THESIS

on

ELECTRICAL MEASUREMENTS.

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Approved:

Redacted for privacy

Dept. of Electrical Engineering.

Redacted for privacy

Dean of School of Engineering.

INTRODUCTION.

The century just closed has been one of great advancement in the application of electric power to commercial purposes. With the first advent of this application, pioneers in the field realized the necessity of measuring devices, both as a means of control and as a means for charging the consumer for the power supplied.

It became necessary to measure volts, amperes and watts, the exact measuring of which was of prime importance to the producer as well as the consumer. The exactness of the readings of these measuring devices, as in all instances where chosen units are involved, had to be referred to a standard. The method of checking commercial instruments against the accepted standard, the errors involved, the selection of instruments to perform this checking that admit of speed as well as accuracy is the object of this thesis.

In large generating stations, a laboratory for checking all of their measuring instruments is an absolute necessity. Switchboard instruments must be checked, the watt-hour meters placed upon the premises of the consumer must undergo a similar operation, all of which calls for portable secondary standards which in turn must be checked repeatedly against absolute primary standards.

The process mentioned involves errors of observation and the errors inherent in the instruments themselves, the latter necessitating a discussion of the type of commercial instruments and the cause for their inherent errors, also a discussion as to the refinement necessary and to show the cases where the correction for the inherent error would be negligible in comparison with the error of observation.

TYPES OF INDICATING ELECTRICAL MEASURING INSTRUMENTS.

Indicating electrical measuring instruments may be divided into three classes viz., first, those adapted for use on direct current only; second, those that may be used on either alternating or direct current; third, those that may be used on alternating current only.

The following are the types:

For use on direct current only,

Permanent magnet moving coil.

For use on alternating and direct current,

Electro-dynamometer.

Electro-magnetic.

Electrostatic.

Hot-wire.

For use on alternating-current only,

Induction.

DIRECT-CURRENT INSTRUMENTS. (Permanent magnet moving coil).

A common form of commercial instrument for measuring direct current and voltage is the permanent magnet, moving coil type, in principle essentially that of the D'Arsonval galvanometer. It consists of a light coil free to turn in the field of a fixed permanent magnet.

Referring to Fig. 1 (A) is a permanent magnet made of steel, B are soft iron pole pieces so shaped to give a uniform radial field in which the coil C moves.

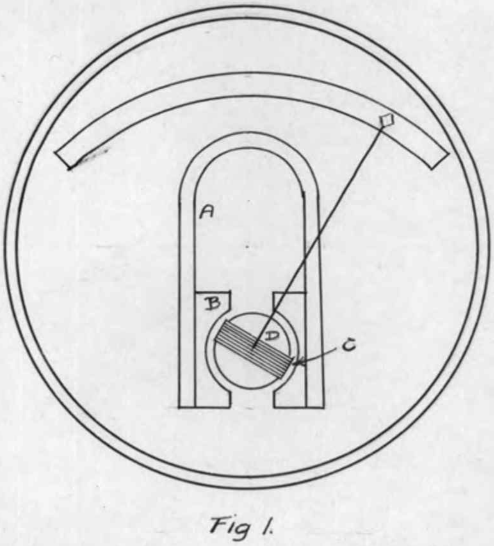


Fig 1.

A soft iron cylinder D fastened to the frame of the instrument is placed inside of the coil. It offers an easier path from pole to pole and also makes the field in the air gap uniform and radial which is of prime importance.

The movement of the coil is opposed by two spiral springs, one at each end of the coil C; the springs are coiled in opposite directions so that one is twisted while the other is untwisted during the movement of the coil. This compensates for possible non-uniformity of the springs. Without the springs the needle would be deflected to the end of the scale with any current.

The force acting on the coil is proportional to the current flowing and the strength of the field in which it is placed. Therefore

$$F = k I H'$$

where H' is the constant field flux between the poles of the permanent magnet and k is a constant. The only quantity that may be varied is I which would produce a corresponding variation in F. The force F is opposed by the action of the spring which follows Hooke's law. Therefore the deflection is proportional to the current flowing.

Since the instrument gives a deflection proportional to the current flowing, it is in reality an ammeter, whether used as such or as a voltmeter. This proportionality to current flowing has the advantage of allowing an open scale or one with equal scale divisions. When used as a voltmeter the moving coil C Fig. 1 is composed of many turns of fine wire and a resistance coil is connected in series with it. The value of this resistance is so chosen that a given maximum voltage will cause the current corresponding to a full scale deflection to flow through the coil.

A given voltmeter may serve for a variety of ranges by varying the amount of resistance in series with it. Some manufacturers of instruments have ranges on their voltmetersfrom 0-3 to 0-750. This is accomplished as shown in Fig. 2 where A is the movable coil and R_1, R_2, R_3, etc. are the resistances in series with it.

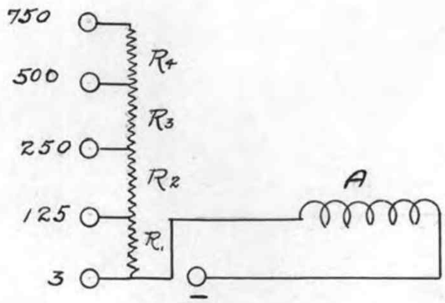
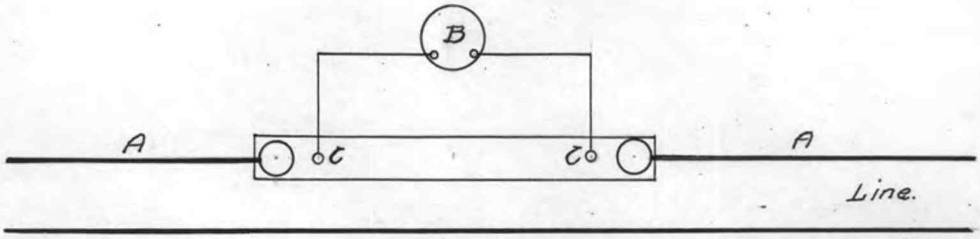


Fig 2.

The permanent magnet, moving coil type of instrument is, as was pointed out, essentially an ammeter and when small currents are to be measured, they may be passed directly through; when the current to be measured exceeds that corresponding to a full scale deflection, it becomes necessary to divert a portion of it through a circuit connected in parallel with the coil. By this means it is possible to measure heavy currents being passed through a suitable low resistance, called a "shunt", from two points on which leads run to the instrument.

The shunt is connected into the line AA Fig. 3 in which it is desired to measure the current.



The instrument B itself is but a millivoltmeter which measures the drop in potential across the terminals CC of the shunt. Assume, for instance, that the resistance of the shunt is 0.01 ohm and the maximum reading of the instrument is 20 millivolts. By Ohm's law the current in the line for a full scale deflection is,

$$I = \frac{E}{R} = \frac{0.020}{0.01} = 2 \text{ amperes.}$$

In commercial instruments, the shunts and millivoltmeters are usually calibrated together, and the scale reads directly in amperes instead of milli-volts.

In order to reduce the voltage necessary for a full scale deflection in a moving-coil instrument for current measurement, it is customary to wind the moving coil with a much smaller number of turns than in the case of the voltmeter. The wire used is considerably larger.

The sources that will cause variation in this type of instrument are due to changing temperature, stray magnetic field, change of magnets and springs with time, overload, mechanical vibration or other service conditions,

ALTERNATING AND DIRECT CURRENT INSTRUMENTS.

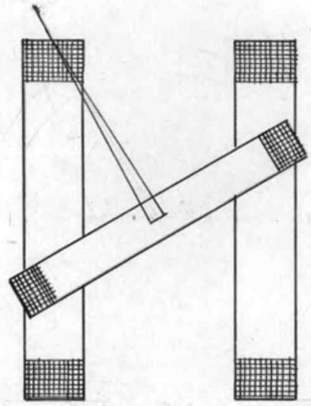
Electrodynamometer.

Electromagnetic.

Electrostatic.

Hot-wire.

ELECTRODYNAMOMETER. The electrodynamometer type is probably the most valuable, all things considered. These instruments have a stationary and movable coil; the two coils, being connected in series, attract each other when a current flows through them. Spiral springs used as a controlling force for the movable coil hold it at a certain angle with the stationary coil when no current is flowing.



When a current is flowing through the coils, the moving coil tends to place itself in a plane parallel to that of the stationary coil, producing a deflection which is read on the scale.

The torque exerted upon the moving system, for a given relative position of the two coils, is proportional to the square of the current strength. Expressed algebraically where T= torque and k a constant

$$T = k \times I(mov.) \times I(sta.)$$

This shows that the deflection of the instrument will vary as the square of the current, which means that the scale will not be uniform as in the case of the permanent magnet moving coil type. Because the coils are connected in series, the instrument becomes equally applicable for direct or alternating current since the current changes simultaneously in both and the attraction between them does not reverse.

The electrodynamometer is the most reliable form of ammeter; when used as a voltmeter, its coils are made of fine wire, and an auxiliary non-inductive resistance is connected in series with the coils. The small wire and consequently more turns in the voltmeter tend to increase the inductance, and the effect of the inductance of a voltmeter is to make an instrument which has been calibrated on direct-current indicate less than the value (effective) of an alternating electromotive force, the error being greater the higher the frequency. This may be shown as follows assumming that the alternating electromotive force is harmonic.

Let I be the direct current in the instrument that flows due to the direct electromotive force E through the resistance R, then the deflection d is proportional to I^2. Also let i be the effective value of alternating current that is caused to flow by e through the impedance Z and give the same deflection d.

If I^2 gives a deflection d and i^2 the same deflection then I^2 must equal i^2. But $I^2 = E^2/R^2$ and $i^2 = e^2/Z^2$. Equating I^2 and i^2 we have,

$$\frac{E^2}{R^2} = \frac{e^2}{Z^2}$$

whence $$e = ZE/R = \sqrt{R^2 + w^2 L^2} \cdot E/R$$

where w= $2\pi f$, f= frequency in cycles per second, L= the inductance in henrys.

Therefore, to produce the same deflection on alternating as on direct current, the alternating electromotive force must be Z/R times as large. The magnitude of the error is evident, taking the data of a Westinghouse voltmeter the inductance of which is 70 millihenrys, 1000 ohms resistance; using a frequency of 60 cycles and a direct electromotive force of 100 volts and substituting in the above equation, we have

$$e = \frac{\sqrt{(1000)^2 + (2 \times \pi \times 60 \times 0.070)^2}}{1000} \times 100 = 100.34 \text{ volts.}$$

Thus it may be seen when the ratio of L/R which is sometimes called the "time constant" is small, the error is negligible.

