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An analysis and modeling method of the diagnostic characteristics of a mechanical or electromechanical system is presented. Diagnosability analysis is especially relevant given the complexities and functional interdependencies of modern-day systems, since improvements in diagnosability can lead to a reduction of a system's life-cycle costs. The diagnosis process of a mechanical system, involving an observation phase and a testing phase, is described, as well as how failure types (the way particular system failure modes occur) impact the diagnostic process. Failure and diagnostic analysis leads to system diagnosability modeling with the Failure Modes and Effects Analysis (FMEA) and component-indication relationship analysis. Finally, methods are developed for translating the diagnosability model into mathematical methods for computing metrics such as distinguishabilty, testability, and Mean Time Between Unscheduled Removals (*MTBUR*). These methods involve the use of matrices to represent the failure and replacement characteristics of the system. Diagnosability metrics are extracted by matrix multiplication. These metrics are useful when comparing the diagnosability of proposed designs or predicting the life-cycle costs of fault isolation.

Diagnostic Analysis for Mechanical Systems

by

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Scott A. Henning, author

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This thesis is dedicated to

Bob and Mary Ann Henning, my loving father and mother,

and Steve and Karen Luten, my encouraging and faithful housemates during my time at Oregon State University.

Diagnostic Analysis for Mechanical Systems

1 Introduction

1.1 The Diagnosability Problem

The complex electromechanical systems that compose modern-day machines are more efficient, cost-effective, reliable than those of only a few years ago. Many systems today are integrated in such a way that components have multiple functions and are managed by sophisticated computer control systems [Manelski 1998]. While the benefits of this evolution in system architecture are numerous, such as increased reliability and simpler, more efficient designs, there is a significant drawback we seek to address in this research project. Because of the many component interdependencies in today's integrated systems, causes of failure are often difficult to distinguish. Thus, because of this increased complexity, more errors are made in the diagnosis and repair of electromechanical systems. This is a problem in *diagnosability*, the system characteristic defined as a measure of the ease of isolating faults in the system.

There are two approaches to alleviating problems with fault isolation. The first is to make improvements to the diagnostic process for systems already designed and in-service. This approach includes developing maintenance and diagnostic procedures and processes and incorporating electronic diagnostics into system design. There has been much research and application in this area of diagnosis. An example lies in the design of the Air Supply and Control System (ASCS) for the Boeing 767-400ER aircraft. Extensive built-in tests (BIT) were incorporated into the design to allow for problems to be easily diagnosed [Boeing].

Less work has been focused on a second approach to the problem, improving inherent system diagnosability. This approach involves looking at the problem during the design stage and asking the questions: *How can this system be improved to make it easier to diagnose? What are ways of measuring this system's diagnosability during design?* In this approach we assume that changes in the structure of the system will affect the efficiency of diagnosing the system's failures. In endeavoring to understand and develop methodologies for improving diagnosability in this sense, we must have a good understanding of the diagnostic process (see section 2).

1.2 Motivation for Pursuing Diagnosability Improvement

Maintaining electromechanical systems is costly in both time and money, and diagnosability problems increase these costs. This fact serves as the primary motivation for exploring diagnosability improvement in systems ranging from airplanes to automobiles to high-tech manufacturing equipment.

The number of maintenance actions on an airplane system (see Figure 1) serves as an illustration of the reality of the diagnosability problem, and as compelling motivation for exploring practical solutions. Figure 1 shows that for the Air Supply and Control System on an airplane, an average of 8.5 maintenance actions were executed to correct each indicated failure condition. Maintenance actions in this particular study can

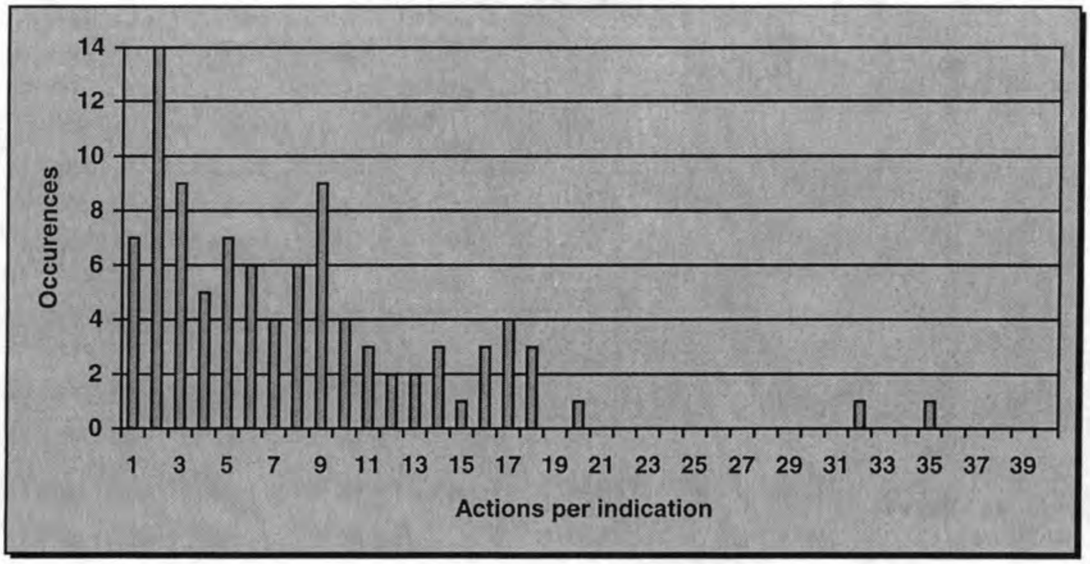

Figure1 Maintenance actions on an airplane system

include replacing, repairing, or checking a component or deferring action until a later time. These statistics are significant because more maintenance actions increase time and labor, and thus costs, and can affect safety because problems on mechanical systems persist longer than necessary.

Thus, the motivation for this research is that the problem of efficiency in fault isolation has been a significant issue in maintenance time and life-cycle costs. The ability to predict the diagnosability of a system early in its design stage would enable the building of systems with more efficient fault isolation, leading to reduced life-cycle costs.

1.3 Background Work in Diagnosability Analysis

Before further developing the diagnosability model in this paper, we will present a review of some previous research in diagnosability.

Pau [1981] explores the links between data analysis and failure diagnosis. He asserts his data analysis methods can achieve several objectives important to diagnosis, including the following:

- Elimination of most redundant observations or tests
- Selection of tests and observations giving the best possible discrimination between failure causes and determination of equipment condition…
- Elimination of imprecise symptoms… [Pau 1981]

These characteristics are important in the development of this paper, including the distinction between observations and tests.

Ruff [1995] introduced the idea of mapping a system's *performance measurements* to system *parameters.* Performance measurements would be indications from lights, gauges, etc. Parameters were usually the system components being measured, such as valves, controllers, or actuators. The complexity of the interdependencies between measurements and parameters was directly related to the diagnosability of the system. Ruff also completed some initial work on evaluating competing designs based on life cycle costs associated with diagnosability.

Clark [1996] extended Ruff's work by establishing some valuable metrics based on performance measurement-parameter relationships. The most significant of these metrics, *Weighted Distinguishability (WD),* represents the complexities of interdependencies between components and indications. The distinguishability metric will be extended in this research, but evaluated from a different perspective (see section 4.1).

Simpson and Sheppard [1994] devote a considerable portion of their book *System Test and Diagnosis* to diagnosability evaluation. They present a highly mathematical and theoretical analysis of diagnosis and testing adapted mainly for electrical applications. In evaluating diagnosability, they develop large matrices of test results and test conclusions to analyze and measure ambiguity and the ability to isolate faults.

Wong [1994] developed methods for minimizing both the time and cost of diagnosis early in the design stage. Wong developed a *checking order index* for each system component, which was calculated by dividing the probability of failure by the average time to check the component. A ranking order of components to be checked could then be established for each possible failure indication. Wong then developed an *expected time to diagnose* for a given indication.

Kurki [1995] researched model-based fault diagnosis, exploring the use of structural and behavioral models in examining fault detection and fault localization processes.

Murphy [1997] developed prediction methods for a system's *Mean Time Between Unscheduled Removals (Unjustified)* ($MTBUR_{unj}$). The $MTBUR_{unj}$ metric is a significant component attribute in doing diagnosability analysis. This present research will broaden the methodology that Murphy began in predicting $MTBUR_{unj}$.

Finally, Fitzpatrick [1999] worked on developing methods for predicting *Mean Time Between Failures (MTBF)* and *Mean Time Between Maintenance Actions (MTBMA)* in addition to $MTBUR_{unj}$.

1.4 Research Goals

The overlying goal of this research is developing a method for measuring system diagnosability, and thus allowing for the comparison of designs and the prediction of life-cycle costs of fault isolation. The methods should allow for designs with optimum diagnosability and minimized diagnostic costs. In the course of pursuing this goal, several original contributions will be made in this paper. The following is a brief outline of these contributions.

First, in section 2, we will present a broader picture of diagnostic phases and failure types for developing a diagnosability model. Previous research in mechanical systems focused on fault isolation based on indication and observation, but not on diagnostic testing. So there is opportunity in this paper to expand the model, accounting for the entire diagnosis process from operational indications to testing procedures. Similarly, previous research developed models for diagnosability assuming only *full* failure. However, in reality failures are often not so clear cut. Failures may also occur partially or intermittently, significantly effecting diagnosability assessment. This research will present an analysis of these additional failure types.

In section 3, we will outline using Failure Modes and Effects Analysis (FMEA) and Fault Tree Analysis (FTA) as tools in the development of a diagnosability model.

