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COMPRESSIVE BUCKLING CURVES FOR SIMPLY SUPPORTED SANDWICH PANELS WITH GLASS-FABRIC-LAMINATE FACINGS AND HONEYCOMB CORES

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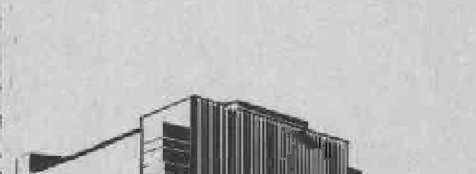

FOREST PRODUCTS LABORATORY
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UNITED STATES DEPARTMENT OF AGRICULTURE
FOREST SERVICE
In Cooperation with the University of Wisconsin

COMPRESSIVE BUCKLING CURVES FOR SIMPLY SUPPORTED SANDWICH PANELS WITH GLASS-FABRIC-LAMINATE FACINGS AND HONEYCOMB CORES[1]

By

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Abstract

In this report are presented curves and formulas for use in calculating the buckling of flat, simply supported panels of sandwich construction under edgewise compressive loads. The curves apply particularly to sandwich panels having glass fabric laminate facings and honeycomb cores.

Introduction

The derivation of formulas for the buckling loads of rectangular sandwich panels subjected to edgewise compression is given in Forest Products Laboratory Report No. 1583-B.[3] These formulas apply to panels having orthotropic facings and cores (fig. 1). They are given also in section 4.2.1.1 of

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[1]This progress report is one of a series (ANC-23, Item 58-1) prepared and distributed by the Forest Products Laboratory under U. S. Navy Bureau of Aeronautics Nos. NAer 01835 and 01853, and U. S. Air Force No. AF (33-66)58-1. Results here reported are preliminary and may be revised as additional data become available.

[2]Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

[3]Erickson, W. S. and March, H. W., "Effects of Shear Deformation in the Core of a Flat Rectangular Sandwich Panel." "Compressive Buckling of Sandwich Panels Having Facings of Unequal Thickness," Forest Products Laboratory Report No. 1583-B, 1950.

ANC Bulletin 23, Part II.[4] These formulas, reduced to apply to sandwich panels with isotropic facings and cores, are given with design curves in section 3.2.1.1 of ANC Bulletin 23. For honeycomb cores it was found that the modulus of rigidity associated with the directions perpendicular to the ribbons of which the honeycomb is made and the length of the cells is roughly 40 percent of the modulus associated with the directions parallel to these ribbons and the length of the cells. Making use of this fact, design curves for sandwich panels having isotropic facings and honeycomb cores were calculated and published in Forest Products Laboratory Report No. 1854.[5]

Upon examination of the design values obtained for the elastic properties of usual glass-fabric laminates, it was found that the combinations of these properties that enter the equations do not vary greatly from one laminate to another; thus useful design curves for the buckling loads of sandwich panels having glass fabric laminate facings and honeycomb cores can be drawn and are presented in this report.

The curves, shown in figures 2 to 11, are drawn only for panels having simply supported edges and buckling into a single half wave. Thus the aspect ratio (b/a) varies from zero to a little beyond unity. The familiar cusps that occur in such curves when the panel breaks into a greater number of half waves are not shown. It is assumed that the minimum values given by the curves apply with sufficient accuracy to all panels with aspect ratios greater than unity.

Formulas Used

The notation in this report is illustrated in figure 1. The lengths of the edges of the panel are denoted by a and b. The load is applied to the edges of length a and acts in the direction parallel to the edges of length b. The natural axes of the facings and core are parallel to the edges of the panel.

The value of the stress in the facings at which buckling occurs is given by:

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[4] U. S. Forest Products Laboratory. Sandwich Construction for Aircraft. Air Force-Navy-Civil Aeronautics Bulletin 23, Part II, Second Edition; 1955.

[5] Norris, Charles B. Compressive Buckling Curves for Sandwich Panels with Isotropic Facings and Isotropic or Orthotropic Cores. Forest Products Laboratory Report No. 1854. Revised January 1958.

$$f_{Fcr} = \frac{\pi^2}{4} \frac{t_{F1} t_{F2}}{a^2} \left[\frac{t + t_C}{t - t_C}\right]^2 \frac{\sqrt{E_{Fa} E_{Fb}}}{\lambda_F} K \tag{1}$$

where t_{F1}, t_{F2}, t_C, and t are the thickness of the two facings, the core, and the total thickness of the panel; E_{Fa} and E_{Fb} are the moduli of elasticity of the facings in the a and b directions and are equal to E_α or E_β, depending upon which way the panel is turned, E_α and E_β being the moduli of elasticity of the facing material in the warp and fill directions; λ_F is unity minus the product of the two Poisson's ratios of the facing material associated with these directions; and:

$$K = K_F + K_M \tag{2}$$

Where

$$K_F = \frac{1}{3}\left(\frac{t_{F1}}{t_{F2}} + \frac{t_{F2}}{t_{F1}} - 1\right)\left(\frac{t - t_C}{t + t_C}\right)^2\left(\frac{\alpha b^2}{a^2} + 2\beta + \frac{a^2}{\alpha b^2}\right) \tag{3}$$

$$K_M = \frac{\frac{\alpha b^2}{a^2} + 2\beta + \frac{a^2}{\alpha b^2} + VA\left(\frac{ra^2}{b^2} + 1\right)}{1 + V\frac{ra^2}{b^2}\left(\frac{\alpha b^2}{a^2} + \gamma\right) + V\left(\frac{a^2}{\alpha b^2} + \gamma\right) + V^2 rA\frac{a^2}{b^2}} \tag{4}$$

$$A = 1 - \beta^2 + \gamma\left(\frac{\alpha b^2}{a^2} + 2\beta + \frac{a^2}{\alpha b^2}\right) \tag{5}$$

$$V = \frac{t_C t_{F1} t_{F2}}{t - t_C} \frac{\pi^2}{a^2} \frac{\sqrt{E_{Fa} E_{Fb}}}{\lambda_F G_{Cbz}} \tag{6}$$

$$r = \frac{G_{Cbz}}{G_{Caz}} \tag{7}$$

$$\alpha = \sqrt{\frac{E_{Fa}}{E_{Fb}}} \tag{8}$$

$$\beta = \alpha\mu_{Fab} + 2\gamma \tag{9}$$

$$\gamma = \frac{G_{Fab}\lambda_F}{\sqrt{E_{Fa}E_{Fb}}} \tag{10}$$

Where G_{Caz} and G_{Cbz} are the moduli of rigidity of the core associated with the axis perpendicular to the surface of the panel and the axes parallel to the edges of lengths a and b, respectively; G_{Fab} is the modulus of rigidity of the facings associated with the axes parallel to the edges of lengths a and b; μ_{Fab} is the Poisson's ratio of the facings associated with the contraction in the a direction and extension in the b direction due to a tensile stress in the b direction.

