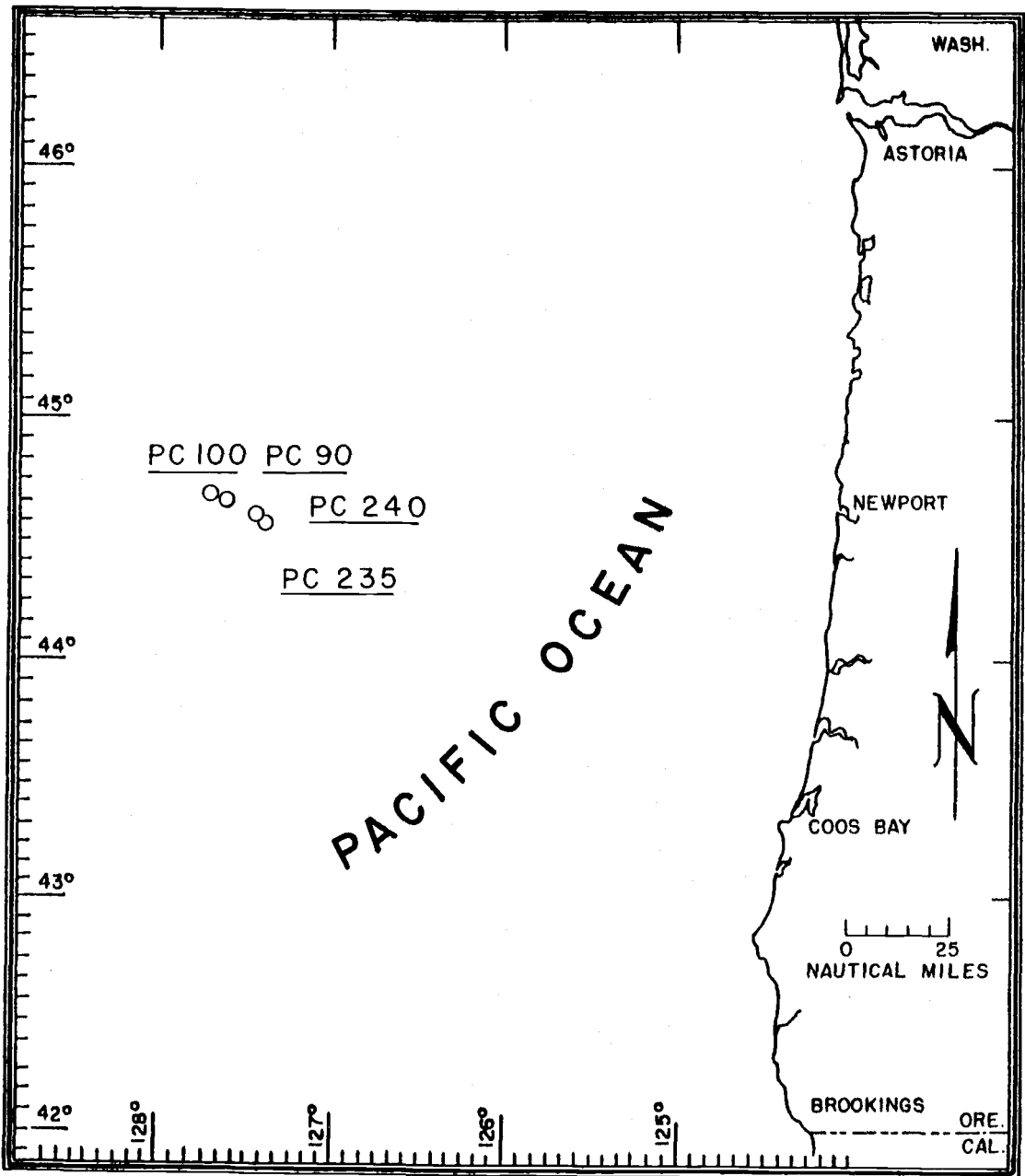


Figure 5. Locations of coring stations for sediment samples



### Thermal Conductivity

Thermal conductivity measurements were made using the needle-probe method described by Von Herzen (1959). The probe used was a type G847A, serial number five, manufactured by Fenwal Electronics Incorporated. The probe dimensions were 0.035 inches in diameter and 2 9/16 inches long. Accuracy of measurements made by this method is said by the supplier, Alpine Geophysical Associates, Inc., to be within ten percent. Repeated measurements made in the course of the present investigation gave a reproducibility of  $\pm$  five percent.

Thermal conductivity measurements were made by inserting the needle probe axially into each four-inch long segment of core. Power was applied to a heating element within the probe, and the change in temperature of the probe with time, given by a thermistor located at the probe center, was recorded. By plotting the probe temperature against the logarithm of the time, a straight line was obtained. The thermal conductivity was determined from the slope of the line.

### Electrical Conductivity

Electrical conductivity was measured by bringing metallic

electrodes of various configurations into contact with the sections of core and measuring the resistive component of the resulting electrode impedance. An Electro-Scientific Industries model 860A Impedance Bridge was used with an alternating current of 1,000 c. p. s. Two different electrode systems were used for the measurements.

Electrode system A consisted of two strips of 0.005 inch thick platinum foil, 18 centimeters long, bent into semicircular cylinders 0.9 centimeter in diameter to provide sufficient strength for insertion axially into the four-inch core sections. The electrodes were spaced three centimeters apart and held parallel by a plexiglas disc. The thermal conductivity needle probe was inserted into the sediment simultaneously with the electrodes at a point midway between them, so that temperature of the sediments could be monitored during measurement by means of the thermistor contained in the needle probe.

Electrode system B consisted of two flat plates which were brought into contact with the ends of the ten-centimeter core sections. Temperatures of the cores were measured by inserting the thermal conductivity needle probe into the sample immediately after completing the electrical measurement. In the case of core PC-100, on which measurements were conducted aboard ship, the two electrodes were made of brass and were brought into direct contact with the ends of the core sections. For cores PC-90, PC-235, and PC-240, the

electrodes were made from 0.0015 inch thick platinum foil. A piece of 5.5 centimeter diameter Whatman no. one filter paper, saturated with 1.0 N potassium chloride solution, was placed between the electrode surface and the surface of the sample, in order to minimize the formation of air bubbles under the electrodes.

Both electrode systems were calibrated by placing them in contact with a 0.1 N potassium chloride solution held in a container made from a section of 2 1/2 inch diameter plastic core liner. For electrode system A, a cell constant was thus obtained, so that resistances measured between the electrodes could be converted to conductivities. For electrode system B, an effective plate area was determined, and this area was used in the formula

$$K_{me} = \frac{l}{R \cdot A} \quad , \quad (17)$$

where A is effective plate area, R is measured resistance at 1,000 C.P.S., and l is the length of sample between the electrodes.

All measured conductivities were brought to an effective temperature of 22° C, using the results of Weyl (1964).

Results using electrode system A are considered accurate to within ten percent. Most of the error was caused by slight bending of the electrodes and resulting poor contact with the sediment. Error for the on-ship configuration of electrode system B is also considered to be ten percent, because of formation of air bubbles

under the electrodes. Where saturated filter paper was used under the electrodes, the major source of error arises from the difficulty of determining the exact length of sediment between the electrodes. The error for this case is estimated at five percent.

### Water Content and Porosity

Water content, as a fraction of the wet weight of the cores, was determined by weighing the wet sections, drying in an oven at temperatures not exceeding 85° C and weighing again. When the weights of the samples did not vary by more than one-tenth of one percent over a twelve-hour period, they were considered dry. A correction to account for a salinity of 34.7 parts per thousand was applied to the measurements to arrive at seawater content. The volumes of the core sections were determined by filling the empty plastic core liners with water and emptying them into a 100-milliliter graduated cylinder. The volume of seawater present divided by the total core volume gave the porosity as a volume fraction. Porosities and water contents given by this method are considered accurate within two percent.