The Kelvin balance is a modification of the electrodynamometer type of instrument, but this balance changes appreciably due to heating, when left in the circuit for any length of time, and have frequency errors which are greater the larger the capacity of the instrument. With thick wires and high frequencies the current will tend to flow near the surface of the wire, and the reading of the instrument for a given current will vary to a certain extent with the frequency.

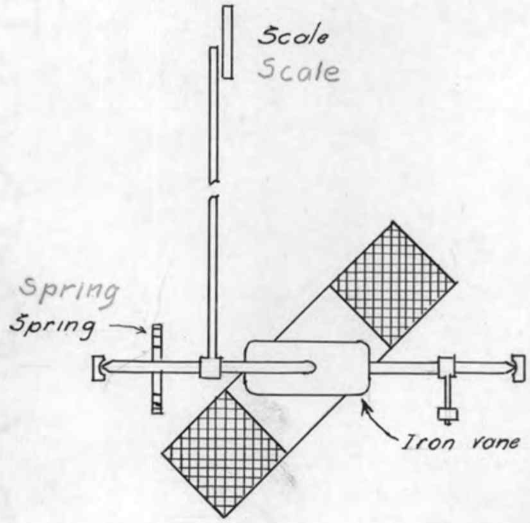
In order to provide a portable and easily operated ammeter of the electrodynamometer type, several European makers arrange the fixed and moving coils in parallel, so that the latter carry only a small part of the current to be measured. In order to avoid differences in the division of the current, due to inductance, the time constants of the two circuits are made small and as nearly equal as possible, by adding non-inductive resistance to each coil. As

Bureau of Standards Bulletin No. 20.

this resistance is of manganin the temperature coefficient of each circuit is reduced; differences in temperature only of the two circuits will introduce error.

ELECTROMAGNETIC INSTRUMENTS. This type depends upon the action of a coil traversed by a current upon one or more pieces of soft iron. In the earlier forms the coil consisted of a solenoid with a plunger moving in the center. In the light of present day practice this is considered very poor design. A more perfect instrument of the same typeis the Thomson inclined-coil.

The coil is placed at about forty five degrees to the direction of the shaft; a soft iron vane is mounted on the shaftin an inclined position. When the coil is energized



it produces a flux parallel to the axis of the coil; the vane tends to move so as to embrace a maximum of lines of force. In doing so it turns the shaft, and the deflection is shown on the scale.

The deflection is proportional to the square of the current in the coil. If I be the current in the coil of the soft iron instrument and M the corresponding magnetic flux in the movable vane, the pull on the plunger is proportional to the product of current times flux, or

$$\text{Torque} = k\ I\ M.$$

The magnetism in the plunger is produced by the current I; with low iron saturation it may be assumed that $M = k'\ I$. Substituting

$$\text{Torque} = k\ k'\ I^2 = K\ I^2.$$

Therefore the electromagnetic as well as the electrodynamometer type will not permit of an evenly divided scale.*

The electromagnetic instrument may be used on either direct or alternating current; when intended for use on alternating-current circuits they should be calibrated on alternating-current, using suitable transfer instruments which may be checked with direct-current standards. The voltmeters and ammeters of this type cannot be accurately checked on direct-current, as even the mean of the reversed readings does not give an accurate test of the performance on alternating-current. When calibrated on alternating current they may be used on direct-current with approximate results, within 2 or 3 per cent.

*See page 12b.

*In most indicating instruments (voltmeters and ammeters) where it is not possible in the construction of the instrument to obtain a uniform and radial field with respect to the moving element throughout its travel, it may be noticed that the scale divisions in midrange are wider than at either end. Remembering that in some instances the deflection is directly proportional to the current and in others the deflection is proportional to the square of the current, it would appear that in the first case the scale divisions should be of equal value and in the xxxx second case increasing in distance between divisions as the high end of the scale is approached.

The foregoing as has been previously stated is not realized in all commercial instruments, particularly the dynamometer and electromagnetis types, and may be accounted for in part by saying that there is only one position in which the moving element embraces the maximum flux, varying on either side as a function of the sine, where the angle is defined as being that between the plane of the moving element and a plane perpendicular to the flux.

In soft iron instruments the openness of the scale may be controlled within certain limits by suitably shaping the moving vane and also its position in the field.

The ammeters are very slightly affected by frequency changes; the voltmeters more so on account of their relatively high time constant. This is evident from the study of the ratio, using small wire to keep down power consumption necessitates a greater number of turns for a given torque which increases the inductance.

The Bureau of Standards Bulletin No. 20 makes the following comment upon the type of instrument under discussion: "Electromagnetic ammeters and voltmeters are practically independent of frequency variations; modern well-made instruments show only a few tenths of one percent change for as large variations in wave form as would be encountered in practice except in the case of very badly designed alternators."

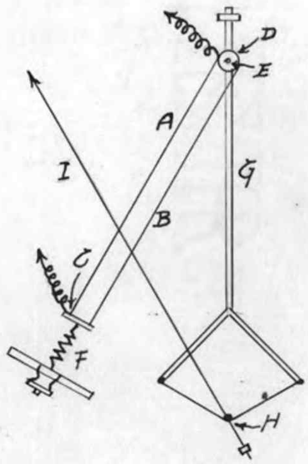
Edgecumbe in "Industrial Electrical Measuring Instruments" gives as the chief disadvantages of moving iron instruments, as usually constructed are: (1) large power consumption as voltmeters (say 4 to 8 watts per 100 volts): (2) temperature error in voltmeters (say 1/3 to 1 per cent per 10° C. rise in temperature.

HOT-WIRE INSTRUMENTS. If a current is passed through a constant resistance, the heat generated is proportional to the square of the current. The energy consumed varies as the square of the current, and the temperature varies proportionately, consequently as a body brought to different temperatures expands and contracts, these variations may be used as a means of measuring current strengths.

Here again the deflection is proportional to the square of the current which results in an open scale at one end and crowded together at the other. On voltmeters this is not objectionable as the range in commercial use is within very narrow limits while the reverse is true for the ammeter.

Major Cardew was the first to use the expansion of a wire which is heated by a current for electrical measuring instruments, The wire used was platinum-silver. There are several instruments on the market of the hot-wire type that give good results, among them being the Hartman and Braun and the Whitney or Roller as it is sometimes called. A description of this well known type is given below as being typical.

The principle on which these instruments operate will be better understood by referring to Fig. 6. A wire AB, of high resistance, low temperature coefficient and non-oxidizable metal is secured at one end to a plate, C, passed around a pulley, D, secured to a shaft, E, and its free end brought back again and mechanically, though not electrically, attached to the same plate, C. Plate, C, is kept under stress by the spring, F, which constantly tends to pull it in a direction at right angles with the axis of the shaft, E,



and is so guided that it can move in that one direction only. To the shaft, E, is likewise secured an arm, G, bifurcated at one end and counterweighted at the other. Between the extremities of the bifurcated ends of the arm, G, is another shaft, H, on which there is a small pulley and to which is attached the needle, I, that gives the desired indications; a fine silk fiber is attached at one end to one of the arms of, G, then passes around the pulley and the staff, H, and finally has its other extremity secured to the other arm, The arms are springy and serve to keep the silk fiber taut. The current to be measured flows through the wire, A, only, entering and leaving as indicated. Evidently, when A is heated by the passage of current, it expands, which, as A and B were originally under the same tension, makes A's tension relatively less than that of B, and equilibrium can be restored only when the pulley, D, rotates sufficiently again to equalize the strain. The rotation of D, of course, carries G with it, and G, in moving, causes the silk fibre to rotate the shaft which carries the needle. If the temperature of the air surrounding the instrument changes, A and B are affected alike, and their resulting equal expansion simply results in a movement of the plate, C, back or forth in its path without any tendency to rotate the pulley.

The defects of the hot-wire instrument are, its large consumption of power, uncertainty of zero and heating when left in the circuit. As the working wire must be run at a fairly elevated temperature to give proper sensibility, it is easily damaged by sudden overloads, which would do little or no damage to other forms, except the possible bending of the pointer.

The good points of the hot-wire instrument, are its independence of frequency, except for very high frequencies such as in wireless telegraphy, wave form and stray magnetic fields; the fact that it may be calibrated on direct-current and used on alternating current with equal accuracy.

ELECTROSTATIC INSTRUMENTS. The range of usefulness of this type of instrument is limited. For high voltages up to 50000 the electrostatic voltmeter has some marked advantages over other types, where the use of potential transformers are not to be considered. It has a small ratio of torque to weight of moving parts, hence frictional errors are hard to avoid. In principle it depends upon the attraction of oppositely charged bodies and the repulsion of similarly charged ones.

ALTERNATING CURRENT INSTRUMENTS ONLY.

INDUCTION TYPE. While the electrodynamometer, soft iron and hot wire instruments may be used on both alternating-current and direct-current, yet their is another type that has come into commercial prominence, known as the "induction meter", that can be used on alternating-current only.

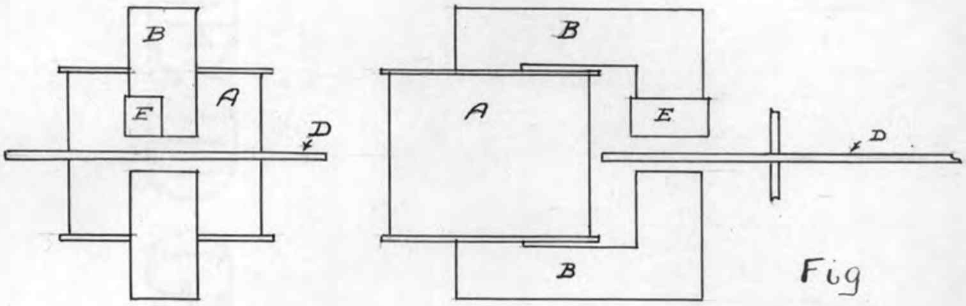
Edgecumbe divides this class of instruments into two classes:

(1) Shielded pole type.

(2) Split circuit type.

It is a well known fact that if a mass of conducting material be placed within the influence of two alternating magnetic fields which vary in intensity and direction at the same rate, but whose maxima and minima do not occur simultaneously, the reaction between the currents thus set up in the mass, and the magnetic fields respectively, will, if the former be suitably journaled, tend to set it in rotation. This principle may be utilized in various ways in the construction of alternating -current instruments.

