INVESTIGATION OF THE ELASTIC BEHAVIOR OF A THIN-SHELL SPHERICAL DOME ROOF

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FOREWORD

Thin-shell structures combine unusual aesthetic values with efficient use of materials and are becoming more and more popular in present day design.

One specific structure, a thin-shell dome designed by Professor Orville Kofoid, was the motivating factor behind the preparation of this paper. This concrete structure, shaped like a spherical triangle, presented design problems not encountered in conventional thin-shell structures.

A basically similar concrete structure was recently completed at Massachusetts Institute of Technology. However, design information for neither of these structures has been made available to the Engineering Profession.

It is the aim of this paper to investigate the elastic behavior of structures of this type as well as the adaptability of wood for their construction.

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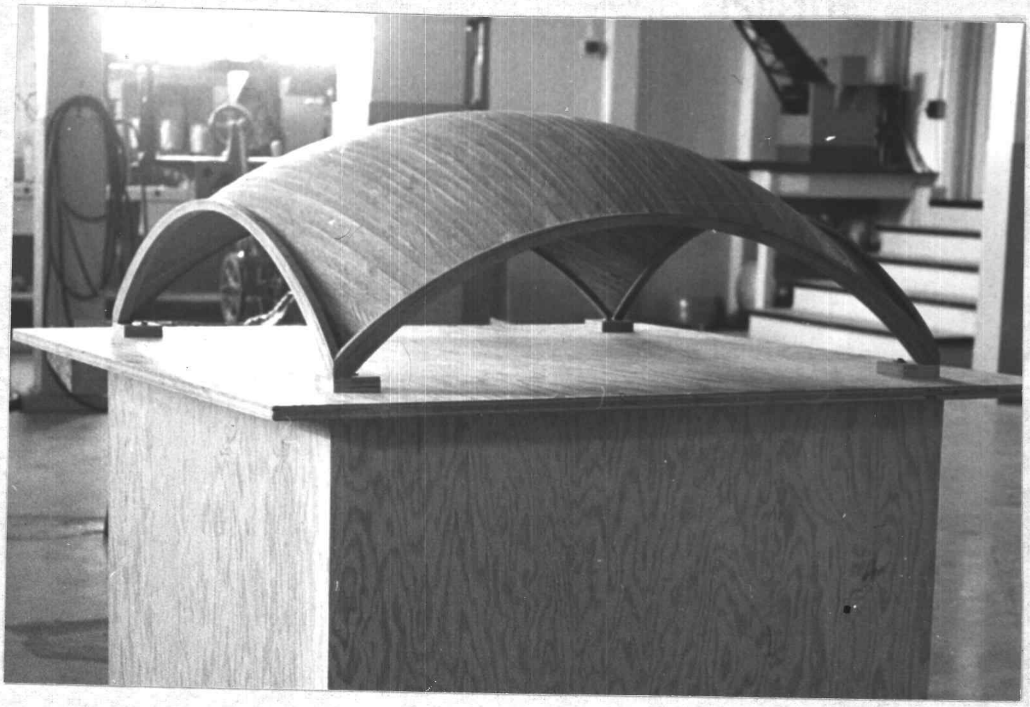

Frontispiece. Completed model.

INVESTIGATION OF THE ELASTIC BEHAVIOR OF A THIN-SHELL SPHERICAL DOME ROOF

I. INTRODUCTION

It has been known for years that thin-shell dome construction is very efficient in carrying large loads over clear areas by the full utilization of the inherent strength of the shell thickness.

Shells are usually considered to be too thin to develop bending moment, yet thick enough to resist buckling. For design purposes, thin-shell domes are assumed to carry only symmetrical loading and the effects of wind pressures are usually ignored.

Existing publications present formulas developed for the design of thin-shell dome roofs on the basis of the above assumptions, applicable only to domes which are circular in plan. Domes not circular in plan are becoming increasingly more popular because of their visual appeal, but at the present time, a search of existing literature has revealed that no information is available concerning the elastic behavior of this type of dome.

The purpose of this project was to analyze in as straightforward a manner as possible, the behavior of a dome roof not circular in plan, when subjected to a uniform live load. This study was facilitated by the construction and testing of a scale model, illustrated in the Frontispiece.

The recently completed auditorium at Massachusetts Institute of Technology is similar to the dome described in this paper except for the sides of the structure, which are only three in number and composed of arcs of great circles of the dome. The M. I. T. dome is actually a spherical triangle occupying one-eighth of the surface of the sphere on whose radius the dome was built. Another concrete dome similar in design to the M. I. T. dome is a three-cornered dome constructed two years ago at Ocean Lake, Oregon, and designed by Mr. Orville Kofoid, Professor of Civil Engineering at Oregon State College.

The dome investigated in this paper is four-sided, the sides being arcs of vertical small circles of the sphere. The dome therefore is not a true spherical polygon. The reason for making the sides vertical was ease in construction, as well as for making a prototype adaptable to fabrication in the form of multiple units.

When planning the model, it was assumed that a prototype structure could be built on a scale of approximately twenty to one, the shell being made of two thicknesses of one-inch lumber nailed and glued at right angles to each other. The model and the prototype would then be completely similar except for the difference between their respective moduli of elasticity. The model was constructed of Phillipine Mahogany because of the desirable working qualities of this wood. The percentage difference between the modulus of elasticity of Mahogany and of Douglas Fir would be negligible in any case.

A structure of this type lends itself to wood construction since, as will be shown later, the deflections, an indication of the stresses, are so small as to be practically negligible. Deflection is often a limiting factor in wood construction, but this model, when subjected to a load corresponding to about twenty-five pounds per square foot on a prototype, deflected a maximum of 0.019 inch, corresponding to a prototype deflection of less than one-half inch in a span of sixty feet. The deflection to span ratio for the prototype under these conditions would be 1 in 1440, far below allowable limits for wood design, with a live load approximately equalling that to which the structure would be normally subjected.

An over-all concept of the dome thickness can be visualized on the basis that an eggshell of comparable size would be approximately four times as thick as the dome.

II. THEORY

The design of a thin-shell dome structure, whether this structure is spherical, conoidal, or elliptical in form, is based on the assumption that the thrust line must follow the contour of the shell for uniform loads. Thus, with a uniform loading, the dome shell is subjected primarily to compressive stresses. These structures, then, are quite adaptable to either concrete or timber construction, since both of these materials can withstand relatively high compressive stresses.

If a zone of a sphere is isolated, and a suitable ring girder placed along the circle of latitude describing the bottom of this zone, the reactions theoretically would be entirely vertical, the horizontal components of the compressive forces in the shell being balanced by the ring tension in the girder. However, if portions of the dome are removed to make the structure a polygon in plan rather than a circle, the ring forces become discontinuous, and vary both in magnitude and direction from point to point along the discontinuous edge. This variation of ring forces (a four-sided dome may be considered as a partial zone of a true dome with irregular ring girders) makes an analytical solution of stresses in a structure of this type very complicated. A graphical summation of the loads acting on the stiffening girder can be used as a more direct solution, but this method is very tedious. For these reasons, and because local bending can be expected in the stiffening girder because of the aforementioned discontinuous ring forces, it was decided to use a scale model for investigation of the action of a dome roof under load.

Since in elastic action stress is proportional to strain, the deflections of a structure under load serve as an indication of the stresses at various points as well as giving a geometric picture of the structure's over-all behavior.

A model, if built precisely, can be correlated to the larger prototype structure through appropriate use of the scale ratio. It was decided to build a model about one-twentieth size. The basic dimensions of the model are shown below, while complete details are given in Plates 1 and 2.

Inside radius of sphere	2'-6"
Thickness of shell	0'-0 1/8"
Inside height at center	1'-2 1/8"
Width, inside to inside of girders	3'-0"
Over-all width	3'-1 1/2"
Inside height of girder at crown	0'-7 7/8"
Inner radius of girder	1'-11 3/4"
Thickness of girder at crown	0'-0 5/8"
Thickness of girder at spring line	0'-1 1/4"

The general design and the selection of the proportions of the dome were guided by practical considerations of a prototype. The square structure (in plan) lends itself to comparatively easy fabrication and to the construction of multiple units for greater area.

The dimensions of the stiffening girders were chosen to be one-twentieth of those of a typical timber arch having a span of sixty feet. Although it was not anticipated that these girders would act as true arches, there was no other information on which to base the size of these girders. By making full-size model drawings of the stiffening girders, the dimensions selected seemed appropriate to the eye.

The thickness of the shell of 1/8 inch corresponds to a prototype shell thickness of 2 1/2 inches. This thickness had proven satisfactory for Mr. Kofoid's design, so it was reasoned that timber, having a considerably smaller dead weight than concrete, would be structurally sound without a doubt at this thickness. The compressive strength of good lumber compares favorably with that of the usual quality of concrete. Although deflections are often critical in wood design, it was assumed that deflections would not be excessive since the shell would be primarily in compression, with no bending moment in the shell itself.

If the model after loading is to remain geometrically similar to the prototype, the deflections of the prototype for the same unit loading must be twenty times those of the model, that is, the deflections must vary directly as the scale ratio. According to deflection formulas from Strength of Materials,

$\delta = KWL^3/I$ where δ = Deflection of the structure,

W = Total applied load,

L = Length of the member,

I = Moment of inertia of the member,

K = Constant if the modulus of elasticity is constant.