## EXPERIMENTAL RESULTS AND ANALYSIS - OCEAN SEDIMENTS

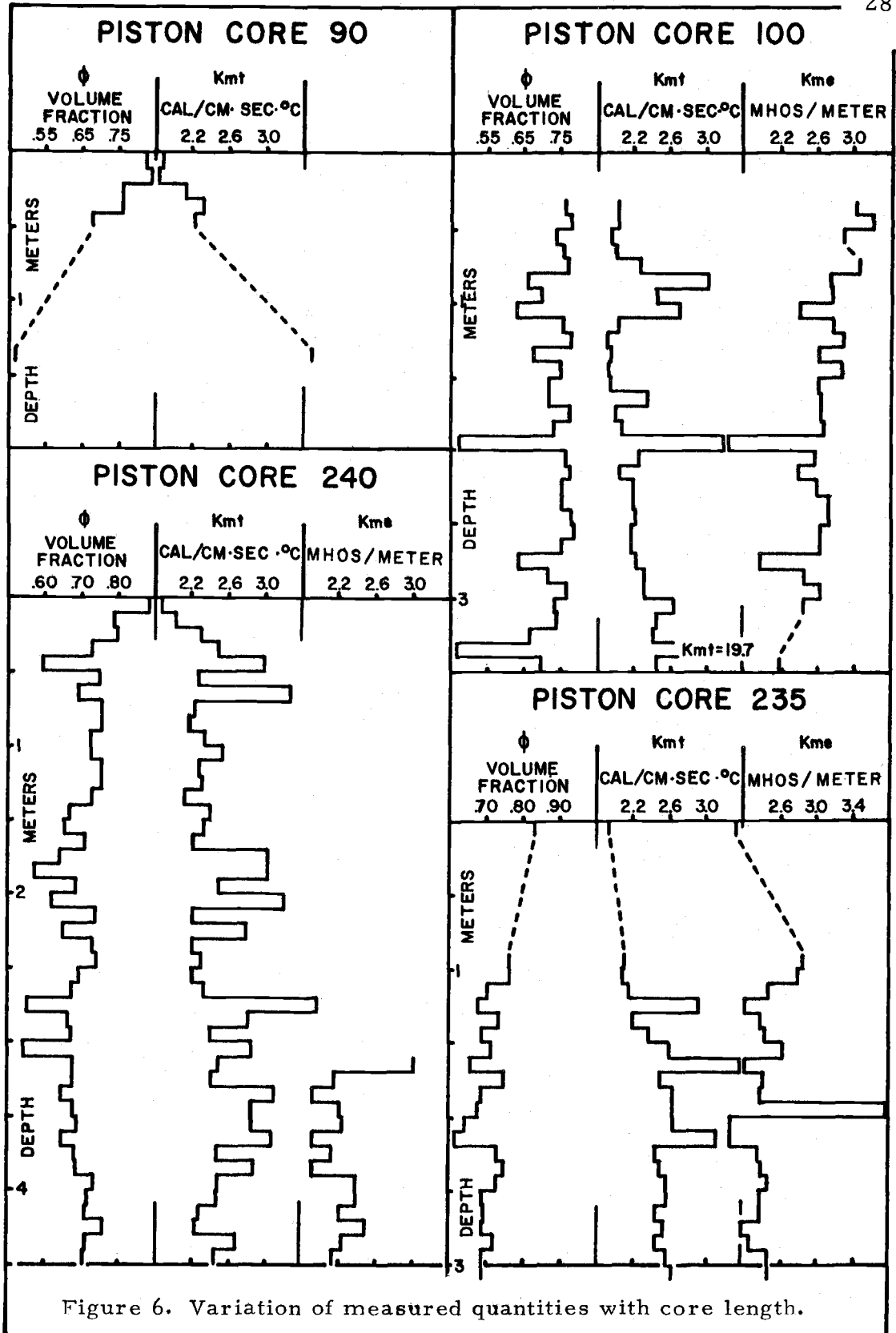
The data gathered using the methods described in the previous section were analyzed using the theoretical models set forth in Section II. Thermal conductivities and porosities of 105 samples were measured, and the electrical conductivity of 64 of these samples was obtained. The variation of porosity, thermal conductivity, and electrical conductivity with depth in the core is shown in Figure 6. The trend appears to be toward a decrease in porosity, an increase in thermal conductivity, and a decrease in electrical conductivity with depth. In addition, measurements published by Ratcliffe (1960) of thermal conductivity and water content for 51 samples, were analyzed.

Porosity and Water Content

Sea-water content,  $\underline{W}$ , is plotted against porosity,  $\underline{\phi}$ , for 106 sediment samples in Figure 7. For each sample the ratio of grain density to sea-water density was calculated using

$$\rho = \frac{\phi (1 - W)}{W(1 - \phi)} \quad , \quad (18)$$

where  $\underline{W}$  is water fraction by weight,  $\underline{\phi}$  is porosity (volume fraction), and  $\underline{\rho} = \frac{\rho_d}{\rho_c}$  is the normalized grain density. The mean density and 95 percent confidence limits for density, determined using Student's



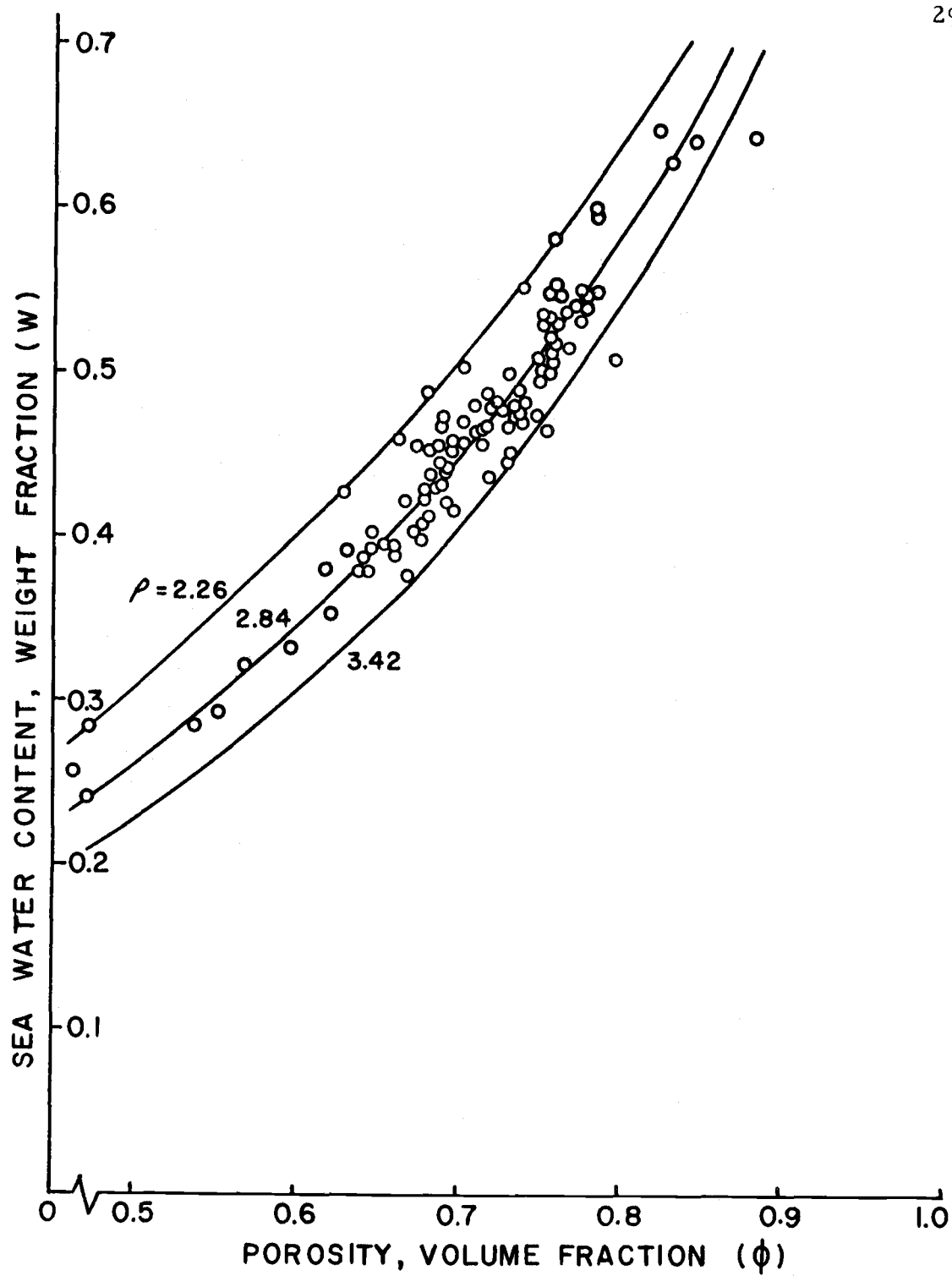


Figure 7. Sea water content and porosity, showing mean grain density and 95% confidence limits

t-distribution, are shown on Figure 7. The mean grain density corresponds to an actual density of 2.87, if a sea-water density of 1.024 is assumed.

### Thermal Conductivity and Porosity

Figure 6, page 28, shows that variations in thermal conductivity in apparently identical sediments are rather extreme. The ratio of maximum to minimum thermal conductivity in core 240 is 1.93; for core 235, 1.76, for core 100, 1.74, and for core 90, 1.91. The ratio of maximum to minimum conductivity in adjacent segments of the same core is 1.54 in the case of core 240, 1.34 for core 235, and 1.51 for core 100.

Figure 8 shows normalized thermal conductivity,  $\frac{K}{K_{mt}}$ , and porosity,  $\phi$ , for 105 ocean sediment samples. The thermal conductivity data of Figure 6 have been normalized by dividing by 1.40 mcal/cm-sec°C, the conductivity of sea water at 22° Centigrade. Values of the grain thermal conductivity,  $\frac{K}{K_{dt}}$ , were calculated for each data point using seven different models. The results of these calculations are given in Table I. Curves corresponding to the mean grain conductivities for five models are shown on Figure 8. It is apparent from Table I and Figure 8 that the Archie, Bruggeman, and geometric models give the closest fit to the observed data. The series