The arrangement of the shielded pole type is shown in Fig. 7; the alternating current to be measured flows through the coil, A, on the laminated iron core, B, and produces in it a pulsating magnetic field. A pivoted aluminum disk, D,



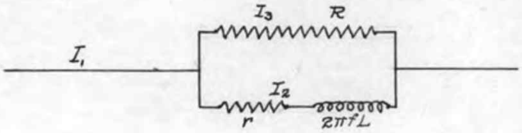
Fig

is subjected to the inductive action of this field in the air gap of the core; the disk moves due to the influence of the eddy currents induced in it, if these currents are unsymmetrical with respectto the flux from the iron core. To produce an unsymmetrical field, a secondary coil, E, or a single strip of copper is placed on one side of the face of the iron core and short-circuited upon itself.

The action is as follows: Let E= the electromotive force across the terminals of the coil A, and assume that the coil has negligible resistance. Then the current I in the coil would be in quadrature with the impressed electromotive force E, the flux Q is in phase with I. This flux Q causes a current to flow in the short-circuited secondary in quadrature with it or displaced 180° from E. The current in the short-circuited secondary would set up a flux Q_2, in phase with it, consequently in quadrature with the fluxQ, which is the desired result, an unsymmetrical field with the fluxes displaced 90°. In reality this condition is never realized as there is resistance in coil A, therefore the flux will not be in quadrature. This unsymmertical field will cause motor action in the disk of the instrument, which is controlled by a spiral spring.

An induction instrument of this arrangement is quite sensitive to any change in frequency. In the case of voltmeters, partial compensation is possible by making the circuit inductive. For any given voltage the flux is inversely proportional to the frequency. The induced currents in the moving element are proportional to the product of flux and frequency, the effect then of frequency is slight.

In the case of ammeters the compensation for a change in frequency is less simple. The usual arrangement is to shunt the magnet winding by a non-inductive resistance, which as the frequency rises, carries a larger portion of the total current. This is at once evident from the figure.

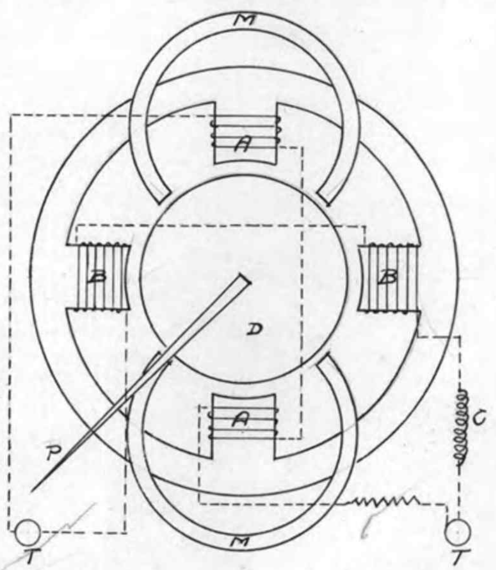


For a correct reading of the instrument for a current, I_1, the current I_2 should remain constant. A rise in frequency will cause I_3 to increase thereby keeping the drop of potential across the instrument constant. This as has been stated before is only a partial solution of the problem.

Split circuit induction ammeters and voltmeters are constructed on similar lines to the induction wattmeter, the phase being split by making one circuit non-inductive, and the other highly inductive. In the case of the induction wattmeters it is imperative that the two fluxes be displaced by 90°, but with ammeters and voltmeters this is immaterial as an unsymmetrical field is all that is necessary.

Referring to Fig. 9 which is diagrammatically the principle of the Ferraris induction voltmeter, the action is as follows:

The drum is acted upon by a rotating* magnetic field produced by four poles AA and BB surrounding it. One



opposite pair of these poles AA is energized by the current to be measured, and the other pair by a shunt current which

* The word rotating is misleading, in reality the flux from BB induces a current in the disk, the flux of which reacts with that of AA to produce rotation.

is displaced in phase from the main current by means of a choking coil C in series with BB.

These main and shunt coils are in parallel between the two terminals T and T of the instrument, and the main coil has a non-inductive resistance R in series with it, which in the case of the ammeter is a low resistance, when the choking coil C is not used owing to the coils themselves having sufficient inductance in an ammeter.

In the induction type instrument an error arises due to the change in temperature of the disk. This is more pronounced in the voltmeters than in the ammeters. In the voltmeters the current flowing should be kept as low as possible and still maintain the required torque. When the temperature of the aluminum disk increases, its resistance is increased, thereby decreasing the current flowing in the disk. This reduces the torque, and consequently the the deflection of the pointer. This is compensated for in meters of low frequencies by shunting a non-inductive resistance around the magnetizing coil. This shunt is selected of such a value that its resistance increases with temperature at the same rate as the resistance of the aluminum disk. Therefore, when the temperature rises, more current flows through the magnetizing coil of the voltmeter, increasing the field strength in the desired proportion.

Portable induction instruments have been used to a considerable extent, their inherent defects being partially compensated for by their practical advantages. They have a nearly closed magnetic circuit, and have fairly strong working fields; they are thus not sensitive to external stray field. The moving element is simple and strong, has no windings, and hence requires no provision for leading current in and out. The scales are long, it being possible to make them cover 300°, or even more. Such instruments may be used for commercial testing on definite frequencies, after calibration under conditions as nearly as possible like those under which they are used. They are not suitable for service where a wide range of frequency is met with.

INDICATING WATTMETERS.

The function of these instruments is to give a direct reading of the power consumed in the circuit in which they are placed. The three types are:

Electrodynamometer.

Induction.

Hot-wire.

Previously the ammeters and voltmeters falling under the above types have been dealt with and the wattmeters were left under a separate heading in the hope of making the description more complete.

THE ELECTRODYNAMOMETER WATTMETER. It was pointed out that the electrodynamometer voltmeter and ammeter would operate satisfactorily on either direct-current or alternating-current, this is also true of the wattmeter.

This instrument has a stationary and movable coil, the stationary coil consists of a few turns of heavy wire or strip connected in series with the circuit , as an ammeter.

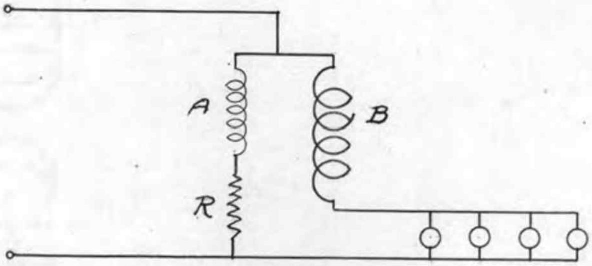


Fig.

The moving coil consists of many turns of fine wire, and is similar to the moving coil in a D'Arsonval type voltmeter; it is connected across the circuit with a high non-inductive resistance R in series with it. The force of attraction between the two coils is proportional to the product of current by voltage, or the power in the circuit.

This wattmeter, when calibrated with direct-current, indicates power accurately when used with alternating-current, providing the inductance of the voltage coil A is negligibly small.

Mr. E. B. Rosa (U.S. Bureau of Standards Bulletin No. 48) gives some of the difficulties met with in using the electrodynamometer for precise measurements. It reads as follows: "Two circuit electrodynamometers are very commonly used as wattmeters, with a high resistance in the potential circuit. For the measuring the power in circuits of relatively high electromotive forceand of large power factor, the effect of the usually small inductance of the coils of the potential circuitand of any slight eddy currents that may be present is negligible. But when the wattmeter is used on circuits of very low electromotive force, where the resistance of the potential circuit must be made small, or when the power is very small, these sources of error may be very important. It is therefore desirable to use the same compensation of resistance in parallel with capacity in precision wattmeters as is used in two circuit ammeters, and also to test for the presence of eddy currents. If these eddy currents are in the metallic part of the instrument(the field conductors being so thoroughly stranded and insulated as to obviate them in the field windings), they may be fully compensated for by a loop in the potential circuit, placed inductively with respect to the field. If, however , they are in the field coils themselves, this compensation for their torque still leaves a possible source of error- namely, in the different distribution of the alternating current in the field coils from that of the direct current employed in calibrating the instrument. The field coils should therefore be carefully protected from eddy currents by making them of insulated strands.

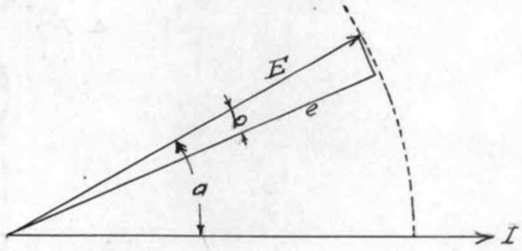
It can be shown for low power factors that the electro-dynamometer type of wattmeter reads high, and when the angle of lag of the current behind the electromotive force in the potential circuit equals the angle of lag of the external circuit, the instrument reads correctly, ignoring of course any error due to eddy currents.

Let W = the true power.

w = wattmeter reading.

a = angle of lag of the current behind the e.m.f. in the external circuit.

b = angle of lag of the current behind the e.m.f. due to the inductance of the potential circuit.



Then the true power $W = E\,I \cos a$. (1)

wattmeter reading $w = e\,I \cos(a - b)$.

$= E \cos b\; I \cos(a-b)$ (2)

Solving for I in (2) and substituting in (1), we have

$$W = \frac{w \cos a}{\cos b \cdot \cos(a-b)}$$

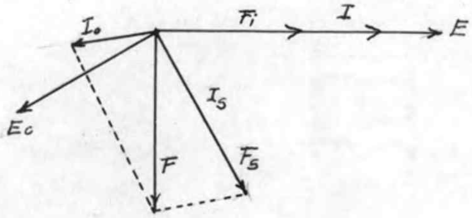
Expanding cos(a-b) and dividing both sides of the equation by $\cos a \cdot \cos^2 b$, we get

$$W = w\,\frac{1+\tan^2 b}{1+\tan a \cdot \tan b} \qquad (3)$$

The wattmeter reading w will equal the true watts W when the expression $(1+\tan^2 b) \div (1+\tan a \cdot \tan b)=1$ which will be the case when b=0 or when a=b. It is evident from equation (3) that w will be larger than W when a is a large angle, the error may be decreased, however, by making angle b as small as possible.

INDUCTION WATTMETERS. In the previous discussion of induction type voltmeters and ammeters, two methods were available for producing the necessary rotating field, (1) the shielded pole, (2) the split circuit. With wattmeters the second method only is available as it is essential for the accuracy of the wattmeter that the potential flux should be exactly 90° out of phase with that due to the current coil.

The working of an induction wattmeter is shown diagrammatically in Fig. 12. Assuming that the potential circuit has no resistance, then it will be pure inductance, and the flux due to the current in the coil A will lag 90° behind



the voltage at its terminals. A current will be induced in the disk, in phase with the voltage (because 90° out of phase with the flux producing it). This induced current will be in phase with the flux due to the current coil B, therefore the resultant torque is proportional to the product of the impressed voltage by the current, or EI, if the load current and voltage are in phase.