Elastic Properties of Glass Fabric Laminates

Table 1 lists the elastic properties of some glass fabric laminates and the calculated values of parameters α, β, and γ that depend upon elastic properties as given by equations 8, 9, and 10. Column 1 of table 1 gives the numbers of the glass fabrics from which the laminates were made. Columns 2 and 3 give values of the secant modulus of elasticity of the laminates in the warp and fill directions. These values were calculated from those given in table 2-1 of ANC Bulletin 17.[6] They are the secant moduli taken to the secondary proportional limit in tension. This secant modulus is proper for use in connection with repeated stresses greater than the primary and less than the secondary propor-

[6] U. S. Forest Products Laboratory. Plastics for Flight Vehicles. Air Force-Navy-Civil Aeronautics Bulletin 17, Revised. In Press.

tional limit.[7] Column 4 gives values of the modulus of elasticity taken at 45 degrees to the warp direction. They were taken directly from table 2-1 of ANC Bulletin 17.[6] The values of Poisson's ratio in column 5 were taken from Forest Products Laboratory Report No. 1860.[8] The values of modulus of rigidity given in column 6 were calculated from the values given in columns 2 to 5 by means of the equations[9]

$$\frac{\mu_{\alpha\beta}}{E_{\alpha}} = \frac{\mu_{\beta\alpha}}{E_{\beta}} \tag{11}$$

and

$$\frac{1}{G_{\alpha\beta}} = \frac{4}{E_{45}} - \frac{1 - \mu_{\alpha\beta}}{E_{\alpha}} - \frac{1 - \mu_{\beta\alpha}}{E_{\beta}} \tag{12}$$

The values of α, β, and γ given in the remaining columns of table 1 are calculated by means of equations 8, 9, and 10.

From examination of values in table 1, it is seen that values of 0.60 for β and 0.20 for γ apply reasonably well for all laminates, and that values of α average about 1.00 except for laminates of 143 fabric, for which α is nearly 0.67 or 1.50. Therefore, curves were calculated for K_M, using values of 0.60 for β, 0.20 for γ and 0.67, 1.00, and 1.50 for α.

Discussion of Design Curves

The curves given in figures 2 to 10 are calculated from equation (4), with β and γ equal to 0.60 and 0.20, hence these curves apply to the glass-fabric laminates listed in table 1. Each family of curves consists of a plot of K_M against b/a for various values of V. The families differ because they apply to different values of α and r. All of the curves apply to panels having simply supported edges that buckle into a single half wave. The curves associated with a greater number of half waves are not shown. Their minimum points

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[7] Erickson, E. C. O., and Norris, C. B., "Tensile Properties of Glass-Fabric Laminates with Laminations Oriented in Any Way," U. S. Forest Products Laboratory Report No. 1853.

[8] Youngs, Robert L., "Poisson's Ratios for Glass-Fabric-Base Laminates," Forest Products Laboratory Report No. 1860.

[9] These equations come from equations 2:12 and 2:26 of ANC-17 Bulletin, Plastics for Aircraft, Part I, Reinforced Plastics, June 1955 edition.

are equal to the minimum points of the curves given. Such curves are shown in figures 2, 3, and 4 of Forest Products Laboratory Report No. 1854[5] for sandwich panels having isotropic facings. It will be noted that these added curves resulting in wave-like cusps add little to the design information.

As the value of $\underline{V}$ increases, the value of the $\underline{K_M}$ intercept decreases and the minimum point of the curve moves to the left. There is a value of $\underline{V}$ for which the minimum point of the curve coincides with the $\underline{K_M}$ intercept. This minimum point is common to the curves associated with all numbers of half waves. Of these curves, the curve for a infinite number of half waves yields the least critical value and is a horizontal straight line. These straight lines are shown on the curve sheets. If $\underline{V}$ is given a value equal to or greater than that associated with these straight lines, the value of $\underline{K_M}$ is $\underline{1/V}$. This is the well-known "shear instability limit" of the critical load.

The curves for $\underline{V}$ equal to zero are, of course, independent of $\underline{r}$. They are repeated on each curve sheet for convenience. They are also plotted to a logarithmic scale in figure 11. As the aspect ratio $\underline{b/a}$ is reduced, the curves approach the asymptotes shown as dashed lines in this figure ; thus values of $\underline{K_{Mo}}$ ($\underline{K_M}$ for $\underline{V}$ equal to zero) may be found for an aspect ratio as small as desired.

These curves allow the calculation of values of $\underline{K_F}$ because the value of the expression in the last parenthesis of equation (3) is equal to the value of $\underline{K_M}$ when $\underline{V}$ is equal to zero ($\underline{K_{Mo}}$); thus the value of $\underline{K}$ may be found according to equation (2). It will be found that the value of $\underline{K}$ is very little greater than $\underline{K_M}$ except for small values of the aspect ratio $\underline{b/a}$ and for very large values of $\underline{V}$.

Table 1.--Elastic properties of glass-fabric laminates

Laminate fabric No.	Modulus of elasticity			Poisson's ratio	Modulus of rigidity calculated	α		β		γ
	Warp direction E_{α}	Fill direction E_{β}	Diagonal direction $E_{45°}$	$\mu_{\alpha\beta}$	$G_{\alpha\beta}$	Loaded in β direction	Loaded in α direction	Loaded in β direction	Loaded in α direction	Loaded in either direction
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	1,000 p.s.i.	1,000 p.s.i.	1,000 p.s.i.		1,000 p.s.i.					
					POLYESTER RESINS					
112	2,420	2,240	1,490	0.12	519	1.04	0.96	0.560	0.570	0.22
116	2,730	2,370	1,600	.12	554	1.07	.94	.547	.564	.22
128	2,520	1,770	1,410	.12	506	1.19	.84	.579	.623	.24
162	2,310	1,460	1,200	.12	460	1.26	.79	.591	.650	.25
164	2,140	1,840	1,570	.12	604	1.08	.93	.714	.733	.30
120	2,660	2,500	1,760	.12	628	1.03	.97	.601	.609	.24
181	2,400	2,230	1,570	.12	559	1.04	.96	.596	.606	.24
182	2,580	2,410	1,640	.12	576	1.03	.97	.577	.585	.23
183	2,160	1,990	1,540	.12	488	1.04	.96	.584	.594	.23
184	2,790	2,220	1,680	.12	602	1.12	.89	.588	.616	.24
143	5,390	1,600	1,570	.25	546	1.84	.54	.484	.808	.17
					EPOXY RESINS					
112	2,550	2,380	1,610	.12	598	1.04	.96	.596	.606	.24
120	2,450	2,410	1,680	.12	602	1.01	.99	.612	.614	.24
181	3,060	2,900	2,200	.12	893	1.03	.97	.709	.717	.29
143	4,760	2,020	1,780	.25	607	1.54	.65	.510	.751	.18