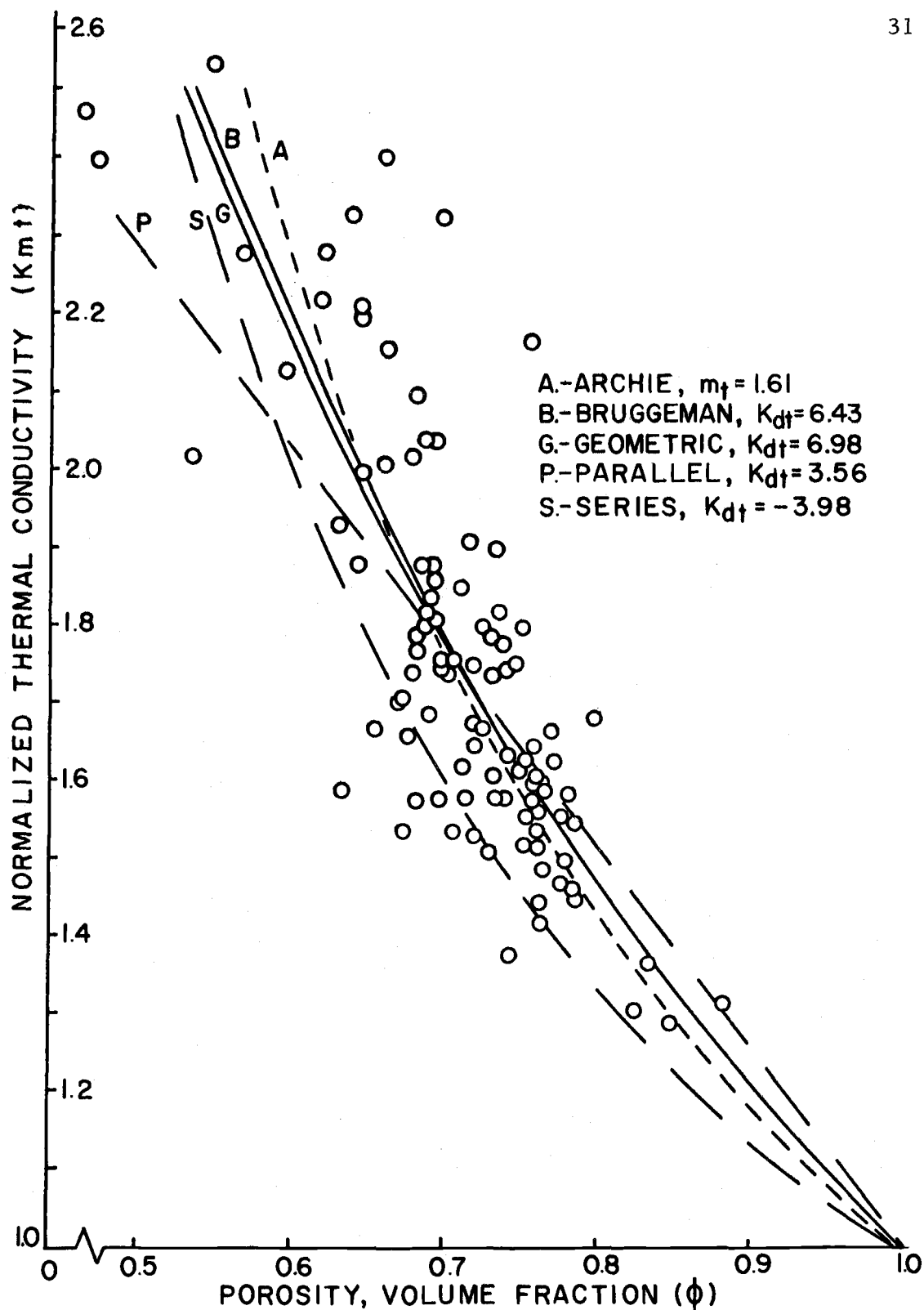


Figure 8. Normalized conductivity and porosity for 105 ocean sediment samples

TABLE I  
ANALYSIS OF THERMAL CONDUCTIVITY  
USING SEVEN MODELS

Model	Parameter	Equation	Sediments 104 Samples		Sediments Smoothed <sup>1</sup>		Ratcliffe 51 Samples <sup>2</sup>		Sandstone 37 Samples <sup>3</sup>	
			Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Series	$K_d$	(2)	-3.98	294%	-2.38	38%	- 5.56	138%	3.06	3930%
Parallel	$K_d$	(1)	3.56	15%	3.61	11%	2.83	11%	7.85	37%
Maxwell (1881)	$K_d$	(3)	-7.68	361%	-9.81	820%	-86.3	764%	- 7.11	730%
Bruggeman (1935)	$K_d$	(5)	6.43	36%	6.41	25%	3.91	16%	9.83	38%
Archie (1942)	m	(6)	-1.61	16%	-1.63	12%	- 1.22	13%	- 1.15	18%
Geometric (1964)	$K_d$	(9)	6.98	31%	7.05	23%	4.31	17%	10.4	37%
Parallel- Tortuosity	$K_d$	(8)	3.16	82% (64 samples)	---	---	---	---	23.4	109%

<sup>1</sup>See page 32

<sup>2</sup>See page 34

<sup>3</sup>See page 57

and Maxwell equations yield negative values of  $\underline{K}_{dt}$ , obviously unrealistic. While Archie's equation appears to give a low range of variation in thermal conductivity values for a given porosity, it should be emphasized that the physical meaning of the parameter  $\underline{m}$  is uncertain in the thermal case. For models other than Archie's, the scatter of data points can be attributed to the variation in thermal conductivity of the individual mineral grains of the sample.

A smoothing process can be applied to the sediment data, in which the conductivity of each core segment, weighted twice, is averaged with the values for the segments on either side of it, weighted once. This process, known as 1-2-1 smoothing, has been used on the present data, and the results are shown in column five of Table I. A reduction in variation of grain conductivity is apparent.

Ratcliffe (1960) published a study of thermal conductivities of various ocean sediments as related to their water content. That study has been used by some workers as a basis for an indirect determination of thermal conductivity when the water content can be determined.

In order to compare Ratcliffe's data with the data of the present study, the water contents given by him were reduced to porosities by assuming a grain density ratio of 2.84 and using the relationship

$$\phi = \frac{\rho W}{1 - W(1 - \rho)} \quad (19)$$

The thermal conductivities given by Ratcliffe were also normalized by dividing by the thermal conductivity of sea water. An analysis similar to that performed on the data of the present study was done on Ratcliffe's data. The results are shown in column six of Table I and in Figure 9.

The grain thermal conductivities determined from Ratcliffe's data are somewhat smaller than those obtained in the present study (see Table I). This circumstance may be due to the nature of the particular sediments considered in the two studies, or it may be caused by the arbitrary choice of grain density ratio made in converting Ratcliffe's water content values to porosities.

The scatter of the data given by Ratcliffe is also somewhat smaller than the scatter of the unsmoothed data obtained during the present study. This may be due to the higher precision of the thermal conductivity measurements made by Ratcliffe, who used a divided-bar apparatus and claims an overall accuracy of three percent. Another factor to consider is the nature of the sediment samples used by Ratcliffe, which were collected on abyssal-plain areas where the composition of the sediments might be expected to be more uniform than that of sediments from the continental slope area, where the samples for the present work were collected.

In order to test the hypothesis that the apparent scatter of the thermal conductivity data is caused by a variation in thermal

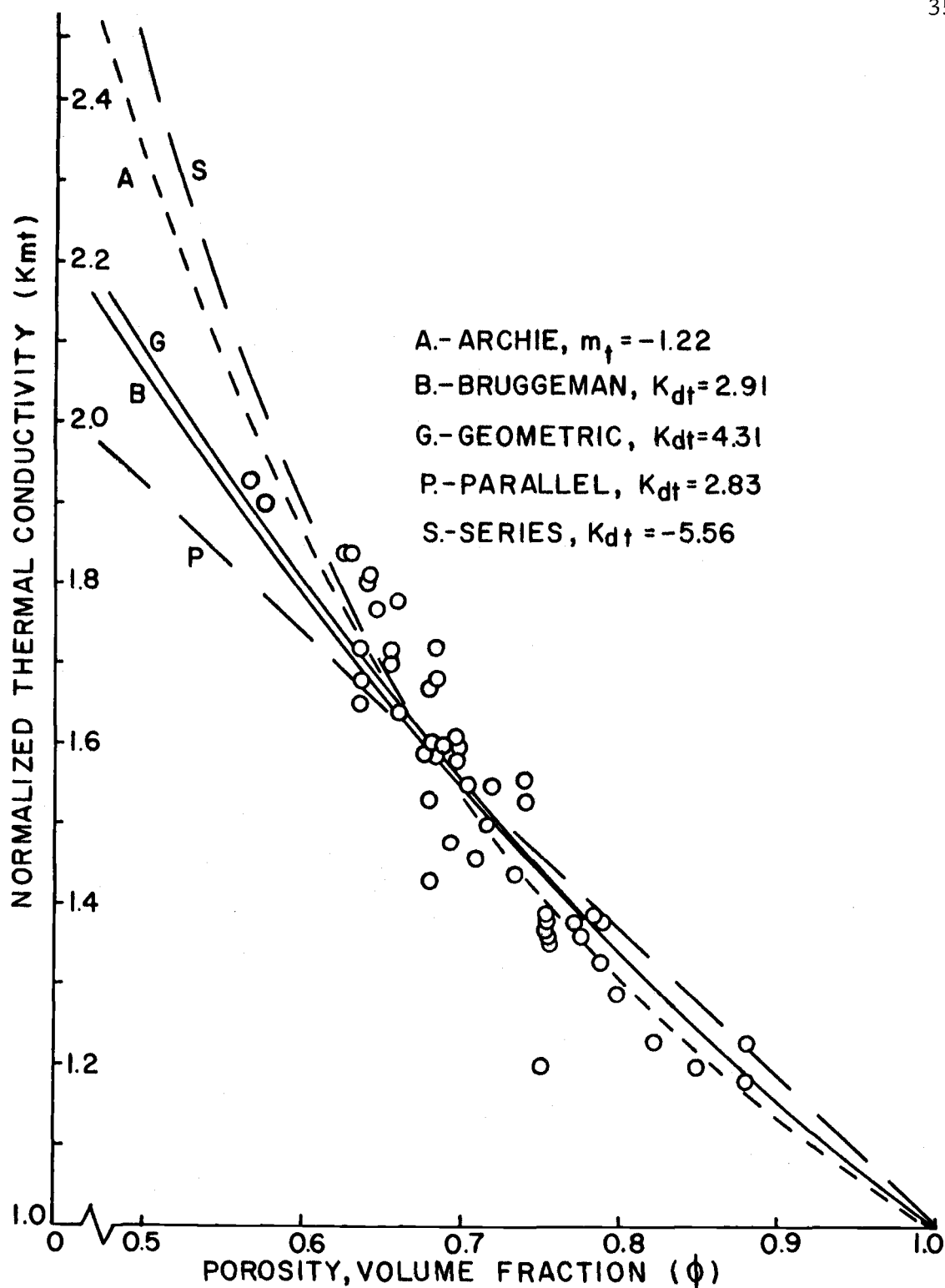


Figure 9. Normalized thermal conductivity and porosity for 51 ocean sediment samples. Data from Ratcliffe (1960).

conductivity of the mineral grains, a rough compositional analysis was performed on twelve selected sediment samples and an average grain conductivity was computed on this basis. To determine the volume composition of the samples, a size analysis was performed on the dried samples by the Department of Soils, Oregon State University, using the pipette method as described by Kilmer and Alexander (1949). The "clay" fraction (particles smaller than two microns diameter) determined by the analysis was assumed to consist entirely of clay minerals. The calcium carbonate content of the "silt" fraction (two microns to fifty microns) was determined by leaching with hydrochloric acid. Heavy minerals were separated from the "sand" fraction (greater than fifty microns diameter) using tetrabromoethane, a heavy liquid, and the percentage obtained was assumed to extend into the silt fraction. The remaining sand grains were microscopically examined and the proportions of quartz and feldspar, rock fragments, and mica thus obtained were also assumed to extend into the silt fraction. A quartz-feldspar ratio of 1.0 and a potash to total feldspar ratio of 0.87 were assumed. Thermal conductivities used are those given by Birch, Schairer, and Spicer (1942) and Beck (1965), divided by 1.40 mcal/cm-sec°C to arrive at a  $\frac{K}{dt}$  value.