As it is impossible to have the potential coil without resistance, the method of bringing the fluxes in quadrature is shown in the vector diagram. Let E represent the electromotive force of the circuit and I, which is in phase with E, the current in the series coils. F_s represents the flux due to the shunt coils which are made as inductive

inductive as possible in order that the flux may be displaced approximately 90° from the flux due to the current I. In reality the current in the potential circuit lags less than 90° due to ohmic resistance. To place the shunt flux in exact quadrature, compensating coils are attached to the poles of the magnetic circuit and short-circuited. An electromotive force E_c is induced in them and a current I_c flows, which owing to inductance is not in phase with E_c. The flux passing through the disk is the resultant of the fluxes F_s and F_c, and by adjusting the resistance of the short-circuited secondary, and consequently the value of I_c, the flux angle between E and I may be made exactly 90°. This is true with the exception that owing to hysteresis and eddy currents in the magnetic circuit of the current coil, its flux will not be exactly in phase with the current. Also due to the copper and iron losses in the pressure coil the resultant flux will not be exactly 90° out of phase with the voltage.

It follows that it is not easy to insure that the flux due to the potential coil shall be exactly 90° out of phase with that due to the current coil. The effect of any such discrepancy on the reading is similar to that discussed in in the case of dynamometer instruments, due to inductance in the potential circuit. The error is small at high power-factors, and increases rapidly as the power-factor is reduced.

The scale of the induction wattmeter may be extended over 300° or more. Edgcumbe in "Industrial Electrical Measuring Instruments" gives the following: A 10°C. rise or fall in temperature will introduce an error of from 1 to 4 per cent; for frequency change of 10% the error is from 3 to 6 per cent; the effect of the changes of wave-form are relatively small although a peaked wave generallymakes the wattmeter read low.

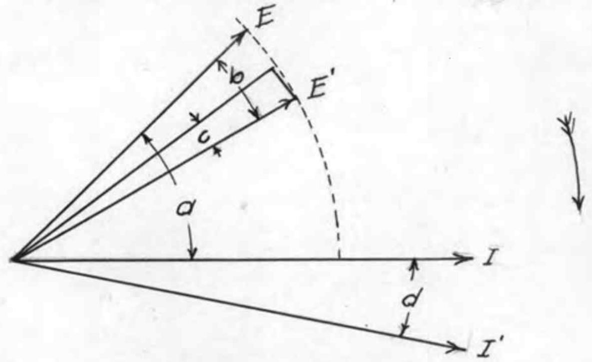
This type of instrument is robust, has a high torque and is used extensively in practice. It can be used on alternating current only and is calibrated through the medium of dynamometer instruments which in turn have been previously standardized on direct-current.

HOT-WIRE WATTMETER. Roller in "Electric and Magnetic Measurements" gives a description of an ingenuous instrument proposed by Bauch. As the hot-wire wattmeter is used to a very limited extent in this country a discussion of it will not be taken up here.

AUXILIARY APPARATUS.

In the measurement of alternating-current it becomes necessary to deal with high voltages. Multipliers may be used with voltmeters and the potential coils of wattmeters, and shunts for current measurements. This necessarily means that there is a high potential on the instruments which is dangerous for the operator. Equally as good results may be obtained by substituting potential transformers for the multipliers and current transformers for the shunts.

It is evident from what follows that in accurate power measurements where potential and current transformers are used that the phase angles of each must be carefully determined.



Let the ratio of transformation in both the potential and current transformers be 1 : 1, and let I represent the current in the primary circuit; E the voltage in the primary circuit which is displaced from I by the angle a. The true power P = EI cos a. Now let a potential transformer be placed in the circuit to excite the volt coil of the wattmeter with the secondary e.m.f. E' of the transformer leading the primary by the angle b. Due to inductance in the potential circuit of a dynamometer wattmeter, the current in the potential coil is displaced from E' by the angle c which is a lag. If W is the wattmeter reading, we have

$$W = E'I\cos(a-b+c) \qquad (1)$$

If a current transformer is placed in the circuit to excite the current coils of the wattmeter, with its secondary current leading the primary by the angle d, then

$$W = E'\cos(a-b+c)I'\cos d. \qquad (2)$$

As the leading current tends to increase the displacement between E' and I, we may write without error,

$$W = E'I'\cos(a-b+c+d) \qquad (3)$$

Thus, we see that a leading current in the current coil of the wattmeter, has the same effect as a lag in the potential coil and may be corrected as such.

From equation (3), $\cos(a-b+c+d) = \frac{W}{E'I'}$ = apparent power-factor.

Dividing equation (1) by (3) we get,

$$\frac{P}{W} = \frac{EI\cos a}{E'I'\cos(a-b+c+d)}$$

or $$P = \frac{W\cos a}{\cos(a-b+c+d)} = \frac{W \times \text{true power-factor}}{\text{apparent power-factor}} \qquad (4)$$

then let the angle of the "apparent power factor = x

$$\cos a = (x+b-c-d)$$

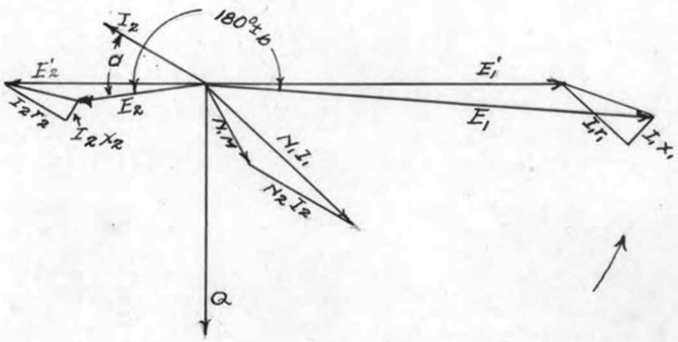
Equation (4) is the expression for the correction to be applied to a wattmeter reading for the true power P, under the conditions stated when a current and potential transformers are used. The reasoning was based on a special case, however, but taking due account of signs whether lagging or leading, the expression given holds true.

Mr. Robinson (Proc. A.I.E.E.vol.28.) states: In order to keep the error due to phase-angle within 1 per cent, if the su of the correction angles is 30 minutes, correction must be applied after the power-factor becomes lower than 0.7. At power-factor 0.5, the error due to this cause would amount to about 1.5 per cent and at power-factor 0.1 to very nearly 9 per cent. To keep within the same 1 per cent limit, as low as power-factor 0.1, the sum of the correction angles must be under 5 minutes.

POTENTIAL TRANSFORMERS. In the use of potential transformers for voltage measurements only, it is essential that the ratio of voltages be determined accurately within the range of voltage upon which this piece of auxiliary apparatus is to be used. The method for making this test will be taken up later under the heading Methods of Calibration.

When a potential transformer is to be used in connection with a wattmeter, it is essential not only to know the ratio of transformation but the phase-angle between the primary and secondary voltages. A lead between the primary and secondary voltage in the potential transformer has the effect of a leading current in the potential circuit of the wattmeter as has been shown.

With the aid of a vector diagram the phase relation between the primary and secondary voltages at once becomes evident.



In the figure the arrow indicates the direction of rotation of the vectors. Q represents the magnetic flux linking both primary and secondary windings. It induces in the secondary winding an electromotive force E_2', and and in the primary winding an electromotive force in the same direction but of different magnitude, fixed by the number of turns. Let N_1 be the number of turns in the primary and M the current, then N_1M are the current turns necessary to produce the flux Q. Denote the secondary current by I_2 lagging behind the secondary voltage E_2 by the angle a.

The secondary terminal electromotive force E_2 , represents what is left after deducting the ohmic drop I_2r_2 and I_2x_2, the secondary reactance. The vector I_2r_2 is parallel to the secondary current I_2, the vector I_2x_2 is perpendicular to the vector I_2. Let N_2 represent the number of secondary turns, then N_1M, N_2I_2 and N_1I_1 (I_1= primary current) may all be drawn to the same scale. Placing N_2I_2, parallel to I_2, at the extremity of N_1M, their vector sum will be equal to N_1I_1.

The electromotive force applied to the primary terminals may be separated into three components; the first balances the induced electromotive force due to the flux Q and is represented by E_1'; the second balances the electromotive force due to ohmic resistance, I_1r_1 (the electromotive force vector parallel to N_1I_1); the third balances the electromotive force due to the reactance of the primary I_1x_1, perpendicular to I_1r_1. The vector sum E_1 of these three components must be the terminal electromotive force of the primary winding.

By inspection of the above figure it may be seen that the angle between the primary and secondary voltages E_1 and E_2 may be 180°± some angle b. Mr. Robinson in the Proceedings of the A.I.E.E. for July 1909 claims that in most commercial transformers that the angle b is leading. It is also evident that the value of the angle a, which is determined by the secondary load whether inductive or non-inductive, has a direct bearing on the ratio of transformation.

In the use of potential transformers with wattmeters it is essential that the ratio of transformation be known exactly for the load placed upon it, that is the regulation of the transformer for different power-factors of secondary load. In the dynamometer type of wattmeter the power-factor would be fairly high as there is little inductance in the potential coil, but in the induction type of instrument the inductance is high owing to the fact that it is necessary that the flux due to the potential coil be in quadrature to that of the current. A low power-factor will cause a decrease in the ratio of transformation.

By deriving an equation for the potential transformer of a 1 : 1 ratio in terms of primary and secondary voltages, primary and secondary resistances and reactances, magnetizing current, power-factor of the external load and the angle between the flux and magnetizing current, the effect of power-factor on the ratio and phase-angle may be studied to better advantage.