Finally, based on this new examination of failures and diagnosis, section 4 will present a new, more mathematically rigorous, method for computing diagnosability metrics, including a prediction of the mean time between unscheduled component removals ($MTBUR$ and $MTBUR_{unj}$). These metrics are important indicators in judging a system's diagnosability.

The scope of the research will involve analyzing systems and their components to the level of the LRU (line replaceable unit). We will not concern ourselves with the inner structure of each LRU and what specifically has failed at that level of detail. (The terms *LRU* and *component* will be used interchangeably in this paper.)

In summary, we will build on previous diagnosability research and introduce new methods in establishing a diagnosability model and diagnosability metrics. This project will broaden the picture of diagnosability so it more accurately reflects the range of circumstances to which it is applied. Ultimately, the methods from this paper will lead to systems with greater ease of fault isolation, and thus a higher likelihood that the correct LRU will be replaced when a failure occurs.

2 Understanding Failure and Diagnosis

This research will investigate two major areas in building a broader diagnostic model. The first is taking into account the two phases of the diagnostic process, observation and testing. The second area is considering failure types. Both areas highlight some unique characteristics of mechanical systems in contrast to electrical systems.

2.1 Diagnostic Phases

2.1.1 Phase 1: Observation

The first phase of diagnosis is *observation*. During this first phase of diagnosis, observable abnormalities in system function and performance are noted (i.e., the presence of a liquid from a leak). Usually observation leads to the conclusion that there is a problem, or some loss of system function, but does not allow for understanding the problem entirely. By observing symptoms, or operational indications (effects), of failures, we are led to conclusions about the nature of a system's failure and its causes. Based on observation we can infer a set of possibly failed candidate components responsible for the observed symptoms. All of the components that are possible causes to a given set of indications are known as an *ambiguity group* [Simpson 1994]. There is ambiguity because each component in the group causes an identical set of indications to occur when they fail (see also section 3.2 on indication sets).

2.1.2 Phase 2: Testing

Our goal in diagnosing failures is narrowing down the possible candidates and choosing the most-likely-failed component for maintenance. Therefore, attempting to minimize an ambiguity group of candidates leads us to the *testing* phase of diagnosis. The purpose of the testing phase is to gain additional information about a problem by running specific tests on system components. Knowledge from observation is used to determine the appropriate tests to conduct (see section 4.2.2). Ideally, the testing phase allows for the complete elimination of ambiguity (the ambiguity group contains *one* component). However, as will be explored in the development of a testability metric, time constraints limit this ideal from always being possible.

2.1.3 Example

Let's use the example of an automobile in illustrating the two diagnostic phases. For phase one, there are many observations one can make in forming conclusions about problems with a car. There may be oil leaking, the "check engine" light flashing, or perhaps an abnormal sound. These are all observations leading to the conclusion that the car engine is not behaving within normal parameters. In other words, there is evidence of some sort of failure based on observation.

For the testing phase, a voltmeter may be used to check electrical connections, or a mechanic could attach a diagnostic computer to the car to determine the cause of the "check engine" light. However, at the end of the process, there may still remain multiple probable causes to the automobile's symptoms. A mechanic would then have to choose the component to replace from the remaining ambiguity group.

2.1.4 Differences Between Mechanical and Electrical Systems

The relationship between the phases of diagnosis and diagnosability in mechanical systems differs from other applications. Most of the prior research into the diagnosis and testing process has been in the area of electrical and computer applications. For example, in Simpson and Sheppard, evaluating diagnosis involves analyzing exhaustive sets of tests on the system [Simpson 1994]. In electrical applications, the observation phase provides less information; more of the diagnostic process is dependent on the testing phase. However, unlike mechanical and electromechanical systems where testing is complex, costly, and time-intensive, large amounts of testing are relatively simple, low cost, and quick in the electrical and computer realm.

Because of the higher time and costs, testing time becomes a more critical constraint in mechanical systems (this constraint will factor into our development of a testability metric later in the paper, see section 4.2.1). Therefore, while the methods developed previously in test and diagnosis study are valuable for studying diagnosability in mechanical applications, we are attempting in this project to more carefully understand the observation and testing phases of diagnosis in mechanical systems. Most specifically, we are interested in how these phases factor into optimizing system diagnosability.

2.2 Failure Types

In developing a broader understanding of the diagnosis process and a more accurate diagnosability evaluation, we will need to take a closer look at the nature of failure and the different ways failures can occur. In general, the failure of a component or system can most simply be expressed as a shortfall between performance and standards [Bignell 1984]. In other words, a system is designed to perform an expected, measurable behavior. When the system unacceptably deviates from this behavior, it has failed to carry out its designed function. This deviation can take the form of either the absence of desired function or the appearance of unwanted side effects [Bignell 1984].

Often failure is modeled in a binary fashion. Either a system or component is failed or not. This binary approach to failure modeling was used in prior diagnosability research, and functions well in electrical and computer applications. Again we encounter an area where there is significant difference in diagnosability modeling in mechanical and electrical systems. Electrical systems are made up of complex connections between relatively simple components that are generally either failed or not. Modeling their failure in more detail would be difficult and unnecessary. However, in mechanical systems, failure is often more complex than this binary simplification. Mechanical systems have fewer components than electrical systems, yet the functionality of individual components may be much more complicated. Failure can vary in severity, from mild to catastrophic; failure can vary in totality, from partial to complete; and, failure can vary in periodicity, from intermittent to continuous. Finally, each failure can stand on its own or be part of a multiple-failure situation. Thus, rather than one failure situation for a given failure mode, there is an entire spectrum of possibilities in varying severity, totality, periodicity, and multiplicity.

For a mechanical or electromechanical system this broader understanding of failure is needed to have a more complete and accurate picture of the system's behavior. Additionally, because the type of failure affects the system's symptoms and testability, the diagnostic process is affected by the nature of a given failure as well. Therefore, it will be significant to account for failure types in modeling diagnosability.

For modeling purposes, it would be difficult to take into account a full continuous spectrum of failure types in each of the four categories above (severity, totality, periodicity, and multiplicity). Thus we will create classifications of failures in each of the failure types. For this research, we will classify each failure mode into three different types based on the above descriptions: *full*, *partial*, and *intermittent*. The failures can be further classified as *mild* or *severe*.[1] First we will define each type of failure, then describe how that type effects diagnosis.

2.2.1 Full failure

A full failure occurs when an LRU completely fails to perform the function for which is was designed. For example, when a valve is stuck in the closed position, it is

[1] In this paper we will not quantify these failure types (i.e., a broader range of totality from none to full); however, this would be a valuable enhancement to failure analysis.

considered a full failure because it is not capable of performing its function to allow a fluid to pass through (i.e., provide airflow).

A full failure is generally more easily diagnosed than partial and intermittent ones. The symptoms are present at all times, both when the operator detects the failure, and when a mechanic checks for it. Additionally, a full failure is likely to be more severe in nature, with a greater deviation from the desired behavior, making it easier to pinpoint the likely cause.

2.2.2 Partial failure

When an LRU is still able to perform its desired function, but to a limited degree, it is partially failed. An example is a pneumatic valve which is designed to provide 1.00 m^3/s of airflow, but is only able to supply 0.50 m^3/s because it cannot fully open. Thus, at times the valve may be able to supply the necessary amount of air. But, if the system requires more than 0.50 m^3/s, the valve can no longer perform its function. Similarly, a valve able to fully open, but not responding properly (i.e., not at the correct pressure schedule), would be considered partially failed.

A partial failure creates ambiguity in system performance. The system may not demonstrate any or all of the effects of full failure. For example, the system response may be "sluggish," but not completely ineffective. Indications such as gauges and built in tests may not show a failure condition, as in the above example if the system requires less than 0.50 m^3/s. Thus, for partial failures, information from the first diagnostic phase defined in the last section (observation of system performance and indications) is not as helpful as for a full failure. The second phase of diagnosis (testing) becomes a more crucial step in isolating a fault. In the valve example, a mechanic would be able to observe the valve opening only partially or in the wrong conditions.

2.2.3 Intermittent Failure

When an LRU is in a failed state at some times, and completely functional at others, it is intermittently failing. The failure could be at random times, or only during certain system operating modes. An example is an airplane-system valve that fails to open during flight, but works properly on the ground.

Intermittent failures can be difficult to test. A failed component can cause a system to perform improperly and display certain indications during operation, yet work perfectly fine when it is tested as a possible cause of the problem. This situation is similar to the plight of a patient who notices certain symptoms at home, but then the symptoms are not present when he or she goes into the doctor to be checked. Intermittent failure can easily misdirect the diagnostic process, because if an LRU is tested to be functioning properly, mechanics will assume it is not the cause of the fault and look at other candidates. Thus, for intermittent failures, information from the testing diagnostic phase is not as helpful as with full failure. Information for the observation phase needs to be gathered when the system is in a failed mode.

2.2.4 Failure severity

Eubanks [1997] describes varying failure severity in an icemaker. If the icemaker is slightly tilted from its proper angle, it will produce non-uniform ice cubes. If the icemaker is highly misaligned, the results are very small ice cubes mixed with large, partially liquid ice cubes. So the specific degree of tilt can lead to different failure modes (i.e., 5-10° misalignment or >10°). We will revisit the icemaker example later in the paper (section 3.3 and 4.4).

Classifying failure severity can sometimes diminish ambiguity in fault diagnosis. "Ambiguity groups can be minimized if we take into consideration the magnitude of parameter changes, in addition to direction and sequence of changes." [Sen 1996] For example, continuing with the pneumatic value from above, a failure may create a state of low over-pressurization or very high over-pressurization. Quantifying how much extra flow is being allowed through the value acts as an additional indication to the failure state of the system.