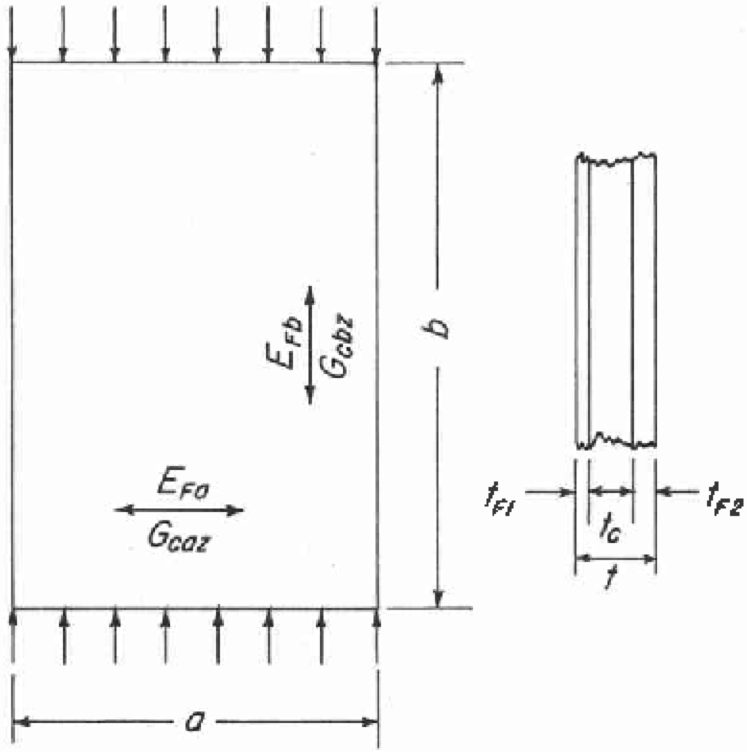

Figure 1. --Notation for dimensions and elastic properties of sandwich panel.

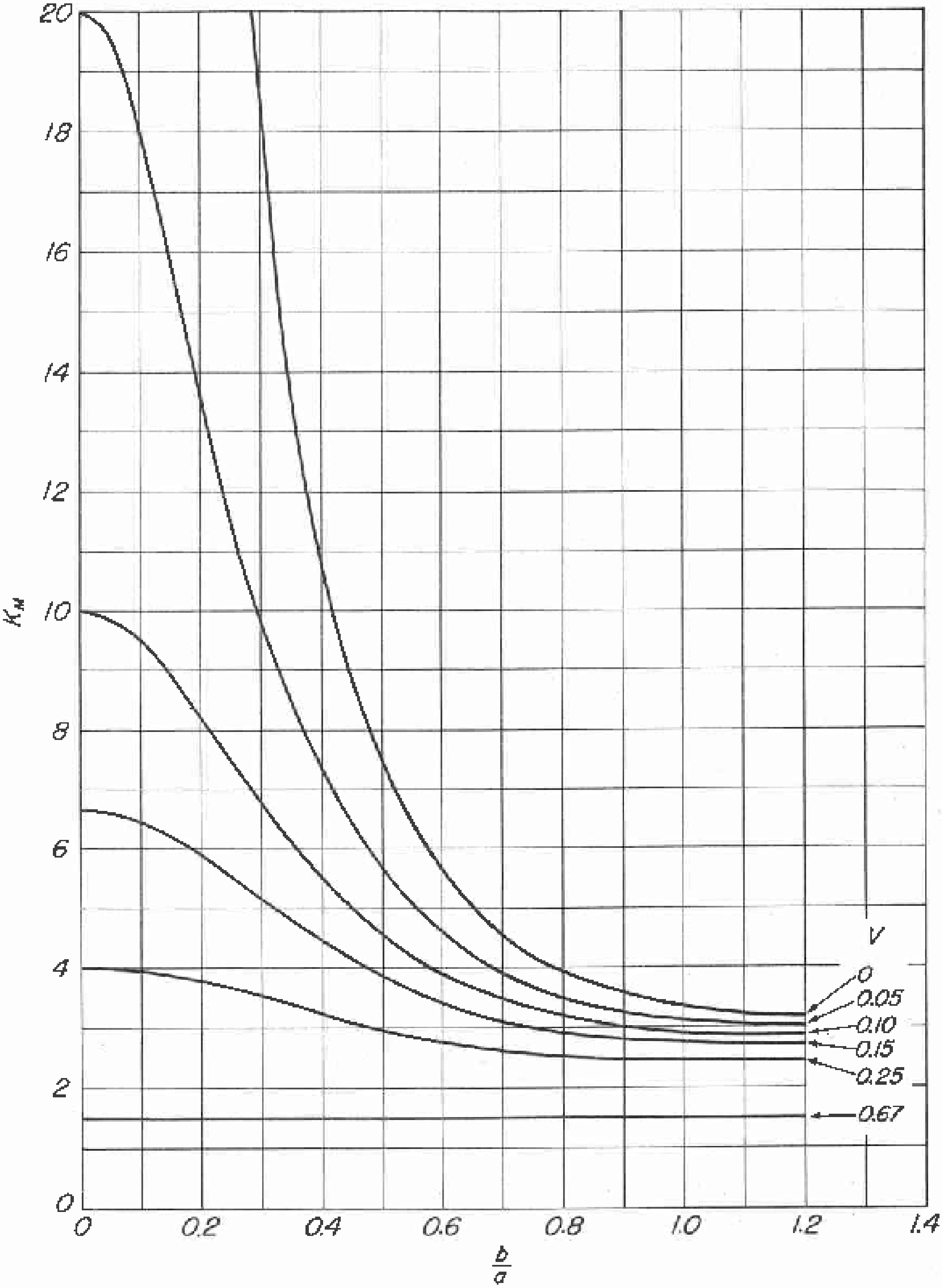

Figure 2. --A plot of K_M against b/a for $\gamma = 0.2$, $\beta = 0.6$, $\alpha = 0.67$, and $r = 0.4$ for various values of V.

