An Analysis of the Slackpulling Forces Encountered in Manual Thinning Carriages
by
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AN ANALYSIS OF THE SLACKPULLING FORCES ENCOUNTERED IN MANUAL THINNING CARRIAGES

## INTRODUCTION

The popularity of gravity-return skyline logging systems is now increasing in the Western United States. Simplicity seems to be the basis for their success. While two drum yarders are most commonly used, some systems, such as the Wyssen, use only a single drum for yarding.

Carriages commonly used with these systems in partial cuts often depend on manpower to pull slack. The carriages either clamp to the skyline or latch to a stop on the skyline. The Köller and Wyssen carriages are examples of the clamping type, while the Maki and Christy latch to a stop on the skyline.

The feasibility of logging with these systems may depend on whether or not the crew is able to pull line to the side to reach the turn of logs. Another consideration for standing skylines, rigged above the reach of the rigging crew, is the amount of weight that must be attached to the hook for it to drop from the carriage.

A quantification of the force required to pull the mainline through the carriage may be useful in the design of carriages that provide some auxilliary means of pulling slack.

## LITERATURE REVIEW

The specific problem addressed in this paper has apparently not been previously covered in the literature.

Sessions and others (10) treated a closely related problem using a simulation model. The simulation gave the amount of slack generated in the mainline by a carriage hitting a fixed stop on a skyline. The additional slack was a function of the kinetic energy in the system at the time of impact.

As an outgrowth of the previous paper, Sessions identified the point where the work done by the mainline dragging on the ground slows the carriage to a stop (11). This point is considered to be the maximum reach for a gravity outhaul logging system.

The use of catenary equations for the solution of skyline logging problems has been covered extensively in several different publications of the Pacific Northwest Forest and Range Experiment Station (2; 3; 4; 5; 6; 7). Particularly valuable as a reference for this paper was Carson's Analysis of the Single Cable Segment (4).

O'Brien (8) gives a general solution method for suspended cable problems in three dimensions.

Wilson, Dykstra and Sessions (14) applied a catenary solution procedure to the problem of calibrating measurements made with a steel tape.

## PROBLEM FORMULATION

## Variables (Figure 1)

```
    a = Acceleration.
    C = Carriage height vertically above the ground.
D1 = Horizontal distance from a point directly under the carriage to the point where the mainline becomes tangent to the ground.
D2 \(=\) Horizontal distance from the top of the headspar to the upper point of tangency (X2, Y2).
\(\varepsilon=\) Magnitude of desired maximum error in a particular iteration process.
\(F=\) Magnitude of the friction force at the point of tangency.
\(\mathrm{g}=\) Acceleration of gravity.
\(\mathrm{H}=\) Height of the headspar.
Hl \(=\) Horizontal component of tension in the cable.
\(L=H o r i z o n t a l\) distance from the top of the headspary to the carriage.
\(\ell=\) Horizontal distance over which the cable is lying on the ground.
\(\mathrm{m}=\) Mass.
\(\mu=\) Coefficient of friction for the cable against the ground.
```



Figure 1. Basic geometry


Figure 2. Tension at carriage
versus kinetic) and the dynamic effects of the line sliding past the equilibrium point (XI, YI).

For the simple case treated first in this paper, the ground slope is assumed to be constant between the headspar and the carriage. Further, the cable is assumed to hit the ground at a point directly beneath the top of the headspar (Figure 4). In terms of the defined variables this means that $T 2=0, X 2=X 3$, and $D 2=0$.

As a tangential problem, the distance, D2, which minimized the tension at the carriage was found. In relative terms, the minimum tension did not differ significantly from the simple case. This problem is discussed in Appendix A.

## Solution Procedure

Three methods were developed for determining the relationship between $T \emptyset$ and Dl. These three procedures will be referred to as the "arc sinh" method; the "tension check" method, and the horizontal distance method.