2.2.5 Multiple failures

Multiple failures occur when two or more components are concurrently in a failed state. How often multiple failures occur is a function of the failure rates of the individual components; usually this is much more infrequent than single failures. Exceptions are in the case where components are highly interdependent and one failure may cause a cascading reaction of subsequent failures. In the analyses and models developed in this paper we will assume that only one component has failed when a failure indication occurs. However, we will make notes about how the processes may be modified to account for multiple failures.

3 Building a Diagnosability Model

The first step in evaluating a design for diagnosability is building a model, which shows the relationships between components of the system and possible failure indications. Sen, et al. [1996] state:

> ...a common modeling paradigm is necessary to represent large systems consisting of electronic, electrical, mechanical and hydraulic subsystems... a test engineer analyses the system, either bottom-up or top-down, identifying various failure source-test dependencies of the system. The resulting model forms the basis for system-level testability analysis and fault diagnosis.

This section describes the system's Failure Modes and Effects Analysis (FMEA) and Fault Tree Analysis (FTA), and how they are used to build the diagnosability model. This model will then be used to calculate the system's diagnosability metrics (see section 4).

3.1 Extracting Information for the Model

The main information sources for building the diagnosability model are the FMEA and the fault tree. These two documents contain different perspectives on the failure characteristics of a system, and together offer a complementary picture of a system's reliability and structure early in the design process. The FMEA is organized in a "bottom-up" approach [Leitch 1995], considering each of the system components and analyzing each possible failure mode for its effects at higher levels. The fault tree has the opposite perspective as a "top-down" analysis [Leitch 1995], and is organized by first considering possible failures and then analyzing all possible causes at lower structural levels. Taken together, the FMEA and fault tree can create a fairly accurate representation of the failure-structure relationships in a system needed for effective diagnosability analysis.[2]

3.1.1 FMEA

The FMEA is a widely used document for failure analysis, and will serve as our primary document for obtaining diagnosability information. From the FMEA designers can gain important insight into a system's structure and information flow early in the design process. The data and relationships in FMEA are also valuable input for predictive analysis such as criticality, operability, manufacturability, maintainability, and the diagnosability we are addressing here [Leitch 66]. We will describe the basic structure of the FMEA document, as well as the enhancements needed for the FMEA to contain all of the relevant and necessary information for diagnosability analysis.

Again, as stated in section 1.4, the FMEA is organized by components (LRUs), which are the smallest level of structure we identify for diagnosability analysis. For each of the components in the sub-system, assembly, etc. being considered, the basic FMEA provides the following information:

---

[2]Family genealogy, which can be analyzed both bottom-up and top-down in "tree" diagrams, is a good analogy to FMEA and the fault tree. A bottom-up family tree will identify parents, grandparents, and great-grandparents, while a top-down family tree will identify brothers and sisters, aunts and uncles, and cousins. Together, like the FMEA and FTA, the two family models present a complete understanding of all family relationships.

- The function of the components
- All of the most likely failure modes of the component
- The failure rates of each mode, or of each component combined with failure mode frequency.
- The failure effects on higher levels in the structure, from sub-assemblies to the whole system. [Leitch 1995]

For the FMEA to be most useful for diagnosability, some sections need to added to the standard form to produce an enhanced FMEA for diagnosability. As stated in earlier sections, we want to consider the broader spectrum of failure characteristics for a system including totality and periodicity, and information available during the diagnosis process including both observation and testing phases.

Therefore, each failure mode can be broken down by failure type, including full, partial, and intermittent. Each failure type will have its own failure rate, i.e. λ_{full}, λ_{part}, and λ_{int}. Each failure mode-type combination will also need to have a description of its failure indications. Additionally, it may be helpful to have replacement time data for each component (see section 4.1.3, replacement matrix), and diagnostic testing information (section 4.2.2).

The diagnosability model can be constructed from the FMEA by making connections between components, failure modes, and indications as described in the FMEA table. This process will be outlined in section 3.2 and 3.3.

3.1.2 Fault Tree

Fault Tree Analysis may also be useful for building our model. Fault trees are widely used not only in reliability analysis, but also in safety analysis because they are able to predict causes of failure beyond mechanical malfunction [Bahr 1997]. For example, they are able to take into account human error and other environmental influences. And while FMEA allows designers to focus on specific components and their failure characteristics, fault trees tend to allow for focusing on a particular failure and the sets of component interactions with can lead to that failure. Thus, there is a new understanding of structural relationship uncovered by looking at the fault tree perspective. Furthermore, and important to diagnosability analysis, the fault tree is a valuable tool in computing failure rates. With knowledge of component failure rates, the fault tree allows the rates to be multiplied or added up the tree to obtain a cumulative failure rate for each failure.

The main disadvantage to the fault tree in building our diagnosability model is the binary nature of the events in the tree. Because we are interested in broadening our look at failure into a wider spectrum and more complete picture, we must be careful not to over-simplify based on the fault tree data [Harms-Ringdahl 1993]. Thus, it is best to use the fault tree as a supplementary data source to the FMEA.

3.2 Diagnosability Model

From the information in the FMEA we can analyze failure indications and establish unique indication sets. These indication sets are linked to system components to form our diagnosability model.

An indication is a measured or observed deviation from the desired behavior or performance of a system. The complexity in the diagnosis process arises because a given indication, or set of indications, does not necessary point to one failed component. The relationship between indications and components is illustrated in Table 1. Here, the lower case "i" represents an individual indication. The upper case "I" represents the set of individual indications which all occur for a given failure. Note that if we were including multiple failures in our model, we would add entries for multiple component failures (i.e., C2C3) along with their corresponding indications.

Component / Failure Mode	**Indications** ( ) = sometimes	**Indication Set**
C1 / FM1	i1	I1
C1 / FM2	i1, i2	I2
C2 / FM1	i1, i2	I2
C3 / FM1	i2, (i1)	I3, (I2)
C3 / FM2	i1	I1

Table 1 Simple Indication Set Illustration

In this simple case, when i1 and i2 appear, there is ambiguity (thus forming an ambiguity group) because either component one or two has failed (or *both* have failed). Here, i1 and i2 form the unique indication set I2. The component-indication diagram this illustration is shown in Figure 2. Each line represents a unique failure mode. Attached to each mode is a particular failure rate λ. Figure 2 represents the critical information needed for our diagnosability model.

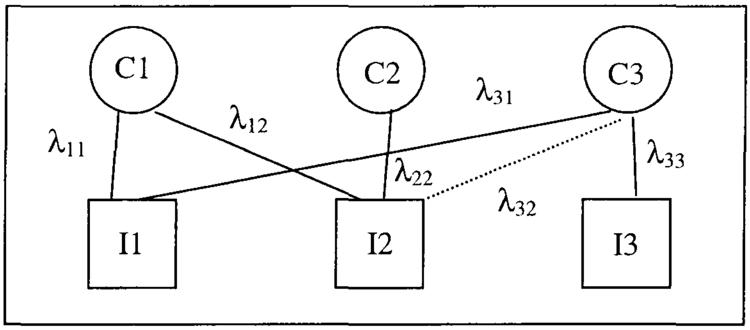

Figure 2 Component-Indication Diagram

Human error or time constraints could cause some indications to be missed. Additionally, certain maintenance procedures could affect diagnosis. For example, some airline procedures (or unofficial maintenance practices) allow for deferring a maintenance action or repeatedly replacing a part that is not likely failed but easily replaceable. Furthermore, human error is often the cause of misdiagnosis (drawing the wrong conclusions from correctly identified indications). These human factors and organizational factors will not be accounted for in our model, and in this paper we will assume that diagnosis is occurring with all indications accurately observed. However, metric values we calculate in the next section may be adjusted to account for these factors.

3.3 Validation Example: Ice-Maker

We will now use some of the modeling methods developed in this section to form a diagnosability model for a validation example.

Eubanks, Kmenta, and Ishii outlined the method for using an *Advanced FMEA* (AFMEA) for modeling a system's behavior. They suggested that their AFMEA could be used for diagnostics prediction [Kmenta 1998]. In fact, Eubanks et al. highlight the inherent relationship between FMEA and diagnosis early in their work. While FMEA builds a model of what behaviors result from given structures, the diagnosis process attempts to accomplish the opposite: seeking the system structure responsible for a given system behavior (or misbehavior) (Figure 3) [Eubanks 1996].

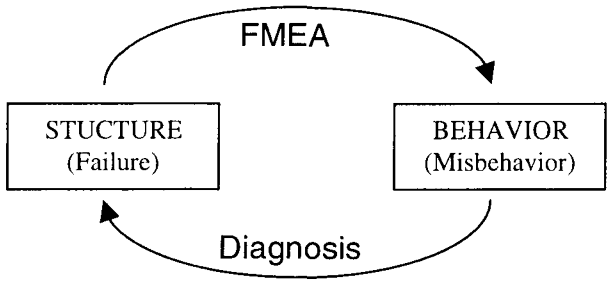

Figure 3 FMEA/Diagnosis Relationship [Eubanks 1996]

Eubanks, et al. [1996, 1997] use an icemaker to illustrate the AFMEA. In order to validate the suggested link between their AFMEA and diagnosis, we will extend the icemaker example by building its diagnosability model. Eubanks presents a function-structure mapping of the icemaker, but for diagnosability analysis we will need to expand this model to include failure indication and component failure rates.