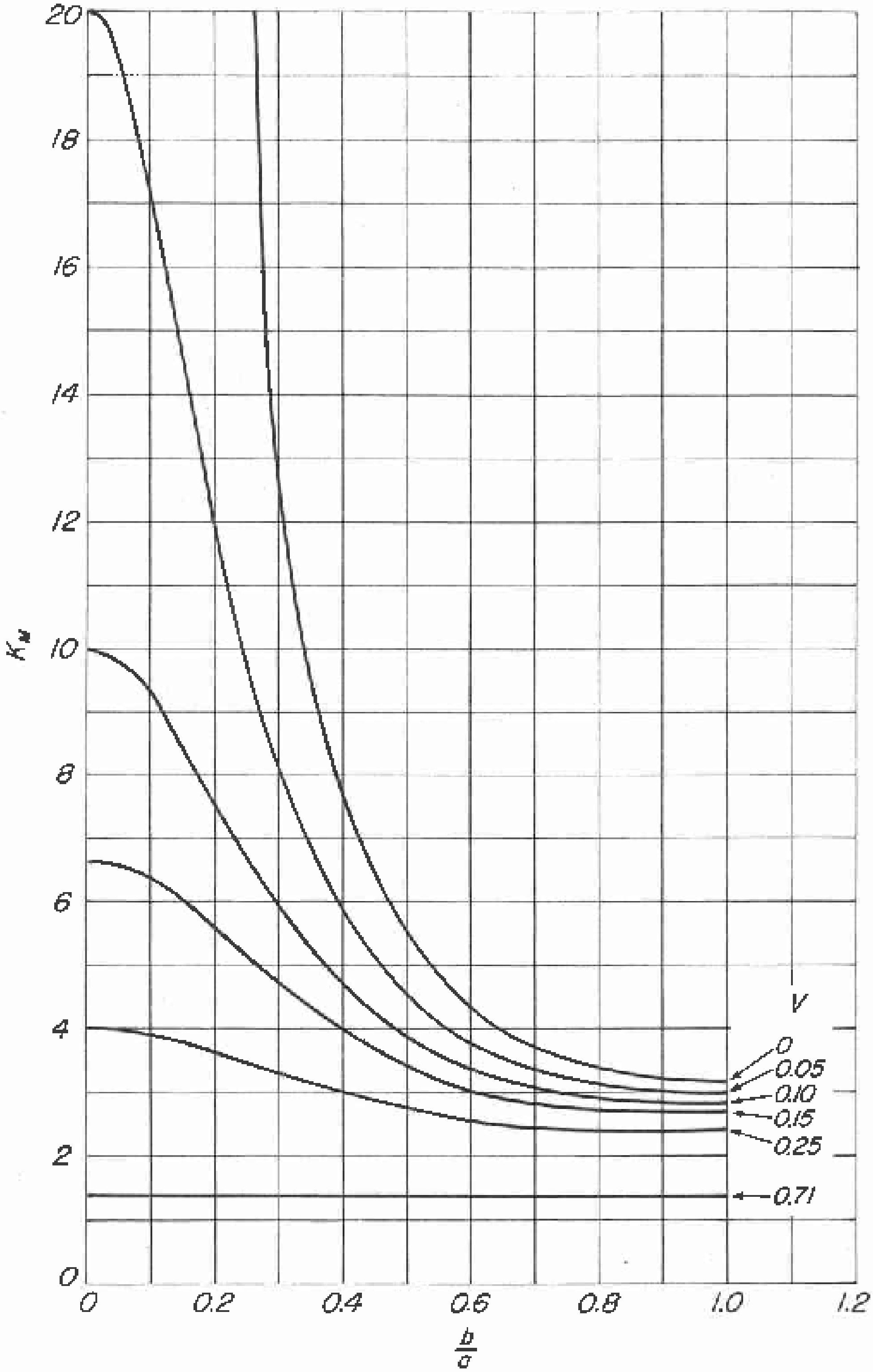

Figure 3. --A plot of K_M against b/a for $\gamma = 0.2$, $\beta = 0.6$, $\alpha = 1.0$, and $r = 0.4$ for various values of V.

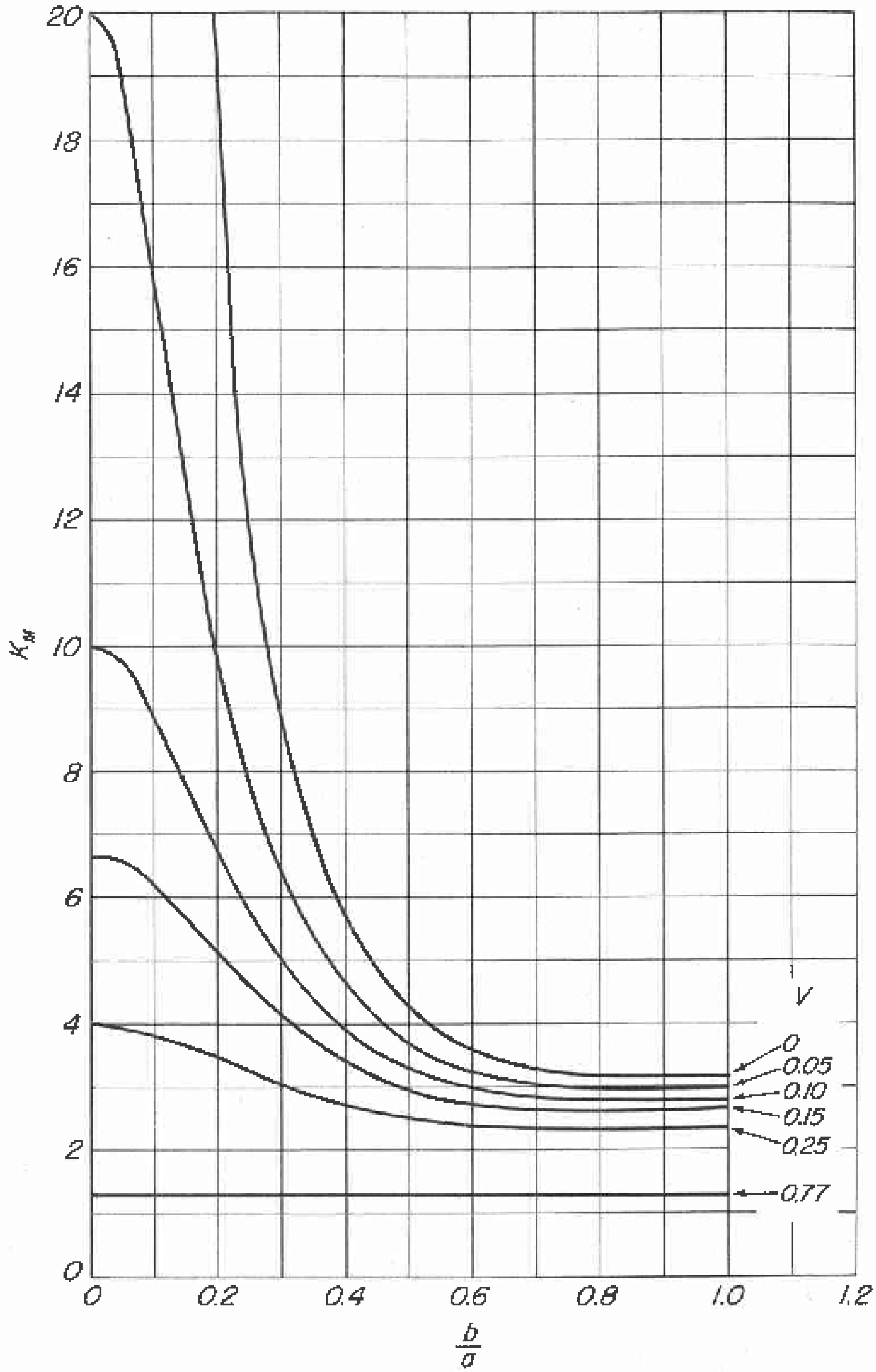

Figure 4.--A plot of K_M against b/a for γ = 0.2, β = 0.6, α = 1.5, and r = 0.4 for various values of V.