In the figure following all of the vectors are resolved into their horizontal and vertical components using the secondary voltage E_2 as the x-axis.

$$E_1^2 = \overline{DB}^2 + \overline{OB}^2 \qquad (1)$$

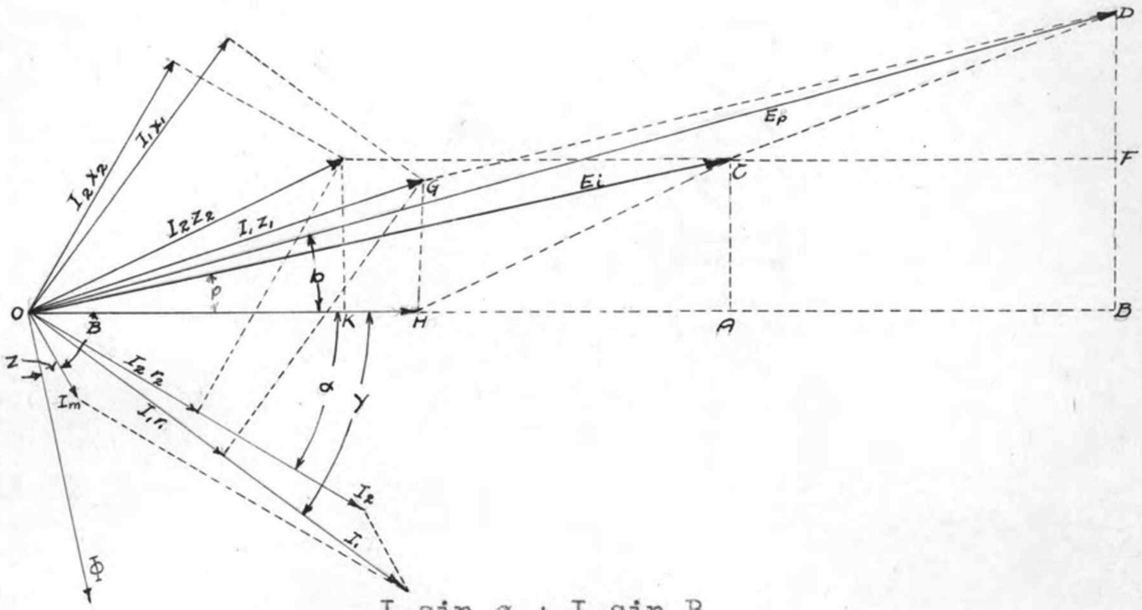
$$= (BF + DF)^2 + (E_2 + AH + AB)^2 \qquad (2)$$

$$BF = AC = I_2 x_2 \sin(90 - \alpha) - I_2 r_2 \sin\alpha.$$

$$= I_2 x_2 \cos\alpha - I_2 r_2 \sin\alpha. \qquad (3)$$

$$DF = GH = I_1 x_1 \sin(90 - y) - I_1 r_1 \sin y.$$

$$= I_1 x_1 \cos y - I_1 r_1 \sin y. \qquad (4)$$



$$\sin y = \frac{I_2 \sin \alpha + I_m \sin B}{I_1}$$

$$\cos y = \frac{I_2 \cos \alpha + I_m \cos B}{I_1}$$

Substituting sin y and cos y in (4) we have,

$$DF = I_2 x_1 \cos\alpha + I_m x_1 \cos B - I_2 r_1 \sin \alpha - I_m r_1 \sin B.$$

$$AH = OK = I_2 x_2 \cos(90 - \alpha) + I_2 r_2 \cos \alpha.$$

$$= I_2 x_2 \sin \alpha + I_2 r_2 \cos \alpha. \qquad (5)$$

$$AB = OH = I_1 x_1 \cos(90 - y) + I_1 r_1 \cos y.$$

$$= I_1 x_1 \sin y + I_1 r_1 \cos y. \qquad (6)$$

Substituting the values for sin y and cos y in (6) it gives,

$$AB = I_2 x_1 \sin \alpha + I_m x_1 \sin B + I_2 r_1 \cos\alpha + I_m r_1 \cos B.$$

Substituting in (2) we have,

$$E_p^2 = \Big[I_2 x_2 \cos \alpha - I_2 r_2 \sin \alpha + I_2 x_1 \cos \alpha + I_m x_1 \cos B - I_2 r_1 \sin \alpha - I_m r_1 \sin B\Big]^2 + \Big[E_2 + I_2 x_2 \sin \alpha + I_2 r_2 \cos \alpha + I_2 x_1 \sin \alpha + I_m x_1 \sin B + I_2 r_1 \cos \alpha + I_m r_1 \cos B\Big]^2$$

$$E_p^2 = \left[I_2 \cos\alpha(x_2 + x_1) - I_2 \sin\alpha(r_2 + r_1) + I_m(x_1 \cos B - r \sin_1 B)\right]^2$$
$$+ \left[E_2 + I_2 \sin\alpha(x_2 + x_1) + I_2 \cos\alpha(r_2 + r_1) + I_m(x_1 \sin B + r_1 \cos B)\right]^2 \quad (7)$$

Angle $B = 90 - (z + p)$.

therefore $\sin B = \cos z \cos p - \sin z \sin p$. (8)

$\cos B = \sin z \cos p + \cos z \sin p$. (9)

$$\sin p = BF \div E = BF \div \sqrt{(E_2 + OK)^2 + (BF)^2}$$
$$= (I_2 x_2 \cos\alpha - I_2 r_2 \sin\alpha) \div \sqrt{(E_2 + I_2 x_2 \sin\alpha + I_2 r_2 \cos\alpha)^2 + (I_2 x_2 \cos\alpha - I_2 r_2 \sin\alpha)^2}$$

$$\cos p = (E_2 + OK) \div \sqrt{(E_2 + OK)^2 + (BF)^2}.$$
$$= (E_2 + I_2 x_2 \sin\alpha + I_2 r_2 \cos\alpha) \div \sqrt{(E_2 + I_2 x_2 \sin\alpha + I_2 r_2 \cos\alpha)^2 + (I_2 x_2 \cos\alpha - I_2 r_2 \sin\alpha)^2}$$

Substituting the expressions derived for sin p and cos p in (8) and (9) and in turn substituting the new value of sin B and cos B in (7) we have,

$$E_p^2 = \Big[I_2 \cos\alpha(x_2 + x_1) - I_2 \sin\alpha(r_2 + r_1) + I_m\Big\{x_1 \sin z(E_2 + I_2 x_2 \sin\alpha + I_2 r_2 \cos\alpha) \div \sqrt{(E_2 + I_2 x_2 \sin\alpha + I_2 r_2 \cos\alpha)^2 + (I_2 x_2 \cos\alpha - I_2 r_2 \sin\alpha)^2}$$
$$- r_1 \cos z(E_2 + I_2 x_2 \sin\alpha + I_2 r_2 \cos\alpha) \div \sqrt{(E_2 + I_2 x_2 \sin\alpha + I_2 r_2 \cos\alpha)^2 + (I_2 x_2 \cos\alpha - I_2 r_2 \sin\alpha)^2}\Big\}\Big]^2$$
$$+ \Big[E_2 + I_2 \sin\alpha(x_2 + x_1) + I_2 \cos\alpha(r_2 + r_1) + I_m \Big\{x_1 \cos z(E_2 + I_2 x_2 \sin\alpha + I_2 r_2 \cos\alpha) \div \sqrt{(E_2 + I_2 x_2 \sin\alpha + I_2 r_2 \cos\alpha)^2 + (I_2 x_2 \cos\alpha - I_2 r_2 \sin\alpha)^2}$$
$$+ + r_1 \sin z(E_2 + I_2 x_2 \sin\alpha + I_2 r_2 \cos\alpha) \div \sqrt{(E_2 + I_2 x_2 \sin\alpha + I_2 r_2 \cos\alpha)^2 + (I_2 x_2 \cos\alpha - I_2 r_2 \sin\alpha)^2}\Big\}\Big]^2 \quad (10)$$

In making substitutions in the above expression it was found more convenient to work out a series of values for sin B and Cos B. In solving for the angle between the primary and secondary voltages, E_1 is the hypotenuse of a right-angled triangle in which the first half of (10) is the side opposite and the second half the side adjacent. Let the side opposite be represented by M and the side adjacent by N therefore,

$$b = \tan^{-1} \frac{M}{N} .$$

To note the changes in ratio and phase-angle for various loads and different power-factors, data was taken from a 100-watt Westinghouse potential transformer from the low voltage side.

Resistance of secondary = 0.6 ohms.

Magnetizing current = 0.4 amperes.

Voltage = 128.

The following assumptions were made:

(1) Ratio = 1:1.

(2) Primary resistance = secondary resistance = 0.6 ohms

(3) Primary reactance = secondary reactance and primary reactance was taken as 2.5 times the primary resistance.

(4) The angle z between the magnetizing current and the flux vector which is constant was assumed to be 14°.

With the above data substitutions were made in the formula derived with different loads (from 0.1 to 1 amp.) at unity power-factor and again with the same loads at a power-factor of 0.5. The results are tabulated below and were plotted.

E_2	E_1	Ratio	I_2	a	b.
128	128.87	.993	0	0	0° 2' lagging.
128	128.99	.992	0.1	0	0° 6' leading.
128	129.23	.990	0.3	0	0° 22' "
128	129.40	.990	0.5	0	0° 37' "
128	129.77	.986	0.75	0	0° 58' "
128	129.80	.985	1.00	0	1° 18' "
128	128.87	.993	0	60°	0° 2' lagging.
128	129.19	.990	0.1	60°	0° 0' 30" "
128	129.83	.985	0.3	60°	0° 2' leading.
128	130.47	.981	0.5	60°	0° 4' "
128	131.27	.975	0.75	60°	0° 7' "
128	132.00	.969	1.00	60°	0° 10' 30" leading.

It is interesting to note that for low power-factors the ratio is affected most with but slight variations in the phase-angle. The reverse is true for unity power-factor, hence with dynamometer type of instruments as a load the phase-angle between the primary and secondary voltages must receive careful attention, while with induction instruments, the ratio if not taken into account will cause serious error.

CURVES
of
POTENTIAL TRANSFORMER
100 Watt.