Eubanks, et al. [1997] bring up an important diagnosability issue in the case of external factors. Often indications in a system are not caused by failure of one of the system's components, but rather due to some external influence. This cause could be an environmental factor (i.e., cold temperature) or from another sub-system interconnected with the one being analyzed. From a diagnosis perspective, this issue is significant because the failure indication may lead to the replacement of a part that is not failed.

The traditional FMEA only accounts for components of the system, not external factors. For the icemaker, Eubanks, et al. [1997] use their behavior modeling to identify the refrigerator's alignment as an external factor affecting the icemaker. External factors may also be extracted from a fault tree analysis. We will incorporate the misalignment into our icemaker FMEA as an "external component."

A complete icemaker FMEA is included a the end of this report. Table 2 summarizes possible failure indications. Notice that i5 and i7 are denoted "not observable." These indications are listed in a separate column of the FMEA, and will not be accounted for in forming indication sets. However, these indications would be helpful in analysis of the testing phase.

i1	No ice the bucket
i2	Ice overflowing
i3	Low ice level in the bucket
i4	Ice layer in bucket and/or fused ice cubes
i5	No water in the mold (*not observable*)
i6	Small or irregular ice cubes
i7	Ice stuck in the mold (*not observable*)
i8	Icemaker not running
i9	Feeler arm in the bucket
i10	Large or partially liquid ice cubes

Table 2 Failure indications for the icemaker

As we explained in section 2, there is a high degree of variation in failure modes, and therefore in component-indication relationships as well. The process of grouping indications into indication sets can be rather subjective, as was the case here. Table 3 summarizes the likely component-indication relationships revealed in the FMEA.[3]

Component	**Indication Sets**	**Component**	**Indication Sets**
C1	I2 I6	C6	I2 I5
C2	I2 I3 I4	C7	I1 I2
C3	I2 I3	C8	I1 I2
C4	I1 I5	C9	I2 I5
C5	I1 I2	E (External)	I5 I7

Table 3 Component-Indication relationships for the icemaker

In the next section we will discuss the computation of diagnosability metrics. The icemaker example will then be continued in section 4.4 where we compute its diagnosability metrics.

[3] Small variations of this model can easily be incorporated into the metric computations of section 4 to experiment with effects of changes in the model.

4 Diagnosability Metrics

We have developed a set of metrics that comprise a complete description of the diagnosability of a system and its components. To derive these metrics, we translate our component-indication model into a matrix model. These matrices can be more easily manipulated mathematically to produce numerical results. First, we will analyze our system based on information only from the observation phase. The result will be our new distinguishability metric. Secondly, we will extend the analysis to the testing phase, and our distinguishability metric will be broadened into the testability metric. Finally, we will evaluate the *MTBUR* of the system and its components, completing a set of metrics, along with the replacement rate matrix, which give valuable insight into the diagnosability of the system.

4.1 Distinguishability

The metric associated with the observation phase of diagnosis we define as distinguishability (D), an estimate of the probability a mechanic, in the initial maintenance attempt, will correctly infer a specific component as the cause of failure, given some failure indication has occurred.[4] The metric comes in several forms: *system*, *indication*, and *component* distinguishability (D_{sys}, D_{ind}, D_{LRU}). Additionally,

[4]Note that this is a different definition of distinguishability from Clark [1996]. While it remains a similar system measure, this new D is specifically a probability of removal rather than an arbitrary index value.

D_{ind} and D_{LRU} can be unweighted or *weighted* (WD_{ind}, WD_{LRU}). The D metrics are all conditional probabilities of a justified removal.[5] Table 4 summarizes the individual definitions.

Metric	Probability of:
D_{sys}	justified removal, given *some* failure indication (or *some* component failed) computed as: $\sum_{i=1}^{n} WD_{ind}(i)$ or $\sum_{j=1}^{m} WD_{LRU}(j)$
$D_{ind}(i)$	justified removal, given *ith* failure indication
$WD_{ind}(i)$	*ith* failure indication and justified removal, given *some* failure indication
$D_{LRU}(j)$	justified removal, given *jth* component failed
$WD_{LRU}(j)$	*jth* component failed and justified removal, given *some* component failed

Table 4 Definitions for Distinguishability metrics

Clark [1996] established a method for evaluating *system* distinguishability, which is important for comparing the overall diagnosability of competing designs. However, in order to evaluate the design of a system, it is helpful to have a metric that evaluates the distinguishability of each individual *component* in a given configuration. D_{LRU} fits this criteria by measuring the overall ability to separate a given component from others in the process of isolating faults. If the number of components mapped to a particular indication decreases, then D_{LRU} will decrease as well. While D_{ind} and D_{LRU} are very similar, the former helps in understanding ease of diagnosis and the latter is geared toward optimizing design.

---

[5] Removing a failed component is *justified*. Removing a working component is *unjustified*.

4.1.1 Susceptibility

Another metric family related to D is susceptibility (S), which are the probabilities of unjustified removals, and summarized in Table 5:

Metric	Probability of:
S_{sys}	unjustified removal, given *some* component removed computed as: $1 - D_{sys}$ or $\sum_{j=1}^{m} WS_{LRU}(j)$
$S_{LRU}(j)$	unjustified removal, given *jth* component removed
$WS_{LRU}(j)$	*jth* component removed and unjustified removal, given *some* component removed

Table 5 Definitions for Susceptibility metrics

The main different between S and D is that D is conditional on a particular *failure* occurring, while S in conditional on a particular *replacement* occurring. Also note that S_{sys} is not unique information from D_{sys}, but only the inverse probability. As we will see later on, S is closely related to $MTBUR_{unj}$.

4.1.2 Example problem: Distinguishability Analysis

To illustrate the method for obtaining these metrics, we will use an simple, abstract sample problem (not representing an actual system). From the FMEA (Table 6), we obtain the component-indication relationships for the system. The component-indication relationships from the FMEA are diagramed in Figure 4.

Component	*Modes (Indication Sets)*
C1	I1 I3 I5
C2	I3 I5
C3	I1 I4 I6
C4	I2
C5	I1 I2 I4 I5

Table 6 Abbreviated FMEA for example problem

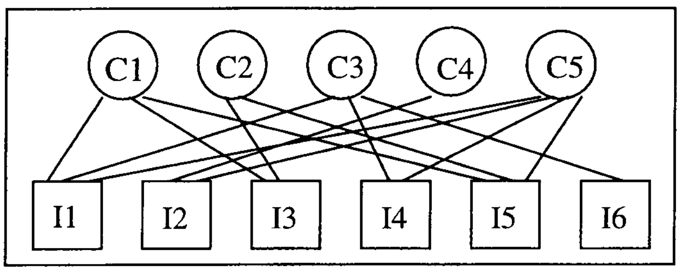

Figure 4 Component-Indication Diagram for example problem

As discussed in the last section, each failure mode (and correlated indication set) has a specific failure rate (λ). Failure rates are the expected frequency of failure over a long period of time. Over short periods, the frequency of failure will vary. These failure rates can be organized as shown in Table 7 for the example problem.

[10^{-4} failures/hour]	C1	C2	C3	C4	C5	Indication Rate	Indication Prob
I1	2	0	4	0	3	9	0.085
I2	0	0	0	2	15	17	0.160
I3	3	2	0	0	0	5	0.047
I4	0	0	17	0	25	42	0.396
I5	10	12	0	0	3	25	0.236
I6	0	0	8	0	0	8	0.075
Comp Rate	15	14	29	2	46	106	
Comp Prob	0.142	0.132	0.274	0.019	0.434		

Table 7 Component-indication failure rate table for example problem

Each row can be summed to obtain an indication rate, $\lambda_{ind}(i)$. Each column can be summed to obtain a component's failure rate (for all failure modes), $\lambda_{LRU}(j)$. The sum of these rates is the system failure rate, λ_{sys}. The indication probability or component failure probability is computed by dividing the indication rate or component failure rate by the system failure rate.[6]

4.1.3 The Failure Rate Matrix λ and Replacement Matrix R

For obtaining our metrics, we will convert the failure rate data into an $n \times m$ matrix, λ (There are n indication sets and m components). Additionally, we will form a $n \times m$ replacement matrix, **R**. The replacement matrix is an important mathematical representation of the predicted maintenance action for each indication. Each row of **R** contains a single 1 and the rest zeroes. The one is placed in the column representing

[6] Initially, for computing distinguishability and testability, it is fine to use relative failure rates; however, when we calculate *MTBUR* we will need absolute failure rates.

the component that will be replaced for the corresponding indication. The replacement component can be determined using one of three criteria:

- Failure rate
- Replacement time
- Checking index (Failure rate / replacement time)

For this example, we will use the highest failure rate to determine the replacement component in each row of **R**. So for indication one, the "1" is placed in the third column because C3 has the highest failure rate. It is important to note that in this analysis we are considering only the *initial* maintenance attempt. We will deal with accounting for subsequent removals in the section on the *MTBUR* metric (section 4.3). Thus we have:

$$\lambda = \begin{bmatrix} \mathbf{2} & \mathbf{0} & \mathbf{4} & \mathbf{0} & \mathbf{3} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{2} & \mathbf{15} \\ \mathbf{3} & \mathbf{2} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{17} & \mathbf{0} & \mathbf{25} \\ \mathbf{10} & \mathbf{12} & \mathbf{0} & \mathbf{0} & \mathbf{3} \\ \mathbf{0} & \mathbf{0} & \mathbf{8} & \mathbf{0} & \mathbf{0} \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