Then $\delta_p = 20\,\delta_m$ where subscripts p and m refer respectively to prototype and model,

and $W_p/L_p = 20\ W_m/L_m$

or $W_p = 20\ L_pW_m/L_m$

$= 400\ W_m.$

In other words, a total load of 100 pounds on the model corresponds to a total load of 40,000 pounds on the prototype. Since the surface area of the model is roughly twelve square feet, a load of 300 pounds on the model would be equivalent to a surface loading on the prototype of approximately 25 pounds per square foot. Apparently, then, a load of about 300 pounds on the model was needed to approximate a comparable prototype loading under typical design conditions.

III. MODEL CONSTRUCTION

The model was built entirely from Phillipine Mahogany. The stiffening girders were laminated from ten strips of 1/8" thickness ripped from 1" by 6" finished material. The bottom two strips were 3/4" wide, while the top eight strips were 1/2" wide. The purpose of the two wider bottom strips was to provide a ledge for anchoring the shell. (See Plate 2.) The stiffening girders were built on an inner radius of 1'-11 3/4", and varied in thickness from 1 1/4" at the spring line to 5/8" at the crown. The tops of the girders were cut on a radius of 2'-2 5/8". Two forms were made of 3/4" plywood, one concave and cut on the outer radius, the other convex and cut on the inner radius. The ten 1/8" strips were covered with glue and placed between these forms, which were clamped with bar clamps until the glue had set. The ends of the girders were then trimmed to the exact size. The top curve was cut with a band saw and finished with a power sander. The girders were then glued to each other at the corners in an assembled position as shown in Figures 1 and 2.

Next, circular ribs were cut on a radius of 2'-6" from 1/4" plywood. Sixteen of these temporary ribs were arranged to outline a portion of a sphere as shown in Plate 1 and in Figures 1 and 2. Each one was fastened

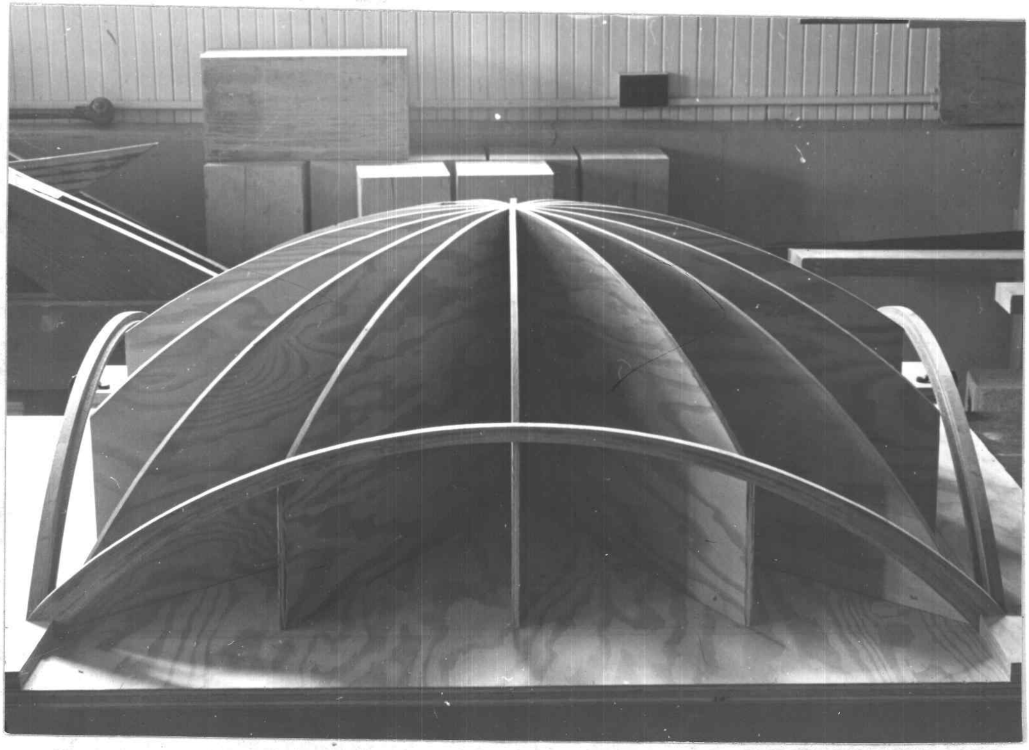

Fig. 1. Stiffening girders and temporary ribs.

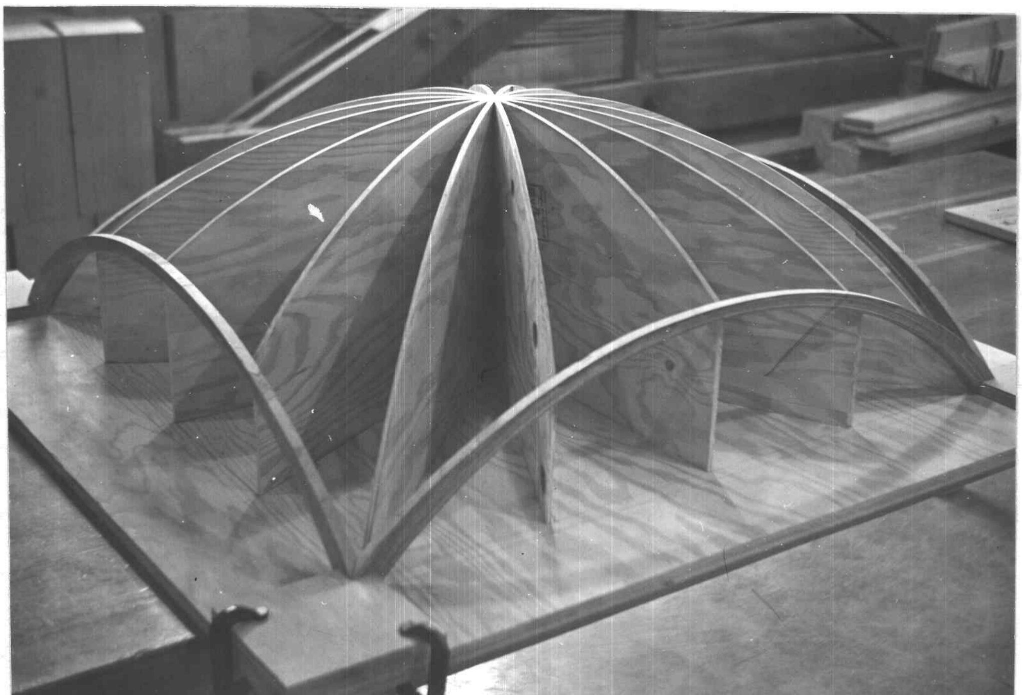

Fig. 2. Stiffening girders and temporary ribs.

Fig. 3. Construction of first layer of shell.

Fig. 4. Construction of first layer of shell.

securely to the 3/4" plywood table and to a 1 3/8" diameter center post.

The shell strips were then ripped from 1" by 6" finished lumber to a size of 1/16" by 3/4". Each one was tapered by hand trimming both edges from the 3/4" width at the center to approximately 5/8" at the ends. Angle cuts at each end of the strips were made in order to meet the ledge of the stiffening girder with a light snap fit for length. (See Plate 2.) The first layer of strips was then placed by gluing the beveled ends of the strips to the ledge of the stiffening girder, and the edge of each strip to the edge of each adjoining strip. Masking tape was used to prevent the glue from sticking to the temporary ribs. C-clamps and small plywood wedges were used to hold the ends of the strips as tightly as possible to the stiffening girders while the glue was setting. This phase of the construction is illustrated in Figures 3 and 4.

When the first layer had been completed (Figures 5 and 6), the second layer was laid 90 degrees to the first as shown in Figures 7 and 8. The same general procedure was followed as for the first layer, except that a special technique had to be developed. The glue, when applied to the bottom and sides of the strip only, caused the bottom side and edges of the strip to expand and create a trough

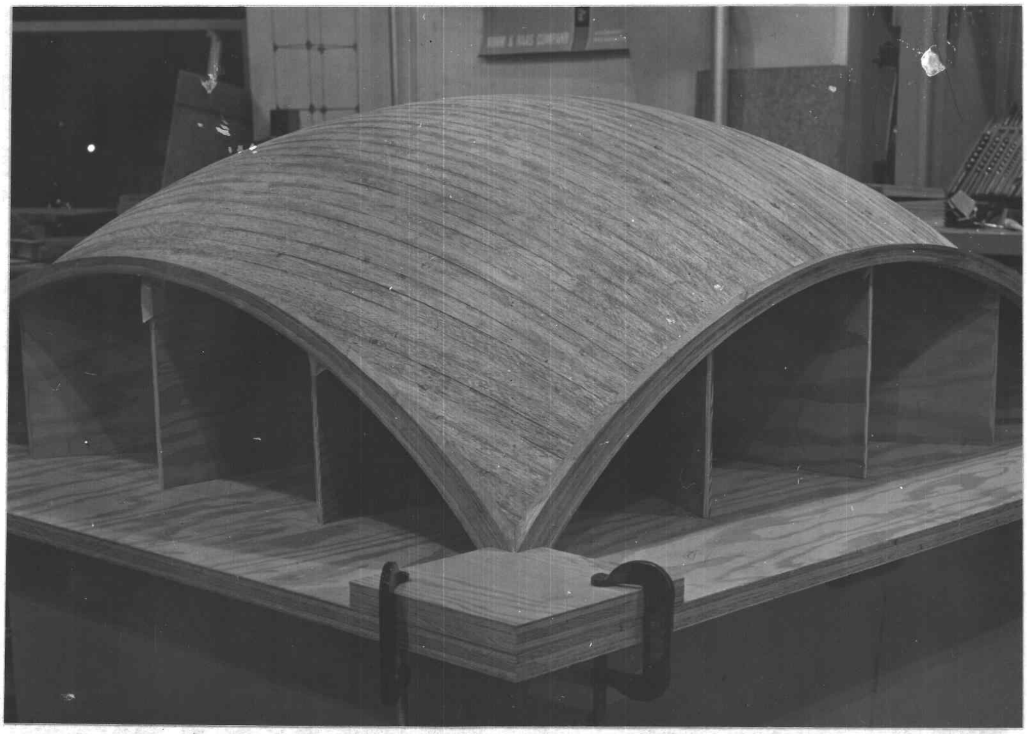

Fig. 5. Model with first layer of shell completed.