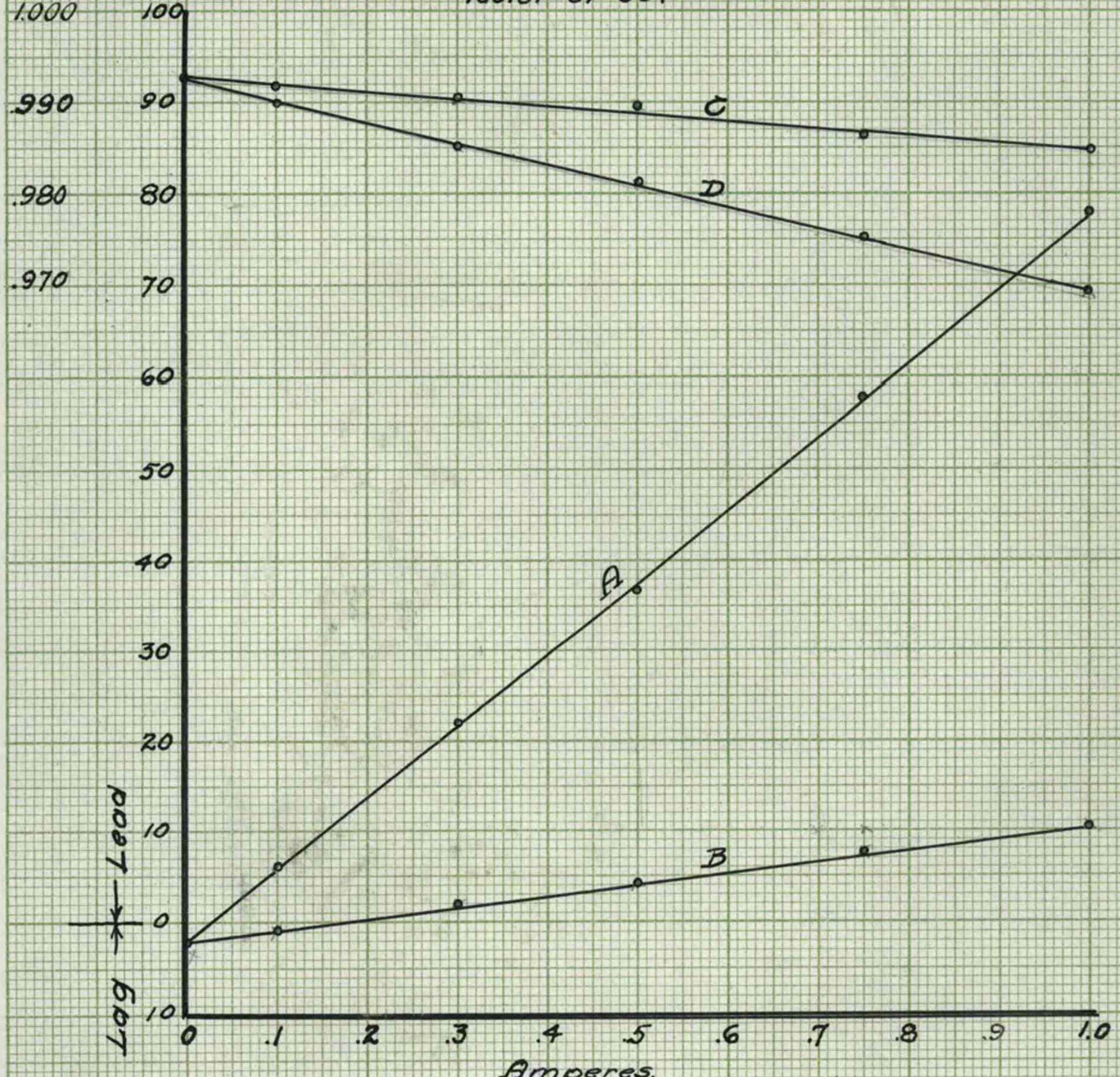
Showing relation between:

"A" Secondary current and phase-angle between primary and secondary voltages for unity P.F.

"B" Secondary current and phase-angle between primary and secondary voltages for a lagging power-factor of 60°.

"C" Ratio of voltage transformation for unity power-factor.

"D" Ratio of voltage transformation for lagging power-factor of 60°.

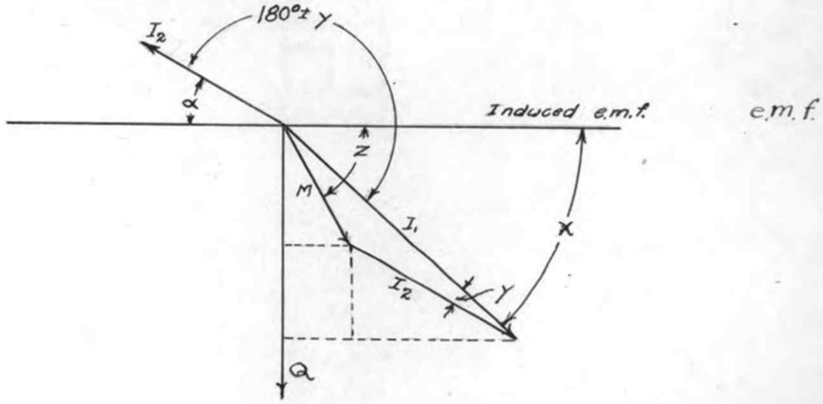


CURRENT TRANSFORMERS. This piece of apparatus, like its mate the potential transformer, is used extensively on alternating-current circuits in connection with measuring instruments; with a variable ratio current transformer one instrument may be employed to measure a wide range of currents.

In the potential transformer the ratio and phase angle between voltages on varying power-factors are required, for current transformers the ratio and phase-angle between currents on varying power-factors are also required, the value of the power-factor in both cases being governed by the impedance in the secondary circuit.

Using the same vector diagram and notation for the current transformer as that for the potential transformer we may neglect I_1r_1 and I_1x_1, thus causing E to coincide with E_1'. Let x be the angle between E_1' and I_1, and z the angle between E_1' and the exciting current M. The angle between I_1 and I_2 will be 180°±y. Without sensible error we may diregard I_2r_2 and I_2x_2, thus bringing E_2 in phase with E_2'. Then let the angle between I_2 and E_2' be α.

Drawing another figure and using flux and induced electromotive force as axes, an expression for the ratio of transformation may be obtained in terms of magnetizing current M, primary current I_1 and secondary current I_2 all expressed in ampere turns.



Then $\frac{I_1^2}{I_2^2} = \frac{(I_2\cos\alpha + M\cos z)^2 + (I_2\sin\alpha + M\sin z)^2}{I_2^2}$ (1)

or ratio = $\frac{\sqrt{(I_2\cos\alpha + M\cos z)^2 + (I_2\sin\alpha + M\sin z)^2}}{I_2}$ (2)

To observe the effect of a change in power-factor, or impedance in the secondary circuit, on the ratio with I_1 and I_2 constant let α=0 and equation (2) becomes,

$$\frac{\sqrt{I_2^2 + 2M(I_2\cos z + M)}}{I_2}$$

then let the power-factor of the secondary circuit equal z and the expression reduces to

$$\frac{I_2 + M}{I_2}$$

thus for the same load and the angle α increasing, the ratio I_1/I_2, increases or reversing the ratio as is usually the case the ratio would decrease. It is evident that as instruments are added to the secondary circuit the impedances will be increased, (therefore the angle α), with a corresponding change in ratio.

The angle between the primary and secondary current, or phase angle, is our next consideration. Referring to the above figure where I_1, I_2 and M are plotted as ampere-turns, we may obtain for analytical purposes an expression for y in terms of magnetizing ampere-turns, secondary ampere-turns and their angles. From the figure it may be seen that,

$$y = \tan^{-1} \frac{M\sin(z-\alpha)}{I_2 + M\cos(z-\alpha)}$$

As the angle α is determined by the impedance of the secondary circuit, let us suppose that this angle were zero, then y would be a maximum with a leading current. Again suppose α=z, then y=0; upon increasing the value of α the resulting relation between I_1 and

I_2 would be lagging and the angle negative with respect to $\alpha=0$. It is evident therefore, that upon making phase angle determinations that two angles of equal magnitude may be obtained and to determine whether it is a lagging or leading angle, a non-inductive load should be placed in the secondary circuit which would make $\alpha=0$ and therefore indicate a leading angle.

It has been shown that a leading current in the secondary with respect to the primary has the same effect as lagging current in the current coils of a wattmeter.

METHODS OF CALIBRATING COMMERCIAL INSTRUMENTS.

In calibrating commercial instruments, they must be compared with precision instruments and the latter must agree with the standard units as legalized by the United States Government. The following are the definitions taken from the Bureau of Standards Bulletin Vol.1, No.1.:

LEGAL DEFINITION OF THE ELECTRICAL UNITS IN THE UNITED STATES.

(Act approved July 12 1894)

Be it enacted,&c., That from and after the passage of this act the legal units of electrical measure in the United States shall be as follows:

First. The unit of resistance shall be what is known as the international ohm, which is substantially equal to one thousand million units of resistance of the centimeter-gram-second system of electromagnetic units, and is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice fourteen and four thousand five hundred and twenty-one ten-thousandths gram in mass, of a constant cross-sectional area, and of the length of one hundred and six and three-tenths centimeters.

Ohm.

Second. The unit of current shall be what is known as the international ampere, which is one-tenth of the unit of current of the centimeter-gram-second system of electro-magnetic units, and is the practical equivalent of the unvarying current,

Ampere.

which, which when passed through a solution of nitrate of silver in water in accordance with standard specifications, deposits silver at the rate of one thousand one hundred and eighteen millionths of a gram per second.

Third. The unit of electro-motive force shall be what is known as the international volt, which is the electro-motive force that, steadily applied to a conductor ehose resistance is one

Volt.

international ohm, will produce a current of one international ampere, and is practically equivalent to one thousand fourteen hundred and thirty-fourths of the electro-motive force between the poles or electrodes of the voltaic cell known as Clark's cell, at a temperature of fifteen degrees centigrade, and is prepared in the manner described in the standard specifications.

Fourth. The unit of quantity shall be what is known as the international coulomb, which is the quantity of electricity transferred by a current of one international ampere in one

Coulomb.

second.

Fifth. The unit of capacity shall be what is known as the international farad, which is the capacity of a condenser charged to a potential of one international volt by one international coulomb of electricity.

Farad.

Sixth. The unit of work shall be the joule, which is equal to ten million units of work in the centimeter-gram-second system, and which is practically equivalent to the energy expended in one second by an international ampere in an international ohm.

Joule.

Seventh. The unit of power shall be the watt, which is equal to ten million units of power in the centimeter-gram-second system, and which is practically equivalent to the work done at the rate of one joule per second.

Watt.

Eighth. The unit of induction shall be the henry, which is the induction in a circuit when the electromotive force induced in this circuit is one international volt while the inducing current varies at the rate of one ampere per second.

Henry.

Sec. 2. That it shall be the duty of the National Academy of Sciences to prescribe and publish, as soon as possible after the passage of this act, such specifications of detail as shall be necessary for the practical application of the definitions of the ampere and volt hereinbefore given, and such specifications shall be the standard specifications herein mentioned.

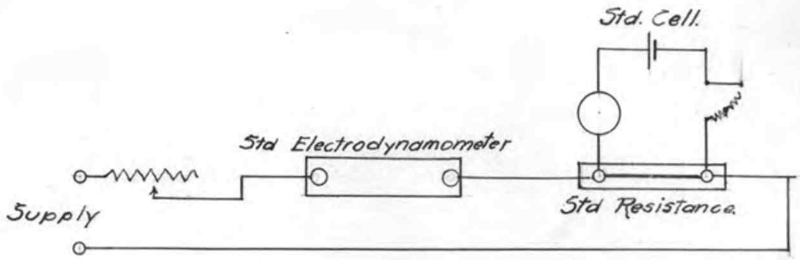
A Committee of the National Academy of Sciences submitted specifications to the above named organization which were accepted and unanimously adopted on February 9, 1895.

From the foregoing we see that the apparatus and conditions for the legal ohm are specified, therefore using this as a basis upon which to work, standards of resistance can be constructed of solid metals, preferably manganin, which has a negligible temperature coefficient.