4.1.4 Computing the Replacement Rate Matrix λ_R

From the λ and **R** matrices we obtain the $m \times m$ (square) replacement rate matrix, $\lambda_{\mathbf{R}}$, by multiplying the transpose of the replacement matrix by the failure rate matrix (Equation 1). Equivalently, the element in the *ith* row and *jth* column of $\lambda_{\mathbf{R}}$ is obtained by taking the dot product of the *ith* column vector of **R** with the *jth* column vector of λ (Equation 1a).

$$\boldsymbol{\lambda}_{\mathbf{R}} = \mathbf{R}^T \boldsymbol{\lambda} \tag{1}$$

$$\text{where} \quad \lambda_{R_{i,j}} = \bar{R}_i \cdot \vec{\lambda}_j \tag{1a}$$

Failed Component

$$\begin{array}{r c} & \begin{array}{ccccc} \mathrm{C1} & \mathrm{C2} & \mathrm{C3} & \mathrm{C4} & \mathrm{C5} \end{array} \\ \begin{array}{r} \mathrm{C1} \\ \mathrm{C2} \\ \text{Replaced Component } \mathrm{C3} \\ \mathrm{C4} \\ \mathrm{C5} \end{array} & \begin{bmatrix} \mathbf{3} & \mathbf{2} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{10} & \mathbf{12} & \mathbf{0} & \mathbf{0} & \mathbf{3} \\ \mathbf{2} & \mathbf{0} & \mathbf{12} & \mathbf{0} & \mathbf{3} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{17} & \mathbf{2} & \mathbf{40} \end{bmatrix} \end{array}$$

UNJUSTIFIED REMOVALS

JUSTIFIED REMOVALS

Figure 5 Replacement Rate Matrix λ_R

The λ_R matrix shown in Figure 5 includes each combination of component failures and replacements. The numbers in the diagonal of the λ_R matrix are the rates for justified removals. The numbers in the off-diagonals are thus unjustified removal rates. From these rates we can easily obtain all our metric values. For example, the failure rate for C1 is 15×10^{-4} failures/hour. The justified removal rate is 3×10^{-4}. Thus, D_{LRU} is computed as 3/15 = 0.200. Table 8 summarizes the metric values for this example problem:

I/C No.	D_{ind}	WD_{ind}	D_{LRU}	WD_{LRU}	S	WS
1	0.444	0.038	0.200	0.028	0.400	0.019
2	0.882	0.142	0.857	0.113	0.520	0.123
3	0.600	0.028	0.414	0.113	0.294	0.047
4	0.595	0.236	0.000	0.000	N/A	0.000
5	0.480	0.113	0.870	0.377	0.322	0.179
6	1.000	0.075				
	D_{sys} =	**0.63**	D_{sys} =	**0.63**	S_{sys} =	**0.37**

Table 8 Metric Values for example problem

D_{sys} and S_{sys} are a system-wide metrics, helpful in determining the system-wide effect of changes in diagnosability. In the current model, our system has a 63% probability of being diagnosed correctly after the observation phase of diagnosis. Table 8 also illuminates the differences between weighted and unweighted metrics. The unweighted metrics range from 0 to 1 for each value, whereas the *sum* of the weighted metrics ranges from 0 to1. The weighted metrics give sense of the importance of the indication or component metric in the perspective of the whole system. For example, indication three has a satisfactory D_{ind} value of 0.600; however, its WD_{ind} value of 0.028 suggests the indication's metric is not as significant as others.

There are a couple of components that Table 8 highlight as good candidates for diagnosability improvement. We notice that component one has a low distinguishability of 0.200; however, this is less significant because its weighting is relatively low. Component three is important to consider: it has only a 41% chance of being diagnosed correctly on the initial attempt, coupled with its significant WD_{LRU} value of 0113.

4.2 Testability

The second metric we will develop, associated with the testing phase of the diagnosis process, is testability *(T)*. Simpson and Sheppard [1994] introduce the following definition for testability:

> ...a design characteristic which allows the status (operable, inoperable, or degraded) of an item to be determined and the isolation of faults within the item to be performed in a timely and efficient manner.

Our testability metric is a measure of the design characteristic to which Simpson and Sheppard refer. Similar to distinguishability, testability will measure the probability a correct component is isolated after diagnostic testing has occurred. Thus, testability broadens the scope of our diagnosability measurements to the testing phase. As it turns out, distinguishability *(D)* is a special case of testability *(T)* where no testing has taken place. Additionally, by examining the changes between *T* and *D* *(ΔT)*, we can ascertain, for the system or an individual indication or component, the ability for ambiguity to be decreased (or discernment increased) through testing procedures.

4.2.1 Critical Time

Crucial to the testing phase of diagnosis is the critical time τ_c for testing. In mechanical systems, testing can be time intensive task. There is a critical point for which the costs of additional testing outweigh the gains in ambiguity reduction. This critical time is apparent in airplane maintenance, where after a certain amount a time a flight will be delayed or even cancelled. We will model a testing phase that is constrained by the critical time. When there is more time available for testing, more tests can be run and thus more ambiguity eliminated. The testability value will therefore be a function of τ_c (Figure 6), where higher τ_c values should produce higher T values. (In reality, the function in Figure 6 would have steps, representing added tests, rather than a smooth curve.)

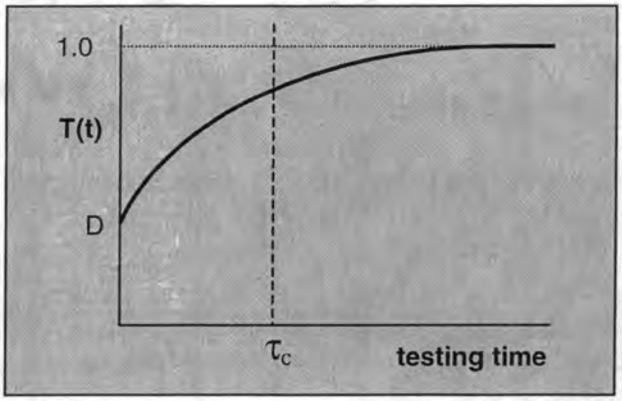

Figure 6 Typical Testability Function

The values of the T family parallel those of D and are summarized in Table 9:

Metric	Probability of:
$T_{sys(}\tau_{c)}$	justified removal, given *some* failure indication and critical time τ_c computed as: $\sum_{i=1}^{n} WT_{ind}(i)$ or $\sum_{j=1}^{m} WT_{LRU}(j)$
$T_{ind}(i,\ \tau_c)$	justified removal, given *ith* failure indication and critical time τ_c
$WT_{ind}(i,\ \tau_c\ )$	*ith* failure indication and justified removal, given *some* failure indication and critical time τ_c
$T_{LRU}(j,\ \tau_c)$	justified removal, given *jth* component failed and critical time τ_c
$WT_{LRU}(j,\ \tau_c)$	*jth* component failed and justified removal, given *some* component failed and critical time τ_c

Table 9 Definitions for Testability metrics

It should be noted that the S metrics will also change values, and continue to provide important insight, when testing is taken into account.

4.2.2 Example Problem Continued: Testability Analysis

We will expand the theoretical example problem begun in the distinguishability section to illustrate testability analysis. The computation of *T* will be similar to *D*, but we will need to analyze the testing options and make some adjustments to the failure rate and replacement matrices.

The first step in computing testability is to catalog the available system tests. Each indication set has an associated catalog of possible tests that verify the candidate components for that indication. Each possible test has an associated testing time (see Table 10 below). The test symbols (T_{ij}) shown in the table have additional subscripts that denote the candidates they verify. If a test is negative, then neither component is failed. If a test is positive, then either or both of the components are failed. (However, we will continue to make the simplifying assumption that only one component has failed.) Our catalog of test needs take into account failure types of the candidate components for each indication. As mentioned in section 2.2.3, intermittent failures may not be verifiable by testing.

Table 10 lists tests relevant to indication one, sorted by the checking index. The checking index is computed by dividing the failure probability by the testing time. The failure probability used for computing the checking index is obtained in one of two ways. For tests that verify one component (i.e., T1 verifies C1), that component's failure probability, given indication one, is used. Because there are only three candidates for indication one, tests that verify either of two candidate components actually verify the remaining candidate (i.e., T35 verifies C1). Thus, the probability for the remaining component is used for the checking index computation.[7]

Test	Failure Rate (λ) [10^{-3}/hour]	Test time (τ) [min]	Checking Index (λ/τ)
T1	2 (verifies C1)	3	0.67
T3	4 (C3)	7	0.57
T23	4 (C3)	10	0.40
T5	3 (C5)	13	0.23
T35	2 (C1)	9	0.22
T45	3 (C5)	14	0.21
T12	2 (C1)	15	0.13

Table 10 Example Test Set and Checking Order for Indication One

The checking order gives the ideal progression of tests for the testing phase of diagnosis. A high checking index means that test should be run first.[8] The testability metric will be a function of which tests can ideally be run which verify the maximum number of components within the critical time restriction.

If we wanted to expand the model further, we could include component removals in the catalog of tests as well. Removals have the same effect as tests. For example, C1 may be removed for testing, or because of misdiagnosis, and found normal. C1 is then verified and the fault lies elsewhere. Thus Table 10 could include removals R1, R3,

[7] These multiple index tests (i.e., T35) are more difficult to compute into the checking index when there are more than three candidate components. Thus, while these tests are included here to show the possibility of these tests, it may be easier to conduct this analysis with only "single-index" tests.

[8] For more information on optimum test sequencing in localizing failure, see Pau Chapter 4.

and R5 with their respective replacement times (which typically are longer than testing times).