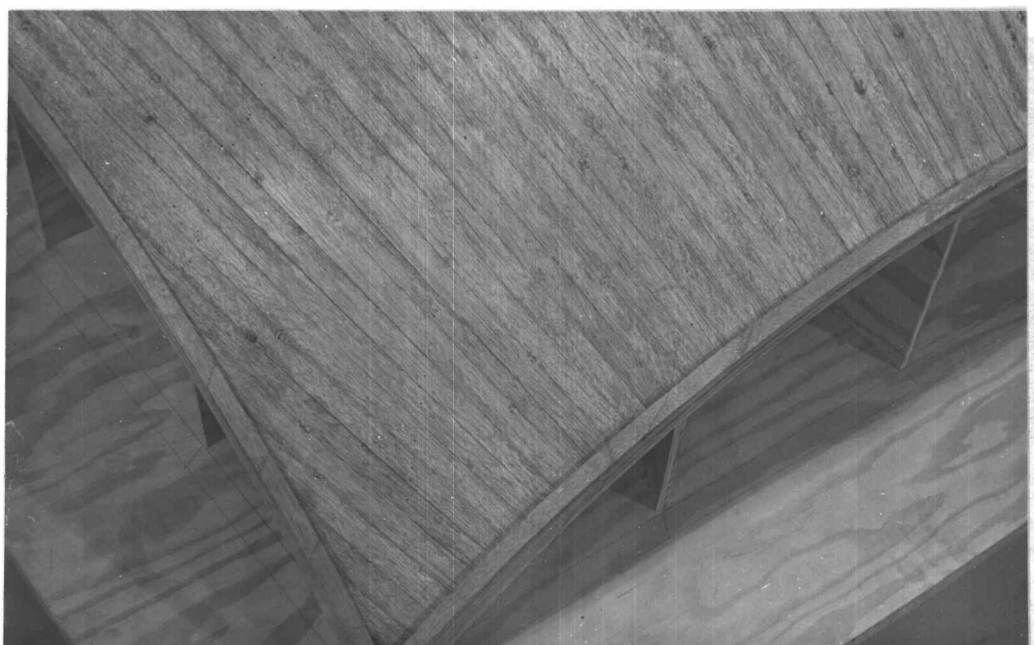

Fig. 6. Closeup of corner of model with first layer of shell completed.

Fig. 7. Construction of second layer of shell.

Fig. 8. Construction of second layer of shell.

along the top that prevailed, even after the glue had set. Study and experimentation proved that coating all surfaces with glue expanded the top and bottom of a strip equally. By using this procedure, all second layer strips were made to lie flat during gluing and after the glue had set. This trouble, of course, was not encountered when placing the first layer.

After the second layer had been completed, the model was removed from the temporary ribs, and triangular fillets were glued along the joint of the stiffening girders with the shell as shown in Plate 2. This was thought to be the most critical region on the model; therefore fillets were incorporated to prevent a premature failure.

Plywood blocks 2 1/4" square by 3/4" thick were then glued to the bottoms of the corners of the girders for the purpose of bolting the model to the table. Abutments were built from two layers of 3/4" plywood and bolted to the table to prevent any translation of the corners when the model was loaded, although as will be pointed out later, some outward deflection did take place. The legs of the table were located directly beneath the corners of the dome, so that the vertical thrust would be transferred directly into the concrete floors of the laboratory. This was done to render the table top free from deflections when the model was loaded. It was thus possible to use the table top as the base for the dial indicators, which were to be used for measuring deflections taking place in the model when loaded. With the exception of mounting these dial indicators, the model was ready for testing.

IV. TEST PROCEDURE

The mountings for the dial indicators used to measure the deflections of the shell and the stiffening girder under load were arranged as shown in Figure 9. These mountings were made from 1/4" and 1/2" steel bar stock, and were securely fastened to the table.

Since the model was symmetrical, and as nearly homogeneous as was possible to obtain with wood construction, deflections were measured at selected points in one octant only. It was assumed that the deflection of a point in this octant would be the same as the deflection of the corresponding point in any of the other octants. Plate 3 indicates the locations of the dial indicators used. Deflections at various corresponding points in other octants (opposite Point 10 for example) were measured periodically to insure that symmetrical loading was being obtained. These measurements verified the assumption that the dome would act symmetrically.

As may be seen on Plate 3, the dial indicators were located on radial lines (in plan) 22.5 degrees apart. Where space was constricted or where a point whose deflection was desired was near the stiffening girder, the deflection of the point in question was left for interpolation. Vertical deflections were also measured for the dome at the crown of the stiffening girder (Point 4) and

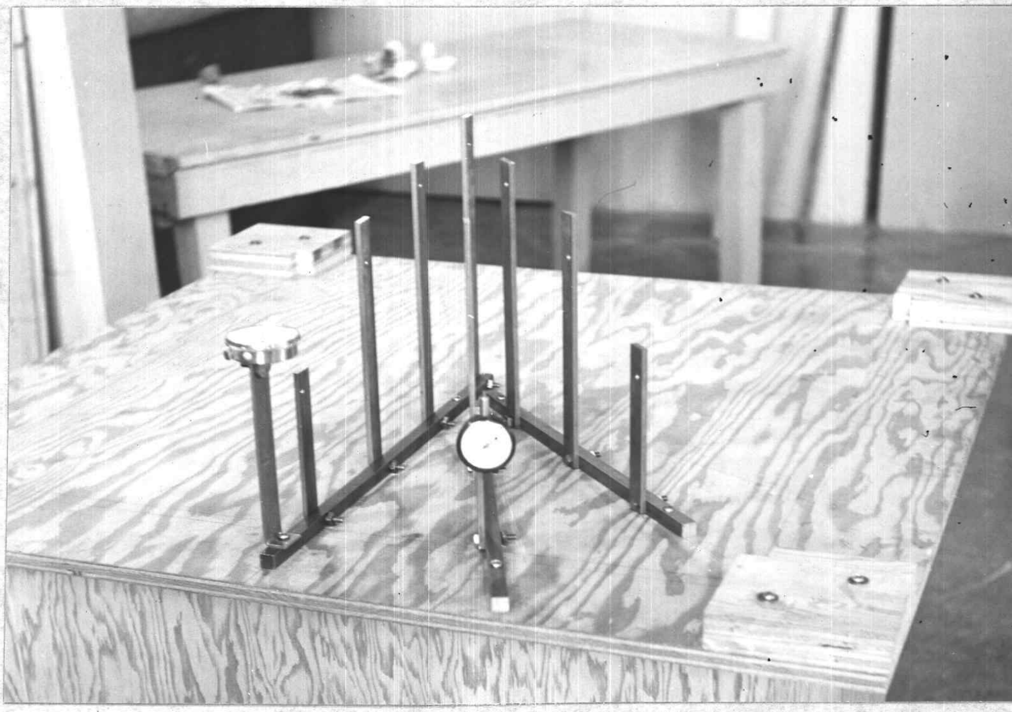

Fig. 9. Arrangement of dial indicator mountings and corner abutments.

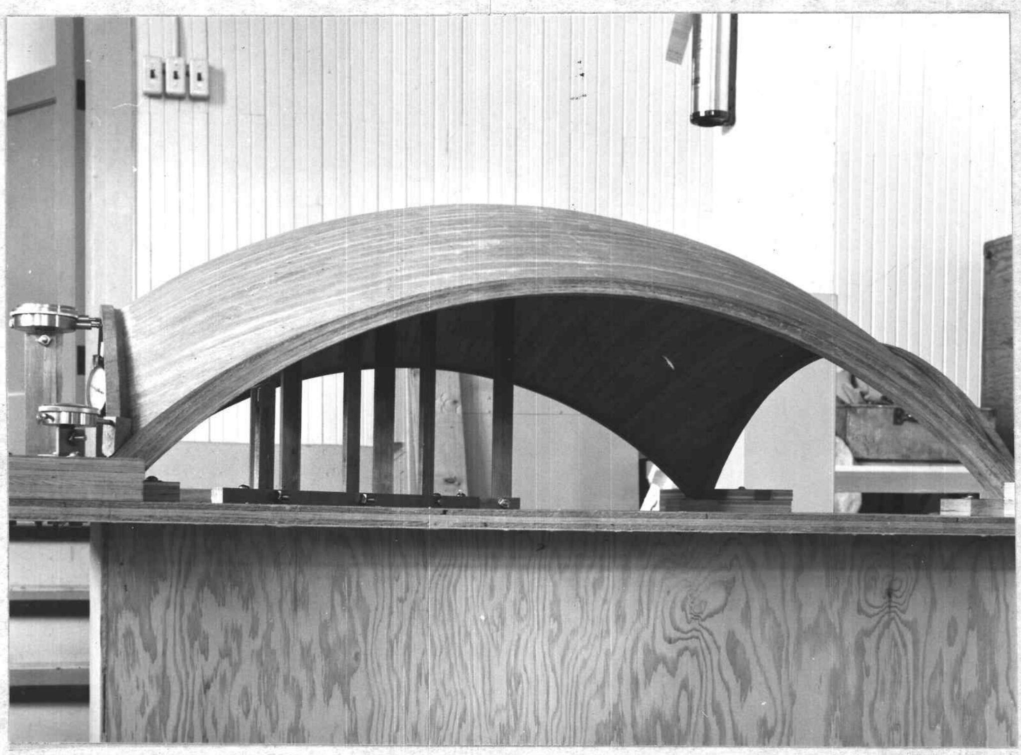

Fig. 10. Model ready for testing.

at the intersection of the intermediate radial profile and the stiffening girder (Point 6). In addition, horizontal deflections were measured at Points 4 and 6 by dials 10 and 11, and at the corner of the dome by dial 12. The purpose of dial 12 was to ascertain whether the corner was subject to any take up during model loading.

