Appendix 1: Solution for steady-state temperature profile under constant flux.

The steady-state governing equation for the system is readily obtained by setting diffusive thermal energy flux equal to advected thermal flux at all locations (which, as a starting point, can be found in many references, including the recent work of Luce, 2013)

$$qC\frac{dT}{dx} = -k\frac{d^2T}{dx^2}$$

A1

Where q is the Darcy flux, C is the heat capacity of water and *k* the thermal conductivity of the saturated media. Defining $\beta = qC/k$ and integrating once, and separating variables yields

$$-dx = \frac{dT}{\beta T + A}$$

A2

Where A is a constant of integration. This is solved by

$$-x = \frac{1}{\beta}ln(\beta T + A) + D$$

A3

Where D is a second constant of integration. This can be solved for T as

$$T = \frac{1}{\beta}exp(\beta(-x - D)) - A$$

A4

The values of A and D can be obtained by imposing the boundary conditions. At the surface, x = 0 and $T(0)=T_s$ and at infinite depth $T(\infty)=T_d$. Starting with infinite depth, we immediately see that $A= -T_d$. With x=0 we see that

$$T_s = \frac{1}{\beta}exp(-\beta D) + T_d$$

A5

or

$$D = -\frac{1}{\beta}ln[\beta(T_s - T_d)]$$

So the solution is simply

$$T = (T_s - T_d)exp(-\beta x) + T_d$$

A6

For example, using the following values results in the temperature profile shown in Figure 1:

q = 5e-7 m/s, (dashed line), 5e-6 (dotted line), and 5e-5 (solid line)
C = 4.18e6 J / m3 K
k = 2.5 W/m K

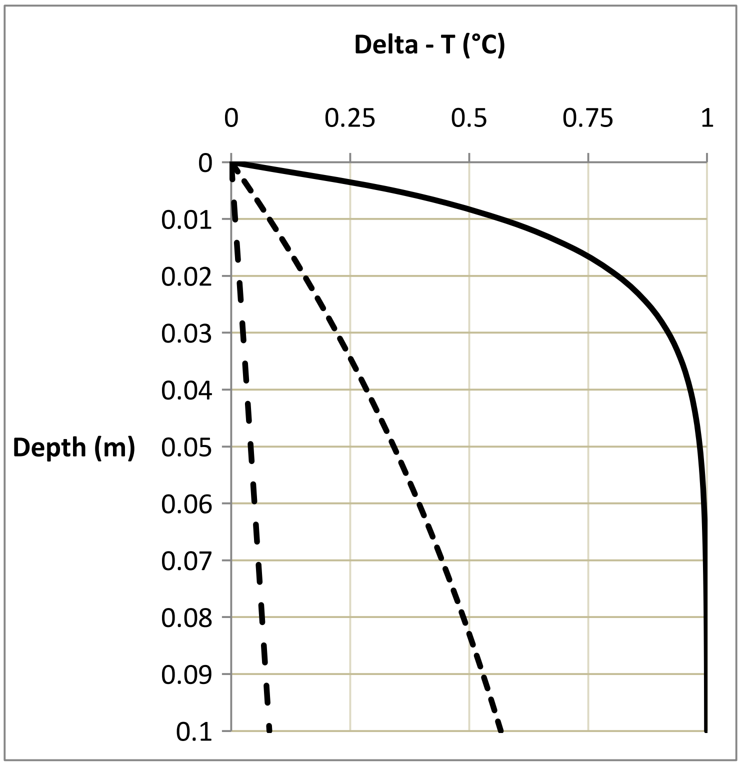

Figure 1. 1-D analytical predictions for temperature as a function of burial depth in sand for three seepage velocities (5e-7 m/s, dashed line; 5e-6, dotted line; and 5e-5, solid line). Delta-T is the fraction of difference between surface (10 °C) and groundwater temperature (22 °C) occurring at each depth. This illustrates how seepage can substantially change the temperature profile and also the potential benefit of burial of sensors.