## Arc Sinh Formulation

Given the catenary in Figure 5, which has the equation $Y=M \cosh \frac{X}{M}$, the following can be established:

$$
\begin{aligned}
& \mathrm{D} 1=\mathrm{XI}-\mathrm{X} \varnothing \\
& \mathrm{X} 1=\mathrm{X} \varnothing+\mathrm{D} 1
\end{aligned}
$$



Figure 5. The catenary

$$
Y=M \cosh \frac{X}{M}
$$

we differentiate with respect to $X$ to obtain

$$
\begin{aligned}
\frac{d Y}{d X} & =M\left(\sinh \frac{X}{M}\right) \frac{1}{M} \\
& =\sinh \frac{X}{M}
\end{aligned}
$$

Since $\frac{d Y}{d X}=\tan \theta$, at the point of tangency

$$
\tan \theta=\sinh \frac{X 1}{M}
$$

Taking the hyperbolic arc sine of both sides we obtain

$$
\frac{X l}{M}=\sinh ^{-1}(\tan \theta)
$$

at the point (X1, Y1). Therefore $\frac{X 1}{M}$ will be a known constant. Let

$$
\begin{equation*}
\frac{\mathrm{XI}}{\mathrm{M}}=\mathrm{K} \tag{2}
\end{equation*}
$$

By combining the equations [1] and [2], we obtain an equation in two unknowns, Dl and M.

$$
\begin{equation*}
K=\sinh ^{-1}\left[\frac{\Delta Y}{2 M \sinh \frac{D I}{2 M}}\right]+\frac{D 1}{2 M} \tag{3}
\end{equation*}
$$

Also,

$$
\begin{equation*}
\Delta Y=(D 1 \tan \theta)-C \tag{4}
\end{equation*}
$$

for a ground slope which is uniform. By substitution,


Figure 6. Flowchart for arc sinh method


Figure 7 . Cable free body diagram

It follows that

$$
\begin{gather*}
X \varnothing=X 1-D 1 \\
Y \varnothing=M \cosh \frac{X \varnothing}{M} \tag{15}
\end{gather*}
$$

If the solution value of Dl was used in the previous process, $T \varnothing$ can be calculated from the equation

$$
\begin{align*}
T \varnothing & =W Y \varnothing \\
& =W M \cosh \frac{X \varnothing}{M} \tag{16}
\end{align*}
$$

The solution procedure entails a search for the value of Dl which matches the calculated value of $T \varnothing$ with the given value. This solution method is outlined in Figure 8.

## Horizontal Distance Formulation

A third solution procedure was contributed by Peters (9). This method is somewhat similar to the tension check method, but is superior in that only one iterative loop is involved.

Initially, a value of Dl is guessed and a value of Tl is obtained via equations [11] and [12].

$$
T 1=W(L-D 1)(\mu-\tan \theta)
$$

Since, by definition, $M$ is equal to the horizontal tension component divided by the line weight per unit length,

$$
\begin{aligned}
M & =\frac{H 1}{W} \\
& =\frac{T 1 \cos \theta}{W}
\end{aligned}
$$

Then, from equation [14]

$$
X 1=M K=M \sinh ^{-1}(\tan \theta)
$$

Using the basic catenary equation,

$$
\mathrm{Y} 1=M \cosh \frac{\mathrm{XI}}{\mathrm{M}}
$$

It then follows that

$$
\begin{aligned}
& Y \emptyset=X 1-D 1 \\
& Y \emptyset=M \cosh \frac{X \varnothing}{M}
\end{aligned}
$$

We now have everything needed to calculate a check value of D1.

$$
\mathrm{D} 1^{\prime}=\frac{(Y 1-Y \phi)+C}{\tan \theta}
$$

When the calculated values of D1 and D1' agree within the specified tolerance, $T d$ can be calculated from the relationship

$$
T \varnothing=W Y \varnothing
$$

This method is outlined in Figure 9.