The model was loaded for the uniform load tests by small paper bags filled with ordinary beach sand to a weight of three pounds apiece. The closely arranged small bags approached a loading that should be considered a uniform loading per square foot of dome surface. The resistance to distortion of the small filled bags defeated attempts toward obtaining a uniform loading per square foot of horizontal projection as is customarily used for a prototype.

Three tests were made with the uniform loading described. In addition, one partial uniform load test was made in which only half of the structure was loaded, and a general information test was made in which a non-uniform load of 450 pounds was applied using large 25 pound sandbags (Figure 12). Deflection was measured only at the center of the model in this test since the loading was nowhere near being uniform.

One partial uniform load test was made for verification of the general assumption that a full uniform

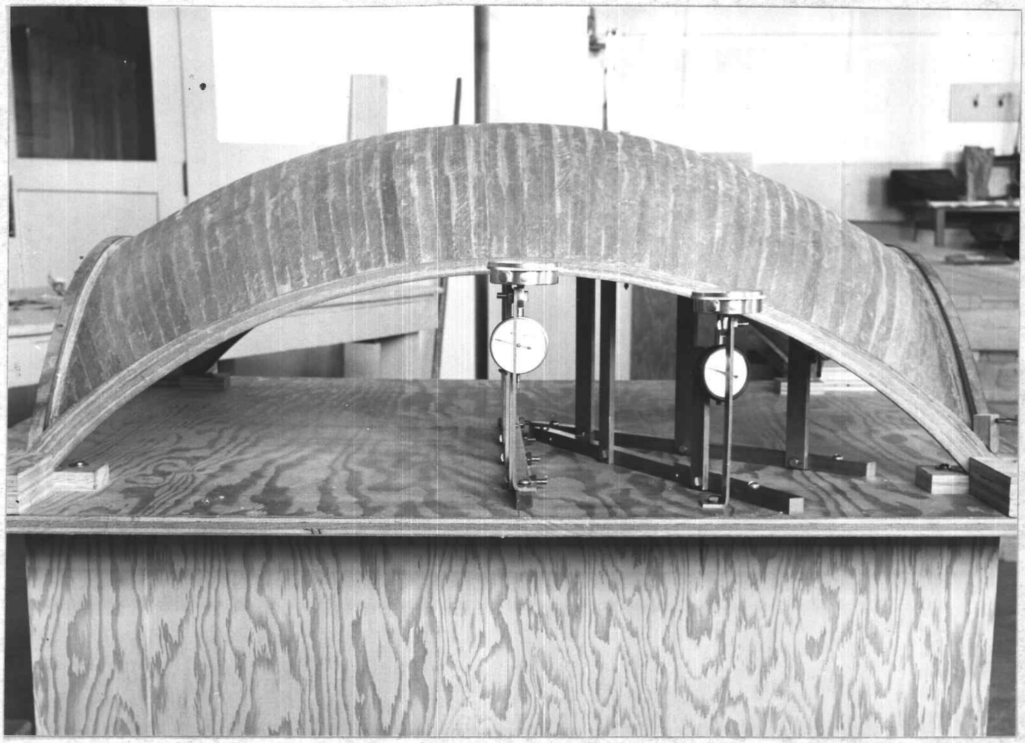

Fig. 11. Model ready for testing.

Fig. 12. Model with 450 pound non-uniform load.

load causes the most severe deflections. Further tests with partial uniform loads for any other purpose would have necessitated more dial indicators than available.

Test number 1 consisted of loading the model with a uniform load of 267 pounds. Test number 2 was the same as test number 1 except that the total load was 270 pounds. Test number 3 was the same as tests number 1 and 2 except that the load was 282 pounds. The discrepancies in the loadings were caused by progressively closer spacing of the sand bags. The largest number that could be uniformly placed on the model was 94 bags (282 pounds) as used in test number 3. The model as loaded for tests number 2 and 3 is illustrated in Figures 13 and 14. Test number 4 was the partial uniform load test of 156 pounds.

Fig. 13. Test number 2. Uniform load of 270 pounds.

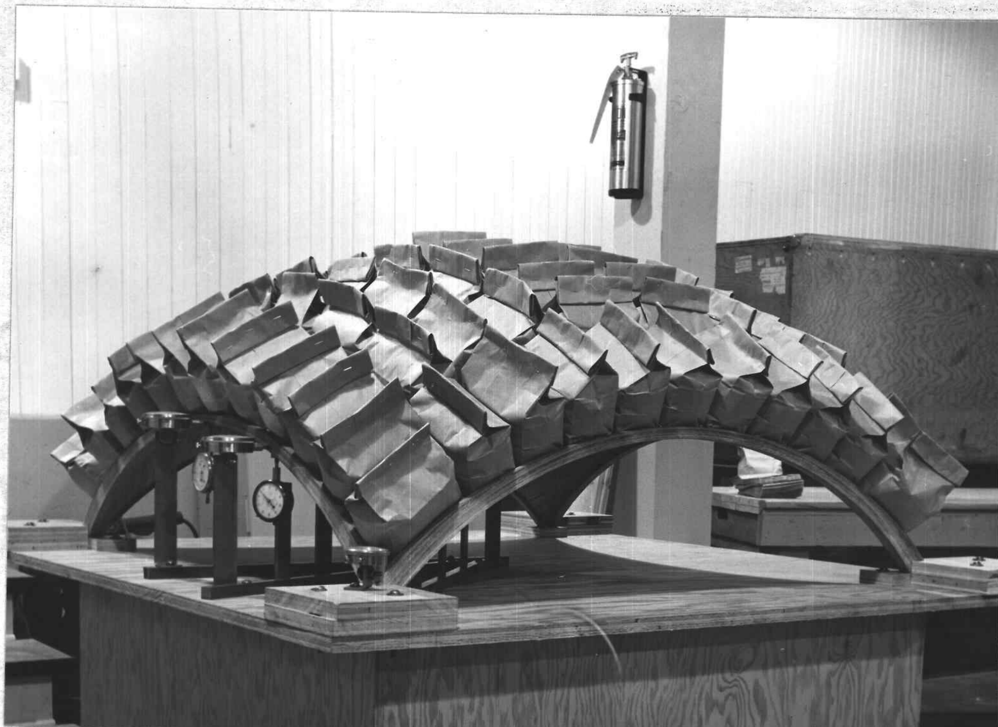

Fig. 14. Test number 3. Uniform load of 282 pounds.

V. RESULTS AND INTERPRETATIONS

Deflections for all tests are tabulated in tables 1 through 4. The deflections for test number 1 are incomplete since one had to be eliminated because of a faulty dial indicator. Deflections for tests number 2 and 3 are very consistent with a maximum difference of only 0.001 inch, the finest graduation of the dial indicators.

Deflections for test number 3 are shown graphically in figures 15 through 18 by profiles of the deflected and undeflected shell at each radial profile indicated on Plate 3, and along the stiffening girder. All deflections produced smooth profile curves, and ideal conditions for a few necessary interpolations. The deflections were plotted to an exaggerated scale because they would not have been apparent if plotted to true scale. The undeflected shell was used as a base line in order to obtain a more correlated picture than would have been possible by plotting from an abstract horizontal base line.

The radial profile, Figure 15, from the center of the model to the crown of the stiffening girder (Section 1-2-3-4 in Plate 3) shows practically uniform vertical deflections, varying from 0.019" at points 1 and 2 to 0.018" at points 3 and 4. These deflections indicate that the shell settled vertically, but that its radius of curvature did