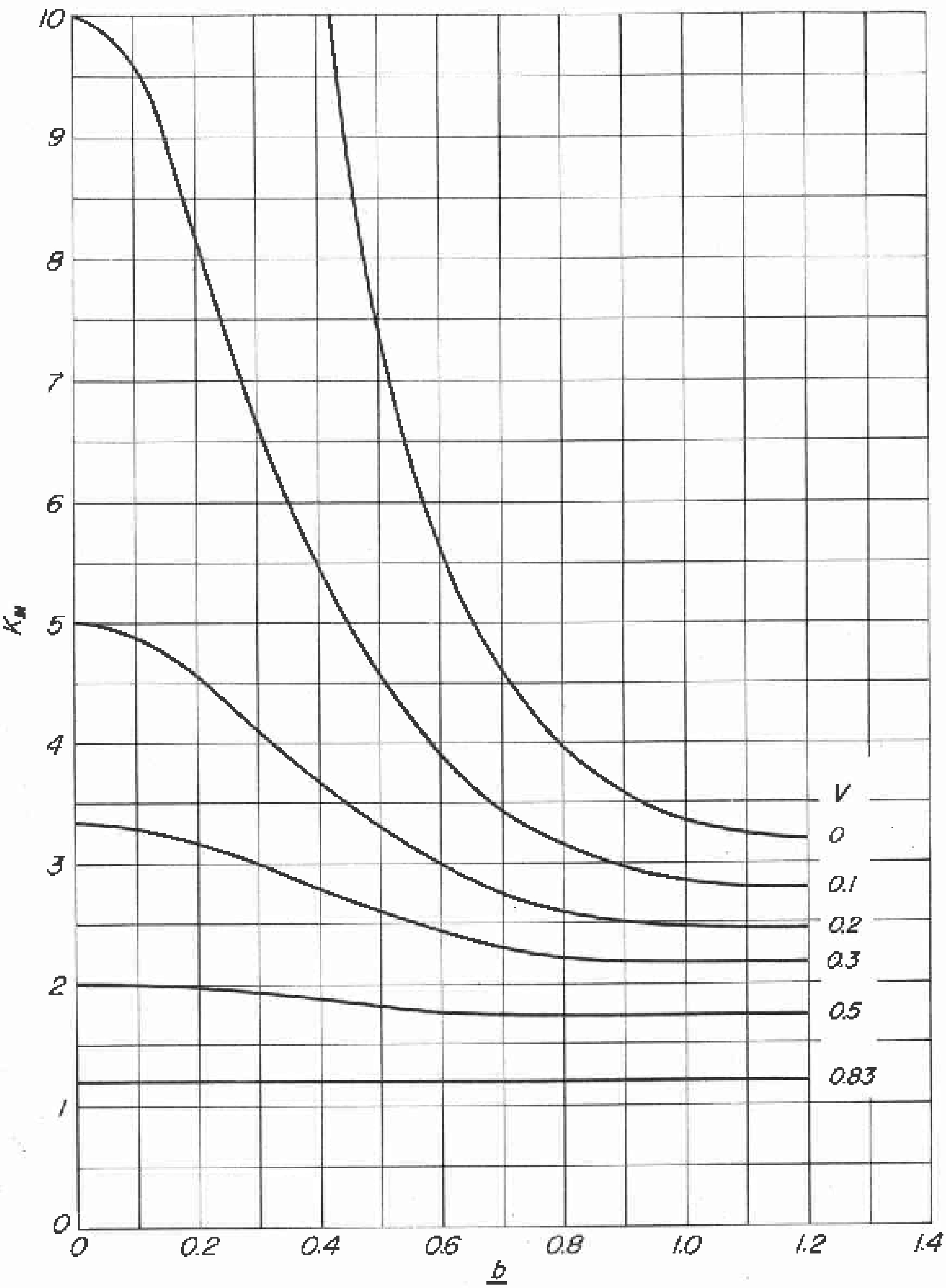

Figure 5. --A plot of K_M against b/a for $\gamma = 0.2$, $\beta = 0.6$, $\alpha = 0.67$, and $r = 1.0$ for various values of V.

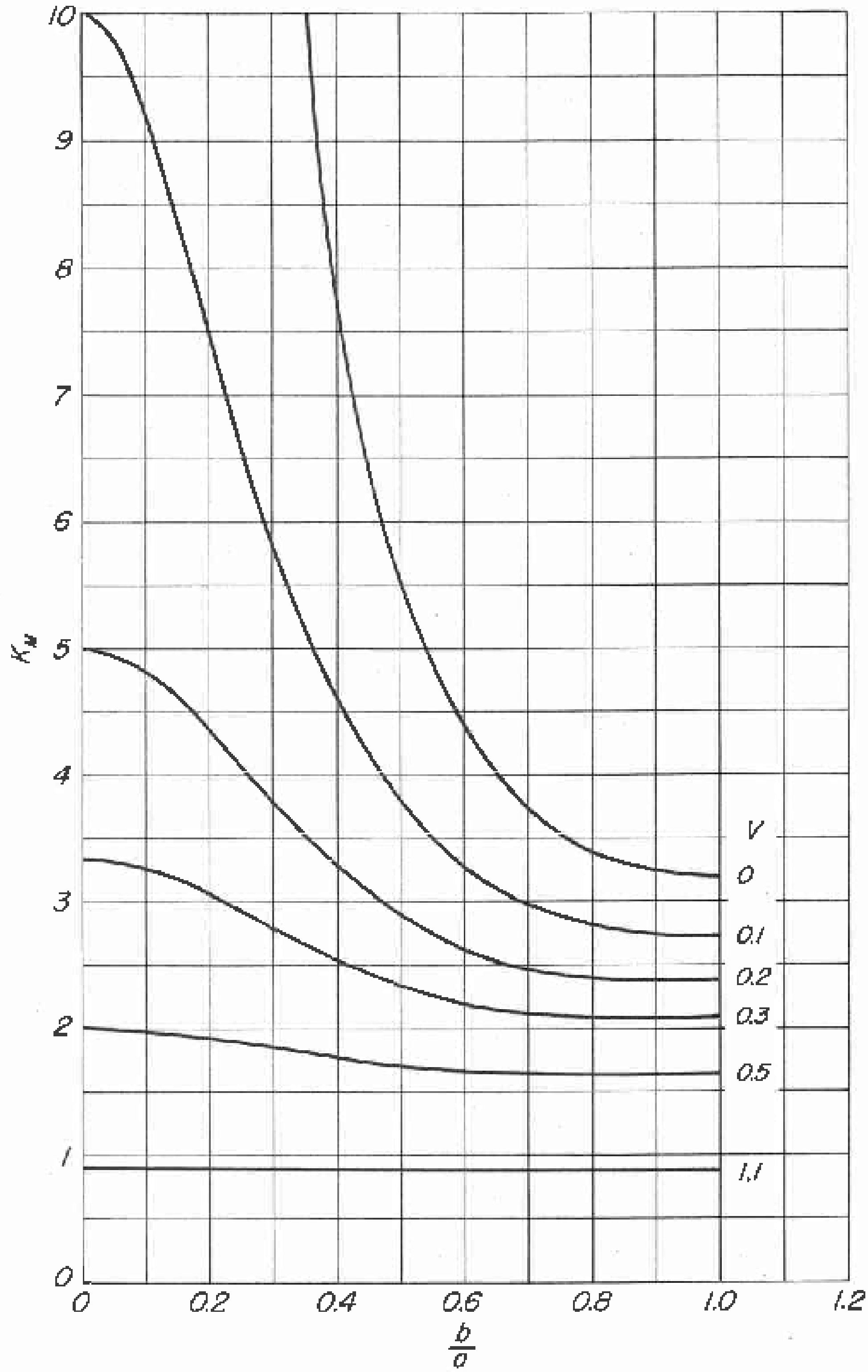

Figure 6.--A plot of K_M against b/a for γ = 0.2, β = 0.6, α = 1.0, and r = 1.0 for various values of V.