## Iteration Methods

Because of the transcendental nature of the basic equations, it was necessary to resort to iterative solution techniques. Iteration schemes considered were the NewtonRaphson, secant, and the half-interval or binary search method.

Programs were written using both the arc-sinh and the horizontal distance solution procedures. The discussion immediately following applies to the arc sinh procedure.

With the arc sinh algorithm, the solution was doubly difficult because it was necessary to iterate for both tension at the carriage, $T \varnothing$, and the horizontal distance from the carriage to the point of tangency, Dl, that corresponded to the chosen value of $T \varnothing$. Unfortunately, an improperly chosen value of $T \|$ could result in total disaster for the Dl iteration. In particular, values of $\mathrm{T} \varnothing$ which are too low result in an equation for which no real roots exist. This problem is illustrated in Figure 10.

In an attempt to use Newton's method (1:51), equation [3] was differentiated. The derivative was very involved. It appeared that it would take at least as long to evaluate the derivative once as to evaluate the original function twice. For this reason, Newton's method was discarded in favor of the secant method.

The secant method (13:178) was adopted for the D1 iteration loop. The method worked well as long as a feasible value of $T \varnothing$ was assumed. The secant method was also tried on the outer loop, but the projection of infeasible TØ values forced abandonment. For reasons not entirely clear, two points within the feasible region sometimes projected a third point that wasn't. This caused the inner secant loop to overflow, underflow, or continue to iterate indefinitely.

To alleviate the problems encountered with the secant method in the outer loop, the half-interval or binary search method was employed. Given two points on either side of the solution value, the binary search method picks a third point halfway between the other two, then selects the interval which contains the solution value and continues the procedure. This slow-but-sure method produced favorable results when coupled with a check statement in the inner loop. The check statement served to increase the value of the lower bound for $T \varnothing$ whenever problems were en-: countered in the inner loop.

For the horizontal distance method, a single loop employing the secant method was used. As expected, this algorithm proved to be the most efficient of the two. The actual computation time required on the HP-9830 for a typical problem was less than half that required for the same problem using the arc sinh algorithm.

closest to the carriage, is treated to determine the solution values of $D 1$ and $T \not \subset$. This is accomplished by adding a constant to the friction force equation.

For $n$ slope intervals, the frictional constant force, F8, is determined as follows:

$$
F 8=\sum_{i=1}^{n-1} L_{i} W\left(\mu-\tan \theta_{i}\right)
$$

where $L_{i}=$ Horizontal length of $i^{\text {th }}$ interval

$$
\theta_{i}=\text { slope angle of } i^{\text {th }} \text { interval }
$$

This sumation procedure for determining the friction force assumes that the additional normal force produced by a line bending around a convex ground section is negligible. This assumption will have the effect of making the estimated slackpulling force less conservative.

From the basic equilibrium equation

$$
T 1=F
$$

we now assemble the expression

$$
T 1=\left(L_{n}-D 1\right)(\mu-\tan \theta) W+F 8
$$

The equation is then balanced, as before, by a trial and error iteration procedure until the solution value of $T \varnothing$ is obtained.
values obtained were .43 for wet ground and .55 for dry ground. Further details of the measurement are included in the appendix. Since it seemed more desirable to obtain the sustained force necessary to pull slack, the coefficient of kinetic friction was used in making the graphs. The computer programs contained in the appendix have provisions for varying the friction coefficient.

A crude estimate of the force a man is capable of exerting in tension on a cable was obtained by attaching a spring scale to an immovable object and testing the strength of several individuals. A somewhat subjective estimate of a "comfortable maximum" pull for one man is 60 pourds.

Using these crude estimates together with the graphs generated by the model, it is possible to establish bounds on feasible distances for hand pulling slack as a function of crew size. This has been done in Figures 12 and 13.

## Sensitivity Analysis

By holding certain variables constant and varying a given variable, the effect of that variable could be examined. Using the observed effects, carriage tension under other conditions could be estimated.