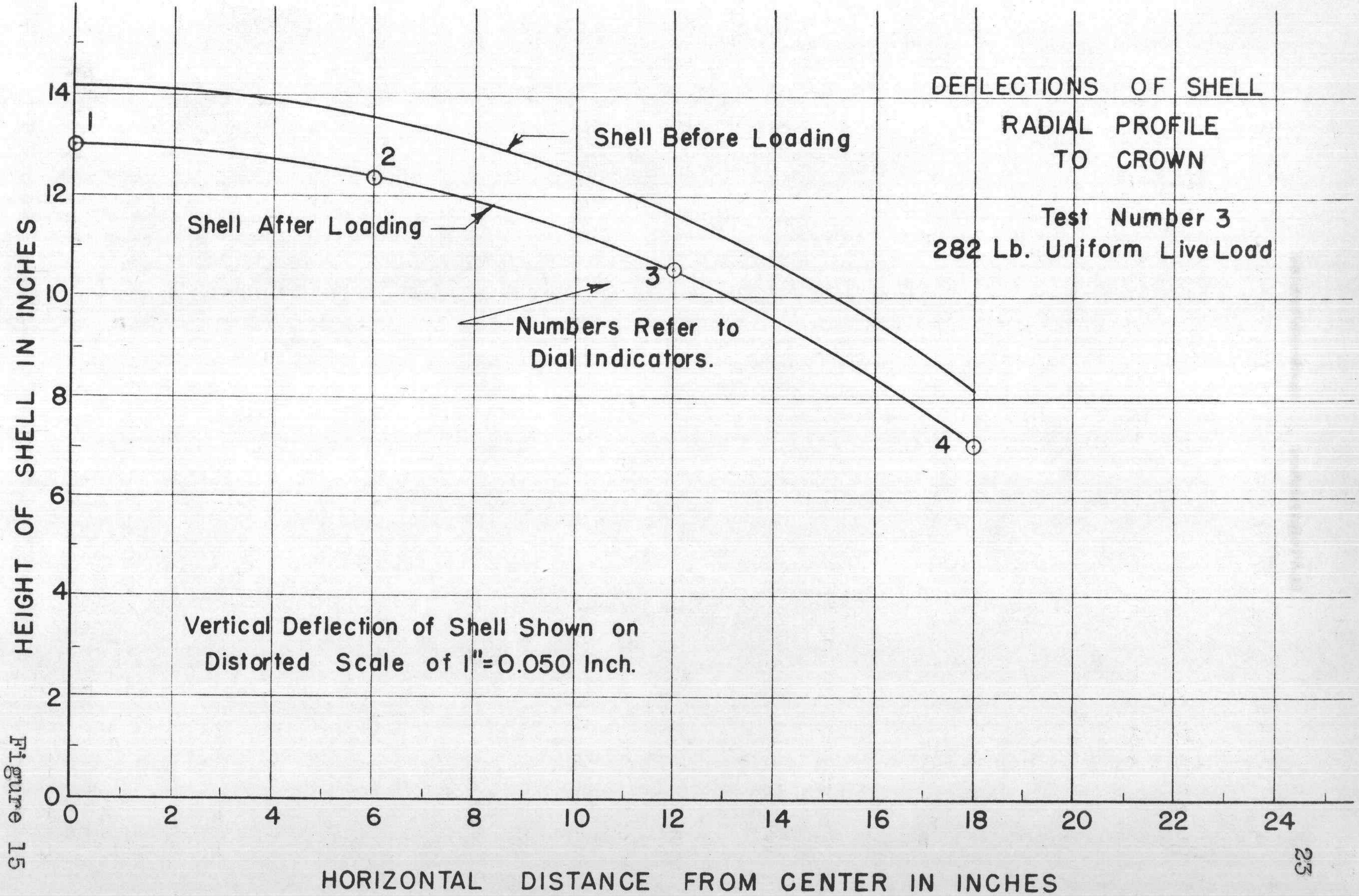
DEFLECTIONS OF SHELL
RADIAL PROFILE
TO CROWN
Test Number 3
282 Lb. Uniform Live Load
Shell Before Loading
Shell After Loading
Numbers Refer to
Dial Indicators.
Vertical Deflection of Shell Shown on
Distorted Scale of 1" = 0.050 Inch.
1
2
3
4
HEIGHT OF SHELL IN INCHES
0
2
4
6
8
10
12
14
16
18
20
22
24
HORIZONTAL DISTANCE FROM CENTER IN INCHES

Figure 15

not change. Since there was no horizontal deflection of the stiffening girder at the crown, it is assumed that the horizontal components of the compressive forces within the shell were opposed by ring tension in the central portion of the stiffening girder, while the vertical components of these compressive forces caused the stiffening girder to deflect vertically, comparable to arch action. By referring to Plate 3, it may be seen that the crown of the stiffening girder marks the point of tangency of the lowest complete circle of latitude on the dome. The fact that no horizontal deflection was observed at this point agrees with the assumption of basic thin-shell dome theory that where a circle of latitude on a dome is continuous and bounded by a ring girder, the reaction at any point along that circle of latitude is entirely vertical.

The radial profile, Figure 16, from the center of the model to the intermediate point of the stiffening girder (Section 1-5-6 in Plate 3) shows a smooth decrease of vertical deflections varying from 0.019" at point 1 to 0.007" at point 6. The interpolated point, assumed to have the same deflection as points 2 and 7 on the same circle of latitude (Plate 3), produced a smooth profile as a check on the interpolation. This profile, including a section beyond the assumed uniform ring tension limits near the girder crown, did not settle uniformly, since the

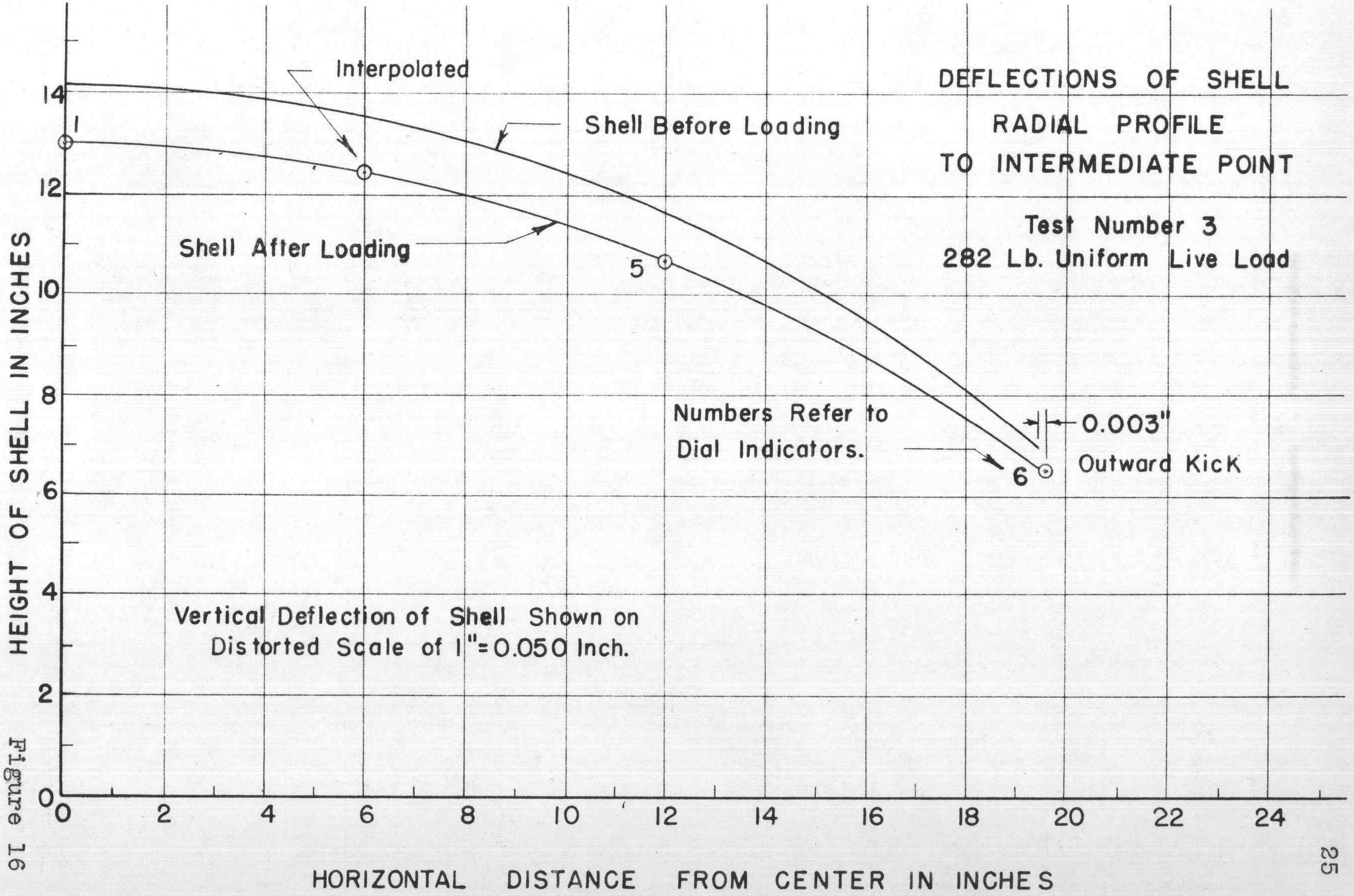
DEFLECTIONS OF SHELL
RADIAL PROFILE
TO INTERMEDIATE POINT
Test Number 3
282 Lb. Uniform Live Load
Interpolated
Shell Before Loading
Shell After Loading
1
5
6
Numbers Refer to
Dial Indicators.
0.003"
Outward Kick
Vertical Deflection of Shell Shown on
Distorted Scale of 1" = 0.050 Inch.
HEIGHT OF SHELL IN INCHES
0
2
4
6
8
10
12
14
0
2
4
6
8
10
12
14
16
18
20
22
24
HORIZONTAL DISTANCE FROM CENTER IN INCHES

Figure 16

vertical deflection at the stiffening girder was only 0.007" as compared with 0.019" at the center of the dome and 0.018" at the crown of the girder. The increased moment of inertia of the stiffening girder together with the fact that the gage point on the girder was nearer the end of the girder undoubtedly contributed to the smaller deflection of the girder at this point. The shell never actually bulged outward. All deflections were downward in all uniform load tests, but varied from minimums near the corners to maximums at the crown. Thus, relatively, the corner areas were bulging, and the crown portions were flattening. There was also an outward deflection of the stiffening girder at point 11 of 0.003". This deflection proved the validity of the assumption that where a point on the shell lies on a circle of latitude that is not continuous around the dome, the horizontal components of the compressive forces within the shell at that point are not balanced by direct ring tension, but are balanced by a stiffening girder that is an equivalent of ring girders which adjust by deflections to the discontinuous ring forces.