The thermal conductivities for each fraction were combined by taking weighted geometric and arithmetic means as follows:

$$K_{dt} = \sum_{i=1}^n K_{dti} N_i \quad (\text{arithmetic mean})$$

$$K_{dt} = \prod_{i=1}^n K_{dti}^{N_i} \quad (\text{geometric mean})$$

where  $\underline{N}_i$  is the volume fraction of the  $\underline{i}$  th mineral and  $\underline{K}_i$  is its thermal conductivity.

The results of this analysis are given in Table II. The grain conductivities calculated by the procedure given above are contrasted in the table with the values derived from the thermal conductivity measurements using the geometric model. If error of the measured values is considered as ten percent and that of the calculated values as twenty-five percent, agreement within experimental error is obtained for eight of the twelve samples using the geometric mean, and for nine of the twelve samples using the arithmetic mean. Calculated values which do not agree with the measured values are indicated by an asterisk (\*) in the table. In general the calculated values which disagree are lower than the measured values.

### Electrical Conductivity and Porosity

Measured values of electrical conductivity are plotted with depth in the core in Figure 6, page 28. The values shown were obtained using electrode system B, which measures electrical conductivity

TABLE II  
COMPOSITIONAL ANALYSIS AND CALCULATED THERMAL CONDUCTIVITIES  
FOR TWELVE OCEAN SEDIMENT SAMPLES

Sample	<u>Sample Analysis by Volume</u>								<u>Calculated and Measured Conductivities</u>		
	% Heavy Min.	% Clay	% Quartz	% K-Felds.	% Na-Felds.	% CaCO <sub>3</sub>	% Mica	% Rock Frag.	Geometric Mean - K <sub>dt</sub>	Arithmetic Mean - K <sub>dt</sub>	Measured Grain Conductivities (Geometric Model) - K <sub>dt</sub>
240- 5	3	20	28	24	4	7	11	4	6.2	7.2	6.5
240-17	16	45	10	9	1	9	--	10	4.6	5.3	5.0
240-19	10	24	17	14	2	8	21	3	5.2	6.1	6.8
240-23	7	42	13	11	2	9	14	2	4.5*	5.3	7.1
240-25	10	47	8	7	1	12	--	16	4.3*	4.9*	8.9
240-31	5	22	15	13	2	11	30	3	4.9	5.8	4.6
235-30	25	47	11	9	1	7	--	--	4.7	5.5	7.2
235-21	25	29	20	18	3	6	--	--	5.9	6.8	5.8
235-16	18	41	4	4	1	7	21	4	4.2*	4.7*	8.3
100-30	16	53	10	8	1	11	--	--	3.5*	5.2*	9.1
100-16	23	41	11	10	2	5	7	2	4.8	5.6	4.5
90- 2	24	60	7	2	--	5	--	1	4.0	4.7	5.2
Assumed Mineral K <sub>dt</sub>	7.2 (1, 2)	2.6 (1)	12 (2)	6.8 (1)	13 (1)	6.1 (1, 2)	3.9 (2)	3.6 (1)	4.7 (Avg.)	5.6 (Avg.)	6.4 (Avg.)

(1) Birch, Schairer and Spicer, 1942

(2) Beck, 1965

\*Does not agree with measured quantity



primarily in the axial direction. A comparison of these conductivities with those obtained using electrode system A, where current flow is primarily radial, shows the existence of a directional characteristic of the electrical conductivity. Figure 10 is a plot of the axial conductivity with the radial conductivity. It can be seen from this figure that the anisotropy is rather pronounced. The conductivity is lower in the axial direction than in the radial direction. This condition may be caused by the parallel arrangement of the long axes of the mineral grains which can occur during sedimentation.

Figure 11 shows normalized electrical conductivity,  $\frac{K}{K_{me}}$ , and porosity,  $\phi$ , for 64 ocean sediment samples. The data from Figure 6 (page 27) have been normalized by dividing by 5.00 mhos/meter, the electrical conductivity of sea water of 34.7 parts per thousand salinity at 22° C, as given by Weyl (1964). Values of the grain thermal conductivity  $K_{dt}$ , cementation factor  $\underline{m}$ , and tortuosity  $\underline{T}$ , were calculated for each of the data points on Figure 11, using seven different models. The results of this calculation are given in Table III, and the curves corresponding to five of the models are shown on Figure 11. From Table III and Figure 11, the geometric, Maxwell, and Archie equations give the best fit of the data. The mean grain conductivity for the series model appears too high; the parallel and Bruggeman equations yield negative mean grain conductivities. The geometric and Maxwell equations yield very similar results. The

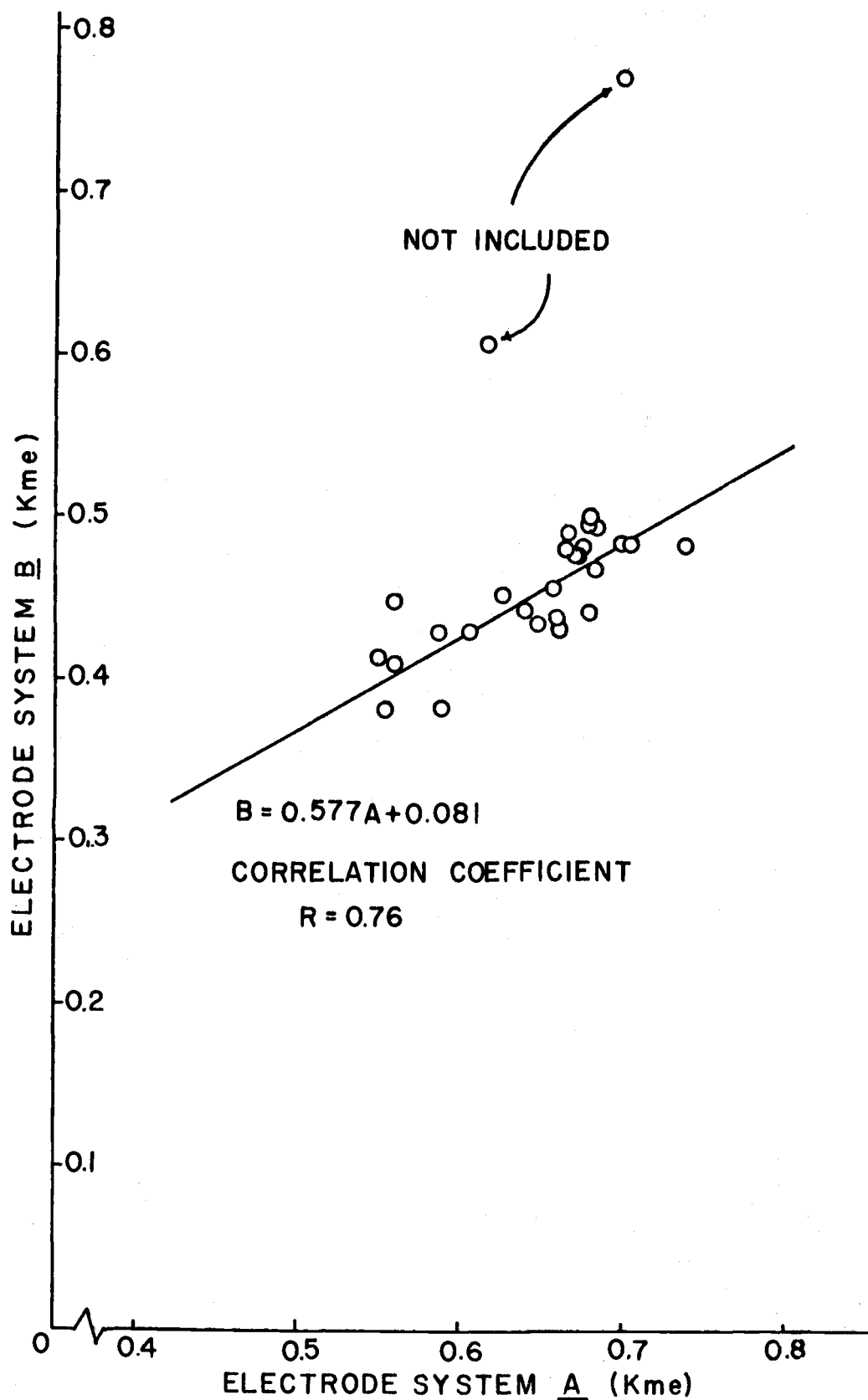


Figure 10. Normalized radial and axial electrical conductivity for 28 ocean sediment samples