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## Friction Coefficient

The effect of changing the friction coefficient is also linear. The method of adjustment is similar to the slope adjustment (see Figure 16).

$$
T \varnothing_{D}=T \varnothing_{G}\left(\frac{\mu_{D}-\tan \theta}{\mu_{G}-\tan \theta}\right)
$$

where

$$
\begin{aligned}
& \mu_{D}=\text { Desired coefficient of friction } \\
& \mu_{G}=\text { Given coefficient of friction }
\end{aligned}
$$

## Combined Adjustment

Because the carriage tension is proportional to the difference between the friction coefficient and the tangent of the: slope angle, the two effects can be estimated using a single equation:

$$
T \varnothing_{D}=T \varnothing_{G}\left(\frac{\mu_{D}-\tan \theta_{D}}{\mu_{G}-\tan \theta_{G}}\right)
$$

## Carriage Height

Carriage height proved to be the surprise variable. From Figure 17 we see that with increasing carriage height, the tension at the carriage decreases and then increases again. The functional variation over the range of commonly encountered values is so small, however, that the effect of this variable can probably be assumed constant. When


## DISCUSSION AND SUGGESTIONS FOR FURTHER RESEARCH

The next logical step in the analysis process is the verification of the model by field testing. Although informal checks were made in the field to lessen the probability of large error, no extensive examination has been made. Because of the weight of the hook used on carriages of this type, a direct measurement of cable tension is sometimes difficult to obtain. An alternative to a direct measurement is the indirect approach of measuring values of Dl.

The weight of the hook adds an additional force component to the cable tension. The resultant of these two vectors is the force that the rigging crew feels when pulling slack to the side (Figure 18). In Figure 18-a, "hook weight of 40 pounds and a line tension of 20 pounds add to give a resultant force of 54 pounds. The component of this force parallel to the ground slope is 46 pounds. In addition to this force, the person pulling slack will have to move a component of his body weight parallel to the slope. Total work done will be equal to the sum of these two components times the distance moved along the slope.

One means of lightening the load is to rig the skyline higher. In this situation (Figure l8-b) the hook weight of 40 pounds and the cable tension of 20 pounds sum to a resultant force of 32 pounds. The component of this force
parallel to the ground slope is 28 pounds, which seems considerably better. Again, however, the slack puller must also move his component of body weight parallel to the slope.

In Figure l8-c, the case is similar to Figure l8-b, except the pull is now being exerted downhill. Although the size of the resultant is nearly the same as in Figure 18-b, the component parallel to the slope, though small, is in the downhill direction. This means that the choker setter merely has to hold the hook away from the ground and gravity does the rest of the work. If the cable tension were much greater, the component of the resultant parallel to the slope would be in the uphill direction. If that were the case, the body weight component could be used effectively to balance the tension component.

For standing skylines using this type of system, there must be enough weight on the hook to pull the line through the carriage when it is above the ground out of reach. The amount of weight required is given by the amount of tension at the carriage $T \emptyset$ which is necessary to pull slack.

Another step in the exploration of the lateral yarding problem is to correlate the required line pull to productivity. Using the information obtained, it would then be possible to identify the economic as well as the physical limits of this type of system. Since the lateral yarding and hooking is characteristically a large part of the total

## CONCLUSION

We have found that the tension required to pull slack can be approximated by the equation:

$$
T G=T \varnothing-W C
$$

where $\quad T \varnothing=\frac{D l^{2} W}{2 C}$
and D1 is found from the equation

$$
D 1=C \sqrt{\mu^{2}+\frac{2 L}{C}(\mu-\tan \theta)+2}-C \mu
$$

Using the model, tension characteristics of a proposed flyer thinning setting can be evaluated prior to actual operation.