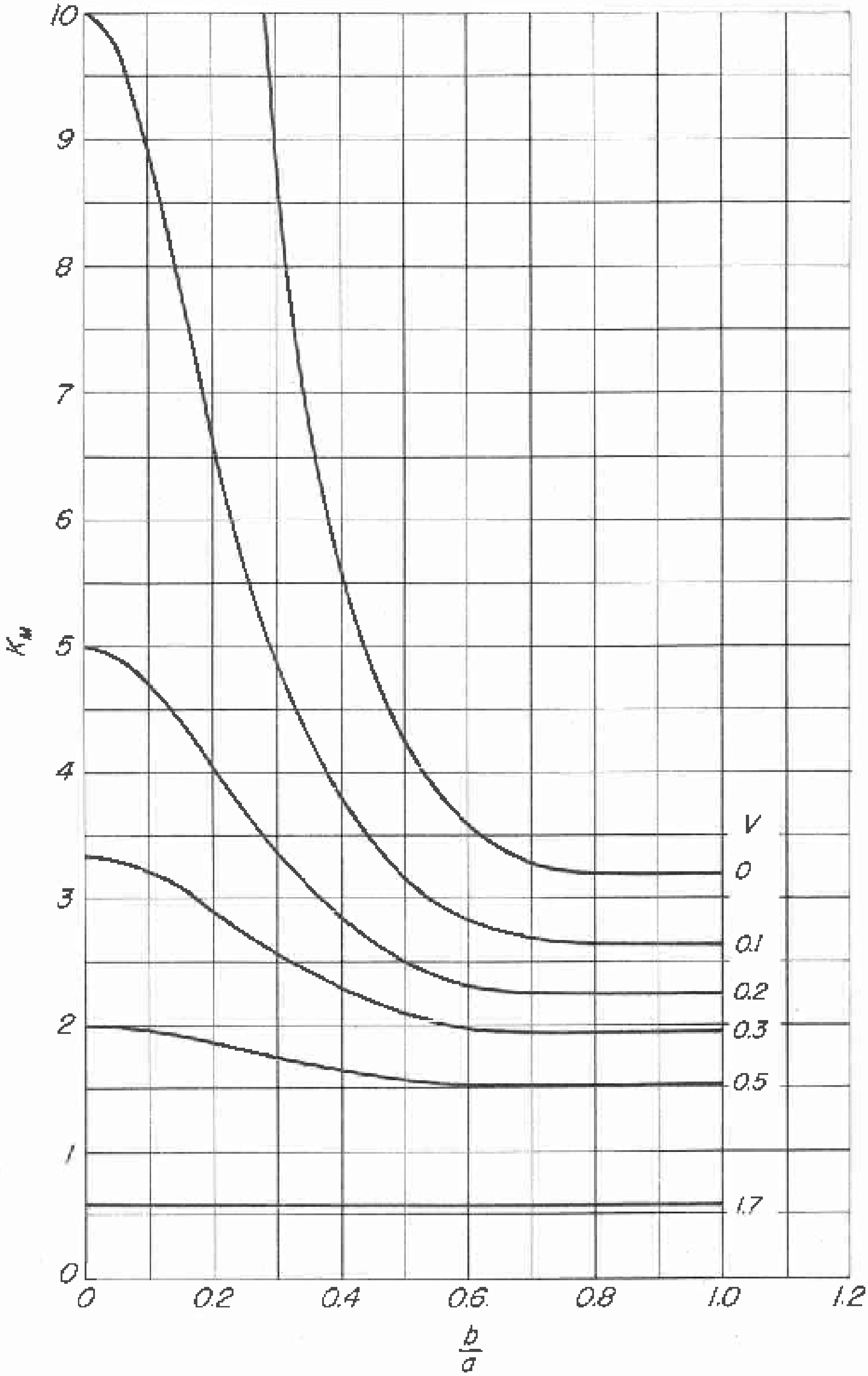

Figure 7. --A plot of K_M against b/a for $\gamma = 0.2$, $\beta = 0.6$, $\alpha = 1.5$, and $r = 1.0$ for various values of V.