The radial profile, Figure 17, from the center of the dome to the corner of the dome shows a relatively smooth variation of vertical deflections from 0.019" at point 1 to 0.000" at the corner. Since any vertical

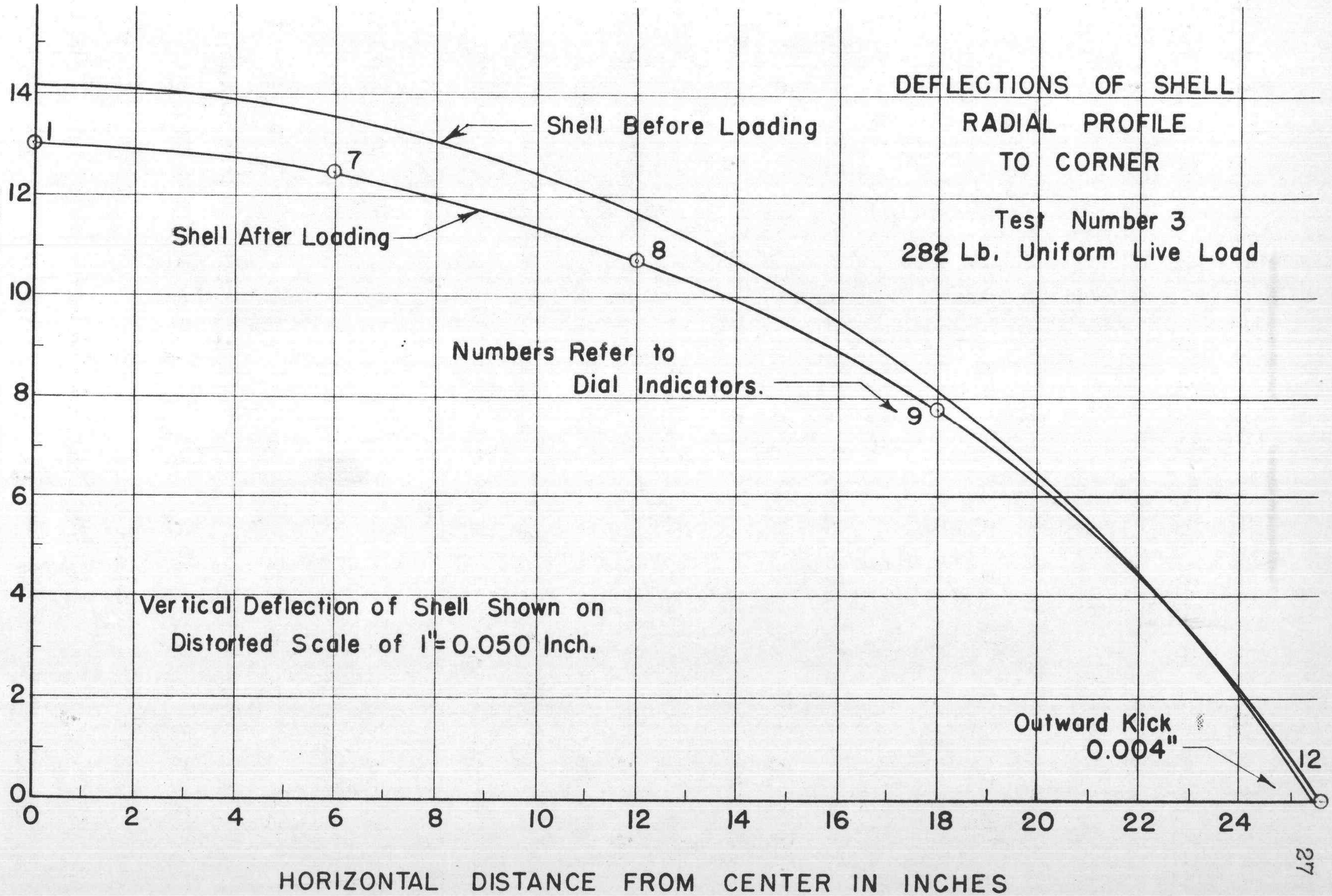
DEFLECTIONS OF SHELL
RADIAL PROFILE
TO CORNER
Test Number 3
282 Lb. Uniform Live Load
Shell Before Loading
Shell After Loading
Numbers Refer to
Dial Indicators.
Vertical Deflection of Shell Shown on
Distorted Scale of 1" = 0.050 Inch.
Outward Kick
0.004"
1
7
8
9
12
HEIGHT OF SHELL IN INCHES
HORIZONTAL DISTANCE FROM CENTER IN INCHES
0
2
4
6
8
10
12
14
16
18
20
22
24

Figure 17

deflection at the corner was impossible, it is reasonable that the shell should have flattened along this profile, in accordance with the horizontal outward movement of the corner. Had the corner not moved outward, it seems evident that the shell should have flattened near the center but in contrast should have shown actual bulging near the corner. The outward deflection of 0.004" at the corner with the model securely bolted down and the abutments previously assumed to prevent any outward deflection, indicates to some extent the inherent strength of the shell. To move the corner outward, the shell had to accumulate and apply a force large enough to move the support. Yet the shell accomplished this without any detrimental distortion to itself. The shell still described a smooth curve after loading and shortened very little indicating low unit stresses within the shell itself. The fact that the deflections along the profile from the center to the corner were smaller than those of the other profiles indicates that the largest magnitude of the relative bulging discussed earlier took place along this profile.

The plot of the vertical deflections along the stiffening girder, Figure 18, shows one-half the girder. The girder, being symmetrical, was justifiably assumed to behave symmetrically under load. The girder flattened under load, the crown deflection being 0.018" and the

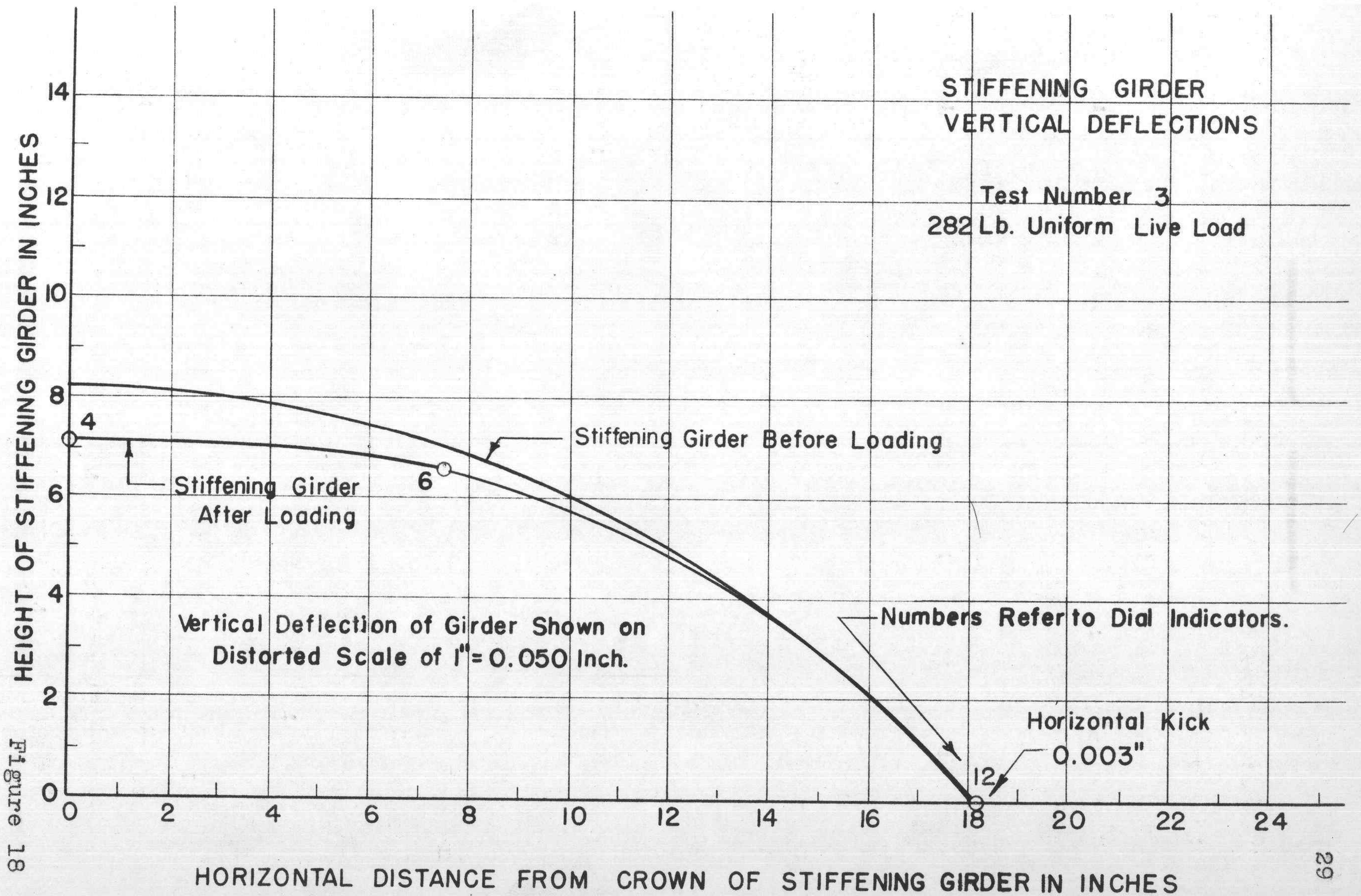
STIFFENING GIRDER
VERTICAL DEFLECTIONS
Test Number 3
282 Lb. Uniform Live Load
HEIGHT OF STIFFENING GIRDER IN INCHES
HORIZONTAL DISTANCE FROM CROWN OF STIFFENING GIRDER IN INCHES
0
2
4
6
8
10
12
14
16
18
20
22
24
4
6
12
Stiffening Girder Before Loading
Stiffening Girder After Loading
Vertical Deflection of Girder Shown on Distorted Scale of 1" = 0.050 Inch.
Numbers Refer to Dial Indicators.
Horizontal Kick 0.003"

Figure 18

intermediate deflection (7.45" from the crown) being 0.007". This flattening is exaggerated by the plotted profile, since by using an even larger scale, the crown of the deflected girder may be made to appear lower than the intermediate point, a physical impossibility. The girder had outward deflections of 0.003" both at intermediate point 11, and at the corner. Had the corner not moved, it is reasonable to assume that the outward deflection of point 11 would have been negligible. Also, these deflections could have been partially caused by the fact that no attempt was made to prevent the load from exerting a horizontal thrust by sliding against the girder. That is, the girder acted as a gutter for the sandbags. Therefore these deflections may be considered negligible, and the girders can be assumed to act primarily as arches in a vertical plane. The vertical components of the loads on these assumed arches are the summations of the externally applied loads along various radial profiles of the shell. However, the discontinuous ring forces, although horizontal also influence the loads on the assumed arches and must be accounted for on some reasonable basis.