4.2.3 Matrix Row-Split Method

To compute the values of T, we will return to the matrix methods developed in section 4.1.3. Because testing introduces multiple replacement possibilities for a given indication, we will have to make some modifications to the failure rate matrix λ and replacement matrix **R**. To illustrate these modifications, we will continue with the testing analysis of indication one in the example problem.

If we have a critical time of eight minutes, there is only time to run one test, T_1.[9] There are two possible outcomes. If the test is positive, then C1 is failed and C1 will be replaced. We will denote this scenario as test outcome T1.1. If the test is negative, then either C3 or C5 are failed. In this case, if failure rate is the replacement criteria, then C3 would be replaced (test outcome T1.2). The first row (corresponding to indication one) of the matrices would be split into two rows for each outcome.

[9] Actually, a mechanic may in this case decide to run only T_3, as this test can be run within the critical time and verifies a component with a higher failure rate.

Matrix Row Split for Indication One (I1 → T1.1/T1.2)	
Failure Rate Matrix λ	**Replacement Matrix R**
$\begin{bmatrix}2 & 0 & 4 & 0 & 3\end{bmatrix} \rightarrow \begin{bmatrix}2 & 0 & 0 & 0 & 0\\ 0 & 0 & 4 & 0 & 3\end{bmatrix}$	$\begin{bmatrix}0 & 0 & 1 & 0 & 0\end{bmatrix} \rightarrow \begin{bmatrix}1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\end{bmatrix}$

Figure 7 Matrix Row Split for Indication One

Notice that each new row of **R** contains a single "1" indicating the component replaced as before. The following is sample computation of the replacement rate matrix $\lambda_{\mathbf{R}}$ from revised matrices for all indications, where each row has been split into test outcomes.

$$\begin{array}{r} \mathbf{T1.1}\\ \mathbf{T1.2}\\ \mathbf{T2.1}\\ \mathbf{T2.2}\\ \mathbf{T3.1}\\ \mathbf{T3.2}\\ \mathbf{T4.1}\\ \mathbf{T5.1}\\ \mathbf{T5.2}\\ \mathbf{T5.3}\\ \mathbf{T6.1} \end{array}\begin{bmatrix} 2 & 0 & 0 & 0 & 0\\ 0 & 0 & 4 & 0 & 3\\ 0 & 0 & 0 & 0 & 15\\ 0 & 0 & 0 & 2 & 0\\ 3 & 0 & 0 & 0 & 0\\ 0 & 2 & 0 & 0 & 0\\ 0 & 0 & 17 & 0 & 25\\ 0 & 12 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 3\\ 10 & 0 & 0 & 0 & 0\\ 0 & 0 & 8 & 0 & 0 \end{bmatrix}^{T} \bullet \begin{bmatrix} 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 1 & 0\\ 1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} = \overset{\lambda_{\mathbf{R}}}{\begin{bmatrix} 15 & 0 & 0 & 0 & 0\\ 0 & 14 & 0 & 0 & 0\\ 0 & 0 & 12 & 0 & 3\\ 0 & 0 & 0 & 2 & 0\\ 0 & 0 & 17 & 0 & 43 \end{bmatrix}}$$

There are now many more values on the diagonal of $\lambda_{\mathbf{R}}$, and therefore more justified removals. In effect, each test outcome moves a value vertically in its column from an off-diagonal to a diagonal position in $\lambda_{\mathbf{R}}$ (compare the new matrix with the one illustrated in Figure 4). Table 11 shows how the testability changes for various critical time values for indication one.

Critical Time [min]	Testability of Indication One	ΔT_{ind}
0	0.444 (D_{ind})	0
8	0.667	0.222
12	1.00	0.556

Table 11 Effect of critical time on testability values

As stated earlier in section 4.2, it is helpful to examine the change between distinguishability and testability. Table 12 shows the how testability analysis effects the diagnosability metrics. (Again, for this example problem, we have increased critical time from zero to eight minutes between computing *D* and *T*). By this table, we can tell that the testing phase has significant impact on indications three and five, along with components one and four. Notice the greatest susceptibility decrease is for component two.

I/C No.	ΔT_{ind}	ΔWT_{ind}	ΔT_{LRU}	ΔWT_{LRU}	ΔS	ΔWS
1	+0.222	+0.019	+0.800	+0.113	-0.400	-0.019
2	+0.118	+0.019	+0.143	+0.019	-0.520	-0.123
3	+0.400	+0.019	0.000	0.000	-0.094	-0.019
4	0.000	0.000	+1.000	+0.019	N/A	0.000
5	+0.520	+0.123	+0.065	+0.028	-0.039	-0.019
6	0.000	0.000				
	ΔT_{sys} =	**+0.18**	ΔT_{sys} =	**+0.18**	ΔS_{sys} =	**-0.18**

Table 12 Metric value changes with the testing phase

Component one, which we flagged has having low distinguishablity in section 4.1.4, had a +0.800 improvement after testing (giving C1 a testability of 1.0). However, C3 had poor diagnosability after the observation stage, and Table 12 indicates there was no improvement in its T_{LRU}. This result would strengthen C3 as candidate for diagnosability improvement.

4.3 Mean Time Between Unscheduled Removals

While the *D* and *T* metrics are valuable for design evaluation, we would also like a metric that lends itself more to the evaluation of the life-cycle costs associated with fault isolation. For this purpose we use Mean Time Between Unscheduled Removals (*MTBUR*). (For example, The Boeing Company has developed a cost model that uses *MTBUR* as one of its inputs [Boeing].) This metric, rather than measuring a *probability* of a removal like *D* or *T*, measures the average *time* between removals.

Because *MTBUR* accounts for all component replacements, we need to rework the failure rate and replacement matrices, which previously only accounted for the initial replacement. We will again utilize the matrix row-split method to make this adjustment. Figure 8 illustrates the changes to the matrices made for indication one, for both distinguishability and testability computations (we denote the rows “IM” and “TM” to correlate with *MTBUR*):

Observation Phase: I1 → IM1.1/1.2/1.3 (I1 split into three rows)	
Failure Rate Matrix λ	**Replacement Matrix R**
$[2\ \ 0\ \ 4\ \ 0\ \ 3] \rightarrow \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$	$[0\ \ 0\ \ 1\ \ 0\ \ 0] \rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$
Test Phase: T1.1/1.2 → TM1.1.1/1.2.1/1.2.2 (Test outcome 1.2 split into two rows)	
$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$

Figure 8 ***MTBUR*** **Matrix Row Split for Indication One**

Simply put, these new matrices describe the replacements made for each failure scenario, based on using the failure rate criteria. Each row of λ now contains only one element (for one failure mode). The corresponding row of **R** describes all the component replacements predicted for the failure (each row can now have multiple "1"s). Figure 8 shows that for indication one, C1's failure leads to replacements of C1, C3, and C5 (based on our criteria, the order of replacement would be C3, C5, C1). In the second row, C3's failure leads to the initial, justified replacement of C3. Finally, the third row represents C5's failure leading to the replacement of C3 and C5.

MTBUR can be calculated directly from the revised λ_R matrix. If each row is summed, we obtain a removal rate for each component (R_{LRU}).

$$MTBUR = \frac{1}{R_{LRU}}$$

MTBUR may be broken up into justified and unjustified removals. While $MTBUR_{just}$ is related to component reliability, $MTBUR_{unj}$ is more closely related to fault isolation problems. For calculating these metrics, we separate R_{LRU} into a justified part (the diagonal matrix element) and an unjustified part (the sum of the off-diagonals for that row). Note the relationship between the *MTBUR* metrics:

$$MTBUR = \left(\frac{1}{MTBUR_{just}} + \frac{1}{MTBUR_{unj}} \right)^{-1}$$

Table 13 summarizes the *MTBUR* results for our example problem. In the table, N/A denotes that the LRU is never replaced according to the model (can be considered MTBUR $\rightarrow \infty$). In the columns for change, a "+" indicates a change from some value to N/A.

	Observation		Testing		Change	
LRU No.	$MTBUR_{unj}$	MTBUR	$MTBUR_{unj}$	MTBUR	$MTBUR_{unj}$	MTBUR
C1	5,000	588	N/A	667	+	+79
C2	769	370	N/A	714	+	+344
C3	2,000	294	3,333	313	+1,333	+19
C4	N/A	5,000	N/A	5,000	N/A	0
C5	323	130	588	159	+265	+29
System	**196**	**64**	**500**	**79**	**+304**	**+15**

Table 13 *MTBUR* Values for the example problem

These *MTBUR* values suggest the impact each individual LRU will have on maintenance costs of fault isolation. Generally, the data shows that *MTBUR* values have improved due to testing. We would conclude, from its low *MTBUR* value, that C5 is a likely LRU for focusing on fault isolation cost improvement.

4.4 Validation Example Continued: Ice-Maker Diagnosability Metrics

Now that we have established the method for computing diagnosability metrics, we will use the information from the icemaker FMEA in section 3.3 to calculate the metrics for the icemaker. The metrics values for the icemaker are listed in appendix, along with the entire spreadsheet tool used to calculate the replacement rate matrix and metric values. For this validation we only took the analysis through the observation phase level. The validation could be expanded further by cataloging tests and using the methods of section 4.2.

For the icemaker, the switch linkage and switch (C2 and C3) have interesting diagnosability characteristics. The switch linkage has a 1.00 distinguishability, weighed at a significant 0.311 over the whole system, suggesting satisfactory component diagnosability. However, the linkage does have the lowest *MTBUR* of 21,008 hours. In contrast, the switch itself has a D_{LRU} of 0.0 an overall 0.119 *misdiagnosis* probability,[10] the highest of all components.