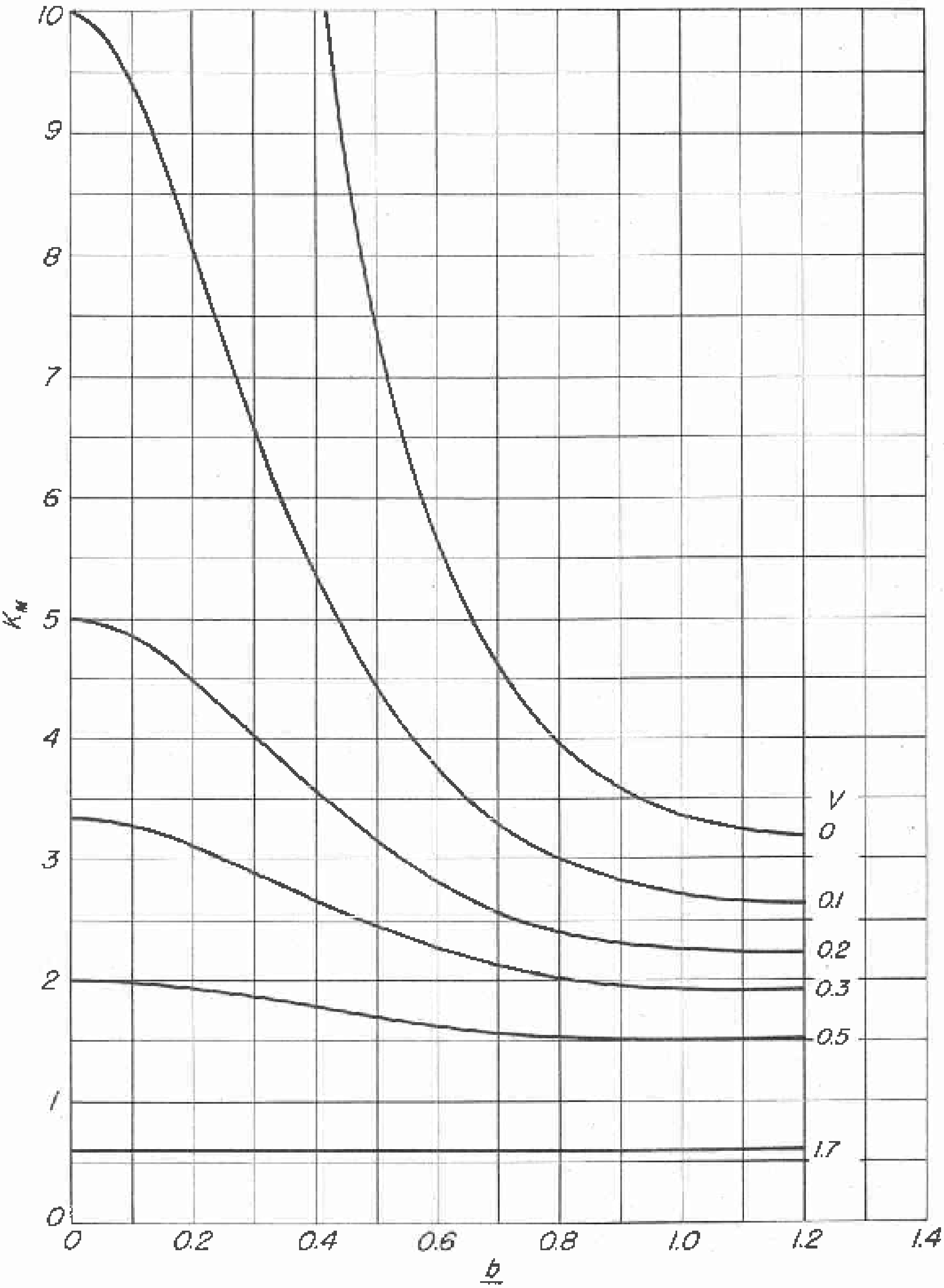

Figure 8.--A plot of K_M against b/a for γ = 0.2, β = 0.6, α = 0.67, and r = 2.5 for various values of V.

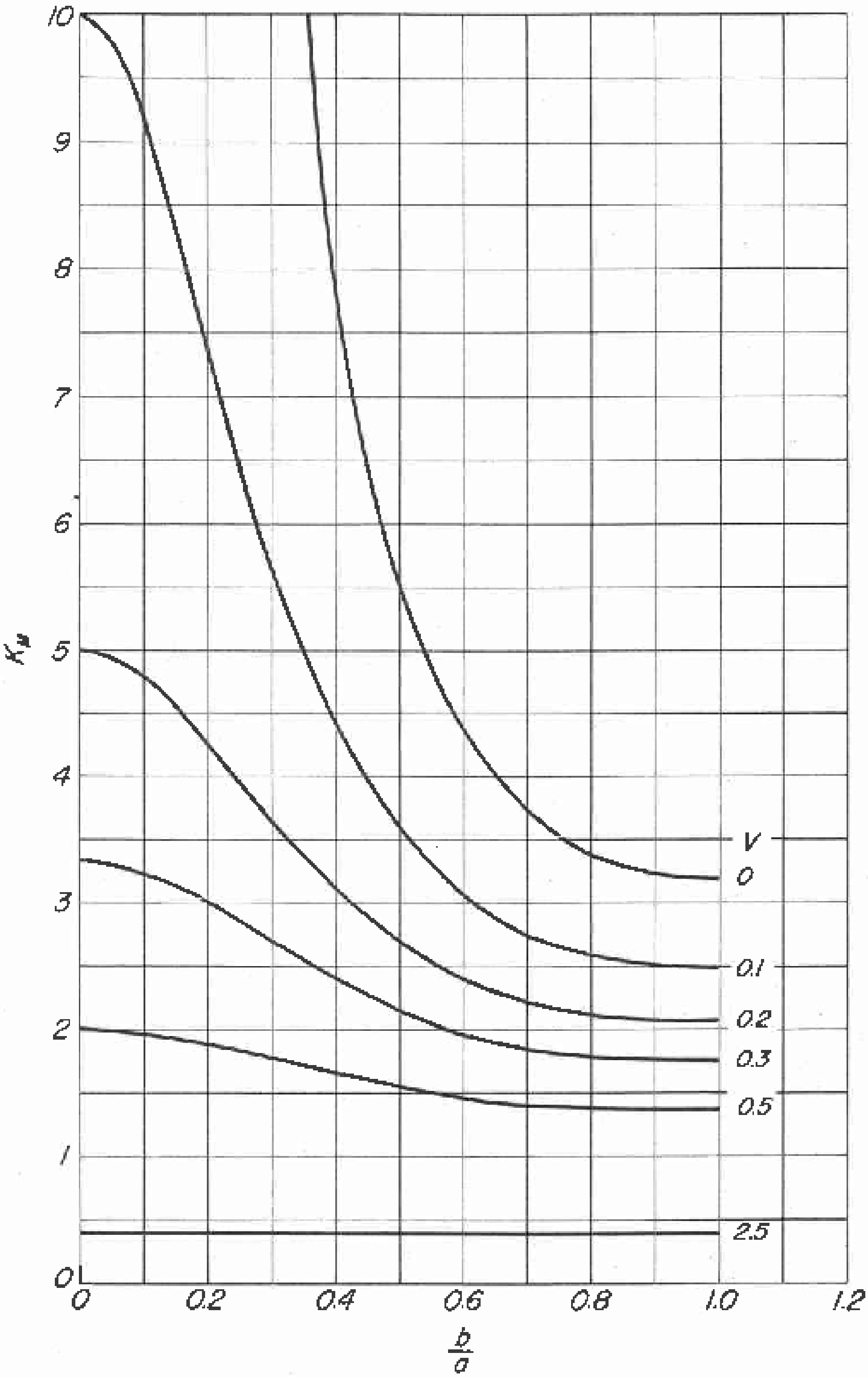

Figure 9.--A plot of K_M against b/a for $\gamma = 0.2$, $\beta = 0.6$, $\alpha = 1.0$, and $r = 2.5$ for various values of V.