In summarizing these results, it should be pointed out that no attempt was made to determine any actual unit stresses in the model under load, but rather to investigate the over-all elastic behavior of the model by observations and a geometry study of the deflected structure. The degree of indeterminacy of the stresses in this type structure puts the determination of stress relationships beyond the scope of this paper. However a knowledge of the basic elastic behavior of the structure is unquestionably necessary if any subsequent investigations are to be undertaken.

VI. CONCLUSIONS

Interpretations of the results of the uniform load tests substantiate the following conclusions.

For uniform loads, the assumptions which are the basis of conventional dome design are valid in regions near the top of the dome.

These same assumptions are not valid near the discontinuous edges of the dome.

The stresses in the shell acting along a meridian are compressive.

The stresses in the shell acting along a circle of latitude may be tension or compression, depending on the location of the circle of latitude along the thrust line.

Along radial profiles, the shell shortens a negligible amount and compressive stresses within the shell are very low.

Along a circle of latitude which passes near the crown of the stiffening girder, the shell relatively bulges in the corner regions and flattens in regions near the girder crown.

All absolute deflections of the shell are downward for full uniform loads.

Internal forces within the shell are transferred into the stiffening girders and abutments without buckling or other detrimental deflections in the shell itself.

The most highly stressed regions of the shell are the corner areas.

The vertical deflections of the shell are a function of the deflections of the stiffening girders. The shell is flexible enough to make minor adjustments to the distortions of the girders and still retain its basic shape without being highly stressed.

The stiffening girders deflect primarily in a vertical plane.

The small outward deflections of the abutments indicate relatively high stresses in the stiffening girders.

The stiffening girders must be rigid enough to withstand all the vertical forces induced by the shell and transfer them into the abutments.

The abutments must be rigid enough to absorb all thrusts induced by the stiffening girders without deflecting.

The full uniform load is more critical than the partial uniform load.

The unit load carried by the model would be the same as that for a prototype with geometrically similar deflections.

Live load deflections would vary directly with the scale ratio between the model and a prototype.

Dead load deflections would vary directly as the cube of the scale ratio between the model and a prototype. The dead load deflections of the model were far too small to be measured by ordinary methods.

The most important design factors in a dome of this type are the anchoring of the shell to the girders, the relative stiffness of these girders, and the rigidity of the abutments.

Interpretations of the elastic behavior of the model apply either to wood or concrete. Glued-laminated wood, when properly fabricated, achieves approximately the same degree of homogeneity as concrete within the working stresses, and is particularly adaptable to this type of structure.

VII. BIBLIOGRAPHY

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2. Molke, E. and J. Kalinka. Principles of concrete shell dome design. Proceedings of the American concrete institute vol. 34: pp.649-708. 1938.

3. Portland cement association. Design of circular domes. n.p. 1948. 8p.

4. Timoshenko, S. Theory of plates and shells. New York, Mcgraw-Hill, 1940. 429p.

VIII. APPENDIX

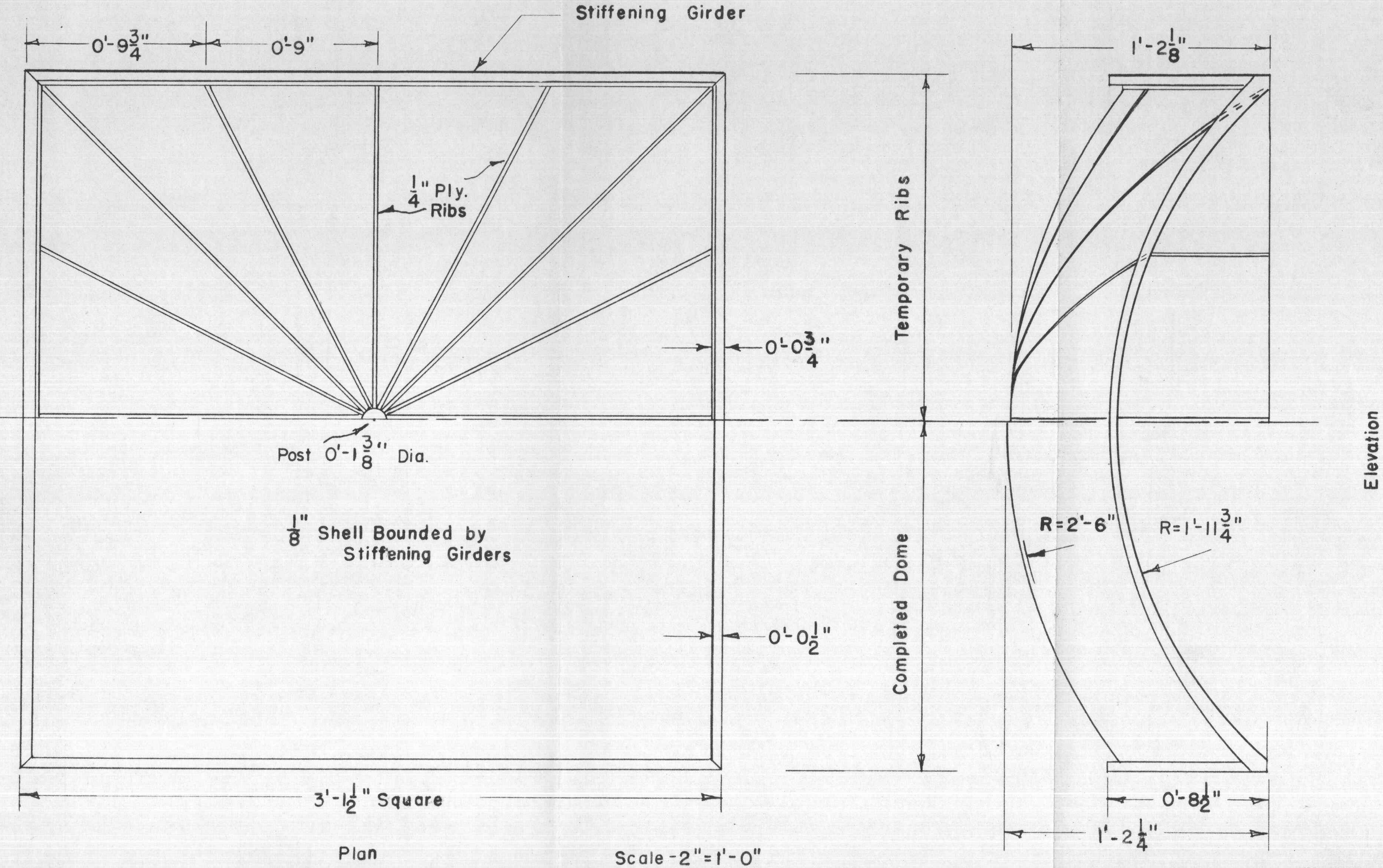

Stiffening Girder
0'-9 3/4"
0'-9"
1/4" Ply. Ribs
0'-0 3/4"
Post 0'-1 3/8" Dia.
1/8" Shell Bounded by Stiffening Girders
0'-0 1/2"
3'-1 1/2" Square
Plan
Scale - 2" = 1'-0"
1'-2 1/8"
Temporary Ribs
Completed Dome
R=2'-6"
R=1'-11 3/4"
Elevation
0'-8 1/2"
1'-2 1/4"