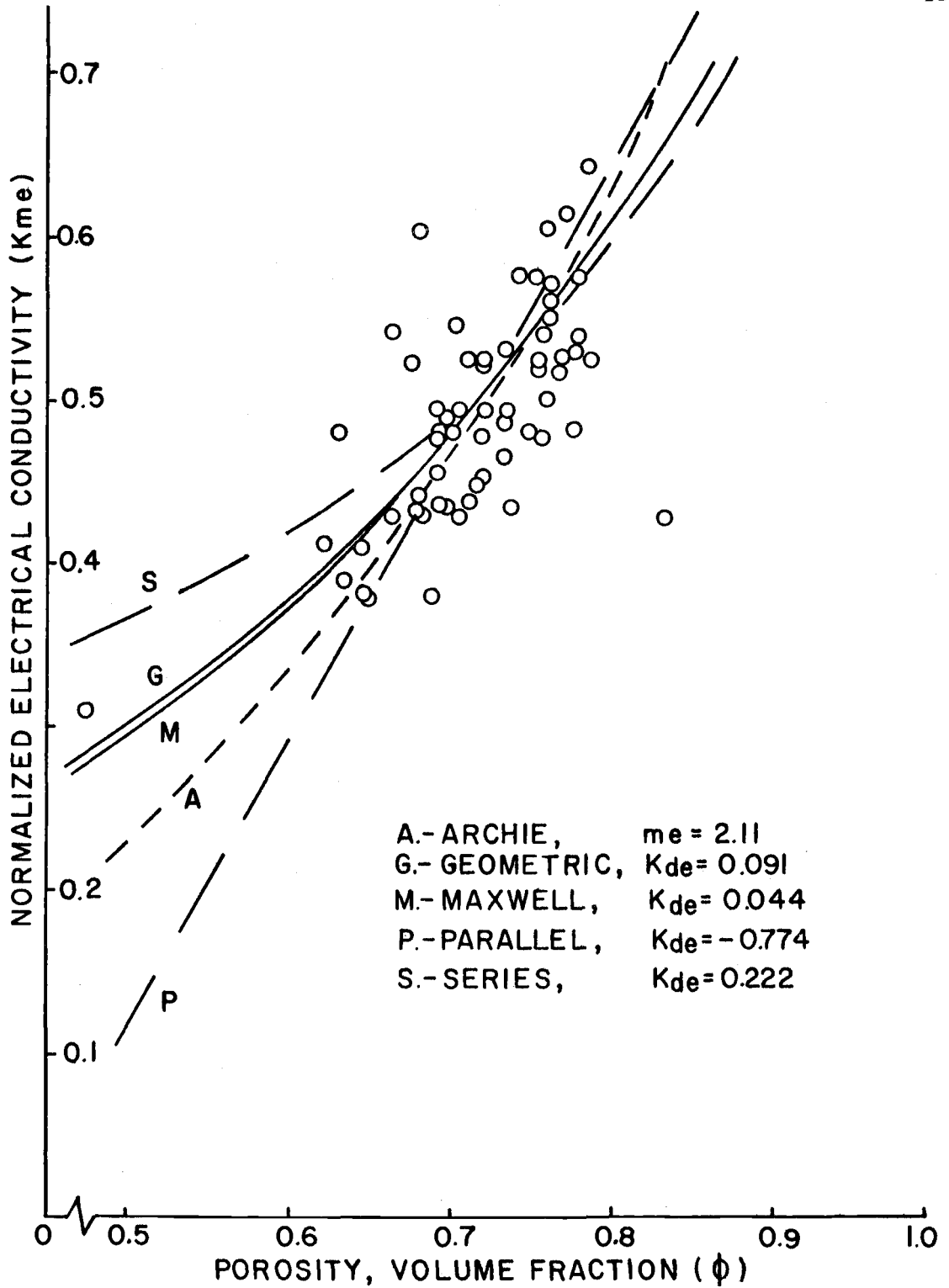


Figure 11. Normalized electrical conductivity and porosity for 64 ocean sediment samples

TABLE III  
ANALYSIS OF ELECTRICAL CONDUCTIVITY DATA  
USING SEVEN MODELS

Model	Parameter	Equation	Sediments 64 Samples		Sediments Smoothed		Sandstones 37 Samples <sup>1</sup>	
			Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Series	$K_d$	(2)	.222	22%	.219	14%	.049	64%
Parallel	$K_d$	(1)	- .774	32%	- .768	24%	- .209	51%
Maxwell (1881)	$K_d$	(3)	.044	160%	.041	113%	- .021	55%
Bruggeman (1935)	$K_d$	(5)	- .156	65%	- .158	43%	- .031	80%
Archie (1942)	m	(6)	2.11	17%	2.10	13%	1.90	11%
Geometric (1964)	$K_d$	(9)	.091	59%	.088	34%	.027	56%
Parallel- Tortuosity	$\tau$	(7)	1.25	6%	--	--	3.71	41%

<sup>1</sup> See page 51.

Archie equation is somewhat unrealistic in this case, since the sediment samples contained large amounts of clay, and thus could be expected to have a non-zero electrical conductivity at zero porosity.

The electrical conductivity data were subjected to the 1-2-1 smoothing process described for the thermal conductivity data. The results of the application of this process are given in column five of Table III.

### Electrical and Thermal Conductivity

Anisotropy must be considered in analyzing the results of the present work. The needle probe measured thermal conductivity in a radial direction, primarily. Electrode system B, used for most of the electrical measurements, gave electrical conductivity in the axial direction. Simmons (1961) has pointed out the possibility of anisotropy in thermal conductivity of ocean sediments. The presence of an electrical conductivity directional characteristic has been demonstrated in this thesis (page 39). At least in the case of electrical conductivity, however, the values in the two directions seem fairly closely related, so that relative results would not be greatly affected by anisotropy, although some additional scatter might be introduced.

Figure 12 shows normalized thermal conductivity,  $\frac{K}{K_{mt}}$ , and normalized electrical conductivity,  $\frac{K}{K_{me}}$ , for 64 ocean sediment

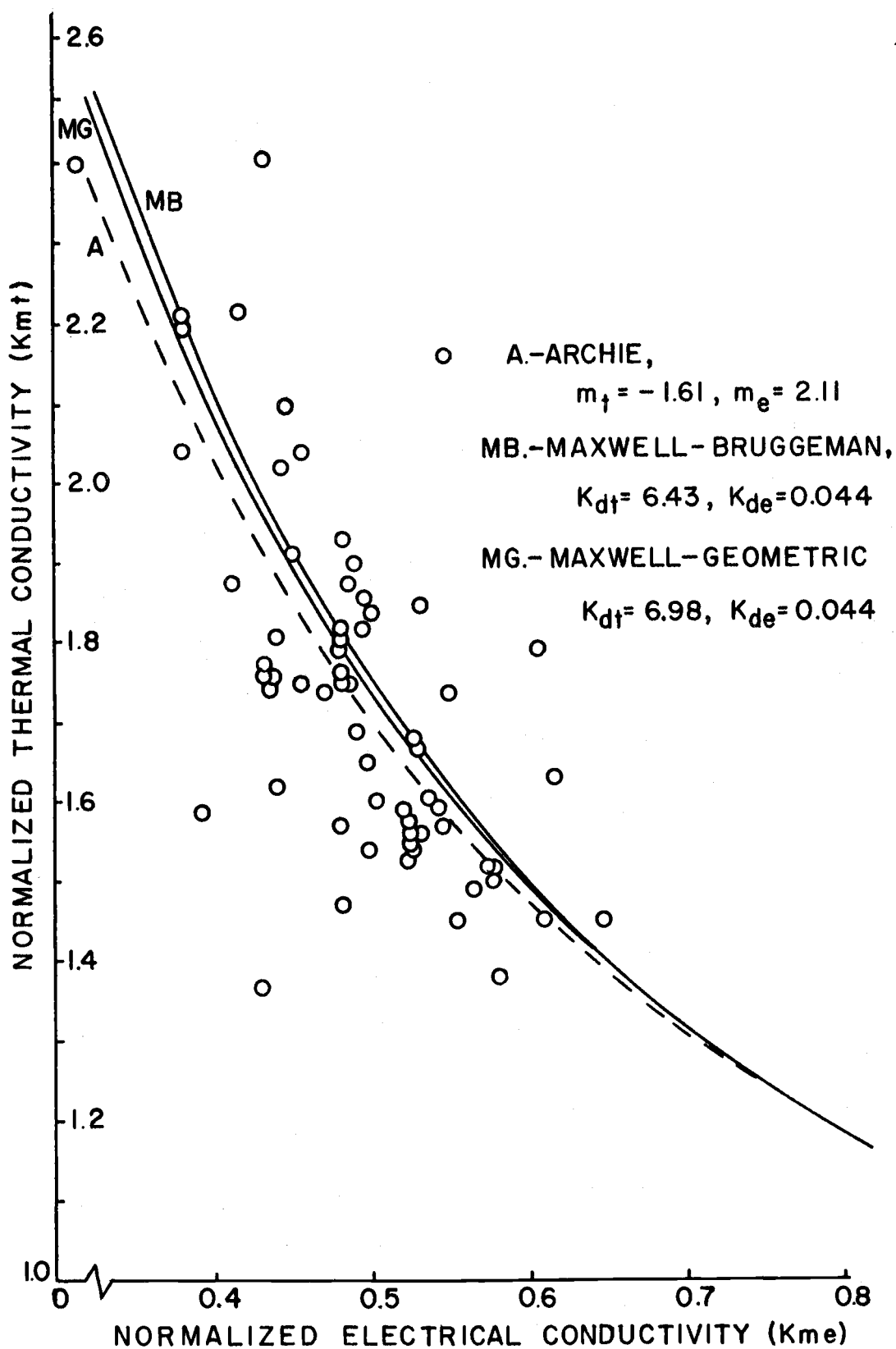


Figure 12. Normalized thermal and electrical conductivity for 64 ocean sediment samples

samples. Theoretical curves for the plot were obtained by eliminating porosity between equations for thermal and electrical conductivity.

The Maxwell model, which seems to fit the electrical data most closely, has been combined with the Bruggeman and Woodside-Messmer models, which seem to give equally good results for thermal conductivity. The Archie equation has not been combined with other models, since the parameter  $\underline{m}$ , as previously explained, is not compatible with the parameter  $\underline{K}_d$  in this case.

Zierfuss and Van der Vliet (1956) found an empirical relationship between the product of thermal conductivity and formation factor ( $\underline{K}_{mt}/\underline{K}_{me}$  in the present notation) and the porosity of rock samples. These quantities are plotted in Figure 13 for the sediment data of the present study. A considerable decrease in scatter of the data is evident when they are plotted in this manner. Since an additional variable, porosity, is present in these plots, the actual uncertainty of obtaining thermal conductivity when electrical conductivity is known, is not decreased by this method.