[10] The misdiagnosis probability was not formally given a name as a metric, but can still be informative-as is the case here.

The difference in C2 and C3 highlights the fact these components have high failure rates combined with the same indication profile. So for the best diagnosability, we want to configure systems so the components with lowest reliability have *different* failure indications. These numbers also show us that components with good distinguishability numbers may still have a low *MTBUR* value because of their low reliability. Thus, it is beneficial to evaluate *all* the metrics before drawing conclusions.

The $\lambda_\mathbf{R}$ matrix also shows us that external factors (E) contribute to the water delivery system's (C6) having the highest susceptibility (*S*) for the icemaker. This fact serves to validate Eubank's assertion that external factors are an important consideration in our modeling.

5 Summary and Conclusions

Our objective in this research was to create a method for modeling a system in way that readily describes the system's diagnosability characteristics, or the ease of isolating faults in the system. After presenting the problem of diagnosability and the motivations for pursuing improvement, we established the setting for developing our model by describing failure and diagnosis. The full spectrum of failure types and complete diagnostic process provided a framework for a more accurate model for diagnosability analysis. We described the use of the FMEA and fault tree for extracting the information needed for our model. Finally, we presented a new process for computing diagnosability metrics by using matrix algebra and the matrix "row-split" method to derive the highly informative replacement rate matrix $\lambda_{\mathbf{R}}$. Important in this process was the new replacement matrix **R**, which described the predicted maintenance actions for given indications. From $\lambda_{\mathbf{R}}$ we were able to extract many diagnosability measurements, including the distinguishability, testability, and *MTBUR* metrics. We successfully validated many of our methodologies with the icemaker mechanism presented by Eubanks [1997]. The new mathematics for computing the metrics are relatively simple compared to previous methods. Changes in the model can be input into a spreadsheet tool, instantly computing updated diagnosability measures.

The methodologies of this paper are generally applicable to many electromechanical systems. However, diagnosability analysis is most beneficial to systems with low reliability, high maintenance costs, and high complexity.[11] The results reveal important characteristics of the system, failure indications, and individual components for improving diagnosis times and minimizing the costs of fault isolation. Observing changes in the metrics disclose the diagnosability effects of design changes.

There is opportunity for future work in several areas. More could be uncovered in the complexity of failures and their relationship to the diagnostic process presented in section 2; these failure and diagnosis theories can be more tightly woven into the modeling and metric computation process, creating a more accurate representation of system diagnosability. For example, failure modes could be weighted in the metric computation by the failure's severity. Failure severity—or the consequences of failure—could also affect the replacement and testing criteria (checking index). Furthermore, the adjustments made in section 4.3 to account for all replacements, creating a modified $\lambda_{\mathbf{R}}$ matrix, could possibly be extended to create more accurate distinguishability and testability metrics. Finally, the methods of this research can be validated more rigorously on a complex system beginning early in its design process.

[11] While the icemaker example served as a simple case for validation in this paper, these criteria suggest it would not benefit as greatly from diagnosability analysis as other systems.

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Appendix

Icemaker FMEA Document

Component	Function	Failure Mode	Failure Type	Failure Rate [per million cycles]	Effect	Sys Effect (observable)	Indication Code () = sometimes [Indication Set]
1. Feeler arm	Sense ice level in bucket	Broken off	Full	3		No ice, feeler arm in bucket at times	1, (9) [2], (6)
2. Switch linkage	Feeler arm – Switch connection	Stuck closed	Full	60		Ice overflow	2 [3]
		Stuck closed	Intermittent	50		Ice overflow	(2) [3]
		Stuck open	Full	80		No ice	1 [2]
		Stuck open	Intermittent	70		Low ice in bucket at times	(3) [4]
3. Switch	Activate/deactivate ice maker	Stuck closed	Full	50		Ice overflow	2 [2]
		Stuck open	Full	50		No ice	1 [3]
4. Mold	Hold water, form ice geometry and size	Crack	Partial	8	Water leak	Small ice, ice layer in bucket	6, (4) [5]
		Hole	Full	1	Water leak, mold empty	No ice, ice layer in bucket	1, 4 [1]
5. Freezer	Freeze water	Not functioning	Full	30	High temp	No ice, water in bucket at times	1, (4) [2], (1)
6. Water Delivery System	Fill mold w/ water	Not functioning	Full	25	No water in mold	No ice	1 [2]
		Slow water	Partial	45		Small ice	6, (3) [5]
7. Mold heating system	Loosen ice	No heat	Full	90	Ice stuck in mold	No ice	1, (4) [2], (1)
8. Ice harvesting system	Remove ice from mold	Not functioning	Full	30	Ice stuck in mold	No ice	1, (4) [2], (1)
9. Ice timer	Allow proper freezing time	Not functioning	Full	40	Ice stuck in mold	No ice	1 [2]
		Too fast	Partial	15	Water leak	Small ice	6, (4) [5]
EXTERNAL: Refrigerator Alignment	Create a consistent water level in the ice mold	Small misalignment	Mild severity	150		Small ice	6 [7]
		Large misalignment	Severe	40	Water leak	Small ice, ice layer in bucket	6, 4 [5]

Worksheet for calculating metrics for example problem

Component-Indication Failure Rate Matrix for example problem

$[10^{-4}\ \text{hour}^{-1}]$	C1	C2	C3	C4	C5	Indication Rate	Indication Prob
I1	2	0	4	0	3	9	0.085
I2	0	0	0	2	15	17	0.160
I3	3	2	0	0	0	5	0.047
I4	0	0	17	0	25	42	0.396
I5	10	12	0	0	3	25	0.236
I6	0	0	8	0	0	8	0.075
Comp Rate	15	14	29	2	46	106	
Comp Prob	0.142	0.132	0.274	0.019	0.434		

Replacement Matrix

	C1	C2	C3	C4	C5	Justified	P(Just)	WP(Just)
I1	0	0	1	0	0	4	0.444	0.038
I2	0	0	0	0	1	15	0.882	0.142
I3	1	0	0	0	0	3	0.600	0.028
I4	0	0	0	0	1	25	0.595	0.236
I5	0	1	0	0	0	12	0.480	0.113
I6	0	0	1	0	0	8	1.000	0.075
						67		**0.63**

replacement probabilities given indication

Replacement Rate Matrix

Rate Replaced	Failed C1	C2	C3	C4	C5	Repl Rate	Justified	P(Just)	WP(Just)	Unjust	P(Unj)	WP(Unj)
C1	**3**	2	0	0	0	5	3	0.600	0.028	2	0.400	0.019
C2	10	**12**	0	0	3	25	12	0.480	0.113	13	0.520	0.123
C3	2	0	**12**	0	3	17	12	0.706	0.113	5	0.294	0.047
C4	0	0	0	**0**	0	0	0	#DIV/0!	0.000	0	#DIV/0!	0.000
C5	0	0	17	2	**40**	59	40	0.678	0.377	19	0.322	0.179
Comp Rate	15	14	29	2	46	106	67		**0.63**	39		**0.37**
Justified	3	12	12	0	40	67						
P(Just)	0.200	0.857	0.414	0.000	0.870							
WP(Just)	0.028	0.113	0.113	0.000	0.377	**0.63**						
Unjustified	12	2	17	2	6	39						
P(Unjust)	0.800	0.143	0.586	1.000	0.130							
WP(Unjust)	0.113	0.019	0.160	0.019	0.057	**0.37**						

given component replaced

given component failed

Testability Failure Rate Matrix

$[10^{-4}\ hour^{-1}]$	C1	C2	C3	C4	C5	Indication Rate
T1.1	2	0	0	0	0	
T1.2	0	0	4	0	3	9
T2.1	0	0	0	0	15	
T2.2	0	0	0	2	0	17
T3.1	3	0	0	0	0	
T3.2	0	2	0	0	0	5
T4.1	0	0	17	0	25	42
T5.1	0	12	0	0	0	
T5.2	0	0	0	0	3	
T5.3	10	0	0	0	0	25
T6.1	0	0	8	0	0	8
Comp Rate	15	14	29	2	46	106
Comp Prob	0.142	0.132	0.274	0.019	0.434	

Worksheet for calculating testability metrics for example problem

	C1	C2	C3	C4	C5	Justified	P(Just)	WP(Just)
T1.1	1	0	0	0	0			
T1.2	0	0	1	0	0	6	0.667	0.057
T2.1	0	0	0	0	1			
T2.2	0	0	0	1	0	17	1.000	0.160
T3.1	1	0	0	0	0			
T3.2	0	1	0	0	0	5	1.000	0.047
T4.1	0	0	0	0	1	25	0.595	0.236
T5.1	0	1	0	0	0			
T5.2	0	0	0	0	1			
T5.3	1	0	0	0	0	25	1.000	0.236
T6.1	0	0	1	0	0	8	1.000	0.075
						86		**0.81**

Rate Replaced	Failed C1	C2	C3	C4	C5	Repl Rate	Justified	P(Just)	WP(Just)	Unjust	P(Unj)	WP(Unj)
C1	**15**	0	0	0	0	15	15	1.000	0.142	0	0.000	0.000
C2	0	**14**	0	0	0	14	14	1.000	0.132	0	0.000	0.000
C3	0	0	**12**	0	3	15	12	0.800	0.113	3	0.200	0.028
C4	0	0	0	**2**	0	2	2	1.000	0.019	0	0.000	0.000
C5	0	0	17	0	**43**	60	43	0.717	0.406	17	0.283	0.160
Comp Rate	15	14	29	2	46	106	86		**0.81**	20		**0.19**
Justified	15	14	12	2	43	86						
P(Just)	1.000	1.000	0.414	1.000	0.935							
WP(Just)	0.142	0.132	0.113	0.019	0.406	**0.81**						
Unjustified	0	0	17	0	3	20						
P(Unjust)	0.000	0.000	0.586	0.000	0.065							
WP(Unjust)	0.000	0.000	0.160	0.000	0.028	**0.19**						