Plate 1

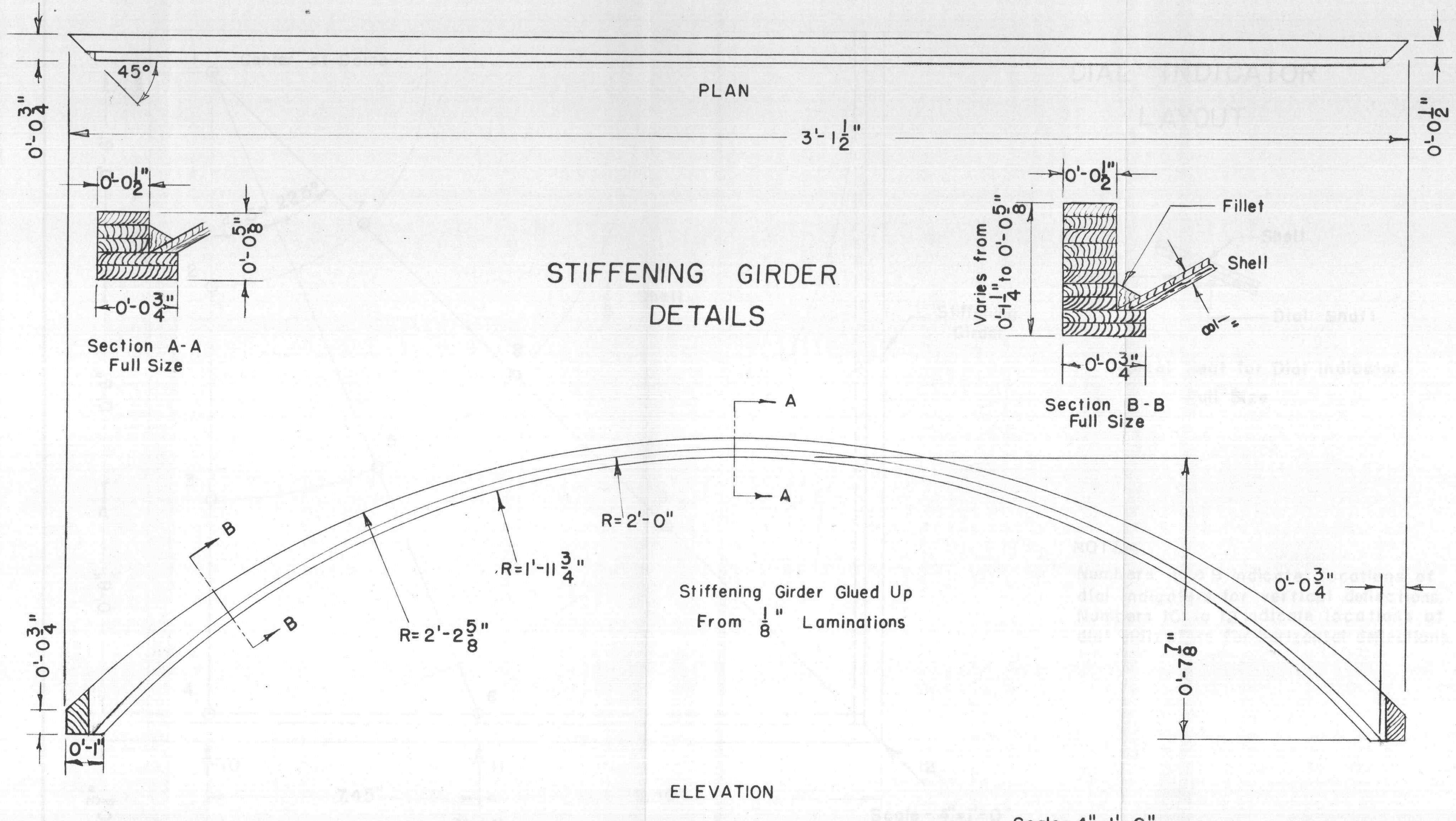
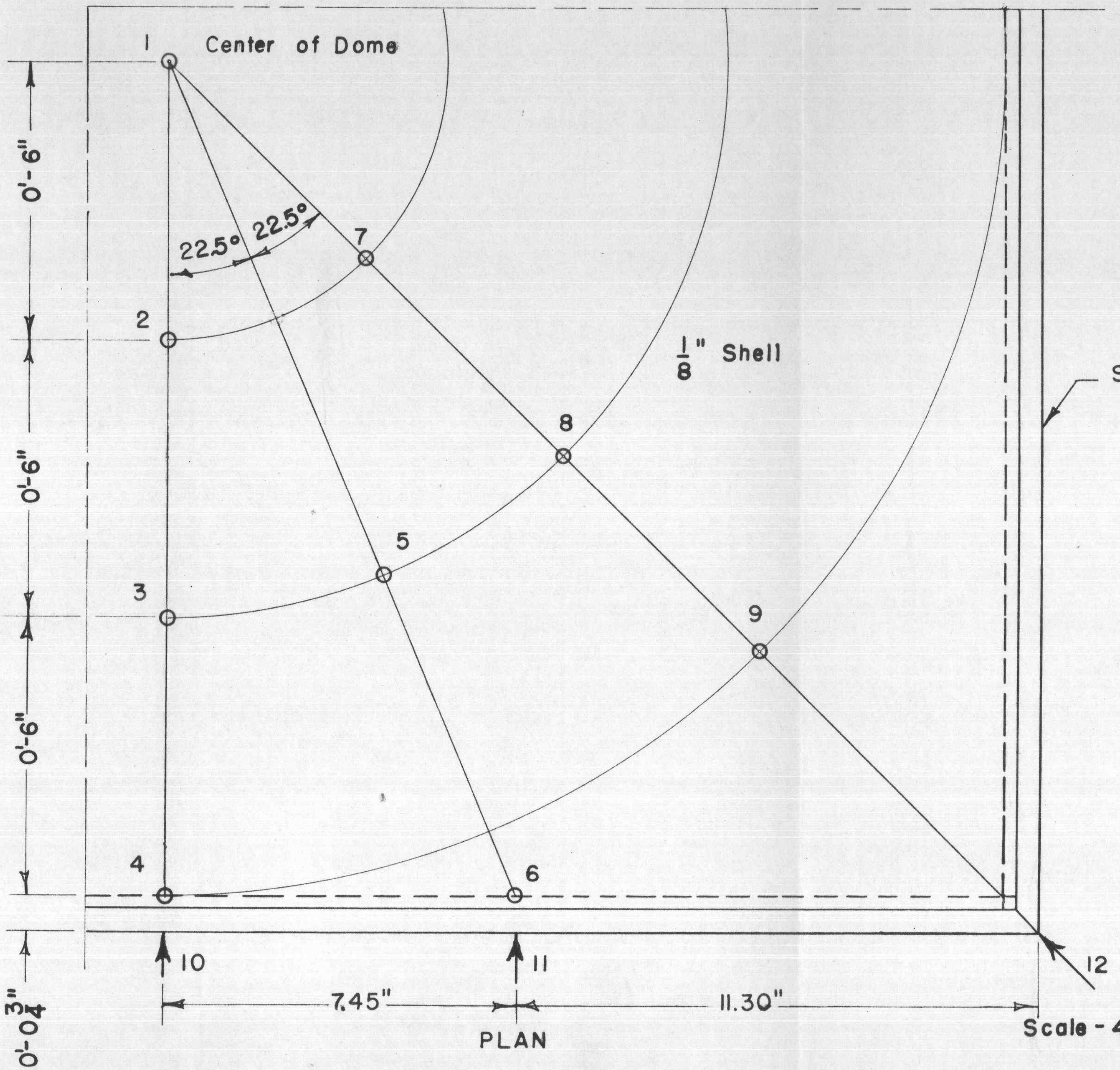

PLAN
45°
0'-0 3/4"
0'-0 1/2"
3'-1 1/2"
0'-0 1/2"
0'-0 5/8"
0'-0 3/4"
Section A-A
Full Size
STIFFENING GIRDER
DETAILS
0'-0 1/2"
Varies from 0'-1 1/4" to 0'-0 5/8"
Fillet
Shell
1/8"
0'-0 3/4"
Section B-B
Full Size
A
A
R=2'-0"
B
B
R=1'-11 3/4"
R=2'-2 5/8"
Stiffening Girder Glued Up
From 1/8" Laminations
0'-0 3/4"
0'-0 3/4"
0'-7 7/8"
0'-1"
ELEVATION
Scale-4"=1'-0"

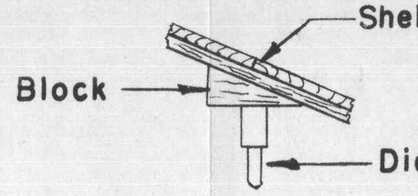
Plate 2

DIAL INDICATOR LAYOUT



PLAN

Scale - 4"=1'-0"



Typical Seat for Dial Indicator
Full Size

NOTE:

Numbers 1 to 9 indicate locations of dial indicators for vertical deflections. Numbers 10 to 12 indicate locations of dial indicators for horizontal deflections.

TABLE 1

LOAD TEST NUMBER 1, 267 LB. UNIFORM LOAD

Dial	Initial reading	Final reading	Deflection
1	26	46	0.020"
2	97	114	0.017"
3	37	52	0.015"
4	82	100	0.018"
5	57	72	0.015"
6	0	3	0.003"
7	29	47	0.018"
8	24	-	-
9	70	73	0.003"
10	0	0	0.000"
11	0	5	0.005"
12	0	5	0.005"

TABLE 2

LOAD TEST NUMBER 2, 270 LB. UNIFORM LOAD

Dial	Initial reading	Final reading	Deflection
1	25	43	0.018"
2	94	112	0.018"
3	35	52	0.017"
4	81	99	0.018"
5	54	69	0.015"
6	0	8	0.008"
7	28	47	0.019"
8	43	58	0.015"
9	65	71	0.006"
10	0	- 1	-0.001"
11	0	3	0.003"
12	0	4	0.004"

TABLE 3

LOAD TEST NUMBER 3, 282 LB. UNIFORM LOAD

Dial	Initial reading	Final reading	Deflection
1	23	42	0.019"
2	94	113	0.019"
3	34	52	0.018"
4	81	99	0.018"
5	52	68	0.016"
6	0	7	0.007"
7	26	44	0.018"
8	41	56	0.015"
9	63	69	0.006"
10	0	0	0.000"
11	0	3	0.003"
12	0	4	0.004"

TABLE 4

LOAD TEST NUMBER 4, 156 LB. PARTIAL UNIFORM LOAD

Dial	Initial reading	Final reading	Deflection
1	18	18	0.000"
2	95	99	0.004"
3	35	38	0.003"
4	81	86	0.005"
5	54	56	0.008"
6	0	0	0.000"
7	28	-	-
8	43	43	0.000"
9	65	59	-0.006"
10	0	6	0.006"
11	0	7	0.007"
12	0	4	0.004"