The legal ampere is defined and the apparatus specified exactly; as this apparatus is too slow and clumsy for practical use, a galvanometer or an electrodynamometer controlled by a torsion head may be calibrated to read directly in amperes.

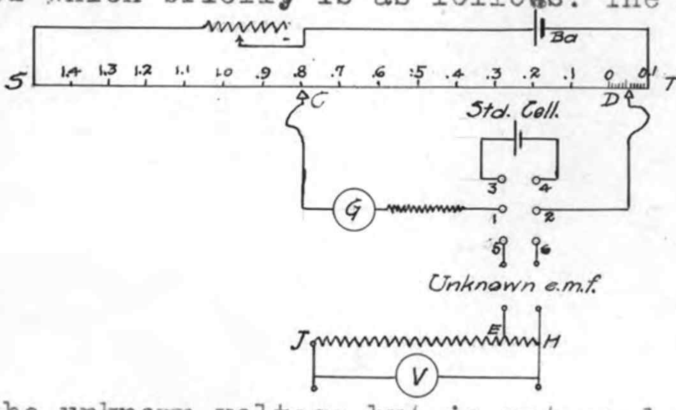
The legal volt is specified and also specifications given for the equivalent in the Clark cell; the electromotive force of the latter changes appreciably with a change of temperature and with age; another cell which has an almost negligible temperature coefficient and better suited for engineering work is the Weston cadmium cell. The normal voltage of this cell is 1.01985 volts, at any temperature between 10°C. and 35°C.

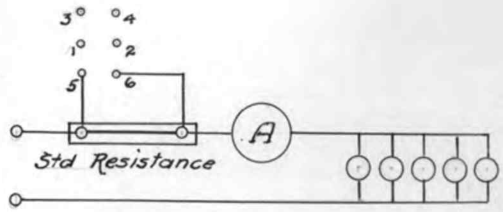
The figure shows the method of checking the three standards: viz, those of current, electromotive force and resistance.



VOLTMETER AND AMMETER CALIBRATION. Having our primary standards calibrated to conform to the legal units, our next step is to check an accurately constructed voltmeter and ammeter against the above mentioned standards. For rapidity and ease, deflection instruments of the electrodynamometer type, which may be used on either alternating or direct current, are usually selected.

The calibration of the selected secondary standards is accomplished by the use of a potentiometer, the operation of which briefly is as follows: The standard cell balances





the unknown voltage but is not used as a source of current. The zero method is used. Therefore it is not necessary to know the galvanometer constant. The errors of this piece of apparatus are due to the inequality of the resistances of the slide wire AB. The potentiometers on the market, however, are so arranged that the uniformity of the resistance of the slide wiremay be checked quickly and accurately.

Referring to the figure which is a set up for a voltmeter calibration, a slide-wire ST is connected in series with one or two storage cells Ba. and a regulating rheostat K; a constant current is thus established in ST. Any desired voltage drop may be had between the contacts C and D by moving them along the slide-wire, or by regulating the rheostatK. The contact C is for crude regulation, D for fine regulation. The drop between C and D is balanced against the standard cell and against the unknown electromotive force in succession. A balance is recognized by the galvanometer needle returning to zero. The ratio of the lengths C-D in the two cases is equal to the ratio of the e.m.f.'s under comparison, provided the current in ST remains constant. By means of the double-throw switch L connections are conveniently changed from the standard cell to the unknown e.m.f., and back.

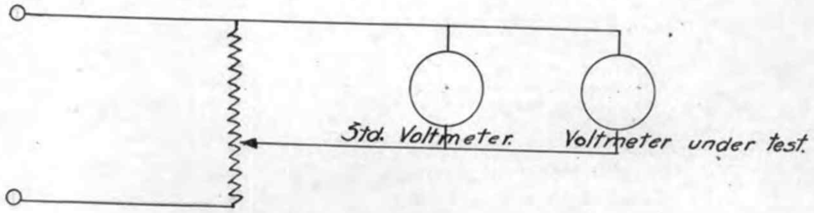
In order to make the arrangement direct-reading, the contacts C and D are set beforehand so as to balance the voltage of the standard cell at the proper divisions of the scale. The switch L is thrown to the left, and the rheostat K adjusted until the galvanometer returns to zero. Then the switch is thrown to the right, and the galvanometer balance again obtained by shifting C and D.

The potentiometer scale usually has a range of 1.5 volts, so that voltages below this limit may be directly compared to that of a standard cell. For voltages above 1.5, a multiplier or volt-box is used. It consists of a high resistance with taps at a known part of it, such as 1/10, 1/100 etc. Again referring to the figure, the unknown e.m.f. is connected across JH; the terminals EH are connected to the terminals of the potentiometer 5 and 6. Then if the unknown voltage be say 110 and the resistance between EH be 1/100 of the total resistance of the volt-box, the voltage drop across EH is 1.1 and can be compared directly to the e.m.f. of the standard cell. By this method voltages from a fraction of a millivolt to several thousand volts can be accurately compared to the e.m.f. of a standard cell.

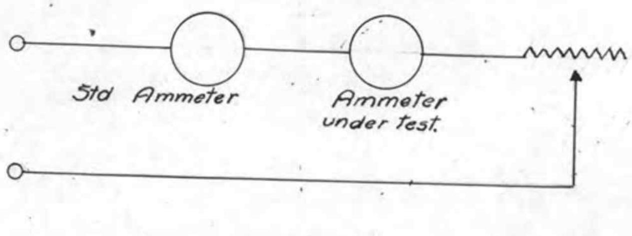
A modification of the set up shown in the figure will have to be made for checking ammeters against a standard cell. The volt-box is replaced by a standard shunt and the connections are those shown to the right of the figure. The resistance of the shunt is accurately known as well as the current capacity and for different values of current they are so selected that the drop of potential across the terminals does not exceed 1.5 volts; if it does not exceed one volt the potentiometer will be direct reading. By selecting shunts with the proper range, currents may be measured from a small fraction of an ampere to many thousand.

With secondary standards calibrated as described above, they are now available for use in checking commercial voltmeters and ammeters for either direct or alternating current.

The figure below shows the usual method of connecting up for a voltmeter calibration for voltages up to 600. Above that on alternating current, potential transformers are used. The exact ratio of transformation must be known and the method of determining it will be taken up later.



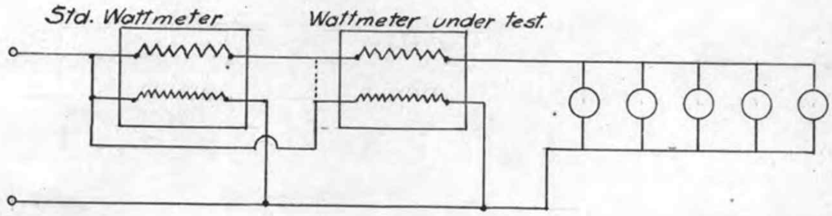
Ammeters for both alternating and direct current may be calibrated as connectedwithin the ordinary range, but for large currents on alternating current circuits current transformers are used. The ammeter is usually calibrated separately and the ratio of the transformer determined.



INDICATING WATTMETERS. In selecting a secondary standard for the measurement of watts, an indicating instrument of the electrodynamometer type is to be preferred, as it may be used on both alternating and direct current. In checking it against primary standards, two potentiometers should be used, one for checking the potential coil and the other for the current coil. By this means the errors are reduced over that by using the product of the voltmeter and ammeter readings of the secondary standards.

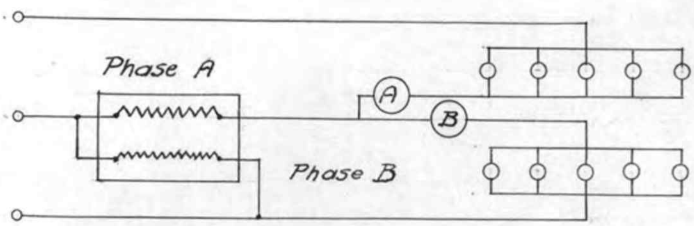
Such an instrument with known corrections could be used for calibrating indicating wattmeters of the induction type, and in conjunction with an accurate stop-watch to check rotating standards and watt-hour meters.

In connecting up the secondary standard with the meter under test, care must be taken that the potential coils of both wattmeters be connected to one point ahead of the instruments as shown. This eliminates the error of the



secondary standard indicating the power consumed in the potential coil of the instrument under test when connected as shown by the dotted lines. On an alternating current circuit with both instruments having a light load, the error would be still more pronounced due to the potential coil of the wattmeter under testbeing inductive. The secondary standard would be measuring an inductive load (highly so in the case of induction instruments), which means that the current and e.m.f. will be displaced thereby causing the inductance error of the electrodynamometer instrument to be more pronounced.

It is necessary to know the performance and accuracy of wattmeters, both indicating and watt-hour instruments on inductive loads, the power-factor ranging from 0.1 to unity. A convenient method of ascertaining this is shown in the figure.



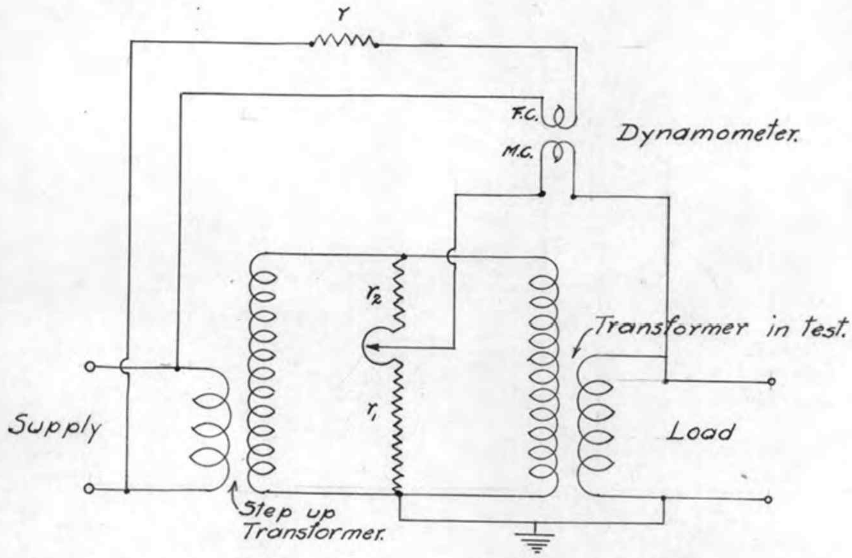
The wattmeter is connected to one phase of a two phase three wire circuit, r is a non-inductive load on phase B, the current flowing through the series coils thereby being in phase with the e.m.f. of phase B. Another resistance R or load is connected across phase A through an ammeter A. Thus we have two currents displaced by 90° flowing through the current coil of the wattmeter. The displacement of the current from the voltage in phase B is denoted by $\tan\alpha = A/B$, where α is the angle of displacement. A small reading on ammeter B should give a reading on the wattmeter, which if the wattmeter is correct should remain the same through any variation of reading of the ammeter A, however, both dynamometer and induction type wattmeters read high on inductive load and this method only serves as an approximate check on the accuracy of the instrument.