It is now apparent that the use of the gravity system with a manual slackpulling carfiage may be limited on flatter terrain. For the successful use of these systems, the logging planner should be aware of the limitation imposed by cable friction.

The value of the variable $T \varnothing$ will give the amount of weight needed on the hook to force it to drop to the ground. Further, $T \varnothing$ may be a useful parameter for the design of carriages which provide mechanical means of pulling slack.

Finally, the model provides a useful guide for the formulation of production models of flyer thinning systems.
written for Statistics 417, Oregon State University. Undated.
11. Sessions, John. Carriage Outhaul Problem. Unpublished paper.
12. Smith, Charles E. Statics. New York: John Wiley and Sons, 1976.
13. Southworth, R.W., and S.L. DeLeeuw. Digital Computation and Numerical Methods. New York: McGrawHill Book Co., 1965.
14. Wilson, Robert L., Dennis P. Dykstra and John Sessions. A Technique for the Solution of the Catenary Problem in Surveying. Unpublished paper.

APPENDICES

## APPENDIX A: VARIATION IN HEADSPAR TENSION

In the early stages of problem formulation, the hypothesis was advanced that the slackpulling force at the carriage could be reduced by increasing the line tension at the headspar enough to raise some of the line off the ground. Raising the line would tend to alleviate the friction force, but would gradually increase the cable tension at the upper point of tangency.

This problem has been formulated as an optimization. Referring to Figure Al, the problem is to find the value of $D 2$ which minimizes the tension at the carriage.

To find the magnitude of the frictional force at any distance D2, the following equation was used:

$$
F=D 2 W \cdot(\mu-\tan \theta)
$$

This problem can be solved in the same manner as the problem of determining the distance Dl given a tension at the carriage, $T \varnothing$. In this case, however, the pertinent variables are tension at the top of the headspar, $T 3$, and horizontal distance to the point where the cable meets the ground, D2. We start with an initially low value of headspar tension and increase it gradually until it is no longer advantageous to do so. In short, we attempt to maximize the function

$$
\begin{equation*}
f_{5}(D 2)=F(D 2)-T 2(D 2) \tag{A1}
\end{equation*}
$$

Where $T 2$ is found using an alogrithm similar to the one discussed previously. The value of $D 2$ which maximizes [A1] will tell where the catenary cable force is a minimum relative to the frictional force. This geometry will minimize slackpulling tension at the carriage.

A program was written to perform the described algorithm. The program was then run to determine the optimal value of D2 for various headspar申 heights, slopes, and line weights. Optimal values of $D 2$ were regressed against tower height for a given coefficient of friction and slope. A linear relationship was observed.

$$
\mathrm{D} 2=\mathrm{a}+\mathrm{b}(\mathrm{H})
$$

Dividing by H,

$$
\frac{D 2}{H}=\frac{a}{H}+b
$$

Since the intercept value, a, was small in relation to H , the first term was neglected to obtain

$$
\frac{\mathrm{D} 2}{\mathrm{H}}=\mathrm{b}
$$

In other words, the optimum value of D2 as a fraction of spar height is a constant for a given slope. Values of $b$ were then regressed against different values of the slope

For standing skylines rigged high in the air, it would be interesting to know. how long it takes for the hook to reach the ground after it is released from the carriage. Since the hook acceleration is proportional to the net force divided by the mass, it is possible to obtain an expression for the hook acceleration as a function of the distance the hook as dropped below the carriage. See Figure Bl.

$$
a=\frac{\text { Force }}{\text { Mass }}
$$

(Newton's second law)

$$
\begin{equation*}
a=\frac{Q+W X-T \varnothing}{\frac{Q}{g}+\frac{W x}{g}} \tag{Bl}
\end{equation*}
$$

where $\quad Q=$ Weight of hook

$$
\begin{aligned}
& x=\text { Instantaneous distance of hook below carriage } \\
& g=\text { Gravitational constant acceleration }
\end{aligned}
$$