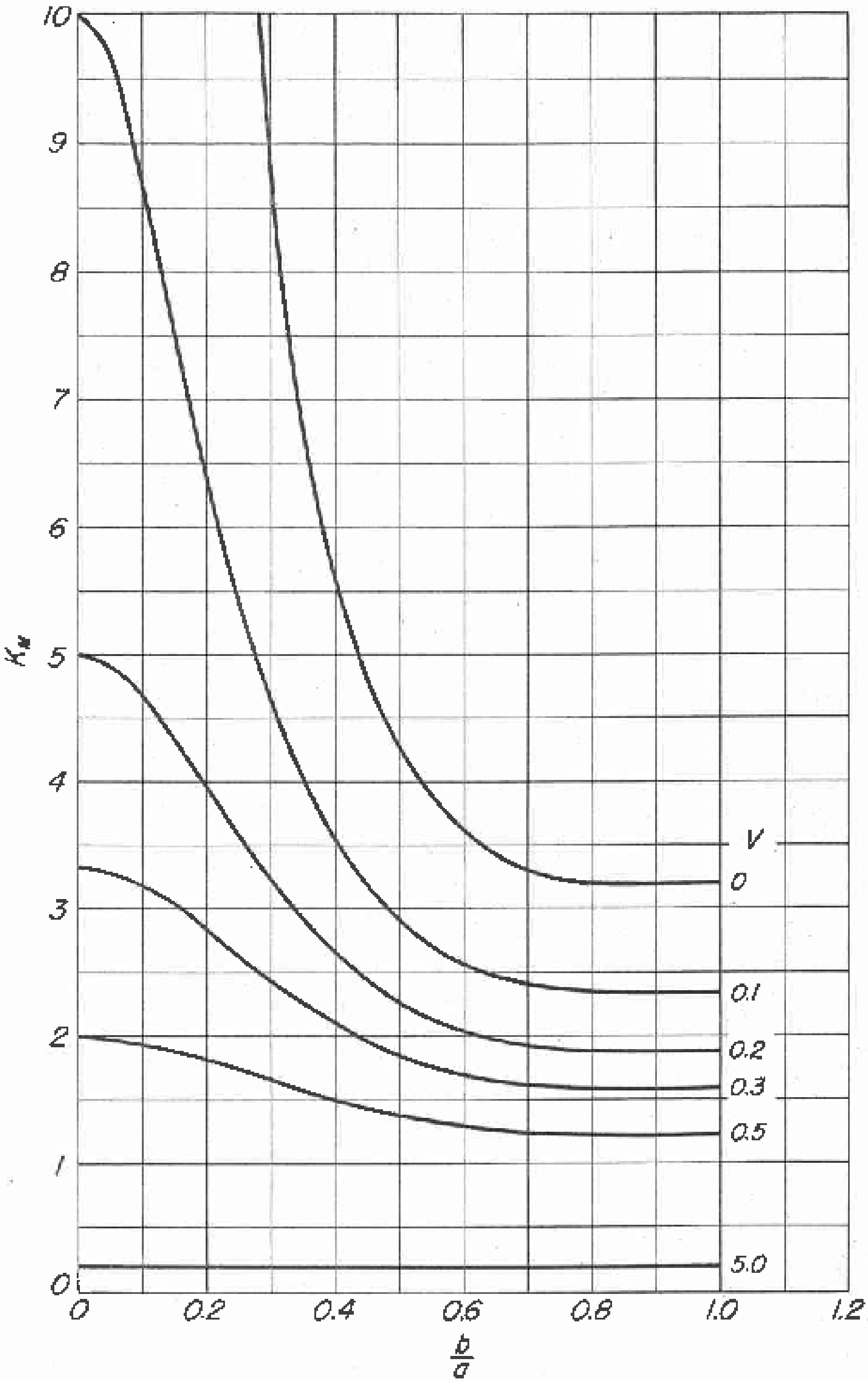

Figure 10. --A plot of K_M against b/a for $\gamma = 0.2$, $\beta = 0.6$, $\alpha = 1.5$, and $r = 2.5$ for various values of V.

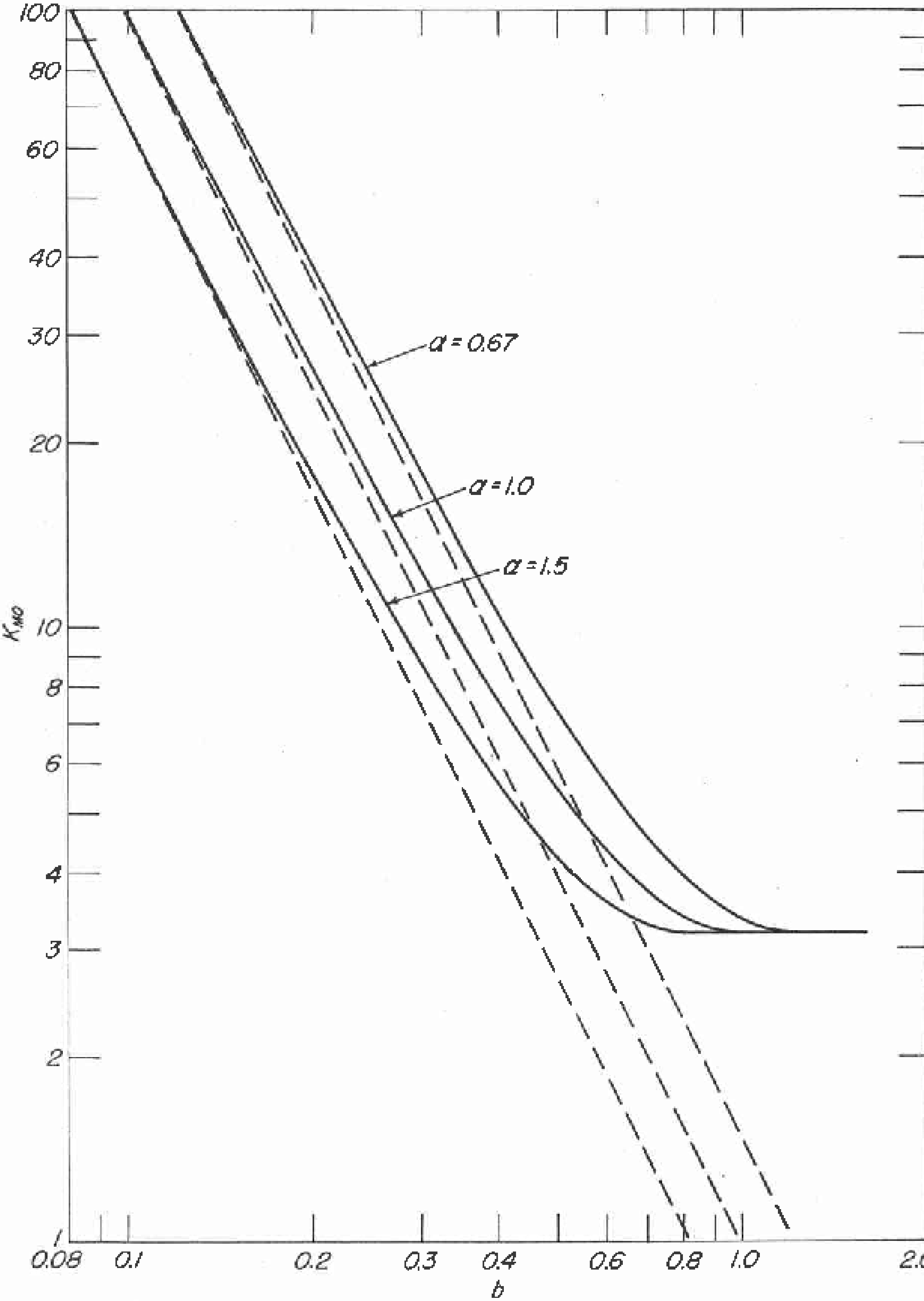

Figure 11. --A plot of K_{Mo} (K_M when V = 0) against b/a for $\gamma = 0.2$, $\beta = 0.6$, and $\alpha = 0.67$, 1.0, and 1.5.