It is of interest now to compare the results of various processes which are sometimes used in determining thermal conductivities of ocean sediments for heat flow purposes. In Table IV are shown the results of various methods. One method sometimes used to obtain heat flow is to take a short core, perhaps one or two feet in length, near the surface of the sediments, and apply thermal conductivities

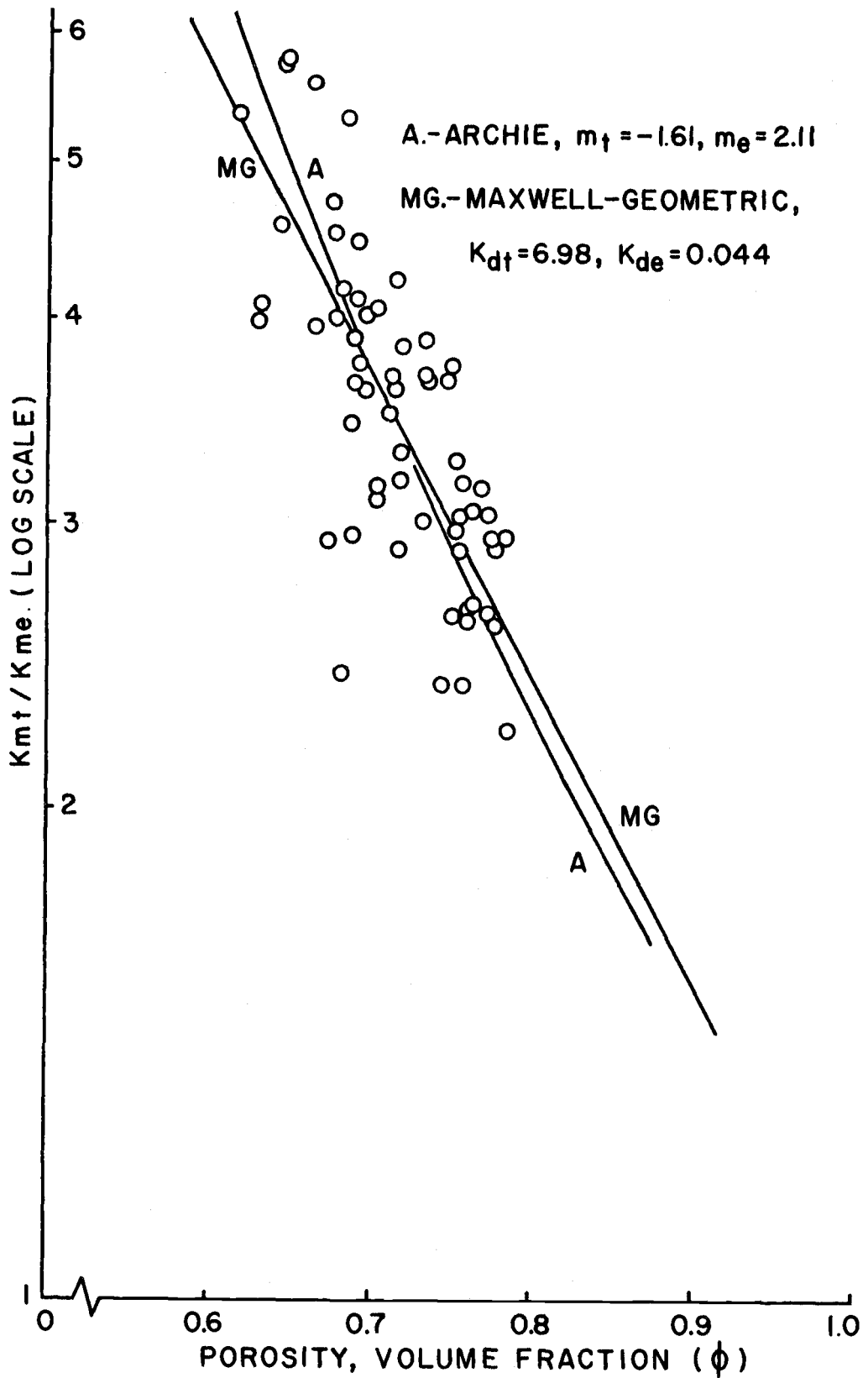


Figure 13. Thermal/electrical conductivity and porosity for 64 ocean sediment samples



TABLE IV

COMPARISON OF ELECTRICAL AND CONVENTIONAL METHODS FOR  
OBTAINING THERMAL CONDUCTIVITY OF OCEAN SEDIMENTS

Core	Sections	Thermal Cond. Avg. for 3 Sections $\frac{\text{m Cal}}{\text{Cm Sec } ^\circ\text{C}}$	Thermal Cond. by Ratcliffe Method Avg. for 3 Sections $\frac{\text{m Cal}}{\text{Cm Sec } ^\circ\text{C}}$	Thermal Conductivity Whole Core Avg. $\frac{\text{m Cal}}{\text{Cm Sec } ^\circ\text{C}}$	Thermal Conductivity Calculated from Electrical Conductivity	
					Maxwell- Geometric Eqn. $K_{dt} = 6.98$ $K_{de} = 0.044$	Archie Eqn. $m_t = -1.61$ $m_e = 2.11$
90	1, 2, 3	1.94	1.72	2.29	--	--
100	4, 5, 6	1.99	1.78	2.29	2.39	2.30
235	10, 11, 12	2.05	1.78	2.52	2.55	2.45
240	1, 2, 3	2.08	1.76	2.56	2.68	2.58

determined from this core to thermal gradient measurements obtained over, perhaps, a ten foot interval. The third column of Table IV shows the results which might be obtained if such a procedure were to be followed with the cores obtained for the present study. The fourth column of the table gives the results which would be obtained if thermal conductivity for such a short core were determined from the water content, using the nomograph given by Ratcliffe (1960). The fifth column gives the average thermal conductivity for the entire core. Columns six and seven are the values which might result from an electrical conductivity measurement, which would give an average value for a large section of the core. These values were obtained by assuming a thermal grain conductivity of 6.98 and an electrical grain conductivity of 0.044 and using the Maxwell-geometric equation (Equation 15), in the case of column six. The values in column seven were determined by using the Archie equation with  $\underline{m}_t = -1.61$  and  $\underline{m}_e = 2.11$ . These parameter values were the averages obtained for all the ocean sediment samples measured. If different values were assumed for these parameters, the values in columns six and seven would be changed. For example, using a thermal grain conductivity of 5.6, calculated from an analysis of mineral content of twelve samples by taking the arithmetic mean (Table II), the values given in column six of Table IV would be decreased by about twenty percent.

The values of thermal conductivity given by the water content (Ratcliffe) method are considerably lower than those given by any other method. The values given by direct measurement of the first few sections of a core are too low, compared with the average for the entire core, since the trend in any core seems to be toward an increased thermal conductivity with depth. The values given by electrical methods, on the other hand, compare favorably with the whole-core average.

## ANALYSIS OF THE DATA GIVEN BY ZIERFUSS AND VAN DER VLIET (1956) FOR CONSOLIDATED ROCKS

Zierfuss and Van der Vliet (1956) performed a series of measurements of porosity, permeability, formation factor, and thermal conductivity of 38 sandstones, two limy sandstones, and ten limestones when saturated with natural gas, oil, and water. Their objective in performing the study was to determine whether an in situ measurement of thermal conductivity (if a simplified method of making such measurements should become available), would be of aid in determining the petroleum reservoir characteristics of these rocks. As a result of the study, an empirical relation between the thermal conductivity-formation factor product and the porosity was developed.

Although Zierfuss and Van der Vliet's objectives were somewhat different from the aims of the present study, the data published by them are directly applicable to the present problem. This section of the thesis is concerned with a theoretical analysis for consolidated rocks similar to that performed for the unconsolidated sediments in the previous section.

### Thermal Conductivity and Porosity

The thermal conductivity data given by Zierfuss and Vander Vliet were normalized by dividing by the thermal conductivity of water and

analyzed using seven different models. The results of the analysis are given in column seven of Table I, page 32. Figure 14 shows the normalized thermal conductivity ( $\underline{K}_{mt}$ ) data plotted against porosity,  $\phi$ . Curves for three of the theoretical models are shown. The geometric and Archie models give the best fits to the data.

An analysis of the volume composition of the rock samples considered here might allow a theoretical calculation of the average grain thermal conductivity such as was performed for unconsolidated sediments in Table II, page 38 of this thesis. Unfortunately, Zierfuss and Van der Vliet do not give a detailed analysis of the composition of the samples which they measured. It is interesting to note, though, that for the data given by them for twelve limestone and limy sandstone samples, the mean grain conductivity as determined by the geometric model is 5.64, with a standard deviation of 0.64. The  $\underline{K}_{dt}$  value for calcium carbonate, derived from data given by Birch, Schairer and Spicer (1942) ranges from 5.55 to 6.31, depending on the orientation of the grains. The excellent agreement of these values furnishes further proof of the usefulness of the geometric model in determining thermal conductivities.

#### Electrical Conductivity and Porosity

An analysis of electrical conductivity and porosity has been made, using seven models, and the results are given in Table III, page 42.