For accurate measurements with a dynamometer wattmeter it is important to ascertain whether the current in the potential coil leads or lags behind the current in the main circuit. Exact measurements of resistance, capacity and inductance should be made and from this the angle obtained by computation. Edgcumbe in "Industrial Electrical Measuring Instruments" claims that in improperly designed instruments that the capacity of the potential coils is quite appreciable.

Mr. Robert W. Paul, instrument maker of London, quotes an article by Mr. R.Beattie in the London Electrician,Vol. 48, page 818, on a means of determining the angle of phase displacement of the current and e.m.f. in the potential coil of a dynamometer wattmeter. His method consisted in doubling the inductance of the potential coil without changing its resistance, and subtracting algebraically from the original reading, the change of the reading thereby produced.

TRANSFORMERS. As has been pointed out previously in the use of current and potential transformers accurate determinations must be made of the ratio and phase-angle, and when once calibrated these pieces of auxiliary apparatus may be classed as precision instruments. Not only must the ratio be determined for one power-factor but for all ranges for which it is expected that the instrument will be used.

Mr.L.T.Robinson, in the Proceedings of the A.I.E.E. for July 1909, takes up in detail the checking of potential and current transformers for ratio and phase-angle under all of the conditions that they would be subjected to in practice.

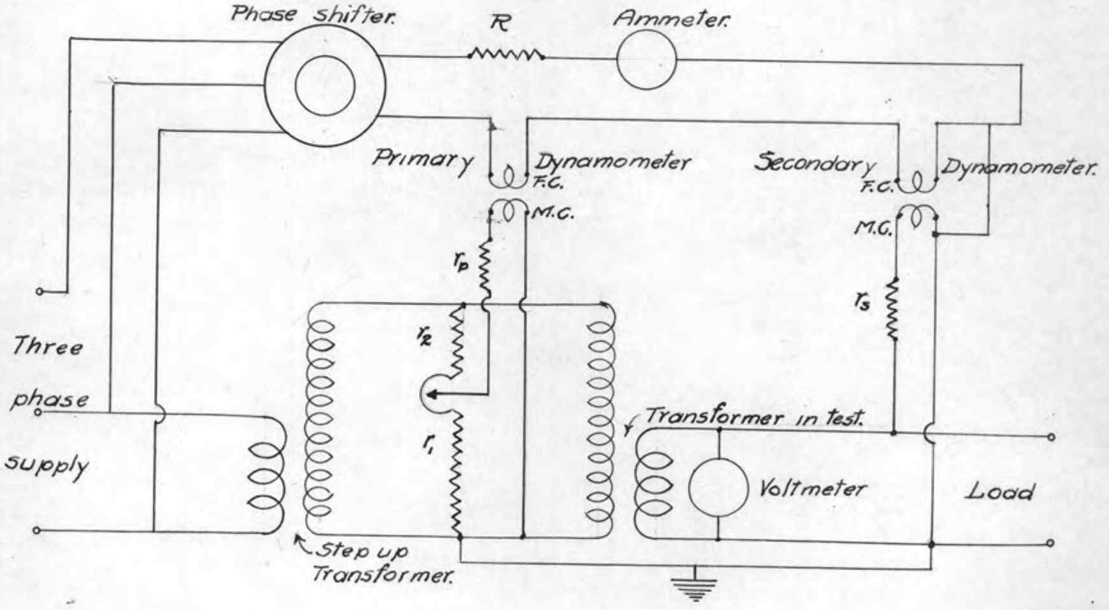


The above figure shows the apparatus for a ratio test of a potential transformer. This is suitable for any frequency and the common voltages met with. The resistances R_1 and R_2 are connected across the primary circuit with taps for various voltages, so that about 10 ohms per volt may be used. The primary and secondary are connected together and to the ground as shown. The polarity is such that the potential rise from the ground along the secondary of the transformer is in the same direction as that from the ground along the main primary resistance. By shifting the moving contact until a zero deflection is obtained, the ratio would be,

$$\frac{R_1 + R_2}{R_2}$$

It may be seen that the ratio is easily found for any value of inductive or non-inductive load up to the capacity of the transformer.

The connections and apparatus for the phase-angle test are given below.



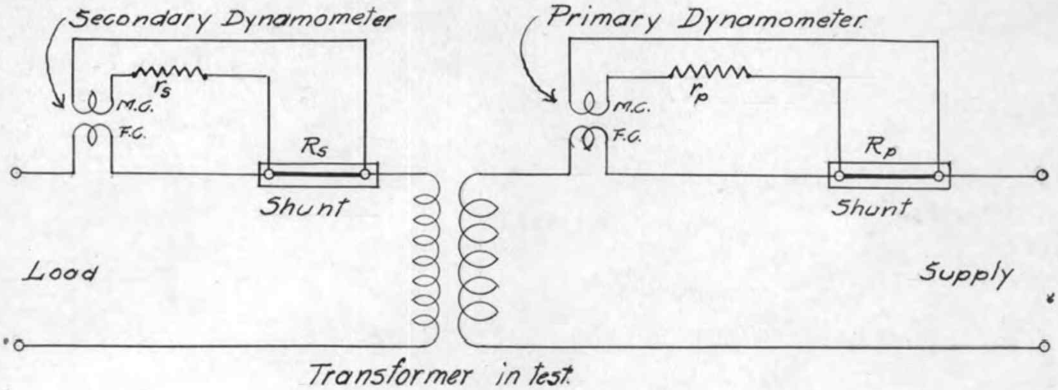
The 3-phase line current is brought to a phase shifter (described in detail by Sharp and Crawford of July 1910 Proceedings of the A.I.E.E.) and to the low voltage side of a step up transformer. A duplicate of the transformer being tested may be used for stepping up the voltage. Current from the secondary of the phase shifter is passed in series through the current coils of two dynamometers, usually of 5 amperes capacity, the amount being controlled by a suitable resistance R in series. The same resistance used in the ratio test and as shown, is connected in multiple withe the primary of the transformer in test, and the drop across a considerable portion of it is used to excite the moving coil of the primary dynamometer. The The secondary voltage of the transformer in test is used to excite the potential coil of the secondary dynamometer through the resistance r_s. The load on the transformer under test is then adjusted to the proper current and power-factor, and the phase is shifted until the primary dynamometer reads zero. The phase-angle between the primary and secondary electromotive force is then,

$$y = \sin^{-1} \frac{W_s}{E \times A}$$

where W_s is the reading of the secondary dynamometer, E is the voltage across the secondary of the transformer in test and if left in must be considered part of the load, A is the reading of the ammeter. Both of the reflecting dynamometers should be accurately calibrated with two potentiometers.

The method described by Mr. Robinson for calibrating current transformers for ratio and phase-angle is similar to that for potential transformers. He gives the one mentioned here as preferable to some four or five other methods for speed and accuracy.

For the ratio test of current transformers, the apparatus is shown in the figure, two reflecting dynamometers with their shunts are required, which admit of accurate calibration on direct current with a potentiometer. The instruments used measure watts lost in the shunts and the deflection is therefore proportional to the square of the current.



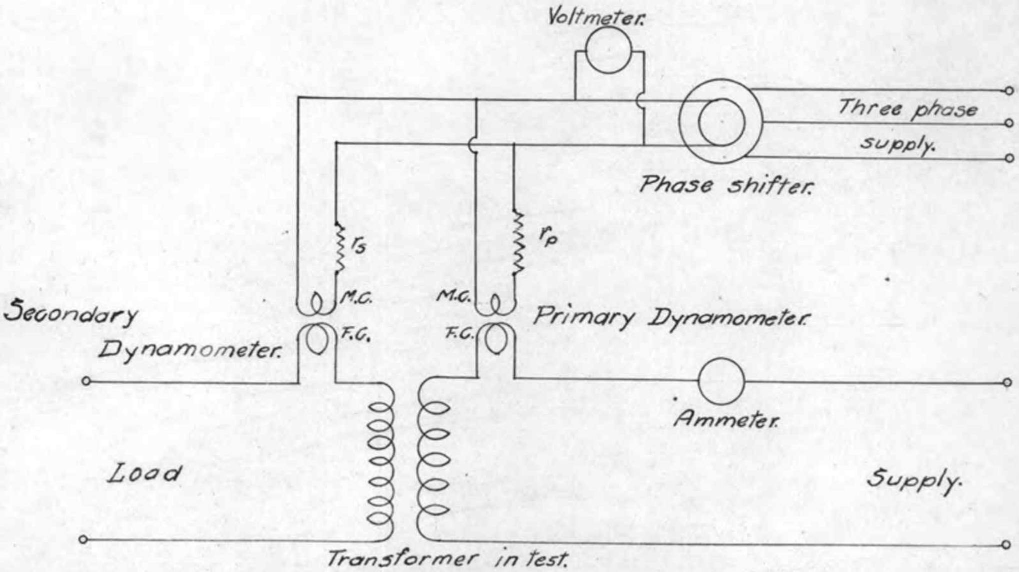
If I_p on calibration gave a deflection W_p, and I_s a deflection W_s; as each is proportional to the square of the current the true ratio would be,

$$\sqrt{W_s} \div \sqrt{W_p}$$

In checking a transformer with the connections as shown, the primary dynamometer is brought to a reading W_p corresponding to I_p and the secondary dynamometer reading noted. The ratio expressed above will then be the ratio of transformation.

Some difficulty may be experienced with the inductance of the shunts with currents of 500 amperes or more. The phase-angle due to this inductance must be determined as it is necessary in order to apply the correction to the dynamometers. In the proceedings of the A.I.E.E. Vol.29, Sharp and Crawford give an accurate method for determining this phase-angle.

For a phase-angle determination of a current transformer the same apparatus is required with the exception of the shunts and a phase shifter in addition.



The current windings of the dynamometers are in the primary and secondary as shown; the potential coils are connected to a phase shifter, this phase-shifter being rotated until the primary dynamometer reads zero. The phase angle then is,

$$B = \sin^{-1} \frac{W_s}{E \times I_s}$$

where I_s is the secondary current in the transformer, W_s

the watts read on the dynamometer and E is the voltage of the phase shifter.