Worksheet for calculating icemaker metrics

Failure Rate Matrix

1.00E-07	C1	C2	C3	C4	C5	C6	C7	C8	C9	E	Indication Rate	Indication Prob
I1	0	0	0	1	10	0	70	20	0	0	101	0.121
I2	1	80	50	0	20	25	20	10	40	0	246	0.294
I3	0	110	50	0	0	0	0	0	0	0	160	0.191
I4	0	70	0	0	0	0	0	0	0	0	70	0.084
I5	0	0	0	8	0	45	0	0	15	40	108	0.129
I6	2	0	0	0	0	0	0	0	0	0	2	0.002
I7	0	0	0	0	0	0	0	0	0	150	150	0.179
Comp Rate	3	260	100	9	30	70	90	30	55	190	837	1.000
Comp Prob	0.004	0.311	0.119	0.011	0.036	0.084	0.108	0.036	0.066	0.227	1.000	

Replacement Matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	E	Justified	Dind P(Just)	WDind WP(Just)
I1	0	0	0	0	0	0	1	0	0	0	70	0.693	0.084
I2	0	1	0	0	0	0	0	0	0	0	80	0.325	0.096
I3	0	1	0	0	0	0	0	0	0	0	110	0.688	0.131
I4	0	1	0	0	0	0	0	0	0	0	70	1.000	0.084
I5	0	0	0	0	0	1	0	0	0	0	45	0.417	0.054
I6	1	0	0	0	0	0	0	0	0	0	2	1.000	0.002
I7	0	0	0	0	0	0	0	0	0	1	150	1.000	0.179
											527		**0.63**

Replacement Rate Matrix

Rate Replaced	Failed C1	C2	C3	C4	C5	C6	C7	C8	C9	E	Repl Rate	Justified	P(Just)	WP(Just)	Unjust	S LRU P(Unj)	WS LRU WP(Unj)
C1	**2**	0	0	0	0	0	0	0	0	0	2	2	1.000	0.002	0	0.000	0.000
C2	1	**260**	100	0	20	25	20	10	40	0	476	260	0.546	0.311	216	0.454	0.258
C3	0	0	**0**	0	0	0	0	0	0	0	0	0	#DIV/0!	0.000	0	#DIV/0!	0.000
C4	0	0	0	**0**	0	0	0	0	0	0	0	0	#DIV/0!	0.000	0	#DIV/0!	0.000
C5	0	0	0	0	**0**	0	0	0	0	0	0	0	#DIV/0!	0.000	0	#DIV/0!	0.000
C6	0	0	0	8	0	**45**	0	0	15	40	108	45	0.417	0.054	63	0.583	0.075
C7	0	0	0	1	10	0	**70**	20	0	0	101	70	0.693	0.084	31	0.307	0.037
C8	0	0	0	0	0	0	0	**0**	0	0	0	0	#DIV/0!	0.000	0	#DIV/0!	0.000
C9	0	0	0	0	0	0	0	0	**0**	0	0	0	#DIV/0!	0.000	0	#DIV/0!	0.000
E	0	0	0	0	0	0	0	0	0	**150**	150	150	1.000	0.179	0	0.000	0.000
Comp Rate	3	260	100	9	30	70	90	30	55	190	837	527		**0.63**	310		**0.37**
Justified	2	260	0	0	0	45	70	0	0	150	527						
P(Just)	0.667	1.000	0.000	0.000	0.000	0.643	0.778	0.000	0.000	0.789		**D LRU**					
WP(Just)	0.002	0.311	0.000	0.000	0.000	0.054	0.084	0.000	0.000	0.179	**0.63**	**WD LRU**					
Unjustified	1	0	100	9	30	25	20	30	55	40	310						
P(Unjust)	0.333	0.000	1.000	1.000	1.000	0.357	0.222	1.000	1.000	0.211							
WP(Unjust)	0.001	0.000	0.119	0.011	0.036	0.030	0.024	0.036	0.066	0.048	**0.37**						

Icemaker Failure Rate Matrix for MTBUR Calculation

1.00E-07	C1	C2	C3	C4	C5	C6	C7	C8	C9	E	Indication Rate	Indication Prob
I1	0	0	0	1	0	0	0	0	0	0	101	0.121
	0	0	0	0	10	0	0	0	0	0		
	0	0	0	0	0	0	70	0	0	0		
	0	0	0	0	0	0	0	20	0	0		
I2	1	0	0	0	0	0	0	0	0	0	246	0.294
	0	80	0	0	0	0	0	0	0	0		
	0	0	50	0	0	0	0	0	0	0		
	0	0	0	0	20	0	0	0	0	0		
	0	0	0	0	0	25	0	0	0	0		
	0	0	0	0	0	0	20	0	0	0		
	0	0	0	0	0	0	0	10	0	0		
	0	0	0	0	0	0	0	0	40	0		
I3	0	110	0	0	0	0	0	0	0	0	160	0.191
	0	0	50	0	0	0	0	0	0	0		
I4	0	70	0	0	0	0	0	0	0	0	70	0.084
I5	0	0	0	8	0	0	0	0	0	0	108	0.129
	0	0	0	0	0	45	0	0	0	0		
	0	0	0	0	0	0	0	0	15	0		
	0	0	0	0	0	0	0	0	0	40		
I6	2	0	0	0	0	0	0	0	0	0	2	0.002
I7	0	0	0	0	0	0	0	0	0	150	150	0.179
Comp Rate	3	260	100	9	30	70	90	30	55	190	837	1.000
Comp Prob	0.004	0.311	0.119	0.011	0.036	0.084	0.108	0.036	0.066	0.227	1.000	

Replacement Matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	E
I1	0	0	0	1	1	0	1	1	0	0
	0	0	0	0	1	0	1	1	0	0
	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	1	1	0	0
I2	1	1	1	0	1	1	1	1	1	0
	0	1	0	0	0	0	0	0	0	0
	0	1	1	0	0	0	0	0	0	0
	0	1	1	0	1	1	0.5	0	1	0
	0	1	1	0	0	1	0	0	1	0
	0	1	1	0	0.5	1	1	0	1	0
	0	1	1	0	1	1	1	1	1	0
	0	1	1	0	0	0	0	0	1	0
I3	0	1	0	0	0	0	0	0	0	0
	0	1	1	0	0	0	0	0	0	0
I4	0	1	0	0	0	0	0	0	0	0
I5	0	0	0	1	0	1	0	0	1	1
	0	0	0	0	0	1	0	0	0	0
	0	0	0	0	0	1	0	0	1	1
	0	0	0	0	0	1	0	0	0	1
I6	1	0	0	0	0	0	0	0	0	0
I7	0	0	0	0	0	0	0	0	0	1

Icemaker Replacement Rate Matrix--ALL REPLACEMENTS

Rate	Failed															S LRU	WS LRU		
Replaced	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**	**C9**	**E**	**Repl Rate**	**Justified**	**P(Just)**	**WP(Just)**	**Unjust**	**P(Unj)**	**WP(Unj)**	**MTBURuj**	**MTBUR**
C1	**3**	0	0	0	0	0	0	0	0	0	3	3	1.000	0.002	0	0.000	0.000	#DIV/0!	3,333,333
C2	1	**260**	100	0	20	25	20	10	40	0	476	260	0.546	0.176	216	0.454	0.146	46,296	21,008
C3	1	0	**100**	0	20	25	20	10	40	0	216	100	0.463	0.068	116	0.537	0.079	86,207	46,296
C4	0	0	0	**9**	0	0	0	0	0	0	9	9	1.000	0.006	0	0.000	0.000	#DIV/0!	1,111,111
C5	1	0	0	1	**30**	0	10	10	0	0	52	30	0.577	0.020	22	0.423	0.015	454,545	192,308
C6	1	0	0	8	20	**70**	20	10	15	40	184	70	0.380	0.047	114	0.620	0.077	87,719	54,348
C7	1	0	0	1	20	0	**90**	30	0	0	142	90	0.634	0.061	52	0.366	0.035	192,308	70,423
C8	1	0	0	1	10	0	0	**30**	0	0	42	30	0.714	0.020	12	0.286	0.008	833,333	238,095
C9	1	0	0	8	20	25	20	10	**55**	0	139	55	0.396	0.037	84	0.604	0.057	119,048	71,942
E	0	0	0	8	0	0	0	0	15	**190**	213	190	0.892	0.129	23	0.108	0.016	434,783	46,948
Comp Rate	10	260	200	36	140	145	180	110	165	230	1476	837		**0.57**	639		**0.43**	**15,649**	**6,775**
Justified	3	260	100	9	30	70	90	30	55	190	837								
P(Just)	0.300	1.000	0.500	0.250	0.214	0.483	0.500	0.273	0.333	0.826		**D LRU**							
WP(Just)	0.002	0.176	0.068	0.006	0.020	0.047	0.061	0.020	0.037	0.129	**0.57**	**WD LRU**							
Unjustified	7	0	100	27	110	75	90	80	110	40	639								
P(Unjust)	0.700	0.000	0.500	0.750	0.786	0.517	0.500	0.727	0.667	0.174									
WP(Unjust)	0.005	0.000	0.068	0.018	0.075	0.051	0.061	0.054	0.075	0.027	**0.43**								