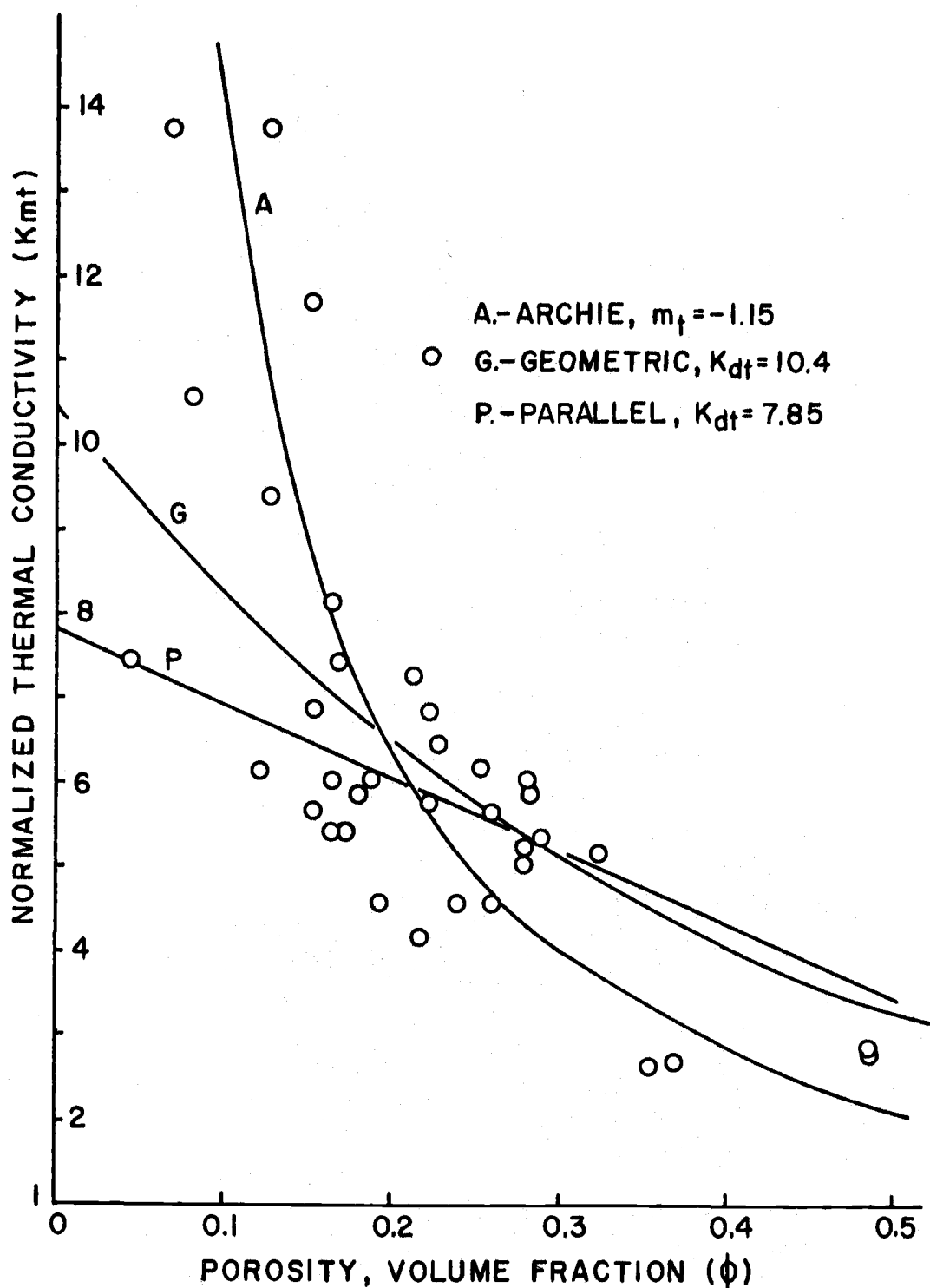


Figure 14. Normalized thermal conductivity and porosity for 37 water-saturated sandstone samples. Data from Zierfuss and Van der Vliet (1956)

Figure 15 shows a plot of normalized electrical conductivity,  $\frac{K}{K_{me}}$ , and porosity,  $\phi$ , including theoretical curves for four different models. The Archie equation gives by far the best fit of the data.

### Electrical and Thermal Conductivity

Figure 16 is a plot of thermal conductivity,  $K_{mt}$ , and electrical conductivity,  $\frac{K}{K_{me}}$ , showing a theoretical curve obtained by combining the Archie equation for electrical conductivity with the geometric equation for thermal conductivity. Considerable scatter is evident in this plot, caused by variations in the thermal conductivities of the individual mineral grains, and variations in the geometry of the electrical conducting path through the rock matrix. The mean grain conductivities obtained in this case are higher than those obtained for ocean sediments, probably because of the smaller amounts of clay (which has a low thermal conductivity) which are encountered in typical sandstones.

Figure 17 is an adaptation of Figure 5 as published by Zierfuss and Van der Vliet. The curve shown is derived from the Archie and geometric models, and replaces the regression polynomial given by Zierfuss and Van der Vliet. They state that the multiplication of formation factor by thermal conductivity reduces scatter of the data because this process compensates for changes in thermal conductivity

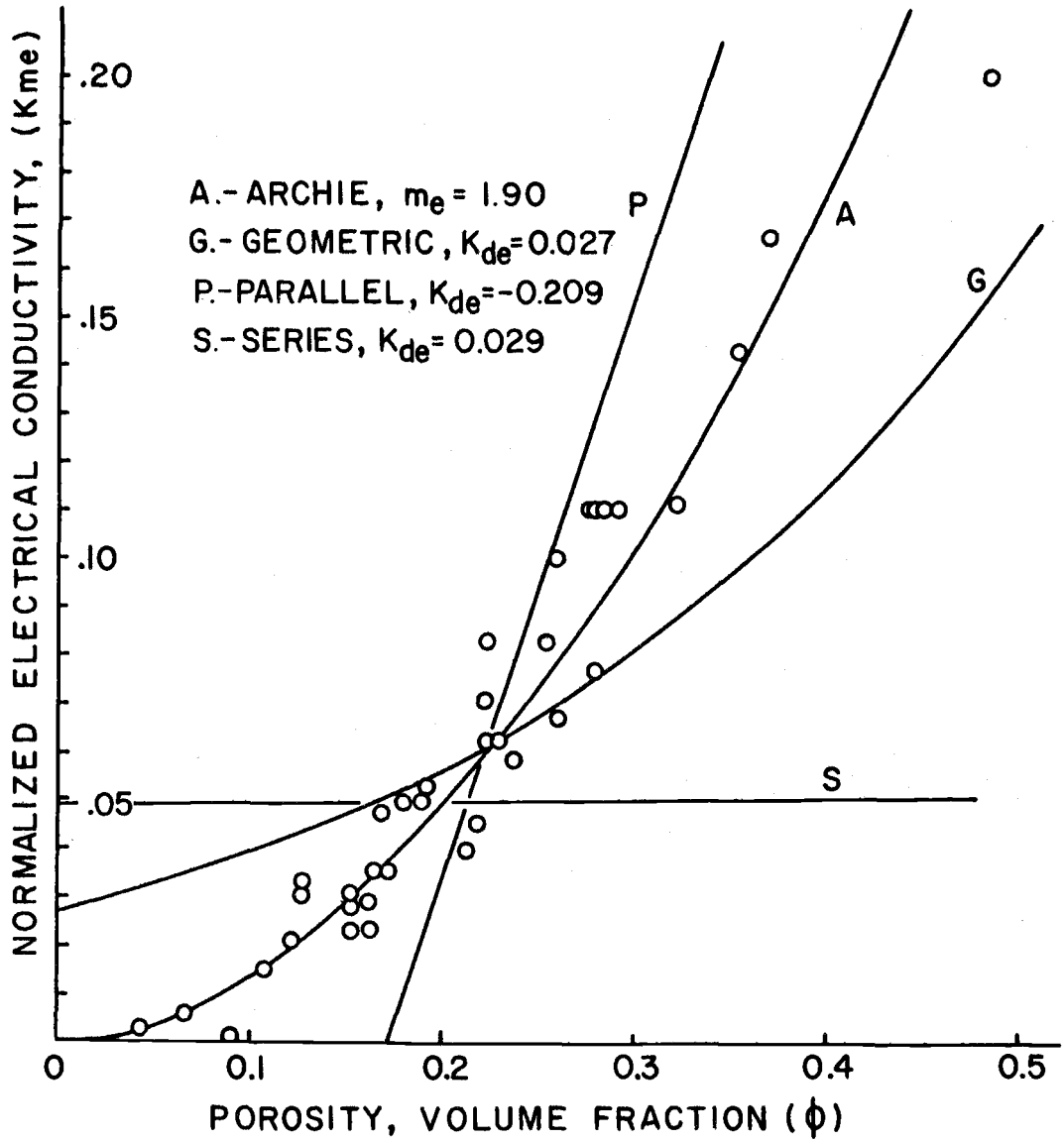


Figure 15. Normalized electrical conductivity and porosity for 38 water-saturated sandstone samples. Data from Zierfuss and Van der Vliet (1956)