By multiplying both sides of the previous expression by the velocity, $V$, and then integrating, we find that the velocity as a function of $x$ is given by

$$
\begin{equation*}
V=\sqrt{2 g x-\frac{2 T \varnothing}{W}\left(\ln \left[\frac{Q}{g}+\frac{W x}{g}\right]-\ln \frac{Q}{g}\right)} \tag{B2}
\end{equation*}
$$

The time for the hook to drop is then given as

$$
T=\int_{0}^{C} \frac{d x}{V}
$$

The solution of the above integral is not readily apparent. Numerical integration techniques could be readily employed to approximate values within acceptable limits of accuracy. From examination of the basic acceleration equation, we can see that as the hook weight increases, the hook acceleration approaches the gravitational acceleration asymptotically.

Static Friction

$$
\begin{aligned}
& \text { Test } A: \vec{x}=70.26, s^{2}=16.93, n=23 \\
& \text { slope }=6 \text { percent } \\
& \text { Test } B: \bar{x}=53.46, s^{2}=21.82, n=24 \\
& \text { slope }=8 \text { percent } \\
& \text { Weighted mean }=\frac{21(.70)+24(.57)}{21+24}=.63
\end{aligned}
$$

## Kinetic Friction

$$
\begin{aligned}
\text { Test } A: & \bar{x}=40.38, s^{2}=9.85, n=16 \\
& \text { slope }=6 \text { percent } \\
\text { Test } B: & \bar{x}=38.32, s^{2}=6.56, n=25 \\
& \text { slope }=8 \text { percent }
\end{aligned}
$$

Both tests produced a mean value of .43

## Conclusions

From the tests, we conclude that the coefficient of friction (kinetic) on dry ground ranges from . 55 to . 86 but that the average is close to . 55 . On wet ground, the coefficient of static friction ranged from .57 to .70 with a mean value of .63. The coefficient of kinetic friction was measured consistently to be .43 on wet ground. The coefficient of static friction was not measured on dry ground.
$\because * * S L A C K F U L L \therefore * *$


```
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EE1!T T =FAN(LI)
E6EG H=T1-EDG; BTH(F1))/W
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675日 %G=1%FHN(%GM)
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75E Y1=H%FHG(N1,H)
-554 %0=%1-D%
7555 %0=M\divFNE(20.OM)
T55t TH=(<1-GO+C)<F1
75ET FS=Пに-I%
790% IF HES(FSN0.001 THEN 3750
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81g% [2=13
3%601FF1=F2
S3515 FE=F2
34015 ETT0 T5G4
3760 TG=| % % 
SEMB FF:IHT L:TB,DS,TI;TD-G%H
SEIG FRIHT
3%G IF DS <= LZ THEH 3900
ESOG FRIHT
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851 FFINT
SSE2 FRIHT
8017% ETOF
900G5 END
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## User Insifredions



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| 2. | Enter |  | 1 |  |
| 3. | Key in P | P | - |  |
| 4 | Enter $P$ |  | $[\square]$ |  |
| 5 | Store and $P$ |  | [a][ |  |
| 6. | Key in L | 1 | $[-][-]$ |  |
| 7 | Enter L. |  | $1][$ |  |
| 8 | Compute horizontal distance from carriage |  | $1 \square$ |  |
|  | to point of tangency, -13 |  | B | D1 |
| 9 | Key in W | 4 | , |  |
| 10 | Compute tension at carriage id, pause, then |  | $\square$ |  |
|  | compute tension at ground T1 |  | 6 | 10. 11 |
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|  |  |  | - |  |
|  |  |  | $\square[-$ |  |
|  |  |  | L |  |
|  |  |  | 1[...] |  |
|  |  |  | $11[\square]$ |  |
|  |  |  | -11] |  |
|  |  |  | -11-3] |  |
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|  |  |  |  |  |
|  |  |  | $\square$ |  |
|  |  |  | 71 |  |