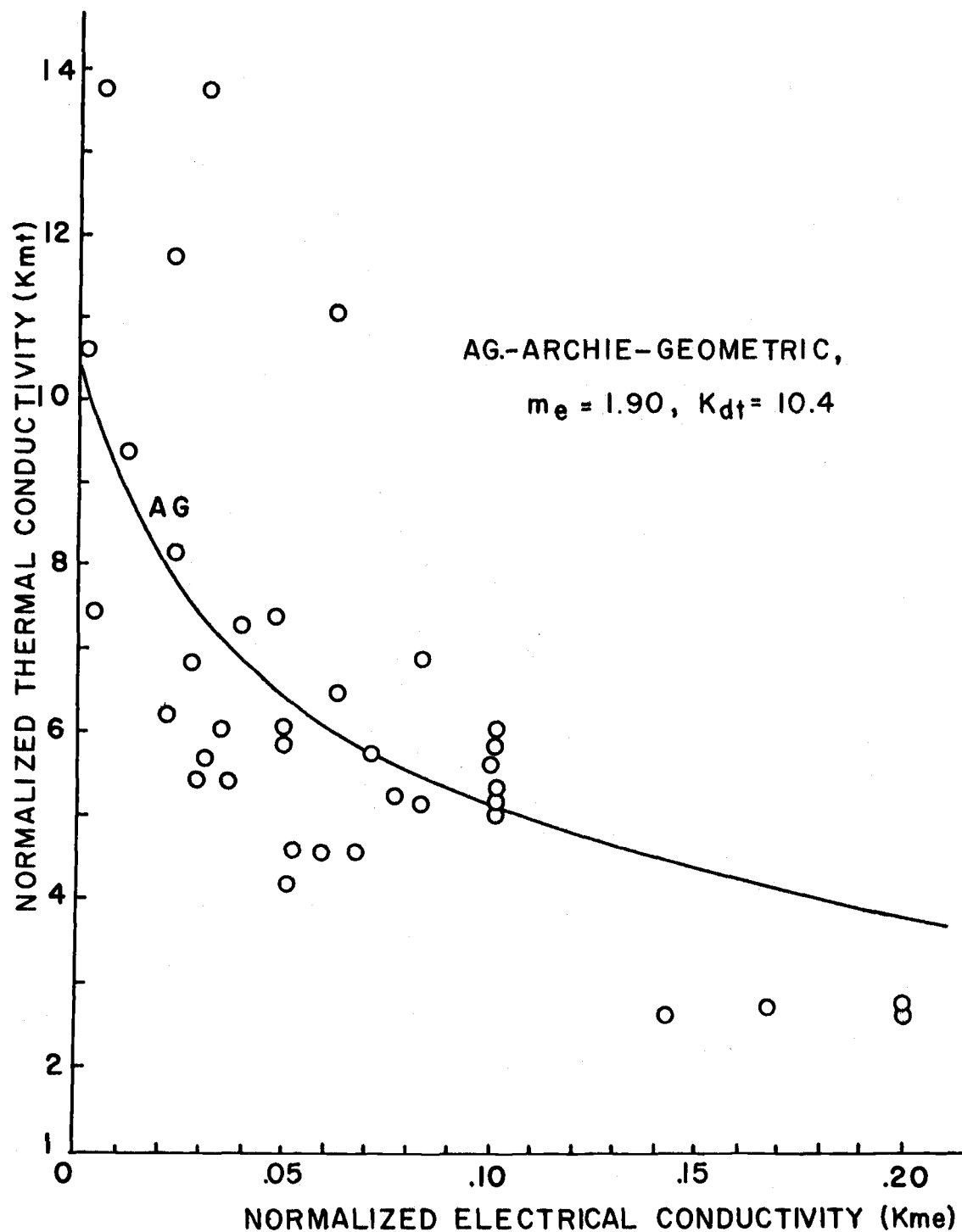


Figure 16. Normalized thermal and electrical conductivity for 37 water-saturated sandstone samples. Data from Zierfuss and Van der Vliet (1956)

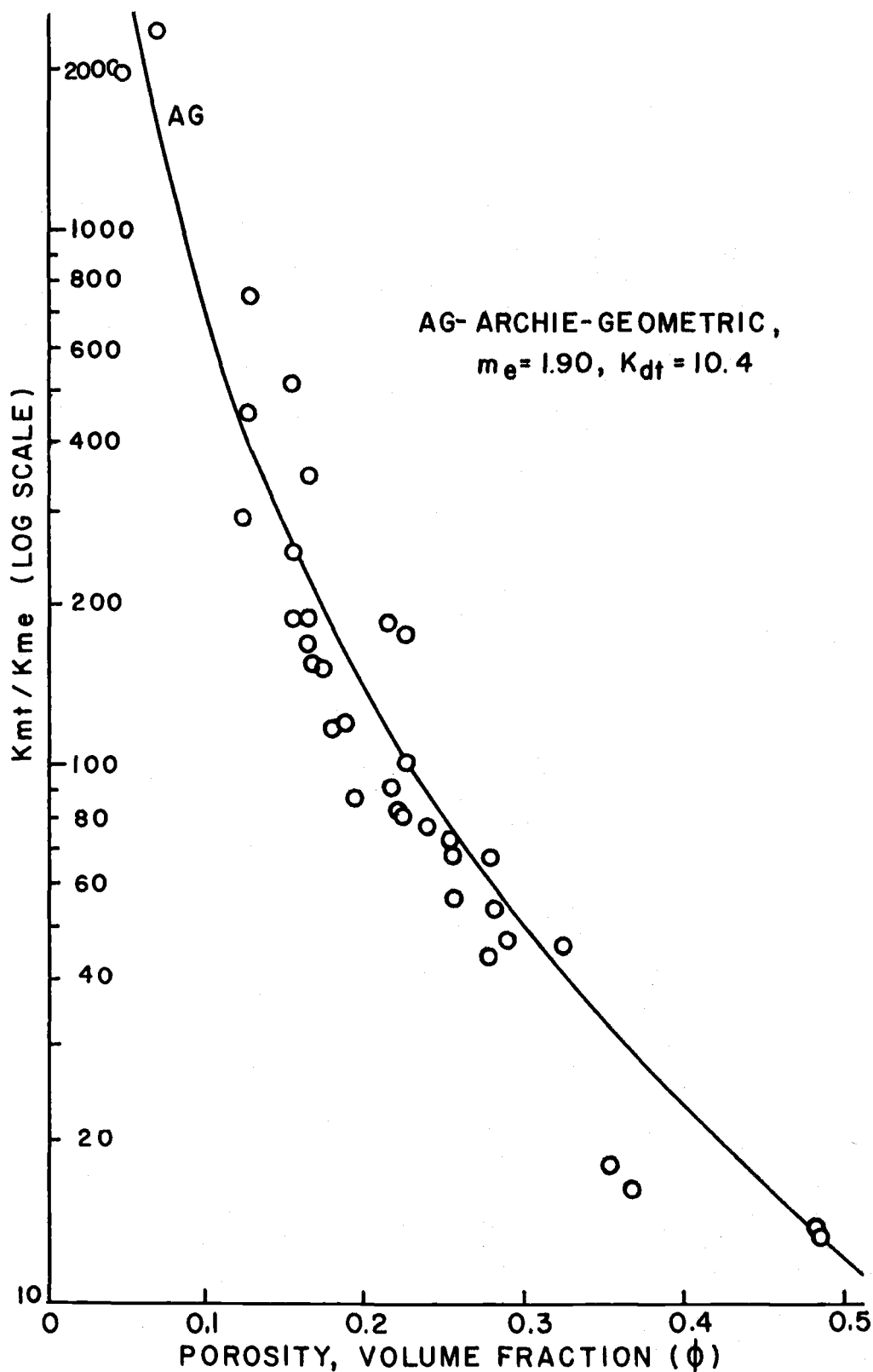


Figure 17. Thermal/electrical conductivity and porosity for 37 water-saturated sandstone samples. Data from Zierfuss and Van der Vliet (1956)

caused by convective heat transfer within the rock pores. This seems reasonable, since the convection would be expected to increase with increasing pore size, while the formation factor decreases with this quantity. A more satisfying reason can be adduced for this reduction in scatter, as follows.

If the parallel model modified to account for tortuosity (Equation (8)) is assumed for both thermal and electrical conductivity, and the electrical conductivity of the rock matrix is assumed to be zero, the following relation for the quantity  $\frac{K_{mt}}{K_{me}}$  is obtained:

$$\frac{K_{mt}}{K_{me}} = \frac{K_{dt}}{\phi} + (1 - K_{dt}) \quad (20)$$

It is seen that the tortuosity, a major source of variation in the thermal-electrical conductivity relationship, is eliminated when the thermal conductivity is divided by the electrical conductivity. This elimination of the tortuosity factor is doubtless the real reason for the reduction in scatter when the data are plotted as in Figures 13 and 17.

## CONCLUSIONS

General

Several theoretical relationships have been derived which can be used to determine the electrical or thermal conductivity of a heterogeneous solid-liquid mixture when the proportions of the constituents and their thermal or electrical properties are known. Use of these relations for a determination of thermal conductivity directly from electrical conductivity demands caution, however, since other factors, namely the variations in grain conductivities and in geometrical characteristics of the materials, introduce unpredictable variations into the relationships.

Ocean Sediments

For determinations of thermal conductivities of unconsolidated sediments from porosity data, the geometric model proposed by Woodside and Messmer (1961) is the most convenient to use and gives results as good as or better than any of the more elaborate equations. Some doubt is cast on the value of the frequent practice of determining thermal conductivity values for heat-flow use by taking a few measurements on samples small in size compared to the size of the depth

interval over which gradients are measured, since wide variations in thermal conductivities of seemingly identical adjacent samples can occur.

In determining electrical conductivities of unconsolidated sediments containing particles which may be electrically conductive, either the geometric model or the Maxwell model may be used. Combining these equations with the thermal conductivity equation gives a useful approximation to the thermal conductivity when electrical conductivity is known.

Even with the uncertainties which exist in the thermal electrical conductivity relation because of grain conductivity variation, an electrical determination of thermal conductivity would probably prove superior to the practice of directly measuring the thermal conductivities of a few small samples for use in heat flow measurements. The improvement would come about because the electrical method would give, in effect, the average thermal conductivity over a large volume of material. It should be emphasized that in order to use the thermal-electrical conductivity relations properly, some estimate of the thermal and electrical conductivities of the mineral grains comprising the sediments should be available.

### Consolidated Rocks

For the determination of the bulk thermal conductivity in consolidated rocks, the geometric equation is again the most useful. Caution must again be urged in using small-scale thermal conductivity measurements for determination of heat flows when gradients are taken over a large interval compared to the size of the thermal conductivity specimens.

For consolidated rocks where the solid constituents are essentially non-conductive but cannot be adequately represented by an aggregation of spherical particles, the Archie equation is most useful in determining electrical conductivity, since it takes into account the varying geometry of the rock matrix. The geometric and Archie equations may be combined to give a theoretical relationship between the thermal conductivity and electrical conductivity of solid rocks.

The problems of variation of grain thermal conductivity and rock matrix geometry are more severe in the case of solid rocks than for ocean sediments. However, the problem of determining thermal conductivities on land from electrical well-logs is considerably simplified, since the mineral content and matrix geometry of many formations traversed by existing wells is known with considerable accuracy. In addition, a method exists (Wyllie, 1963), whereby porosity and mineral composition can be uniquely determined through

a combination of the results from the neutron, continuous velocity, and gamma-gamma density logs. In theory, then, it should be possible to obtain a continuous estimate of in situ thermal conductivity by the use of well-log data alone. The existence of such a profile would be an invaluable aid to those workers wishing to obtain accurate land heat-flow measurements, and the method could expand greatly the available number of such measurements.

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